Assignment 3

Varenya Upadhyaya EP20BTECH11026

Download all python codes from:

https://github.com/varenya27/AI1103/blob/main/Assignment3/codes

and all latex-tikz codes from:

https://github.com/varenya27/AI1103/blob/main/Assignment3/main.tex

Problem

Let (X,Y) be the coordinates of a point chosen at random inside the disc $x^2 + y^2 \le r^2$ where $r \ge 0$. The probability that $Y \ge mX$ is

(a) $\frac{1}{2^r}$ (c) $\frac{1}{2}$

(b) $\frac{1}{2^m}$ (d) $\frac{1}{2^{r+m}}$

Solution

We know that the point (X, Y) satisfies the equation

$$x^2 + y^2 \le r^2 \tag{0.0.1}$$

Let a random variable $Z \in \{0, 1\}$ denote the possible outcomes of the experiment

Equation satisfied by (X,Y)	Z
$y - mx < 0$	0
$y - mx \ge 0$	1

TABLE I: Outcome of the Experiment

The coordinates (X, Y) can be parametrized as follows:

$$X = a\sin\theta \tag{0.0.2}$$
$$Y = a\cos\theta \tag{0.0.3}$$

where $a \in [0, r]$ and $\theta \in [0, 2\pi]$.

$$Y \ge mX \tag{0.0.4}$$
$$\implies a\sin\theta \ge ma\cos\theta \tag{0.0.5}$$
$$\implies \tan\theta \ge m \tag{0.0.6}$$
$$\implies \theta \in [\arctan m, \pi + \arctan m] \tag{0.0.7}$$

The graph of the distribution function of θ will be a horizontal line above the x axis in $[0, 2\pi]$.

$$\Pr(0 \le \theta \le 2\pi) = 1 \tag{0.0.8}$$
$$\implies f(\theta) \times 2\pi = 1 \tag{0.0.9}$$

The value of the constant function will be $\frac{1}{2\pi}$

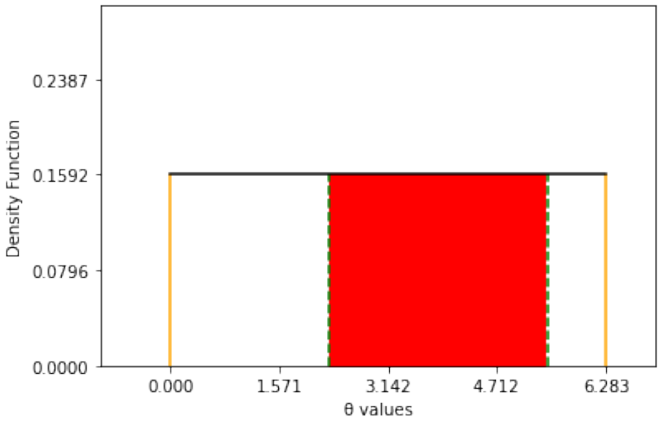

Fig. 0: Distribution function of θ

The shaded region represents the required probability.

$$\Pr(\arctan m \le \theta \le \arctan m + \pi) = \int_{\arctan m}^{\pi+\arctan m} f(\theta)\, d\theta \tag{0.0.10}$$
$$= \int_{\arctan m}^{\pi+\arctan m} \frac{1}{2\pi}\, d\theta \tag{0.0.11}$$
$$= \frac{\pi}{2\pi} \tag{0.0.12}$$
$$= \frac{1}{2} \tag{0.0.13}$$

$\therefore$ option (c) is correct.