Exercise Sheet 1

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1 Logical statements, Sets

a) If we will deny the $(\neg B \implies \neg A)$, we will get $\neg(\neg B \implies \neg A)$,
then $\neg\neg A \implies \neg\neg B,\ \ \neg\neg A = A,\ \ \neg\neg B = B,$
$\neg\neg A \implies \neg\neg B \iff A \implies B$

b) I think it's true and can prove with logic table.

A	B	$A \longrightarrow B$	$\neg A \vee B$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

c) $A^c = \{\forall a \notin A\}$,then
A $\cap A^c = \{\forall \tilde{a} \in A and \in A^c\} \implies A \cap A^c = \emptyset$

$d)$A $\times B = \{(a, b) : a \in A, b \in B\}$.
If $A \times B = \emptyset, then A = \emptyset \vee B = \emptyset$

2 Functions, Maps

a) The limit is function .
If x=1, $\lim_{n\to\infty} f_n(x)$=1.
If 0$\leq x < 1$ $\lim_{n\to+\infty} f_n(x)$=0.
If 0$\leq x < 1$ $\lim_{n\to-\infty} f_n(x)$=$+\infty$.

$b)$

2.1 Chain rule

$\frac{d}{dx}\left[f\left(u\right)\right] = \frac{d}{du}\left[f\left(u\right)\right]\frac{du}{dx}$

2.2 Composite Functions Definition

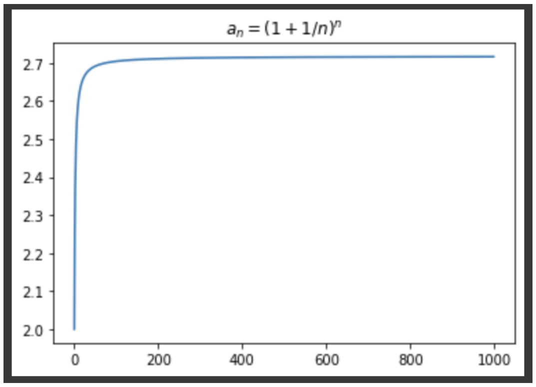
If f: X $\to Y and\ g : Y \to U,\ g \circ f : X \to U, and\ g \circ f(x) = g(f(x)), \forall x \in X.$

2.3 Associative property of composite functions

If we assign y = f(x), z = g(y), F = g $\circ f, so\ that\ F(x) = g(f(x)) = g(y) = z$
$and\ G = h \circ g\ then$
$h \circ (g \circ f)(x) = h \circ F(x) = h(F(x)) = h(z)$
$(h \circ g) \circ f(x) = G \circ f(x) = G(f(x)) = G(y) = h \circ g(y) = h(g(y)) = h(z)$

3 Sequences, Series

Yes it is monotone increasing because $a_{n+1} - a_n > 0$, but $\lim_{n\to -\infty} a_n =$ e.



a) This is my Google Colab: https://colab.research.google.com/drive/1QpQm5Fg7hyQF0fxyY7lOIsI60su91T3k?usp=sharing

b)This is a harmonic series,(in mathematics, the harmonic series is the infinite series formed by summing all positive unit fractions),

$$\sum_{n=1}^{\infty} \frac{1}{n}$$

I tried prove this with Cauchy's integrate indication.
f(x) $= \frac{1}{x}$, x $\in [1, +\infty]$, $\int_1^{+\infty} \frac{1}{x}\, dx = \lim_{n\to +\infty} \int_1^n \frac{1}{x}\, dx = \lim_{n\to +\infty} (\ln |x|) = +\infty$

c)I solve this similar b).
$f(x) = \frac{1}{x^2}, x \in [1, +\infty]$, $\int_1^{+\infty} \frac{1}{x^2}\, dx = \lim_{n\to +\infty} \int_1^n \frac{1}{x^2}\, dx = \lim_{n\to +\infty} (-\frac{1}{n} + 1) = 1$

4 Limes, Convergence

a) $\lim_{n\to\infty}\{a_n\} = a, \forall \varepsilon > 0 \ \exists \ n > n_0, |a_n - a| < \varepsilon$
$\lim_{n\to\infty}\{b_n\} = b, \forall \varepsilon > 0 \ \exists \ n > n_0, |b_n - b| < \varepsilon, \ b \neq 0$
$\lim_{n\to\infty} \frac{a_n}{b_n} = k, \ \forall \varepsilon > 0 \ \exists \ n > n_0, |\frac{a_n}{b_n} - k| \leq |\frac{a}{b} - k| < \varepsilon$

b) $\lim_{n\to\infty}\{a_n\} = a, \forall \varepsilon > 0 \ \exists \ n > n_0, |a_n - a| < \varepsilon_1$
$\lim_{n\to\infty}\{b_n\} = b, \forall \varepsilon > 0 \ \exists \ n > n_0, |b_n - b| < \varepsilon_2$
$If \ a_n < b_n, then \ a + \varepsilon_1 < b + \varepsilon_2 \Longrightarrow a < b$

$c)x \in (0, +\infty),$
$\lim_{x\to 0}(x\ln(x)) = \lim_{x\to 0}(\frac{\ln(x)}{1/x}) = \lim_{x\to 0}(\frac{d/dx(\ln(x))}{d/dx(1/x)}) = \lim_{x\to 0}(-\frac{1/x}{1/x^2}) = -\lim_{x\to 0}(x) = 0$
$If \lim_{x\to 0}(x\ln(x)) = 0, then \ \forall \varepsilon > 0 \ \exists \ \delta > 0, |x - 0| < \delta \Longrightarrow |x\ln(x) - 0| < \varepsilon$
$x\ln(x) \leq x^2 < \delta^2 = \varepsilon,$
$\delta = \sqrt{\varepsilon}, \ I find \ \delta from \ which \ |x\ln(x) - 0| < \delta$

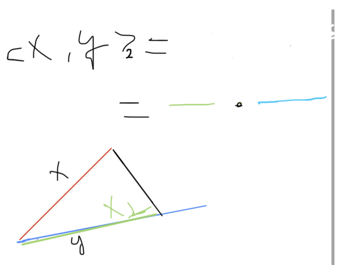
$d)At \ first \ x = e^{\ln x}$
$\lim_{x\to 0+} x^x = \lim_{x\to 0+} e^{x\ln x} = e^{\lim_{x\to 0+} x\ln x}$, $and \lim_{x\to 0+}(x\ln(x)) = 0 \ (c)$, $\lim_{x\to 0+} x^x = e^0 = 1$

5 Norms, Scalar products

a) $\sqrt{<x,x>_2} = \sqrt{{x_1}^2 + {x_2}^2} = ||x||$
$1.Definiteness\ \sqrt{{x_1}^2 + {x_2}^2} = 0,\ iff\ x = 0$
$2.Homogeneity\ ||\lambda x|| = \lambda ||x||,\ (insert\ in\ \sqrt{{x_1}^2 + {x_2}^2})$
$3.Triangleinequality ||x + x|| \leq ||x|| + ||x||$

$b)\ \sum_{i=0}^{N} (2x_i) \cdot y_i =< 2x_i, y_i >_N$
$1.Linearity < 2x_i + z, y_i >_N = \sum_{i=0}^{N} (2x_i + z) \cdot y_i = \sum_{i=0}^{N} (2x_i) \cdot (y_i + z) =< 2x_i, y_i + z >_N$
$< \lambda(2x_i), y_i >_N = \sum_{i=0}^{N} (\lambda 2x_i) \cdot y_i = \sum_{i=0}^{N} (2x_i) \cdot (\lambda y_i) = \lambda \sum_{i=0}^{N} (2x_i) \cdot y_i =< 2x_i, \lambda y_i >_N$
$2.Symmetry < 2x_i, y_i >_N = \sum_{i=0}^{N} (2x_i) \cdot y_i = \sum_{i=0}^{N} y_i \cdot (2x_i) =< y_i, 2x_i >_N$
$3.Positivedefinition < x, x >_N = \sum_{i=0}^{N} (2x_i) \cdot x_i \geq 0,\ and < x, x >_N = 0\ iff\ x = 0.$

$c) As\ far\ as\ I\ understand\ it's\ about\ x_\perp = \frac{<x,y>_2}{||y||}$



6 Python task

a) This is my Python task: https://colab.research.google.com/drive/1mUnWaJvfiYvIjkmluijQS2sVNLCmbi-V?usp=sharing