

# Classical and Quantum Optics

## Assignment-2 Answers

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### Problem 1

A beam with a photon flux of 1000 photons  $s^{-1}$  is incident on a detector with a quantum efficiency of 20%. If the time interval of the counter is set to 10s, calculate the average and standard deviation of the photocount number for the following scenarios:

- (a) the light has Poissonian statistics;
- (b) the light has super-Poissonian statistics with  $\Delta n = 2 \times \Delta n_{\text{Poisson}}$  ;
- (c) the light is in a photon number state.

### Answer

$$\phi = 1000/\text{s}, \eta = 20\%, t = 10\text{s}$$

$$(a) \bar{n} = \frac{L\phi}{c}$$

$$\text{But, } L = ct \Rightarrow \bar{n} = t\phi$$

$$\text{Then, } \bar{n} = 10000$$

$$\Delta n = \sqrt{\bar{n}} = 100$$

The photocount number is given by,

$$(\Delta N)^2 = \eta^2 (\Delta n)^2 + \eta(1 - \eta)\bar{n}$$

$$\Rightarrow,$$

$$\Delta N = 44.721$$

.

$$\bar{N} = \eta\bar{n} = 2000$$

$$(b) \text{ Given } \Delta n = 2 \times \Delta n_{\text{Poisson}}. \text{ i.e., } \Delta n = 89.442$$

We know that, for super poissonian statistics,

$$(\Delta n)^2 = \bar{n} + \bar{n}^2$$

.

$$\Rightarrow$$

$$\bar{n} + \bar{n}^2 = 4\bar{n}_{\text{Poisson}}^2 = 8000$$

By solving this,

$$\bar{n} = 88.944$$

$$\Delta N = 18.28, \bar{N} = 17.79$$

(c) In photon number state,  $\Delta n = 0$ . That means, there is no variation from mean value. Then,  
 $\bar{n} = 10000$

$\Rightarrow$

$$\bar{N} = 2000, \Delta N = 40$$

## Problem 2

Calculate the values of  $g^{(2)}(0)$  for a monochromatic light wave with a square wave intensity modulation of  $\pm 20\%$ .

## Problem 3

The 632.8 nm line of a neon discharge lamp is Doppler-broadened with a linewidth of 1.5GHz. Sketch the second-order correlation function  $g^{(2)}(\tau)$  for  $\tau$  in range 0-1 ns.

## Problem 4

For the coherent states  $|\alpha\rangle$  with  $\alpha=5$ , calculate

- (a) the mean photon number;
- (b) the standard deviation in the photon number;
- (c) the quantum uncertainty in the optical phase.

## Problem 5

A ruby laser operating at 693 nm emits pulses of energy 1mJ. Calculate the quantum uncertainty in the phase of the laser light.

## Problem 6

For the coherent state  $|\alpha\rangle$  with  $\alpha=|\alpha|e^{i\phi}$ , show that  $\langle \alpha | \hat{X}_1 | \alpha \rangle = |\alpha| \cos \phi$  and  $\langle \alpha | \hat{X}_2 | \alpha \rangle = |\alpha| \sin \phi$ . Show further that  $\Delta X_1 = \Delta X_2 = \frac{1}{2}$ .

## Problem 7

Prove that for two coherent states  $|\alpha\rangle$  and  $|\beta\rangle$ ,

$$| \langle \alpha | \beta \rangle |^2 = \exp(-|\alpha - \beta|^2)$$