Assignment 5

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Download all latex-tikz codes from:

https://github.com/varenya27/AI1103/blob/main/Assignment5/main.tex

Problem

Men arrive in a queue according to a Poisson process with rate λ_1 and women arrive in the same queue according to another Poisson process with rate λ_2. The arrivals of men and women are independent. The probability that the first arrival is a man is

(a) $\dfrac{\lambda_1}{\lambda_1+\lambda_2}$ (b) $\dfrac{\lambda_2}{\lambda_1+\lambda_2}$ (c) $\dfrac{\lambda_1}{\lambda_2}$ (d) $\dfrac{\lambda_2}{\lambda_1}$

Solution

Let X and Y be two discrete random variables that represent the number of men and women arriving in the queue in a given time interval, and E be the event that a man arrives first. For a time interval of t units:

Random variable	Event occurring	Rate	Poisson parameter
X	Men arriving in the queue	λ_1	$\lambda_1 \times t$
Y	Women arriving in the queue	λ_2	$\lambda_2 \times t$
$X+Y$	Any person arriving	$\lambda_3 = \lambda_1 + \lambda_2$	$\lambda_3 \times t$

TABLE I: Outcome of the Experiment

For a Poisson distribution, with parameter λ, we know that,

$$\Pr(X=k) = \frac{e^{-\lambda}\lambda^k}{k!} \tag{0.0.1}$$

Let $(0,t)$ be the time interval in which the first person arrives.

$$\Pr(X+Y=1) = \frac{e^{-\lambda_3 t}(\lambda_3 t)^1}{1!} \tag{0.0.2}$$
$$= e^{-\lambda_3 t}\lambda_3 t \tag{0.0.3}$$

The condition is that the first person arriving has to be a man,

$$\Pr(X=1) = \frac{e^{-\lambda_1 t}(\lambda_1 t)^1}{1!} \tag{0.0.4}$$
$$= e^{-\lambda_1 t}\lambda_1 t \tag{0.0.5}$$
$$\Pr(Y=0) = \frac{e^{-\lambda_2 t}(\lambda_2 t)^0}{0!} \tag{0.0.6}$$
$$= e^{-\lambda_2 t} \tag{0.0.7}$$

The required probability can be calculated as,

$$\Pr(E) = \Pr((X=1)(Y=0) \mid X+Y=1) \tag{0.0.8}$$
$$= \frac{\Pr(X=1)\times\Pr(Y=0)}{\Pr(X+Y=1)} \tag{0.0.9}$$
$$= \frac{e^{-\lambda_1 t}\lambda_1 t \times e^{-\lambda_2 t}}{e^{-\lambda_3 t}\times\lambda_3 t} \tag{0.0.10}$$
$$= \frac{e^{-(\lambda_1+\lambda_2)t}}{e^{-\lambda_3 t}} \times \frac{\lambda_1 t}{\lambda_3 t} \tag{0.0.11}$$

Using $\lambda_3 = \lambda_1 + \lambda_2$ in (0.0.11)

$$\Pr(E) = \frac{e^{-(\lambda_1+\lambda_2)t}}{e^{-(\lambda_1+\lambda_2)t}} \times \frac{\lambda_1}{\lambda_1+\lambda_2} \tag{0.0.12}$$
$$= \frac{\lambda_1}{\lambda_1+\lambda_2} \tag{0.0.13}$$

Thus, option (a) is correct.