

The Hawking-Penrose Singularity Theorems

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October 26, 2024

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1 Introduction

The theory of general relativity revolutionized our understanding of space and time, describing gravity as a consequence of the curvature of space-time. Singularities play a pivotal role in cosmology and black hole physics. This report explores the contributions of Stephen Hawking and Roger Penrose, whose singularity theorem provides fundamental insights into the nature of these regions.

This report is inspired by and follows the series of lectures "Geodesics and Singularity theorems in General Relativity" by Professor Sunil Mukhi. I will try to derive certain results that were left to the reader of the original lectures that will hopefully add to the understanding of the Singularity theorems.

2 Causality in Space-Time

I will define and derive certain results that will be needed further into the report. These include the Schwarzschild solution of the Einstein Field equations, the equation of a geodesic in space-time and will take a deeper look into causality. Once the stage has been set, I will introduce the concept of globally hyperbolic space-time and look into its properties.

2.1 Basic Definitions and derivation of Schwarzschild metric

2.2 Globally Hyperbolic Space-Times and its properties

2.3 Importance of Causality in Singularity Theorems

3 Geodesics and Focal Points

In this section, I will take a deeper dive into geodesics in space-time and introduce the idea of a focal point. Towards the end, I will derive the Raychaudhuri equation.

3.1 Geodesics in Space-Time

3.2 Focal Points and Their Role

3.3 The Raychaudhuri Equation

4 The Hawking-Penrose Singularity Theorem

With all the pieces set, I will state and explain the consequences of Hawking's Singularity theorem and formalize the definition of a singularity. Then Penrose's Singularity theorem for will be stated and its consequences will be studied. Towards the end of the report I will try to examine the consequences of relaxing them.

4.1 Hawking's Theorem

4.2 Penrose's Theorem

4.3 Energy Conditions

References

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