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Geometry: Combinatorics & AlgorithmsHomework Assignment 1 (due: Thursday October 27, 14:00) HS22

General Information

This is the first of the two graded homework assignments. You have two weeks to send your solution, typeset in LaTeX as a PDF, to meghana.mreddy@inf.ethz.ch and yanhwang@student.ethz.ch. Your grade of this assignment counts 20% towards your final grade. It is not allowed to hand in your solution in groups.

Writing Guidelines

In the end, a proof should be correct, but writing correct proofs is not easy. One goal of these assignments is to help you in mastering this art. Writing a proof is a stepwise reflection during which you constantly question the validity of your moves. The meaning of proofs diminishes if you don't work out the details. In fact, many errors hide behind phrases like "it can easily be shown that...", or "clearly, it holds that..."; even more hide behind omissions of steps that the author thought too obvious to mention at all! Worse still, these errors are very hard to spot. If all "obvious" steps had been documented in detail, then it would be much easier to see the failure and, if possible, to correct them.

Therefore, we require you to provide detailed proofs, both as a self-check and as a nicety to the verifiers. This allows us to confirm their correctness (the ideal outcome), or to clearly see why and where they fail. In particular,

- All your algorithms must be accompanied by a correctness proof and a runtime analysis.
- When you use results from the lecture notes or weekly exercises, you should refer to them and justify the conditions.
- If you use any material beyond what were covered/referenced in the lecture, you should provide adequate reference.

Of course, there are some "common sense" that you do not need to explain (e.g., n numbers can be sorted in $O(n \log n)$ time). If you are unclear about the extent of common sense, feel free to ask the TAs.

Finally, for drawing illustrations we strongly recommend Ipe, which can create PDF figures with LaTeX support.

Task 1 (20 points)

In this task, we consider a special class of embeddings of planar graphs, namely *CS-embeddings*. CS-embeddings are plane embeddings that satisfy the following two conditions:

(1) all vertices lie on the x-axis;

(2) each edge is drawn as a x-monotone simple curve whose interior crosses the x-axis at most once.

We call the edges whose interior do not cross the x-axis *C-edges*, and the ones whose interior cross the x-axis once *S-edges*.

(a) Show that a graph admits a CS-embedding containing only C-edges, with each edge drawn above the x-axis, if and only if it is outerplanar.

(b) Show that a graph $G = (V, E)$ admits a CS-embedding containing only C-edges if and only if it is a subgraph of a hamiltonian planar graph, i.e., there exists an edge set $E' \supseteq E$ such that (V, E') is planar and contains a hamiltonian cycle.

(c) Show that the planar graph $G_0 = (V_0, E_0)$ depicted in Figure 1 is not hamiltonian.

Hint: Observe that G_0 can be constructed by starting with the maximal plane graph depicted in Figure 2 and adding a vertex of degree 3 in each face (bounded or not).

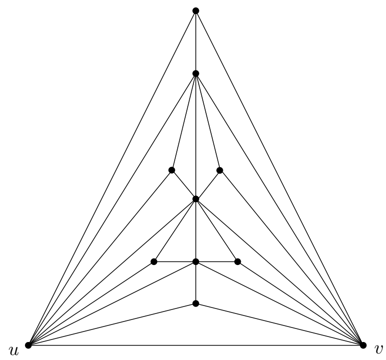

Figure 1: A plane embedding of the graph G_0

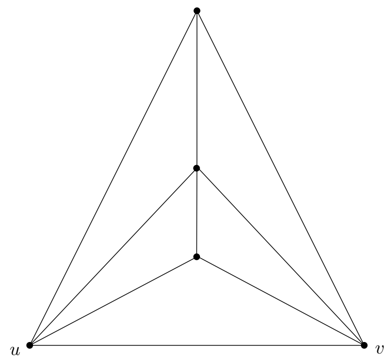

Figure 2: The graph G_0 can be constructed from this graph by adding a vertex of degree 3 in each face.

(d) Show that its edge set E_0 can be partitioned into two edge sets E_1 and E_2 such that (V_0, E_1) and (V_0, E_2) are outerplanar graphs.

Remark: The moral of the story is that since G_0 is maximal planar but not hamiltonian, it does not admit a CS-embedding containing only C-edges. However, (V_0, E_1) can be drawn only with C-edges above the x-axis, and (V_0, E_2) can be drawn only with C-edges below the x-axis. The thing is that not both can be done for the same ordering of the vertices in V_0 along the x-axis.

Task 2 (40 points)

In the lecture, we have mentioned several linear time algorithms that determine if a graph G is planar. These algorithms, however, have the drawback of being fairly complex and you are not allowed to use them here. In this task, we look at the easier problem of determining whether G is *maximal* planar in linear time.

(a) Give an algorithm that takes as input a maximal planar graph G on n vertices, represented with adjacency lists, and fixed vertices $v_1, v_2, v_n \in V(G)$ that form a facial triangle of G, and computes a canonical ordering $v_1, v_2, \ldots, v_n$ of G in linear time.

Remark: recall that we discussed an algorithm in the lecture that computes a canonical ordering of a plane *graph. The twist in this exercise is that the input graph is* not *embedded.*

(b) What can happen if the algorithm you gave in part (a) is given a graph G and vertices v_1, v_2, v_n that do not satisfy the precondition of your algorithm?

(c) Suppose that we give an *arbitrary* graph G (i.e., we do not know if it is maximal planar or not) to the algorithm you gave in part (a). Suppose furthermore that your algorithm returns some vertex ordering r. Describe how to determine in linear time if r is a valid canonical ordering of G.

Hint: try to construct an embedding from r *and see if and where it fails.*

(d) Give a linear time algorithm that computes an embedding of an arbitrary input graph G if G is maximal planar and returns FAIL otherwise. *Hint: guess the outer face.*

Task 3 (40 points)

Choose one of the topics below to investigate. For your chosen topic, find the relevant research papers, surveys or textbooks that deal with this problem and find out what is known about it. What are the main results? What are the related open problems? You are not supposed to read papers in detail, but rather try to gain an overview.

1. Upward planarity
2. Pseudoline arrangements
3. Lombardi drawings
4. Linkless embeddings
5. Combinatorial holes in planar point sets
6. Clustered planarity
7. Delaunay triangulation for moving points
8. Higher order Delaunay triangulations
9. Simultaneous embeddings with fixed edges
10. Order types of point sets
11. Pseudo-triangulations
12. Delaunay refinement meshing
13. Contact representations of graphs
14. Combinatorial flips in triangulations
15. Geometric spanners
16. Universal point sets
17. Unit disk graphs
18. Counting planar triangulations
19. Depth measures on point sets
20. k-sets
21. Shortest paths in polygons
22. Geometric thickness
23. k-planar graphs
24. quasi-planar graphs
25. Computational geometry for imprecise data
26. Thrackles
27. Compatible Geometric Graphs
28. Queue layouts
29. Book embeddings
30. Geometric set cover problems
31. Minimum convex partitions
32. Art gallery problems
33. Simple topological graphs
34. Maximum traveling salesman problem
35. LP-type problems
36. Fan-planar graphs
37. Hadwiger-Nelson problem
38. Tilings of the plane
39. Geometric packing problems
40. VC-dimension and ε-nets
41. RAC-drawings
42. Terrain visibility graphs
43. Confluent drawings
44. String graphs
45. The geometric k-center problem

Below are some databases for scientific literature. Note that many resources require ETH network, so you might need to set up a VPN if you search at home.

- http://zbmath.org,
- http://www.ams.org/mathscinet/,
- http://scholar.google.com,
- http://www.scopus.com.

Hand in a short report (of between 1 and 2 pages, not longer) about the problem and what you have found out. Your report should contain:

- an informal description of the problem using your own words (as you would explain it to a friend),
- a precise definition of the problem (in which even the most nitpicking reader is not able to find anything unclear),
- the important results regarding this problem (with enough explanations to clarify the difference between results, but without going into unnecessary details),
- the current state of the problem (how much has already been solved, what remains open),
- a complete list of references (include appropriate reference for every result you state; if you only have 3 references, you probably should search more thoroughly).

The choice of your topic should be discussed with the assistants, so that no topic is assigned twice. The topics are assigned on a first come first served basis. You should send to the assistants a sufficiently long list of topics you like (say around seven topics), ordered by preference. You have until Tuesday October 18, 14:00 to do so.
Remark: Keep in mind that for the second homework set you will have to give a five minute presentation about your topic.