1

Assignment 3

Varenya Upadhyaya EP20BTECH11026

Download all python codes from:

https://github.com/varenya27/AI1103/blob/main/ Assignment3/codes

and all latex-tikz codes from:

https://github.com/varenya27/AI1103/blob/main/ Assignment3/main.tex

PROBLEM

Let (X,Y) be the coordinates of a point chosen at random inside the disc $x^2 + y^2 \le r^2$ where $r \ge 0$. The probability that $Y \ge mX$ is

(a)
$$\frac{1}{2^r}$$

(c)
$$\frac{1}{2}$$

(b)
$$\frac{1}{2^m}$$

(d)
$$\frac{1}{2^{r+m}}$$

SOLUTION

We know that the equation

$$x^2 + y^2 \le r^2 \tag{1}$$

represents a disc of radius r centred at the origin, while

$$y - mx > 0 \tag{2}$$

represents the region above a line passing through the origin with a slope m. Also, the line y = mx is a diameter to the circle $x^2 + y^2 = r^2$.

(X,Y) is a point selected on the disc. Let a random variable $Z \in \{0,1\}$ represent the possible outcomes of the experiment of selecting a point on the disc.

Equation satisfied by (X,Y)	Z
y - mx < 0	0
$y - mx \ge 0$	1
Table I	

OUTCOME OF THE EXPERIMENT

Since the given line is a diameter of the circle, the number of points on either sides will be equal.

$$\Rightarrow n(Z=0) = n(Z=1) \tag{3}$$

The required probability can be calculated as follows:

$$P(Z=1) = \frac{n(Z=1)}{n(Z=1) + n(Z=0)}$$
 (4)

$$\Rightarrow P(Z=1) = \frac{1}{2} \tag{5}$$

: option (c) is correct.

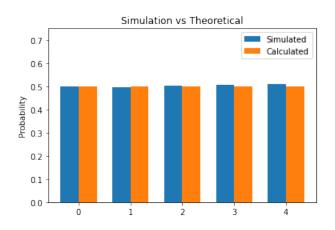


Figure 1. Comparison between the practical and calculated values of the probability