EE3900 - Assignment 2

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1 Problem

2.32 (c) Consider the difference equation

$$y[n] - \frac{1}{2}y[n-1] = x[n] \tag{1.1}$$

and suppose that

$$x[n] = \left(\frac{1}{3}\right)^n u[n] \tag{1.2}$$

Assume that the solution y[n] consists of the sum of a particular solution $y_p[n]$ to (1.1) and a homogeneous solution $y_h[n]$ satisfying the equation

$$y_h[n] - \frac{1}{2}y_h[n-1] = 0 \tag{1.3}$$

(c) Suppose that the LTI system described by (1.1) and initially at rest has as its input the signal specified by (1.2). Since $x[n] = 0$ for $n < 0$, we have that $y[n] = 0$ for $n < 0$. Also, from parts (a) and (b) we have that y[n] has the form

$$y[n] = A\left(\frac{1}{2}\right)^n + B\left(\frac{1}{3}\right)^n \tag{1.4}$$

for $n \geq 0$. In order to solve for the unknown constant A, we must specify a value for y[n] for some $n \geq 0$. Use the condition of initial rest and (1.1) and (1.2) to determine $y[0]$. From this value determine the constant A. The result of this calculation yields the solution to the difference equation (1.1) under the condition of initial rest, when the input is given by (1.2).

2 Solution

From the condition of initial rest, we have

$$y[n] = 0, n < 0 \tag{2.1}$$

$$\implies y[0] - \frac{1}{2}y[-1] = x[0] \tag{2.2}$$

$$\implies y[0] = \left(\frac{1}{3}\right)^0 u[0] = 1 \tag{2.3}$$

Thus, for $n \geq 0$:

$$y_p[n] - \frac{1}{2}y_p[n-1] = \left(\frac{1}{3}\right)^n \tag{2.4}$$

$$y_p[n] = B\left(\frac{1}{3}\right)^n \tag{2.5}$$

$$\implies B\left(\frac{1}{3}\right)^n - \frac{1}{2}B\left(\frac{1}{3}\right)^{n-1} = \left(\frac{1}{3}\right)^n \tag{2.6}$$

$$\implies B\left(1 - \frac{3}{2}\right) = 1 \tag{2.7}$$

$$\implies B = -2 \tag{2.8}$$

Using (2.8) and (2.3) in(1.4):

$$y[0] = A + B \tag{2.9}$$

$$\implies A = 1 + 2 \tag{2.10}$$

$$\implies A = 3 \tag{2.11}$$

The solution for the difference equation is thus:

$$y[n] = 3\left(\frac{1}{2}\right)^n u[n] - 2\left(\frac{1}{3}\right)^n u[n] \tag{2.12}$$