Computational Maths - Assignment 2

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Assignment 2 of Computational Mathematics.

Question 1

Using MATLAB with the given script gave me an answer of logical 1.

$X = (1\tilde{}= 0)|(2 > 2)\&(7 < 4);$

Here, the first bit is asking if 1 is not equal to 0 (~=)., which it is, so answer is true i.e. 1

The symbol — is logical OR

The symbol & is logical AND

$(2 > 2)\&(7 < 4)$ is equivalent to (0 & 0), because both are false and thus 0.

So, X = 1 OR 0

X = 1

Answer is A

Question 2

Upon running the given script, the result were:

-7.7417

-0.2583

The script:

p=[1 8 2];

r=roots(p);

where p is polynomial represented by a row vector with elements 1,8,2. [1 8 2] represents $x^2 + 8x + 2$. Using the quadratic formula, we get results very close to matlab's.

Answer is E

Question 3

a=12/1*15/1; = 180

b= a/a*a; = 180

c=tand(30)+1/3; = 0.9107

d=1+c; = 1.9107

e = a-b*c+d; = 17.9876

Answer is E

Question 4

Use the Power Method and start with vector $x = [1, -0.8, 0.9]^T$. Perform 7 iterations.

Starting with $i = 1$,$x_1 = [1, -0.8, 0.9]^T$.
With the power method, the vector $[x]_2$ is first calculated by $[x]_2 = [a][x]_1$ (step 2) and is then normalized (step 3):

$$[x]_2 = [a][x]_1 = \begin{bmatrix} -7 & 13 & -16 \\ 13 & -10 & 13 \\ -16 & 13 & -7 \end{bmatrix} \begin{bmatrix} 1 \\ -0.8 \\ 0.9 \end{bmatrix} = \begin{bmatrix} -31.8 \\ 32.7 \\ -32.7 \end{bmatrix} = 32.7 \begin{bmatrix} -0.9725 \\ 1 \\ -1 \end{bmatrix}$$

This is done by taking 32.7 out and dividing each element of the matrix $\begin{bmatrix} -31.8 \\ 32.7 \\ -32.7 \end{bmatrix}$ by 32.7, giving the result $= 32.7 \begin{bmatrix} -0.9725 \\ 1 \\ -1 \end{bmatrix}$

Now with $i = 2$

$$[x]_3 = [a][x]_2 = \begin{bmatrix} -7 & 13 & -16 \\ 13 & -10 & 13 \\ -16 & 13 & -7 \end{bmatrix} \begin{bmatrix} -0.9725 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 35.8075 \\ -35.6425 \\ 35.56 \end{bmatrix} = -35.6425 \begin{bmatrix} -1.005 \\ 1 \\ -0.9977 \end{bmatrix}$$

Now with $i = 3$

$$[x]_4 = [a][x]_3 = \begin{bmatrix} -7 & 13 & -16 \\ 13 & -10 & 13 \\ -16 & 13 & -7 \end{bmatrix} \begin{bmatrix} -1.005 \\ 1 \\ -0.9977 \end{bmatrix} = \begin{bmatrix} 35.9982 \\ -36.0351 \\ 36.0639 \end{bmatrix} = -36.0351 \begin{bmatrix} -0.999 \\ 1 \\ -1 \end{bmatrix}$$

Now with $i = 4$

$$[x]_5 = [a][x]_4 = \begin{bmatrix} -7 & 13 & -16 \\ 13 & -10 & 13 \\ -16 & 13 & -7 \end{bmatrix} \begin{bmatrix} -0.999 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 35.993 \\ -35.987 \\ 35.984 \end{bmatrix} = -35.987 \begin{bmatrix} -1 \\ 1 \\ -0.999 \end{bmatrix}$$

Now with $i = 5$

$$[x]_6 = [a][x]_5 = \begin{bmatrix} -7 & 13 & -16 \\ 13 & -10 & 13 \\ -16 & 13 & -7 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -0.999 \end{bmatrix} = \begin{bmatrix} 35.984 \\ -35.987 \\ 35.993 \end{bmatrix} = -35.987 \begin{bmatrix} -0.999 \\ 1 \\ -1// \end{bmatrix}$$

Now with $i = 6$

$$[x]_7 = [a][x]_6 = \begin{bmatrix} -7 & 13 & -16 \\ 13 & -10 & 13 \\ -16 & 13 & -7 \end{bmatrix} \begin{bmatrix} -0.999 \\ 1 \\ -1// \end{bmatrix} = \begin{bmatrix} 35.993 \\ -35.987 \\ 35.984 \end{bmatrix} = -35.987 \begin{bmatrix} -1 \\ 1 \\ -0.999 \end{bmatrix}$$

Now with $i = 7$

$$[x]_8 = [a][x]_7 = \begin{bmatrix} -7 & 13 & -16 \\ 13 & -10 & 13 \\ -16 & 13 & -7 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -0.999 \end{bmatrix} = \begin{bmatrix} 35.984 \\ -35.987 \\ 35.993 \end{bmatrix} = -35.987 \begin{bmatrix} -0.999 \\ 1 \\ -1// \end{bmatrix}$$

Now with $i = 8$

$$[x]_9 = [a][x]_8 = \begin{bmatrix} -7 & 13 & -16 \\ 13 & -10 & 13 \\ -16 & 13 & -7 \end{bmatrix} \begin{bmatrix} -0.999 \\ 1 \\ -1// \end{bmatrix} = \begin{bmatrix} 35.993 \\ -35.987 \\ 35.984 \end{bmatrix} = -35.987 \begin{bmatrix} -1 \\ 1 \\ -0.999 \end{bmatrix}$$

After 8 iterations, I have gotten -36, [-1, 1, -1], leading me to answer E. None of the above. It is close to answer C so perhaps some of my signs are wrong but I will answer E.

Question 5

T_k	0	1	2	3	4	5	6	7
$S_k)$	1.15	2.32	3.32	4.53	5.65	6.97	8.02	9.23

Testing the linear relationship between T and S
$S = aT + b$
$S_x = \sum_{i=1}^{8} X_i = 28$
$S_y = \sum_{i=1}^{8} Y_i = 41.19$
$S_{xx} = \sum_{i=1}^{8} X_i^2 = 140$
$S_{xy} = \sum_{i=1}^{8} X_i Y_i = 0 + 2.32 + 6.64 + 13.59 + 22.6 + 34.85 + 48.12 + 64.61 = 192.73$

$$a_1 = \frac{nS_{xy} - S_x S_y}{nS_{xx} - (S_x)^2} = \frac{(8 * 192.73) - (28 * 41.19)}{(8 * 140) - (28)^2} = -1.138$$

$$a_0 = \frac{S_{xx} S_y - S_{xy} S_x}{nS_{xx} - (S_x)^2} = \frac{(140 * 41.19) - (192.73 * 28)}{(8 * 140 - (28)^2} = 1.1016$$

The equation of best fit is $y = a_1 x + a_0$
$S = -1.138T + 1.1016$
The answer is E

Question 6

(1, 5.12), (3, 3), (6, 2.48), (9, 2.34), (15, 2.18)

$$f(x) = \alpha e^{\beta x}$$

For $y = be^{mx}, b = \alpha, m = \beta$
$ln(y) = mx + ln(b)$
In $Y = a_1 X + a_0$,

- $Y = ln(y)$
- $X = x$

- $a_1 = m$
- $a_0 = ln(b)$

The values for the linear least squares regression are x_i and $ln(yi)$
$S_x = \sum_{i=1}^{5} X_i = 1 + 3 + 6 + 9 + 15 = 34$
$S_y = \sum_{i=1}^{5} ln(Y_i) = ln(5.12) + ln(3) + ln(2.48) + ln(2.34) + ln(2.18) = 5.269$
$S_{xx} = \sum_{i=1}^{5} X_i^2 = 352$
$S_{xy} = \sum_{i=1}^{5} X_i ln(Y_i) = 1.633 + 3.296 + 5.4496 + 7.651 + 11.6899 = 29.7195$

$$a_0 = \frac{S_{xx}S_y - S_{xy}S_x}{nS_{xx} - (S_x)^2} = 1.3977$$

$$a_1 = \frac{nS_{xy} - S_xS_y}{nS_{xx} - (S_x)^2} = -0.050577$$

Approximately 1.3980, -0.050601
Answer is A
Question 7
(1, 5.12), (3, 3), (6, 2.48), (9, 2.34), (15, 2.18)

$$ln(y) = mln(x) + ln(b)$$

$$Y = a_1X + a_0$$

$$g(x) = \alpha + \frac{\beta}{x}$$

Take g(x) as:

$$y = \beta(\frac{1}{x}) + a$$

Convert it to normal form by using X = 1/x to get $Y = \beta X + \alpha$
Treat X_i's as $\frac{1}{x_i}$
$ln(y) = mx + ln(b)$
$Y = a_1X + a_0$,
The values for the linear least squares regression are $1/x_i$ and yi
$S_x = \sum_{i=1}^{5} 1/X_i = (1/1 + 1/3 + 1/6 + 1/9 + 1/15) = 1.6778$
$S_y = \sum_{i=1}^{5} Y_i = 5.12 + 3 + 2.48 + 2.34 + 2.18 = 15.12$
$S_{xx} = \sum_{i=1}^{5} (\frac{1}{X_i})^2 = 1 + 1/9 + 1/36 + 1/81 + 1/225 = 1.155679$
$S_{xy} = \sum_{i=1}^{5} \frac{1}{X_i} Y_i = 5.12 + 1 + 0.413 + 0.26 + 0.1453 = 6.9383$

$$a_0 = \frac{S_{xx}S_y - S_{xy}S_x}{nS_{xx} - (S_x)^2} = 1.9683$$

$$a_1 = \frac{nS_{xy} - S_xS_y}{nS_{xx} - (S_x)^2} = 3.1461$$

Approximately 1.9681, 3.1468
Answer is A
Question 8
Newton's Interpolating Polynomial

$$f(x) = log_4(cos(x))$$

$x_1 = 0.5$
$x_2 = 1.0$
$x_3 = 1.5$

$$f(0.5) = log_4(cos(0.5)) = -0.0942 = y_1$$

$$f(1.0) = log_4(cos(1.0)) = -0.4444 = y_2$$

$$f(1.5) = log_4(cos(1.0)) = -1.911 = y_3$$

Evaluate $P_2(x)$ at $x = 1.3$

$$P_2(x) = a_1 + a_2(1.3 - 0.5) + a_3(1.3 - 0.5)(1.3 - 1.0)$$

$$P_2(x) = a_1 + a_2(0.8) + a_3(0.8)(0.3)$$

Sub in the two points to get:

$$y_2 = y_1 + a_2(x_2 - x_1)$$

or

$$a_2 = \frac{-0.4444 - (-0.0942)}{1.0 - 0.5}$$

$$a_2 = -0.7004$$

Subbing the third point:

$$y_3 = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x_3 - x_1) + a_3(x_3 - x_1)(x_3 - x_2)$$

$$-1.911 = -0.0942 + \frac{-0.4444 - (-0.0942)}{1 - 0.5}(1.5 - 0.5) + a_3(1.5 - 0.5))(1.5 - 1.0)$$

$$-1.911 = -0.0942 + (-0.7004)(1.5 - 0.5) + a_3(1.5 - 0.5))(1.5 - 1.0)$$

$$-1.911 = -0.7946 + a_3(1.5 - 0.5))(1.5 - 1.0)$$

$$-1.911 = -0.7946 + a_3(0.5)$$

$$-1.911 + 0.7946 = a_3(0.5)$$

$$-1.1164 = a_3(0.5)$$

$$-2.2328$$

This is totally wrong lol. I did it in lagrange and got D so im not sure what happened

Question 9

Evaluate the Lagrange interpolating polynomial

$$f(x) = x^3 log_2(x)$$

$x_1 = 2$
$x_2 = 3$
$x_3 = 7$
To get y values:

$$f(2) = (x)^3 log_2(x) = y_1 = 8$$

$$f(3) = (x)^3 log_2(x) = y_2 = 42.794$$

$$f(7) = (x)^3 log_2(x) = y_3 = 962.9227$$

Evaluate $P_2(x)$ at $x = 5$

$$P_2(x) = \frac{(x - x_2)(x - x_3)}{(x_1 - x_2)(x_1 - x_3)} y_1 + \frac{(x - x_1)(x - x_3)}{(x_2 - x_1)(x_2 - x_3)} y_2 + \frac{(x - x_1)(x - x_2)}{(x_3 - x_1)(x_3 - x_2)} y_3$$

$$P_2(5) = \frac{(5 - 3)(5 - 7)}{(2 - 3)(2 - 7)} y_1 + \frac{(5 - 2)(5 - 7)}{(3 - 2)(3 - 7)} y_2 + \frac{(5 - 2)(5 - 3)}{(7 - 2)(7 - 3)} y_3$$

$$P_2(5) = (-0.8)y_1 + (1.5)y_2 + (0.3)y_3$$

$$P_2(5) = (-0.8)(8) + (1.5)(42.794) + (0.3)(962.9227)$$

$$P_2(5) = 346.66781$$

Answer is E

Question 10

Evaluate the Lagrange interpolating polynomial, then derive it to get the acceleration

From the textbook: The coefficients a1, and a2 are the same in the first-order and second-order polynomials. This means that if two points are given and a first-order Newton's polynomial is fit to pass through those points, and then a third point is added, the polynomial can be changed to be of second-order and pass through the three points by only determining the value of one additional coefficient.

This seems relevant to the fact that there are 4 data points but the scientist used a second order equation.

$$f(x) = x^3 log_2(x)$$

$x_1 = 9$
$x_2 = 15$
$x_3 = 20$
$y_1 = 21$
$y_2 = 32$
$y_3 = 48$
1:

$$f(x) = a_1 + a_2(x - x_1)$$

$$\frac{f(x) - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

Solve for f(x):
2:

$$f(x) = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$$

Comparing equations 1 and 2, we get:
$a_1 = y_1$ and $a_2 \frac{y_2 - y_1}{x_2 - x_1}$
$a_1 = 21$
$a_2 = \frac{32-21}{15-9} = 1.833$

$$f(x) = a_1 + a_2(x - x_1) + a_3(x - x_1)(x - x_2)$$

$$y_3 = y_1 + \frac{y_2 - y_1}{x_2 - x_1}(x_3 - x_1) + a_3(x_3 - x_1)(x_3 - x_2)$$

Rearranged:

$$a_3 = \frac{\frac{y_3 - y_2}{x_3 - x_2} - \frac{y_2 - y_1}{x_2 - x_1}}{(x_3 - x_1)}$$

$$a_3 = 0.124$$

We now have a second order with 3 points. Sub in our x = 18

$$f(x) = a_1 + a_2(x - x_1) + a_3(x - x_1)(x - x_2)$$

Differentiated:

$$f'(x) = a_3(x - x_2) + a_3(x - x_1) + a_2$$

$$f'(18) = 0.124(18 - 15) + 0.124(18 - 9) + 1.833$$

$$f'(18) = 3.321$$

Answer is E