Astroparticle Physics
Lecture 2

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This lecture

- Discovery of cosmic rays
  - The nuanced history
- Cosmic Ray Physics in India
  - Homi Bhabha’s role

- On the way
  - Understand elementary particle physics
  - Cross sections
  - Natural Units
  - Standard model particle content, quantum numbers

Cirkel-Bartelt : Living Rev Relativ. 2008; 11(1): 2. https://dx.doi.org/10.12942%2Flrr-2008-2

Astroparticle Physics, a History

Victor Hess 1911-1912 Cosmic Rays

Ionization of air : Already known wrt Becquerel's discovery of radioactivity. Gold leaf electroscope

1911 : Hess, electroscope. As opposed to lower ionization, at higher altitudes he found higher ionization.

Millikan 1913 : Ionization reduced underwater, increased on mountains.

Wilson cloud chambers : Secondary showers (1900-1950+)

1938: Auger, long distance coincidence between low and high altitudes.

1933 : Positron discovered through cosmic ray studies in cloud chambers.

1944 : Synchrotron

1947 : Fermi acceleration publications

Hadrons			Leptons	
	Baryons	Mesons	Charged	Neutral
u, d:	p, n, Δ	π, η, ρ, ω ...	$e^{\pm}$	ν_L
s:	Λ, Σ, Ξ, Ω	K	$\mu^{\pm}$	ν_M
c:	Λ_c, Σ_c, Ξ_c, Ω_c	D, D_s, η_c, J/ψ, χ_c		
b:	Λ_b, Σ_b, Ξ_b, Ω_b	B, B_s, B_c, Υ, χ_b	$\tau^{\pm}$	ν_H
t:	–	–		

This short table gives the name, the quantum numbers (where known), and the status of baryons in the Review. Only the baryons with 3- or 4-star status are included in the Baryon Summary Table. Due to insufficient data or uncertain interpretation, the other entries in the table are not established baryons. The names with masses are of baryons that decay strongly. The spin-parity J^P (when known) is given with each particle. For the strongly decaying particles, the J^P values are considered to be part of the names.

p	$1/2^+$	****	$\Delta(1232)$	$3/2^+$	****	Σ^+	$1/2^+$	****	Ξ^0	$1/2^+$	****	Λ_c^+	$1/2^+$	****
n	$1/2^+$	****	$\Delta(1600)$	$3/2^+$	***	Σ^0	$1/2^+$	****	Ξ^-	$1/2^+$	****	$\Lambda_c(2595)^+$	$1/2^-$	***
$N(1440)$	$1/2^+$	****	$\Delta(1620)$	$1/2^-$	****	Σ^-	$1/2^+$	****	$\Xi(1530)$	$3/2^+$	****	$\Lambda_c(2625)^+$	$3/2^-$	***
$N(1520)$	$3/2^-$	****	$\Delta(1700)$	$3/2^-$	****	$\Sigma(1385)$	$3/2^+$	****	$\Xi(1620)$		*	$\Lambda_c(2765)^+$		*
$N(1535)$	$1/2^-$	****	$\Delta(1750)$	$1/2^+$	*	$\Sigma(1480)$		*	$\Xi(1690)$		***	$\Lambda_c(2880)^+$	$5/2^+$	***
$N(1650)$	$1/2^-$	****	$\Delta(1900)$	$1/2^-$	**	$\Sigma(1560)$		**	$\Xi(1820)$	$3/2^-$	***	$\Lambda_c(2940)^+$		***
$N(1675)$	$5/2^-$	****	$\Delta(1905)$	$5/2^+$	****	$\Sigma(1580)$	$3/2^-$	*	$\Xi(1950)$		***	$\Sigma_c(2455)$	$1/2^+$	****
$N(1680)$	$5/2^+$	****	$\Delta(1910)$	$1/2^+$	****	$\Sigma(1620)$	$1/2^-$	*	$\Xi(2030)$	$\geq \frac{5}{2}^?$	***	$\Sigma_c(2520)$	$3/2^+$	***
$N(1685)$		*	$\Delta(1920)$	$3/2^+$	***	$\Sigma(1660)$	$1/2^+$	***	$\Xi(2120)$		*	$\Sigma_c(2800)$		***
$N(1700)$	$3/2^-$	***	$\Delta(1930)$	$5/2^-$	***	$\Sigma(1670)$	$3/2^-$	****	$\Xi(2250)$		**	Ξ_c^+	$1/2^+$	***
$N(1710)$	$1/2^+$	***	$\Delta(1940)$	$3/2^-$	**	$\Sigma(1690)$		**	$\Xi(2370)$		**	Ξ_c^0	$1/2^+$	***
$N(1720)$	$3/2^+$	****	$\Delta(1950)$	$7/2^+$	****	$\Sigma(1730)$	$3/2^+$	*	$\Xi(2500)$		*	$\Xi_c'^+$	$1/2^+$	***
$N(1860)$	$5/2^+$	**	$\Delta(2000)$	$5/2^+$	**	$\Sigma(1750)$	$1/2^-$	***				$\Xi_c'^0$	$1/2^+$	***
$N(1875)$	$3/2^-$	***	$\Delta(2150)$	$1/2^-$	*	$\Sigma(1770)$	$1/2^+$	*	Ω^-	$3/2^+$	****	$\Xi_c(2645)$	$3/2^+$	***
$N(1880)$	$1/2^+$	**	$\Delta(2200)$	$7/2^-$	*	$\Sigma(1775)$	$5/2^-$	****	$\Omega(2250)^-$		***	$\Xi_c(2790)$	$1/2^-$	***
$N(1895)$	$1/2^-$	**	$\Delta(2300)$	$9/2^+$	**	$\Sigma(1840)$	$3/2^+$	*	$\Omega(2380)^-$		**	$\Xi_c(2815)$	$3/2^-$	***
$N(1900)$	$3/2^+$	***	$\Delta(2350)$	$5/2^-$	*	$\Sigma(1880)$	$1/2^+$	**	$\Omega(2470)^-$		**	$\Xi_c(2930)$		*
$N(1990)$	$7/2^+$	**	$\Delta(2390)$	$7/2^+$	*	$\Sigma(1900)$	$1/2^-$	*				$\Xi_c(2980)$		***
$N(2000)$	$5/2^+$	**	$\Delta(2400)$	$9/2^-$	**	$\Sigma(1915)$	$5/2^+$	****				$\Xi_c(3055)$		**
$N(2040)$	$3/2^+$	*	$\Delta(2420)$	$11/2^+$	****	$\Sigma(1940)$	$3/2^+$	*				$\Xi_c(3080)$		***
$N(2060)$	$5/2^-$	**	$\Delta(2750)$	$13/2^-$	**	$\Sigma(1940)$	$3/2^-$	***				$\Xi_c(3123)$		*
$N(2100)$	$1/2^+$	*	$\Delta(2950)$	$15/2^+$	**	$\Sigma(2000)$	$1/2^-$	*				Ω_c^0	$1/2^+$	***
$N(2120)$	$3/2^-$	**				$\Sigma(2030)$	$7/2^+$	****				$\Omega_c(2770)^0$	$3/2^+$	***
$N(2190)$	$7/2^-$	****	Λ	$1/2^+$	****	$\Sigma(2070)$	$5/2^+$	*						
$N(2220)$	$9/2^+$	****	$\Lambda(1405)$	$1/2^-$	****	$\Sigma(2080)$	$3/2^+$	**				Ξ_{cc}^+		*
$N(2250)$	$9/2^-$	****	$\Lambda(1520)$	$3/2^-$	****	$\Sigma(2100)$	$7/2^-$	*						
$N(2300)$	$1/2^+$	**	$\Lambda(1600)$	$1/2^+$	***	$\Sigma(2250)$		***				Λ_b^0	$1/2^+$	***
$N(2570)$	$5/2^-$	**	$\Lambda(1670)$	$1/2^-$	****	$\Sigma(2455)$		**				$\Lambda_b(5912)^0$	$1/2^-$	***
$N(2600)$	$11/2^-$	***	$\Lambda(1690)$	$3/2^-$	****	$\Sigma(2620)$		**				$\Lambda_b(5920)^0$	$3/2^-$	***
$N(2700)$	$13/2^+$	**	$\Lambda(1710)$	$1/2^+$	*	$\Sigma(3000)$		*				Σ_b	$1/2^+$	***
			$\Lambda(1800)$	$1/2^-$	***	$\Sigma(3170)$		*				Σ_b^*	$3/2^+$	***
			$\Lambda(1810)$	$1/2^+$	***							Ξ_b^0, Ξ_b^-	$1/2^+$	***
			$\Lambda(1820)$	$5/2^+$	****							$\Xi_b(5945)^0$	$3/2^+$	***
			$\Lambda(1830)$	$5/2^-$	****							Ω_b^-	$1/2^+$	***
			$\Lambda(1890)$	$3/2^+$	****									
			$\Lambda(2000)$		*									
			$\Lambda(2020)$	$7/2^+$	*									
			$\Lambda(2050)$	$3/2^-$	*									
			$\Lambda(2100)$	$7/2^-$	****									
			$\Lambda(2110)$	$5/2^+$	***									
			$\Lambda(2325)$	$3/2^-$	*									
			$\Lambda(2350)$	$9/2^+$	***									
			$\Lambda(2585)$		**									

**** Existence is certain, and properties are at least fairly well explored.

*** Existence ranges from very likely to certain, but further confirmation is desirable and/or quantum numbers, branching fractions, etc. are not well determined.

** Evidence of existence is only fair.

* Evidence of existence is poor.

http://pdg.lbl.gov

See also the table of suggested $q\overline{q}$ quark-model assignments in the Quark Model section.

• Indicates particles that appear in the preceding Meson Summary Table. We do not regard the other entries as being established.

LIGHT UNFLAVORED ($S = C = B = 0$)

	$I^G(J^{PC})$		$I^G(J^{PC})$
• $\pi^\pm$	$1^-(0^-)$	• $\phi(1680)$	$0^-(1^{--})$
• π^0	$1^-(0^{-+})$	• $\rho_3(1690)$	$1^+(3^{--})$
• η	$0^+(0^{-+})$	• $\rho(1700)$	$1^+(1^{--})$
• $f_0(500)$	$0^+(0^{++})$	$a_2(1700)$	$1^-(2^{++})$
• $\rho(770)$	$1^+(1^{--})$	• $f_0(1710)$	$0^+(0^{++})$
• $\omega(782)$	$0^-(1^{--})$	$\eta(1760)$	$0^+(0^{-+})$
• $\eta'(958)$	$0^+(0^{-+})$	• $\pi(1800)$	$1^-(0^{-+})$
• $f_0(980)$	$0^+(0^{++})$	$f_2(1810)$	$0^+(2^{++})$
• $a_0(980)$	$1^-(0^{++})$	$X(1835)$	$?^?(?^{-+})$
• $\phi(1020)$	$0^-(1^{--})$	$X(1840)$	$?^?(?^{??})$
• $h_1(1170)$	$0^-(1^{+-})$	• $\phi_3(1850)$	$0^-(3^{--})$
• $b_1(1235)$	$1^+(1^{+-})$	$\eta_2(1870)$	$0^+(2^{-+})$
• $a_1(1260)$	$1^-(1^{++})$	• $\pi_2(1880)$	$1^-(2^{-+})$
• $f_2(1270)$	$0^+(2^{++})$	$\rho(1900)$	$1^+(1^{--})$
• $f_1(1285)$	$0^+(1^{++})$	$f_2(1910)$	$0^+(2^{++})$
• $\eta(1295)$	$0^+(0^{-+})$	• $f_2(1950)$	$0^+(2^{++})$
• $\pi(1300)$	$1^-(0^{-+})$	$\rho_3(1990)$	$1^+(3^{--})$
• $a_2(1320)$	$1^-(2^{++})$	• $f_2(2010)$	$0^+(2^{++})$
• $f_0(1370)$	$0^+(0^{++})$	$f_0(2020)$	$0^+(0^{++})$
$h_1(1380)$	$?^-(1^{+-})$	• $a_4(2040)$	$1^-(4^{++})$
• $\pi_1(1400)$	$1^-(1^{-+})$	• $f_4(2050)$	$0^+(4^{++})$
• $\eta(1405)$	$0^+(0^{-+})$	$\pi_2(2100)$	$1^-(2^{-+})$
• $f_1(1420)$	$0^+(1^{++})$	$f_0(2100)$	$0^+(0^{++})$
• $\omega(1420)$	$0^-(1^{--})$	$f_2(2150)$	$0^+(2^{++})$
$f_2(1430)$	$0^+(2^{++})$	$\rho(2150)$	$1^+(1^{--})$
• $a_0(1450)$	$1^-(0^{++})$	• $\phi(2170)$	$0^-(1^{--})$
• $\rho(1450)$	$1^+(1^{--})$	$f_0(2200)$	$0^+(0^{++})$
• $\eta(1475)$	$0^+(0^{-+})$	$f_J(2220)$	$0^+(2^{++}$
• $f_0(1500)$	$0^+(0^{++})$		or $4^{++})$
$f_1(1510)$	$0^+(1^{++})$	$\eta(2225)$	$0^+(0^{-+})$
• $f_2'(1525)$	$0^+(2^{++})$	$\rho_3(2250)$	$1^+(3^{--})$
$f_2(1565)$	$0^+(2^{++})$	• $f_2(2300)$	$0^+(2^{++})$
$\rho(1570)$	$1^+(1^{--})$	$f_4(2300)$	$0^+(4^{++})$
$h_1(1595)$	$0^-(1^{+-})$	$f_0(2330)$	$0^+(0^{++})$
• $\pi_1(1600)$	$1^-(1^{-+})$	• $f_2(2340)$	$0^+(2^{++})$
$a_1(1640)$	$1^-(1^{++})$	$\rho_5(2350)$	$1^+(5^{--})$
$f_2(1640)$	$0^+(2^{++})$	$a_6(2450)$	$1^-(6^{++})$
• $\eta_2(1645)$	$0^+(2^{-+})$	$f_6(2510)$	$0^+(6^{++})$
• $\omega(1650)$	$0^-(1^{--})$		
• $\omega_3(1670)$	$0^-(3^{--})$		
• $\pi_2(1670)$	$1^-(2^{-+})$		

OTHER LIGHT

Further States

STRANGE ($S = \pm1$, $C = B = 0$)

	$I(J^P)$
• $K^\pm$	$1/2(0^-)$
• K^0	$1/2(0^-)$
• K_S^0	$1/2(0^-)$
• K_L^0	$1/2(0^-)$
$K_0^*(800)$	$1/2(0^+)$
• $K^*(892)$	$1/2(1^-)$
• $K_1(1270)$	$1/2(1^+)$
• $K_1(1400)$	$1/2(1^+)$
• $K^*(1410)$	$1/2(1^-)$
• $K_0^*(1430)$	$1/2(0^+)$
• $K_2^*(1430)$	$1/2(2^+)$
$K(1460)$	$1/2(0^-)$
$K_2(1580)$	$1/2(2^-)$
$K(1630)$	$1/2(?^?)$
$K_1(1650)$	$1/2(1^+)$
• $K^*(1680)$	$1/2(1^-)$
• $K_2(1770)$	$1/2(2^-)$
• $K_3^*(1780)$	$1/2(3^-)$
• $K_2(1820)$	$1/2(2^-)$
$K(1830)$	$1/2(0^-)$
$K_0^*(1950)$	$1/2(0^+)$
$K_2^*(1980)$	$1/2(2^+)$
• $K_4^*(2045)$	$1/2(4^+)$
$K_2(2250)$	$1/2(2^-)$
$K_3(2320)$	$1/2(3^+)$
$K_5^*(2380)$	$1/2(5^-)$
$K_4(2500)$	$1/2(4^-)$
$K(3100)$	$?^?(?^{??})$

CHARMED ($C = \pm1$)

	$I(J^P)$
• $D^\pm$	$1/2(0^-)$
• D^0	$1/2(0^-)$
• $D^*(2007)^0$	$1/2(1^-)$
• $D^*(2010)^\pm$	$1/2(1^-)$
• $D_0^*(2400)^0$	$1/2(0^+)$
$D_0^*(2400)^\pm$	$1/2(0^+)$
• $D_1(2420)^0$	$1/2(1^+)$
$D_1(2420)^\pm$	$1/2(?^?)$
$D_1(2430)^0$	$1/2(1^+)$
• $D_2^*(2460)^0$	$1/2(2^+)$
• $D_2^*(2460)^\pm$	$1/2(2^+)$
$D(2550)^0$	$1/2(0^-)$
$D(2600)$	$1/2(?^?)$
$D^*(2640)^\pm$	$1/2(?^?)$
$D(2750)$	$1/2(?^?)$

CHARMED, STRANGE ($C = S = \pm1$)

	$I(J^P)$
• $D_s^\pm$	$0(0^-)$
• $D_s^{*\pm}$	$0(?^?)$
• $D_{s0}^*(2317)^\pm$	$0(0^+)$
• $D_{s1}(2460)^\pm$	$0(1^+)$
• $D_{s1}(2536)^\pm$	$0(1^+)$
• $D_{s2}(2573)$	$0(?^?)$
• $D_{s1}^*(2700)^\pm$	$0(1^-)$
$D_{sJ}^*(2860)^\pm$	$0(?^?)$
$D_{sJ}(3040)^\pm$	$0(?^?)$

BOTTOM ($B = \pm1$)

• $B^\pm$	$1/2(0^-)$
• B^0	$1/2(0^-)$
• $B^\pm/B^0$ ADMIXTURE	
• $B^\pm/B^0/B_s^0/b$ baryon ADMIXTURE	
V_{cb} and V_{ub} CKM Matrix Elements	
• B^*	$1/2(1^-)$
$B_J^*(5732)$	$?(?^?)$
• $B_1(5721)^0$	$1/2(1^+)$
• $B_2^*(5747)^0$	$1/2(2^+)$

BOTTOM, STRANGE ($B = \pm1$, $S = \mp1$)

• B_s^0	$0(0^-)$
• B_s^*	$0(1^-)$
• $B_{s1}(5830)^0$	$0(1^+)$
• $B_{s2}^*(5840)^0$	$0(2^+)$
$B_{sJ}^*(5850)$	$?(?^?)$

BOTTOM, CHARMED ($B = C = \pm1$)

• $B_c^\pm$	$0(0^-)$

$c\overline{c}$

	$I^G(J^{PC})$
• $\eta_c(1S)$	$0^+(0^{-+})$
• $J/\psi(1S)$	$0^-(1^{--})$
• $\chi_{c0}(1P)$	$0^+(0^{++})$
• $\chi_{c1}(1P)$	$0^+(1^{++})$
• $h_c(1P)$	$?^?(1^{+-})$
• $\chi_{c2}(1P)$	$0^+(2^{++})$
• $\eta_c(2S)$	$0^+(0^{-+})$
• $\psi(2S)$	$0^-(1^{--})$
• $\psi(3770)$	$0^-(1^{--})$
$X(3823)$	$?^?(?^{?-})$
• $X(3872)$	$0^+(1^{++})$
• $X(3900)^\pm$	$?(1^+)$
$X(3900)^0$	$?(?^?)$
• $\chi_{c0}(2P)$	$0^+(0^{++})$
• $\chi_{c2}(2P)$	$0^+(2^{++})$
$X(3940)$	$?^?(?^{??})$
$X(4020)^\pm$	$?(?^?)$
• $\psi(4040)$	$0^-(1^{--})$
$X(4050)^\pm$	$?(?^?)$
$X(4140)$	$0^+(?^{?+})$
• $\psi(4160)$	$0^-(1^{--})$
$X(4160)$	$?^?(?^{??})$
$X(4250)^\pm$	$?(?^?)$
• $X(4260)$	$?^?(1^{--})$
$X(4350)$	$0^+(?^{?+})$
• $X(4360)$	$?^?(1^{--})$
• $\psi(4415)$	$0^-(1^{--})$
$X(4430)^\pm$	$?(1^+)$
• $X(4660)$	$?^?(1^{--})$

$b\overline{b}$

	$I^G(J^{PC})$
$\eta_b(1S)$	$0^+(0^{-+})$
• $\Upsilon(1S)$	$0^-(1^{--})$
• $\chi_{b0}(1P)$	$0^+(0^{++})$
• $\chi_{b1}(1P)$	$0^+(1^{++})$
• $h_b(1P)$	$?^?(1^{+-})$
• $\chi_{b2}(1P)$	$0^+(2^{++})$
$\eta_b(2S)$	$0^+(0^{-+})$
• $\Upsilon(2S)$	$0^-(1^{--})$
• $\Upsilon(1D)$	$0^-(2^{--})$
• $\chi_{b0}(2P)$	$0^+(0^{++})$
• $\chi_{b1}(2P)$	$0^+(1^{++})$
$h_b(2P)$	$?^?(1^{+-})$
• $\chi_{b2}(2P)$	$0^+(2^{++})$
• $\Upsilon(3S)$	$0^-(1^{--})$
• $\chi_b(3P)$	$?^?(?^{?+})$
• $\Upsilon(4S)$	$0^-(1^{--})$
$X(10610)^\pm$	$1^+(1^+)$
$X(10610)^0$	$1^+(1^+)$
$X(10650)^\pm$	$?^+(1^+)$
• $\Upsilon(10860)$	$0^-(1^{--})$
• $\Upsilon(11020)$	$0^-(1^{--})$

	Spin	# bar.	# lept.	Q elec.	T, T_3 weak[1]	C strong
Leptons:						
ν_L, ν_M, ν_H	$1/2$	0	$+1$	0	$1/2, +1/2$	0
e^-, μ^-, τ^-	$1/2$	0	$+1$	-1	$1/2, -1/2$	0
Quarks:						
u, c, t	$1/2$	$+1/3$	0	$+2/3$	$1/2, +1/2$	R, G, B
d, s, b	$1/2$	$+1/3$	0	$-1/3$	$1/2, -1/2$	R, G, B
Gauge Bosons:						
γ	1	0	0	0	0	0
Z, $W^\pm$	1	0	0	$0, \pm 1$	$1, (0, \pm 1)$	0
Gluons	1	0	0	0	0	$C\bar{C}$
Vacuum:						
Higgs	0	0	0	0	0	0

[1] Mind helicities and mixtures!

- Total baryon number:

$$\begin{aligned} \#(\text{baryons}) - \#(\text{antibaryons}) &= \text{const} \\ \#(\text{q}) - \#(\bar{\text{q}}) &= \text{const} \end{aligned}$$

- Total lepton number:

$$\begin{aligned} \#(\text{leptons}) - \#(\text{antileptons}) &= \text{const} \\ \#(l^-, \nu_l) - \#(l^+, \bar{\nu}_l) &= \text{const} \; ; \; l = \text{e}, \mu, \tau \end{aligned}$$

ν_e, ν_μ, ν_τ are mixed states of the particles ν_L, ν_M, ν_H.

- Charges of all types:

Electric charge Q ; Weak isospin (T, T_3) ; Color $C = (R, G, B)$

- Flavor: conserved by strong and electromagnetic, but not by weak interactions.

Force	Acts on	Strength	Range	Boson
Strong	Quarks and particles containing quarks	10^4	$\sim 10^{-14}$ m	g
Electromagnetic	Electrically charged particles	10^2	∞	γ
Weak	All particles	10^{-2}	$\sim 10^{-17}$ m	$W^{\pm}, Z$
Gravitational	All particles	10^{-34}	∞	?

Note: strength depends on distance or momentum transfer!

BOSONS

force carriers
spin = 0, 1, 2, …

Unified Electroweak spin = 1

Name	Mass GeV/c^2	Electric charge
γ photon	0	0
W^-	80.39	−1
W^+ W bosons	80.39	+1
Z^0 Z boson	91.188	0

Strong (color) spin = 1

Name	Mass GeV/c^2	Electric charge
g gluon	0	0

Higgs Boson spin = 0

Name	Mass GeV/c^2	Electric charge
H Higgs	126	0

Validity: $v \ll c$

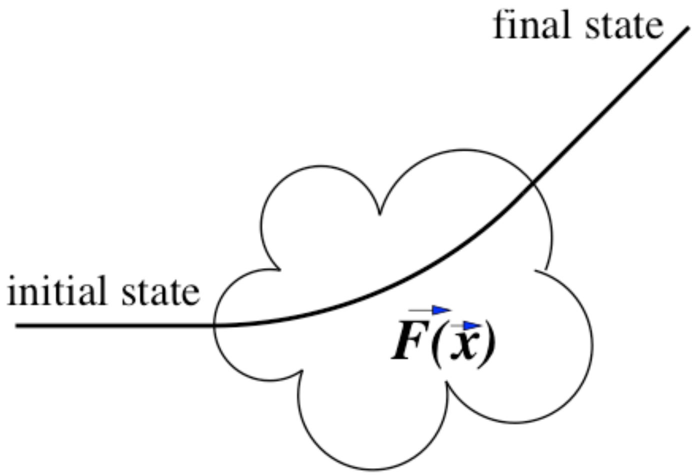
$\Delta x \gg 10^{-10}$m

- Evolution according to Newton's law:

$$m\frac{d^2\vec{x}}{dt^2} = \sum_i \vec{F}_i$$

- Forces and energy derive from a potential:

$$\vec{F} = q\vec{E} \quad ; \quad \vec{E} = -\vec{\nabla} V \quad ; \quad E_{pot} = qV$$



Validity: $v \ll c$
$\Delta x \leq 10^{-10}$m

- Probability amplitude:

$$\psi(t, \vec{x})$$

- Probability to find a particle at $(t, \vec{x})$:

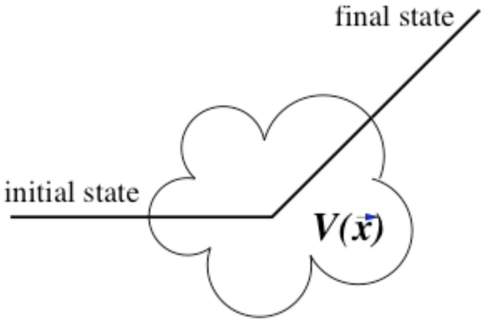
$$0 \leq \rho = |\psi|^2 \leq 1$$

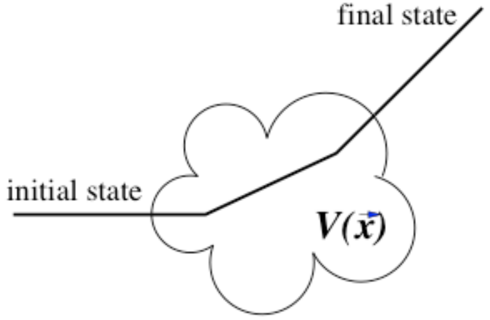
- Schrödinger equation:

$$\frac{p^2}{2m} = E \quad \Rightarrow \quad -\frac{1}{2m}\vec{\nabla}^2\psi = i\frac{\partial}{\partial t}\psi$$

- Interaction with the potentiel, perturbative approach:

$$i\frac{\partial}{\partial t}\psi + \frac{1}{2m}\vec{\nabla}^2\psi = V\psi$$

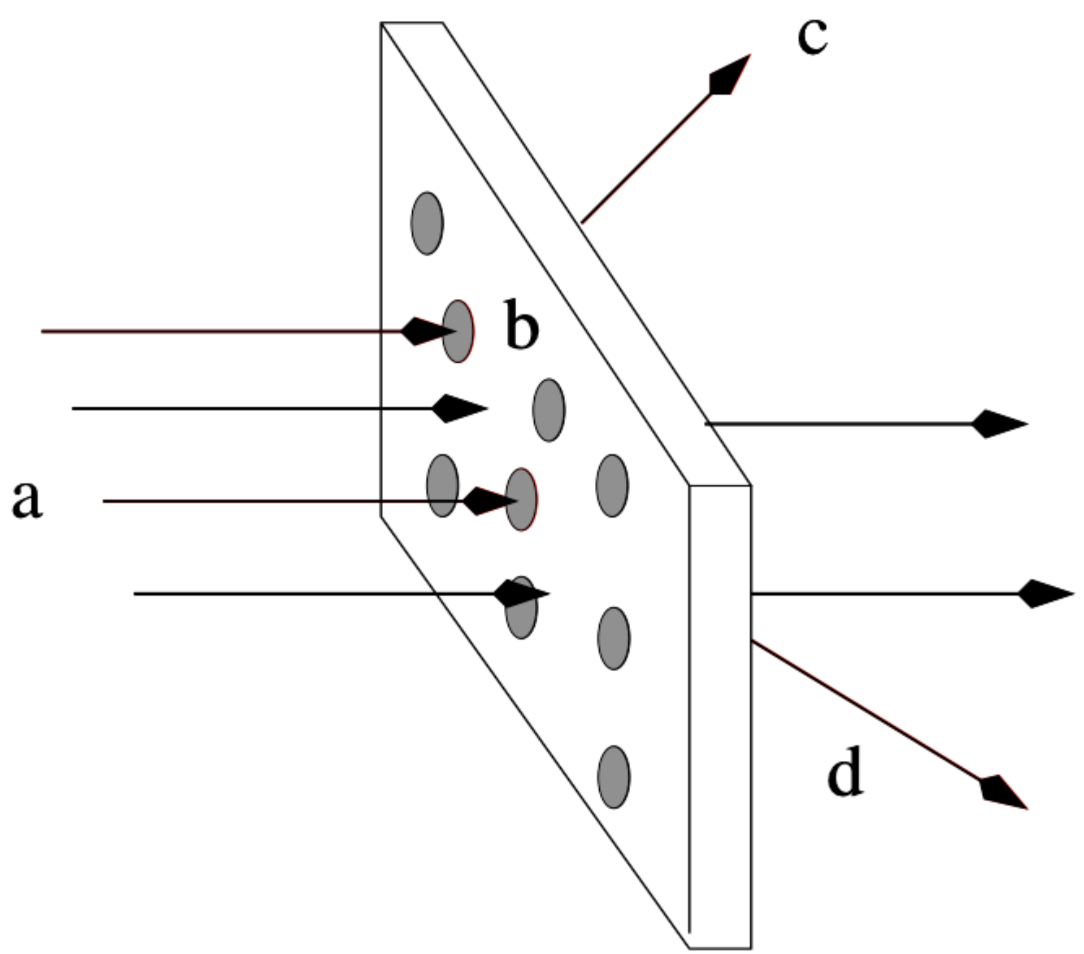




- Number of particles conserved:

$$\rho = (\psi^*\psi) \quad ; \quad \vec{j} = -\frac{i}{2m}(\psi^*\vec{\nabla}\psi - \psi\vec{\nabla}\psi^*) \quad ; \quad \partial\rho/\partial t + \vec{\nabla}\vec{j} = 0$$

$$\boxed{\text{Flux} = \frac{\#\ \text{particles a}}{\text{s} \cdot \text{m}^2}}$$

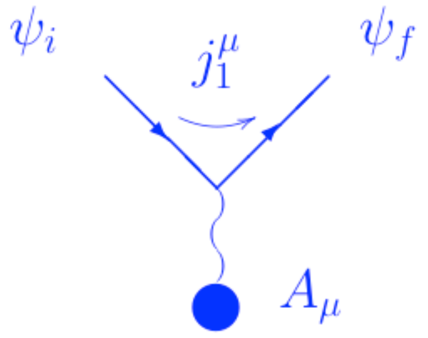


$$\boxed{p_{int} = n\sigma}$$

Validity: ???

- Relativistic equation of motion:

$$E^2 - \vec{p}^2 = m^2 \quad \Rightarrow \quad -\frac{\partial^2}{\partial t^2}\psi + \vec{\nabla}^2\psi = m^2\psi$$

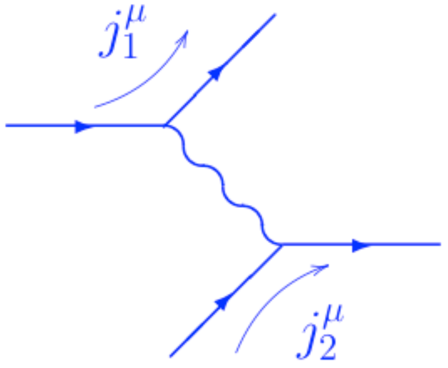


- Conserved electromagnetic current:

$$j_1^\mu = -2ep^\mu$$

- Potential generated by a second current according to Maxwell's equations:

$$A^\mu = -\frac{1}{q^2}j_2^\mu$$



- Propagator $1/q^2 = (E_\gamma^2 - p_\gamma^2)^{-1}$ describes probability amplitude for the exchange of a photons between the two currents.

- Relativistic equation of motion:

$$E^2 - \vec{p}^2 = m^2 \quad \longrightarrow \quad -\frac{\partial^2}{\partial t^2}\psi + \vec{\nabla}^2\psi = m^2\psi$$

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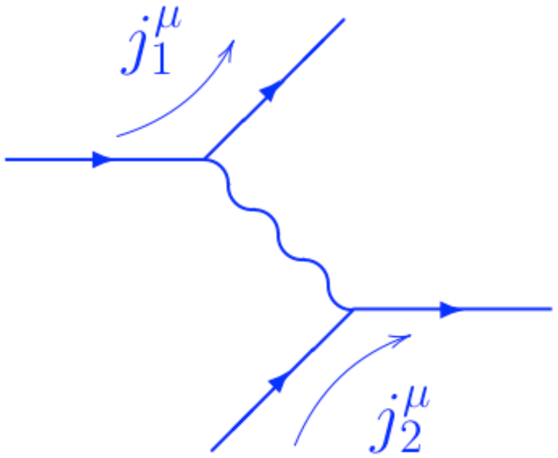
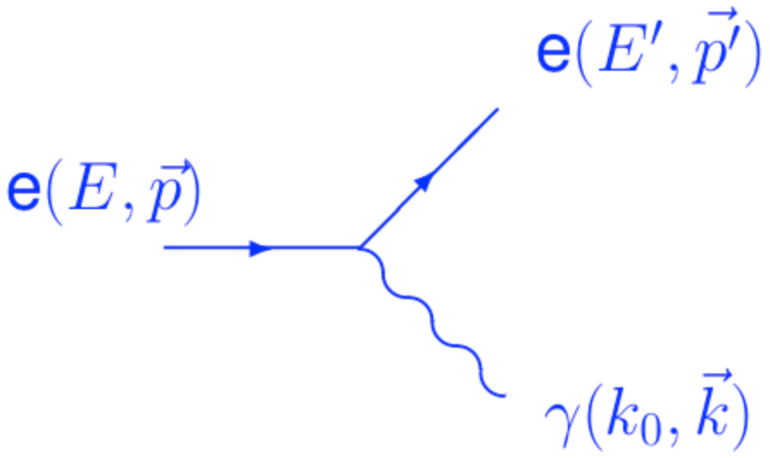


Electron emits a photon:

$$
\begin{aligned}
E^2 - \vec{p}^2 &= (E' + k_0)^2 - (\vec{p}' + \vec{k})^2 \\
{m_e}^2 &= {m_e}^2 + {m_\gamma}^2 + 2E'k_0 - 2\vec{p}'\vec{k}
\end{aligned}
$$



With $m_\gamma = 0$, $k_0 = |\vec{k}|$: $E' \leq |\vec{p}'| \Rightarrow$ contradiction with $m_e \neq 0$!

Conclusion:

- The electromagnetic force is transmitted by virtual photons, with $k_0^2 - \vec{k}^2 > 0$.
- Virtual photons have all the same properties as real ones, except that they have non-zero mass.

Probability $p \in \mathbb{R}$:

$$0 \leq p \leq 1$$

Frequentist approach:

$$p = \frac{\#\,\text{successes}}{\#\,\text{trials}}$$

Events A and B independent:

$$p(A \wedge B) \;=\; p(A) \cdot p(B)$$

Events A and B incompatible:

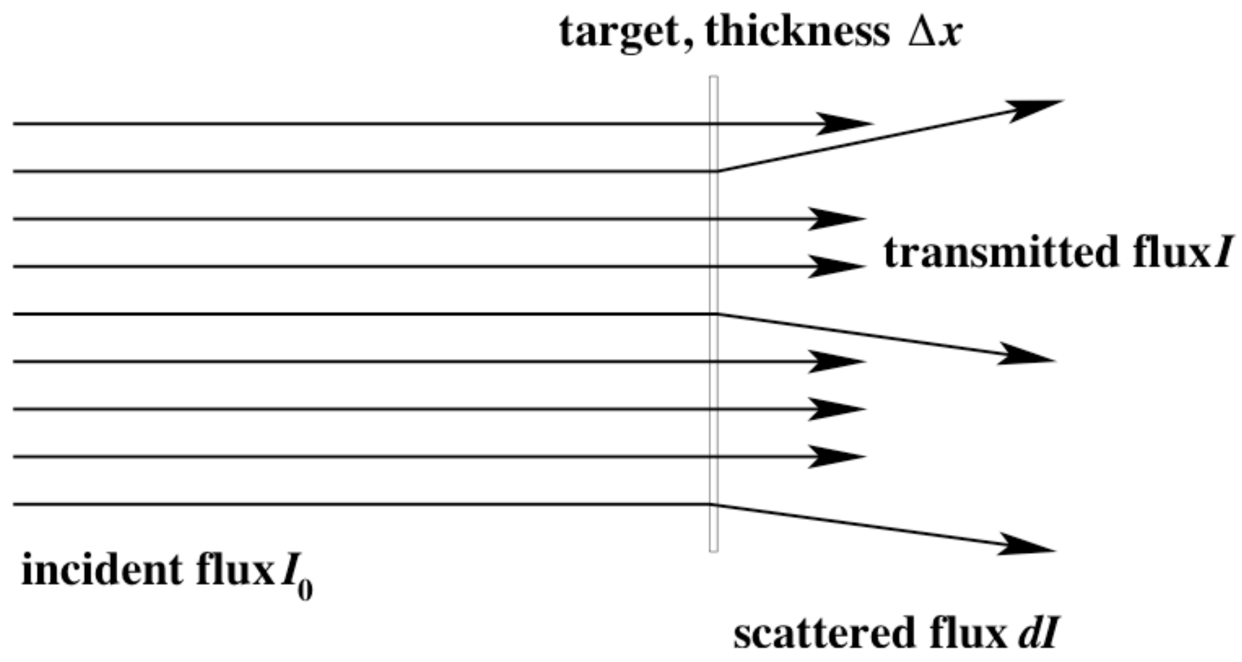
$$p(A \vee B) \;=\; p(A) + p(B)$$

$$\frac{-dI}{I} = \sigma n = \sigma \rho\, dx \quad ; \quad I = I_0 e^{-\sigma\rho\Delta x}$$

$$[\sigma] = \text{barn} = 10^{-28}\text{m}^2$$

- n: surface density of targets
- ρ: volume density of targets

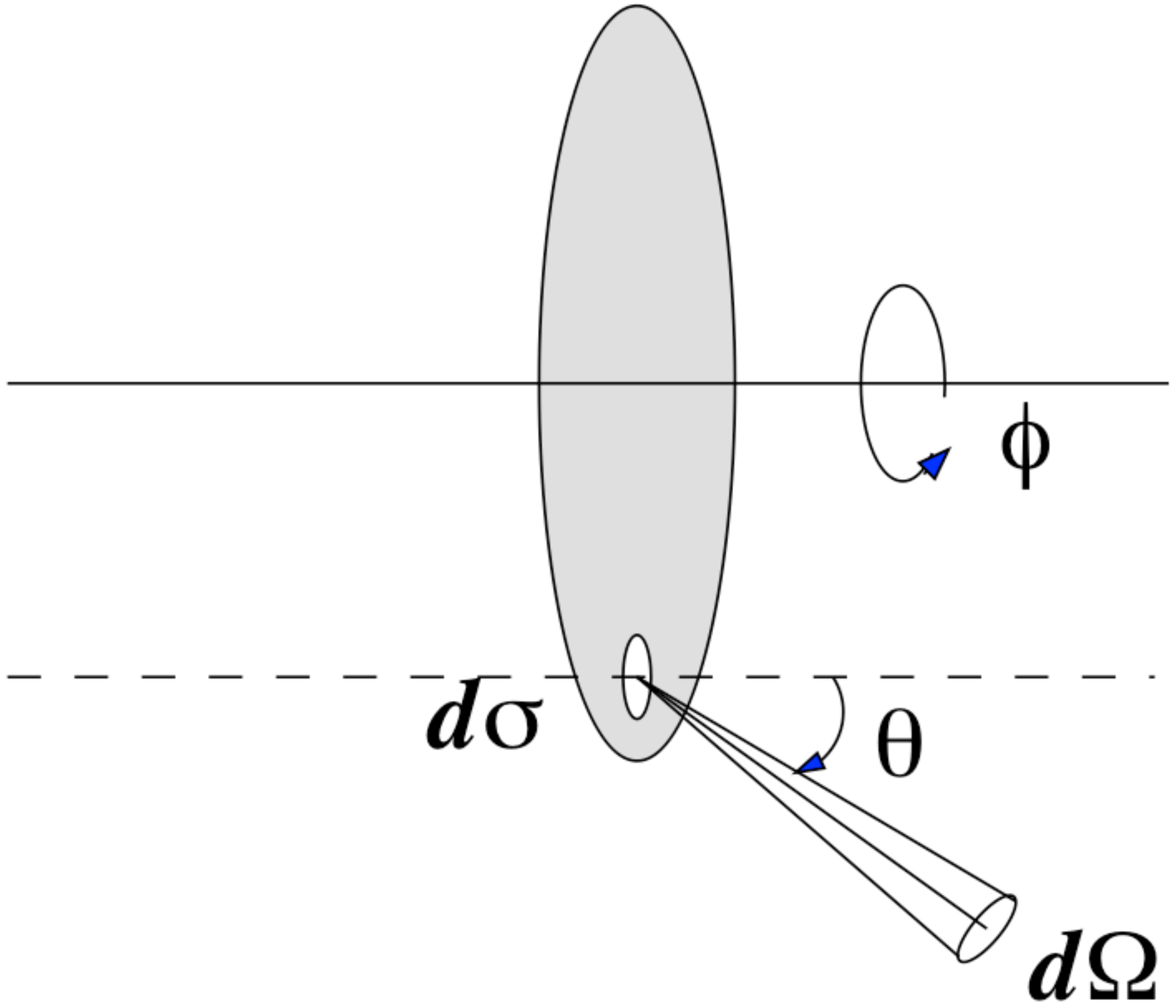
$$\sigma_{tot} = \sigma_{el} + \sigma_{inel} + \sigma_{abs}$$

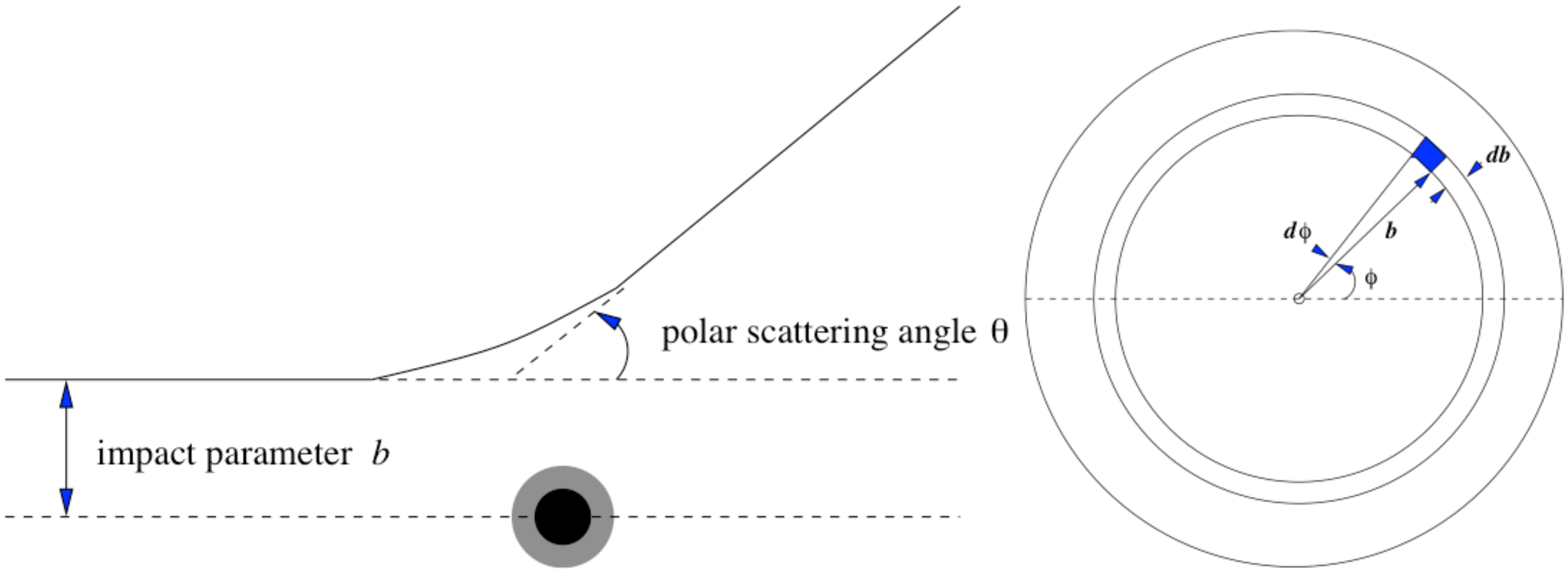


Differential cross section:

$$\frac{-dI}{I} = (\rho \Delta x) \frac{d\sigma(\theta, \phi)}{d\Omega} d\Omega$$

$$\sigma = \int_0^{2\pi} \int_0^{\pi} \frac{d\sigma(\theta, \phi)}{d\Omega} \sin\theta \, d\theta \, d\phi$$

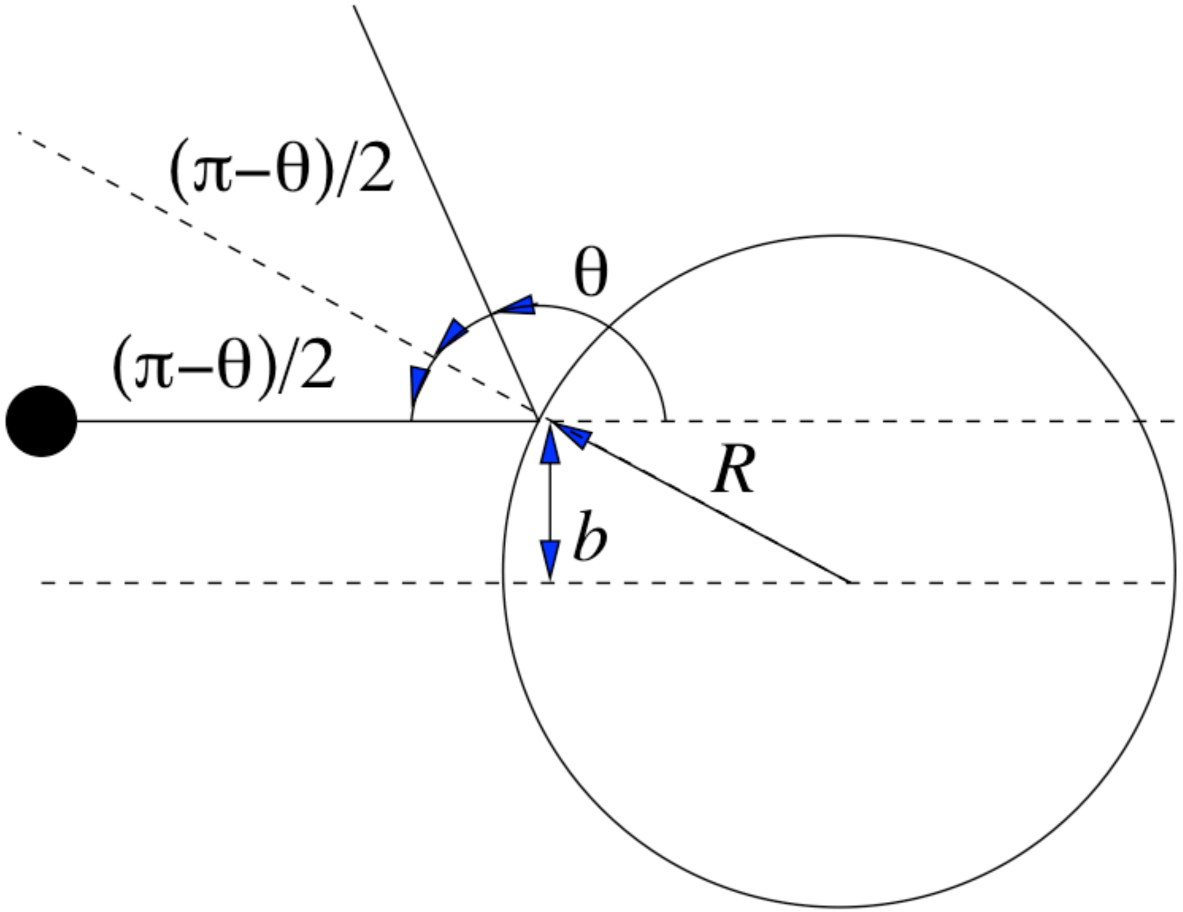




$$\frac{-dI}{I} = (\rho\Delta x)\,(b\;db\;d\phi) = (\rho\Delta x)\frac{d\sigma(\theta,\phi)}{d\Omega}\,d\Omega$$

$$\int \frac{-dI}{I}\,d\phi = (\rho\Delta x)\,(2\pi b\;db) = (\rho\Delta x)\;2\pi\frac{d\sigma(\theta)}{d\Omega}\sin\theta\;d\theta$$

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right|$$



Impact parameter:

$$b = R\sin\left(\frac{\pi-\theta}{2}\right) = R\cos\left(\frac{\theta}{2}\right)$$

$$\frac{db}{d\theta} = \frac{R}{2}\sin\left(\frac{\theta}{2}\right)$$

$$\frac{d\sigma(\theta)}{d\Omega} = \frac{b}{\sin\theta}\left|\frac{db}{d\theta}\right| = \frac{R^2}{4}$$

Total cross section:

$$\sigma = \pi R^2$$