Indian Institute of Technology Dharwad, Karnataka, India

EE 221 / EE 201: Introduction to Probability / Data Analysis (first half)
Project (Autumn 2021)
Due date: On or before 20th September 2021, 23:59:59 IST

Instructions

1. Write a short note (typed) on the approach to the problem. If you find typing equations difficult, then insert equations as images of hand-written ones.
2. As usual, discussion is allowed. But copying is not allowed. Any kind of copying, like copying of solutions, copying the project code by simply changing variable names, etc. will result in zero marks for this entire project component.

Questions

1. (25 points) **It's still about coin toss:** Your friend has a biased coin at his home and challenges you to determine its bias, i.e., the probability of heads. You both have a personal communication system which can transmit/receive bits. You ask your friend to toss the coin 10000 times, send a 0 when a tail comes up and a 5 when a head comes up. He/she agrees because whatever you receive is anyway corrupted by noise, which has a Gaussian PDF with mean μ and variance σ^2. Since you have taken this course and can use some tricks to estimate the noise parameters, you put an additional request to your friend and ask him/her to simply send 1000 zeros before sending the coin toss results. Using these 11000 samples of data, characterize the noise and then find the bias of the coin (**rounded to the first decimal place**).
2. (25 points) **Disturbing distributions:** You have been given the data (50000 samples) of a random variable Z. You know that $Z = X + 10Y$, where X is a uniform random variable between -3 and 3. You also know that

$$Y = \sum_{i=1}^{k} W_i,$$

where $k \in \{2, 3, 4\}$, W_i's are independent and identically distributed (i.i.d.) and belong to one of the following:

- Exponential distribution characterized by its mean $\frac{1}{\lambda}$.
- Rayleigh distribution characterized by σ (`https://en.wikipedia.org/wiki/Rayleigh_distribution`)
- Half-normal distribution characterized by σ (`https://en.wikipedia.org/wiki/Half-normal_distribution`).

Come up with a mechanism to find k and the distribution of W_i along with the characterizing parameter **rounded to the nearest integer** (mean if it is an exponential distribution and σ otherwise). Justify your mechanism analytically.