

Assignment 6

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Download all python codes from:

<https://github.com/varenya27/AI1103/blob/main/Assignment6/codes>

and all latex-tikz codes from:

<https://github.com/varenya27/AI1103/blob/main/Assignment6/main.tex>

Now,

$$F(x) = \Pr(X \leq x) \quad (2.0.7)$$

$$\Rightarrow 1 - F(x) = \Pr(X > x) \quad (2.0.8)$$

In the random sample of size n , Y_1 can take n values. For $Y_1 = x$, all the other terms must be greater than x . Let $Q(x)$ be the pdf for Y_1 :

$$Y_1 = \min\{X_1, X_2, \dots, X_n\} \quad (2.0.9)$$

$$Q(x) = \Pr(Y_1 = x) \quad (2.0.10)$$

Assume that X_k is the minimum in the random sample for $k \in \{1, 2, \dots, n\}$

$$Q(x) = n \times \Pr(X_k = x) \times \Pr(X_1 > x, X_2 > x, \dots, X_{k-1} > x) \\ \times \Pr(X_{k+1} > x, \dots, X_n > x)$$

$$= n \times \Pr(X_k = x) \times \left[\prod_{i=1}^{k-1} \Pr(X_i > x) \right] \\ \times \left[\prod_{i=k+1}^n \Pr(X_i > x) \right]$$

$$= n \times f(x) \times [1 - F(x)]^{n-1} \\ = n(f(x))^n \quad (2.0.11)$$

Thus,

$$Q(x) = \begin{cases} n(e^{-n(x-\theta)}) & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases} \quad (2.0.12)$$

Fig. 0 and Fig. 0 are the plots for the function in (2.0.12) when $\theta = 20$. It can be seen that as n approaches infinity, the function attains a spike at $x = \theta$

The required distribution is Z_n :

$$Z_n = n(Y_1 - \theta) \quad (2.0.13)$$

$$= \begin{cases} n(n(e^{-n(x-\theta)}) - \theta) & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases} \quad (2.0.14)$$

1 PROBLEM

Let Y_1 denote the first order statistic in a random sample of size n from a distribution that has the pdf,

$$f(x) = \begin{cases} e^{-(x-\theta)} & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Obtain the distribution of $Z_n = n(Y_1 - \theta)$

2 SOLUTION

The first order statistic for any sample is the minimum of the given sample. In order to calculate the distribution for Y_1 , we need the cumulative distribution function $F(x)$:

$$F(x) = \int_{-\infty}^x f(t) dt \quad (2.0.1)$$

$$= \int_{-\infty}^{\theta} 0 dt + \int_{\theta}^x e^{\theta-t} dt \quad (2.0.2)$$

$$= \left(e^{\theta-t} \right)_x^{\theta} \quad (2.0.3)$$

$$= 1 - e^{\theta-x} \quad (2.0.4)$$

$$= 1 - f(x) \quad (2.0.5)$$

This gives:

$$F(x) = \begin{cases} 1 - e^{-(x-\theta)} & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases} \quad (2.0.6)$$

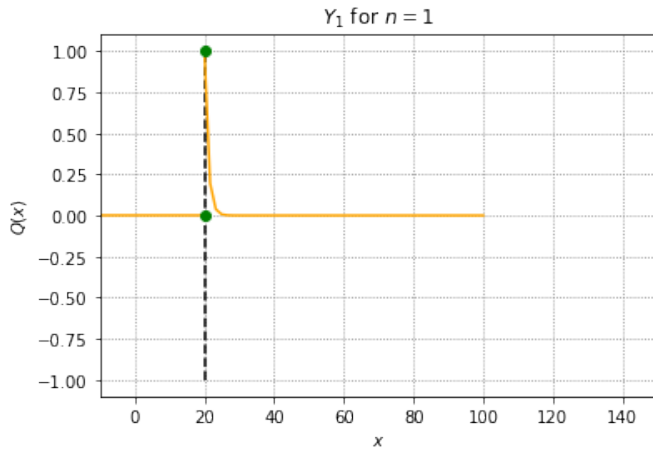


Fig. 0: Distribution when $n = 1$

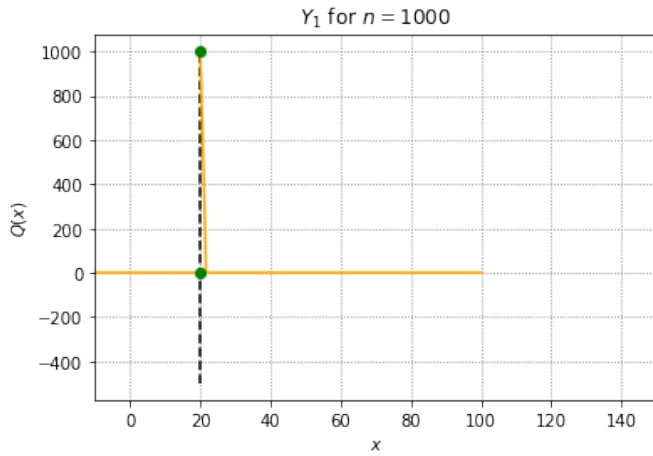


Fig. 0: Distribution as n increases to 1000

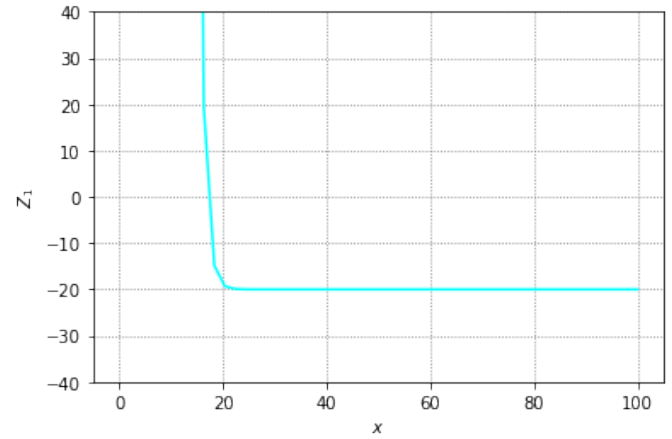


Fig. 0: Z_n when $n = 1$

Fig. 0 and Fig. 0 are the plots for the distribution in (2.0.14) for $\theta = 20$

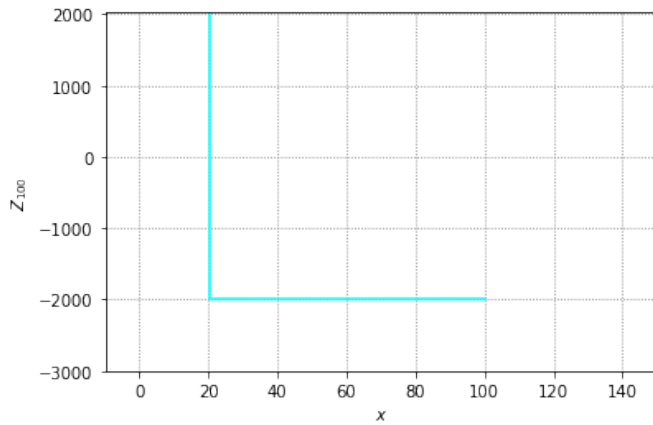


Fig. 0: Z_n when $n = 100$