ME685 Programming Assignment-1

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1 Formulation report

1.1 The Problem

Solve the following linear unsteady problem numerically on a grid of 21-101 points

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}; 0 < x < L = 1 \tag{1}$$

$$T(x,0) = 0; T(0,t) = 1; T(1,t) = 0 \tag{2}$$

Use an implicit finite difference formulation. Use GAUSSIAN ELIMINATION (specifically, TDMA) for solving the system of linear algebraic equations. Plot temperature profiles in the physical domain for select time instants including steady state. The analytical solution of the PDE given above is given as

$$T(x,t) = (1-x) + \sum_{n=1}^{\infty} C_n * sin(n\pi x) exp(-n^2\pi^2 t) \tag{3}$$

$$C_n = 2\int_0^1 (x-1) sin(n\pi x) dx \tag{4}$$

Write a computer program that evaluates the analytical solution. The integral may be evaluated using numerical integration with 1001 points.

Plot the variation of true error as a function at time at points x = 0.25, 0.5 and 0.75.

1.2 The Solution

Equation (1) may be written numerically as:

$$\frac{T_i^{t+1} - T_i^t}{\Delta t} = \frac{T_{i+1}^{t+1} - 2T_i^{t+1} + T_{i-1}^{t+1}}{\Delta x^2}$$

which may also be written as:

$$T_i^t = \lambda(-T_{i+1}^{t+1} + 2T_i^{t+1} - T_{i-1}^{t+1}) + T_i^{t+1} \lambda = \frac{\Delta t}{\Delta x^2}$$

This equation may be solved by using the matrix form $Ax = B$ where, A is the tri-diagonal matrix: $\begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ -\lambda & 1+2\lambda & -\lambda & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & \dots & 0 & 0 & 1 \end{bmatrix}$,

$$B = \begin{bmatrix} T_0^t \\ T_1^t \\ T_2^t \\ \vdots \\ T_N^t \end{bmatrix}, And \;\; x = \begin{bmatrix} T_0^{t+1} \\ T_1^{t+1} \\ T_2^{t+1} \\ \vdots \\ T_N^{t+1} \end{bmatrix}$$

Since, A is a Tri-Diagonal matrix, this system of equations may be solved using TDMA or, Tri-Diagonal Matrix Algorithm.

I have written the code to solve this problem in C++. For plotting graphs, I have used a C++ wrapper of the Matplotlib library taken from here. The rest of the code is my own.