Assignment 3

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Download all python codes from:

https://github.com/varenya27/AI1103/blob/main/Assignment3/codes

and all latex-tikz codes from:

https://github.com/varenya27/AI1103/blob/main/Assignment3/main.tex

Problem

Let (X,Y) be the coordinates of a point chosen at random inside the disc $x^2 + y^2 \leq r^2$ where $r \geq 0$. The probability that $Y \geq mX$ is

(a) $\frac{1}{2^r}$ (c) $\frac{1}{2}$

(b) $\frac{1}{2^m}$ (d) $\frac{1}{2^{r+m}}$

Solution

We know that the point (X, Y) satisfies the equation

$$x^2 + y^2 \leq r^2 \tag{0.0.1}$$

Let a random variable $Z \in \{0, 1\}$ denote the possible outcomes of the experiment

Equation satisfied by (X,Y)	Z
$y - mx < 0$	0
$y - mx \geq 0$	1

TABLE I: Outcome of the Experiment

The coordinates (X, Y) can be parametrized as follows:

$$X = a \sin\theta \tag{0.0.2}$$
$$Y = a \cos\theta \tag{0.0.3}$$

where $a \in [0, r]$ and $\theta \in [0, 2\pi]$.

$$Y \geq mX \tag{0.0.4}$$
$$\implies a \sin\theta \geq ma \cos\theta \tag{0.0.5}$$
$$\implies \tan\theta \geq m \tag{0.0.6}$$
$$\implies \theta \in [\arctan m, \pi + \arctan m] \tag{0.0.7}$$

Let **A** denote the set $[\arctan m, \pi + \arctan m]$ and **B** denote the $[0, 2\pi]$. Then,

$$n(Z = 1) = n(\mathbf{A}) \tag{0.0.8}$$
$$n(Z = 1) + n(Z = 0) = n(\mathbf{B}) \tag{0.0.9}$$

The required probability can then be calculated as

$$\Pr(Z = 1) = \frac{n(Z = 1)}{n(Z = 1) + n(Z = 0)} \tag{0.0.10}$$
$$= \frac{n(\mathbf{A})}{n(\mathbf{B})} \tag{0.0.11}$$
$$= \frac{\pi + \arctan m - \arctan m}{2\pi - 0} \tag{0.0.12}$$
$$= \frac{1}{2} \tag{0.0.13}$$

$\therefore$ option (c) is correct.