Assignment 2

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PROBLEM

If P and Q are two random events, then the following is true:
 (a) Independence of P and Q implies that probability $(P \cap Q) = 0$
 (b) Probability $(P \cup Q) \leq$ Probability (P) + Probability(Q)
 (c) If P and Q are mutually exclusive, then they must be independent
 (d) Probability $(P \cap Q) \leq$ Probability (P)

SOLUTION

For two random events A and B that are independent, we know that,

$$Pr(AB) = Pr(A)Pr(B) \qquad (1)$$

and for two mutually exclusive events C and D,

$$Pr(CD) = 0 \qquad (2)$$

(a) Independence of P and Q implies that the occurrence of one is unaffected by the other.

$$\Rightarrow Pr(PQ) = Pr(P)Pr(Q) \qquad (3)$$

The given option will be true only when either Pr(P) or Pr(Q) will be zero, therefore, (a) is incorrect.

(b) From set theory,

$$A \cup B = A + B - A \cap B \qquad (4)$$

$$\Rightarrow Pr(P + Q) = Pr(P) \\ + Pr(Q) - Pr(PQ) \quad (5)$$

$$\Rightarrow Pr(P + Q) \leq Pr(P) + Pr(Q) \quad (6)$$

thus, (b) is incorrect.

(c) Two events can be both mutually exclusive and independent only when one of them have a zero probability. Since it isn't necessary that $Pr(P) = 0$ or $Pr(Q) = 0$, (c) is incorrect.

(d) The set P will have either have the same or more elements than the set $P \cap Q$

$$Pr(PQ) \leq Pr(P) \qquad (7)$$

(d) is correct.

Thus, the only correct option is (d).