

Assignment 3

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Download all python codes from:

<https://github.com/varenya27/AI1103/blob/main/Assignment3/codes>

and all latex-tikz codes from:

<https://github.com/varenya27/AI1103/blob/main/Assignment3/main.tex>

The required probability can be calculated as follows:

$$P(Z = 1) = \frac{n(Z = 1)}{n(Z = 1) + n(Z = 0)} \quad (4)$$

$$\Rightarrow P(Z = 1) = \frac{1}{2} \quad (5)$$

\therefore option (c) is correct.

PROBLEM

Let (X, Y) be the coordinates of a point chosen at random inside the disc $x^2 + y^2 \leq r^2$ where $r \geq 0$. The probability that $Y \geq mX$ is

- (a) $\frac{1}{2^r}$ (c) $\frac{1}{2}$
 (b) $\frac{1}{2^m}$ (d) $\frac{1}{2^{r+m}}$

SOLUTION

We know that the equation

$$x^2 + y^2 \leq r^2 \quad (1)$$

represents a disc of radius r centred at the origin, while

$$y - mx \geq 0 \quad (2)$$

represents the region above a line passing through the origin with a slope m . Also, the line $y = mx$ is a diameter to the circle $x^2 + y^2 = r^2$.

(X, Y) is a point selected on the disc. Let a random variable $Z \in \{0, 1\}$ represent the possible outcomes of the experiment of selecting a point on the disc.

| Equation satisfied by (X, Y) | Z |
|--------------------------------|-----|
| $y - mx < 0$ | 0 |
| $y - mx \geq 0$ | 1 |

Table I

OUTCOME OF THE EXPERIMENT

Since the given line is a diameter of the circle, the number of points on either sides will be equal.

$$\Rightarrow n(Z = 0) = n(Z = 1) \quad (3)$$

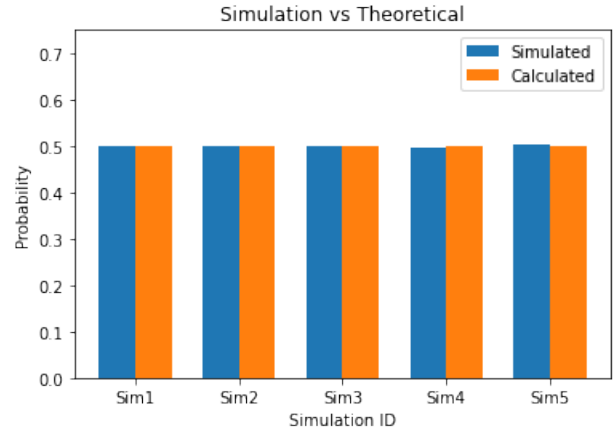


Figure 1. Comparison between the practical and calculated values of the probability