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PROJECT REPORT

"Simulation and Modelling of Compound Helicopter Unmanned Aerial Vehicle"

Submitted by:

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CERTIFICATE

This is to certify that the project entitled **"Simulation and Modelling of Compound Helicopter Unmanned Aerial Vehicle"** submitted by **Vardhaman Jain (2130304)** as a part of Summer Undergraduate Research and Graduate Excellence 2021 offered by the Indian Institute of Technology, Kanpur, is a bonafide record of the work done by her under my guidance and supervision at the Indian Institute of Technology, Kanpur from 14th June 2021 to 22nd August 2021 in the online mode.

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Vardhaman Jain

Abstract

The compound helicopter is a hybrid of a helicopter and a fixed-wing aircraft developed to make use of the features of a rotary-wing aircraft, like hover and low-speed performance, as well as the high-speed capabilities of a fixed-wing aircraft. Incorporating an auxiliary propulsion system reduces the load on the main rotor required to produce forward thrust during high-speed flight. An auxiliary lifting device contributes towards the total force necessary to counter the weight of the aircraft. Thus, the load on the main rotor reduces further. The incorporation is in the form of a wing and propellors on each side.

As it incorporates one main rotor and two propellers, the aerodynamic modelling of a rotor becomes essential. For this, Blade Element Momentum Theory (BEMT) is used, which calculates the required thrust and power generated by fixed pitch or variable pitch propellor using MATLAB and how the results are affected because of the induced tip vortices, which can be corrected by incorporating the Prandtl's tip loss factor.

Contents

Chapter 1
Introduction

Unmanned aerial vehicles (UAVs) are finding broader applications including surveillance, rescue, navigation, formation, coordination, among others. Helicopter-type UAVs offer the best option in many open and built-up areas for their maneuverability through narrow alleys and sharp bends, and their ability to hover in place if, for example, there is a need to have a close look at a place of interest. As with any vehicular design, control design for helicopters is non-trivial. For example, ensuring stability in helicopter flight is a challenging problem for nonlinear control design and development, as the helicopter represents an underactuated system. Helicopter systems are statically unstable without closed-loop control, especially when hovering. In addition, helicopter dynamics are highly nonlinear and strongly coupled such that disturbances along a single degree of freedom can easily propagate to the other degrees of freedom and lead to loss of performance, even destabilization.

Why "Compound" Helicopter?

In a conventional helicopter, the main rotor is the sole source to produce forward thrust, moments for translational flight, and necessary lift to support the weight of the aircraft, making them mechanically more complicated. The tail rotor balances the main rotor torque and responsible for directional control.

Even with excellent vertical flight abilities, the helicopters suffer from poor performance at high-level cruise speeds due to the aerodynamic limitations of the main rotor. In translational flight, the rotating blade experiences different velocities at different blade sections and azimuth, causing asymmetry of lift and increased vibration. At high speeds, the advancing tip of the blade reaches drag divergence Mach number and rotor root in retracting side experiences stall. Further effects like high parasite drag experienced by the hub and other airframe components reduce the overall lift to drag ratio of helicopters. These phenomena truncate the rotor performances and form a barrier to achieving higher speeds.

By incorporating a fixed wing and propellors with the conventional helicopter, it reduces the load on the main rotor which providing lift and thrust. Also, this "compounding" methodology enhances the range and maximum speed.

This concept of "compounding" is not new and has been tested and developed in the past which dates to 1950s. Prototypes like Sikorsky X2 and Eurocopter X^3 proves how the compounding method is not only feasible but also how it improved its high-speed performance significantly.

Figure 1: Sikorsky X2

Figure 2: Eurocopter X^3

Chapter 2
Aerodynamic modelling of Helicopter rotor

The rotor of a helicopter provides three functions:

1)The generation of a vertical lifting force (thrust) in opposition to the helicopter's weight.
2)The generation of a horizontal propulsive force for forward fight; and,
3)A means of generating forces and moments to control the attitude and position of the helicopter in three-dimensional space.
All three of these functions must be under the full control of the pilot. Unlike a fixed-wing aircraft where these functions are separated, the helicopter rotor alone must provide all three functions.
A normal rotor with variable pitch can be used for an axial or hovering flight where the pitch angle is varied to produce the desired thrust. For a fixed-pitch rotor, we can change only the RPM of the rotor.
For hovering flight, the performance prediction of a rotor blade can be made with the help of **Blade Element Momentum Theory (BEMT)** by writing a computer program on MATLAB, which gives us the required thrust and power generated by the given rotor and RPM.
The blade element momentum theory for hovering rotors is a hybrid method that combines the basic principles from both blade element and momentum theory approaches.

Momentum Theory for hovering flight

In momentum theory for hovering flight, the rotor field flow is azimuthally axisymmetric. Momentum theory is the application of the conservation of mass, energy, and momentum of the air that passes through the hovering rotor and gains momentum.

It is assumed that the flow through the rotor is one-dimensional, incompressible, quasi-static and inviscid so that the fluid properties only changes vertically and stay constant with time.

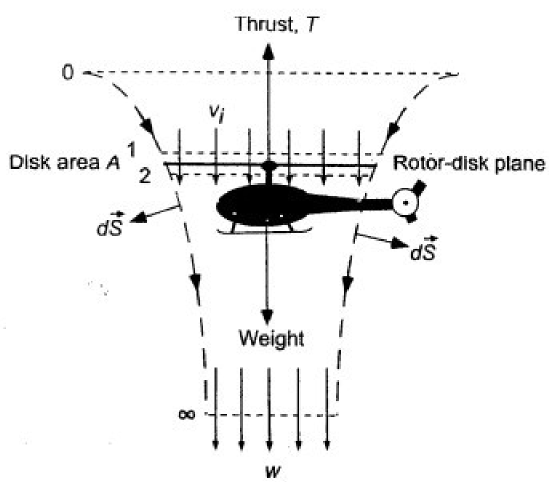

Figure 3: Flow model for momentum theory analysis of rotor in hovering flight

For the assumptions given above, the following set of equations are used in momentum theory for an elemental area dS (considering the rotating rotor as a solid disc), ρ is the density of air, and V is the velocity of air.

Conservation of Mass :

$$\iint_S \rho \vec{V} \cdot \vec{dS} = 0$$

Conservation of Momentum:

$$\vec{F} = \iint_S p\vec{dS} + \iint (\rho \vec{V} \cdot \vec{dS})\vec{V}$$

Conservation of Energy:

$$W = \iint_S \frac{1}{2}(\rho \vec{V} \vec{dS})|\vec{V}|^2 + \iint_S p(\vec{V} \cdot \vec{ds})$$

After solving these equations for the locations 0,1,2 and at a very large distance from the rotor for the hovering flight of the helicopter, we can obtain the thrust and power required.

Blade Element Theory

The blade element theory assumes that each blade section acts as a quasi-2D airfoil to produce aerodynamic forces and moments. The rotor performance is then obtained by integrating the section airloads at every element of the blade, which is then averaged over a rotor revolution.

The resultant velocity U at every radial location of the blade is given as

$$U = \sqrt{U_T^2 + U_P^2}.$$

Where U_T is the in-plane component of velocity and U_p is out of plane component of velocity (Figure 4).

Using the relative inflow angle ϕ, the effective angle of attack α can be given as:

$$\alpha = \theta - \phi = \theta - \frac{U_P}{U_T}.$$

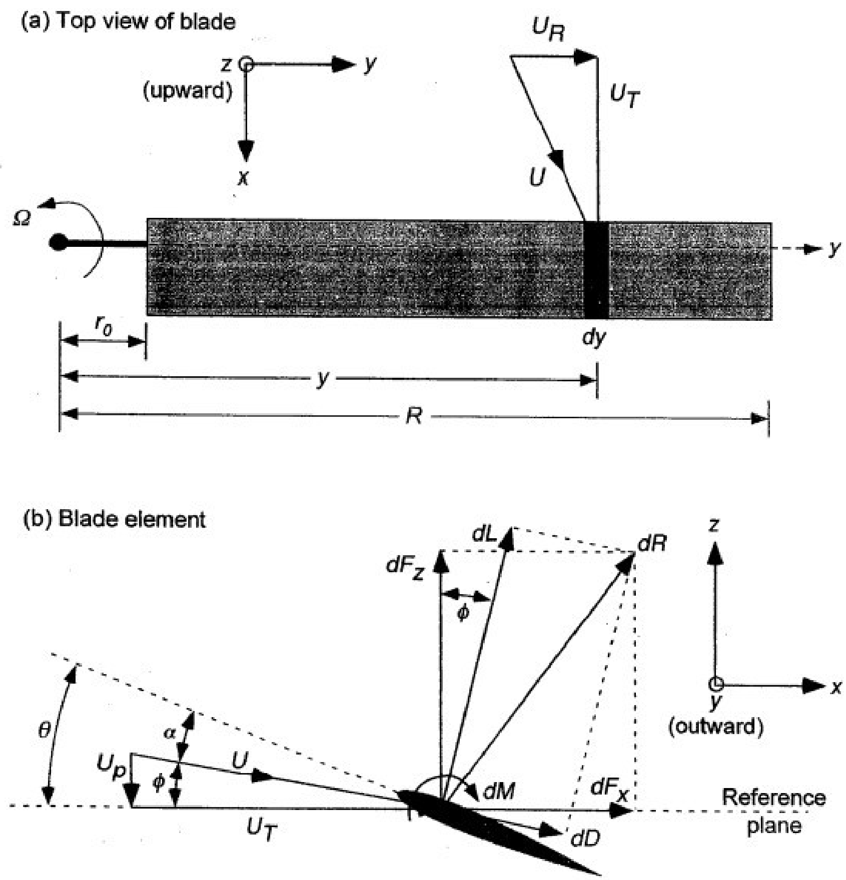

Figure 4: Incident velocities and aerodynamic environment of the blade element

By writing the incremental lift *dL* and drag *dD* on the blade element in terms of coefficient of lift Cl and coefficient of drag Cd and integrating over the length of blade, we can obtain the coefficient of thrust and coefficient of power which is given as

$$C_T = \frac{1}{2}\sigma \int_0^1 C_l r^2 \, dr$$

$$C_Q \equiv C_P = \frac{1}{2}\sigma \int_0^1 (\phi C_l + C_d) r^3 \, dr = \frac{1}{2}\sigma \int_0^1 (\lambda C_l r^2 + C_d r^3) \, dr$$

C_Q is the torque coefficient which is same as C_p

Where $\sigma = \frac{\mathrm{Nb\ c}}{\pi \mathrm{R}}$ is the solidity of the rotor disk, such that N_b is the number of rotor blades, c is the chord length

of the rectangular blade, R is the radius of the blade.

$\lambda = \phi r$, where r is the non-dimensional radius ranging from 0 to 1.

From C_l vs α plot, from steady linearized aerodynamics C_l can be written as

$$C_l = C_{l\alpha} * \alpha$$

And,

$$C_d = C_{do}$$

So the equations can be re-written as

$$C_T = \frac{1}{2}\int_0^1 \sigma C_{l_\alpha}(\theta r^2 - \lambda r)dr$$

$$C_Q = \int_0^1 \lambda dC_T + \int_0^1 \frac{1}{2}\sigma C_d r^3 dr$$

C_Q, i.e., C_p can be written as the sum of induced power coefficient C_{pi} and profile power coefficient C_{pp} such that,

$$C_{Pi} = \int_0^1 \lambda \ dC_T$$

$$C_{PP} = \int_0^1 \frac{1}{2} \sigma \ C_d r^3 dr$$

Thrust and power can be the given by averaging the result over the revolution of the blade such that

$$\text{Thrust} = C_T \rho A(\Omega R)^2$$

$$\text{Power} = C_P \rho A(\Omega R)^3$$

where A is the area of the rotor disc and Ω is the angular velocity of the rotor.

Blade Element Momentum Theory

The blade element momentum theory is a hybrid method that combines the basic principles of from both blade element and momentum theory. The blade element theory considers the inflow λ to be uniform, not varying along the radius of the blade.

BEMT allows the inflow distribution along the blade to be estimated.

In BEMT, incremental thrust on an annulus of the rotor disc is taken using both momentum theory and blade element theory and equated to find the inflow distribution.

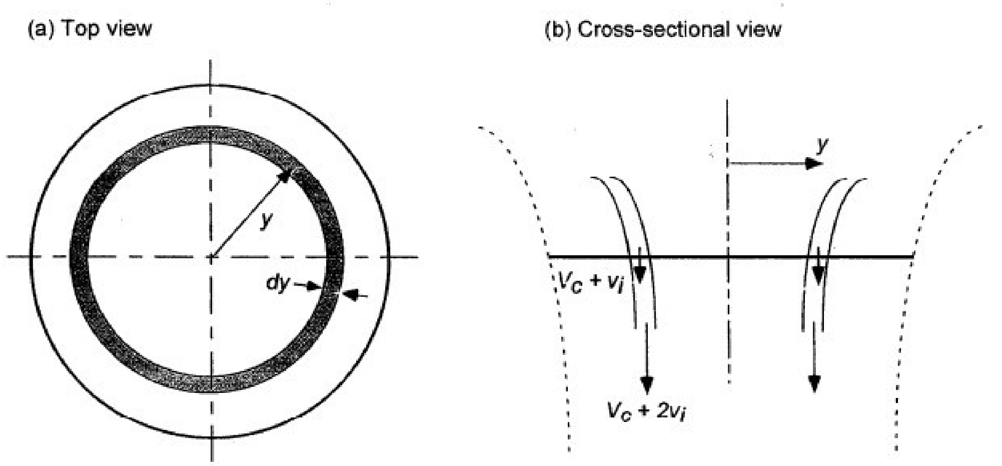

Figure 5: Annulus of rotor disc for momentum analysis on the hovering rotor

From Momentum Theory,

$$dC_T = 4\lambda(\lambda - \lambda_c)rdr$$

From Blade Element Theory,

$$dC_T = \frac{1}{4}\sigma C_{l_\alpha}(\theta r^2 - \lambda r)dr$$

On equating the above equations and solving the quadratic for λ and $\lambda_c=0$ for hovering flight, we get

$$\lambda(r) = \frac{\sigma C_{l_\alpha}}{16}\left(\sqrt{1+\frac{32\theta r}{\sigma C_{l_\alpha}}}-1\right)$$

This inflow is used in C_T and C_P calculations obtained from Blade Element Theory.

Chapter 3

Implementation and Results

Using the blade element momentum theory, a computer program is written to implement the numerical solution for finding out the thrust and power produced by a rotor.

For the given rotor characteristics:

Rotor Geometry	Given Data
Blade Radius, R	0.355m
Blade Chord, c	0.032m
Number of blades, Nb	2
RPM	1500
Density	1.225 kg/m^3
Drag Coeffiect, Cd_0	0.01

The following plots are obtained:

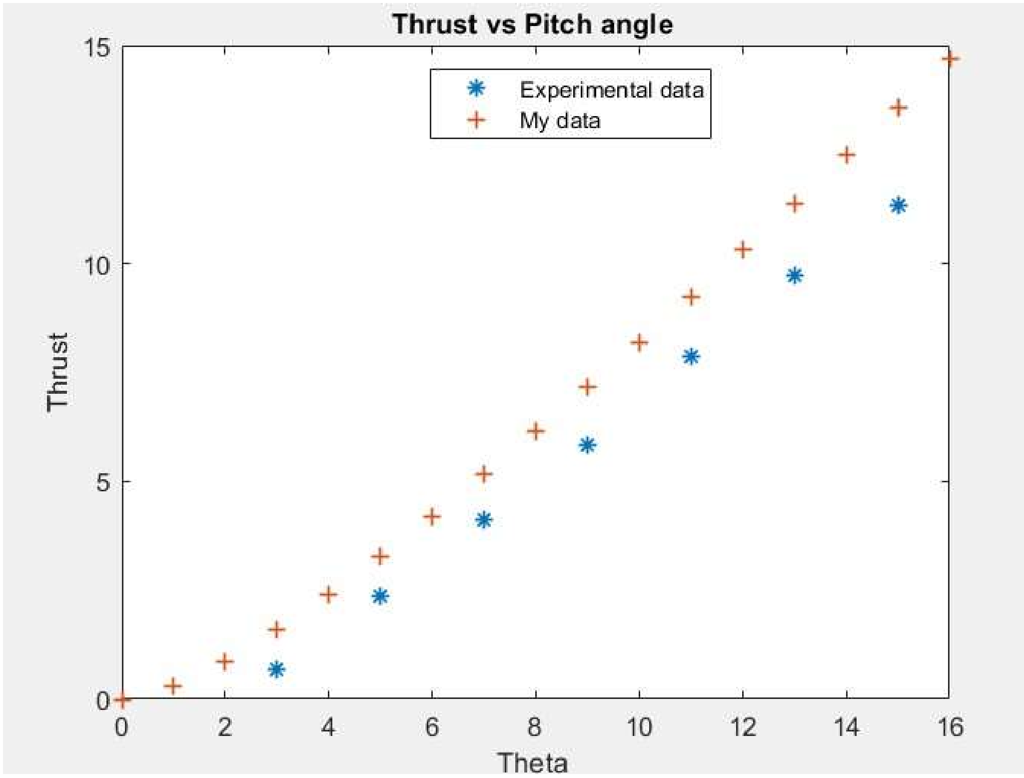

Figure 6: Plot for Thrust vs Pitch angle Ɵ

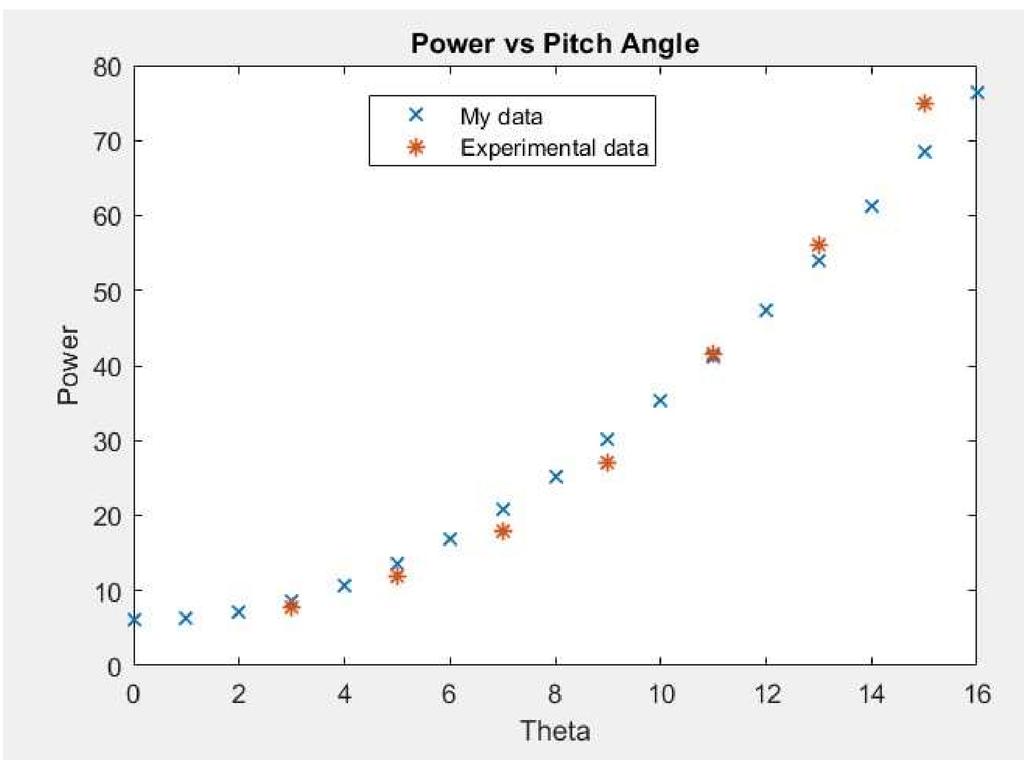

Figure 7: Plot for Power vs Pitch angle Ɵ

The above plots show the variation of thrust and power with pitch angle compared with the experimental data.

We assumed that the angle of attack lies in the linear region of the C_l vs α plot for obtaining the figure 6 and 7.

For more accurate results, we incorporate the following lookup table data for airfoil NACA0012

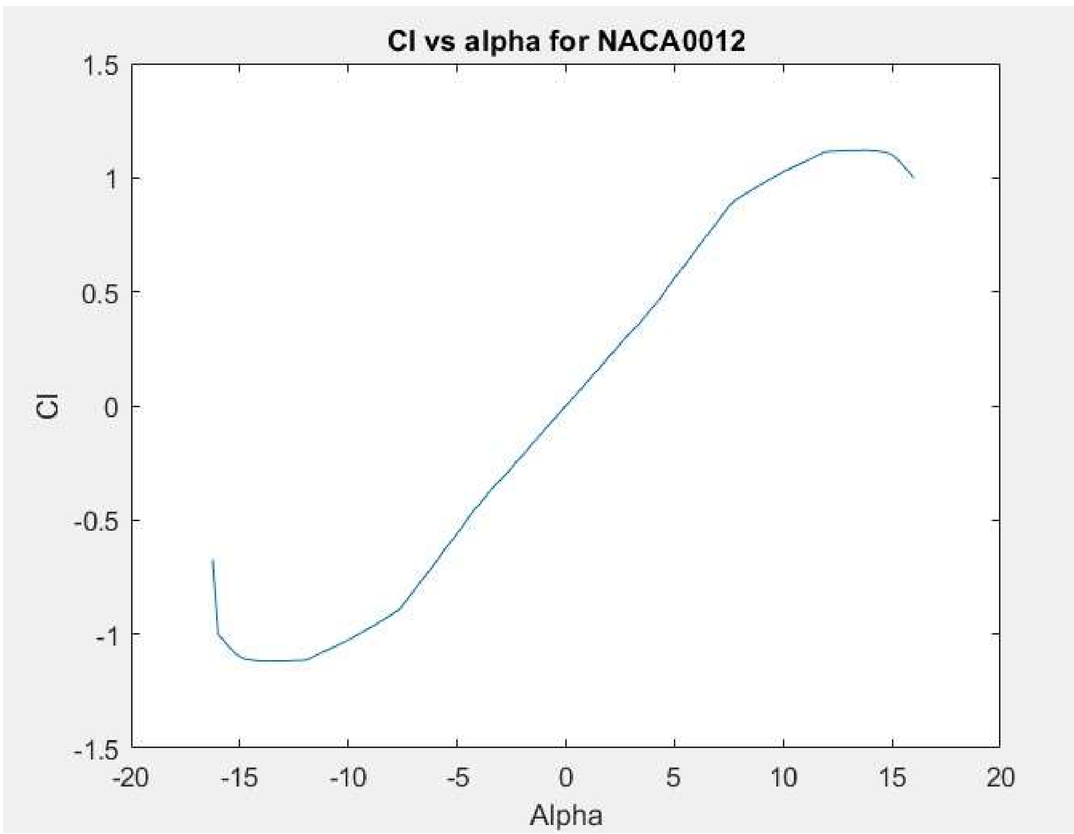

Figure 8

The following plots are obtained after using this data:

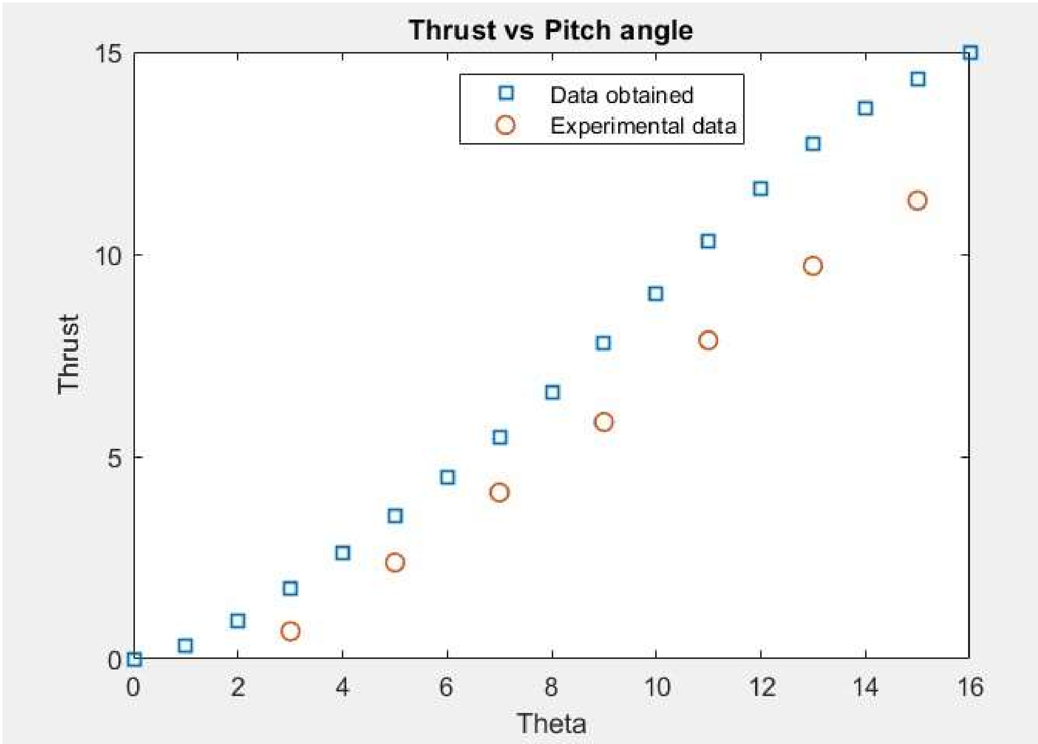

Figure 9: Thrust vs Pitch angle Ө (lookup table)

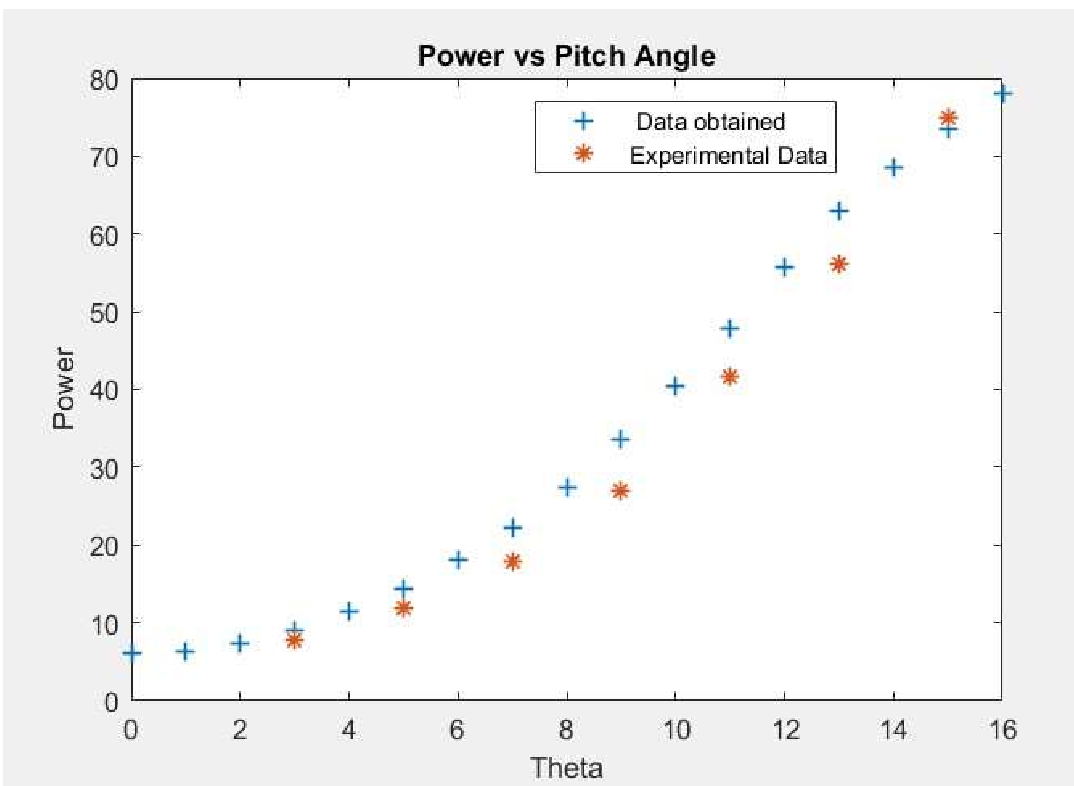

Figure 10: Power vs Pitch angle Ө (look up table)

Induced tip loss and correction

The lifting capacity of the rotor reduces at region near the tip because of the formation of trailed tip vortices which produces high local inflow over the tip region. This is known as tip-loss as it represents the a loss in the value of lift relative to what the value would have been without the tip vortices.

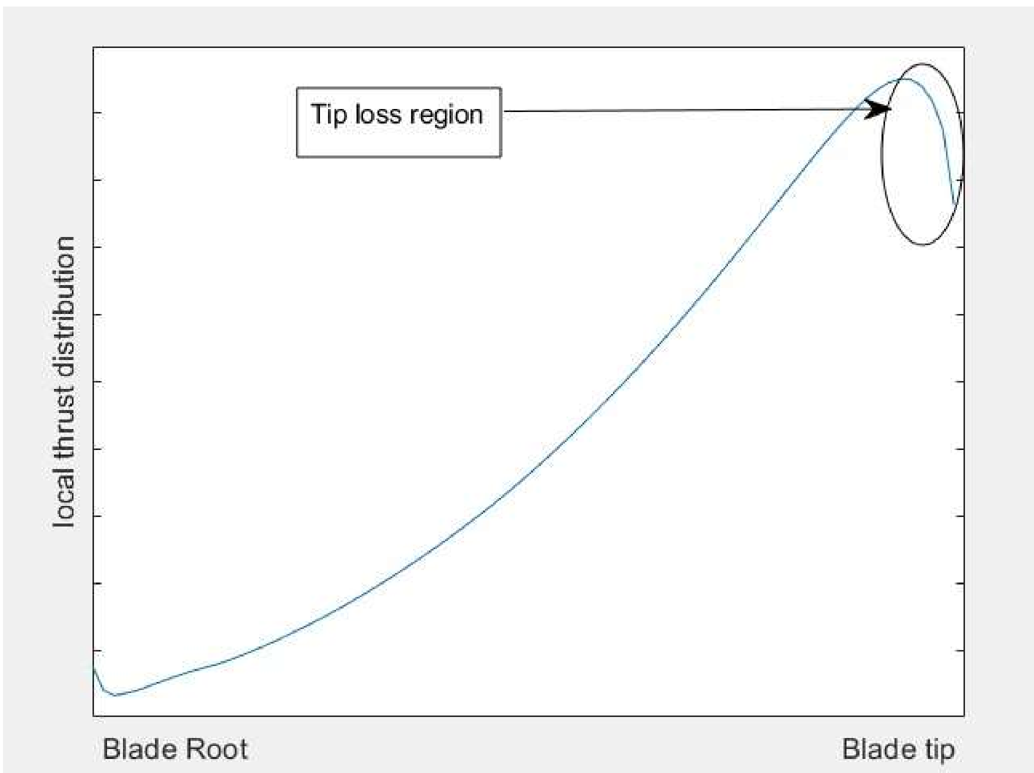

Figure 11: Plot depicting the tip loss effect near the blade tip

Prandtl's Tip Loss Function

Prandtl provided a solution to this tip loss by introducing a tip loss function **F,** which can be interpreted as a correction factor or the reduction factor which is applied to the change in fluid velocity as it passes through the control volume.

The function F can be expressed as:

$$F = \left(\frac{2}{\pi}\right) \cos^{-1}\left(\exp(-f)\right),$$

Where, **f** can be given in terms of radial position and number of blades as

$$f = \frac{N_b}{2}\left(\frac{1-r}{r\phi}\right)$$

This function can be then incorporated in the BEMT code by changing the thrust coefficient obtained from the momentum theory for hovering motor as:

$$dC_T = 4F\lambda^2 r\, dr.$$

Which is then equated with the thrust coefficient obtained from BET, to get the new inflow, and is used to calculate the thrust and power.

After tip-loss correction, we attain the following plots:

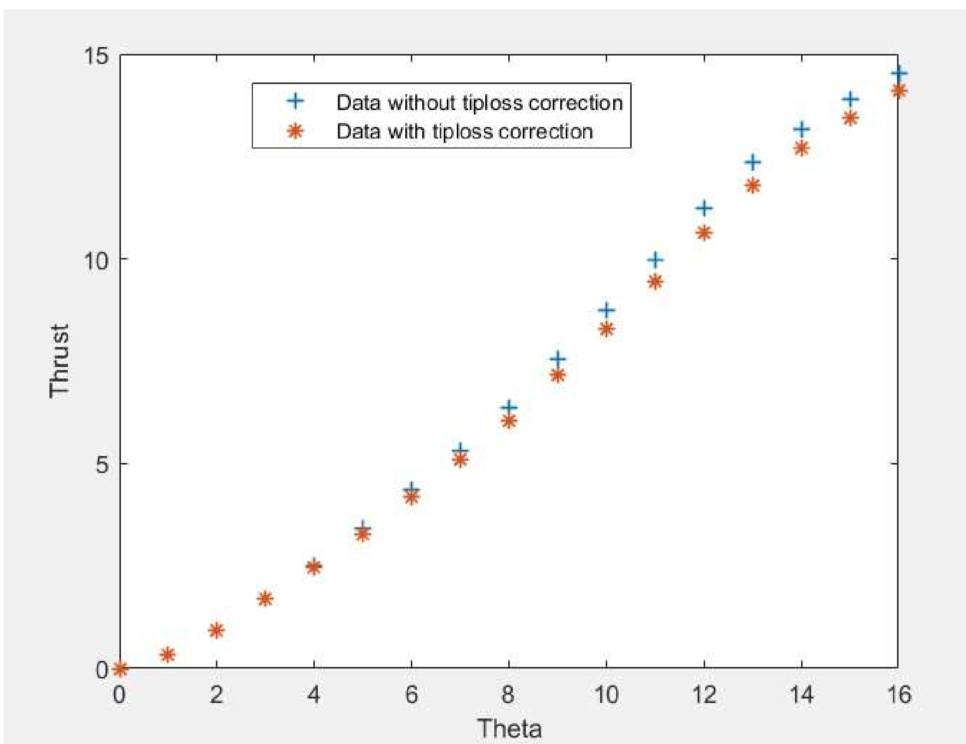

Figure 12: Plot for Thrust vs Pitch angle for values, with and without tip-loss correction

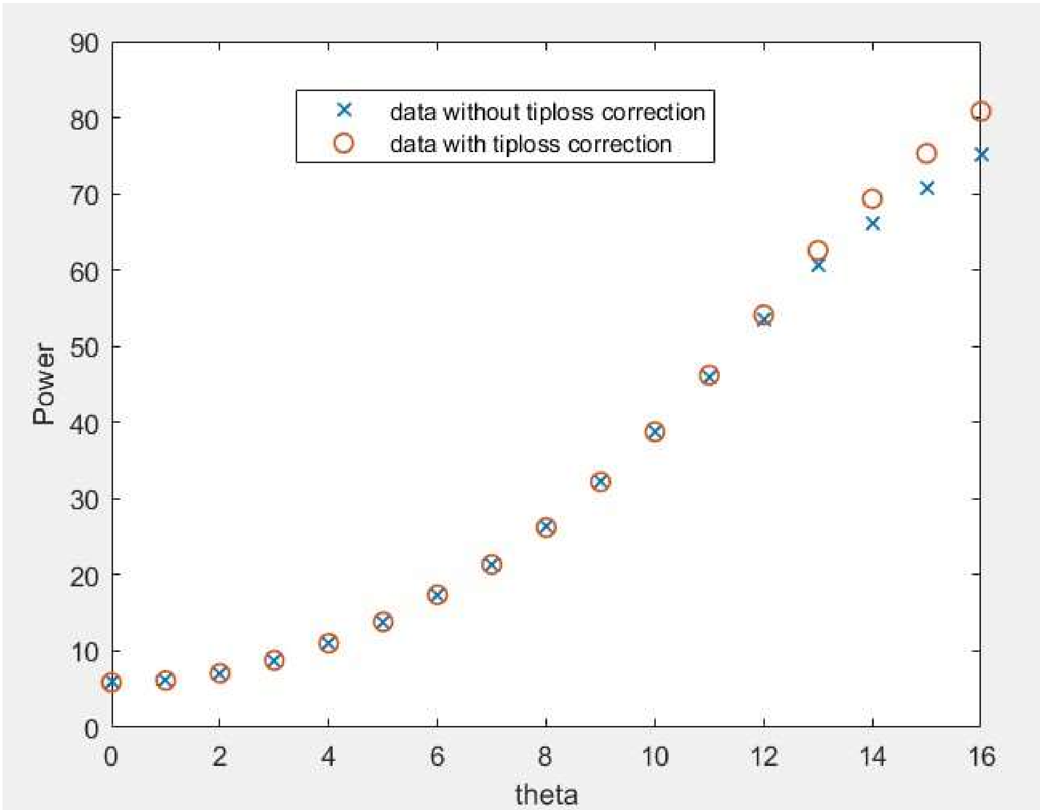

Figure 13: Plot for Power vs Pitch angle for values with and without tip-loss correction

We find that the thrust and power produced with the tip loss correction factor do not vary much from the values without tip loss correction at lower pitch angles ϴ but we can see the deviation at higher pitch angles which gives use the more accurate results.

Conclusion

Till now, we have covered majorly on the rotor aerodynamics for hovering flight, which depicts how different parameters like the number of blades, rotor radius and chord, type of airfoil used, the angular velocity of the rotor, and pitch angle affect the power and thrust produced by it.

We also see how the thrust and power values varies with the methods used for computing them.

Blade Element Momentum Theory produces results that are in vicinity and follow the similar trend of the experimental data. This makes the BEMT quite accurate and precise, which can be used for different rotors and propellors for determining which would be the best fit considering the prototype or the parts we are using to produce the desired thrust and power results.

Future Work

With this, the following work remains to be done in the future:

- The conventional and compound helicopter configurations involve many individual components such as main rotor, tail rotor, fixed-pitch propeller, wing, fuselage and empennage. Building a mathematical model for all these parts would be the first task.
- Building the flight mechanics model for compound helicopter using minimum complexity simulation model which will make the computation easy and make the model more flexible.
- Then a flight control law is developed for the compound helicopter UAV which is validated using the flight mechanics model.

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