Classical and Quantum Optics

Assignment-2 Answers

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Problem 1

A beam with a photon flux of 1000 photons s^{-1} is incident on a detector with a quantum efficiencyj of 20%. If the time interval of the counter is set to 10s, calculate the average and standard deviation of the photocount number for the followin g scenarios:

(a) the light has Poissonian statistics;

(b) the light has super-Poissonian statistics with Δ n=2× Δ n$_{\text{Poisson}}$;

(c) the light is in a photon number state.

Answer

$\phi = 1000/\text{s}$, η=20%, t=10s

(a) $\bar{n} = \eta \frac{L\phi}{c}$

But, $L = ct \Rightarrow \bar{n} = \eta t \phi$

$\therefore \bar{n} = 2000.$

Since light follows poisson statistics, $\Delta n = \sqrt{\bar{n}}$.

Then, $\Delta n = 44.721$.

(b) Given $\Delta n = 2 \times \Delta n_{Poisson}$. i.e., $\Delta n = 89.442$

We know that, for super poissonian statistics,

$$(\Delta n)^2 = \bar{n} + \bar{n}^2$$

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$\Rightarrow$

$$\bar{n} + \bar{n}^2 = 4\bar{n}_{Poiss}^2 = 8000$$

By solving this,

$$\bar{n} = 88.944$$

(c) In phton number state, $\Delta n = 0$. That means, there is no variation from mean value. Then, $\bar{n} = 2000$

Problem 2

Calculate the values of $g^{(2)}(0)$ for a monochromatic light wave with a square wave intensity modulation of $\pm$ 20%.

Problem 3

The 632.8 nm line of a neon discharge lamp is Doppler-broadened with a linewidth of 1.5GHz. Sketch the second-order correlation function $g^{(2)}(\tau)$ for τ in range 0-1 ns.

Problem 4

For the coherent states $|\alpha>$ with α=5, calculate

(a) the mean photon number;

(b) the standard deviation in the photon number;

(c) the quantum uncertinity in the optical phase.

Problem 5

A ruby laser operating at 693 nm emits pulses of energy 1mJ. Calculate the quantum uncertinity in the phase of the laser light.

Problem 6

For the coherent state $|\alpha>$ with $\alpha=|\alpha|e^{i\phi}$, show that $<\alpha|\hat{X}_1|\alpha>=|\alpha|\cos\phi$ and $<\alpha|\hat{X}_2|\alpha>=|\alpha|\sin\phi$. Show further that $\Delta X_1 = \Delta X_2 = \frac{1}{2}$.

Problem 7

Prove that for two coherent states $|\alpha>$ and $|\beta>$,

$$|<\alpha|\beta>|^2 = \exp(-|\alpha-\beta|^2)$$