# Introduction to Python for Mathematics
# A Crash Course

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Mathematics

August 19, 2024





## Contents







## Getting started

- Log In
- Start Anaconda Navigator (use search box)
- untick box, click OK, or whatever you choose.
- Press 'LAUNCH' for jupyter notebook (not jupyter lab)
- Choose 'python 3' from 'new' menu on right
- In the first cell type 2+3 followed by SHIFT-RETURN.

Did you get 5?





# Leontief Input-Output Problems in Economics





### Input Output Problems in Economics

Consider the tourist economy in a small resort. The major industries are

A: Accommodation — rentals, hotels, B & B's, . . .
F: Food & Drink — restaurants, kiosks, pubs, take away, . . .
E: Entertainment — theatre, cinema, nightclubs, . . .
T: Transportation — buses, trains, taxis, ferries, . . .

The turnover of each of these industries will contain cash inputs from thenselves and the others, as well as from external demand like tourists, other industrial and commercial sectors, etc.

### Reference

*Mathematics for Economics and Business*, Ian Jacques, Prentice Hall, 4 ed. 2003.





## Let's consider just A and F. Suppose that . . .

Each £1 of A's turnover requires an input of 10p of its own turnover plus 30p of F's.

Each £1 of F's turnover requires an input of 20p of its own turnover plus 50p of A's.

So, assuming these proportions are constant across all turnover levels, if we want A to turn over £50,000 and F to turnover £40,000 then:

For A: £50,000 requires £5000 (0.1 of £50,000) from itself, plus £15,000 (0.3 of £50,000) from F.

For F: £40,000 requires £20,000 (0.5 of £40,000) from A, plus £8000 (0.2 of £40,000) from itself.

Get the idea? Easily generalised to more industries. Look carefully:

You can see matrices at work here. Let's figure it out. . .





## Look at it mathematically. . .

Each £1 of A's turnover requires an input of 10p of its own turnover plus 30p of F's.

Each £1 of F's turnover requires an input of 20p of its own turnover plus 50p of A's.

Let $x_1$ denote A's turnover and $x_2$ denote F's turnover.

Also $d_1$ (resp. $d_2$) denote external demand for A (resp. F). Then:

$$x_1 = \overbrace{0.1x_1 + 0.5x_2 + d_1}^{\text{payments to A}}$$
$$x_2 = \underbrace{0.3x_1 + 0.2x_2 + d_2}_{\text{payments to F}}$$

Can you see the matrix now?





## Leontief's input-output model

Let $x_1$ denote A's turnover and $x_2$ denote F's turnover.

Let $d_1$ (resp. $d_2$) denote external demand for A (resp. F). Then:

$$x_1 = 0.1x_1 + 0.5x_2 + d_1$$
$$x_2 = 0.3x_1 + 0.2x_2 + d_2$$

This can be written as $\boldsymbol{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{d}$ for

$$\boldsymbol{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad \boldsymbol{d} = \begin{pmatrix} d_1 \\ d_2 \end{pmatrix} \quad \text{and} \quad \boldsymbol{A} = \begin{pmatrix} 0.1 & 0.5 \\ 0.3 & 0.2 \end{pmatrix}.$$

In this model $\boldsymbol{A}$ is called the matrix of technical coeffcients, or the technology matrix. (Developed by Wassily Leontief.)

$\boldsymbol{A}$'s columns give the inputs needed for one unit of output.

## Using the model

The Leontief input-output model

$$\boldsymbol{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{d}$$

can be used to answer three key questions:

1. Given total output $\boldsymbol{x}$, how much is available for demand $\boldsymbol{d}$?
   Find $\boldsymbol{d} = (\boldsymbol{I} - \boldsymbol{A})\boldsymbol{x}$
2. How much total output $\boldsymbol{x}$ is required to satisfy a given level of demand $\boldsymbol{d}$?
   Solve $(\boldsymbol{I} - \boldsymbol{A})\boldsymbol{x} = \boldsymbol{d}$
3. How should output $\boldsymbol{x}$ change if demand changes by $\Delta\boldsymbol{d}$?
   Solve $(\boldsymbol{I} - \boldsymbol{A})\Delta\boldsymbol{x} = \Delta\boldsymbol{d}$

We'll do these by hand first, and then with python.

Remember that $\begin{pmatrix} a & b \\ c & d \end{pmatrix}^{-1} = \frac{1}{\det}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$





Use $\boldsymbol{x} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{d}$ with $\boldsymbol{A} = \begin{pmatrix} 0.1 & 0.5 \\ 0.3 & 0.2 \end{pmatrix}$ to answer these:

1. Given total output $\boldsymbol{x} = \begin{pmatrix} 50000 \\ 40000 \end{pmatrix}$, how much is available for demand $\boldsymbol{d}$?
   Find $\boldsymbol{d} = (\boldsymbol{I} - \boldsymbol{A})\boldsymbol{x} = \begin{pmatrix} 0.9 & -0.5 \\ -0.3 & 0.8 \end{pmatrix}\begin{pmatrix} 50000 \\ 40000 \end{pmatrix} = \begin{pmatrix} 25000 \\ 17000 \end{pmatrix}$
2. How much total output $\boldsymbol{x}$ is required to satisfy a given level of demand $\boldsymbol{d} = \begin{pmatrix} 35000 \\ 29000 \end{pmatrix}$?
   Solve $(\boldsymbol{I} - \boldsymbol{A})\boldsymbol{x} = \boldsymbol{d} \Longrightarrow \boldsymbol{x} = \frac{10}{57}\begin{pmatrix} 8 & 5 \\ 3 & 9 \end{pmatrix}\begin{pmatrix} 35000 \\ 29000 \end{pmatrix} \approx \begin{pmatrix} 74561 \\ 64210 \end{pmatrix}$
3. How should output $\boldsymbol{x}$ change if demand changes by $\Delta\boldsymbol{d} = \begin{pmatrix} 2500 \\ 1900 \end{pmatrix}$?
   Solve $(\boldsymbol{I} - \boldsymbol{A})\Delta\boldsymbol{x} = \Delta\boldsymbol{d} \Longrightarrow \Delta\boldsymbol{x} = \frac{10}{57}\begin{pmatrix} 8 & 5 \\ 3 & 9 \end{pmatrix}\begin{pmatrix} 2500 \\ 1900 \end{pmatrix} \approx \begin{pmatrix} 5175 \\ 4315 \end{pmatrix}$

Now let's do this in python...





## The key commands

```
import numpy as np                          # import numerical python
A = np.array([[0.1, 0.5],[0.3, 0.2]])       # our technology matrix
Id = np.eye(2)                              # 2 by 2 identity matrix
d = np.array([[35000],[29000]])             # demand vector d
print(np.linalg.solve(Id-A, d))             # solve (I-A) x = d for x
Dd = np.array([[2500],[1900]])              # change in demand vector, Dd
print(np.linalg.solve(Id-A, Dd))            # solve (I-A) Dx = Dd for Dx
```

Note you can start a new cell whenever you like.

Do so frequently.





## Exercise - use python

Given the technology matrix $\boldsymbol{A} = \begin{pmatrix} 0.3 & 0.2 \\ 0.1 & 0.6 \end{pmatrix}$...

1. Given total output $\boldsymbol{x} = \begin{pmatrix} 70000 \\ 90000 \end{pmatrix}$, how much is available for demand $\boldsymbol{d}$?
   Find $\boldsymbol{d} = (\boldsymbol{I} - \boldsymbol{A})\boldsymbol{x}$
2. How much total output $\boldsymbol{x}$ is required to satisfy a given level of demand $\boldsymbol{d} = \begin{pmatrix} 52000 \\ 48000 \end{pmatrix}$?
   Solve $(\boldsymbol{I} - \boldsymbol{A})\boldsymbol{x} = \boldsymbol{d}$
3. How should output $\boldsymbol{x}$ change if demand changes by $\Delta\boldsymbol{d} = \begin{pmatrix} 5200 \\ 4100 \end{pmatrix}$?
   Solve $(\boldsymbol{I} - \boldsymbol{A})\Delta\boldsymbol{x} = \Delta\boldsymbol{d}$





## Exercise

We started with a tourist economy in a small resort with the major industries: A (Accommodation), F (Food & Drink), E (Entertainment) and T (Transportation).

The technology matrix for this economy is

$$\boldsymbol{A} = \begin{pmatrix} 0.15 & 0.12 & 0.05 & 0.03 \\ 0.17 & 0.16 & 0.04 & 0.04 \\ 0.03 & 0.08 & 0.18 & 0.22 \\ 0.07 & 0.18 & 0.03 & 0.19 \end{pmatrix}$$

1. How much is left for demand $\boldsymbol{d}$ with a total output $\boldsymbol{x} = \begin{pmatrix} 89000, & 55000, & 47000, & 76000 \end{pmatrix}^T$?
2. How much total output $\boldsymbol{x}$ is needed for demand $\boldsymbol{d} = \begin{pmatrix} 55000, & 24000, & 18000, & 40000 \end{pmatrix}^T$?
3. How should output $\boldsymbol{x}$ change if demand changes by $\Delta\boldsymbol{d} = \begin{pmatrix} -5000, & 350, & 2300, & -500 \end{pmatrix}^T$?





## Finishing up

There is much much much more to python, and numpy.

We're just scratching the surface

As is the nature of a crash course

Let's look now at how to plot graphs





# 2D Plotting



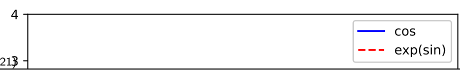


## Plotting in 2D

We jump straight in.

Plot $\cos(2\pi x)$ in solid blue and $\exp(\sin(4\pi x))$ in dashed red for $x \in [-1, 5]$

```
import matplotlib.pyplot as plt
import numpy as np
x = np.arange(-1,5,0.01)
y1, y2 = np.cos(2*np.pi*x), np.exp(np.sin(4*np.pi*x))
plt.plot(x,y1, 'b-')
plt.plot(x,y2, 'r--')
plt.axis([-2, 6, -2, 4])
plt.legend(['cos', 'exp(sin)'])
plt.xlabel(r'$x_1$'); plt.ylabel('$y_1$ and $y_2$')
plt.savefig('my2Dplot.png', dpi=600)
plt.savefig('my2Dplot.eps', dpi=600)
```



## Exercise

In python. . .

Plot $2^{3\sin(3\pi x)}$ in solid dash-dot blue and $\ln\left(1.2+\sin(3\pi x)\right)$ in dotted red for $x \in [-4, 3]$

Hint: for the line-styles use

```
plt.plot(x,y1, 'b-.')
plt.plot(x,y2, 'r:')
```





## How Anonymous is Anonymised Data?





## How anonymous is anonymized data?

A university collects answers to personal questions from all of its students.

Each student's answer has their name, date of birth, gender and department.

On average a department has 250 students in each year.

We assume the UK setup where students attend for three years.

The names are erased: how anonymous are the resulting data?





## Here's part of the dataset

| NAME | D.O.B. | GENDER | DEPARTMENT |
|---|---|---|---|
| ⋮ | ⋮ | ⋮ | ⋮ |
| Ringo Starr | 26/07/43 | M | Music |
| Al Gebra | 12/08/05 | F | Maths |
| Sandie Shaw | 16/02/38 | F | Puppetry |
| Michael Mouse | 17/04/92 | F | Computing |
| Mr Pink | 4/12/56 | M | Criminology |
| L.O. Gear | 11/9/23 | F | Engineering |
| Donkey Kong | 23/10/73 | M | Video Games |
| ⋮ | ⋮ | ⋮ | ⋮ |

The names are erased — can one line dentify the person? Is this enough information to identify the person?

How anonymous is a dataset like this?





## Let's simulate

We assume that all students are born within a three year window, and that each department has 750 students across its three years.

- There are $365 \times 3 = 1095$ possible birthdays
- For each there are at least 2 possible genders
- So, for a given department, there are $d = 1095 \times 2 = 2190$ possible entries among $N = 750$ students.
- Can you see why anonymity might not be assured?

Think about a line of $N = 2190$ empty buckets. Now throw $N = 750$ balls at random into the buckets.

Most will stay empty. Some will have just one ball — the 'loners'.

The proportion of $N$ having just one ball estimates the probabilty that line of anonymised data occurs just once in the department.





## Planning the code

We are going to generate a list of $d = 2190$ zeros.

We'll then generate $N = 750$ random integers $z \in \{1, 2, \dots, 2190\}$.

For each $z$ we'll add one to the $z^{\text{th}}$ item in the list, $d$.

In the end, the $n^{\text{th}}$ item in the list, $d$, tells us how many students share that same data.

We want to find the 'loners' — the buckets with only one item in them.





## Background and Exercise

- There is about a $70\%$ probability that a student can be identified from this anonymised data.
- A 'near exact' solution is $\exp(-N/d) \approx 71\%$.

My main reference is John D Cook:

`www.johndcook.com/blog/2018/12/07/simulating-zipcode-sex-birthdate/`

Which itself references the paper *Only You, Your Doctor, and Many Others May Know*, Latanya Sweeney: `https://techscience.org/a/2015092903/`

Exercise: What is the probability that a line of anonymised data can be narrowed down to at most two students? Or three? Or four?





## The End

That's it!

Thanks for listening

There's lots more to learn — as ever!

Good Luck!