

Assignment 2

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Download all latex-tikz codes from:

<https://github.com/varennya27/AI1103/blob/main/Assignment2/main.tex>

PROBLEM

If P and Q are two random events, then the following is true:

- (a) Independence of P and Q implies that probability $(P \cap Q) = 0$
- (b) Probability $(P \cup Q) \leq \text{Probability}(P) + \text{Probability}(Q)$
- (c) If P and Q are mutually exclusive, then they must be independent
- (d) Probability $(P \cap Q) \leq \text{Probability}(P)$

SOLUTION

For two random events A and B that are independent, we know that,

$$Pr(A \cap B) = Pr(A)Pr(B) \quad (1)$$

and for two mutually exclusive events C and D,

$$Pr(C \cap D) = 0 \quad (2)$$

- (a) Independence of P and Q implies that the occurrence of one is unaffected by the other.

$$\Rightarrow Pr(P \cap Q) = Pr(P)Pr(Q) \quad (3)$$

The given option will be true only when either $Pr(P)$ or $Pr(Q)$ will be zero, therefore, (a) is incorrect.

- (b) From set theory,

$$A \cup B = A + B - A \cap B \quad (4)$$

$$\Rightarrow Pr(P + Q) = Pr(P) + Pr(Q) - Pr(P \cap Q) \quad (5)$$

$$\Rightarrow Pr(P + Q) \leq Pr(P) + Pr(Q) \quad (6)$$

thus, (b) is incorrect.

- (c) Two events can be both mutually exclusive and independent only when one of them have a zero probability. Since it isn't necessary that $Pr(P) = 0$ or $Pr(Q) = 0$, (c) is incorrect.

- (d) The set P will have either have the same or more elements than the set $P \cap Q$

$$Pr(P \cap Q) \leq Pr(P) \quad (7)$$

- (d) is correct.

Thus, the only correct option is (d).