Scalar-Tensor theory of Gravity: Project Outline

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1 General Relativity: The Classical theory of Gravity

- Formulated by Einstein in 1915; describes gravity as spacetime curvature.
- Equivalence Priciple
- Predictions validated by numerous experiment.

1.1 Einstein-Hilbert Action

- Action is given by

$$S = \frac{1}{16\pi} \int d^4x \sqrt{-g} R + S_M[\Psi, g_{\mu\nu}] \tag{1}$$

where g is the metric, R is the Ricci scalar and S_M is the matter action (Ψ denotes the matter fields)

1.2 Derivation of Einstein Field-equations from Einstein-Hilbert Action

- Application of the variational principle to derive equations of motion.

2 Modified Theories of GR

- Motivation: Address limitations of GR in extreme conditions (low and high energy scales)

2.1 Lovelock's theorem

Theorem: *In four spacetime dimensions the only divergence-free symmetric rank-2 tensor constructed solely from the metric $g_{\mu\nu}$ and its derivatives up to second differential order, and preserving diffeomorphism invariance, is the Einstein tensor plus a cosmological term.*

- Provides a framework for understanding which assumptions can be relaxed or modified to develop alternative theories of gravity.

2.2 Classification of Modified-GR theories

[1]

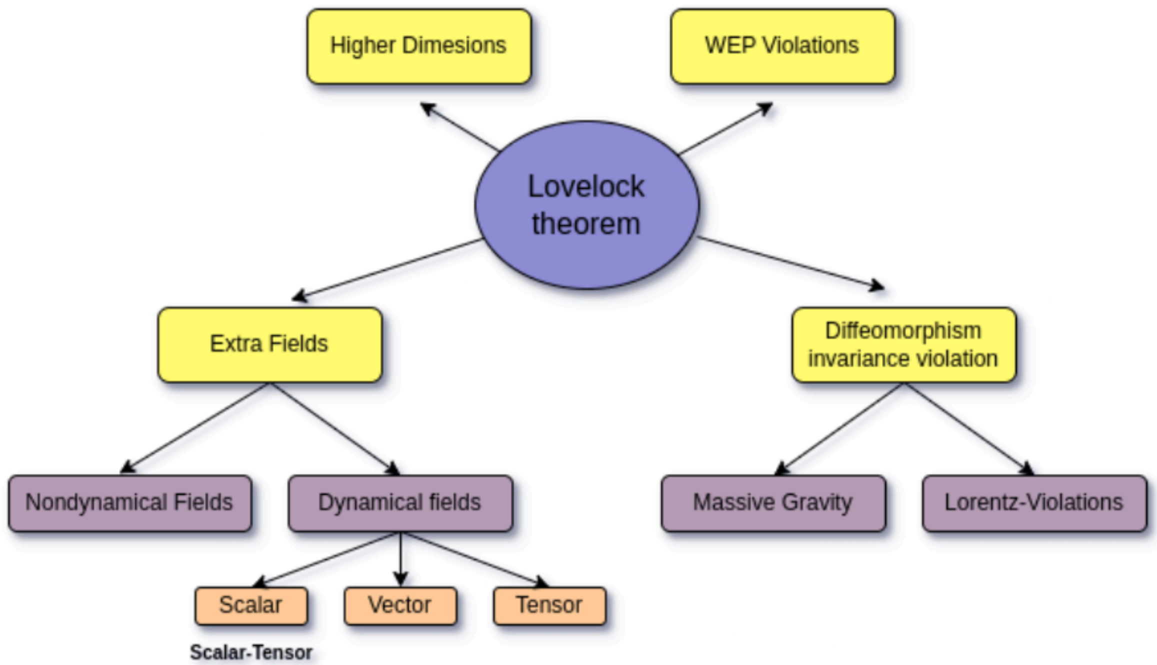

Figure 1: Classification of modified theories of gravity governed by Lovelock's theorem

3 Scalar-Tensor theory of Gravity

- Introduces scalar fields alongside the tensor field of GR.

3.1 Action

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}[\phi R - \frac{\omega(\phi)}{\phi}g_{\mu\nu}(\partial_\mu\phi)((\partial_\nu\phi) - U(\phi)] + S_M[\Psi, g_{\mu\nu}] \tag{2}$$

where U and ω are the arbitrary functions of the introduced scalar field ϕ.

- Brans-Dicke gravity extension will be considered for this project wherein $\omega(\phi) = \omega$ is constant and $U(\phi) = 0$

3.2 Derivation of Field-equations from the action

- Application of the variational principle to derive equations of motion.

4 Comparison of Field-equations in ST and GR

- Examine similarities and differences between GR and scalar-tensor field equations.
- Discuss how modifications affect predictions and observable phenomena.

5 Observational Evidence for Scalar-Tensor Theory

- Brief discussion based on literature on how scalar-tensor theories can be tested against observations. For ex- dipolar radiation gravitational waves signature in black hole-neutron star binary case[2].

References

[1] Berti, E., et al. Testing general relativity with present and future astrophysical observations. *Classical and Quantum Gravity 32*, 24 (Dec. 2015), 243001.

[2] Ma, S., Varma, V., Stein, L. C., Foucart, F., Duez, M. D., Kidder, L. E., Pfeiffer, H. P., and Scheel, M. A. Numerical simulations of black hole-neutron star mergers in scalar-tensor gravity. *Physical Review D 107*, 12 (June 2023).