Assignment 6

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Download all python codes from:

https://github.com/varenya27/AI1103/blob/main/Assignment6/codes

and all latex-tikz codes from:

https://github.com/varenya27/AI1103/blob/main/Assignment6/main.tex

Problem

Let Y_1 denote the first order statistic in a random sample of size n from a distribution that has the pdf,

$$f(x) = \begin{cases} e^{-(x-\theta)} & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases}$$

Obtain the distribution of $Z_n = n(Y_1 - \theta)$

Solution

The first order statistic for any sample is the first term in the sample once the terms have been placed in ascending order (i.e.. the minimum of the given sample) In order to calculate the distribution for Y_1, we need the the cumulative distribution function of x:
Let $F(x)$ denote the cdf

$$F(x) = \int_{-\infty}^{x} f(t)\,dt \tag{0.0.1}$$

$$= \int_{-\infty}^{\theta} 0\,dt + \int_{\theta}^{x} e^{\theta-t}\,dt \tag{0.0.2}$$

$$= \left(e^{\theta-t}\right)_x^\theta \tag{0.0.3}$$

$$= 1 - e^{\theta-x} \tag{0.0.4}$$

$$= 1 - f(x) \tag{0.0.5}$$

This gives:

$$F(x) = \begin{cases} 1 - e^{-(x-\theta)} & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases} \tag{0.0.6}$$

The cdf $F(x)$ represents the probability that a random variable has a value lesser than x. Consequently, $1 - F(x)$ represents the probability that a random variable has a value greater than or equal to x.
Let $Q(x)$ represent the distribution for Y_1 in a sample of size n:

$$Q(x) = n \times (1 - F(x))^{n-1} \times f(x) \tag{0.0.7}$$

$$= n(f(x))^n \tag{0.0.8}$$

Thus,

$$Q(x) = \begin{cases} n(e^{-n(x-\theta)}) & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases} \tag{0.0.9}$$

The required distribution is Z_n:

$$Z_n = n(Y_1 - \theta) \tag{0.0.10}$$

$$= \begin{cases} n(n(e^{-n(x-\theta)}) - \theta) & \text{when } \theta < x < \infty \\ 0 & \text{otherwise} \end{cases} \tag{0.0.11}$$