Assignment 6

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Download all python codes from:

```
https://github.com/varenya27/AI1103/blob/main/
    Assignment6/codes
```

and all latex-tikz codes from:

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https://github.com/varenya27/AI1103/blob/main/
    Assignment6/main.tex
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1 Problem

Let Y_1 denote the first order statistic in a random sample of size n from a distribution that has the pdf,

$$f(x) = \begin{cases} e^{-(x-\theta)} & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Obtain the distribution of $Z_n = n(Y_1 - \theta)$

2 Solution

The first order statistic for any sample is the the minimum of the given sample In order to calculate the distribution for Y_1, we need the the cumulative distribution function $F(x)$ of x:

$$F(x) = \int_{-\infty}^{x} f(t)\,dt \tag{2.0.1}$$

$$= \int_{-\infty}^{\theta} 0\,dt + \int_{\theta}^{x} e^{\theta-t}\,dt \tag{2.0.2}$$

$$= \left(e^{\theta-t}\right)_x^{\theta} \tag{2.0.3}$$

$$= 1 - e^{\theta-x} \tag{2.0.4}$$

$$= 1 - f(x) \tag{2.0.5}$$

This gives:

$$F(x) = \begin{cases} 1 - e^{-(x-\theta)} & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases} \tag{2.0.6}$$

$1 - F(x)$ represents the probability that a random variable has a value greater than or equal to x. Let $Q(x)$ represent the distribution for Y_1 in a sample of size n:

$$Q(x) = n \times (1 - F(x))^{n-1} \times f(x) \tag{2.0.7}$$

$$= n(f(x))^n \tag{2.0.8}$$

Thus,

$$Q(x) = \begin{cases} n(e^{-n(x-\theta)}) & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases} \tag{2.0.9}$$

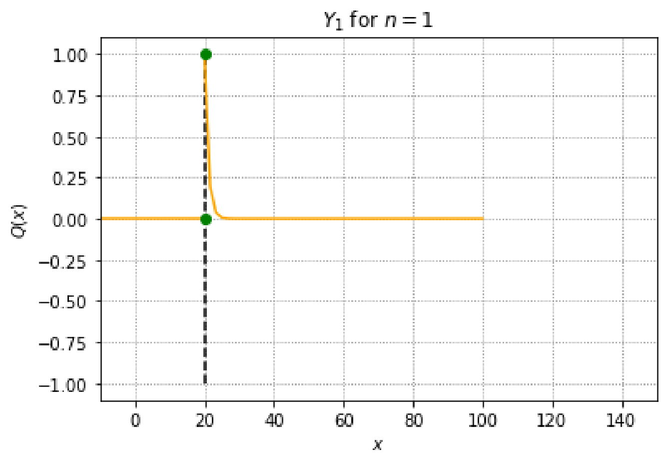

Fig. 0: Distribution when $n = 1$

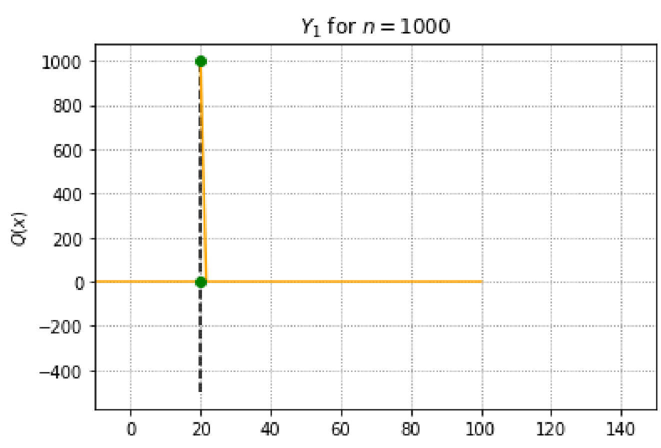

Fig. 0: Distribution as n increases to 1000

Fig. 0 and Fig. 0 are the plots for the function in (2.0.9) when $\theta = 20$. It can be seen that as n approaches infinity, the function becomes a vertical line at $x = \theta$

The required distribution is Z_n:

$$Z_n = n(Y_1 - \theta) \tag{2.0.10}$$

$$= \begin{cases} n(n(e^{-n(x-\theta)}) - \theta) & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases} \tag{2.0.11}$$

Fig. 0 and Fig. 0 are the plots for the distribution in (2.0.11) for $\theta = 20$

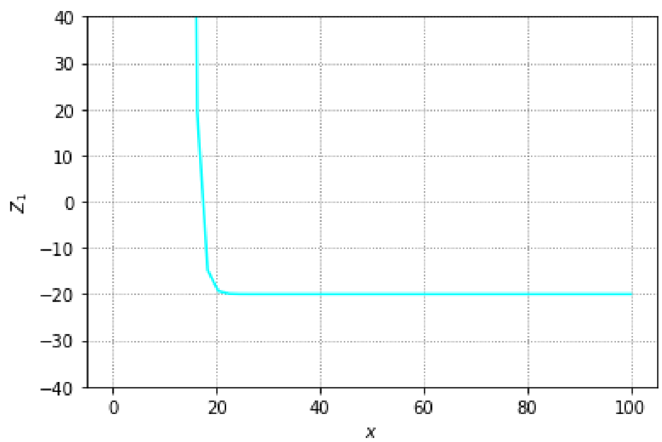

Fig. 0: Z_n when $n = 1$

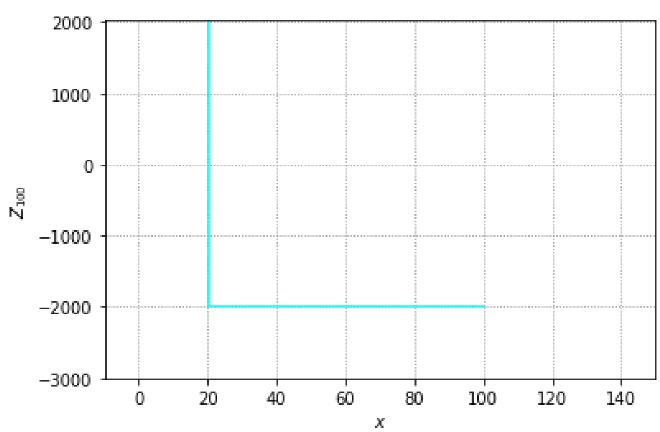

Fig. 0: Z_n when $n = 100$