INSTRUCTOR'S SOLUTIONS MANUAL

to accompany

Physics Fifth Edition

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PREFACE

This volume contains the complete solutions to those problems in the text that are marked with an **SSM** icon. There are about 600 such problems, which represent 25% of the total number of problems in the text.

The solutions are worked out with great care, and all the steps are included. Most of the solutions are comprised of two parts, a REASONING part followed by a SOLUTION part. In the REASONING part we explain what motivates our procedure for solving the problem, before any algebraic or numerical work is done. During the SOLUTION part, numerical calculations are performed, and the answer to the problem is obtained.

For ease of use and consistency, the notation, terminology, laws, theorems, and definitions employed in this manual are the same as those in the text.

Some of the solutions are preceded with a **WWW** icon. These solutions are also available on the World Wide Web at http://www.wiley.com/college/cutnell.

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1. **SSM** ***REASONING AND SOLUTION*** We use the fact that 0.200 g = 1 carat and that, under the conditions stated, 1000 g has a weight of 2.205 lb to construct the two conversion factors: (0.200 g)/(1 carat) = 1 and (2.205 lb)/(1000 g) = 1. Then,

$$(3106 \text{ carats}) \left(\frac{0.200 \text{ g}}{1 \text{ carat}}\right) \left(\frac{2.205 \text{ lb}}{1000 \text{ g}}\right) = \boxed{1.37 \text{ lb}}$$

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2. ***REASONING*** We use the facts that 1 mi = 5280 ft, 1 m = 3.281 ft, and 1 yd = 3 ft. With these facts we construct three conversion factors: (5280 ft)/(1 mi) = 1, (1 m)/(3.281 ft) = 1, and (3 ft)/(1 yd) = 1.

SOLUTION By multiplying by the given distance d of the fall by the appropriate conversion factors we find that

$$d = (6\text{ mi})\left(\frac{5280\text{ ft}}{1\text{ mi}}\right)\left(\frac{1\text{ m}}{3.281\text{ ft}}\right) + (551\text{ yd})\left(\frac{3\text{ ft}}{1\text{ yd}}\right)\left(\frac{1\text{ m}}{3.281\text{ ft}}\right) = \boxed{10\,159\text{ m}}$$

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3. ***REASONING AND SOLUTION***

a. 1 minute = 60 seconds, 1 hour = 3600 seconds.
(35 minutes) [60 seconds/(1 minute)] = 2100 seconds.
Hence, 1 hour 35 minutes = 3600 seconds + 2100 seconds = $\boxed{5700 \text{ s}}$

b. 1 day = 24 hours, 1 hour = 3600 s
1 day = (24 hours) [3600 seconds/(1 hour)] = $\boxed{86\,400 \text{ s}}$

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4. ***REASONING*** We use the fact that 1 mi = 1.609 km to construct the following conversion factor: (1 mi)/(1.609 km) = 1.

SOLUTION By multiplying the given train speed v by this conversion factor we find that

$$v = \left(515.2\ \frac{\text{km}}{\text{h}}\right)\left(\frac{1\ \text{mi}}{1.609\ \text{km}}\right) = \boxed{320.2\ \text{mi/h}}$$

5. **SSM** ***REASONING*** When converting between units, we write down the units explicitly in the calculations and treat them like any algebraic quantity. We construct the appropriate conversion factor (equal to unity) so that the final result has the desired units.

SOLUTION

a. Since 1.0×10^3 grams = 1.0 kilogram, it follows that the appropriate conversion factor is $(1.0 \times 10^3 \text{ g})/(1.0 \text{ kg}) = 1$. Therefore,

$$\left(5 \times 10^{-6} \text{ kg}\right)\left(\frac{1.0 \times 10^3 \text{ g}}{1.0 \text{ kg}}\right) = \boxed{5 \times 10^{-3} \text{ g}}$$

b. Since 1.0×10^3 milligrams = 1.0 gram,

$$\left(5 \times 10^{-3} \text{ g}\right)\left(\frac{1.0 \times 10^3 \text{ mg}}{1.0 \text{ g}}\right) = \boxed{5 \text{ mg}}$$

c. Since 1.0×10^6 micrograms = 1.0 gram,

$$\left(5 \times 10^{-3} \text{ g}\right)\left(\frac{1.0 \times 10^6 \ \mu\text{g}}{1.0 \text{ g}}\right) = \boxed{5 \times 10^3 \ \mu\text{g}}$$

6. ***REASONING*** This problem involves using unit conversions to determine the number of magnums in one jeroboam. The necessary relationships are

$$1.0 \text{ magnum} = 1.5 \text{ liters}$$
$$1.0 \text{ jeroboam} = 0.792 \text{ U. S. gallons}$$
$$1.00 \text{ U. S. gallon} = 3.785 \times 10^{-3} \text{ m}^3 = 3.785 \text{ liters}$$

These relationships may be used to construct the appropriate conversion factors.

SOLUTION By multiplying one jeroboam by the appropriate conversion factors we can determine the number of magnums in a jeroboam as shown below:

$$(1.0 \text{ jeroboam})\left(\frac{0.792 \text{ gallons}}{1.0 \text{ jeroboam}}\right)\left(\frac{3.785 \text{ liters}}{1.0 \text{ gallon}}\right)\left(\frac{1.0 \text{ magnum}}{1.5 \text{ liters}}\right) = \boxed{2.0 \text{ magnums}}$$

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7. ***REASONING AND SOLUTION***

a. $F = [M][L]/[T]^2$; $ma = [M][L]/[T]^2 = [M][L]/[T]^2$ so $F = ma$ $\boxed{\text{is dimensionally correct}}$.

b. $x = [L]$; $at^3 = ([L]/[T]^2)[T]^3 = [L][T]$ so $x = (1/2)at^3$ $\boxed{\text{is } \textit{not} \text{ dimensionally correct}}$.

c. $E = [M][L]^2/[T]^2$; $mv = [M][L]/[T]$ so $E = (1/2)mv$ $\boxed{\text{is } \textit{not} \text{ dimensionally correct}}$.

d. $E = [M][L]^2/[T]^2$; $max = [M]([L]/[T]^2)[L] = [M][L]^2/[T]^2$ so $E = max$
$\boxed{\text{is dimensionally correct}}$.

e. $v = [L]/[T]$; $(Fx/m)^{1/2} = \{([M][L]/[T]^2)([L]/[M])\}^{1/2} = \{[L]^2/[T]^2\}^{1/2} = [L]/[T]$ so
$v = (Fx/m)^{1/2}$ $\boxed{\text{is dimensionally correct}}$.

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8. ***REASONING AND SOLUTION*** Since $v = \frac{1}{3} z x t^2$, it follows that $z = \frac{3v}{x t^2}$. Since $x = [\mathrm{L}]$, $v = [\mathrm{L}]/[\mathrm{T}]$, and $t = [\mathrm{T}]$, we have

$$\frac{v}{x t^2} = \frac{[\mathrm{L}]/[\mathrm{T}]}{[\mathrm{L}][\mathrm{T}]^2} = \boxed{\frac{1}{[\mathrm{T}]^3}}$$

Note that the numerical factor "3" does not contribute to the dimensions.

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9. $\boxed{\text{SSM}}$ ***REASONING*** The volume of water at a depth d beneath the rectangle is equal to the area of the rectangle multiplied by d. The area of the rectangle = (1.20 nautical miles) × (2.60 nautical miles) = 3.12 (nautical miles)^2. Since 6076 ft = 1 nautical mile and 0.3048 m = 1 ft, the conversion factor between nautical miles and meters is

$$\left(\frac{6076\ \text{ft}}{1\ \text{nautical mile}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)=\frac{1.852\times10^{3}\ \text{m}}{1\ \text{nautical mile}}$$

SOLUTION The area of the rectangle of water in m^2 is, therefore,

$$\left[3.12\ (\text{nautical miles})^2\right]\left(\frac{1.852\times10^{3}\ \text{m}}{1\ \text{nautical mile}}\right)^2=1.07\times10^{7}\ \text{m}^2$$

Since 1 fathom = 6 ft, and 1 ft = 0.3048 m, the depth d in meters is

$$(16.0\ \text{fathoms})\left(\frac{6\ \text{ft}}{1\ \text{fathom}}\right)\left(\frac{0.3048\ \text{m}}{1\ \text{ft}}\right)=2.93\times10^{1}\ \text{m}$$

The volume of water beneath the rectangle is

$$(1.07\times10^{7}\ \text{m}^2)\,(2.93\times10^{1}\ \text{m})=\boxed{3.13\times10^{8}\ \text{m}^3}$$

10. ***REASONING AND SOLUTION*** By multiplying the quantity 1.78×10^{-3} kg/(s · m) by the appropriate conversions factors, we can convert the quantity to units of poise (P). The necessary relationships are

$$1.0 \text{ kg} = 1.0 \times 10^{3} \text{ g}$$
$$1.0 \text{ m} = 1.0 \times 10^{2} \text{ cm}$$
$$1 \text{ P} = 1 \text{ g/(s} \cdot \text{cm)}$$

Therefore,

$$\left(1.78 \times 10^{-3} \ \frac{\text{kg}}{\text{s} \cdot \text{m}}\right)\left(\frac{1.0 \times 10^{3} \text{ g}}{1.0 \text{ kg}}\right)\left(\frac{1.0 \text{ m}}{1.0 \times 10^{2} \text{ cm}}\right)\left(\frac{1.0 \text{ P}}{1.0 \text{ g/(s} \cdot \text{cm)}}\right) = \boxed{1.78 \times 10^{-2} \text{ P}}$$

11. **SSM** **WWW** ***REASONING*** The shortest distance between the two towns is along the line that joins them. This distance, h, is the hypotenuse of a right triangle whose other sides are $h_o = 35.0$ km and $h_a = 72.0$ km, as shown in the figure below.

SOLUTION The angle θ is given by $\tan\theta = h_o / h_a$ so that

$$\theta = \tan^{-1}\left(\frac{35.0 \text{ km}}{72.0 \text{ km}}\right) = \boxed{25.9° \text{ S of W}}$$

We can then use the Pythagorean theorem to find h.

$$h = \sqrt{h_o^2 + h_a^2} = \sqrt{(35.0 \text{ km})^2 + (72.0 \text{ km})^2} = \boxed{80.1 \text{ km}}$$

12. ***REASONING AND SOLUTION*** The angle θ is given by $\tan\theta = h_o/h_a$ so

$$\theta = \tan^{-1}(12.0\text{ m}/100.0\text{ m}) = \boxed{6.84°}$$

13. ***REASONING AND SOLUTION*** In the diagram below, $\theta = 14.6^{\circ}$ and $h = 2830$ m.

We know that $\sin\theta = H/h$ and therefore

$H = h \sin\theta$

$= (2830 \text{ m}) \sin 14.6^{\circ} = \boxed{713 \text{ m}}$

14. ***REASONING AND SOLUTION*** The following figure (*not* drawn to scale) shows the geometry of the situation, when the observer is a distance r from the base of the arch.

The angle θ is related to r and h by $\tan\theta = h/r$. Solving for r, we find

$$r = \frac{h}{\tan\theta} = \frac{192\text{ m}}{\tan 2.0°} = 5.5\times10^{3}\text{ m} = \boxed{5.5\text{ km}}$$

15. ***REASONING AND SOLUTION*** One half of the tree forms a right triangle as shown.

$$H = \frac{1.00\text{ m}}{\tan 15.0^\circ} = \boxed{3.73\text{ m}}$$

16. ***REASONING AND SOLUTION*** Since the center of the circle and the center of the triangle coincide, a radius of the circle bisects a 60° angle of the triangle, as the drawing indicates. Therefore, we have

$$D = R \cos 30.0°$$

But the length of a side of the triangle is $2D$, so that

$$2D = 2R \cos 30.0°$$

$$= 2(0.25 \text{ m}) \cos 30.0° = \boxed{0.43 \text{ m}}$$

$\theta = 30.0°$

17. ***REASONING AND SOLUTION*** Consider the following views of the cube.

The length, L, of the diagonal of the bottom face of the cube can be found using the Pythagorean theorem to be

$$L^2 = a^2 + a^2 = 2(0.281\ \text{nm})^2 = 0.158\ \text{nm}^2 \quad \text{or} \quad L = 0.397\ \text{nm}$$

The required distance c is also found using the Pythagorean theorem.

$$c^2 = L^2 + a^2 = (0.397\ \text{nm})^2 + (0.281\ \text{nm})^2 = 0.237\ \text{nm}^2$$

Then,

$$\boxed{c = 0.487\ \text{nm}}$$

18. ***REASONING AND SOLUTION***

$$\theta = \tan^{-1}(a/L) = \tan^{-1}(0.281/0.397) = \boxed{35.3°}$$

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19. $\boxed{\text{SSM}}$ ***REASONING*** The law of cosines is

$$a^2 = b^2 + c^2 - 2bc\cos\theta$$

where a is the side opposite angle θ, and b and c are the other two sides. Solving for θ, we have

$$\theta = \cos^{-1}\left[(b^2 + c^2 - a^2)/(2bc)\right]$$

SOLUTION For a = 190 cm, b = 95 cm, c = 150 cm,

$$\theta = \cos^{-1}\left[\frac{(95\text{ cm})^2 + (150\text{ cm})^2 - (190\text{ cm})^2}{2(95\text{ cm})(150\text{ cm})}\right] = 99°$$

Thus, the angle opposite the side of length 190 cm is $\boxed{\text{99 degrees}}$.

Similarly, when a = 95 cm, b =190 cm, c = 150 cm, we find that the angle opposite the side of length 95 cm is $\boxed{\theta = 3.0 \times 10^1 \text{ degrees}}$.

Finally, when a = 150 cm, b = 190 cm, c = 95 cm, we find that the angle opposite the side of length 150 cm is $\boxed{\theta = 51 \text{ degrees}}$.

20. ***REASONING AND SOLUTION*** Consider the bottom face of a tetrahedron which is an equilateral triangle.

The distance, x, from a vertex to the intersection of the perpendicular bisectors is

$$x = \frac{L/2}{\cos 30^{o}} = \frac{L}{\sqrt{3}}$$

Now consider any right triangle in the tetrahedron whose sides are H, L, and x. The Pythagorean theorem gives

$$L^2 = H^2 + x^2 = H^2 + L^2/3.$$

Then

$$H^2 = L^2 - L^2/3 = 2\,L^2/3 \text{ so that } H/L = \boxed{\sqrt{2/3}}$$

21. ***REASONING AND SOLUTION*** Since the initial force and the resultant force point along the east/west line, the second force must also point along the east/west line. The direction of the second force is not specified; it could point either due east or due west.

If the second force points $\boxed{\text{due east}}$, both forces point in the same direction and the magnitude of the resultant force is the sum of the two magnitudes: $F_1 + F_2 = F_R$. Therefore

$$F_2 = F_R - F_1 = 400\text{ N} - 200\text{ N} = \boxed{200\text{ N}}$$

If the second force points $\boxed{\text{due west}}$, the two forces point in opposite directions, and the magnitude of the resultant force is the difference of the two magnitudes: $F_2 - F_1 = F_R$. Therefore

$$F_2 = F_R + F_1 = 400\text{ N} + 200\text{ N} = \boxed{600\text{ N}}$$

22. ***REASONING*** The shortest distance between the tree and the termite mound is equal to the magnitude of the chimpanzee's displacement **r**.

SOLUTION

a. From the Pythagorean theorem, we have

$$r = \sqrt{(51\text{ m})^2 + (39\text{ m})^2} = \boxed{64\text{ m}}$$

b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{39\text{ m}}{51\text{ m}}\right) = \boxed{37^\circ\text{ south of east}}$$

23. SSM WWW ***REASONING*** When a vector is multiplied by – 1, the magnitude of the vector remains the same, but the direction is reversed. Vector subtraction is carried out in the same manner as vector addition except that one of the vectors has been multiplied by –1.

SOLUTION Since both vectors point north, they are colinear. Therefore their magnitudes may be added by the rules of ordinary algebra.

a. Taking north as the positive direction, we have

$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) = +2.43 \text{ km} + (-7.74 \text{ km}) = -5.31 \text{ km}$$

The minus sign in the answer for $\mathbf{A} - \mathbf{B}$ indicates that the direction is south so that

$$\boxed{\mathbf{A} - \mathbf{B} = 5.31 \text{ km, south}}$$

b. Similarly,

$$\mathbf{B} - \mathbf{A} = \mathbf{B} + (-\mathbf{A}) = +7.74 \text{ km} + (-2.43 \text{ km}) = +5.31 \text{ km}$$

The plus sign in the answer for $\mathbf{B} - \mathbf{A}$ indicates that the direction is north, so that

$$\boxed{\mathbf{B} - \mathbf{A} = 5.31 \text{ km, north}}$$

24. ***REASONING*** The triple jump consists of a double jump (assumed to end on a square that we label C) followed by a single jump. The single jump is perpendicular to the double jump, so that the length ℓ_{AC} of the double jump, the length ℓ_{CB} of the single jump, and the magnitude ℓ_{AB} of the total displacement form a right triangle. Thus, we have

$$\ell_{AB} = \sqrt{\ell_{AC}^2 + \ell_{CB}^2}$$

SOLUTION The diagonal of one square on the checkerboard has a length d of

$$d = \sqrt{(4.0\text{ cm})^2 + (4.0\text{ cm})^2} = 5.66\text{ cm}$$

Since $\ell_{AC} = 4\,d$ and $\ell_{CB} = 2\,d$, it follows that

$$\ell_{AB} = \sqrt{(4d)^2 + (2d)^2} = d\sqrt{20} = (5.66\text{ cm})\sqrt{20} = \boxed{25\text{ cm}}$$

25. SSM ***REASONING*** Using the component method, we find the components of the resultant **R** that are due east and due north. The magnitude and direction of the resultant **R** can be determined from its components, the Pythagorean theorem, and the tangent function.

SOLUTION The first four rows of the table below give the components of the vectors **A**, **B**, **C**, and **D**. Note that east and north have been taken as the positive directions; hence vectors pointing due west and due south will appear with a negative sign.

Vector	*East/West Component*	*North/South Component*
A	+ 2.00 km	0
B	0	+ 3.75 km
C	– 2.50 km	0
D	0	–3.00 km
R = A + B + C + D	– 0.50 km	+ 0.75 km

The fifth row in the table gives the components of **R**. The magnitude of **R** is given by the Pythagorean theorem as

$$R = \sqrt{(-0.50\text{ km})^2 + (+0.75\text{ km})^2} = \boxed{0.90\text{ km}}$$

The angle θ that **R** makes with the direction due west is

$$\theta = \tan^{-1}\left(\frac{0.75\text{ km}}{0.50\text{ km}}\right) = \boxed{56^\circ\text{ north of west}}$$

26. ***REASONING*** The performer walks out on the wire a distance d, and the vertical distance to the net is h. Since these two distances are perpendicular, the magnitude of the displacement is given by the Pythagorean theorem as $s = \sqrt{d^2 + h^2}$. Values for s and h are given, so we can solve this expression for the distance d. The angle that the performer's displacement makes below the horizontal can be found using trigonometry.

SOLUTION

a. Using the Pythagorean theorem, we find that

$$s = \sqrt{d^2 + h^2} \quad \text{or} \quad d = \sqrt{s^2 - h^2} = \sqrt{(26.7\text{ ft})^2 - (25.0\text{ ft})^2} = \boxed{9.4\text{ ft}}$$

b. The angle θ that the performer's displacement makes below the horizontal is given by

$$\tan\theta = \frac{h}{d} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{h}{d}\right) = \tan^{-1}\left(\frac{25.0\text{ ft}}{9.4\text{ ft}}\right) = \boxed{69°}$$

27. **SSM** **WWW** ***REASONING AND SOLUTION*** The single force needed to produce the same effect is equal to the resultant of the forces provided by the two ropes. The figure below shows the force vectors drawn to scale and arranged tail to head. The magnitude and direction of the resultant can be found by direct measurement using the scale factor shown in the figure.

a. From the figure, the magnitude of the resultant is **5600 N**.

b. The single rope should be directed **along the dashed line** in the text drawing.

28. ***REASONING*** The displacement vector **R** for a straight-line dash between the starting and finishing points can be found by drawing the vectors **A**, **B**, and **C** to scale in both magnitude and orientation, in a tail-to-head fashion, and connecting the tail of **A** to the head of **C**. The magnitude of the resultant can be found by measuring its length and making use of the scale factor. Similarly, the direction of the resultant is found by measuring the angle θ it makes with the side of the court.

SOLUTION By construction and measurement, the magnitude of the resultant **R** is $\boxed{20\text{ m}}$. The angle θ made with the side of the court is $\boxed{15°}$.

29. ***REASONING*** The displacement **Q** needed for the bear to return to its starting point is the vector that is equal in magnitude and opposite in direction to the actual displacement **R** of the bear.

SOLUTION The first two rows of the following table gives components of the individual displacements, **A** and **B**, of the bear. The third row gives the components of $\mathbf{R} = \mathbf{A} + \mathbf{B}$. The directions due east and due north have been chosen to be positive.

Displacement	*East/West Component*	*North/South Component*
A	–1563 m	0
B	(– 3348 m) cos 32°= –2839 m	(– 3348 m) sin 32°= 1774 m
R = A + B	–4402 m	1774 m

Thus, the components of the displacement **Q** needed for the bear to *return to its starting point* are an eastward component of +4402 m and a southward component of –1774 m.

a. From the Pythagorean theorem, we have

$$Q = \sqrt{(4402\text{ m})^2 + (-1774\text{ m})^2} = \boxed{4746\text{ m}}$$

b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{1774\text{ m}}{4402\text{ m}}\right) = \boxed{21.9°,\text{ south of east}}$$

30. ***REASONING*** a. Since the two displacement vectors **A** and **B** have directions due south and due east, they are perpendicular. Therefore, the resultant vector **R = A + B** has a magnitude given by the Pythagorean theorem: $R^2 = A^2 + B^2$. Knowing the magnitudes of **R** and **A**, we can calculate the magnitude of **B**. The direction of the resultant can be obtained using trigonometry.

b. For the vector **R′ = A – B** we note that the subtraction can be regarded as an addition in the following sense: **R′ = A + (–B)**. The vector **–B** points due west, opposite the vector **B**, so the two vectors are once again perpendicular and the magnitude of **R′** again is given by the Pythagorean theorem. The direction again can be obtained using trigonometry.

SOLUTION a. The drawing shows the two vectors and the resultant vector. According to the Pythagorean theorem, we have

$$R^2 = A^2 + B^2$$

$$B = \sqrt{R^2 - A^2}$$

$$B = \sqrt{(3.75\text{ km})^2 - (2.50\text{ km})^2}$$

$$= \boxed{2.8\text{ km}}$$

Using trigonometry, we can see that the direction of the resultant is

$$\tan\theta = \frac{B}{A} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{2.8\text{ km}}{2.50\text{ km}}\right) = \boxed{48^\circ\text{ east of south}}$$

b. Referring to the drawing and following the same procedure as in part a, we find

$$R'^2 = A^2 + (-B)^2 \quad \text{or} \quad B = \sqrt{R'^2 - A^2} = \sqrt{(3.75\text{ km})^2 - (2.50\text{ km})^2} = \boxed{2.8\text{ km}}$$

$$\tan\theta = \frac{B}{A} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{2.8\text{ km}}{2.50\text{ km}}\right) = \boxed{48^\circ\text{ west of south}}$$

31. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** In order to determine which vector has the largest x and y components, we calculate the magnitude of the x and y components explicitly and compare them. In the calculations, the symbol u denotes the units of the vectors.

$$A_x = (100.0 \text{ u}) \cos 90.0° = 0.00 \text{ u} \qquad A_y = (100.0 \text{ u}) \sin 90.0° = 1.00 \times 10^2 \text{ u}$$

$$B_x = (200.0 \text{ u}) \cos 60.0° = 1.00 \times 10^2 \text{ u} \qquad B_y = (200.0 \text{ u}) \sin 60.0° = 173 \text{ u}$$

$$C_x = (150.0 \text{ u}) \cos 0.00° = 150.0 \text{ u} \qquad C_y = (150.0 \text{ u}) \sin 0.00° = 0.00 \text{ u}$$

a. $\boxed{\textbf{C} \text{ has the largest } x \text{ component}}$.

b. $\boxed{\textbf{B} \text{ has the largest } y \text{ component}}$.

32. ***REASONING AND SOLUTION***

a. From the Pythagorean theorem, we have

$$F = \sqrt{(150\text{ N})^2 + (130\text{ N})^2} = \boxed{2.0 \times 10^2\text{ N}}$$

b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{130\text{ N}}{150\text{ N}}\right) = \boxed{41°}$$

33. ***REASONING AND SOLUTION*** The east and north components are, respectively

a. $$A_e = A\cos\theta = (155\text{ km})\cos 18.0° = \boxed{147\text{ km}}$$

b. $$A_n = A\sin\theta = (155\text{ km})\sin 18.0° = \boxed{47.9\text{ km}}$$

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34. ***REASONING AND SOLUTION***

$$v_e = v \cos\theta = (5.00 \text{ m/s}) \cos 35.0° = \boxed{4.10 \text{ m/s}}$$

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35. **SSM** ***REASONING*** The x and y components of **r** are mutually perpendicular; therefore, the magnitude of **r** can be found using the Pythagorean theorem. The direction of **r** can be found using the definition of the tangent function.

SOLUTION According to the Pythagorean theorem, we have

$$r = \sqrt{x^2 + y^2} = \sqrt{(-125\text{ m})^2 + (-184\text{ m})^2} = \boxed{222\text{ m}}$$

The angle θ is

$$\theta = \tan^{-1}\left(\frac{184\text{ m}}{125\text{ m}}\right) = \boxed{55.8°}$$

36. ***REASONING AND SOLUTION*** The horizontal component of the plane's velocity is

$$v_x = v \cos \theta = (180 \text{ m/s}) \cos 34° = \boxed{150 \text{ m/s}}$$

37. ***REASONING AND SOLUTION***

a. $A_x = A \cos 30.0° = (750 \text{ units})(0.866) = \boxed{650 \text{ units}}$

$A_y = A \sin 30.0° = (750 \text{ units})(0.500) = \boxed{380 \text{ units}}$

b. $A_{x'} = A \cos 40.0° = (750 \text{ units})(0.643) = \boxed{570 \text{ units}}$

$A_{y'} = -A \sin 40.0° = -(750 \text{ units})(0.766) = \boxed{-480 \text{ units}}$

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38. ***REASONING*** The force vector **F** points at an angle of θ above the $+x$ axis. Therefore, its x and y components are given by $F_x = F\cos\theta$ and $F_y = F\sin\theta$.

SOLUTION a. The magnitude of the vector can be obtained from the y component as follows:

$$F_y = F\sin\theta \quad \text{or} \quad F = \frac{F_y}{\sin\theta} = \frac{290\text{ N}}{\sin 52°} = \boxed{370\text{ N}}$$

b. Now that the magnitude of the vector is known, the x component of the vector can be calculated as follows:

$$F_x = F\cos\theta = (370\text{ N})\cos 52° = \boxed{+230\text{ N}}$$

39. SSM WWW ***REASONING AND SOLUTION*** The force **F** can be first resolved into two components; the z component F_z and the projection onto the x-y plane, F_p as shown in the figure below on the left. According to that figure,

$$F_p = F \sin 54.0° = (475 \text{ N}) \sin 54.0° = 384 \text{ N}.$$

The projection onto the x-y plane, F_p, can then be resolved into x and y components.

a. From the figure on the right,

$$F_x = F_p \cos 33.0° = (384 \text{ N}) \cos 33.0° = \boxed{322 \text{ N}}$$

b. Also from the figure on the right,

$$F_y = F_p \sin 33.0° = (384 \text{ N}) \sin 33.0° = \boxed{209 \text{ N}}$$

c. From the figure on the left,

$$F_z = F \cos 54.0° = (475 \text{ N}) \cos 54.0° = \boxed{279 \text{ N}}$$

40. ***REASONING AND SOLUTION*** The first three rows of the following table gives the components of each of the three individual displacements. The fourth row gives the components of the resultant displacement. The directions due east and due north have been taken as the positive directions.

Displacement	***East/West Component***	***North/South Component***
A	–52 paces	0
B	–(42 paces) cos 30.0° = –36 paces	(42 paces) sin 30.0° = 21 paces
C	0	25 paces
R = A + B + C	–88 paces	46 paces

a. Therefore, the magnitude of the displacement in the direction due north is $\boxed{\text{46 paces}}$.

b. Similarly, the magnitude of the displacement in the direction due west is $\boxed{\text{88 paces}}$.

41. SSM ***REASONING*** The individual displacements of the golf ball, **A**, **B**, and **C** are shown in the figure. Their resultant, **R,** is the displacement that would have been needed to "hole the ball" on the very first putt. We will use the component method to find **R**.

SOLUTION The components of each displacement vector are given in the table below.

Vector	***x* Components**	***y* Components**
A	(5.0 m) cos 0° = 5.0 m	(5.0 m) sin 0° = 0
B	(2.1 m) cos 20.0° = 2.0 m	(2.1 m) sin 20.0° = 0.72 m
C	(0.50 m) cos 90.0° = 0	(0.50 m) sin 90.0° = 0.50 m
$\mathbf{R} = \mathbf{A} + \mathbf{B} + \mathbf{C}$	7.0 m	1.22 m

The resultant vector **R** has magnitude

$$R = \sqrt{(7.0\ \text{m})^2 + (1.22\ \text{m})^2} = \boxed{7.1\ \text{m}}$$

and the angle θ is

$$\theta = \tan^{-1}\left(\frac{1.22\ \text{m}}{7.0\ \text{m}}\right) = 9.9°$$

Thus, the required direction is $\boxed{9.9° \text{ north of east}}$.

42. ***REASONING*** Using the component method for vector addition, we will find the x component of the resultant force vector by adding the x components of the individual vectors. Then we will find the y component of the resultant vector by adding the y components of the individual vectors. Once the x and y components of the resultant are known, we will use the Pythagorean theorem to find the magnitude of the resultant and trigonometry to find its direction. We will take east as the $+x$ direction and north as the $+y$ direction.

SOLUTION The x component of the resultant force **F** is

$$F_x = \underbrace{(2240\text{ N})\cos 34.0°}_{F_{Ax}} + \underbrace{(3160\text{ N})\cos 90.0°}_{F_{Bx}} = (2240\text{ N})\cos 34.0°$$

The y component of the resultant force **F** is

$$F_y = \underbrace{-(2240\text{ N})\sin 34.0°}_{F_{Ay}} + \underbrace{(-3160\text{ N})}_{F_{By}}$$

Using the Pythagorean theorem, we find that the magnitude of the resultant force is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{\left[(2240\text{ N})\cos 34.0°\right]^2 + \left[-(2240\text{ N})\sin 34.0° - 3160\text{ N}\right]^2} = \boxed{4790\text{ N}}$$

Using trigonometry, we find that the direction of the resultant force is

$$\theta = \tan^{-1}\left[\frac{(2240\text{ N})\sin 34.0° + 3160\text{ N}}{(2240\text{ N})\cos 34.0°}\right] = \boxed{67.2°\text{ south of east}}$$

43. ***REASONING AND SOLUTION*** The horizontal component of the resultant vector is

$$R_h = A_h + B_h + C_h = (0.00\text{ m}) + (15.0\text{ m}) + (18.0\text{ m})\cos 35.0° = 29.7\text{ m}.$$

Similarly, the vertical component is

$$R_v = A_v + B_v + C_v = (5.00\text{ m}) + (0.00\text{ m}) + (-18.0\text{ m})\sin 35.0° = -5.32\text{ m}.$$

The magnitude of the resultant vector is

$$R = \sqrt{R_h^2 + R_v^2} = \sqrt{(29.7\text{ m})^2 + (-5.32\text{ m})^2} = \boxed{30.2\text{ m}}$$

The angle θ is obtained from

$$\theta = \tan^{-1}[(5.32\text{ m})/(29.7\text{ m})] = \boxed{10.2°}$$

44. ***REASONING*** If we let the directions due east and due north be the positive directions, then the desired displacement **A** has components

$$A_E = (4.8\ \text{km})\cos 42° = 3.57\ \text{km}$$

$$A_N = (4.8\ \text{km})\sin 42° = 3.21\ \text{km}$$

while the actual displacement **B** has components

$$B_E = (2.4\ \text{km})\cos 22° = 2.23\ \text{km}$$

$$B_N = (2.4\ \text{km})\sin 22° = 0.90\ \text{km}$$

Therefore, the research team must go

$$3.57\ \text{km} - 2.23\ \text{km} = 1.34\ \text{km, eastward}$$

and

$$3.21\ \text{km} - 0.90\ \text{km} = 2.31\ \text{km, northward}$$

to reach the research station.

SOLUTION

a. From the Pythagorean theorem, we find that the magnitude of the displacement vector required to bring the team to the research station is

$$R = \sqrt{(1.34\ \text{km})^2 + (2.31\ \text{km})^2} = \boxed{2.7\ \text{km}}$$

b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{2.31\ \text{km}}{1.34\ \text{km}}\right) = \boxed{6.0 \times 10^1\ \text{degrees, north of east}}$$

45. ***REASONING*** The magnitude of the resultant force in part (b) of the drawing is given by

$$F_A + F_B \cos 20.0° + F_C \cos 20.0° = F_A + 2F\cos 20.0°$$

where, since $F_B \sin 20.0° = -F_C \sin 20.0°$, the y components add to zero. As stated in the problem, the magnitude of the resultant force acting on the elephant in part (b) of the drawing is twice that in part (a); therefore,

$$F_A + 2F\cos 20.0° = 2F_A \qquad \text{or} \qquad 2F\cos 20.0° = F_A$$

SOLUTION Solving for the ratio F/F_A, we have

$$\frac{F}{F_A} = \frac{1}{2(\cos 20.0°)} = \boxed{0.532}$$

46. ***REASONING AND SOLUTION*** Since the resultant of the three vectors is zero, each component of the resultant must be equal to zero.

The x components of the first two forces are

$$F_{1x} = (166\text{ N})\cos 60.0° = 83.0\text{ N}$$

$$F_{2x} = (284\text{ N})\cos 30.0° = 246\text{ N}$$

The x component of the resultant of the three vectors is, therefore,

$$(83\text{ N}) + (246\text{ N}) + F_{3x} = 0$$

or $F_{3x} = -329$ N. The minus sign indicates that $\mathbf{F}_{3x}$ points in the negative x direction.

The y components of the first two forces are

$$F_{1y} = (166\text{ N})\sin 60.0° = 144\text{ N}$$

$$F_{2y} = (284\text{ N})\sin 30.0° = 142\text{ N}$$

The y component of the resultant of the three vectors is, therefore,

$$(144\text{ N}) + (142\text{ N}) + F_{3y} = 0$$

or $F_{3y} = -286$ N. The minus sign indicates that $\mathbf{F}_{3y}$ points in the negative y direction.

The magnitude of $\mathbf{F}_3$ is, from the Pythagorean theorem,

$$\sqrt{(329\text{ N})^2 + (286\text{ N})^2} = \boxed{436\text{ N}}$$

The angle made by $\mathbf{F}_3$ with the negative x axis is

$$\theta = \tan^{-1}\left[\frac{286\text{ N}}{392\text{ N}}\right] = \boxed{41.0°}$$

47. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** We take due north to be the direction of the $+y$ axis. Vectors **A** and **B** are the components of the resultant, **C**. The angle that **C** makes with the x axis is then $\theta = \tan^{-1}(B/A)$. The symbol u denotes the units of the vectors.

a. Solving for B gives

$$B = A \tan\theta = (6.00 \text{ u}) \tan 60.0° = \boxed{10.4 \text{ u}}$$

b. The magnitude of **C** is

$$C = \sqrt{A^2 + B^2} = \sqrt{(6.00 \text{ u})^2 + (10.4 \text{ u})^2} = \boxed{12.0 \text{ u}}$$

48. ***REASONING*** We know that the three displacement vectors have a resultant of zero, so that $\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$. This means that the sum of the x components of the vectors and the sum of the y components of the vectors are separately equal to zero. From these two equations we will be able to determine the magnitudes of vectors **B** and **C**. The directions east and north are, respectively, the $+x$ and $+y$ directions.

SOLUTION Setting the sum of the x components of the vectors and the sum of the y components of the vectors separately equal to zero, we have

$$\underbrace{(1550\text{ m})\cos 25.0°}_{A_x} + \underbrace{B\sin 41.0°}_{B_x} + \underbrace{(-C\cos 35.0°)}_{C_x} = 0$$

$$\underbrace{(1550\text{ m})\sin 25.0°}_{A_y} + \underbrace{(-B\cos 41.0°)}_{B_y} + \underbrace{C\sin 35.0°}_{C_y} = 0$$

These two equations contain two unknown variables, B and C. They can be solved simultaneously to show that

a. $B = \boxed{5550\text{ m}}$ and b. $C = \boxed{6160\text{ m}}$

49. $\boxed{\text{SSM}}$ ***REASONING*** Since the finish line is coincident with the starting line, the net displacement of the sailboat is zero. Hence the sum of the components of the displacement vectors of the individual legs must be zero. In the drawing in the text, the directions to the right and upward are taken as positive.

SOLUTION In the horizontal direction $R_h = A_h + B_h + C_h + D_h = 0$

$$R_h = (3.20 \text{ km}) \cos 40.0° - (5.10 \text{ km}) \cos 35.0° - (4.80 \text{ km}) \cos 23.0° + D \cos \theta = 0$$

$$D \cos \theta = 6.14 \text{ km}. \qquad (1)$$

In the vertical direction $R_V = A_V + B_V + C_V + D_V = 0$.

$$R_V = (3.20 \text{ km}) \sin 40.0° + (5.10 \text{ km}) \sin 35.0° - (4.80 \text{ km}) \sin 23.0° - D \sin \theta = 0.$$

$$D \sin \theta = 3.11 \text{ km} \qquad (2)$$

Dividing (2) by (1) gives

$$\tan \theta = (3.11 \text{ km})/(6.14 \text{ km}) \quad \text{or} \quad \theta = \boxed{26.9°}$$

Solving (1) gives

$$D = (6.14 \text{ km})/\cos 26.9° = \boxed{6.88 \text{ km}}$$

50. ***REASONING*** The following table shows the components of the individual displacements and the components of the resultant. The directions due east and due north are taken as the positive directions.

Displacement	***East/West Component***	***North/South Component***
(1)	–27.0 cm	0
(2)	–(23.0 cm) cos 35.0° = –18.84 cm	–(23.0 cm) sin 35.0° = –13.19 cm
(3)	(28.0 cm) cos 55.0° = 16.06 cm	–(28.0 cm) sin 55.0° = –22.94 cm
(4)	(35.0 cm) cos 63.0° = 15.89 cm	(35.0 cm) sin 63.0° = 31.19 cm
Resultant	–13.89 cm	–4.94 cm

SOLUTION

a. From the Pythagorean theorem, we find that the magnitude of the resultant displacement vector is

$$R = \sqrt{(13.89\text{ cm})^2 + (4.94\text{ cm})^2} = \boxed{14.7\text{ cm}}$$

b. The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{4.94\text{ cm}}{13.89\text{ cm}}\right) = \boxed{19.6°,\text{ south of west}}$$

51. ***REASONING AND SOLUTION*** The observer's eye, the treetop and a point on the tree trunk 1.83 m above the ground are the vertices of a right triangle. The height of the tree, H is

$$H = 1.83 \text{ m} + (32.0 \text{ m}) \tan 20.0° = \boxed{13.5 \text{ m}}$$

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52. ***REASONING*** a. Since the two force vectors **A** and **B** have directions due west and due north, they are perpendicular. Therefore, the resultant vector **F = A + B** has a magnitude given by the Pythagorean theorem: $F^2 = A^2 + B^2$. Knowing the magnitudes of **A** and **B**, we can calculate the magnitude of **F**. The direction of the resultant can be obtained using trigonometry.

b. For the vector **F′ = A – B** we note that the subtraction can be regarded as an addition in the following sense: **F′ = A + (–B)**. The vector **–B** points due south, opposite the vector **B**, so the two vectors are once again perpendicular and the magnitude of **F′** again is given by the Pythagorean theorem. The direction again can be obtained using trigonometry.

SOLUTION a. The drawing shows the two vectors and the resultant vector. According to the Pythagorean theorem, we have

$$F^2 = A^2 + B^2$$

$$F = \sqrt{A^2 + B^2}$$

$$F = \sqrt{(445\text{ N})^2 + (325\text{ N})^2}$$

$$= \boxed{551\text{ N}}$$

Using trigonometry, we can see that the direction of the resultant is

$$\tan\theta = \frac{B}{A} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{325\text{ N}}{445\text{ N}}\right) = \boxed{36.1^\circ\text{ north of west}}$$

b. Referring to the drawing and following the same procedure as in part a, we find

$$F'^2 = A^2 + (-B)^2 \quad \text{or} \quad F' = \sqrt{A^2 + (-B)^2} = \sqrt{(445\text{ N})^2 + (-325\text{ N})^2} = \boxed{551\text{ N}}$$

$$\tan\theta = \frac{B}{A} \quad \text{or} \quad \theta = \tan^{-1}\left(\frac{325\text{ N}}{445\text{ N}}\right) = \boxed{36.1^\circ\text{ south of west}}$$

53. **SSM** ***REASONING*** The ostrich's velocity vector **v** and the desired components are shown in the figure at the right. The components of the velocity in the directions due west and due north are $\mathbf{v}_W$ and $\mathbf{v}_N$, respectively. The sine and cosine functions can be used to find the components.

SOLUTION

a. According to the definition of the sine function, we have for the vectors in the figure

$$\sin\theta = \frac{v_N}{v} \quad \text{or} \quad v_N = v\sin\theta = (17.0\ \text{m/s})\sin 68° = \boxed{15.8\ \text{m/s}}$$

b. Similarly,

$$\cos\theta = \frac{v_W}{v} \quad \text{or} \quad v_W = v\cos\theta = (17.0\ \text{m/s})\cos 68.0° = \boxed{6.37\ \text{m/s}}$$

54. ***REASONING AND SOLUTION*** x has the dimensions of [L], v has the dimensions of [L]/[T], and a has the dimensions of [L]/[T]^2. The equation under consideration is $v^n = 2ax$.

The dimensions of the right hand side are $\frac{[L]}{[T]^2}[L] = \frac{[L]^2}{[T]^2}$, while the dimensions of the left hand side are $\left(\frac{[L]}{[T]}\right)^n = \frac{[L]^n}{[T]^n}$. The right side will equal the left side only when $\boxed{n = 2}$.

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55. ***REASONING AND SOLUTION*** The x-components of the vectors are

$$+A_x = A\ \cos(180° - 20.0°) = (5.00\text{ m})\cos 160° = -4.70\text{ m}$$
$$+B_x = B\ \cos(60.0°) = (5.00\text{ m})\cos 60.0° = 2.50\text{ m}$$
$$+C_x = C\ \cos(270°) = 0.00\text{ m}$$

The x-component of the resultant vector is then

$$+R_x = -4.70\text{ m} + 2.50\text{ m} + 0.00\text{ m} = -2.20\text{ m}$$

Similarly, the y-components of the vectors are

$$+A_y = A\ \sin 160° = 1.71\text{ m}$$
$$+B_y = B\ \sin 60.0° = 4.33\text{ m}$$
$$+C_y = C\ \sin 270° = -4.00\text{ m}$$

The y-component of the resultant vector is then

$$R_y = 1.71\text{ m} + 4.33\text{ m} - 4.00\text{ m} = 2.04\text{ m}$$

Therefore,

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-2.20\text{ m})^2 + (2.04\text{ m})^2} = \boxed{3.00\text{ m}}$$

$$\theta = \tan^{-1}(R_y/R_x) = \boxed{42.8° \text{ above the negative } x \text{ axis}}$$

56. ***REASONING AND SOLUTION*** The frog's net displacement is three hops forward and one hop to the right. Since each hop covers a distance of 28 cm, the forward displacement, A, has magnitude 84 cm and the displacement to the right, **B**, has magnitude 28 cm. Therefore, the magnitude of the frog's displacement **R** is

$$R = \sqrt{A^2 + B^2} = \sqrt{(84\text{ cm})^2 + (28\text{ cm})^2} = \boxed{89\text{ cm}}$$

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57. **SSM** ***REASONING AND SOLUTION*** A single rope must supply the resultant of the two forces. Since the forces are perpendicular, the magnitude of the resultant can be found from the Pythagorean theorem.

a. Applying the Pythagorean theorem,

$$F = \sqrt{(475\text{ N})^2 + (315\text{ N})^2} = \boxed{5.70 \times 10^2\text{ N}}$$

b. The angle θ that the resultant makes with the westward direction is

$$\theta = \tan^{-1}\left(\frac{315\text{ N}}{475\text{ N}}\right) = 33.6°$$

Thus, the rope must make an angle of $\boxed{33.6° \text{ south of west}}$.

58. ***REASONING AND SOLUTION***

The resultant **A** + **B** has the same magnitude as **A** but points in the opposite direction. Therefore the resultant is -**A** so that

$$\mathbf{A} + \mathbf{B} = -\mathbf{A} \quad \text{or} \quad \mathbf{B} = -2\mathbf{A}.$$

a. Consequently, the x component of **B** is related to the x component of **A** as follows:

$$B_x = -2A_x = -2[-(636\text{ m})\cos 40.0°] = \boxed{974\text{ m}}$$

b. Similarly,

$$B_y = -2A_y = -2[(636\text{ m})\sin 40.0°] = \boxed{-818\text{ m}}$$

59. ***REASONING AND SOLUTION***

a. Using the data in problem 55, we find the following components for the vectors **A** and **B**:

$$+A_x = A\ \cos(180° - 20.0°) = (5.00\text{ m})\cos 160° = -4.70\text{ m}$$
$$+B_x = B\ \cos(60.0°) = (5.00\text{ m})\cos 60.0° = 2.50\text{ m}$$

$$+A_y = A\ \sin 160° = 1.71\text{ m}$$
$$+B_y = B\ \sin 60.0° = 4.33\text{ m}$$

The components of the sum are

$$R_x = (A + B)_x = -4.70\text{ m} + 2.50\text{ m} = -2.20\text{ m}$$
$$R_y = (A + B)_y = 1.71\text{ m} + 4.33\text{ m} = 6.04\text{ m}$$

The magnitude of the sum is then

$$R = \boxed{6.43\text{ m}}$$

The direction of the sum is given by the angle

$$\theta = \tan^{-1}(R_y/R_x) = \boxed{70.0° \text{ above the negative } x \text{ axis}}$$

b. The components of the difference are

$$R_x = -4.70\text{ m} - 2.50\text{ m} = -7.20\text{ m}$$
$$R_y = 1.71\text{ m} - 4.33\text{ m} = -2.62\text{ m}$$

The magnitude of the difference is

$$R_d = \boxed{7.66\text{ m}}$$

The direction is given by the angle

$$\theta = \tan^{-1}(2.62/7.20) = \boxed{20.0° \text{ below the negative } x \text{ axis}}$$

60. ***REASONING AND SOLUTION*** The following figure is a scale diagram of the forces drawn tail-to-head. The scale factor is shown in the figure. The head of $\mathbf{F}_3$ touches the tail of $\mathbf{F}_1$, because the resultant of the three forces is zero.

a. From the figure, $\mathbf{F}_3$ must have a magnitude of $\boxed{78\text{ N}}$ if the resultant force acting on the ball is zero.

b. Measurement with a protractor indicates that the angle $\boxed{\theta = 34°}$.

61. ***REASONING AND SOLUTION*** The figures below are scale diagrams of the forces drawn tail-to-head. The scale factor is shown in the figure.

a. From the figure on the left, we see that $\boxed{F_A - F_B = 142\ \text{N},\ \theta = 67°\ \text{south of east}}$.

b. Similarly, from the figure on the right, $\boxed{F_B - F_A = 142\ \text{N},\ \theta = 67°\ \text{north of west}}$.

62. ***REASONING*** The drawing at the right shows the location of each deer A, B, and C. From the problem statement it follows that

$$b = 62 \text{ m}$$

$$c = 95 \text{ m}$$

$$\gamma = 180° - 51° - 77° = 52°$$

Applying the law of cosines (given in Appendix E) to the geometry in the figure, we have

$$a^2 - 2ab\cos\gamma + (b^2 - c^2) = 0$$

which is an expression that is quadratic in a. It can be simplified to $Aa^2 + Ba + C = 0$, with

$$A = 1$$

$$B = -2b\cos\gamma = -2(62 \text{ m})\cos 52° = -76 \text{ m}$$

$$C = (b^2 - c^2) = (62 \text{ m})^2 - (95 \text{ m})^2 = -5181 \text{ m}^2$$

This quadratic equation can be solved for the desired quantity a.

SOLUTION Suppressing units, we obtain from the quadratic formula

$$a = \frac{-(-76) \pm \sqrt{(-76)^2 - 4(1)(-5181)}}{2(1)} = 1.2 \times 10^2 \text{ m} \quad \text{and} \quad -43 \text{ m}$$

Discarding the negative root, which has no physical significance, we conclude that the distance between deer A and C is $\boxed{1.2 \times 10^2 \text{ m}}$.

63. ***REASONING AND SOLUTION*** If **D** is the unknown vector, then **A** + **B** + **C** + **D** = 0 requires that $D_E = -(A_E + B_E + C_E)$ or

$$D_E = (113\text{ u})\cos 60.0° - (222\text{ u})\cos 35.0° - (177\text{ u})\cos 23.0° = \boxed{-288\text{ units}}$$

The minus sign indicates that D_E has a direction of due west.

Also, $D_N = -(A_N + B_N + C_N)$ or

$$D_N = (113\text{ u})\sin 60.0° + (222\text{ u})\sin 35.0° - (177\text{ u})\sin 23.0° = \boxed{156\text{ units}}$$

1. SSM ***REASONING AND SOLUTION***

a. The total distance traveled is found by adding the distances traveled during each segment of the trip.

$$6.9 \text{ km} + 1.8 \text{ km} + 3.7 \text{ km} = \boxed{12.4 \text{ km}}$$

b. All three segments of the trip lie along the east-west line. Taking east as the positive direction, the individual displacements can then be added to yield the resultant displacement.

$$6.9 \text{ km} + (-1.8 \text{ km}) + 3.7 \text{ km} = +8.8 \text{ km}$$

The displacement is positive, indicating that it points due east. Therefore,

$$\text{Displacement of the whale} = \boxed{8.8 \text{ km, due east}}$$

2. ***REASONING AND SOLUTION***

a. The distance traveled is equal to three-fourths of the circumference of the circular lake.

$$\frac{3}{4} \times \text{ circumference of lake } = \frac{3}{4}(2\pi r) = \frac{3}{2}\pi(1.50 \text{ km}) = \boxed{7.07 \text{ km}}$$

b. The displacement R of the couple is shown in the figure at the right. The displacement forms the hypotenuse of a right triangle with sides equal to the radius of the circle. From the Pythagorean theorem,

$$R = \sqrt{r^2 + r^2} = \sqrt{2r^2} = \sqrt{2(1.50 \text{ km})^2} = \boxed{2.12 \text{ km}}$$

The angle θ that R makes with due east is

$$\theta = \tan^{-1}\left(\frac{r}{r}\right) = \tan^{-1}(1) = \boxed{45.0° \text{ north of east}}$$

3. **SSM** ***REASONING AND SOLUTION*** The moving plane can be seen through the window of the stationary plane for the time that it takes the moving plane to travel its own length. To find the time, we write Equation 2.1 as

$$\text{Elapsed time} = \frac{\text{Distance}}{\text{Average speed}} = \frac{36\text{ m}}{45\text{ m/s}} = \boxed{0.80\text{ s}}$$

4. ***REASONING*** The average speed is the distance traveled divided by the elapsed time (Equation 2.1). Since the average speed and distance are known, we can use this relation to find the time.

SOLUTION The time it takes for the continents to drift apart by 1500 m is

$$\text{Elapsed time} = \frac{\text{Distance}}{\text{Average speed}} = \frac{1500\,\text{m}}{\left(3\,\dfrac{\text{cm}}{\text{y}}\right)\left(\dfrac{1\ \text{m}}{100\ \text{cm}}\right)} = \boxed{5\times10^{4}\ \text{y}}$$

5. ***REASONING AND SOLUTION*** In 12 minutes the sloth travels a distance of

$$x_s = v_s t = (0.037 \text{ m/s})(12 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 27 \text{ m}$$

while the tortoise travels a distance of

$$x_t = v_t t = (0.076 \text{ m/s})(12 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 55 \text{ m}$$

The tortoise goes farther than the sloth by an amount that equals 55 m – 27 m = $\boxed{28 \text{ m}}$

6. ***REASONING*** Equation 2.1 gives the average speed as the distance traveled divided by the elapsed time. The distance is the circumference of the circular path, $2\pi r$, where r is the radius of the earth

SOLUTION

a. The average speed (in m/s) of the person standing on the equator is

$$\text{Average speed} = \frac{\text{Distance}}{\text{Elapsed time}} = \frac{2\pi r}{t} = \frac{2\pi\left(6.38\times10^{6}\ \text{m}\right)}{\left(24\ \text{h}\right)\left(\dfrac{3600\ \text{s}}{1\ \text{h}}\right)} = \boxed{464\ \text{m/s}}$$

b. Since 1 mi/h = 0.447 m/s, the average speed (in mi/h) is

$$\text{Average speed} = \left(464\ \text{m/s}\right)\left(\frac{1\ \text{mi/h}}{0.447\ \text{m/s}}\right) = \boxed{1040\ \text{mi/h}}$$

7. ***REASONING*** In order for the bear to catch the tourist over the distance d, the bear must reach the car at the same time as the tourist. During the time t that it takes for the tourist to reach the car, the bear must travel a total distance of $d + 26$ m. From Equation 2.1,

$$v_{\text{tourist}} = \frac{d}{t} \quad (1) \qquad \text{and} \qquad v_{\text{bear}} = \frac{d + 26\text{ m}}{t} \quad (2)$$

Equations (1) and (2) can be solved simultaneously to find d.

SOLUTION Solving Equation (1) for t and substituting into Equation (2), we find

$$v_{\text{bear}} = \frac{d + 26\text{ m}}{d / v_{\text{tourist}}} = \frac{(d + 26\text{ m})v_{\text{tourist}}}{d}$$

$$v_{\text{bear}} = \left(1 + \frac{26\text{ m}}{d}\right)v_{\text{tourist}}$$

Solving for d yields:

$$d = \frac{26\text{ m}}{\dfrac{v_{\text{bear}}}{v_{\text{tourist}}} - 1} = \frac{26\text{ m}}{\dfrac{6.0\text{ m/s}}{4.0\text{ m/s}} - 1} = \boxed{52\text{ m}}$$

8. ***REASONING AND SOLUTION*** Her average speed is the total distance she falls divided by the total time of the fall:

$$v = \frac{x}{t} = \frac{625\ \text{m} + 356\ \text{m}}{15.0\ \text{s} + 142\ \text{s}} = \boxed{6.25\ \text{m/s}}$$

The direction of the velocity is $\boxed{\text{downward}}$.

9. **SSM** **WWW** ***REASONING*** Since the woman runs for a known distance at a known constant speed, we can find the time it takes for her to reach the water from Equation 2.1. We can then use Equation 2.1 to determine the total distance traveled by the dog in this time.

SOLUTION The time required for the woman to reach the water is

$$\text{Elapsed time} = \frac{d_{\text{woman}}}{v_{\text{woman}}} = \left(\frac{4.0\ \text{km}}{2.5\ \text{m/s}}\right)\left(\frac{1000\ \text{m}}{1.0\ \text{km}}\right) = 1600\ \text{s}$$

In 1600 s, the dog travels a total distance of

$$d_{\text{dog}} = v_{\text{dog}}t = (4.5\ \text{m/s})(1600\ \text{s}) = \boxed{7.2 \times 10^{3}\ \text{m}}$$

10. ***REASONING AND SOLUTION*** The average speed during the third part of the trip is

$$v_3 = \frac{x_3}{t_3}$$

where x_3 = 13 km and t_3 must be determined. The total time taken for the trip is

$$t = \frac{x}{v} = \frac{60.0 \text{ km}}{40.0 \text{ km/h}} = 1.50 \text{ h}$$

The time taken for the first part of the trip is

$$t_1 = \frac{x_1}{v_1} = \frac{15 \text{ km}}{25 \text{ km/h}} = 0.60 \text{ h}$$

Similarly, the time taken for the second part of the trip is

$$t_2 = \frac{x_2}{v_2} = \frac{32 \text{ km}}{62 \text{ km/h}} = 0.52 \text{ h}$$

The time for the third part is then

$$t_3 = 1.50 \text{ h} - 0.60 \text{ h} - 0.52 \text{ h} = 0.38 \text{ h}$$

and the velocity is

$$v_3 = \frac{13 \text{ km}}{0.38 \text{ h}} = \boxed{34 \text{ km/h}}$$

The direction is $\boxed{\text{north}}$.

11. ***REASONING AND SOLUTION*** The upper edge of the wall will disappear after the train has traveled the distance d in the figure below.

The distance d is equal to the length of the window plus the base of the 12° right triangle of height 0.90 m.

The base of the triangle is given by

$$b = \frac{0.90 \text{ m}}{\tan 12°} = 4.2 \text{ m}$$

Thus, $d = 2.0 \text{ m} + 4.2 \text{ m} = 6.2 \text{ m}$.

The time required for the train to travel 6.2 m is, from the definition of average speed,

$$t = \frac{x}{v} = \frac{6.2 \text{ m}}{3.0 \text{ m/s}} = \boxed{2.1 \text{ s}}$$

12. ***REASONING AND SOLUTION*** The magnitude of the car's acceleration can be found from Equation 2.4 ($v = v_0 + at$) as

$$a = \frac{v - v_0}{t} = \frac{26.8 \text{ m/s} - 0 \text{ m/s}}{3.275 \text{ s}} = \boxed{8.18 \text{ m/s}^2}$$

13. SSM WWW *REASONING AND SOLUTION*

a. The magnitude of the acceleration can be found from Equation 2.4 ($v = v_0 + at$) as

$$a = \frac{v - v_0}{t} = \frac{3.0 \text{ m/s} - 0 \text{ m/s}}{2.0 \text{ s}} = \boxed{1.5 \text{ m/s}^2}$$

b. Similarly the magnitude of the acceleration of the car is

$$a = \frac{v - v_0}{t} = \frac{41.0 \text{ m/s} - 38.0 \text{ m/s}}{2.0 \text{ s}} = \boxed{1.5 \text{ m/s}^2}$$

c. Assuming that the acceleration is constant, the displacement covered by the car can be found from Equation 2.9 ($v^2 = v_0^2 + 2ax$):

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(41.0 \text{ m/s})^2 - (38.0 \text{ m/s})^2}{2(1.5 \text{ m/s}^2)} = 79 \text{ m}$$

Similarly, the displacement traveled by the jogger is

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(3.0 \text{ m/s})^2 - (0 \text{ m/s})^2}{2(1.5 \text{ m/s}^2)} = 3.0 \text{ m}$$

Therefore, the car travels 79 m – 3.0 m = $\boxed{76 \text{ m}}$ further than the jogger.

14. ***REASONING*** The average acceleration is defined by Equation 2.4 as the change in velocity divided by the elapsed time. We can find the elapsed time from this relation because the acceleration and the change in velocity are given. Since the acceleration of the spacecraft is constant, it is equal to the average acceleration.

SOLUTION

a. The time Δt that it takes for the spacecraft to change its velocity by an amount $\Delta v = +2700$ m/s is

$$\Delta t = \frac{\Delta v}{a} = \frac{+2700 \text{ m/s}}{+9.0 \dfrac{\text{m/s}}{\text{day}}} = \boxed{3.0 \times 10^2 \text{ days}}$$

b. Since 24 hr = 1 day and 3600 s = 1 hr, the acceleration of the spacecraft (in m/s^2) is

$$a = \frac{\Delta v}{t} = \frac{+9.0 \text{ m/s}}{(1 \text{ day})\left(\dfrac{24 \text{ hr}}{1 \text{ day}}\right)\left(\dfrac{3600 \text{ s}}{1 \text{ hr}}\right)} = \boxed{+1.04 \times 10^{-4} \text{ m/s}^2}$$

15. **SSM** ***REASONING AND SOLUTION*** The initial velocity of the runner can be found by solving Equation 2.4 ($v = v_0 + at$) for v_0. Taking west as the positive direction, we have

$$v_0 = v - at = (+5.36 \text{ m/s}) - (+0.640 \text{ m/s}^2)(3.00 \text{ s}) = +3.44 \text{ m/s}$$

Therefore, the initial velocity of the runner is $\boxed{3.44 \text{ m/s, due west}}$.

16. ***REASONING AND SOLUTION*** Since $v = v_0 + at$, the acceleration of the boat is

$$a = \frac{v - v_0}{t} = \frac{3.2 \text{ m/s} - 0 \text{ m/s}}{2.0 \text{ s}} = 1.6 \text{ m/s}^2$$

After an additional 3.0 s, the boat has traveled for a total time of 5.0 s starting from rest. Its speed is, therefore,

$$v = v_0 + at = 0 \text{ m/s} + (1.6 \text{ m/s})(5.0 \text{ s}) = \boxed{8.0 \text{ m/s}}$$

Alternatively, one could apply equation 2.4 to the 3.0 s interval. At the beginning of the 3.0 s interval, the speed of the boat v_0 is 3.2 m/s. Therefore,

$$v = v_0 + at = 3.2 \text{ m/s} + (1.6 \text{ m/s})(3.0 \text{ s}) = \boxed{8.0 \text{ m/s}}$$

17. ***REASONING*** According to Equation 2.4, the average acceleration of the car for the first ten seconds after the engine cuts out is

$$\bar{a}_1 = \frac{v_{1f} - v_{10}}{\Delta t_1} \qquad (1)$$

and the average acceleration of the car during the next five seconds is

$$\bar{a}_2 = \frac{v_{2f} - v_{20}}{\Delta t_2} = \frac{v_{2f} - v_{1f}}{\Delta t_2} \qquad (2)$$

The velocity of the car at the end of the initial ten-second interval, v_{1f}, can be found by solving Equations (1) and (2) simultaneously.

SOLUTION Dividing Equation (1) by Equation (2), we have

$$\frac{\bar{a}_1}{\bar{a}_2} = \frac{(v_{1f} - v_{10})/\Delta t_1}{(v_{2f} - v_{1f})/\Delta t_2} = \frac{(v_{1f} - v_{10})\Delta t_2}{(v_{2f} - v_{1f})\Delta t_1}$$

Solving for v_{1f}, we obtain

$$v_{1f} = \frac{\bar{a}_1 \Delta t_1 v_{2f} + \bar{a}_2 \Delta t_2 v_{10}}{\bar{a}_1 \Delta t_1 + \bar{a}_2 \Delta t_2} = \frac{(\bar{a}_1/\bar{a}_2)\Delta t_1 v_{2f} + \Delta t_2 v_{10}}{(\bar{a}_1/\bar{a}_2)\Delta t_1 + \Delta t_2}$$

$$v_{1f} = \frac{1.67(10.0\text{ s})(+24.5\text{ m/s}) + (5.0\text{ s})(+31.0\text{ m/s})}{1.67(10.0\text{ s}) + 5.0\text{ s}} = \boxed{26.0\text{ m/s}}$$

18. ***REASONING AND SOLUTION*** Both motorcycles have the same velocity v at the end of the four second interval. Now

$$v = v_{0A} + a_A t$$

for motorcycle A and

$$v = v_{0B} + a_B t$$

for motorcycle B. Subtraction of these equations and rearrangement gives

$$v_{0A} - v_{0B} = (4.0\ \text{m/s}^2 - 2.0\ \text{m/s}^2)(4\ \text{s}) = \boxed{+8.0\ \text{m/s}}$$

The positive result indicates that motorcycle A was initially traveling faster.

19. **SSM** ***REASONING*** The average acceleration is defined by Equation 2.4 as the change in velocity divided by the elapsed time. We can find the elapsed time from this relation because the acceleration and the change in velocity are given.

SOLUTION

a. The time Δt that it takes for the VW Beetle to change its velocity by an amount $\Delta v = v - v_0$ is (and noting that 0.4470 m/s = 1 mi/h)

$$\Delta t = \frac{v - v_0}{a} = \frac{(60.0\ \text{mi/h})\left(\dfrac{0.4470\ \text{m/s}}{1\ \text{mi/h}}\right) - 0\ \text{m/s}}{2.35\ \text{m/s}^2} = \boxed{11.4\ \text{s}}$$

b. From Equation 2.4, the acceleration (in m/s^2) of the dragster is

$$a = \frac{v - v_0}{t - t_0} = \frac{(60.0\ \text{mi/h})\left(\dfrac{0.4470\ \text{m/s}}{1\ \text{mi/h}}\right) - 0\ \text{m/s}}{0.600\ \text{s} - 0\ \text{s}} = \boxed{44.7\ \text{m/s}^2}$$

20. ***REASONING AND SOLUTION*** Since $v = v_0 + at$, the acceleration is given by $a = (v - v_0)/t$. Since the direction of travel is in the negative direction throughout the problem, all velocities will be negative.

a.
$$a = \frac{(-29.0\text{ m/s}) - (-27.0\text{ m/s})}{5.0\text{ s}} = \boxed{-0.40\text{ m/s}^2}$$

Since the acceleration is negative, it is in the same direction as the velocity and the car is speeding up.

b.
$$a = \frac{(-23.0\text{ m/s}) - (-27.0\text{ m/s})}{5.0\text{ s}} = \boxed{+0.80\text{ m/s}^2}$$

Since the acceleration is positive, it is in the opposite direction to the velocity and the car is slowing down or decelerating.

21. ***REASONING AND SOLUTION*** The average acceleration of the basketball player is $\bar{a} = v/t$, so

$$x = \frac{1}{2}\bar{a}t^2 = \frac{1}{2}\left(\frac{6.0\ \text{m/s}}{1.5\ \text{s}}\right)(1.5\ \text{s})^2 = \boxed{4.5\ \text{m}}$$

22. ***REASONING AND SOLUTION***

a. From Equation 2.4, the definition of average acceleration, the magnitude of the average acceleration of the skier is

$$\bar{a} = \frac{v - v_0}{t - t_0} = \frac{8.0 \text{ m/s} - 0 \text{ m/s}}{5.0 \text{ s}} = \boxed{1.6 \text{ m/s}^2}$$

b. With x representing the displacement traveled along the slope, Equation 2.7 gives:

$$x = \tfrac{1}{2}(v_0 + v)t = \tfrac{1}{2}(8.0 \text{ m/s} + 0 \text{ m/s})(5.0 \text{ s}) = \boxed{2.0 \times 10^1 \text{ m}}$$

23. **SSM** ***REASONING AND SOLUTION*** The average acceleration of the plane can be found by solving Equation 2.9 $\left(v^2 = v_0^2 + 2ax\right)$ for a. Taking the direction of motion as positive, we have

$$a = \frac{v^2 - v_0^2}{2x} = \frac{(+6.1\text{ m/s})^2 - (+69\text{ m/s})^2}{2(+750\text{ m})} = \boxed{-3.1\text{ m/s}^2}$$

The minus sign indicates that the direction of the acceleration is opposite to the direction of motion, and the plane is slowing down.

24. ***REASONING*** We know the initial and final velocities of the blood, as well as its displacement. Therefore, Equation 2.9 $\left(v^2 = v_0^2 + 2ax\right)$ can be used to find the acceleration of the blood. The time it takes for the blood to reach it final velocity can be found by using Equation 2.7 $\left[t = \dfrac{x}{\frac{1}{2}\left(v_0 + v\right)}\right]$.

SOLUTION

a. The acceleration of the blood is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{\left(26\text{ cm/s}\right)^2 - \left(0\text{ cm/s}\right)^2}{2\left(2.0\text{ cm}\right)} = \boxed{1.7\times10^2\text{ cm/s}^2}$$

b. The time it takes for the blood, starting from 0 cm/s, to reach a final velocity of +26 cm/s is

$$t = \frac{x}{\frac{1}{2}\left(v_0 + v\right)} = \frac{2.0\text{ cm}}{\frac{1}{2}\left(0\text{ cm/s} + 26\text{ cm/s}\right)} = \boxed{0.15\text{ s}}$$

25. ***REASONING AND SOLUTION*** Solving Equation 2.7 for the time and setting the initial velocity to zero gives

$$t = \frac{2x}{v} = \frac{2(1.2\text{ m})}{14\text{ m/s}} = \boxed{0.17\text{ s}}$$

26. ***REASONING AND SOLUTION*** The acceleration can be found from Equation 2.9 if the speed v of the car as it crosses the second strip is known. The average speed $\bar{v}$ of the car must be 21 m/s, so

$$\bar{v} = \tfrac{1}{2}\left(v_o + v\right)$$

gives

$$v = 2(21 \text{ m/s}) - 33 \text{ m/s} = 9 \text{ m/s}$$

Also,

$$a = \frac{v^2 - v_0^2}{2x} = \frac{\left(9 \text{ m/s}\right)^2 - \left(33 \text{ m/s}\right)^2}{2\left(110 \text{ m}\right)} = \boxed{-4.6 \text{ m/s}^2}$$

27. **SSM** **WWW** ***REASONING*** Since the belt is moving with constant velocity, the displacement ($x_0 = 0$ m) covered by the belt in a time t_{belt} is giving by Equation 2.2 (with x_0 assumed to be zero) as

$$x = v_{\text{belt}} t_{\text{belt}} \qquad (1)$$

Since Clifford moves with constant acceleration, the displacement covered by Clifford in a time t_{Cliff} is, from Equation 2.8,

$$x = v_0 t_{\text{Cliff}} + \tfrac{1}{2} a t_{\text{Cliff}}^2 = \tfrac{1}{2} a t_{\text{Cliff}}^2 \qquad (2)$$

The speed v_{belt} with which the belt of the ramp is moving can be found by eliminating x between Equations (1) and (2).

SOLUTION Equating the right hand sides of Equations (1) and (2), and noting that $t_{\text{Cliff}} = \frac{1}{4} t_{\text{belt}}$, we have

$$v_{\text{belt}} t_{\text{belt}} = \tfrac{1}{2} a (\tfrac{1}{4} t_{\text{belt}})^2$$

$$v_{\text{belt}} = \tfrac{1}{32} a\, t_{\text{belt}} = \tfrac{1}{32}(0.37 \text{ m/s}^2)(64 \text{ s}) = \boxed{0.74 \text{ m/s}}$$

28. ***REASONING AND SOLUTION*** The speed of the car at the end of the first (402 m) phase can be obtained as follows:

$$v_1^2 = v_o^2 + 2a_1x_1$$

$$v_1 = \sqrt{2(17.0\ \text{m/s}^2)(402\ \text{m})}$$

The speed after the second phase (3.50×10^2 m) can be obtained in a similar fashion.

$$v_2^2 = v_{02}^2 + 2a_2x_2$$

$$v_2 = \sqrt{v_1^2 + 2(-\ 6.10\ \text{m/s}^2)(3.50 \text{ x } 10^2\ \text{m})}$$

$$v_2 = \boxed{96.9\ \text{m/s}}$$

29. ***REASONING*** For the car traveling on the dry and icy roads, the initial velocity, the final velocity, and the acceleration are known. Therefore, we may use Equation 2.9 $\left(v^2 = v_0^2 - 2ax\right)$ to find the distance x the car travels on each surface before coming to a halt.

SOLUTION The distance x_{dry} that the car travels on the dry road is

$$x_{\text{dry}} = \frac{v^2 - v_0^2}{2a} = \frac{(0\ \text{m/s})^2 - (32.0\ \text{m/s})^2}{2\left(-3.00\ \text{m/s}^2\right)} = 171\ \text{m}$$

The distance x_{icy} that the car travels on the icy road is

$$x_{\text{dry}} = \frac{v^2 - v_0^2}{2a} = \frac{(0\ \text{m/s})^2 - (32.0\ \text{m/s})^2}{2\left(-8.00\ \text{m/s}^2\right)} = 64\ \text{m}$$

The car travels further on the icy road compared to the dry road by an amount

$$x_{\text{icy}} - x_{\text{dry}} = 171\ \text{m} - 64\ \text{m} = \boxed{107\ \text{m}}$$

30. ***REASONING AND SOLUTION***

a. The velocity at the end of the first (7.00 s) period is

$$v_1 = v_0 + a_1 = (2.01\ \text{m/s}^2)(7.00\ \text{s})$$

At the end of the second period the velocity is

$$v_2 = v_1 + a_2t_2 = v_1 + (0.518\ \text{m/s}^2)(6.00\ \text{s})$$

And the velocity at the end of the third (8.00 s) period is

$$v_3 = v_2 + a_3t_3 = v_2 + (-1.49\ \text{m/s}^2)(8.00\ \text{s}) = \boxed{5.26\ \text{m/s}}$$

b. The displacement for the first time period is found from

$$x_1 = v_0t_1 + 1/2\ a_1t_1^2$$

$$x_1 = (0\ \text{m/s})(7.00\ \text{s}) + 1/2\ (2.01\ \text{m/s}^2)(7.00\ \text{s})^2 = 49.2\ \text{m}$$

Similarly, $x_2 = 93.7$ m and $x_3 = 89.7$ m, so the total displacement of the boat is

$$x = x_1 + x_2 + x_3 = \boxed{233\ \text{m}}$$

31. ***REASONING AND SOLUTION*** The distance covered by the cab driver during the two phases of the trip must satisfy the relation

$$x_1 + x_2 = 2.00 \text{ km} \tag{1}$$

where x_1 and x_2 are the displacements of the acceleration and deceleration phases of the trip, respectively. The quantities x_1 and x_2 can be determined from Equation 2.9 $\left(v^2 = v_0^2 + 2ax\right)$:

$$x_1 = \frac{v_1^2 - (0 \text{ m/s})^2}{2a_1} = \frac{v_1^2}{2a_1} \quad \text{and} \quad x_2 = \frac{(0 \text{ m/s})^2 - v_{02}^2}{2a_2} = -\frac{v_{02}^2}{2a_2}$$

with $v_{02} = v_1$ and $a_2 = -3a_1$. Thus,

$$\frac{x_1}{x_2} = \frac{v_1^2/(2a_1)}{-v_1^2/(-6a_1)} = 3$$

so that

$$x_1 = 3x_2 \tag{2}$$

Combining (1) and (2), we have,

$$3x_2 + x_2 = 2.00 \text{ km}$$

Therefore, $x_2 = 0.50 \text{ km}$, and from Equation (1), $x_1 = 1.50 \text{ km}$. Thus, the length of the acceleration phase of the trip is $x_1 = \boxed{1.50 \text{ km}}$, while the length of the deceleration phase is $x_2 = \boxed{0.50 \text{ km}}$.

32. ***REASONING AND SOLUTION*** From the definition of average velocity, the time required for the first car to reach the next exit 1.8 km (1.8×10^3 m) away is

$$t = \frac{x_1}{v_1} = \frac{1.8 \times 10^3 \text{ m}}{27 \text{ m/s}} = 67 \text{ s}$$

The second car has a constant acceleration, so its displacement during a time interval t is

$$x = v_0 t + \tfrac{1}{2} a t^2$$

Noting that the second car starts from rest (v_0 = 0 m/s), the acceleration required for the second car to travel 1.8 km in 67 s is

$$a = \frac{2x}{t^2} = \frac{2(1.8 \times 10^3 \text{ m})}{(67 \text{ s})^2} = \boxed{0.81 \text{ m/s}^2}$$

The car must speed up, so this acceleration must be $\boxed{\text{in the same direction as the velocity}}$.

33. ***REASONING*** The players collide when they have the same x coordinate relative to a common origin. For convenience, we will place the origin at the starting point of the first player. From Equation 2.8, the x coordinate of each player is given by

$$x_1 = v_{01}t_1 + \tfrac{1}{2}a_1t_1^2 = \tfrac{1}{2}a_1t_1^2 \qquad (1)$$

and

$$x_2 = d + v_{02}t_2 + \tfrac{1}{2}a_2t_2^2 = d + \tfrac{1}{2}a_2t_2^2 \qquad (2)$$

where d = +48 m is the initial position of the second player. When $x_1 = x_2$, the players collide at time $t = t_1 = t_2$.

SOLUTION

a. Equating Equations (1) and (2) when $t_1 = t_2 = t$, we have

$$\tfrac{1}{2}a_1t^2 = d + \tfrac{1}{2}a_2t^2$$

We note that a_1 = +0.50 m/s^2, while a_2 = –0.30 m/s^2, since the first player accelerates in the +x direction and the second player in the –x direction. Solving for t, we have

$$t = \sqrt{\frac{2d}{(a_1 - a_2)}} = \sqrt{\frac{2(48\text{ m})}{(0.50\text{ m/s}^2) - (-0.30\text{ m/s}^2)}} = \boxed{11\text{ s}}$$

b. From Equation (1),

$$x_1 = \tfrac{1}{2}a_1t_1^2 = \tfrac{1}{2}(0.50\text{ m/s}^2)(11\text{ s})^2 = \boxed{3.0 \times 10^1\text{ m}}$$

34. ***REASONING AND SOLUTION*** As the plane decelerates through the intersection, it covers a total distance equal to the length of the plane plus the width of the intersection, so

$$x = 59.7 \text{ m} + 25.0 \text{ m} = 84.7 \text{ m}$$

The speed of the plane as it enters the intersection can be found from Equation 2.9. Solving Equation 2.9 for v_0 gives

$$v_0 = \sqrt{v^2 - 2ax} = \sqrt{(45.0 \text{ m})^2 - 2(-5.70 \text{ m/s}^2)(84.7 \text{ m})} = 54.7 \text{ m/s}$$

The time required to traverse the intersection can then be found from Equation 2.4. Solving Equation 2.4 for t gives

$$t = \frac{v - v_0}{a} = \frac{45.0 \text{ m/s} - 54.7 \text{ m/s}}{-5.70 \text{ m/s}^2} = \boxed{1.7 \text{ s}}$$

35. **SSM** ***REASONING*** As the train passes through the crossing, its motion is described by Equations 2.4 ($v = v_0 + at$) and 2.7 $\left[x = \frac{1}{2}(v + v_0)t\right]$, which can be rearranged to give

$$v - v_0 = at \quad \text{and} \quad v + v_0 = \frac{2x}{t}$$

These can be solved simultaneously to obtain the speed v when the train reaches the end of the crossing. Once v is known, Equation 2.4 can be used to find the time required for the train to reach a speed of 32 m/s.

SOLUTION Adding the above equations and solving for v, we obtain

$$v = \frac{1}{2}\left(at + \frac{2x}{t}\right) = \frac{1}{2}\left[(1.6\ \text{m/s}^2)(2.4\ \text{s}) + \frac{2(20.0\ \text{m})}{2.4\ \text{s}}\right] = 1.0 \times 10^1\ \text{m/s}$$

The motion from the end of the crossing until the locomotive reaches a speed of 32 m/s requires a time

$$t = \frac{v - v_0}{a} = \frac{32\ \text{m/s} - 1.0 \times 10^1\ \text{m/s}}{1.6\ \text{m/s}^2} = \boxed{14\ \text{s}}$$

36. ***REASONING AND SOLUTION*** During the acceleration phase of the motion, the sprinter runs a distance

$$x_1 = (1/2)\, at_1^2 \qquad (1)$$

acquiring a final speed

$$v = at_1 \qquad (2)$$

During the rest of the race he runs a distance

$$x_2 = vt_2 \qquad (3)$$

Adding Equations (1) and (3) gives

$$x = (1/2)\, at_1^2 + vt_2$$

where $x = x_1 + x_2$.

Substituting from Equation (2) and noting that the total time is $t = t_1 + t_2$, we have

$$(1/2)\, at_1^2 - att_1 + x = 0$$

or (suppressing the units)

$$1.34t_1^2 - 32.2t_1 + 100.0 = 0$$

Solving for t_1 gives 20 s and 3.7 s. The first is obviously not a physically realistic solution, since it is larger than the total time for the race so, $t_1 = 3.7$ s. Using this value in Equation (1) gives

$$x_1 = (1/2)(2.68\ \text{m/s}^2)(3.7\ \text{s})^2 = \boxed{18\ \text{m}}$$

37. **SSM** ***REASONING AND SOLUTION*** When air resistance is neglected, free fall conditions are applicable. The final speed can be found from Equation 2.9;

$$v^2 = v_0^2 + 2ay$$

where v_0 is zero since the stunt man falls from rest. If the origin is chosen at the top of the hotel and the upward direction is positive, then the displacement is $y = -99.4$ m. Solving for v, we have

$$v = -\sqrt{2ay} = -\sqrt{2(-9.80\ \text{m/s}^2)(-99.4\ \text{m})} = -44.1\ \text{m/s}$$

The speed at impact is the magnitude of this result or $\boxed{44.1\ \text{m/s}}$.

38. ***REASONING AND SOLUTION***

a. Once the pebble has left the slingshot, it is subject only to the acceleration due to gravity. Since the downward direction is negative, the acceleration of the pebble is $\boxed{-9.80\ \text{m/s}^2}$.

b. The displacement y traveled by the pebble as a function of the time t can be found from Equation 2.8. Using Equation 2.8, we have

$$y = v_0 t + \tfrac{1}{2} a_y t^2 = (-9.0\ \text{m/s})(0.50\ \text{s}) + \tfrac{1}{2}\left[(-9.80\ \text{m/s}^2)(0.50\ \text{s})^2\right] = -5.7\ \text{m}$$

Thus, after 0.50 s, the pebble is $\boxed{5.7\ \text{m}}$ beneath the cliff-top.

39. ***REASONING AND SOLUTION*** In a time t the card will undergo a vertical displacement y given by

$$y = \frac{1}{2}\ at^2$$

where $a = -9.80\ m/s^2$. When $t = 60.0\ ms = 6.0 \times 10^{-2}\ s$, the displacement of the card is 0.018 m, and the distance is the magnitude of this value or $\boxed{d_1 = 0.018\ m}$.

Similarly, when t = 120 ms, $\boxed{d_2 = 0.071\ m}$, and when t = 180 ms, $\boxed{d_3 = 0.16\ m}$.

40. ***REASONING*** The initial velocity and the elapsed time are given in the problem. Since the rock returns to the same place from which it was thrown, its displacement is zero ($y = 0$ m). Using this information, we can employ Equation 2.8 $\left(y = v_0 t + \frac{1}{2} a t^2\right)$ to determine the acceleration a due to gravity.

SOLUTION Solving Equation 2.8 for the acceleration yields

$$a = \frac{2(y - v_0 t)}{t^2} = \frac{2\left[0\text{ m} - (+15\text{ m/s})(20.0\text{ s})\right]}{(20.0\text{ s})^2} = \boxed{-1.5\text{ m/s}^2}$$

41. **SSM** ***REASONING AND SOLUTION*** Since the balloon is released from rest, its initial velocity is zero. The time required to fall through a vertical displacement y can be found from Equation 2.8 $\left(y = v_0 t + \frac{1}{2}at^2\right)$ with $v_0 = 0$ m/s. Assuming upward to be the positive direction, we find

$$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-6.0\text{ m})}{-9.80\text{ m/s}^2}} = \boxed{1.1\text{ s}}$$

42. ***REASONING*** The initial velocity of the compass is +2.50 m/s. The initial position of the compass is 3.00 m and its final position is 0 m when it strikes the ground. The displacement of the compass is the final position minus the initial position, or $y = -3.00$ m. As the compass falls to the ground, its acceleration is the acceleration due to gravity, a = –9.80 m/s^2. Equation 2.8 $\left(y = v_0 t + \frac{1}{2} a t^2\right)$ can be used to find how much time elapses before the compass hits the ground.

SOLUTION Starting with Equation 2.8, we use the quadratic equation to find the elapsed time.

$$t = \frac{-v_0 \pm \sqrt{v_0^2 - 4\left(\frac{1}{2}a\right)(-y)}}{2\left(\frac{1}{2}a\right)} = \frac{-(2.50\text{ m/s}) \pm \sqrt{(2.50\text{ m/s})^2 - 4\left(-4.90\text{ m/s}^2\right)\left[-(-3.00\text{ m})\right]}}{2\left(-4.90\text{ m/s}^2\right)}$$

There are two solutions to this quadratic equation, t_1 = $\boxed{1.08\text{ s}}$ and t_2 = –0.568 s. The second solution, being a negative time, is discarded.

43. ***REASONING AND SOLUTION***

a.

$$v^2 = v_0^2 + 2ay$$

$$v = \pm\sqrt{(1.8\ \text{m/s})^2 + 2\left(-9.80\ \text{m/s}^2\right)(-3.0\ \text{m})} = \pm 7.9\ \text{m/s}$$

The minus is chosen, since the diver is now moving down. Hence, $\boxed{v = -7.9\ \text{m/s}}$.

b. The diver's velocity is zero at his highest point. The position of the diver relative to the board is

$$y = -\frac{v_0^2}{2a} = -\frac{(1.8\ \text{m/s})^2}{2\left(-9.80\ \text{m/s}^2\right)} = 0.17\ \text{m}$$

The position above the water is 3.0 m + 0.17 m = $\boxed{3.2\ \text{m}}$.

44. ***REASONING AND SOLUTION*** The figure at the right shows the paths taken by the pellets fired from gun **A** and gun **B**. The two paths differ by the extra distance covered by the pellet from gun **A** as it rises and falls back to the edge of the cliff. When it falls back to the edge of the cliff, the pellet from gun A will have the same speed as the pellet fired from gun B, as conceptual example 18 discusses. Therefore, the flight time of pellet **A** will be greater than that of **B** by the amount of time that it takes for pellet **A** to cover the extra distance.

The time required for pellet **A** to return to the cliff edge after being fired can be found from Equation 2.4: $v = v_0 + at$.

If "up" is taken as the positive direction then $v_0 = +30.0$ m/s and $v = -30.0$ m/s. Solving Equation 2.4 for t gives

$$t = \frac{v - v_0}{a} = \frac{(-30.0 \text{ m/s}) - (+30.0 \text{ m/s})}{-9.80 \text{ m/s}^2} = \boxed{6.12 \text{ s}}$$

Notice that this result is *independent* of the height of the cliff.

45. **SSM** ***REASONING AND SOLUTION*** Equation 2.8 can be used to determine the displacement that the ball covers as it falls halfway to the ground. Since the ball falls from rest, its initial velocity is zero. Taking down to be the negative direction, we have

$$y = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}at^2 = \frac{1}{2}(-9.80\ \text{m/s}^2)(1.2\ \text{s}) = -7.1\ \text{m}$$

In falling all the way to the ground, the ball has a displacement of $y = -14.2\ \text{m}$. Solving Equation 2.8 with this displacement then yields the time

$$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-14.2\ \text{m})}{-9.80\ \text{m/s}^2}} = \boxed{1.7\ \text{s}}$$

46. ***REASONING*** Equation 2.9 $\left(v^2 = v_0^2 + 2ay\right)$ can be used to find out how far above the cliff's edge the pellet would have gone if the gun had been fired straight upward, provided that we can determine the initial speed imparted to the pellet by the gun. This initial speed can be found by applying Equation 2.9 to the downward motion of the pellet described in the problem statement.

SOLUTION If we assume that upward is the positive direction, the initial speed of the pellet is, from Equation 2.9,

$$v_0 = \sqrt{v^2 - 2ay} = \sqrt{(-27\ \text{m/s})^2 - 2(-9.80\ \text{m/s}^2)(-15\ \text{m})} = 20.9\ \text{m/s}$$

Equation 2.9 can again be used to find the maximum height of the pellet if it were fired straight up. At its maximum height, $v = 0$ m/s, and Equation 2.9 gives

$$y = \frac{-v_0^2}{2a} = \frac{-(20.9\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = \boxed{22\ \text{m}}$$

47. ***REASONING*** The initial speed of the ball can be determined from Equation 2.9 $\left(v^2 = v_0^2 + 2ay\right)$. Once the initial speed of the ball is known, Equation 2.9 can be used a second time to determine the height above the launch point when the speed of the ball has decreased to one half of its initial value.

SOLUTION When the ball has reached its maximum height, its velocity is zero. If we take upward as the positive direction, we have from Equation 2.9

$$v_0 = \sqrt{v^2 - 2ay} = \sqrt{(0\text{ m/s})^2 - 2(-9.80\text{ m/s}^2)(16\text{ m})} = 18\text{ m/s}$$

When the speed of the ball has decreased to one half of its initial value, $v = \frac{1}{2}v_0$, and Equation 2.9 gives

$$y = \frac{v^2 - v_0^2}{2a} = \frac{(\frac{1}{2}v_0)^2 - v_0^2}{2a} = \frac{v_0^2}{2a}\left(\frac{1}{4} - 1\right) = \frac{(18\text{ m/s})^2}{2(-9.80\text{ m/s}^2)}\left(\frac{1}{4} - 1\right) = \boxed{12\text{ m}}$$

48. ***REASONING AND SOLUTION*** During the time t that it takes the stone to fall, the raft travels a distance of 6.00 m – 2.00 m = 4.00 m, so its speed is

$$v = \frac{4.00 \text{ m}}{t}$$

The stone falls downward for a distance of 90.0 m, so its displacement is y = –90.0 m, where the downward direction is taken to be the negative direction. Equation 2.8 $\left(y = v_0 t + \frac{1}{2} a t^2\right)$ can be used to find the time of fall. Setting v_0 = 0 m/s, and solving Equation 2.8 for the time t, we have

$$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-90.0 \text{ m})}{-9.80 \text{ m/s}^2}} = 4.29 \text{ s}$$

The speed of the raft is

$$v = \frac{x}{t} = \frac{4.00 \text{ m}}{4.29 \text{ s}} = \boxed{0.932 \text{ m/s}}$$

49. ***REASONING AND SOLUTION*** The time required for the first arrow to reach its maximum height can be determined from Equation 2.4 ($v = v_0 + at$). Taking upward as the positive direction, we have

$$t = \frac{v - v_0}{a} = \frac{0 \text{ m/s} - 25.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = 2.55 \text{ s}$$

Since both arrows reach their maximum height at the same time, the second arrow reaches its maximum height

$$2.55 \text{ s} - 1.20 \text{ s} = 1.35 \text{ s}$$

after being fired. The initial speed of the second arrow can then be found from Equation 2.4:

$$v_0 = v - at = 0 \text{ m/s} - (-9.80 \text{ m/s}^2)(1.35 \text{ s}) = \boxed{13.2 \text{ m/s}}$$

50. ***REASONING AND SOLUTION*** The initial speed of either student can be found from Equation (2.9): $v^2 = v_0^2 + 2ay$. Since the speed of either student is zero when the student is at her highest point, v = 0 m/s and

$$v_0 = \sqrt{-2ay}$$

Since Anne bounces twice as high as Joan, $y_A = 2y_J$, and $v_{0A} = \sqrt{2}\ v_{0J}$.

The time it takes for either student to reach the highest point in her trajectory can be found from Equation 2.4: $v = at + v_0$. Solving for t with v = 0 m/s gives

$$t = \frac{-v_o}{a}$$

The total time in the air is twice this value. Therefore,

$$\frac{t_A}{t_J} = \frac{-2v_{0A}/a}{-2v_{0J}/a} = \frac{v_{0A}}{v_{0J}} = \frac{\sqrt{2}v_{0J}}{v_{0J}} = \boxed{\sqrt{2}}$$

51. $\boxed{\text{SSM}}$ ***REASONING*** Once the man sees the block, the man must get out of the way in the time it takes for the block to fall through an additional 12.0 m. The velocity of the block at the instant that the man looks up can be determined from Equation 2.9. Once the velocity is known at that instant, Equation 2.8 can be used to find the time required for the block to fall through the additional distance.

SOLUTION When the man first notices the block, it is 14.0 m above the ground and its displacement from the starting point is $y = 14.0\text{ m} - 53.0\text{ m}$. Its velocity is given by Equation 2.9 $\left(v^2 = v_0^2 + 2ay\right)$. Since the block is moving down, its velocity has a negative value,

$$v = -\sqrt{v_0 + 2ay} = -\sqrt{(0\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(14.0\text{ m} - 53.0\text{ m})} = -27.7\text{ m/s}$$

The block then falls the additional 12.0 m to the level of the man's head in a time t which satisfies Equation 2.8:

$$y = v_0 t + \tfrac{1}{2}at^2$$

where $y = -12.0$ m and $v_0 = -27.7$ m/s. Thus, t is the solution to the quadratic equation

$$4.90t^2 + 27.7t - 12.0 = 0$$

where the units have been suppressed for brevity. From the quadratic formula, we obtain

$$t = \frac{-27.7 \pm \sqrt{(27.7)^2 - 4(4.90)(-12.0)}}{2(4.90)} = 0.40\text{ s} \quad \text{or} \quad -6.1\text{ s}$$

The negative solution can be rejected as nonphysical, and the time it takes for the block to reach the level of the man is $\boxed{0.40\text{ s}}$.

52. ***REASONING AND SOLUTION***

a. We can use Equation 2.9 to obtain the speed acquired as she falls through the distance H. Taking downward as the positive direction, we find

$$v^2 = v_0^2 + 2ay = (0\text{ m/s})^2 + 2aH \qquad \text{or} \qquad v = \sqrt{2aH}$$

To acquire a speed of twice this value or $2\sqrt{2aH}$, she must fall an additional distance H'. According to Equation 2.9 $\left(v^2 = v_0^2 + 2ay\right)$, we have

$$\left(2\sqrt{2aH}\right)^2 = \left(\sqrt{2aH}\right)^2 + 2aH' \qquad \text{or} \qquad 4(2aH) = 2aH + 2aH'$$

The acceleration due to gravity a can be eliminated algebraically from this result, giving

$$4H = H + H' \qquad \text{or} \qquad \boxed{H' = 3H}$$

b. In the previous calculation the acceleration due to gravity was eliminated algebraically. Thus, a value other than 9.80 m/s^2 would $\boxed{\text{not have affected the answer to part (a)}}$.

53. SSM WWW ***REASONING AND SOLUTION*** The stone requires a time, t_1, to reach the bottom of the hole, a distance y below the ground. Assuming downward to be the positive direction, the variables are related by Equation 2.8 with $v_0 = 0$ m/s:

$$y = \frac{1}{2}at_1^2 \qquad (1)$$

The sound travels the distance y from the bottom to the top of the hole in a time t_2. Since the sound does not experience any acceleration, the variables y and t_2 are related by Equation 2.8 with $a = 0$ m/s^2 and v_{sound} denoting the speed of sound:

$$y = v_{\text{sound}}\, t_2 \qquad (2)$$

Equating the right hand sides of Equations (1) and (2) and using the fact that the total elapsed time is $t = t_1 + t_2$, we have

$$\tfrac{1}{2}at_1^2 = v_{\text{sound}}\, t_2 \qquad \text{or} \qquad \tfrac{1}{2}at_1^2 = v_{\text{sound}}(t - t_1)$$

Rearranging gives

$$\frac{1}{2}at_1^2 + v_{\text{sound}}t_1 - v_{\text{sound}}t = 0$$

Substituting values and suppressing units for brevity, we obtain the following quadratic equation for t_1:

$$4.90t_1^2 + 343t_1 - 514 = 0$$

From the quadratic formula, we obtain

$$t_1 = \frac{-343 \pm \sqrt{(343)^2 - 4(4.90)(-514)}}{2(4.90)} = 1.47 \text{ s} \quad \text{or} \quad -71.5 \text{ s}$$

The negative time corresponds to a nonphysical result and is rejected. The depth of the hole is then found using Equation 2.8 with the value of t_1 obtained above:

$$y = v_0t_1 + \tfrac{1}{2}at_1^2 = (0 \text{ m/s})(1.47 \text{ s}) + \tfrac{1}{2}(9.80 \text{ m/s}^2)(1.47 \text{ s})^2 = \boxed{10.6 \text{ m}}$$

54. ***REASONING AND SOLUTION*** The position of the ball relative to the top of the building at any time t is

$$y = v_0 t + \tfrac{1}{2} a t^2 = v_0 t + \tfrac{1}{2}\left(-9.80 \text{ m/s}^2\right) t^2$$

For the particular time that the ball arrives at the bottom of the building, y = –25.0 m. The equation becomes

$$4.90t^2 - 12.0t - 25.0 = 0$$

The quadratic equation yields solutions t = 3.79 s and –1.34 s. The negative solution is rejected as being non physical. During time t the person has run a distance $x = vt$ so

$$v = \frac{x}{t} = \frac{31.0 \text{ m}}{3.79 \text{ s}} = \boxed{8.18 \text{ m/s}}$$

55. ***REASONING AND SOLUTION*** We measure the positions of the balloon and the pellet relative to the ground and assume up to be positive. The balloon has no acceleration, since it travels at a constant velocity v_B, so its displacement in time t is $v_B t$. Its position above the ground, therefore, is

$$y_B = H_0 + v_B t$$

where H_0 = 12 m. The pellet moves under the influence of gravity (a = –9.80 m/s^2), so its position above the ground is given by Equation 2.8 as

$$y_P = v_0 t + \frac{1}{2}at^2$$

But $y_P = y_B$ at time t, so that

$$v_0 t + \frac{1}{2}at^2 = H_0 + v_B t$$

Rearranging this result and suppressing the units gives

$$\tfrac{1}{2}at^2 + (v_0 - v_B)t - H_0 = \tfrac{1}{2}(-9.80)t^2 + (30.0 - 7.0)t - 12.0 = 0$$

$$4.90t^2 - 23.0t + 12.0 = 0$$

$$t = \frac{23.0 \pm \sqrt{23.0^2 - 4(4.90)(12.0)}}{2(4.90)} = 4.09 \text{ s} \quad \text{or} \quad 0.602 \text{ s}$$

Substituting each of these values in the expression for y_B gives

$$y_B = 12.0 \text{ m} + (7.0 \text{ m/s})(4.09 \text{ s}) = \boxed{41 \text{ m}}$$

$$y_B = 12.0 \text{ m} + (7.0 \text{ m/s})(0.602 \text{ s}) = \boxed{16 \text{ m}}$$

56. ***REASONING AND SOLUTION*** Assuming that down is positive, the distance that the tile falls in going from the roof top to the top of the window is found to be

$$y = \frac{v^2}{2a}$$

where v is the velocity of the tile at the top of the window. The tile travels an additional distance h in traversing the window in time t.

$$h = v_0 t + (1/2)\, at^2$$

Solving for v_0 yields,

$$v_0 = \frac{h}{t} - (1/2)at = \frac{1.6\text{ m}}{0.20\text{ s}} - (1/2)(9.80\text{ m/s}^2)(0.20\text{ s}) = 7.0\text{ m/s}.$$

Then,

$$y = \frac{(7.0\text{ m/s})^2}{2\left(9.80\text{ m/s}^2\right)} = \boxed{2.5\text{ m}}$$

57. $\boxed{\text{SSM}}$ ***REASONING*** In order to construct the graph, the time for each segment of the trip must be determined.

SOLUTION From the definition of average velocity

$$\Delta t = \frac{\Delta x}{\bar{v}}$$

Therefore,

$$\Delta t_1 = \left(\frac{10.0 \text{ km}}{15.0 \text{ km/h}}\right)\left(\frac{60 \text{ min}}{1.0 \text{ h}}\right) = 40 \text{ min}$$

$$\Delta t_2 = \left(\frac{15.0 \text{ km}}{10.0 \text{ km/h}}\right)\left(\frac{60 \text{ min}}{1.0 \text{ h}}\right) = 90 \text{ min}$$

Thus, the second time interval ends $40 \text{ min} + 90 \text{ min} = 130 \text{ min}$ after the trip begins.

$$\Delta t_3 = \left(\frac{15.0 \text{ km}}{5.0 \text{ km/h}}\right)\left(\frac{60 \text{ min}}{1.0 \text{ h}}\right) = 180 \text{ min}$$

and the third time interval ends $130 \text{ min} + 180 \text{ min} = 310 \text{ min}$ after the trip begins.

Note that the slope of each segment of the graph gives the average velocity during that interval.

58. ***REASONING AND SOLUTION*** The average acceleration for each segment is the slope of that segment.

$$a_A = \frac{40 \text{ m/s} - 0 \text{ m/s}}{21 \text{ s} - 0 \text{ s}} = \boxed{1.9 \text{ m/s}^2}$$

$$a_B = \frac{40 \text{ m/s} - 40 \text{ m/s}}{48 \text{ s} - 21 \text{ s}} = \boxed{0 \text{ m/s}^2}$$

$$a_C = \frac{80 \text{ m/s} - 40 \text{ m/s}}{60 \text{ s} - 48 \text{ s}} = \boxed{3.3 \text{ m/s}^2}$$

59. ***REASONING*** The average velocity for each segment is the slope of the line for that segment.

SOLUTION Taking the direction of motion as positive, we have from the graph for segments A, B, and C,

$$v_A = \frac{10.0\text{ km} - 40.0\text{ km}}{1.5\text{ h} - 0.0\text{ h}} = \boxed{-2.0 \times 10^1\ \text{km/h}}$$

$$v_B = \frac{20.0\text{ km} - 10.0\text{ km}}{2.5\text{ h} - 1.5\text{ h}} = \boxed{1.0 \times 10^1\ \text{km/h}}$$

$$v_C = \frac{40.0\text{ km} - 20.0\text{ km}}{3.0\text{ h} - 2.5\text{ h}} = \boxed{40\ \text{km/h}}$$

60. ***REASONING AND SOLUTION***

a. The sign of the average velocity during a segment corresponds to the sign of the *slope* of the segment. The slope, and hence the average velocity, is *positive* for segments A and D, *negative* for segment C, and ZERO for segment B.

b.

$$v_A = \frac{1.00\text{ km} - 0\text{ km}}{0.20\text{ s} - 0\text{ s}} = \boxed{5.0\text{ km/h}}$$

$$v_B = \frac{1.00\text{ km} - 1.00\text{ km}}{0.40\text{ s} - 0.20\text{ s}} = \boxed{0.0\text{ km/h}}$$

$$v_C = \frac{0.25\text{ km} - 1.00\text{ km}}{0.60\text{ s} - 0.40\text{ s}} = \boxed{-3.8\text{ km/h}}$$

$$v_D = \frac{1.25\text{ km} - 0.25\text{ km}}{1.00\text{ s} - 0.60\text{ s}} = \boxed{2.5\text{ km/h}}$$

61. **SSM** ***REASONING*** The average acceleration is given by Equation 2.4: $\bar{a} = (v_C - v_A)/\Delta t$. The velocities v_A and v_C can be found from the slopes of the position-time graph for segments A and C.

SOLUTION The average velocities in the segments A and C are

$$v_A = \frac{24\text{ km} - 0\text{ km}}{1.0\text{ h} - 0\text{ h}} = 24\text{ km/h}$$

$$v_C = \frac{27\text{ km} - 33\text{ km}}{3.5\text{ h} - 2.2\text{ h}} = -5\text{ km/h}$$

From the definition of average acceleration,

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_C - v_A}{\Delta t} = \frac{(-5\text{ km/h}) - (24\text{ km/h})}{3.5\text{ h} - 0\text{ h}} = \boxed{-8.3\text{ km/h}^2}$$

62. ***REASONING AND SOLUTION*** The runner is at the position $x = 0$ m when time $t = 0$ s; the finish line is 100 m away. During each ten-second segment, the runner has a constant velocity and runs half the remaining distance to the finish line. Therefore, from $t = 0$ s to $t = 10.0$ s, the position of the runner changes from $x = 0$ m to $x = 50.0$ m. From $t = 10.0$ s to $t = 20.0$ s, the position of the runner changes from $x = 50.0$ m to $x = 50.0 \text{ m} + 25.0 \text{ m} = 75.0$ m. From $t = 20.0$ s to $t = 30.0$ s, the position of the runner changes from $x = 75.0$ m to $x = 75.0 \text{ m} + 12.5 \text{ m} = 87.5$ m. Finally, from $t = 30.0$ s to $t = 40.0$ s, the position of the runner changes from $x = 87.5$ m to $x = 87.5 \text{ m} + 6.25 \text{ m} = 93.8$ m. This data can be used to construct the position-time graph. Since the runner has a constant velocity during each ten-second segment, we can find the velocity during each segment from the slope of the position-time graph for that segment.

a. The following figure shows the position-time graph for the first forty seconds.

b. The slope of each segment of the position-time graph is calculated as follows:

$$[0.00 \text{ s to } 10.0 \text{ s}] \qquad v = \frac{\Delta x}{\Delta t} = \frac{50.0 \text{ m} - 0.00 \text{ m}}{10.0 \text{ s} - 0 \text{ s}} = 5.00 \text{ m/s}$$

$$[10.0 \text{ s to } 20.0 \text{ s}] \qquad v = \frac{\Delta x}{\Delta t} = \frac{75.0 \text{ m} - 50.0 \text{ m}}{20.0 \text{ s} - 10.0 \text{ s}} = 2.50 \text{ m/s}$$

$$[20.0 \text{ s to } 30.0 \text{ s}] \qquad v = \frac{\Delta x}{\Delta t} = \frac{87.5 \text{ m} - 75.0 \text{ m}}{30.0 \text{ s} - 20.0 \text{ s}} = 1.25 \text{ m/s}$$

$$[30.0 \text{ s to } 40.0 \text{ s}] \qquad v = \frac{\Delta x}{\Delta t} = \frac{93.8 \text{ m} - 87.5 \text{ m}}{40.0 \text{ s} - 30.0 \text{ s}} = 0.625 \text{ m/s}$$

Therefore, the velocity-time graph is:

5.00
4.00
3.00
2.00
1.00
0.00
Velocity v (m/s)
Time t (s)
0.00
10.0
20.0
30.0
40.0

63. ***REASONING*** The two runners start one hundred meters apart and run toward each other. As stated in the problem, each runs ten meters during the first second and, during each second thereafter, each runner runs ninety percent of the distance he ran in the previous second. While the velocity of each runner changes from second to second, it remains constant during any one second. The following table shows the distance covered during each second for one of the runners, and the position at the end of each second (assuming that he begins at the origin) for the first eight seconds.

Time t (s)	Distance covered (m)	Position x (m)
0.00		0.00
1.00	10.00	10.00
2.00	9.00	19.00
3.00	8.10	27.10
4.00	7.29	34.39
5.00	6.56	40.95
6.00	5.90	46.85
7.00	5.31	52.16
8.00	4.78	56.94

The following graph is the position-time graph constructed from the data in the table above.

a. Since the two runners are running toward each other in exactly the same way, they will meet halfway between their respective starting points. That is, they will meet at $x = 50.0$ m. According to the graph, therefore, this position corresponds to a time of $\boxed{6.6 \text{ s}}$.

b. Since the runners collide during the seventh second, the speed at the instant of collision can be found by taking the slope of the position-time graph for the seventh second. The speed of either runner in the interval from $t = 6.00$ s to $t = 7.00$ s is

$$v = \frac{\Delta x}{\Delta t} = \frac{52.16 \text{ m} - 46.85 \text{ m}}{7.00 \text{ s} - 6.00 \text{ s}} = 5.3 \text{ m/s}$$

Therefore, at the moment of collision, the speed of either runner is $\boxed{5.3 \text{ m/s}}$.

64. ***REASONING AND SOLUTION*** From the definition of average acceleration, the magnitude of the average acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{112 \text{ m/s}}{20.0 \text{ s}} = \boxed{5.60 \text{ m/s}^2}$$

The acceleration is in the direction of the *change in velocity*. Since the velocity of the jetliner is increasing due east, the direction of the acceleration is $\boxed{\text{due east}}$.

65. **SSM** ***REASONING AND SOLUTION*** The speed of the penny as it hits the ground can be determined from Equation 2.9: $v^2 = v_0^2 + 2ay$. Since the penny is dropped from rest, $v_0 = 0$ m/s. Solving for v, with downward taken as the positive direction, we have

$$v = \sqrt{2(9.80 \text{ m/s}^2)(427 \text{ m})} = \boxed{91.5 \text{ m/s}}$$

66. ***REASONING AND SOLUTION*** The slower runner has an average speed of

$$v = \frac{x}{t} = \frac{1609 \text{ m}}{232.72 \text{ s}} = 6.914 \text{ m/s}$$

and has only covered a distance of

$$x = (6.914 \text{ m/s})(232.46 \text{ s}) = 1607 \text{ m}$$

when the faster runner has finished the race. Hence, the runners are separated at the end of the race by $\boxed{2 \text{ m}}$.

67. **SSM** ***REASONING*** Since the velocity and acceleration of the motorcycle point in the same direction, their numerical values will have the same algebraic sign. For convenience, we will choose them to be positive. The velocity, acceleration, and the time are related by Equation 2.4: $v = v_0 + at$.

SOLUTION

a. Solving Equation 2.4 for t we have

$$t = \frac{v - v_0}{a} = \frac{(+31\ \text{m/s}) - (+21\ \text{m/s})}{+2.5\ \text{m/s}^2} = \boxed{4.0\ \text{s}}$$

b. Similarly,

$$t = \frac{v - v_0}{a} = \frac{(+61\ \text{m/s}) - (+51\ \text{m/s})}{+2.5\ \text{m/s}^2} = \boxed{4.0\ \text{s}}$$

68. ***REASONING*** The minimum time that a player must wait before touching the basketball is the time required for the ball to reach its maximum height. The initial and final velocities are known, as well as the acceleration due to gravity, so Equation 2.4 ($v = v_0 + at$) can be used to find the time.

SOLUTION Solving Equation 2.4 for the time yields

$$t = \frac{v - v_0}{a} = \frac{0\ \text{m/s} - 4.6\ \text{m/s}}{-9.8\ \text{m/s}^2} = \boxed{0.47\ \text{s}}$$

69. ***REASONING*** The distance traveled by the Space Shuttle is equal to its speed multiplied by the time. The number of football fields is equal to this distance divided by the length L of one football field.

SOLUTION The number of football fields is

$$\text{Number} = \frac{x}{L} = \frac{vt}{L} = \frac{\left(7.6\times10^{3}\ \text{m/s}\right)\left(110\times10^{-3}\,\text{s}\right)}{91.4\ \text{m}} = \boxed{9.1}$$

70. ***REASONING AND SOLUTION*** The velocity of the automobile for each stage is given by Equation 2.4: $v = v_0 + at$. Therefore,

$$v_1 = v_0 + a_1 t = 0 \text{ m/s} + a_1 t \quad \text{and} \quad v_2 = v_1 + a_2 t$$

Since the magnitude of the car's velocity at the end of stage 2 is 2.5 times greater than it is at the end of stage 1, $v_2 = 2.5v_1$. Thus, rearranging the result for v_2, we find

$$a_2 = \frac{v_2 - v_1}{t} = \frac{2.5v_1 - v_1}{t} = \frac{1.5v_1}{t} = \frac{1.5(a_1 t)}{t} = 1.5a_1 = 1.5(3.0 \text{ m/s}^2) = \boxed{4.5 \text{ m/s}^2}$$

71. ***REASONING AND SOLUTION*** Due to the symmetry of free fall, the arrow has the same speed at the 30.0 m level coming down as it had going up at that level. On going up to the 30.0 m level the speed is found from

$$v^2 = v_o^2 + 2ay$$

$$v = \sqrt{(50.0 \text{ m/s})^2 + 2(-9.80 \text{ m/s}^2)(30.0 \text{ m})}$$

$$v = 43.7 \text{ m/s}$$

Hence the velocity is $\boxed{43.7 \text{ m/s, downward}}$.

72. ***REASONING*** Let the total distance between the first and third sign be equal to $2d$. Then, the time t_A is given by

$$t_A = \frac{d}{v_{55}} + \frac{d}{v_{35}} = \frac{d(v_{35} + v_{55})}{v_{55}v_{35}} \qquad (1)$$

Equation 2.7 $\left[x = \frac{1}{2}(v_0 + v)t\right]$ can be written as $t = 2x/(v_0 + v)$, so that

$$t_B = \frac{2d}{v_{55} + v_{35}} + \frac{2d}{v_{35} + v_{25}} = \frac{2d\left[(v_{35} + v_{25}) + (v_{55} + v_{35})\right]}{(v_{55} + v_{35})(v_{35} + v_{25})} \qquad (2)$$

SOLUTION Dividing Equation (2) by Equation (1) and suppressing units, we obtain

$$\frac{t_B}{t_A} = \frac{2v_{55}v_{35}\left[(v_{35} + v_{25}) + (v_{55} + v_{35})\right]}{(v_{55} + v_{35})^2(v_{35} + v_{25})} = \frac{2(55)(35)[(35+25)+(55+35)]}{(55+35)^2(35+25)} = \boxed{1.2}$$

73. **SSM** ***REASONING AND SOLUTION***

a. The total displacement traveled by the bicyclist for the entire trip is equal to the sum of the displacements traveled during each part of the trip. The displacement traveled during each part of the trip is given by Equation 2.2: $\Delta x = \bar{v}\Delta t$. Therefore,

$$\Delta x_1 = (7.2 \text{ m/s})(22 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 9500 \text{ m}$$

$$\Delta x_2 = (5.1 \text{ m/s})(36 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 11\,000 \text{ m}$$

$$\Delta x_3 = (13 \text{ m/s})(8.0 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = 6200 \text{ m}$$

The total displacement traveled by the bicyclist during the entire trip is then

$$\Delta x = 9500 \text{ m} + 11\,000 \text{ m} + 6200 \text{ m} = \boxed{2.67 \times 10^4 \text{ m}}$$

b. The average velocity can be found from Equation 2.2.

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{2.67 \times 10^4 \text{ m}}{(22 \text{ min} + 36 \text{ min} + 8.0 \text{ min})}\left(\frac{1 \text{min}}{60 \text{ s}}\right) = \boxed{6.74 \text{ m/s, due north}}$$

74. ***REASONING AND SOLUTION*** Let west be the positive direction. The average velocity of the backpacker is

$$v = \frac{x_w + x_e}{t_w + t_e} \quad \text{where} \quad t_w = \frac{x_w}{v_w} \quad \text{and} \quad t_e = \frac{x_e}{v_e}$$

Combining these equations and solving for x_e (suppressing the units) gives

$$x_e = \frac{-(1 - v/v_w)x_w}{(1 - v/v_e)} = \frac{-[1-(1.34\text{ m/s})/(2.68\text{ m/s})](6.44\text{ km})}{1-(1.34\text{ m/s})/(0.447\text{ m/s})} = -0.81\text{ km}$$

The distance traveled is the magnitude of x_e, or $\boxed{0.81\text{ km}}$.

75. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The stone will reach the water (and hence the log) after falling for a time t, where t can be determined from Equation 2.8: $y = v_0 t + \frac{1}{2}at^2$. Since the stone is dropped from rest, $v_0 = 0$ m/s. Assuming that downward is positive and solving for t, we have

$$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(75\text{ m})}{9.80\text{ m/s}^2}} = 3.9\text{ s}$$

During that time, the displacement of the log can be found from Equation 2.8. Since the log moves with constant velocity, $a = 0\text{ m/s}^2$, and v_0 is equal to the velocity of the log.

$$x = v_0 t = (5.0\text{ m/s})(3.9\text{ s}) = 2.0 \times 10^1\text{ m}$$

Therefore, the horizontal distance between the log and the bridge when the stone is released is $\boxed{2.0 \times 10^1\text{ m}}$.

76. ***REASONING*** When the second-place cyclist catches the leader, the displacement x_{2nd} of the second-place cyclist is 10.0 m greater than the displacement x_{leader} of the leader, so $x_{2nd} = x_{leader} + 10.0$ m. The initial velocity and acceleration of the second-place cyclist are known ($v_0 = +9.50$ m/s, $a = +1.20$ m/s^2), as well as those of the leader ($v_0 = +11.1$ m/s, $a = 0$ m/s^2). Equation 2.8 may be used to provide a relationship between these variables and the displacement x.

SOLUTION Substituting Equation 2.8 into each side of the relation $x_{2nd} = x_{leader} + 10.0$ m, we have that

$$\underbrace{v_0 t + \tfrac{1}{2} a t^2}_{x_{2nd}} = \underbrace{v_0 t + \tfrac{1}{2} a t^2}_{x_{leader}} + 10.0 \text{ m}$$

$$(9.50 \text{ m/s})t + \tfrac{1}{2}(1.20 \text{ m/s}^2)t^2 = (11.1 \text{ m/s})t + \tfrac{1}{2}(0 \text{ m/s}^2)t^2 + 10.0 \text{ m}$$

Rearranging the terms of this equation so it is in quadratic form, we have

$$\tfrac{1}{2}(1.20 \text{ m/s}^2)t^2 - (1.60 \text{ m/s})t - 10.0 \text{ m} = 0$$

This equation can be solved using the quadratic formula, with the result that $t = \boxed{5.63 \text{ s}}$

77. ***REASONING AND SOLUTION*** The car enters the speedway with a speed of

$$v_{o1} = a_1 t_1 = (6.0 \text{ m/s}^2)(4.0 \text{ s}) = 24 \text{ m/s}$$

After an additional time, t, it will have traveled a distance of

$$x = v_{o1} t + 1/2\, a_1 t^2$$

to overtake the other car. This second car travels the same distance $x = v_{o2} t$. Equating and solving for t yields

$$t = \frac{2(v_{02} - v_{01})}{a_1} = \frac{2(70.0 \text{ m/s} - 24 \text{ m/s})}{6.0 \text{ m/s}^2} = \boxed{15 \text{ s}}$$

78 ***REASONING AND SOLUTION*** The balls pass at a time t when both are at a position y above the ground. Applying Equation 2.8 to the ball that is dropped from rest, we have

$$y = 24\text{ m} + v_{01}t + \tfrac{1}{2}at^2 = 24\text{ m} + (0\text{ m/s})t + \tfrac{1}{2}at^2 \qquad (1)$$

Note that we have taken into account the fact that $y = 24$ m when $t = 0$ s in Equation (1). For the second ball that is thrown straight upward,

$$y = v_{02}t + \frac{1}{2}at^2 \qquad (2)$$

Equating Equations (1) and (2) for y yields

$$24\text{ m} + \frac{1}{2}at^2 = v_{02}t + \frac{1}{2}at^2 \qquad \text{or} \qquad 24\text{ m} = v_{02}t$$

Thus, the two balls pass at a time t, where

$$t = \frac{24\text{ m}}{v_{02}}$$

The initial speed v_{02} of the second ball is exactly the same as that with which the first ball hits the ground. To find the speed with which the first ball hits the ground, we take upward as the positive direction and use Equation 2.9 $\left(v^2 = v_0^2 + 2ay\right)$. Since the first ball is dropped from rest, we find that

$$v_{02} = v = \sqrt{2ay} = \sqrt{2(-9.80\text{ m/s}^2)(-24\text{ m})} = 21.7\text{ m/s}$$

Thus, the balls pass after a time

$$t = \frac{24\text{ m}}{21.7\text{ m/s}} = 1.11\text{ s}$$

At a time $t = 1.11$ s, the position of the first ball according to Equation (1) is

$$y = 24\text{ m} + \frac{1}{2}(-9.80\text{ m/s}^2)(1.11\text{ s})^2 = 24\text{ m} - 6.0\text{ m}$$

which is $\boxed{\text{6.0 m below the top of the cliff}}$.

79. **SSM** ***REASONING*** Since the car is moving with a constant velocity, the displacement of the car in a time t can be found from Equation 2.8 with $a = 0\ \text{m/s}^2$ and v_0 equal to the velocity of the car: $x_{\text{car}} = v_{\text{car}}t$. Since the train starts from rest with a constant acceleration, the displacement of the train in a time t is given by Equation 2.8 with $v_0 = 0\ \text{m/s}$:

$$x_{\text{train}} = \tfrac{1}{2}a_{\text{train}}t^2$$

At a time t_1, when the car just reaches the front of the train, $x_{\text{car}} = L_{\text{train}} + x_{\text{train}}$, where L_{train} is the length of the train. Thus, at time t_1,

$$v_{\text{car}}t_1 = L_{\text{train}} + \tfrac{1}{2}a_{\text{train}}t_1^2 \qquad (1)$$

At a time t_2, when the car is again at the rear of the train, $x_{\text{car}} = x_{\text{train}}$. Thus, at time t_2

$$v_{\text{car}}t_2 = \tfrac{1}{2}a_{\text{train}}t_2^2 \qquad (2)$$

Equations (1) and (2) can be solved simultaneously for the speed of the car v_{car} and the acceleration of the train a_{train}.

SOLUTION

a. Solving Equation (2) for a_{train} we have

$$a_{\text{train}} = \frac{2v_{\text{car}}}{t_2} \qquad (3)$$

Substituting this expression for a_{train} into Equation (1) and solving for v_{car}, we have

$$v_{\text{car}} = \frac{L_{\text{train}}}{t_1\left(1 - \dfrac{t_1}{t_2}\right)} = \frac{92\ \text{m}}{(14\ \text{s})\left(1 - \dfrac{14\ \text{s}}{28\ \text{s}}\right)} = \boxed{13\ \text{m/s}}$$

b. Direct substitution into Equation (3) gives the acceleration of the train:

$$a_{\text{train}} = \frac{2v_{\text{car}}}{t_2} = \frac{2\ (13\ \text{m/s})}{28\ \text{s}} = \boxed{0.93\ \text{m/s}^2}$$

80. ***REASONING AND SOLUTION*** During the first phase of the acceleration,

$$a_1 = \frac{v}{t_1}$$

During the second phase of the acceleration,

$$v = (6.4 \text{ m/s}) - (1.1 \text{ m/s}^2)(2.0 \text{ s}) = 4.2 \text{ m/s}$$

Then

$$a_1 = \frac{4.2 \text{ m/s}}{3.0 \text{ s}} = \boxed{1.4 \text{ m/s}^2}$$

1. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The horizontal and vertical components of the plane's velocity are related to the speed of the plane by the Pythagorean theorem: $v^2 = v_h^2 + v_v^2$. Solving for v_h we have

$$v_h = \sqrt{v^2 - v_v^2} = \sqrt{(245\text{ m/s})^2 - (40.6\text{ m/s})^2} = \boxed{242\text{ m/s}}$$

2. ***REASONING AND SOLUTION***

$$x = \Delta r \cos\theta = (162\text{ km})\cos 62.3° = \boxed{75.3\text{ km}}$$

$$y = \Delta r \sin\theta = (162\text{ km})\sin 62.3° = \boxed{143\text{ km}}$$

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3. ***REASONING AND SOLUTION*** The horizontal displacement is

$$x = 19\ 600\ \text{m} - 11\ 200\ \text{m} = 8400\ \text{m}.$$

The vertical displacement is

$$y = 4900\ \text{m} - 3200\ \text{m} = 1700\ \text{m}.$$

The magnitude of the displacement is therefore,

$$\Delta r = \sqrt{x^2 + y^2} = \sqrt{(8400\ \text{m})^2 + (1700\ \text{m})^2} = \boxed{8600\ \text{m}}$$

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4. ***REASONING AND SOLUTION*** The increase in altitude represents $v_y = 6.80$ m/s. The movement of the shadow represents $v_x = 15.5$ m/s. The magnitude of the glider's velocity is therefore

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.5 \text{ m/s})^2 + (6.80 \text{ m/s})^2} = \boxed{16.9 \text{ m/s}}$$

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5. **SSM** ***REASONING*** The displacement of the elephant seal has two components; 460 m due east and 750 m downward. These components are mutually perpendicular; hence, the Pythagorean theorem can be used to determine their resultant.

SOLUTION From the Pythagorean theorem,

$$R^2 = (460\text{ m})^2 + (750\text{ m})^2$$

Therefore,

$$R = \sqrt{(460\text{ m})^2 + (750\text{ m})^2} = \boxed{8.8 \times 10^2\text{ m}}$$

6. ***REASONING*** Trigonometry indicates that the x and y components of the dolphin's velocity are related to the launch angle θ according to $\tan\theta = v_y / v_x$.

SOLUTION Using trigonometry, we find that the y component of the dolphin's velocity is

$$v_y = v_x \tan\theta = v_x \tan 35° = (7.7\ \text{m/s}) \tan 35° = \boxed{5.4\ \text{m/s}}$$

7. ***REASONING AND SOLUTION***

a. From the definition of average acceleration (Equation 3.2), we have

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{12\ \text{m/s} - 0}{36\ \text{s}} = \boxed{0.33\ \text{m/s}^2}$$

The figure at the right shows the acceleration vector and its components along the north-south line and the east-west line

b. Assuming that north and east are the positive directions, the component of the horse's acceleration along the north-south line is

$$a_y = \left(0.33\ \text{m/s}^2\right) \sin 29° = \boxed{0.16\ \text{m/s}^2}$$

c. Similarly, the component of the horse's acceleration along the east-west line is

$$a_y = -\left(0.33\ \text{m/s}^2\right) \cos 29° = \boxed{-0.29\ \text{m/s}^2}$$

8. ***REASONING*** To determine the horizontal and vertical components of the launch velocity, we will use trigonometry. To do so, however, we need to know both the launch angle and the magnitude of the launch velocity. The launch angle is given. The magnitude of the launch velocity can be determined from the given acceleration and the definition of acceleration given in Equation 3.2.

SOLUTION According to Equation 3.2, we have

$$a = \frac{v - v_0}{t - t_0} \quad \text{or} \quad 340\ \text{m/s}^2 = \frac{v - 0\ \text{m/s}}{0.050\ \text{s}} \quad \text{or} \quad v = \left(340\ \text{m/s}^2\right)\left(0.050\ \text{s}\right)$$

Using trigonometry, we find the components to be

$$v_x = v\cos 51° = \left(340\ \text{m/s}^2\right)\left(0.050\ \text{s}\right)\cos 51° = \boxed{11\ \text{m/s}}$$

$$v_y = v\sin 51° = \left(340\ \text{m/s}^2\right)\left(0.050\ \text{s}\right)\sin 51° = \boxed{13\ \text{m/s}}$$

9. **SSM** **WWW** ***REASONING AND SOLUTION*** The escalator is the hypotenuse of a right triangle formed by the lower floor and the vertical distance between floors, as shown in the figure. The angle θ at which the escalator is inclined above the horizontal is related to the length L of the escalator and the vertical distance between the floors by the sine function:

$$\sin\theta = \frac{6.00\text{ m}}{L} \qquad (1)$$

The length L of the escalator can be found from the right triangle formed by the components of the shopper's displacement up the escalator and to the right of the escalator.

From the Pythagorean theorem, we have

$$L^2 + (9.00\text{ m})^2 = (16.0\text{ m})^2$$

so that

$$L = \sqrt{(16.0\text{ m})^2 - (9.00\text{ m})^2} = 13.2\text{ m}$$

From Equation (1) we have

$$\theta = \sin^{-1}\left(\frac{6.00\text{ m}}{13.2\text{ m}}\right) = \boxed{27.0°}$$

10. ***REASONING*** The component method can be used to determine the magnitude and direction of the bird watcher's displacement. Once the displacement is known, Equation 3.1 can be used to find the average velocity.

SOLUTION The following table gives the components of the individual displacements of the bird watcher. The last entry gives the components of the bird watcher's resultant displacement. Due east and due north have been chosen as the positive directions.

Displacement	***East/West Component***	***North/South Component***
A	0.50 km	0
B	0	–0.75 km
C	–(2.15 km) cos 35.0° = –1.76 km	(2.15 km) sin 35.0° = 1.23 km
r = A + B + C	–1.26 km	0.48 km

a. From the Pythagorean theorem, we have

$$r = \sqrt{(-1.26\text{ km})^2 + (0.48\text{ km})^2} = \boxed{1.35\text{ km}}$$

The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{0.48\text{ km}}{1.26\text{ km}}\right) = \boxed{21°,\text{ north of west}}$$

b. From Equation 3.1, the average velocity is

$$\bar{\mathbf{v}} = \frac{\Delta\mathbf{r}}{\Delta t} = \frac{1.35\text{ km}}{2.50\text{ h}} = \boxed{0.540\text{ km/h, }21°\text{ north of west}}$$

Note that the direction of the average velocity is, by definition, the same as the direction of the displacement.

11. ***REASONING AND SOLUTION***

a. Average speed is defined as the total distance d covered divided by the time Δt required to cover the distance. The total distance covered by the earth is one-fourth the circumference of its circular orbit around the sun:

$$d = \frac{1}{4} \times 2\pi\,(1.50 \times 10^{11}\ \text{m}) = 2.36 \times 10^{11}\ \text{m}$$

$$\bar{v} = \frac{d}{\Delta t} = \frac{2.36 \times 10^{11}\ \text{m}}{7.89 \times 10^{6}\ \text{s}} = \boxed{2.99 \times 10^{4}\ \text{m/s}}$$

b. The average velocity is defined as the displacement divided by the elapsed time.

In moving one-fourth of the distance around the sun, the earth completes the displacement shown in the figure at the right. From the Pythagorean theorem, the magnitude of this displacement is

$$\Delta r = \sqrt{r^2 + r^2} = \sqrt{2}\, r$$

Thus, the magnitude of the average velocity is

$$\bar{v} = \frac{\Delta r}{\Delta t} = \frac{\sqrt{2} \times 1.50 \times 10^{11}\ \text{m}}{7.89 \times 10^{6}\ \text{s}} = \boxed{2.69 \times 10^{4}\ \text{m/s}}$$

12. ***REASONING*** The magnitude and direction of the initial velocity $\mathbf{v_0}$ can be obtained using the Pythagorean theorem and trigonometry, once the x and y components of the initial velocity v_{0x} and v_{0y} are known. These components can be calculated using Equations 3.3a and 3.3b.

SOLUTION Using Equations 3.3a and 3.3b, we obtain the following results for the velocity components:

$$v_{0x} = v_x - a_x t = 3775 \text{ m/s} - \left(5.10 \text{ m/s}^2\right)\left(565 \text{ s}\right) = 893.5 \text{ m/s}$$

$$v_{0y} = v_y - a_y t = 4816 \text{ m/s} - \left(7.30 \text{ m/s}^2\right)\left(565 \text{ s}\right) = 691.5 \text{ m/s}$$

Using the Pythagorean theorem and trigonometry, we find

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{\left(893.5 \text{ m/s}\right)^2 + \left(691.5 \text{ m/s}\right)^2} = \boxed{1130 \text{ m/s}}$$

$$\theta = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right) = \tan^{-1}\left(\frac{691.5 \text{ m/s}}{893.5 \text{ m/s}}\right) = \boxed{37.7°}$$

13. ***REASONING AND SOLUTION*** The vertical motion consists of the ball rising for a time, t, stopping and returning to the ground in another time, t. For the upward portion

$$t = \frac{v_{oy} - v_y}{g} = \frac{v_{oy}}{g}$$

Note: $v_y = 0$ since the ball stops at the top. Now

$$v_{oy} = v_o \sin \theta_o = (25.0 \text{ m/s}) \sin 60.0° = 21.7 \text{ m/s}$$

$$t = (21.7 \text{ m/s})/(9.80 \text{ m/s}^2) = 2.21 \text{ s}$$

The required "hang time" is $2t = \boxed{4.42 \text{ s}}$.

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14. ***REASONING*** The vertical component of the ball's velocity $\mathbf{v_0}$ changes as the ball approaches the opposing player. It changes due to the acceleration of gravity. However, the horizontal component does not change, assuming that air resistance can be neglected. Hence, the horizontal component of the ball's velocity when the opposing player fields the ball is the same as it was initially.

SOLUTION Using trigonometry, we find that the horizontal component is

$$v_x = v_0 \cos\theta = (15 \text{ m/s})\cos 55° = \boxed{8.6 \text{ m/s}}$$

15. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** As shown in Example 2, the time required for the package to hit the ground is given by $t = \sqrt{2y/a_y}$ and is independent of the plane's horizontal velocity. Thus, the time needed for the package to hit the ground is still $\boxed{14.6\text{ s}}$.

16. ***REASONING AND SOLUTION*** Using $v_y = 0$ and

$$v_{oy} = v_o \sin\theta = (11 \text{ m/s}) \sin 65^\circ = 1.0 \times 10^1 \text{ m/s}$$

and $v_y^2 = v_{oy}^2 + 2a_y y$, we have

$$y = \frac{-v_{oy}^2}{2a_y} = \frac{-(1.0 \text{ X } 10^1 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{5.1 \text{ m}}$$

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17. **SSM** **WWW** ***REASONING*** Once the diver is airborne, he moves in the x direction with constant velocity while his motion in the y direction is accelerated (at the acceleration due to gravity). Therefore, the magnitude of the x component of his velocity remains constant at 1.20 m/s for all times t. The magnitude of the y component of the diver's velocity after he has fallen through a vertical displacement y can be determined from Equation 3.6b: $v_y^2 = v_{0y}^2 + 2a_y y$. Since the diver runs off the platform horizontally, $v_{0y} = 0$. Once the x and y components of the velocity are known for a particular vertical displacement y, the speed of the diver can be obtained from $v = \sqrt{v_x^2 + v_y^2}$.

SOLUTION For convenience, we will take downward as the positive y direction. After the diver has fallen 10.0 m, the y component of his velocity is, from Equation 3.6b,

$$v_y = \sqrt{v_{0y}^2 + 2a_y y} = \sqrt{0^2 + 2(9.80\ \text{m/s}^2)(10.0\ \text{m})} = 14.0\ \text{m/s}$$

Therefore,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1.20\ \text{m/s})^2 + (14.0\ \text{m/s})^2} = \boxed{14.1\ \text{m/s}}$$

18. ***REASONING AND SOLUTION*** The components of the initial velocity are

$$v_{0x} = v_0 \cos\theta = (22 \text{ m/s}) \cos 40.0° = 17 \text{ m/s}$$
$$v_{0y} = v_0 \sin\theta = (22 \text{ m/s}) \sin 40.0° = 14 \text{ m/s}$$

a. Solving Equation 3.6b for y gives

$$y = \frac{v_y^2 - v_{0y}^2}{2a_y}$$

When the football is at the maximum height $y = H$, and the football is momentarily at rest, so $v_y = 0$. Thus,

$$H = \frac{0 - v_{0y}^2}{2a_y} = \frac{0 - (14 \text{ m/s})^2}{2(-1.62 \text{ m/s}^2)} = \boxed{6.0 \times 10^1 \text{ m}}$$

b. When the ball strikes the ground, $y = 0$; therefore, the time of flight can be determined from Equation 3.5b with $y = 0$.

$$y = v_{0y}t + \tfrac{1}{2} a_y t^2$$

or

$$0 = [(14 \text{ m/s}) + \tfrac{1}{2}(-1.62 \text{ m/s}^2)t]\, t$$

$$t = 17 \text{ s.}$$

The range is

$$x = R = v_{0x}t = (17 \text{ m/s})(17 \text{ s}) = \boxed{290 \text{ m}}$$

19. ***REASONING*** As shown in Example 7 in the text, the range R of a projectile is given by $R = v_{0x}t$, where t is the total flight time of the projectile. In the absence of air resistance, the time required for the projectile to rise is equal to the time required for the projectile to fall; therefore, the total flight time t is equal to twice the time t_H required for a projectile to reach its maximum vertical displacement H. The time t_H can be found from Equation 3.3b ($v_y = v_{0y} + a_y t$) by setting $v_y = 0$ and solving for $t = t_H$.

SOLUTION

a. With upward taken as the positive direction, the time t_H for the greyhound to reach its maximum vertical displacement H is given by Equation 3.3b as

$$t_H = -\frac{v_{0y}}{a_y} = -\frac{v_0 \sin 31°}{a_y} = -\frac{(10.0 \text{ m/s}) \sin 31°}{-9.80 \text{ m/s}^2} = 0.526 \text{ s}$$

The range of the leap is, therefore,

$$R = v_{0x}t = (v_0 \cos 31°)t = \left[(10.0 \text{ m/s}) \cos 31°\right]\left[2(0.526 \text{ s})\right] = \boxed{9.0 \text{ m}}$$

b. As discussed above, in the absence of air resistance, the time for the projectile to rise is equal to the time for the projectile to fall, so the greyhound is in the air for $2(0.526 \text{ s}) = \boxed{1.1 \text{ s}}$.

20. ***REASONING AND SOLUTION*** The ball travels a distance $x = v_{0x}t$, so that it takes

$$t = x/v_{0x} = (115\text{ m})/(41\text{ m/s}) = 2.80\text{ s}$$

to reach home plate. Since the runner takes 3.50 s to reach home, the $\boxed{\text{ball arrives first}}$ by

$$3.50\text{ s} - 2.80\text{ s} = \boxed{0.70\text{ s}}$$

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21. **SSM** ***REASONING AND SOLUTION*** The water exhibits projectile motion. The x component of the motion has zero acceleration while the y component is subject to the acceleration due to gravity. In order to reach the highest possible fire, the displacement of the hose from the building is x, where, according to Equation 3.5a (with $a_x = 0$),

$$x = v_{0x}t = (v_0 \cos\theta)t$$

with t equal to the time required for the water the reach its maximum vertical displacement. The time t can be found by considering the vertical motion. From Equation 3.3b,

$$v_y = v_{0y} + a_y t$$

When the water has reached its maximum vertical displacement, $v_y = 0$. Taking up and to the right as the positive directions, we find that

$$t = \frac{-v_{0y}}{a_y} = \frac{-v_0 \sin\theta}{a_y}$$

and

$$x = (v_0 \cos\theta)\left(\frac{-v_0 \sin\theta}{a_y}\right)$$

Therefore, we have

$$x = -\frac{v_0^2 \cos\theta \sin\theta}{a_y} = -\frac{(25.0 \text{ m/s})^2 \cos 35.0° \sin 35.0°}{-9.80 \text{ m/s}^2} = \boxed{30.0 \text{ m}}$$

22. ***REASONING*** Since we know the launch angle $\theta = 15.0°$, the launch speed v_0 can be obtained using trigonometry, which gives the y component of the launch velocity as $v_{0y} = v_0 \sin \theta$. Solving this equation for v_0 requires a value for v_{0y}, which we can obtain from the vertical height of $y = 13.5$ m by using Equation 3.6b from the equations of kinematics.

SOLUTION From Equation 3.6b we have

$$v_y^2 = v_{0y}^2 + 2a_y y$$

$$(0 \text{ m/s})^2 = v_{0y}^2 + 2(-9.80 \text{ m/s}^2)(13.5 \text{ m}) \quad \text{or} \quad v_{0y} = \sqrt{2(9.80 \text{ m/s}^2)(13.5 \text{ m})}$$

Using trigonometry, we find

$$v_0 = \frac{v_{0y}}{\sin 15.0°} = \frac{\sqrt{2(9.80 \text{ m/s}^2)(13.5 \text{ m})}}{\sin 15.0°} = \boxed{62.8 \text{ m/s}}$$

23. ***REASONING AND SOLUTION*** The time required for the car to fall to the ground is given by

$$t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-54\text{ m})}{9.80\text{ m/s}^2}} = 3.3\text{ s.}$$

During this time, the car traveled a horizontal distance of 130 m. Using $a_x = 0$ gives

$$v_{ox} = x/t = (130\text{ m})/(3.3\text{ s}) = \boxed{39\text{ m/s}}$$

24. ***REASONING*** When the skier leaves the ramp, she exhibits projectile motion . Since we know the maximum height attained by the skier, we can find her launch speed v_0 using Equation 3.6b , $v_y^2 = v_{0y}^2 + 2a_y y$, where $v_{0y} = v_0 \sin 63°$.

SOLUTION At the highest point in her trajectory, $v_y = 0$. Solving Equation 3.6b for v_{0y} we obtain, taking upward as the positive direction,

$$v_{0y} = v_0 \sin 63° = \sqrt{-2a_y y} \quad \text{or} \quad v_0 = \frac{\sqrt{-2a_y y}}{\sin 63°} = \frac{\sqrt{-2(-9.80\ \text{m/s}^2)(13\ \text{m})}}{\sin 63°} = \boxed{18\ \text{m/s}}$$

25. $\boxed{\text{SSM}}$ ***REASONING*** Since the magnitude of the velocity of the fuel tank is given by $v = \sqrt{v_x^2 + v_y^2}$, it is necessary to know the velocity components v_x and v_y just before impact. At the instant of release, the empty fuel tank has the same velocity as that of the plane. Therefore, the magnitudes of the initial velocity components of the fuel tank are given by $v_{0x} = v_0 \cos\theta$ and $v_{0y} = v_0 \sin\theta$, where v_0 is the speed of the plane at the instant of release. Since the x motion has zero acceleration, the x component of the velocity of the plane remains equal to v_{0x} for all later times while the tank is airborne. The y component of the velocity of the tank after it has undergone a vertical displacement y is given by Equation 3.6b.

SOLUTION

a. Taking up as the positive direction, the velocity components of the fuel tank just before it hits the ground are

$$v_x = v_{0x} = v\cos\theta = (135 \text{ m/s}) \cos 15° = 1.30 \times 10^2 \text{ m/s}$$

From Equation 3.6b, we have

$$v_y = -\sqrt{v_{0y}^2 + 2a_y y} = -\sqrt{(v_0 \sin\theta)^2 + 2a_y y}$$

$$= -\sqrt{\left[(135 \text{ m/s}) \sin 15.0°\right]^2 + \left[2(-9.80 \text{ m/s}^2)(-2.00 \times 10^3 \text{ m})\right]} = -201 \text{ m/s}$$

Therefore, the magnitude of the velocity of the fuel tank just before impact is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(1.30 \times 10^2 \text{ m/s})^2 + (201 \text{ m/s})^2} = \boxed{239 \text{ m/s}}$$

The velocity vector just before impact is inclined at an angle ϕ above the horizontal. This angle is

$$\phi = \tan^{-1}\left(\frac{201 \text{ m/s}}{1.30 \times 10^2 \text{ m/s}}\right) = \boxed{57.1°}$$

b. As shown in Conceptual Example 9, once the fuel tank in part a rises and falls to the same altitude at which it was released, its motion is identical to the fuel tank in part b. Therefore, the velocity of the fuel tank in part b just before impact is $\boxed{\text{239 m/s at an angle of 57.1° above the horizontal}}$.

26. ***REASONING AND SOLUTION*** The maximum vertical displacement y attained by a projectile is given by Equation 3.6b ($v_y^2 = v_{0y}^2 + 2a_y y$) with $v_y = 0$:

$$y = -\frac{v_{0y}^2}{2a_y}$$

In order to use Equation 3.6b, we must first estimate his initial speed v_{0y}. When Jordan has reached his maximum vertical displacement, $v_y = 0$, and $t = 1.00$ s. Therefore, according to Equation 3.3b ($v_y = v_{0y} + a_y t$), with upward taken as positive, we find that

$$v_{0y} = -a_y t = -\ (-9.80\ \text{m/s}^2)\ (1.00\ \text{s}) = 9.80\ \text{m/s}$$

Therefore, Jordan's maximum jump height is

$$y = -\frac{(9.80\ \text{m/s})^2}{2(-9.80\ \text{m/s}^2)} = \boxed{4.90\ \text{m}}$$

This result far exceeds Jordan's maximum jump height, so the claim that he can remain in the air for two full seconds is false.

27. ***REASONING AND SOLUTION*** The time of flight of the motorcycle is given by

$$t = \frac{2v_o \sin\theta_o}{g} = \frac{2(33.5\text{ m/s})\sin 18.0^o}{9.80\text{ m/s}^2} = 2.11\text{ s.}$$

The horizontal distance traveled by the motorcycle is then

$$x = v_o \cos\theta_o\, t = (33.5\text{ m/s})(\cos 18.0°)(2.11\text{ s}) = 67.2\text{ m.}$$

The daredevil can jump over (67.2 m)/(2.74 m/bus) = 24.5 buses. In even numbers, this means $\boxed{\text{24 buses}}$.

28. ***REASONING AND SOLUTION*** The coordinates of the bullet when it hits the target are $y = -(1/2)gt^2$ and $x = v_0t$. The first of these yields the time of flight

$$t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-0.025\text{ m})}{9.80\text{ m/s}^2}} = 0.071\text{ s}.$$

The horizontal distance traveled is then

$$x = (670\text{ m/s})(0.071\text{ s}) = \boxed{48\text{ m}}$$

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29. $\boxed{\text{SSM}}$ ***REASONING*** The upward direction is chosen as positive. Since the ballast bag is released from rest relative to the balloon, its initial velocity relative to the ground is equal to the velocity of the balloon relative to the ground, so that $v_{0y} = 3.0$ m/s. Time required for the ballast to reach the ground can be found by solving Equation 3.5b for *t*.

SOLUTION Using Equation 3.5b, we have

$$\tfrac{1}{2}a_y t^2 + v_{0y}t - y = 0 \quad \text{or} \quad \tfrac{1}{2}(-9.80\ \text{m/s}^2)t^2 + (3.0\ \text{m/s})t - (-9.5\ \text{m}) = 0$$

This equation is quadratic in *t*, and *t* may be found from the quadratic formula. Using the quadratic formula, suppressing the units, and discarding the negative root, we find

$$t = \frac{-3.0 \pm \sqrt{(3.0)^2 - 4(-4.90)(9.5)}}{2(-4.90)} = \boxed{1.7\ \text{s}}$$

30. ***REASONING*** The rocket will clear the top of the wall by an amount that is the height of the rocket as it passes over the wall minus the height of the wall. To find the height of the rocket as it passes over the wall, we separate the rocket's projectile motion into its horizontal and vertical parts and treat each one separately. From the horizontal part we will obtain the time of flight until the rocket reaches the location of the wall. Then, we will use this time along with the acceleration due to gravity in the equations of kinematics to determine the height of the rocket as it passes over the wall.

SOLUTION We begin by finding the horizontal and vertical components of the launch velocity

$$v_{0x} = v_0 \cos 60.0° = (75.0 \text{ m/s}) \cos 60.0°$$

$$v_{0y} = v_0 \sin 60.0° = (75.0 \text{ m/s}) \sin 60.0°$$

Using v_{0x}, we can obtain the time of flight, since the distance to the wall is known to be 27.0 m:

$$t = \frac{27.0 \text{ m}}{v_{0x}} = \frac{27.0 \text{ m}}{(75.0 \text{ m/s}) \cos 60.0°} = 0.720 \text{ s}$$

The height of the rocket as it clears the wall can be obtained from Equation 3.5b, in which we take upward to be the positive direction. The amount by which the rocket clears the wall can then be obtained:

$$y = v_{0y} t + \tfrac{1}{2} a_y t^2$$

$$y = (75.0 \text{ m/s})(\sin 60.0°)(0.720 \text{ s}) + \tfrac{1}{2}(-9.80 \text{ m/s}^2)(0.720 \text{ s})^2 = 44.2 \text{ m}$$

$$\text{clearance} = 44.2 \text{ m} - 11.0 \text{ m} = \boxed{33.2 \text{ m}}$$

31. ***REASONING*** The drawing at the right shows the velocity vector **v** of the water at a point below the top of the falls. The components of the velocity are also shown. The angle θ is given by $\tan\theta = v_y / v_x$, so that

$$v_y = -v_x \tan\theta = -(2.7 \text{ m/s}) \tan 75°$$

Here we have used the fact that the horizontal velocity component v_x remains unchanged at its initial value of 2.7 m/s as the water falls. Knowing the y component of the velocity, we can use Equation 3.6b, ($v_y^2 = v_{0y}^2 + 2a_y y$) to find the vertical distance y.

SOLUTION Taking $v_{0y} = 0$, and taking upward as the positive direction, we have from Equation 3.5b that

$$y = \frac{v_y^2}{2a_y} = \frac{[-(2.7 \text{ m/s}) \tan 75°]^2}{2(-9.80 \text{ m/s}^2)} = -5.2 \text{ m}$$

Therefore, the velocity vector of the water points downward at a 75° angle below the horizontal at a vertical distance of $\boxed{5.2 \text{ m}}$ below the edge.

32. ***REASONING*** For each stone we will treat the horizontal and vertical parts of the motion separately. Since both stones strike the ground at the same place, they have the same range R. In each case, the time of flight t is the same, and the range is the horizontal velocity component times the time. The vertical parts of the motion are different for each stone. Taking upward to be the positive direction, we note that stone A has a displacement of –H, where H is the height of the cliff, since it is initially on top the cliff and lands on the ground below. Stone B has a displacement of zero, however, because it is initially on the ground beneath the cliff and returns to the same level from which it is launched. We will use Equation 3.5b and the acceleration due to gravity to describe the vertical part of the motion for each stone.

SOLUTION a. Since the ranges of the two stones are the same, we have

$$R = v_{0xB}t = v_{0xA}t \quad \text{or} \quad v_{0xB} = v_{0xA} \quad \text{or} \quad v_{0xB} - v_{0xA} = \boxed{0\ \text{m/s}}$$

b. Applying Equation 3.5 $\left(y = v_{0y}t + \frac{1}{2}a_y t^2\right)$ to each stone, we have

$$\underbrace{-H = v_{0yA}t + \tfrac{1}{2}a_y t^2}_{\text{Stone A}} \quad \text{and} \quad \underbrace{0 = v_{0yB}t + \tfrac{1}{2}a_y t^2}_{\text{Stone B}}$$

From the equation for stone B, we see that $\frac{1}{2}a_y t^2 = -v_{0yB}t$. Substituting this result into the equation for stone A gives

$$-H = v_{0yA}t - v_{0yB}t \quad \text{or} \quad v_{0yB} - v_{0yA} = \frac{H}{t} = \frac{12\ \text{m}}{3.0\ \text{s}} = \boxed{4.0\ \text{m/s}}$$

33. SSM WWW ***REASONING*** The horizontal distance covered by stone **1** is equal to the distance covered by stone **2** after it passes point **P** in the following diagram. Thus, the distance Δx between the points where the stones strike the ground is equal to x_2, the horizontal distance covered by stone **2** when it reaches **P**. In the diagram, we assume up and to the right are positive.

SOLUTION If t_P is the time required for stone **2** to reach **P**, then

$$x_2 = v_{0x}t_P = (v_0 \cos\theta)t_P$$

For the vertical motion of stone **2**, $v_y = v_0 \sin\theta + a_y t$. Solving for t gives

$$t = \frac{v_y - v_0 \sin\theta}{a_y}$$

When stone **2** reaches **P**, $v_y = -v_0 \sin\theta$, so the time required to reach **P** is

$$t_P = \frac{-2v_0 \sin\theta}{a_y}$$

Then,

$$x_2 = v_{0x}t_P = (v_0 \cos\theta)\left(\frac{-2v_0 \sin\theta}{a_y}\right)$$

$$x_2 = \frac{-2v_0^2 \sin\theta \cos\theta}{a_y} = \frac{-2(13.0\ \text{m/s})^2 \sin\ 30.0° \cos 30.0°}{-9.80\ \text{m/s}^2} = \boxed{14.9\ \text{m}}$$

34. ***REASONING*** As discussed in Conceptual Example 4, the horizontal velocity component of the bullet does not change from its initial value and is equal to the horizontal velocity of the car. The same thing is true here for the tomato. In other words, regardless of its vertical position relative to the ground, the tomato always remains above you as you travel in the convertible. From the symmetry of free fall motion, we know that when you catch the tomato, its velocity will be 11 m/s straight downward. The time t required to catch the tomato can be found by solving Equation 3.3b $(v_y = v_{0y} + a_y t)$ with $v_y = -v_{0y}$. Once t is known, the distance that the car moved can be found from $x = v_x t$.

SOLUTION Taking upward as the positive direction, we find the flight time of the tomato to be

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-2v_{0y}}{a_y} = \frac{-2(11\text{ m/s})}{-9.80\text{ m/s}^2} = 2.24\text{ s}$$

Thus, the car moves through a distance of

$$x = v_x t = (25\text{ m/s})(2.24\text{ s}) = \boxed{56\text{ m}}$$

35. ***REASONING AND SOLUTION*** At $x = 29.0$ m we have

$$v_x = v_{0x} = v_0 \cos \theta_0 = (19.0 \text{ m/s}) \cos 32.0° = 16.1 \text{ m/s}$$

For the time of flight

$$t = x/v_{0x} = (29.0 \text{ m})/(16.1 \text{ m/s}) = 1.80 \text{ s}$$

Thus,

$$v_y = v_{0y} - gt = v_0 \sin \theta_0 - gt$$

$$= (19.0 \text{ m/s}) \sin 32.0° - (9.80 \text{ m/s}^2)(1.80 \text{ s}) = -7.6 \text{ m/s}$$

Then,

$$v = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(16.1 \text{ m/s})^2 + (-7.6 \text{ m/s})^2} = \boxed{17.8 \text{ m/s}}$$

36. ***REASONING AND SOLUTION*** On impact

$$v_x = v \cos 75.0° = (8.90 \text{ m/s}) \cos 75.0° = 2.30 \text{ m/s}$$

and

$$v_{0y}^2 = v_y^2 + 2gy = (8.90 \text{ m/s})^2 \sin^2 75.0° + 2(9.80 \text{ m/s}^2)(-3.00 \text{ m})$$

so

$$v_{0y} = 3.89 \text{ m/s}.$$

The magnitude of the diver's initial velocity is

$$v_o = \sqrt{(2.30 \text{ m/s})^2 + (3.89 \text{ m/s})^2} = \boxed{4.52 \text{ m/s}}$$

The angle the initial velocity vector makes with the horizontal is

$$\theta_0 = \tan^{-1} (v_{oy} / v_{ox}) = \boxed{59.4°}$$

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37. **SSM** ***REASONING AND SOLUTION*** The horizontal displacement of the jumper is given by Equation 3.5a with $a_x = 0$: $x = v_{0x}t = (v_0 \cos\theta)t$. The vertical component of the jumper's velocity at any time t is given by Equation 3.3b: $v_y = v_{0y} + a_y t$. At the instant that the jumper lands, $v_y = -v_{0y}$. Therefore, the vertical component of the jumper's velocity is

$$-v_{0y} = v_{0y} + a_y t$$

Solving for t and assuming that upward is the positive direction gives

$$t = \frac{-2v_{0y}}{a_y} = \frac{-2v_0 \sin\theta}{a_y}$$

Substituting this expression for t into the expression for x gives

$$x = (v_0 \cos\theta)t = v_0 \cos\theta \left(\frac{-2v_0 \sin\theta}{a_y} \right)$$

or

$$x = \frac{-2v_0^2 \sin\theta \cos\theta}{a_y}$$

Solving for v_0 gives

$$v_0 = \sqrt{\frac{-x\,a_y}{2\cos\theta \sin\theta}} = \sqrt{\frac{-(8.7\text{ m})(-9.80\text{ m/s}^2)}{2\cos 23^\circ \sin 23^\circ}} = \boxed{11\text{ m/s}}$$

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38. ***REASONING AND SOLUTION*** In the horizontal direction

$$v_x = v_{0x} = v_0 \cos\theta = (25 \text{ m/s}) \cos 38^\circ = 2.0 \times 10^1 \text{ m/s}$$

Also, the time of flight is

$$t = x/v_{0x} = (55 \text{ m})/(2.0 \times 10^1 \text{ m/s}) = 2.8 \text{ s}$$

In the vertical direction

$$v_y = v_{0y} - gt \quad \text{so} \quad v_{0y} = v_y + gt = -v_0 \sin 38.0^\circ + gt$$

$$v_{0y} = (-25 \text{ m/s}) \sin 38.0^\circ + (9.80 \text{ m/s}^2)(2.8 \text{ s}) = 12 \text{ m/s}$$

Thus,

$$v_0 = \sqrt{\left(2.0 \times 10^1 \text{ m/s}\right)^2 + \left(12 \text{ m/s}\right)^2} = \boxed{23 \text{ m/s}}$$

The direction of the initial velocity is given by

$$\theta_0 = \tan^{-1}\left(\frac{v_{0y}}{v_{0x}}\right) = \boxed{31^\circ}$$

39. ***REASONING AND SOLUTION*** The horizontal position of the impact point is $x_1 = v_{0x}t_1$ for one stone and $x_2 = v_{0x}t_2$ for the other stone.

The vertical positions are $y_1 = -(1/2)gt_1^2$ and $y_2 = -(1/2)gt_2^2$. Dividing the x equations gives

$$x_2/x_1 = t_2/t_1 = 2$$

Dividing the y equations gives

$$y_2/y_1 = t_2^2/t_1^2 = 4$$

So one building is $\boxed{\text{four times}}$ as tall as the other.

40. ***REASONING*** Using the data given in the problem, we can find the maximum flight time t of the ball using Equation 3.5b ($y = v_{0y}t + \frac{1}{2}a_y t^2$). Once the flight time is known, we can use the definition of average velocity to find the minimum speed required to cover the distance x in that time.

SOLUTION Equation 3.5b is quadratic in t and can be solved for t using the quadratic formula. According to Equation 3.5b, the maximum flight time is (with upward taken as the positive direction)

$$t = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 - 4\left(\frac{1}{2}\right)a_y(-y)}}{2\left(\frac{1}{2}\right)a_y} = \frac{-v_{0y} \pm \sqrt{v_{0y}^2 + 2a_y y}}{a_y}$$

$$= \frac{-(15.0\text{ m/s})\sin 50.0° \pm \sqrt{\left[(15.0\text{ m/s})\sin 50.0°\right]^2 + 2(-9.80\text{ m/s}^2)(2.10\text{ m})}}{-9.80\text{ m/s}^2}$$

$$= 0.200\text{ s} \quad \text{and} \quad 2.145\text{ s}$$

where the first root corresponds to the time required for the ball to reach a vertical displacement of $y = +2.10$ m as it travels upward, and the second root corresponds to the time required for the ball to have a vertical displacement of $y = +2.10$ m as the ball travels upward and then downward. The desired flight time t is 2.145 s.

During the 2.145 s, the horizontal distance traveled by the ball is

$$x = v_x t = (v_0 \cos\theta)t = \left[(15.0\text{ m/s})\cos 50.0°\right](2.145\text{ s}) = 20.68\text{ m}$$

Thus, the opponent must move $20.68\text{ m} - 10.0\text{ m} = 10.68\text{ m}$ in $2.145\text{ s} - 0.30\text{ s} = 1.845\text{ s}$. The opponent must, therefore, move with a minimum average speed of

$$\bar{v}_{\min} = \frac{10.68\text{ m}}{1.845\text{ s}} = \boxed{5.79\text{ m/s}}$$

41. $\boxed{\text{SSM}}$ ***REASONING*** The angle θ can be found from

$$\theta = \tan^{-1}\left(\frac{2400\text{ m}}{x}\right) \qquad (1)$$

where x is the horizontal displacement of the flare. Since $a_x = 0$, it follows that $x = (v_0 \cos 30.0°)t$. The flight time t is determined by the vertical motion. In particular, the time t can be found from Equation 3.5b. Once the time is known, x can be calculated.

SOLUTION From Equation 3.5b, assuming upward is the positive direction, we have

$$y = -(v_0 \sin 30.0°)t + \tfrac{1}{2}a_y t^2$$

which can be rearranged to give the following equation that is quadratic in t:

$$\tfrac{1}{2}a_y t^2 - (v_0 \sin 30.0°)t - y = 0$$

Using $y = -2400$ m and $a_y = -9.80$ m/s^2 and suppressing the units, we obtain the quadratic equation

$$4.9t^2 + 120t - 2400 = 0$$

Using the quadratic formula, we obtain $t = 13$ s. Therefore, we find that

$$x = (v_0 \cos 30.0°)t = (240\text{ m/s})(\cos 30.0°)(13\text{ s}) = 2700\text{ m}$$

Equation (1) then gives

$$\theta = \tan^{-1}\left(\frac{2400\text{ m}}{2700\text{ m}}\right) = \boxed{42°}$$

42. ***REASONING AND SOLUTION*** As shown in the following figure, the water hits the sunbather at $x = 4.4$ m and $y = -1.4$ m.

The time required for the water to reach the sunbather is $t = x/v_{0x}$.

$$y = v_{0y}t - \tfrac{1}{2}gt^2 = (v_0 \sin\theta)\left(\frac{x}{v_0 \cos\theta}\right) - \tfrac{1}{2}g\left(\frac{x}{v_0 \cos\theta}\right)^2 = x\tan\theta - \frac{gx^2}{2v_0^2\cos^2\theta}$$

Solving for v_o yields

$$v_0 = \sqrt{\frac{-gx^2}{2(y - x\tan\theta)\cos^2\theta}} = \sqrt{\frac{-(9.80\ \text{m/s}^2)(4.4\ \text{m})^2}{2[-1.4-(4.4\ \text{m})\tan 25°]\cos^2 25°}} = \boxed{5.8\ \text{m/s}}$$

43. **SSM** ***REASONING*** Since the horizontal motion is not accelerated, we know that the x component of the velocity remains constant at 340 m/s. Thus, we can use Equation 3.5a (with $a_x = 0$) to determine the time that the bullet spends in the building before it is embedded in the wall. Since we know the vertical displacement of the bullet after it enters the building, we can use the flight time in the building and Equation 3.5b to find the y component of the velocity of the bullet as it enters the window. Then, Equation 3.6b can be used (with $v_{0y} = 0$) to determine the vertical displacement y of the bullet as it passes between the buildings. We can determine the distance H by adding the magnitude of y to the vertical distance of 0.50 m within the building.

Once we know the vertical displacement of the bullet as it passes between the buildings, we can determine the time t_1 required for the bullet to reach the window using Equation 3.4b. Since the motion in the x direction is not accelerated, the distance D can then be found from $D = v_{0x}t_1$.

SOLUTION Assuming that the direction to the right is positive, we find that the time that the bullet spends in the building is (according to Equation 3.5a)

$$t = \frac{x}{v_{0x}} = \frac{6.9 \text{ m}}{340 \text{ m/s}} = 0.0203 \text{ s}$$

The vertical displacement of the bullet after it enters the building is, taking down as the negative direction, equal to –0.50 m. Therefore, the vertical component of the velocity of the bullet as it passes through the window is, from Equation 3.5b,

$$v_{0\,y(\text{window})} = \frac{y - \frac{1}{2}a_y t^2}{t} = \frac{y}{t} - \frac{1}{2}a_y t = \frac{-0.50 \text{ m}}{0.0203 \text{ s}} - \frac{1}{2}(-9.80 \text{ m/s}^2)(0.0203 \text{ s}) = -24.5 \text{ m/s}$$

The vertical displacement of the bullet as it travels between the buildings is (according to Equation 3.6b with $v_{0y} = 0$)

$$y = \frac{v_y^2}{2a_y} = \frac{(-24.5 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = -30.6 \text{ m}$$

Therefore, the distance H is

$$H = (30.6 \text{ m}) + (0.50 \text{ m}) = \boxed{31 \text{ m}}$$

The time for the bullet to reach the window, according to Equation 3.4b, is

$$t_1 = \frac{2y}{v_{0y} + v_y} = \frac{2y}{v_y} = \frac{2(-30.6 \text{ m})}{(-24.5 \text{ m/s})} = 2.50 \text{ s}$$

Hence, the distance D is given by

$$D = v_{0x}t_1 = (340 \text{ m/s})(2.50 \text{ s}) = \boxed{850 \text{ m}}$$

44. ***REASONING*** We will treat the horizontal and vertical parts of the motion separately. The range R is the product of the horizontal component of the initial velocity v_{0x} and the time of flight t. The time of flight can be obtained from the vertical part of the motion by using Equation 3.5b $\left(y = v_{0y}t + \frac{1}{2}a_y t^2\right)$ and the fact that the displacement y in the vertical direction is zero, since the projectile is launched from and returns to ground level. The expression for the range obtained in this way can then be applied to obtain the desired launch angle for doubling the range.

SOLUTION The range of the projectile is

$$R = v_{0x}t = \left(v_0 \cos\theta\right)t$$

Using Equation 3.5b, we obtain the time of flight as

$$y = 0 = v_{0y}t + \tfrac{1}{2}a_y t^2 \quad \text{or} \quad t = -\frac{2v_{0y}}{a_y} = -\frac{2v_0 \sin\theta}{a_y}$$

Substituting this expression for t into the range expression gives

$$R = \left(v_0 \cos\theta\right)\left(\frac{-2v_0 \sin\theta}{a_y}\right) = -\frac{2v_0^2 \cos\theta \sin\theta}{a_y} = -\frac{v_0^2 \sin 2\theta}{a_y}$$

where we have used the fact that $2 \cos\theta \sin\theta = \sin 2\theta$. We can now apply this expression for the range to the initial range R_1 for θ_1 and the range R_2 for θ_2:

$$R_1 = -\frac{v_0^2 \sin 2\theta_1}{a_y} \quad \text{or} \quad R_2 = -\frac{v_0^2 \sin 2\theta_2}{a_y}$$

Dividing these two expressions gives

$$\frac{R_2}{R_1} = \frac{-\left(v_0^2 \sin 2\theta_2\right)/a_y}{-\left(v_0^2 \sin 2\theta_1\right)/a_y} = \frac{\sin 2\theta_2}{\sin 2\theta_1} = 2$$

where we have used the fact that $R_2/R_1/ = 2$. Since $\theta_1 = 12.0°$, we find that

$$\sin 2\theta_2 = 2.00 \sin 2\theta_1 = 2.00 \sin 2\left(12.0°\right) = 0.813$$

$$\theta_2 = \frac{\sin^{-1} 0.813}{2.00} = \boxed{27.2°}$$

45. ***REASONING*** We can use information about the motion of clown A and the collision to determine the initial velocity components for clown B. Once the initial velocity components are known, the launch speed v_{0B} and the launch angle θ_B for clown B can be determined.

SOLUTION From Equation 3.5b $\left(y = v_{0y}t + \frac{1}{2}a_y t^2\right)$ we can find the time of flight until the collision. Taking upward as positive and noting for clown A that v_{0y} = (9.00 m/s) sin 75.0° = 8.693 m/s, we have

$$1.00\ \text{m} = (8.693\ \text{m/s})t + \tfrac{1}{2}(-9.80\ \text{m/s}^2)t^2$$

Rearranging this result and suppressing the units gives

$$4.90t^2 - 8.693t + 1.00 = 0$$

The quadratic equation reveals that

$$t = \frac{8.693 \pm \sqrt{(-8.693)^2 - 4(4.90)(1.00)}}{2(4.90)} = 1.650\ \text{s} \quad \text{or} \quad 0.1237\ \text{s}$$

Using these values for t with the magnitudes v_{0xA} and v_{0xB} of the horizontal velocity components for each clown, we note that the horizontal distances traveled by each clown must add up to 6.00 m. Thus,

$$v_{0xA}t + v_{0xB}t = 6.00\ \text{m} \quad \text{or} \quad v_{0xB} = \frac{6.00\ \text{m}}{t} - v_{0xA}$$

Using v_{0xA} = (9.00 m/s) cos 75.0° = 2.329 m/s, we find

$$v_{0xB} = \frac{6.00\ \text{m}}{1.650\ \text{s}} - 2.329\ \text{m/s} = 1.307\ \text{m/s} \quad \text{or} \quad v_{0xB} = \frac{6.00\ \text{m}}{0.1237\ \text{s}} - 2.329\ \text{m/s} = 46.175\ \text{m/s}$$

The vertical component of clown B's velocity is v_{0yB} and must be the same as that for clown A, since each clown has the same vertical displacement of 1.00 m at the same time. Hence, v_{0yB} = 8.693 m/s (see above). The launch speed of clown B, finally, is $v_{0B} = \sqrt{v_{0xB}^2 + v_{0yB}^2}$. Thus, we find

$$v_{0B} = \sqrt{(1.307\ \text{m/s})^2 + (8.693\ \text{m/s})^2} = 8.79\ \text{m/s}$$

or

$$v_{0B} = \sqrt{(46.175\ \text{m/s})^2 + (8.693\ \text{m/s})^2} = 47.0\ \text{m/s}$$

For these two possible launch speeds, we find the corresponding launch angles using the following drawings, neither of which is to scale"

$$\theta = \tan^{-1}\left(\frac{8.693\ \text{m/s}}{1.307\ \text{m/s}}\right) = 81.5° \qquad \theta = \tan^{-1}\left(\frac{8.693\ \text{m/s}}{46.175\ \text{m/s}}\right) = 10.7°$$

Since the problem states that $\theta_B > 45°$, the solution is $\boxed{v_{0\,B} = 8.79\ \text{m/s} \quad \text{and} \quad \theta_B = 81.5°}$.

46. ***REASONING AND SOLUTION*** Let H be the initial height of the can above the muzzle of the rifle. Relative to its initial position the vertical coordinate of the bullet at time, t, is

$$y = v_{oy}t - (1/2)gt^2$$

Relative to the can's initial position the vertical coordinate of the can at the *same* time is $y' = (1/2)gt^2$.

NOTE: $y = H - y'$ if the bullet hits the can. Then $y = v_{oy}t - y'$. The horizontal distance traveled by the bullet in time, t, is $x = v_{ox}t$.

Solving for t and substituting gives

$$y = (v_{oy}/v_{ox})x - y'$$

Now $v_{oy}/v_{ox} = \tan\theta$ and it is seen from the figure that $H = x \tan\theta$ if the bullet is to hit the can so $y = H - y'$. Hence, both objects are at the same place at the same time, and the bullet will always strike the can.

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47. $\boxed{\text{SSM}}$ ***REASONING*** The velocity $\mathbf{v}_{AB}$ of train **A** relative to train **B** is the vector sum of the velocity $\mathbf{v}_{AG}$ of train **A** relative to the ground and the velocity $\mathbf{v}_{GB}$ of the ground relative to train **B**, as indicated by Equation 3.7: $\mathbf{v}_{AB} = \mathbf{v}_{AG} + \mathbf{v}_{GB}$. The values of $\mathbf{v}_{AG}$ and $\mathbf{v}_{BG}$ are given in the statement of the problem. We must also make use of the fact that $\mathbf{v}_{GB} = -\mathbf{v}_{BG}$.

SOLUTION

a. Taking east as the positive direction, the velocity of **A** relative to **B** is, according to Equation 3.7,

$$\mathbf{v}_{AB} = \mathbf{v}_{AG} + \mathbf{v}_{GB} = \mathbf{v}_{AG} - \mathbf{v}_{BG} = (+13\text{ m/s}) - (-28\text{ m/s}) = \boxed{+41\text{ m/s}}$$

The positive sign indicates that the direction of $\mathbf{v}_{AB}$ is $\boxed{\text{due east}}$.

b. Similarly, the velocity of **B** relative to **A** is

$$\mathbf{v}_{BA} = \mathbf{v}_{BG} + \mathbf{v}_{GA} = \mathbf{v}_{BG} - \mathbf{v}_{AG} = (-28\text{ m/s}) - (+13\text{ m/s}) = \boxed{-41\text{ m/s}}$$

The negative sign indicates that the direction of $\mathbf{v}_{BA}$ is $\boxed{\text{due west}}$.

48. ***REASONING*** The time it takes for John to pass Chad is the distance between them divided by John's speed v_{JC} relative to Chad. We need, then to find John's velocity relative to Chad.

SOLUTION In computing the necessary relative velocity, we define the following symbols:

$\mathbf{v}_{\mathbf{JC}}$ = John's velocity relative to Chad
$\mathbf{v}_{\mathbf{JG}}$ = John's velocity relative to the ground
$\mathbf{v}_{\mathbf{GC}}$ = The ground's velocity relative to Chad
$\mathbf{v}_{\mathbf{CG}}$ = Chad's velocity relative to the ground

John's velocity relative to Chad is

$$\mathbf{v}_{\mathbf{JC}} = \mathbf{v}_{\mathbf{JG}} + \mathbf{v}_{\mathbf{GC}} = \mathbf{v}_{\mathbf{JG}} - \mathbf{v}_{\mathbf{CG}} = (4.50\ \text{m/s, north}) - (4.00\ \text{m/s, north}) = 0.50\ \text{m/s, north}$$

Here we have made use of the fact that $\mathbf{v}_{\mathbf{GC}} = -\mathbf{v}_{\mathbf{CG}}$. The time it takes for John to pass Chad is

$$t = \frac{95\ \text{m}}{v_{JC}} = \frac{95\ \text{m}}{0.50\ \text{m/s}} = \boxed{190\ \text{s}}$$

49. ***REASONING AND SOLUTION*** The speed of a person relative to the ground is

$$v = x/t = (105 \text{ m})/(75 \text{ s}) = 1.4 \text{ m/s}$$

If the person walks at this rate relative to the conveyor belt, his speed relative to the ground is

$$v' = 1.4 \text{ m/s} + 2.0 \text{ m/s} = 3.4 \text{ m/s}$$

Thus,

$$t' = x/v' = (105 \text{ m})/(3.4 \text{ m/s}) = \boxed{31 \text{ s}}$$

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50. ***REASONING*** The time it takes for the passenger to walk the distance on the boat is the distance divided by the passenger's speed v_{PB} relative to the boat. The time it takes for the passenger to cover the distance on the water is the distance divided by the passenger's speed v_{PW} relative to the water. The passenger's velocity relative to the boat is given. However, we need to determine the passenger's velocity relative to the water.

SOLUTION a. In determining the velocity of the passenger relative to the water, we define the following symbols:

$\mathbf{v_{PW}}$ = Passenger's velocity relative to the water

$\mathbf{v_{PB}}$ = Passenger's velocity relative to the boat

$\mathbf{v_{BW}}$ = Boat's velocity relative to the water

The passenger's velocity relative to the water is

$$\mathbf{v}_{\mathbf{PW}} = \mathbf{v}_{\mathbf{PB}} + \mathbf{v}_{\mathbf{BW}} = (1.5\ \text{m/s, north}) + (5.0\ \text{m/s, south}) = \boxed{3.5\ \text{m/s, south}}$$

b. The time it takes for the passenger to walk a distance of 27 m on the boat is

$$t = \frac{27\ \text{m}}{v_{PB}} = \frac{27\ \text{m}}{1.5\ \text{m/s}} = \boxed{18\ \text{s}}$$

c. The time it takes for the passenger to cover a distance of 27 m on the water is

$$t = \frac{27\ \text{m}}{v_{PW}} = \frac{27\ \text{m}}{3.5\ \text{m/s}} = \boxed{7.7\ \text{s}}$$

51. **SSM** ***REASONING*** The velocity $\mathbf{v}_{SG}$ of the swimmer relative to the ground is the vector sum of the velocity $\mathbf{v}_{SW}$ of the swimmer relative to the water and the velocity $\mathbf{v}_{WG}$ of the water relative to the ground as shown at the right: $\mathbf{v}_{SG} = \mathbf{v}_{SW} + \mathbf{v}_{WG}$.

The component of $\mathbf{v}_{SG}$ that is parallel to the width of the river determines how fast the swimmer is moving across the river; this parallel component is v_{SW}. The time for the swimmer to cross the river is equal to the width of the river divided by the magnitude of this velocity component.

The component of $\mathbf{v}_{SG}$ that is parallel to the direction of the current determines how far the swimmer is carried down stream; this component is v_{WG}. Since the motion occurs with constant velocity, the distance that the swimmer is carried downstream while crossing the river is equal to the magnitude of v_{WG} multiplied by the time it takes for the swimmer to cross the river.

SOLUTION

a. The time t for the swimmer to cross the river is

$$t = \frac{\text{width}}{v_{SW}} = \frac{2.8 \times 10^3 \text{ m}}{1.4 \text{ m/s}} = \boxed{2.0 \times 10^3 \text{ s}}$$

b. The distance x that the swimmer is carried downstream while crossing the river is

$$x = v_{WG}t = (0.91 \text{ m/s})(2.0 \times 10^3 \text{ s}) = \boxed{1.8 \times 10^3 \text{ m}}$$

52. ***REASONING*** As indicated by Equation 3.7, the velocity of the student relative to the ground is

$$\mathbf{v}_{SG} = \mathbf{v}_{SE} + \mathbf{v}_{EG}$$

where $\mathbf{v}_{SE}$ is the velocity of the student relative to the escalator and $\mathbf{v}_{EG}$ is the velocity of the escalator relative to the ground. Thus, we have

$$\mathbf{v}_{SE} = \mathbf{v}_{SG} - \mathbf{v}_{EG}$$

SOLUTION Taking the direction in which the student runs as the positive direction, we find that

$$v_{SE} = v_{SG} - v_{EG} = \frac{30.0 \text{ m}}{11 \text{ s}} - (-1.8 \text{ m/s}) = \boxed{4.5 \text{ m/s}}$$

The student must exceed this speed of 4.5 m/s to beat the record.

53. ***REASONING*** Let $\mathbf{v}_{HB}$ represent the velocity of the hawk relative to the balloon and $\mathbf{v}_{BG}$ represent the velocity of the balloon relative to the ground. Then, as indicated by Equation 3.7, the velocity of the hawk relative to the ground is $\mathbf{v}_{HG} = \mathbf{v}_{HB} + \mathbf{v}_{BG}$. Since the vectors $\mathbf{v}_{HB}$ and $\mathbf{v}_{BG}$ are at right angles to each other, the vector addition can be carried out using the Pythagorean theorem.

SOLUTION Using the drawing at the right, we have from the Pythagorean theorem,

$$v_{HG} = \sqrt{v_{HB}^2 + v_{BG}^2}$$

$$= \sqrt{(2.0\ \text{m/s})^2 + (6.0\ \text{m/s})^2} = \boxed{6.3\ \text{m/s}}$$

The angle θ is

$$\theta = \tan^{-1}\left(\frac{v_{HB}}{v_{BG}}\right) = \tan^{-1}\left(\frac{2.0\ \text{m/s}}{6.0\ \text{m/s}}\right) = \boxed{18°,\ \text{north of east}}$$

54. ***REASONING AND SOLUTION*** In the book's notation, $v_{AG} = 8.70$ m/s and $v_{PA} = 22.6$ m/s are the components of the plane's velocity vector. Hence,

$$v_{PG} = \sqrt{v_{AG}^2 + v_{PA}^2} = \sqrt{(8.70 \text{ m/s})^2 + (22.6 \text{ m/s})^2} = \boxed{24.2 \text{ m/s}}$$

Additionally,

$$\theta = \tan^{-1}(8.70/22.6) = \boxed{21.1°, \text{ south of east}}$$

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55. ***REASONING AND SOLUTION*** The velocity of the raindrops relative to the train is given by

$$v_{RT} = v_{RG} + v_{GT} .$$

where v_{RG} is the velocity of the raindrops relative to the ground and v_{GT} is the velocity of the ground relative to the train.

Since the train moves horizontally, and the rain falls vertically, the velocity vectors are related as shown in the figure at the right. Then

$$v_{GT} = v_{RG} \tan \theta = (5.0 \text{ m/s}) (\tan 25°) = 2.3 \text{ m/s}.$$

The train is moving at a speed of $\boxed{2.3 \text{ m/s}}$

56. ***REASONING AND SOLUTION*** In the book's notation, $\mathbf{v}_{PW}$, $\mathbf{v}_{BW}$ and $\mathbf{v}_{PB}$ form a triangle for which the law of cosines gives

$$v_{PW}^2 = v_{BW}^2 + v_{PB}^2 - 2v_{BW}v_{PB}\cos(180.0° - 35.1°)$$

$$= (5.12\text{ m/s})^2 + (2.71\text{ m/s})^2 + 2(5.12\text{ m/s})(2.71\text{ m/s})\cos 35.1°$$

so

$$v_{PW} = \boxed{7.50\text{ m/s}}$$

The law of sines gives

$$\sin\theta = (v_{BW}/v_{PW})\sin 35.1° \quad \text{or} \quad \theta = \boxed{23.1°\text{, north of east}}$$

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57. **SSM** ***REASONING*** The velocity $\mathbf{v}_{PM}$ of the puck relative to Mario is the vector sum of the velocity $\mathbf{v}_{PI}$ of the puck relative to the ice and the velocity $\mathbf{v}_{IM}$ of the ice relative to Mario as indicated by Equation 3.7: $\mathbf{v}_{PM} = \mathbf{v}_{PI} + \mathbf{v}_{IM}$. The values of $\mathbf{v}_{MI}$ and $\mathbf{v}_{PI}$ are given in the statement of the problem. In order to use the data, we must make use of the fact that $\mathbf{v}_{IM} = -\mathbf{v}_{MI}$, with the result that $\mathbf{v}_{PM} = \mathbf{v}_{PI} - \mathbf{v}_{MI}$.

SOLUTION The first two rows of the following table give the east/west and north/south components of the vectors $\mathbf{v}_{PI}$ and $-\mathbf{v}_{MI}$. The third row gives the components of their resultant $\mathbf{v}_{PM} = \mathbf{v}_{PI} - \mathbf{v}_{MI}$. Due east and due north have been taken as positive.

Vector	*East/West Component*	*North/South Component*
$\mathbf{v}_{PI}$	–(11.0 m/s) sin 22° = –4.1 m/s	–(11.0 m/s) cos 22° = –10.2 m/s
$-\mathbf{v}_{MI}$	0	+7.0 m/s
$\mathbf{v}_{PM} = \mathbf{v}_{PI} - \mathbf{v}_{MI}$	–4.1 m/s	–3.2 m/s

Now that the components of $\mathbf{v}_{PM}$ are known, the Pythagorean theorem can be used to find the magnitude.

$$v_{PM} = \sqrt{(-4.1\ \text{m/s})^2 + (-3.2\ \text{m/s})^2} = \boxed{5.2\ \text{m/s}}$$

The direction of $\mathbf{v}_{PM}$ is found from

$$\phi = \tan^{-1}\left(\frac{4.1\ \text{m/s}}{3.2\ \text{m/s}}\right) = \boxed{52°\ \text{west of south}}$$

58. ***REASONING AND SOLUTION*** Let x represent the width or the distance between opposite banks of the river. If we assume that the twins travel with constant velocity, then the time required to cross the river can be found from $v_p = x/t$, where v_p is the component of the velocity that is parallel to the width of the river. For either twin, the required time is, therefore,

$$t = \frac{x}{v_p}$$

The figure below shows the velocity vectors of the two twins relative to the water. The water is moving to the right.

From the figure, it is clear that for twin **A**, $v_p = v_{AW}$, while for twin **B**, $v_p = v_{BW} \sin 55°$. Therefore,

$$\frac{t_A}{t_B} = \frac{x/v_{AW}}{x/(v_{BW} \sin 55°)} = \frac{v_{BW} \sin 55°}{v_{AW}}$$

Since both twins can row at the same speed relative to the water, $v_{AW} = v_{BW}$, so

$$\frac{t_A}{t_B} = \sin 55° = \boxed{0.82}$$

59. SSM WWW ***REASONING*** The velocity $\mathbf{v}_{OW}$ of the object relative to the water is the vector sum of the velocity $\mathbf{v}_{OS}$ of the object relative to the ship and the velocity $\mathbf{v}_{SW}$ of the ship relative to the water, as indicated by Equation 3.7: $\mathbf{v}_{OW} = \mathbf{v}_{OS} + \mathbf{v}_{SW}$. The value of $\mathbf{v}_{SW}$ is given in the statement of the problem. We can find the value of $\mathbf{v}_{OS}$ from the fact that we know the position of the object relative to the ship at two different times. The initial position is $\mathbf{r}_{OS1}$, and the final position is $\mathbf{r}_{OS2}$. Since the object moves with constant velocity,

$$\mathbf{v}_{OS} = \frac{\Delta\mathbf{r}_{OS}}{\Delta t} = \frac{\mathbf{r}_{OS2} - \mathbf{r}_{OS1}}{\Delta t} \qquad (1)$$

SOLUTION The first two rows of the following table give the east/west and north/south components of the vectors $\mathbf{r}_{OS2}$ and $-\mathbf{r}_{OS1}$. The third row of the table gives the components of $\Delta\mathbf{r}_{OS} = \mathbf{r}_{OS2} - \mathbf{r}_{OS1}$. Due east and due north have been taken as positive.

Vector	*East/West Component*	*North/South Component*
$\mathbf{r}_{OS2}$	$-(1120\text{ m})\cos 57.0°$ $= -6.10\times10^{2}\text{ m}$	$-(1120\text{ m})\sin 57.0°$ $= -9.39\times10^{2}\text{ m}$
$-\mathbf{r}_{OS1}$	$-(2310\text{ m})\cos 32.0°$ $= -1.96\times10^{3}\text{ m}$	$+(2310\text{ m})\sin 32.0°$ $= 1.22\times10^{3}\text{ m}$
$\Delta\mathbf{r}_{OS} = \mathbf{r}_{OS2} - \mathbf{r}_{OS1}$	$-2.57\times10^{3}\text{ m}$	$2.81\times10^{2}\text{ m}$

Now that the components of $\Delta\mathbf{r}_{OS}$ are known, the Pythagorean theorem can be used to find the magnitude.

$$\Delta r_{OS} = \sqrt{(-2.57\times10^{3}\text{ m})^2 + (2.81\times10^{2}\text{ m})^2} = 2.59\times10^{3}\text{ m}$$

The direction of $\Delta\mathbf{r}_{OS}$ is found from

$$\phi = \tan^{-1}\left(\frac{2.81\times10^{2}\text{ m}}{2.57\times10^{3}\text{ m}}\right) = 6.24°$$

Therefore, from Equation (1),

$$\mathbf{v}_{OS} = \frac{\Delta\mathbf{r}_{OS}}{\Delta t} = \frac{\mathbf{r}_{OS2} - \mathbf{r}_{OS1}}{\Delta t} = \frac{2.59\times10^{3}\text{ m}}{360\text{ s}} = 7.19\text{ m/s, } 6.24°\text{ north of west}$$

Now that $\mathbf{v}_{OS}$ is known, we can find $\mathbf{v}_{OW}$, as indicated by Equation 3.7: $\mathbf{v}_{OW} = \mathbf{v}_{OS} + \mathbf{v}_{SW}$. The following table summarizes the vector addition:

Vector	*East/West Component*	*North/South Component*
$\mathbf{v}_{OS}$	$-(7.19\text{ m/s})\cos 6.24° = -7.15\text{ m/s}$	$(7.19\text{ m/s})\sin 6.24° = 0.782\text{ m/s}$
$\mathbf{v}_{SW}$	+4.20 m/s	0 m/s
$\mathbf{v}_{OW} = \mathbf{v}_{OS} + \mathbf{v}_{SW}$	–2.95 m/s	0.782 m/s

Now that the components of $\mathbf{v}_{OW}$ are known, the Pythagorean theorem can be used to find the magnitude.

$$v_{OW} = \sqrt{(-2.95\text{ m/s})^2 + (0.782\text{ m/s})^2} = \boxed{3.05\text{ m/s}}$$

The direction of $\mathbf{v}_{OW}$ is found from

$$\phi = \tan^{-1}\left(\frac{0.782\text{ m/s}}{2.95\text{ m/s}}\right) = \boxed{14.8°\text{ north of west}}$$

60. ***REASONING AND SOLUTION*** The batter ends up at third base, 27.4 m from home plate, where he started. Therefore, the magnitude of his displacement is simply $\boxed{27.4 \text{ m}}$.

61. ***REASONING AND SOLUTION*** Use the information concerning the x motion to find the time of flight of the ball

$$x = v_{ox}t \quad \text{or} \quad t = x/v_{ox} = (19.6\text{ m})/(28.0\text{ m/s}) = 0.700\text{ s}$$

The motion in the y direction is therefore subject to

$$y = v_{oy}t - (1/2)gt^2 = 0 - (1/2)(9.80\text{ m/s}^2)(0.700\text{ s})^2 = -2.40\text{ m}$$

The height of the tennis ball is $\boxed{2.40\text{ m}}$.

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62. ***REASONING AND SOLUTION***

a. The bullet strikes the ground when its y coordinate is – 1.6 m. We know

$$v_{oy} = v_o \sin 0° = 0$$

$$t = \sqrt{\frac{-2y}{g}} = \sqrt{\frac{-2(-1.6\text{ m})}{9.80\text{ m/s}^2}} = \boxed{0.57\text{ s}}$$

b. During this time the bullet travels a horizontal distance, x, with $a_x = 0$.

$$x = v_{ox}t = (v_o \cos \theta_o)t = (1100\text{ m/s})(\cos 0°)(0.57\text{ s}) = \boxed{630\text{ m}}$$

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63. **SSM** ***REASONING*** The time that the ball spends in the air is determined by its vertical motion. The time required for the ball to reach the lake can be found by solving Equation 3.5b for t. The motion of the golf ball is characterized by constant velocity in the x direction and accelerated motion (due to gravity) in the y direction. Thus, the x component of the velocity of the golf ball is constant, and the y component of the velocity at any time t can be found from Equation 3.3b. Once the x and y components of the velocity are known for a particular time t, the speed can be obtained from $v = \sqrt{v_x^2 + v_y^2}$.

SOLUTION

a. Since the ball rolls off the cliff horizontally, $v_{0y} = 0$. If the origin is chosen at top of the cliff and upward is assumed to be the positive direction, then the vertical component of the ball's displacement is $y = -15.5$ m. Thus, Equation 3.5b gives

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-15.5\text{ m})}{(-9.80\text{ m/s}^2)}} = \boxed{1.78\text{ s}}$$

b. Since there is no acceleration in the x direction, $v_x = v_{0x} = 11.4$ m/s.
The y component of the velocity of the ball just before it strikes the water is, according to Equation 3.3b,

$$v_y = v_{0y} + a_y t = \left[0 + (-9.80\text{ m/s}^2)(1.78\text{ s})\right] = -17.4\text{ m/s}$$

The speed of the ball just before it strikes the water is, therefore,

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(11.4\text{ m/s})^2 + (-17.4\text{ m/s})^2} = \boxed{20.8\text{ m/s}}$$

64. ***REASONING AND SOLUTION***

a. The x component of velocity is

$$v_x = v_{ox} + a_x t = 5480 \text{ m/s} + (1.20 \text{ m/s}^2)(842 \text{ s}) = \boxed{6490 \text{ m/s}}$$

b. For the y component

$$v_y = v_{oy} + a_y t = 0 \text{ m/s} + (8.40 \text{ m/s}^2)(842 \text{ s}) = \boxed{7070 \text{ m/s}}$$

65. **SSM** ***REASONING*** The speed of the fish at any time t is given by $v = \sqrt{v_x^2 + v_y^2}$, where v_x and v_y are the x and y components of the velocity at that instant. Since the horizontal motion of the fish has zero acceleration, $v_x = v_{0x}$ for all times t. Since the fish is dropped by the eagle, v_{0x} is equal to the horizontal speed of the eagle and $v_{0y} = 0$. The y component of the velocity of the fish for any time t is given by Equation 3.3b with $v_{0y} = 0$. Thus, the speed at any time t is given by $v = \sqrt{v_{0x}^2 + (a_y t)^2}$.

SOLUTION

a. The initial speed of the fish is $v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{v_{0x}^2 + 0^2} = v_{0x}$. When the fish's speed doubles, $v = 2v_{0x}$. Therefore,

$$2v_{0x} = \sqrt{v_{0x}^2 + (a_y t)^2} \qquad \text{or} \qquad 4v_{0x}^2 = v_{0x}^2 + (a_y t)^2$$

Assuming that downward is positive and solving for t, we have

$$t = \sqrt{3}\frac{v_{0x}}{a_y} = \sqrt{3}\left(\frac{6.0\ \text{m/s}}{9.80\ \text{m/s}^2}\right) = \boxed{1.1\ \text{s}}$$

b. When the fish's speed doubles again, $v = 4v_{0x}$. Therefore,

$$4v_{0x} = \sqrt{v_{0x}^2 + (a_y t)^2} \qquad \text{or} \qquad 16v_{0x}^2 = v_{0x}^2 + (a_y t)^2$$

Solving for t, we have

$$t = \sqrt{15}\frac{v_{0x}}{a_y} = \sqrt{15}\left(\frac{6.0\ \text{m/s}}{9.80\ \text{m/s}^2}\right) = 2.37\ \text{s}$$

Therefore, the additional time for the speed to double again is $(2.4\ \text{s}) - (1.1\ \text{s}) = \boxed{1.3\ \text{s}}$.

66. ***REASONING*** We begin by considering the flight time of the ball on the distant planet. Once the flight time is known, we can determine the maximum height and the range of the ball.

The range of a projectile is proportional to the time that the projectile is in the air. Therefore, the flight time on the distant plant 3.5 times larger than on earth. The flight time can be found from Equation 3.3b ($v_y = v_{0y} + a_y t$). When the ball lands, it is at the same level as the tee; therefore, from the symmetry of the motion $v_y = -v_{0y}$. Taking upward and to the right as the positive directions, we find that the flight time on earth would be

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-2v_{0y}}{a_y} = \frac{-2v_0 \sin\theta}{a_y} = \frac{-2(45\text{ m/s})\sin 29°}{-9.80\text{ m/s}^2} = 4.45\text{ s}$$

Therefore, the flight time on the distant planet is 3.5 × (4.45 s) = 15.6 s. From the symmetry of the problem, we know that this is twice the amount of time required for the ball to reach its maximum height, which, consequently, is 7.80 s.

SOLUTION

a. The height y of the ball at any instant is given by Equation 3.4b as the product of the average velocity component in the y direction $\frac{1}{2}\left(v_{0y} + v_y\right)$ and the time t: $y = \frac{1}{2}\left(v_{0y} + v_y\right)t$. Since the maximum height H is reached when the final velocity component in the y direction is zero ($v_y = 0$), we find that

$$H = \tfrac{1}{2}v_{0y}t = \tfrac{1}{2}v_0(\sin 29°)t = \tfrac{1}{2}(45\text{ m/s})(\sin 29°)(7.80\text{ s}) = \boxed{85\text{ m}}$$

b. The range of the ball on the distant planet is

$$x = v_{0x}t = v_0(\cos 29°)t = (45\text{ m/s})(\cos 29°)(15.6\text{ s}) = \boxed{610\text{ m}}$$

67. ***REASONING*** The time t_H required for the soccer ball to reach its maximum vertical displacement H can be found from Equation 3.3b ($v_y = v_{0y} + a_y t$) by setting $v_y = 0$ and solving for $t = t_H$ to obtain

$$t_H = -\frac{v_{0y}}{a_y} = -\frac{v_0 \sin\theta}{a_y}$$

According to Equation 3.4b $\left[y = \frac{1}{2}\left(v_{0y} + v_y\right)t\right]$, the maximum vertical displacement of the ball is, therefore,

$$H = y_{\max} = \tfrac{1}{2} v_{0y} t_H = \tfrac{1}{2}\left(v_0 \sin\theta\right) t_H$$

Combining the expressions for t_H and H, we find that the maximum height H depends on the angle θ according to the relation

$$H = \tfrac{1}{2}(v_0 \sin\theta)\left(\frac{-v_0 \sin\theta}{a_y}\right) = -\frac{v_0^2 \sin^2\theta}{2a_y}$$

SOLUTION Let the maximum height when $\theta_1 = 11.0°$ be designated by H_1. Then, for a different angle θ_2, we can form the ratio

$$\frac{H_1}{H_2} = \frac{v_0^2 \sin^2\theta_1 / (2a_y)}{v_0^2 \sin^2\theta_2 / (2a_y)} = \left(\frac{\sin\theta_1}{\sin\theta_2}\right)^2$$

Setting $H_2 = 2H_1$ (we are doubling the maximum height), and solving for θ_2, we obtain

$$\frac{H_1}{2H_1} = \frac{1}{2} = \left(\frac{\sin 11.0°}{\sin\theta_2}\right)^2 \qquad \text{or} \qquad \sin\theta_2 = \sqrt{2}\,\sin 11.0° = 0.270$$

$$\theta_2 = \sin^{-1} 0.270 = \boxed{15.7°}$$

68. ***REASONING*** As shown in the drawing, the angle that the velocity vector makes with the horizontal is given by

$$\tan\theta = \frac{v_y}{v_x}$$

where, from Equation 3.3b,

$$v_y = v_{0y} + a_y t = v_0 \sin\theta_0 + a_y t$$

and, from Equation 3.3a (since $a_x = 0$),

$$v_x = v_{0x} = v_0 \cos\theta_0$$

Therefore,

$$\tan\theta = \frac{v_y}{v_x} = \frac{v_0 \sin\theta_0 + a_y t}{v_0 \cos\theta_0}$$

SOLUTION Solving for *t*, we find

$$t = \frac{v_0\left(\cos\theta_0 \ \tan\theta - \sin\theta_0\right)}{a_y} = \frac{(29\ \text{m/s})(\cos 36° \ \tan 18° - \sin 36°)}{-9.80\ \text{m/s}^2} = \boxed{0.96\ \text{s}}$$

69. **SSM** ***REASONING*** The velocity $\mathbf{v}_{PG}$ of the plane relative to the ground is the vector sum of the velocity $\mathbf{v}_{PA}$ of the plane relative to the air and the velocity $\mathbf{v}_{AG}$ of the air relative to the ground, as indicated by Equation 3.7: $\mathbf{v}_{PG} = \mathbf{v}_{PA} + \mathbf{v}_{AG}$, or $\mathbf{v}_{AG} = \mathbf{v}_{PG} - \mathbf{v}_{PA}$. We are given $\mathbf{v}_{PA}$ and can find $\mathbf{v}_{PG}$ from the given displacement and travel time. Thus,

$$\mathbf{v}_{PG} = \frac{81.0 \times 10^3 \text{ m}}{9.00 \times 10^2 \text{ s}} = 90.0 \text{ m/s, } 45° \text{ west of south}$$

SOLUTION The first two rows of the following table give the east/west and north/south components of the vectors $\mathbf{v}_{PG}$ and $-\mathbf{v}_{PA}$. The third row gives the components of $\mathbf{v}_{AG} = \mathbf{v}_{PG} - \mathbf{v}_{PA}$. Due east and due north have been chosen as the positive directions.

Vector	*East/West Component*	*North/South Component*
$\mathbf{v}_{PG}$	–(90.0 m/s) sin 45° = –63.6 m/s	–(90.0 m/s) cos 45° = –63.6 m/s
$-\mathbf{v}_{PA}$	0 m/s	+57.8 m/s
$\mathbf{v}_{AG} = \mathbf{v}_{PG} - \mathbf{v}_{PA}$	–63.6 m/s	–5.8 m/s

The magnitude of the velocity of the wind with respect to the ground can be obtained using the Pythagorean theorem:

$$v_{AG} = \sqrt{(-63.6 \text{ m/s})^2 + (-5.8 \text{ m/s})^2} = \boxed{63.9 \text{ m/s}}$$

The direction of $\mathbf{v}_{AG}$ is found from

$$\phi = \tan^{-1}\left(\frac{63.6 \text{ m/s}}{5.8 \text{ m/s}}\right) = \boxed{85° \text{ west of south}}$$

70. ***REASONING AND SOLUTION*** In the book's notation $\mathbf{v}_{SW} = \mathbf{v}_{SP} + \mathbf{v}_{PW}$. The vectors form the sides of a triangle for which the law of cosines gives

$$v_{SP}^2 = v_{SW}^2 + v_{PW}^2 - 2v_{SW}v_{PW}\cos 45°$$

$$= (1.0\text{ m/s})^2 + (8.5\text{ m/s})^2 - 2(1.0\text{ m/s})(8.5\text{ m/s})\cos 45°, \quad \text{so} \quad v_{SP} = \boxed{7.8\text{ m/s}}$$

The law of sines gives the directional angle to be

$$\sin\theta = (v_{SW}/v_{SP})\sin 45° \quad \text{or} \quad \theta = 5.2° \text{ east of south}$$

This corresponds to $180.0° - 5.2° = \boxed{174.8° \text{ east of north}}$.

71. **SSM** ***REASONING AND SOLUTION*** In the absence of air resistance, the bullet exhibits projectile motion. The x component of the motion has zero acceleration while the y component of the motion is subject to the acceleration due to gravity. The horizontal distance traveled by the bullet is given by Equation 3.5a (with $a_x = 0$):

$$x = v_{0x}t = (v_0 \cos\theta)t$$

with t equal to the time required for the bullet to reach the target. The time t can be found by considering the vertical motion. From Equation 3.3b,

$$v_y = v_{0y} + a_y t$$

When the bullet reaches the target, $v_y = -v_{0y}$. Assuming that up and to the right are the positive directions, we have

$$t = \frac{-2v_{0y}}{a_y} = \frac{-2v_0 \sin\theta}{a_y} \qquad \text{and} \qquad x = (v_0 \cos\theta)\left(\frac{-2v_0 \sin\theta}{a_y}\right)$$

Using the fact that $2\sin\theta\cos\theta = \sin 2\theta$, we have

$$x = -\frac{2v_0^2 \cos\theta \sin\theta}{a_y} = -\frac{v_0^2 \sin 2\theta}{a_y}$$

Thus, we find that

$$\sin 2\theta = -\frac{x\,a_y}{v_0^2} = -\frac{(91.4\text{ m})(-9.80\text{ m/s}^2)}{(427\text{ m/s})^2} = 4.91 \times 10^{-3}$$

and

$$2\theta = 0.281° \quad \text{or} \quad 2\theta = 180.000° - 0.281° = 179.719°$$

Therefore,

$$\theta = \boxed{0.141° \text{ and } 89.860°}$$

72. ***REASONING AND SOLUTION*** While flying west, the airplane has a ground speed of

$$v_{PG} = 2.40 \times 10^2 \text{ m/s} - 57.8 \text{ m/s} = 182 \text{ m/s}$$

and requires time $t_W = x/(182 \text{ m/s})$ to reach the turn-around point. While flying east the airplane has a ground speed of

$$v_{PG} = 2.40 \times 10^2 \text{ m/s} + 57.8 \text{ m/s} = 298 \text{ m/s}$$

and requires $t_E = x/(298 \text{ m/s})$ to return home. Now the total time for the trip is $t = t_W + t_E = 6.00 \text{ h} = 2.16 \times 10^4 \text{ s}$, so

$$x/182 + x/298 = 2.16 \times 10^4 \quad \text{or} \quad x = \boxed{2.44 \times 10^6 \text{ m} = 2440 \text{ km}}$$

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73. ***REASONING AND SOLUTION*** The ball is caught at a position of $y = +0.914$ m and $x = v_F t + 1.10 \times 10^2$ m where t is the time of flight and v_F is the speed of the center fielder.

$$v_F = \frac{x - 1.10 \times 10^2 \text{ m}}{t} = v_{ox} - \frac{1.10 \times 10^2 \text{ m}}{t}$$

The time, t, can be found from

$$y = v_{oy}t - (1/2)\,gt^2 = (v_o \sin\theta_o)t - (1/2)\,gt^2 \quad \text{or} \quad 0.914 \text{ m} = (36.6 \text{ m/s})(\sin 50.0°)t - (4.9 \text{ m/s}^2)t^2$$

Rearrangement gives

$$4.90\,t^2 - 28.0t + 0.914 = 0 \quad \text{where the units have been suppressed.}$$

The quadratic formula gives $t = 5.68$ s. Hence

$$v_F = (36.6 \text{ m/s}) \cos 50.0^o - (1.10 \times 10^2 \text{ m})/(5.68 \text{ s}) = \boxed{4.2 \text{ m/s}}$$

1. ***REASONING AND SOLUTION*** According to Newton's second law, the acceleration is $a = \Sigma F/m$. Since the pilot and the plane have the same acceleration, we can write

$$\left(\frac{\Sigma F}{m}\right)_{\text{PILOT}} = \left(\frac{\Sigma F}{m}\right)_{\text{PLANE}} \qquad \text{or} \qquad (\Sigma F)_{\text{PILOT}} = m_{\text{PILOT}}\left(\frac{\Sigma F}{m}\right)_{\text{PLANE}}$$

Therefore, we find

$$(\Sigma F)_{\text{PILOT}} = (78\text{ kg})\left(\frac{3.7\times10^{4}\text{ N}}{3.1\times10^{4}\text{ kg}}\right) = \boxed{93\text{ N}}$$

2. ***REASONING AND SOLUTION*** The acceleration of the bicycle/rider combination is

$$a = \frac{\Sigma F}{m} = \frac{9.78\ \text{N}}{13.1\ \text{kg} + 81.7\ \text{kg}} = \boxed{0.103\ \text{m/s}^2}$$

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3. **SSM** ***REASONING*** According to Newton's second law of motion, the net force applied to the fist is equal to the mass of the fist multiplied by its acceleration. The data in the problem gives the final velocity of the fist and the time it takes to acquire that velocity. The average acceleration can be obtained directly from these data using the definition of average acceleration given in Equation 2.4.

SOLUTION The magnitude of the average net force applied to the fist is, therefore,

$$\sum \overline{F} = ma = m\left(\frac{\Delta v}{\Delta t}\right) = (0.70\text{ kg})\left(\frac{8.0\text{ m/s} - 0\text{ m/s}}{0.15\text{ s}}\right) = \boxed{37\text{ N}}$$

4. ***REASONING AND SOLUTION*** Using Equation 2.4 and assuming that $t_0 = 0$ s, we have for the required time that

$$t = \frac{v - v_0}{a}$$

Since $\Sigma F = ma$, it follows that

$$t = \frac{v - v_0}{\Sigma F / m} = \frac{m(v - v_0)}{\Sigma F} = \frac{(5.0\text{ kg})\left[(4.0 \times 10^3\text{ m/s}) - (0\text{ m/s})\right]}{4.9 \times 10^5\text{ N}} = \boxed{4.1 \times 10^{-2}\text{ s}}$$

5. $\boxed{\text{SSM}}$ ***REASONING*** The net force acting on the ball can be calculated using Newton's second law. Before we can use Newton's second law, however, we must use Equation 2.9 from the equations of kinematics to determine the acceleration of the ball.

SOLUTION According to Equation 2.9, the acceleration of the ball is given by

$$a = \frac{v^2 - v_0^2}{2x}$$

Thus, the magnitude of the net force on the ball is given by

$$\sum F = ma = m\left(\frac{v^2 - v_0^2}{2x}\right) = (0.058\text{ kg})\left[\frac{(45\text{ m/s})^2 - (0\text{ m/s})^2}{2(0.44\text{ m})}\right] = \boxed{130\text{ N}}$$

6. ***REASONING AND SOLUTION*** The acceleration is obtained from

$$x = v_0 t + \frac{1}{2} a t^2$$

where $v_0 = 0$ m/s. So

$$a = 2x/t^2$$

Newton's second law gives

$$\Sigma F = ma = m\left(\frac{2x}{t^2}\right) = (72\text{ kg})\left[\frac{2(18\text{ m})}{(0.95\text{ s})^2}\right] = \boxed{2900\text{ N}}$$

7. ***REASONING*** According to Newton's second law, Equation 4.1, the average net force $\Sigma\overline{F}$ is equal to the product of the object's mass m and the average acceleration $\overline{a}$. The average acceleration is equal to the change in velocity divided by the elapsed time (Equation 2.4), where the change in velocity is the final velocity v minus the initial velocity v_0.

SOLUTION The average net force exerted on the car and riders is

$$\Sigma\overline{F} = m\overline{a} = m\frac{v - v_0}{t - t_0} = \left(5.5 \times 10^3 \text{ kg}\right)\frac{45 \text{ m/s} - 0 \text{ m/s}}{7.0 \text{ s}} = \boxed{3.5 \times 10^4 \text{ N}}$$

8. ***REASONING AND SOLUTION*** From Equation 2.9,

$$v^2 = v_0^2 + 2ax$$

Since the arrow starts from rest, $v_0 = 0$ m/s. In both cases, x is the same so

$$\frac{v_1^2}{v_2^2} = \frac{2a_1x}{2a_2x} = \frac{a_1}{a_2} \quad \text{or} \quad \frac{v_1}{v_2} = \sqrt{\frac{a_1}{a_2}}$$

Since F = ma, it follows that a = F/m. The mass of the arrow is unchanged, and

$$\frac{v_1}{v_2} = \sqrt{\frac{F_1}{F_2}}$$

or

$$v_2 = v_1\sqrt{\frac{F_2}{F_1}} = v_1\sqrt{\frac{2F_1}{F_1}} = (25.0\ \text{m/s})\sqrt{2} = \boxed{35.4\ \text{m/s}}$$

9. **SSM** **WWW** ***REASONING*** Let due east be chosen as the positive direction. Then, when both forces point due east, Newton's second law gives

$$\underbrace{F_A + F_B}_{\Sigma F} = ma_1 \tag{1}$$

where $a_1 = 0.50\ \text{m/s}^2$. When F_A points due east and F_B points due west, Newton's second law gives

$$\underbrace{F_A - F_B}_{\Sigma F} = ma_2 \tag{2}$$

where $a_2 = 0.40\ \text{m/s}^2$. These two equations can be used to find the magnitude of each force.

SOLUTION

a. Adding Equations 1 and 2 gives

$$F_A = \frac{m(a_1 + a_2)}{2} = \frac{(8.0\ \text{kg})(0.50\ \text{m/s}^2 + 0.40\ \text{m/s}^2)}{2} = \boxed{3.6\ \text{N}}$$

b. Subtracting Equation 2 from Equation 1 gives

$$F_B = \frac{m(a_1 - a_2)}{2} = \frac{(8.0\ \text{kg})(0.50\ \text{m/s}^2 - 0.40\ \text{m/s}^2)}{2} = \boxed{0.40\ \text{N}}$$

10. ***REASONING AND SOLUTION***

$$F_E = F \cos \theta = (720 \text{ N}) \cos 38° = \boxed{570 \text{ N}}$$

$$F_N = F \sin \theta = (720 \text{ N}) \sin 38° = \boxed{440 \text{ N}}$$

11. ***REASONING*** According to Newton's second law, the acceleration has the same direction as the net force and a magnitude given by $a = \Sigma F/m$.

SOLUTION Since the two forces are perpendicular, the magnitude of the net force is given by the Pythagorean theorem as $\Sigma F = \sqrt{(40.0\text{ N})^2 + (60.0\text{ N})^2}$. Thus, according to Newton's second law, the magnitude of the acceleration is

$$a = \frac{\Sigma F}{m} = \frac{\sqrt{(40.0\text{ N})^2 + (60.0\text{ N})^2}}{4.00\text{ kg}} = \boxed{18.0\text{ m/s}^2}$$

The direction of the acceleration vector is given by

$$\theta = \tan^{-1}\left(\frac{60.0\text{ N}}{40.0\text{ N}}\right) = \boxed{56.3^\circ \text{ above the } +x \text{ axis}}$$

12. ***REASONING AND SOLUTION*** The free body diagram is shown at the right, where

$$F_1 = 45.0 \text{ N}$$
$$F_2 = 25.0 \text{ N}$$
$$\theta = 65.0°$$

When F_1 is replaced by its x and y components, we obtain the free body diagram in the second drawing.

If we choose the right as the positive direction, then

$$F_1 \cos\theta - F_2 = ma_x$$

or

$$a_x = \frac{F_1 \cos\theta - F_2}{m}$$

$$a_x = \frac{(45.0 \text{ N})\cos 65.0° - (25.0 \text{ N})}{5.00 \text{ kg}} = \boxed{-1.20 \text{ m/s}^2}$$

The minus sign indicates that the acceleration is to the $\boxed{\text{left}}$.

13. **SSM** ***REASONING*** According to Newton's second law ($\sum \mathbf{F} = m\mathbf{a}$), the acceleration of the object is given by $\mathbf{a} = \sum \mathbf{F} / m$, where $\sum \mathbf{F}$ is the net force that acts on the object. We must first find the net force that acts on the object, and then determine the acceleration using Newton's second law.

SOLUTION The following table gives the x and y components of the two forces that act on the object. The third row of that table gives the components of the net force.

Force	*x-Component*	*y-Component*
$\mathbf{F}_1$	40.0 N	0 N
$\mathbf{F}_2$	(60.0 N) cos 45.0° = 42.4 N	(60.0 N) sin 45.0° = 42.4 N
$\sum \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$	82.4 N	42.4 N

The magnitude of $\sum \mathbf{F}$ is given by the Pythagorean theorem as

$$\sum F = \sqrt{(82.4\text{ N})^2 + (42.4)^2} = 92.7\text{ N}$$

The angle θ that $\sum \mathbf{F}$ makes with the $+x$ axis is

$$\theta = \tan^{-1}\left(\frac{42.4\text{ N}}{82.4\text{ N}}\right) = 27.2°$$

According to Newton's second law, the magnitude of the acceleration of the object is

$$a = \frac{\sum F}{m} = \frac{92.7\text{ N}}{3.00\text{ kg}} = \boxed{30.9\text{ m/s}^2}$$

Since Newton's second law is a vector equation, we know that the direction of the right hand side must be equal to the direction of the left hand side. In other words, the direction of the acceleration a is the same as the direction of the net force $\sum \mathbf{F}$. Therefore, the direction of the acceleration of the object is $\boxed{27.2° \text{ above the } +x \text{ axis}}$.

14. ***REASONING*** Equations 3.5a $\left(x = v_{0x}t + \frac{1}{2}a_x t^2\right)$ and 3.5b $\left(y = v_{0y}t + \frac{1}{2}a_y t^2\right)$ give the displacements of an object under the influence of constant accelerations a_x and a_y. We can add these displacements as vectors to find the magnitude and direction of the resultant displacement. To use Equations 3.5a and 3.5b, however, we must have values for a_x and a_y. We can obtain these values from Newton's second law, provided that we combine the given forces to calculate the x and y components of the net force acting on the duck, and it is here that our solution begins.

SOLUTION Let the directions due east and due north, respectively, be the $+x$ and $+y$ directions. Then, the components of the net force are

$$\Sigma F_x = 0.10\text{ N} + (0.20\text{ N})\cos 52° = 0.2231\text{ N}$$

$$\Sigma F_y = -(0.20\text{ N})\sin 52° = -0.1576\text{ N}$$

According to Newton's second law, the components of the acceleration are

$$a_x = \frac{\Sigma F_x}{m} = \frac{0.2231\text{ N}}{2.5\text{ kg}} = 0.08924\text{ m/s}^2$$

$$a_y = \frac{\Sigma F_y}{m} = \frac{-0.1576\text{ N}}{2.5\text{ kg}} = -0.06304\text{ m/s}^2$$

From Equations 3.5a and 3.5b, we now obtain the displacements in the x and y directions:

$$x = v_{0x}t + \frac{1}{2}a_x t^2 = (0.11\text{ m/s})(3.0\text{ s}) + \frac{1}{2}(0.08924\text{ m/s}^2)(3.0\text{ s})^2 = 0.7316\text{ m}$$

$$y = v_{0y}t + \frac{1}{2}a_y t^2 = (0\text{ m/s})(3.0\text{ s}) + \frac{1}{2}(-0.06304\text{ m/s}^2)(3.0\text{ s})^2 = -0.2837\text{ m}$$

The magnitude of the resultant displacement is

$$r = \sqrt{x^2 + y^2} = \sqrt{(0.7316\text{ m})^2 + (-0.2837\text{ m})^2} = \boxed{0.78\text{ m}}$$

The direction of the resultant displacement is

$$\theta = \tan^{-1}\left(\frac{0.2837\text{ m}}{0.7316\text{ m}}\right) = \boxed{21°\text{ south of east}}$$

15. ***REASONING AND SOLUTION*** Combining Newton's second law with $v = v_0 + at$, we have

$$\Sigma F = ma = m\left(\frac{v - v_0}{t}\right) = (0.38\text{ kg})\left[\frac{(-2.0\text{ m/s}) - (+2.1\text{ m/s})}{3.3 \times 10^{-3}\text{ s}}\right] = \boxed{-470\text{ N}}$$

The answer is negative, indicating that the force is directed away from the cushion.

16. ***REASONING*** For both the tug and the asteroid, Equation 2.8 $\left(x = v_0 t + \frac{1}{2}at^2\right)$ applies with $v_0 = 0$ m/s, since both are initially at rest. In applying this equation, we must be careful and use the proper acceleration for each object. Newton's second law indicates that the acceleration is given by $a = \Sigma F/m$. In this expression, we note that the magnitudes of the net forces acting on the tug and the asteroid are the same, according to Newton's action-reaction law. The masses of the tug and the asteroid are different, however. Thus, the distance traveled for either object is given by, where we use for ΣF only the magnitude of the pulling force

$$x = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{\Sigma F}{m}\right)t^2$$

SOLUTION Let L be the initial distance between the tug and the asteroid. When the two objects meet, the distances that each has traveled must add up to equal L. Therefore,

$$L = x_T + x_A = \frac{1}{2}a_T t^2 + \frac{1}{2}a_A t^2$$

$$L = \frac{1}{2}\left(\frac{\Sigma F}{m_T}\right)t^2 + \frac{1}{2}\left(\frac{\Sigma F}{m_A}\right)t^2 = \frac{1}{2}\Sigma F\left(\frac{1}{m_T} + \frac{1}{m_A}\right)t^2$$

Solving for the time t gives

$$t = \sqrt{\frac{2L}{\Sigma F\left(\frac{1}{m_T} + \frac{1}{m_A}\right)}} = \sqrt{\frac{2(450\text{ m})}{(490\text{ N})\left(\frac{1}{3500\text{ kg}} + \frac{1}{6200\text{ kg}}\right)}} = \boxed{64\text{ s}}$$

17. SSM WWW ***REASONING*** We first determine the acceleration of the boat. Then, using Newton's second law, we can find the net force $\sum \mathbf{F}$ that acts on the boat. Since two of the three forces are known, we can solve for the unknown force $\mathbf{F}_W$ once the net force $\sum \mathbf{F}$ is known.

SOLUTION Let the direction due east be the positive x direction and the direction due north be the positive y direction. The x and y components of the initial velocity of the boat are then

$$v_{0x} = (2.00 \text{ m/s}) \cos 15.0° = 1.93 \text{ m/s}$$

$$v_{0y} = (2.00 \text{ m/s}) \sin 15.0° = 0.518 \text{ m/s}$$

Thirty seconds later, the x and y velocity components of the boat are

$$v_x = (4.00 \text{ m/s}) \cos 35.0° = 3.28 \text{ m/s}$$

$$v_y = (4.00 \text{ m/s}) \sin 35.0° = 2.29 \text{ m/s}$$

Therefore, according to Equations 3.3a and 3.3b, the x and y components of the acceleration of the boat are

$$a_x = \frac{v_x - v_{0x}}{t} = \frac{3.28 \text{ m/s} - 1.93 \text{ m/s}}{30.0 \text{ s}} = 4.50 \times 10^{-2} \text{ m/s}^2$$

$$a_y = \frac{v_y - v_{0y}}{t} = \frac{2.29 \text{ m/s} - 0.518 \text{ m/s}}{30.0 \text{ s}} = 5.91 \times 10^{-2} \text{ m/s}^2$$

Thus, the x and y components of the net force that act on the boat are

$$\sum F_x = ma_x = (325 \text{ kg})(4.50 \times 10^{-2} \text{ m/s}^2) = 14.6 \text{ N}$$

$$\sum F_y = ma_y = (325 \text{ kg})(5.91 \times 10^{-2} \text{ m/s}^2) = 19.2 \text{ N}$$

The following table gives the x and y components of the net force $\sum \mathbf{F}$ and the two known forces that act on the boat. The fourth row of that table gives the components of the unknown force $\mathbf{F}_W$.

Force	*x-Component*	*y-Component*
$\sum \mathbf{F}$	14.6 N	19.2 N
$\mathbf{F}_1$	(31.0 N) cos 15.0° = 29.9 N	(31.0 N) sin 15.0° = 8.02 N
$\mathbf{F}_2$	–(23.0 N) cos 15.0° = –22.2 N	–(23.0 N) sin 15.0° = –5.95 N
$\mathbf{F}_W = \sum \mathbf{F} - \mathbf{F}_1 - \mathbf{F}_2$	14.6 N – 29.9 N + 22.2 N = 6.9 N	19.2 N – 8.02 N + 5.95 N = 17.1 N

The magnitude of $\mathbf{F}_W$ is given by the Pythagorean theorem as

$$F_W = \sqrt{(6.9\text{ N})^2 + (17.1\text{ N})^2} = \boxed{18.4\text{ N}}$$

The angle θ that $\mathbf{F}_W$ makes with the x axis is

$$\theta = \tan^{-1}\left(\frac{17.1\text{ N}}{6.9\text{ N}}\right) = 68°$$

Therefore, the direction of $\mathbf{F}_W$ is $\boxed{68°,\text{ north of east}}$.

18. ***REASONING AND SOLUTION*** Newton's law of gravitation gives

$$F = \frac{Gm_1m_2}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.50\times10^{-3}\ \text{kg}\right)\left(870.0\times10^{-3}\ \text{kg}\right)}{\left(0.100\ \text{m}\right)^2} = \boxed{8.70\times10^{-12}\ \text{N}}$$

19. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION***

a. According to Equation 4.5, the weight of the space traveler of mass $\boxed{m = 115 \text{ kg}}$ on earth is

$$W = mg = (115 \text{ kg})\,(9.80 \text{ m/s}^2) = \boxed{1.13 \times 10^3 \text{ N}}$$

b. In interplanetary space where there are no nearby planetary objects, the gravitational force exerted on the space traveler is zero and $g = 0 \text{ m/s}^2$. Therefore, the weight is $\boxed{W = 0 \text{ N}}$. Since the mass of an object is an intrinsic property of the object and is independent of its location in the universe, the mass of the space traveler is still $\boxed{m = 115 \text{ kg}}$.

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20. ***REASONING AND SOLUTION*** The forces that act on the rock are shown at the right. Newton's second law (with the direction of motion as positive) is

$$\Sigma F = mg - R = ma$$

Solving for the acceleration a gives

$$a = \frac{mg - R}{m} = \frac{(45\text{ kg})(9.80\text{ m/s}^2) - (250\text{ N})}{45\text{ kg}} = \boxed{4.2\text{ m/s}^2}$$

21. ***REASONING*** The magnitude of the gravitational force that each part exerts on the other is given by Newton's law of gravitation as $F = Gm_1m_2/r^2$. To use this expression, we need the masses m_1 and m_2 of the parts, whereas the problem statement gives the weights W_1 and W_2. However, the weight is related to the mass by $W = mg$, so that for each part we know that $m = W/g$.

SOLUTION The gravitational force that each part exerts on the other is

$$F = \frac{Gm_1m_2}{r^2} = \frac{G(W_1/g)(W_2/g)}{r^2}$$

$$= \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(11\,000\ \text{N})(3400\ \text{N})}{\left(9.80\ \text{m/s}^2\right)^2(12\ \text{m})^2} = \boxed{1.8\times10^{-7}\ \text{N}}$$

22. ***REASONING*** Each particle experiences two gravitational forces, one due to each of the remaining particles. To get the net gravitational force, we must add the two contributions, taking into account the directions. The magnitude of the gravitational force that any one particle exerts on another is given by Newton's law of gravitation as $F = Gm_1m_2/r^2$. Thus, for particle A, we need to apply this law to its interaction with particle B and with particle C. For particle B, we need to apply the law to its interaction with particle A and with particle C. Lastly, for particle C, we must apply the law to its interaction with particle A and with particle B. In considering the directions, we remember that the gravitational force between two particles is always a force of attraction.

SOLUTION We begin by calculating the magnitude of the gravitational force for each pair of particles:

$$F_{AB} = \frac{Gm_Am_B}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(363\ \text{kg})(517\ \text{kg})}{(0.500\ \text{m})^2} = 5.007\times10^{-5}\ \text{N}$$

$$F_{BC} = \frac{Gm_Bm_C}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(517\ \text{kg})(154\ \text{kg})}{(0.500\ \text{m})^2} = 8.497\times10^{-5}\ \text{N}$$

$$F_{AC} = \frac{Gm_Am_C}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(363\ \text{kg})(154\ \text{kg})}{(0.500\ \text{m})^2} = 6.629\times10^{-6}\ \text{N}$$

In using these magnitudes we take the direction to the right as positive.

a. Both particles B and C attract particle A to the right, the net force being

$$F_A = F_{AB} + F_{AC} = 5.007\times10^{-5}\ \text{N} + 6.629\times10^{-6}\ \text{N} = \boxed{5.67\times10^{-5}\ \text{N, right}}$$

b. Particle C attracts particle B to the right, while particle A attracts particle B to the left, the net force being

$$F_B = F_{BC} - F_{AB} = 8.497\times10^{-5}\ \text{N} - 5.007\times10^{-5}\ \text{N} = \boxed{3.49\times10^{-5}\ \text{N, right}}$$

c. Both particles A and B attract particle C to the left, the net force being

$$F_C = F_{AC} + F_{BC} = 6.629\times10^{-6}\ \text{N} + 8.497\times10^{-5}\ \text{N} = \boxed{9.16\times10^{-5}\ \text{N, left}}$$

23. **SSM** ***REASONING AND SOLUTION***

a. Combining Equations 4.4 and 4.5, we see that the acceleration due to gravity on the surface of Saturn can be calculated as follows:

$$g_{\text{Saturn}} = G\frac{M_{\text{Saturn}}}{r_{\text{Saturn}}^2} = \left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\frac{\left(5.67\times10^{26}\ \text{kg}\right)}{\left(6.00\times10^{7}\ \text{m}\right)^2} = \boxed{10.5\ \text{m/s}^2}$$

b. The ratio of the person's weight on Saturn to that on earth is

$$\frac{W_{\text{Saturn}}}{W_{\text{earth}}} = \frac{mg_{\text{Saturn}}}{mg_{\text{earth}}} = \frac{g_{\text{Saturn}}}{g_{\text{earth}}} = \frac{10.5\ \text{m/s}^2}{9.80\ \text{m/s}^2} = \boxed{1.07}$$

24. ***REASONING AND SOLUTION*** The magnitude of the net force acting on the moon is found by the Pythagorean theorem to be

$$F = \sqrt{F_{SM}^2 + F_{EM}^2}$$

Newton's law of gravitation applied to the sun-moon (the units have been suppressed)

$$F_{SM} = G\frac{m_S m_M}{r_{SM}^2} = (6.67 \times 10^{-11})\frac{(1.99 \times 10^{30})(7.35 \times 10^{22})}{(1.50 \times 10^{11}\ \text{m})^2} = 4.34 \times 10^{20}\text{N}.$$

A similar application to the earth-moon gives

$$\Phi_{EM} = \Gamma\frac{\mu_E \mu_M}{\rho_{EM}^2} = (6.67 \times 10^{-11})\frac{(5.98 \times 10^{24})(7.35 \times 10^{22})}{(3.85 \times 10^{8}\ \text{m})^2} = 1.98 \times 10^{20}\text{N}.$$

The net force on the moon is then

$$F = \sqrt{\left(4.34 \times 10^{20}\ \text{N}\right)^2 + \left(1.98 \times 10^{20}\ \text{N}\right)^2} = \boxed{4.77 \times 10^{20}\ \text{N}}$$

25. ***REASONING*** According to Equation 4.4, the weights of an object of mass m on the surfaces of planet A (mass = M_A, radius = R) and planet B (mass = M_B , radius = R) are

$$W_A = \frac{GM_A m}{R^2} \quad \text{and} \quad W_B = \frac{GM_B m}{R^2}$$

The difference between these weights is given in the problem.

SOLUTION The difference in weights is

$$W_A - W_B = \frac{GM_A m}{R^2} - \frac{GM_B m}{R^2} = \frac{Gm}{R^2}\left(M_A - M_B\right)$$

Rearranging this result, we find

$$M_A - M_B = \frac{\left(W_A - W_B\right)R^2}{Gm} = \frac{(3620\ \text{N})\left(1.33 \times 10^7\ \text{m}\right)^2}{(6.67 \times 10^{-11}\ \text{N} \cdot \text{m}^2/\text{kg}^2)(5450\ \text{kg})} = \boxed{1.76 \times 10^{24}\ \text{kg}}$$

26. ***REASONING*** The acceleration due to gravity at the surface of a planet is given by Equations 4.4 and 4.5 as $g_{\text{Planet}} = GM/r^2$, where M is the mass of the planet and r is its radius. By applying this relation to the unknown planet and to the earth, and noting the ratios of the masses and radii, we will be able to find the acceleration due to gravity on the surface of the planet.

SOLUTION Taking the ratio of g_{Planet} to g_{Earth}, and noting that $M_{\text{Planet}}/M_{\text{Earth}} = 0.10$ and $r_{\text{Earth}}/r_{\text{Planet}} = 1/0.50$, we have that

$$\frac{g_{\text{Planet}}}{g_{\text{Earth}}} = \frac{\dfrac{GM_{\text{Planet}}}{r_{\text{Planet}}^2}}{\dfrac{GM_{\text{Earth}}}{r_{\text{Earth}}^2}} = \left(\frac{M_{\text{Planet}}}{M_{\text{Earth}}}\right)\left(\frac{r_{\text{Earth}}}{r_{\text{Planet}}}\right)^2 = (0.10)\left(\frac{1}{0.50}\right)^2 = 0.40$$

Therefore, the acceleration due to gravity on the planet is

$$g_{\text{Planet}} = (0.40)g_{\text{Earth}} = (0.40)\left(9.80\ \text{m/s}^2\right) = \boxed{3.9\ \text{m/s}^2}$$

27. **SSM** ***REASONING AND SOLUTION*** According to Equations 4.4 and 4.5, the weight of an object of mass m at a distance r from the *center* of the earth is

$$mg = \frac{GM_E m}{r^2}$$

In a circular orbit that is 3.59×10^7 m above the surface of the earth (radius $= 6.38 \times 10^6$ m, mass $= 5.98 \times 10^{24}$ kg), the total distance from the center of the earth is $r = 3.59 \times 10^7 \text{ m} + 6.38 \times 10^6 \text{ m}$. Thus the acceleration g due to gravity is

$$g = \frac{GM_E}{r^2} = \frac{(6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}{(3.59 \times 10^7 \text{ m} + 6.38 \times 10^6 \text{ m})^2} = \boxed{0.223 \text{ m/s}^2}$$

28. ***REASONING AND SOLUTION*** The figure at the right shows the three spheres with sphere 3 being the sphere of unknown mass. Sphere 3 feels a force $\mathbf{F}_{31}$ due to the presence of sphere 1, and a force $\mathbf{F}_{32}$ due to the presence of sphere 2. The net force on sphere 3 is the resultant of $\mathbf{F}_{31}$ and $\mathbf{F}_{32}$.

Note that since the spheres form an equilateral triangle, each interior angle is 60°. Therefore, both $\mathbf{F}_{31}$ and $\mathbf{F}_{32}$ make a 30° angle with the vertical line as shown.

Furthermore, $\mathbf{F}_{31}$ and $\mathbf{F}_{32}$ have the same magnitude given by

$$F = \frac{GMm_3}{r^2}$$

where M is the mass of either sphere 1 or 2 and m_3 is the mass of sphere 3. The components of the two forces are shown in the following drawings:

Clearly, the horizontal components of the two forces add to zero. Thus, the net force on sphere 3 is the resultant of the vertical components of $\mathbf{F}_{31}$ and $\mathbf{F}_{32}$:

$$F_3 = 2F\cos\theta = 2\frac{GMm_3}{r^2}\cos\theta.$$

The acceleration of sphere 3 is given by Newton's second law:

$$a_3 = \frac{F_3}{m_3} = 2\frac{GM}{r^2}\cos\theta = 2\frac{(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2})(2.80\,\mathrm{kg})}{(1.20\,\mathrm{m})^2}\cos 30.0°$$

$$= \boxed{2.25\times10^{-10}\ \mathrm{m/s^2}}$$

29. ***REASONING*** According to Equation 4.4, the weights of an object of mass m on the surface of a planet (mass = M, radius = R) and at a height H above the surface are

$$\underbrace{W = \frac{GMm}{R^2}}_{\text{On surface}} \quad \text{and} \quad \underbrace{W_H = \frac{GMm}{(R+H)^2}}_{\text{At height } H \text{ above surface}}$$

The fact that W is one percent less than W_H tells us that the $W_H/W = 0.9900$, which is the starting point for our solution.

SOLUTION The ratio W_H/W is

$$\frac{W_H}{W} = \frac{\dfrac{GMm}{(R+H)^2}}{\dfrac{GMm}{R^2}} = \frac{R^2}{(R+H)^2} = \frac{1}{(1+H/R)^2} = 0.9900$$

Solving for H/R gives

$$1 + \frac{H}{R} = \sqrt{\frac{1}{0.9900}} \quad \text{or} \quad \frac{H}{R} = \boxed{0.0050}$$

30. ***REASONING*** The gravitational force that the sun exerts on a person standing on the earth is given by Equation 4.3 as $F_{\text{sun}} = GM_{\text{sun}}m / r^2_{\text{sun-earth}}$, where M_{sun} is the mass of the sun, m is the mass of the person, and $r_{\text{sun-earth}}$ is the distance from the sun to the earth. Likewise, the gravitational force that the moon exerts on a person standing on the earth is given by $F_{\text{moon}} = GM_{\text{moon}}m / r^2_{\text{moon-earth}}$, where M_{moon} is the mass of the moon and $r_{\text{moon-earth}}$ is the distance from the moon to the earth. These relations will allow us to determine whether the sun or the moon exerts the greater gravitational force on the person.

SOLUTION Taking the ratio of F_{sun} to F_{moon}, and using the mass and distance data found on the inside of the front cover of the text, we have that

$$\frac{F_{\text{sun}}}{F_{\text{moon}}} = \frac{\dfrac{GM_{\text{sun}}m}{r^2_{\text{sun-earth}}}}{\dfrac{GM_{\text{moon}}m}{r^2_{\text{moon-earth}}}} = \left(\frac{M_{\text{sun}}}{M_{\text{moon}}}\right)\left(\frac{r_{\text{moon-earth}}}{r_{\text{sun-earth}}}\right)^2$$

$$= \left(\frac{1.99 \times 10^{30}\ \text{kg}}{7.35 \times 10^{22}\ \text{kg}}\right)\left(\frac{3.85 \times 10^{8}\ \text{m}}{1.50 \times 10^{11}\ \text{m}}\right)^2 = \boxed{178}$$

Therefore, the sun exerts the greater gravitational force.

31. SSM WWW ***REASONING AND SOLUTION*** There are two forces that act on the balloon; they are, the combined weight of the balloon and its load, Mg, and the upward buoyant force F_B. If we take upward as the positive direction, then, initially when the balloon is motionless, Newton's second law gives $F_B - Mg = 0$. If an amount of mass m is dropped overboard so that the balloon has an upward acceleration, Newton's second law for this situation is

$$F_B - (M - m)g = (M - m)a$$

But $F_B = mg$, so that

$$Mg - (M - m)g = mg = (M - m)a$$

Solving for the mass m that should be dropped overboard, we obtain

$$m = \frac{Ma}{g + a} = \frac{(310\text{ kg})(0.15\text{ m/s}^2)}{9.80\text{ m/s}^2 + 0.15\text{ m/s}^2} = \boxed{4.7\text{ kg}}$$

32. ***REASONING AND SOLUTION*** The acceleration due to gravity at the surface of the neutron star is

$$a = \frac{Gm}{r^2} = \frac{(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2})(2.0\times10^{30}\,\mathrm{kg})}{(5.0\times10^{3}\,\mathrm{m})^{\,2}} = 5.3\times10^{12}\ \mathrm{m/s^2}$$

Since the gravitational force is assumed to be constant, the acceleration will be constant and the speed of the object can be calculated from $v^2 = v_0^2 + 2ay$, with $v_0 = 0\ \mathrm{m/s}$ since the object falls from rest. Solving for v yields

$$v = \sqrt{2ay} = \sqrt{2\left(5.3\times10^{12}\ \mathrm{m/s^2}\right)\left(0.010\ \mathrm{m}\right)} = \boxed{3.3\times10^{5}\ \mathrm{m/s}}$$

---

33. ***REASONING*** We place the third particle (mass = m_3) as shown in the following drawing:

The magnitude of the gravitational force that one particle exerts on another is given by Newton's law of gravitation as $F = Gm_1m_2/r^2$. Before the third particle is in place, this law indicates that the force on each particle has a magnitude $F_{\text{before}} = Gm2m/L^2$. After the third particle is in place, each of the first two particles experiences a greater net force, because the third particle also exerts a gravitational force on them.

SOLUTION For the particle of mass m, we have

$$\frac{F_{\text{after}}}{F_{\text{before}}} = \frac{\dfrac{Gmm_3}{D^2} + \dfrac{Gm2m}{L^2}}{\dfrac{Gm2m}{L^2}} = \frac{L^2 m_3}{2mD^2} + 1$$

For the particle of mass $2m$, we have

$$\frac{F_{\text{after}}}{F_{\text{before}}} = \frac{\dfrac{G2mm_3}{(L-D)^2} + \dfrac{Gm2m}{L^2}}{\dfrac{Gm2m}{L^2}} = \frac{L^2 m_3}{m(L-D)^2} + 1$$

Since $F_{\text{after}}/F_{\text{before}} = 2$ for both particles, we have

$$\frac{L^2 m_3}{2mD^2} + 1 = \frac{L^2 m_3}{m(L-D)^2} + 1 \quad \text{or} \quad 2D^2 = (L-D)^2$$

Expanding and rearranging this result gives $D^2 + 2LD - L^2 = 0$, which can be solved for D using the quadratic formula:

$$D = \frac{-2L \pm \sqrt{(2L)^2 - 4(1)(-L^2)}}{2(1)} = 0.414L \quad \text{or} \quad -2.414L$$

The negative solution is discarded because the third particle lies on the $+x$ axis between m and $2m$. Thus, $D = \boxed{0.414\,L}$.

34. ***REASONING*** In each case the object is in equilibrium. According to Equation 4.9b, $\Sigma F_y = 0$, the net force acting in the y (vertical) direction must be zero. The net force is composed of the weight of the object(s) and the normal force exerted on them.

SOLUTION

a. There are three vertical forces acting on the crate: an upward normal force $+F_N$ that the floor exerts, the weight $-m_1g$ of the crate, and the weight $-m_2g$ of the person standing on the crate. Since the weights act downward, they are assigned negative numbers. Setting the sum of these forces equal to zero gives

$$\underbrace{F_N + (-m_1g) + (-m_2g)}_{\Sigma F_y} = 0$$

The magnitude of the normal force is

$$F_N = m_1g + m_2g = (35\text{ kg} + 65\text{ kg})(9.80\text{ m/s}^2) = \boxed{980\text{ N}}$$

b. There are only two vertical forces acting on the person: an upward normal force $+F_N$ that the crate exerts and the weight $-m_2g$ of the person. Setting the sum of these forces equal to zero gives

$$\underbrace{F_N + (-m_2g)}_{\Sigma F_y} = 0$$

The magnitude of the normal force is

$$F_N = m_2g = (65\text{ kg})(9.80\text{ m/s}^2) = \boxed{640\text{ N}}$$

35. **SSM** ***REASONING AND SOLUTION*** According to Equation 3.3b, the acceleration of the astronaut is $a_y = (v_y - v_{0y})/t = v_y/t$. The apparent weight and the true weight of the astronaut are related according to Equation 4.6. Direct substitution gives

$$\underbrace{F_N}_{\substack{\text{Apparent}\\\text{weight}}} = \underbrace{mg}_{\substack{\text{True}\\\text{weight}}} + ma_y = m\,(g + a_y) = m\left(g + \frac{v_y}{t}\right)$$

$$= (57\text{ kg})\left(9.80\text{ m/s}^2 + \frac{45\text{ m/s}}{15\text{ s}}\right) = \boxed{7.3\times10^2\text{ N}}$$

36. ***REASONING AND SOLUTION***

a. The apparent weight of the person is

$$F_N = mg + ma$$

$$= (95.0\ \text{kg})(9.80\ \text{m/s}^2 + 1.80\ \text{m/s}^2) = \boxed{1.10 \times 10^3\ \text{N}}$$

b. $$F_N = (95.0\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{931\ \text{N}}$$

c. $$F_N = (95.0\ \text{kg})(9.80\ \text{m/s}^2 - 1.30\ \text{m/s}^2) = \boxed{808\ \text{N}}$$

37. ***REASONING AND SOLUTION*** The block will move only if the applied force is greater than the maximum static frictional force acting on the block. That is, if

$$F > \mu_s F_N = \mu_s mg = (0.650)(45.0\text{ N}) = 29.2\text{ N}$$

The applied force is given to be F = 36.0 N which is greater than the maximum static frictional force, so $\boxed{\text{the block will move}}$.

The block's acceleration is found from Newton's second law.

$$a = \frac{\Sigma F}{m} = \frac{F - f_k}{m} = \frac{F - \mu_k mg}{m} = \boxed{3.72\text{ m/s}^2}$$

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38. ***REASONING*** It is the static friction force that accelerates the cup when the plane accelerates. The maximum magnitude that this force can have will determine the maximum acceleration, according to Newton's second law.

SOLUTION According to Newton's second law, we have

$$\Sigma F = f_s^{MAX} = \mu_s F_N = \mu_s mg = ma$$

In this result, we have used the fact that the magnitude of the normal force is $F_N = mg$, since the plane is flying horizontally and the normal force acting on the cup balances the cup's weight. Solving for the acceleration gives

$$a = \mu_s g = (0.30)\left(9.80 \text{ m/s}^2\right) = \boxed{2.9 \text{ m/s}^2}$$

39. $\boxed{\text{SSM}}$ ***REASONING*** In order to start the crate moving, an external agent must supply a force that is at least as large as the maximum value $f_s^{MAX} = \mu_s F_N$, where μ_s is the coefficient of static friction (see Equation 4.7). Once the crate is moving, the magnitude of the frictional force is very nearly constant at the value $f_k = \mu_k F_N$, where μ_k is the coefficient of kinetic friction (see Equation 4.8). In both cases described in the problem statement, there are only two vertical forces that act on the crate; they are the upward normal force $\mathbf{F}_N$, and the downward pull of gravity (the weight) mg. Furthermore, the crate has no vertical acceleration in either case. Therefore, if we take upward as the positive direction, Newton's second law in the vertical direction gives $F_N - mg = 0$, and we see that, in both cases, the magnitude of the normal force is $F_N = mg$.

SOLUTION

a. Therefore, the applied force needed to start the crate moving is

$$f_s^{MAX} = \mu_s mg = (0.760)(60.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{447\text{ N}}$$

b. When the crate moves in a straight line at constant speed, its velocity does not change, and it has zero acceleration. Thus, Newton's second law in the horizontal direction becomes $\mathbf{P} - \mathbf{f}_k = 0$, where P is the required pushing force. Thus, the applied force required to keep the crate sliding across the dock at a constant speed is

$$P = f_k = \mu_k mg = (0.410)(60.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{241\text{ N}}$$

40. ***REASONING AND SOLUTION***

a. $$f_k = \mu_k mg = (0.61)(92\text{ kg})(9.80\text{ m/s}^2) = \boxed{550\text{ N}}$$

b. From kinematics

$$v_0 = -at = -\left(\frac{f_k}{m}\right)t = -\left(\frac{-550\text{ N}}{92\text{ kg}}\right)(1.2\text{ s}) = \boxed{7.2\text{ m/s}}$$

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41. ***REASONING*** In each of the three cases under consideration the kinetic frictional force is given by $f_k = \mu_k F_N$. However, the normal force F_N varies from case to case. To determine the normal force, we use Equation 4.6 ($F_N = mg + ma$) and thereby take into account the acceleration of the elevator. The normal force is greatest when the elevator accelerates upward (a positive) and smallest when the elevator accelerates downward (a negative).

SOLUTION

a. When the elevator is stationary, its acceleration is $a = 0$ m/s^2. Using Equation 4.6, we can express the kinetic frictional force as

$$f_k = \mu_k F_N = \mu_k (mg + ma) = \mu_k m(g + a)$$

$$= (0.360)(6.00 \text{ kg})\left[(9.80 \text{ m/s}^2) + (0 \text{ m/s}^2)\right] = \boxed{21.2 \text{ N}}$$

b. When the elevator accelerates upward, $a = +1.20$ m/s^2. Then,

$$f_k = \mu_k F_N = \mu_k (mg + ma) = \mu_k m(g + a)$$

$$= (0.360)(6.00 \text{ kg})\left[(9.80 \text{ m/s}^2) + (1.20 \text{ m/s}^2)\right] = \boxed{23.8 \text{ N}}$$

c. When the elevator accelerates downward, $a = -1.20$ m/s^2. Then,

$$f_k = \mu_k F_N = \mu_k (mg + ma) = \mu_k m(g + a)$$

$$= (0.360)(6.00 \text{ kg})\left[(9.80 \text{ m/s}^2) + (-1.20 \text{ m/s}^2)\right] = \boxed{18.6 \text{ N}}$$

42. ***REASONING*** The initial and final velocities of the car are known, so we may use Equation 2.9 ($v_x^2 = v_{0x}^2 + 2a_x x$) of the equations of kinematics to determine the stopping displacement x, provided the acceleration a_x can be found. The acceleration is related to the net force ΣF_x acting on the car by Equation 4.2, $\sum F_x = ma_x$, where m is its mass. The only force acting on the car in the x-direction is the static frictional force that acts on the tires. The minimum stopping distance occurs when the static frictional force is a maximum, so the net force can be expressed with the aid of Equation 4.7 as $\sum F_x = -f_s^{\text{MAX}} = -\mu_s F_N$, where μ_s is the coefficient of static friction and F_N is the magnitude of the normal force that acts on the tires. The minus sign indicates that the force points to the left, along the $-x$ axis. By combining these relations, will be able to find the displacement of the car.

SOLUTION Solving Equation 2.9 for x and substituting Newton's second law $(a_x = \sum F_x/m)$ into this equation yields

$$x = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{v_x^2 - v_{0x}^2}{2(\Sigma F_x / m)}$$

The net force acting on the car is equal to the maximum static frictional force. The maximum static frictional force is given by $-f_s^{\text{MAX}} = -\mu_s F_N$. With this substitution, the displacement becomes:

$$x = \frac{v_x^2 - v_{0x}^2}{2(\Sigma F_x / m)} = \frac{v_x^2 - v_{0x}^2}{2(-\mu_s F_N / m)}$$

There are two forces acting on the car in the vertical, or y, direction, the downward-acting weight $-mg$ and the upward-acting normal force $+F_N$. Since the car is traveling on a horizontal road and does not leave it, the net force in the vertical direction must be zero, so that $F_N - mg = 0$, or $F_N = mg$. Thus, the displacement can be written as (notice that the mass m algebraically cancels out of this expression)

$$x = \frac{v_x^2 - v_{0x}^2}{2(-\mu_s F_N / m)} = \frac{v_x^2 - v_{0x}^2}{-2\mu_s g}$$

The stopping displacements with and without antilock brakes are

$$x_{\text{with}} = \frac{v_x^2 - v_{0x}^2}{-2\mu_s g} = \frac{(0\text{ m/s})^2 - (17.9\text{ m/s})^2}{-2(0.764)(9.80\text{ m/s}^2)} = 21.4\text{ m}$$

$$x_{\text{without}} = \frac{v_x^2 - v_{0x}^2}{-2\mu_s g} = \frac{(0\text{ m/s})^2 - (17.9\text{ m/s})^2}{2(-0.615)(9.80\text{ m/s}^2)} = 26.6\text{ m}$$

The difference between these two displacements is 26.6 m – 21.4 m = $\boxed{5.2\text{ m}}$.

43. **SSM** ***REASONING*** If we assume that kinetic friction is the only horizontal force that acts on the skater, then, since kinetic friction is a resistive force, it acts opposite to the direction of motion and the skater slows down. According to Newton's second law ($\sum \mathbf{F} = m\mathbf{a}$), the magnitude of the deceleration is $a = f_k / m$.

The magnitude of the frictional force that acts on the skater is, according to Equation 4.8, $f_k = \mu_k F_N$ where μ_k is the coefficient of kinetic friction between the ice and the skate blades. There are only two vertical forces that act on the skater; they are the upward normal force $\mathbf{F}_N$ and the downward pull of gravity (the weight) mg. Since the skater has no vertical acceleration, Newton's second law in the vertical direction gives (if we take upward as the positive direction) $F_N - mg = 0$. Therefore, the magnitude of the normal force is $F_N = mg$ and the magnitude of the deceleration is given by

$$a = \frac{f_k}{m} = \frac{\mu_k F_N}{m} = \frac{\mu_k mg}{m} = \mu_k g$$

SOLUTION

a. Direct substitution into the previous expression gives

$$a = \mu_k g = (0.100)(9.80 \text{ m/s}^2) = \boxed{0.980 \text{ m/s}^2}$$

Since the skater is slowing down, the

$\boxed{\text{direction of the acceleration must be opposite to the direction of motion}}$.

b. The displacement through which the skater will slide before he comes to rest can be obtained from Equation 2.9 ($v^2 - v_0^2 = 2ax$). Since the skater comes to rest, v = 0 m/s. If we take the direction of motion of the skater as the positive direction, then, solving for x, we obtain

$$x = \frac{-v_0^2}{2a} = \frac{-(7.60 \text{ m/s})^2}{2(-0.980 \text{ m/s}^2)} = \boxed{29.5 \text{ m}}$$

44. ***REASONING*** The free-body diagrams for the large cube (mass = M) and the small cube (mass = m) are shown in the following drawings. In the case of the large cube, we have omitted the weight and the normal force from the surface, since the play no role in the solution (although they do balance).

In these diagrams, note that the two blocks exert a normal force on each other; the large block exerts the force F_N on the smaller block, while the smaller block exerts the force $-F_N$ on the larger block. In accord with Newton's third law these forces have opposite directions and equal magnitudes F_N. Under the influence of the forces shown, the two blocks have the same acceleration a. We begin our solution by applying Newton's second law to each one.

SOLUTION According to Newton's second law, we have

$$\underbrace{\Sigma F = P - F_N = Ma}_{\text{Large block}} \qquad \underbrace{F_N = ma}_{\text{Small block}}$$

Substituting $F_N = ma$ into the large-block expression and solving for P gives

$$P = (M + m)\, a$$

For the smaller block to remain in place against the larger block, the static frictional force must balance the weight of the smaller block, so that $f_s^{\text{MAX}} = mg$. But f_s^{MAX} is given by $f_s^{\text{MAX}} = \mu_s F_N$, where, from the Newton's second law, we know that $F_N = ma$. Thus, we have $\mu_s ma = mg$ or $a = g/\mu_s$. Using this result in the expression for P gives

$$P = (M+m)a = \frac{(M+m)g}{\mu_s} = \frac{(25\text{ kg} + 4.0\text{ kg})(9.80\text{ m/s}^2)}{0.71} = \boxed{4.0 \times 10^2\text{ N}}$$

45. ***REASONING*** The free-body diagram for the box is shown in the following drawing on the left. On the right the same drawing is repeated, except that the pushing force P is resolved into its horizontal and vertical components.

Since the block is moving at a constant velocity, it has no acceleration, and Newton's second law indicates that the net vertical and net horizontal forces must separately be zero.

SOLUTION Taking upward and to the right as the positive directions, we write the zero net vertical and horizontal forces as follows:

$$\underbrace{F_N - mg - P\sin\theta = 0}_{\text{Vertical}} \qquad \underbrace{P\cos\theta - f_k = 0}_{\text{Horizontal}}$$

From the equation for the horizontal forces, we have $P\cos\theta = f_k$. But the kinetic frictional force is $f_k = \mu_k F_N$. Furthermore, from the equation for the vertical forces, we have $F_N = mg + P\sin\theta$. With these substitutions, we obtain

$$P\cos\theta = f_k = \mu_k F_N = \mu_k (mg + P\sin\theta)$$

Solving for P gives

$$P = \frac{\mu_k mg}{\cos\theta - \mu_k \sin\theta}$$

The necessary pushing force becomes infinitely large when the denominator in this expression is zero. Hence, we find that $\cos\theta - \mu_k \sin\theta = 0$, which can be rearranged to show that

$$\frac{\sin\theta}{\cos\theta} = \tan\theta = \frac{1}{\mu_k} \qquad \text{or} \qquad \theta = \tan^{-1}\left(\frac{1}{0.41}\right) = \boxed{68°}$$

46. ***REASONING AND SOLUTION***

a. In the horizontal direction the thrust, F, is balanced by the resistive force, f_r, of the water. That is,

$$\bullet F_x = 0$$

or

$$f_r = F = \boxed{7.40 \times 10^5 \text{ N}}$$

b. In the vertical direction, the weight, mg, is balanced by the buoyant force, F_b. So

$$\bullet F_y = 0$$

gives

$$F_b = mg = (1.70 \times 10^8 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{1.67 \times 10^9 \text{ N}}$$

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47. **SSM** ***REASONING AND SOLUTION*** There are three vertical forces that act on the lantern. The two upward forces of tension exerted by the wires, and the downward pull of gravity (the weight). If we let upward be the positive direction, and let T represent the tension in *one* of the wires, then Equation 4.9b gives

$$\sum F_y = 2T - mg = 0$$

Solving for T gives

$$T = \frac{mg}{2} = \frac{(12.0 \text{ kg})(9.80 \text{ m/s}^2)}{2} = \boxed{58.8 \text{ N}}$$

48. ***REASONING AND SOLUTION*** The free body diagram for the plane is shown below to the left. The figure at the right shows the forces resolved into components parallel to and perpendicular to the line of motion of the plane.

If the plane is to continue at constant velocity, the resultant force must still be zero after the fuel is jettisoned. Therefore (using the directions of T and L to define the positive directions),

$$T - R - W(\sin\theta) = 0 \qquad (1)$$
$$L - W(\cos\theta) = 0 \qquad (2)$$

From Example 13, before the fuel is jettisoned, the weight of the plane is 86 500 N, the thrust is 103 000 N, and the lift is 74 900 N. The force of air resistance is the same before and after the fuel is jettisoned and is given in Example 13 as R = 59 800 N.

After the fuel is jettisoned, W = 86 500 N – 2800 N = 83 700 N

From Equation (1) above, the thrust after the fuel is jettisoned is

$$T = R + W(\sin\theta) = [(59\,800\text{ N}) + (83\,700\text{ N})(\sin 30.0°)] = 101\,600\text{ N}$$

From Equation (2), the lift after the fuel is jettisoned is

$$L = W(\cos\theta) = (83\,700\text{ N})(\cos 30.0°) = 72\,500\text{ N}$$

a. The pilot must, therefore, reduce the thrust by

$$103\,000\text{ N} - 101\,600\text{ N} = \boxed{1400\text{ N}}$$

b. The pilot must reduce the lift by

$$74\,900\text{ N} - 72\,500\text{ N} = \boxed{2400\text{ N}}$$

49. **SSM** ***REASONING AND SOLUTION*** The figure at the right shows the forces that act on the wine bottle. Newton's second law applied in the horizontal and vertical directions gives

$$\Sigma F_y = F_1 \cos 45.0° + F_2 \cos 45.0° - W = 0 \qquad (1)$$

$$\Sigma F_x = F_2 \sin 45.0° - F_1 \sin 45.0° = 0 \qquad (2)$$

From Equation (2), we see that $F_1 = F_2$. According to Equation (1), we have

$$F_1 = \frac{W}{2 \cos 45.0°} = \frac{mg}{2 \cos 45.0°}$$

Therefore,

$$F_1 = F_2 = \frac{(1.40 \text{ kg})(9.80 \text{ m/s}^2)}{2 \cos 45.0°} = \boxed{9.70 \text{ N}}$$

50. ***REASONING*** The drawing shows the I-beam and the three forces that act on it, its weight **W** and the tension **T** in each of the cables. Since the I-beam is moving upward at a constant velocity, its acceleration is zero and it is in vertical equilibrium. According to Equation 4.9b, $\sum F_y = 0$, the net force in the vertical (or y) direction must be zero. This relation will allow us to find the magnitude of the tension.

SOLUTION Taking up as the $+y$ direction, Equation 4.9b becomes

$$\underbrace{+T\sin 70.0° + T\sin 70.0° - 8.00\times 10^3 \text{ N}}_{\Sigma F_y} = 0$$

Solving this equation for the tension gives $T = \boxed{4260 \text{ N}}$.

51. ***REASONING*** The book is kept from falling as long as the total static frictional force balances the weight of the book. The forces that act on the book are shown in the following free-body diagram, where P is the pressing force applied by each hand.

In this diagram, note that there are two pressing forces, one from each hand. Each hand also applies a static frictional force, and, therefore, two static frictional forces are shown. The maximum static frictional force is related in the usual way to a normal force F_N, but in this problem the normal force is provided by the pressing force, so that $F_N = P$.

SOLUTION Since the frictional forces balance the weight, we have

$$2f_s^{MAX} = 2(\mu_s F_N) = 2(\mu_s P) = W$$

Solving for P, we find that

$$P = \frac{W}{2\mu_s} = \frac{31\text{ N}}{2(0.40)} = \boxed{39\text{ N}}$$

52. ***REASONING AND SOLUTION*** Newton's second law applied in the vertical and horizontal directions gives

$$L\cos 21.0° - W = 0 \qquad (1)$$
$$L\sin 21.0° - R = 0 \qquad (2)$$

a. Equation (1) gives

$$L = \frac{W}{\cos 21.0°} = \frac{53\ 800\ \text{N}}{\cos 21.0°} = \boxed{57\ 600\ \text{N}}$$

b. Equation (2) gives

$$R = L\sin 21.0° = (57\ 600\ \text{N})\sin 21.0° = \boxed{20\ 600\ \text{N}}$$

53. **SSM** ***REASONING*** In order for the object to move with constant velocity, the net force on the object must be zero. Therefore, the north/south component of the third force must be equal in magnitude and opposite in direction to the 80.0 N force, while the east/west component of the third force must be equal in magnitude and opposite in direction to the 60.0 N force. Therefore, the third force has components: 80.0 N due south and 60.0 N due east. We can use the Pythagorean theorem and trigonometry to find the magnitude and direction of this third force.

SOLUTION The magnitude of the third force is

$$F_3 = \sqrt{(80.0\ \text{N})^2 + (60.0\ \text{N})^2} = \boxed{1.00 \times 10^2\ \text{N}}$$

The direction of F_3 is specified by the angle θ where

$$\theta = \tan^{-1}\left(\frac{80.0\ \text{N}}{60.0\ \text{N}}\right) = \boxed{53.1°,\ \text{south of east}}$$

54. ***REASONING*** The free-body diagram in the drawing at the right shows the forces that act on the clown (weight = W). In this drawing, note that P denotes the pulling force. Since the rope passes around three pulleys, forces of magnitude P are applied both to the clown's hands and his feet. The normal force due to the floor is F_N, and the maximum static frictional force is f_s^{MAX}. At the instant just before the clown's feet move, the net vertical and net horizontal forces are zero, according to Newton's second law, since there is no acceleration at this instant.

SOLUTION According to Newton's second law, with upward and to the right chosen as the positive directions, we have

$$\underbrace{F_N + P - W = 0}_{\text{Vertical forces}} \qquad \text{and} \qquad \underbrace{f_s^{MAX} - P = 0}_{\text{Horizontal forces}}$$

From the horizontal-force equation we find $P = f_s^{MAX}$. But $f_s^{MAX} = \mu_s F_N$. From the vertical-force equation, the normal force is $F_N = W - P$. With these substitutions, it follows that

$$P = f_s^{MAX} = \mu_s F_N = \mu_s (W - P)$$

Solving for P gives

$$P = \frac{\mu_s W}{1 + \mu_s} = \frac{(0.53)(890\text{ N})}{1 + 0.53} = \boxed{310\text{ N}}$$

55. ***REASONING*** The drawing shows the bicycle (represented as a circle) moving down the hill. Since the bicycle is moving at a constant velocity, its acceleration is zero and it is in equilibrium. Choosing the x axis to be parallel to the hill, Equation 4.9a states that $\sum F_x = 0$, so the net force along the x axis is zero. This relation will allow us to find the value of the numerical constant c that appears in the expression for $\mathbf{f}_{\text{air}}$.

SOLUTION Taking up the hill as the $+x$ direction, the x-component of the weight is $-mg \sin 8.0°$, and the force due to air resistance is $+f_{\text{air}} = +cv$, where the plus sign indicates that this force points opposite to the motion of the bicycle, or up the hill. Equation 4.9a can be written as

$$\underbrace{+cv - mg \sin 8.0°}_{\Sigma F_x} = 0$$

Solving this equation for the constant c gives

$$c = \frac{mg \sin 8.0°}{v} = \frac{(85\text{ kg})(9.80\text{ m/s}^2)\sin 8.0°}{8.9\text{ m/s}} = \boxed{13\text{ kg/s}}$$

56. ***REASONING AND SOLUTION*** If the +x axis is taken in the direction of motion, $\bullet F_x = 0$ gives

$$F - f_k - mg \sin\theta = 0$$

where

$$f_k = \mu_k F_N$$

Then

$$F - \mu_k F_N - mg \sin\theta = 0 \qquad (1)$$

Also, $\bullet F_y = 0$ gives

$$F_N - mg \cos\theta = 0$$

so

$$F_N = mg \cos\theta \qquad (2)$$

Substituting Equation (2) into Equation (1) and solving for F yields

$$F = mg(\sin\theta + \mu_k \cos\theta)$$

$$F = (55.0\text{ kg})(9.80\text{ m/s}^2)[\sin 25.0° + (0.120)\cos 25.0°] = \boxed{286\text{ N}}$$

57. **SSM** ***REASONING*** There are four forces that act on the chandelier; they are the forces of tension T in each of the three wires, and the downward force of gravity mg. Under the influence of these forces, the chandelier is at rest and, therefore, in equilibrium. Consequently, the sum of the x components as well as the sum of the y components of the forces must each be zero. The figure below shows the free-body diagram for the chandelier and the force components for a suitable system of x, y axes. Note that the free-body diagram only shows one of the forces of tension; the second and third tension forces are not shown. The triangle at the right shows the geometry of one of the cords, where ℓ is the length of the cord, and d is the distance from the ceiling.

We can use the forces in the y direction to find the magnitude T of the tension in any one wire.

SOLUTION Remembering that there are three tension forces, we see from the free-body diagram that

$$3T\sin\theta = mg \qquad \text{or} \qquad T = \frac{mg}{3\sin\theta} = \frac{mg}{3(d/\ell)} = \frac{mg\ell}{3d}$$

Therefore, the magnitude of the tension in any one of the cords is

$$T = \frac{(44\text{ kg})(9.80\text{ m/s}^2)(2.0\text{ m})}{3(1.5\text{ m})} = \boxed{1.9\times10^2\text{ N}}$$

58. ***REASONING AND SOLUTION*** Let the tension in wire 1 be T_1 and the tension in wire 2 be T_2. The sum of the vertical forces acting on the point where the wires join must be zero.

$$T_1 \sin 43.0° + T_2 \sin 55.0° - mg = 0 \qquad (1)$$

Similarly, the horizontal forces must add to zero so

$$-T_1 \cos 43.0° + T_2 \cos 55.0° = 0 \qquad (2)$$

Solving Equation (2) for T_1, substituting into Equation (1), and rearranging yields

$$T_2 = \boxed{317 \text{ N}}$$

Using this result in Equation (2) gives $T_1 = \boxed{249 \text{ N}}$

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59. ***REASONING*** The tree limb is in equilibrium, so we will apply Newton's second law to it. This law will allow us to determine the tension in both segments of the wire.

SOLUTION Let $\mathbf{T_1}$ be the tension in the shorter segment of the wire, $\mathbf{T_2}$ be the tension in the longer segment, and **W** be the weight of the limb (see the drawing). In the horizontal direction, $\bullet F_x = 0$ (Equation 4.9a) gives

$$\underbrace{-T_1 \cos\theta_1 + T_2 \cos\theta_2}_{\Sigma F_x} = 0 \qquad (1)$$

where $\theta_1 = \tan^{-1}\left(\dfrac{0.200\text{ m}}{0.800\text{ m}}\right) = 14.0°$ and $\theta_2 = \tan^{-1}\left(\dfrac{0.200\text{ m}}{3.20\text{ m}}\right) = 3.58°$. In the vertical direction, $\bullet F_y = 0$ (Equation 4.9b) gives

$$\underbrace{+T_1 \sin\theta_1 + T_2 \sin\theta_2 - W}_{\Sigma F_y} = 0 \qquad (2)$$

a. Solving Equation (1) for T_2 in terms of T_1, and substituting the result into Equation (2) yields

$$T_1 = \frac{W}{\sin\theta_1 + (\sin\theta_2)(\cos\theta_1)/(\cos\theta_2)} = \frac{151\text{ N}}{\sin 14.0° + (\sin 3.58°)(\cos 14.0°)/(\cos 3.58°)} = \boxed{499\text{ N}}$$

b. Substituting $T_1 = 499$ N into Equation (1) and solving for T_2 gives

$$T_2 = T_1(\cos\theta_1)/(\cos\theta_2) = (499\text{ N})(\cos 14.0°)/(\cos 3.58°) = \boxed{485\text{ N}}$$

60. ***REASONING AND SOLUTION***

a. If the block is not to slide down the wall, then the vertical forces acting on the block must sum to zero.

$$F \cos 40.0° - mg + \mu_s F_N = 0$$

Additionally, the horizontal forces must sum to zero.

$$F \sin 40.0° - F_N = 0$$

Eliminating F_N gives

$$F = \frac{mg}{\cos 40.0° + \mu_s \sin 40.0°} = \boxed{79.0 \text{ N}}$$

b. The above analysis applies to the case where the block is starting to slide up the wall except, that the frictional force will be in the opposite direction. Hence,

$$F = \frac{mg}{\cos 40.0° - \mu_s \sin 40.0°} = \boxed{219 \text{ N}}$$

61. **SSM** ***REASONING*** The following figure shows the crate on the incline and the free body diagram for the crate. The diagram at the far right shows all the forces resolved into components that are parallel and perpendicular to the surface of the incline. We can analyze the motion of the crate using Newton's second law. The coefficient of friction can be determined from the resulting equations.

SOLUTION Since the crate is at rest, it is in equilibrium and its acceleration is zero in all directions. If we take the direction down the incline as positive, Newton's second law indicates that

$$\sum F_x = P\cos\theta + mg\sin\theta - f_s^{\text{MAX}} = 0$$

According to Equation 4.7, $f_s^{\text{MAX}} = \mu_s F_N$. Therefore, we have

$$P\cos\theta + mg\sin\theta - \mu_s F_N = 0 \qquad (1)$$

The expression for the normal force can be found from analyzing the forces that are perpendicular to the incline. Taking up to be positive, we have

$$\sum F_y = P\sin\theta + F_N - mg\cos\theta = 0 \qquad \text{or} \qquad F_N = mg\cos\theta - P\sin\theta$$

Equation (1) then becomes

$$P\cos\theta + mg\sin\theta - \mu_s(mg\cos\theta - P\sin\theta) = 0$$

Solving for the coefficient of static friction, we find that

$$\mu_s = \frac{P\cos\theta + mg\sin\theta}{mg\cos\theta - P\sin\theta} = \frac{(535\text{ N})\cos 20.0° + (225\text{ kg})(9.80\text{ m/s}^2)\sin 20.0°}{(225\text{ kg})(9.80\text{ m/s}^2)\cos 20.0° - (535\text{ N})\sin 20.0°} = \boxed{0.665}$$

62. ***REASONING*** The weight of the part of the washcloth off the table is $m_{off}g$. At the instant just before the washcloth begins to slide, this weight is supported by a force that has magnitude equal to f_s^{MAX}, which is the static frictional force that the table surface applies to the part of the washcloth on the table. This force is transmitted "around the bend" in the washcloth hanging over the edge by the tension forces between the molecules of the washcloth, in much the same way that a force applied to one end of a rope is transmitted along the rope as it passes around a pulley.

SOLUTION Since the static frictional supports the weight of the washcloth off the table, we have $f_s^{MAX} = m_{off}g$. The static frictional force is $f_s^{MAX} = \mu_s F_N$. The normal force F_N is applied by the table to the part of the washcloth on the table and has a magnitude equal to the weight of that part of the washcloth. This is so, because the table is assumed to be horizontal and the part of the washcloth on it does not accelerate in the vertical direction. Thus, we have

$$f_s^{MAX} = \mu_s F_N = \mu_s m_{on} g = m_{off} g$$

The magnitude g of the acceleration due to gravity can be eliminated algebraically from this result, giving $\mu_s m_{on} = m_{off}$. Dividing both sides by $m_{on} + m_{off}$ gives

$$\mu_s \left(\frac{m_{on}}{m_{on} + m_{off}} \right) = \frac{m_{off}}{m_{on} + m_{off}} \qquad \text{or} \qquad \mu_s f_{on} = f_{off}$$

where we have used f_{on} and f_{off} to denote the fractions of the washcloth on and off the table, respectively. Since $f_{on} + f_{off} = 1$, we can write the above equation on the left as

$$\mu_s \left(1 - f_{off}\right) = f_{off} \qquad \text{or} \qquad f_{off} = \frac{\mu_s}{1 + \mu_s} = \frac{0.40}{1 + 0.40} = \boxed{0.29}$$

63. ***REASONING AND SOLUTION*** Newton's second law, $\bullet\mathbf{F} = m\mathbf{a}$, implies that the acceleration **a** and the net force are in the same direction. This is $\boxed{64° \text{ N of E}}$. The magnitude of the net force is

$$\bullet F = ma = (350 \text{ kg})(0.62 \text{ m/s}^2) = \boxed{220 \text{ N}}$$

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64. ***REASONING*** Suppose the bobsled is moving along the $+x$ direction. There are two forces acting on it that are parallel to its motion; a force $+F_x$ propelling it forward and a force of –450 N that is resisting its motion. The net force is the sum of these two forces. According to Newton's second law, Equation 4.2a, the net force is equal to the mass of the bobsled times its acceleration. Since the mass and acceleration are known, we can use the second law to determine the magnitude of the propelling force.

SOLUTION

a. Newton's second law states that

$$\underbrace{+F_x - 450\text{ N}}_{\Sigma F_x} = ma_x \tag{4.2a}$$

Solving this equation for F_x gives

$$F_x = ma_x + 450\text{ N} = (270\text{ kg})(2.4\text{ m/s}^2) + 450\text{ N} = \boxed{1100\text{ N}}$$

b. The magnitude of the net force that acts on the bobsled is

$$\Sigma F_x = ma_x = (270\text{ kg})(2.4\text{ m/s}^2) = \boxed{650\text{ N}} \tag{4.2a}$$

65. $\boxed{\text{SSM}}$ ***REASONING*** If we assume that the acceleration is constant, we can use Equation 2.4 ($v = v_0 + at$) to find the acceleration of the car. Once the acceleration is known, Newton's second law ($\sum \mathbf{F} = m\mathbf{a}$) can be used to find the magnitude and direction of the net force that produces the deceleration of the car.

SOLUTION The average acceleration of the car is, according to Equation 2.4,

$$a = \frac{v - v_0}{t} = \frac{17.0 \text{ m/s} - 27.0 \text{ m/s}}{8.00 \text{ s}} = -1.25 \text{ m/s}^2$$

where the minus sign indicates that the direction of the acceleration is opposite to the direction of motion; therefore, the acceleration points due west.

According to Newton's Second law, the net force on the car is

$$\sum F = ma = (1380 \text{ kg})(-1.25 \text{ m/s}^2) = -1730 \text{ N}$$

The magnitude of the net force is $\boxed{1730 \text{ N}}$. From Newton's second law, we know that the direction of the force is the same as the direction of the acceleration, so the force also points $\boxed{\text{due west}}$.

66. ***REASONING AND SOLUTION*** From Newton's second law and the equation: $v = v_0 + at$, we have

$$F = ma = m\frac{v - v_0}{t}$$

a. When the skier accelerates from rest (v_0 = 0 m/s) to a speed of 11 m/s in 8.0 s, the required net force is

$$F = m\frac{v - v_0}{t} = (73\text{ kg})\frac{(11\text{ m/s}) - 0\text{ m/s}}{8.0\ s} = \boxed{1.0 \times 10^2\text{ N}}$$

b. When the skier lets go of the tow rope and glides to a halt (v = 0 m/s) in 21 s, the net force acting on the skier is

$$F = m\frac{v - v_0}{t} = (73\text{ kg})\frac{0\text{ m/s} - (11\text{ m/s})}{21\text{ s}} = -38\text{ N}$$

The magnitude of the net force is $\boxed{38\text{ N}}$.

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67. ***REASONING AND SOLUTION*** Newton's second law applied to block 1 (422 N) gives

$$T = m_1 a_1$$

Similarly, for block 2 (185 N)

$$T - m_2 g = m_2 a_2$$

If the string is not to break or go slack, both blocks must have accelerations of the same magnitude.

Then $a_1 = a$ and $a_2 = -a$. The above equations become

$$T = m_1 a \qquad (1)$$
$$T - m_2 g = -m_2 a \qquad (2)$$

a. Substituting Equation (1) into Equation (2) and solving for a yields

$$a = \frac{m_2 g}{m_1 + m_2} = \boxed{2.99 \text{ m/s}^2}$$

b. Using this value in Equation (1) gives

$$T = m_1 a = \boxed{129 \text{ N}}$$

68. ***REASONING*** Let's assume that the rocket is moving in the $+y$ direction. There are three forces acting on it: the upward thrust of $+T$, the force $-f_{\text{air}}$ of air resistance, and the weight $-mg$ of the rocket. The net force is the sum of these three forces. According to Newton's second law, Equation 4.2b, the net force is equal to the mass of the rocket times its acceleration. We can use the second law to determine the acceleration.

SOLUTION
Newton's second law states that

$$\underbrace{T - mg - f_{\text{air}}}_{\Sigma F_y} = ma_y \tag{4.2b}$$

Solving this equation for a_y gives

$$a_y = \frac{+T - mg - f_{\text{air}}}{m}$$

$$= \frac{7.5 \times 10^6 \text{ N} - \left(4.5 \times 10^5 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right) - 2.4 \times 10^6 \text{ N}}{4.5 \times 10^5 \text{ kg}} = \boxed{+1.5 \text{ m/s}^2}$$

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69. $\boxed{\text{SSM}}$ $\boxed{\text{WWW}}$ ***REASONING*** The speed of the skateboarder at the bottom of the ramp can be found by solving Equation 2.9 ($v^2 = v_0^2 + 2ax$ where x is the distance that the skater moves down the ramp) for v. The figure at the right shows the free-body diagram for the skateboarder. The net force $\sum F$, which accelerates the skateboarder down the ramp, is the component of the weight that is parallel to the incline: $\sum F = mg \sin\theta$. Therefore, we know from Newton's second law that the acceleration of the skateboarder down the ramp is

$$a = \frac{\sum F}{m} = \frac{mg \sin\theta}{m} = g \sin\theta$$

SOLUTION Thus, the speed of the skateboarder at the bottom of the ramp is

$$v = \sqrt{v_0^2 + 2ax} = \sqrt{v_0^2 + 2gx \sin\theta} = \sqrt{(2.6 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(6.0 \text{ m}) \sin 18°} = \boxed{6.6 \text{ m/s}}$$

70. ***REASONING AND SOLUTION***

a. A free-body diagram for the crate gives

$$T_B - W_C = m_C a$$

$$T_B = W_C + m_C a$$

$$T_B = 1510\text{ N} + \left(\frac{1510\text{ N}}{9.80\text{ m/s}^2}\right)\left(0.620\text{ m/s}^2\right) = \boxed{1610\text{ N}}$$

b. An analysis of the free-body diagram for the platform yields

$$T_A - T_B - W_W = m_W a$$

$$T_A = T_B + W_W + m_W a$$

$$T_A = 1610\text{ N} + \left(\frac{965\text{ N}}{9.80\text{ m/s}^2}\right)\left(0.620\text{ m/s}^2\right) + 965\text{ N}$$

$$= \boxed{2640\text{ N}}$$

71. **SSM** ***REASONING*** We can use the appropriate equation of kinematics to find the acceleration of the bullet. Then Newton's second law can be used to find the average net force on the bullet.

SOLUTION According to Equation 2.4, the acceleration of the bullet is

$$a = \frac{v - v_0}{t} = \frac{715 \text{ m/s} - 0 \text{ m/s}}{2.50 \times 10^{-3} \text{ s}} = 2.86 \times 10^5 \text{ m/s}^2$$

Therefore, the net average force on the bullet is

$$\sum F = ma = (15 \times 10^{-3} \text{ kg})(2.86 \times 10^5 \text{ m/s}^2) = \boxed{4290 \text{ N}}$$

72. ***REASONING AND SOLUTION*** Newton's second law gives

$$F \cos 35.0° - f_k = ma$$

Then

$$a = \left(\frac{70.0 \text{ N}}{15.0 \text{ kg}}\right) \cos 35.0° - \frac{37.8 \text{ N}}{15.0 \text{ kg}} = \boxed{1.30 \text{ m/s}^2}$$

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73. ***REASONING AND SOLUTION*** The acceleration needed so that the craft touches down with zero velocity is

$$a = \frac{v^2 - v_0^2}{2s} = \frac{-(18.0\ \text{m/s})^2}{2(-165\ \text{m})} = 0.982\ \text{m/s}^2.$$

Newton's second law applied in the vertical direction gives

$$F - mg = ma$$

Then

$$F = m(a + g) = (1.14 \times 10^4\ \text{kg})(0.982\ \text{m/s}^2 + 1.60\ \text{m/s}^2) = \boxed{29\ 400\ \text{N}}$$

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74. ***REASONING*** The free-body diagrams for Robin (mass = m) and for the chandelier (mass = M) are given at the right. The tension T in the rope applies an upward force to both. Robin accelerates upward, while the chandelier accelerates downward, each acceleration having the same magnitude. Our solution is based on separate applications of Newton's second law to Robin and the chandelier.

SOLUTION Applying Newton's second law, we find

$$\underbrace{T - mg = ma}_{\text{Robin Hood}} \qquad \text{and} \qquad \underbrace{T - Mg = -Ma}_{\text{Chandelier}}$$

In these applications we have taken upward as the positive direction, so that Robin's acceleration is a, while the chandelier's acceleration is $-a$. Solving the Robin-Hood equation for T gives

$$T = mg + ma$$

Substituting this expression for T into the Chandelier equation gives

$$mg + ma - Mg = -Ma \qquad \text{or} \qquad a = \left(\frac{M - m}{M + m}\right)g$$

a. Robin's acceleration is

$$a = \left(\frac{M - m}{M + m}\right)g = \left[\frac{(220\text{ kg}) - (82\text{ kg})}{(220\text{ kg}) + (82\text{ kg})}\right](9.80\text{ m/s}^2) = \boxed{4.5\text{ m/s}^2}$$

b. Substituting the value of a into the expression for T gives

$$T = mg + ma = (82\text{ kg})(9.80\text{ m/s}^2 + 4.5\text{ m/s}^2) = \boxed{1200\text{ N}}$$

75. **SSM** ***REASONING*** The shortest time to pull the person from the cave corresponds to the maximum acceleration, a_y, that the rope can withstand. We first determine this acceleration and then use kinematic Equation 3.5b ($y = v_{0y}t + \frac{1}{2}a_y t^2$) to find the time t.

SOLUTION As the person is being pulled from the cave, there are two forces that act on him; they are the tension T in the rope that points vertically upward, and the weight of the person mg that points vertically downward. Thus, if we take upward as the positive direction, Newton's second law gives $\sum F_y = T - mg = ma_y$. Solving for a_y, we have

$$a_y = \frac{T}{m} - g = \frac{T}{W/g} - g = \frac{569\text{ N}}{(5.20 \times 10^2\text{ N})/(9.80\text{ m/s}^2)} - 9.80\text{ m/s}^2 = 0.92\text{ m/s}^2$$

Therefore, from Equation 3.5b with $v_{0y} = 0$ m/s, we have $y = \frac{1}{2}a_y t^2$. Solving for t, we find

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(35.1\text{ m})}{0.92\text{ m/s}^2}} = \boxed{8.7\text{ s}}$$

76. ***REASONING AND SOLUTION*** If the +x axis is taken to be parallel to and up the ramp, then $\bullet F_x = ma_x$ gives

$$T - f_k - mg \sin 30.0° = ma_x$$

where $f_k = \mu_k F_N$. Hence,

$$T = ma_x + \mu_k F_N + mg \sin 30.0° \qquad (1)$$

Also, $\bullet F_y = ma_y$ gives

$$F_N - mg \cos 30.0° = 0$$

since no acceleration occurs in this direction. Then

$$F_N = mg \cos 30.0° \qquad (2)$$

Substitution of Equation (2) into Equation (1) yields

$$T = ma_x + \mu_k mg \cos 30.0° + mg \sin 30.0°$$

$$T = (205 \text{ kg})(0.800 \text{ m/s}^2) + (0.900)(205 \text{ kg})(9.80 \text{ m/s}^2)\cos 30.0° + (205 \text{ kg})(9.80 \text{ m/s}^2)\sin 30.0° = \boxed{2730 \text{ N}}$$

77. ***REASONING AND SOLUTION*** The static frictional force accelerates the crate so that it does not slip against the bed of the truck. The maximum force that friction can supply is

$$f_s^{max} = \mu_s F_N = \mu_s mg$$

Newton's second law gives $f_s^{max} = ma$, so

$$a = \mu_s g = (0.40)(9.80\ \text{m/s}^2) = \boxed{3.9\ \text{m/s}^2}$$

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78. ***REASONING*** Newton's second law, Equation 4.2a, can be used to find the tension in the coupling between the cars, since the mass and acceleration are known. The tension in the coupling between the 30th and 31st cars is responsible for providing the acceleration for the 20 cars from the 31st to the 50th car. The tension in the coupling between the 49th and 50th cars is responsible only for pulling one car, the 50th.

SOLUTION

a. The tension T between the 30th and 31st cars is

$$T_x = (\text{Mass of 20 cars})a_x \qquad (4.2a)$$

$$= (20 \text{ cars})\left(6.8 \times 10^3 \text{ kg/car}\right)\left(8.0 \times 10^{-2} \text{ m/s}^2\right) = \boxed{1.1 \times 10^4 \text{ N}}$$

b. The tension T between the 49th and 50th cars is

$$T_x = (\text{Mass of 1 car})a_x \qquad (4.2a)$$

$$= (1 \text{ car})\left(6.8 \times 10^3 \text{ kg/car}\right)\left(8.0 \times 10^{-2} \text{ m/s}^2\right) = \boxed{5.4 \times 10^2 \text{ N}}$$

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79. **SSM** ***REASONING*** The box comes to a halt because the kinetic frictional force and the component of its weight parallel to the incline oppose the motion and cause the box to slow down. The distance that the box travels up the incline can be can be found by solving Equation 2.9 ($v^2 = v_0^2 + 2ax$) for x. Before we use this approach, however, we must first determine the acceleration of the box as it travels along the incline.

SOLUTION The figure above shows the free-body diagram for the box. It shows the resolved components of the forces that act on the box. If we take the direction up the incline as the positive x direction, then, Newton's second law gives

$$\sum F_x = -mg\sin\theta - f_k = ma_x \qquad \text{or} \qquad -mg\sin\theta - \mu_k F_N = ma_x$$

where we have used Equation 4.8, $f_k = \mu_k F_N$. In the y direction we have

$$\sum F_y = F_N - mg\cos\theta = 0 \qquad \text{or} \qquad F_N = mg\cos\theta$$

since there is no acceleration in the y direction. Therefore, the equation for the motion in the x direction becomes

$$-mg\sin\theta - \mu_k mg\cos\theta = ma_x \qquad \text{or} \qquad a_x = -g(\sin\theta + \mu_k\cos\theta)$$

According to Equation 2.9, with this value for the acceleration and the fact that $v = 0$ m/s, the distance that the box slides up the incline is

$$x = -\frac{v_0^2}{2a} = \frac{v_0^2}{2g(\sin\theta + \mu_k\cos\theta)} = \frac{(1.50\text{ m/s})^2}{2(9.80\text{ m/s}^2)[\sin 15.0° + (0.180)\cos 15.0°]} = \boxed{0.265\text{ m}}$$

80. ***REASONING AND SOLUTION*** If the x axis is taken parallel to the slope with +x down the slope, then $\bullet F_x = ma_x$ gives

$$F_w + mg \sin\theta - f_k = ma_x$$

where $f_k = \mu_k F_N$ and F_w is the force exerted by the wind on the person and sled.
$\bullet F_y = ma_y$ gives

$$F_N - mg \cos\theta = 0$$

since there is no acceleration of the sled in this direction. Hence,

$$F_N = mg \cos\theta$$

Substitution of this into the above result gives

$$a_x = F_w/m + g \sin\theta - \mu_k g \cos\theta$$

$$a_x = (105\text{ N})/(65.0\text{ kg}) + (9.80\text{ m/s}^2)\sin 30.0° - (0.150)(9.80\text{ m/s}^2)\cos 30.0° = 5.24\text{ m/s}^2$$

The time required for the sled to travel a distance, x, subject to this acceleration is found from

$$x = v_o t + (1/2)a_x t^2$$

Therefore, and using the fact that $v_0 = 0$ m/s, the time is

$$t = \sqrt{\frac{2x}{a_x}} = \sqrt{\frac{2(175\text{ m})}{5.24\text{ m/s}^2}} = \boxed{8.17\text{ s}}$$

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81. ***REASONING AND SOLUTION*** The figure to the left below shows the forces that act on the sports car as it accelerates up the hill. The figure to the right below shows these forces resolved into components parallel to and perpendicular to the line of motion. Forces pointing up the hill will be taken as positive.

a. The acceleration will be a maximum when $P = f_s^{max}$. From the forces along the line of motion:

$$f_s^{max} - mg \sin\theta = ma$$

The force f_s^{max} is equal to $\mu_s F_N$. The normal force can be found from the forces perpendicular to the line of motion:

$$F_N = mg \cos\theta$$

Then

$$\mu_s(mg \cos\theta) - mg \sin\theta = ma$$

and

$$a = g(\mu_s \cos\theta - \sin\theta) = (9.80\ \text{m/s}^2)(0.88\cos 18° - \sin 18°) = \boxed{5.2\ \text{m/s}^2}$$

b. When the car is being driven downhill, P (= f_s^{max}) now points down the hill in the same direction as (mg sin θ). Taking the *direction of motion* as positive, we have

$$f_s^{max} + mg \sin\theta = ma$$

Following the same steps as above we obtain

$$\mu_s(mg \cos\theta) + mg \sin\theta = ma$$

and

$$a = g(\mu_s \cos\theta + \sin\theta) = (9.80\ \text{m/s}^2)(0.88\cos 18° + \sin 18°) = \boxed{11\ \text{m/s}^2}$$

82. ***REASONING AND SOLUTION*** The distance required for the truck to stop is found from

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(0\text{ m/s})^2 - v_0^2}{2a}$$

The acceleration of the truck is needed. The frictional force decelerates the crate. The maximum force that friction can supply is

$$f_s^{max} = \mu_s F_N = \mu_s mg$$

Newton's second law requires that

$$f_s^{max} = -ma \text{ so } a = -\mu_s g$$

Now the stopping distance is

$$x = \frac{v_0^2}{2\mu_s g} = \frac{(25\text{ m/s})^2}{2(0.650)(9.80\text{ m/s}^2)} = \boxed{49.1\text{ m}}$$

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83. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The penguin comes to a halt on the horizontal surface because the kinetic frictional force opposes the motion and causes it to slow down. The time required for the penguin to slide to a halt ($v = 0$ m/s) after entering the horizontal patch of ice is, according to Equation 2.4,

$$t = \frac{v - v_0}{a_x} = \frac{-v_0}{a_x}$$

We must, therefore, determine the acceleration of the penguin as it slides along the horizontal patch.

For the penguin sliding on the horizontal patch of ice, we find from free-body diagram B and Newton's second law in the x direction (motion to the right is taken as positive) that

$$\sum F_x = -f_{k2} = ma_x \qquad \text{or} \qquad a_x = \frac{-f_{k2}}{m} = \frac{-\mu_k F_{N2}}{m}$$

In the y direction in free-body diagram B, we have $\sum F_y = F_{N2} - mg = 0$, or $F_{N2} = mg$. Therefore, the acceleration of the penguin is

$$a_x = \frac{-\mu_k mg}{m} = -\mu_k g \qquad (1)$$

Equation (1) indicates that, in order to find the acceleration a_x, we must find the coefficient of kinetic friction.

We are told in the problem statement that the coefficient of kinetic friction between the penguin and the ice is the same for the incline as for the horizontal patch. Therefore, we can use the motion of the penguin on the incline to determine the coefficient of friction and use it in Equation (1).

For the penguin sliding down the incline, we find from free-body diagram A and Newton's second law (taking the direction of motion as positive) that

$$\sum F_x = mg\sin\theta - f_{k1} = ma_x = 0 \qquad \text{or} \qquad f_{k1} = mg\sin\theta \qquad (2)$$

Here, we have used the fact that the penguin slides down the incline with a constant velocity, so that it has zero acceleration. From Equation 4.8, we know that $f_{k1} = \mu_k F_{N1}$. Applying Newton's second law in the direction perpendicular to the incline, we have

$$\sum F_y = F_{N1} - mg\cos\theta = 0 \qquad \text{or} \qquad F_{N1} = mg\cos\theta$$

Therefore, $f_{k1} = \mu_k mg\cos\theta$, so that according to Equation (2), we find

$$f_{k1} = \mu_k mg\cos\theta = mg\sin\theta$$

Solving for the coefficient of kinetic friction, we have

$$\mu_k = \frac{\sin\theta}{\cos\theta} = \tan\theta$$

Finally, the time required for the penguin to slide to a halt after entering the horizontal patch of ice is

$$t = \frac{-v_0}{a_x} = \frac{-v_0}{-\mu_k g} = \frac{v_0}{g\tan\theta} = \frac{1.4\text{ m/s}}{(9.80\text{ m/s}^2)\tan\ 6.9°} = \boxed{1.2\text{ s}}$$

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84. ***REASONING AND SOLUTION***

a. The rope exerts a tension, T, acting upward on each block. Applying Newton's second law to the lighter block (block 1) gives

$$T - m_1 g = m_1 a$$

Similarly, for the heavier block (block 2)

$$T - m_2 g = -m_2 a$$

Subtracting the second equation from the first and rearranging yields

$$a = \left(\frac{m_2 - m_1}{m_2 + m_1}\right) g = \boxed{3.68 \text{ m/s}^2}$$

b. The tension in the rope is now 908 N since the tension is the reaction to the applied force exerted by the hand. Newton's second law applied to the block is

$$T - m_1 g = m_1 a$$

Solving for a gives

$$a = \frac{T}{m_1} - g = \frac{(908 \text{ N})}{42.0 \text{ kg}} - 9.80 \text{ m/s}^2 = \boxed{11.8 \text{ m/s}^2}$$

c. In the first case, the inertia of BOTH blocks affects the acceleration whereas, in the second case, only the lighter block's inertia remains.

85. ***REASONING AND SOLUTION***

a. The left mass (mass 1) has a tension T_1 pulling it up. Newton's second law gives

$$T_1 - m_1 g = m_1 a \qquad (1)$$

The right mass (mass 3) has a different tension, T_3, trying to pull it up. Newton's second for it is

$$T_3 - m_3 g = - m_3 a \qquad (2)$$

The middle mass (mass 2) has both tensions acting on it along with friction. Newton's second law for its horizontal motion is

$$T_3 - T_1 - \mu_k m_2 g = m_2 a \qquad (3)$$

Solving Equation (1) and Equation (2) for T_1 and T_3, respectively, and substituting into Equation (3) gives

$$a = \frac{(m_3 - m_1 - \mu_k m_2)g}{m_1 + m_2 + m_3}.$$

Hence,

$$a = \boxed{0.60 \text{ m/s}^2}$$

b. From part a

$$T_1 = m_1(g + a) = \boxed{104 \text{ N}} \qquad \text{and} \qquad T_3 = m_3(g - a) = \boxed{230 \text{ N}}$$

86. ***REASONING AND SOLUTION***

a. The static frictional force is responsible for accelerating the top block so that it does not slip against the bottom one. The maximum force that can be supplied by friction is

$$f_s^{max} = \mu_s F_N = \mu_s m_1 g$$

Newton's second law requires that $f_s^{max} = m_1 a$, so

$$a = \mu_s g$$

The force necessary to cause BOTH blocks to have this acceleration is

$$F = (m_1 + m_2)a = (m_1 + m_2)\mu_s g$$

$$F = (5.00\text{ kg} + 12.0\text{ kg})(0.600)(9.80\text{ m/s}^2) = \boxed{1.00 \times 10^2\text{ N}}$$

b. The maximum acceleration that the two block combination can have before slipping occurs is

$$a = F/(17.0\text{ kg})$$

Newton's second law applied to the 5.00 kg block is

$$F - \mu_s m_1 g = m_1 a = (5.00\text{ kg})(F)/(17.0\text{ kg})$$

Hence

$$F = \boxed{41.6\text{ N}}$$

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87. **SSM** ***REASONING AND SOLUTION***

a. According to Equation 4.4, the weight of an object of mass m on the surface of Mars would be given by

$$W = \frac{GM_{\mathrm{M}}m}{R_{\mathrm{M}}^2}$$

where M_{M} is the mass of Mars and R_{M} is the radius of Mars. On the surface of Mars, the weight of the object can be given as $W = mg$ (see Equation 4.5), so

$$mg = \frac{GM_{\mathrm{M}}m}{R_{\mathrm{M}}^2} \quad \text{or} \quad g = \frac{GM_{\mathrm{M}}}{R_{\mathrm{M}}^2}$$

Substituting values, we have

$$g = \frac{(6.67\times10^{-11}\,\mathrm{N\cdot m^2/kg^2})(6.46\times10^{23}\ \mathrm{kg})}{(3.39\times10^{6}\ \mathrm{m})^2} = \boxed{3.75\ \mathrm{m/s^2}}$$

b. According to Equation 4.5,

$$W = mg = (65\ \mathrm{kg})(3.75\ \mathrm{m/s^2}) = \boxed{2.4\times10^{2}\ \mathrm{N}}$$

---

88. ***REASONING AND SOLUTION*** Since the skier is pulled at constant velocity, the net force on the skier is zero. The free body diagram is shown at the right, where P is the horizontal pulling force and R is the resistive force exerted on the skier by the water. The positive directions are to the right and up in the diagram.

a. $\Sigma F_x = P - R = 0$ or $R = P = \boxed{350 \text{ N}}$

b. $\Sigma F_y = F_W - mg = 0$

or $F_W = mg = (92 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{9.0 \times 10^2 \text{ N}}$

89. ***REASONING AND SOLUTION*** Consider a free body diagram for the stuntman with the x-axis parallel to the ground and the +y-axis vertically upward. The motion is along the +x-axis. Newton's second law written for no motion along the y-axis is $\bullet F_y = 0$ or

$$F_N - mg = 0$$

This gives the normal force to be

$$F_N = mg = (109\text{ kg})(9.80\text{ m/s}^2)$$

Newton's second law for uniform motion in the x direction is $\bullet F_x = 0$ or

$$T - f_k = 0$$

Then

$$T = f_k = \mu_k F_N = (0.870)F_N = \boxed{929\text{ N}}$$

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90. ***REASONING*** The magnitude of the gravitational force exerted on the satellite by the earth is given by Equation 4.3 as $F = Gm_{\text{satellite}}m_{\text{earth}} / r^2$, where r is the distance between the satellite and the center of the earth. This expression also gives the magnitude of the gravitational force exerted on the earth by the satellite. According to Newton's second law, the magnitude of the earth's acceleration is equal to the magnitude of the gravitational force exerted on it divided by its mass. Similarly, the magnitude of the satellite's acceleration is equal to the magnitude of the gravitational force exerted on it divided by its mass.

SOLUTION

a. The magnitude of the gravitational force exerted on the satellite when it is a distance of two earth radii from the center of the earth is

$$F = \frac{Gm_{\text{satellite}}m_{\text{earth}}}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(425\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left[\left(2\right)\left(6.38\times10^{6}\ \text{m}\right)\right]^2} = \boxed{1.04\times10^{3}\ \text{N}}$$

b. The magnitude of the gravitational force exerted on the earth when it is a distance of two earth radii from the center of the satellite is

$$F = \frac{Gm_{\text{satellite}}m_{\text{earth}}}{r^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(425\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)}{\left[\left(2\right)\left(6.38\times10^{6}\ \text{m}\right)\right]^2} = \boxed{1.04\times10^{3}\ \text{N}}$$

c. The acceleration of the satellite can be obtained from Newton's second law.

$$a_{\text{satellite}} = \frac{F}{m_{\text{satellite}}} = \frac{1.04\times10^{3}\ \text{N}}{425\ \text{kg}} = \boxed{2.45\ \text{m/s}^2}$$

d. The acceleration of the earth can also be obtained from Newton's second law.

$$a_{\text{earth}} = \frac{F}{m_{\text{earth}}} = \frac{1.04\times10^{3}\ \text{N}}{5.98\times10^{24}\ \text{kg}} = \boxed{1.74\times10^{-22}\ \text{m/s}^2}$$

91. **SSM** ***REASONING AND SOLUTION*** Four forces act on the sled. They are the pulling force P, the force of kinetic friction $\mathbf{f}_k$, the weight mg of the sled, and the normal force $\mathbf{F}_N$ exerted on the sled by the surface on which it slides. The following figures show free-body diagrams for the sled. In the diagram on the right, the forces have been resolved into their x and y components.

Since the sled is pulled at constant velocity, its acceleration is zero, and Newton's second law in the direction of motion is (with right chosen as the positive direction)

$$\sum F_x = P\cos\theta - f_k = ma_x = 0$$

From Equation 4.8, we know that $f_k = \mu_k F_N$, so that the above expression becomes

$$P\cos\theta - \mu_k F_N = 0 \qquad (1)$$

In the vertical direction,

$$\sum F_y = P\sin\theta + F_N - mg = ma_y = 0 \qquad (2)$$

Solving Equation (2) for the normal force, and substituting into Equation (1), we obtain

$$P\cos\theta - \mu_k\left(mg - P\sin\theta\right) = 0$$

Solving for μ_k, the coefficient of kinetic friction, we find

$$\mu_k = \frac{P\cos\theta}{mg - P\sin\theta} = \frac{(80.0\text{ N})\cos 30.0°}{(20.0\text{ kg})(9.80\text{ m/s}^2) - (80.0\text{N})\sin 30.0°} = \boxed{0.444}$$

92. ***REASONING*** The skydiver is falling along the $-y$ direction. There are two forces acting on him: the upward-acting force $+f_{\text{air}}$ of air resistance, and his weight, $-mg$. The net force is the sum of these forces. According to Newton's second law, Equation 4.2b, the net force is equal to the mass of the skydiver times his acceleration. We can use the second law to determine the acceleration.

SOLUTION
Newton's second law states that

$$\underbrace{+f_{\text{air}} - mg}_{\Sigma F_y} = ma_y \tag{4.2b}$$

Solving this equation for a_y and noting that $f_{\text{air}} = \frac{1}{3}mg$, we have

$$a_y = \frac{-mg + \frac{1}{3}mg}{m} = -\tfrac{2}{3}g = -6.5\ \text{m/s}^2$$

The magnitude of the skydiver's acceleration is $\boxed{6.5\ \text{m/s}^2}$.

b. When the skydiver falls at a constant velocity, his acceleration is zero, and he is in equilibrium. According to Equation 4.9b, $\Sigma F_y = 0$, and the net force acting on him must be zero.

$$\underbrace{f_{\text{air}} - mg}_{\Sigma F_y} = 0$$

The force of air resistance is

$$f_{\text{air}} = mg = (110\ \text{kg})(9.8\ \text{m/s}^2) = \boxed{1100\ \text{N, upward}}$$

93. ***REASONING AND SOLUTION***

a. In the vertical direction $\bullet F_y = ma_y$ gives

$$T - mg = ma_y$$

so

$$T = ma_y + mg = mg(1 + a_y/g)$$

$$T = (822\ \text{N})\left(1 + \frac{1.10\ \text{m/s}^2}{9.80\ \text{m/s}^2}\right) = \boxed{914\ \text{N}}$$

b. The acceleration of the man is zero if his velocity is constant. From part a

$$T = mg = \boxed{822\ \text{N}}$$

94. ***REASONING AND SOLUTION*** The apparent weight is

$$F_N = m_w(g + a)$$

We need to find the acceleration, a. Newton's second law applied to the elevator gives

$$T - (m_w + m_e)g = (m_w + m_e)a$$

Solving for a gives

$$a = \frac{T}{m_w + m_e} - g = \frac{9410\text{ N}}{60.0\text{ kg} + 815\text{ kg}} - 9.80\text{ m/s}^2 = 0.954\text{ m/s}^2.$$

Now the apparent weight is

$$F_N = 60.0\text{ kg}(9.80\text{ m/s}^2 + 0.954\text{ m/s}^2) = \boxed{645\text{ N}}$$

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95. **SSM** ***REASONING AND SOLUTION***

a. Each cart has the same mass and acceleration; therefore, the net force acting on any one of the carts is, according to Newton's second law

$$\sum F = ma = (26\text{ kg})(0.050\text{ m/s}^2) = \boxed{1.3\text{ N}}$$

b. The fifth cart must essentially push the sixth, seventh, eight, ninth and tenth cart. In other words, it must exert on the sixth cart a total force of

$$\sum F = ma = 5(26\text{ kg})(0.050\text{ m/s}^2) = \boxed{6.5\text{ N}}$$

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96. ***REASONING AND SOLUTION*** The acceleration required is

$$a = \frac{v^2 - v_0^2}{2x} = \frac{-(15.0\ \text{m/s})^2}{2(50.0\ \text{m})} = -2.25\ \text{m/s}^2$$

Newton's second law then gives the magnitude of the net force as

$$F = ma = (1580\ \text{kg})(2.25\ \text{m/s}^2) = \boxed{3560\ \text{N}}$$

---

97. ***REASONING AND SOLUTION***

a. Newton's second law applied to the spacecraft gives

$$a = \frac{\Sigma F}{m} = \frac{2240 \text{ N}}{3.50 \times 10^4 \text{ kg}} = \boxed{0.0640 \text{ m/s}^2}$$

This is a + acceleration since the thrust is in the direction of motion.

b. The distance traveled is found to be

$$x = \frac{v^2 - v_0^2}{2a} = \frac{(2310 \text{ m/s})^2 - (1820 \text{ m/s})^2}{2\left(0.0640 \text{ m/s}^2\right)} = 1.58 \times 10^7 \text{ m} = \boxed{1.58 \times 10^4 \text{ km}}$$

---

98. ***REASONING*** As discussed in Conceptual Example 7, the same net force is required on the moon as on the earth. This net force is given by Newton's second law as $\Sigma F = ma$, where the mass m is the same in both places. Thus, from the given mass and acceleration, we can calculate the net force. On the moon, the net force comes about due to the drive force and the opposing frictional force. Since the drive force is given, we can find the frictional force.

SOLUTION Newton's second law, with the direction of motion taken as positive, gives

$$\Sigma F = ma \qquad \text{or} \qquad (1430\ \text{N}) - f = \left(5.90 \times 10^3\ \text{kg}\right)\left(0.220\ \text{m/s}^2\right)$$

Solving for the frictional force f, we find

$$f = (1430\ \text{N}) - \left(5.90 \times 10^3\ \text{kg}\right)\left(0.220\ \text{m/s}^2\right) = \boxed{130\text{N}}$$

---

99. **SSM** ***REASONING AND SOLUTION*** The system is shown in the drawing. We will let $m_1 = 21.0\text{ kg}$, and $m_2 = 45.0\text{ kg}$. Then, m_1 will move upward, and m_2 will move downward. There are two forces that act on each object; they are the tension T in the cord and the weight mg of the object. The forces are shown in the free-body diagrams at the far right.

We will take up as the positive direction. If the acceleration of m_1 is a, then the acceleration of m_2 must be $-a$.

From Newton's second law, we have for m_1

$$\sum F_y = T - m_1 g = m_1 a \tag{1}$$

and for m_2

$$\sum F_y = T - m_2 g = -m_2 a \tag{2}$$

a. Eliminating T between these two equations, we obtain

$$a = \frac{m_2 - m_1}{m_2 + m_1} g = \left(\frac{45.0\text{ kg} - 21.0\text{ kg}}{45.0\text{ kg} + 21.0\text{ kg}}\right)(9.80\text{ m/s}^2) = \boxed{3.56\text{ m/s}^2}$$

b. Eliminating a between Equations (1) and (2), we find

$$T = \frac{2m_1 m_2}{m_1 + m_2} g = \left[\frac{2(21.0\text{ kg})(45.0\text{ kg})}{21.0\text{ kg} + 45.0\text{ kg}}\right](9.80\text{ m/s}^2) = \boxed{281\text{ N}}$$

100. ***REASONING AND SOLUTION*** The deceleration produced by the frictional force is

$$a = -\frac{f_k}{m} = \frac{-\mu_k mg}{m} = -\mu_k g$$

The time it takes for the car to come to a halt is given by Equation 2.4 as

$$t = \frac{v - v_0}{a} = \frac{v - v_0}{-\mu_k g} = \frac{0\text{ m/s} - 16.1\text{ m/s}}{-(0.720)(9.80\text{ m/s}^2)} = \boxed{2.28\text{ s}}$$

---

101. ***REASONING AND SOLUTION*** Take the x-axis of a free body diagram for the block to be parallel to the incline with +x down the incline. Newton's second law for uniform motion in the +x direction is $\bullet F_x = 0$, so

$$mg \sin \theta - f_k = 0$$

where $f_k = \mu_k F_N$. Then

$$mg \sin \theta = \mu_k F_N$$

Similarly, the equation for the y direction is $\bullet F_y = 0$, so

$$F_N - mg \cos \theta = 0 \qquad \text{or} \qquad mg \cos \theta = F_N$$

Division of the equations gives $\tan \theta = \mu_k$, hence $\mu_k = \tan 11.3° = \boxed{0.200}$

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102. ***REASONING*** Since the boxes are at rest, they are in equilibrium. According to Equation 4.9b, the net force in the vertical, or y, direction is zero, $\Sigma F_y = 0$. There are two unknowns in this problem, the normal force that the table exerts on box 1 and the tension in the rope that connects boxes 2 and 3. To determine these unknowns we will apply the relation $\Sigma F_y = 0$ twice, once to the boxes on the left of the pulley and once to the box on the right.

SOLUTION There are four forces acting on the two blocks on the left. The blocks are in equilibrium, so that the net force must be zero. Choosing the $+y$ direction as being the upward direction, we have that

$$\underbrace{-W_1 - W_2 + F_N + T}_{\Sigma F_y} = 0$$

where W_1 and W_2 are the magnitudes of the weights of the boxes, F_N is the magnitude of the normal force that the table exerts on box 1, and T is the magnitude of the tension in the rope. We know the weights, but do not know the tension. To find the tension, we note that the block on the right is also in equilibrium, so that the net force acting on it must be zero.

$$\underbrace{-W_3 + T}_{\Sigma F_y} = 0 \qquad \text{so that} \qquad T = W_3$$

Substituting this expression for T back into the first equation and solving for the normal force gives

$$F_N = W_1 + W_2 - W_3 = 55\text{ N} + 35\text{ N} - 28\text{ N} = \boxed{62\text{ N}}$$

---

103. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** Three forces act on the man. They are two upward forces of tension of equal magnitude T, and the force of gravity $m\mathbf{g}$. Therefore, if we take up as the positive direction, Newton's second law gives

$$\sum F = 2T - mg = ma$$

Solving for the acceleration a, we find

$$a = \frac{2T - mg}{m} = \frac{2T}{m} - g = \frac{2(358\text{ N})}{72.0\text{ kg}} - 9.80\text{ m/s}^2 = \boxed{0.14\text{ m/s}^2}$$

104. ***REASONING*** According to Newton's second law, the acceleration of the probe is $a = \Sigma F/m$. Using this value for the acceleration in Equation 2.8 and noting that the probe starts from rest ($v_0 = 0$ m/s), we can write the distance traveled by the probe as

$$x = v_0 t + \frac{1}{2}at^2 = \frac{1}{2}\left(\frac{\Sigma F}{m}\right)t^2$$

This equation is the basis for our solution.

SOLUTION Since each engine produces the same amount of force or thrust T, the net force is $\Sigma F = 2T$ when the engines apply their forces in the same direction and $\Sigma F = \sqrt{T^2 + T^2} = \sqrt{2}T$ when they apply their forces perpendicularly. Thus, we write the distances traveled in the two situations as follows:

$$\underbrace{x = \frac{1}{2}\left(\frac{2T}{m}\right)t^2}_{\text{Engines fired in same direction}} \quad \text{and} \quad \underbrace{x = \frac{1}{2}\left(\frac{\sqrt{2}\,T}{m}\right)t_\perp^2}_{\text{Engines fired perpendicularly}}$$

Since the distances are the same, we have

$$\frac{1}{2}\left(\frac{2T}{m}\right)t^2 = \frac{1}{2}\left(\frac{\sqrt{2}\,T}{m}\right)t_\perp^2 \quad \text{or} \quad \sqrt{2}\,t^2 = t_\perp^2$$

The firing time when the engines apply their forces perpendicularly is, then,

$$t_\perp = \left(\sqrt[4]{2}\right)t = \left(\sqrt[4]{2}\right)(28\text{ s}) = \boxed{33\text{ s}}$$

105. ***REASONING*** The tension in each coupling bar is responsible for accelerating the objects behind it. The masses of the cars are m_1, m_2, and m_3. We can use Newton's second law to express the tension in each coupling bar, since friction is negligible:

$$\underbrace{T_A = (m_1 + m_2 + m_3)a}_{\text{Coupling bar A}} \qquad \underbrace{T_B = (m_2 + m_3)a}_{\text{Coupling bar B}} \qquad \underbrace{T_C = m_3 a}_{\text{Coupling bar C}}$$

In these expressions $a = 0.12\ \text{m/s}^2$ remains constant. Consequently, the tension in a given bar will change only if the total mass of the objects accelerated by that bar changes as a result of the luggage transfer. Using Δ (Greek capital delta) to denote a change in the usual fashion, we can express the changes in the above tensions as follows:

$$\underbrace{\Delta T_A = [\Delta(m_1 + m_2 + m_3)]a}_{\text{Coupling bar A}} \qquad \underbrace{\Delta T_B = [\Delta(m_2 + m_3)]a}_{\text{Coupling bar B}} \qquad \underbrace{\Delta T_C = (\Delta m_3)a}_{\text{Coupling bar C}}$$

SOLUTION

a. Moving luggage from car 2 to car 1 does not change the total mass $m_1 + m_2 + m_3$, so $\Delta(m_1 + m_2 + m_3) = 0$ kg and $\boxed{\Delta T_A = 0\ \text{N}}$.

The transfer from car 2 to car 1 causes the total mass $m_2 + m_3$ to decrease by 39 kg, so $\Delta(m_2 + m_3) = -39$ kg and

$$\Delta T_B = [\Delta(m_2 + m_3)]a = (-39\ \text{kg})(0.12\ \text{m/s}^2) = \boxed{-4.7\ \text{N}}$$

The transfer from car 2 to car 1 does not change the mass m_3, so $\Delta m_3 = 0$ kg and $\boxed{\Delta T_C = 0\ \text{N}}$.

b. Moving luggage from car 2 to car 3 does not change the total mass $m_1 + m_2 + m_3$, so $\Delta(m_1 + m_2 + m_3) = 0$ kg and $\boxed{\Delta T_A = 0\ \text{N}}$.

The transfer from car 2 to car 3 does not change the total mass $m_2 + m_3$, so $\Delta(m_2 + m_3) = 0$ kg and $\boxed{\Delta T_B = 0\ \text{N}}$.

The transfer from car 2 to car 3 causes the mass m_3 to increase by 39 kg, so $\Delta m_3 = +39$ kg and

$$\Delta T_C = (\Delta m_3)a = (+39\ \text{kg})(0.12\ \text{m/s}^2) = \boxed{+4.7\ \text{N}}$$

106. ***REASONING AND SOLUTION*** Since both motions are characterized by constant acceleration, it follows that

$$\frac{y_J}{y_E} = \frac{\frac{1}{2}a_J t_J^2}{\frac{1}{2}a_E t_E^2}$$

where the subscripts designate those quantities that pertain to Jupiter and Earth. Since both objects fall the same distance, the above ratio is equal to unity. Solving for the ratio of the times yields

$$\frac{t_J}{t_E} = \sqrt{\frac{a_E}{a_J}} = \sqrt{\frac{GM_E / R_E^2}{GM_J / R_J^2}} = \frac{R_J}{R_E}\sqrt{\frac{M_E}{M_J}} = (11.2)\sqrt{\frac{1}{318}} = \boxed{0.628}$$

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107. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The free-body diagram is shown at the right. The forces that act on the picture are the pressing force P, the normal force $\mathbf{F}_N$ exerted on the picture by the wall, the weight mg of the picture, and the force of static friction $\mathbf{f}_s^{MAX}$. The maximum magnitude for the frictional force is given by Equation 4.7: $f_s^{MAX} = \mu_s F_N$. The picture is in equilibrium, and, if we take the directions to the right and up as positive, we have in the x direction

$$\sum F_x = P - F_N = 0 \qquad \text{or} \qquad P = F_N$$

and in the y direction

$$\sum F_y = f_s^{MAX} - mg = 0 \qquad \text{or} \qquad f_s^{MAX} = mg$$

Therefore,

$$f_s^{MAX} = \mu_s F_N = mg$$

But since $F_N = P$, we have

$$\mu_s P = mg$$

Solving for P, we have

$$P = \frac{mg}{\mu_s} = \frac{(1.10\text{ kg})(9.80\text{ m/s}^2)}{0.660} = \boxed{16.3\text{ N}}$$

108. ***REASONING AND SOLUTION***

a. The force acting on the sphere which accelerates it is the horizontal component of the tension in the string. Newton's second law for the horizontal motion of the sphere gives

$$T \sin \theta = ma$$

The vertical component of the tension in the string supports the weight of the sphere so

$$T \cos \theta = mg$$

Eliminating T from the above equations results in $\boxed{a = g \tan \theta}$.

b. $$a = g \tan \theta = \left(9.80 \text{ m/s}^2\right) \tan 10.0° = \boxed{1.73 \text{ m/s}^2}$$

c. Rearranging the result of part a and setting $a = 0 \text{ m/s}^2$ gives

$$\theta = \tan^{-1}(a/g) = \boxed{0°}$$

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109. ***REASONING AND SOLUTION***

a. Newton's second law for block 1 (10.0 kg) is

$$T = m_1 a \qquad (1)$$

Block 2 (3.00 kg) has two ropes attached each carrying a tension T. Also, block 2 only travels half the distance that block 1 travels in the same amount of time so its acceleration is only half of block 1's acceleration. Newton's second law for block 2 is then

$$2T - m_2 g = -(1/2)m_2 a \qquad (2)$$

Solving Equation (1) for a, substituting into Equation (2), and rearranging gives

$$T = \frac{\frac{1}{2} m_2 g}{1 + \frac{1}{4}\left(m_2 / m_1\right)} = \boxed{13.7 \text{ N}}$$

b. Using this result in Equation (1) yields

$$a = \frac{T}{m_1} = \frac{13.7 \text{ N}}{10.0 \text{ kg}} = \boxed{1.37 \text{ m/s}^2}$$

110. ***REASONING*** The diagram at the right shows the two applied forces that act on the crate. These two forces, plus the kinetic frictional force $\mathbf{f}_k$ constitute the net force that acts on the crate. Once the net force has been determined, Newtons' second law, $\Sigma\mathbf{F} = m\mathbf{a}$ (Equation 4.1) can be used to find the acceleration of the crate.

SOLUTION The sum of the two applied forces is $\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2$. The x-component of this sum is $F_x = F_1 \cos 55.0° + F_2$ = (88.0 N) cos 55.0° + 54.0 N = 104 N. The y-component of **F** is $F_y = F_1 \sin 55.0°$ = (88.0 N) sin 55.0° = 72.1 N. The magnitude of **F** is

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(104\text{ N})^2 + (72.1\text{ N})^2} = 127\text{ N}$$

Since the crate starts from rest, it moves along the direction of **F**. The kinetic frictional force $\mathbf{f}_k$ opposes the motion, so it points opposite to **F**. The net force acting on the crate is

the sum of **F** and $\mathbf{f}_k$. The magnitude a of the crate's acceleration is equal to the magnitude ΣF of the net force divided by the mass m of the crate

$$a = \frac{\Sigma F}{m} = \frac{-f_k + F}{m} \qquad (4.1)$$

According to Equation 4.8, the magnitude f_k of the kinetic frictional force is given by $f_k = \mu_k F_N$, where F_N is the magnitude of the normal force. In this situation, F_N is equal to the magnitude of the crate's weight, so $F_N = mg$. Thus, the x-component of the acceleration is

$$a = \frac{-\mu_k mg + F}{m} = \frac{-(0.35)(25.0\text{ kg})(9.80\text{ m/s}^2) + 127\text{ N}}{25.0\text{ kg}} = \boxed{1.65\text{ m/s}^2}$$

The crate moves along the direction of **F**, whose x and y components have been determined previously. Therefore, the acceleration is also along **F**. The angle ϕ that **F** makes with the x-axis can be found using the inverse tangent function.

$$\varphi = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \tan^{-1}\left(\frac{F_1 \sin 55.0°}{F_1 \cos 55.0° + F_2}\right)$$

$$= \tan^{-1}\left[\frac{(88.0\text{ N})\sin 55.0°}{(88.0\text{ N})\cos 55.0° + 54.0\text{ N}}\right] = \boxed{34.6° \text{ above the } x \text{ axis}}$$

111. **SSM** ***REASONING*** When the bicycle is coasting straight down the hill, the forces that act on it are the normal force $\mathbf{F}_N$ exerted by the surface of the hill, the force of gravity mg, and the force of air resistance R. When the bicycle climbs the hill, there is one additional force; it is the applied force that is required for the bicyclist to climb the hill at constant speed. We can use our knowledge of the motion of the bicycle down the hill to find R. Once R is known, we can analyze the motion of the bicycle as it climbs the hill.

SOLUTION The figure to the left below shows the free-body diagram for the forces during the downhill motion. The hill is inclined at an angle θ above the horizontal. The figure to the right shows these forces resolved into components parallel to and perpendicular to the line of motion.

Since the bicyclist is traveling at a constant velocity, his acceleration is zero. Therefore, according to Newton's second law, we have $\sum F_x = 0$ and $\sum F_y = 0$. Taking the direction up the hill as positive, we have $\sum F_x = R - mg \sin\theta = 0$, or

$$R = mg \sin\theta = (80.0 \text{ kg})(9.80 \text{ m/s}^2) \sin 15.0° = 203 \text{ N}$$

When the bicyclist climbs the same hill at constant speed, an applied force P must push the system up the hill, and the force of air resistance will oppose the motion by pointing down the hill.

The figure at the right shows the resolved forces that act on the system during the uphill motion.

Using the same sign convention as above, we have $\sum F_x = P - mg \sin\theta - R = 0$, or

$$P = R + mg \sin\theta = 203 \text{ N} + 203 \text{ N} = \boxed{406 \text{ N}}$$

1. $\boxed{\text{SSM}}$ ***REASONING*** The speed of the plane is given by Equation 5.1: $v = 2\pi r / T$, where T is the period or the time required for the plane to complete one revolution.

SOLUTION Solving Equation 5.1 for T we have

$$T = \frac{2\pi r}{v} = \frac{2\pi(2850\text{ m})}{110\text{ m/s}} = \boxed{160\text{ s}}$$

2. ***REASONING AND SOLUTION*** The period is $T = 2\pi r/v$ so the radius of the track is

$$r = vT/(2\pi) = (17 \text{ m/s})(118 \text{ s})/(2\pi) = \boxed{320 \text{ m}}$$

---

3. ***REASONING AND SOLUTION*** The time for one revolution, the period T, can be found from

$$T = 2\pi\, r/v = 2\pi\,(0.053\text{ m})/(21\text{ m/s}) = \boxed{0.016\text{ s}}$$

4. ***REASONING AND SOLUTION*** Since the speed of the object on and off the circle remains constant at the same value, the object always travels the same distance in equal time intervals, both on and off the circle. Furthermore since the object travels the distance *OA* in the same time it would have moved from *O* to *P* on the circle, we know that the distance *OA* is equal to the distance along the arc of the circle from *O* to *P*.

The circumference of the circle is $2\pi r = 2\pi(3.6\text{ m}) = 22.6\text{ m}$. The arc *OP* subtends an angle of $\theta = 25°$; therefore, since any circle contains 360°, the arc *OP* is 25/360 or 6.9 per cent of the circumference of the circle. Thus,

$$OP = (22.6\text{ m})(0.069) = 1.6\text{ m}$$

and, from the argument given above, we conclude that the distance *OA* is $\boxed{1.6\text{ m}}$.

5. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** In each case, the magnitude of the centripetal acceleration is given by Equation 5.2, $a_c = v^2 / r$. Therefore,

$$\frac{a_{cA}}{a_{cB}} = \frac{v_A^2 / r_A}{v_B^2 / r_B}$$

Since each boat experiences the same centripetal acceleration, $a_{cA} / a_{cB} = 1$. Solving for the ratio of the speeds gives

$$\frac{v_A}{v_B} = \sqrt{\frac{r_A}{r_B}} = \sqrt{\frac{120 \text{ m}}{240 \text{ m}}} = \boxed{0.71}$$

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6. ***REASONING*** The centripetal acceleration is given by Equation 5.2 as $a_c = v^2/r$. The value of the radius r is given, so to determine a_c we need information about the speed v. But the speed is related to the period T by $v = (2\pi r)/T$, according to Equation 5.1. We can substitute this expression for the speed into Equation 5.2 and see that

$$a_c = \frac{v^2}{r} = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

SOLUTION To use the expression obtained in the reasoning, we need a value for the period T. The period is the time for one revolution. Since the container is turning at 2.0 revolutions per second, the period is T = (1 s)/(2.0 revolutions) = 0.50 s. Thus, we find that the centripetal acceleration is

$$a_c = \frac{4\pi^2 r}{T^2} = \frac{4\pi^2 (0.12\text{ m})}{(0.50\text{ s})^2} = \boxed{19\text{ m/s}^2}$$

7. ***REASONING AND SOLUTION*** Let s represent the length of the path of the pebble after it is released. From Conceptual Example 2, we know that the pebble will fly off tangentially. Therefore, the path s is per-perpendicular to the radius r of the circle. Thus, the distances r, s, and d form a right triangle with hypotenuse d as shown in the figure at the right. From the figure we see that

$$\cos\alpha = \frac{r}{d} = \frac{r}{10r} = \frac{1}{10}$$

or

$$\alpha = \cos^{-1}\left(\frac{1}{10}\right) = 84°$$

Furthermore, from the figure, we see that $\alpha + \theta + 35° = 180°$. Therefore,

$$\theta = 145° - \alpha = 145° - 84° = \boxed{61°}$$

8. ***REASONING AND SOLUTION***

a. At the equator a person travels in a circle whose radius equals the radius of the earth, $r = R_e = 6.38 \times 10^6$ m, and whose period of rotation is T = 1 d = 86 400 s. We have

$$v = 2\pi R_e/T = \boxed{464 \text{ m/s}}$$

The centripetal acceleration is

$$a_c = \frac{v^2}{r} = \frac{(464 \text{ m/s})^2}{6.38 \times 10^6 \text{ m}} = \boxed{3.37 \times 10^{-2} \text{ m/s}^2}$$

b. At 30.0° latitude a person travels in a circle of radius,

$$r = R_e \cos 30.0° = 5.53 \times 10^6 \text{ m}$$

Thus,

$$v = 2\pi r/T = \boxed{402 \text{ m/s}} \quad \text{and} \quad a_c = v^2/r = \boxed{2.92 \times 10^{-2} \text{ m/s}^2}$$

9. **SSM** ***REASONING*** The magnitude of the centripetal acceleration of any point on the helicopter blade is given by Equation 5.2, $a_C = v^2 / r$, where r is the radius of the circle on which that point moves. From Equation 5.1: $v = 2\pi r / T$. Combining these two expressions, we obtain

$$a_C = \frac{4\pi^2 r}{T^2}$$

All points on the blade move with the same period T.

SOLUTION The ratio of the centripetal acceleration at the end of the blade (point 1) to that which exists at a point located 3.0 m from the center of the circle (point 2) is

$$\frac{a_{C1}}{a_{C2}} = \frac{4\pi^2 r_1 / T^2}{4\pi^2 r_2 / T^2} = \frac{r_1}{r_2} = \frac{6.7\text{ m}}{3.0\text{ m}} = \boxed{2.2}$$

10. ***REASONING AND SOLUTION*** The sample makes one revolution in time, T, as given by $T = 2\pi r/v$. The speed is

$$v^2 = ra_c = (5.00 \times 10^{-2}\ \text{m})(6.25 \times 10^3)(9.80\ \text{m/s}^2) \text{ so that } v = 55.3\ \text{m/s}$$

The period is

$$T = 2\pi\ (5.00 \times 10^{-2}\ \text{m})/(55.3\ \text{m/s}) = 5.68 \times 10^{-3}\ \text{s} = 9.47 \times 10^{-5}\ \text{min}$$

The number of revolutions per minute = $1/T = \boxed{10\ 600\ \text{rev/min}}$.

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11. $\boxed{\text{SSM}}$ ***REASONING*** In Example 3, it was shown that the magnitudes of the centripetal acceleration for the two cases are

$$[\text{Radius} = 33\ \text{m}] \qquad a_C = 35\ \text{m/s}^2$$
$$[\text{Radius} = 24\ \text{m}] \qquad a_C = 48\ \text{m/s}^2$$

According to Newton's second law, the centripetal force is $F_C = ma_C$ (see Equation 5.3).

SOLUTION a. Therefore, when the sled undergoes the turn of radius 33 m,

$$F_C = ma_C = (350\ \text{kg})(35\ \text{m/s}^2) = \boxed{1.2 \times 10^4\ \text{N}}$$

b. Similarly, when the radius of the turn is 24 m,

$$F_C = ma_C = (350\ \text{kg})(48\ \text{m/s}^2) = \boxed{1.7 \times 10^4\ \text{N}}$$

12. ***REASONING*** The centripetal force that acts on the skater is $F_c = mv^2/r$, according to Equation 5.3. This expression can be solved directly to determine the mass m.

SOLUTION Solving Equation 5.3 for the mass m gives

$$m = \frac{F_c r}{v^2} = \frac{(460\ \text{N})(31\ \text{m})}{(14\ \text{m/s})^2} = \boxed{73\ \text{kg}}$$

13. ***REASONING AND SOLUTION*** The centripetal force is provided by the maximum force of static friction, $f_s^{max} = mv^2/r$. The new frictional force, f_s', is one-third the original value so $f_s' = mv'^2/r = f_s^{max}/3 = mv^2/3r$. Solving for the new velocity, v', we obtain

$$v' = \frac{(v)}{\sqrt{3}} = \frac{(21\ \text{m/s})}{\sqrt{3}} = \boxed{12\ \text{m/s}}$$

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14. ***REASONING*** The person feels the centripetal force acting on his back. This force is $F_C = mv^2/r$, according to Equation 5.3. This expression can be solved directly to determine the radius r of the chamber.

SOLUTION Solving Equation 5.3 for the radius r gives

$$r = \frac{mv^2}{F_C} = \frac{(83\text{ kg})(3.2\text{ m/s})^2}{560\text{ N}} = \boxed{1.5\text{ m}}$$

15. ***REASONING AND SOLUTION***

a. In terms of the period of the motion, the centripetal force is written as

$$F_c = 4\pi^2 mr/T^2 = 4\pi^2 (0.0120 \text{ kg})(0.100 \text{ m})/(0.500 \text{ s})^2 = \boxed{0.189 \text{ N}}$$

b. The centripetal force varies as the square of the speed. Thus, doubling the speed would increase the centripetal force $\boxed{\text{by a factor of } 2^2 = 4}$.

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16. ***REASONING AND SOLUTION*** Initially, the stone executes uniform circular motion in a circle of radius r which is equal to the radius of the tire. At the instant that the stone flies out of the tire, the force of static friction just exceeds its maximum value $f_s^{\text{MAX}} = \mu_s F_N$ (see Equation 4.7). The force of static friction that acts on the stone from one side of the tread channel is, therefore,

$$f_s^{\text{MAX}} = 0.90\ (1.8\ \text{N}) = 1.6\ \text{N}$$

and the magnitude of the total frictional force that acts on the stone just before it flies out is $2 \times 1.6\ \text{N} = 3.2\ \text{N}$. If we assume that only static friction supplies the centripetal force, then, $F_c = 3.2\ \text{N}$. Solving Equation 5.3 ($F_c = mv^2/r$) for the radius r, we have

$$r = \frac{mv^2}{F_c} = \frac{6.0 \times 10^{-3}\ \text{kg}\ (13\ \text{m/s})^2}{3.2\ \text{N}} = \boxed{0.31\ \text{m}}$$

17. SSM WWW ***REASONING*** Let v_0 be the initial speed of the ball as it begins its projectile motion. Then, the centripetal force is given by Equation 5.3: $F_C = mv_0^2/r$. We are given the values for m and r; however, we must determine the value of v_0 from the details of the projectile motion after the ball is released.

In the absence of air resistance, the x component of the projectile motion has zero acceleration, while the y component of the motion is subject to the acceleration due to gravity. The horizontal distance traveled by the ball is given by Equation 3.5a (with $a_x = 0$):

$$x = v_{0x}t = (v_0 \cos\theta)t$$

with t equal to the flight time of the ball while it exhibits projectile motion. The time t can be found by considering the vertical motion. From Equation 3.3b,

$$v_y = v_{0y} + a_y t$$

After a time t, $v_y = -v_{0y}$. Assuming that up and to the right are the positive directions, we have

$$t = \frac{-2v_{0y}}{a_y} = \frac{-2v_0 \sin\theta}{a_y}$$

and

$$x = (v_0 \cos\theta)\left(\frac{-2v_0 \sin\theta}{a_y}\right)$$

Using the fact that $2\sin\theta\cos\theta = \sin 2\theta$, we have

$$x = -\frac{2v_0^2 \cos\theta \sin\theta}{a_y} = -\frac{v_0^2 \sin 2\theta}{a_y} \qquad (1)$$

Equation (1) (with upward and to the right chosen as the positive directions) can be used to determine the speed v_0 with which the ball begins its projectile motion. Then Equation 5.3 can be used to find the centripetal force.

SOLUTION Solving equation (1) for v_0, we have

$$v_0 = \sqrt{\frac{-x\,a_y}{\sin 2\theta}} = \sqrt{\frac{-(86.75\text{ m})(-9.80\text{ m/s}^2)}{\sin 2(41°)}} = 29.3\text{ m/s}$$

Then, from Equation 5.3,

$$F_C = \frac{mv_0^2}{r} = \frac{(7.3\text{ kg})(29.3\text{ m/s})^2}{1.8\text{ m}} = \boxed{3500\text{ N}}$$

18. ***REASONING AND SOLUTION***

a. The vertical component of the tension in the cable supports the weight of the chair and occupants. Newton's second law applied to the vertical gives

$$T \cos 65.0° - mg = 0$$

Then

$$T = mg/\cos 65.0° = \boxed{5.1 \times 10^3 \text{ N}}$$

b. The horizontal component of the tension is the centripetal force so

$$T \sin 65.0° = mv^2/r$$

The radius r is found from the drawing: r = (12.0 m) sin 65.0° = 10.9 m. Now,

$$v = \sqrt{\frac{rT \sin 65.0^o}{m}} = \sqrt{\frac{(10.9 \text{ m})(5.1 \text{ x } 10^3 \text{ N}) \sin 65.0^o}{220 \text{ kg}}} = \boxed{15 \text{ m/s}}$$

19. ***REASONING AND SOLUTION*** The centripetal acceleration of the block is

$$a_c = v^2/r = (28\ \text{m/s})^2/(150\ \text{m}) = 5.2\ \text{m/s}^2$$

The angle θ can be obtained from

$$\theta = \tan^{-1}\left(\frac{a_C}{g}\right) = \tan^{-1}\left(\frac{5.2\ \text{m/s}^2}{9.80\ \text{m/s}^2}\right) = \boxed{28°}$$

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20. ***REASONING AND SOLUTION*** Assuming that there is no friction between the tires and the ice, Equation 5.4 applies.

$$\tan\theta = v^2/rg = (25\text{ m/s})^2/(150\text{ m})(9.80\text{ m/s}^2) = 0.43$$

or

$$\boxed{\theta = 23°}$$

---

21. **SSM** ***REASONING AND SOLUTION*** Equation 5.4 gives the relationship between the speed v the angle of banking, and the radius of curvature. Solving for v, we obtain

$$v = \sqrt{r\,g\,\tan\theta} = \sqrt{(120\text{ m})(9.80\text{ m/s}^2)\tan 18°} = \boxed{2.0\times 10^1\text{ m/s}}$$

22. ***REASONING*** For an unbanked curve static friction supplies the centripetal force. Therefore, we can obtain an expression for the speed by equating the force of static friction to the centripetal force, which is given by Equation 5.3. For a banked curve without friction, Equation 5.4 relates the banking angle, the speed, and the radius of the curve. By equating the speeds for the unbanked and banked curves we will be able to obtain a value for the banking angle.

SOLUTION The force of static friction is given by Equation 4.7 as $f_s^{\text{MAX}} = \mu_s F_N$, where μ_s is the coefficient of static friction and F_N is the normal force. Since the car does not accelerate in the vertical direction, the normal force and the weight of the car balance, so that $F_N = mg$. Thus, the force of static friction is $f_s^{\text{MAX}} = \mu_s mg$. Substituting this expression for the centripetal force in Equation 5.3, we find for an unbanked curve that

$$F_c = \mu_s mg = \frac{mv^2}{r} \qquad \text{or} \qquad v = \sqrt{\mu_s gr}$$

For a banked curve, we have from Equation 5.4 that

$$\tan\theta = \frac{v^2}{rg} \qquad \text{or} \qquad v = \sqrt{rg\tan\theta}$$

Equating the two expressions for the speed, we find that

$$\sqrt{rg\tan\theta} = \sqrt{\mu_s gr} \qquad \text{or} \qquad \tan\theta = \mu_s$$

$$\theta = \tan^{-1}(\mu_s) = \tan^{-1}(0.88) = \boxed{41°}$$

23. ***REASONING*** The angle θ at which a friction-free curve is banked depends on the radius r of the curve and the speed v with which the curve is to be negotiated, according to Equation 5.4: $\tan\theta = v^2/(rg)$. For known values of θ and r, the safe speed is

$$v = \sqrt{rg\ \tan\theta}$$

Before we can use this result, we must determine $\tan\theta$ for the banking of the track.

SOLUTION The drawing at the right shows a cross-section of the track. From the drawing we have

$$\tan\theta = \frac{18\ \text{m}}{53\ \text{m}} = 0.34$$

a. Therefore, the smallest speed at which cars can move on this track without relying on friction is

$$v_{\text{min}} = \sqrt{(112\ \text{m})\ (9.80\ \text{m/s}^2)\ 0.34} = \boxed{19\ \text{m/s}}$$

b. Similarly, the largest speed is

$$v_{\text{max}} = \sqrt{(165\ \text{m})\ (9.80\ \text{m/s}^2)\ 0.34} = \boxed{23\ \text{m/s}}$$

24. ***REASONING AND SOLUTION***

a. The horizontal component of the lift, L, is the centripetal force which holds the plane in the circle,

$$L \sin \theta = mv^2/r$$

The vertical component of the lift supports the weight of the plane,

$$L \cos \theta = mg$$

Dividing the first equation by the second yields the required result.

$$\boxed{\tan\theta = \frac{v^2}{rg}}$$

b. Substituting values yields

$$\theta = \tan^{-1}\left[\frac{(195 \text{ m/s})^2}{(8250 \text{ m})(9.80 \text{ m/s}^2)}\right] = \boxed{25.2°}$$

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25. **SSM** **WWW** ***REASONING*** Refer to Figure 5.10 in the text. The horizontal component of the lift **L** is the centripetal force that holds the plane in the circle. Thus,

$$L \sin\theta = \frac{mv^2}{r} \tag{1}$$

The vertical component of the lift supports the weight of the plane; therefore,

$$L \cos\theta = mg \tag{2}$$

Dividing the first equation by the second gives

$$\tan\theta = \frac{v^2}{rg} \tag{3}$$

Equation (3) can be used to determine the angle θ of banking. Once θ is known, then the magnitude of L can be found from either equation (1) or equation (2).

SOLUTION Solving equation (3) for θ gives

$$\theta = \tan^{-1}\left[\frac{(123\ \text{m/s})^2}{(3810\ \text{m})(9.80\ \text{m/s}^2)}\right] = 22.1°$$

The lifting force is, from equation (2),

$$L = \frac{mg}{\cos\theta} = \frac{(2.00\times10^5\ \text{kg})(9.80\ \text{m/s}^2)}{\cos 22.1°} = \boxed{2.12\times10^6\ \text{N}}$$

26. ***REASONING*** The centripetal force F_c required to keep an object of mass m that moves with speed v on a circle of radius r is $F_c = mv^2/r$ (Equation 5.3). From Equation 5.1, we know that $v = 2\pi r/T$, where T is the period or the time for the suitcase to go around once. Therefore, the centripetal force can be written as

$$F_c = \frac{m(2\pi r/T)^2}{r} = \frac{4m\pi^2 r}{T^2} \qquad (1)$$

This expression can be solved for T. However, we must first find the centripetal force that acts on the suitcase.

SOLUTION Three forces act on the suitcase. They are the weight $m\mathbf{g}$ of the suitcase, the force of static friction $\mathbf{f}_s^{MAX}$, and the normal force $\mathbf{F}_N$ exerted on the suitcase by the surface of the carousel. The following figure shows the free body diagram for the suitcase.

In this diagram, the y axis is oriented along the vertical direction. The force of gravity acts, then, in the $-y$ direction. The centripetal force that causes the suitcase to move on its circular path is provided by the net force in the $+x$ direction in the diagram. From the diagram, we can see that only the forces $\mathbf{F}_N$ and $\mathbf{f}_s^{MAX}$ have horizontal components. Thus, we have $F_c = f_s^{MAX}\cos\theta - F_N\sin\theta$, where the minus sign indicates that the x component of $\mathbf{F}_N$ points to the left in the diagram. Using Equation 4.7 for the maximum static frictional force, we can write this result as in equation (2).

$$F_c = \mu_s F_N \cos\theta - F_N \sin\theta = F_N(\mu_s \cos\theta - \sin\theta) \qquad (2)$$

If we apply Newton's second law in the y direction, we see from the diagram that

$$F_N \cos\theta + f_s^{MAX}\sin\theta - mg = ma_y = 0 \qquad \text{or} \qquad F_N\cos\theta + \mu_s F_N \sin\theta - mg = 0$$

where we again have used Equation 4.7 for the maximum static frictional force. Solving for the normal force, we find

$$F_N = \frac{mg}{\cos\theta + \mu_s \sin\theta}$$

Using this result in equation (2), we obtain the magnitude of the centripetal force that acts on the suitcase:

$$F_c = F_N(\mu_s \cos\theta - \sin\theta) = \frac{mg(\mu_s \cos\theta - \sin\theta)}{\cos\theta + \mu_s \sin\theta}$$

With this expression for the centripetal force, equation (1) becomes

$$\frac{mg(\mu_s \cos\theta - \sin\theta)}{\cos\theta + \mu_s \sin\theta} = \frac{4m\pi^2 r}{T^2}$$

Solving for the period T, we find

$$T = \sqrt{\frac{4\pi^2 r(\cos\theta + \mu_s \sin\theta)}{g(\mu_s \cos\theta - \sin\theta)}} = \sqrt{\frac{4\pi^2 (11.0\text{ m})(\cos 36.0° + 0.760 \sin 36.0°)}{(9.80\text{ m/s}^2)(0.760 \cos 36.0° - \sin 36.0°)}} = \boxed{45\text{ s}}$$

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27. **SSM** **WWW** ***REASONING*** Equation 5.5 gives the orbital speed for a satellite in a circular orbit around the earth. It can be modified to determine the orbital speed around any planet **P** by replacing the mass of the earth M_E by the mass of the planet M_P: $v = \sqrt{GM_P / r}$.

SOLUTION The ratio of the orbital speeds is, therefore,

$$\frac{v_2}{v_1} = \frac{\sqrt{GM_P / r_2}}{\sqrt{GM_P / r_1}} = \sqrt{\frac{r_1}{r_2}}$$

Solving for v_2 gives

$$v_2 = v_1 \sqrt{\frac{r_1}{r_2}} = (1.70 \times 10^4 \text{ m/s}) \sqrt{\frac{5.25 \times 10^6 \text{ m}}{8.60 \times 10^6 \text{ m}}} = \boxed{1.33 \times 10^4 \text{ m/s}}$$

28. ***REASONING AND SOLUTION*** The normal force exerted by the wall on each astronaut is the centripetal force needed to keep him in the circular path, i.e., $F_c = mv^2/r$. Rearranging and letting $F_c = (1/2)mg$ yields

$$r = 2v^2/g = 2(35.8 \text{ m/s})^2/(9.80 \text{ m/s}^2) = \boxed{262 \text{ m}}$$

---

29. ***REASONING AND SOLUTION*** We have for Jupiter $v^2 = GM_J/r$ where

$$r = 6.00 \times 10^5 \text{ m} + 7.14 \times 10^7 \text{ m} = 7.20 \times 10^7 \text{ m}$$

Thus,

$$v = \sqrt{\frac{(6.67 \text{ x } 10^{-11} \text{ Nm}^2/\text{kg}^2)(1.90 \text{ x } 10^{27} \text{ kg})}{7.20 \text{ x } 10^7 \text{ m}}} = \boxed{4.20 \text{ x } 10^4 \text{ m/s}}$$

---

30. ***REASONING AND SOLUTION*** The period of a satellite is given by

$$T = \sqrt{\frac{4\pi^2 r^3}{GM_E}} = \sqrt{\frac{4\pi^2 [(2)(6.38 \times 10^6 \text{ m})]^3}{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}} = \boxed{1.43 \times 10^4 \text{ s}}$$

31. ***REASONING AND SOLUTION***

a. We have

$$v = \frac{2\pi r}{T} = \frac{2\pi\left(1.50 \times 10^{11}\ \text{m}\right)}{3.15 \times 10^{7}\ \text{s}} = \boxed{2.99 \times 10^{4}\ \text{m/s}}$$

b. For the earth-sun system

$$M_S = \frac{v^2 r}{G} = \frac{\left(2.99 \times 10^{4}\ \text{m/s}\right)^2 \left(1.50 \times 10^{11}\ \text{m}\right)}{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2} = \boxed{2.01 \times 10^{30}\ \text{kg}}$$

---

32. ***REASONING AND SOLUTION*** The period of rotation is given by $T^2 = 4\pi^2 r^3/GM$. Comparing the Earth and Venus yields

$$(T_V/T_E)^2 = (r_V/r_E)^3 \text{ so that } T_V/T_E = 0.611.$$

The earth's orbital period is 365 days so

$$T_V = (0.611)(365 \text{ days}) = \boxed{223 \text{ days}}$$

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33. $\boxed{\text{SSM}}$ ***REASONING*** The true weight of the satellite when it is at rest on the planet's surface can be found from Equation 4.4: $W = (GM_P m)/r^2$ where M_P and m are the masses of the planet and the satellite, respectively, and r is the radius of the planet. However, before we can use Equation 4.4, we must determine the mass M_P of the planet.

The mass of the planet can be found by replacing M_E by M_P in Equation 5.6 and solving for M_P. When using Equation 5.6, we note that r corresponds to the radius of the circular orbit *relative to the center of the planet*.

SOLUTION The period of the satellite is $T = 2.00\text{ h} = 7.20 \times 10^3\text{ s}$. From Equation 5.6,

$$M_P = \frac{4\pi^2 r^3}{GT^2} = \frac{4\pi^2\left[(4.15\times10^6\text{ m}) + (4.1\times10^5\text{ m})\right]^3}{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(7.20\times10^3\text{ s})^2} = 1.08\times10^{24}\text{ kg}$$

Using Equation 4.4, we have

$$W = \frac{GM_P m}{r^2} = \frac{(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2)(1.08\times10^{24}\text{ kg})(5850\text{ kg})}{(4.15\times10^6\text{ m})^2} = \boxed{2.45\times10^4\text{ N}}$$

34. ***REASONING*** Equation 5.2 for the centripetal acceleration applies to both the plane and the satellite, and the centripetal acceleration is the same for each. Thus, we have

$$a_c = \frac{v_{\text{plane}}^2}{r_{\text{plane}}} = \frac{v_{\text{satellite}}^2}{r_{\text{satellite}}} \quad \text{or} \quad v_{\text{plane}} = \left(\sqrt{\frac{r_{\text{plane}}}{r_{\text{satellite}}}}\right) v_{\text{satellite}}$$

The speed of the satellite can be obtained directly from Equation 5.5.

SOLUTION Using Equation 5.5, we can express the speed of the satellite as

$$v_{\text{satellite}} = \sqrt{\frac{Gm_E}{r_{\text{satellite}}}}$$

Substituting this expression into the expression obtained in the reasoning for the speed of the plane gives

$$v_{\text{plane}} = \left(\sqrt{\frac{r_{\text{plane}}}{r_{\text{satellite}}}}\right) v_{\text{satellite}} = \left(\sqrt{\frac{r_{\text{plane}}}{r_{\text{satellite}}}}\right)\sqrt{\frac{Gm_E}{r_{\text{satellite}}}} = \frac{\sqrt{r_{\text{plane}} Gm_E}}{r_{\text{satellite}}}$$

$$v_{\text{plane}} = \frac{\sqrt{(15\ \text{m})(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(5.98\times10^{24}\ \text{kg})}}{6.7\times10^{6}\ \text{m}} = \boxed{12\ \text{m/s}}$$

35. ***REASONING AND SOLUTION***

a. The centripetal acceleration of a point on the rim of chamber A is the artificial acceleration due to gravity,

$$a_A = v_A^2/r_A = 10.0 \text{ m/s}^2.$$

A point on the rim of chamber A moves with a speed $v_A = 2\pi r_A/T$ where T is the period of revolution, 60.0 s. Substituting the second equation into the first and rearranging yields

$$r_A = a_A T^2/(4\pi^2) = \boxed{912 \text{ m}}$$

b. Now

$$r_B = r_A/4.00 = \boxed{228 \text{ m}}$$

c. A point on the rim of chamber B has a centripetal acceleration $a_B = v_B^2/r_B$. The point moves with a speed $v_B = 2\pi r_B/T$. Substituting the second equation into the first yields

$$a_B = \frac{4\pi^2 r_B}{T^2} = \frac{4\pi^2 (228 \text{ m})}{(60.0 \text{ s})^2} = \boxed{2.50 \text{ m/s}^2}$$

36. ***REASONING*** As the pilot goes around the vertical circle, the magnitude of the normal force that is exerted on the pilot changes. The magnitude of the normal force is largest at the bottom of the circle where the normal points upward and the weight $m\mathbf{g}$ points downward (see Figure 5.21). The resultant of the normal force and the weight provide the centripetal force that keeps the pilot moving on the vertical circle. If we take upward as the positive direction in Figure 5.21, then Equation 5.3 ($F_c = mv^2/r$) can be written

$$F_N - mg = \frac{mv^2}{r}$$

When the normal force exerted on the pilot by his seat is equal to three times his weight, $F_N = 3mg$, and the above expression becomes

$$3mg - mg = \frac{mv^2}{r} \qquad \text{or} \qquad 2g = \frac{v^2}{r}$$

SOLUTION Solving for r we obtain

$$r = \frac{v^2}{2g} = \frac{(230\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = \boxed{2.7 \times 10^3\ \text{m}}$$

37. **SSM** ***REASONING*** This situation is similar to the loop-the-loop trick discussed in Section 5.7 of the text. When the plane passes over the top of a vertical circle of radius r such that the passengers experience apparent weightlessness, the centripetal force is provided entirely by the true weight mg. Thus, $mg = mv^2 / r$.

SOLUTION Solving the above expression for r gives

$$r = \frac{v^2}{g} = \frac{(215\ \text{m/s})^2}{9.80\ \text{m/s}^2} = \boxed{4.72 \times 10^3\ \text{m}}$$

38. ***REASONING*** The centripetal force is the name given to the net force pointing toward the center of the circular path. At point 3 at the top the net force pointing toward the center of the circle consists of the normal force and the weight, both pointing toward the center. At point 1 at the bottom the net force consists of the normal force pointing upward toward the center and the weight pointing downward or away from the center. In either case the centripetal force is given by Equation 5.3 as $F_c = mv^2/r$.

SOLUTION At point 3 we have

$$F_c = F_N + mg = \frac{mv_3^2}{r}$$

At point 1 we have

$$F_c = F_N - mg = \frac{mv_1^2}{r}$$

Subtracting the second equation from the first gives

$$2mg = \frac{mv_3^2}{r} - \frac{mv_1^2}{r}$$

Rearranging gives

$$v_3^2 = 2gr + v_1^2$$

Thus, we find that

$$v_3 = \sqrt{2(9.80\ \text{m/s}^2)(3.0\ \text{m}) + (15\ \text{m/s})^2} = \boxed{17\ \text{m/s}}$$

39. **SSM** ***REASONING*** As the motorcycle passes over the top of the hill, it will experience a centripetal force, the magnitude of which is given by Equation 5.3: $F_C = mv^2 / r$. The centripetal force is provided by the net force on the cycle + driver system. At that instant, the net force on the system is composed of the normal force, which points upward, and the weight, which points downward. Taking the direction toward the center of the circle (downward) as the positive direction, we have $F_C = mg - F_N$. This expression can be solved for F_N, the normal force.

SOLUTION

a. The magnitude of the centripetal force is

$$F_C = \frac{mv^2}{r} = \frac{(342\text{ kg})(25.0\text{ m/s})^2}{126\text{ m}} = \boxed{1.70 \times 10^3\text{ N}}$$

b. The magnitude of the normal force is

$$F_N = mg - F_C = (343\text{ kg})(9.80\text{ m/s}^2) - 1.70 \times 10^3\text{ N} = \boxed{1.66 \times 10^3\text{ N}}$$

40. ***REASONING AND SOLUTION*** When the stone is whirled in a horizontal circle, the centripetal force is provided by the tension in the string and is given by

$$T_h = \frac{mv^2}{r}$$

When the stone is whirled in a vertical circle, the maximum tension occurs when the stone is at the lowest point in its path.

The diagram at the right shows that forces that act on the stone in this case. The centripetal force is provided by the resultant of T_v and the weight of the stone:

$$T_v - mg = \frac{mv^2}{r}$$

The tension in the string for this case is

$$T_v = \frac{mv^2}{r} + mg$$

Since the maximum tension in the string for the case of vertical motion is 10.0% larger than that in the case of horizontal motion,

$$T_v = (1.000 + 0.100)\, T_h$$

or

$$\frac{mv^2}{r} + mg = (1.100)\,\frac{mv^2}{r}$$

Solving for v yields

$$v = \sqrt{\frac{gr}{0.100}} = \sqrt{\frac{(9.80\ \text{m/s}^2)\,(0.950\ \text{m})}{0.100}} = \boxed{9.65\ \text{m/s}}$$

41. ***REASONING AND SOLUTION*** In general, when the roller-blader is at the top of the crest, there are two forces that act on her. They are her weight $m\mathbf{g}$, which acts downward, and the normal force $\mathbf{F}_{\mathrm{N}}$ exerted on the roller-blader by the ground. If we take upward as the positive direction, then, according to Equation 5.3 ($F_{\mathrm{c}} = mv^2/r$) the condition for the roller-blader to follow the circular crest of the hill is,

$$F_{\mathrm{N}} - mg = -\frac{mv^2}{r}$$

If the roller-blader goes fast enough to lose contact with the ground, $F_{\mathrm{N}} = 0$, and the above expression becomes

$$mg = \frac{mv^2}{r}$$

Solving for v gives

$$v = \sqrt{gr} = \sqrt{(9.80\ \mathrm{m/s^2})(20.0\ \mathrm{m})} = \boxed{14.0\ \mathrm{m/s}}$$

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42. ***REASONING*** The drawing at the right shows the two forces that act on a piece of clothing *just before* it loses contact with the wall of the cylinder. At that instant the centripetal force is provided by the normal force $\mathbf{F}_N$ and the radial component of the weight. From the drawing, the radial component of the weight is given by

$$mg\cos\phi = mg\cos(90° - \theta) = mg\sin\theta$$

Therefore, with inward taken as the positive direction, Equation 5.3 ($F_c = mv^2/r$) gives

$$F_N + mg\sin\theta = \frac{mv^2}{r}$$

At the instant that a piece of clothing loses contact with the surface of the drum, $F_N = 0$, and the above expression becomes

$$mg\sin\theta = \frac{mv^2}{r}$$

According to Equation 5.1, $v = 2\pi r/T$, and with this substitution we obtain

$$g\sin\theta = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

This expression can be solved for the period T. Since the period is the required time for one revolution, the number of revolutions per second can be found by calculating $1/T$.

SOLUTION Solving for the period, we obtain

$$T = \sqrt{\frac{4\pi^2 r}{g\sin\theta}} = 2\pi\sqrt{\frac{r}{g\sin\theta}} = 2\pi\sqrt{\frac{0.32\text{ m}}{(9.80\text{ m/s}^2)\sin 70.0°}} = 1.17\text{ s}$$

Therefore, the number of revolutions per second that the cylinder should make is

$$\frac{1}{T} = \frac{1}{1.17\text{ s}} = \boxed{0.85\text{ rev/s}}$$

43. **SSM** ***REASONING AND SOLUTION*** The magnitude of the centripetal force on the ball is given by Equation 5.3: $F_C = mv^2 / r$. Solving for v, we have

$$v = \sqrt{\frac{F_C r}{m}} = \sqrt{\frac{(0.028\ \text{N})(0.25\ \text{m})}{0.015\ \text{kg}}} = \boxed{0.68\ \text{m/s}}$$

44. ***REASONING AND SOLUTION*** The radius of a synchronous satellite is calculated in Example 11 as $r = 4.23 \times 10^7$ m. The speed is therefore,

$$v = \sqrt{\frac{GM_E}{r}} = \sqrt{\frac{(6.67 \times 10^{-11}\ \text{Nm}^2/\text{kg}^2)(5.98 \times 10^{24}\ \text{kg})}{4.23 \times 10^7\ \text{m}}} = \boxed{3070\ \text{m/s}}$$

---

45. ***REASONING AND SOLUTION***

a. We know

$$a_c = v^2/r = (1.4\ \text{m/s})^2/(0.039\ \text{m}) = \boxed{5.0 \times 10^1\ \text{m/s}^2}$$

b. Between sprockets, the chain is straight and travels at a constant speed, so its acceleration is $\boxed{\text{zero}}$.

c. For the front sprocket

$$a_c = v^2/r = (1.4\ \text{m/s})^2/(0.10\ \text{m}) = \boxed{2.0 \times 10^1\ \text{m/s}^2}$$

---

46. ***REASONING AND SOLUTION*** The force **P** supplied by the man will be largest when the partner is at the lowest point in the swing. The diagram at the right shows the forces acting on the partner in this situation. The centripetal force necessary to keep the partner swinging along the arc of a circle is provided by the resultant of the force supplied by the man and the weight of the partner. From the figure

$$P - mg = \frac{mv^2}{r}$$

Therefore,

$$P = \frac{mv^2}{r} + mg$$

Since the weight of the partner, W, is equal to mg, it follows that $m = (W/g)$ and

$$P = \frac{(W/g)v^2}{r} + W = \frac{[(475\text{ N})/(9.80\text{ m/s}^2)]\,(4.00\text{ m/s})^2}{(6.50\text{ m})} + (475\text{ N}) = \boxed{594\text{ N}}$$

47. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** Since the magnitude of the centripetal acceleration is given by Equation 5.2, $a_C = v^2 / r$, we can solve for r and find that

$$r = \frac{v^2}{a_C} = \frac{(98.8\ \text{m/s})^2}{3.00(9.80\ \text{m/s}^2)} = \boxed{332\ \text{m}}$$

48. ***REASONING*** The centripetal force is the name given to the net force pointing toward the center of the circular path. At the lowest point the net force consists of the tension in the arm pointing upward toward the center and the weight pointing downward or away from the center. In either case the centripetal force is given by Equation 5.3 as $F_c = mv^2/r$.

SOLUTION (a) The centripetal force is

$$F_c = \frac{mv^2}{r} = \frac{(9.5\text{ kg})(2.8\text{ m/s})^2}{0.85\text{ m}} = \boxed{88\text{ N}}$$

(b) Using T to denote the tension in the arm, at the bottom of the circle we have

$$F_c = T - mg = \frac{mv^2}{r}$$

$$T = mg + \frac{mv^2}{r} = (9.5\text{kg})(9.80\text{ m/s}^2) + \frac{(9.5\text{ kg})(2.8\text{ m/s})^2}{0.85\text{ m}} = \boxed{181\text{ N}}$$

49. **SSM** ***REASONING AND SOLUTION*** Since the tension serves the same purpose as the normal force at point 1 in Figure 5.21, we have, using the equation for the situation at point 1 with F_{N1} replaced by T,

$$\frac{mv^2}{r} = T - mg$$

Solving for T gives

$$T = \frac{mv^2}{r} + mg = m\left(\frac{v^2}{r} + g\right) = (2100\text{ kg})\left[\frac{(7.6\text{ m/s})^2}{15\text{ m}} + (9.80\text{ m/s}^2)\right] = \boxed{2.9 \times 10^4\text{ N}}$$

50. ***REASONING AND SOLUTION*** The period of the moon's motion (approximately the length of a month) is given by

$$T = \sqrt{\frac{4\pi^2 r^3}{G M_E}} = \sqrt{\frac{4\pi^2 (3.85 \times 10^8 \text{ m})^3}{(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(5.98 \times 10^{24} \text{ kg})}}$$

$$= 2.38 \times 10^6 \text{s} = \boxed{27.5 \text{ days}}$$

51. ***REASONING*** The centripetal acceleration for any point that is a distance r from the center of the disc is, according to Equation 5.2, $a_c = v^2/r$. From Equation 5.1, we know that $v = 2\pi r/T$ where T is the period of the motion. Combining these two equations, we obtain

$$a_c = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

SOLUTION Using the above expression for a_c, the ratio of the centripetal accelerations of the two points in question is

$$\frac{a_2}{a_1} = \frac{4\pi^2 r_2/T_2^2}{4\pi^2 r_1/T_1^2} = \frac{r_2/T_2^2}{r_1/T_1^2}$$

Since the disc is rigid, all points on the disc must move with the same period, so $T_1 = T_2$. Making this cancellation and solving for a_2, we obtain

$$a_2 = a_1 \frac{r_2}{r_1} = \left(120\ \text{m/s}^2\right)\left(\frac{0.050\ \text{m}}{0.030\ \text{m}}\right) = \boxed{2.0 \times 10^2\ \ \text{m/s}^2}$$

Note that even though $T_1 = T_2$, it is not true that $v_1 = v_2$. Thus, the simplest way to approach this problem is to express the centripetal acceleration in terms of the period T which cancels in the final step.

52. ***REASONING AND SOLUTION*** The centripetal acceleration for any point on the blade a distance r from center of the circle, according to Equation 5.2, is $a_c = v^2/r$. From Equation 5.1, we know that $v = 2\pi r/T$ where T is the period of the motion. Combining these two equations, we obtain

$$a_c = \frac{(2\pi r/T)^2}{r} = \frac{4\pi^2 r}{T^2}$$

a. Since the turbine blades rotate at 617 rev/s, all points on the blades rotate with a period of $T = (1/617)\text{ s} = 1.62 \times 10^{-3}\text{ s}$. Therefore, for a point with $r = 0.020\text{ m}$, the magnitude of the centripetal acceleration is

$$a_c = \frac{4\pi^2(0.020\text{ m})}{(1.62 \times 10^{-3}\text{ s})^2} = \boxed{3.0 \times 10^5\text{ m/s}^2}$$

b. Expressed as a multiple of g, this centripetal acceleration is

$$a_c = \left(3.0 \times 10^5\text{ m/s}^2\right)\left(\frac{1.00\,g}{9.80\text{ m/s}^2}\right) = \boxed{3.1 \times 10^4\ g}$$

53. **SSM** **WWW** ***REASONING*** If the effects of gravity are not ignored in Example 5, the plane will make an angle θ with the vertical as shown in figure **A** below. The figure **B** shows the forces that act on the plane, and figure **C** shows the horizontal and vertical components of these forces.

From figure **C** we see that the resultant force in the horizontal direction is the horizontal component of the tension in the guideline and provides the centripetal force. Therefore,

$$T \sin \theta = \frac{mv^2}{r}$$

From figure **A**, the radius r is related to the length L of the guideline by $r = L \sin\theta$; therefore,

$$T \sin \theta = \frac{mv^2}{L \sin \theta} \qquad (1)$$

The resultant force in the vertical direction is zero: $T\cos\theta - mg = 0$, so that

$$T\cos\theta = mg \qquad (2)$$

From equation (2) we have

$$T = \frac{mg}{\cos \theta} \qquad (3)$$

Equation (3) contains two unknown, T and θ. First we will solve equations (1) and (3) simultaneously to determine the value(s) of the angle θ. Once θ is known, we can calculate the tension using equation (3).

SOLUTION Substituting equation (3) into equation (1):

$$\left(\frac{mg}{\cos \theta}\right) \sin \theta = \frac{mv^2}{L \sin \theta}$$

Thus,

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v^2}{gL} \tag{4}$$

Using the fact that $\cos^2\theta + \sin^2\theta = 1$, equation (4) can be written

$$\frac{1 - \cos^2\theta}{\cos\theta} = \frac{v^2}{gL}$$

or

$$\frac{1}{\cos\theta} - \cos\theta = \frac{v^2}{gL}$$

This can be put in the form of an equation that is quadratic in $\cos\theta$. Multiplying both sides by $\cos\theta$ and rearranging yields:

$$\cos^2\theta + \frac{v^2}{gL}\cos\theta - 1 = 0 \tag{5}$$

Equation (5) is of the form

$$ax^2 + bx + c = 0 \tag{6}$$

with $x = \cos\theta$, $a = 1$, $b = v^2/(gL)$, and $c = -1$. The solution to equation (6) is found from the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

When $v = 19.0$ m/s, $b = 2.17$. The positive root from the quadratic formula gives $x = \cos\theta = 0.391$. Substitution into equation (3) yields

$$T = \frac{mg}{\cos\theta} = \frac{(0.900\text{ kg})(9.80\text{ m/s}^2)}{0.391} = \boxed{23\text{ N}}$$

When $v = 38.0$ m/s, $b = 8.67$. The positive root from the quadratic formula gives $x = \cos\theta = 0.114$. Substitution into equation (3) yields

$$T = \frac{mg}{\cos\theta} = \frac{(0.900\text{ kg})(9.80\text{ m/s}^2)}{0.114} = \boxed{77\text{ N}}$$

54. ***REASONING AND SOLUTION***

a. The centripetal force is provided by $\boxed{\text{the normal force exerted on the rider by the wall}}$.

b. Newton's second law applied in the horizontal direction gives

$$F_N = mv^2/r = (55.0\text{ kg})(10.0\text{ m/s})^2/(3.30\text{ m}) = \boxed{1670\text{ N}}$$

c. Newton's second law applied in the vertical direction gives $\mu_S F_N - mg = 0$ or

$$\mu_s = (mg)/F_N = \boxed{0.323}$$

---

55. ***REASONING AND SOLUTION*** If F is the force on mass 2, then 2F is the force on mass 1, and we have

$$\text{For mass 1:} \quad 2F = m_1 v_1^2/r_1 \qquad\qquad \text{For mass 2:} \quad F = m_2 v_2^2/r_2$$

Dividing the equations and rearranging gives

$$m_2/m_1 = (1/2)(r_2/r_1)(v_1/v_2)^2 = (v_1/v_2)^2 \text{ since } r_2 = 2r_1$$

The period of revolution is the same for both masses so $v_1 = 2\pi r_1/T$ and $v_2 = 2\pi r_2/T$. Dividing these gives $v_1/v_2 = r_1/r_2 = 1/2$. Now

$$m_2/m_1 = (1/2)^2 = \boxed{\frac{1}{4}}$$

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1. **SSM** ***REASONING AND SOLUTION*** We will assume that the tug-of-war rope remains parallel to the ground, so that the force that moves team B is in the same direction as the displacement. According to Equation 6.1, the work done by team A is

$$W = (F\cos\theta)s = (1100\ \text{N})(\cos 0°)(2.0\ \text{m}) = \boxed{2.2 \times 10^3\ \text{J}}$$

2. ***REASONING*** The work done by a constant force is given by Equation 6.1 as $W = (F\cos\theta)s$, where F is the magnitude of the force, s is the magnitude of the displacement, and θ is the angle between the force and the displacement. When the amplifier is being lowered at a constant velocity, it is in equilibrium so the net force in the vertical direction is zero. Thus, the magnitude of the force **F** exerted by the student is equal to the magnitude of the amplifier's weight m**g**. Since the amplifier is moving down, the angle between the force **F** and the displacement **s** is $\theta = 180°$.

SOLUTION

a. The work done by the student in lowering the amplifier is

$$W = (F\cos\theta)s = (mg\cos\theta)s$$

$$= (10.5\text{ kg})(9.80\text{ m/s}^2)(\cos 180°)(1.82\text{ m} - 0.75\text{ m}) = \boxed{-1.10\times10^2\text{ J}}$$

b. The work done by the gravitational force (the weight m**g** of the amplifier) is

$$W = (mg\cos\theta)s$$

$$= (10.5\text{ kg})(9.80\text{ m/s}^2)(\cos 0°)(1.82\text{ m} - 0.75\text{ m}) = \boxed{+1.10\times10^2\text{ J}}$$

3. ***REASONING AND SOLUTION*** According to Equation 6.1, $W = Fs \cos \theta$, the work is

a. $$W = (94.0\ \text{N})(35.0\ \text{m}) \cos 25.0° = \boxed{2980\ \text{J}}$$

b. $$W = (94.0\ \text{N})(35.0\ \text{m}) \cos 0° = \boxed{3290\ \text{J}}$$

4. ***REASONING AND SOLUTION*** Each locomotive does work

$$W = Ts \cos \theta = (5.00 \times 10^3 \text{ N})(2.00 \times 10^3 \text{ m}) \cos 20.0° = 9.40 \times 10^6 \text{ J}$$

The net work is then

$$W_T = 2W = \boxed{1.88 \times 10^7 \text{ J}}$$

5. **SSM** ***REASONING AND SOLUTION*** Solving Equation 6.1 for the angle θ, we obtain

$$\theta = \cos^{-1}\left(\frac{W}{F\,s}\right) = \cos^{-1}\left[\frac{1.10\times10^{3}\ \text{J}}{(30.0\ \text{N})(50.0\ \text{m})}\right] = \boxed{42.8^\circ}$$

6. ***REASONING AND SOLUTION***

a. In both cases, the lift force **L** is perpendicular to the displacement of the plane, and, therefore, does no work. As shown in the drawing in the text, when the plane is in the dive, there is a component of the weight **W** that points in the direction of the displacement of the plane. When the plane is climbing, there is a component of the weight that points opposite to the displacement of the plane. Thus, since the thrust **T** is the same for both cases, the net force in the direction of the displacement is greater for the case where the plane is diving. Since the displacement **s** is the same in both cases, $\boxed{\text{more net work is done during the dive}}$.

b. The work done during the dive is $W_{\text{dive}} = (T + W\cos 75°)\,s$, while the work done during the climb is $W_{\text{climb}} = (T + W\cos 115°)\,s$. Therefore, the difference between the net work done during the dive and the climb is

$$W_{\text{dive}} - W_{\text{climb}} = (T + W\cos 75°)\,s - (T + W\cos 115°)\,s = Ws\,(\cos 75° - \cos 115°)$$

$$= (5.9\times 10^4\ \text{N})(1.7\times 10^3\ \text{m})(\cos 75° - \cos 115°) = \boxed{6.8\times 10^7\ \text{J}}$$

7. ***REASONING*** The drawing shows three of the forces that act on the cart: **F** is the pushing force that the shopper exerts, **f** is the frictional force that opposes the motion of the cart, and $m\mathbf{g}$ is its weight. The displacement **s** of the cart is also shown. Since the cart moves at a constant velocity along the $+x$ direction, it is in equilibrium. The net force acting on it in this direction is zero, $\Sigma F_x = 0$. This relation can be used to find the magnitude of the pushing force. The work done by a constant force is given by Equation 6.1 as $W = (F\cos\theta)s$, where F is the magnitude of the force, s is the magnitude of the displacement, and θ is the angle between the force and the displacement.

SOLUTION

a. The x-component of the net force is zero, $\Sigma F_x = 0$, so that

$$\underbrace{F\cos 29.0° - f}_{\Sigma F_x} = 0 \tag{4.9a}$$

The magnitude of the force that the shopper exerts is $F = \dfrac{f}{\cos 29.0°} = \dfrac{48.0\text{ N}}{\cos 29.0°} = \boxed{54.9\text{ N}}$.

b. The work done by the pushing force **F** is

$$W = (F\cos\theta)s = (54.9\text{ N})(\cos 29.0°)(22.0\text{ m}) = \boxed{1060\text{ J}} \tag{6.1}$$

c. The angle between the frictional force and the displacement is 180°, so the work done by the frictional force **f** is

$$W = (f\cos\theta)s = (48.0\text{ N})(\cos 180.0°)(22.0\text{ m}) = \boxed{-1060\text{ J}}$$

d. The angle between the weight of the cart and the displacement is 90°, so the work done by the weight $m\mathbf{g}$ is

$$W = (mg\cos\theta)s = (16.0\text{ kg})(9.80\text{ m/s}^2)(\cos 90°)(22.0\text{ m}) = \boxed{0\text{ J}}$$

8. ***REASONING AND SOLUTION***

a. The work done by the applied force is

$$W = Fs\cos\theta = (2.40 \times 10^2\ \text{N})(8.00\ \text{m})\cos 20.0° = \boxed{1.80 \times 10^3\ \text{J}}$$

b. The work done by the frictional force is $W_f = f_k\, s\ \cos\theta$, where

$$f_k = \mu_s(mg - F\sin\theta)$$

$$= (0.200)\left[(85.0\ \text{kg})(9.80\ \text{m/s}^2) - (2.40\times10^2\ \text{N})(\sin 20.0°)\right] = 1.50\times10^2\ \text{N}$$

Now

$$W_f = (1.50 \times 10^2\ \text{N})(8.00\ \text{m})\cos 180° = \boxed{-1.20 \times 10^3\ \text{J}}$$

9. **SSM** ***REASONING AND SOLUTION*** According to Equation 6.1, the work done by the husband and wife are, respectively,

$$[\text{Husband}] \qquad W_{\text{H}} = (F_{\text{H}} \cos \theta_{\text{H}})s$$

$$[\text{Wife}] \qquad W_{\text{W}} = (F_{\text{W}} \cos \theta_{\text{W}})s$$

Since both the husband and the wife do the same amount of work,

$$(F_{\text{H}} \cos \theta_{\text{H}})s = (F_{\text{W}} \cos \theta_{\text{W}})s$$

Since the displacement has the same magnitude s in both cases, the magnitude of the force exerted by the wife is

$$F_{\text{W}} = F_{\text{H}} \frac{\cos \theta_{\text{H}}}{\cos \theta_{\text{W}}} = (67\ \text{N}) \frac{\cos\ 58°}{\cos\ 38°} = \boxed{45\ \text{N}}$$

10. ***REASONING AND SOLUTION*** The work done by the force P is

$$W_P = Ps \cos 30.0° = 0.866\, P\, s$$

Similarly, the work done by the frictional force is

$$W_f = f_k\, s \cos 180° = -f_k\, s$$

where

$$f_k = \mu_k\, (mg - P \sin 30.0°)$$

Then

$$W_f = -(0.200)[(1.00 \times 10^2 \text{ kg})(9.80 \text{ m/s}^2) - 0.500P]s = (0.100P - 196)s$$

The net work is zero if

$$W_P + W_f = 0$$

or

$$0.866\, Ps + (0.100P - 196)s = 0$$

Solving for P gives $\boxed{P = 203 \text{ N}}$.

11. ***REASONING AND SOLUTION*** The net work done on the car is

$$W_T = W_F + W_f + W_g + W_N$$

$$W_T = Fs \cos 0.0° + fs \cos 180° - mgs \sin 5.0° + F_N s \cos 90°$$

Rearranging this result gives

$$F = \frac{W_T}{s} + f + mg \sin 5.0°$$

$$= \frac{150 \times 10^3 \text{ J}}{290 \text{ m}} + 524 \text{ N} + (1200 \text{ kg})(9.80 \text{ m/s}^2) \sin 5.0° = \boxed{2.07 \times 10^3 \text{ N}}$$

12. ***REASONING AND SOLUTION*** The work done on the arrow by the bow is given by

$$W = Fs \cos 0° = Fs$$

This work is converted into kinetic energy according to the work energy theorem.

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2$$

Solving for v_f, we find that

$$v_f = \sqrt{\frac{2W}{m} + v_0^2} = \sqrt{\frac{2(65\text{ N})(0.90\text{ m})}{75 \times 10^{-3}\text{ kg}} + (0\text{ m/s})^2} = \boxed{39\text{ m/s}}$$

13. **SSM** ***REASONING*** The work done to launch either object can be found from Equation 6.3, the work-energy theorem, $W = \mathrm{KE_f} - \mathrm{KE_0} = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_0^2$.

SOLUTION

a. The work required to launch the hammer is

$$W = \tfrac{1}{2}mv_\mathrm{f}^2 - \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}m\left(v_\mathrm{f}^2 - v_0^2\right) = \tfrac{1}{2}(7.3\text{ kg})\left[(29\text{ m/s})^2 - (0\text{ m/s})^2\right] = \boxed{3.1\times10^3\text{ J}}$$

b. Similarly, the work required to launch the bullet is

$$W = \tfrac{1}{2}m\left(v_\mathrm{f}^2 - v_0^2\right) = \tfrac{1}{2}(0.0026\text{ kg})\left[(410\text{ m/s})^2 - (0\text{ m/s})^2\right] = \boxed{2.2\times10^2\text{ J}}$$

14. ***REASONING*** The average kinetic energy is given by Equation 6.2 as $\overline{\text{KE}} = \frac{1}{2}m\bar{v}^2$. The mass of the aircraft carrier is known, and its average speed $\bar{v}$ can be obtained from Equation 2.1 as the distance traveled divided by the elapsed time.

SOLUTION The average speed of the aircraft carrier is equal to the distance s it travels divided by the elapsed time t, or $\bar{v} = s/t$. Therefore, the average kinetic energy is

$$\overline{\text{KE}} = \tfrac{1}{2}m\bar{v}^2 = \tfrac{1}{2}m\left(\frac{s}{t}\right)^2$$

$$= \tfrac{1}{2}\left(8.35\times10^7\ \text{kg}\right)\left[\frac{208\times10^3\ \text{m}}{(3.50\ \text{h})\left(\dfrac{3600\ \text{s}}{1\ \text{h}}\right)}\right]^2 = \boxed{1.14\times10^{10}\ \text{J}}$$

15. **SSM** **WWW** ***REASONING AND SOLUTION*** The work required to bring each car up to speed is, from the work-energy theorem, $W = \mathrm{KE_f} - \mathrm{KE_0} = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_0^2$. Therefore,

$$W_\mathrm{B} = \tfrac{1}{2}m\left(v_\mathrm{f}^2 - v_0^2\right) = \tfrac{1}{2}(1.20\times10^3\ \mathrm{kg})\left[(40.0\ \mathrm{m/s})^2 - (0\ \mathrm{m/s})^2\right] = 9.60\times10^5\ \mathrm{J}$$

$$W_\mathrm{B} = \tfrac{1}{2}m\left(v_\mathrm{f}^2 - v_0^2\right) = \tfrac{1}{2}(2.00\times10^3\ \mathrm{kg})\left[(40.0\ \mathrm{m/s})^2 - (0\ \mathrm{m/s})^2\right] = 1.60\times10^6\ \mathrm{J}$$

The *additional* work required to bring car B up to speed is, therefore,

$$W_\mathrm{B} - W_\mathrm{A} = (1.6\times10^6\ \mathrm{J}) - (9.60\times10^5\ \mathrm{J}) = \boxed{6.4\times10^5\ \mathrm{J}}$$

16. ***REASONING AND SOLUTION*** From the work-energy theorem, Equation 6.3,

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2 = \frac{1}{2}m\left[v_f^2 - v_0^2\right]$$

a. $W = \frac{1}{2}(7420\text{ kg})\left[(8450\text{ m/s})^2 - (2820\text{ m/s})^2\right] = \boxed{2.35\times10^{11}\text{ J}}$

b. $\text{W} = \frac{1}{2}(7420\text{ kg})\left[(2820\text{ m/s})^2 - (8450\text{ m/s})^2\right] = \boxed{-2.35\times10^{11}\text{ J}}$

17. ***REASONING AND SOLUTION***

a. The work-energy theorem gives

$$W = (1/2)mv_f^2 - (1/2)mv_o^2 = (1/2)(45 \times 10^{-3}\ \text{kg})(41\ \text{m/s})^2 = \boxed{38\ \text{J}}$$

b. From the definition of work

$$W = Fs \cos 0°$$

so

$$F = W/s = (38\ \text{J})/(1.0 \times 10^{-2}\ \text{m}) = \boxed{3.8 \times 10^3\ \text{N}}$$

18. ***REASONING AND SOLUTION*** The work energy theorem, $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$, gives

$$v_f = \sqrt{\frac{2W}{m} + v_0^2}$$

where

$$W = Fs\cos 180° = -(4.0 \times 10^5 \text{ N})(2500 \times 10^3 \text{ m}) = -1.0 \times 10^{12} \text{ J}$$

Now

$$v_f = \sqrt{\frac{2\left(-1.0 \times 10^{12} \text{ J}\right)}{5.0 \times 10^4 \text{ kg}} + \left(11\,000 \text{ m/s}\right)^2} = \boxed{9 \times 10^3 \text{ m/s}}$$

19. **SSM** ***REASONING*** According to the work-energy theorem, the kinetic energy of the sled increases in each case because work is done on the sled. The work-energy theorem is given by Equation 6.3: $W = \mathrm{KE_f} - \mathrm{KE_0} = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_0^2$. The work done on the sled is given by Equation 6.1: $W = (F\cos\theta)s$. The work done in each case can, therefore, be expressed as

$$W_1 = (F\cos 0°)s = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_0^2 = \Delta\mathrm{KE}_1$$

and

$$W_2 = (F\cos 62°)s = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_0^2 = \Delta\mathrm{KE}_2$$

The fractional increase in the kinetic energy of the sled when $\theta = 0°$ is

$$\frac{\Delta\mathrm{KE}_1}{\mathrm{KE}_0} = \frac{(F\cos 0°)s}{\mathrm{KE}_0} = 0.38$$

Therefore,

$$Fs = (0.38)\ \mathrm{KE}_0 \qquad (1)$$

The fractional increase in the kinetic energy of the sled when $\theta = 62°$ is

$$\frac{\Delta\mathrm{KE}_2}{\mathrm{KE}_0} = \frac{(F\cos 62°)s}{\mathrm{KE}_0} = \frac{Fs}{\mathrm{KE}_0}(\cos 62°) \qquad (2)$$

Equation (1) can be used to substitute in Equation (2) for Fs.

SOLUTION Combining Equations (1) and (2), we have

$$\frac{\Delta\mathrm{KE}_2}{\mathrm{KE}_0} = \frac{Fs}{\mathrm{KE}_0}(\cos 62°) = \frac{(0.38)\ \mathrm{KE}_0}{\mathrm{KE}_0}(\cos 62°) = (0.38)(\cos 62°) = 0.18$$

Thus, the sled's kinetic energy would increase by $\boxed{18\ \%}$.

20. ***REASONING*** Since the person has an upward acceleration, there must be a net force acting in the upward direction. The net force ΣF_y is related to the acceleration a_y by Newton's second law, $\Sigma F_y = ma_y$, where m is the mass of the person. This relation will allow us to determine the tension in the cable. The work done by the tension and the person's weight can be found directly from the definition of work, Equation 6.1.

SOLUTION

a. The free-body diagram at the right shows the two forces that act on the person. Applying Newton's second law, we have

$$\underbrace{T - mg}_{\Sigma F_y} = ma_y$$

Solving for the magnitude of the tension in the cable yields

$$T = m(a_y + g) = (79\text{ kg})(0.70\text{ m/s}^2 + 9.80\text{ m/s}^2) = \boxed{8.3 \times 10^2\text{ N}}$$

b. The work done by the tension in the cable is

$$W_\text{T} = (T\cos\theta)s = (8.3 \times 10^2\text{ N})(\cos 0°)(11\text{ m}) = \boxed{9.1 \times 10^3\text{ J}} \qquad (6.1)$$

c. The work done by the person's weight is

$$W_\text{W} = (mg\cos\theta)s = (79\text{ kg})(9.8\text{ m/s}^2)(\cos 180°)(11\text{ m}) = \boxed{-8.5 \times 10^3\text{ J}} \qquad (6.1)$$

d. The work-energy theorem relates the work done by the two forces to the change in the kinetic energy of the person. The work done by the two forces is $W = W_\text{T} + W_\text{W}$:

$$\underbrace{\text{W}_\text{T} + \text{W}_\text{W}}_{W} = \tfrac{1}{2}mv_\text{f}^2 - \tfrac{1}{2}mv_0^2 \qquad (6.3)$$

Solving this equation for the final speed of the person gives

$$v_\text{f} = \sqrt{v_0^2 + \frac{2}{m}(W_\text{T} + W_\text{W})}$$

$$= \sqrt{(0\text{ m/s})^2 + \frac{2}{79\text{ kg}}(9.1 \times 10^3\text{ J} - 8.5 \times 10^3\text{ J})} = \boxed{4\text{ m/s}}$$

21. ***REASONING*** It is useful to divide this problem into two parts. The first part involves the skier moving on the snow. We can use the work-energy theorem to find her speed when she comes to the edge of the cliff. In the second part she leaves the snow and falls freely toward the ground. We can again employ the work-energy theorem to find her speed just before she lands.

SOLUTION The drawing at the right shows the three forces that act on the skier as she glides on the snow. The forces are: her weight $m\mathbf{g}$, the normal force $\mathbf{F}_N$, and the kinetic frictional force $\mathbf{f}_k$. Her displacement is labeled as $\mathbf{s}$. The work-energy theorem, Equation 6.3, is

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2$$

where W is the work done by the net external force that acts on the skier. The work done by each force is given by Equation 6.1, $W = (F\cos\theta)s$, so the work-energy theorem becomes

$$\underbrace{(mg\cos 65.0^\circ)s + (f_k\cos 180^\circ)s + (F_N\cos 90^\circ)s}_{W} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2$$

Since cos 90° = 0, the third term on the left side can be eliminated. The magnitude f_k of the kinetic frictional force is given by Equation 4.8 as $f_k = \mu_k F_N$. The magnitude F_N of the normal force can be determined by noting that the skier does not leave the surface of the slope, so $a_y = 0$ m/s^2. Thus, we have that $\Sigma F_y = 0$, so

$$\underbrace{F_N - mg\cos 25.0^\circ}_{\Sigma F_y} = 0 \qquad \text{or} \qquad F_N = mg\cos 25.0^\circ$$

The magnitude of the kinetic frictional force becomes $f_k = \mu_k F_N = \mu_k mg\cos 25.0^\circ$. Substituting this result into the work-energy theorem, we find that

$$\underbrace{(mg\cos 65.0^\circ)s + (\mu_k mg\cos 25.0^\circ)(\cos 180^\circ)s}_{W} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2$$

Algebraically eliminating the mass m of the skier from every term, setting cos 180° = –1 and $v_0 = 0$ m/s, and solving for the final speed v_f, gives

$$v_f = \sqrt{2gs(\cos 65.0^\circ - \mu_k\cos 25.0^\circ)}$$

$$= \sqrt{2(9.80\text{ m/s}^2)(10.4\text{ m})[\cos 65.0^\circ - (0.200)\cos 25.0^\circ]} = 7.01\text{ m/s}$$

The drawing at the right shows her displacement **s** during free fall. Note that the displacement is a vector that starts where she leaves the slope and ends where she touches the ground. The only force acting on her during the free fall is her weight $m\mathbf{g}$. The work-energy theorem, Equation 6.3, is

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2$$

The work W is that done by her weight, so the work-energy theorem becomes

$$\underbrace{(mg\cos\theta)s}_{W} = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2$$

In this expression θ is the angle between her weight (which points vertically downward) and her displacement. Note from the drawing that $s\cos\theta = 3.50$ m. Algebraically eliminating the mass m of the skier from every term in the equation above and solving for the final speed v_f gives

$$v_f = \sqrt{v_0^2 + 2g(s\cos\theta)}$$

$$= \sqrt{(7.01\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(3.50\text{ m})} = \boxed{10.9\text{ m/s}}$$

22. ***REASONING AND SOLUTION*** Since the boat travels with constant speed, the net force on the boat is zero. Therefore, the drive force must be equal in magnitude and opposite in direction to the resistive force due to the water, F_R. Since the force due to the water is resistive, it always points opposite to the direction of motion of the boat and $\theta = 180°$. After the engine is shut off, the resistive force due to the water does negative work on the boat thereby bringing it to a halt.

a. From the work-energy theorem,

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2$$

where W is the work done by the resistive force due to the water. Since the boat coasts to a halt, $v_f = 0$ m/s, and we have

$$(F_R \cos\theta)s = \tfrac{1}{2}m(0 \text{ m/s})^2 - \tfrac{1}{2}mv_0^2$$

Solving for s gives

$$s = -\frac{mv_0^2}{2F_R \cos\theta} = -\frac{(480 \text{ kg})(8.9 \text{ m/s})^2}{2(760 \text{ N})(\cos 180°)} = \boxed{25 \text{ m}}$$

b. The time required for the boat to come to a halt can be found from Equation 2.4:

$$v = v_0 + at$$

where $v = 0$ m/s since the final speed of the boat is zero. Solving for t yields

$$t = \frac{(0 \text{ m/s}) - v_0}{a} = -\frac{v_0}{F/m} = -\frac{v_0 m}{F} = -\frac{(8.9 \text{ m/s})(480 \text{ kg})}{(-760 \text{ N})} = \boxed{5.6 \text{ s}}$$

Notice that the value substituted for F is negative. This is because $\mathbf{F}$ and $\mathbf{v}_0$ point in opposite directions, and the value substituted for v_0 is positive.

23. **SSM** **WWW** ***REASONING*** When the satellite goes from the first to the second orbit, its kinetic energy changes. The net work that the external force must do to change the orbit can be found from the work-energy theorem: $W = \mathrm{KE_f} - \mathrm{KE_0} = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_0^2$. The speeds v_f and v_0 can be obtained from Equation 5.5 for the speed of a satellite in a circular orbit of radius r. Given the speeds, the work energy theorem can be used to obtain the work.

SOLUTION According to Equation 5.5, $v = \sqrt{GM_\mathrm{E}/\mathrm{r}}$. Substituting into the work-energy theorem, we have

$$W = \tfrac{1}{2}mv_\mathrm{f}^2 - \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}m\left(v_\mathrm{f}^2 - v_0^2\right) = \tfrac{1}{2}m\left[\left(\sqrt{\frac{GM_\mathrm{E}}{r_\mathrm{f}}}\right)^2 - \left(\sqrt{\frac{GM_\mathrm{E}}{r_0}}\right)^2\right] = \frac{GM_\mathrm{E}m}{2}\left(\frac{1}{r_\mathrm{f}} - \frac{1}{r_0}\right)$$

Therefore,

$$\mathrm{W} = \frac{(6.67\times10^{-11}\ \mathrm{N\cdot m^2/kg^2})(5.98\times10^{24}\ \mathrm{kg})(6200\ \mathrm{kg})}{2}$$

$$\times\left(\frac{1}{7.0\times10^{6}\ \mathrm{m}} - \frac{1}{3.3\times10^{7}\ \mathrm{m}}\right) = \boxed{1.4\times10^{11}\ \mathrm{J}}$$

24. ***REASONING AND SOLUTION*** The net work done on the plane can be found from the work-energy theorem:

$$W = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}m\left[v_f^2 - v_0^2\right]$$

The tension in the guideline provides the centripetal force required to keep the plane moving in a circular path. Since the tension in the guideline becomes four times greater,

$$T_f = 4\,T_0$$

or

$$\frac{mv_f^2}{r_f} = 4\frac{mv_0^2}{r_0}$$

Solving for v_f^2 gives

$$v_f^2 = \frac{4v_0^2 r_f}{r_0}$$

Substitution of this expression into the work-energy theorem gives

$$W = \tfrac{1}{2}m\left[\frac{4v_0^2 r_f}{r_0} - v_0^2\right] = \tfrac{1}{2}mv_0^2\left[4\left(\frac{r_f}{r_0}\right) - 1\right]$$

Therefore,

$$W = \tfrac{1}{2}(0.90\text{ kg})(22\text{ m/s})^2\left[4\left(\frac{14\text{ m}}{16\text{ m}}\right) - 1\right] = \boxed{5.4\times10^2\text{ J}}$$

25. ***REASONING AND SOLUTION***

$$PE = mgh = (55.0\ \text{kg})(9.80\ \text{m/s}^2)(443\text{m}) = \boxed{2.39 \times 10^5\ \text{J}} \tag{6.5}$$

26. ***REASONING*** The work done by the weight of the basketball is given by Equation 6.1 as $W = (F\cos\theta)s$, where $F = mg$ is the magnitude of the weight, θ is the angle between the weight and the displacement, and s is the magnitude of the displacement. The drawing shows that the weight and displacement are parallel, so that $\theta = 0°$. The potential energy of the basketball is given by Equation 6.5 as PE = mgh, where h is the height of the ball above the ground.

SOLUTION

a. The work done by the weight of the basketball is

$$W = (F\cos\theta)s = \text{mg}(\cos 0°)(h_0 - h_f) = (0.60\text{ kg})(9.80\text{ m/s}^2)(6.1\text{ m} - 1.5\text{ m}) = \boxed{27\text{ J}}$$

b. The potential energy of the ball, relative to the ground, when it is released is

$$\text{PE}_0 = mgh_0 = (0.60\text{ kg})(9.80\text{ m/s}^2)(6.1\text{ m}) = \boxed{36\text{ J}} \qquad (6.5)$$

c. The potential energy of the ball, relative to the ground, when it is caught is

$$\text{PE}_f = mgh_f = (0.60\text{ kg})(9.80\text{ m/s}^2)(1.5\text{ m}) = \boxed{8.8\text{ J}} \qquad (6.5)$$

d. The change in the ball's gravitational potential energy is

$$\Delta\text{PE} = \text{PE}_f - \text{PE}_0 = 8.8\text{ J} - 36\text{ J} = -27\text{ J}$$

We see that the change in the gravitational potential energy is equal to –27 J = $\boxed{-W}$, where W is the work done by the weight of the ball (see part a).

27. **SSM** ***REASONING*** During each portion of the trip, the work done by the resistive force is given by Equation 6.1, $W = (F\cos\theta)s$. Since the resistive force always points opposite to the displacement of the bicyclist, $\theta = 180°$; hence, on each part of the trip, $W = (F\cos 180°)s = -Fs$. The work done by the resistive force during the round trip is the algebraic sum of the work done during each portion of the trip.

SOLUTION

a. The work done during the round trip is, therefore,

$$W_{\text{total}} = W_1 + W_2 = -F_1 s_1 - F_2 s_2$$

$$= -(3.0\text{ N})(5.0\times10^3\text{ m}) - (3.0\text{ N})(5.0\times10^3\text{ m}) = \boxed{-3.0\times10^4\text{ J}}$$

b. Since the work done by the resistive force over the closed path is *not* zero, we can conclude that $\boxed{\text{the resistive force is } \textit{not} \text{ a conservative force}}$.

28. ***REASONING AND SOLUTION***

a. From the definition of work, $W = Fs \cos \theta$. For upward motion $\theta = 180°$ so

$$W = -mgs = -(71.1\text{ N})(2.13\text{ m} - 1.52\text{ m}) = \boxed{-43\text{ J}}$$

b. The change in potential energy is

$$\Delta\text{PE} = mgh_f - mgh_0 = (71.1\text{ N})(2.13\text{ m} - 1.52\text{ m}) = \boxed{+43\text{ J}}$$

29. ***REASONING AND SOLUTION***

The vertical height of the skier is $h = s \sin 14.6°$ so

$$\Delta PE = mgs \sin 14.6°$$

$$= (75.0 \text{ kg})(9.80 \text{ m/s}^2)(2830 \text{ m}) \sin 14.6°$$

$$= \boxed{5.24 \times 10^5 \text{ J}}$$

30. ***REASONING AND SOLUTION*** The change in the pole-vaulter's potential energy is

$$\mathrm{PE_f} - \mathrm{PE_0} = mgh_\mathrm{f} - mgh_0$$

Let $h_\mathrm{f} = 0$ m, so that $h_0 = 5.80$ m. Then,

$$mg = \frac{\mathrm{PE_f} - \mathrm{PE_0}}{h_\mathrm{f} - h_0} = \frac{0\ \mathrm{J} - 3.70\times10^3\ \mathrm{J}}{0\ \mathrm{m} - 5.80\ \mathrm{m}} = \boxed{6.38\times10^2\ \mathrm{N}}$$

31. **SSM** ***REASONING*** The only nonconservative force that acts on Rocket man is the force generated by the propulsion unit. Thus, Equation 6.7b can be used to determine the net work done by this force.

SOLUTION From Equation 6.7b, we have

$$W_{\text{nc}} = \Delta\text{KE} + \Delta\text{PE} = \left(\tfrac{1}{2}mv_{\text{f}}^2 - \tfrac{1}{2}mv_0^2\right) + \left(mgh_{\text{f}} - mgh_0\right)$$

Since rocket man starts from rest, $v_0 = 0$ m/s. If we take $h_0 = 0$ m on the ground, we have

$$W_{\text{nc}} = \tfrac{1}{2}mv_{\text{f}}^2 + mgh_{\text{f}} = m\left(\tfrac{1}{2}v_{\text{f}}^2 + gh_{\text{f}}\right)$$

$$= (136\text{ kg})\left[\tfrac{1}{2}(5.0\text{ m/s})^2 + (9.80\text{ m/s}^2)(16\text{ m})\right] = \boxed{2.3\times10^4\text{ J}}$$

32. ***REASONING*** The only two forces that act on the gymnast are his weight and the force exerted on his hands by the high bar. The latter is the (non-conservative) reaction force to the force exerted on the bar by the gymnast, as predicted by Newton's third law. This force, however, does no work because it points perpendicular to the circular path of motion. Thus, $W_{\text{nc}} = 0$ J, and we can apply the principle of conservation of mechanical energy.

SOLUTION The conservation principle gives

$$\underbrace{\frac{1}{2}mv_{\text{f}}^2 + mgh_{\text{f}}}_{E_{\text{f}}} = \underbrace{\frac{1}{2}mv_0^2 + mgh_0}_{E_0}$$

Since the gymnast's speed is momentarily zero at the top of the swing, $v_0 = 0$ m/s. If we take $h_{\text{f}} = 0$ m at the bottom of the swing, then $h_0 = 2r$, where r is the radius of the circular path followed by the gymnast's waist. Making these substitutions in the above expression and solving for v_{f}, we obtain

$$v_{\text{f}} = \sqrt{2gh_0} = \sqrt{2g(2r)} = \sqrt{2(9.80\ \text{m/s}^2)(2 \times 1.1\ \text{m})} = \boxed{6.6\ \text{m/s}}$$

33. ***REASONING*** Since air resistance is being neglected, the only force that acts on the falling water is the conservative gravitational force (its weight). Since the height of the falls and the speed of the water at the bottom are known, we may use the conservation of mechanical energy to find the speed of the water at the top of the falls.

SOLUTION The conservation of mechanical energy, as expressed by Equation 6.9b, states that

$$\underbrace{\tfrac{1}{2}mv_{\mathrm{f}}^2 + mgh_{\mathrm{f}}}_{E_{\mathrm{f}}} = \underbrace{\tfrac{1}{2}mv_0^2 + mgh_0}_{E_{\mathrm{f}}}$$

The mass m can be eliminated algebraically from this equation, since it appears as a factor in every term. Solving for v_0 gives

$$v_0 = \sqrt{v_{\mathrm{f}}^2 + 2g\left(h_{\mathrm{f}} - h_0\right)}$$

$$= \sqrt{\left(25.8\ \mathrm{m/s}\right)^2 + 2\left(9.80\ \mathrm{m/s^2}\right)\left(0\ \mathrm{m} - 33.2\ \mathrm{m}\right)} = \boxed{3.9\ \mathrm{m/s}}$$

34. ***REASONING AND SOLUTION*** The conservation of energy gives

$$\tfrac{1}{2}mv_{\mathrm{f}}^2 + mgh_{\mathrm{f}} = \tfrac{1}{2}mv_0^2 + mgh_0$$

Rearranging gives

$$h_{\mathrm{f}} - h_0 = \frac{(14.0\ \mathrm{m/s})^2 - (13.0\ \mathrm{m/s})^2}{2(9.80\ \mathrm{m/s}^2)} = \boxed{1.4\ \mathrm{m}}$$

35. $\boxed{\text{SSM}}$ ***REASONING*** No forces other than gravity act on the rock since air resistance is being ignored. Thus, the net work done by nonconservative forces is zero, $W_{\text{nc}} = 0$ J. Consequently, the principle of conservation of mechanical energy holds, so the total mechanical energy remains constant as the rock falls.

If we take h = 0 m at ground level, the gravitational potential energy at any height h is, according to Equation 6.5, $\text{PE} = mgh$. The kinetic energy of the rock is given by Equation 6.2: $\text{KE} = \frac{1}{2}mv^2$. In order to use Equation 6.2, we must have a value for v^2 at each desired height h. The quantity v^2 can be found from Equation 2.9 with $v_0 = 0$ m/s, since the rock is released from rest. At a height h, the rock has fallen through a distance $(20.0\text{ m}) - h$, and according to Equation 2.9, $v^2 = 2ay = 2a[(20.0\text{ m}) - h]$. Therefore, the kinetic energy at any height h is given by $\text{KE} = ma[(20.0\text{ m}) - h]$. The total energy of the rock at any height h is the sum of the kinetic energy and potential energy at the particular height.

SOLUTION The calculations are performed below for $h = 10.0$ m. The table that follows also shows the results for h = 20.0 m and h = 0 m.

$$\text{PE} = mgh = (2.00\text{ kg})(9.80\text{ m/s}^2)(10.0\text{ m}) = \boxed{196\text{ J}}$$

$$\text{KE} = ma[(20.0\text{ m}) - h] = (2.00\text{ kg})(9.80\text{ m/s}^2)(20.0\text{ m} - 10.0\text{ m}) = \boxed{196\text{ J}}$$

$$E = \text{KE} + \text{PE} = 196\text{ J} + 196\text{ J} = \boxed{392\text{ J}}$$

h **(m)**	**KE (J)**	**PE (J)**	**E (J)**
20.0	0	392	392
10.0	196	196	392
0	392	0	392

Note that in each case, the value of E is the same, because mechanical energy is conserved.

36. ***REASONING*** Since air resistance is being neglected, the only force that acts on the golf ball is the conservative gravitational force (its weight). Since the maximum height of the trajectory and the initial speed of the ball are known, the conservation of mechanical energy can be used to find the kinetic energy of the ball at the top of the highest point. The conservation of mechanical energy can also be used to find the speed of the ball when it is 8.0 m below its highest point.

SOLUTION

a. The conservation of mechanical energy, Equation 6.9b, states that

$$\underbrace{\tfrac{1}{2}mv_{\mathrm{f}}^2}_{\mathrm{KE_f}} + mgh_{\mathrm{f}} = \tfrac{1}{2}mv_0^2 + mgh_0$$

Solving this equation for the final kinetic energy, $\mathrm{KE_f}$, yields

$$\begin{aligned}\mathrm{KE_f} &= \tfrac{1}{2}mv_0^2 + mg\left(h_0 - h_{\mathrm{f}}\right)\\ &= \tfrac{1}{2}(0.0470\ \mathrm{kg})(52.0\ \mathrm{m/s})^2 + (0.0470\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(0\ \mathrm{m} - 24.6\ \mathrm{m}) = \boxed{52.2\ \mathrm{J}}\end{aligned}$$

b. The conservation of mechanical energy, Equation 6.9b, states that

$$\underbrace{\tfrac{1}{2}mv_{\mathrm{f}}^2 + mgh_{\mathrm{f}}}_{E_{\mathrm{f}}} = \underbrace{\tfrac{1}{2}mv_0^2 + mgh_0}_{E_{\mathrm{f}}}$$

The mass m can be eliminated algebraically from this equation, since it appears as a factor in every term. Solving for v_{f}, and noting that the final height of the ball is $h_{\mathrm{f}} = 24.6\ \mathrm{m} - 8.0\ \mathrm{m} = 16.6\ \mathrm{m}$, we have that

$$v_{\mathrm{f}} = \sqrt{v_0^2 + 2g\left(h_0 - h_{\mathrm{f}}\right)} = \sqrt{(52.0\ \mathrm{m/s})^2 + 2\left(9.80\ \mathrm{m/s^2}\right)(0\ \mathrm{m} - 16.6\ \mathrm{m})} = \boxed{48.8\ \mathrm{m/s}}$$

37. ***REASONING AND SOLUTION***

Mechanical energy is conserved if no friction acts. $E_f = E_0$ or

$$\tfrac{1}{2}mv_f^2 + mgh = \tfrac{1}{2}mv_0^2$$

Taking h_0 = 0 m and rearranging yields

$$v_f = \sqrt{v_0^2 - 2gh} = \sqrt{(11.0\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(5.0\text{ m})} = \boxed{4.8\text{ m/s}}$$

38. ***REASONING AND SOLUTION*** The hook is let go when it is $h_0 = 1.5$ m above the ground and traveling vertically upward. Its initial speed is

$$v_0 = \frac{2\pi r}{T} = \frac{2\pi(1.5\text{ m})}{\frac{1}{3}\text{s}} = 28\text{ m/s}$$

The conservation of energy requires

$$\tfrac{1}{2}mv_\text{f}^2 + mgh_\text{f} = \tfrac{1}{2}mv_0^2 + mgh_0$$

At maximum height $v_\text{f} = 0$ m/s, so

$$h_\text{f} = \frac{v_0^2}{2g} + h_0 = \frac{(28\text{ m/s})^2}{2(9.80\text{ m/s}^2)} + 1.5\text{ m} = \boxed{42\text{ m}}$$

39. **SSM** **WWW** ***REASONING*** Gravity is the only force acting on the vaulters, since friction and air resistance are being ignored. Therefore, the net work done by the nonconservative forces is zero, and the principle of conservation of mechanical energy holds.

SOLUTION Let $E_{2\text{f}}$ represent the total mechanical energy of the second vaulter at the ground, and E_{20} represent the total mechanical energy of the second vaulter at the bar. Then, the principle of mechanical energy is written for the second vaulter as

$$\underbrace{\frac{1}{2}mv_{2\text{f}}^2 + mgh_{2\text{f}}}_{E_{2\text{f}}} = \underbrace{\frac{1}{2}mv_{20}^2 + mgh_{20}}_{E_{20}}$$

Since the mass m of the vaulter appears in every term of the equation, m can be eliminated algebraically. The quantity $h_{20} = h$, where h is the height of the bar. Furthermore, when the vaulter is at ground level, $h_{2f} = 0$ m. Solving for v_{20} we have

$$v_{20} = \sqrt{v_{2\text{f}}^2 - 2gh} \qquad (1)$$

In order to use Equation (1), we must first determine the height h of the bar. The height h can be determined by applying the principle of conservation of mechanical energy to the first vaulter on the ground and at the bar. Using notation similar to that above, we have

$$\underbrace{\frac{1}{2}mv_{1\text{f}}^2 + mgh_{1\text{f}}}_{E_{1\text{f}}} = \underbrace{\frac{1}{2}mv_{10}^2 + mgh_{10}}_{E_{10}}$$

where E_{10} corresponds to the total mechanical energy of the first vaulter at the bar. The height of the bar is, therefore,

$$h = h_{10} = \frac{v_{1\text{f}}^2 - v_{10}^2}{2g} = \frac{(8.90\text{ m/s})^2 - (1.00\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = 3.99\text{ m}$$

The speed at which the second vaulter clears the bar is, from Equation (1),

$$v_{20} = \sqrt{v_{2\text{f}}^2 - 2gh} = \sqrt{(9.00\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(3.99\text{ m})} = \boxed{1.7\text{ m/s}}$$

40. ***REASONING AND SOLUTION*** Since energy is conserved, the ball will have the same speed at the bottom of its swing whether it is moving toward or away from the crane. As the cable swings to an angle of 32.0°, the ball will rise a distance

$$h = L\,(1 - \cos 32.0°)$$

where L is the length of the cable. The conservation of energy then gives

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}mv_f^2 + mgh$$

Solving for v_f gives

$$v_f = \sqrt{v_0^2 - 2gL(1 - \cos 32.0°)}$$

$$= \sqrt{(6.50\text{ m/s})^2 - 2(9.80\text{ m/s}^2)(10.0\text{ m})(1 - \cos 32.0°)} = \boxed{3.53\text{ m/s}}$$

41. **SSM** ***REASONING*** Friction and air resistance are being ignored. The normal force from the slide is perpendicular to the motion, so it does no work. Thus, no net work is done by nonconservative forces, and the principle of conservation of mechanical energy applies.

SOLUTION Applying the principle of conservation of mechanical energy to the swimmer at the top and the bottom of the slide, we have

$$\underbrace{\frac{1}{2}mv_f^2 + mgh_f}_{E_f} = \underbrace{\frac{1}{2}mv_0^2 + mgh_0}_{E_0}$$

If we let h be the height of the bottom of the slide above the water, $h_f = h$, and $h_0 = H$. Since the swimmer starts from rest, $v_0 = 0$ m/s, and the above expression becomes

$$\frac{1}{2}v_f^2 + gh = gH$$

Solving for H, we obtain

$$H = h + \frac{v_f^2}{2g}$$

Before we can calculate H, we must find v_f and h. Since the velocity in the horizontal direction is constant,

$$v_f = \frac{\Delta x}{\Delta t} = \frac{5.00 \text{ m}}{0.500 \text{ s}} = 10.0 \text{ m/s}$$

The vertical displacement of the swimmer after leaving the slide is, from Equation 3.5b (with down being negative),

$$y = \tfrac{1}{2}a_y t^2 = \tfrac{1}{2}(-9.80 \text{ m/s}^2)(0.500 \text{ s})^2 = -1.23 \text{ m}$$

Therefore, h = 1.23 m. Using these values of v_f and h in the above expression for H, we find

$$H = h + \frac{v_f^2}{2g} = 1.23 \text{ m} + \frac{(10.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{6.33 \text{ m}}$$

42. ***REASONING AND SOLUTION*** If air resistance is ignored, the only nonconservative force that acts on the skier is the normal force exerted on the skier by the snow. Since this force is always perpendicular to the direction of the displacement, the work done by the normal force is zero. We can conclude, therefore, that mechanical energy is conserved.

$$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_f^2 + mgh_f$$

Since the skier starts from rest $v_0 = 0$ m/s. Let h_f define the zero level for heights, then the final gravitational potential energy is zero. This gives

$$mgh_0 = \frac{1}{2}mv_f^2 \qquad (1)$$

At the crest of the second hill, the two forces that act on the skier are the normal force and the weight of the skier. The resultant of these two forces provides the necessary centripetal force to keep the skier moving along the circular arc of the hill. When the skier *just loses contact* with the snow, the normal force is zero and the weight of the skier must provide the necessary centripetal force.

$$mg = \frac{mv_f^2}{r} \qquad \text{so that} \qquad v_f^2 = gr \qquad (2)$$

Substituting this expression for v_f^2 into Equation (1) gives

$$mgh_0 = \frac{1}{2}mgr$$

Solving for h_0 gives

$$h_0 = \frac{r}{2} = \frac{36\text{ m}}{2} = \boxed{18\text{ m}}$$

43. ***REASONING AND SOLUTION*** At the bottom of the circular path of the swing, the centripetal force is provided by the tension in the rope less the weight of the swing and rider. That is,

$$\underbrace{\frac{mv^2}{r}}_{F_c} = T - mg$$

Solving for the mass yields

$$m = \frac{T}{\dfrac{v^2}{r} + g}$$

The energy of the swing is conserved if friction is ignored. The initial energy, E_0, when the swing is released is completely potential energy and is $E_0 = mgh_0$, where

$$h_0 = r\,(1 - \cos 60.0°) = \tfrac{1}{2}r$$

is the vertical height of the swing. At the bottom of the path the swing's energy is entirely kinetic and is

$$E_f = \tfrac{1}{2}mv^2$$

Setting $E_o = E_f$ and solving for v gives

$$v = \sqrt{2gh_0} = \sqrt{gr}$$

The expression for the mass now becomes

$$m = \frac{T}{2g} = \frac{8.00 \times 10^2 \text{ N}}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{40.8 \text{ kg}}$$

44. ***REASONING AND SOLUTION*** If air resistance is ignored, the only nonconservative force that acts on the person is the normal force exerted on the person by the surface. Since this force is always perpendicular to the direction of the displacement, the work done by the normal force is zero. We can conclude, therefore, that mechanical energy is conserved.

$$\frac{1}{2}mv_0^2 + mgh_0 = \frac{1}{2}mv_f^2 + mgh_f \qquad (1)$$

where the final state pertains to the position where the person leaves the surface. Since the person starts from rest $v_0 = 0$ m/s. Since the radius of the surface is r, $h_0 = r$, and $h_f = r\cos\theta_f$ where θ_f is the angle at which the person leaves the surface.

Equation (1) becomes

$$mgr = \frac{1}{2}mv_f^2 + mg(r\cos\theta_f) \qquad (2)$$

In general, as the person slides down the surface, the two forces that act on him are the normal force $\mathbf{F}_N$ and the weight m**g**. The centripetal force required to keep the person moving in the circular path is the resultant of $\mathbf{F}_N$ and the radial component of the weight, mg cos θ.

When the person leaves the surface, the normal force is zero, and the radial component of the weight provides the centripetal force.

$$mg\cos\theta_f = \frac{mv_f^2}{r} \qquad \Rightarrow \qquad v_f^2 = gr\cos\theta_f \quad (3)$$

Substituting this expression for v_f^2 into Equation (2) gives

$$mgr = \tfrac{1}{2}mg(r\cos\theta_f) + mg(r\cos\theta_f)$$

Solving for θ_f gives

$$\theta_f = \cos^{-1}\left(\frac{2}{3}\right) = \boxed{48°}$$

45. **SSM** ***REASONING AND SOLUTION*** The work-energy theorem can be used to determine the change in kinetic energy of the car according to Equation 6.8:

$$W_{\text{nc}} = \underbrace{\left(\tfrac{1}{2}mv_{\text{f}}^2 + mgh_{\text{f}}\right)}_{E_{\text{f}}} - \underbrace{\left(\tfrac{1}{2}mv_0^2 + mgh_0\right)}_{E_0}$$

The nonconservative forces are the forces of friction and the force due to the chain mechanism. Thus, we can rewrite Equation 6.8 as

$$W_{\text{friction}} + W_{\text{chain}} = \left(\text{KE}_{\text{f}} + mgh_{\text{f}}\right) - \left(\text{KE}_0 + mgh_0\right)$$

We will measure the heights from ground level. Solving for the change in kinetic energy of the car, we have

$$\text{KE}_{\text{f}} - \text{KE}_0 = W_{\text{friction}} + W_{\text{chain}} - mgh_{\text{f}} + mgh_0$$

$$= -2.00\times10^4\ \text{J} + 3.00\times10^4\ \text{J} - (375\ \text{kg})(9.80\ \text{m/s}^2)(20.0\ \text{m})$$
$$+ (375\ \text{kg})(9.80\ \text{m/s}^2)(5.00\ \text{m}) = \boxed{-4.51\times10^4\ \text{J}}$$

46. ***REASONING AND SOLUTION***

The work done by air resistance is equal to the energy lost by the ball.

$$W_{nc} = E_f - E_0$$

$$W_{nc} = \left(\tfrac{1}{2}mv_f^2 + mgh_f\right) - \left(\tfrac{1}{2}mv_0^2 + mgh_0\right)$$

$$W_{nc} = \tfrac{1}{2}(0.600\text{ kg})(4.20\text{ m/s})^2 + (0.600\text{ kg})(9.80\text{ m/s}^2)(3.10\text{ m})$$
$$-\tfrac{1}{2}(0.600\text{ kg})(7.20\text{ m/s})^2 - (0.600\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m}) = \boxed{-3.8\text{ J}}$$

47. ***REASONING AND SOLUTION***

a. The work done by non-conservative forces is given by Equation 6.7b as

$$W_{nc} = \Delta KE + \Delta PE \qquad \text{so} \qquad \Delta PE = W_{nc} - \Delta KE$$

Now

$$\Delta KE = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}(55.0\text{ kg})[(6.00\text{ m/s})^2 - (1.80\text{ m/s})^2] = 901\text{ J}$$

and

$$\Delta PE = 80.0\text{ J} - 265\text{ J} - 901\text{ J} = \boxed{-1086\text{ J}}$$

b. $\Delta PE = mg\,(h - h_0)$ so

$$h - h_0 = \frac{\Delta PE}{mg} = \frac{-1086\text{ J}}{(55.0\text{ kg})(9.80\text{ m/s}^2)} = -2.01\text{ m}$$

Thus, the skater's vertical position has changed by $\boxed{2.01\text{ m}}$, and the skater is $\boxed{\text{below the starting point}}$.

48. ***REASONING***

a. Since there is no air friction, the only force that acts on the projectile is the conservative gravitational force (its weight). The initial and final speeds of the ball are known, so the conservation of mechanical energy can be used to find the maximum height that the projectile attains.

b. When air resistance, a nonconservative force, is present, it does negative work on the projectile and slows it down. Consequently, the projectile does not rise as high as when there is no air resistance. The work-energy theorem, in the form of Equation 6.6, may be used to find the work done by air friction. Then, using the definition of work, Equation 6.1, the average force due to air resistance can be found.

SOLUTION

a. The conservation of mechanical energy, as expressed by Equation 6.9b, states that

$$\underbrace{\tfrac{1}{2}mv_{\mathrm{f}}^2 + mgh_{\mathrm{f}}}_{E_{\mathrm{f}}} = \underbrace{\tfrac{1}{2}mv_0^2 + mgh_0}_{E_0}$$

The mass m can be eliminated algebraically from this equation since it appears as a factor in every term. Solving for the final height h_{f} gives

$$h_{\mathrm{f}} = \frac{\tfrac{1}{2}\left(v_0^2 - v_{\mathrm{f}}^2\right)}{g} + h_0$$

Setting $h_0 = 0$ m and $v_{\mathrm{f}} = 0$ m/s, the final height, in the absence of air resistance, is

$$h_{\mathrm{f}} = \frac{v_{\mathrm{o}}^2 - v_{\mathrm{f}}^2}{2g} = \frac{(18.0\ \mathrm{m/s})^2 - (0\ \mathrm{m/s})^2}{2\left(9.80\ \mathrm{m/s^2}\right)} = \boxed{16.5\ \mathrm{m}}$$

b. The work-energy theorem is

$$W_{\mathrm{nc}} = \left(\tfrac{1}{2}mv_{\mathrm{f}}^2 - \tfrac{1}{2}mv_0^2\right) + \left(mgh_{\mathrm{f}} - mgh_0\right) \qquad (6.6)$$

where W_{nc} is the nonconservative work done by air resistance. According to Equation 6.1, the work can be written as $W_{\mathrm{nc}} = \left(\bar{F}_{\mathrm{R}} \cos 180°\right)s$, where $\bar{F}_{\mathrm{R}}$ is the average force of air resistance. As the projectile moves upward, the force of air resistance is directed downward, so the angle between the two vectors is $\theta = 180°$ and $\cos\theta = -1$. The magnitude s of the displacement is the difference between the final and initial heights, $s = h_{\mathrm{f}} - h_0 = 11.8$ m. With these substitutions, the work-energy theorem becomes

$$-\bar{F}_{\mathrm{R}}\, s = \tfrac{1}{2}m\left(v_{\mathrm{f}}^2 - v_{\mathrm{o}}^2\right) + mg\left(h_{\mathrm{f}} - h_0\right)$$

Solving for $\bar{F}_{\mathrm{R}}$ gives

$$\bar{F}_{\mathrm{R}} = \frac{\frac{1}{2}m\left(v_{\mathrm{f}}^2 - v_{\mathrm{o}}^2\right) + mg\left(h_{\mathrm{f}} - h_0\right)}{-s}$$

$$= \frac{\frac{1}{2}(0.750\ \mathrm{kg})\left[(0\ \mathrm{m/s})^2 - (18.0\ \mathrm{m/s})^2\right] + (0.750\ \mathrm{kg})\left(9.80\ \mathrm{m/s^2}\right)(11.8\ \mathrm{m})}{-(11.8\ \mathrm{m})} = \boxed{2.9\ \mathrm{N}}$$

49. ***REASONING AND SOLUTION*** The force exerted by the bat on the ball is the only non-conservative force acting. The work due to this force is

$$W_{\text{nc}} = \frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_0^2 + mg\left(h_{\text{f}} - h_0\right)$$

Taking $h_0 = 0$ m at the level of the bat, $v_0 = 40.0$ m/s just before the bat strikes the ball and v_{f} to be the speed of the ball at $h_{\text{f}} = 25.0$ m, we have

$$v_{\text{f}} = \sqrt{\frac{2W_{\text{nc}}}{m} + v_0^2 - 2gh_{\text{f}}}$$

$$= \sqrt{\frac{2(70.0\text{ J})}{0.140\text{ kg}} + \left(40.0\text{ m/s}\right)^2 - 2(9.80\text{ m/s}^2)(25.0\text{ m})} = \boxed{45.9\text{ m/s}}$$

50. ***REASONING***

a. Because the kinetic frictional force, a nonconservative force, is present, it does negative work on the skier. The work-energy theorem, in the form of Equation 6.6, may be used to find the work done by this force.

b. Once the work done by the kinetic frictional force is known, the magnitude of the kinetic frictional force can be determined by using the definition of work, Equation 6.1, since the magnitude of the skier's displacement is known.

SOLUTION

a. The work W_{nc} done by the kinetic frictional force is related to the object's kinetic and potential energies by Equation 6.6:

$$W_{\text{nc}} = \left(\tfrac{1}{2}mv_{\text{f}}^2 - \tfrac{1}{2}mv_0^2\right) + \left(mgh_{\text{f}} - mgh_0\right)$$

The initial height of the skier at the bottom of the hill is $h_0 = 0$ m, and the final height is $h_{\text{f}} = s \sin 25°$ (see the drawing). Thus, the work is

$$W_{\text{nc}} = \left(\tfrac{1}{2}mv_{\text{f}}^2 - \tfrac{1}{2}mv_0^2\right) + mg\left(s \sin 25° - h_0\right)$$

$$= \tfrac{1}{2}(63\text{ kg})(4.4\text{ m/s})^2 - \tfrac{1}{2}(63\text{ kg})(6.6\text{ m/s})^2$$

$$+ (63\text{ kg})\left(9.80\text{ m/s}^2\right)\left[(1.9\text{ m})\sin 25° - 0\text{ m}\right] = \boxed{-270\text{ J}}$$

b. The work done by the kinetic frictional force is, according to Equation 6.1, $W_{\text{nc}} = \left(f_{\text{k}} \cos 180°\right)s$, where f_{k} is the magnitude of the kinetic frictional force, and s is the magnitude of the skier's displacement. The displacement of the skier is up the hill and the kinetic frictional force is directed down the hill, so the angle between the two vectors is $\theta = 180°$ and $\cos\theta = -1$. Solving the equation above for f_{k}, we have

$$f_{\text{k}} = \frac{W_{\text{nc}}}{-s} = \frac{-270\text{ J}}{-1.9\text{ m}} = \boxed{140\text{ N}}$$

51. ***REASONING*** The work-energy theorem, as expressed in Equation 6.7b, is

$$W_{nc} = \Delta \text{KE} + \Delta \text{PE}$$

and can be applied to method 1 and method 2. In either method, the speed is constant, so that ΔKE = 0 J. In addition, we take the height at ground level to be $h_0 = 0$ m, so that $h_f = h$ and $\Delta \text{PE} = mgh_f - mgh_0 = mgh$. Thus, the work-energy theorem becomes

$$W_{\text{nc}} = mgh \qquad (1)$$

SOLUTION In method 1, the nonconservative work that you do is the only nonconservative work present, so that Equation (1) becomes

$$W_{\text{nc}} = W_{\text{you, 1}} = mgh$$

In method 2, there are two contributions to the nonconservative work, so that Equation (1) becomes

$$W_{\text{nc}} = -485 \text{ J} + W_{\text{you, 2}} = mgh$$

Since $W_{\text{you, 2}} = 2W_{\text{you, 1}} = 2mgh$, we have

$$-485 \text{ J} + 2mgh = mgh \qquad \text{or} \qquad 485 \text{ J} = mgh$$

But h = (3.00 m) sin θ, so it follows that

$$485 \text{ J} = mg(3.00 \text{ m})\sin\theta$$

Therefore,

$$\theta = \sin^{-1}\left[\frac{485 \text{ J}}{mg(3.00 \text{ m})}\right] = \sin^{-1}\left[\frac{485 \text{ J}}{(45.0 \text{ kg})(9.80 \text{ m/s}^2)(3.00 \text{ m})}\right] = \boxed{21.5°}$$

52. ***REASONING AND SOLUTION***

a. The lost energy is

$$E_{\text{lost}} = E_0 - E_{\text{f}}$$

The ball dropped from rest, so its initial energy is purely potential. The ball is momentarily at rest at the highest point in its rebound, so its final energy is also purely potential. Then

$$E_{\text{lost}} = mgh_0 - mgh_{\text{f}} = (0.60\text{ kg})\left(9.80\text{ m/s}^2\right)\left[(1.22\text{ m}) - (0.69\text{ m})\right] = \boxed{3.1\text{ J}}$$

b. The work done by the player must compensate for this loss of energy.

$$E_{\text{lost}} = (F\cos\theta)\,s \quad \Rightarrow \quad F = \frac{E_{\text{lost}}}{(\cos\theta)\,s} = \frac{3.1\text{ J}}{(\cos 0°)(0.13\text{ m})} = \boxed{24\text{ N}}$$

53. **SSM** ***REASONING AND SOLUTION*** According to the work-energy theorem as given in Equation 6.8, we have

$$W_{\text{nc}} = \left(\tfrac{1}{2}mv_{\text{f}}^2 + mgh_{\text{f}}\right) - \left(\tfrac{1}{2}mv_0^2 + mgh_0\right)$$

The metal piece starts at rest and is at rest just as it barely strikes the bell, so that $v_{\text{f}} = v_0 = 0$ m/s. In addition, $h_{\text{f}} = h$ and $h_0 = 0$ m, while $W_{\text{nc}} = 0.25\left(\tfrac{1}{2}Mv^2\right)$, where M and v are the mass and speed of the hammer. Thus, the work-energy theorem becomes

$$0.25\left(\tfrac{1}{2}Mv^2\right) = mgh$$

Solving for the speed of the hammer, we find

$$v = \sqrt{\frac{2mgh}{0.25M}} = \sqrt{\frac{2(0.400\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m})}{0.25\ (9.00\text{ kg})}} = \boxed{4.17\text{ m/s}}$$

54. ***REASONING AND SOLUTION*** For the actual motion of the rocket

$$\Delta \text{KE} = W_{\text{nc}} - \Delta \text{PE} = W_{\text{nc}} - mgh \tag{6.7b}$$

$$\Delta \text{KE} = -8.00 \times 10^2 \text{ J} - (3.00 \text{ kg})(9.80 \text{ m/s}^2)(1.00 \times 10^2 \text{ m}) = -3740 \text{ J}$$

Since $\Delta \text{KE} = \text{KE}_f - \text{KE}_0 = -\text{KE}_0$, the initial kinetic energy of the rocket is then

$$\text{KE}_0 = 3740 \text{ J}$$

If the rocket were launched with this initial kinetic energy and no air resistance, the potential energy of the rocket at the top of its trajectory would be

$$\text{PE} = mgh = 3740 \text{ J}$$

Hence,

$$h = \frac{3740 \text{ J}}{(3.00 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{127 \text{ m}}$$

55. ***REASONING*** We can find the power produced by the hero by using Equation 6.10a, $\overline{P} = W/t$. In order to use this expression, we must first determine the work done by the hero. In order to calculate the work, we must first find the net force. The net force can be found from Newton's second law.

SOLUTION There are two forces that act on the villain while he is being lifted upward. They are the upward vertical force **F** exerted on the villain by the hero, and the downward weight $m\mathbf{g}$. With upward taken as positive, Newton's second law gives $F - mg = ma = 0$, since the villain moves straight upward at constant speed. Therefore, $F = mg$, and the work done by the hero in lifting the villain straight up through a vertical distance h is $W = Fh = mgh$. Therefore, the power produced by the hero is

$$\overline{P} = \frac{W}{t} = \frac{mgh}{t} = \frac{(91\text{ kg})(9.80\text{ m/s}^2)(1.2\text{ m})}{0.51\text{ s}} = \boxed{2.1\times10^3\text{ W}}$$

56. ***REASONING AND SOLUTION*** The work done by the person in turning the crank one revolution is

$$W = F(2\pi r) = (22\text{ N})(2\pi)(0.28\text{ m}) = 39\text{ J}$$

The power expended is then

$$\overline{P} = \frac{W}{t} = \frac{39\text{ J}}{1.3\text{ s}} = \boxed{3.0 \times 10^{1}\text{ W}} \qquad (6.10a)$$

57. **SSM** ***REASONING AND SOLUTION*** One kilowatt · hour is the amount of work or energy generated when one kilowatt of power is supplied for a time of one hour. From Equation 6.10a, we know that $W = \bar{P}t$. Using the fact that $1\text{ kW} = 1.0 \times 10^3\text{ J/s}$ and that $1\text{h} = 3600\text{ s}$, we have

$$1.0\text{ kWh} = (1.0 \times 10^3\text{ J/s})(1\text{ h}) = (1.0 \times 10^3\text{ J/s})(3600\text{ s}) = \boxed{3.6 \times 10^6\text{ J}}$$

58. ***REASONING AND SOLUTION*** The work done by the crane is the work done against gravity, $W = mgh$, so

$$t = \frac{mgh}{P} = \frac{\left(3.00 \times 10^2 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(10.0 \text{ m}\right)}{4.00 \times 10^2 \text{ W}} = \boxed{73.5 \text{ s}}$$

59. ***REASONING*** The average power developed by the cheetah is equal to the work done by the cheetah divided by the elapsed time (Equation 6.10a). The work, on the other hand, can be related to the change in the cheetah's kinetic energy by the work-energy theorem, Equation 6.3.

SOLUTION

a. The average power is

$$\bar{P} = \frac{W}{t} \tag{6.10a}$$

where W is the work done by the cheetah. This work is related to the change in the cheetah's kinetic energy by Equation 6.3, $W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$, so the average power is

$$\bar{P} = \frac{W}{t} = \frac{\frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2}{t}$$

$$= \frac{\frac{1}{2}(110\text{ kg})(27\text{ m/s})^2 - \frac{1}{2}(110\text{ kg})(0\text{ m/s})^2}{4.0\text{ s}} = \boxed{1.0\times10^4\text{ W}}$$

b. The power, in units of horsepower (hp), is

$$\bar{P} = \left(1.0\times10^4\text{ W}\right)\left(\frac{1\text{ hp}}{745.7\text{ W}}\right) = \boxed{13\text{ hp}}$$

60. ***REASONING AND SOLUTION***

a. The power developed by the engine is

$$P = Fv = (2.00 \times 10^2 \text{ N})(20.0 \text{ m/s}) = \boxed{4.00 \times 10^3 \text{ W}}$$

b. The force required of the engine in order to maintain a constant speed up the slope is

$$F = F_a + mg \sin 37.0°$$

The power developed by the engine is then

$$P = Fv = (F_a + mg \sin 37.0°)v \tag{6.11}$$

$$P = [2.00 \times 10^2 \text{ N} + (2.50 \times 10^2 \text{ kg})(9.80 \text{ m/s}^2)\sin 37.0°](20.0 \text{ m/s}) = \boxed{3.35 \times 10^4 \text{ W}}$$

61. **SSM** ***REASONING AND SOLUTION*** In the drawings below, the positive direction is to the right. When the boat is not pulling a skier, the engine must provide a force F_1 to overcome the resistive force of the water F_R. Since the boat travels with constant speed, these forces must be equal in magnitude and opposite in direction.

$$-F_R + F_1 = ma = 0$$

or

$$F_R = F_1 \qquad (1)$$

When the boat is pulling a skier, the engine must provide a force F_2 to balance the resistive force of the water F_R and the tension in the tow rope T.

$$-F_R + F_2 - T = ma = 0$$

or

$$F_2 = F_R + T \qquad (2)$$

The average power is given by $\overline{P} = F\overline{v}$, according to Equation 6.11 in the text. Since the boat moves with the same speed in both cases, $v_1 = v_2$, and we have

$$\frac{\overline{P}_1}{F_1} = \frac{\overline{P}_2}{F_2} \qquad \text{or} \qquad F_2 = F_1\frac{\overline{P}_2}{\overline{P}_1}$$

Using Equations (1) and (2), this becomes

$$F_1 + T = F_1\frac{\overline{P}_2}{\overline{P}_1}$$

Solving for T gives

$$T = F_1\left(\frac{\overline{P}_2}{\overline{P}_1} - 1\right)$$

The force F_1 can be determined from $\overline{P}_1 = F_1 v_1$, thereby giving

$$T = \frac{\overline{P}_1}{v_1}\left(\frac{\overline{P}_2}{\overline{P}_1} - 1\right) = \frac{7.50\times10^4\ \text{W}}{12\ \text{m/s}}\left(\frac{8.30\times10^4\ \text{W}}{7.50\times10^4\ \text{W}} - 1\right) = \boxed{6.7\times10^2\ \text{N}}$$

62. ***REASONING AND SOLUTION*** The following drawings show the free-body diagrams for the car in going both up and down the hill. The force $\mathbf{F}_R$ is the combined force of air resistance and friction, and the forces $\mathbf{F}_U$ and $\mathbf{F}_D$ are the forces supplied by the engine in going uphill and downhill respectively.

Going up the hill

Going down the hill

Writing Newton's second law in the direction of motion for the car as it goes uphill, taking uphill as the positive direction, we have

$$F_U - F_R - mg\sin\theta = ma = 0$$

Solving for F_U, we have

$$F_U = F_R + mg\sin\theta$$

Similarly, when the car is going downhill, Newton's second law in the direction of motion gives

$$F_R - F_D - mg\sin\theta = ma = 0$$

so that

$$F_D = F_R - mg\sin\theta$$

Since the car needs 47 hp more to sustain the constant uphill velocity than the constant downhill velocity, we can write

$$\overline{P}_U = \overline{P}_D + \Delta P$$

where $\overline{P}_U$ is the power needed to sustain the constant uphill velocity, $\overline{P}_D$ is the power needed to sustain the constant downhill velocity, and $\Delta P = 47\text{ hp}$. In terms of SI units,

$$\Delta P = (47\text{ hp})\left(\frac{746\text{ W}}{1\text{ hp}}\right) = 3.51\times10^4\text{ W}$$

Using Equation 6.11 ($\overline{P} = F\overline{v}$), the equation $\overline{P}_U = \overline{P}_D + \Delta P$ can be written as

$$F_U\overline{v} = F_D\overline{v} + \Delta P$$

Using the expressions for F_U and F_D, we have

$$(F_R + mg\sin\theta)\overline{v} = (F_R - mg\sin\theta)\overline{v} + \Delta P$$

Solving for θ, we find

$$\theta = \sin^{-1}\left(\frac{\Delta P}{2mg\overline{v}}\right) = \sin^{-1}\left(\frac{3.51\times10^{4}\ \text{W}}{2(1900\ \text{kg})(9.80\ \text{m/s}^{2})(27\ \text{m/s})}\right) = \boxed{2.0^{\circ}}$$

63. **SSM** ***REASONING*** The area under the force-versus-displacement graph over any displacement interval is equal to the work done over that displacement interval.

SOLUTION

a. Since the area under the force-versus-displacement graph from $s = 0$ to 0.50 m is greater for bow 1, it follows that $\boxed{\text{bow 1 requires more work to draw the bow fully}}$.

b. The work required to draw bow 1 is equal to the area of the triangular region under the force-versus-displacement curve for bow 1. Since the area of a triangle is equal one-half times the base of the triangle times the height of the triangle, we have

$$W_1 = \frac{1}{2}(0.50\text{m})(350\text{ N}) = 88\text{ J}$$

For bow 2, we note that each small square under the force-versus-displacement graph has an area of

$$(0.050\text{ m})(40.0\text{ N}) = 2.0\text{ J}$$

We estimate that there are approximately 31.3 squares under the force-versus-displacement graph for bow 2; therefore, the total work done is

$$(31.3\text{ squares})\left(\frac{2.0\text{ J}}{\text{square}}\right) = 63\text{ J}$$

Therefore, the additional work required to stretch bow 1 as compared to bow 2 is

$$88\text{ J} - 63\text{ J} = \boxed{25\text{ J}}$$

64. ***REASONING AND SOLUTION*** The work done during each interval is equal to the area under the force vs. displacement curve over that interval.

a. Since the area under the curve from 0 to 1.0 m is triangular, the area under the curve over this interval is (1/2) × height × base:

$$W_{01} = \frac{1}{2}(+6.0\text{ N})(1.0\text{ m} - 0\text{ m}) = \boxed{3.0\text{ J}}$$

b. Since there is no area under the curve, the $\boxed{\text{work done is zero}}$. This is reasonable since there is no net force acting on the object during this interval.

c. In the interval from 2.0 to 4.0 m, the area under the curve has two distinct regions. From 2.0 to 3.0 m, the area is triangular in shape, while from 3.0 to 4.0 m the region is rectangular. The total work done from 2.0 to 4.0 m is the sum of the areas of these two regions.

$$\boxed{\text{work done is zero}}$$

and

$$W_{34} = (-6.0\text{ N})(4.0\text{ m} - 3.0\text{ m}) = -6.0\text{ J}$$

Thus, the total work done in the interval from 2.0 to 4.0 m is

$$W_{24} = W_{23} + W_{34} = (-3.0\text{ J}) + (-6.0\text{ J}) = \boxed{-9.0\text{ J}}$$

Notice in the interval from 2.0 to 4.0 m, the area under the curve (and hence the work done) is *negative* since the force is negative in that interval.

65. ***REASONING AND SOLUTION***

a. The work done is equal to the colored area under the graph. Therefore, the work done by the net force on the skater from 0 to 3.0 m is

$$\begin{bmatrix}\text{Area under graph}\\ \text{from 0 to 3.0 m}\end{bmatrix} = (31\text{ N} - 0\text{ N})(3.0\text{ m} - 0\text{ m}) = \boxed{93\text{ J}}$$

b. From 3.0 m to 6.0 m, the net force component $F\cos\theta$ is zero, and there is no area under the curve. Therefore, $\boxed{\text{no work is done}}$.

c. The speed of the skater will increase from 1.5 m/s when $s = 0$ m to a final value v_f at $s = 3.0$ m. Since the force drops to zero at $s = 3.0$ m, the speed remains constant at the value v_f from $s = 3.0$ m to $s = 6.0$ m; therefore, the speed at $s = 6.0$ m is v_f. We can find v_f from the work-energy theorem (Equation 6.3). According to the work-energy theorem, the work done on the skater is given by

$$W = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_0^2$$

Solving for v_f, we find

$$v_f = \sqrt{\frac{2W}{m} + v_0^2} = \sqrt{\frac{2(93\text{ J})}{65\text{ kg}} + (1.5\text{ m/s})^2} = \boxed{2.3\text{ m/s}}$$

66. ***REASONING*** The area under the force-versus-displacement graph over any displacement interval is equal to the work done over that interval. From Example 16, we know that the total work done in drawing back the string of the bow is 60.5 J, corresponding to a total area under the curve of 242 squares (each square represents 0.250 J of work). The percentage of the total work done over any displacement interval is, therefore,

$$\frac{\text{Work done during interval}}{\text{Total work done during all intervals}} \times 100 = \frac{\text{Number of squares under curve during interval}}{\text{Total number of squares under entire curve}} \times 100$$

SOLUTION

a. We estimate that there are 130 squares under the curve from $s = 0$ to $0.306\ \text{m}$; therefore, the percentage of work done during the interval in question is

$$\frac{130\ \text{squares}}{242\ \text{squares}} \times 100 = \boxed{54\%}$$

b. Similarly, we estimate that there are 112 squares under the curve in the interval from $s = 0.306\ \text{m}$ to $0.500\ \text{m}$; therefore, the percentage of work done during the interval in question is

$$\frac{112\ \text{squares}}{242\ \text{squares}} \times 100 = \boxed{46\%}$$

67. ***REASONING AND SOLUTION***

a. The net work done is equal to the area under the curve. For the first 10 m, the area is triangular and the area under the curve is equal to (1/2) × height × base; therefore

$$W_{0,10} = \tfrac{1}{2}(10.0\text{ N})(10.0\text{ m} - 0\text{ m}) = 5.00\times10^{1}\text{ J}$$

From 10 to 20 m the region is rectangular and the area under the curve is

$$W_{10,20} = (10.0\text{ N})(20.0\text{ m} - 10.0\text{ m}) = 1.00\times10^{2}\text{ J}$$

Thus, the total work done by the force is

$$\mathrm{W}_{0,10} + \mathrm{W}_{10,20} = (5.00\times10^{1}\text{ J}) + (1.00\times10^{2}\text{ J}) = \boxed{1.50\times10^{2}\text{ J}}$$

b. From the work-energy theorem,

$$W = \tfrac{1}{2}mv_{\mathrm{f}}^{2} - \tfrac{1}{2}mv_{0}^{2}$$

Since the object is initially at rest, $v_0 = 0$ m/s. Solving for v_{f} gives

$$v_{\mathrm{f}} = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2(1.50\times10^{2}\text{ J})}{6.00\text{ kg}}} = \boxed{7.07\text{ m/s}}$$

68. ***REASONING*** The work done by the tension in the cable is given by Equation 6.1 as $W = (T \cos\theta)s$. Since the elevator is moving upward at a constant velocity, it is in equilibrium, and the magnitude T of the tension must be equal to the magnitude mg of the elevator's weight; $T = mg$.

SOLUTION

a. The tension and the displacement vectors point in the same direction (upward), so the angle between them is $\theta = 0°$. The work done by the tension is

$$W = (T \cos\theta)s = (mg \cos\theta)s \qquad (6.1)$$

$$= (1200\ \text{kg})(9.80\ \text{m/s}^2)\cos 0°\,(35\ \text{m}) = \boxed{4.1 \times 10^5\ \text{J}}$$

b. The weight and the displacement vectors point in opposite directions, so the angle between them is $\theta = 180°$. The work done by the weight is

$$W = (mg \cos\theta)s = (1200\ \text{kg})(9.80\ \text{m/s}^2)\cos 180°\,(35\ \text{m}) = \boxed{-4.1 \times 10^5\ \text{J}} \qquad (6.1)$$

69. **SSM** ***REASONING AND SOLUTION*** Since we are ignoring friction and air resistance, the net work done by the nonconservative forces is zero, and the principle of conservation of mechanical energy holds. Let E_0 be the total mechanical energy of the vaulter at take-off and E_f be the total mechanical energy as he clears the bar. The principle of conservation of mechanical energy gives

$$\underbrace{\frac{1}{2}mv_f^2 + mgh_f}_{E_f} = \underbrace{\frac{1}{2}mv_0^2 + mgh_0}_{E_0}$$

For maximum height h_f, the vaulter just clears the bar with zero speed, $v_f = 0$ m/s. If we let $h_0 = 0$ m at ground level, we have

$$mgh_f = \frac{1}{2}mv_0^2$$

Solving for h_f, we find

$$h_f = \frac{v_0^2}{2g} = \frac{(9.00\text{ m/s})^2}{2(9.80\text{ m/s}^2)} = \boxed{4.13\text{ m}}$$

70. ***REASONING AND SOLUTION*** The work done by the pitcher is

$$W_{\text{nc}} = F\,(\pi\, r)$$

Assuming that only the pitcher does work on the ball

$$W_{\text{nc}} = \tfrac{1}{2}mv_{\text{f}}^2 - \tfrac{1}{2}mv_0^2 + mgh_{\text{f}} - mgh_0$$

Then,

$$v_{\text{f}} = \sqrt{\frac{2W_{\text{nc}}}{m} + v_0^2 - 2g\left(h_{\text{f}} - h_0\right)}$$

$$= \sqrt{\frac{2(28\text{ N})\pi\left(0.51\text{ m}\right)}{0.25\text{ kg}} + \left(12\text{ m/s}\right)^2 - 2(9.80\text{ m/s}^2)(-1.02\text{ m})} = \boxed{23\text{ m/s}}$$

71. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The work done by the retarding force is given by Equation 6.1: $W = (F\cos\theta)s$. Since the force is a retarding force, it must point opposite to the direction of the displacement, so that $\theta = 180°$. Thus, we have

$$W = (F\cos\theta)s = (3.0 \times 10^3 \text{ N})(\cos 180°)(850 \text{ m}) = \boxed{-2.6 \times 10^6 \text{ J}}$$

The work done by this force is $\boxed{\text{negative}}$, because the retarding force is directed opposite to the direction of the displacement of the truck.

72. ***REASONING AND SOLUTION*** The work done by nonconservative forces on the balloon is

$$W_{\text{nc}} = \Delta\text{KE} + \Delta\text{PE} = +9.70 \times 10^4 \text{ J}$$

The change in kinetic energy of the balloon is

$$\Delta\text{KE} = \tfrac{1}{2}mv_{\text{f}}^2 = \tfrac{1}{2}(5.00 \times 10^2 \text{ kg})(8.00 \text{ m/s})^2 = +1.60 \times 10^4 \text{ J}$$

The change in the potential energy is

$$\Delta\text{PE} = mgh = (5.00 \times 10^2 \text{ kg})(9.80 \text{ m/s}^2)h = (4.90 \times 10^3 \text{ kg·m/s}^2)h$$

Hence,

$$h = \frac{9.70 \times 10^4 \text{ J} - 1.60 \times 10^4 \text{ J}}{4.90 \times 10^3 \text{ kg} \cdot \text{m/s}^2} = \boxed{16.5 \text{ m}}$$

73. ***REASONING AND SOLUTION*** The applied force does work

$$W_P = Ps \cos 0° = (150 \text{ N})(7.0 \text{ m}) = \boxed{1.0 \times 10^3 \text{ J}}$$

The frictional force does work

$$W_f = f_k s \cos 180° = -\mu_k F_N s$$

where $F_N = mg$, so

$$W_f = -(0.25)(55 \text{ kg})(9.80 \text{ m/s}^2)(7.0 \text{ m}) = \boxed{-940 \text{ J}}$$

$\boxed{\text{The normal force and gravity do no work}}$, since they both act at a 90° angle to the displacement.

74. ***REASONING*** The average power generated by the tension in the cable is equal to the work done by the tension divided by the elapsed time (Equation 6.10a). The work can be related to the change in the lift's kinetic and potential energies via the work-energy theorem.

SOLUTION

a. The average power is

$$\bar{P} = \frac{W_{\text{nc}}}{t} \tag{6.10a}$$

where W_{nc} is the work done by the nonconservative tension force. This work is related to the lift's kinetic and potential energies by Equation 6.6, $W_{\text{nc}} = \left(\frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_0^2\right) + \left(mgh_{\text{f}} - mgh_0\right)$, so the average power is

$$\bar{P} = \frac{\text{W}_{\text{nc}}}{t} = \frac{\left(\frac{1}{2}mv_{\text{f}}^2 - \frac{1}{2}mv_0^2\right) + \left(mgh_{\text{f}} - mgh_0\right)}{t}$$

Since the lift moves at a constant speed, $v_0 = v_{\text{f}}$, the average power becomes

$$\bar{P} = \frac{mg\left(h_{\text{f}} - h_0\right)}{t} = \frac{(4\text{ skiers})\left(65\ \frac{\text{kg}}{\text{skier}}\right)\left(9.80\text{ m/s}^2\right)(140\text{ m})}{(2\text{ min})\left(\frac{60\text{ s}}{1\text{ min}}\right)} = \boxed{3.0 \times 10^3\ \text{W}}$$

75. **SSM** ***REASONING*** The work-energy theorem can be used to determine the net work done on the car by the nonconservative forces of friction and air resistance. The work-energy theorem is, according to Equation 6.8,

$$W_{\text{nc}} = \left(\tfrac{1}{2}mv_{\text{f}}^2 + mgh_{\text{f}}\right) - \left(\tfrac{1}{2}mv_0^2 + mgh_0\right)$$

The nonconservative forces are the force of friction, the force of air resistance, and the force provided by the engine. Thus, we can rewrite Equation 6.8 as

$$W_{\text{friction}} + W_{\text{air}} + W_{\text{engine}} = \left(\tfrac{1}{2}mv_{\text{f}}^2 + mgh_{\text{f}}\right) - \left(\tfrac{1}{2}mv_0^2 + mgh_0\right)$$

This expression can be solved for $W_{\text{friction}} + W_{\text{air}}$.

SOLUTION We will measure the heights from sea level, where $h_0 = 0$ m. Since the car starts from rest, $v_0 = 0$ m/s. Thus, we have

$$\begin{aligned} W_{\text{friction}} + W_{\text{air}} &= m\left(\tfrac{1}{2}v_{\text{f}}^2 + gh_{\text{f}}\right) - W_{\text{engine}} \\ &= (1.50\times10^3\ \text{kg})\left[\tfrac{1}{2}(27.0\ \text{m/s})^2 + (9.80\ \text{m/s}^2)(2.00\times10^2\ \text{m})\right] - (4.70\times10^6\ \text{J}) \\ &= \boxed{-1.21\times10^6\ \text{J}} \end{aligned}$$

76. ***REASONING AND SOLUTION*** Relative to her final position in the water, the diver initially had energy mgh_0. All of this energy was dissipated as work in traveling through 3.00 m of water.

$$W_{\text{nc}} = \tfrac{1}{2}mv_{\text{f}}^{\,2} - \tfrac{1}{2}mv_0^{\,2} + mgh_{\text{f}} - mgh_0$$

Thus,

$$W_{\text{nc}} = Fs\cos 180° = mgh_{\text{f}} - mgh_0$$

Then,

$$F = \frac{(55.0\text{ kg})\left(9.80\text{ m/s}^2\right)(-13.0\text{ m})}{(3.00\text{ m})\cos 180°} = \boxed{2340\text{ N}}$$

77. ***REASONING AND SOLUTION*** The conservation of energy applied between point A and the top of the trajectory gives

$$\mathrm{KE_A} + mgh_\mathrm{A} = mgh$$

where $h = 4.00$ m. Rearranging, we find

$$\mathrm{KE_A} = mg(h - h_\mathrm{A})$$

or

$$v_\mathrm{A} = \sqrt{2g\left(h - h_\mathrm{A}\right)} = \sqrt{2\left(9.80\ \mathrm{m/s^2}\right)\left(4.00\ \mathrm{m} - 3.00\ \mathrm{m}\right)} = \boxed{4.43\ \mathrm{m/s}}$$

78. ***REASONING AND SOLUTION*** When the car is at the top of the track the centripetal force consists of the full weight of the car.

$$mv^2/r = mg$$

Applying the conservation of energy between the bottom and the top of the track gives

$$(1/2)mv^2 + mg(2r) = (1/2)mv_0^2$$

Using both of the above equations

$$v_0^2 = 5gr$$

so

$$r = v_0^2/(5g) = (4.00\ \text{m/s})^2/(49.0\ \text{m/s}^2) = \boxed{0.327\ \text{m}}$$

79. **SSM** **WWW** ***REASONING*** After the wheels lock, the only nonconservative force acting on the truck is friction. The work done by this conservative force can be determined from the work-energy theorem. According to Equation 6.8,

$$W_{\text{nc}} = E_{\text{f}} - E_0 = \left(\tfrac{1}{2}mv_{\text{f}}^2 + mgh_{\text{f}}\right) - \left(\tfrac{1}{2}mv_0^2 + mgh_0\right) \tag{1}$$

where W_{nc} is the work done by friction. According to Equation 6.1, $W = (F\cos\theta)s$; since the force of kinetic friction points opposite to the displacement, $\theta = 180°$. According to Equation 4.8, the kinetic frictional force has a magnitude of $f_{\text{k}} = \mu_{\text{k}}F_{\text{N}}$, where μ_{k} is the coefficient of kinetic friction and F_{N} is the magnitude of the normal force. Thus,

$$W_{\text{nc}} = (f_{\text{k}}\ \cos\theta)s = \mu_{\text{k}}F_{\text{N}}\ (\cos 180°)s = -\mu_{\text{k}}F_{\text{N}}s \tag{2}$$

Since the truck is sliding down an incline, we refer to the free-body diagram in order to determine the magnitude of the normal force F_{N}. The free-body diagram at the right shows the three forces that act on the truck. They are the normal force $\mathbf{F}_{\text{N}}$, the force of kinetic friction $\mathbf{f}_{\text{k}}$, and the weight $m\mathbf{g}$. The weight has been resolved into its components, and these vectors are shown as dashed arrows. From the free body diagram and the fact that the truck does not accelerate perpendicular to the incline, we can see that the magnitude of the normal force is given by

$$F_{\text{N}} = mg\cos 15°$$

Therefore, Equation (2) becomes

$$W_{\text{nc}} = -\mu_{\text{k}}(mg\cos 15°)s \tag{3}$$

Combining Equations (1), (2), and (3), we have

$$-\mu_{\text{k}}\left(mg\ \cos 15°\right)s\ = \left(\tfrac{1}{2}mv_{\text{f}}^2 + mgh_{\text{f}}\right) - \left(\tfrac{1}{2}mv_0^2 + mgh_0\right) \tag{4}$$

This expression may be solved for s.

SOLUTION When the car comes to a stop, $v_{\text{f}} = 0$ m/s. If we take $h_{\text{f}} = 0$ m at ground level, then, from the drawing at the right, we see that $h_0 = s\ \sin 15°$. Equation (4) then gives

$$s = \frac{v_0^2}{2g(\mu_{\text{k}}\cos 15° - \sin 15°)} = \frac{(11.1\text{ m/s})^2}{2(9.80\text{ m/s}^2)[(0.750)(\cos 15°) - \sin 15°]} = \boxed{13.5\text{ m}}$$

1. SSM ***REASONING AND SOLUTION*** The impulse is given directly by Equation 7.1:

$$\textbf{Impulse} = \bar{\mathbf{F}}\,\Delta t = (+1400\text{ N})(7.9\times 10^{-3}\text{ s}) = \boxed{+11\text{ N}\cdot\text{s}}$$

The plus sign indicates that the direction of the impulse is the same as that of $\bar{\mathbf{F}}$.

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2. ***REASONING AND SOLUTION*** The change in momentum is equal to the impulse

$$\Delta p = \bar{F}\Delta t = (+680\ \text{N})(7.2\ \text{s}) = \boxed{+4.9 \times 10^{3}\ \text{kg} \cdot \text{m/s}}$$

3. ***REASONING AND SOLUTION*** According to the impulse-momentum theorem (see Equation 7.4), $\bar{\mathbf{F}}\Delta t = m\mathbf{v}_f - m\mathbf{v}_0$, where $\bar{\mathbf{F}}$ is the average net force. Taking the direction of motion (downward) as the negative direction and solving for the average net force $\bar{\mathbf{F}}$, we obtain

$$\bar{\mathbf{F}} = \frac{m(\mathbf{v}_f - \mathbf{v}_0)}{\Delta t} = \frac{(62.0\text{ kg})[-1.10\text{ m/s} - (-5.50\text{ m/s})]}{1.65\text{ s}} = \boxed{+165\text{ N}}$$

where the plus sign indicates that the force acts $\boxed{\text{upward}}$.

4. ***REASONING AND SOLUTION*** The magnitude of each arrow's momentum is

$$p = mv = (0.100\ \text{kg})(30.0\ \text{m/s}) = 3.00\ \text{kg}\cdot\text{m/s}$$

The vector momenta of the arrows are perpendicular, so the magnitude of the total momentum is given by the Pythagorean theorem

$$P_{\text{T}} = \sqrt{p_1^2 + p_2^2} = \sqrt{(3.00\ \text{kg}\cdot\text{m/s})^2 + (3.00\ \text{kg}\cdot\text{m/s})^2} = \boxed{4.24\ \text{kg}\cdot\text{m/s}}$$

The angle of the total momentum vector as measured from due east is

$$\theta = \tan^{-1}\left(\frac{-p_2}{p_1}\right) = \tan^{-1}(-1.00) = \boxed{45.0^\circ \text{ south of east}}$$

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5. SSM ***REASONING AND SOLUTION*** The impulse can be found from Equation 7.4, the impulse-momentum theorem:

$$\underbrace{\overline{\mathbf{F}}\,\Delta t}_{\text{Impulse}} = \underbrace{m\mathbf{v}_{\text{f}}}_{\substack{\text{Final}\\ \text{momentum}}} - \underbrace{m\mathbf{v}_{0}}_{\substack{\text{Initial}\\ \text{momentum}}}$$

$$= m(\mathbf{v}_{\text{f}} - \mathbf{v}_{0}) = (0.35\text{ kg})\left[(-21\text{ m/s}) - (+4.0\text{ m/s})\right] = \boxed{-8.7\text{ kg}\cdot\text{m/s}}$$

The minus sign indicates that the direction of the impulse is the same as that of the final velocity.

6. ***REASONING AND SOLUTION***

a. According to Equation 7.4, the impulse-momentum theorem, $\overline{\mathbf{F}}\Delta t = m\mathbf{v}_{\mathrm{f}} - m\mathbf{v}_0$. Since the goalie catches the puck, $v_{\mathrm{f}} = 0$. Solving for the average force exerted on the puck, we have

$$\overline{\mathbf{F}} = \frac{m(\mathbf{v}_{\mathrm{f}} - \mathbf{v}_0)}{\Delta t} = \frac{(0.17\ \mathrm{kg})[0 - (+65\ \mathrm{m/s})]}{5.0 \times 10^{-3}\ \mathrm{s}} = -2.2 \times 10^3\ \mathrm{N}$$

By Newton's third law, the force exerted on the goalie by the puck is equal in magnitude, but opposite in direction, to the force exerted on the puck by the goalie. Thus, the average force exerted on the goalie is $\boxed{+2.2 \times 10^3\ \mathrm{N}}$.

b. If, instead of catching the puck, the goalie slaps it with his stick and returns the puck straight back to the player with a velocity of –65 m/s, then the average force exerted on the puck by the goalie is

$$\overline{\mathbf{F}} = \frac{m(\mathbf{v}_{\mathrm{f}} - \mathbf{v}_0)}{\Delta t} = \frac{(0.17\ \mathrm{kg})[(-65\ \mathrm{m/s}) - (+65\ \mathrm{m/s})]}{5.0 \times 10^{-3}\ \mathrm{s}} = -4.4 \times 10^3\ \mathrm{N}$$

The average force exerted on the goalie by the puck is thus $\boxed{+4.4 \times 10^3\ \mathrm{N}}$.

The answer in part (b) is twice that in part (a). This is consistent with the conclusion of Conceptual Example 3. The change in the momentum of the puck is greater when the puck rebounds from the stick. Thus, the puck exerts a greater impulse, and hence a greater force, on the goalie.

7. ***REASONING AND SOLUTION*** Impulse = $F\Delta t = m\Delta v$, so that the average force exerted on the skater is

$$F = m\Delta v/\Delta t = (46\text{ kg})[-1.2\text{ m/s} - 0\text{ m/s}]/(0.80\text{ s}) = -69\text{ N}.$$

The force that the skater exerts on the wall is $\boxed{+69\text{ N}}$.

---

8. ***REASONING*** We will apply the impulse momentum theorem as given in Equation 7.3 to solve this problem. From this theorem we know that, for a given change in momentum, greater forces are associated with shorter time intervals. Therefore, we expect that the force in the stiff-legged case will be greater than in the knees-bent case.

SOLUTION a. Assuming that upward is the positive direction, we find from the impulse-momentum theorem that

$$\overline{F} = \frac{mv_{\mathrm{f}} - mv_{\mathrm{o}}}{\Delta t} = \frac{(75\ \mathrm{kg})(0\ \mathrm{m/s}) - (75\ \mathrm{kg})(-6.4\ \mathrm{m/s})}{2.0 \times 10^{-3}\ \mathrm{s}} = \boxed{2.4 \times 10^{5}\ \mathrm{N}}$$

b. Again using the impulse-momentum theorem, we find that

$$\overline{F} = \frac{mv_{\mathrm{f}} - mv_{\mathrm{o}}}{\Delta t} = \frac{(75\ \mathrm{kg})(0\ \mathrm{m/s}) - (75\ \mathrm{kg})(-6.4\ \mathrm{m/s})}{0.10\ \mathrm{s}} = \boxed{4.8 \times 10^{3}\ \mathrm{N}}$$

c. The average net force acting on the man is $\overline{F} = F_{\mathrm{Ground}} - mg$, where F_{Ground} is the average upward force exerted on the man by the ground and $-mg$ is the downward-acting weight of the man. It follows, then, that $F_{\mathrm{Ground}} = \overline{F} + mg$. Therefore,

Stiff - legged

$$F_{\mathrm{Ground}} = \overline{F} + mg$$

$$= 2.4 \times 10^{5}\ \mathrm{N} + (75\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = \boxed{2.4 \times 10^{5}\ \mathrm{N}}$$

Knees bent

$$F_{\mathrm{Ground}} = \overline{F} + mg$$

$$= 4.8 \times 10^{3}\ \mathrm{N} + (75\ \mathrm{kg})(9.80\ \mathrm{m/s^2}) = \boxed{5.5 \times 10^{3}\ \mathrm{N}}$$

9. SSM WWW ***REASONING*** The impulse applied to the golf ball by the floor can be found from Equation 7.4, the impulse-momentum theorem: $\overline{\mathbf{F}}\,\Delta t = m\mathbf{v}_{\mathrm{f}} - m\mathbf{v}_0$. Only the vertical component of the ball's momentum changes during impact with the floor. In order to use Equation 7.4 directly, we must first find the vertical components of the initial and final velocities. We begin, then, by finding these velocity components.

SOLUTION The figures below show the initial and final velocities of the golf ball.

If we take up as the positive direction, then the vertical components of the initial and final velocities are, respectively, $\mathbf{v}_{0y} = -v_0 \cos 30.0°$ and $\mathbf{v}_{\mathrm{f}y} = v_{\mathrm{f}} \cos 30.0°$. Then, from Equation 7.4 the magnitude of the impulse is

$$\overline{F}\Delta t = m(v_{\mathrm{f}y} - v_{0y}) = m\left[(v_{\mathrm{f}} \cos 30.0°) - (-v_0 \cos 30.0°)\right]$$

Since $v_0 = v_{\mathrm{f}} = 45\ \mathrm{m/s}$,

$$\overline{F}\Delta t = 2mv_0 \cos 30.0° = 2(0.047\ \mathrm{kg})(45\ \mathrm{m/s})(\cos 30.0°) = \boxed{3.7\ \mathrm{N \cdot s}}$$

10. ***REASONING*** The height from which the basketball was dropped can be found by solving Equation 3.6b ($v_y^2 = v_{0y}^2 + 2a_y y$) for the displacement y. Since the ball was dropped from rest, $v_{0y} = 0$. From the definition of momentum (Equation 7.2), it follows that the speed v_y of the ball just before striking the floor is given by $v_y = p/m$.

SOLUTION Therefore, we have

$$y = \frac{v_y^2}{2a_y} = \frac{(p/m)^2}{2a_y} = \frac{\left[(3.1\ \text{kg}\cdot\text{m/s})\ /\ (0.60\ \text{kg})\right]^2}{2\ (9.80\ \text{m/s}^2)} = \boxed{1.4\ \text{m}}$$

11. ***REASONING AND SOLUTION*** In time t, a mass m of water strikes the blade. The impulse acting on the water is

$$\overline{F}\,t = -mv - (+mv)$$

$$\overline{F} = -2(m/t)v = -2(25.0\text{ kg/s})(18.0\text{ m/s}) = -9.00 \times 10^2\text{ N}$$

The magnitude of this force is $\boxed{9.00 \times 10^2\text{ N}}$.

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12. ***REASONING*** This is a problem in vector addition, and we will use the component method for vector addition. Using this method, we will add the components of the individual momenta in the direction due north to obtain the component of the vector sum in the direction due north. We will obtain the component of the vector sum in the direction due east in a similar fashion from the individual components in that direction. For each jogger the momentum is the mass times the velocity.

SOLUTION Assuming that the directions north and east are positive, the components of the joggers' momenta are as shown in the following table:

	Direction due east	Direction due north
85 kg jogger	$(85\text{ kg})(2.0\text{ m/s}) = 170\text{ kg}\cdot\text{m/s}$	0 kg•m/s
55 kg jogger	$(55\text{ kg})(3.0\text{ m/s})\cos 32° = 140\text{ kg}\cdot\text{m/s}$	$(55\text{ kg})(3.0\text{ m/s})\sin 32° = 87\text{ kg}\cdot\text{m/s}$
Total	310 kg•m/s	87 kg•m/s

Using the Pythagorean theorem, we find that the magnitude of the total momentum is

$$\sqrt{(310\text{ kg}\cdot\text{m/s})^2 + (87\text{ kg}\cdot\text{m/s})^2} = \boxed{322\text{ kg}\cdot\text{m/s}}$$

The total momentum vector points north of east by an angle θ, which is given by

$$\theta = \tan^{-1}\left(\frac{87\text{ kg}\cdot\text{m/s}}{310\text{ kg}\cdot\text{m/s}}\right) = \boxed{16°}$$

13. ***REASONING AND SOLUTION*** According to the impulse-momentum theorem (Equation 7.4)

$$\overline{F}\Delta t = m(v_f - v_0) \tag{1}$$

Conservation of mechanical energy can be used to relate the velocities to the heights. If the floor is used to define the zero level for heights, we have

$$mgh_0 = \tfrac{1}{2}mv_B^2$$

where h_0 is the height of the ball when it is dropped and v_B is the speed of the ball just before it strikes the ground. Solving for v_B gives

$$v_B = \sqrt{2gh_0} \tag{2}$$

Similarly,

$$mgh_f = \tfrac{1}{2}mv_A^2$$

where h_f is the maximum height of the ball when it rebounds and v_A is the speed of the ball just after it strikes the ground. Solving for v_A gives

$$v_A = \sqrt{2gh_f} \tag{3}$$

Substituting equations (2) and (3) into equation (1), where $-v_B = v_0$, and $v_A = v_f$ gives (taking "upward" as the positive direction)

$$\overline{F}\Delta t = m\sqrt{2g}\left[\sqrt{h_f} - (-\sqrt{h_0})\right]$$

$$= (0.500\text{ kg})\sqrt{2(9.80\text{ m/s}^2)}\left[\sqrt{0.700\text{ m}} - (-\sqrt{1.20\text{ m}})\right] = \boxed{+4.28\text{ N}\cdot\text{s}}$$

Since the impulse is positive, it is directed $\boxed{\text{upward}}$.

14. ***REASONING AND SOLUTION*** The excess weight of the truck is due to the force exerted on the truck by the sand. Newton's third law requires that this force be equal in magnitude to the force exerted on the sand by the truck. In time, t, a mass m of sand falls into the truck bed and comes to rest. The impulse is

$$\overline{F}t = mv \qquad \text{so} \qquad \overline{F} = \frac{mv}{t}$$

The sand gains a speed, v, in falling a height, h, so

$$v = \sqrt{2gh} = \sqrt{2\left(9.80\ \text{m/s}^2\right)\left(2.00\ \text{m}\right)} = 6.26\ \text{m/s}$$

$$\overline{F} = \left(55.0\ \text{kg/s}\right)\left(6.26\ \text{m/s}\right) = \boxed{344\ \text{N}}$$

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15. ***REASONING AND SOLUTION*** The momentum is zero before the beat. Conservation of momentum requires that it is also zero after the beat; thus

$$0 = m_p v_p + m_b v_b$$

so that

$$v_p = -(m_b/m_p)v_b = -(0.050\ \text{kg}/85\ \text{kg})(0.25\ \text{m/s}) = \boxed{-1.5 \times 10^{-4}\ \text{m/s}}$$

16. ***REASONING AND SOLUTION*** The momentum of the spaceship is transferred to the rocket, so $(m_s + m_r)v_s = m_r v_r$, and the rocket's velocity is

$$v_r = \frac{(m_s + m_r)v_s}{m_r} = \frac{(4.0 \times 10^6 \text{ kg} + 1300 \text{ kg})(230 \text{ m/s})}{1300 \text{ kg}} = \boxed{7.1 \times 10^5 \text{ m/s}}$$

17. SSM ***REASONING*** Batman and the boat with the criminal constitute the system. Gravity acts on this system as an external force; however, gravity acts vertically, and we are concerned only with the horizontal motion of the system. If we neglect air resistance and friction, there are no external forces that act horizontally; therefore, the total linear momentum in the horizontal direction is conserved. When Batman collides with the boat, the horizontal component of his velocity is zero, so the statement of conservation of linear momentum in the horizontal direction can be written as

$$\underbrace{(m_1 + m_2)v_f}_{\substack{\text{Total horizontal momentum}\\ \text{after collision}}} = \underbrace{m_1 v_{01} + 0}_{\substack{\text{Total horizontal momentum}\\ \text{before collision}}}$$

Here, m_1 is the mass of the boat, and m_2 is the mass of Batman. This expression can be solved for v_f, the velocity of the boat after Batman lands in it.

SOLUTION Solving for v_f gives

$$v_f = \frac{m_1 v_{01}}{m_1 + m_2} = \frac{(510\text{ kg})(+11\text{ m/s})}{510\text{ kg} + 91\text{ kg}} = \boxed{+9.3\text{ m/s}}$$

The plus sign indicates that the boat continues to move in its initial direction of motion; it does not recoil.

18. ***REASONING*** Let m be Al's mass, which means that Jo's mass is 168 kg – m. Since friction is negligible and since the downward-acting weight of each person is balanced by the upward-acting normal force from the ice, the net external force acting on the two-person system is zero. Therefore, the system is isolated, and the conservation of linear momentum applies. The initial total momentum must be equal to the final total momentum.

SOLUTION Applying the principle of conservation of linear momentum and assuming that the direction in which Al moves is the positive direction, we find

$$\underbrace{m(0\ \text{m/s})+(168\ \text{kg}-m)(0\ \text{m/s})}_{\text{Initial total momentum}}=\underbrace{m(0.90\ \text{m/s})+(168\ \text{kg}-m)(-1.2\ \text{m/s})}_{\text{Final total momentum}}$$

Solving this equation for m and suppressing the units in the interests of clarity, we find

$$0=m(0.90)-(168)(1.2)+m(1.2)$$

$$m=\frac{(168)(1.2)}{0.90+1.2}=\boxed{96\ \text{kg}}$$

19. ***REASONING*** For the system consisting of the female character, the gun and the bullet, the sum of the external forces is zero, because the weight of each object is balanced by a corresponding upward (normal) force, and we are ignoring friction. The female character, the gun and the bullet, then, constitute an isolated system, and the principle of conservation of linear momentum applies.

SOLUTION a. The total momentum of the system before the gun is fired is zero, since all parts of the system are at rest. Momentum conservation requires that the total momentum remains zero after the gun has been fired.

$$\underbrace{m_1 v_{f1} + m_2 v_{f2}}_{\substack{\text{Total momentum after}\\ \text{gun is fired}}} = \underbrace{0}_{\substack{\text{Total momentum before}\\ \text{gun is fired}}}$$

where the subscripts 1 and 2 refer to the woman (plus gun) and the bullet, respectively. Solving for v_{f1}, the recoil velocity of the woman (plus gun), gives

$$v_{f1} = -\frac{m_2 v_{f2}}{m_1} = \frac{-(0.010\text{ kg})(720\text{ m/s})}{51\text{ kg}} = \boxed{-0.14\text{ m/s}}$$

b. Repeating the calculation for the situation in which the woman shoots a blank cartridge, we have

$$v_{f1} = -\frac{m_2 v_{f2}}{m_1} = \frac{-(5.0\times10^{-4}\text{ kg})(720\text{ m/s})}{51\text{ kg}} = \boxed{-7.1\times10^{-3}\text{ m/s}}$$

In both cases, the minus sign means that the bullet and the woman move in opposite directions when the gun is fired. The total momentum of the system remains zero, because momentum is a vector quantity, and the momenta of the bullet and the woman have equal magnitudes, but opposite directions.

20. ***REASONING AND SOLUTION*** First, find the initial speed of Adolf by using the conservation of mechanical energy, $m_A g h_A = \frac{1}{2} m_A v_A^2$.

$$v_A = \sqrt{2gh_A} = \sqrt{2(9.80\ \text{m/s}^2)(0.65\ \text{m})} = 3.6\ \text{m/s}$$

Now use the conservation of momentum $\left(m_E v_E = -m_A v_A\right)$ to find the velocity of Ed, so that

$$v_E = -\frac{m_A}{m_E} v_A = -\frac{120\ \text{kg}}{78\ \text{kg}}(3.6\ \text{m/s}) = -5.5\ \text{m/s}$$

The conservation of mechanical energy allows us to find the height reached by Ed,

$$h_E = \frac{v_E^2}{2g} = \frac{\left(-5.5\ \text{m/s}\right)^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{1.5\ \text{m}}$$

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21. **SSM** ***REASONING AND SOLUTION*** The comet piece and Jupiter constitute an isolated system, since no external forces act on them. Therefore, the head-on collision obeys the conservation of linear momentum:

$$\underbrace{(m_{\text{comet}} + m_{\text{Jupiter}})v_{\text{f}}}_{\substack{\text{Total momentum}\\ \text{after collision}}} = \underbrace{m_{\text{comet}}v_{\text{comet}} + m_{\text{Jupiter}}v_{\text{Jupiter}}}_{\substack{\text{Total momentum}\\ \text{before collision}}}$$

where v_{f} is the common velocity of the comet piece and Jupiter after the collision. We assume initially that Jupiter is moving in the $+x$ direction so $v_{\text{Jupiter}} = +1.3\times10^{4}\ \text{m/s}$. The comet piece must be moving in the opposite direction so $v_{\text{comet}} = -6.0\times10^{4}\ \text{m/s}$. The final velocity v_{f} of Jupiter and the comet piece can be written as $v_{\text{f}} = v_{\text{Jupiter}} + \Delta v$ where Δv is the change in velocity of Jupiter due to the collision. Substituting this expression into the conservation of momentum equation gives

$$(m_{\text{comet}} + m_{\text{Jupiter}})(v_{\text{Jupiter}} + \Delta v) = m_{\text{comet}}v_{\text{comet}} + m_{\text{Jupiter}}v_{\text{Jupiter}}$$

Multiplying out the left side of this equation, algebraically canceling the term $m_{\text{Jupiter}}v_{\text{Jupiter}}$ from both sides of the equation, and solving for Δv yields

$$\Delta v = \frac{m_{\text{comet}}(v_{\text{comet}} - v_{\text{Jupiter}})}{m_{\text{comet}} + m_{\text{Jupiter}}}$$

$$= \frac{(4.0\times10^{12}\ \text{kg})(-6.0\times10^{4}\ \text{m/s} - 1.3\times10^{4}\ \text{m/s})}{4.0\times10^{12}\ \text{kg} + 1.9\times10^{27}\ \text{kg}} = -1.5\times10^{-10}\ \text{m/s}$$

The change in Jupiter's speed is $\boxed{1.5\times10^{-10}\ \text{m/s}}$.

22. ***REASONING AND SOLUTION*** Momentum is conserved during the breakup. Applying the conservation of momentum along the original line of motion gives

$$mv_1 \cos 30.0° + mv_2 \cos 60.0° = 2mv_o \quad \text{or} \quad 0.866v_1 + 0.500v_2 = 1.00 \times 10^2$$

A similar application along a line perpendicular to the original direction of motion gives

$$mv_1 \sin 30.0° - mv_2 \sin 60.0° = 0 \quad \text{or} \quad 0.500v_1 - 0.866v_2 = 0 \quad \text{or} \quad v_1 = 1.73v_2.$$

Substitution of the first equation into the second yields the following results

a. The magnitude of the velocity of the first piece is $\boxed{v_1 = 86.5 \text{ m/s}}$.

b. Similarly, the magnitude of the velocity of the second piece is $\boxed{v_2 = 50.0 \text{ m/s}}$.

23. SSM ***REASONING*** We will define the system to be the platform, the two people and the ball. Since the ball travels nearly horizontally, the effects of gravity are negligible. Momentum is conserved. Since the initial momentum of the system is zero, it must remain zero as the ball is thrown and caught. While the ball is in motion, the platform will recoil in such a way that the total momentum of the system remains zero. As the ball is caught, the system must come to rest so that the total momentum remains zero. The distance that the platform moves before coming to rest again can be determined by using the expressions for momentum conservation and the kinematic description for this situation.

SOLUTION While the ball is in motion, we have

$$MV + mv = 0 \tag{1}$$

where M is the combined mass of the platform and the two people, V is the recoil velocity of the platform, m is the mass of the ball, and v is the velocity of the ball.

The distance that the platform moves is given by

$$x = Vt \tag{2}$$

where t is the time that the ball is in the air. The time that the ball is in the air is given by

$$t = \frac{L}{v - V} \tag{3}$$

where L is the length of the platform, and the quantity $(v - V)$ is the velocity of the ball relative to the platform. Remember, both the ball and the platform are moving while the ball is in the air. Combining equations (2) and (3) gives

$$x = \left(\frac{V}{v - V}\right) L \tag{4}$$

From equation (1) the ratio of the velocities is $V/v = -(m/M)$. Equation (4) then gives

$$x = -\frac{(V/v)L}{1 - (V/v)} = \frac{(-m/M)L}{1 + (m/M)} = -\frac{mL}{M + m} = -\frac{(6.0\text{ kg})(2.0\text{ m})}{118\text{ kg} + 6.0\text{ kg}} = -0.097\text{ m}$$

The minus sign indicates that displacement of the platform is in the opposite direction to the displacement of the ball. The distance moved by the platform is the magnitude of this displacement, or $\boxed{0.097\text{ m}}$.

24. ***REASONING AND SOLUTION*** Since no net external force acts in the horizontal direction, the total horizontal momentum of the system is conserved regardless of which direction the mass is thrown. The momentum of the system before the mass is thrown off the wagon is $m_W v_A$, where m_W and v_A are the mass and velocity of the wagon, respectively. When ten percent of the wagon's mass is thrown forward, the wagon is brought to a halt so its final momentum is zero. The momentum of the mass thrown forward is $0.1\, m_W(v_A + v_M)$, where v_M is the velocity of the mass relative to the wagon and $(v_A + v_M)$ is the velocity of the mass relative to the ground. The conservation of momentum gives

$$\underbrace{0.1 m_W \left(v_A + v_M\right)}_{\substack{\text{Momentum after mass}\\ \text{is thrown forward}}} = \underbrace{m_W v_A}_{\substack{\text{Momentum before mass}\\ \text{is thrown forward}}}$$

Solving for v_M yields

$$v_M = 9 v_A \tag{1}$$

When the direction in which the mass is thrown is reversed, the velocity of the mass relative to the ground is now $(v_A - v_M)$. The momentum of the mass and wagon is, therefore,

$$\underbrace{0.1 m_W \left(v_A - v_M\right)}_{\text{Momentum of the mass}} + \underbrace{0.9 m_W v_B}_{\text{Momentum of the wagon}}$$

where v_B is the velocity of the wagon. The conservation of momentum gives

$$\underbrace{0.1 m_W \left(v_A - v_M\right) + 0.9 m_W v_B}_{\substack{\text{Momentum after}\\ \text{mass is thrown off}}} = \underbrace{m_W v_A}_{\substack{\text{Momentum before}\\ \text{mass is thrown off}}} \tag{2}$$

Substituting equation (1) into equation (2) and solving for v_B/v_A gives $\boxed{\dfrac{v_B}{v_A} = 2}$

25. ***REASONING AND SOLUTION*** The collision is an inelastic one, with the total linear momentum being conserved.

$$m_1 v_1 = (m_1 + m_2)V$$

The mass m_2 of the receiver is

$$m_2 = \frac{m_1 v_1}{V} - m_1 = \frac{(115\text{ kg})(4.5\text{ m/s})}{2.6\text{ m/s}} - 115\text{ kg} = \boxed{84\text{ kg}}$$

26. ***REASONING*** Since friction is negligible and since the downward-acting weight of each person is balanced by the upward-acting normal force from the sidewalk, the net external force acting on the two-person system is zero. Therefore, the system is isolated, and the conservation of linear momentum applies. The initial total momentum must be equal to the final total momentum.

SOLUTION Applying the principle of conservation of linear momentum and assuming that the direction in which Kevin is moving is the positive direction, we find

$$\underbrace{(87\ \text{kg})v_{\text{Kevin}} + (22\ \text{kg})(0\ \text{m/s})}_{\text{Initial total momentum}} = \underbrace{(87\ \text{kg} + 22\ \text{kg})(2.4\ \text{m/s})}_{\text{Final total momentum}}$$

$$v_{\text{Kevin}} = \frac{(87\ \text{kg} + 22\ \text{kg})(2.4\ \text{m/s})}{87\ \text{kg}} = \boxed{3.0\ \text{m/s}}$$

27. SSM ***REASONING*** Since all of the collisions are elastic, the total mechanical energy of the ball is conserved. However, since gravity affects its vertical motion, its linear momentum is *not* conserved. If h_f is the maximum height of the ball on its final bounce, conservation of energy gives

$$\underbrace{\frac{1}{2}mv_f^2 + mgh_f}_{E_f} = \underbrace{\frac{1}{2}mv_0^2 + mgh_0}_{E_0}$$

$$h_f = \frac{v_0^2 - v_f^2}{2g} + h_0$$

SOLUTION In order to use this expression, we must obtain the values for the velocities v_0 and v_f. The initial velocity has only a horizontal component, $v_0 = v_{0x}$. The final velocity also has only a horizontal component since the ball is at the top of its trajectory, $v_f = v_{fx}$. No forces act in the horizontal direction so the momentum of the ball in this direction is conserved, hence $v_0 = v_f$. Therefore

$$h_f = h_0 = \boxed{3.00\text{ m}}$$

28. ***REASONING AND SOLUTION***

a. The total momentum of the system is conserved so $(m_B + m_W)v_f = m_B v_{0B}$ or

$$v_f = \frac{m_B\, v_{0B}}{m_B + m_W} = \frac{(8.00 \times 10^{-3}\ \text{kg})(8.00 \times 10^{2}\ \text{m/s})}{8.00 \times 10^{-3}\ \text{kg} + 3.00\ \text{kg}} = \boxed{2.13\ \text{m/s, upward}}$$

b. Mechanical energy is conserved as the bullet-block system rises, so $mgh = (1/2)mv_f^2$. The system rises to a height

$$h = v_f^2/(2g) = (2.13\ \text{m/s})^2/(19.6\ \text{m/s}^2) = \boxed{0.231\ \text{m}}$$

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29. ***REASONING*** The velocity of the second ball just after the collision can be found from Equation 7.8b (see Example 7). In order to use Equation 7.8b, however, we must know the velocity of the first ball just before it strikes the second ball. Since we know the impulse delivered to the first ball by the pool stick, we can use the impulse-momentum theorem (Equation 7.4) to find the velocity of the first ball just before the collision.

SOLUTION According to the impulse-momentum theorem, $\overline{F}\Delta t = mv_{\mathrm{f}} - mv_0$, and setting $v_0 = 0$ and solving for v_{f}, we find that the velocity of the first ball after it is struck by the pool stick and just before it hits the second ball is

$$v_{\mathrm{f}} = \frac{\overline{F}\Delta t}{m} = \frac{+1.50\ \mathrm{N \cdot s}}{0.165\ \mathrm{kg}} = 9.09\ \mathrm{m/s}$$

Substituting values into Equation 7.8b (with $v_{01} = 9.09$ m/s), we have

$$v_{\mathrm{f2}} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{01} = \left(\frac{2m}{m + m}\right) v_{01} = v_{01} = \boxed{+9.09\ \mathrm{m/s}}$$

30. ***REASONING AND SOLUTION*** The conservation of momentum gives

$$m_b v_b + m_p v_p = m_p v_0$$

Therefore,

$$v_p = v_0 - \frac{m_b}{m_p} v_b = 715 \text{ m/s} - \frac{2.00 \text{ kg}}{0.150 \text{ kg}}(40.0 \text{ m/s}) = \boxed{182 \text{ m/s}}$$

31. SSM WWW ***REASONING*** The system consists of the two balls. The total linear momentum of the two-ball system is conserved because the net external force acting on it is zero. The principle of conservation of linear momentum applies whether or not the collision is elastic.

$$\underbrace{m_1 v_{f1} + m_2 v_{f2}}_{\substack{\text{Total momentum}\\ \text{after collision}}} = \underbrace{m_1 v_{01} + 0}_{\substack{\text{Total momentum}\\ \text{before collision}}}$$

When the collision is elastic, the kinetic energy is also conserved during the collision

$$\underbrace{\tfrac{1}{2} m_1 v_{f1}^2 + \tfrac{1}{2} m_2 v_{f2}^2}_{\substack{\text{Total kinetic energy}\\ \text{after collision}}} = \underbrace{\tfrac{1}{2} m_1 v_{01}^2 + 0}_{\substack{\text{Total kinetic energy}\\ \text{before collision}}}$$

SOLUTION

a. The final velocities for an elastic collision are determined by simultaneously solving the above equations for the final velocities. The procedure is discussed in Example 7 in the text, and leads to Equations 7.8a and 7.8b. According to Equation 7.8:

$$v_{f1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{01} \qquad \text{and} \qquad v_{f2} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{01}$$

Let the initial direction of motion of the 5.00-kg ball define the positive direction. Substituting the values given in the text, these equations give

$$[5.00\text{-kg ball}] \qquad v_{f1} = \left(\frac{5.00\text{ kg} - 7.50\text{ kg}}{5.00\text{ kg} + 7.50\text{ kg}}\right)(2.00\text{ m/s}) = \boxed{-0.400\text{ m/s}}$$

$$[7.50\text{-kg ball}] \qquad v_{f2} = \left(\frac{2(5.00\text{ kg})}{5.00\text{ kg} + 7.50\text{ kg}}\right)(2.00\text{ m/s}) = \boxed{+1.60\text{ m/s}}$$

The signs indicate that, after the collision, the 5.00-kg ball reverses its direction of motion, while the 7.50-kg ball moves in the direction in which the 5.00-kg ball was initially moving.

b. When the collision is completely inelastic, the balls stick together, giving a composite body of mass $m_1 + m_2$ which moves with a velocity v_f. The statement of conservation of linear momentum then becomes

$$\underbrace{(m_1 + m_2) v_f}_{\substack{\text{Total momentum}\\ \text{after collision}}} = \underbrace{m_1 v_{01} + 0}_{\substack{\text{Total momentum}\\ \text{before collision}}}$$

The final velocity of the two balls after the collision is, therefore,

$$v_f = \frac{m_1 v_{01}}{m_1 + m_2} = \frac{(5.00\text{ kg})(2.00\text{ m/s})}{5.00\text{ kg} + 7.50\text{ kg}} = \boxed{+0.800\text{ m/s}}$$

32. ***REASONING*** The net external force acting on the two-puck system is zero (the weight of each ball is balanced by an upward normal force, and we are ignoring friction due to the layer of air on the hockey table). Therefore, the two pucks constitute an isolated system, and the principle of conservation of linear momentum applies.

SOLUTION Conservation of linear momentum requires that the total momentum is the same before and after the collision. Since linear momentum is a vector, the x and y components must be conserved separately. Using the drawing in the text, momentum conservation in the x direction yields

$$m_A v_{0A} = m_A v_{fA}(\cos\ 65°) + m_B v_{fB}(\cos\ 37°) \tag{1}$$

while momentum conservation in the y direction yields

$$0 = m_A v_{fA}(\sin\ 65°) - m_B v_{fB}(\sin\ 37°) \tag{2}$$

Solving equation (2) for v_{fB}, we find that

$$v_{fB} = \frac{m_A v_{fA}(\sin 65°)}{m_B(\sin 37°)} \tag{3}$$

Substituting equation (3) into Equation (1) leads to

$$m_A v_{0A} = m_A v_{fA}(\cos\ 65°) + \left(\frac{m_A v_{fA}(\sin 65°)}{\sin 37°}\right)(\cos\ 37°)$$

a. Solving for v_{fA} gives

$$v_{fA} = \frac{v_{0A}}{\cos 65° + \left(\dfrac{\sin 65°}{\tan 37°}\right)} = \frac{+5.5\text{ m/s}}{\cos 65° + \left(\dfrac{\sin 65°}{\tan 37°}\right)} = \boxed{3.4\text{ m/s}}$$

b. From equation (3), we find that

$$v_{fB} = \frac{(0.025\text{ kg})\ (3.4\text{ m/s})\ (\sin 65°)}{(0.050\text{ kg})\ (\sin 37°)} = \boxed{2.6\text{ m/s}}$$

33. ***REASONING AND SOLUTION*** Define the system to be the bullet and the wooden block. The physics of this problem can be divided into two parts. First, there is the completely inelastic collision of the bullet and the block. Second, there is the subsequent motion of the system as it moves vertically upward.

In this problem, the external force of gravity acts on the system during the collision. The force of gravity will change the momentum of the system by a negligibly small amount since the collision occurs over an extremely short time interval. Thus, to a very good approximation, we can say that the sum of the external forces acting on the system is negligible and the momentum of the system is conserved during the collision.

Let M and V represent the mass and the velocity of the block, respectively, just before the collision. Let m and v represent the mass and the velocity of the bullet, respectively, just before the collision. Let v_A represent the velocity of the combined mass of the system just after the collision. Then momentum conservation gives

$$MV + mv = (M + m)\, v_A \qquad (1)$$

As a result of the collision, the block (with the bullet in it) rises and comes to a momentary halt at the top of the building. This is the exact reverse of the motion of the block in falling. Therefore, the velocity v_A must have the same magnitude as V, but the opposite direction. In other words, $v_A = -V$, so that equation (1) becomes:

$$MV + mv = (M + m)(-V) \qquad (2)$$

Rearranging equation (2) gives

$$mv = -(2M + m)V \qquad (3)$$

But the block acquires the velocity V by falling from rest for a time t. According to the equations of kinematics, we have V = at (Equation 2.4), where $a = -9.80\ \text{m/s}^2$. Using this expression for V in equation (3), we find that

$$t = \frac{mv}{-(2M + m)a} = \frac{(0.010\ \text{kg})(750\ \text{m/s})}{-[2(2.0\ \text{kg}) + 0.010\ \text{kg}](-9.80\ \text{m/s}^2)} = \boxed{0.19\ \text{s}}$$

34. ***REASONING***

We assume that the collision time is so short that the effect of gravity during the collision can be ignored. Since no other external forces act on the three-bullet system, the principle of conservation of linear momentum applies. The drawing shows the set-up.

SOLUTION

Applying the principle to the x components of the momentum, we have

$$\left(4.50\times10^{-3}\ \text{kg}\right)v_1\cos30.0°+\left(4.50\times10^{-3}\ \text{kg}\right)\left(-v_2\cos30.0°\right)=0$$

$$v_1-v_2=0 \quad \text{or} \quad v_1=v_2$$

Applying the principle to the y components of the momentum, we have

$$\left(4.50\times10^{-3}\ \text{kg}\right)v_1\sin30.0°+\left(4.50\times10^{-3}\ \text{kg}\right)\left(v_2\sin30.0°\right)$$
$$+\left(2.50\times10^{-3}\ \text{kg}\right)\left(-575\ \text{m/s}\right)=0$$

Since $v_1 = v_2 = v$, this result simplifies and can be solved for the speed as follows:

$$\left(4.50\times10^{-3}\ \text{kg}\right)v\sin30.0°+\left(4.50\times10^{-3}\ \text{kg}\right)\left(v\sin30.0°\right)+\left(2.50\times10^{-3}\ \text{kg}\right)\left(-575\ \text{m/s}\right)=0$$

$$2\left(4.50\times10^{-3}\ \text{kg}\right)v\sin30.0°+\left(2.50\times10^{-3}\ \text{kg}\right)\left(-575\ \text{m/s}\right)=0$$

$$v=\frac{\left(2.50\times10^{-3}\ \text{kg}\right)\left(575\ \text{m/s}\right)}{2\left(4.50\times10^{-3}\ \text{kg}\right)\sin30.0°}=\boxed{319\ \text{m/s}}$$

35. SSM ***REASONING*** The two skaters constitute the system. Since the net external force acting on the system is zero, the total linear momentum of the system is conserved. In the x direction (the east/west direction), conservation of linear momentum gives $P_{fx} = P_{0x}$, or

$$(m_1 + m_2)v_f \cos\theta = m_1 v_{01}$$

Note that since the skaters hold onto each other, they move away with a common velocity v_f. In the y direction, $P_{fy} = P_{0y}$, or

$$(m_1 + m_2)v_f \sin\theta = m_2 v_{02}$$

These equations can be solved simultaneously to obtain both the angle θ and the velocity v_f.

SOLUTION

a. Division of the equations above gives

$$\theta = \tan^{-1}\left(\frac{m_2 v_{02}}{m_1 v_{01}}\right) = \tan^{-1}\left[\frac{(70.0\text{ kg})(7.00\text{ m/s})}{(50.0\text{ kg})(3.00\text{ m/s})}\right] = \boxed{73.0°}$$

b. Solution of the first of the momentum equations gives

$$v_f = \frac{m_1 v_{01}}{(m_1 + m_2)\cos\theta} = \frac{(50.0\text{ kg})(3.00\text{ m/s})}{(50.0\text{ kg} + 70.0\text{ kg})(\cos 73.0°)} = \boxed{4.28\text{ m/s}}$$

36. ***REASONING AND SOLUTION*** Momentum is conserved in the horizontal direction during the "collision". Let the coal be object 1 and the car be object 2. Then

$$(m_1 + m_2)v_f = m_1 v_1 \cos 25.0° + m_2 v_2$$

$$v_f = \frac{m_1 v_1 \cos 25.0° + m_2 v_2}{m_1 + m_2} = \frac{(150\text{ kg})(0.80\text{ m/s})\cos 25.0° + (440\text{ kg})(0.50\text{ m/s})}{150\text{ kg} + 440\text{ kg}} = \boxed{0.56\text{ m/s}}$$

The direction of the final velocity is $\boxed{\text{to the right}}$.

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37. ***REASONING AND SOLUTION***

a. The total momentum of the sled and person is conserved since no external force acts in the horizontal direction. $(m_p + m_s)v_f = m_p v_{op}$ gives

$$v_f = \frac{m_p\, v_{op}}{m_p + m_s} = \frac{(60.0\text{ kg})(3.80\text{ m/s})}{60.0\text{ kg} + 12.0\text{ kg}} = \boxed{3.17\text{ m/s}}$$

The direction of v_f is the same as the direction of v_{op}.

b. The frictional force acting on the sled is $f_k = -\mu_k F_N = -\mu_k mg$. Newton's second law then gives the acceleration to be

$$a = f_k/m = -\mu_k g\ .$$

The acceleration is constant so $a = -v_f^2/(2x)$ from the equations of kinematics. Equating these two expressions and solving yields

$$\mu_k = \frac{v_f^2}{2gx} = \frac{(3.17\text{ m/s})^2}{2(9.80\text{ m/s}^2)(30.0\text{ m})} = \boxed{0.0171}$$

38. ***REASONING AND SOLUTION*** Initially, the ball has a total mechanical energy given by $E_o = mgh_o$. After one bounce it reaches the top of its trajectory with an energy of

$$E_1 = 0.900\ E_o = 0.900\ mgh_o$$

After two bounces it has energy

$$E_2 = (0.900)^2 mgh_o$$

After N bounces it has a remaining energy of

$$E_N = (0.900)^N mgh_o$$

In order to just reach the sill the ball must have $E_N = mgh$ where h = 2.44 m. Hence,

$$(0.900)^N mgh_o = mgh \quad \text{or} \quad (0.900)^N = h/h_o$$

Taking the log of both sides gives

$$N \log(0.900) = \log(h/h_o).$$

Then

$$N = \frac{\log\left(\dfrac{2.44\ \text{m}}{6.10\ \text{m}}\right)}{\log(0.900)} = 8.7$$

The ball can make $\boxed{\text{8 bounces}}$ and still reach the sill.

39. SSM ***REASONING*** The two balls constitute the system. The tension in the wire is the only nonconservative force that acts on the ball. The tension does no work since it is perpendicular to the displacement of the ball. Since $W_{nc} = 0$, the principle of conservation of mechanical energy holds and can be used to find the speed of the 1.50-kg ball just before the collision. Momentum is conserved during the collision, so the principle of conservation of momentum can be used to find the velocities of both balls just after the collision. Once the collision has occurred, energy conservation can be used to determine how high each ball rises.

SOLUTION

a. Applying the principle of energy conservation to the 1.50-kg ball, we have

$$\underbrace{\tfrac{1}{2}mv_f^2 + mgh_f}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + mgh_0}_{E_0}$$

If we measure the heights from the lowest point in the swing, $h_f = 0$, and the expression above simplifies to

$$\tfrac{1}{2}mv_f^2 = \tfrac{1}{2}mv_0^2 + mgh_0$$

Solving for v_f, we have

$$v_f = \sqrt{v_0^2 + 2gh_0} = \sqrt{(5.00\text{ m/s})^2 + 2(9.80\text{ m/s}^2)(0.300\text{ m})} = \boxed{5.56\text{ m/s}}$$

b. If we assume that the collision is elastic, then the velocities of both balls just after the collision can be obtained from Equations 7.8a and 7.8b:

$$v_{f1} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)v_{01} \qquad \text{and} \qquad v_{f2} = \left(\frac{2m_1}{m_1 + m_2}\right)v_{01}$$

Since v_{01} corresponds to the speed of the 1.50-kg ball just before the collision, it is equal to the quantity v_f calculated in part (a). With the given values of $m_1 = 1.50\text{ kg}$ and $m_2 = 4.60\text{ kg}$, and the value of $v_{01} = 5.56\text{ m/s}$ obtained in part (a), Equations 7.8a and 7.8b yield the following values:

$$\boxed{v_{f1} = -2.83\text{ m/s}} \qquad \text{and} \qquad \boxed{v_{f2} = +2.73\text{ m/s}}$$

The minus sign in v_{f1} indicates that the first ball reverses its direction as a result of the collision.

c. If we apply the conservation of mechanical energy to either ball after the collision we have

$$\underbrace{\tfrac{1}{2}mv_f^2 + mgh_f}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + mgh_0}_{E_0}$$

where v_0 is the speed of the ball just after the collision, and h_f is the final height to which the ball rises. For either ball, $h_0 = 0$, and when either ball has reached its maximum height, $v_f = 0$. Therefore, the expression of energy conservation reduces to

$$gh_{\mathrm{f}} = \tfrac{1}{2}v_0^2 \qquad \text{or} \qquad h_{\mathrm{f}} = \frac{v_0^2}{2g}$$

Thus, the heights to which each ball rises after the collision are

$$[1.50\text{-kg ball}] \qquad h_{\mathrm{f}} = \frac{v_0^2}{2g} = \frac{(2.83\ \mathrm{m/s})^2}{2(9.80\ \mathrm{m/s^2})} = \boxed{0.409\ \mathrm{m}}$$

$$[4.60\text{-kg ball}] \qquad h_{\mathrm{f}} = \frac{v_0^2}{2g} = \frac{(2.73\ \mathrm{m/s})^2}{2(9.80\ \mathrm{m/s^2})} = \boxed{0.380\ \mathrm{m}}$$

40. ***REASONING AND SOLUTION*** During the elastic collision both the total kinetic energy and the total momentum of the balls are conserved. The conservation of momentum gives

$$mv_{f1} + mv_{f2} = mv_{01} + mv_{02}$$

where ball 1 has $\mathbf{v}_{01} = +7.0$ m/s and ball 2 has $\mathbf{v}_{02} = -4.0$ m/s. Hence,

$$v_{f1} + v_{f2} = v_{01} + v_{02} = 3.0 \text{ m/s}.$$

The conservation of kinetic energy gives

$$(1/2)mv_{f1}^2 + (1/2)mv_{f2}^2 = (1/2)mv_{01}^2 + (1/2)mv_{02}^2 \text{ or}$$
$$v_{f1}^2 + v_{f2}^2 = v_{01}^2 + v_{02}^2 = 65 \text{ m}^2/\text{s}^2.$$

Solving the first equation for v_{f1} and substituting into the second gives

$$2v_{f2}^2 - (6.0 \text{ m/s})v_{f2} - 56 \text{ m}^2/\text{s}^2 = 0.$$

The quadratic formula yields two solutions, $v_{f2} = 7.0$ m/s and $v_{f2} = -4.0$ m/s.

The first equation now gives $v_{f1} = -4.0$ m/s and $v_{f1} = +7.0$ m/s.

Hence, the 7.0 m/s ball has a final velocity of $\boxed{-4.0 \text{ m/s}}$, opposite its original direction.

The 4.0 m/s ball has a final velocity of $\boxed{+7.0 \text{ m/s}}$, opposite its original direction.

41. SSM ***REASONING AND SOLUTION*** The location of the center of mass for a two-body system is given by Equation 7.10:

$$x_{\text{cm}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$$

where the subscripts "1" and "2" refer to the earth and the moon, respectively. For convenience, we will let the center of the earth be coincident with the origin so that $x_1 = 0$ and $x_2 = d$, the center-to-center distance between the earth and the moon. Direct calculation then gives

$$x_{\text{cm}} = \frac{m_2 d}{m_1 + m_2} = \frac{(7.35 \times 10^{22}\ \text{kg})(3.85 \times 10^{8}\ \text{m})}{5.98 \times 10^{24}\ \text{kg} + 7.35 \times 10^{22}\ \text{kg}} = \boxed{4.67 \times 10^{6}\ \text{m}}$$

42. ***REASONING AND SOLUTION*** The velocity of the center of mass is given by Equation 7.11,

$$v_{\text{cm}} = \frac{m_1 v_1 + m_2 v_2}{m_1 + m_2} = \frac{(m_1 / m_2) v_1 + v_2}{(m_1 / m_2) + 1}$$

a. When the masses are equal, $m_1 / m_2 = 1$, and we have

$$v_{\text{cm}} = \frac{v_1 + v_2}{2} = \frac{9.70 \text{ m/s} + (-11.8 \text{ m/s})}{2} = \boxed{-1.05 \text{ m/s}}$$

b. When the mass of the ball moving at 9.70 m/s is twice the mass of the other ball, we have $m_1/m_2 = 2$, and the velocity of the center of mass is

$$v_{\text{cm}} = \frac{2v_1 + v_2}{2 + 1} = \frac{2(9.70 \text{ m/s}) + (-11.8 \text{ m/s})}{3} = \boxed{+2.53 \text{ m/s}}$$

43. ***REASONING AND SOLUTION*** Equation 7.10 gives the center of mass of this two-atom system as

$$x_{\text{cm}} = \frac{m_{\text{c}}x_{\text{c}} + m_{\text{o}}x_{\text{o}}}{m_{\text{c}} + m_{\text{o}}}$$

If we take the origin at the center of the carbon atom, then $x_{\text{c}} = 0$, and we have

$$x_{\text{cm}} = \frac{m_{\text{o}}x_{\text{o}}}{m_{\text{c}} + m_{\text{o}}} = \frac{x_{\text{o}}}{(m_{\text{c}}/m_{\text{o}}) + 1} = \frac{1.13 \times 10^{-10}\ \text{m}}{(0.750 m_{\text{o}}/m_{\text{o}})\ +\ 1} = \boxed{6.46 \times 10^{-11}\ \text{m}}$$

44. ***REASONING*** The location of the center of mass is given by Equation 7.10. This equation contains the two unknown masses. To solve for the masses, we need an additional equation, which we can obtain from the fact that the total mass is given as 10.0 g.

SOLUTION Using Equation 7.10, and suppressing the units in the interests of clarity, we have

$$x_{\text{cm}} = \frac{m_{\text{A}} x_{\text{A}} + m_{\text{B}} x_{\text{B}}}{m_{\text{A}} + m_{\text{B}}} \quad \text{or} \quad 3.4 = \frac{m_{\text{A}}(2.0) + m_{\text{B}}(4.0)}{m_{\text{A}} + m_{\text{B}}}$$

We know also that the two particles have a total mass of 10.0 g, so that $m_{\text{A}} + m_{\text{B}} = 10.0$, where we again have suppressed the units. Solving this equation for m_{A} and substituting into the center-of-mass expression gives

$$3.4 = \frac{(10.0 - m_{\text{B}})(2.0) + m_{\text{B}}(4.0)}{10.0} \quad \text{or} \quad 3.4(10.0) = (10.0 - m_{\text{B}})(2.0) + m_{\text{B}}(4.0)$$

$$34 = 20 - m_{\text{B}}(2.0) + m_{\text{B}}(4.0) \quad \text{or} \quad 14 = m_{\text{B}}(2.0)$$

$$m_{\text{B}} = \frac{14}{2.0} = \boxed{7.0\text{ g}} \quad \text{and} \quad m_{\text{A}} = 10.0 - m_{\text{B}} = 10.0 - 7.0 = \boxed{3.0\text{ g}}$$

45. SSM WWW ***REASONING*** The system consists of the lumberjack and the log. For this system, the sum of the external forces is zero. This is because the weight of the system is balanced by the corresponding normal force (provided by the buoyant force of the water) and the water is assumed to be frictionless. The lumberjack and the log, then, constitute an isolated system, and the principle of conservation of linear momentum holds.

SOLUTION

a. The total linear momentum of the system before the lumberjack begins to move is zero, since all parts of the system are at rest. Momentum conservation requires that the total momentum remains zero during the motion of the lumberjack.

$$\underbrace{m_1 v_{f1} + m_2 v_{f2}}_{\substack{\text{Total momentum}\\ \text{just before the jump}}} = \underbrace{0}_{\text{Initial momentum}}$$

Here the subscripts "1" and "2" refer to the first log and lumberjack, respectively. Let the direction of motion of the lumberjack be the positive direction. Then, solving for v_{1f} gives

$$v_{f1} = -\frac{m_2 v_{f2}}{m_1} = -\frac{(98\text{ kg})(+3.6\text{ m/s})}{230\text{ kg}} = \boxed{-1.5\text{ m/s}}$$

The minus sign indicates that the first log recoils as the lumberjack jumps off.

b. Now the system is composed of the lumberjack, just before he lands on the second log, and the second log. Gravity acts on the system, but for the short time under consideration while the lumberjack lands, the effects of gravity in changing the linear momentum of the system are negligible. Therefore, to a very good approximation, we can say that the linear momentum of the system is very nearly conserved. In this case, the initial momentum is not zero as it was in part (a); rather the initial momentum of the system is the momentum of the lumberjack just before he lands on the second log. Therefore,

$$\underbrace{m_1 v_{f1} + m_2 v_{f2}}_{\substack{\text{Total momentum}\\ \text{just after lumberjack lands}}} = \underbrace{m_1 v_{01} + m_2 v_{02}}_{\text{Initial momentum}}$$

In this expression, the subscripts "1" and "2" now represent the second log and lumberjack, respectively. Since the second log is initially at rest, $v_{01} = 0$. Furthermore, since the lumberjack and the second log move with a common velocity, $v_{f1} = v_{f2} = v_f$. The statement of momentum conservation then becomes

$$m_1 v_f + m_2 v_f = m_2 v_{02}$$

Solving for v_f, we have

$$v_f = \frac{m_2 v_{02}}{m_1 + m_2} = \frac{(98\text{ kg})(+3.6\text{ m/s})}{230\text{ kg} + 98\text{ kg}} = \boxed{+1.1\text{ m/s}}$$

The positive sign indicates that the system moves in the same direction as the original direction of the lumberjack's motion.

46. ***REASONING AND SOLUTION*** We will define the system to be comprised of (1) the bullet in a standard cartridge, and (2) the rest of the cartridge, the gun and the person firing it. If air resistance is ignored, there are no external forces in the (horizontal) direction of motion; thus, the momentum of the system must be conserved during the firing process. Initially, the two-part system has zero total momentum. From momentum conservation the total momentum must be zero after the gun is fired.

Let m represent the mass of the bullet, v_b represent the velocity of the bullet, M represent the mass of the person, the gun and the rest of the cartridge, and v_C, the recoil velocity of the person, the gun and the rest of the cartridge. Then

$$0 = Mv_C + m_b v_b$$

which gives

$$v_C = -\frac{m_b v_b}{M} \qquad (1)$$

Similarly, when a blank cartridge of mass m_{bc} is fired with a forward velocity of v_{bc}, momentum conservation gives

$$0 = Mv_B + m_{bc} v_{bc}$$

where v_B is the recoil velocity of the person, the gun and the rest of the cartridge in this situation. Solving for v_B gives

$$v_B = -\frac{m_{bc} v_{bc}}{M} \qquad (2)$$

Using equations (1) and (2) we obtain the ratio

$$\frac{v_B}{v_C} = \frac{-(m_{bc} v_{bc}/M)}{-(m_b v_b/M)} = \frac{m_{bc} v_{bc}}{m_b v_b} = \frac{(0.10\text{ g})(55\text{ m/s})}{(14\text{ g})(280\text{ m/s})} = \boxed{1.4 \times 10^{-3}}$$

Thus, when the blank is fired, the recoil is 1.4×10^{-3} times the recoil when a bullet is fired. Clearly, the recoil is much less when the blank is fired compared with the recoil when a bullet is fired.

47. SSM ***REASONING AND SOLUTION*** Bonzo and Ender constitute an isolated system. Therefore, the principle of conservation of linear momentum holds. Since both members of the system are initially stationary, the initial linear momentum is zero, and must remain zero during and after any interaction between Bonzo and Ender.

a. Since the total momentum of the system is conserved, and the total momentum of the system is zero, Bonzo and Ender must have equal but opposite linear momenta. Bonzo must have the larger mass, since he flies off with the smaller recoil velocity.

b. Conservation of linear momentum gives $0 = m_{\text{Bonzo}} v_{\text{Bonzo}} + m_{\text{Ender}} v_{\text{Ender}}$. Solving for the ratio of the masses, we have

$$\frac{m_{\text{Bonzo}}}{m_{\text{Ender}}} = -\frac{v_{\text{Ender}}}{v_{\text{Bonzo}}} = -\frac{-2.5\ \text{m/s}}{1.5\ \text{m/s}} = \boxed{1.7}$$

48. ***REASONING AND SOLUTION***

a. $p_E = m_E v_E = \left(5.98 \times 10^{24}\ \text{kg}\right)\left(29.9 \times 10^{3}\ \text{m/s}\right) = \boxed{1.79 \times 10^{29}\ \text{kg} \cdot \text{m/s}}$

b. $\boxed{\text{No.}}$ The earth's velocity is changing direction as the earth travels in its nearly circular orbit around the sun. The sun's gravitational force causes the Earth's linear momentum to change.

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49. ***REASONING AND SOLUTION*** According to Equation 7.2, the linear momentum of an object of mass m moving with velocity $\mathbf{v}$ is $\mathbf{p} = m\mathbf{v}$. In order for the automobile to have the same momentum as the train,

$$m_{\text{auto}} v_{\text{auto}} = m_{\text{train}} v_{\text{train}}$$

Solving for v_{auto}, the mass of the automobile, we have

$$v_{\text{auto}} = \frac{m_{\text{train}} v_{\text{train}}}{m_{\text{auto}}} = \frac{\left(4.5 \times 10^5 \text{ kg}\right)\left(1.4 \text{ m/s}\right)}{1800 \text{ kg}} = \boxed{3.5 \times 10^2 \text{ m/s}}$$

50. ***REASONING AND SOLUTION*** From Equation 7.4: $\bar{F}\Delta t = m(v_f - v_0)$. Solving for $\bar{F}$ (with *up* taken to be the positive direction) gives

$$\bar{F} = \left(\frac{m}{\Delta t}\right)(v_f - v_0) = (0.060\ \text{kg/s})\left[(+15\ \text{m/s}) - (-15\ \text{m/s})\right] = +1.8\ \text{N}$$

This is the average force exerted on the hail by the roof of the car. The positive sign indicates that this force points upward. From Newton's third law, the average force exerted by the hail on the roof is equal in magnitude and opposite in direction to this force. Therefore,

$$\text{Force on roof} = \boxed{-1.8\ \text{N}}$$

The negative sign indicates that this force $\boxed{\text{points downward}}$.

51. SSM ***REASONING*** The two-stage rocket constitutes the system. The forces that act to cause the separation during the explosion are, therefore, forces that are internal to the system. Since no external forces act on this system, it is isolated and the principle of conservation of linear momentum applies:

$$\underbrace{m_1 v_{f1} + m_2 v_{f2}}_{\substack{\text{Total momentum}\\ \text{after separation}}} = \underbrace{(m_1 + m_2) v_0}_{\substack{\text{Total momentum}\\ \text{before separation}}}$$

where the subscripts "1" and "2" refer to the lower and upper stages, respectively. This expression can be solved for v_{f1}.

SOLUTION Solving for v_{f1} gives

$$v_{f1} = \frac{(m_1 + m_2) v_0 - m_2 v_{f2}}{m_1}$$

$$= \frac{[2400\text{ kg} + 1200\text{ kg}](4900\text{ m/s}) - (1200\text{ kg})(5700\text{ m/s})}{2400\text{ kg}} = \boxed{+4500\text{ m/s}}$$

Since v_{f1} is positive $\boxed{\text{its direction is the same as the rocket before the explosion}}$.

52. ***REASONING AND SOLUTION*** The velocity of the center of mass of a system is given by Equation 7.11. Using the data and the results obtained in Example 5, we obtain the following:

a. The velocity of the center of mass of the two-car system before the collision is

$$\left(v_{\text{cm}}\right)_{\text{before}} = \frac{m_1 v_{01} + m_2 v_{02}}{m_1 + m_2}$$

$$= \frac{(65\times10^3\ \text{kg})(+0.80\ \text{m/s}) + (92\times10^3\ \text{kg})(+1.2\ \text{m/s})}{65\times10^3\ \text{kg} + 92\times10^3\ \text{kg}} = \boxed{+1.0\ \text{m/s}}$$

b. The velocity of the center of mass of the two-car system after the collision is

$$\left(v_{\text{cm}}\right)_{\text{after}} = \frac{m_1 v_{\text{f}} + m_2 v_{\text{f}}}{m_1 + m_2} = v_{\text{f}} = \boxed{+1.0\ \text{m/s}}$$

c. The answer in part (b) $\boxed{\text{should be the same}}$ as the common velocity v_{f}. Since the cars are coupled together, every point of the two-car system, including the center of mass, must move with the same velocity.

53. SSM ***REASONING*** No net external force acts on the plate parallel to the floor; therefore, the component of the momentum of the plate that is parallel to the floor is conserved as the plate breaks and flies apart. Initially, the total momentum parallel to the floor is zero. After the collision with the floor, the component of the total momentum parallel to the floor must remain zero. The drawing in the text shows the pieces in the plane parallel to the floor just after the collision. Clearly, the linear momentum in the plane parallel to the floor has two components; therefore the linear momentum of the plate must be conserved in each of these two mutually perpendicular directions. Using the drawing in the text, with the positive directions taken to be up and to the right, we have

[x direction] $$-m_1 v_1 (\sin 25.0°) + m_2 v_2 (\cos 45.0°) = 0$$

[y direction] $$m_1 v_1 (\cos 25.0°) + m_2 v_2 (\sin 45.0°) - m_3 v_3 = 0$$

These equations can be solved simultaneously for the masses m_1 and m_2.

SOLUTION Using the values given in the drawing for the velocities after the plate breaks, we have, suppressing units,

$$-1.27 m_1 + 1.27 m_2 = 0 \qquad (1)$$

$$2.72 m_1 + 1.27 m_2 - 3.99 = 0 \qquad (2)$$

Subtracting (1) from (2) gives $\boxed{m_1 = 1.00 \text{ kg}}$. Substituting this value into either (1) or (2) then yields $\boxed{m_2 = 1.00 \text{ kg}}$.

54. ***REASONING AND SOLUTION*** The conservation of momentum law applied in the horizontal direction gives

$$m_c v_{fc} + m_s v_{fs} \cos 30.0° = 0$$

so that

$$v_{fc} = \frac{-m_s v_{fs} \cos 30.0°}{m_c} = \frac{-(5.00 \text{ kg})(8.00 \text{ m/s})\cos 30.0°}{105 \text{ kg}} = -0.330 \text{ m/s}$$

The magnitude of the velocity is $\boxed{0.330 \text{ m/s}}$. The minus sign indicates that the direction is

$\boxed{\text{opposite the horizontal velocity component of the stone}}$.

55. ***REASONING AND SOLUTION*** The total linear momentum of the two-car system is conserved because no net external force acts on the system during the collision (we are ignoring friction *during the collision*). Momentum conservation gives us

$$\underbrace{(m_1 + m_2)v_f}_{\substack{\text{Total momentum}\\ \text{after collision}}} = \underbrace{m_1 v_{01} + m_2 v_{02}}_{\substack{\text{Total momentum}\\ \text{after collision}}}$$

where $v_{02} = 0$ since the 1800-kg car is stationary before the collision.

a. Solving for v_f, we find that the velocity of the two cars just after the collision is

$$v_f = \frac{m_1 v_{01} + m_2 v_{02}}{m_1 + m_2} = \frac{2300\text{ kg }(+16\text{ m/s}) + 0}{2300\text{ kg} + 1800\text{ kg}} = \boxed{+9.0\text{ m/s}}$$

The plus sign indicates that the velocity of the two cars just after the collision is in the same direction as the direction of the velocity of the 2300-kg car before the collision.

b. According to the impulse-momentum theorem (Equation 7.4)

$$\underbrace{\overline{F}\Delta t}_{\substack{\text{Impulse}\\ \text{due to}\\ \text{friction}}} = \underbrace{(m_1 + m_2)v_{\text{final}}}_{\text{Final momentum}} - \underbrace{(m_1 + m_2)v_{\text{after}}}_{\substack{\text{Total momentum}\\ \text{just after collision}}}$$

where $v_{\text{final}} = 0$ since the cars come to a halt, and $v_{\text{after}} = v_f = +9.0\text{ m/s}$. Therefore, we have

$$\overline{F}\Delta t = 0 - (m_1 + m_2)v_{\text{after}} = -(2300\text{ kg} + 1800\text{ kg})(9.0\text{ m/s}) = \boxed{-3.7\times10^4\text{ N}\cdot\text{s}}$$

The minus sign indicates that the impulse due to friction acts opposite to the direction of motion of the locked, two-car system. This is reasonable since the velocity of the cars is decreasing in magnitude as the cars skid to a halt.

c. Using the same notation as in part (b) above, we have, from Equation 2.9,

$$v_{\text{final}}^2 = v_{\text{after}}^2 + 2ax$$

where $v_{\text{final}} = 0$ and $v_{\text{after}} = v_f = +9.0\text{ m/s}$. From Newton's second law we have that $a = -f_k/(m_1 + m_2)$, where f_k is the force of kinetic friction that acts on the cars as they skid to a halt. Therefore,

$$0 = v_f^2 + 2\left(\frac{-f_k}{m_1 + m_2}\right)x \qquad \text{or} \qquad x = \frac{(m_1 + m_2)v_f^2}{2f_k}$$

According to Equation 4.8, $f_k = \mu_k F_N$, where μ_k is the coefficient of kinetic friction and F_N is the magnitude of the normal force that acts on the two-car system. There are only two vertical forces that act on the system; they are the upward normal force $\mathbf{F}_N$ and the weight

$(m_1 + m_2)\boldsymbol{g}$ of the cars. Taking upward as the positive direction and applying Newton's second law in the vertical direction, we have $F_N - (m_1 + m_2)g = (m_1 + m_2)a_y = 0$, or $F_N = (m_1 + m_2)g$. Therefore, $f_k = \mu_k (m_1 + m_2)g$, and we have

$$x = \frac{(m_1 + m_2)v_f^2}{2\mu_k (m_1 + m_2)g} = \frac{v_f^2}{2\mu_k g} = \frac{(9.0\ \text{m/s})^2}{2(0.80)(9.80\ \text{m/s}^2)} = \boxed{5.2\ \text{m}}$$

56. ***REASONING AND SOLUTION*** The drawing below shows the relative positions and distances of the individual atoms that make up the nitric acid molecule.

Since the molecules are distributed in a plane, we must specify both the x and y components of the center of mass. Generalizing Equation 7.10 to account for all five atoms, we find that the x coordinate of the center of mass is given by

$$x_{\text{cm}} = \frac{m_{\text{H}} x_{\text{H}} + m_{\text{O}} x_{\text{O1}} + m_{\text{N}} x_{\text{N}} + 2 m_{\text{O}} x_{\text{O2}}}{m_{\text{H}} + m_{\text{N}} + 3 m_{\text{O}}}$$

where the subscripts on the masses refer to the name of the atom. The distances x_{H}, x_{O1}, x_{N}, and x_{O2} refer to the x distances of the atoms from the coordinate origin. The distance x_{O1} represents the location of the oxygen atom that is collinear with the hydrogen atom and the nitrogen atom, while x_{O2} refers to the x component of the location of the oxygen atoms that are off the x axis. For convenience, we take the origin of coordinates at the location of the hydrogen atom, so $x_{\text{H}} = 0$, $x_{\text{O1}} = x_{\text{A}}$, $x_{\text{N}} = x_{\text{A}} + x_{\text{B}}$, and $x_{\text{O2}} = x_{\text{A}} + x_{\text{B}} + d\cos 65.0°$, where the distances x_{A}, x_{B}, and d are identified in the drawing above. Thus, we have

$$x_{\text{cm}} = \frac{m_{\text{H}}(0) + m_{\text{O}} x_{\text{A}} + m_{\text{N}}(x_{\text{A}} + x_{\text{B}}) + 2 m_{\text{O}}(x_{\text{A}} + x_{\text{B}} + d\cos 65.0°)}{m_{\text{H}} + m_{\text{N}} + 3 m_{\text{O}}}$$

From the drawing we have

$$d\cos 65.0° = \left(1.22 \times 10^{-10}\ \text{m}\right)\cos 65.0° = 5.16 \times 10^{-11}\ \text{m}$$

Substituting values, we obtain

$$x_{\text{cm}} = \frac{\left[\begin{array}{l}(26.6 \times 10^{-27}\ \text{kg})(1.00 \times 10^{-10}\ \text{m}) + (23.3 \times 10^{-27}\ \text{kg})(2.41 \times 10^{-10}\ \text{m}) \\ \qquad + 2(26.6 \times 10^{-27}\ \text{kg})(2.93 \times 10^{-10}\ \text{m})\end{array}\right]}{1.67 \times 10^{-27}\ \text{kg} + 23.3 \times 10^{-27}\ \text{kg} + 3(26.6 \times 10^{-27}\ \text{kg})} = 2.28 \times 10^{-10}\ \text{m}$$

Using similar reasoning, we deduce that the y coordinate of the center of mass of the molecule is given by

$$y_{\text{cm}} = \frac{m_{\text{H}} y_{\text{H}} + m_{\text{O}} y_{\text{O1}} + m_{\text{N}} y_{\text{N}} + m_{\text{O}} y_{\text{O2 above}} + m_{\text{O}} y_{\text{O2 below}}}{m_{\text{H}} + m_{\text{N}} + 3m_{\text{O}}}$$

where $y_{\text{O2 above}}$ is the y coordinate of the oxygen atom located above the H–O–N line and $y_{\text{O2 below}}$ is the y coordinate of the oxygen atom located below the H–O–N line.

However, since the coordinate origin was chosen at the location of the hydrogen atom, the hydrogen atom, one of the oxygen atoms, and the nitrogen atom are on the x axis and have zero y coordinates, so that $y_{\text{H}} = y_{\text{O1}} = y_{\text{N}} = 0$. Furthermore, since the remaining two oxygen atoms are located symmetrically on either side of the H–O–N line, we see that $y_{\text{O2 above}} = -y_{\text{O2 below}}$. Thus, the y coordinate of the center of mass of the nitric acid molecule is $y_{\text{cm}} = 0$. Therefore, the center of mass lies along the x axis at

$$x_{\text{cm}} = \boxed{2.28 \times 10^{-10}\ \text{m}}$$

57. SSM WWW ***REASONING*** The cannon and the shell constitute the system. Since no external force hinders the motion of the system after the cannon is unbolted, conservation of linear momentum applies in that case. If we assume that the burning gun powder imparts the same kinetic energy to the system in each case, we have sufficient information to develop a mathematical description of this situation, and solve it for the velocity of the shell fired by the loose cannon.

SOLUTION For the case where the cannon is unbolted, momentum conservation gives

$$\underbrace{m_1 v_{f1} + m_2 v_{f2}}_{\substack{\text{Total momentum}\\ \text{after shell is fired}}} = \underbrace{0}_{\substack{\text{Initial momentum}\\ \text{of system}}} \tag{1}$$

where the subscripts "1" and "2" refer to the cannon and shell, respectively. In both cases, the burning gun power imparts the same kinetic energy to the system. When the cannon is bolted to the ground, only the shell moves and the kinetic energy imparted to the system is

$$\text{KE} = \tfrac{1}{2} m_{\text{shell}} v_{\text{shell}}^2 = \tfrac{1}{2}(85.0\text{ kg})(551\text{ m/s})^2 = 1.29 \times 10^7\text{ J}$$

The kinetic energy imparted to the system when the cannon is unbolted has the same value and can be written using the same notation as equation (1):

$$\text{KE} = \tfrac{1}{2} m_1 v_{f1}^2 + \tfrac{1}{2} m_2 v_{f2}^2 \tag{2}$$

Solving equation (1) for v_{f1}, the velocity of the cannon after the shell is fired, and substituting the resulting expression into Equation (2) gives

$$\text{KE} = \frac{m_2^2 v_{f2}^2}{2\text{m}_1} + \tfrac{1}{2} m_2 v_{f2}^2 \tag{3}$$

Solving equation (3) for v_{f2} gives

$$v_{f2} = \sqrt{\frac{2\,\text{KE}}{m_2\left(\dfrac{m_2}{m_1}+1\right)}} = \sqrt{\frac{2(1.29 \times 10^7\text{ J})}{(85\text{ kg})\left(\dfrac{85\text{ kg}}{5.80 \times 10^3\text{ kg}}+1\right)}} = \boxed{+547\text{ m/s}}$$

1. **SSM** ***REASONING AND SOLUTION*** Since there are 2π radians per revolution, and it is stated in the problem that there are 100 grads in one-quarter of a circle, we find that the number of grads in one radian is

$$(1.00\text{ rad})\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{100\text{ grad}}{0.250\text{ rev}}\right)=\boxed{63.7\text{ grad}}$$

2. ***REASONING*** The average angular velocity is equal to the angular displacement divided by the elapsed time (Equation 8.2). Thus, the angular displacement of the engine is equal to the product of the average angular velocity and the elapsed time.

SOLUTION

a. Since 2π rad = 1 rev and 1 min = 60 s, the angular speed (in rad/s) of the engine is

$$\left(\frac{2500 \text{ rev}}{\text{min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right) = \boxed{260 \text{ rad/s}}$$

b. The average angular velocity $\bar{\omega}$ of the engine is equal to the angular displacement $\Delta\theta$ divided by the elapsed time Δt. The angular displacement is

$$\Delta\theta = \bar{\omega}\Delta t = (260 \text{ rad/s})(5.1 \text{ min})\left(\frac{60 \text{ s}}{1 \text{ min}}\right) = \boxed{8.0 \times 10^4 \text{ rad}} \qquad (8.2)$$

3. **SSM** ***REASONING AND SOLUTION*** Equation 8.2 gives the desired result. Since the answer is to be expressed in rad/s, we must convert revolutions to radians.

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \left(\frac{3.5 \text{ rev}}{1.7 \text{ s}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = \boxed{13 \text{ rad/s}}$$

4. ***REASONING AND SOLUTION*** In one revolution the pulsar turns through 2π radians. The average angular speed of the pulsar is, from Equation 8.2,

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{0.033 \text{ s}} = \boxed{1.9 \times 10^2 \text{ rad/s}}$$

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5. ***REASONING AND SOLUTION*** Using Equation 8.4 and the appropriate conversion factors, the average angular acceleration of the CD in rad/s^2 is

$$\bar{\alpha} = \frac{\Delta\omega}{\Delta t} = \left(\frac{210 \text{ rev/min} - 480 \text{ rev/min}}{74 \text{ min}}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right)\left(\frac{1 \text{ min}}{60 \text{ s}}\right)^2 = -6.4\times10^{-3} \text{ rad/s}^2$$

The magnitude of the average angular acceleration is $\boxed{6.4\times10^{-3} \text{ rad/s}^2}$.

---

6. ***REASONING AND SOLUTION***

a. The angle θ is given by Equation 8.6 as $\theta = \bar{\omega} t$. Thus, the time is

$$t = \frac{\theta}{\bar{\omega}} = \frac{0.61\text{ rad}}{650\text{ rad/s}} = \boxed{9.4 \times 10^{-4}\text{ s}}$$

b. The distance s through which the tip of the string moves is

$$s = r\theta = (0.21\text{ m})(0.61\text{ rad}) = \boxed{0.13\text{ m}} \qquad (8.1)$$

7. **SSM** ***REASONING AND SOLUTION*** Equation 8.4 gives the desired result. Assuming $t_0 = 0$ s, the final angular velocity is

$$\omega = \omega_0 + \alpha t = 0 \text{ rad/s} + (328 \text{ rad/s}^2)(1.50 \text{ s}) = \boxed{492 \text{ rad/s}}$$

---

8. ***REASONING AND SOLUTION*** We can use Equation 8.4, assuming, as usual, that $t_0 = 0$ s. The time is

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{5.2 \text{ rad/s} - 0 \text{ rad/s}}{4.0 \text{ rad/s}^2} = \boxed{1.3 \text{ s}}$$

---

9. ***REASONING AND SOLUTION*** The angular displacements of the astronauts are equal.

$$\text{For A} \qquad \theta = s_A/r_A \tag{8.1}$$

$$\text{For B} \qquad \theta = s_B/r_B$$

Equating these two equations for θ and solving for s_B gives

$$s_B = (r_B/r_A)s_A = [(1.10 \times 10^3 \text{ m})/(3.20 \times 10^2 \text{ m})](2.40 \times 10^2 \text{ m}) = \boxed{825 \text{ m}}$$

---

10. ***REASONING*** The average angular velocity is defined as the angular displacement divided by the elapsed time (Equation 8.2). Therefore, the angular displacement is equal to the product of the average angular velocity and the elapsed time The elapsed time is given, so we need to determine the average angular velocity. We can do this by using the graph of angular velocity versus time that accompanies the problem.

SOLUTION The angular displacement $\Delta\theta$ is related to the average angular velocity $\bar{\omega}$ and the elapsed time Δt by Equation 8.2, $\Delta\theta = \bar{\omega}\,\Delta t$. The elapsed time is given as 8.0 s. To obtain the average angular velocity, we need to extent the graph that accompanies this problem from a time of 5.0 s to 8.0 s. It can be seen from the graph that the angular velocity increases by +3.0 rad/s during each second. Therefore, when the time increases from 5.0 to 8.0 s, the angular velocity increases from +6.0 rad/s to 6 rad/s + 3×(3.0 rad/s) = +15 rad/s. A graph of the angular velocity from 0 to 8.0 s is shown at the right. The average angular velocity during this time is equal to one half the sum of the initial and final angular velocities:

$$\bar{\omega} = \tfrac{1}{2}\left(\omega_0 + \omega\right) = \tfrac{1}{2}\left(-9.0\text{ rad/s} + 15\text{ rad/s}\right) = +3.0\text{ rad/s}$$

The angular displacement of the wheel from 0 to 8.0 s is

$$\Delta\theta = \bar{\omega}\,\Delta t = \left(+3.0\text{ rad/s}\right)\left(8.0\text{ s}\right) = \boxed{+24\text{ rad}}$$

11. SSM ***REASONING*** The time required for the bullet to travel the distance d is equal to the time required for the discs to undergo an angular displacement of 0.240 rad. The time can be found from Equation 8.2; once the time is known, the speed of the bullet can be found using Equation 2.2.

SOLUTION From the definition of average angular velocity:

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t}$$

the required time is

$$\Delta t = \frac{\Delta\theta}{\bar{\omega}} = \frac{0.240 \text{ rad}}{95.0 \text{ rad/s}} = 2.53 \times 10^{-3} \text{ s}$$

Note that $\bar{\omega} = \omega$ because the angular speed is constant. The (constant) speed of the bullet can then be determined from the definition of average speed:

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{d}{\Delta t} = \frac{0.850 \text{ m}}{2.53 \times 10^{-3} \text{ s}} = \boxed{336 \text{ m/s}}$$

12. ***REASONING AND SOLUTION*** The people meet at time t. At this time the magnitudes of their angular displacements must total 2π rad.

$$\theta_1 + \theta_2 = 2\pi \text{ rad}$$

Then

$$\omega_1 t + \omega_2 t = 2\pi \text{ rad}$$

$$t = \frac{2\pi \text{ rad}}{\omega_1 + \omega_2} = \frac{2\pi \text{ rad}}{1.7\times10^{-3} \text{ rad/s} + 3.4\times10^{-3} \text{ rad/s}} = \boxed{1200 \text{ s}}$$

13. ***REASONING AND SOLUTION*** The baton will make four revolutions in a time t given by

$$t = \frac{\theta}{\omega}$$

Half of this time is required for the baton to reach its highest point. The magnitude of the initial vertical velocity of the baton is then

$$v_0 = g\left(\tfrac{1}{2}t\right) = g\left(\frac{\theta}{2\omega}\right)$$

With this initial velocity the baton can reach a height of

$$h = \frac{v_0^2}{2g} = \frac{g\theta^2}{8\omega^2} = \frac{\left(9.80\ \text{m/s}^2\right)\left(8\pi\ \text{rad}\right)^2}{8\left[\left(1.80\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\right]^2} = \boxed{6.05\ \text{m}}$$

14. ***REASONING AND SOLUTION***

The figure at the right shows the relevant angles and dimensions for either one of the celestial bodies under consideration.

a. Using the figure above

$$\theta_{\text{moon}} = \frac{s_{\text{moon}}}{r_{\text{moon}}} = \frac{3.48 \times 10^{6}\ \text{m}}{3.85 \times 10^{8}\ \text{m}} = \boxed{9.04 \times 10^{-3}\ \text{rad}}$$

$$\theta_{\text{sun}} = \frac{s_{\text{sun}}}{r_{\text{sun}}} = \frac{1.39 \times 10^{9}\ \text{m}}{1.50 \times 10^{11}\ \text{m}} = \boxed{9.27 \times 10^{-3}\ \text{rad}}$$

b. Since the sun subtends a slightly larger angle than the moon, as measured by a person standing on the earth, the sun cannot be completely blocked by the moon. Therefore, $\boxed{\text{a "total" eclipse of the sun is not really total}}$.

c. The relevant geometry is shown below.

The apparent circular area of the sun as measured by a person standing on the earth is given by: $A_{\text{sun}} = \pi R_{\text{sun}}^2$, where R_{sun} is the radius of the sun. The apparent circular area of the sun that is blocked by the moon is $A_{\text{blocked}} = \pi R_{\text{b}}^2$, where R_{b} is shown in the figure above. Also from the figure above, it follows that

$$R_{\text{sun}} = (1/2)\, s_{\text{sun}} \quad \text{and} \quad R_{\text{b}} = (1/2)\, s_{\text{b}}$$

Therefore, the fraction of the apparent circular area of the sun that is blocked by the moon is

$$\frac{A_{\text{blocked}}}{A_{\text{sun}}} = \frac{\pi R_{\text{b}}^2}{\pi R_{\text{sun}}^2} = \frac{\pi (s_{\text{b}}/2)^2}{\pi (s_{\text{sun}}/2)^2} = \left(\frac{s_{\text{b}}}{s_{\text{sun}}}\right)^2 = \left(\frac{\theta_{\text{moon}}\, r_{\text{sun}}}{\theta_{\text{sun}}\, r_{\text{sun}}}\right)^2$$

$$= \left(\frac{\theta_{\text{moon}}}{\theta_{\text{sun}}}\right)^2 = \left(\frac{9.04 \times 10^{-3}\ \text{rad}}{9.27 \times 10^{-3}\ \text{rad}}\right)^2 = 0.951$$

The moon blocks out $\boxed{\text{95.1 percent}}$ of the apparent circular area of the sun.

15. ***REASONING AND SOLUTION*** The ball has a time of flight given by the kinematics equation $0\text{ m} = v_0 t - \frac{1}{2} a_y t^2$. Solving for the time gives

$$t = \frac{2v_0}{g} = \frac{2(19\text{ m/s})(\sin 55°)}{9.80\text{ m/s}^2} = 3.2\text{ s}$$

The ball then makes (7.7 rev/s)(3.2 s) = $\boxed{25\text{ rev}}$

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16. ***REASONING AND SOLUTION***

a. Since the skater comes to a stop, her final angular speed is zero. Using Equation 8.8, $\omega^2 = \omega_0^2 + 2\alpha\theta$, and solving for α, we obtain

$$\alpha = -\frac{\omega_0^2}{2\theta} = -\frac{(15\text{ rad/s})^2}{2(5.1\text{ rad})} = \boxed{-22\text{ rad/s}^2}$$

b. Using Equation 8.4 ($\omega = \omega_0 + \alpha t$) with $\omega = 0$ rad/s, we have

$$t = -\frac{\omega_0}{\alpha} = -\frac{15\text{ rad/s}}{-22\text{ rad/s}^2} = \boxed{0.68\text{ s}}$$

---

17. SSM ***REASONING AND SOLUTION***

a. From Equation 8.7 we obtain

$$\theta = \omega_0 t + \tfrac{1}{2}\alpha t^2 = (5.00 \text{ rad/s})(4.00 \text{ s}) + \tfrac{1}{2}(2.50 \text{ rad/s}^2)(4.00 \text{ s})^2 = \boxed{4.00 \times 10^1 \text{ rad}}$$

b. From Equation 8.4, we obtain

$$\omega = \omega_0 + \alpha t = 5.00 \text{ rad/s} + (2.50 \text{ rad/s}^2)(4.00 \text{ s}) = \boxed{15.0 \text{ rad/s}}$$

---

18. ***REASONING AND SOLUTION*** From Equation 8.6, $\theta = \frac{1}{2}(\omega_0 + \omega)t$.

Solving for t gives

$$t = \frac{2\theta}{\omega_0 + \omega} = \frac{2(85.1 \text{ rad})}{18.5 \text{ rad/s} + 14.1 \text{ rad/s}} = \boxed{5.22 \text{ s}}$$

19. SSM WWW *REASONING AND SOLUTION*

a. Since the flywheel comes to rest, its final angular velocity is zero. Furthermore, if the initial angular velocity ω_0 is assumed to be a positive number and the flywheel decelerates, the angular acceleration must be a negative number. Solving Equation 8.8 for θ, we obtain

$$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(0 \text{ rad/s})^2 - (220 \text{ rad/s})^2}{2(-2.0 \text{ rad/s}^2)} = \boxed{1.2 \times 10^4 \text{ rad}}$$

b. The time required for the flywheel to come to rest can be found from Equation 8.4. Solving for t, we obtain

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 \text{ rad/s} - 220 \text{ rad/s}}{-2.0 \text{ rad/s}^2} = \boxed{1.1 \times 10^2 \text{ s}}$$

20. ***REASONING*** The angle θ (in radians) through which the pulley rotates is given by Equation 8.1 and is equal to the length s of the string wrapped around the circumference divided by the radius r of the pulley. Once the angle has been determined, and since the initial angular velocity and the angular acceleration are known, Equation 8.8 from the equations of rotational kinematics can be employed to find the final angular velocity.

SOLUTION

a. The angle θ through which the pulley rotates is

$$\theta = \frac{s}{r} = \frac{9.0\text{ m}}{0.060\text{ m}} = \boxed{150\text{ rad}} \tag{8.1}$$

b. The final angular velocity can be obtained from Equation 8.8, since ω_0, α, and θ are known.

$$\omega = \pm\sqrt{\omega_0^2 + 2\alpha\theta} = \pm\sqrt{(0\text{ rad/s})^2 + 2\left(+17\text{ rad/s}^2\right)(150\text{ rad})} = \pm 71\text{ rad/s}$$

The wheel starts from rest and has a positive angular acceleration. Therefore, the final angular velocity must also be positive, so that $\omega = \boxed{+71\text{ rad/s}}$.

21. ***REASONING AND SOLUTION***

a. $$\theta = \tfrac{1}{2}(\omega + \omega_0)t = \tfrac{1}{2}(1420 \text{ rad/s} + 420 \text{ rad/s})(5.00 \text{ s}) = \boxed{4.60 \times 10^3 \text{ rad}}$$

b. $$\alpha = \frac{\omega - \omega_0}{t} = \frac{1420 \text{ rad/s} - 420 \text{ rad/s}}{5.00 \text{ s}} = \boxed{2.00 \times 10^2 \text{ rad/s}^2}$$

22. ***REASONING*** The equations of kinematics for rotational motion cannot be used directly to find the angular displacement, because the final angular velocity (not the initial angular velocity), the acceleration, and the time are known. We will combine two of the equations, Equations 8.4 and 8.6 to obtain an expression for the angular displacement that contains the three known variables.

SOLUTION The angular displacement of each wheel is equal to the average angular velocity multiplied by the time

$$\theta = \underbrace{\tfrac{1}{2}\left(\omega_0 + \omega\right)}_{\bar{\omega}} t \tag{8.6}$$

The initial angular velocity ω_0 is not known, but it can be found in terms of the angular acceleration and time, which are known. The angular acceleration is defined as (with $t_0 = 0$ s)

$$\bar{\alpha} = \frac{\omega - \omega_0}{t} \quad \text{or} \quad \omega_0 = \omega - \alpha t \tag{8.4}$$

Substituting this expression for ω_0 into Equation 8.6 gives

$$\theta = \tfrac{1}{2}\left[\underbrace{\left(\omega - \alpha t\right)}_{\omega_0} + \omega\right] t = \omega t - \tfrac{1}{2}\alpha t^2$$

$$= \left(+74.5 \text{ rad/s}\right)\left(4.50 \text{ s}\right) - \tfrac{1}{2}\left(+6.70 \text{ rad/s}^2\right)\left(4.50 \text{ s}\right)^2 = \boxed{+267 \text{ rad}}$$

23. SSM ***REASONING*** The time required for the change in the angular velocity to occur can be found by solving Equation 8.4 for t. In order to use Equation 8.4, however, we must know the initial angular velocity ω_0. Equation 8.6 can be used to find the initial angular velocity.

SOLUTION From Equation 8.6 we have

$$\theta = \frac{1}{2}(\omega_0 + \omega)t$$

Solving for ω_0 gives

$$\omega_0 = \frac{2\theta}{t} - \omega$$

Since the angular displacement θ is zero, $\omega_0 = -\omega$. Solving Equation 8.4 for t, and using the fact that $\omega_0 = -\omega$, gives

$$t = \frac{2\omega}{\alpha} = \frac{2(-25.0 \text{ rad/s})}{-4.00 \text{ rad/s}^2} = \boxed{12.5 \text{ s}}$$

24. ***REASONING*** The angular displacement for the interval from 0 to 25.0 s can be obtained by breaking the interval into two segments. During the first segment, from 0 to 15.0 s, the wheel has an constant angular acceleration. During the second segment, from 15.0 to 25.0 s, the angular velocity remains constant so the angular acceleration is zero. We will find the angular displacement for each segment by using the equations of kinematics for rotational motion. The angular displacement for the entire interval is the sum of the two displacements.

SOLUTION The angular acceleration α_1 during the first segment can be found, since we know that the angular velocity changes from 0 to +7.50 rad/s during a time of 5.00 s:

$$\alpha_1 = \frac{\omega - \omega_0}{t - t_0} = \frac{+7.50 \text{ rad/s} - 0 \text{ rad/s}}{5.00 \text{ s} - 0 \text{ s}} = +1.50 \text{ rad/s}^2 \qquad (8.4)$$

The angular displacement θ_1 for the first segment (0 to 15.0 s) is

$$\theta_1 = \omega_{0,1}\, t_1 + \tfrac{1}{2}\alpha_1 t_1^2 = (0 \text{ rad/s})(15.0 \text{ s}) + \tfrac{1}{2}\left(+1.50 \text{ rad/s}^2\right)(15.0 \text{ s})^2 = +169 \text{ rad} \qquad (8.7)$$

The angular velocity ω_1 at the end of the first segment is

$$\omega_1 = \omega_{0,1} + \alpha_1 t_1 = 0 \text{ rad/s} + \left(+1.50 \text{ rad/s}^2\right)(15.0 \text{ s}) = +22.5 \text{ rad/s} \qquad (8.4)$$

The angular displacement θ_2 for the second segment (15.0 to 25.0 s) is

$$\theta_2 = \omega_{0,2}\, t + \tfrac{1}{2}\alpha_2 t^2 = (+22.5 \text{ rad/s})(10.0 \text{ s}) + \tfrac{1}{2}\left(0 \text{ rad/s}^2\right)(10.0 \text{ s})^2 = +225 \text{ rad} \qquad (8.7)$$

where we have used the fact that the final angular velocity ω_1 of the first segment becomes the initial angular velocity $\omega_{0,2}$ for the second segment.

The total displacement is $\theta_1 + \theta_2$ = 169 rad + 225 rad = $\boxed{+394 \text{ rad}}$.

25. ***REASONING AND SOLUTION*** The angular acceleration is found for the first circumstance.

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\theta} = \frac{\left(3.14 \times 10^4 \text{ rad/s}\right)^2 - \left(1.05 \times 10^4 \text{ rad/s}\right)^2}{2\left(1.88 \times 10^4 \text{ rad}\right)} = 2.33 \times 10^4 \text{ rad/s}^2$$

For the second circumstance

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{7.85 \times 10^4 \text{ rad/s} - 0 \text{ rad/s}}{2.33 \times 10^4 \text{ rad/s}^2} = \boxed{3.37 \text{ s}}$$

---

26. ***REASONING*** According to Equation 3.5b, the time required for the diver to reach the water, assuming free-fall conditions, is $t = \sqrt{2y/a_y}$. If we assume that the "ball" formed by the diver is rotating at the instant that she begins falling vertically, we can use Equation 8.2 to calculate the number of revolutions made on the way down.

SOLUTION Taking upward as the positive direction, the time required for the diver to reach the water is

$$t = \sqrt{\frac{2(-8.3\text{ m})}{-9.80\text{ m/s}^2}} = 1.3\text{ s}$$

Solving Equation 8.2 for $\Delta\theta$, we find

$$\Delta\theta = \bar{\omega}\Delta t = (1.6\text{ rev/s})(1.3\text{ s}) = \boxed{2.1\text{ rev}}$$

27. SSM WWW ***REASONING*** The angular displacement of the child when he catches the horse is, from Equation 8.2, $\theta_c = \omega_c t$. In the same time, the angular displacement of the horse is, from Equation 8.7 with $\omega_0 = 0$ rad/s, $\theta_h = \frac{1}{2}\alpha t^2$. If the child is to catch the horse $\theta_c = \theta_h + (\pi/2)$.

SOLUTION Using the above conditions yields

$$\tfrac{1}{2}\alpha t^2 - \omega_c t + \tfrac{\pi}{2} = 0$$

or (suppressing the units)

$$\tfrac{1}{2}(0.0100)t^2 - 0.250t + \tfrac{\pi}{2} = 0$$

The quadratic formula yields t = 7.37 s and 42.6 s; therefore, the shortest time needed to catch the horse is $\boxed{t = 7.37 \text{ s}}$.

28. ***REASONING AND SOLUTION***

a. According to Equation 8.9, the tangential speed of the sun is

$$v_T = r\omega = (2.2\times10^{20}\ \text{m})(1.2\times10^{-15}\ \text{rad/s}) = \boxed{2.6\times10^{5}\ \text{m/s}}$$

b. According to Equation 8.2, $\bar{\omega} = \Delta\theta / \Delta t$. Since the angular speed of the sun is constant, $\bar{\omega} = \omega$. Solving for Δt, we have

$$\Delta t = \frac{\Delta\theta}{\omega} = \left(\frac{2\pi\ \text{rad}}{1.2\times10^{-15}\ \text{rad/s}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1\ \text{day}}{24\ \text{h}}\right)\left(\frac{1\ \text{y}}{365.25\ \text{day}}\right) = \boxed{1.7\times10^{8}\ \text{y}}$$

29. ***REASONING AND SOLUTION***

a. In one lap, the car undergoes an angular displacement of 2π radians. Therefore, from the definition of average angular speed

$$\bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{18.9 \text{ s}} = \boxed{0.332 \text{ rad/s}}$$

b. Equation 8.9 relates the average angular speed $\bar{\omega}$ of an object moving in a circle to its average linear speed $\bar{v}_T$ tangent to the path of motion:

$$\bar{v}_T = r\bar{\omega}$$

Solving for r gives

$$r = \frac{\bar{v}_T}{\bar{\omega}} = \frac{42.6 \text{ m/s}}{0.332 \text{ rad/s}} = \boxed{128 \text{ m}}$$

Notice that the unit "rad," being dimensionless, does not appear in the final answer.

30. ***REASONING*** The angular speed ω of the tornado is related to its tangential speed v_T and radius r by Equation 8.9. However, to use this equation we must convert miles to meters and hours to seconds, so the final answer will be in units of rad/s.

SOLUTION We note that 1609 m = 1 mi and 1 h = 3600 s. The angular speed of the tornado is

$$\omega = \frac{v_T}{r} = \frac{\left(\frac{251\text{ mi}}{\text{h}}\right)\left(\frac{1609\text{ m}}{1\text{ mi}}\right)\left(\frac{1\text{ h}}{3600\text{ s}}\right)}{29\text{ m}} = 3.9\text{ rad/s} \tag{8.9}$$

This angular speed can be expressed in revolutions per second by noting that 1 rev = 2π rad:

$$\omega = \left(\frac{3.9\text{ rad}}{\text{s}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{0.62\text{ rev/s}}$$

31. SSM ***REASONING AND SOLUTION*** Equation 8.9 gives the desired result.

$$v_T = r\omega = (2.00 \times 10^{-3}\ \text{m})(7.85 \times 10^{4}\ \text{rad/s}) = \boxed{157\ \text{m/s}}$$

32. ***REASONING AND SOLUTION***

a. A person living in Ecuador makes one revolution (2π rad) every 23.9 hr (8.60×10^4 s). The angular speed of this person is $\omega = (2\pi \text{ rad})/(8.60 \times 10^4 \text{ s}) = 7.31 \times 10^{-5}$ rad/s. According to Equation 8.9, the tangential speed of the person is, therefore,

$$v_T = r\omega = \left(6.38 \times 10^6 \text{ m}\right)\left(7.31 \times 10^{-5} \text{ rad/s}\right) = \boxed{4.66 \times 10^2 \text{ m/s}}$$

b. The relevant geometry is shown in the drawing at the right. Since the tangential speed is one-third of that of a person living in Ecuador, we have,

$$\frac{v_T}{3} = r_\theta \omega$$

or

$$r_\theta = \frac{v_T}{3\omega} = \frac{4.66 \times 10^2 \text{ m/s}}{3\left(7.31 \times 10^{-5} \text{ rad/s}\right)} = 2.12 \times 10^6 \text{ m}$$

The angle θ is, therefore,

$$\theta = \cos^{-1}\left(\frac{2.12 \times 10^6 \text{ m}}{6.38 \times 10^6 \text{ m}}\right) = \boxed{70.6°}$$

33. ***REASONING AND SOLUTION*** The average tangential speed $\bar{v}_T$ of points on the rim of the reel is given by Equation 8.9,

$$\bar{v}_T = r\,\bar{\omega} = (2.5 \times 10^{-2}\ \text{m})(1.9\ \text{rad/s}) = 0.048\ \text{m/s}$$

where $\bar{\omega}$ is the average angular speed. The average tangential speed must equal the average linear speed $\bar{v}$ of the tape. The length L of tape that passes around the reel in a time Δt can be found from the definition of average velocity:

$$\bar{v} = \frac{\Delta x}{\Delta t}$$

Solving for Δx and setting $\Delta x = L$ gives

$$L = \bar{v}\Delta t = (0.048\ \text{m/s})(7.1\ \text{s}) = \boxed{0.34\ \text{m}}$$

34. ***REASONING*** The tangential speed v_T of a point on the "equator" of the baseball is given by Equation 8.9 as $v_T = r\omega$, where r is the radius of the baseball and ω is its angular speed. The radius is given in the statement of the problem. The (constant) angular speed is related to that angle θ through which the ball rotates by Equation 8.2 as $\omega = \theta/t$, where we have assumed for convenience that $\theta_0 = 0$ rad when $t_0 = 0$ s. Thus, the tangential speed of the ball is

$$v_T = r\omega = r\left(\frac{\theta}{t}\right)$$

The time t that the ball is in the air is equal to the distance x it travels divided by its linear speed v, $t = x/v$, so the tangential speed can be written as

$$v_T = r\left(\frac{\theta}{t}\right) = r\left(\frac{\theta}{\frac{x}{v}}\right) = \frac{r\theta v}{x}$$

SOLUTION The tangential speed of a point on the equator of the baseball is

$$v_T = \frac{r\theta v}{x} = \frac{(3.67\times10^{-2}\ \text{m})(49.0\ \text{rad})(42.5\ \text{m/s})}{16.5\ \text{m}} = \boxed{4.63\ \text{m/s}}$$

35. SSM ***REASONING AND SOLUTION***

a. From Equation 8.9, and the fact that 1 revolution = 2π radians, we obtain

$$v_T = r\omega = (0.0568\text{ m})\left(3.50\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \boxed{1.25\text{ m/s}}$$

b. Since the disk rotates at *constant* tangential speed,

$$v_{T1} = v_{T2} \qquad \text{or} \qquad \omega_1 r_1 = \omega_2 r_2$$

Solving for ω_2, we obtain

$$\omega_2 = \frac{\omega_1 r_1}{r_2} = \frac{(3.50\text{ rev/s})(0.0568\text{ m})}{0.0249\text{ m}} = \boxed{7.98\text{ rev/s}}$$

36. ***REASONING AND SOLUTION*** The figure below shows the initial and final states of the system.

INITIAL CONFIGURATION **FINAL CONFIGURATION**

a. From the principle of conservation of mechanical energy:

$$E_0 = E_f$$

Initially the system has only gravitational potential energy. If the level of the hinge is chosen as the zero level for measuring heights, then: $E_0 = mgh_0 = mgL$. Just before the object hits the floor, the system has only kinetic energy.

Therefore

$$mgL = \frac{1}{2}mv^2$$

Solving for v gives

$$v = \sqrt{2gL}$$

From Equation 8.9, $v_T = r\omega$. Solving for ω gives $\omega = v_T/r$. As the object rotates downward, it travels in a circle of radius L. Its speed just before it strikes the floor is its tangential speed. Therefore,

$$\omega = \frac{v_T}{r} = \frac{v}{L} = \frac{\sqrt{2gL}}{L} = \sqrt{\frac{2g}{L}} = \sqrt{\frac{2(9.80\text{ m/s}^2)}{1.50\text{ m}}} = \boxed{3.61\text{ rad/s}}$$

b. From Equation 8.10:

$$a_T = r\alpha$$

Solving for α gives $\alpha = a_T/r$. Just before the object hits the floor, its tangential acceleration is the acceleration due to gravity. Thus,

$$\alpha = \frac{a_T}{r} = \frac{g}{L} = \frac{9.80\text{ m/s}^2}{1.50\text{ m}} = \boxed{6.53\text{ rad/s}^2}$$

37. ***REASONING*** The golf ball must travel a distance equal to its diameter in a maximum time equal to the time required for one blade to move into the position of the previous blade.

SOLUTION The time required for the golf ball to pass through the opening between two blades is given by $\Delta t = \Delta\theta / \omega$, with $\omega = 1.25 \text{ rad/s}$ and $\Delta\theta = (2\pi \text{ rad})/16 = 0.393 \text{ rad}$. Therefore, the ball must pass between two blades in a maximum time of

$$\Delta t = \frac{0.393 \text{ rad}}{1.25 \text{ rad/s}} = 0.314 \text{ s}$$

The minimum speed of the ball is

$$v = \frac{\Delta x}{\Delta t} = \frac{4.50 \times 10^{-2} \text{ m}}{0.314 \text{ s}} = \boxed{1.43 \times 10^{-1} \text{ m/s}}$$

38. ***REASONING AND SOLUTION*** The centripetal acceleration of any point on the fan blade a distance r from the rotation axis is given by Equation 8.11, $a_c = r\omega^2$.

Thus, for any given point a distance r from the rotation axis,

$$\frac{a_1}{a_2} = \frac{r\,\omega_1^2}{r\,\omega_2^2} = \left(\frac{\omega_1}{\omega_2}\right)^2 = \left(\frac{440\text{ rev/min}}{110\text{ rev/min}}\right)^2 = \boxed{16}$$

In general, Equation 8.11 is only valid if the angular displacement is expressed in radians. In this problem, however, we are dealing with the *ratio* of two values of ω, so it is not necessary to convert the angular speeds to rad/s. The conversions factors will cancel.

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39. SSM ***REASONING*** The top of the racket has both tangential and centripetal acceleration components given by Equations 8.10 and 8.11, respectively: $a_T = r\alpha$ and $a_c = r\omega^2$. The total acceleration of the top of the racket is the resultant of these two components. Since these acceleration components are mutually perpendicular, their resultant can be found by using the Pythagorean theorem.

SOLUTION Employing the Pythagorean theorem, we obtain

$$a = \sqrt{a_T^2 + a_c^2} = \sqrt{(r\alpha)^2 + (r\omega^2)^2} = r\sqrt{\alpha^2 + \omega^4}$$

Therefore,

$$a = (1.5\text{ m})\sqrt{(160\text{ rad/s}^2)^2 + (14\text{ rad/s})^4} = \boxed{380\text{ m/s}^2}$$

40. ***REASONING***

a. According to Equation 8.2, the average angular speed is equal to the magnitude of the angular displacement divided by the elapsed time. The magnitude of the angular displacement is one revolution, or 2π rad. The elapsed time is one year, expressed in seconds.

b. The tangential speed of the earth in its orbit is equal to the product of its orbital radius and its orbital angular speed (Equation 8.9).

c. Since the earth is moving on a nearly circular orbit, it has a centripetal acceleration that is directed toward the center of the orbit. The magnitude a_c of the centripetal acceleration is given by Equation 8.11 as $a_c = r\omega^2$.

SOLUTION

a. The average angular speed is

$$\omega = \bar{\omega} = \frac{\Delta\theta}{\Delta t} = \frac{2\pi \text{ rad}}{3.16 \times 10^7 \text{ s}} = \boxed{1.99 \times 10^{-7} \text{ rad/s}} \tag{8.2}$$

b. The tangential speed of the earth in its orbit is

$$v_T = r\omega = \left(1.50 \times 10^{11} \text{ m}\right)\left(1.99 \times 10^{-7} \text{ rad/s}\right) = \boxed{2.98 \times 10^4 \text{ m/s}} \tag{8.9}$$

c. The centripetal acceleration of the earth due to its circular motion around the sun is

$$a_c = r\omega^2 = \left(1.50 \times 10^{11} \text{ m}\right)\left(1.99 \times 10^{-7} \text{ rad/s}\right)^2 = \boxed{5.94 \times 10^{-3} \text{ m/s}^2} \tag{8.11}$$

$\boxed{\text{The acceleration is directed toward the center of the orbit.}}$

41. SSM ***REASONING*** The tangential acceleration $\mathbf{a}_T$ of the speedboat can be found by using Newton's second law, $\mathbf{F}_T = m\mathbf{a}_T$, where $\mathbf{F}_T$ is the net tangential force. Once the tangential acceleration of the boat is known, Equation 2.4 can be used to find the tangential speed of the boat 2.0 s into the turn. With the tangential speed and the radius of the turn known, Equation 5.2 can then be used to find the centripetal acceleration of the boat.

SOLUTION

a. From Newton's second law, we obtain

$$a_T = \frac{F_T}{m} = \frac{550\text{ N}}{220\text{ kg}} = \boxed{2.5\text{ m/s}^2}$$

b. The tangential speed of the boat 2.0 s into the turn is, according to Equation 2.4,

$$v_T = v_{0T} + a_T t = 5.0\text{ m/s} + (2.5\text{ m/s}^2)(2.0\text{ s}) = 1.0 \times 10^1\text{ m/s}$$

The centripetal acceleration of the boat is then

$$a_c = \frac{v_T^2}{r} = \frac{(1.0 \times 10^1\text{ m/s})^2}{32\text{ m}} = \boxed{3.1\text{ m/s}^2}$$

42. ***REASONING AND SOLUTION*** The linear speed of all points on either sprocket, as well as the linear speed of all points on the chain, must be equal (otherwise the chain would bunch up or break).

a. Therefore, according to Equation 8.9, the linear speed of the chain as it moves between the sprockets is

$$v_T = r\omega = (9.00\ \text{cm})(9.40\ \text{rad/s}) = \boxed{84.6\ \text{cm/s}}$$

b. The centripetal acceleration of the chain as it passes around the rear sprocket is, according to Equation 8.11, $a_c = r_{rear}\omega^2$, where $\omega = v_T / r_{rear}$. Therefore,

$$a_c = r_{rear}\left(\frac{v_T}{r_{rear}}\right)^2 = \frac{v_T^2}{r_{rear}} = \frac{(84.6\ \text{cm/s})^2}{5.10\ \text{cm}} = \boxed{1.40 \times 10^3\ \text{cm/s}^2}$$

43. ***REASONING AND SOLUTION*** The tangential acceleration of point A is $a_{TA} = r_A\alpha$ and of point B is $a_{TB} = r_B\alpha$. Eliminating α gives

$$r_A/r_B = a_{TA}/a_{TB}$$

We are given that $a_{TA} = 2a_{TB}$, so $r_A/r_B = 2$.

From the figure in the text it is seen that

$$r_A = \sqrt{L_1^2 + L_2^2}$$

and

$$r_B = L_1$$

which gives

$$(L_2/L_1)^2 = (r_A/r_B)^2 - 1$$

Then

$$\frac{L_1}{L_2} = \boxed{\frac{1}{\sqrt{3}}}$$

---

44. ***REASONING AND SOLUTION*** Since the magnitude of the tangential acceleration equals the magnitude of the centripetal acceleration $a_T = a_c$, or $r\alpha = r\omega^2$. Thus, $\alpha = \omega^2$. The time at which these two accelerations are equal is $t = \omega/\alpha$. Combining these last two relations yields

$$t = \frac{1}{\sqrt{\alpha}} = \frac{1}{\sqrt{22.0\ \text{rad/s}^2}} = \boxed{0.213\,\text{s}}$$

45. SSM ***REASONING*** The tangential acceleration and the centripetal acceleration of a point at a distance r from the rotation axis are given by Equations 8.10 and 8.11, respectively: $a_T = r\alpha$ and $a_c = r\omega^2$. After the drill has rotated through the angle in question, $a_c = 2a_T$, or

$$r\omega^2 = 2r\alpha$$

This expression can be used to find the angular acceleration α. Once the angular acceleration is known, Equation 8.8 can be used to find the desired angle.

SOLUTION Solving the expression obtained above for α gives

$$\alpha = \frac{\omega^2}{2}$$

Solving Equation 8.8 for θ (with ω_0 = 0 rad/s since the drill starts from rest), and using the expression above for the angular acceleration α gives

$$\theta = \frac{\omega^2}{2\alpha} = \frac{\omega^2}{2(\omega^2/2)} = \left(\frac{\omega^2}{2}\right)\left(\frac{2}{\omega^2}\right) = \boxed{1.00 \text{ rad}}$$

Note that since both Equations 8.10 and 8.11 require that the angles be expressed in radians, the final result for θ is in radians.

46. ***REASONING AND SOLUTION*** The bike would travel with the same speed as a point on the wheel $v = r\omega$. It would then travel a distance

$$x = vt = r\omega t = (0.45\text{ m})(9.1\text{ rad/s})(35\text{ min})\left(\frac{60\text{ s}}{1\text{ min}}\right) = \boxed{8.6\times10^{3}\text{ m}}$$

---

47. SSM WWW ***REASONING AND SOLUTION*** From Equation 2.4, the linear acceleration of the motorcycle is

$$a = \frac{v - v_0}{t} = \frac{22.0 \text{ m/s} - 0 \text{ m/s}}{9.00 \text{ s}} = 2.44 \text{ m/s}^2$$

Since the tire rolls without slipping, the linear acceleration equals the tangential acceleration of a point on the outer edge of the tire: $a = a_T$. Solving Equation 8.13 for α gives

$$\alpha = \frac{a_T}{r} = \frac{2.44 \text{ m/s}^2}{0.280 \text{ m}} = \boxed{8.71 \text{ rad/s}^2}$$

48. ***REASONING AND SOLUTION***

a. If the wheel does not slip, a point on the rim rotates about the axle with a speed

$$v_T = v = 15.0 \text{ m/s}$$

For a point on the rim

$$\omega = v_T/r = (15.0 \text{ m/s})/(0.330 \text{ m}) = \boxed{45.5 \text{ rad/s}}$$

b. $$v_T = r\omega = (0.175 \text{ m})(45.5 \text{ rad/s}) = \boxed{7.96 \text{ m/s}}$$

---

49. ***REASONING AND SOLUTION***

a. $\omega_A = v/r = (0.381 \text{ m/s})/(0.0508 \text{ m}) = \boxed{7.50 \text{ rad/s}}$

b. $\omega_B = v/r = (0.381 \text{ m/s})(0.114 \text{ m}) = 3.34 \text{ rad/s}$

$$\overline{\alpha} = \frac{\omega_B - \omega_A}{t} = \frac{3.34 \text{ rad/s} - 7.50 \text{ rad/s}}{2.40 \times 10^3 \text{ s}} = \boxed{-1.73 \times 10^{-3} \text{ rad/s}^2}$$

The angular velocity is $\boxed{\text{decreasing}}$.

---

50. ***REASONING*** The angle through which the tire rotates is equal to its average angular velocity $\bar{\omega}$ multiplied by the elapsed time t, $\theta = \bar{\omega} t$. According to Equation 8.6, this angle is related to the initial and final angular velocities of the tire by

$$\theta = \bar{\omega} t = \tfrac{1}{2}(\omega_0 + \omega)t$$

The tire is assumed to roll at a constant angular velocity, so that $\omega_0 = \omega$ and $\theta = \omega t$. Since the tire is rolling, its angular speed is related to its linear speed v by Equation 8.12, $v = r\omega$, where r is the radius of the tire. The angle of rotation then becomes

$$\theta = \omega t = \left(\frac{v}{r}\right)t$$

The time t that it takes for the tire to travel a distance x is equal to $t = x/v$, according to Equation 2.1. Thus, the angle that the tire rotates through is

$$\theta = \left(\frac{v}{r}\right)t = \left(\frac{v}{r}\right)\left(\frac{x}{v}\right) = \frac{x}{r}$$

SOLUTION Since 1 rev = 2π rad, the angle (in revolutions) is

$$\theta = \frac{x}{r} = \frac{96\,000 \times 10^3 \text{ m}}{0.31 \text{ m}} = 3.1 \times 10^8 \text{ rad}$$

$$\theta = \left(3.1 \times 10^8 \text{ rad}\right)\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{4.9 \times 10^7 \text{ rev}}$$

51. SSM ***REASONING AND SOLUTION*** According to Equation 8.9, the crank handle has an angular speed of

$$\omega = \frac{v_T}{r} = \frac{1.20 \text{ m/s}}{\frac{1}{2}(0.400 \text{ m})} = 6.00 \text{ rad/s}$$

The crank barrel must have the same angular speed as the handle; therefore, the tangential speed of a point on the barrel is, again using Equation 8.9,

$$v_T = r\omega = (5.00 \times 10^{-2} \text{ m})(6.00 \text{ rad/s}) = 0.300 \text{ m/s}$$

If we assume that the rope does not slip, then all points on the rope must move with the same speed as the tangential speed of any point on the barrel. Therefore the linear speed with which the bucket moves down the well is $\boxed{v = 0.300 \text{ m/s}}$.

52. ***REASONING AND SOLUTION*** The angular speed of the ball can be found from Equation 8.12, $v = r\omega$. Solving for ω gives

$$\omega = \frac{v}{r} = \frac{3.60 \text{ m/s}}{0.200 \text{ m}} = 18.0 \text{ rad/s}$$

The time it takes for the ball to fall through the vertical distance y can be found from Equation 3.5b:

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$

Solving for t (with $v_{0y} = 0$ m/s, and taking "up" to be positive) gives

$$t = \sqrt{\frac{2y}{a_y}} = \sqrt{\frac{2(-2.10 \text{ m})}{(-9.80 \text{ m/s}^2)}} = 0.655 \text{ s}$$

Since the angular speed of the ball remains constant while it is falling,

$$\theta = \omega t = (18.0 \text{ rad/s})(0.655 \text{ s}) = \boxed{11.8 \text{ rad}}$$

53. ***REASONING AND SOLUTION***

a. If the rope is not slipping on the cylinder, then the tangential speed of the teeth on the larger gear (gear 1) is 2.50 m/s. The angular speed of gear 1 is then

$$\omega_1 = v/r_1 = (2.50 \text{ m/s})/(0.300 \text{ m}) = \boxed{8.33 \text{ rad/s}}$$

The direction of the larger gear is $\boxed{\text{counterclockwise}}$.

b. The gears are in contact and do not slip. This requires that the teeth on both gears move with the same tangential speed.

$$v_{T1} = v_{T2}$$

or

$$\omega_1 r_1 = \omega_2 r_2$$

So

$$\omega_2 = \left(\frac{r_1}{r_2}\right)\omega_1 = \left(\frac{0.300 \text{ m}}{0.170 \text{ m}}\right)(8.33 \text{ rad/s}) = \boxed{14.7 \text{ rad/s}}$$

The direction of the smaller gear is $\boxed{\text{clockwise}}$.

54. ***REASONING AND SOLUTION*** The angular speed of the front sprocket in "rad/s" is

$$\omega_{\text{front}} = \left(1.50\,\frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right) = 9.42\ \text{rad/s}$$

Since the front and rear sprockets are connected by the chain, the tangential speed of points on the rim of the front sprocket must equal the tangential speed of points on the rear sprocket. Therefore,

$$(v_T)_{\text{rear}} = (v_T)_{\text{front}} = r_{\text{front}}\,\omega_{\text{front}} = (0.125\ \text{m})(9.42\ \text{rad/s}) = 1.18\ \text{m/s}$$

The angular speed of the rear sprocket is

$$\omega_{\text{rear}} = \frac{(v_T)_{\text{rear}}}{r_{\text{rear}}} = \frac{1.18\ \text{m/s}}{0.0350\ \text{m}} = 33.7\ \text{rad/s}$$

Since the rear wheel and the rear sprocket turn together as a single unit, the angular speed of the rear wheel (as well as the front wheel) is $\omega_{\text{wheel}} = \omega_{\text{rear}} = 33.7$ rad/s.

From Equation 8.12, the linear speed of the bicycle is

$$v = r_{\text{wheel}}\,\omega_{\text{wheel}} = (0.400\ \text{m})(33.7\ \text{rad/s}) = \boxed{13.5\ \text{m/s}}$$

55. ***REASONING AND SOLUTION*** The distance d traveled by the axle of the wheel is given by Equation 8.1 as $d = r\,\theta$, where r is the distance from the center of the circular hill to the axle ($r = 9.00\text{ m} - 0.400\text{ m} = 8.60\text{ m}$) and $\theta = 0.960$ rad. Thus,

$$d = (8.60\text{ m})(0.960\text{ rad}) = 8.26\text{ m}$$

According to the discussion in Section 8.6, the distance d traveled by the axle equals the circular arc length s along the outer edge of the rotating wheel; $d = s$. But $s = r_w\,\theta_w$, where r_w is the radius of the wheel and θ_w is the angle through which the wheel rotates; so, $d = s = r_w\,\theta_w$. Solving for θ_w,

$$\theta_w = \frac{d}{r_w} = \frac{8.26\text{ m}}{0.400\text{ m}} = \boxed{20.6\text{ rad}}$$

56. ***REASONING***

a. Since the tire is rolling on the runway, the angular speed ω of each tire is related to its linear speed v and radius r by Equation 8.12, $\omega = v/r$.

b. There are three known variables in this problem: the initial angular velocity ω_0 (= 0 rad/s), the angular acceleration α, and the final angular velocity ω (from part a). Therefore, we can use Equation 8.8 from the equations of kinematics for rotational motion to find the angular displacement θ of each tire.

SOLUTION

a. The tire is rolling. When its linear speed is 72 m/s, its angular speed is

$$\omega = \frac{v}{r} = \frac{72\ \text{m/s}}{0.42\ \text{m}} = \boxed{170\ \text{rad/s}} \tag{8.12}$$

b. The angular displacement of a tire can be found directly from Equation 8.8, $\omega^2 = \omega_0^2 + 2\alpha\theta$, since the three rotational variables in this equation are known. Solving for θ gives

$$\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(170\ \text{rad/s})^2 - (0\ \text{rad/s})^2}{2\left(1.9\ \text{rad/s}^2\right)} = \boxed{7.6 \times 10^3\ \text{rad}}$$

57. SSM ***REASONING*** Since the car is traveling with a constant speed, its tangential acceleration must be zero. The radial or centripetal acceleration of the car can be found from Equation 5.2. Since the tangential acceleration is zero, the total acceleration of the car is equal to its radial acceleration.

SOLUTION

a. Using Equation 5.2, we find that the car's radial acceleration, and therefore its total acceleration, is

$$a = a_{\mathrm{R}} = \frac{v_{\mathrm{T}}^2}{r} = \frac{(75.0\ \mathrm{m/s})^2}{625\ \mathrm{m}} = \boxed{9.00\ \mathrm{m/s}^2}$$

b. The direction of the car's total acceleration is the same as the direction of its radial acceleration. That is, the direction is $\boxed{\text{radially inward}}$.

58. ***REASONING AND SOLUTION*** The linear speed at A must be the same as it is at every other point on the chain. Thus, from Equation 8.9 ($v_T = r\omega$), we have

$$\omega = \frac{v_T}{r} = \frac{7.7 \text{ m/s}}{3.5 \times 10^{-2} \text{ m}} \left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{35 \text{ rev/s}}$$

59. SSM ***REASONING AND SOLUTION*** Since the angular speed of the fan decreases, the sign of the angular acceleration must be opposite to the sign for the angular velocity. Taking the angular velocity to be positive, the angular acceleration, therefore, must be a negative quantity. Using Equation 8.4 we obtain

$$\omega_0 = \omega - \alpha t = 83.8 \text{ rad/s} - (-42.0 \text{ rad/s}^2)(1.75 \text{ s}) = \boxed{157.3 \text{ rad/s}}$$

60. ***REASONING AND SOLUTION***

a. The tangential acceleration of the train is given by Equation 8.10 as

$$a_T = r\alpha = (2.00\times10^2 \text{ m})(1.50\times10^{-3} \text{ rad/s}^2) = 0.300 \text{ m/s}^2$$

The centripetal acceleration of the train is given by Equation 8.11 as

$$a_c = r\omega^2 = (2.00\times10^2 \text{ m})(0.0500 \text{ rad/s})^2 = 0.500 \text{ m/s}^2$$

The magnitude of the total acceleration is found from the Pythagorean theorem to be

$$a = \sqrt{a_T^2 + a_c^2} = \boxed{0.583 \text{ m/s}^2}$$

b. The total acceleration vector makes an angle relative to the radial acceleration of

$$\theta = \tan^{-1}\left(\frac{a_T}{a_c}\right) = \tan^{-1}\left(\frac{0.300 \text{ m/s}^2}{0.500 \text{ m/s}^2}\right) = \boxed{31.0°}$$

61. ***REASONING AND SOLUTION***

a. $\omega = \omega_0 + \alpha t = 0 \text{ rad/s} + (3.00 \text{ rad/s}^2)(18.0 \text{ s}) = \boxed{54.0 \text{ rad/s}}$

b. $\theta = (1/2)(\omega_0 + \omega)t = (1/2)(0 \text{ rad/s} + 54.0 \text{ rad/s})(18.0 \text{ s}) = \boxed{486 \text{ rad}}$

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62. ***REASONING***

a. The tangential speed v_T of the sun as it orbits about the center of the Milky Way is related to the orbital radius r and angular speed ω by Equation 8.9, $v_T = r\omega$. Before we use this relation, however, we must first convert r to meters from light-years.

b. The centripetal force is the net force required to keep an object, such as the sun, moving on a circular path. According to Newton's second law of motion, the magnitude F_c of the centripetal force is equal to the product of the object's mass m and the magnitude a_c of its centripetal acceleration (see Section 5.3): $F_c = ma_c$. The magnitude of the centripetal acceleration is expressed by Equation 8.11 as $a_c = r\omega^2$, where r is the radius of the circular path and ω is the angular speed of the object.

SOLUTION

a. The radius of the sun's orbit about the center of the Milky Way is

$$r = \left(2.3 \times 10^4 \text{ light-years}\right)\left(\frac{9.5 \times 10^{15} \text{ m}}{1 \text{ light-year}}\right) = 2.2 \times 10^{20} \text{ m}$$

The tangential speed of the sun is

$$v_T = r\omega = \left(2.2 \times 10^{20} \text{ m}\right)\left(1.1 \times 10^{-15} \text{ rad/s}\right) = \boxed{2.4 \times 10^5 \text{ m/s}} \qquad (8.9)$$

b. The magnitude of the centripetal force that acts on the sun is

$$\underbrace{F_c}_{\substack{\text{Centripetal}\\ \text{force}}} = ma_c = m r \omega^2$$

$$= \left(1.99 \times 10^{30} \text{ kg}\right)\left(2.2 \times 10^{20} \text{ m}\right)\left(1.1 \times 10^{-15} \text{ rad/s}\right)^2 = \boxed{5.3 \times 10^{20} \text{ N}}$$

63. SSM WWW ***REASONING*** Assuming that the belt does not slip on the platter or the shaft pulley, the tangential speed of points on the platter and shaft pulley must be equal; therefore,

$$r_s\omega_s = r_p\omega_p$$

SOLUTION Solving the above expression for ω_s gives

$$\omega_s = \frac{r_p\omega_p}{r_s} = \frac{(3.49\ \text{rad/s})(0.102\ \text{m})}{1.27\times10^{-2}\ \text{m}} = \boxed{28.0\ \text{rad/s}}$$

64. ***REASONING AND SOLUTION***

a. If the propeller is to appear stationary, each blade must move through an angle of 120° or $2\pi/3$ rad between flashes. The time required is

$$t = \frac{\theta}{\omega} = \frac{(2\pi/3)\text{ rad}}{(16.7\text{ rev/s})\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)} = \boxed{2.00\times10^{-2}\text{ s}}$$

b. The next shortest time occurs when each blade moves through an angle of 240°, or $4\pi/3$ rad, between successive flashes. This time is twice that found in part a, or $\boxed{4.00\times10^{-2}\text{ s}}$.

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65. ***REASONING AND SOLUTION*** The time that it takes the fan to rotate from rest to the beginning of the 11 s period is

$$t_1 = \frac{\omega_1}{\alpha}$$

Kinematics applied to the 11 s period gives

$$\omega_1 = \frac{\theta}{t_2} - \tfrac{1}{2}\alpha t_2 = \frac{285 \text{ rad}}{11.0 \text{ s}} - \tfrac{1}{2}\left(2.00 \text{ rad/s}^2\right)\left(11.0 \text{ s}\right) = 14.9 \text{ rad/s}$$

Then

$$t_1 = (14.9 \text{ rad/s})/(2.00 \text{ rad/s}^2) = \boxed{7.45 \text{ s}}$$

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66. ***REASONING*** The drawing shows a top view of the race car as it travels around the circular turn. Its acceleration **a** has two perpendicular components: a centripetal acceleration $\mathbf{a}_c$ that arises because the car is moving on a circular path and a tangential acceleration $\mathbf{a}_T$ due to the fact that the car has an angular acceleration and its angular velocity is increasing. We can determine the magnitude of the centripetal acceleration from Equation 8.11 as $a_c = r\omega^2$, since both r and ω are given in the statement of the problem. As the drawing shows, we can use trigonometry to determine the magnitude a of the total acceleration, since the angle (35.0°) between **a** and $\mathbf{a}_c$ is given.

SOLUTION Since the vectors $\mathbf{a}_c$ and **a** are one side and the hypotenuse of a right triangle, we have that

$$a = \frac{a_c}{\cos 35.0°}$$

The magnitude of the centripetal acceleration is given by Equation 8.11 as $a_c = r\omega^2$, so the magnitude of the total acceleration is

$$a = \frac{a_c}{\cos 35.0°} = \frac{r\omega^2}{\cos 35.0°} = \frac{(23.5\text{ m})(0.571\text{ rad/s})^2}{\cos 35.0°} = \boxed{9.35\text{ m/s}^2}$$

67. SSM ***REASONING AND SOLUTION*** By inspection, the distance traveled by the "axle" or the center of the moving quarter is

$$d = 2\pi(2r) = 4\pi r$$

where r is the radius of the quarter. The distance d traveled by the "axle" of the moving quarter must be equal to the circular arc length s along the outer edge of the quarter. This arc length is $s = r\theta$, where θ is the angle through which the quarter rotates. Thus,

$$4\pi r = r\theta$$

so that $\theta = 4\pi \text{ rad}$. This is equivalent to

$$(4\pi \text{ rad})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{2 \text{ revolutions}}$$

---

68. ***REASONING AND SOLUTION*** The stone leaves the circular path with a horizontal speed

$$v_0 = v_T = r\omega$$

so $\omega = v_0/r$. We are given that $r = x/30$ so $\omega = 30v_0/x$. Kinematics gives $x = v_0 t$ and $h = (1/2)\, gt^2$. Using the above yields

$$\omega = 30\sqrt{\frac{g}{2h}} = 30\sqrt{\frac{9.80\ \text{m/s}^2}{2(20.0\ \text{m})}} = \boxed{14.8\ \text{rad/s}}$$

---

1. **SSM** ***REASONING AND SOLUTION*** Solving Equation 9.1 for the lever arm ℓ, we obtain

$$\ell = \frac{\tau}{F} = \frac{3.0\ \text{N}\cdot\text{m}}{12\ \text{N}} = \boxed{0.25\ \text{m}}$$

2. ***REASONING*** The torque is given by Equation 9.1, $\tau = F\ell$, where F is the magnitude of the applied force and ℓ is the lever arm. From the figure in the text, the lever arm is given by $\ell = (0.28\text{ m})\sin 50.0°$. Since both τ and ℓ are known, Equation 9.1 can be solved for F.

SOLUTION Solving Equation 9.1 for F, we have

$$F = \frac{\tau}{\ell} = \frac{45\text{ N}\cdot\text{m}}{(0.28\text{ m})\sin 50.0°} = \boxed{2.1\times 10^{2}\text{ N}}$$

---

3. ***REASONING AND SOLUTION*** The torque about the center of the cable car is $\tau = FL$, where L is the lever arm.

$$\tau = FL = 2(185\text{ N})(4.60\text{ m}) = \boxed{1.70 \times 10^3\text{ N}\cdot\text{m}}$$

4. ***REASONING*** To calculate the torques, we need to determine the lever arms for each of the forces. These lever arms are shown in the following drawings:

SOLUTION

a. Using Equation 9.1, we find that the magnitude of the torque due to the weight **W** is

$$\tau_W = W\ell_W = (10\,200\ \text{N})(2.5\ \text{m})\sin 32° = \boxed{13\,500\ \text{N}\cdot\text{m}}$$

b. Using Equation 9.1, we find that the magnitude of the torque due to the thrust **T** is

$$\tau_T = T\ell_T = (62\,300\ \text{N})(2.5\ \text{m})\cos 32° = \boxed{132\,000\ \text{N}\cdot\text{m}}$$

5. **SSM** ***REASONING*** The torque on either wheel is given by Equation 9.1, $\tau = F\ell$, where F is the magnitude of the force and ℓ is the lever arm. Regardless of how the force is applied, the lever arm will be proportional to the radius of the wheel.

SOLUTION The ratio of the torque produced by the force in the truck to the torque produced in the car is

$$\frac{\tau_{\text{truck}}}{\tau_{\text{car}}} = \frac{F\ell_{\text{truck}}}{F\ell_{\text{car}}} = \frac{Fr_{\text{truck}}}{Fr_{\text{car}}} = \frac{r_{\text{truck}}}{r_{\text{car}}} = \frac{0.25\ \text{m}}{0.19\ \text{m}} = \boxed{1.3}$$

6. ***REASONING*** To calculate the torque, we will need the lever arm of the 110 N force. The drawing identifies the lever arm ℓ.

SOLUTION According to Equation 9.1, the magnitude of the torque is

$$\tau = W\ell = (110\text{ N})(3.0\text{ m})\cos 15° = \boxed{320\text{ N}\cdot\text{m}}$$

7. ***REASONING AND SOLUTION*** The torque due to F_1 is

$$\tau_1 = -F_1 L_1 = -(20.0\ \text{N})(0.500\ \text{m}) = -10.0\ \text{N}\cdot\text{m}\ \ (\text{CW})$$

The torque due to F_2 is

$$\tau_2 = (F_2 \cos 30.0°)L_2 = (35.0\ \text{N})(1.10\ \text{m}) \cos 30.0° = 33.3\ \text{N}\cdot\text{m}\ \ (\text{CCW})$$

The net torque is therefore,

$$\Sigma\tau = \tau_1 + \tau_2 = -10.0\ \text{N}\cdot\text{m} + 33.3\ \text{N}\cdot\text{m} = \boxed{23.3\ \text{N}\cdot\text{m},\ \text{counterclockwise}}$$

---

8. ***REASONING AND SOLUTION*** The torque produced by a force of magnitude F is given by Equation 9.1, $\tau = F\ell$, where ℓ is the lever arm. In each case, the torque produced by the couple is equal to the sum of the individual torques produced by each member of the couple.

a. When the axis passes through point A, the torque due to the force at A is zero. The lever arm for the force at C is L. Therefore, taking counterclockwise as the positive direction, we have

$$\tau = \tau_A + \tau_C = 0 + \tau_C = \boxed{FL}$$

b. Each force produces a counterclockwise rotation. The magnitude of each force is F and each force has a lever arm of $L/2$. Taking counterclockwise as the positive direction, we have

$$\tau = \tau_A + \tau_C = F\left(\frac{L}{2}\right) + F\left(\frac{L}{2}\right) = \boxed{FL}$$

c. When the axis passes through point C, the torque due to the force at C is zero. The lever arm for the force at A is L. Therefore, taking counterclockwise as the positive direction, we have

$$\tau = \tau_A + \tau_C = \tau_A + 0 = \boxed{FL}$$

Note that the value of the torque produced by the couple is the same in all three cases; in other words, when the couple acts on the tire wrench, the couple produces a torque that does *not* depend on the location of the axis.

9. SSM ***REASONING*** Since the meter stick does not move, it is in equilibrium. The forces and the torques, therefore, each add to zero. We can determine the location of the 6.00 N force, by using the condition that the sum of the torques must vectorially add to zero.

SOLUTION If we take counterclockwise torques as positive, then the torque of the first force about the pin is

$$\tau_1 = F_1 \ell_1 = (2.00\ \text{N})(1.00\ \text{m}) = 2.00\ \text{N}\cdot\text{m}$$

The torque due to the second force is

$$\tau_2 = -F_2\ (\sin 30.0°)\ \ell_2 = -(6.00\ \text{N})(\sin 30.0°)\ \ell_2 = -(3.00\ \text{N})\ \ell_2$$

The torque τ_2 is negative because the force F_2 tends to produce a clockwise rotation about the pinned end. Since the net torque is zero, we have

$$2.00\ \text{N}\cdot\text{m} + [-(3.00\ \text{N})\ell_2] = 0$$

Thus,

$$\ell_2 = \frac{2.00\ \text{N}\cdot\text{m}}{3.00\ \text{N}} = \boxed{0.667\ \text{m}}$$

10. ***REASONING AND SOLUTION*** The net torque about the axis in text drawing (a) is

$$\Sigma\tau = \tau_1 + \tau_2 = F_1 b - F_2 a = 0.$$

Considering that $F_2 = 3F_1$, we have $b - 3a = 0$. The net torque in drawing (b) is then

$$\Sigma\tau = F_1(1.00\text{ m} - a) - F_2 b = 0 \quad \text{or} \quad 1.00\text{ m} - a - 3b = 0.$$

Solving the first equation for b, substituting into the second equation and rearranging, gives

$$a = \boxed{0.100\text{ m}} \quad \text{and} \quad b = \boxed{0.300\text{ m}}$$

---

11. ***REASONING AND SOLUTION***

a. Taking torques about an axis through the rear axle of the car gives

$$2F_f(2.54\text{ m}) - (1160\text{ kg})(9.80\text{ m/s}^2)(1.52\text{ m}) = 0 \quad \text{or} \quad F_f = \boxed{3.40 \times 10^3\text{ N}}$$

b. Taking torques about an axis through the front axle gives

$$(1160\text{ kg})(9.80\text{ m/s}^2)(1.02\text{ m}) - 2F_r(2.54\text{ m}) = 0 \quad \text{or} \quad F_r = \boxed{2.28 \times 10^3\text{ N}}$$

---

12. ***REASONING*** The drawing shows the forces acting on the person. It also shows the lever arms for a rotational axis perpendicular to the plane of the paper at the place where the person's toes touch the floor. Since the person is in equilibrium, the sum of the forces must be zero. Likewise, we know that the sum of the torques must be zero.

SOLUTION Taking upward to be the positive direction, we have

$$F_{\text{FEET}} + F_{\text{HANDS}} - W = 0$$

Remembering that counterclockwise torques are positive and using the axis and the lever arms shown in the drawing, we find

$$W\ell_{\text{W}} - F_{\text{HANDS}}\ell_{\text{HANDS}} = 0$$

$$F_{\text{HANDS}} = \frac{W\ell_{\text{W}}}{\ell_{\text{HANDS}}} = \frac{(584\ \text{N})(0.840\ \text{m})}{1.250\ \text{m}} = 392\ \text{N}$$

Substituting this value into the balance-of-forces equation, we find

$$F_{\text{FEET}} = W - F_{\text{HANDS}} = 584\ \text{N} - 392\ \text{N} = 192\ \text{N}$$

The force on each hand is half the value calculated above, or $\boxed{196\ \text{N}}$. Likewise, the force on each foot is half the value calculated above, or $\boxed{96\ \text{N}}$.

13. SSM ***REASONING*** The minimum value for the coefficient of static friction between the ladder and the ground, so that the ladder does not slip, is given by Equation 4.7:

$$f_s^{\text{MAX}} = \mu_s F_N$$

SOLUTION From Example 4, the magnitude of the force of static friction is $G_x = 727$ N. The magnitude of the normal force applied to the ladder by the ground is $G_y = 1230$ N. The minimum value for the coefficient of static friction between the ladder and the ground is

$$\mu_s = \frac{f_s^{\text{MAX}}}{F_N} = \frac{G_x}{G_y} = \frac{727\text{ N}}{1230\text{ N}} = \boxed{0.591}$$

14. ***REASONING*** When the board just begins to tip, three forces act on the board. They are the weight W of the board, the weight W_P of the person, and the force F exerted by the right support.

Since the board will rotate around the right support, the lever arm for this force is zero, and the torque exerted by the right support is zero. The lever arm for the weight of the board is equal to one-half the length of the board minus the overhang length: $2.5\text{ m} - 1.1\text{ m} = 1.4\text{ m}$.

The lever arm for the weight of the person is x. Therefore, taking counterclockwise torques as positive, we have

$$-W_P x + W(1.4\text{ m}) = 0$$

This expression can be solved for x.

SOLUTION Solving the expression above for x, we obtain

$$x = \frac{W(1.4\text{ m})}{W_P} = \frac{(225\text{ N})(1.4\text{ m})}{450\text{ N}} = \boxed{0.70\text{ m}}$$

15. SSM ***REASONING*** The figure at the right shows the door and the forces that act upon it. Since the door is uniform, the center of gravity, and, thus, the location of the weight **W,** is at the geometric center of the door.

Let **U** represent the force applied to the door by the upper hinge, and **L** the force applied to the door by the lower hinge. Taking forces that point to the right and forces that point up as positive, we have

$$\sum F_x = L_x - U_x = 0 \quad \text{or} \quad L_x = U_x \qquad (1)$$

$$\sum F_y = L_y - W = 0 \quad \text{or} \quad L_y = W = 140 \text{ N} \qquad (2)$$

Taking torques about an axis perpendicular to the plane of the door and through the lower hinge, with counterclockwise torques being positive, gives

$$\sum \tau = -W\left(\frac{d}{2}\right) + U_x h = 0 \qquad (3)$$

Equations (1), (2), and (3) can be used to find the force applied to the door by the hinges; Newton's third law can then be used to find the force applied by the door to the hinges.

SOLUTION

a. Solving Equation (3) for U_x gives

$$U_x = \frac{Wd}{2h} = \frac{(140 \text{ N})(0.81 \text{ m})}{2(2.1 \text{ m})} = 27 \text{ N}$$

From Equation (1), $L_x = 27$ N.

Therefore, the upper hinge exerts on the door a horizontal force of $\boxed{\text{27 N to the left}}$.

b. From Equation (1), $L_x = 27$ N, we conclude that the bottom hinge exerts on the door a horizontal force of $\boxed{\text{27 N to the right}}$.

c. From Newton's third law, the door applies a force of $\boxed{\text{27 N}}$ on the upper hinge. This force $\boxed{\text{points horizontally to the right}}$, according to Newton's third law.

d. Let **D** represent the force applied by the door to the lower hinge. Then, from Newton's third law, the force exerted on the lower hinge by the door has components $D_x = -27$ N and $D_y = -140$ N.

From the Pythagorean theorem,

$$D = \sqrt{D_x^2 + D_y^2} = \sqrt{(-27\text{ N})^2 + (-140\text{ N})^2} = \boxed{143\text{ N}}$$

The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{-140\text{ N}}{-27\text{ N}}\right) = 79°$$

The force is directed $\boxed{79° \text{ below the horizontal}}$.

16. ***REASONING AND SOLUTION*** The net torque about an axis through the contact point between the tray and the thumb is

$$\Sigma\tau = F(0.0400\text{ m}) - (0.250\text{ kg})(9.80\text{ m/s}^2)(0.320\text{ m}) - (1.00\text{ kg})(9.80\text{ m/s}^2)(0.180\text{ m}) - (0.200\text{ kg})(9.80\text{ m/s}^2)(0.140\text{ m}) = 0$$

$$\boxed{F = 70.6\text{ N, up}}$$

Similarly, the net torque about an axis through the point of contact between the tray and the finger is

$$\Sigma\tau = T(0.0400\text{ m}) - (0.250\text{ kg})(9.80\text{ m/s}^2)(0.280\text{ m}) - (1.00\text{ kg})(9.80\text{ m/s}^2)(0.140\text{ m}) - (0.200\text{ kg})(9.80\text{ m/s}^2)(0.100\text{ m}) = 0$$

$$\boxed{T = 56.4\text{ N, down}}$$

17. ***REASONING AND SOLUTION*** When the arm is in equilibrium, the sum of the torques is zero. Taking torques about the elbow joint with counterclockwise torques taken as positive, we have

$$-F\ell_{\mathrm{F}} + M\ell_{\mathrm{M}} = 0$$

Solving for M,

$$M = F\left(\frac{\ell_{\mathrm{F}}}{\ell_{\mathrm{M}}}\right) = (170\ \mathrm{N})\left(\frac{0.30\ \mathrm{m}}{0.051\ \mathrm{m}}\right) = \boxed{1.0 \times 10^{3}\ \mathrm{N}}$$

The direction of the force is to the $\boxed{\text{left}}$.

---

18. ***REASONING*** Although this arrangement of body parts is vertical, we can apply Equation 9.3 to locate the overall center of gravity by simply replacing the horizontal position x by the vertical position y, as measured relative to the floor.

SOLUTION Using Equation 9.3, we have

$$y_{\text{cg}} = \frac{W_1 y_1 + W_2 y_2 + W_3 y_3}{W_1 + W_2 + W_3}$$

$$= \frac{(438\ \text{N})(1.28\ \text{m}) + (144\ \text{N})(0.760\ \text{m}) + (87\ \text{N})(0.250\ \text{m})}{438\ \text{N} + 144\ \text{N} + 87\ \text{N}} = \boxed{1.03\ \text{m}}$$

19. ***REASONING***

The jet is in equilibrium, so the sum of the external forces is zero, and the sum of the external torques is zero. We can use these two conditions to evaluate the forces exerted on the wheels.

SOLUTION

a. Let F_f be the magnitude of the normal force that the ground exerts on the front wheel. Since the net torque acting on the plane is zero, we have (using an axis through the points of contact between the rear wheels and the ground)

$$\Sigma\tau = -W\,\ell_w + F_f\,\ell_f = 0$$

where W is the weight of the plane, and ℓ_w and ℓ_f are the lever arms for the forces W and F_f, respectively. Thus,

$$\Sigma\tau = -(1.00 \times 10^6\ \text{N})(15.0\ \text{m} - 12.6\ \text{m}) + F_f\,(15.0\ \text{m}) = 0$$

Solving for F_f gives $F_f = \boxed{1.60 \times 10^5\ \text{N}}$.

b. Setting the sum of the vertical forces equal to zero yields

$$\Sigma F_y = F_f + 2F_r - W = 0$$

where the factor of 2 arises because there are two rear wheels. Substituting in the data,

$$\Sigma F_y = 1.60 \times 10^5\ \text{N} + 2F_r - 1.00 \times 10^6\ \text{N} = 0$$

$$F_r = \boxed{4.20 \times 10^5\ \text{N}}$$

20. ***REASONING AND SOLUTION***

a. Taking torques about an axis through the bolt we have

$$-(4.5 \times 10^3 \text{ N})(2.50 \text{ m}) - (12.0 \times 10^3 \text{ N})(3.50 \text{ m}) + T(5.00 \text{ m}) \sin 25.0° = 0$$

Solving for T yields

$$T = \boxed{2.52 \times 10^4 \text{ N}}$$

b. Applying Newton's second law in the horizontal direction gives $-T \cos 25.0° + F_h = 0$, so

$$F_h = 2.28 \times 10^4 \text{ N}$$

In the vertical direction $F_v - W_L - W_B + T \sin 25.0° = 0$, and

$$F_v = 5.85 \times 10^3 \text{ N}$$

Now the force exerted on the beam by the bolt is

$$F = \sqrt{F_h^2 + F_v^2} = \boxed{2.35 \times 10^4 \text{ N}}$$

---

21. SSM WWW ***REASONING AND SOLUTION*** The figure below shows the massless board and the forces that act on the board due to the person and the scales.

a. Applying Newton's second law to the vertical direction gives

$$315\text{ N} + 425\text{ N} - W = 0 \qquad \text{or} \qquad \boxed{\mathbf{W} = 7.40 \times 10^2\text{ N, downward}}$$

b. Let x be the position of the center of gravity relative to the scale at the person's head. Taking torques about an axis through the contact point between the scale at the person's head and the board gives

$$(315\text{ N})(2.00\text{ m}) - (7.40 \times 10^2\text{ N})x = 0 \qquad \text{or} \qquad \boxed{x = 0.851\text{ m}}$$

22. ***REASONING AND SOLUTION*** The net torque about an axis through the knee joint is

$$\Sigma\tau = M(\sin 25.0°)(0.100\text{ m}) - (44.5\text{ N})(\cos 30.0°)(0.250\text{ m}) = 0$$

$$\boxed{M = 228\text{ N}}$$

23. SSM ***REASONING*** Since the man holds the ball motionless, the ball and the arm are in equilibrium. Therefore, the net force, as well as the net torque about any axis, must be zero.

SOLUTION

a. Using Equation 9.1, the net torque about an axis through the elbow joint is

$$\Sigma\tau = M(0.0510\text{ m}) - (22.0\text{ N})(0.140\text{ m}) - (178\text{ N})(0.330\text{ m}) = 0$$

Solving this expression for M gives $\boxed{M = 1.21 \times 10^3\text{ N}}$.

b. The net torque about an axis through the center of gravity is

$$\Sigma\tau = -(1210\text{ N})(0.0890\text{ m}) + F(0.140\text{ m}) - (178\text{ N})(0.190\text{ m}) = 0$$

Solving this expression for F gives $\boxed{F = 1.01 \times 10^3\text{ N}}$. Since the forces must add to give a net force of zero, we know that the direction of **F** is $\boxed{\text{downward}}$.

24. ***REASONING*** If we assume that the system is in equilibrium, we know that the vector sum of all the forces, as well as the vector sum of all the torques, that act on the system must be zero.

The figure below shows a free body diagram for the boom. Since the boom is assumed to be uniform, its weight $\mathbf{W}_B$ is located at its center of gravity, which coincides with its geometrical center. There is a tension **T** in the cable that acts at an angle θ to the horizontal, as shown. At the hinge pin *P*, there are two forces acting. The vertical force **V** that acts on the end of the boom prevents the boom from falling down. The horizontal force **H** that also acts at the hinge pin prevents the boom from sliding to the left. The weight $\mathbf{W}_L$ of the wrecking ball (the "load") acts at the end of the boom.

By applying the equilibrium conditions to the boom, we can determine the desired forces.

SOLUTION The directions upward and to the right will be taken as the positive directions. In the *x* direction we have

$$\sum F_x = H - T\cos\theta = 0 \tag{1}$$

while in the *y* direction we have

$$\sum F_y = V - T\sin\theta - W_L - W_B = 0 \tag{2}$$

Equations (1) and (2) give us two equations in three unknown. We must, therefore, find a third equation that can be used to determine one of the unknowns. We can get the third equation from the torque equation.

In order to write the torque equation, we must first pick an axis of rotation and determine the lever arms for the forces involved. Since both *V* and *H* are unknown, we can eliminate them from the torque equation by picking the rotation axis through the point *P* (then both *V* and *H* have zero lever arms). If we let the boom have a length *L*, then the lever arm for $\mathbf{W}_L$ is $L\cos\phi$, while the lever arm for $\mathbf{W}_B$ is $(L/2)\cos\phi$. From the figure, we see that the lever arm for **T** is $L\sin(\phi-\theta)$. If we take counterclockwise torques as positive, then the torque equation is

$$\sum \tau = -W_B\left(\frac{L\cos\phi}{2}\right) - W_L L\cos\phi + TL\sin(\phi-\theta) = 0$$

Solving for *T*, we have

$$T = \frac{\frac{1}{2}W_B + W_L}{\sin(\phi - \theta)}\cos\phi \tag{3}$$

a. From Equation (3) the tension in the support cable is

$$T = \frac{\frac{1}{2}(3600\text{ N}) + 4800\text{ N}}{\sin(48° - 32°)}\cos 48° = \boxed{1.6 \times 10^4\text{ N}}$$

b. The force exerted on the lower end of the hinge at the point P is the vector sum of the forces **H** and **V**. According to Equation (1),

$$H = T\cos\theta = \left(1.6 \times 10^4\text{ N}\right)\cos 32° = 1.4 \times 10^4\text{ N}$$

and, from Equation (2)

$$V = W_L + W_B + T\sin\theta = 4800\text{ N} + 3600\text{ N} + \left(1.6 \times 10^4\text{ N}\right)\sin 32° = 1.7 \times 10^4\text{ N}$$

Since the forces **H** and **V** are at right angles to each other, the magnitude of their vector sum can be found from the Pythagorean theorem:

$$F_P = \sqrt{H^2 + V^2} = \sqrt{(1.4 \times 10^4\text{ N})^2 + (1.7 \times 10^4\text{ N})^2} = \boxed{2.2 \times 10^4\text{ N}}$$

25. ***REASONING AND SOLUTION*** Taking torques on the right leg about the apex of the "A" gives

$$\Sigma\tau = -(120\text{ N})\,L\cos 60.0° - FL\cos 30.0° + T(2L)\cos 60.0° = 0$$

where F is the horizontal component of the force exerted by the crossbar and T is the tension in the right string. Due to symmetry, the tension in the left string is also T. Newton's second law applied to the vertical gives

$$2T - 2W = 0 \qquad \text{so} \qquad T = 120\text{ N}$$

Substituting T = 120 N into the first equation yields $\boxed{F = 69\text{ N}}$.

---

26. ***REASONING*** The drawing shows the forces acting on the board, which has a length L. Wall 2 exerts a normal force P_2 on the lower end of the board. The maximum force of static friction that wall 2 can apply to the lower end of the board is $\mu_s P_2$ and is directed upward in the drawing. The weight W acts downward at the board's center. Wall 1 applies a normal force P_1 to the upper end of the board. We take upward and to the right as our positive directions.

SOLUTION Then, since the horizontal forces balance to zero, we have

$$P_1 - P_2 = 0 \tag{1}$$

The vertical forces also balance to zero:

$$\mu_s P_2 - W = 0 \tag{2}$$

Using an axis through the lower end of the board, we now balance the torques to zero:

$$W\left(\frac{L}{2}\right)(\cos\theta) - P_1 L(\sin\theta) = 0 \tag{3}$$

Rearranging Equation (3) gives

$$\tan\theta = \frac{W}{2P_1} \tag{4}$$

But $W = \mu_s P_2$ according to Equation (2), and $P_2 = P_1$ according to Equation (1). Therefore, $W = \mu_s P_1$, which can be substituted in Equation (4) to show that

$$\tan\theta = \frac{\mu_s P_1}{2P_1} = \frac{\mu_s}{2} = \frac{0.98}{2}$$

or

$$\theta = \tan^{-1}(0.49) = 26°$$

From the drawing at the right,

$$\cos\theta = \frac{d}{L}$$

Therefore, the longest board that can be propped between the two walls is

$$L = \frac{d}{\cos\theta} = \frac{1.5\text{ m}}{\cos 26°} = \boxed{1.7\text{ m}}$$

27. ***REASONING AND SOLUTION*** The weight W of the left side of the ladder, the normal force F_N of the floor on the left leg of the ladder, the tension T in the crossbar, and the reaction force R due to the right-hand side of the ladder, are shown in the figure below. In the vertical direction $-W + F_N = 0$, so that

$$F_N = W = mg = (10.0\text{ kg})(9.80\text{ m/s}^2) = 98.0\text{ N}$$

In the horizontal direction it is clear that R = T.
The net torque about the base of the ladder is

$$\Sigma\tau = -T\,[(1.00\text{ m})\sin 75.0^{\circ}] - W\,[(2.00\text{ m})\cos 75.0^{\circ}] + R\,[(4.00\text{ m})\sin 75.0^{\circ}] = 0$$

Substituting for W and using R = T, we obtain

$$T = \frac{(98.0\text{ N})(2.00\text{ m})\cos 75.0^{\circ}}{(3.00\text{ m})\sin 75.0^{\circ}} = \boxed{17.5\text{ N}}$$

28. ***REASONING*** The moment of inertia of the stool is the sum of the individual moments of inertia of its parts. According to Table 9.1, a circular disk of radius R has a moment of inertia of $I_{\text{disk}} = \frac{1}{2} M_{\text{disk}} R^2$ with respect to an axis perpendicular to the disk center. Each thin rod is attached perpendicular to the disk at its outer edge. Therefore, each particle in a rod is located at a perpendicular distance from the axis that is equal to the radius of the disk. This means that each of the rods has a moment of inertia of $I_{\text{rod}} = M_{\text{rod}} R^2$.

SOLUTION Remembering that the stool has three legs, we find that the its moment of inertia is

$$I_{\text{stool}} = I_{\text{disk}} + 3I_{\text{rod}} = \tfrac{1}{2} M_{\text{disk}} R^2 + 3M_{\text{rod}} R^2$$

$$= \tfrac{1}{2}(1.2\ \text{kg})(0.16\ \text{m})^2 + 3(0.15\ \text{kg})(0.16\ \text{m})^2 = \boxed{0.027\ \text{kg}\cdot\text{m}^2}$$

29. **SSM** ***REASONING AND SOLUTION*** For a rigid body rotating about a fixed axis, Newton's second law for rotational motion is given by Equation 9.7, $\sum \tau = I\alpha$, where I is the moment of inertia of the body and α is the angular acceleration expressed in rad/s^2. Equation 9.7 gives

$$I = \frac{\sum \tau}{\alpha} = \frac{10.0 \text{ N} \cdot \text{m}}{8.00 \text{ rad/s}^2} = \boxed{1.25 \text{ kg} \cdot \text{m}^2}$$

30. ***REASONING AND SOLUTION*** We know

$$\Sigma\tau = I\alpha = (0.16\ \text{kg}\cdot\text{m}^2)(7.0\ \text{rad/s}^2) = 1.1\ \text{kg}\cdot\text{m}^2/\text{s}^2 = \boxed{1.1\ \text{N}\cdot\text{m}}$$

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31. ***REASONING AND SOLUTION***

a. The net torque on the disk about the axle is

$$\Sigma\tau = F_1R - F_2R = (0.314\text{ m})(90.0\text{ N} - 125\text{ N}) = \boxed{-11\text{ N}\cdot\text{m}}$$

b. The angular acceleration is given by $\alpha = \Sigma\tau/I$. From Table 9.1, the moment of inertia of the disk is

$$I = (1/2)\,MR^2 = (1/2)(24.3\text{ kg})(0.314\text{ m})^2 = 1.20\text{ kg}\cdot\text{m}^2$$

$$\alpha = (-11\text{ N}\cdot\text{m})/(1.20\text{ kg}\cdot\text{m}^2) = \boxed{-9.2\text{ rad/s}^2}$$

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32. ***REASONING*** The rotational analog of Newton's second law is given by Equation 9.7, $\sum \tau = I\alpha$. Since the person pushes on the outer edge of one pane of the door with a force **F** that is directed perpendicular to the pane, the torque exerted on the door has a magnitude of FL, where the lever arm L is equal to the width of one pane of the door. Once the moment of inertia is known, Equation 9.7 can be solved for the angular acceleration α.

The moment of inertia of the door relative to the rotation axis is $I = 4I_P$, where I_P is the moment of inertia for one pane. According to Table 9.1, we find $I_P = \frac{1}{3}ML^2$, so that the rotational inertia of the door is $I = \frac{4}{3}ML^2$.

SOLUTION Solving Equation 9.7 for α, and using the expression for I determined above, we have

$$\alpha = \frac{FL}{\frac{4}{3}ML^2} = \frac{F}{\frac{4}{3}ML} = \frac{68\text{ N}}{\frac{4}{3}(85\text{ kg})(1.2\text{ m})} = \boxed{0.50\text{ rad/s}^2}$$

33. **SSM** ***REASONING*** The figure below shows eight particles, each one located at a different corner of an imaginary cube. As shown, if we consider an axis that lies along one edge of the cube, two of the particles lie on the axis, and for these particles $r = 0$. The next four particles closest to the axis have $r = \ell$, where ℓ is the length of one edge of the cube. The remaining two particles have $r = d$, where d is the length of the diagonal along any one of the faces. From the Pythagorean theorem, $d = \sqrt{\ell^2 + \ell^2} = \ell\sqrt{2}$.

According to Equation 9.6, the moment of inertia of a system of particles is given by $I = \sum mr^2$.

SOLUTION Direct application of Equation 9.6 gives

$$I = \sum mr^2 = 4(m\ell^2) + 2(md^2) = 4(m\ell^2) + 2(2m\ell^2) = 8m\ell^2$$

or

$$I = 8(0.12\text{ kg})(0.25\text{ m})^2 = \boxed{0.060\text{ kg}\cdot\text{m}^2}$$

34. ***REASONING AND SOLUTION***

a. We know that $\alpha = (\omega - \omega_o)/t$. But $\omega = (1/2)\omega_o$, so

$$\alpha = -(1/2)\omega_o/t = -(1/2)(88.0 \text{ rad/s})/(5.00 \text{ s}) = -8.80 \text{ rad/s}^2$$

The magnitude of α is $\boxed{8.80 \text{ rad/s}^2}$.

b. $\Sigma\tau = I\alpha$ and $FR = I\alpha$. The magnitude of the frictional force is

$$F = I\alpha/R = (0.850 \text{ kg}\cdot\text{m}^2)(8.80 \text{ rad/s}^2)/(0.0750 \text{ m}) = \boxed{99.7 \text{ N}}$$

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35. ***REASONING AND SOLUTION*** We know $\Sigma\tau = I\alpha$, where

$$I = (1/2)\ mr^2 = (1/2)(0.400\ \text{kg})(0.130\ \text{m})^2 = 3.38 \times 10^{-3}\ \text{kg}\cdot\text{m}^2$$

Also, $\omega = \omega_0 + \alpha t$, so that $\alpha = (85.0\ \text{rad/s} - 262\ \text{rad/s})/(18.0\ \text{s}) = -9.83\ \text{rad/s}^2$. Thus, the net torque is

$$\Sigma\tau = I\alpha = (3.38 \times 10^{-3}\ \text{kg}\cdot\text{m}^2)(-9.83\ \text{rad/s}^2) = \boxed{-3.32 \times 10^{-2}\ \text{N}\cdot\text{m}}$$

36. ***REASONING*** The force applied to the baton creates a net torque that gives the rod an angular acceleration, according to Newton's second law as expressed for rotational motion ($\Sigma\tau = I\alpha$). From the data given, we will calculate the moment of inertia I and the angular acceleration α. Newton's second law, then, will allow us to obtain the net torque $\Sigma\tau$. From the net torque, we will be able to determine the force, since the magnitude of the torque in this case is just the magnitude of the force times the lever arm of 2.0 cm.

SOLUTION The momentum of inertia of the baton is the sum of the individual moments of inertia of its parts. For a thin rod of length L rotating about an axis perpendicular to the rod at its center, the moment of inertia is $I_{\text{rod}} = \frac{1}{12} M_{\text{rod}} L^2$. For each ball (assumed to be very small compared to the length of the rod), the moment of inertia is $I_{\text{ball}} = M_{\text{ball}} (L/2)^2$. The total moment of inertia, then, is

$$I_{\text{baton}} = I_{\text{rod}} + 2I_{\text{ball}} = \tfrac{1}{12} M_{\text{rod}} L^2 + 2M_{\text{ball}} \left(\tfrac{1}{2} L\right)^2 = \tfrac{1}{12} M_{\text{rod}} L^2 + \tfrac{1}{2} M_{\text{ball}} L^2$$

Using Equation 8.4 from the equations of kinematics for rotational motion, we can determine the angular acceleration as

$$\alpha = \frac{\omega - \omega_0}{t}$$

where ω and ω_0 are, respectively, the final and initial angular velocities. Since only one force of magnitude F creates the torque and since Equation 9.1 gives the magnitude of the torque as the magnitude of the force times the lever arm ℓ, Newton's second law becomes $\Sigma\tau = F\ell = I\alpha$. Substituting our expressions for the moment of inertia and angular acceleration of the baton into the second law gives

$$F\ell = I_{\text{baton}}\alpha = \left(\tfrac{1}{12} M_{\text{rod}} L^2 + \tfrac{1}{2} M_{\text{ball}} L^2\right)\left(\frac{\omega - \omega_0}{t}\right)$$

In Newton's second law the angular acceleration α must be expressed in rad/s^2. Therefore, the values for the angular speeds ω and ω_0 must be expressed in rad/s. Recognizing that 2.0 rev/s is 4.0 π rad/s and solving for F, we find that

$$F = \frac{\left(\tfrac{1}{12} M_{\text{rod}} L^2 + \tfrac{1}{2} M_{\text{ball}} L^2\right)}{\ell}\left(\frac{\omega - \omega_0}{t}\right)$$

$$= \frac{\tfrac{1}{12}(0.13\text{ kg})(0.81\text{ m})^2 + \tfrac{1}{2}(0.13\text{ kg})(0.81\text{ m})^2}{0.020\text{ m}}\left(\frac{4\pi\text{ rad/s} - 0\text{ rad/s}}{0.40\text{ s}}\right) = \boxed{78\text{ N}}$$

37. SSM *REASONING AND SOLUTION*

a. From Table 9.1 in the text, the moment of inertia of the rod relative to an axis that is perpendicular to the rod at one end is given by

$$I_{\text{rod}} = \tfrac{1}{3}ML^2 = \tfrac{1}{3}(2.00\text{ kg})(2.00\text{ m})^2 = \boxed{2.67\text{ kg}\cdot\text{m}^2}$$

b. The moment of inertia of a point particle of mass m relative to a rotation axis located a perpendicular distance r from the particle is $I = mr^2$. Suppose that all the mass of the rod were located at a single point located a perpendicular distance R from the axis in part (a). If this point particle has the same moment of inertia as the rod in part (a), then the distance R, the radius of gyration, is given by

$$R = \sqrt{\frac{I}{m}} = \sqrt{\frac{2.67\text{ kg}\cdot\text{m}^2}{2.00\text{ kg}}} = \boxed{1.16\text{ m}}$$

38. ***REASONING AND SOLUTION*** The final angular speed of the arm is $\omega = v_T/r$, where $r = 0.28$ m. The angular acceleration needed to produce this angular speed is $\alpha = (\omega - \omega_0)/t$. The net torque required is $\Sigma\tau = I\alpha$. This torque is due solely to the force M, so that $\Sigma\tau = ML$. Thus,

$$M = \frac{\Sigma\tau}{L} = \frac{I\left(\dfrac{\omega - \omega_0}{t}\right)}{L}$$

Setting $\omega_0 = 0$ and $\omega = v_T/r$, the force becomes

$$M = \frac{I\left(\dfrac{v_T}{rt}\right)}{L} = \frac{Iv_T}{Lrt} = \frac{(0.065\ \text{kg}\cdot\text{m}^2)(5.0\ \text{m/s})}{(0.025\ \text{m})(0.28\ \text{m})(0.10\ \text{s})} = \boxed{460\ \text{N}}$$

39. SSM ***REASONING*** The angular acceleration of the bicycle wheel can be calculated from Equation 8.4. Once the angular acceleration is known, Equation 9.7 can be used to find the net torque caused by the brake pads. The normal force can be calculated from the torque using Equation 9.1.

SOLUTION The angular acceleration of the wheel is, according to Equation 8.4,

$$\alpha = \frac{\omega - \omega_0}{t} = \frac{3.7 \text{ rad/s} - 13.1 \text{ rad/s}}{3.0 \text{ s}} = -3.1 \text{ rad/s}^2$$

If we assume that all the mass of the wheel is concentrated in the rim, we may treat the wheel as a hollow cylinder. From Table 9.1, we know that the moment of inertia of a hollow cylinder of mass m and radius r about an axis through its center is $I = mr^2$. The net torque that acts on the wheel due to the brake pads is, therefore,

$$\sum \tau = I\alpha = (mr^2)\alpha \tag{1}$$

From Equation 9.1, the net torque that acts on the wheel due to the action of the two brake pads is

$$\sum \tau = -2 f_k \ell \tag{2}$$

where f_k is the kinetic frictional force applied to the wheel by each brake pad, and $\ell = 0.33$ m is the lever arm between the axle of the wheel and the brake pad (see the drawing in the text). The factor of 2 accounts for the fact that there are two brake pads. The minus sign arises because the net torque must have the same sign as the angular acceleration. The kinetic frictional force can be written as (see Equation 4.8)

$$f_k = \mu_k F_N \tag{3}$$

where μ_k is the coefficient of kinetic friction and F_N is the magnitude of the normal force applied to the wheel by each brake pad.

Combining Equations (1), (2), and (3) gives

$$-2(\mu_k F_N)\ell = (mr^2)\alpha$$

$$F_N = \frac{-mr^2\alpha}{2\mu_k \ell} = \frac{-(1.3 \text{ kg})(0.33 \text{ m})^2(-3.1 \text{ rad/s}^2)}{2(0.85)(0.33 \text{ m})} = \boxed{0.78 \text{ N}}$$

40. ***REASONING AND SOLUTION*** The moment of inertia of a solid cylinder about an axis coinciding with the cylinder axis which contains the center of mass is $I_{cm} = (1/2)\,MR^2$. The parallel axis theorem applied to an axis on the surface (h = R) gives $I = I_{cm} + MR^2$, so that

$$\boxed{I = \frac{3}{2}MR^2}$$

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41. ***REASONING AND SOLUTION*** For door A, $\theta = (1/2)\,\alpha_A t_A^2$. For door B, $\theta = (1/2)\,\alpha_B t_B^2$. Equating gives

$$t_B = t_A \sqrt{\frac{\alpha_A}{\alpha_B}}$$

We need the angular accelerations. Applying Newton's second law for rotation to door A

$$FL = (1/3)ML^2\alpha_A \quad \text{and to door B} \quad F(1/2)L = (1/12)ML^2\alpha_B$$

Division of the previous two equations yields $\alpha_A/\alpha_B = 1/2$. Now

$$t_B = t_A/\sqrt{2} = (3.00\text{ s})/(1.41) = \boxed{2.12\text{ s}}$$

42. ***REASONING*** The drawing shows the drum, pulley, and the crate, as well as the tensions in the cord

Let T_1 represent the magnitude of the tension in the cord between the drum and the pulley. Then, the net torque exerted on the drum must be, according to Equation 9.7, $\Sigma\tau = I_1\alpha_1$, where I_1 is the moment of inertia of the drum, and α_1 is its angular acceleration. If we assume that the cable does not slip, then Equation 9.7 can be written as

$$\underbrace{-T_1 r_1 + \tau}_{\Sigma\tau} = \underbrace{\left(m_1 r_1^2\right)}_{I_1} \underbrace{\left(\frac{a}{r_1}\right)}_{\alpha_1} \tag{1}$$

where τ is the counterclockwise torque provided by the motor, and a is the acceleration of the cord (a = 1.2 m/s^2). This equation cannot be solved for τ directly, because the tension T_1 is not known.

We next apply Newton's second law for rotational motion to the pulley in the drawing:

$$\underbrace{+T_1 r_2 - T_2 r_2}_{\Sigma\tau} = \underbrace{\left(\tfrac{1}{2} m_2 r_2^2\right)}_{I_2} \underbrace{\left(\frac{a}{r_2}\right)}_{\alpha_2} \tag{2}$$

where T_2 is the magnitude of the tension in the cord between the pulley and the crate, and I_2 is the moment of inertial of the pulley.

Finally, Newton's second law for translational motion ($\Sigma F_y = m\,a$) is applied to the crate, yielding

$$\underbrace{+T_2 - m_3 g}_{\Sigma F_y} = m_3 a \tag{3}$$

SOLUTION Solving Equation (1) for T_1 and substituting the result into Equation (2), then

solving Equation (2) for T_2 and substituting the result into Equation (3), results in the following value for the torque

$$\tau = r_1\left[a\left(m_1 + \tfrac{1}{2}m_2 + m_3\right) + m_3 g\right]$$

$$= (0.76\text{ m})\left[(1.2\text{ m/s}^2)\left(150\text{ kg} + \tfrac{1}{2}130\text{ kg} + 180\text{ kg}\right) + (180\text{ kg})(9.80\text{ m/s}^2)\right] = \boxed{1700\text{ N}\cdot\text{m}}$$

43. SSM ***REASONING*** The kinetic energy of the flywheel is given by Equation 9.9. The moment of inertia of the flywheel is the same as that of a solid disk, and, according to Table 9.1 in the text, is given by $I = \frac{1}{2}MR^2$. Once the moment of inertia of the flywheel is known, Equation 9.9 can be solved for the angular speed ω in rad/s. This quantity can then be converted to rev/min.

SOLUTION Solving Equation 9.9 for ω, we obtain,

$$\omega = \sqrt{\frac{2(\mathrm{KE_R})}{I}} = \sqrt{\frac{2(\mathrm{KE_R})}{\frac{1}{2}MR^2}} = \sqrt{\frac{4(1.2\times10^9\ \mathrm{J})}{(13\ \mathrm{kg})(0.30\ \mathrm{m})^2}} = 6.4\times10^4\ \mathrm{rad/s}$$

Converting this answer into rev/min, we find that

$$\omega = \left(6.4\times10^4\ \mathrm{rad/s}\right)\left(\frac{1\ \mathrm{rev}}{2\pi\ \mathrm{rad}}\right)\left(\frac{60\ \mathrm{s}}{1\ \mathrm{min}}\right) = \boxed{6.1\times10^5\ \mathrm{rev/min}}$$

44. ***REASONING*** The rotational kinetic energy of each object is given by Equation 9.9, $\mathrm{KE_R} = \frac{1}{2} I\omega^2$. To solve this problem, we need only set the two kinetic energies equal, being sure to use the proper moment of inertia for each object, as given in Table 9.1.

SOLUTION According to Table 9.1 the moment of inertia of the solid sphere is $I_{\text{solid}} = \frac{2}{5} MR^2$, while the moment of inertia of the shell is $I_{\text{shell}} = \frac{2}{3} MR^2$. Using these expressions in Equation 9.9 for the rotational kinetic energy, we obtain

$$\underbrace{\tfrac{1}{2} I_{\text{solid}} \omega_{\text{solid}}^2}_{\substack{\text{Rotational KE}\\ \text{of solid sphere}}} = \underbrace{\tfrac{1}{2} I_{\text{shell}} \omega_{\text{shell}}^2}_{\substack{\text{Rotational KE}\\ \text{of spherical shell}}}$$

$$\tfrac{1}{2}\left(\tfrac{2}{5} MR^2\right)\omega_{\text{solid}}^2 = \tfrac{1}{2}\left(\tfrac{2}{3} MR^2\right)\omega_{\text{shell}}^2$$

$$\tfrac{2}{5}\omega_{\text{solid}}^2 = \tfrac{2}{3}\omega_{\text{shell}}^2$$

$$\omega_{\text{solid}} = \sqrt{\tfrac{5}{3}}\,\omega_{\text{shell}} = \sqrt{\tfrac{5}{3}}\,(7.0\ \text{rad/s}) = \boxed{9.0\ \text{rad/s}}$$

45. **SSM** ***REASONING AND SOLUTION***

a. The tangential speed of each object is given by Equation 8.9, $v_T = r\omega$. Therefore,

For object 1: $v_{T1} = (2.00\text{ m})(6.00\text{ rad/s}) = \boxed{12.0\text{ m/s}}$

For object 2: $v_{T2} = (1.50\text{ m})(6.00\text{ rad/s}) = \boxed{9.00\text{ m/s}}$

For object 3: $v_{T3} = (3.00\text{ m})(6.00\text{ rad/s}) = \boxed{18.0\text{ m/s}}$

b. The total kinetic energy of this system can be calculated by computing the sum of the kinetic energies of each object in the system. Therefore,

$$\text{KE} = \tfrac{1}{2}m_1v_1^2 + \tfrac{1}{2}m_2v_2^2 + \tfrac{1}{2}m_3v_3^2$$

$$\text{KE} = \tfrac{1}{2}\left[(6.00\text{ kg})(12.0\text{ m/s})^2 + (4.00\text{ kg})(9.00\text{ m/s})^2 + (3.00\text{ kg})(18.0\text{ m/s})^2\right] = \boxed{1.08\times10^3\text{ J}}$$

c. The total moment of inertia of this system can be calculated by computing the sum of the moments of inertia of each object in the system. Therefore,

$$I = \sum mr^2 = m_1r_1^2 + m_2r_2^2 + m_3r_3^2$$

$$I = (6.00\text{ kg})(2.00\text{ m})^2 + (4.00\text{ kg})(1.50\text{ m})^2 + (3.00\text{ kg})(3.00\text{ m})^2 = \boxed{60.0\text{ kg}\cdot\text{m}^2}$$

d. The rotational kinetic energy of the system is, according to Equation 9.9,

$$\text{KE}_R = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}(60.0\text{ kg}\cdot\text{m}^2)(6.00\text{ rad/s})^2 = \boxed{1.08\times10^3\text{ J}}$$

This agrees, as it should, with the result for part (b).

46. ***REASONING***

a. The kinetic energy is given by Equation 9.9 as $\mathrm{KE_R} = \frac{1}{2}I\omega^2$. Assuming the earth to be a uniform solid sphere, we find from Table 9.1 that the moment of inertia is $I = \frac{2}{5}MR^2$. The mass and radius of the earth is $M = 5.98 \times 10^{24}$ kg and $R = 6.38 \times 10^6$ m (see the inside of the text's front cover). The angular speed ω must be expressed in rad/s, and we note that the earth turns once around its axis each day, which corresponds to 2π rad/day.

b. The kinetic energy for the earth's motion around the sun can be obtained from Equation 9.9 as $\mathrm{KE_R} = \frac{1}{2}I\omega^2$. Since the earth's radius is small compared to the radius of the earth's orbit ($R_{\text{orbit}} = 1.50 \times 10^{11}$ m, see the inside of the text's front cover), the moment of inertia in this case is just $I = MR_{\text{orbit}}^2$. The angular speed ω of the earth as it goes around the sun can be obtained from the fact that it makes one revolution each year, which corresponds to 2π rad/year.

SOLUTION

a. According to Equation 9.9, we have

$$\mathrm{KE_R} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(\tfrac{2}{5}MR^2\right)\omega^2$$

$$= \tfrac{1}{2}\left[\tfrac{2}{5}\left(5.98\times10^{24}\ \text{kg}\right)\left(6.38\times10^{6}\ \text{m}\right)^2\right]\left[\left(\frac{2\pi\ \text{rad}}{1\ \text{day}}\right)\left(\frac{1\ \text{day}}{24\ \text{h}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\right]^2$$

$$= \boxed{2.57\times10^{29}\ \text{J}}$$

b. According to Equation 9.9, we have

$$\mathrm{KE_R} = \tfrac{1}{2}I\omega^2 = \tfrac{1}{2}\left(MR_{\text{orbit}}^2\right)\omega^2$$

$$= \tfrac{1}{2}\left(5.98\times10^{24}\ \text{kg}\right)\left(1.50\times10^{11}\ \text{m}\right)^2\left[\left(\frac{2\pi\ \text{rad}}{1\ \text{yr}}\right)\left(\frac{1\ \text{yr}}{365\ \text{day}}\right)\left(\frac{1\ \text{day}}{24\ \text{h}}\right)\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\right]^2$$

$$= \boxed{2.67\times10^{33}\ \text{J}}$$

47. ***REASONING AND SOLUTION*** The conservation of energy gives for the cube $(1/2)\,mv_c^2 = mgh$, so $v_c = \sqrt{2gh}$. The conservation of energy gives for the marble $(1/2)\,mv_m^2 + (1/2)\,I\omega^2 = mgh$. Since the marble rolls without slipping: $\omega = v/R$. For a solid sphere we know $I = (2/5)\,mR^2$. Combining the last three equations gives

$$v_m = \sqrt{\frac{10gh}{7}}$$

Thus,

$$\frac{v_c}{v_m} = \sqrt{\frac{7}{5}} = \boxed{1.18}$$

48. ***REASONING AND SOLUTION*** The rotational kinetic energy of the shell is $KE_r = (1/2)\, I\omega^2$ and the translational kinetic energy is $KE_t = (1/2)\, Mv^2$. Now, $v = R\omega$ since the sphere rolls without slipping, so

$$KE_t = (1/2)\, MR^2\omega^2$$

The desired fraction is

$$KE_r/KE = I/(I + MR^2) = \boxed{2/5}$$

49. SSM ***REASONING AND SOLUTION*** The only force that does work on the cylinders as they move up the incline is the conservative force of gravity; hence, the total mechanical energy is conserved as the cylinders ascend the incline. We will let $h = 0$ on the horizontal plane at the bottom of the incline. Applying the principle of conservation of mechanical energy to the solid cylinder, we have

$$mgh_s = \tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I_s\omega_0^2 \qquad (1)$$

where, from Table 9.1, $I_s = \tfrac{1}{2}mr^2$. In this expression, v_0 and ω_0 are the initial translational and rotational speeds, and h_s is the final height attained by the solid cylinder. Since the cylinder rolls without slipping, the rotational speed ω_0 and the translational speed v_0 are related according to Equation 8.12, $\omega_0 = v_0 / r$. Then, solving Equation (1) for h_s, we obtain

$$h_s = \frac{3v_0^2}{4g}$$

Repeating the above for the hollow cylinder and using $I_h = mr^2$ we have

$$h_h = \frac{v_0^2}{g}$$

The height h attained by each cylinder is related to the distance s traveled along the incline and the angle θ of the incline by

$$s_s = \frac{h_s}{\sin\theta} \qquad \text{and} \qquad s_h = \frac{h_h}{\sin\theta}$$

Dividing these gives

$$\frac{s_s}{s_h} = \boxed{3/4}$$

50. ***REASONING AND SOLUTION*** The conservation of energy gives

$$mgh + (1/2)\,mv^2 + (1/2)\,I\omega^2 = (1/2)\,mv_o^2 + (1/2)\,I\omega_o^2$$

If the ball rolls without slipping, $\omega = v/R$ and $\omega_o = v_o/R$. We also know $I = (2/5)\,mR^2$. Substitution of the last two equations into the first and rearrangement gives

$$v = \sqrt{v_0^2 - \tfrac{10}{7}gh} = \sqrt{(3.50\ \text{m/s})^2 - \tfrac{10}{7}(9.80\ \text{m/s}^2)(0.760\ \text{m})} = \boxed{1.3\ \text{m/s}}$$

51. ***REASONING*** We first find the speed v_0 of the ball when it becomes airborne using the conservation of mechanical energy. Once v_0 is known, we can use the equations of kinematics to find its range x.

SOLUTION When the tennis ball starts from rest, its total mechanical energy is in the form of gravitational potential energy. The gravitational potential energy is equal to mgh if we take $h = 0$ at the height where the ball becomes airborne. Just before the ball becomes airborne, its mechanical energy is in the form of rotational kinetic energy and translational kinetic energy. At this instant its total energy is $\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2$. If we treat the tennis ball as a thin-walled spherical shell of mass m and radius r, and take into account that the ball rolls down the hill without slipping, its total kinetic energy can be written as

$$\frac{1}{2}mv_0^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv_0^2 + \frac{1}{2}\left(\frac{2}{3}mr^2\right)\left(\frac{v_0}{r}\right)^2 = \frac{5}{6}mv_0^2$$

Therefore, from conservation of mechanical energy, he have

$$mgh = \frac{5}{6}mv_0^2 \quad \text{or} \quad v_0 = \sqrt{\frac{6gh}{5}}$$

The range of the tennis ball is given by $x = v_{0x}t = v_0(\cos\theta)\,t$, where t is the flight time of the ball. From Equation 3.3b, we find that the flight time t is given by

$$t = \frac{v - v_{0y}}{a_y} = \frac{(-v_{0y}) - v_{0y}}{a_y} = -\frac{2v_0 \sin\theta}{a_y}$$

Therefore, the range of the tennis ball is

$$x = v_{0x}t = v_0(\cos\theta)\left(-\frac{2v_0 \sin\theta}{a_y}\right)$$

If we take upward as the positive direction, then using the fact that $a_y = -g$ and the expression for v_0 given above, we find

$$x = \left(\frac{2\cos\theta\sin\theta}{g}\right)v_0^2 = \left(\frac{2\cos\theta\sin\theta}{g}\right)\left(\sqrt{\frac{6gh}{5}}\right)^2 = \frac{12}{5}h\cos\theta\sin\theta$$

$$= \frac{12}{5}(1.8\text{ m})(\cos 35°)(\sin 35°) = \boxed{2.0\text{ m}}$$

52. ***REASONING AND SOLUTION*** The angular momentum of the satellite is conserved if only the gravitational force of the earth is acting on it. We have $L_A = L_P$ or $mv_A r_A = mv_P r_P$. Now

$$r_P = (v_A/v_P)r_A = (1/1.20)(1.30 \times 10^7 \text{ m}) = \boxed{1.08 \times 10^7 \text{ m}}$$

53. **SSM** **WWW** ***REASONING*** Let the two disks constitute the system. Since there are no external torques acting on the system, the principle of conservation of angular momentum applies. Therefore we have $L_{\text{initial}} = L_{\text{final}}$, or

$$I_A \omega_A + I_B \omega_B = (I_A + I_B)\omega_{\text{final}}$$

This expression can be solved for the moment of inertia of disk B.

SOLUTION Solving the above expression for I_B, we obtain

$$I_B = I_A \left(\frac{\omega_{\text{final}} - \omega_A}{\omega_B - \omega_{\text{final}}} \right) = (3.4\ \text{kg}\cdot\text{m}^2) \left[\frac{-2.4\ \text{rad/s} - 7.2\ \text{rad/s}}{-9.8\ \text{rad/s} - (-2.4\ \text{rad/s})} \right] = \boxed{4.4\ \text{kg}\cdot\text{m}^2}$$

54. ***REASONING AND SOLUTION*** Angular momentum is conserved, $I\omega = I_0\omega_0$, so

$$\omega = \left(\frac{I_0}{I}\right)\omega_0 = \left(\frac{5.40\ \text{kg}\cdot\text{m}^2}{3.80\ \text{kg}\cdot\text{m}^2}\right)(5.00\ \text{rad/s}) = \boxed{7.11\ \text{rad/s}}$$

55. ***REASONING AND SOLUTION***

a. Angular momentum is conserved, so that $I\omega = I_o\omega_o$, where

$$I = I_o + 10m_bR_b^2 = 2100\ \text{kg}\cdot\text{m}^2$$

$$\omega = (I_o/I)\omega_o = [(1500\ \text{kg}\cdot\text{m}^2)/(2100\ \text{kg}\cdot\text{m}^2)](0.20\ \text{rad/s}) = \boxed{0.14\ \text{rad/s}}$$

b. A net external torque must be applied in a direction that is opposite to the angular deceleration caused by the baggage dropping onto the carousel.

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56. ***REASONING AND SOLUTION***

The conservation of angular momentum applies about the indicated axis

$$I_d\omega_d + M_pR_p^2\omega_p = 0 \quad \text{where} \quad \omega_p = v_p/R_p$$

Now $I_d = (1/2)\, M_dR_d^2$. Combining and solving yields

$$\omega_d = -2(M_p/M_d)(R_p/R_d^2)v_p = -2\left(\frac{40.0\ \text{kg}}{1.00\times10^2\ \text{kg}}\right)\left[\frac{1.25\ \text{m}}{(2.00\ \text{m})^2}\right](2.00\ \text{m/s}) = -0.500\ \text{rad/s}$$

The magnitude of ω_d is $\boxed{0.500\ \text{rad/s}}$. The negative sign indicates that the disk rotates in a direction opposite to the motion of the person.

57. SSM ***REASONING*** Let the space station and the people within it constitute the system. Then as the people move radially from the outer surface of the cylinder toward the axis, any torques that occur are internal torques. Since there are no external torques acting on the system, the principle of conservation of angular momentum can be employed.

SOLUTION Since angular momentum is conserved,

$$I_{\text{final}}\omega_{\text{final}} = I_0\omega_0$$

Before the people move from the outer rim, the moment of inertia is

$$I_0 = I_{\text{station}} + 500m_{\text{person}}r_{\text{person}}^2$$

or

$$I_0 = 3.00\times10^9\ \text{kg}\cdot\text{m}^2 + (500)(70.0\ \text{kg})(82.5\ \text{m})^2 = 3.24\times10^9\ \text{kg}\cdot\text{m}^2$$

If the people all move to the center of the space station, the total moment of inertia is

$$I_{\text{final}} = I_{\text{station}} = 3.00\times10^9\ \text{kg}\cdot\text{m}^2$$

Therefore,

$$\frac{\omega_{\text{final}}}{\omega_0} = \frac{I_0}{I_{\text{final}}} = \frac{3.24\times10^9\ \text{kg}\cdot\text{m}^2}{3.00\times10^9\ \text{kg}\cdot\text{m}^2} = 1.08$$

This fraction represents a percentage increase of $\boxed{\text{8 percent}}$.

58. ***REASONING*** When the modules pull together, they do so by means of forces that are internal. These pulling forces, therefore, do not create a net external torque, and the angular momentum of the system is conserved. In other words, it remains constant. We will use the conservation of angular momentum to obtain a relationship between the initial and final angular speeds. Then, we will use Equation 8.9 ($v = r\omega$) to relate the angular speeds ω_0 and ω_f to the tangential speeds v_0 and v_f.

SOLUTION Let L be the initial length of the cable between the modules and ω_0 be the initial angular speed. Relative to the center-of-mass axis, the initial momentum of inertia of the two-module system is $I_0 = 2M(L/2)^2$, according to Equation 9.6. After the modules pull together, the length of the cable is $L/2$, the final angular speed is ω_f, and the momentum of inertia is $I_f = 2M(L/4)^2$. The conservation of angular momentum indicates that

$$\underbrace{I_f\omega_f}_{\substack{\text{Final angular}\\ \text{momentum}}} = \underbrace{I_0\omega_0}_{\substack{\text{Initial angular}\\ \text{momentum}}}$$

$$\left[2M\left(\frac{L}{4}\right)^2\right]\omega_f = \left[2M\left(\frac{L}{2}\right)^2\right]\omega_0$$

$$\omega_f = 4\omega_0$$

According to Equation 8.9, $\omega_f = v_f/(L/4)$ and $\omega_0 = v_0/(L/2)$. With these substitutions, the result that $\omega_f = 4\omega_0$ becomes

$$\frac{v_f}{L/4} = 4\left(\frac{v_0}{L/2}\right) \quad \text{or} \quad v_f = 2v_0 = 2(17\ \text{m/s}) = \boxed{34\ \text{m/s}}$$

59. ***REASONING AND SOLUTION*** After the mass has moved inward to its final path the centripetal force acting on it is $T = 105$ N.

Its centripetal acceleration is

$$a_c = v^2/R = T/m$$

Now

$$v = \omega R \quad \text{so} \quad R = T/(\omega^2 m)$$

The centripetal force is parallel to the line of action (the string), so the force produces no torque on the object. Hence, angular momentum is conserved.

$$I\omega = I_o\omega_o \quad \text{so that} \quad \omega = (I_o/I)\omega_o = (R_o^2/R^2)\omega_o$$

Substituting and simplifying

$$R^3 = (mR_o^4\,\omega_o^2)/T, \text{ so that } R = \boxed{0.573 \text{ m}}$$

60. ***REASONING AND SOLUTION*** The block will just start to move when the centripetal force on the block just exceeds f_s^{max}. Thus, if r_f is the smallest distance from the axis at which the block stays at rest when the angular speed of the block is ω_f, then $\mu_s F_N = mr_f \omega_f^2$, or

$$\mu_s mg = mr_f \omega_f^2$$

$$\mu_s g = r_f \omega_f^2 \qquad (1)$$

Since there are no external torques acting on the system, angular momentum will be conserved.

$$I_0 \omega_0 = I_f \omega_f$$

where $I_0 = mr_0^2$, and $I_f = mr_f^2$. Making these substitutions yields

$$r_0^2 \omega_0 = r_f^2 \omega_f \qquad (2)$$

Solving Equation (2) for ω_f and substituting into Equation (1) yields:

$$\mu_s g = r_f \omega_0^2 \frac{r_0^4}{r_f^4}$$

Solving for r_f gives

$$r_f = \left(\frac{\omega_0^2 r_0^4}{\mu_s g} \right)^{1/3} = \left[\frac{(2.2 \text{ rad/s})^2 (0.30 \text{ m})^4}{(0.75)(9.80 \text{ m/s}^2)} \right]^{1/3} = \boxed{0.17 \text{ m}}$$

61. SSM WWW ***REASONING*** The maximum torque will occur when the force is applied perpendicular to the diagonal of the square as shown. The lever arm ℓ is half the length of the diagonal. From the Pythagorean theorem, the lever arm is, therefore,

$$\ell = \tfrac{1}{2}\sqrt{(0.40\text{ m})^2 + (0.40\text{ m})^2} = 0.28\text{ m}$$

Since the lever arm is now known, we can use Equation 9.1 to obtain the desired result directly.

SOLUTION Equation 9.1 gives

$$\tau = F\ell = (15\text{ N})(0.28\text{ m}) = \boxed{4.2\text{ N}\cdot\text{m}}$$

62. ***REASONING AND SOLUTION*** For a solid disk we know $I = (1/2)MR^2$ so $M = 2I/R^2$.

Newton's second law for rotation yields $I = (\Sigma\tau)/\alpha$, so $M = 2(\Sigma\tau)/(\alpha R^2)$.

$\alpha = (\omega - \omega_o)/t = (-3.49 \text{ rad/s})/(15.0 \text{ s}) = -0.233 \text{ rad/s}^2$, so that

$$M = \frac{2\left(-6.20 \times 10^{-3} \text{ N} \cdot \text{m}\right)}{(-0.233 \text{ rad/s}^2)(0.150 \text{ m})^2} = \boxed{2.37 \text{ kg}}$$

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63. ***REASONING AND SOLUTION*** The sum of the forces must be zero:

$$\Sigma F_y = F_W + F_C - mg = 0$$

Therefore,

$$F_W + F_C = mg$$
$$= (71.0\text{ kg})(9.80\text{ m/s}^2) = 696\text{ N}$$

The sum of the torques must also be zero. Taking the axis for torques at the point just below the boulder, we have

$$\Sigma\tau = F_C\,(0.60\text{ m}) - F_W\,(1.40\text{ m}) = 0.$$

a. Substituting for F_W yields

$$F_C(0.60\text{ m}) - (696 - F_C)(1.40\text{ m}) = 0$$

Solving for F_C gives $F_C = \boxed{487\text{ N}}$.

b. Also, $F_W = 696\text{ N} - F_C = \boxed{209\text{ N}}$

64. ***REASONING AND SOLUTION*** Use conservation of angular momentum, $I_o\omega_o = (I_o + I)\,\omega$, where

$$I_o = 1.2 \times 10^{-3}\ \text{kg}\cdot\text{m}^2 \quad \text{and} \quad \omega_o = 0.40\ \text{rad/s}$$

The moment of inertia of the bug is

$$I = mL^2 = (5.0 \times 10^{-3}\ \text{kg})(0.20\ \text{m})^2 = 2.0 \times 10^{-4}\ \text{kg}\cdot\text{m}^2$$

The final angular velocity is

$$\omega = I_o\omega_o/(I_o + I) = \boxed{0.34\ \text{rad/s}}$$

65. SSM ***REASONING AND SOLUTION***

a. The rim of the bicycle wheel can be treated as a hoop. Using the expression given in Table 9.1 in the text, we have

$$I_{\text{hoop}} = MR^2 = (1.20\text{ kg})(0.330\text{ m})^2 = \boxed{0.131\text{ kg}\cdot\text{m}^2}$$

b. Any one of the spokes may be treated as a long, thin rod that can rotate about one end. The expression in Table 9.1 gives

$$I_{\text{rod}} = \tfrac{1}{3}ML^2 = \tfrac{1}{3}(0.010\text{ kg})(0.330\text{ m})^2 = \boxed{3.6\times10^{-4}\text{ kg}\cdot\text{m}^2}$$

c. The total moment of inertia of the bicycle wheel is the sum of the moments of inertia of each constituent part. Therefore, we have

$$I = I_{\text{hoop}} + 50I_{\text{rod}} = 0.131\text{ kg}\cdot\text{m}^2 + 50(3.6\times10^{-4}\text{ kg}\cdot\text{m}^2) = \boxed{0.149\text{ kg}\cdot\text{m}^2}$$

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66. ***REASONING AND SOLUTION*** The net torque about an axis through the elbow joint is

$$\Sigma\tau = (111\text{ N})(0.300\text{ m}) - (22.0\text{ N})(0.150\text{ m}) - (0.0250\text{ m})M = 0 \quad \text{or} \quad \boxed{M = 1.20 \times 10^3\text{ N}}$$

67. SSM ***REASONING*** The drawing shows the forces acting on the board, which has a length L. The ground exerts the vertical normal force **V** on the lower end of the board. The maximum force of static friction has a magnitude of $\mu_S V$ and acts horizontally on the lower end of the board. The weight **W** acts downward at the board's center. The vertical wall applies a force **P** to the upper end of the board, this force being perpendicular to the wall since the wall is smooth (i.e., there is no friction along the wall). We take upward and to the right as our positive directions. Then, since the horizontal forces balance to zero, we have

$$\mu_S V - P = 0 \tag{1}$$

The vertical forces also balance to zero giving

$$V - W = 0 \tag{2}$$

Using an axis through the lower end of the board, we express the fact that the torques balance to zero as

$$PL \sin\theta - W\left(\frac{L}{2}\right)\cos\theta = 0 \tag{3}$$

Equations (1), (2), and (3) may then be combined to yield an expression for θ.

SOLUTION Rearranging Equation (3) gives

$$\tan\theta = \frac{W}{2P} \tag{4}$$

But, $P = \mu_S V$ according to Equation (1), and $W = V$ according to Equation (2). Substituting these results into Equation (4) gives

$$\tan\theta = \frac{V}{2\mu_s V} = \frac{1}{2\mu_s}$$

Therefore,

$$\theta = \tan^{-1}\left(\frac{1}{2\mu_s}\right) = \tan^{-1}\left[\frac{1}{2(0.650)}\right] = \boxed{37.6°}$$

68. ***REASONING AND SOLUTION***

a. The net torque about an axis through the point of contact between the floor and her shoes is

$$\Sigma\tau = -(5.00 \times 10^2 \text{ N})(1.10 \text{ m})\sin 30.0° + F_N(\cos 30.0°)(1.50 \text{ m}) = 0$$

$$\boxed{F_N = 212 \text{ N}}$$

b. Newton's second law applied in the horizontal direction gives $F_h - F_N = 0$, so

$$\boxed{F_h = 212 \text{ N}}$$

c. Newton's second law in the vertical direction gives $F_v - W = 0$, so $\boxed{F_v = 5.00 \times 10^2 \text{ N}}$

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69. ***REASONING AND SOLUTION*** Since the change occurs without the aid of external torques, the angular momentum of the system is conserved: $I_f\omega_f = I_0\omega_0$. Solving for ω_f gives

$$\omega_f = \omega_0\left(\frac{I_0}{I_f}\right)$$

where for a rod of mass M and length L, $I_0 = \frac{1}{12}ML^2$. To determine I_f, we will treat the arms of the "u" as point masses with mass M/4 a distance L/4 from the rotation axis. Thus,

$$I_f = \left[\frac{1}{12}\left(\frac{M}{2}\right)\left(\frac{L}{2}\right)^2\right] + \left[2\left(\frac{M}{4}\right)\left(\frac{L}{4}\right)^2\right] = \frac{1}{24}ML^2$$

and

$$\omega_f = \omega_0\left[\frac{\frac{1}{12}ML^2}{\frac{1}{24}ML^2}\right] = (7.0\text{ rad/s})(2) = \boxed{14\text{ rad/s}}$$

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70. ***REASONING AND SOLUTION*** The conservation of energy gives $(1/2)\,I\omega^2 = mg[(1/2)\,L]$. Now the tip of the rod has, as it hits the ground, a speed of $v = L\omega$, so $\omega = v/L$. The moment of inertia of the rod is given by $I = (1/3)\,mL^2$. Substituting the last two equations into the first equation and simplifying yields

$$v = \sqrt{3gL} = \sqrt{3\left(9.8\ \mathrm{m/s^2}\right)\left(2.00\ \mathrm{m}\right)} = \boxed{7.67\ \mathrm{m/s}}$$

71. SSM WWW ***REASONING AND SOLUTION*** Consider the left board, which has a length L and a weight of (356 N)/2 = 178 N. Let $\mathbf{F}_V$ be the upward normal force exerted by the ground on the board. This force balances the weight, so $F_V = 178$ N. Let $\mathbf{f}_S$ be the force of static friction, which acts horizontally on the end of the board in contact with the ground. $\mathbf{f}_S$ points to the right. Since the board is in equilibrium, the net torque acting on the board through any axis must be zero. Measuring the torques with respect to an axis through the apex of the triangle formed by the boards, we have

$$+ (178\text{ N})(\sin 30.0°)\left(\frac{L}{2}\right) + f_S(L\cos 30.0°) - F_V\,(L\sin 30.0°) = 0$$

or

$$44.5\text{ N} + f_s\cos 30.0° - F_V\ \sin 30.0° = 0$$

so that

$$f_s = \frac{(178\text{ N})(\sin 30.0°) - 44.5\text{ N}}{\cos 30.0°} = \boxed{51.4\text{ N}}$$

72. ***REASONING AND SOLUTION*** Newton's law applied to the 11.0-kg object gives

$$T_2 - (11.0\text{ kg})(9.80\text{ m/s}^2) = (11.0\text{ kg})(4.90\text{ m/s}^2) \quad \text{or} \quad T_2 = 162\text{ N}$$

A similar treatment for the 44.0-kg object yields

$$T_1 - (44.0\text{ kg})(9.80\text{ m/s}^2) = (44.0\text{ kg})(-4.90\text{ m/s}^2) \quad \text{or} \quad T_1 = 216\text{ N}$$

For an axis about the center of the pulley

$$T_2 r - T_1 r = I(-\alpha) = (1/2)\,Mr^2\,(-a/r)$$

Solving for the mass M we obtain

$$M = (-2/a)(T_2 - T_1) = [-2/(4.90\text{ m/s}^2)](162\text{ N} - 216\text{ N}) = \boxed{22.0\text{ kg}}$$

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1. **SSM** ***REASONING AND SOLUTION*** Using Equation 10.1, we first determine the spring constant:

$$k = \frac{F_{\text{Applied}}}{x} = \frac{89.0 \text{ N}}{0.0191 \text{ m}} = 4660 \text{ N/m}$$

Again using Equation 10.1, we find that the force needed to compress the spring by 0.0508 m is

$$F_{\text{Applied}} = kx = (4660 \text{ N/m})(0.0508 \text{ m}) = \boxed{237 \text{ N}}$$

2. ***REASONING*** The weight of the person causes the spring in the scale to compress. The amount x of compression, according to Equation 10.1, depends on the magnitude F_{Applied} of the applied force and the spring constant k.

SOLUTION

a. Since the applied force is equal to the person's weight, the spring constant is

$$k = \frac{F_{\text{Applied}}}{x} = \frac{670\ \text{N}}{0.79 \times 10^{-2}\ \text{m}} = \boxed{8.5 \times 10^{4}\ \text{N/m}} \qquad (10.1)$$

b. When another person steps on the scale, it compresses by 0.34 cm. The weight (or applied force) that this person exerts on the scale is

$$F_{\text{Applied}} = k\,x = \left(8.5 \times 10^{4}\ \text{N/m}\right)\left(0.34 \times 10^{-2}\ \text{m}\right) = \boxed{290\ \text{N}} \qquad (10.1)$$

3. ***REASONING AND SOLUTION*** $F_{\text{Applied}} = kx$ for either expansion or compression. Therefore,

a. $$F_{\text{Applied}} = kx = (248\ \text{N/m})(0.0300\ \text{m}) = \boxed{7.44\ \text{N}} \tag{10.1}$$

b. $$F_{\text{Applied}} = kx = (248\ \text{N/m})(0.0300\ \text{m}) = \boxed{7.44\ \text{N}}$$

4. ***REASONING AND SOLUTION*** The effective spring constant is, according to Equation 10.1 and the data from the graph,

$$k = \frac{F_{\text{Applied}}}{x} = \frac{160\ \text{N}}{0.24\ \text{m}} = \boxed{6.7 \times 10^{2}\ \text{N/m}}$$

5. **SSM** ***REASONING AND SOLUTION*** According to Newton's second law, the force required to accelerate the trailer is $F = ma$. The displacement of the spring is given by Equation 10.2; solving Equation 10.2 for x and using $F = ma$, we obtain

$$x = -\frac{F}{k} = -\frac{ma}{k} = -\frac{(92\text{ kg})(0.30\text{ m/s}^2)}{2300\text{ N/m}} = -0.012\text{ m}$$

The amount that the spring stretches is $\boxed{0.012\text{ m}}$.

6. ***REASONING*** The weight of the block causes the spring to stretch. The amount x of stretching, according to Equation 10.1, depends on the magnitude F_{Applied} of the applied force and the spring constant k.

SOLUTION

a. The applied force is equal to the weight of the block. The amount x that the spring stretches is equal to the length of the stretched spring minus the length of the unstretched spring. The spring constant is

$$k = \frac{F_{\text{Applied}}}{x} = \frac{4.50\ \text{N}}{0.350\ \text{m} - 0.200\ \text{m}} = \boxed{30.0\ \text{N/m}} \qquad (10.1)$$

b. When a block of unknown weight is attached to the spring, it stretches it by 0.500 m – 0.200 m. The weight (or applied force) of the block is

$$F_{\text{Applied}} = kx = (30.0\ \text{N/m})(0.500\ \text{m} - 0.200\ \text{m}) = \boxed{9.00\ \text{N}} \qquad (10.1)$$

7. ***REASONING AND SOLUTION*** The two forces that act on the 5.0-kg object are the upward force **F** of the spring and the downward weight $m\mathbf{g}$ of the object. Accordingly, Newton's second law gives (with up taken as the positive direction), $F - mg = ma$, where a is the acceleration of the elevator. Substituting this into Equation 10.2 and solving for x, we find

$$x = -\frac{F}{k} = -\frac{m(g+a)}{k} = -\frac{(5.0\text{ kg})(9.80\text{ m/s}^2 + 0.60\text{ m/s}^2)}{830\text{ N/m}} = -6.3\times10^{-2}\text{ m}$$

The amount that the spring stretches is $\boxed{6.3\times10^{-2}\text{ m}}$.

8. ***REASONING AND SOLUTION*** The figure at the right shows the original situation before the spring is cut. The weight, W, of the object stretches the string by an amount x.

Applying F = kx to this situation gives

$$W = kx \qquad (1)$$

The figure at the right shows the situation after the spring is cut into two segments of equal length.

Let k' represent the spring constant of each half of the spring after it is cut. Now the weight, W, of the object stretches *each* segment by an amount x'.

Applying F = kx to this situation gives

$$W = k'x' + k'x' = 2k'x' \qquad (2)$$

Combining Equations (1) and (2) yields

$$kx = 2k'x'$$

From Conceptual Example 2, we know that k' = 2k so that

$$kx = 2(2k)x'$$

Solving for x' gives

$$x' = \frac{x}{4} = \frac{0.160\ \text{m}}{4} = \boxed{0.040\ \text{m}}$$

9. ***REASONING AND SOLUTION*** In order to find the amount the spring stretches we need to calculate the force, F = kx, acting on the spring. Since the board is in equilibrium, the net torque acting on it is zero. Taking the axis of rotation to be at the corner and assuming the board has a length L, the net torque is

$$\Sigma\tau = FL \sin 50.0° - mg(L/2) \cos 50.0° = 0$$

Solving for F gives

$$F = \frac{mg \cos 50.0°}{2 \sin 50.0°} = \frac{mg}{2 \tan 50.0°}$$

Therefore,

$$x = \frac{F}{k} = \frac{mg}{2k \tan 50.0°} = \frac{(10.1 \text{ kg})(9.80 \text{ m/s}^2)}{2(176 \text{ N/m}) \tan 50.0°} = \boxed{0.236 \text{ m}}$$

10. ***REASONING AND SOLUTION*** From the figure we see that the maximum frictional force, f_s^{max}, must balance the force of gravity, mg, on the block; $f_s^{max} = mg$. Since $f_s^{max} = \mu_s F_N$, where F_N is the normal force due to the spring pushing perpendicular to the block (i.e., $F_N = kx$) we have that

$$f_s^{max} = \mu_s F_N = \mu_s(kx)$$

Therefore, $\mu_s(kx) = mg$, and we can write

$$\mu_s = \frac{mg}{kx} = \frac{(2.3\text{ kg})(9.80\text{ m/s}^2)}{(480\text{ N/m})(0.051\text{ m})} = \boxed{0.92}$$

11. SSM ***REASONING*** When the ball is whirled in a horizontal circle of radius r at speed v, the centripetal force is provided by the restoring force of the spring. From Equation 5.3, the magnitude of the centripetal force is mv^2/r, while the magnitude of the restoring force is kx (see Equation 10.2). Thus,

$$\frac{mv^2}{r} = kx \qquad (1)$$

The radius of the circle is equal to $(L_0 + \Delta L)$, where L_0 is the unstretched length of the spring and ΔL is the amount that the spring stretches. Equation (1) becomes

$$\frac{mv^2}{L_0 + \Delta L} = k\Delta L \qquad (1')$$

If the spring were attached to the ceiling and the ball were allowed to hang straight down, motionless, the net force must be zero: $mg - kx = 0$, where $-kx$ is the restoring force of the spring. If we let Δy be the displacement of the spring in the vertical direction, then

$$mg = k\Delta y$$

Solving for Δy, we obtain

$$\Delta y = \frac{mg}{k} \qquad (2)$$

SOLUTION According to equation (1') above, the spring constant k is given by

$$k = \frac{mv^2}{\Delta L(L_0 + \Delta L)}$$

Substituting this expression for k into equation (2) gives

$$\Delta y = \frac{mg\Delta L(L_0 + \Delta L)}{mv^2} = \frac{g\Delta L(L_0 + \Delta L)}{v^2}$$

or

$$\Delta y = \frac{(9.80\ \text{m/s}^2)(0.010\ \text{m})(0.200\ \text{m} + 0.010\ \text{m})}{(3.00\ \text{m/s})^2} = \boxed{2.29 \times 10^{-3}\ \text{m}}$$

12. ***REASONING AND SOLUTION*** From the diagram we see that the kinetic frictional force on the bottom block (#1) is given by

$$F_{f1} = \mu_k(m_1 + m_2)g$$

and the maximum static frictional force on the top block (#2) is

$$F_{f2} = \mu_s m_2 g$$

Newton's second law applied to the bottom block gives

$$F - F_{f1} - kx = 0$$

and to the top block gives

$$F_{f2} - kx = 0$$

a. To find the compression, x, we have

$$x = F_{f2}/k = \mu_s m_2 g/k = (0.900)(15.0\text{ kg})(9.80\text{ m/s}^2)/(325\text{ N/m}) = \boxed{0.407\text{ m}}$$

b. $$F = kx + \mu_k(m_1 + m_2)g$$

$$F = (325\text{ N/m})(0.407\text{ m}) + (0.600)(45.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{397\text{ N}}$$

13. ***REASONING AND SOLUTION*** In phase 1 of the block's motion (uniform acceleration) we find that the net force on the block is $F_1 - f_k = ma$ where the force of friction is $f_k = \mu_k mg$. Therefore $F_1 = m(a + \mu_k g)$, which is just the force exerted by the spring on the block, i.e., $F_1 = kx_1$. So we have

$$kx_1 = m(a + \mu_k g) \qquad (1)$$

We can find the acceleration using

$$a = \frac{v_f - v_0}{t} = \frac{5.00 \text{ m/s} - 0 \text{ m/s}}{0.500 \text{ s}} = \boxed{10.0 \text{ m/s}^2}$$

In phase 2 of the block's motion (constant speed) $a = 0$ m/s^2, so the force exerted by the spring is

$$k\,x_2 = m\mu_k g \qquad (2)$$

so that

$$\mu_k = \frac{k\,x_2}{m\,g}$$

Using this expression for μ_k in Equation (1) we obtain

$$k\,x_1 = m\left[a + \left(\frac{kx_2}{mg}\right)g\right]$$

a. Solving for k gives

$$k = \frac{ma}{x_1 - x_2} = \frac{(15.0 \text{ kg})(10.0 \text{ m/s}^2)}{0.200 \text{ m} - 0.0500 \text{ m}} = \boxed{1.00 \times 10^3 \text{ N/m}}$$

b. Substituting this value for k into Equation (2) , we have

$$\mu_k = \frac{k\,x_2}{m\,g} = \frac{(1.00 \times 10^3 \text{ N/m})(0.0500 \text{ m})}{(15.0 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{0.340}$$

14. ***REASONING*** The period of the engine is the time for it to make one revolution. This time is the total time required to make one thousand revolutions divided by 1000. The frequency f of the rotational motion is given by Equation 10.5 as the reciprocal of the period. The angular frequency ω is the frequency f times 2π (Equation 10.6).

SOLUTION

a. The period of the engine, which is the time to complete one revolution, is

$$T = \frac{\text{Total time}}{\text{Number of revolutions}} = \frac{50.0 \times 10^{-3}\ \text{s}}{1000} = \boxed{50.0 \times 10^{-6}\ \text{s}}$$

b. The frequency f of the rotational motion is related to the period T by

$$f = \frac{1}{T} = \frac{1}{50.0 \times 10^{-6}\ \text{s}} = \boxed{2.00 \times 10^{4}\ \text{Hz}} \tag{10.5}$$

c. The angular frequency ω of the rotational motion is related to the frequency f by

$$\omega = 2\pi f = 2\pi\left(2.00 \times 10^{4}\ \text{Hz}\right) = \boxed{1.26 \times 10^{5}\ \text{rad/s}} \tag{10.6}$$

15. **SSM** ***REASONING AND SOLUTION*** According to Equations 10.6 and 10.11, $2\pi f = \sqrt{k/m}$. According to the data in the problem, the frequency of vibration of either spring is

$$f = \frac{5.0 \text{ cycles}}{3.0 \text{ s}} = 1.67 \text{ Hz}$$

Squaring both sides of the equation $2\pi f = \sqrt{k/m}$ and solving for *k*, we obtain

$$k = 4\pi^2 f^2 m = 4\pi^2 (1.67 \text{ Hz})^2 (320 \text{ kg}) = \boxed{3.5 \times 10^4 \text{ N/m}}$$

16. ***REASONING AND SOLUTION*** From Conceptual Example 2, we know that when the spring is cut in half, the spring constant for each half is twice as large as that of the original spring. In this case, the spring is cut into four shorter springs. Thus, each of the four shorter springs with 25 coils has a spring constant of $4 \times 420 \text{ N/m} = 1680 \text{ N/m}$.

The angular frequency of simple harmonic motion is given by Equation 10.11:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1680 \text{ N/m}}{46 \text{ kg}}} = \boxed{6.0 \text{ rad/s}}$$

17. ***REASONING AND SOLUTION***

a. Since the object oscillates between $\pm 0.080\text{ m}$, the amplitude of the motion is $\boxed{0.080\text{ m}}$.

b. From the graph, the period is $T = 4.0\text{ s}$. Therefore, according to Equation 10.4,

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4.0\text{ s}} = \boxed{1.6\text{ rad/s}}$$

c. Equation 10.11 relates the angular frequency to the spring constant: $\omega = \sqrt{k/m}$. Solving for k we find

$$k = \omega^2 m = (1.6\text{ rad/s})^2\,(0.80\text{ kg}) = \boxed{2.0\text{ N/m}}$$

d. At $t = 1.0\text{ s}$, the graph shows that the spring has its maximum displacement. At this location, the object is momentarily at rest, so that its speed is $\boxed{v = 0\text{ m/s}}$.

e. The acceleration of the object at $t = 1.0\text{ s}$ is a maximum, and its magnitude is

$$a_{\text{max}} = A\omega^2 = (0.080\text{ m})(1.6\text{ rad/s})^2 = \boxed{0.20\text{ m/s}^2}$$

18. ***REASONING*** The amplitude of simple harmonic motion is the distance from the equilibrium position to the point of maximum height. The angular frequency ω is related to the period T of the motion by Equation 10.6. The maximum speed attained by the person is the product of the amplitude and the angular speed (Equation 10.8).

SOLUTION

a. Since the distance from the equilibrium position to the point of maximum height is the amplitude A of the motion, we have that $A = 45.0\text{ cm} = \boxed{0.450\text{ m}}$.

b. The angular frequency is inversely proportional to the period of the motion:

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{1.90\text{ s}} = \boxed{3.31\text{ rad/s}} \qquad (10.6)$$

c. The maximum speed v_{max} attained by the person on the trampoline depends on the amplitude A and the angular frequency ω of the motion:

$$v_{\text{max}} = A\omega = (0.450\text{ m})(3.31\text{ rad/s}) = \boxed{1.49\text{ m/s}} \qquad (10.8)$$

19. SSM WWW ***REASONING AND SOLUTION*** According to Equation 10.6, $\omega = 2\pi f$. The maximum speed and maximum acceleration of the atoms may be calculated from Equations 10.8 and 10.10, respectively.

a. Combining Equations 10.6 and 10.8, we obtain

$$v_{\max} = \omega A = (2\pi f)A = 2\pi(2.0\times10^{12}\ \text{Hz})(1.1\times10^{-11}\ \text{m}) = \boxed{140\ \text{m/s}}$$

b. Combining Equations 10.6 and 10.10, we obtain

$$a_{\max} = \omega^2 A = (4\pi^2 f^2)A = 4\pi^2(2.0\times10^{12}\ \text{Hz})^2(1.1\times10^{-11}\ \text{m}) = \boxed{1.7\times10^{15}\ \text{m/s}^2}$$

20. ***REASONING AND SOLUTION*** For mass 1 we have

$$f_1 = \frac{1}{2\pi}\sqrt{\frac{k}{m_1}} \tag{10.11}$$

For mass 2 we have

$$f_2 = \frac{1}{2\pi}\sqrt{\frac{k}{m_1 + m_2}}$$

Therefore,

$$\frac{f_1}{f_2} = \sqrt{\frac{m_1 + m_2}{m_1}} = 3.00$$

Squaring and solving yields

$$\frac{m_1 + m_2}{m_1} = 9.00 \quad \text{or} \quad m_1 + m_2 = 9.00\ m_1$$

Solving for the ratio m_2/m_1 gives $m_1/m_2 = \boxed{8.00}$.

21. SSM ***REASONING*** The frequency of vibration of the spring is related to the added mass m by Equations 10.6 and 10.11:

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{1}$$

The spring constant can be determined from Equation 10.1.

SOLUTION Since the spring stretches by 0.018 m when a 2.8-kg object is suspended from its end, the spring constant is, according to Equation 10.1,

$$k = \frac{F_{\text{Applied}}}{x} = \frac{mg}{x} = \frac{(2.8\text{ kg})(9.80\text{ m/s}^2)}{0.018\text{ m}} = 1.52\times10^3\text{ N/m}$$

Solving Equation (1) for m, we find that the mass required to make the spring vibrate at 3.0 Hz is

$$m = \frac{k}{4\pi^2 f^2} = \frac{1.52\times10^3\text{ N/m}}{4\pi^2(3.0\text{ Hz})^2} = \boxed{4.3\text{ kg}}$$

22. ***REASONING AND SOLUTION***

a. $x = A \cos \omega t$ where $\omega = 2\pi f = 4\pi$ rad/s

$$x = (0.500 \text{ m}) \cos [(4\pi \text{ rad/s})(0.0500 \text{ s})] = \boxed{0.405 \text{ m}} \tag{10.3}$$

b. $v = -A\omega \sin \omega t$

$$v = -(0.500 \text{ m}) (4\pi \text{ rad/s}) \sin [(4\pi \text{ rad/s})(0.0500 \text{ s})] = -3.69 \text{ m/s} \tag{10.7}$$

The magnitude of v is $\boxed{3.69 \text{ m/s}}$

c. $a = -A\omega^2 \cos \omega t$

$$a = -(0.500 \text{ m}) (4\pi \text{ rad/s})^2 \cos [(4\pi \text{ rad/s})(0.0500 \text{ s})] = -63.9 \text{ m/s}^2 \tag{10.9}$$

The magnitude of a is $\boxed{63.9 \text{ m/s}^2}$

23. ***REASONING AND SOLUTION*** The cup slips when the force of static friction is overcome. So $F = ma = \mu_s mg$, where the acceleration is the maximum value for the simple harmonic motion, i.e., so that

$$a_{max} = A\omega^2 = A(2\pi f)^2 \tag{10.10}$$

$$\mu_s = A(2\pi f)^2/g = (0.0500\text{ m})4\pi^2(2.00\text{ Hz})^2/(9.80\text{ m/s}^2) = \boxed{0.806}$$

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24. ***REASONING AND SOLUTION***

a. $PE = (1/2)\,kx^2 = (1/2)\,(425\text{ N/m})(0.470\text{ m})^2 = \boxed{46.9\text{ J}}$ (10.13)

b. Assuming that mechanical energy is conserved, KE = PE gives

$$v = \sqrt{\frac{2(PE)}{m}} = \sqrt{\frac{2(46.9\text{ J})}{0.0300\text{ kg}}} = \boxed{55.9\text{ m/s}}$$

25. SSM ***REASONING*** The only force that acts on the block along the line of motion is the force due to the spring. Since the force due to the spring is a conservative force, the principle of conservation of mechanical energy applies. Initially, when the spring is unstrained, all of the mechanical energy is kinetic energy, $(1/2)mv_0^2$. When the spring is fully compressed, all of the mechanical energy is in the form of elastic potential energy, $(1/2)kx_{\text{max}}^2$, where x_{max}, the maximum compression of the spring, is the amplitude A. Therefore, the statement of energy conservation can be written as

$$\tfrac{1}{2}mv_0^2 = \tfrac{1}{2}kA^2$$

This expression may be solved for the amplitude A.

SOLUTION Solving for the amplitude A, we obtain

$$A = \sqrt{\frac{mv_0^2}{k}} = \sqrt{\frac{(1.00\times10^{-2}\ \text{kg})(8.00\ \text{m/s})^2}{124\ \text{N/m}}} = \boxed{7.18\times10^{-2}\ \text{m}}$$

26. ***REASONING*** As the block falls, only two forces act on it: its weight and the elastic force of the spring. Both of these forces are conservative forces, so the falling block obeys the principle of conservation of mechanical energy. We will use this conservation principle to determine the spring constant of the spring. Once the spring constant is known, Equation 10.11, $\omega = \sqrt{k/m}$, may be used to find the angular frequency of the block's vibrations.

SOLUTION

a. The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0, or $E_f = E_0$ (Equation 6.9a). The expression for the total mechanical energy of an object oscillating on a spring is given by Equation 10.14. Thus, the conservation of total mechanical energy can be written as

$$\underbrace{\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}kx_f^2}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgh_0 + \tfrac{1}{2}kx_0^2}_{E_0}$$

Before going any further, let's simplify this equation by noting which variables are zero. Since the block starts and ends at rest, $v_f = v_0 = 0$ m/s. The block does not rotate, so its angular speed is zero, $\omega_f = \omega_0 = 0$ rad/s. Initially, the spring is unstretched, so that $x_0 = 0$ m. Setting these terms equal to zero in the equation above gives

$$mgh_f + \tfrac{1}{2}kx_f^2 = mgh_0$$

Solving this equation for the spring constant k, we have that

$$k = \frac{mg(h_0 - h_f)}{\frac{1}{2}x_f^2} = \frac{(0.450\text{ kg})(9.80\text{ m/s}^2)(0.150\text{ m})}{\frac{1}{2}(0.150\text{ m})^2} = \boxed{58.8\text{ N/m}}$$

b. The angular frequency ω of the block's vibrations depends on the spring constant k and the mass m of the block:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{58.8\text{ N/m}}{0.450\text{ kg}}} = \boxed{11.4\text{ rad/s}} \qquad (10.11)$$

27. ***REASONING AND SOLUTION*** Assuming that mechanical energy is conserved

$$(1/2)\,kx_0^2 = (1/2)\,mv_f^2 + mgh_f$$

or

$$v_f = \sqrt{\frac{kx_0^2}{m} - 2gh_f} = \sqrt{\frac{(675\text{ N/m})(0.0650\text{ m})^2}{0.0585\text{ kg}} - 2\left(9.80\text{ m/s}^2\right)(0.300\text{ m})} = \boxed{6.55\text{ m/s}}$$

28. ***REASONING*** As the block falls, only two forces act on it: its weight and the elastic force of the spring. Both of these forces are conservative forces, so the falling block obeys the principle of conservation of mechanical energy. We will use this conservation principle to determine the initial speed of the block.

SOLUTION

The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0, or $E_f = E_0$ (Equation 6.9a). The expression for the total mechanical energy of an object is given by Equation 10.14. Thus, the conservation of total mechanical energy can be written as

$$\underbrace{\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}kx_f^2}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgh_0 + \tfrac{1}{2}kx_0^2}_{E_0}$$

We can simplify this equation by noting which variables are zero. Since the block comes to a momentarily halt, $v_f = 0$ m/s. The block does not rotate, so its angular speed is zero, $\omega_f = \omega_0 = 0$ rad/s. Initially, the spring is unstretched, so that $x_0 = 0$ m. Setting these terms equal to zero in the equation above gives

$$mgh_f + \tfrac{1}{2}kx_f^2 = \tfrac{1}{2}mv_0^2 + mgh_0$$

Solving this equation for the initial speed v_0 of the block gives

$$v_0 = \sqrt{\frac{kx_f^2}{m} - 2g(h_0 - h_f)} = \sqrt{\frac{(58\text{ N/m})(0.15\text{ m})^2}{0.25\text{ kg}} - 2(9.8\text{ m/s}^2)(0.15\text{ m})} = \boxed{1.5\text{ m/s}}$$

29. SSM ***REASONING AND SOLUTION*** If we neglect air resistance, only the conservative forces of the spring and gravity act on the ball. Therefore, the principle of conservation of mechanical energy applies.

When the 2.00 kg object is hung on the end of the vertical spring, it stretches the spring by an amount x, where

$$x = \frac{F}{k} = \frac{mg}{k} = \frac{(2.00\text{ kg})(9.80\text{ m/s}^2)}{50.0\text{ N/m}} = 0.392\text{ m} \tag{10.1}$$

This position represents the equilibrium position of the system with the 2.00-kg object suspended from the spring. The object is then pulled down another 0.200 m and released from rest ($v_0 = 0\text{ m/s}$). At this point the spring is stretched by an amount of $0.392\text{ m} + 0.200\text{m} = 0.592\text{ m}$. This point represents the zero reference level ($h = 0\text{ m}$) for the gravitational potential energy.

***h* = 0 m:** The kinetic energy, the gravitational potential energy, and the elastic potential energy at the point of release are:

$$\text{KE} = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}m(0\text{ m/s})^2 = \boxed{0\text{ J}}$$

$$\text{PE}_{\text{gravity}} = mgh = mg(0\text{ m}) = \boxed{0\text{ J}}$$

$$\text{PE}_{\text{elastic}} = \frac{1}{2}kx_0^2 = \frac{1}{2}(50.0\text{ N/m})(0.592\text{ m})^2 = \boxed{8.76\text{ J}}$$

The total mechanical energy E_0 at the point of release is the sum of the three energies above: $E_0 = \boxed{8.76\text{ J}}$.

***h* = 0.200 m:** When the object has risen a distance of $h = 0.200\text{ m}$ above the release point, the spring is stretched by an amount of $0.592\text{ m} - 0.200\text{ m} = 0.392\text{ m}$. Since the total mechanical energy is conserved, its value at this point is still $E = \boxed{8.76\text{ J}}$. The gravitational and elastic potential energies are:

$$\text{PE}_{\text{gravity}} = mgh = (2.00\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) = \boxed{3.92\text{ J}}$$

$$\text{PE}_{\text{elastic}} = \frac{1}{2}kx^2 = \frac{1}{2}(50.0\text{ N/m})(0.392\text{ m})^2 = \boxed{3.84\text{ J}}$$

Since $\text{KE} + \text{PE}_{\text{gravity}} + \text{PE}_{\text{elastic}} = E$,

$$\text{KE} = \text{E} - \text{PE}_{\text{gravity}} - \text{PE}_{\text{elastic}} = 8.76\text{ J} - 3.92\text{ J} - 3.84\text{ J} = \boxed{1.00\text{ J}}$$

***h* = 0.400 m:** When the object has risen a distance of $h = 0.400\text{ m}$ above the release point, the spring is stretched by an amount of $0.592\text{ m} - 0.400\text{ m} = 0.192\text{ m}$. At this point, the total mechanical energy is still $E = \boxed{8.76\text{ J}}$. The gravitational and elastic potential energies are:

$$\mathrm{PE}_{\text{gravity}} = mgh = (2.00\ \text{kg})(9.80\ \text{m/s}^2)(0.400\ \text{m}) = \boxed{7.84\ \text{J}}$$

$$\mathrm{PE}_{\text{elastic}} = \frac{1}{2}kx^2 = \frac{1}{2}(50.0\ \text{N/m})(0.192\ \text{m})^2 = \boxed{0.92\ \text{J}}$$

The kinetic energy is

$$\mathrm{KE} = \mathrm{E} - \mathrm{PE}_{\text{gravity}} - \mathrm{PE}_{\text{elastic}} = 8.76\ \text{J} - 7.84\ \text{J} - 0.92\ \text{J} = \boxed{0\ \text{J}}$$

The results are summarized in the table below:

h	**KE**	**$\mathrm{PE}_{\text{grav}}$**	**$\mathrm{PE}_{\text{elastic}}$**	***E***
0.000 m	0.00 J	0.00 J	8.76 J	8.76 J
0.200 m	1.00 J	3.92 J	3.84 J	8.76 J
0.400 m	0.00 J	7.84 J	0.92 J	8.76 J

30. ***REASONING AND SOLUTION*** When the "ram spring" is fully compressed the energy stored in it is

$$E_0 = \tfrac{1}{2}kx_0^2 + mgh_0$$

When the ram hits the staple the energy can be expressed as

$$E_f = \tfrac{1}{2}kx_f^2 + \tfrac{1}{2}mv_f^2$$

Using conservation of energy we can equate E_0 and E_f and solve for the final speed

$$v_f = \sqrt{\frac{k\left(x_0^2 - x_f^2\right)}{m} + 2gh_0}$$

$$= \sqrt{\frac{\left(3.4\times10^4\ \text{N/m}\right)\left[\left(0.035\ \text{m}\right)^2 - \left(0.010\ \text{m}\right)^2\right]}{0.150\ \text{kg}} + 2\left(9.80\ \text{m/s}^2\right)\left(0.025\ \text{m}\right)} = \boxed{16\ \text{m/s}}$$

31. ***REASONING*** Since the incline is frictionless, the only forces that act on the block are the force of gravity and the conservative spring force; therefore, we can apply the conservation of mechanical energy to the motion of the block. If we let $x_0 = 0.31$ m represent the initial position of the block and the position $x_f = 0.14$ m represent the final position of the block, conservation of energy gives us

$$\underbrace{\frac{1}{2}kx_0^2 + mgh_0}_{E_0} = \underbrace{\frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 + mgh_f}_{E_f}$$

This expression can be solved for v_f. We must first relate the distances to the angle θ.

From the figure at the right, we see that

$$h_0 - h_f = \sin\theta(x_0 - x_f)$$

SOLUTION Solving for v_f, we have

$$v_f = \sqrt{\frac{k}{m}(x_0^2 - x_f^2) + 2g(h_0 - h_f)} = \sqrt{\frac{k}{m}(x_0^2 - x_f^2) + 2g\sin\theta(x_0 - x_f)}$$

$$v_f = \sqrt{\frac{310\text{ N/m}}{1.7\text{ kg}}\left[(0.31\text{ m})^2 - (0.14\text{ m})^2\right] + 2(9.80\text{ m/s}^2)(\sin 30.0°)(0.31\text{ m} - 0.14\text{ m})} = \boxed{4.0\text{ m/s}}$$

32. ***REASONING AND SOLUTION***

a. Now look at conservation of energy before and after the split

Before: $$(1/2)\, mv_{max}^2 = (1/2)\, kA^2$$

so

$$A = v_{max}\sqrt{\frac{m}{k}}$$

After: $$(1/2)\,(m/2)v'^2 = (1/2)\,(m/2)(v_{max})^2 = (1/2)\, kA'^2$$

so

$$A' = v_{max}\sqrt{\frac{m}{2k}}$$

Therefore, $A' = A/\sqrt{2} = (5.08 \times 10^{-2}\ \text{m})/\sqrt{2} = \boxed{3.59 \times 10^{-2}\ \text{m}}$

Similarly, for the frequency, we can show that

$$f' = f\sqrt{2} = (3.00\ \text{Hz})\sqrt{2} = \boxed{4.24\ \text{Hz}}$$

b. If the block splits at one of the extreme positions, the amplitude of the SHM would not change, so it would remain as $\boxed{5.08 \times 10^{-2}\ \text{m}}$

The frequency would be

$$f' = f\sqrt{2} = (3.00\ \text{Hz})\sqrt{2} = \boxed{4.24\ \text{Hz}}$$

33. SSM ***REASONING AND SOLUTION*** The amount by which the spring stretches due to the weight of the 1.1-kg object can be calculated using Equation 10.1, where the force F is equal to the weight of the object. The position of the object when the spring is stretched is the equilibrium position for the vertical harmonic motion of the object-spring system.

a. Solving Equation 10.1 for x with F equal to the weight of the object gives

$$x = \frac{F}{k} = \frac{mg}{k} = \frac{(1.1\text{ kg})(9.80\text{ m/s}^2)}{120\text{ N/m}} = \boxed{9.0 \times 10^{-2}\text{ m}}$$

b. The object is then pulled down another 0.20 m and released from rest ($v_0 = 0$ m/s). At this point the spring is stretched by an amount of $0.090\text{ m} + 0.20\text{ m} = 0.29\text{ m}$. We will let this point be the zero reference level ($h = 0$ m) for the gravitational potential energy.

The kinetic energy, the gravitational potential energy, and the elastic potential energy at the point of release are:

$$\text{KE} = \tfrac{1}{2}mv_0^2 = \tfrac{1}{2}m(0\text{ m/s})^2 = 0\text{ J}$$

$$\text{PE}_{\text{gravity}} = mgh = mg(0\text{ m}) = 0\text{ J}$$

$$\text{PE}_{\text{elastic}} = \frac{1}{2}kx_0^2 = \frac{1}{2}(120\text{ N/m})(0.29\text{ m})^2 = 5.0\text{ J}$$

The total mechanical energy E_0 is the sum of these three energies, so $E_0 = 5.0\text{ J}$. When the object has risen a distance of $h = 0.20\text{ m}$ above the release point, the spring is stretched by an amount of $0.29\text{ m} - 0.20\text{ m} = 0.090\text{ m}$. Since the total mechanical energy is conserved, its value at this point is still 5.0 J. Thus,

$$E = \text{KE} + \text{PE}_{\text{gravity}} + \text{PE}_{\text{elastic}}$$

$$E = \frac{1}{2}mv^2 + mgh + \frac{1}{2}kx^2$$

$$5.0\text{ J} = \frac{1}{2}(1.1\text{ kg})v^2 + (1.1\text{ kg})(9.80\text{ m/s}^2)(0.20\text{ m}) + \frac{1}{2}(120\text{ N/m})(0.090\text{ m})^2$$

Solving for v yields $v = \boxed{2.1\text{ m/s}}$.

34. ***REASONING AND SOLUTION*** We need to employ conservation of energy and momentum for the blocks and spring. Conservation of energy yields

$$(1/2)\, m_1 v_1^2 + (1/2)\, m_2 v_2^2 = (1/2)\, kx^2$$

and from conservation of momentum we have

$$0 = m_1 v_1 + m_2 v_2$$

or

$$v_1 = -(m_2/m_1) v_2$$

Using this relation in the conservation of energy equation yields

$$(1/2)\, m_1 (m_2/m_1)^2 v_2^2 + (1/2)\, m_2 v_2^2 = (1/2)\, kx^2$$

Solving for v_2

$$v_2 = \sqrt{\frac{kx^2 m_1}{m_2 (m_1 + m_2)}} = \sqrt{\frac{(1170\ \text{N/m})(0.152\ \text{m})^2 (14.6\ \text{kg})}{(29.2\ \text{kg})(14.6\ \text{kg} + 29.2\ \text{kg})}} = \boxed{0.556\ \text{m/s}}$$

Then

$$v_1 = -(m_2/m_1) v_2 = -2(0.556\ \text{m/s}) = -1.11\ \text{m/s}$$

The speed of the block is $\boxed{1.11\ \text{m/s}}$.

35. ***REASONING AND SOLUTION*** Conservation of mechanical energy gives $(1/2)\ kx^2 = (1/2)\ mv^2$. This gives $k/m = v^2/x^2$. Also,

$$\omega = \sqrt{\frac{k}{m}} \tag{10.11}$$

so

$$\omega = v/x = (2.40\ \text{m/s})/(0.0800\ \text{m}) = \boxed{3.00\times10^1\ \text{rad/s}}$$

36. ***REASONING AND SOLUTION*** Use conservation of energy to find the speed of point A (take the pivot to have zero gravitational PE).

$$E_{up} = mgh = E_{down} = (1/2)\, I\omega^2 + (1/2)\, kx^2$$

where the moment of inertia of the bar is $I = (1/3)mL^2$, L = bar length and $\omega = v/L$. Substituting these into the energy equation and solving for v (Note that x = 0.224 m – 0.100 m = 0.124 m and that h = L/2 from the diagram),

$$v = \sqrt{\frac{3\left(mgL - kx^2\right)}{m}} = \sqrt{\frac{3\left[(0.750\text{ kg})(9.80\text{ m/s}^2)(0.200\text{ m}) - (25.0\text{ N/m})(0.124\text{ m})^2\right]}{0.750\text{ kg}}} = \boxed{2.08\text{ m/s}}$$

37. SSM ***REASONING*** Using the principle of conservation of mechanical energy, the initial elastic potential energy stored in the elastic bands must be equal to the sum of the kinetic energy and the gravitational potential energy of the performer at the point of ejection:

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv_0^2 + mgh$$

where v_0 is the speed of the performer at the point of ejection and, from the figure at the right, $h = x \sin\theta$.

Thus,

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}mv_0^2 + mgx\sin\theta \qquad (1)$$

From the horizontal motion of the performer

$$v_{0x} = v_0 \cos\theta \qquad (2)$$

where

$$v_{0x} = \frac{s_x}{t} \qquad (3)$$

and $s_x = 50.0\text{ m}$. Combining equations (2) and (3) gives

$$v_0 = \frac{s_x}{t(\cos\theta)}$$

Equation (1) becomes:

$$\tfrac{1}{2}kx^2 = \tfrac{1}{2}m\frac{s_x^2}{t^2\cos^2\theta} + mgx\sin\theta$$

This expression can be solved for k, the spring constant of the firing mechanism.

SOLUTION Solving for k yields:

$$k = m\left(\frac{s_x}{x\,t\cos\theta}\right)^2 + \frac{2\,mg(\sin\theta)}{x}$$

$$k = (70.0\text{ kg})\left(\frac{50.0\text{ m}}{(3.00\text{ m})(4.00\text{ s})(\cos 40.0°)}\right)^2$$

$$+ \frac{2(70.0\text{ kg})(9.80\text{ m/s}^2)(\sin 40.0°)}{3.00\text{ m}} = \boxed{2.36\times10^3\text{ N/m}}$$

38. ***REASONING AND SOLUTION*** Using

$$f = 1/T = 1/(0.250\ \text{s}) = 4.00\ \text{Hz}$$

and also

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

we can find the ratio

$$k/m = 4\pi^2 f^2 = 632\ \text{N/(kg·m)}$$

With the object resting on the spring, F = kx = mg so that,

$$x = \frac{g}{\frac{k}{m}} = 0.0155\ \text{m}.$$

When the mass leaves the spring, potential energy of the spring has been converted to gravitational energy, i.e.,

$$(1/2)\ kx'^2 = mgh$$

Where

$$x' = 0.0500\ \text{m} + 0.0155\ \text{m} = 0.0655\ \text{m}$$

Solving for h we get

$$h = \left(\frac{k}{m}\right)\left(\frac{x'^2}{2g}\right) = [632\ \text{N/(kg}\cdot\text{m)}]\left[\frac{(0.0655\ \text{m})^2}{2(9.80\ \text{m/s}^2)}\right] = \boxed{0.138\ \text{m}}$$

39. ***REASONING AND SOLUTION*** Applying Equation 10.16 and recalling that frequency and period are related by f = 1/T,

$$2\pi f = \frac{2\pi}{T} = \sqrt{\frac{g}{L}}$$

where L is the length of the pendulum. Thus,

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Solving for L gives

$$L = g\left(\frac{T}{2\pi}\right)^2 = \left(9.80\ \text{m/s}^2\right)\left(\frac{9.2\ \text{s}}{2\pi}\right)^2 = \boxed{21\ \text{m}}$$

---

40. ***REASONING AND SOLUTION*** The period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

The gravitational acceleration changes, but we want the period to remain the same, therefore we can write

$$2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{L'}{g'}}$$

Squaring each side of the equation and solving for L', the new length, we obtain

$$L' = L(g'/g) = (1.00\text{ m})(9.78\text{ m/s}^2)/(9.83\text{ m/s}^2) = \boxed{0.995\text{ m}}$$

41. SSM ***REASONING AND SOLUTION*** Recall that the relationship between frequency f and period T is $f = 1/T$. Then, according to Equations 10.6 and 10.16, the period of the simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where L is the length of the pendulum. Solving for g, we obtain

$$g = \frac{4\pi^2 L}{T^2} = \frac{4\pi^2(1.2\text{ m})}{(2.8\text{ s})^2} = \boxed{6.0\text{ m/s}^2}$$

42. ***REASONING*** As the ball swings down, it reaches it greatest speed at the lowest point in the motion. One complete cycle of the pendulum has four parts: the downward motion in which the ball attains its greatest speed at the lowest point, the subsequent upward motion in which the ball slows down and then momentarily comes to rest. The ball then retraces its motion, finally ending up where it originally began. The time it takes to reach the lowest point is one-quarter of the period of the pendulum, or $t = (1/4)T$. The period is related to the angular frequency ω of the pendulum by Equation 10.4, $T = 2\pi/\omega$. Thus, the time for the ball to reach its lowest point is

$$t = \tfrac{1}{4}T = \frac{1}{4}\left(\frac{2\pi}{\omega}\right)$$

The angular frequency ω of the pendulum depends on its length L and the acceleration g due to gravity through the relation $\omega = \sqrt{g/L}$ (Equation 10.16). Thus, the time is

$$t = \frac{1}{4}\left(\frac{2\pi}{\omega}\right) = \frac{1}{4}\left(\frac{2\pi}{\sqrt{\frac{g}{L}}}\right) = \frac{\pi}{2}\sqrt{\frac{L}{g}}$$

SOLUTION After the ball is released, the time that has elapsed before it attains its greatest speed is

$$t = \frac{\pi}{2}\sqrt{\frac{L}{g}} = \frac{\pi}{2}\sqrt{\frac{0.65\text{ m}}{9.80\text{ m/s}^2}} = \boxed{0.40\text{ s}}$$

43. SSM ***REASONING*** For small-angle displacements, the frequency of simple harmonic motion for a physical pendulum is determined by $2\pi f = \sqrt{mgL/I}$, where L is the distance between the axis of rotation and the center of gravity of the rigid body of moment of inertia I. Since the frequency f and the period T are related by $f = 1/T$, the period of pendulum A is given by

$$T_A = 2\pi\sqrt{\frac{I}{mgL}}$$

Since the pendulum is made from a thin, rigid, uniform rod, its moment of inertia is given by $I = (1/3)md^2$, where d is the length of the rod. Since the rod is uniform, its center of gravity lies at its geometric center, and $L = d/2$ Therefore, the period of pendulum A is given by

$$T_A = 2\pi\sqrt{\frac{2d}{3g}}$$

For the simple pendulum we have

$$T_B = 2\pi\sqrt{\frac{d}{g}}$$

SOLUTION The ratio of the periods is, therefore,

$$\frac{T_A}{T_B} = \frac{2\pi\sqrt{2d/(3g)}}{2\pi\sqrt{d/g}} = \sqrt{\frac{2}{3}} = \boxed{0.816}$$

44. ***REASONING AND SOLUTION*** As the object oscillates back and forth at the bottom of the bowl, it is subject to a restoring torque. The restoring torque on the object is given by $\tau = -mgR\sin\theta$. When the angle θ is small enough so that the object oscillates in simple harmonic motion, we can replace $\sin\theta$ with the angle θ itself, provided that the angle θ is expressed in radians. Thus, we have for small angles

$$\tau \approx -\underbrace{mgR}_{k'}\,\theta$$

The quantity mgR has a constant value k' and is analogous to the spring constant. As usual in rotational motion, the moment of inertia I appears in place of the mass m in equations, thus, we find, in analogy with $\omega = 2\pi f = \sqrt{k/m}$,

$$\omega = 2\pi f = \sqrt{\frac{k'}{I}} = \sqrt{\frac{mgR}{mR^2}} = \boxed{\sqrt{\frac{g}{R}}}$$

45. ***REASONING AND SOLUTION*** The period of the sphere (a physical pendulum) is

$$T_s = 2\pi\sqrt{\frac{I}{mgL}}$$

where for a solid sphere I = (7/5) MR^2 (see Table 9.1) and L = R. Therefore,

$$T_s = 2\pi\sqrt{\frac{7R}{5g}}$$

For the simple pendulum we know that

$$T_o = 2\pi\sqrt{\frac{L}{g}}$$

We want $T_s = T_o$ which leads to

$$7R/5g = L/g \quad \text{or} \quad \boxed{L = (7/5)R}$$

46. ***REASONING*** When the tow truck pulls the car out of the ditch, the cable stretches and a tension exists in it. This tension is the force that acts on the car. The amount ΔL that the cable stretches depends on the tension F, the length L_0 and cross-sectional area A of the cable, as well as Young's modulus Y for steel. All of these quantities are given in the statement of the problem, except for Young's modulus, which can be found by consulting Table 10.1.

SOLUTION Solving Equation 10.17, $F = Y\left(\frac{\Delta L}{L_0}\right)A$, for the change in length, we have

$$\Delta L = \frac{F L_0}{A Y} = \frac{(890\ \text{N})(9.1\ \text{m})}{\pi\left(0.50\times10^{-2}\ \text{m}\right)^2\left(2.0\times10^{11}\ \text{N/m}^2\right)} = \boxed{5.2\times10^{-4}\ \text{m}}$$

47. SSM WWW ***REASONING AND SOLUTION*** The amount of compression can be obtained from Equation 10.17,

$$F = Y\left(\frac{\Delta L}{L_0}\right)A$$

where F is the magnitude of the force on the stand due to the weight of the statue. Solving for ΔL gives

$$\Delta L = \frac{FL_0}{YA} = \frac{mgL_0}{YA} = \frac{(3500\text{ kg})(9.80\text{ m/s}^2)(1.8\text{ m})}{(2.3\times10^{10}\text{ N/m}^2)(7.3\times10^{-2}\text{ m}^2)} = \boxed{3.7\times10^{-5}\text{ m}}$$

48. ***REASONING*** The shear stress is equal to the magnitude of the shearing force exerted on the bar divided by the cross sectional area of the bar. The vertical deflection ΔY of the right end of the bar is given by Equation 10.18 [$F = S(\Delta Y / L_0)A$].

SOLUTION

a. The stress is

$$\frac{F}{A} = \frac{mg}{A} = \frac{(160\text{ kg})(9.80\text{ m/s}^2)}{3.2 \times 10^{-4}\text{ m}^2} = \boxed{4.9 \times 10^6\text{ N/m}^2}$$

b. The vertical deflection ΔY of the right end of the bar is

$$\Delta Y = \left(\frac{F}{A}\right)\frac{L_0}{S} = (4.9 \times 10^6\text{ N/m}^2)\frac{0.10\text{ m}}{8.1 \times 10^{10}\text{ N/m}^2} = \boxed{6.0 \times 10^{-6}\text{ m}}$$

49. **SSM** ***REASONING AND SOLUTION*** If we assume that each block is subjected to one-fourth of the applied force F, then, according to Equation 10.18, the force on each block is given by

$$\frac{F}{4} = S\left(\frac{\Delta X}{L_0}\right)A$$

Solving for ΔX, we obtain

$$\Delta X = \frac{FL_0}{4SA} = \frac{(32\text{ N})(0.030\text{ m})}{4(2.6\times10^{6}\text{ N/m}^2)(1.2\times10^{-3}\text{ m}^2)} = \boxed{7.7\times10^{-5}\text{ m}}$$

50. ***REASONING***

a. According to the discussion in Section 10.8, the stress is the magnitude of the force per unit area required to cause an elastic deformation. We can determine the maximum stress that will fracture the femur by dividing the magnitude of the compressional force by the cross-sectional area of the femur.

b. The strain is defined in Section 10.8 as the change in length of the femur divided by its original length. Equation 10.17 shows how the strain $\Delta L/L_0$ is related to the stress F/A and Young's modulus Y.

SOLUTION

a. The maximum stress is equal to the maximum compressional force divided by the cross-sectional area of the femur:

$$\text{Maximum stress} = \frac{F}{A} = \frac{6.8 \times 10^4 \text{ N}}{4.0 \times 10^{-4} \text{ m}^2} = \boxed{1.7 \times 10^8 \text{ N/m}^2}$$

b. The strain $\Delta L/L$ can be found by rearranging Equation 10.17:

$$\underbrace{\frac{\Delta L}{L_0}}_{\text{Strain}} = \frac{1}{Y}\underbrace{\left(\frac{F}{A}\right)}_{\text{Stress}} = \left(\frac{1}{9.4 \times 10^9 \text{ N/m}^2}\right)\left(1.7 \times 10^8 \text{ N/m}^2\right) = \boxed{1.8 \times 10^{-2}}$$

51. ***REASONING AND SOLUTION*** We know that

$$\Delta P = -B(\Delta V/V_0) = -(2.6 \times 10^{10}\ \text{N/m}^2)(-1.0 \times 10^{-10}\ \text{m}^3)/(1.0 \times 10^{-6}\ \text{m}^3) \qquad (10.20)$$

$$\Delta P = 2.6 \times 10^6\ \text{N/m}^2$$

Since the pressure increases by 1.0×10^4 N/m^2 per meter of depth, the depth is

$$\frac{2.6 \times 10^6\ \text{N/m}^2}{1.0 \times 10^4\ \dfrac{\text{N/m}^2}{\text{m}}} = \boxed{260\ \text{m}}$$

52. ***REASONING AND SOLUTION*** Y = Stress/Strain where Stress = F/A and Strain = $\Delta L/L_0$.

a. Stress = $\dfrac{(1800\text{ kg})(9.80\text{ m/s}^2)}{\pi(6.0\times10^{-3}\text{ m})^2} = \boxed{1.6\times10^8\text{ N/m}^2}$

b. Strain = $(8.0\times10^{-3}\text{ m})/(15\text{ m}) = \boxed{5.3\times10^{-4}}$

c. Y = Stress/Strain = $(1.6\times10^8\text{ N/m}^2)/(5.3\times10^{-4}) = \boxed{3.0\times10^{11}\text{ N/m}^2}$

53. SSM ***REASONING AND SOLUTION*** Equation 10.20 gives the desired result. Solving for $\Delta V / V_0$, we obtain

$$\frac{\Delta V}{V_0} = -\frac{\Delta P}{B} = -\frac{-1.01 \times 10^5 \text{ Pa}}{7.1 \times 10^{10} \text{ N/m}^2} = \boxed{1.4 \times 10^{-6}}$$

54. ***REASONING*** Both cylinders experience the same compressive force **F**. The magnitude of this force is related to the change in length of each cylinder according to Equation 10.17: $F = Y(\Delta L / L_0)A$. Each cylinder will decrease in length; the total decrease in the length of the stack is equal to the sum of the decrease in length of each cylinder.

SOLUTION The length of the copper cylinder decreases by

$$\Delta L_{\text{copper}} = \frac{FL_0}{YA} = \frac{FL_0}{Y(\pi r^2)} = \frac{(6500\ \text{N})(3.0 \times 10^{-2}\ \text{m})}{(1.1 \times 10^{11}\ \text{N/m}^2)\pi(0.25 \times 10^{-2}\ \text{m})^2} = 9.0 \times 10^{-5}\ \text{m}$$

Similarly, the length of the brass decreases by

$$\Delta L_{\text{brass}} = \frac{(6500\ \text{N})(5.0 \times 10^{-2}\ \text{m})}{(9.0 \times 10^{10}\ \text{N/m}^2)\pi(0.25 \times 10^{-2}\ \text{m})^2} = 1.8 \times 10^{-4}\ \text{m}$$

Therefore, the amount by which the length of the stack decreases is $\boxed{2.7 \times 10^{-4}\ \text{m}}$.

55. ***REASONING AND SOLUTION*** From the drawing we have $\Delta x = 3.0$ mm and

$$A = 2\pi r \Delta x = 2\pi (1.00 \times 10^{-2}\ \text{m})(3.0 \times 10^{-3}\ \text{m})$$

We now have Stress = F/A. Therefore,

$$F = (\text{Stress})A = (3.5 \times 10^{8}\ \text{Pa})[2\pi(1.00 \times 10^{-2}\ \text{m})(3.0 \times 10^{-3}\ \text{m})] = \boxed{6.6 \times 10^{4}\ \text{N}}$$

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56. ***REASONING*** When the object is placed on top of the pipe, the pipe is compressed due to the weight of the object. The amount ΔL of compression depends on the weight F, the length L_0 and cross-sectional area A of the pipe, as well as Young's modulus Y for steel. All of these quantities, except for the weight, are given in the statement of the problem. Young's modulus can be found by consulting Table 10.1.

SOLUTION Using Equation 10.17, the weight F of the object is

$$F = Y\left(\frac{\Delta L}{L_0}\right)A = \left(2.0\times10^{11}\ \text{N/m}^2\right)\left(\frac{5.7\times10^{-7}\ \text{m}}{3.6\ \text{m}}\right)\pi\left(65\times10^{-2}\ \text{m}\right)^2 = \boxed{4.2\times10^{4}\ \text{N}}$$

57. ***REASONING AND SOLUTION*** $\Delta P = -B(\Delta V/V_0)$ where $\Delta V/V_0$ is the volume strain. Therefore,

$$\Delta V/V_0 = -\Delta P/B = -(4.7 \times 10^5 \text{ Pa})/(1.7 \times 10^9 \text{ Pa}) = \boxed{-2.8 \times 10^{-4}} \qquad (10.20)$$

58. ***REASONING AND SOLUTION*** In this problem we want the strain = $\Delta L/L_0$ = 0.01. $F = Y(\Delta L/L_0)A$, where F = mg. The area of one piece of mohair is πr^2, so the total area of N identical pieces is $N\pi r^2$. Substituting for the force and area in the equation above and solving for N yields

$$N = \frac{mg}{Y\left(\frac{\Delta L}{L_0}\right)\pi r^2} = \frac{(75\text{ kg})(9.80\text{ m/s}^2)}{(2.9\times10^9\text{ N/m}^2)(0.010)\pi(31\times10^{-6}\text{ m})^2} = \boxed{8400\text{ pieces}}$$

59. SSM *REASONING AND SOLUTION*

a. When the jeep is suspended motionless, the tension T in the cable must be equal in magnitude and opposite in direction to the weight mg of the jeep. Therefore, from Equation 10.17,

$$mg = Y\left(\frac{\Delta L}{L_0}\right)A$$

Solving for ΔL, we obtain

$$\Delta L = \frac{mgL_0}{YA} = \frac{mgL_0}{Y(\pi r^2)} = \frac{(2100\text{ kg})(9.80\text{ m/s}^2)(48\text{ m})}{(2.0\times10^{11}\text{ Pa})\pi(5.0\times10^{-3}\text{ m})^2} = \boxed{6.3\times10^{-2}\text{ m}}$$

b. When the jeep is hoisted upward with an acceleration a, Newton's second law gives

$$T - mg = ma \qquad \text{or} \qquad T = m(a+g)$$

Therefore, Equation 10.17 becomes

$$m(a+g) = Y\left(\frac{\Delta L}{L_0}\right)A$$

and solving for ΔL gives

$$\Delta L = \frac{m(a+g)L_0}{Y(\pi r^2)} = \frac{(2100\text{ kg})(1.5\text{ m/s}^2 + 9.80\text{ m/s}^2)(48\text{ m})}{(2.0\times10^{11}\text{ Pa})\pi(5.0\times10^{-3}\text{ m})^2} = \boxed{7.3\times10^{-2}\text{ m}}$$

60. ***REASONING AND SOLUTION*** The applied force, $F_{applied}$, may be resolved into two components, one which is parallel to the area of the top surface (a shearing force) and one which is perpendicular to the top surface (a tensile force).

a. The change in the height of the block is caused by the component of the applied force that is perpendicular to the top surface. Since this component is a tensile force, the change in the height of the block can be found from Equation 10.17 with $F = F_{applied}(\sin\theta)$, $L_0 = H_0$, and $\Delta L = \Delta H$:

$$F_{applied}(\sin\theta) = Y\left(\frac{\Delta H}{H_0}\right)A$$

From Table 10.1, the Young's modulus of copper is 1.1×10^{11} N/m^2. The area, A of the top surface is $A = (5.0 \times 10^{-2}\text{ m})(3.0 \times 10^{-2}\text{ m}) = 1.5 \times 10^{-3}\text{ m}^2$. Solving for ΔH gives:

$$\Delta H = \frac{F_{applied}(\sin\theta)H_0}{YA} = \frac{(1800\text{ N})(\sin 25°)(0.040\text{ m})}{(1.1\times10^{11}\text{ N/m}^2)(1.5\times10^{-3}\text{ m}^2)} = \boxed{1.8\times10^{-7}\text{ m}}$$

b. The shear deformation of the block is caused by the component of the applied force that is tangent to the top surface, and can be determined from Equation 10.18 with $F = F_{applied}(\cos\theta)$, $L_0 = 0.040$ m, and $A = 1.5 \times 10^{-3}$ m^2.

$$F_{applied}(\cos\theta) = S\left(\frac{\Delta X}{L_0}\right)A$$

From Table 10.2, the shear modulus of copper is 4.2×10^{10} N/m^2. Solving for ΔX gives

$$\Delta X = \frac{F_{applied}(\cos\theta)L_0}{SA} = \frac{(1800\text{ N})(\cos 25°)(0.040\text{ m})}{(4.2\times10^{10}\text{ N/m}^2)(1.5\times10^{-3}\text{ m}^2)} = \boxed{1.0\times10^{-6}\text{ m}}$$

61. SSM WWW ***REASONING*** The strain in the wire is given by $\Delta L / L_0$. From Equation 10.17, the strain is therefore given by

$$\frac{\Delta L}{L_0} = \frac{F}{YA} \qquad (1)$$

where F must be equal to the magnitude of the centripetal force that keeps the stone moving in the circular path of radius R.

SOLUTION Combining Equation (1) with Equation 5.3 for the magnitude of the centripetal force, we obtain

$$\frac{\Delta L}{L_0} = \frac{F}{Y(\pi r^2)} = \frac{(mv^2 / R)}{Y(\pi r^2)} = \frac{(8.0\text{ kg})(12\text{ m/s})^2 /(4.0\text{ m})}{(2.0\times10^{11}\text{ Pa})\pi(1.0\times10^{-3}\text{ m})^2} = \boxed{4.6\times10^{-4}}$$

62. ***REASONING AND SOLUTION*** We can write $F = Y(\Delta L/L_0)A$ and $F' = Y(\Delta L'/L_0')/A'$. Since ΔL and $\Delta L'$ are equal, solve for ΔL and $\Delta L'$ above and set them equal. We thus obtain

$$FL_0/YA = F'L_0'/YA'$$

Now use the fact that $L_0' = (1/2)\ L_0$ to get $F' = 2F(A'/A)$. To find (A'/A) use the fact that the volume of the wire must be conserved, i.e., $V = AL_0 = V' = A'L_0'$, which leads to

$$(A'/A) = (L_0/L_0') = 2$$

Finally,

$$F' = 2F(A'/A) = \boxed{4F}$$

63. ***REASONING AND SOLUTION*** Strain = $\Delta L/L_0 = F/YA$ where $F = mg$ and $A = \pi r^2$. Setting the strain for the spider web equal to the strain for the wire

$$F/YA = F'/Y'A' \quad \text{so that} \quad F/Yr^2 = F'/Y'r'^2$$

Thus,

$$r'^2 = F'Yr^2/FY'$$

$$r' = \sqrt{\frac{(95\text{ kg})(9.80\text{ m/s}^2)(4.5\times10^9\text{ Pa})(13\times10^{-6}\text{ m})^2}{(1.0\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)(6.9\times10^{10}\text{ Pa})}} = \boxed{1.0\times10^{-3}\text{ m}}$$

64. ***REASONING AND SOLUTION*** We know $F = S(\Delta X/L_0)A$ where the force is due to friction, i.e.,

$$F = f_s^{max} = \mu_s mg = (0.90)(7.2 \times 10^{-2}\ \text{kg})(9.80\ \text{m/s}^2) = 0.64\ \text{N}$$

Therefore,

a. Stress = $F/A = (0.64\ \text{N})/(0.030\ \text{m})^2 = \boxed{710\ \text{N/m}^2}$

b. Strain = $\Delta X/L_0 = F/AS = (710\ \text{N/m}^2)/(2.0 \times 10^{10}\ \text{N/m}^2) = \boxed{3.5 \times 10^{-8}}$

c. ΔX = (Strain) $L_0 = (3.5 \times 10^{-8})(1.0 \times 10^{-2}\ \text{m}) = \boxed{3.5 \times 10^{-10}\ \text{m}}$

65. SSM WWW ***REASONING*** Equation 10.20 can be used to find the fractional change in volume of the brass sphere when it is exposed to the Venusian atmosphere. Once the fractional change in volume is known, it can be used to calculate the fractional change in radius.

SOLUTION According to Equation 10.20, the fractional change in volume is

$$\frac{\Delta V}{V_0} = -\frac{\Delta P}{B} = -\frac{8.9 \times 10^6 \text{ Pa}}{6.7 \times 10^{10} \text{ Pa}} = -1.33 \times 10^{-4}$$

The initial volume of the sphere is $V_0 = (4/3)\pi r^3$. If we assume that the change in the radius of the sphere is very small relative to the initial radius, we can think of the sphere's change in volume as the addition or subtraction of a spherical shell of volume ΔV, whose radius is r and whose thickness is Δr. Then, the change in volume of the sphere is equal to the volume of the shell and is given by $\Delta V = (4\pi r^2)\Delta r$. Combining the expressions for V_0 and ΔV, and solving for $\Delta r / r$, we have

$$\frac{\Delta r}{r} = \frac{1}{3}\frac{\Delta V}{V_0}$$

Therefore,

$$\frac{\Delta r}{r} = \frac{1}{3}(-1.33 \times 10^{-4}) = \boxed{-4.4 \times 10^{-5}}$$

66. ***REASONING*** The change in length of the wire is, According to Equation 10.17, $\Delta L = FL_0 / YA$, where the force F is equal to the tension T in the wire. The tension in the wire can be found by applying Newton's second law to the two crates.

SOLUTION The drawing shows the free-body diagrams for the two crates. Taking up as the positive direction, Newton's second law for each of the two crates gives

$$T - m_1 g = m_1 a \qquad (1)$$

$$T - m_2 g = -m_2 a \qquad (2)$$

Solving Equation (2) for a, we find $a = -\left(\dfrac{T - m_2 g}{m_2}\right)$. Substituting into Equation (1) gives

$$T - 2m_1 g + \frac{m_1}{m_2} T = 0$$

Solving for T we find

$$T = \frac{2m_1 m_2 g}{m_1 + m_2} = \frac{2(3.0\text{ kg})(5.0\text{ kg})(9.80\text{ m/s}^2)}{3.0\text{ kg} + 5.0\text{ kg}} = 37\text{ N}$$

Therefore, the change in length of the wire, according to Equation 10.17, is

$$\Delta L = \frac{(37\text{ N})(1.5\text{ m})}{(2.0\times10^{11}\text{ N/m}^2)(1.3\times10^{-5}\text{ m}^2)} = \boxed{2.1\times10^{-5}\text{ m}}$$

67. ***REASONING AND SOLUTION*** From the figure we see that

a. Amplitude = radius = $\boxed{5.08 \times 10^{-2}\ \text{m}}$

b. $T = 2\pi / \omega = 2\pi / (126\ \text{rad/s}) = \boxed{5.00 \times 10^{-2}\ \text{s}}$ (10.4)

c. $v_{max} = A\omega = (5.08 \times 10^{-2}\ \text{m})(126\ \text{rad/s}) = \boxed{6.40\ \text{m/s}}$ (10.8)

68. ***REASONING*** The force F that the spring exerts on the block just before it is released is equal to $-kx$, according to Equation 10.2. Here k is the spring constant and x is the displacement of the spring from its equilibrium position. Once the block has been released, it oscillates back and forth with an angular frequency given by Equation 10.11 as $\omega = \sqrt{k/m}$, where m is the mass of the block. The maximum speed that the block attains during the oscillatory motion is $v_{\text{max}} = A\omega$ (Equation 10.8). The magnitude of the maximum acceleration that the block attains is $a_{\text{max}} = A\omega^2$ (Equation 10.10).

SOLUTION

a. The force F exerted on the block by the spring is

$$F = -kx = -(82.0\ \text{N/m})(0.120\ \text{m}) = \boxed{-9.84\ \text{N}} \qquad (10.2)$$

b. The angular frequency ω of the resulting oscillatory motion is

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{82.0\ \text{N/m}}{0.750\ \text{kg}}} = \boxed{10.5\ \text{rad/s}} \qquad (10.11)$$

c. The maximum speed v_{max} is the product of the amplitude and the angular frequency:

$$v_{\text{max}} = A\omega = (0.120\ \text{m})(10.5\ \text{rad/s}) = \boxed{1.26\ \text{m/s}} \qquad (10.8)$$

d. The magnitude a_{max} of the maximum acceleration is

$$a_{\text{max}} = A\omega^2 = (0.120\ \text{m})(10.5\ \text{rad/s})^2 = \boxed{13.2\ \text{m/s}^2} \qquad (10.10)$$

69. ***REASONING AND SOLUTION*** The period of a simple pendulum is given by

$$T = 2\pi\sqrt{\frac{L}{g}}$$

Solving for L gives

$$L = \frac{T^2 g}{4\pi^2} = \frac{(2.0\text{ s})^2(9.80\text{ m/s}^2)}{4\pi^2} = \boxed{0.99\text{ m}}$$

70. ***REASONING AND SOLUTION*** The maximum acceleration is 25g, i.e., $a_{max} = 25(9.80\ \text{m/s}^2)$. We know that $a_{max} = A\omega^2$, where $\omega = 2\pi f$, so

$$A = \frac{25\left(9.80\ \text{m/s}^2\right)}{4\pi^2\left(9.5\ \text{Hz}\right)^2} = \boxed{0.069\ \text{m}}$$

71. SSM ***REASONING AND SOLUTION*** The shearing stress is equal to the force per unit area applied to the rivet. Thus, when a shearing stress of 5.0×10^8 Pa is applied to each rivet, the force experienced by each rivet is

$$F = (\text{Stress})A = (\text{Stress})(\pi r^2) = (5.0 \times 10^8 \text{ Pa})\left[\pi(5.0 \times 10^{-3} \text{ m})^2\right] = 3.9 \times 10^4 \text{ N}$$

Therefore, the maximum tension T that can be applied to each beam, assuming that each rivet carries one-fourth of the total load, is $\boxed{4F = 1.6 \times 10^5 \text{ N}}$.

72. ***REASONING AND SOLUTION*** Since $F = kx$,

$$x = \frac{F}{k} = \frac{9.6\ \text{N}}{44\ \text{N/m}} = \boxed{0.22\ \text{m}}$$

73. ***REASONING AND SOLUTION*** $F = S(\Delta X/L_0)A$ for the shearing force. From the figure we also see that $\tan\theta = (\Delta X/L_0)$ so that

$$\theta = \tan^{-1}\left(\frac{F}{SA}\right) = \tan^{-1}\left[\frac{6.0\times10^{6}\ \text{N}}{\left(4.2\times10^{10}\ \text{N/m}^2\right)\left(0.090\,\text{m}^2\right)}\right] = \boxed{0.091^\circ}$$

74. ***REASONING AND SOLUTION***

a. When the block passes through the position x = 0 m, its velocity is a maximum and can be found from Equation 10.8: $v_{max} = A\omega$.

We can find the angular frequency ω from the following reasoning: When the mass is given a displacement x, one spring is stretched by an amount x, while the other is compressed by an amount x. The total restoring force on the mass is, therefore,

$$F = -k_1x - k_2x = -(k_1 + k_2)x$$

Comparison with Equation 10.2 shows that the two spring system has an effective spring constant $k_{eff} = k_1 + k_2$. Thus, from Equation 10.11

$$\omega = \sqrt{\frac{k_{eff}}{m}} = \sqrt{\frac{k_1 + k_2}{m}}$$

Combining this with Equation 10.8 we obtain

$$v_{max} = A\sqrt{\frac{k_1 + k_2}{m}} = (0.070\ \text{m})\sqrt{\frac{650\ \text{N/m} + 450\ \text{N/m}}{3.0\ \text{kg}}} = \boxed{1.3\ \text{m/s}}$$

b. The angular frequency of the system is

$$\omega = \sqrt{\frac{k_1 + k_2}{m}} = \sqrt{\frac{650\ \text{N/m} + 450\ \text{N/m}}{3.0\ \text{kg}}} = \boxed{19\ \text{rad/s}}$$

75. **SSM** ***REASONING AND SOLUTION*** The force that acts on the block is given by Newton's Second law, $F = ma$. Since the block has a constant acceleration, the acceleration is given by Equation 2.8 with $v_0 = 0$ m/s; that is, $a = 2d/t^2$, where d is the distance through which the block is pulled. Therefore, the force that acts on the block is given by

$$F = ma = \frac{2md}{t^2}$$

The force acting on the block is the restoring force of the spring. Thus, according to Equation 10.2, $F = -kx$, where k is the spring constant and x is the displacement. Solving Equation 10.2 for x and using the expression above for F, we obtain

$$x = -\frac{F}{k} = -\frac{2md}{kt^2} = -\frac{2(7.00\text{ kg})(4.00\text{ m})}{(415\text{ N/m})(0.750\text{ s})^2} = -0.240\text{ m}$$

The amount that the spring stretches is $\boxed{0.240\text{ m}}$.

76. ***REASONING AND SOLUTION***

a. Strain = $\Delta L/L_0 = F/YA$. In this case the area subjected to the compression is given by

$$A = \pi\,(r_{out}^{\,2} - r_{in}^{\,2}) = 2.64 \times 10^{-4}\ \text{m}^2$$

and the force is F = mg. Therefore,

$$\text{Strain} = \frac{(63\ \text{kg})(9.80\ \text{m/s}^2)}{(9.4 \times 10^9\ \text{Pa})(2.64 \times 10^{-4}\ \text{m}^2)} = \boxed{2.5 \times 10^{-4}}$$

b. $\Delta L = \text{Strain} \times L_0 = (2.5 \times 10^{-4})(0.30\ \text{m}) = \boxed{7.5 \times 10^{-5}\ \text{m}}$

77. ***REASONING AND SOLUTION*** When the centrifuge is in motion, the aluminum rod will exert a centripetal force **F'** on the specimen holder. By Newton's third law, the specimen holder will exert a force **F** on the rod that is equal in magnitude and opposite in direction to **F'**. The force **F** causes the aluminum rod to stretch according to Equation 10.17:

$$F = Y\left(\frac{\Delta L}{L_0}\right)A$$

Since **F** is the reaction force to the centripetal force **F'**, the magnitude of F is given by $F = mr\omega^2$. Then,

$$mr\omega^2 = Y\left(\frac{\Delta L}{L_0}\right)A$$

Since the original length of the arm is L_0 and ΔL is its increase in length when it is in motion, the radius of the circular motion is $r = (L_0 + \Delta L) \cong L_0$. In other words, we will assume that the change in length of the rod is small compared to its original length. Then,

$$mL_0\omega^2 = Y\left(\frac{\Delta L}{L_0}\right)A$$

From Table 10.1, the Young's modulus for aluminum is 6.9×10^{10} N/m^2. Solving for ΔL gives:

$$\Delta L = \frac{mL_0^2\omega^2}{YA} = \frac{(0.58\text{ kg})(0.10\text{ m})^2(180\text{ rad/s})^2}{(6.9\times10^{10}\text{ N/m}^2)(1.3\times10^{-4}\text{ m}^2)} = \boxed{2.1\times10^{-5}\text{ m}}$$

Notice that this increase in length is small compared to the original length of the arm as it was assumed in the approximation $r = (L_0 + \Delta L) \cong L_0$.

78. ***REASONING AND SOLUTION*** The natural frequency of the suspension system is given by Equation 10.11:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{1.50 \times 10^6 \text{ N/m}}{215 \text{ kg}}} = 83.5 \text{ rad/s}$$

Thus, the wheel will resonate when its angular speed is 83.5 rad/s. This corresponds to a linear speed of

$$v = r\omega = (0.400 \text{ m})(83.5 \text{ rad/s}) = \boxed{33.4 \text{ m/s}}$$

79. **SSM** ***REASONING AND SOLUTION*** If we assume that the wire has a circular cross-section, then Equation 10.17 becomes

$$F = Y\left(\frac{\Delta L}{L_0}\right)(\pi r^2)$$

where F is the tension in the wire and r is the radius of the wire. Solving for r gives

$$r = \sqrt{\frac{FL_0}{\pi Y \Delta L}} \qquad (1)$$

Young's modulus for steel is given in Table 10.1 as $Y = 2.0 \times 10^{11}$ N/m^2. The change in length ΔL can be determined from the figure at the right. If L_0 represents the length of the wire when it is horizontal, and L its length when stretched, then the figure shows that

$$\cos 26° = \frac{L_0/2}{L/2} \quad \text{or} \quad \mathrm{L} = \frac{\mathrm{L}_0}{\cos 26°} = 1.11L_0 \qquad (2)$$

Therefore,

$$\Delta L = L - L_0 = 0.11L_0$$

The tension in the wire can be determined from the free-body diagram at the right. Applying Newton's second law to the forces in the vertical direction yields

$$2F \sin 26° - W = 0 \quad \text{or} \quad F = \frac{W}{2 \sin 26°} \qquad (3)$$

Substituting Equations (2) and (3) into Equation (1) yields

$$r = \sqrt{\frac{W L_0}{2\pi(\sin 26°)Y(0.11L_0)}} = \sqrt{\frac{(96\ \mathrm{N})L_0}{2\pi(\sin 26°)(2.0\times10^{11}\ \mathrm{N/m^2})(0.11L_0)}} = \boxed{4.0\times10^{-5}\ \mathrm{m}}$$

80. ***REASONING AND SOLUTION*** The bullet (mass m) moves with speed v, strikes the block (mass M) in an inelastic collision and the two move together with a final speed V. We first need to employ the conservation of linear momentum to the collision to obtain an expression for the final speed:

$$mv = (m + M)V \qquad \text{or} \qquad V = \frac{mv}{m + M}$$

The block/bullet system now compresses the spring by an amount x. During the compression the total mechanical energy is conserved so that

$$\tfrac{1}{2}(m + M)V^2 = \tfrac{1}{2}kx^2$$

Substituting the expression for V into this equation, we obtain

$$\tfrac{1}{2}(m + M)\left(\frac{mv}{m + M}\right)^2 = \tfrac{1}{2}kx^2$$

Solving this expression for v gives

$$v = \sqrt{\frac{k\,x^2\,(M + m)}{m^2}} = \sqrt{\frac{(845\text{ N/m})(0.200\text{ m})^2\,(2.51\text{ kg})^2}{(0.0100\text{ kg})^2}} = \boxed{921\text{ m/s}}$$

81. ***REASONING AND SOLUTION*** The frequency f of the simple harmonic motion is given by Equations 10.6 and 10.11 as $f = (1/2\pi)\sqrt{k/m}$. If we compare Equation 10.15, which governs the stretching and compression of a solid rod with Equation 10.1, we find that x is analogous to ΔL and k is analogous to the term YA/L_0:

$$F = \underbrace{\frac{YA}{L_0}}_{k}\underbrace{\Delta L}_{x}$$

Assuming that the rod has a circular cross-section, its area A is equal to πr^2, and we have

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{YA}{L_0 m}} = \frac{1}{2\pi}\sqrt{\frac{Y(\pi r^2)}{L_0 m}}$$

$$= \frac{1}{2\pi}\sqrt{\frac{(1.1\times 10^{11}\ \text{N/m}^2)\pi(3.0\times 10^{-3}\ \text{m})^2}{(2.0\ \text{m})(9.0\ \text{kg})}} = \boxed{66\ \text{Hz}}$$

82. ***REASONING*** As the climber falls, only two forces act on him: his weight and the elastic force of the nylon rope. Both of these forces are conservative forces, so the falling climber obeys the conservation of mechanical energy. We will use this conservation law to determine how much the rope is stretched when it breaks his fall and momentarily brings him to rest.

SOLUTION

The conservation of mechanical energy states that the final total mechanical energy E_f is equal to the initial total mechanical energy E_0, or $E_f = E_0$ (Equation 6.9a). The expression for the total mechanical energy of an object oscillating on a spring is given by Equation 10.14. Thus, the conservation of total mechanical energy can be written as

$$\underbrace{\tfrac{1}{2}mv_f^2 + \tfrac{1}{2}I\omega_f^2 + mgh_f + \tfrac{1}{2}kx_f^2}_{E_f} = \underbrace{\tfrac{1}{2}mv_0^2 + \tfrac{1}{2}I\omega_0^2 + mgh_0 + \tfrac{1}{2}kx_0^2}_{E_0}$$

Before going any further, let's simplify this equation by noting which variables are zero. Since the climber starts and ends at rest, $v_f = v_0 = 0$ m/s. The climber does not rotate, so his angular speed is zero, $\omega_f = \omega_0 = 0$ rad/s. Initially, the spring is unstretched, so that $x_0 = 0$ m. Setting these terms to zero in the equation above gives

$$mgh_f + \tfrac{1}{2}kx_f^2 = mgh_0$$

Rearranging the terms in this equation, we have

$$\tfrac{1}{2}kx_f^2 + mg\underbrace{(h_f - h_0)}_{x_f - 0.750\text{ m}} = 0$$

Note that the climber falls a distance of 0.750 m before the rope starts to stretch, and x_f is the displacement of the stretched rope. Since x_f points downward, it is considered to be negative. Thus, $x_f - 0.750$ m is the total downward displacement of the falling climber, which is also equal to $h_f - h_0$. With this substitution, the equation above becomes

$$\tfrac{1}{2}kx_f^2 + mgx_f - mg(0.750\text{ m}) = 0$$

This is a quadratic equation in the variable x_f. Using the quadratic formula gives

$$x_f = \frac{mg \pm \sqrt{(mg)^2 - 4\left(\frac{1}{2}k\right)\left[-mg(0.750\text{ m})\right]}}{2\left(\frac{1}{2}k\right)}$$

$$x_f = \frac{(86.0\text{ kg})\left(9.80\text{ m/s}^2\right)}{1.20\times10^3\text{ N/m}}$$

$$\pm\frac{\sqrt{\left[(86.0\text{ kg})\left(9.80\text{ m/s}^2\right)\right]^2 - 4\left(\frac{1}{2}\right)\left(1.20\times10^3\text{ N/m}\right)\left[-(86.0\text{ kg})\left(9.80\text{ m/s}^2\right)(0.750\text{ m})\right]}}{1.20\times10^3\text{ N/m}}$$

There are two answers, x_f = +0.54 m and –1.95 m. Since the rope is stretched in the downward direction, which we have taken to be the negative direction, the displacement is –1.95 m. Thus, the amount that the rope is stretched is $x_f = \boxed{1.95\text{ m}}$.

83. $\boxed{\text{SSM}}$ ***REASONING*** The angular frequency for simple harmonic motion is given by Equation 10.11 as $\omega = \sqrt{k/m}$. Since the frequency f is related to the angular frequency ω by
$f = \omega/2\pi$, and f is related to the period T by $f = 1/T$, the period of the motion is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{k}{m}}$$

SOLUTION

a. When $m_1 = m_2 = 3.0$ kg, we have that

$$T_1 = T_2 = 2\pi\sqrt{\frac{3.0\text{ kg}}{120\text{ N/m}}} = 0.99\text{ s}$$

Both particles will pass through the position $x = 0$ m for the first time one-quarter of the way through one cycle, or

$$\Delta t = \tfrac{1}{4}T_1 = \tfrac{1}{4}T_2 = \frac{0.99\text{ s}}{4} = \boxed{0.25\text{ s}}$$

b. $T_1 = 0.99$ s, as in part (a) above, while

$$T_2 = 2\pi\sqrt{\frac{27.0\text{ kg}}{120\text{ N/m}}} = 3.0\text{ s}$$

Each particle will pass through the position $x = 0$ m every odd-quarter of a cycle, $\tfrac{1}{4}T, \tfrac{3}{4}T, \tfrac{5}{4}T, \ldots$ Thus, the two particles will pass through $x = 0$ m when

3.0-kg particle $\quad t = \tfrac{1}{4}T_1, \tfrac{3}{4}T_1, \tfrac{5}{4}T_1, \ldots$

27.0-kg particle $\quad t = \tfrac{1}{4}T_2, \tfrac{3}{4}T_2, \tfrac{5}{4}T_2, \ldots$

Since $T_2 = 3T_1$, we see that both particles will be at $x = 0$ m simultaneously when $t = \tfrac{3}{4}T_1$, or $t = \tfrac{1}{4}T_2 = \tfrac{3}{4}T_1$. Thus,

$$t = \tfrac{3}{4}T_1 = \tfrac{3}{4}(0.99\text{ s}) = \boxed{0.75\text{ s}}$$

84. ***REASONING*** If we compare Equation 10.17, which governs the stretching and compression of a solid cylinder, with Hooke's law (Equation 10.2), we find that x is analogous to ΔL and k is analogous to the term YA/L_0:

$$F = \underbrace{\left(\frac{YA}{L_0}\right)}_{k} \underbrace{\Delta L}_{x}$$

SOLUTION

a. Solving for k we have

$$k = \frac{YA}{L_0} = \frac{Y(\pi r^2)}{L_0} = \frac{(3.1\times10^6\ \text{N/m}^2)\pi(0.091\times10^{-2}\ \text{m})^2}{2.5\times10^{-2}\ \text{m}} = \boxed{3.2\times10^2\ \text{N/m}}$$

b. The work done by the variable force is equal to the area under the F-versus-x curve. The amount x of stretch is

$$x = \frac{F}{k} = \frac{3.0\times10^{-2}\ \text{N}}{3.2\times10^2\ \text{N/m}} = 9.4\times10^{-5}\ \text{m}$$

The work done is

$$W = \tfrac{1}{2}Fx = \tfrac{1}{2}(3.0\times10^{-2}\ \text{N})(9.4\times10^{-5}\ \text{m}) = \boxed{1.4\times10^{-6}\ \text{J}}$$

1. SSM ***REASONING AND SOLUTION*** Equation 11.1 can be solved for the mass m. In order to use Equation 11.1, however, we must know the volume of the ice. Since the lake is circular, the volume of the ice is equal to the circular area of the lake $(A = \pi r^2)$ multiplied by the thickness d of the ice.

$$m = \rho V = \rho(\pi r^2)d = (917 \text{ kg/m}^3)\pi(480 \text{ m})^2(0.010 \text{ m}) = \boxed{6.6 \times 10^6 \text{ kg}}$$

2. ***REASONING AND SOLUTION***

a. We will treat the neutron star as spherical in shape, so that its volume is given by the familiar formula, $V = \frac{4}{3}\pi r^3$. Then, according to Equation 11.1, the density of the neutron star described in the problem statement is

$$\rho = \frac{m}{V} = \frac{m}{\frac{4}{3}\pi r^3} = \frac{3m}{4\pi r^3} = \frac{3(2.7\times10^{28}\ \text{kg})}{4\pi(1.2\times10^3\ \text{m})^3} = \boxed{3.7\times10^{18}\ \text{kg/m}^3}$$

b. If a dime of volume $2.0\times10^{-7}\ \text{m}^3$ were made of this material, it would weigh

$$W = mg = \rho V g = (3.7\times10^{18}\ \text{kg/m}^3)(2.0\times10^{-7}\ \text{m}^3)(9.80\ \text{m/s}^2) = 7.3\times10^{12}\ \text{N}$$

This weight corresponds to

$$7.3\times10^{12}\ \text{N}\left(\frac{1\ \text{lb}}{4.448\ \text{N}}\right) = \boxed{1.6\times10^{12}\ \text{lb}}.$$

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3. ***REASONING AND SOLUTION*** The weight of the concrete in the highway is

$$W = mg \tag{1}$$

where the mass m of the concrete can be found from the density of concrete and the volume of the roadbed. By definition, density is the mass per unit volume: $\rho = m/V$, where the volume is $V = \ell wd$. Therefore,

$$m = \rho(\ell wd) \tag{2}$$

Combining Equations (1) and (2) and obtaining the density of concrete from Table 11.1, we find that

$$W = \rho(\ell wd)g = (2200\ \text{kg/m}^3)(150 \times 10^3\ \text{m})(7.0\ \text{m})(0.30\ \text{m})(9.80\ \text{m/s}^2) = \boxed{6.8 \times 10^9\ \text{N}}$$

4. ***REASONING*** The density of the brass ball is given by Equation 11.1: $\rho = m/V$. Since the ball is spherical, its volume is given by the expression $V = (4/3)\pi r^3$, so that the density may be written as

$$\rho = \frac{m}{V} = \frac{3m}{4\pi r^3}$$

This expression can be solved for the radius r; but first, we must eliminate the mass m from the equation since its value is not specified explicitly in the problem. We can determine an expression for the mass m of the brass ball by analyzing the forces on the ball.

The only two forces that act on the ball are the upward tension **T** in the wire and the downward weight $m\mathbf{g}$ of the ball. If we take up as the positive direction, and we apply Newton's second law, we find that $T - mg = 0$, or $m = T/g$. Therefore, the density can be written as

$$\rho = \frac{3m}{4\pi r^3} = \frac{3T}{4\pi r^3 g}$$

This expression can now be solved for r.

SOLUTION We find that

$$r^3 = \frac{3T}{4\pi\rho g}$$

or, taking the cube root of both sides,

$$r = \sqrt[3]{\frac{3T}{4\pi\rho g}} = \sqrt[3]{\frac{3(120\text{ N})}{4\pi(8470\text{ kg/m}^3)(9.80\text{ m/s}^2)}} = \boxed{7.0\times10^{-2}\text{ m}}$$

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5. SSM ***REASONING*** Equation 11.1 can be used to find the volume occupied by 1.00 kg of silver. Once the volume is known, the area of a sheet of silver of thickness d can be found from the fact that the volume is equal to the area of the sheet times its thickness.

SOLUTION Solving Equation 11.1 for V, the volume of 1.00 kg of silver is

$$V = \frac{m}{\rho} = \frac{1.00 \text{ kg}}{10\,500 \text{ kg/m}^3} = 9.52 \times 10^{-5} \text{ m}^3$$

The area of the silver, is, therefore,

$$A = \frac{V}{d} = \frac{9.52 \times 10^{-5} \text{ m}^3}{3.00 \times 10^{-7} \text{ m}} = \boxed{317 \text{ m}^2}$$

6. ***REASONING*** According to the definition of density ρ given in Equation 11.1, the mass m of a substance is $m = \rho V$, where V is the volume. We will use this equation and the fact that the mass of the water and the gold are equal to find our answer. To convert from a volume in cubic meters to a volume in gallons, we refer to the inside of the front cover of the text to find that 1 gal = 3.785×10^{-3} m^3.

REASONING Using Equation 11.1, we find that

$$\rho_{\text{Water}} V_{\text{Water}} = \rho_{\text{Gold}} V_{\text{Gold}} \quad \text{or} \quad V_{\text{Water}} = \frac{\rho_{\text{Gold}} V_{\text{Gold}}}{\rho_{\text{Water}}}$$

Using the fact that $1\ \text{gal} = 3.785 \times 10^{-3}\ \text{m}^3$ and densities for gold and water from Table 11.1, we find

$$V_{\text{Water}} = \frac{\rho_{\text{Gold}} V_{\text{Gold}}}{\rho_{\text{Water}}}$$

$$= \frac{(19\,300\ \text{kg/m}^3)(0.15\ \text{m})(0.050\ \text{m})(0.050\ \text{m})}{(1000\ \text{kg/m}^3)}\left(\frac{1\ \text{gal}}{3.785 \times 10^{-3}\ \text{m}^3}\right) = \boxed{1.9\ \text{gal}}$$

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7. ***REASONING AND SOLUTION*** If the concrete were solid, it would have a mass of

$$M = \rho V = (2200\ \text{kg/m}^3)(0.025\ \text{m}^3) = 55\ \text{kg}$$

The mass of concrete removed to make the hole is then 55 kg – 33 kg = 22 kg. This corresponds to a volume $V = (22\ \text{kg})/(2200\ \text{kg/m}^3) = 0.010\ \text{m}^3$. Since the hole is spherical $V = (4/3)\pi r^3$ so

$$r = \sqrt[3]{\frac{3V}{4\pi}} = \sqrt[3]{\frac{3(0.010\ \text{m}^3)}{4\pi}} = \boxed{0.13\ \text{m}}$$

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8. ***REASONING AND SOLUTION*** The volume of helium enclosed by the sphere is $V = M_H/\rho_H$. Since $V = (4/3)\pi R^3$ we have

$$R^3 = (3/4)M_H/(\pi\rho_H) \tag{1}$$

The volume of silver is $V_S = M_S/\rho_S$

$$(4/3)\pi(R + T)^3 - (4/3)\pi R^3 = M_S/\rho_S$$

$$3R^2T + 3RT^2 + T^3 = 3/(4)M_S/(\pi\rho_S) \tag{2}$$

Dividing (2) by (1) and remembering that $M_H = M_S$ gives

$$3T/R + 3T^2/R^2 + T^3/R^3 = \rho_H/\rho_S$$

If T/R is small, then the squared and cubed ratios may be neglected and

$$T/R = (1/3)(\rho_H/\rho_S) = \boxed{5.68 \times 10^{-6}}$$

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9. SSM WWW ***REASONING*** The total mass of the solution is the sum of the masses of its constituents. Therefore,

$$\rho_s V_s = \rho_w V_w + \rho_g V_g \tag{1}$$

where the subscripts s, w, and g refer to the solution, the water, and the ethylene glycol, respectively. The volume of the water can be written as $V_w = V_s - V_g$. Making this substitution for V_w, Equation (1) above can be rearranged to give

$$\frac{V_g}{V_s} = \frac{\rho_s - \rho_w}{\rho_g - \rho_w} \tag{2}$$

Equation (2) can be used to calculate the relative volume of ethylene glycol in the solution.

SOLUTION The density of ethylene glycol is given in the problem. The density of water is given in Table 11.1 as 1.000×10^3 kg/m^3. The specific gravity of the solution is given as 1.0730. Therefore, the density of the solution is

$$\begin{aligned}\rho_s &= (\text{specific gravity of solution}) \times \rho_w \\ &= (1.0730)(1.000 \times 10^3 \text{ kg/m}^3) = 1.0730 \times 10^3 \text{ kg/m}^3\end{aligned}$$

Substituting the values for the densities into Equation (2), we obtain

$$\frac{V_g}{V_s} = \frac{\rho_s - \rho_w}{\rho_g - \rho_w} = \frac{1.0730 \times 10^3 \text{ kg/m}^3 - 1.000 \times 10^3 \text{ kg/m}^3}{1116 \text{ kg/m}^3 - 1.000 \times 10^3 \text{ kg/m}^3} = 0.63$$

Therefore, the volume percentage of ethylene glycol is $\boxed{63\%}$.

10. ***REASONING*** Since the inside of the box is completely evacuated, there is no air to exert an upward force on the lid from the inside. Furthermore, since the weight of the lid is negligible, there is only one force that acts on the lid; the downward force caused by the air pressure on the outside of the lid. In order to pull the lid off the box, one must supply a force that is at least equal in magnitude and opposite in direction to the force exerted on the lid by the outside air.

SOLUTION According to Equation 11.3, pressure is defined as $P = F/A$; therefore, the magnitude of the force on the lid due to the air pressure is

$$F = (0.85 \times 10^5 \text{ N/m}^2)(1.3 \times 10^{-2} \text{ m}^2) = \boxed{1.1 \times 10^3 \text{ N}}$$

11. ***REASONING AND SOLUTION***

a. We know that $F_T = PA_T = P\pi R^2$, so that

$$F_T = (1.40 \times 10^5 \text{ Pa})\pi(3.20 \times 10^{-2} \text{ m})^2 = \boxed{4.50 \times 10^2 \text{ N}}$$

b. $$F_p = (1.40 \times 10^5 \text{ Pa})(3.80 \times 10^{-4} \text{ m}^2) = \boxed{53.2 \text{ N}}$$

---

12. ***REASONING*** Pressure is the magnitude of the force applied perpendicularly to a surface divided by the area of the surface, according to Equation 11.3. The force magnitude, therefore, is equal to the pressure times the area.

SOLUTION According to Equation 11.3, we have

$$F = PA = \left(8.0 \times 10^4 \text{ lb/in.}^2\right)\left[(6.1 \text{ in.})(2.6 \text{ in.})\right] = \boxed{1.3 \times 10^6 \text{ lb}}$$

13. SSM ***REASONING AND SOLUTION*** Equation 11.3 gives the desired result, where F is equal to the weight of the woman. Assuming that the heel has a circular cross section, its area is $A = \pi r^2$, where r is the radius.

$$P = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(50.0\text{ kg})(9.80\text{ m/s}^2)}{\pi(6.00\times10^{-3}\text{ m})^2} = \boxed{4.33\times10^6\text{ Pa}}$$

14. ***REASONING*** According to Equation 11.3, each side of the window will experience a force of magnitude $F = PA$ where P is the pressure on the window and A is its area. We can find the magnitude of the net force on the window by adding the forces on either side of the window.

SOLUTION Before the pressure inside your apartment has had time to change, the outward force on the window has a magnitude of $F_1 = P_1 A$, where $P_1 = 1.013 \times 10^5$ Pa is the outward pressure on the window. Similarly, the inward force on the window has a magnitude of $F_2 = P_2 A$, where $P_2 = 0.960 \times 10^5$ Pa is the inward pressure on the window.

If we take outward as the positive direction, the net force on the window is

$$F_{\text{net}} = F_1 - F_2 = P_1 A - P_2 A = (P_1 - P_2) A$$

$$= \left(1.013 \times 10^5 \text{ Pa} - 0.960 \times 10^5 \text{ Pa}\right)(2.0 \text{ m} \times 3.1 \text{ m}) = +3.3 \times 10^4 \text{ N}$$

The plus sign indicates that the net force exerted on the window points outward. The magnitude of this force is $\boxed{3.3 \times 10^4 \text{ N}}$.

15. ***REASONING AND SOLUTION*** The least number of bricks are used when the surface area is the smallest. The area of the brick in contact with the ground has a surface area of

$$A = (0.0570\ \text{m})(0.0890\ \text{m}) = 5.07 \times 10^{-3}\ \text{m}^2$$

The pressure exerted by a single brick is

$$P = F/A = (17.8\ \text{N})/(5.07 \times 10^{-3}\ \text{m}^2) = 3.51 \times 10^3\ \text{N/m}^2$$

In order to equal or exceed atmospheric pressure

$$\text{Number of bricks} = (1.013 \times 10^5\ \text{N/m}^2)/(3.51 \times 10^3\ \text{N/m}^2) = \boxed{29\ \text{bricks}}$$

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16. ***REASONING*** Pressure is the magnitude of the force applied perpendicularly to a surface divided by the area of the surface, according to Equation 11.3. To find the force that the suitcase applies to the floor, we first find the upward normal force F_N that the floor applies to the suitcase. By Newton's action-reaction law, the force that the suitcase applies to the floor has the same magnitude as F_N, but points downward. In determining F_N, we will use Newton's second law to account for the fact that the suitcase is accelerating.

SOLUTION Assuming that upward is the positive direction, we apply Newton's second law as follows:

$$\underbrace{\Sigma F}_{\text{Net force}} = \underbrace{F_N}_{\substack{\text{Normal}\\ \text{force}}} - \underbrace{mg}_{\text{Weight}} = ma \quad \text{or} \quad F_N = mg + ma$$

According to Equation 11.3, the pressure, then, is

$$P = \frac{F_N}{A} = \frac{m(g+a)}{A} = \frac{(16\text{ kg})(9.80\text{ m/s}^2 + 1.5\text{ m/s}^2)}{(0.50\text{ m})(0.15\text{ m})} = \boxed{2.4\times10^3\text{ Pa}}$$

17. **SSM** ***REASONING AND SOLUTION*** Both the cylinder and the hemisphere have circular cross sections. According to Equation 11.3, the pressure exerted on the ground by the hemisphere is

$$P = \frac{W_h}{A_h} = \frac{W_h}{\pi r_h^2}$$

where W_h and r_h are the weight and radius of the hemisphere. Similarly, the pressure exerted on the ground by the cylinder is

$$P = \frac{W_c}{A_c} = \frac{W_c}{\pi r_c^2}$$

where W_c and r_c are the weight and radius of the cylinder. Since each object exerts the same pressure on the ground, we can equate the right-hand sides of the expressions above to obtain

$$\frac{W_h}{\pi r_h^2} = \frac{W_c}{\pi r_c^2}$$

Solving for ${r_h}^2$, we obtain

$$r_h^2 = r_c^2 \frac{W_h}{W_c} \tag{1}$$

The weight of the hemisphere is

$$W_h = \rho g V_h = \rho g\left[\tfrac{1}{2}\left(\tfrac{4}{3}\pi r_h^3\right)\right] = \tfrac{2}{3}\rho g \pi r_h^3$$

where ρ and V_h are the density and volume of the hemisphere, respectively. The weight of the cylinder is

$$W_c = \rho g V_c = \rho g \pi r_c^2 h$$

where ρ and V_c are the density and volume of the cylinder, respectively, and h is the height of the cylinder. Substituting these expressions for the weights into Equation (1) gives

$$r_h^2 = r_c^2 \frac{W_h}{W_c} = r_c^2 \frac{\frac{2}{3}\rho g \pi r_h^3}{\rho g \pi r_c^2 h}$$

Solving for r_h gives

$$r_h = \tfrac{3}{2}h = \tfrac{3}{2}(0.500\text{ m}) = \boxed{0.750\text{ m}}$$

18. ***REASONING AND SOLUTION*** The figures at the right show all the forces that act on each mass. T is the tension in the rope, F_B is the upward force exerted on mass 1 due to the pressure beneath the piston, and F_0 is the downward force exerted on mass 1 by the atmosphere.

Applying Newton's second law to mass 1, and taking "up" as the positive direction, we have

$$F_B + T - m_1 g - F_0 = m_1 a \qquad (1)$$

Similarly for mass 2 we have

$$T - m_2 g = -m_2 a \qquad (2)$$

Subtracting Equation (2) from Equation (1) gives

$$F_B + m_2 g - m_1 g - F_0 = (m_1 + m_2)\, a \qquad (3)$$

Solving Equation (3) for F_B gives

$$F_B = m_1(g + a) + m_2(a - g) + F_0 \qquad (4)$$

We must now find the acceleration a and the force F_0. From kinematics, we recall that $y = v_{0y}t + \frac{1}{2}a_y t^2$. Solving for a_y and setting $v_{oy} = 0$, we have

$$a_y = \frac{2y}{t^2} = \frac{2(1.25\text{ m})}{(3.30\text{ s})^2} = 0.230\text{ m/s}^2$$

From the definition of pressure (P = F/A), the force F_0 is

$$F_0 = P_0 A = P_0(\pi r^2) = \pi(1.013 \times 10^5\text{ N/m}^2)(2.50 \times 10^{-2}\text{ m})^2 = 1.99 \times 10^2\text{ N}$$

Substituting these values into Equation (4) gives:

$$F_B = (0.500\text{ kg})(9.80\text{ m/s}^2 + 0.230\text{ m/s}^2) + (9.50\text{ kg})(0.230\text{ m/s}^2 - 9.80\text{ m/s}^2) + 1.99 \times 10^2\text{ N} = 113\text{ N}$$

Thus, the pressure beneath the piston is

$$P_B = \frac{F_B}{A} = \frac{F_B}{\pi r^2} = \frac{113\text{ N}}{\pi(2.50 \times 10^{-2}\text{ m})^2} = \boxed{5.75 \times 10^4\text{ Pa}}$$

19. **SSM** ***REASONING AND SOLUTION*** Using Equation 11.4, we have

$$P_{\text{heart}} - P_{\text{brain}} = \rho g h = (1.000 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(12 \text{ m}) = \boxed{1.2 \times 10^5 \text{ Pa}}$$

20. ***REASONING*** The drawing at the right shows the situation. As discussed in Conceptual Example 6, the job of the pump is to draw air out of the pipe that dips down into the water. The atmospheric pressure in the well then pushes the water upward into the pipe. In the drawing, the best the pump can do is to remove all of the air, in which case, the pressure P_1 at the top of the water in the pipe is zero. The pressure P_2 at the bottom of the pipe at point *A* is the same as that at the point *B*, namely, it is equal to atmospheric pressure (1.013×10^5 Pa), because the two points are at the same elevation, and point *B* is open to the atmosphere. Equation 11.4, $P_2 = P_1 + \rho g h$ can be applied to obtain the maximum depth *h* of the well.

SOLUTION Setting $P_1 = 0$, and solving Equation 11.4 for *h*, we have

$$h = \frac{P_1}{\rho g} = \frac{1.013 \times 10^5 \text{ Pa}}{(1.000 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{10.3 \text{ m}}$$

21. ***REASONING*** The magnitude of the force that would be exerted on the window is given by Equation 11.3, $F = PA$, where the pressure can be found from Equation 11.4: $P_2 = P_1 + \rho gh$. Since P_1 represents the pressure at the surface of the water, it is equal to atmospheric pressure, P_{atm}. Therefore, the magnitude of the force is given by

$$F = (P_{\text{atm}} + \rho gh)A$$

where, if we assume that the window is circular with radius *r*, its area *A* is given by $A = \pi r^2$.

SOLUTION

a. Thus, the magnitude of the force is

$$F = \left[1.013 \times 10^5 \text{ Pa} + (1025 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(11\,000 \text{ m})\right]\pi(0.10 \text{ m})^2 = \boxed{3.5 \times 10^6 \text{ N}}$$

b. The weight of a jetliner whose mass is 1.2×10^5 kg is

$$W = mg = (1.2 \times 10^5 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{1.2 \times 10^6 \text{ N}}$$

Therefore, the force exerted on the window at a depth of 11 000 m is about three times greater than the weight of a jetliner!

22. ***REASONING*** As the depth h increases, the pressure increases according to Equation 11.4 ($P_2 = P_1 + \rho gh$). In this equation, P_1 is the pressure at the shallow end, P_2 is the pressure at the deep end, and ρ is the density of water (1.00×10^3 kg/m^3, see Table 11.1). We seek a value for the pressure at the deep end minus the pressure at the shallow end.

SOLUTION Using Equation 11.4, we find

$$P_{\text{Deep}} = P_{\text{Shallow}} + \rho gh \quad \text{or} \quad P_{\text{Deep}} - P_{\text{Shallow}} = \rho gh$$

The drawing at the right shows that a value for h can be obtained from the 15-m length of the pool by using the tangent of the 11° angle:

$$\tan 11° = \frac{h}{15\text{ m}} \quad \text{or} \quad h = (15\text{ m})\tan 11°$$

$$\begin{aligned} P_{\text{Deep}} - P_{\text{Shallow}} &= \rho g(15\text{ m})\tan 11° \\ &= (1.00 \times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(15\text{ m})\tan 11° = \boxed{2.9 \times 10^4\text{ Pa}} \end{aligned}$$

23. **SSM** ***REASONING*** Since the faucet is closed, the water in the pipe may be treated as a static fluid. The gauge pressure P_2 at the faucet on the first floor is related to the gauge pressure P_1 at the faucet on the second floor by Equation 11.4, $P_2 = P_1 + \rho gh$.

SOLUTION

a. Solving Equation 11.4 for P_1, we find the gauge pressure at the second-floor faucet is

$$P_1 = P_2 - \rho gh = 1.90 \times 10^5 \text{ Pa} - (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(6.50 \text{ m}) = \boxed{1.26 \times 10^5 \text{ Pa}}$$

b. If the second faucet were placed at a height h above the first-floor faucet so that the gauge pressure P_1 at the second faucet were zero, then no water would flow from the second faucet, even if it were open. Solving Equation 11.4 for h when P_1 equals zero, we obtain

$$h = \frac{P_2 - P_1}{\rho g} = \frac{1.90 \times 10^5 \text{ Pa} - 0}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{19.4 \text{ m}}$$

24. ***REASONING AND SOLUTION*** The pump must generate an upward force to counteract the weight of the column of water above it. Therefore, $F = mg = (\rho hA)g$. The required pressure is then

$$P = F/A = \rho gh = (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(71 \text{ m}) = \boxed{7.0 \times 10^5 \text{ Pa}}$$

25. ***REASONING AND SOLUTION*** The gauge pressure of the solution at the location of the vein is

$$P = \rho g h = (1030\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.610\ \text{m}) = 6.16 \times 10^3\ \text{Pa}$$

Now

$$1.013 \times 10^5\ \text{Pa} = 760\ \text{mm Hg} \qquad \text{so} \qquad 1\ \text{Pa} = 7.50 \times 10^{-3}\ \text{mm Hg}$$

Then

$$P = \left(6.16 \times 10^3\ \text{Pa}\right)\left(\frac{7.50 \times 10^{-3}\ \text{mm Hg}}{1\ \text{Pa}}\right) = \boxed{46.2\ \text{mm Hg}}$$

26. ***REASONING AND SOLUTION*** The difference in pressure between the ground and the roof is $\Delta P = \rho g h$, so

$$h = \frac{(13.0 \text{ mm Hg})\left(\dfrac{133 \text{ Pa}}{1 \text{ mm Hg}}\right)}{(1.29 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{137 \text{ m}}$$

Note that we have used the fact that 133 Pa = 1 mm Hg as a conversion factor.

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27. $\boxed{\textbf{SSM}}$ ***REASONING*** Let the length of the tube be denoted by L, and let the length of the liquid be denoted by ℓ. When the tube is whirled in a circle at a constant angular speed about an axis through one end, the liquid collects at the other end and experiences a centripetal force given by (see Equation 8.11, and use $F = ma$) $F = mr\omega^2 = mL\omega^2$.

Since there is no air in the tube, this is the only radial force experienced by the liquid, and it results in a pressure of

$$P = \frac{F}{A} = \frac{mL\omega^2}{A}$$

where A is the cross-sectional area of the tube. The mass of the liquid can be expressed in terms of its density ρ and volume V: $m = \rho V = \rho A\ell$. The pressure may then be written as

$$P = \frac{\rho A\ell L\omega^2}{A} = \rho\ell L\omega^2 \qquad (1)$$

If the tube were completely filled with liquid and allowed to hang vertically, the pressure at the bottom of the tube (that is, where $h = L$) would be given by

$$P = \rho gL \qquad (2)$$

SOLUTION According to the statement of the problem, the quantities calculated by Equations (1) and (2) are equal, so that $\rho\ell L\omega^2 = \rho gL$. Solving for ω gives

$$\omega = \sqrt{\frac{g}{\ell}} = \sqrt{\frac{9.80\ \text{m/s}^2}{0.0100\ \text{m}}} = \boxed{31.3\ \text{rad/s}}$$

28. ***REASONING*** In each case the pressure at point A in Figure 11.12 is atmospheric pressure and the pressure in the tube above the top of the liquid column is P. In Equation 11.4 ($P_2 = P_1 + \rho gh$), this means that $P_2 = P_{\text{atmosphere}}$ and $P_1 = P$. With this identification of the pressures and a value of 13 600 kg/m^3 for the density of mercury (see Table 11.1), Equation 11.4 provides a solution to the problem.

SOLUTION Rearranging Equation 11.4, we have $P_2 - P_1 = \rho gh$. Applying this expression to each setup gives

$$\left(P_2 - P_1\right)_{\text{Mercury}} = P_{\text{Atmosphere}} - P = \left(\rho gh\right)_{\text{Mercury}}$$

$$\left(P_2 - P_1\right)_{\text{Unknown}} = P_{\text{Unknown}} - P = \left(\rho gh\right)_{\text{Unknown}}$$

Since the left side of each of these equations is the same, we have

$$\rho_{\text{Mercury}} g h_{\text{Mercury}} = \rho_{\text{Unknown}} g h_{\text{Unknown}}$$

$$\rho_{\text{Unknown}} = \rho_{\text{Mercury}} \left(\frac{h_{\text{Mercury}}}{h_{\text{Unknown}}}\right) = \left(13\,600 \text{ kg/m}^3\right)\left(\frac{h_{\text{Mercury}}}{16 h_{\text{Mercury}}}\right) = \boxed{850 \text{ kg/m}^3}$$

29. ***REASONING AND SOLUTION*** The mercury, being more dense, will flow from the right container into the left container until the pressure is equalized. Then the pressure at the bottom of the left container will be $P = \rho_w g h_w + \rho_m g h_{mL}$ and the pressure at the bottom of the right container will be $P = \rho_m g h_{mR}$. Equating gives

$$\rho_w g h_w + \rho_m g(h_{mL} - h_{mR}) = 0 \qquad (1)$$

Both liquids are incompressible and immiscible so

$$h_w = 1.00 \text{ m} \quad \text{and} \quad h_{mL} + h_{mR} = 1.00 \text{ m}$$

Using these in (1) and solving for h_{mL} gives, $h_{mL} = (1/2)(1.00 - \rho_w/\rho_m) = 0.46$ m.

So the fluid level in the left container is 1.00 m + 0.46 m = $\boxed{1.46 \text{ m}}$ from the bottom.

30. ***REASONING AND SOLUTION*** The force exerted on the top surface is

$$F_2 = P_2A_2 = P_{atm}\pi R_2^{\ 2}$$

The force exerted on the bottom surface is $F_1 = P_1A_1 = (P_{atm} + \rho gh)\pi R_1^{\ 2}$. Equating and rearranging yields

$$R_2^{\ 2} = R_1^{\ 2}(1 + \rho gh/P_{atm})$$

or

$$R_2^{\ 2} = 1.485\ R_1^{\ 2} \qquad (1)$$

Consider a right triangle formed by drawing a vertical line from a point on the circumference of the bottom circle to the plane of the top circle so that two sides are equal to $R_2 - R_1$ and h. Then $\tan 30.0° = (R_2 - R_1)/h$.

a. Now

$$R_1 = R_2 - 2.887\ \text{m} \qquad (2)$$

Substituting (2) into (1) and rearranging yields

$$0.485\ R_2^{\ 2} - 8.57\ R_2 + 12.38 = 0$$

which has two roots, namely, $R_2 = 16.1$ m and 1.59 m. The value $R_2 = 1.59$ m leads to a negative value for R_1. Clearly, a radius cannot be negative, so we can eliminate the root $R_2 = 1.59$ m, and we conclude that $\boxed{R_2 = 16.1\ \text{m}}$.

b. Now that R_2 is known, Equation (2) gives $\boxed{R_1 = 13.2\ \text{m}}$.

31. $\boxed{\textbf{SSM}}$ ***REASONING*** According to Equation 11.4, the initial pressure at the bottom of the pool is $P_0 = \left(P_{\text{atm}}\right)_0 + \rho g h$, while the final pressure is $P_{\text{f}} = \left(P_{\text{atm}}\right)_{\text{f}} + \rho g h$. Therefore, the change in pressure at the bottom of the pool is

$$\Delta P = P_{\text{f}} - P_0 = \left[\left(P_{\text{atm}}\right)_{\text{f}} + \rho g h\right] - \left[\left(P_{\text{atm}}\right)_0 + \rho g h\right] = \left(P_{\text{atm}}\right)_{\text{f}} - \left(P_{\text{atm}}\right)_0$$

According to Equation 11.3, $F = PA$, the change in the force at the bottom of the pool is

$$\Delta F = (\Delta P)\,A = \left[\left(P_{\text{atm}}\right)_{\text{f}} - \left(P_{\text{atm}}\right)_0\right] A$$

SOLUTION Direct substitution of the data given in the problem into the expression above yields

$$\Delta F = \left(765\text{ mm Hg} - 755\text{ mm Hg}\right)(12\text{ m})(24\text{ m})\left(\frac{133\text{ Pa}}{1.0\text{ mm Hg}}\right) = \boxed{3.8 \times 10^5\text{ N}}$$

Note that the conversion factor 133 Pa = 1.0 mm Hg is used to convert mm Hg to Pa.

32. ***REASONING*** Pressure is the magnitude of the force applied perpendicularly to a surface divided by the area of the surface, according to Equation 11.3.

SOLUTION According to Equation 11.3, the pressure is

$$P = \frac{F}{A} = \frac{F}{\pi r^2} = \frac{21\ 600\ \text{N}}{\pi(0.090\ \text{m})^2} = \boxed{8.5 \times 10^5\ \text{Pa}}$$

This is about eight times atmospheric pressure.

33. ***REASONING AND SOLUTION*** We know that $F_2 = (A_2/A_1)F_1$. Since the pistons are circular in cross-section

$$F_2 = \frac{R_2^2}{R_1^2} F_1 = \left(\frac{5.1 \times 10^{-2}\ \text{m}}{6.4 \times 10^{-3}\ \text{m}} \right)^2 (330\ \text{N}) = \boxed{2.1 \times 10^4\ \text{N}}$$

34. ***REASONING*** Equation 11.5 gives the force F_2 of the output plunger in terms of the force F_1 applied to the input piston as $F_2 = F_1(A_2/A_1)$, where A_2 and A_1 are the corresponding areas. In this problem the chair begins to rise when the output force just barely exceeds the weight, so $F_2 = 2100$ N. We are given the input force as 55 N. We seek the ratio of the radii, so we will express the area of each circular cross section as πr^2 when we apply Equation 11.5.

SOLUTION According to Equation 11.5, we have

$$\frac{A_2}{A_1} = \frac{F_2}{F_1} \quad \text{or} \quad \frac{\pi r_2^2}{\pi r_1^2} = \frac{F_2}{F_1}$$

Solving for the ratio of the radii yields

$$\frac{r_2}{r_1} = \sqrt{\frac{F_2}{F_1}} = \sqrt{\frac{2100\ \text{N}}{55\ \text{N}}} = \boxed{6.2}$$

35. **SSM** ***REASONING*** The pressure P' exerted on the bed of the truck by the plunger is $P' = P - P_{\text{atm}}$. According to Equation 11.3, $F = P'A$, so the force exerted on the bed of the truck can be expressed as $F = \left(P - P_{\text{atm}}\right)A$. If we assume that the plunger remains perpendicular to the floor of the load bed, the torque that the plunger creates about the axis shown in the figure in the text is

$$\tau = F\ell = \left(P - P_{\text{atm}}\right)A\ell = \left(P - P_{\text{atm}}\right)(\pi r^2)\ell$$

SOLUTION Direct substitution of the numerical data into the expression above gives

$$\tau = \left(3.54 \times 10^6 \text{ Pa} - 1.01 \times 10^5 \text{ Pa}\right)\pi(0.150 \text{ m})^2(3.50 \text{ m}) = \boxed{8.50 \times 10^5 \text{ N} \cdot \text{m}}$$

36. ***REASONING*** Since the piston and the plunger are at the same height, Equation 11.5, $F_2 = F_1(A_2 / A_1)$, applies, and we can find an expression for the force exerted on the spring. Then Equation 10.1, $F = kx$, can be used to determine the amount of compression of the spring.

SOLUTION Substituting the right hand side of Equation 11.5 into Equation 10.1, we find that

$$F_1\left(\frac{A_2}{A_1}\right) = kx$$

From the drawing in the text, we see that the force on the right piston must be equal in magnitude to the weight of the rock, or $F_1 = mg$. Therefore,

$$mg\left(\frac{A_2}{A_1}\right) = kx$$

Solving for x, we obtain

$$x = \frac{mg}{k}\left(\frac{A_2}{A_1}\right) = \frac{(40.0\text{ kg})(9.80\text{ m/s}^2)}{1600\text{ N/m}}\left(\frac{15\text{ cm}^2}{65\text{ cm}^2}\right) = \boxed{5.7\times10^{-2}\text{ m}}$$

37. ***REASONING AND SOLUTION*** From Pascal's principle, the pressure in the brake fluid at the master cylinder is equal to the pressure in the brake fluid at the plungers: $P_C = P_P$, or

$$\frac{F_C}{A_C} = \frac{F_P}{A_P} \qquad \text{or} \qquad F_P = F_C \frac{A_P}{A_C} = F_C \frac{\pi r_P^2}{\pi r_C^2} = F_C \left(\frac{r_P}{r_C}\right)^2$$

The torque on the pedal is equal to the torque that is applied to the master cylinder so that

$$F\ell = F_C \ell_C \qquad \text{or} \qquad F_C = F\frac{\ell}{\ell_C}$$

Combining the expression for F_C with the expression for F_P above, we have

$$F_P = F\frac{\ell}{\ell_C}\left(\frac{r_p}{r_C}\right)^2 = (9.00\text{ N})\left(\frac{0.150\text{ m}}{0.0500\text{ m}}\right)\left(\frac{1.90\times10^{-2}\text{ m}}{9.50\times10^{-3}\text{ m}}\right)^2 = \boxed{108\text{ N}}$$

38. ***REASONING*** When the body builder holds the steel barbell above her head, there are two forces that act on the barbell. They are the upward force exerted by the body builder and the downward pull of gravity (the weight). Therefore, the minimum force required for the bodybuilder to lift the barbell is equal to the weight of the barbell. If she lifted the barbell under water, there is an additional upward buoyant force that acts on the barbell. Since the buoyant force points upward, it reduces the force required by the bodybuilder in lifting the barbell. According to Archimedes principle (Equation 11.6) the magnitude of the buoyant force on any object submerged in a fluid equals the weight of the fluid that the object displaces. We can use Archimedes principle to determine the magnitude of the buoyant force.

SOLUTION According to Equation 11.1, $\rho = m/V$, and the volume of the barbell is

$$V_{\text{barbell}} = \frac{m_{\text{barbell}}}{\rho_{\text{steel}}}$$

This must be equal to the volume of the water displaced, so that the mass of water displaced is

$$m_{\text{water}} = \rho_{\text{water}} V_{\text{water}} = \rho_{\text{water}} V_{\text{barbell}} = \rho_{\text{water}} \frac{m_{\text{barbell}}}{\rho_{\text{steel}}} = m_{\text{barbell}} \left(\frac{\rho_{\text{water}}}{\rho_{\text{steel}}} \right)$$

The weight of the water that is displaced by the barbell is, therefore,

$$W_{\text{disp}} = m_{\text{water}} g = m_{\text{barbell}} g \left(\frac{\rho_{\text{water}}}{\rho_{\text{steel}}} \right) = (29\text{ kg})(9.80\text{ m/s}^2) \left(\frac{1.000 \times 10^3\text{ kg/m}^3}{7860\text{ kg/m}^3} \right) = 36\text{ N}$$

According to Archimedes principle, this is equal to the magnitude of the buoyant force on the barbell. Therefore, if we take upward as the positive direction, the net force on the barbell is

$$F_{\text{net}} = 36\text{ N} - m_{\text{barbell}} g = 36\text{ N} - (29\text{ kg})(9.80\text{ m/s}^2) = -2.5 \times 10^2\text{ N}$$

Thus, the bodybuilder would have to exert a force of magnitude $\boxed{2.5 \times 10^2\text{ N}}$ if she lifted the barbell under water.

39. SSM ***REASONING*** According to Archimedes principle, the buoyant force that acts on the block is equal to the weight of the water that is displaced by the block. The block displaces an amount of water V, where V is the volume of the block. Therefore, the weight of the water displaced by the block is $W = mg = (\rho_{\text{water}} V)g$.

SOLUTION The buoyant force that acts on the block is, therefore,

$$F = \rho_{\text{water}} V g = (1.00 \times 10^3 \text{ kg/m}^3)(0.10 \text{ m} \times 0.20 \text{ m} \times 0.30 \text{ m})(9.80 \text{ m/s}^2) = \boxed{59 \text{ N}}$$

40. ***REASONING*** We can use Archimedes principle to determine the density of the object. Once the density is known, the identity of the composition of the object can be determined by comparing its density with those listed in Table 11.1.

SOLUTION According to Archimedes Principle, the magnitude of the buoyant force on the object is equal to the weight of the displaced ethyl alcohol, $F_B = W_{ethyl}$. Since the object floats motionless, it is in equilibrium, and the magnitude of the buoyant force on the object must be equal to its weight, $F_B = W_{object}$. Therefore, we have $W_{object} = W_{ethyl}$, or

$$m_{object} g = m_{ethyl} g$$

$$\rho_{object} V_{object} g = \rho_{ethyl} V_{ethyl} g$$

Since the object floats with 68.2% of its volume submerged, we have $V_{ethyl} = (0.682) V_{object}$, and the above expression becomes

$$\rho_{object} V_{object} = \rho_{ethyl} (0.682) V_{object}$$

Solving for ρ_{object} we find that the density of the object is

$$\rho_{object} = (0.682)\rho_{ethyl} = (0.682)(806 \text{ kg/m}^3) = 550 \text{ kg/m}^3$$

Comparison with the densities given in Table 11.1 suggests that the object is made of $\boxed{\text{yellow pine}}$.

41. ***REASONING AND SOLUTION*** The upward buoyant force on the raft must equal the weight of the raft plus the weight of the swimmers, $F_B = W_r + W_s$. Thus,

$$\rho_w g V_r = \rho_p g V_r + M_s g$$

$$M_s = (\rho_w - \rho_p)V_r = (1.00 \times 10^3 \text{ kg/m}^3 - 550 \text{ kg/m}^3)(4.0 \text{ m})(4.0 \text{ m})(0.30 \text{ m})$$

$$\boxed{M_s = 2.2 \times 10^3 \text{ kg}}$$

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42. ***REASONING*** The paperweight weighs less in water than in air, because of the buoyant force F_B of the water. The buoyant force points upward, while the weight points downward, leading to an effective weight in water of $W_{\text{In water}} = W - F_B$. There is also a buoyant force when the paperweight is weighed in air, but it is negligibly small. Thus, from the given weights, we can obtain the buoyant force, which is the weight of the displaced water, according to Archimedes' principle. From the weight of the displaced water and the density of water, we can obtain the volume of the water, which is also the volume of the completely immersed paperweight.

SOLUTION We have

$$W_{\text{In water}} = W - F_B \quad \text{or} \quad F_B = W - W_{\text{In water}}$$

According to Archimedes' principle, the buoyant force is the weight of the displaced water, which is mg, where m is the mass of the displaced water. Using Equation 11.1, we can write the mass as the density times the volume or $m = \rho V$. Thus, for the buoyant force, we have

$$F_B = W - W_{\text{In water}} = \rho V g$$

Solving for the volume and using $\rho = 1.00 \times 10^3\ \text{kg/m}^3$ for the density of water (see Table 11.1), we find

$$V = \frac{W - W_{\text{In water}}}{\rho g} = \frac{6.9\ \text{N} - 4.3\ \text{N}}{\left(1.00 \times 10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{2.7 \times 10^{-4}\ \text{m}^3}$$

43. SSM WWW ***REASONING AND SOLUTION*** Under water, the weight of the person with empty lungs is $W_{\text{empty}} = W - \rho_{\text{water}} g V_{\text{empty}}$, where W is the weight of the person in air and V_{empty} is the volume of the empty lungs. Similarly, when the person's lungs are partially full under water, the weight of the person is $W_{\text{full}} = W - \rho_{\text{water}} g V_{\text{full}}$. Subtracting the second equation from the first equation and rearranging gives

$$V_{\text{full}} - V_{\text{empty}} = \frac{W_{\text{empty}} - W_{\text{full}}}{\rho_{\text{water}}\, g} = \frac{40.0\text{ N} - 20.0\text{ N}}{(1.00 \times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)} = \boxed{2.04 \times 10^{-3}\text{ m}^3}$$

44. ***REASONING AND SOLUTION*** The buoyant force exerted by the water must at least equal the weight of the logs plus the weight of the people,

$$F_B = W_L + W_P$$

$$\rho_w g V = \rho_L g V + W_P$$

Now the volume of logs needed is

$$V = \frac{M_P}{\rho_W - \rho_L} = \frac{320\text{ kg}}{1.00 \times 10^3\text{ kg/m}^3 - 725\text{ kg/m}^3} = 1.16\text{ m}^3$$

The volume of one log is

$$V_L = \pi(8.00 \times 10^{-2}\text{ m})^2(3.00\text{ m}) = 6.03 \times 10^{-2}\text{ m}^3$$

The number of logs needed is

$$N = V/V_L = (1.16)/(6.03 \times 10^{-2}) = 19.2$$

Therefore, $\boxed{\text{at least 20 logs are needed}}$.

45. ***REASONING AND SOLUTION*** The figure at the right shows the two forces that initially act on the box. Since the box is not accelerated, the two forces must have zero resultant: $F_{B0} - W_{box} = 0$. Therefore,

$$F_{B0} = W_{box} \qquad (1)$$

From Archimedes' principle, the buoyant force on the box is equal to the weight of the water that is displaced by the box:

$$F_{B0} = W_{disp} \qquad (2)$$

Combining (1) and (2) we have $W_{box} = W_{disp}$, or $m_{box}\, g = \rho_{water} g V_{disp}$. Therefore,

$$m_{box} = \rho_{water} V_{disp}$$

Since the box floats with one-third of its height beneath the water, $V_{disp} = (1/3)V_{box}$, or $V_{disp} = (1/3)L^3$. Therefore,

$$m_{box} = \frac{\rho_{water} L^3}{3} \qquad (3)$$

The figure at the right shows the three forces that act on the box after water is poured into the box. The box begins to sink when

$$W_{box} + W_{water} \geq F_B \qquad (4)$$

The box just begins to sink when the equality is satisfied. From Archimedes' principle, the buoyant force on the system is equal to the weight of the water that is displaced by the system: $F_B = W_{displaced}$.

The equality in (4) can be written as

$$m_{box} g + m_{water} g = m_{displaced} g \qquad (5)$$

When the box begins to sink, the volume of the water displaced is equal to the volume of the box; Equation (5) then becomes

$$m_{box} + \rho_{water} V_{water} = \rho_{water} V_{box}.$$

The volume of water in the box at this instant is $V_{water} = L^2 h$, where h is the depth of the water in the box. Thus, the equation above becomes

$$m_{box} + \rho_{water} L^2 h = \rho_{water} L^3$$

Using Equation (3) for the mass of the box, we obtain

$$\frac{\rho_{water} L^3}{3} + \rho_{water} L^2 h = \rho_{water} L^3$$

Solving for h gives

$$h = \frac{2}{3}L = \frac{2}{3}(0.30\ \text{m}) = \boxed{0.20\ \text{m}}$$

46. ***REASONING*** The fraction of the specimen's apparent volume that is solid will be given by

$$\frac{V_{\text{solid}}}{V_{\text{total}}} = \frac{V_{\text{solid}}}{V_{\text{solid}} + V_{\text{hollow}}} \tag{1}$$

where V_{solid} is the volume of the solid part of the specimen and V_{hollow} is the volume of the hollow part. Since the specimen weighs twice as much in air as it does in water, $W_{\text{air}} = 2W_{\text{water}}$. Furthermore, we are told that the solid part of the specimen has a density of $5.0 \times 10^3 \text{ kg/m}^3$; this is five times greater than the density of water ρ_{fluid}, so $\rho_{\text{solid}} = 5\rho_{\text{fluid}}$. Using this information and Archimedes principle, we can obtain a value for this fraction.

SOLUTION Let M represent the mass of the rock. Then, according to Equation 11.1, the volume of the solid part of the rock is given by $V_{\text{solid}} = M / \rho_{\text{solid}}$.

From Archimedes principle, we know that when then rock is submerged in water, the buoyant force on the rock will be equal to the weight of the water that is displaced by the rock so that

$$F_{\text{B}} = W_{\text{fluid displaced}} = \rho_{\text{fluid}} g V_{\text{fluid displaced}}$$

We know that the rock will displace a volume of water that is equal to the total volume of the rock, so the volume of water displaced is equal to V_{total}, and we have

$$W_{\text{fluid displaced}} = \rho_{\text{fluid}} g V_{\text{total}}$$

Since the weight of the rock in air is twice that in water, we can write

$$\underbrace{2\left(Mg - \overbrace{\rho_{\text{fluid}} g V_{\text{total}}}^{\text{Buoyant force}}\right)}_{\text{Weight of rock in water}} = \underbrace{Mg}_{\substack{\text{Weight}\\ \text{of rock}\\ \text{in air}}}$$

or $Mg = 2\rho_{\text{fluid}} g V_{\text{total}}$. Substituting in the relation $M = \rho_{\text{solid}} V_{\text{solid}}$ and solving for the ratio of the volumes, we obtain

$$\frac{V_{\text{solid}}}{V_{\text{total}}} = 2\frac{\rho_{\text{fluid}}}{\rho_{\text{solid}}} = 2\left(\frac{1.00 \times 10^3 \text{ kg/m}^3}{5.0 \times 10^3 \text{ kg/m}^3}\right) = \boxed{0.40}$$

47. **SSM** ***REASONING*** The height of the cylinder that is in the oil is given by $h_{\text{oil}} = V_{\text{oil}} / (\pi r^2)$, where V_{oil} is the volume of oil displaced by the cylinder and r is the radius of the cylinder. We must, therefore, find the volume of oil displaced by the cylinder. After the oil is poured in, the buoyant force that acts on the cylinder is equal to the sum of the weight of the water displaced by the cylinder and the weight of the oil displaced by the cylinder. Therefore, the magnitude of the buoyant force is given by $F = \rho_{\text{water}} g V_{\text{water}} + \rho_{\text{oil}} g V_{\text{oil}}$. Since the cylinder floats in the fluid, the net force that acts on the cylinder must be zero. Therefore, the buoyant force that supports the cylinder must be equal to the weight of the cylinder, or

$$\rho_{\text{water}} g V_{\text{water}} + \rho_{\text{oil}} g V_{\text{oil}} = mg$$

where m is the mass of the cylinder. Substituting values into the expression above leads to

$$V_{\text{water}} + (0.725) V_{\text{oil}} = 7.00 \times 10^{-3}\ \text{m}^3 \qquad (1)$$

From the figure in the text, $V_{\text{cylinder}} = V_{\text{water}} + V_{\text{oil}}$. Substituting values into the expression for V_{cylinder} gives

$$V_{\text{water}} + V_{\text{oil}} = 8.48 \times 10^{-3}\ \text{m}^3 \qquad (2)$$

Subtracting Equation (1) from Equation (2) yields $V_{\text{oil}} = 5.38 \times 10^{-3}\ \text{m}^3$.

SOLUTION The height of the cylinder that is in the oil is, therefore,

$$h_{\text{oil}} = \frac{V_{\text{oil}}}{\pi r^2} = \frac{5.38 \times 10^{-3}\ \text{m}^3}{\pi (0.150\ \text{m})^2} = \boxed{7.6 \times 10^{-2}\ \text{m}}$$

48. ***REASONING AND SOLUTION*** The figure at the right shows the forces that act on the balloon as it holds the passengers and the ballast stationary above the earth. W_0 is the combined weight of the balloon, the load of passengers, and the ballast. The quantity F_B is the buoyant force provided by the air outside the balloon and is given by

$$F_B = \rho_{air} g V_{balloon} \qquad (1)$$

Since the balloon is stationary, it follows that $F_B - W_0 = 0$, or

$$F_B = W_0 \qquad (2)$$

Then the ballast is dropped overboard, the balloon accelerates upward through a distance y in a time t with acceleration a_y, where (from kinematics) $a_y = \frac{2y}{t^2}$.

The figure at the right shows the forces that act on the balloon while it accelerates upward.

Applying Newton's second law, we have $F_B - W = ma_y$, where W is the weight of the balloon and the load of passengers. Replacing m by (W/g) we have

$$F_B - W = \frac{W}{g} a_y$$

Thus,

$$F_B = W + \frac{W}{g}\left(\frac{2y}{t^2}\right) = W\left(1 + \frac{2y}{gt^2}\right)$$

Solving for W gives

$$W = \frac{F_B}{\left(1 + \frac{2y}{gt^2}\right)} = F_B\left(\frac{gt^2}{gt^2 + 2y}\right)$$

The amount of ballast that must be thrown overboard is therefore [using Equations (1) and (2)]

$$\Delta W = W_0 - W = F_B - F_B\left(\frac{gt^2}{gt^2 + 2y}\right) = \left(1 - \frac{gt^2}{gt^2 + 2y}\right)\rho_{air} g V_{balloon}$$

$$\Delta W = r_{air}\, g\left(\tfrac{4}{3}\rho r_{balloon}^3\right)\left(1 - \frac{gt^2}{gt^2 + 2y}\right)$$

$$\Delta W = (1.29\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\left[\tfrac{4}{3}\pi(6.25\ \text{m})^3\right]\left[1 - \frac{(9.80\ \text{m/s}^2)(15.0\ \text{s})^2}{(9.80\ \text{m/s}^2)(15.0\ \text{s})^2 + 2(105\ \text{m})}\right]$$

$$= \boxed{1120\ \text{N}}$$

49. ***REASONING AND SOLUTION*** Only the weight of the block compresses the spring. Applying Hooke's law gives $W = kx$. The spring is stretched by the buoyant force acting on the block minus the weight of the block. Hooke's law again gives $F_B - W = 2kx$. Eliminating kx gives $F_B = 3W$. Now $F_B = \rho_W gV$, so that the volume of the block is

$$V = 3M/\rho_W = 3(8.00\text{ kg})/(1.00 \times 10^3\text{ kg/m}^3) = 2.40 \times 10^{-2}\text{ m}^3$$

The volume of wood in the block is

$$V_W = M/\rho_b = (8.00\text{ kg})/(840\text{ kg/m}^3) = 9.52 \times 10^{-3}\text{ m}^3$$

The volume of the block that is hollow is $V - V_W = 1.45 \times 10^{-2}\text{ m}^3$. The percentage of the block that is hollow is then

$$100(1.45 \times 10^{-2})/(2.40 \times 10^{-2}) = \boxed{60.3\ \%}$$

50. ***REASONING AND SOLUTION*** The volume flow rate is given by

$$Q = Av = \pi r^2 v = \pi(0.305\text{ m})^2(1.22\text{ m/s}) = 0.356\text{ m}^3/\text{s}$$

The number of gallons that flows in one day (8.64×10^4 s) is

$$\left(0.356\text{ m}^3/\text{s}\right)\left(\frac{1.0\text{ gal}}{3.79 \times 10^{-3}\text{ m}^3}\right)\left(8.64 \times 10^4\text{ s}\right) = \boxed{8.12 \times 10^6\text{ gal}}$$

51. **SSM** ***REASONING AND SOLUTION*** The mass flow rate Q_{mass} is the amount of fluid mass that flows per unit time. Therefore,

$$Q_{\text{mass}} = \frac{m}{t} = \frac{\rho V}{t} = \frac{(1030\ \text{kg/m}^3)(9.5\times10^{-4}\ \text{m}^3)}{6.0\ \text{h}}\left(\frac{1.0\ \text{h}}{3600\ \text{s}}\right) = \boxed{4.5\times10^{-5}\ \text{kg/s}}$$

52. ***REASONING*** The length L of the side of the square can be obtained, if we can find a value for the cross-sectional area A of the ducts. The area is related to the volume flow rate Q and the air speed v by Equation 11.10 ($Q = Av$). The volume flow rate can be obtained from the volume V of the room and the replacement time t as $Q = V/t$.

SOLUTION For a square cross section with sides of length L, we have $A = L^2$. And we know that the volume flow rate is $Q = V/t$. Therefore, using Equation 11.10 gives

$$Q = Av \quad \text{or} \quad \frac{V}{t} = L^2 v$$

Solving for L shows that

(a) ***Air speed = 3.0 m / s***

$$L = \sqrt{\frac{V}{tv}} = \sqrt{\frac{120\ \text{m}^3}{(1200\ \text{s})(3.0\ \text{m/s})}} = \boxed{0.18\ \text{m}}$$

(b) ***Air speed = 5.0 m / s***

$$L = \sqrt{\frac{V}{tv}} = \sqrt{\frac{120\ \text{m}^3}{(1200\ \text{s})(5.0\ \text{m/s})}} = \boxed{0.14\ \text{m}}$$

53. ***REASONING AND SOLUTION***

a. The volume flow rate is given by

$$Q = Av = (2.0 \times 10^{-4}\ \text{m}^2)(0.35\ \text{m/s}) = \boxed{7.0 \times 10^{-5}\ \text{m}^3/\text{s}}$$

b. We know that $A_1v_1 = A_2v_2$ so that

$$v_2 = v_1(A_1/A_2) = (0.35\ \text{m/s})(2.0 \times 10^{-4}\ \text{m}^2)/(0.28\ \text{m}^2) = \boxed{2.5 \times 10^{-4}\ \text{m/s}}$$

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54. ***REASONING AND SOLUTION*** Let r_h represent the inside radius of the hose, and r_p the radius of the plug, as suggested by the figure below.

Then, from Equation 11.9, $A_1v_1 = A_2v_2$, we have $\pi r_h^2 v_1 = (\pi r_h^2 - \pi r_p^2)v_2$, or

$$\frac{v_1}{v_2} = \frac{\left(r_h^2 - r_p^2\right)}{r_h^2} = 1 - \left(\frac{r_p}{r_h}\right)^2 \qquad \text{or} \qquad \frac{r_p}{r_h} = \sqrt{1 - \frac{v_1}{v_2}}$$

According to the problem statement, $v_2 = 3v_1$, or

$$\frac{r_p}{r_h} = \sqrt{1 - \frac{v_1}{3v_1}} = \sqrt{\frac{2}{3}} = \boxed{0.816}$$

55. SSM ***REASONING AND SOLUTION***

a. The volume flow rate is given by Equation 11.10. Assuming that the line has a circular cross section, $A = \pi r^2$, we have

$$Q = Av = (\pi r^2)v = \pi(0.0065\text{ m})^2\,(1.2\text{ m/s}) = \boxed{1.6\times10^{-4}\ \text{m}^3/\text{s}}$$

b. The volume flow rate calculated in part (a) above is the flow rate for all twelve holes. Therefore, the volume flow rate through one of the twelve holes is

$$Q_{\text{hole}} = \frac{Q}{12} = (\pi r_{\text{hole}}^2)v_{\text{hole}}$$

Solving for v_{hole} we have

$$v_{\text{hole}} = \frac{Q}{12\pi r_{\text{hole}}^2} = \frac{1.6\times10^{-4}\ \text{m}^3/\text{s}}{12\pi(4.6\times10^{-4}\ \text{m})^2} = \boxed{2.0\times10^{1}\ \text{m/s}}$$

56. ***REASONING*** We apply Bernoulli's equation as follows:

$$\underbrace{P_S + \tfrac{1}{2}\rho v_S^2 + \rho g y_S}_{\text{At surface of vaccine in reservoir}} = \underbrace{P_O + \tfrac{1}{2}\rho v_O^2 + \rho g y_O}_{\text{At opening}}$$

SOLUTION The vaccine's surface in the reservoir is stationary during the inoculation, so that $v_S = 0$ m/s. The vertical height between the vaccine's surface in reservoir and the opening can be ignored, so $y_S = y_O$. With these simplifications Bernoulli's equation becomes

$$P_S = P_O + \tfrac{1}{2}\rho v_O^2$$

Solving for the speed at the opening gives

$$v_O = \sqrt{\frac{2\left(P_S - P_O\right)}{\rho}} = \sqrt{\frac{2\left(4.1\times 10^6\ \text{Pa}\right)}{1100\ \text{kg/m}^3}} = \boxed{86\ \text{m/s}}$$

57. SSM ***REASONING AND SOLUTION***

a. Using Equation 11.12, the form of Bernoulli's equation with $y_1 = y_2$, we have

$$P_1 - P_2 = \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1.29\ \text{kg/m}^3}{2}\left[(15\ \text{m/s})^2 - 0\right] = \boxed{150\ \text{Pa}}$$

b. The pressure inside the roof is greater than the pressure on the outside. Therefore, there is a net outward force on the roof. If the wind speed is sufficiently high, some roofs are "blown outward."

58. ***REASONING AND SOLUTION*** Using Equation 11.12, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$, we obtain $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$. With $v_1 = 0$ and $v_2 = 27$ m/s, we have

$$P_1 - P_2 = \frac{1}{2}(1.29\ \text{kg/m}^3)(27\ \text{m/s})^2 = \boxed{4.7 \times 10^2\ \text{Pa}}$$

59. $\boxed{\textbf{SSM}}$ ***REASONING AND SOLUTION*** Let the speed of the air below and above the wing be given by v_1 and v_2, respectively. According to Equation 11.12, the form of Bernoulli's equation with $y_1 = y_2$, we have

$$P_1 - P_2 = \tfrac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1.29\ \text{kg/m}^3}{2}\left[(251\ \text{m/s})^2 - (225\ \text{m/s})^2\right] = 7.98\times10^3\ \text{Pa}$$

From Equation 11.3, the lifting force is, therefore,

$$F = \left(P_1 - P_2\right)A = (7.98\times10^3\ \text{Pa})(24.0\ \text{m}^2) = \boxed{1.92\times10^5\ \text{N}}$$

60. ***REASONING*** If we can find a value for the speed v_1 of the water entering the hold, the volume flow rate Q can be obtained from Equation 11.10 ($Q = Av_1$), since the area A of the hole is known. The place where the water enters the hold is just like point 1 in Figure 11.36*a*, whereas the lake surface, 2.0 m above, is just like point 2 in the figure. Therefore, Torricelli's theorem applies and gives the speed of the water leaking into the hold as $v_1 = \sqrt{2gh}$, where h = 2.0 m.

SOLUTION Using Equation 11.10 together with Torricelli's theorem, we find

$$Q = Av_1 \quad \text{and} \quad v_1 = \sqrt{2gh}$$

$$Q = A\sqrt{2gh} = \left(8.0\times10^{-3}\ \text{m}^2\right)\sqrt{2\left(9.80\ \text{m/s}^2\right)\left(2.0\ \text{m}\right)} = 5.0\times10^{-2}\ \text{m}^3/\text{s}$$

Converting cubic meters to gallons, we have

$$Q = \left(5.0\times10^{-2}\ \frac{\text{m}^3}{\text{s}}\right)\left(\frac{1\ \text{gal}}{3.785\times10^{-3}\ \text{m}^3}\right) = \boxed{13\ \text{gal/s}}$$

61. ***REASONING AND SOLUTION***

a. Using Bernoulli's equation we have

$$P_1 = P_2 + \rho gh$$
$$= (1.01 \times 10^5 \text{ Pa}) + (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(15.0 \text{ m}) = \boxed{2.48 \times 10^5 \text{ Pa}}$$

b. The pressure becomes equal to atmospheric pressure when the valve is opened, so $\boxed{P_1 = 1.01 \times 10^5 \text{ Pa}}$.

c. The volume flow rate is given by Q = Av. We can find the speed v by using Bernoulli's equation, $P_2 + \rho gh = P_1 + (1/2)\rho v^2$, with $P_2 = P_1$. The result is

$$v = \sqrt{2gh} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(15.0 \text{ m}\right)} = 17.1 \text{ m/s}$$

The volume flow rate is $Q = Av = (2.00 \times 10^{-2} \text{ m}^2)(17.1 \text{ m/s}) = \boxed{0.342 \text{ m}^3\text{/s}}$

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62. ***REASONING AND SOLUTION*** First find the velocity at the lower point using the equation of continuity, $A_1v_1 = A_2v_2$.

$$v_2 = v_1(A_1/A_2) = v_1(r_1/r_2)^2 = v_1(2r_2/r_2)^2 = 4v_1 = 4(0.50\text{ m/s}) = 2.0\text{ m/s}$$

Now use Bernoulli's equation, $(1/2)\rho v_1^2 + \rho gy = (1/2)\rho v_2^2$, so

$$y = \tfrac{1}{2}\left(v_2^2 - v_1^2\right)/g = \tfrac{1}{2}\left[(2.0\text{ m/s})^2 - (0.50\text{ m/s})^2\right]/\left(9.80\text{ m/s}^2\right) = \boxed{0.19\text{ m}}$$

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63. ***REASONING AND SOLUTION*** Using Equation 11.12, $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$, we obtain $P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}\rho v_2^2$, where $v_1 = 0$. Since pressure is force per unit area, we have

$$\frac{F_{\text{net}}}{A} = \frac{1}{2}\rho v_2^2$$

Solving for v_2 we have

$$v_2 = \sqrt{\frac{2F_{\text{net}}}{\rho A}} = \sqrt{\frac{2(21\ 000\ \text{N})}{(1.29\ \text{kg/m}^3)(4.0\ \text{m} \times 5.5\ \text{m})}} = \boxed{38\ \text{m/s}}$$

64. ***REASONING*** In level flight the lift force must balance the plane's weight W, so its magnitude is also W. The lift force arises because the pressure P_B beneath the wings is greater than the pressure P_T on top of the wings. The lift force, then, is the pressure difference times the effective wing surface area A, so that $W = (P_B - P_T)A$. The area is given, and we can determine the pressure difference by using Bernoulli's equation.

SOLUTION According to Bernoulli's equation, we have

$$P_B + \tfrac{1}{2}\rho v_B^2 + \rho g y_B = P_T + \tfrac{1}{2}\rho v_T^2 + \rho g y_T$$

Since the flight is level, the height is constant and $y_B = y_T$, where we assume that the wing thickness may be ignored. Then, Bernoulli's equation simplifies and may be rearranged as follows:

$$P_B + \tfrac{1}{2}\rho v_B^2 = P_T + \tfrac{1}{2}\rho v_T^2 \quad \text{or} \quad P_B - P_T = \tfrac{1}{2}\rho v_T^2 - \tfrac{1}{2}\rho v_B^2$$

Recognizing that $W = (P_B - P_T)A$, we can substitute for the pressure difference from Bernoulli's equation to show that

$$W = \left(P_B - P_T\right)A = \tfrac{1}{2}\rho\left(v_T^2 - v_B^2\right)A$$

$$= \tfrac{1}{2}\left(1.29\ \text{kg/m}^3\right)\left[(62.0\ \text{m/s})^2 - (54.0\ \text{m/s})^2\right]\left(16\ \text{m}^2\right) = \boxed{9600\ \text{N}}$$

We have used a value of 1.29 kg/m^3 from Table 11.1 for the density of air. This is an approximation, since the density of air decreases with increasing altitude above sea level.

65. $\boxed{\textbf{SSM}}$ ***REASONING*** Since the pressure difference is known, Bernoulli's equation can be used to find the speed v_2 of the gas in the pipe. Bernoulli's equation also contains the unknown speed v_1 of the gas in the Venturi meter; therefore, we must first express v_1 in terms of v_2. This can be done by using Equation 11.9, the equation of continuity.

SOLUTION

a. From the equation of continuity (Equation 11.9) it follows that $v_1 = \left(A_2 / A_1\right)v_2$. Therefore,

$$v_1 = \frac{0.0700\ \text{m}^2}{0.0500\ \text{m}^2} v_2 = (1.40)v_2$$

Substituting this expression into Bernoulli's equation (Equation 11.12), we have

$$P_1 + \tfrac{1}{2}\rho(1.40\ v_2)^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

Solving for v_2, we obtain

$$v_2 = \sqrt{\frac{2(P_2 - P_1)}{\rho\left[(1.40)^2 - 1\right]}} = \sqrt{\frac{2(120\ \text{Pa})}{(1.30\ \text{kg/m}^3)\left[(1.40)^2 - 1\right]}} = \boxed{14\ \text{m/s}}$$

b. According to Equation 11.10, the volume flow rate is

$$Q = A_2 v_2 = (0.0700\ \text{m}^2)(14\ \text{m/s}) = \boxed{0.98\ \text{m}^3/\text{s}}$$

66. ***REASONING AND SOLUTION*** As seen in the figure, the lower pipe is at the level of zero potential energy.

If the flow rate is uniform in both pipes, we have $(1/2)\rho v_1^2 = (1/2)\rho v_2^2 + \rho gh$ (since $P_1 = P_2$) and $A_1 v_1 = A_2 v_2$. We can solve for v_1, i.e., $v_1 = v_2(A_2/A_1)$, and plug into the previous expression to find v_2, so that

$$v_2 = \sqrt{\frac{2gh}{\left(\frac{A_2}{A_1}\right)^2 - 1}} = \sqrt{\frac{2\left(9.80\ \text{m/s}^2\right)\left(10.0\ \text{m}\right)}{\left[\frac{\pi\left(0.0400\ \text{m}\right)^2}{\pi\left(0.0200\ \text{m}\right)^2}\right]^2 - 1}} = 3.61\ \text{m/s}$$

The volume flow rate is then given by

$$Q = A_2 v_2 = \pi r_2^2 v_2 = \pi(0.0400\ \text{m})^2(3.61\ \text{m/s}) = \boxed{1.81 \times 10^{-2}\ \text{m}^3/\text{s}}$$

67. ***REASONING AND SOLUTION*** We have

$$P_1 + (1/2)\rho v_1^2 + \rho g y_1 = P_2 + (1/2)\rho v_2^2 + \rho g y_2$$

Taking $v_1 = 0$, $y_2 = 0$, $P_1 = 1.01 \times 10^5$ Pa, and noting that v_2 is a maximum when $P_2 = 0$, we can solve for v_2. We thus obtain,

$$v_2 = \sqrt{\frac{2P_1}{\rho} + 2gy_1} = \sqrt{\frac{2\left(1.01 \times 10^5 \text{ Pa}\right)}{1.00 \times 10^3 \text{ kg/m}^3} + 2\left(9.80 \text{ m/s}^2\right)\left(12 \text{ m}\right)} = \boxed{21 \text{ m/s}}$$

68. ***REASONING AND SOLUTION***

a. Since the volume flow rate, $Q = Av$, is the same at each point, and since v is greater at the lower point, $\boxed{\text{the upper hole must have the larger area}}$.

b. Call the upper hole number 1 and the lower hole number 2 (the surface of the water is position 0). Take the zero level of potential energy at the bottom hole and then write Bernoulli's equation as

$$P_1 + (1/2)\rho v_1^2 + \rho gh = P_2 + (1/2)\rho v_2^2 = P_0 + \rho g(2h)$$

in which $P_1 = P_2 = P_0$ and from which we obtain

$$v_1 = \sqrt{2gh} \quad \text{and} \quad v_2 = \sqrt{4gh} \quad \text{or} \quad v_2 / v_1 = \sqrt{2}$$

Using the fact that $Q_1 = A_1v_1 = Q_2 = A_2v_2$, we have $v_2/v_1 = A_1/A_2$. But since the ratio of the areas is $A_1/A_2 = r_1^2/r_2^2$, we can write that

$$\frac{r_1}{r_2} = \sqrt{\frac{A_1}{A_2}} = \sqrt{\frac{v_2}{v_1}} = \sqrt[4]{2} = \boxed{1.19}$$

69. **SSM** ***REASONING AND SOLUTION*** Bernoulli's equation (Equation 11.12) is

$$P_1 + \tfrac{1}{2}\rho v_1^2 = P_2 + \tfrac{1}{2}\rho v_2^2$$

If we let the right hand side refer to the air above the plate, and the left hand side refer to the air below the plate, then $v_1 = 0$, since the air below the plate is stationary. We wish to find v_2 for the situation illustrated in part *b* of the figure shown in the text. Solving the equation above for v_2 (with $v_2 = v_{2\text{b}}$ and $v_1 = 0$) gives

$$v_{2\text{b}} = \sqrt{\frac{2(P_1 - P_2)}{\rho}} \tag{1}$$

In Equation (1), P_1 is atmospheric pressure and P_2 must be determined. We must first consider the situation in part *a* of the text figure.

The figure at the right shows the forces that act on the rectangular plate in part *a* of the text drawing. F_1 is the force exerted on the plate from the air below the plate, and F_2 is the force exerted on the plate from the air above the plate. Applying Newton's second law, we have (taking "up" to be the positive direction),

$$F_1 - F_2 - mg = 0$$

$$F_1 - F_2 = mg$$

Thus, the difference in pressures exerted by the air on the plate in part *a* of the drawing is

$$P_1 - P_2 = \frac{F_1 - F_2}{A} = \frac{mg}{A} \tag{2}$$

where *A* is the area of the plate. From Bernoulli's equation (Equation 11.12) we have, with $v_2 = v_{2\text{a}}$ and $\text{v}_{1\text{a}} = 0$,

$$P_1 - P_2 = \tfrac{1}{2}\rho v_{2\text{a}}^2 \tag{3}$$

where $v_{2\text{a}}$ is the speed of the air along the top of the plate in part *a* of the text drawing. Combining Equations (2) and (3) we have

$$\frac{mg}{A} = \tfrac{1}{2}\rho v_{2\text{a}}^2 \tag{4}$$

The figure below, on the left, shows the forces that act on the plate in part *b* of the text drawing. The notation is the same as that used when the plate was horizontal (part *a* of the text figure). The figure at the right below shows the same forces resolved into components along the plate and perpendicular to the plate.

Applying Newton's second law we have $F_1 - F_2 - mg\sin\theta = 0$, or

$$F_1 - F_2 = mg\sin\theta$$

Thus, the difference in pressures exerted by the air on the plate in part *b* of the text figure is

$$P_1 - P_2 = \frac{F_1 - F_2}{A} = \frac{mg\sin\theta}{A}$$

Using Equation (4) above,

$$P_1 - P_2 = \tfrac{1}{2}\rho v_{2\mathrm{a}}^2 \sin\theta$$

Thus, Equation (1) becomes

$$v_{2\mathrm{b}} = \sqrt{\frac{2\left(\tfrac{1}{2}\rho v_{2\mathrm{a}}^2 \sin\theta\right)}{\rho}}$$

Therefore,

$$v_{2\mathrm{b}} = \sqrt{v_{2\mathrm{a}}^2 \sin\ \theta} = \sqrt{(11.0\ \mathrm{m/s})^2 \sin 30.0°} = \boxed{7.78\ \mathrm{m/s}}$$

70. ***REASONING*** Bernoulli's equation does not apply to this problem, because the fluid is not ideal. It has a viscosity, and Poiseuille's law, as given in Equation 11.14, must be used instead.

SOLUTION According to Equation 11.14, we have

$$Q = \frac{\pi R^4 \left(P_{\text{Input}} - P_{\text{Output}} \right)}{8 \eta L} \quad \text{or} \quad P_{\text{Input}} - P_{\text{Output}} = \frac{Q 8 \eta L}{\pi R^4}$$

Solving for the input pressure gives

$$P_{\text{Input}} = P_{\text{Output}} + \frac{Q 8 \eta L}{\pi R^4}$$

$$= 1.013 \times 10^5 \text{ Pa} + \frac{\left(5.3 \times 10^{-5} \text{ m}^3/\text{s}\right) 8 \left(0.14 \text{ Pa} \cdot \text{s}\right) \left(37 \text{ m}\right)}{\pi \left(6.0 \times 10^{-3} \text{ m}\right)^4} = \boxed{6.4 \times 10^5 \text{ Pa}}$$

71. **SSM** **WWW** ***REASONING AND SOLUTION***

a. If the water behaves as an ideal fluid, and since the pipe is horizontal and has the same radius throughout, the speed and pressure of the water are the same at all points in the pipe. Since the right end of the pipe is open to the atmosphere, the pressure at the right end is atmospheric pressure; therefore, the pressure at the left end is also atmospheric pressure, or $\boxed{1.01\times10^5\ \text{Pa}}$.

b. If the water is treated as a viscous fluid, the volume flow rate Q is described by Poiseuille's law (Equation 11.14):

$$Q=\frac{\pi R^4(P_2-P_1)}{8\eta L}$$

Let P_1 represent the pressure at the right end of the pipe, and let P_2 represent the pressure at the left end of the pipe. Solving for P_2 (with P_1 equal to atmospheric pressure), we obtain

$$P_2=\frac{8\eta LQ}{\pi R^4}+P_1$$

Therefore,

$$P_2=\frac{8(1.00\times10^{-3}\ \text{Pa}\cdot\text{s})(1.3\ \text{m})(9.0\times10^{-3}\ \text{m}^3/\text{s})}{\pi(6.4\times10^{-3}\ \text{m})^4}+1.013\times10^5\ \text{Pa}=\boxed{1.19\times10^5\ \text{Pa}}$$

72. ***REASONING AND SOLUTION*** We have $Q = \pi R^4(P_2 - P_1)/(8\eta L)$, so the difference in pressure $\Delta P = (P_2 - P_1)$ is $\Delta P = 8\eta LQ/(\pi R^4)$. Therefore,

$$\Delta P = \frac{8\left(4 \times 10^{-3}\ \text{Pa}\cdot\text{s}\right)\left(0.10\ \text{m}\right)\left(1.0 \times 10^{-7}\ \text{m}^3/\text{s}\right)}{\pi\left(1.5 \times 10^{-3}\ \text{m}\right)^4} = \boxed{20\ \text{Pa}}$$

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73. ***REASONING*** In this problem, we are treating air as a viscous fluid. According to Poiseuille's law, a fluid with viscosity η flowing through a pipe of radius R and length L has a volume flow rate Q given by Equation 11.14: $Q = \pi R^4 (P_2 - P_1)/(8\eta L)$. This expression can be solved for the quantity $P_2 - P_1$, the difference in pressure between the ends of the air duct. First, however, we must determine the volume flow rate Q of the air.

SOLUTION Since the fan forces air through the duct such that $280\ \text{m}^3$ of air is replenished every ten minutes, the volume flow rate is

$$Q = \left(\frac{280\ \text{m}^3}{10.0\ \text{min}}\right)\left(\frac{1.0\ \text{min}}{60\ \text{s}}\right) = 0.467\ \text{m}^3/\text{s}$$

The difference in pressure between the ends of the air duct is, according to Poiseuille's law,

$$P_2 - P_1 = \frac{8\eta L Q}{\pi R^4} = \frac{8(1.8\times10^{-5}\ \text{Pa}\cdot\text{s})(5.5\ \text{m})(0.467\ \text{m}^3/\text{s})}{\pi(7.2\times10^{-2}\ \text{m})^4} = \boxed{4.4\ \text{Pa}}$$

74. ***REASONING*** This is an application of Poiseuille's law, as given in Equation 11.14. According to this law, a pressure difference $P_2 - P_1$ is needed to make the viscous blood flow through the needle.

SOLUTION The pressure at the input of the needle is $P_2 = P_{\text{Atmosphere}} + \rho gh$, as given by Equation 11.4, whereas the pressure at the output of the needle is $P_1 = P_{\text{Atmosphere}}$. Therefore, we have

$$P_2 - P_1 = P_{\text{Atmosphere}} + \rho gh - P_{\text{Atmosphere}} = \rho gh$$

Using Poiseuille's law, we find

$$Q = \frac{\pi R^4 \left(P_2 - P_1\right)}{8\eta L} = \frac{\pi R^4 \rho gh}{8\eta L}$$

Solving for h gives

$$h = \frac{Q 8\eta L}{\pi R^4 \rho g} = \frac{\left(4.5\times10^{-8}\ \text{m}^3/\text{s}\right)8\left(4.0\times10^{-3}\ \text{Pa}\cdot\text{s}\right)\left(0.030\ \text{m}\right)}{\pi\left(2.5\times10^{-4}\ \text{m}\right)^4\left(1060\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{0.34\ \text{m}}$$

75. **SSM** ***REASONING AND SOLUTION***

a. Using Stoke's law, the viscous force is

$$F = 6\pi\eta R v = 6\pi(1.00\times10^{-3}\ \text{Pa}\cdot\text{s})(5.0\times10^{-4}\ \text{m})(3.0\ \text{m/s}) = \boxed{2.8\times10^{-5}\ \text{N}}$$

b. When the sphere reaches its terminal speed, the net force on the sphere is zero, so that the magnitude of **F** must be equal to the magnitude of $m\mathbf{g}$, or $F = mg$. Therefore, $6\pi\eta R v_{\text{T}} = mg$, where v_{T} is the terminal speed of the sphere. Solving for v_{T}, we have

$$v_{\text{T}} = \frac{mg}{6\pi\eta R} = \frac{(1.0\times10^{-5}\ \text{kg})(9.80\ \text{m/s}^2)}{6\pi(1.00\times10^{-3}\ \text{Pa}\cdot\text{s})(5.0\times10^{-4}\ \text{m})} = \boxed{1.0\times10^{1}\ \text{m/s}}$$

76. ***REASONING AND SOLUTION*** Since water is a viscous fluid, its behavior is described by Poiseuille's law (Equation 11.14):

$$Q = \frac{\pi R^4 (P_2 - P_1)}{8\eta L}$$

Solving for R gives:

$$R = \left[\frac{8\eta L Q}{\pi (P_2 - P_1)}\right]^{1/4}$$

In this expression P_2 is the pressure just below the left vertical tube, P_1 is the pressure just below the right vertical tube, and L is the horizontal distance between the centers of the vertical tubes. The difference in pressures is given by

$$P_2 - P_1 = \rho g(y_2 - y_1) = \rho g \Delta y$$

Therefore,

$$\mathrm{R} = \left[\frac{8\eta \mathrm{LQ}}{\pi(\mathrm{P}_2 - \mathrm{P}_1)}\right]^{1/4}$$

$$= \left[\frac{8(1.00\times10^{-3}\ \mathrm{Pa\cdot s})(0.70\ \mathrm{m})(0.014\ \mathrm{m^3/s})}{\pi(1.000\times10^{3}\ \mathrm{kg/m^3})(9.80\ \mathrm{m/s^2})(0.045\ \mathrm{m})}\right]^{1/4} = \boxed{1.5\times10^{-2}\ \mathrm{m}}$$

77. ***REASONING AND SOLUTION*** We know that $P_2 = P_1 + \rho gh$ so that

$$h = \frac{P_2 - P_1}{\rho g} = \frac{3(1.01 \times 10^5 \text{ Pa}) - 1.01 \times 10^5 \text{ Pa}}{\left(1.00 \times 10^3 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)} = \boxed{20.6 \text{ m}}$$

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78. ***REASONING AND SOLUTION*** Using Bernoulli's equation

$$\Delta P = P_1 - P_2 = (1/2)\rho v_2^2 - (1/2)\rho v_1^2 = (1/2)(1.29\ \text{kg/m}^3)[(8.5\ \text{m/s})^2 - (1.1\ \text{m/s})^2]$$

$$\boxed{\Delta P = 46\ \text{Pa}}$$

The air flows from high pressure to low pressure (from lower to higher velocity), so it $\boxed{\text{enters at } B \text{ and exits at } A}$.

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79. SSM ***REASONING*** The buoyant force exerted on the balloon by the air must be equal in magnitude to the weight of the balloon and its contents (load and hydrogen). The magnitude of the buoyant force is given by $\rho_{\text{air}} V g$. Therefore,

$$\rho_{\text{air}} V g = W_{\text{load}} + \rho_{\text{hydrogen}} V g$$

where, since the balloon is spherical, $V = (4/3)\pi r^3$. Making this substitution for V and solving for r, we obtain

$$r = \left[\frac{3W_{\text{load}}}{4\pi g(\rho_{\text{air}} - \rho_{\text{hydrogen}})} \right]^{1/3}$$

SOLUTION Direct substitution of the data given in the problem yields

$$r = \left[\frac{3(5750\ \text{N})}{4\pi(9.80\ \text{m/s}^2)\,(1.29\ \text{kg/m}^3 - 0.0899\ \text{kg/m}^3)} \right]^{1/3} = \boxed{4.89\ \text{m}}$$

80. ***REASONING*** The absolute pressure P_2 at any vertical distance h from the surface of a fluid of density ρ is given by Equation 11.4 as $P_2 = P_1 + \rho g h$, where P_1 is the absolute pressure at the surface. As discussed in the text, the quantity $P_2 - P_1$ is called the *gauge pressure* .

SOLUTION Solving for h, we have

$$h = \frac{P_2 - P_1}{\rho g} = \frac{1200 \text{ Pa}}{(1.0 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{1.2 \times 10^{-1} \text{ m}}$$

81. ***REASONING AND SOLUTION*** The weight of the gold is $W = Mg = \rho Vg$. Therefore,

$$W = (19\ 300\ \text{kg/m}^3)(0.30\ \text{m})(0.30\ \text{m})(0.20\ \text{m})(9.80\ \text{m/s}^2) = \boxed{3400\ \text{N}}$$

Since 1 N = 0.225 lb, W = (3400 N)[(0.225 lb)/(1 N)] = 760 lb. In other words, the movie pirate would have to be capable of carrying a 760-lb chest.

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82. ***REASONING AND SOLUTION*** 14.0 karat gold is (14.0)/(24.0) gold or 58.3%. The weight of the gold in the necklace is then (1.27 N)(0.583) = 0.740 N. This corresponds to a volume given by $V = M/\rho = W/(\rho g)$. Thus,

$$V = \frac{0.740\ \text{N}}{\left(19\ 300\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{3.91 \times 10^{-6}\ \text{m}^3}$$

83. $\boxed{\textbf{SSM}}$ ***REASONING AND SOLUTION*** Using Equation 11.10 for the volume flow rate, we have $Q = Av$, where A is the cross-sectional area of the pipe and v is the speed of the fluid. Since the pipe has a circular cross section, $A = \pi r^2$. Therefore, solving Equation 11.10 for v, we have

$$v = \frac{Q}{A} = \frac{Q}{\pi r^2} = \frac{1.50\ \text{m}^3/\text{s}}{\pi(0.500\ \text{m})^2} = \boxed{1.91\ \text{m/s}}$$

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84. ***REASONING AND SOLUTION*** The Reynold's number, Re, can be written $\text{Re} = 2\bar{v}\rho R/\eta$. To find the average velocity $\bar{v}$,

$$\bar{v} = \frac{(\text{Re})\eta}{2\rho R} = \frac{(2000)(4 \times 10^{-3}\ \text{Pa}\cdot\text{s})}{2(1060\ \text{kg/m}^3)(8.0 \times 10^{-3}\ \text{m})} = \boxed{0.5\ \text{m/s}}$$

85. ***REASONING AND SOLUTION***

a. The force exerted on the piston (surface area $A = \pi r^2 = 1.96 \times 10^{-3}\ m^2$) is

$$F = P\,A = (1.013 \times 10^5\ N/m^2)(1.96 \times 10^{-3}\ m^2) = 199\ N$$

The spring is therefore compressed by an amount x, where

$$x = F/k = (199\ N)/(3600\ N/m) = \boxed{0.055\ m}$$

b. The work done by the spring is

$$W = (1/2)\,kx^2 = (1/2)(3600\ N/m)(0.055\ m)^2 = \boxed{5.4\ J}$$

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86. ***REASONING AND SOLUTION*** The upward buoyant force of the water on the iceberg must equal the weight of the iceberg if it floats, $F_B = W$, so that $\rho_w g V = \rho_i g V_i$. Now

$$V/V_i = \rho_i/\rho_w = (917\ \text{kg/m}^3)/(1025\ \text{kg/m}^3) = 0.895$$

The percent of the volume of the iceberg which is submerged is $\boxed{89.5\%}$.

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87. SSM ***REASONING AND SOLUTION***

a. The pressure at the level of house A is given by Equation 11.4 as $P = P_{\text{atm}} + \rho g h$. Now the height h consists of the 15.0 m and the diameter d of the tank. We first calculate the radius of the tank, from which we can infer d. Since the tank is spherical, its full mass is given by $M = \rho V = \rho[(4/3)\pi r^3]$. Therefore,

$$r^3 = \frac{3M}{4\pi\rho} \quad \text{or} \quad r = \left(\frac{3M}{4\pi\rho}\right)^{1/3} = \left[\frac{3(5.25\times10^5\ \text{kg})}{4\pi(1.000\times10^3\ \text{kg/m}^3)}\right]^{1/3} = 5.00\ \text{m}$$

Therefore, the diameter of the tank is 10.0 m, and the height h is given by

$$h = 10.0\ \text{m} + 15.0\ \text{m} = 25.0\ \text{m}$$

According to Equation 11.4, the gauge pressure in house A is, therefore,

$$P - P_{\text{atm}} = \rho g h = (1.000\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(25.0\ \text{m}) = \boxed{2.45\times10^5\ \text{Pa}}$$

b. The pressure at house B is $P = P_{\text{atm}} + \rho g h$, where

$$h = 15.0\ \text{m} + 10.0\ \text{m} - 7.30\ \text{m} = 17.7\ \text{m}$$

According to Equation 11.4, the gauge pressure in house B is

$$P - P_{\text{atm}} = \rho g h = (1.000\times10^3\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(17.7\ \text{m}) = \boxed{1.73\times10^5\ \text{Pa}}$$

88. ***REASONING AND SOLUTION***

a. The mass flow rate is equal to $\rho A v$:

$$\text{Mass flow rate} = (1.00 \times 10^3 \text{ kg/m}^3)\pi(0.080 \text{ m})^2 (3.0 \text{ m/s})\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 2.2 \times 10^5 \text{ kg/hr}$$

The mass that flows in one hour is $(2.2 \times 10^5 \text{ kg/hr})(1.0 \text{ hr}) = \boxed{2.2 \times 10^5 \text{ kg}}$.

b. Using $A_1 v_1 = A_2 v_2$, we have $\pi r_1^2 v_1 = 3\pi r_2^2 v_2$ (since there are 3 hoses). Solving for v_2, we obtain

$$v_2 = (1/3)(r_1/r_2)^2 v_1 = (1/3)[(0.080 \text{ m})/(0.020 \text{ m})]^2(3.0 \text{ m/s}) = \boxed{16 \text{ m/s}}$$

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89. **SSM** ***REASONING AND SOLUTION*** The weight of the coin in air is equal to the sum of the weights of the silver and copper that make up the coin:

$$W_{\text{air}} = m_{\text{coin}} g = \left(m_{\text{silver}} + m_{\text{copper}}\right) g = \left(\rho_{\text{silver}} V_{\text{silver}} + \rho_{\text{copper}} V_{\text{copper}}\right) g \tag{1}$$

The weight of the coin in water is equal to its weight in air minus the buoyant force exerted on the coin by the water. Therefore,

$$W_{\text{water}} = W_{\text{air}} - \rho_{\text{water}} (V_{\text{silver}} + V_{\text{copper}}) g \tag{2}$$

Solving Equation (2) for the sum of the volumes gives

$$V_{\text{silver}} + V_{\text{copper}} = \frac{W_{\text{air}} - W_{\text{water}}}{\rho_{\text{water}} g} = \frac{m_{\text{coin}} g - W_{\text{water}}}{\rho_{\text{water}} g}$$

or

$$V_{\text{silver}} + V_{\text{copper}} = \frac{(1.150 \times 10^{-2}\ \text{kg})(9.80\ \text{m/s}^2) - 0.1011\ \text{N}}{(1.000 \times 10^{3}\ \text{kg/m}^3)(9.80\ \text{m/s}^2)} = 1.184 \times 10^{-6}\ \text{m}^3$$

From Equation (1) we have

$$V_{\text{silver}} = \frac{m_{\text{coin}} - \rho_{\text{copper}} V_{\text{copper}}}{\rho_{\text{silver}}} = \frac{1.150 \times 10^{-2}\ \text{kg} - (8890\ \text{kg/m}^3)\text{V}_{\text{copper}}}{10\ 500\ \text{kg/m}^3}$$

or

$$V_{\text{silver}} = 1.095 \times 10^{-6}\ \text{m}^3 - (0.8467) V_{\text{copper}} \tag{3}$$

Substitution of Equation (3) into Equation (2) gives

$$W_{\text{water}} = W_{\text{air}} - \rho_{\text{water}} \left[1.095 \times 10^{-6}\ \text{m}^3 - (0.8467) V_{\text{copper}} + V_{\text{copper}}\right] g$$

Solving for V_{copper} gives $V_{\text{copper}} = 5.806 \times 10^{-7}\ \text{m}^3$. Substituting into Equation (3) gives

$$V_{\text{silver}} = 1.095 \times 10^{-6}\ \text{m}^3 - (0.8467)(5.806 \times 10^{-7}\ \text{m}^3) = 6.034 \times 10^{-7}\ \text{m}^3$$

From Equation 11.1, the mass of the silver is

$$m_{\text{silver}} = \rho_{\text{silver}} V_{\text{silver}} = (10\ 500\ \text{kg/m}^3)(6.034 \times 10^{-7}\ \text{m}^3) = \boxed{6.3 \times 10^{-3}\ \text{kg}}$$

90. ***REASONING AND SOLUTION*** The pressure at the bottom of the container is

$$P = P_{atm} + \rho_w g h_w + \rho_m g h_m$$

We want $P = 2P_{atm}$, and we know $h = h_w + h_m = 1.00$ m. Using the above and rearranging gives

$$h_m = \frac{P_{atm} - \rho_w gh}{(\rho_m - \rho_w)g} = \frac{1.01 \times 10^5 \text{ Pa} - (1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(1.00 \text{ m})}{(13.6 \times 10^3 \text{ kg/m}^3 - 1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{0.74 \text{ m}}$$

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91. **SSM** **WWW** ***REASONING*** According to Equation 11.4, the pressure P_{mercury} at a point 7.10 cm below the ethyl alcohol-mercury interface is

$$P_{\text{mercury}} = P_{\text{interface}} + \rho_{\text{mercury}} g h_{\text{mercury}} \tag{1}$$

where $P_{\text{interface}}$ is the pressure at the alcohol-mercury interface, and $h_{\text{mercury}} = 0.0710\ \text{m}$. The pressure at the interface is

$$P_{\text{interface}} = P_{\text{atm}} + \rho_{\text{ethyl}} g h_{\text{ethyl}} \tag{2}$$

Equation (2) can be used to find the pressure at the interface. This value can then be used in Equation (1) to determine the pressure 7.10 cm below the interface.

SOLUTION Direct substitution of the numerical data into Equation (2) yields

$$P_{\text{interface}} = 1.01 \times 10^5\ \text{Pa} + (806\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(1.10\ \text{m}) = 1.10 \times 10^5\ \text{Pa}$$

Therefore, the pressure 7.10 cm below the ethyl alcohol-mercury interface is

$$P_{\text{mercury}} = 1.10 \times 10^5\ \text{Pa} + (13\ 600\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.0710\ \text{m}) = \boxed{1.19 \times 10^5\ \text{Pa}}$$

92. ***REASONING AND SOLUTION*** The mass of aluminum required to make the can is

$$m_{Al} = m_{total} - m_{soda}$$

where the mass of the soda is

$$m_{soda} = \rho_{soda} V_{soda} = (1.000 \times 10^3 \text{ kg/m}^3)(3.54 \times 10^{-4} \text{ m}^3) = 0.354 \text{ kg}$$

Therefore, the mass of the aluminum used to make the can is

$$m_{Al} = 0.416 \text{ kg} - 0.354 \text{ kg} = 0.062 \text{ kg}$$

Using the definition of density, $\rho = m/V$, we have

$$V_{Al} = \frac{m_{Al}}{\rho_{Al}} = \frac{0.062 \text{ kg}}{2700 \text{ kg/m}^3} = \boxed{2.3 \times 10^{-5} \text{ m}^3}$$

93. ***REASONING AND SOLUTION***

a. Taking the nozzle as position 1 and the top of the tank as position 2 we have, using Bernoulli's equation, $P_1 + (1/2)\rho v_1^2 = P_2 + \rho gh$, we can solve for v_1 so that

$$v_1^2 = 2[(P_2 - P_1)/\rho + gh]$$

$$v_1 = \sqrt{2[(5.00 \times 10^5 \text{ Pa}) / (1.00 \times 10^3 \text{ kg/m}^3) + (9.80 \text{ m/s}^2)(4.00 \text{ m})]} = \boxed{32.8 \text{ m/s}}$$

b. To find the height of the water use $(1/2)\rho v_1^2 = \rho gh$ so that

$$h = (1/2)v_1^2/g = (1/2)(32.8 \text{ m/s})^2/(9.80 \text{ m/s}^2) = \boxed{54.9 \text{ m}}$$

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94. ***REASONING AND SOLUTION*** The upward buoyant force must equal the weight of the shell if it is floating, $\rho_w gV = W$. The submerged volume of the shell is $V = (4/3)\pi R_2^3$, where R_2 is its outer radius. Now $R_2^3 = (3/4)m/(\rho_w \pi)$ gives

$$R_2 = \sqrt[3]{\frac{3m}{4\pi\rho_w}} = \sqrt[3]{\frac{3(1.00\text{ kg})}{4\pi(1.00\times10^3\text{ kg/m}^3)}} = \boxed{6.20\times10^{-2}\text{ m}}$$

The weight of the shell is $W = \rho_g g(V_2 - V_1)$ so $R_1^3 = R_2^3 - (3/4)m/(\pi\rho_g)$, and

$$R_1 = \sqrt[3]{\frac{3m}{4\pi}\left(\frac{1}{\rho_w} - \frac{1}{\rho_g}\right)} = \sqrt[3]{\frac{3(1.00\text{ kg})}{4\pi}\left(\frac{1}{1.00\times10^3\text{ kg/m}^3} - \frac{1}{2.60\times10^3\text{ kg/m}^3}\right)}$$

$$= \boxed{5.28\times10^{-2}\text{ m}}$$

95. ***REASONING AND SOLUTION*** The force exerted by a pressure is F = PA and is perpendicular to the surface. The force on the outside of each half of the roof is

$$F = (10.0 \text{ mm Hg})\left(\frac{133 \text{ N/m}^2}{1 \text{ mm Hg}}\right)(14.5 \text{ m} \times 4.21 \text{ m}) = 8.11 \times 10^4 \text{ N}$$

Adding the forces vectorially gives for the vertical force

$$F_v = 2F \cos 30.0° = 1.41 \times 10^5 \text{ N}$$

and for the horizontal force $F_h = 0$ N. The net force is then $\boxed{1.41 \times 10^5 \text{ N}}$

The direction of the force is $\boxed{\text{downward}}$ since the horizontal components of the forces on the halves cancel.

96. ***REASONING AND SOLUTION***

a. At the surface of the water (position 1) and at the exit of the hose (position 2) the pressures are equal ($P_1 = P_2$) to the atmospheric pressure. If the hose exit defines $y = 0$, we have from Bernoulli's equation $(1/2)\rho v_1^2 + \rho g y = (1/2)\rho v_2^2$. If we take the velocity at the surface of the water to be zero (i.e., $v_1 = 0$) we find that

$$\boxed{v_2 = \sqrt{2gy}}$$

b. The siphon will stop working when $v_2 = 0$, or $\boxed{y = 0}$, i.e., when the end of the hose is at the water level in the tank.

c. At point A we have

$$P_A + \rho g(h + y) + (1/2)\rho v_A^2 = P_0 + (1/2)\rho v_2^2$$

But $v_A = v_2$ so that

$$\boxed{P_A = P_0 - \rho g(y + h)}$$

1. **SSM** ***REASONING AND SOLUTION*** The difference between these two averages, expressed in Fahrenheit degrees, is

$$98.6\ ^\circ\text{F} - 98.2\ ^\circ\text{F} = 0.4\ \text{F}^\circ$$

Since 1 C° is equal to $\frac{9}{5}$ F°, we can make the following conversion

$$(0.4\ \text{F}^\circ)\left(\frac{1\ \text{C}^\circ}{(9/5)\ \text{F}^\circ}\right) = \boxed{0.2\ \text{C}^\circ}$$

2. ***REASONING AND SOLUTION***

a. The Kelvin temperature and the temperature on the Celsius scale are related by Equation 12.1: $T = T_c + 273.15$, where T is the Kelvin temperature and T_c is the Celsius temperature. Therefore, a temperature of 77 K on the Celsius scale is

$$T_c = T - 273.15 = 77\text{ K} - 273.15\text{ K} = \boxed{-196\ ^\circ\text{C}}$$

b. The temperature of –196 °C is 196 Celsius degrees *below* the ice point of 0 °C. Since $1\text{ C}^\circ = \frac{9}{5}\text{ F}^\circ$, this number of Celsius degrees corresponds to

$$196\text{ C}^\circ\left(\frac{\frac{9}{5}\text{ F}^\circ}{1\text{ C}^\circ}\right) = 353\text{ F}^\circ$$

Subtracting 353 Fahrenheit degrees from the ice point of 32.0 °F on the Fahrenheit scale gives a Fahrenheit temperature of $\boxed{-321\ ^\circ\text{F}}$.

3. ***REASONING AND SOLUTION***

a. A temperature of 50.0 °F is 18.0 Fahrenheit degrees above the ice point of 32.0 °F. Since $1\ \text{C}° = \frac{9}{5}\ \text{F}°$, the difference of 18.0 F° is equivalent to

$$18.0\ \text{F}° \left(\frac{1\ \text{C}°}{\frac{9}{5}\ \text{F}°} \right) = 10.0\ \text{C}°$$

Adding 10.0 Celsius degrees to the ice point of 0 °C on the Celsius scale gives a Celsius temperature of $\boxed{10.0\ °\text{C}}$. Similar reasoning shows that a temperature of 104 °F is equivalent to a Celsius temperature of $\boxed{40.0\ °\text{C}}$.

b. The Kelvin temperature and the temperature on the Celsius scale are related by Equation 12.1: $T = T_c + 273.15$, where T is the Kelvin temperature and T_c is the Celsius temperature. Therefore, a temperature of 10.0 °C is equivalent to a Kelvin temperature of $\boxed{283.15\ \text{K}}$, and a temperature of 40.0 °C is equivalent to $\boxed{313.15\ \text{K}}$.

4. ***REASONING*** We will first convert the temperature from the Kelvin scale to the Celsius scale by using Equation 12.1, $T_c = T - 273.15$, where T_c is the Celsius temperature and T is the Kelvin temperature. Then we will convert from the Celsius scale to the Fahrenheit scale by multiplying the Celsius temperature by a factor of 9/5 and then adding 32 to the result.

SOLUTION The temperature on the Celsius scale is

$$T_c = T - 273.15 = (310.6 - 273.15)\,°\text{C} = 37.4\,°\text{C} \tag{12.1}$$

The temperature of 37.5 °C is equivalent to a Fahrenheit temperature of

$$\text{Temperature} = (37.4)\left(\frac{9}{5}\right) + 32.0 = \boxed{99.3\,°\text{F}}$$

5. **SSM** ***REASONING AND SOLUTION*** The temperature of –273.15 °C is 273.15 Celsius degrees *below* the ice point of 0 °C. This number of Celsius degrees corresponds to

$$(273.15\ \text{C}^\circ)\left(\frac{(9/5)\ \text{F}^\circ}{1\ \text{C}^\circ}\right) = 491.67\ \text{F}^\circ$$

Subtracting 491.67 F° from the ice point of 32.00 °F on the Fahrenheit scale gives a Fahrenheit temperature of $\boxed{-459.67\ ^\circ\text{F}}$

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6. ***REASONING AND SOLUTION*** The Rankine and Fahrenheit degrees are the same size, since the difference between the steam point and ice point temperatures is the same for both. The difference in the ice points of the two scales is 491.67 – 32.00 = 459.67. To get Rankine from Fahrenheit this amount must be added, so $\boxed{T_R = T_F + 459.67}$

7. SSM ***REASONING*** The temperature at any pressure can be determined from the equation of the graph in Figure 12.4. Since the gas pressure is 5.00×10^3 Pa when the temperature is 0.00 °C, and the pressure is zero when the temperature is –273.15 °C, the slope of the line is given by

$$\frac{\Delta P}{\Delta T} = \frac{5.00 \times 10^3 \text{ Pa} - 0.00 \text{ Pa}}{0.00 \text{ °C} - (-273.15 \text{ °C})} = 18.3 \text{ Pa/C°}$$

Therefore, the equation of the line is $\Delta P = (18.3 \text{ Pa/C°})\Delta T$, or

$$P - P_0 = (18.3 \text{ Pa/C°})(T - T_0)$$

where $P_0 = 5.00 \times 10^3$ Pa when $T_0 = 0.00$ °C.

SOLUTION Solving the equation above for *T*, we obtain

$$T = \frac{P - P_0}{18.3 \text{ Pa/C°}} + T_0 = \frac{2.00 \times 10^3 \text{ Pa} - 5.00 \times 10^3 \text{ Pa}}{18.3 \text{ Pa/C°}} + 0.00 \text{ °C} = \boxed{-164 \text{ °C}}$$

8. ***REASONING AND SOLUTION*** If the voltage is proportional to the temperature difference between the junctions, then

$$\frac{V_1}{\Delta T_1} = \frac{V_2}{\Delta T_2} \quad \text{or} \quad \Delta T_2 = \frac{V_2}{V_1}\Delta T_1$$

Thus,

$$T_2 - 0.0\ °\text{C} = \frac{1.90\times10^{-3}\ \text{V}}{4.75\times10^{-3}\ \text{V}}(110.0\ °\text{C} - 0.0\ °\text{C})$$

Solving for T_2 yields $\boxed{T_2 = 44.0\ °\text{C}}$.

9. ***REASONING AND SOLUTION*** The linear expansion of the aircraft carrier is

$$\Delta L = \alpha L_0 \Delta T = (12 \times 10^{-6}\ \text{C}^{\circ -1})(370\ \text{m})(21\ ^\circ\text{C} - 2.0\ ^\circ\text{C}) = \boxed{0.084\ \text{m}} \tag{12.2}$$

10. ***REASONING*** The change in length ΔL of the pipe is proportional to the coefficient of linear expansion α for steel, the original length L_0 of the pipe, and the change in temperature ΔT. The coefficient of linear expansion for steel can be found in Table 12.1.

SOLUTION The change in length of the pipe is

$$\Delta L = \alpha L_0 \Delta T = \left[1.2 \times 10^{-5}\ (\text{C}^\circ)^{-1}\right](65\ \text{m})\left[18\ ^\circ\text{C} - (-45\ ^\circ\text{C})\right] = \boxed{4.9 \times 10^{-2}\ \text{m}} \qquad (12.2)$$

11. SSM WWW ***REASONING AND SOLUTION*** The steel in the bridge expands according to Equation 12.2, $\Delta L = \alpha L_0 \Delta T$. Solving for L_0, we find that the approximate length of the Golden Gate bridge is

$$L_0 = \frac{\Delta L}{\alpha \Delta T} = \frac{0.53 \text{ m}}{\left[12 \times 10^{-6} \ (\text{C}°)^{-1}\right](32\,°\text{C} - 2\,°\text{C})} = \boxed{1500 \text{ m}}$$

12. ***REASONING AND SOLUTION***

a. The radius of the hole will be $\boxed{\text{larger}}$ when the plate is heated, because the hole expands as if it were made of copper.

b. The expansion of the radius is $\Delta r = \alpha r_0 \Delta T$. The fractional change in the radius is

$$\Delta r/r_0 = \alpha \Delta T = (17 \times 10^{-6}\ \text{C}^{\circ -1})(110\ °\text{C} - 11\ °\text{C}) = \boxed{0.0017}$$

13. ***REASONING AND SOLUTION*** The fractional change in the length of the beam is given by $\Delta L = \alpha L_0 \Delta T$ to be $\Delta L/L_0 = \alpha \Delta T$

$$\frac{\Delta L}{L_0} = \left(12 \times 10^{-6}\ \text{C}^{\circ -1}\right)\left[105\,^{\circ}\text{F} - \left(-15\,^{\circ}\text{F}\right)\right]\left(\frac{1\ \text{C}^{\circ}}{\frac{9}{5}\text{F}^{\circ}}\right) = \boxed{8.0 \times 10^{-4}}$$

14. ***REASONING*** The height L_0 of the Eiffel Tower at the lower temperatue can be determined from Equation 12.2, $L_0 = \Delta L / (\alpha \Delta T)$, where ΔL is the increase in the height, α is the coefficient of linear expansion for steel, and ΔT is the change in temperature. The coefficient of linear expansion for steel can be found in Table 12.1.

SOLUTION The height of the Eiffel Tower at the lower temperature is

$$L_0 = \frac{\Delta L}{\alpha \Delta T} = \frac{18 \times 10^{-2}\ \text{m}}{\left[1.2 \times 10^{-5}\ (\text{C}^\circ)^{-1}\right]\left[41\ ^\circ\text{C} - (-9\ ^\circ\text{C})\right]} = \boxed{3.0 \times 10^{2}\ \text{m}} \tag{12.2}$$

15. SSM ***REASONING AND SOLUTION*** Assuming that the rod expands linearly with heat, we first calculate the quantity $\Delta L / \Delta T$ using the data given in the problem.

$$\frac{\Delta L}{\Delta T} = \frac{8.47 \times 10^{-4}\ \text{m}}{100.0\ °\text{C} - 25.0\ °\text{C}} = 1.13 \times 10^{-5}\ \text{m/C°}$$

Therefore, when the rod is cooled from 25.0 °C, it will shrink by

$$\begin{aligned}\Delta L &= (1.13 \times 10^{-5}\ \text{m/C°})\ \Delta T \\ &= (1.13 \times 10^{-5}\ \text{m/C°})\ (0.00\ °\text{C} - 25.0\ °\text{C}) = \boxed{-2.82 \times 10^{-4}\ \text{m}}\end{aligned}$$

16. ***REASONING AND SOLUTION*** From Equation 10.17, the stress, F/A, required to change the length L_0 of a rod by an amount ΔL is

$$\text{Stress} = Y\frac{\Delta L}{L_0} \tag{1}$$

where Y is Young's modulus of the rod. If the rod were free to contract as it cooled, its length would change by an amount $\Delta L = \alpha L_0 \Delta T$, as predicted by Equation 12.2. Since the rod is fastened securely at both ends to an immovable support, the rod will stretch by this amount as it is cooled. Thus, Equation (1) becomes

$$\text{Stress} = Y\frac{\alpha L_0 \Delta T}{L_0} = Y\alpha\Delta T \tag{2}$$

When the tensile stress applied at the ends reaches 2.3×10^7 N/m^2, the rod will rupture. The corresponding temperature change can be found by solving Equation (2) for ΔT:

$$\Delta T = \frac{\text{Stress}}{Y\alpha} = \frac{2.3 \times 10^7 \text{ N/m}^2}{(1.1 \times 10^{11} \text{ N/m}^2)[17 \times 10^{-6} \ (\text{C}°)^{-1}]} = \boxed{12 \text{ C}°}$$

17. ***REASONING AND SOLUTION*** $\Delta L = \alpha L_0 \Delta T$ gives for the expansion of the aluminum

$$\Delta L_A = \alpha_A L_A \Delta T \tag{1}$$

and for the expansion of the brass

$$\Delta L_B = \alpha_B L_B \Delta T \tag{2}$$

Adding (1) and (2) and solving for ΔT gives

$$\Delta T = \frac{\Delta L_A - \Delta L_B}{\alpha_A L_A + \alpha_B L_B} = \frac{1.3 \times 10^{-3}\ \text{m}}{\left(23 \times 10^{-6}\ \text{C}^{\circ -1}\right)(1.0\ \text{m}) + \left(19 \times 10^{-6}\ \text{C}^{\circ -1}\right)(2.0\ \text{m})} = 21\,\text{C}^{\circ}$$

The desired temperature is then

$$T = 28\ ^{\circ}\text{C} + 21\ \text{C}^{\circ} = \boxed{49\ ^{\circ}\text{C}}$$

18. ***REASONING AND SOLUTION*** The initial diameter of the sphere, d_s, is

$$d_s = (5.0 \times 10^{-4})d_r + d_r \tag{1}$$

where d_r is the initial diameter of the ring. Applying $\Delta L = \alpha L_0 \Delta T$ to the diameter of the sphere gives

$$\Delta d_s = \alpha_s d_s \Delta T \tag{2}$$

and to the ring gives

$$\Delta d_r = \alpha_r d_r \Delta T \tag{3}$$

If the sphere is just to fit inside the ring, we must have

$$d_s + \Delta d_s = d_r + \Delta d_r$$

Using (2) and (3) in this expression and solving for ΔT gives

$$\Delta T = \frac{d_r - d_s}{\alpha_s d_s - \alpha_r d_r}$$

Using (1) yields

$$\Delta T = \frac{-5.0 \times 10^{-4}}{\left(29 \times 10^{-6}\ \text{C}^{\circ -1}\right)\left(5.0 \times 10^{-4} + 1\right) - 12 \times 10^{-6}\ \text{C}^{\circ -1}} = \boxed{-29\ \text{C}^{\circ}}$$

The final temperature is

$$T_f = 70.0\ ^{\circ}\text{C} - 29\ \text{C}^{\circ} = \boxed{41\ ^{\circ}\text{C}}$$

19. SSM WWW ***REASONING AND SOLUTION*** Recall that $\omega = 2\pi / T$, Equation 10.6, where ω is the angular frequency of the pendulum and T is the period. Using this fact and Equation 10.16, we know that the period of the pendulum before the temperature rise is given by $T_1 = 2\pi\sqrt{L_0 / g}$, where L_0 is the length of the pendulum. After the temperature has risen, the period becomes (using Equation 12.2), $T_2 = 2\pi\sqrt{\left[L_0 + \alpha L_0 \Delta T\right] / g}$. Dividing these expressions and solving for T_2 we have

$$T_2 = T_1\sqrt{1 + \alpha\Delta T} = (2.0000\text{ s})\sqrt{1 + (19 \times 10^{-6}\text{ /C}^\circ)\ (140\text{ C}^\circ)} = \boxed{2.0027\text{ s}}$$

20. ***REASONING*** The force F_{applied} to compress a spring by an amount ΔL is given by Equation 10.1 as $F_{\text{applied}} = k\,\Delta L$, where k is the spring constant of the spring. The change in length of the spring is related to its initial length L_0 and the change in temperature ΔT by Equation 12.2, $\Delta L = \alpha\ L_0 \Delta T$, where α is the coefficient of linear expansion for brass. Thus, the applied force is

$$F_{\text{Applied}} = k\,\Delta L = k\,\alpha\,L_0\,\Delta T$$

SOLUTION Using the value of α for brass from Table 12.1, we have

$$F_{\text{Applied}} = k\,\alpha\,L_0\,\Delta T = \left(1.3\times10^{4}\ \text{N/m}\right)\left[1.9\times10^{-5}\ (\text{C}^\circ)^{-1}\right](0.18\ \text{m})(135\ ^\circ\text{C} - 21\ ^\circ\text{C}) = \boxed{5.1\ \text{N}}$$

21. ***REASONING AND SOLUTION*** When the cube is heated, its length changes from L_0 to L where $L = L_0 + \Delta L$, and $\Delta L = \alpha L_0 \Delta T$. Thus, the length of each edge of the heated cube is given by $L = L_0 + \alpha L_0 \Delta T = L_0(1 + \alpha\Delta T)$. The area of each face of the cube increases from A_0 to A where $A_0 = L_0^2$, and $A = L^2$. Therefore,

$$A = L_0^2(1 + \alpha\Delta T)^2 = L_0^2\ (1 + 2\alpha\Delta T + \alpha^2\Delta T^2)$$

Since α is on the order of 10^{-6} for most common solids, the term involving α^2 is much smaller than the term involving α; thus, it may be ignored.

$$A \cong L_0^2(1 + 2\alpha\Delta T) = A_0(1 + 2\alpha\Delta T)$$

The increase in area of each face of the cube is given by

$$\Delta A = A - A_0 = 2\alpha A_0 \Delta T$$

$$= 2\left[23 \times 10^{-6}\ \text{C}^{\circ -1}\right](0.050\ \text{m})^2(145\ ^\circ\text{C} - 25\ ^\circ\text{C}) = 1.38 \times 10^{-5}\ \text{m}^2$$

Since the cube has six faces, the total increase in the surface area of the cube is $6(1.38 \times 10^{-5}\ \text{m}^2)$, or $\boxed{8.3 \times 10^{-5}\ \text{m}^2}$.

22. ***REASONING AND SOLUTION*** Let $L_0 = 0.50$ m and L be the true length of the line at 40.0 °C. The ruler has expanded an amount

$$\Delta L_r = L - L_0 = \alpha_r L_0 \Delta T_r \tag{1}$$

The copper plate must shrink by an amount

$$\Delta L_p = L_0 - L = \alpha_p L \Delta T_p \tag{2}$$

Eliminating L from Equations (1) and (2), and solving for ΔT_p, gives

$$\Delta T_p = \frac{-\alpha_r \Delta T_r}{\alpha_p (1 + \alpha_r \Delta T_r)} = 14\ \text{C°}.$$

Therefore, $T_p = 40.0\ °\text{C} - 14\ \text{C°} = \boxed{26\ °\text{C}}$.

23. SSM ***REASONING*** The change in length of the wire is the sum of the change in length of each of the two segments: $\Delta L = \Delta L_{\text{al}} + \Delta L_{\text{st}}$. Using Equation 12.2 to express the ΔLs

$$\alpha L_0 \Delta T = \alpha_{\text{al}} L_{0\text{al}} \Delta T + \alpha_{\text{st}} L_{0\text{st}} \Delta T$$

Dividing both sides by L_0 and algebraically canceling ΔT gives

$$\alpha = \alpha_{\text{al}} \left(\frac{L_{0\text{al}}}{L_0} \right) + \alpha_{\text{st}} \left(\frac{L_{0\text{st}}}{L_0} \right)$$

The length of the steel segment of the wire is given by $L_{0\text{st}} = L_0 - L_{0\text{al}}$. Making this substitution leads to

$$\alpha = \alpha_{\text{al}} \left(\frac{L_{0\text{al}}}{L_0} \right) + \alpha_{\text{st}} \left(\frac{L_0 - L_{0\text{al}}}{L_0} \right)$$

$$= \alpha_{\text{al}} \left(\frac{L_{0\text{al}}}{L_0} \right) + \alpha_{\text{st}} \left(\frac{L_0}{L_0} \right) - \alpha_{\text{st}} \left(\frac{L_{0\text{al}}}{L_0} \right)$$

This expression can be solved for the desired quantity, $L_{0\text{al}} / L_0$.

SOLUTION Solving for the ratio $(L_{0\text{al}} / L_0)$ gives

$$\frac{L_{0\text{al}}}{L_0} = \frac{\alpha - \alpha_{\text{st}}}{\alpha_{\text{al}} - \alpha_{\text{st}}} = \frac{19 \times 10^{-6} (\text{C}^\circ)^{-1} - 12 \times 10^{-6} (\text{C}^\circ)^{-1}}{23 \times 10^{-6} (\text{C}^\circ)^{-1} - 12 \times 10^{-6} (\text{C}^\circ)^{-1}} = \boxed{0.6}$$

24. ***REASONING AND SOLUTION*** Let L_0 be the length of the wire before heating. After heating the wire will have stretched an amount $\Delta L_W = \alpha_W L_0 \Delta T$, while the gap in the concrete "stretches" an amount $\Delta L_C = \alpha_C L_0 \Delta T$.

The net change in length of the wire is then $\Delta L_C - \Delta L_W = (\alpha_C - \alpha_W) L_0 \Delta T$. The additional stress needed to produce this change is

$$\text{Stress} = Y(\Delta L_C - \Delta L_W)/ L_0 = Y (\alpha_C - \alpha_W)\Delta T$$

The additional tension in the wire is

$$T = (\text{Stress})(\pi R^2) = Y(\alpha_c - \alpha_w)\Delta T(\pi R^2)$$

$$= (6.9 \times 10^{10}\ \text{N/m}^2)[12 \times 10^{-6}\ \text{C}^{\circ -1} - 23 \times 10^{-6}\ \text{C}^{\circ -1}](185\,^\circ\text{C} - 35\,^\circ\text{C})\pi(3.0 \times 10^{-4}\ \text{m})^2$$

$$= -32\ \text{N}$$

The new tension in the wire is then $50.0\ \text{N} - 32\ \text{N} = \boxed{18\ \text{N}}$

25. ***REASONING AND SOLUTION*** The volume V_0 of an object changes by an amount ΔV when its temperature changes by an amount ΔT; the mathematical relationship is given by Equation 12.3: $\Delta V = \beta V_0 \Delta T$. Thus, the volume of the kettle at 24 °C can be found by solving Equation 12.3 for V_0. According to Table 12.1, the coefficient of volumetric expansion for copper is $51 \times 10^{-6}\ (\text{C}°)^{-1}$. Solving Equation 12.3 for V_0, we have

$$V_0 = \frac{\Delta V}{\beta \Delta T} = \frac{1.2 \times 10^{-5}\ \text{m}^3}{[51 \times 10^{-6}\ (\text{C}°)^{-1}](100\ °\text{C} - 24\ °\text{C})} = \boxed{3.1 \times 10^{-3}\ \text{m}^3}$$

26. ***REASONING AND SOLUTION*** The change in the volume of the water is

$$\Delta V = \beta V_0 \Delta T = (207 \times 10^{-6}\ C^{\circ -1})(110\ m^3)(27\ °C - 17\ °C) = \boxed{0.23\ m^3} \qquad (12.3)$$

27. **SSM** ***REASONING AND SOLUTION*** According to Equation 12.3, $\Delta V = \beta V_0 \Delta T$. Therefore,

$$\frac{\Delta V_{\text{air}}}{\Delta V_{\text{water}}} = \frac{\beta_{\text{air}} V_0 \Delta T}{\beta_{\text{water}} V_0 \Delta T} = \frac{\beta_{\text{air}}}{\beta_{\text{water}}} = \frac{3.7 \times 10^{-3} (\text{C}^\circ)^{-1}}{207 \times 10^{-6} (\text{C}^\circ)^{-1}} = \boxed{18}$$

28. ***REASONING*** The change in volume of the cavity is proportional to the coefficient of volume expansion β for aluminum, the original volume V_0 of the cavity, and the change in temperature ΔT . The original volume of the spherical cavity is $V_0 = \frac{4}{3}\pi r^3$, where r is its radius. The coefficient of volume expansion for aluminum can be found in Table 12.1.

SOLUTION

a. The change in the volume of the spherical cavity is

$$\Delta V = \beta V_0 \Delta T \tag{12.3}$$

$$= \left[6.9 \times 10^{-5}\ (\text{C}°)^{-1}\right]\left[\tfrac{4}{3}\pi(5.0\ \text{cm})^3\right](282\ °\text{C} - 23\ °\text{C}) = \boxed{9.4\ \text{cm}^3}$$

b. Because the temperature increases, $\boxed{\text{the cavity becomes larger}}$.

29. ***REASONING AND SOLUTION*** The interior volume of the shell will expand as if it were solid silver. Then $\Delta V = \beta V_0 \Delta T = \beta (4/3)\pi r^3 \Delta T$

$$\Delta V = (57 \times 10^{-6}\ C^{\circ -1})(4/3)(\pi)(1.5 \times 10^{-2}\ m)^3(135\ °C - 25\ °C) = \boxed{8.9 \times 10^{-8}\ m^3}$$

30. ***REASONING AND SOLUTION*** Both the water and pipe expand as the temperature increases. For the expansion of the water

$$\Delta V_w = \beta_w V_0 \Delta T$$

and for the expansion of the pipe

$$\Delta V_p = \beta_c V_0 \Delta T$$

The initial volume of the pipe and water is $V_0 = \pi r^2 L$ The reservoir then needs a capacity of

$$\Delta V = \Delta V_w - \Delta V_p = (\beta_w - \beta_c) V_0 \Delta T$$

$$\Delta V = (207 \times 10^{-6}\ C^{\circ -1} - 51 \times 10^{-6}\ C^{\circ -1})\pi (9.5 \times 10^{-3}\ m)^2 (76\ m)(54\ C^{\circ}) = \boxed{1.8 \times 10^{-4}\ m^3}$$

31. **SSM** ***REASONING AND SOLUTION*** The cider will expand according to Equation 12.3; therefore, the change in volume of the cider is

$$\Delta V = \beta V_0 \Delta T = \left[280 \times 10^{-6}\ (\text{C}°)^{-1}\right] 1.0\ \text{gal})(22\ \text{C}°) = 6.2 \times 10^{-3}\ \text{gal}$$

At a cost of two dollars per gallon, this amounts to

$$\left(6.2 \times 10^{-3}\ \text{gal}\right)\left(\frac{\$\ 2.00}{\text{gal}}\right) = \$\ 0.01 \quad \text{or} \quad \boxed{\text{one penny}}$$

32. ***REASONING AND SOLUTION*** Both the coffee and beaker expand as the temperature increases. For the expansion of the coffee

$$\Delta V_c = \beta_w V_0 \Delta T$$

and for the expansion of the beaker

$$\Delta V_b = \beta_b V_0 \Delta T$$

The excess expansion of the coffee, hence the amount which spills, is

$$\Delta V = \Delta V_c - \Delta V_b = (\beta_w - \beta_b) V_0 \Delta T$$

$$\Delta V = [207 \times 10^{-6} \ (C°)^{-1} - 9.9 \times 10^{-6} \ (C°)^{-1}](0.50 \times 10^{-3} \ m^3)(92 \ °C - 18 \ °C)$$

$$= \boxed{7.3 \times 10^{-6} \ m^3}$$

33. SSM ***REASONING*** In order to keep the water from expanding as its temperature increases from 15 to 25 °C, the atmospheric pressure must be increased to compress the water as it tries to expand. The magnitude of the pressure change ΔP needed to compress a substance by an amount ΔV is, according to Equation 10.20, $\Delta P = B(\Delta V / V_0)$. The ratio $\Delta V / V_0$ is, according to Equation 12.3, $\Delta V / V_0 = \beta \Delta T$. Combining these two equations yields

$$\Delta P = B \beta \Delta T$$

SOLUTION The change in atmospheric pressure that is required to keep the water from expanding is, therefore,

$$\Delta P = (2.2 \times 10^9 \text{ N/m}^2)\left[207 \times 10^{-6} \text{ (C°)}^{-1}\right](25 \text{ °C} - 15 \text{ °C})$$

$$= 4.6 \times 10^6 \text{ Pa}\left(\frac{1 \text{ atm}}{1.01 \times 10^5 \text{ Pa}}\right) = \boxed{45 \text{ atm}}$$

34. ***REASONING*** The change in volume of the cylindrical mercury column is proportional to the coefficient of volume expansion β for mercury, its original volume V_0, and the change ΔT in temperature. The volume of a cylinder of radius r and height ΔL is $\Delta V = \pi r^2 \Delta L$. The coefficient of volume expansion for mercury can be found in Table 12.1.

SOLUTION The change in the volume of the mercury is

$$\underbrace{\pi r^2 \Delta L}_{\Delta V} = \beta V_0 \Delta T \tag{12.3}$$

Solving for ΔL gives

$$\Delta L = \frac{\beta V_0 \Delta T}{\pi r^2} = \frac{\left[1.82 \times 10^{-4}\ (\mathrm{C}^\circ)^{-1}\right]\left(45\ \mathrm{mm}^3\right)\left(1.0\ \mathrm{C}^\circ\right)}{\pi\left(1.7 \times 10^{-2}\ \mathrm{mm}\right)^2} = \boxed{9.0\ \mathrm{mm}}$$

35. ***REASONING AND SOLUTION*** The density of mercury is given by Equation 11.1: $\rho = m/V$. Therefore, If ρ_0 represents the density of mercury at 0 °C, and ρ represents its density at 166 °C, then the ratio of the density of mercury at 166 °C to that at 0 °C is

$$\frac{\rho}{\rho_0} = \frac{m/V}{m/V_0} = \frac{V_0}{V}$$

and the density at 166 °C is given by

$$\rho = \rho_0 \frac{V_0}{V} \tag{1}$$

The volume V_0 of an object changes by an amount ΔV when its temperature changes by an amount ΔT; the mathematical relationship is given by Equation 12.3: $\Delta V = \beta V_0 \Delta T$ where $\Delta V = V - V_0$ so that $V = V_0 + \Delta V = V_0 + \beta V_0 \Delta T$. Therefore, Equation (1) becomes

$$\rho = \rho_0 \frac{V_0}{V_0 + \beta V_0 \Delta T} = \rho_0 \frac{V_0}{V_0(1 + \beta \Delta T)} = \frac{\rho_0}{1 + \beta \Delta T}$$

and we find that

$$\rho = \frac{13\ 600\ \text{kg/m}^3}{1 + \left[182 \times 10^{-6}\ (\text{C}°)^{-1}\right](166\ °\text{C} - 0\ °\text{C})} = \boxed{13\ 200\ \text{kg/m}^3}$$

36. ***REASONING AND SOLUTION***

a. The apparent weight of the sphere will be larger $\boxed{\text{after}}$ it cools. This is because the sphere shrinks while cooling, displacing less water, and hence decreasing the buoyant force acting on it.

b. The submerged weight of the sphere before cooling is

$$W_0 = \rho g V_0 - \rho_W g V_0$$

The submerged weight after cooling is

$$W = \rho g V_0 - \rho_W g V$$

The weight difference is

$$\Delta W = W - W_0 = -\rho_W g \Delta V$$

where $\Delta V = V - V_0$. The volume change is $\Delta V = \beta V_0 \Delta T$. Now $\Delta W = -\rho_W g \beta V_0 \Delta T$. The original volume of the sphere is

$$V_0 = (4/3)\pi r^3 = (4/3)(\pi)(0.50\text{ m})^3 = 0.52\text{ m}^3$$

Now

$$\Delta W = -(1.0 \times 10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(69 \times 10^{-6}\text{ C}^{\circ -1})(0.52\text{ m}^3)(-5.0 \times 10^1\text{ C}^\circ) = \boxed{18\text{ N}}$$

37. SSM WWW ***REASONING*** The cavity that contains the liquid in either Pyrex thermometer expands according to Equation 12.3, $\Delta V_g = \beta_g V_0 \Delta T$. On the other hand, the volume of mercury expands by an amount $\Delta V_m = \beta_m V_0 \Delta T$, while the volume of alcohol expands by an amount $\Delta V_a = \beta_a V_0 \Delta T$. Therefore, the net change in volume for the mercury thermometer is

$$\Delta V_m - \Delta V_g = (\beta_m - \beta_g) V_0 \Delta T$$

while the net change in volume for the alcohol thermometer is

$$\Delta V_a - \Delta V_g = (\beta_a - \beta_g) V_0 \Delta T$$

In each case, this volume change is related to a movement of the liquid into a cylindrical region of the thermometer with volume $\pi r^2 h$, where r is the radius of the region and h is the height of the region. For the mercury thermometer, therefore,

$$h_m = \frac{(\beta_m - \beta_g) V_0 \Delta T}{\pi r^2}$$

Similarly, for the alcohol thermometer

$$h_a = \frac{(\beta_a - \beta_g) V_0 \Delta T}{\pi r^2}$$

These two expressions can be combined to give the ratio of the heights, h_a/h_m.

SOLUTION Division of the two expressions for the heights of the liquids in the thermometers gives

$$\frac{h_a}{h_m} = \frac{\beta_a - \beta_g}{\beta_m - \beta_g} = \frac{1200 \times 10^{-6}\ (\text{C}°)^{-1} - 9.9 \times 10^{-6}\ (\text{C}°)^{-1}}{182 \times 10^{-6}\ (\text{C}°)^{-1} - 9.9 \times 10^{-6}\ (\text{C}°)^{-1}} = 6.9$$

Therefore, the degree marks are $\boxed{\text{6.9 times further apart}}$ on the alcohol thermometer than on the mercury thermometer.

38. ***REASONING AND SOLUTION*** When the temperature is 0.0 °C, $P = \rho_0 g h_0$, and when the temperature is 38.0 °C, $P = \rho g h$. Equating and solving for h gives $h = (\rho_0/\rho)h_0$. Now $\rho_0/\rho = V/V_0$, since the mass of mercury in the tube remains constant. Then $h = (V/V_0)h_0$. Now

$$\Delta V = V - V_0 = \beta V_0 \Delta T \quad \text{or} \quad V/V_0 = 1 + \beta\Delta T$$

Therefore,

$$h = (1 + \beta\Delta T)h_0 = [1 + (182 \times 10^{-6}\ \text{C}^{\circ -1})(38.0\ °\text{C} - 0.0\ °\text{C})](0.760\ \text{m}) = \boxed{0.765\ \text{m}}$$

39. ***REASONING AND SOLUTION*** The copper vessel and the water exchange an equal amount of heat with the stone, i.e., $Q_{Cu} + Q_w = Q_{st}$ which becomes

$$(cm\Delta T)_{Cu} + (cm\Delta T)_w = (cm\Delta T)_{st}$$

Solving for c of the stone,

$$c = \frac{[387\ \text{J/(kg}\cdot\text{C}^\circ)](0.15\ \text{kg}) + [4186\ \text{J/(kg}\cdot\text{C}^\circ)](0.080\ \text{kg})}{(0.030\ \text{kg})(95.0\ ^\circ\text{C} - 28.5\ ^\circ\text{C})}(3.5\ \text{C}^\circ) = \boxed{690\ \text{J/(kg}\cdot\text{C}^\circ)}$$

40. ***REASONING AND SOLUTION*** The heat released by the blood is given by $Q = cm\Delta T$. Then

$$\Delta T = \frac{Q}{cm} = \frac{2000\ \text{J}}{[4186\ \text{J/(kg}\cdot\text{C}^\circ)](0.6\ \text{kg})} = 0.8\ \text{C}^\circ$$

Therefore,

$$T_f = T_i - \Delta T = \boxed{36.2\ ^\circ\text{C}}$$

41. SSM ***REASONING*** Let the system be comprised only of the metal forging and the oil. Then, according to the principle of energy conservation, the heat lost by the forging equals the heat gained by the oil, or $Q_{\text{metal}} = Q_{\text{oil}}$. According to Equation 12.4, the heat lost by the forging is $Q_{\text{metal}} = c_{\text{metal}} m_{\text{metal}} (T_{0\text{metal}} - T_{\text{eq}})$, where T_{eq} is the final temperature of the system at thermal equilibrium. Similarly, the heat gained by the oil is given by $Q_{\text{oil}} = c_{\text{oil}} m_{\text{oil}} (T_{\text{eq}} - T_{0\text{oil}})$.

SOLUTION

$$Q_{\text{metal}} = Q_{\text{oil}}$$

$$c_{\text{metal}} m_{\text{metal}} (T_{0\text{metal}} - T_{\text{eq}}) = c_{\text{oil}} m_{\text{oil}} (T_{\text{eq}} - T_{0\text{oil}})$$

Solving for $T_{0\text{metal}}$, we have

$$T_{0\text{metal}} = \frac{c_{\text{oil}} m_{\text{oil}} (T_{\text{eq}} - T_{0\text{oil}})}{c_{\text{metal}} m_{\text{metal}}} + T_{\text{eq}}$$

or

$$T_{0\text{metal}} = \frac{[2700\ \text{J/(kg}\cdot\text{C°)}](710\ \text{kg})(47\ °\text{C} - 32\ °\text{C})}{[430\ \text{J/(kg}\cdot\text{C°)}](75\ \text{kg})} + 47\ °\text{C} = \boxed{940\ °\text{C}}$$

42. ***REASONING*** Since there is no heat lost or gained by the system, the heat lost by the water in cooling down must be equal to the heat gained by the thermometer in warming up. The heat Q lost or gained by a substance is given by Equation 12.4 as $Q = cm\Delta T$, where c is the specific heat capacity, m is the mass, and ΔT is the change in temperature. Thus, we have that

$$\underbrace{c_{H_2O}\, m_{H_2O}\, \Delta T_{H_2O}}_{\text{Heat lost by water}} = \underbrace{c_{\text{therm}}\, m_{\text{therm}}\, \Delta T_{\text{therm}}}_{\text{Heat gained by thermometer}}$$

We can use this equation to find the temperature of the water before the insertion of the thermometer.

SOLUTION Solving the equation above for ΔT_{H_2O}, and using the value of c_{H_2O} from Table 12.2, we have

$$\Delta T_{H_2O} = \frac{c_{\text{therm}}\, m_{\text{therm}}\, \Delta T_{\text{therm}}}{c_{H_2O}\, m_{H_2O}}$$

$$= \frac{\left[815\ \text{J/}(\text{kg}\cdot\text{C}^\circ)\right](31.0\ \text{g})(41.5\ ^\circ\text{C} - 12.0\ ^\circ\text{C})}{\left[4186\ \text{J/}(\text{kg}\cdot\text{C}^\circ)\right](119\ \text{g})} = 1.50\ \text{C}^\circ$$

The temperature of the water before the insertion of the thermometer was

$$T = 41.5\ ^\circ\text{C} + 1.50\ \text{C}^\circ = \boxed{43.0\ ^\circ\text{C}}$$

43. ***REASONING AND SOLUTION*** From Equation 12.4, $Q = cm\Delta T$, where $\Delta T = T - T_0$. Making this substitution and solving for T gives

$$T = \frac{Q + cmT_0}{cm} = \frac{Q}{cm} + T_0 = \frac{(4.2 \times 10^5 \text{ J/h})(0.50 \text{ h})}{[4186 \text{ J/(kg} \cdot \text{C°)}](1.0 \times 10^3 \text{ kg})} + 27.00\,°\text{C} = \boxed{27.05\,°\text{C}}$$

44. ***REASONING AND SOLUTION*** We require that the heat gained by the cold water equals the heat lost by the hot water, i.e.,

$$Q_{cw} = Q_{hw}$$

Therefore,

$$c_{cw}m_{cw}\Delta T_{cw} = c_{hw}m_{hw}\Delta T_{hw}$$

Since the specific heat capacity is that of water in each case, $c_{cw} = c_{hw}$, then

$$m_{cw}(36.0\ °C - 13.0\ °C) = m_{hw}(49.0\ °C - 36.0\ °C)$$

Then

$$m_{cw} = (0.57)m_{hw}$$

We also know that $m_{cw} + m_{hw} = 191$ kg. Substituting for m_{cw}, and solving for m_{hw}, we have

$$m_{hw} = \frac{191\ \text{kg}}{1.57} = \boxed{121\ \text{kg}}$$

45. SSM ***REASONING*** According to Equation 12.4, the heat required to warm the pool can be calculated from $Q = cm\Delta T$. In order to use Equation 12.4, we must first determine the mass of the water in the pool. Equation 11.1 indicates that the mass can be calculated from $m = \rho V$, where ρ is the density of water and V is the volume of water in the pool.

SOLUTION Combining these two expressions, we have $Q = c\rho V\Delta T$, or

$$Q = [4186\ \text{J/(kg}\cdot\text{C°)}](1.00\times10^{3}\ \text{kg/m}^{3})(12.0\ \text{m}\times 9.00\ \text{m}\times 1.5\ \text{m})\ (27\ °\text{C} - 15\ °\text{C}) = 8.14\times10^{9}\ \text{J}$$

Using the fact that $1\ \text{kWh} = 3.6\times10^{6}\ \text{J}$, the cost of using electrical energy to heat the water in the pool at a cost of \$0.10 per kWh is

$$(8.14\times10^{9}\ \text{J})\left(\frac{\$0.10}{3.6\times10^{6}\ \text{J}}\right) = \boxed{\$230.}$$

46. ***REASONING*** The heat Q that must be added to the water is given by Equation 12.4 as $Q = cm\Delta T$, where c is the specific heat capacity, m is the mass, and ΔT is the change in temperature. The mass of the water is related to its density ρ and volume V by $m = \rho V$ (Equation 11.1). Thus, the heat that must be added can be written as

$$Q = cm\Delta T = c(\rho V)\Delta T$$

Solving this equation for the volume V of water, and converting cubic meters to barrels, we have

$$V = \frac{Q}{c\rho\Delta T}$$

$$= \left(\frac{2.4\times10^{8}\ \text{J}}{\left[4186\ \text{J/(kg}\cdot\text{C}^\circ)\right]\left(1.0\times10^{3}\ \text{kg/m}^{3}\right)\left(17.0\ \text{C}^\circ\right)}\right)\left(\frac{1\ \text{barrel}}{0.16\ \text{m}^{3}}\right) = \boxed{21\ \text{barrels}}$$

47. ***REASONING AND SOLUTION*** We wish to convert 2.0% of the heat Q into gravitational potential energy, i.e., (0.020)Q = mgh. Thus,

$$mg = \frac{(0.020)Q}{h} = \frac{(0.020)(110\text{ Calories})\left(\frac{4186\text{ J}}{1\text{ Calorie}}\right)}{2.1\text{ m}} = \boxed{4.4 \times 10^3\text{ N}}$$

48. ***REASONING*** The change ΔV in volume of the water in the swimming pool is given by $\Delta V = \beta V_0 \Delta T$, Equation 12.3, where β is the coefficient of volume expansion for water, V_0 is the original volume of the water, and ΔT is the change in its temperature. The change in temperature depends on the heat Q added to the water by $\Delta T = Q/cm$, Equation 12.4, where c is the specific heat capacity and m is the mass. Substituting this expression for ΔT into the equation for ΔT gives

$$\Delta V = \beta V_0 \left(\frac{Q}{cm} \right) = \frac{\beta Q}{c\left(\frac{m}{V_0} \right)}$$

But m/V_0 is equal to the density ρ of water (Equation 11.1), so the change in volume of the water is

$$\Delta V = \frac{\beta Q}{c\rho}$$

$$= \frac{\left[2.07 \times 10^{-4}\ (\mathrm{C}^\circ)^{-1} \right] \left(2.00 \times 10^{9}\ \mathrm{J} \right)}{\left[4186\ \mathrm{J}/(\mathrm{kg} \cdot \mathrm{C}^\circ) \right] \left(1.00 \times 10^{3}\ \mathrm{kg/m^3} \right)} = \boxed{9.89 \times 10^{-2}\ \mathrm{m^3}}$$

49. SSM ***REASONING AND SOLUTION*** According to Equation 12.4, the total heat per kilogram required to raise the temperature of the water is

$$\frac{Q}{m} = c\Delta T = [4186\ \text{J/(kg}\cdot\text{C}^\circ)]\ (32.0\,^\circ\text{C}) = 1.34\times10^5\ \text{J/kg}$$

The mass flow rate, $\Delta m/\Delta t$, is given by Equations 11.7 and 11.10 as $\Delta m/\Delta t = \rho A v = \rho Q_\text{V}$, where ρ and Q_V are the density and volume flow rate, respectively. We have, therefore,

$$\frac{\Delta m}{\Delta t} = (1.000\times10^3\ \text{kg/m}^3)(5.0\times10^{-6}\ \text{m}^3/\text{s}) = 5.0\times10^{-3}\ \text{kg/s}$$

Therefore, the minimum power rating of the heater must be

$$(1.34\times10^5\ \text{J/kg})(5.0\times10^{-3}\ \text{kg/s}) = 6.7\times10^2\ \text{J/s} = \boxed{6.7\times10^2\ \text{W}}$$

50. ***REASONING AND SOLUTION*** The heat lost by the steel rod is $Q = cm\Delta T = c\rho V_0 \Delta T$. The rod contracts according to the equation: $\Delta L = \alpha L_0 \Delta T$. We also know that $F = AY(\Delta L/ L_0)$, Equation 10.17, so that $F = AY\alpha\Delta T$.

Combining expressions yields,

$$F = \frac{\alpha QY}{c\rho L_0} = \frac{(12 \times 10^{-6}\ C^{\circ -1})(3300\ J)(2.0 \times 10^{11}\ Pa)}{[452\ J/(kg \cdot C^{\circ})(7860\ kg/m^3)(2.0\ m)} = \boxed{1.1 \times 10^3\ N}$$

51. SSM ***REASONING*** Heat Q_1 must be added to raise the temperature of the aluminum in its solid phase from 130 °C to its melting point at 660 °C. According to Equation 12.4, $Q_1 = cm\Delta T$. Once the solid aluminum is at its melting point, additional heat Q_2 must be supplied to change its phase from solid to liquid. The additional heat required to melt or liquefy the aluminum is $Q_2 = mL_f$, where L_f is the latent heat of fusion of aluminum. Therefore, the total amount of heat which must be added to the aluminum in its solid phase to liquefy it is

$$Q_{\text{total}} = Q_1 + Q_2 = m(c\Delta T + L_f)$$

SOLUTION Substituting values, we obtain

$$Q_{\text{total}} = (0.45\text{ kg})\left([9.00\times10^2\text{ J/(kg}\cdot\text{C°)}](660\text{ °C} - 130\text{ °C}) + 4.0\times10^5\text{ J/kg}\right) = \boxed{3.9\times10^5\text{ J}}$$

52. ***REASONING AND SOLUTION*** The block of ice must undergo a change in temperature, followed by a change in phase, and another change in temperature,

$$Q = Q_{ice} + Q_{ice\text{-}to\text{-}water} + Q_{water}$$

$$Q = (cm\Delta T)_{ice} + mL_f + (cm\Delta T)_{water}$$

$$4.11 \times 10^6 \text{ J} = [2.00 \times 10^3 \text{ J/(kg·C°)}](10.0 \text{ kg})(10.0 \text{ C°}) + (10.0 \text{ kg})(33.5 \times 10^4 \text{ J/kg})$$

$$+ [4186 \text{ J/(kg·C°)}](10.0 \text{ kg})T_f$$

Solving for T_f we obtain, $\boxed{T_f = 13 \text{ °C}}$

53. ***REASONING AND SOLUTION*** We want the same amount of heat removed from the water as from the ethyl alcohol, i.e.,

$$Q_{water} = Q_{alcohol}$$

or

$$(mL_f)_{water} = (mL_f)_{alcohol}$$

Then

$$m_{alcohol} = m_{water}\frac{(L_f)_{water}}{(L_f)_{alcohol}} = (3.0\text{ kg})\frac{33.5\times10^4\text{ J/kg}}{10.8\times10^4\text{ J/kg}} = \boxed{9.3\text{ kg}}$$

54. ***REASONING*** As the body perspires, heat Q must be added to change the water from the liquid to the gaseous state. The amount of heat depends on the mass m of the water and the latent heat of vaporization L_v, $Q = mL_v$ (Equation 12.5).

SOLUTION The mass of water lost to perspiration is

$$m = \frac{Q}{L_v} = \frac{(240 \text{ Calories})\left(\dfrac{4186 \text{ J}}{1 \text{ Calorie}}\right)}{2.42 \times 10^6 \text{ J/kg}} = \boxed{0.42 \text{ kg}}$$

55. SSM ***REASONING*** From the principle of conservation of energy, the heat lost by the coin must be equal to the heat gained by the liquid nitrogen. The heat lost by the coin is, from Equation 12.4, $Q = c_{\text{coin}} m_{\text{coin}} \Delta T_{\text{coin}}$. If the liquid nitrogen is at its boiling point, –195.8 °C, then the heat gained by the nitrogen will cause it to change phase from a liquid to a vapor. The heat gained by the liquid nitrogen is $Q = m_{\text{nitrogen}} L_{\text{v}}$, where m_{nitrogen} is the mass of liquid nitrogen that vaporizes, and L_{v} is the latent heat of vaporization for nitrogen.

SOLUTION

$$Q_{\substack{\text{lost by}\\ \text{coin}}} = Q_{\substack{\text{gained by}\\ \text{nitrogen}}}$$

$$c_{\text{coin}} m_{\text{coin}} \Delta T_{\text{coin}} = m_{\text{nitrogen}} L_{\text{v}}$$

Solving for the mass of the nitrogen that vaporizes

$$m_{\text{nitrogen}} = \frac{c_{\text{coin}} m_{\text{coin}} \Delta T_{\text{coin}}}{L_{\text{v}}}$$

$$= \frac{[235\ \text{J/(kg}\cdot\text{C°)}](1.5\times10^{-2}\ \text{kg})[25\ \text{°C} - (-195.8\ \text{°C})]}{2.00\times10^{5}\ \text{J/kg}} = \boxed{3.9\times10^{-3}\ \text{kg}}$$

56. ***REASONING AND SOLUTION*** The heat required to evaporate the water is $Q = mL_v$, and to lower the temperature of the jogger we have $Q = m_j c\Delta T$. Equating these two expressions and solving for the mass of the water, m, we have

$$m = m_j c\Delta T/L_v$$

$$m = (75\text{ kg})[3500\text{ J/(kg}\cdot\text{C}^\circ)](1.5\text{ C}^\circ)/(2.42 \times 10^6\text{ J/kg}) = \boxed{0.16\text{ kg}}$$

57. ***REASONING AND SOLUTION*** The heat required is $Q = mL_f + cm\Delta T$, where $m = \rho V$. Thus,

$$Q = \rho V L_f + c\rho V \Delta T$$

$$Q = (917\ \text{kg/m}^3)(4.50 \times 10^{-4}\ \text{m})(1.25\ \text{m}^2)\{33.5 \times 10^4\ \text{J/kg}$$

$$+ [2.00 \times 10^3\ \text{J/(kg}\cdot\text{C°)}](12.0\ \text{C°})\} = \boxed{1.85 \times 10^5\ \text{J}}$$

58. ***REASONING*** The average power $\overline{P}$ entering the cooler is equal to the energy Q entering the cooler divided by the time t during which it enters, $\overline{P} = Q/t$ (Equation 6.10b). This energy turns ice at 0 °C into liquid water at 0 °C, so the amount of energy required is $Q = mL_f$, Equation 12.5, where m is the mass and L_f is the latent heat of fusion for water. Thus, the average power entering the cooler can be expressed as

$$\overline{P} = \frac{Q}{t} = \frac{m L_f}{t}$$

SOLUTION Using the value of the latent heat of fusion for water from Table 12.3, the average power is

$$\overline{P} = \frac{m L_f}{t} = \frac{(2.4\text{ kg})\left(3.35 \times 10^5\text{ J/kg}\right)}{(8.0\text{ h})\left(\dfrac{3600\text{ s}}{1\text{ h}}\right)} = \boxed{28\text{ W}}$$

59. ***REASONING AND SOLUTION***

a. $Q = mL_V = \rho V L_V$

$Q = (1.000 \times 10^3 \text{ kg/m}^3)(2.59 \times 10^6 \text{ m}^2)(0.0254 \text{ m})(22.6 \times 10^5 \text{ J/kg}) = \boxed{1.49 \times 10^{14} \text{ J}}$

b. Number of homes $= (1.49 \times 10^{14} \text{ J})\left(\dfrac{1 \text{ home}}{1.50 \times 10^{11} \text{ J}}\right) = \boxed{993 \text{ homes}}$

60. ***REASONING AND SOLUTION*** The energy released by the water is

$$Q = (cm\Delta T)_{water} + mL_f$$

$$Q = [4186\ \text{J/(kg·C°)}](840\ \text{kg})(10.0\ \text{C°}) + (840\ \text{kg})(3.35 \times 10^5\ \text{J/kg}) = 3.2 \times 10^8\ \text{J}$$

The 2.0-kW heater provides 2.0×10^3 J/s, so that the time is

$$\left(\frac{3.2 \times 10^8\ \text{J}}{2.0 \times 10^3\ \text{J/s}}\right)\left(\frac{1\ \text{hr}}{3600\ \text{s}}\right) = \boxed{44\ \text{hr}}$$

61. SSM ***REASONING*** According to the statement of the problem, the initial state of the system is comprised of the ice and the steam. From the principle of energy conservation, the heat lost by the steam equals the heat gained by the ice, or $Q_{\text{steam}} = Q_{\text{ice}}$. When the ice and the steam are brought together, the steam immediately begins losing heat to the ice. An amount $Q_{1(\text{lost})}$ is released as the temperature of the steam drops from 130 °C to 100 °C, the boiling point of water. Then an amount of heat $Q_{2(\text{lost})}$ is released as the steam condenses into liquid water at 100 °C. The remainder of the heat lost by the "steam" $Q_{3(\text{lost})}$ is the heat that is released as the water at 100 °C cools to the equilibrium temperature of T_{eq} = 50.0 °C. According to Equation 12.4, $Q_{1(\text{lost})}$ and $Q_{3(\text{lost})}$ are given by

$$Q_{1(\text{lost})} = c_{\text{steam}} m_{\text{steam}} (T_{\text{steam}} - 100.0\ ^\circ\text{C}) \quad \text{and} \quad Q_{3(\text{lost})} = c_{\text{water}} m_{\text{steam}} (100.0\ ^\circ\text{C} - T_{\text{eq}})$$

$Q_{2(\text{lost})}$ is given by $Q_{2(\text{lost})} = m_{\text{steam}} L_{\text{v}}$, where L_{v} is the latent heat of vaporization of water. The total heat lost by the steam has three effects on the ice. First, a portion of this heat $Q_{1(\text{gained})}$ is used to raise the temperature of the ice to its melting point at 0.00 °C. Then, an amount of heat $Q_{2(\text{gained})}$ is used to melt the ice completely (we know this because the problem states that after thermal equilibrium is reached the liquid phase is present at 50.0 °C). The remainder of the heat $Q_{3(\text{gained})}$ gained by the "ice" is used to raise the temperature of the resulting liquid at 0.0 °C to the final equilibrium temperature. According to Equation 12.4, $Q_{1(\text{gained})}$ and $Q_{3(\text{gained})}$ are given by

$$Q_{1(\text{gained})} = c_{\text{ice}} m_{\text{ice}} (0.00\ ^\circ\text{C} - T_{\text{ice}}) \quad \text{and} \quad Q_{3(\text{gained})} = c_{\text{water}} m_{\text{ice}} (T_{\text{eq}} - 0.00\ ^\circ\text{C})$$

$Q_{2(\text{gained})}$ is given by $Q_{2(\text{gained})} = m_{\text{ice}} L_{\text{f}}$, where L_{f} is the latent heat of fusion of ice.

SOLUTION

$$Q_{\text{steam}} = Q_{\text{ice}}$$

$$Q_{1(\text{lost})} + Q_{2(\text{lost})} + Q_{3(\text{lost})} = Q_{1(\text{gained})} + Q_{2(\text{gained})} + Q_{3(\text{gained})}$$

or

$$c_{\text{steam}} m_{\text{steam}} (T_{\text{steam}} - 100.0\ ^\circ\text{C}) + m_{\text{steam}} L_{\text{v}} + c_{\text{water}} m_{\text{steam}} (100.0\ ^\circ\text{C} - T_{\text{eq}})$$

$$= c_{\text{ice}} m_{\text{ice}} (0.00\ ^\circ\text{C} - T_{\text{ice}}) + m_{\text{ice}} L_{\text{f}} + c_{\text{water}} m_{\text{ice}} (T_{\text{eq}} - 0.00\ ^\circ\text{C})$$

Solving for the ratio of the masses gives

$$\frac{m_{\text{steam}}}{m_{\text{ice}}} = \frac{c_{\text{ice}} (0.00\ ^\circ\text{C} - T_{\text{ice}}) + L_{\text{f}} + c_{\text{water}} (T_{\text{eq}} - 0.00\ ^\circ\text{C})}{c_{\text{steam}} (T_{\text{steam}} - 100.0\ ^\circ\text{C}) + L_{\text{v}} + c_{\text{water}} (100.0\ ^\circ\text{C} - T_{\text{eq}})}$$

$$= \frac{\left[2.00 \times 10^3\ \text{J/(kg}\cdot\text{C}^\circ)\right] \left[0.0\ ^\circ\text{C} - (-10.0^\circ\text{C})\right] + 33.5 \times 10^4\ \text{J/kg} + \left[4186\ \text{J/(kg}\cdot\text{C}^\circ)\right](50.0\ ^\circ\text{C} - 0.0\ ^\circ\text{C})}{\left[2020\ \text{J/(kg}\cdot\text{C}^\circ)\right](130\ ^\circ\text{C} - 100.0\ ^\circ\text{C}) + 22.6 \times 10^5\ \text{J/kg} + \left[4186\ \text{J/(kg}\cdot\text{C}^\circ)\right](100.0\ ^\circ\text{C} - 50.0\ ^\circ\text{C})}$$

or

$$\frac{m_{\text{steam}}}{m_{\text{ice}}} = \boxed{0.223}$$

62. ***REASONING*** The mass $m_{\text{remaining}}$ of the liquid water that remains at 100 °C is equal to the original mass m minus the mass $m_{\text{vaporized}}$ of the liquid water that has been vaporized. The heat Q required to vaporize this mass of liquid is given by Equation 12.5 as $Q = m\,L_{\text{v}}$, where L_{v} is the latent heat of vaporization for water. Thus, we have

$$m_{\text{remaining}} = m - m_{\text{vaporized}} = m - \frac{Q}{L_{\text{v}}}$$

The heat required to vaporize the water comes from the heat that is removed from the water at 0 °C when it changes phase from the liquid state to ice. This heat is also given by Equation 12.5 as $Q = mL_{\text{f}}$, where L_{f} is the latent heat of fusion for water. Thus, the remaining mass of liquid water can be written as

$$m_{\text{remaining}} = m - \frac{Q}{L_{\text{v}}} = m - \frac{mL_{\text{f}}}{L_{\text{v}}} = m\left(1 - \frac{L_{\text{f}}}{L_{\text{v}}}\right)$$

SOLUTION Using the values of L_{f} and L_{v} from Table 12.3, the mass of the liquid water at 100 °C that remains is

$$m_{\text{remaining}} = m\left(1 - \frac{L_{\text{f}}}{L_{\text{v}}}\right) = (2.00\text{ g})\left(1 - \frac{3.35\times10^{5}\text{ J/kg}}{2.26\times10^{6}\text{ J/kg}}\right) = \boxed{1.70\text{ g}}$$

63. ***REASONING AND SOLUTION*** If we assume that all the heat generated by kinetic friction goes into the block of ice, then from energy conservation,

$$Q_{\text{friction}} = Q_{\substack{\text{gained}\\ \text{by ice}}} \tag{1}$$

Also from energy conservation, $Q_{\text{friction}} = -W_{\text{friction}}$. From the work-energy theorem, the work done by friction is given by

$$W_{\text{friction}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2$$

Since the ice is at 0 °C, any heat that it absorbs will be used to change the phase (i.e., melt) of the ice. Thus, Equation (1) becomes

$$-\left[\tfrac{1}{2}m_{\text{block}}v^2 - \tfrac{1}{2}m_{\text{block}}v_0^2\right] = m_{\text{melted}}L_{\text{f}} \tag{2}$$

Solving Equation (2) for the mass of the ice that melts gives

$$m_{\text{melted}} = \frac{\frac{1}{2}m_{\text{block}}(v_0^2 - v^2)}{L_{\text{f}}} = \frac{\frac{1}{2}(35\text{ kg})[(6.5\text{ m/s})^2 - (4.8\text{ m/s})^2]}{(33.5\times10^4\text{ J/kg})} = \boxed{1.0\times10^{-3}\text{ kg}}$$

64. ***REASONING AND SOLUTION*** In order to convert steam into ice, the steam must first be liquefied to water at 100.0 °C, the temperature of the water must be lowered to 0.0 °C, and then the water must be solidified at 0.0 °C. Thus, the amount of heat that must be removed is

$$Q = mL_v + cm\Delta T + mL_f \tag{1}$$

where L_v and L_f are the latent heat of vaporization of water and the latent heat of fusion of water, respectively, and ΔT = (100.0 °C – 0.0 °C). If this energy where then used to accelerate the ice, we have from energy conservation $\Delta(KE) = Q$, or

$$\frac{1}{2}m(v^2 - v_0^2) = m(L_v + c\Delta T + L_f) \tag{2}$$

Solving Equation (2) for v, and noting that $v_0 = 0$ m/s, we have

$$v = \sqrt{2(L_v + c\Delta T + L_f)}$$

$$v = \sqrt{2\{(22.6\times10^5\ \text{J/kg}) + [4186\ \text{J/(kg}\cdot\text{C°)}](100\ \text{C°}) + (33.5\times10^4\ \text{J/kg})\}} = \boxed{2.46\times10^3\ \text{m/s}}$$

65. SSM ***REASONING*** In order to melt, the bullet must first heat up to 327.3 °C (its melting point) and then undergo a phase change. According to Equation 12.4, the amount of heat necessary to raise the temperature of the bullet to 327.3 °C is $Q = cm(327.3\ ^\circ\text{C} - 30.0\ ^\circ\text{C})$, where m is the mass of the bullet. The amount of heat required to melt the bullet is given by $Q_{\text{melt}} = mL_{\text{f}}$, where L_{f} is the latent heat of fusion of lead.

The lead bullet melts completely when it comes to a sudden halt; all of the kinetic energy of the bullet is converted into heat; therefore,

$$\text{KE} = Q + Q_{\text{melt}}$$

$$\frac{1}{2}mv^2 = cm(327.3\ ^\circ\text{C} - 30.0\ ^\circ\text{C}) + mL_{\text{f}}$$

This expression can be solved for v, the minimum speed of the bullet for such an event to occur.

SOLUTION Solving for v, we find that the minimum speed of the lead bullet is

$$v = \sqrt{2L_{\text{f}} + 2c\,(327.3\ ^\circ\text{C} - 30.0\ ^\circ\text{C})}$$

$$v = \sqrt{2(2.32 \times 10^4\ \text{J/kg}) + 2[128\ \text{J/(kg}\cdot\text{C}^\circ)](327.3\ ^\circ\text{C} - 30.0\ ^\circ\text{C})} = \boxed{3.50 \times 10^2\ \text{m/s}}$$

66. ***REASONING AND SOLUTION*** The steel band must be heated so that it can expand to fit the wheel. The diameter of the band must increase in length by an amount $\Delta L = 6.00 \times 10^{-4}$ m. Also, $\Delta L = \alpha L_0 \Delta T$ so that

$$\Delta T = \Delta L/\alpha\, L_0 = (6.00 \times 10^{-4}\ \text{m})/[(12 \times 10^{-6}/\text{C}°)(1.00\ \text{m})] = 5.0 \times 10^{1}\ \text{C}°$$

The heat to expand the steel band comes from the heat released from the steam as it changes to water. Therefore $Q_{stm} = Q_{sb}$, or

$$m_{stm}L_v + (cm_{stm}\Delta T)_{water} = (cm_{sb}\Delta T)$$

Now

$$m_{stm}(22.6 \times 10^{5}\ \text{J/kg}) + [4186\ \text{J/(kg}\cdot\text{C}°)]m_{stm}(100.0\ °\text{C} - 70.0\ °\text{C})$$

$$= [452\ \text{J/(kg}\cdot\text{C}°)](25.0\ \text{kg})(50.0\ \text{C}°)$$

Solving for the mass of the steam we obtain $\boxed{m_{stm} = 0.237\ \text{kg}}$.

67. SSM ***REASONING AND SOLUTION*** From inspection of the graph that accompanies this problem, a pressure of 3.5×10^6 Pa corresponds to a temperature of 0 °C. Therefore, liquid carbon dioxide exists in equilibrium with its vapor phase at $\boxed{0\ ^\circ\text{C}}$ when the pressure is 3.5×10^6 Pa.

68. ***REASONING AND SOLUTION*** Boiling occurs when the vapor pressure of the substance at a particular temperature equals the atmospheric pressure at that temperature. According to the vapor pressure curve for carbon dioxide that accompanies problem 67, a temperature of 30 °C corresponds to a pressure of 7.4×10^6 Pa. Therefore, we can conclude that a pressure of $\boxed{7.4 \times 10^6 \text{ Pa}}$ would be required for carbon dioxide to boil at a temperature of 30 °C.

69. ***REASONING AND SOLUTION*** The pressure inside the container is due to the weight on the piston in addition to the pressure of the atmosphere. The 120-kg mass produces a pressure of

$$P = \frac{F}{A} = \frac{mg}{\pi r^2} = \frac{(120\text{ kg})(9.80\text{ m/s}^2)}{\pi(0.061\text{ m})^2} = 1.0 \times 10^5\text{ Pa}$$

If we include atmospheric pressure (1.01×10^5 Pa), the total pressure inside the container is

$$P_{total} = P + P_{atm} = 2.0 \times 10^5\text{ Pa}$$

Examination of the vaporization curve in Figure 12.31 shows that the temperature corresponding to this pressure at equilibrium is $\boxed{T = 120\text{ °C}}$.

70. ***REASONING AND SOLUTION*** The water will boil if the vapor pressure of the water is equal to the ambient pressure. The pressure at a depth h in the water can be determined from Equation 11.4: $P_2 = P_1 + \rho gh$. When h = 10.3 m,

$$P_2 = (1.01 \times 10^5 \text{ Pa}) + [(1.000 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(10.3 \text{ m})] = 2.02 \times 10^5 \text{ Pa}$$

The vapor pressure curve in Figure 12.31 shows that the vapor pressure of water is equal to 2.02×10^5 Pa at a temperature of 123 °C. Thus, the water at that depth has a temperature of $\boxed{T = 123 \text{ °C}}$.

71. ***REASONING AND SOLUTION*** At a temperature of 30.0 °C the equilibrium vapor pressure is 4200 Pa (as seen from the diagram).

$$\text{Percent relative humidity} = \left(\frac{\text{Partial pressure}}{\text{Equilibrium vapor pressure}}\right) \times 100 \qquad (12.6)$$

The partial pressure is equal to

$$\text{Partial pressure} = \frac{(\text{Percent relative humidity})(\text{Equilibrium vapor pressure})}{100}$$

$$= \frac{(56.0\%)(4200\ \text{Pa})}{100} = \boxed{2400\ \text{Pa}}$$

72. ***REASONING AND SOLUTION*** The percent relative humidity at 10 °C is

$$\begin{array}{c}\text{Percent}\\ \text{relative}\\ \text{humidity}\\ \text{at 10 °C}\end{array} = \frac{\begin{array}{c}\text{Partial pressure}\\ \text{of water vapor at 10 °C}\end{array}}{\begin{array}{c}\text{Equilibrium vapor}\\ \text{pressure of water}\\ \text{at 10 °C}\end{array}} \times 100 \qquad (12.6)$$

According to the statement of the problem, the partial pressure of water vapor at 10 °C is equal to 26% of the equilibrium vapor pressure at 30 °C. According to the vapor pressure curve that accompanies problem 71, the equilibrium vapor pressure at 30 °C is 4200 Pa. Therefore, 26% of this value is (0.26)(4200 Pa), so we have

$$\begin{array}{c}\text{Percent}\\ \text{relative}\\ \text{humidity}\\ \text{at 10 °C}\end{array} = \frac{(0.26)(4200\text{ Pa})}{\begin{array}{c}\text{Equilibrium vapor}\\ \text{pressure of water}\\ \text{at 10 °C}\end{array}} \times 100$$

Again using the graph in problem 71, we find that the equilibrium vapor pressure of water at 10 °C is 1300 Pa, so the percent relative humidity at 10 °C is

$$\begin{array}{c}\text{Percent}\\ \text{relative}\\ \text{humidity}\\ \text{at 10 °C}\end{array} = \frac{(0.26)(4200\text{ Pa})}{1300\text{ Pa}} \times 100 = \boxed{84\%}$$

73. SSM ***REASONING AND SOLUTION*** From the vapor pressure curve that accompanies problem 71, it is seen that the partial pressure of water vapor in the atmosphere at 10 °C is about 1400 Pa, and that the equilibrium vapor pressure at 30 °C is about 4200 Pa. The relative humidity is, from Equation 12.6,

$$\begin{matrix}\text{Percent}\\ \text{relative}\\ \text{humidity}\end{matrix} = \left(\frac{1400\ \text{Pa}}{4200\ \text{Pa}}\right) \times 100 = \boxed{33\%}$$

74. ***REASONING AND SOLUTION*** Equation 12.6 defines the relative humidity as

$$\text{Percent relative humidity} = \frac{\text{Partial pressure of water vapor}}{\text{Equilibrium vapor pressure of water at the existing temeprature}} \times 100$$

According to the vapor pressure curve for problem 71, the equilibrium vapor pressure of water at 36°C is 5.8×10^3 Pa. Since the water condenses on the coils when the temperature of the coils is 30 °C, the relative humidity at 30 °C is 100 percent. From Equation 12.6 this implies that the partial pressure of water vapor in the air must be equal to the equilibrium vapor pressure of water at 30 °C. From the vapor pressure curve, this pressure is 4.4×10^3 Pa. Thus, the relative humidity in the room is

$$\text{Percent relative humidity} = \frac{4.4 \times 10^3 \text{ Pa}}{5.8 \times 10^3 \text{ Pa}} \times 100 = \boxed{76\%}$$

75. SSM ***REASONING*** We must first find the equilibrium temperature T_{eq} of the iced tea. Once this is known, we can use the vapor pressure curve that accompanies problem 71 to find the partial pressure of water vapor at that temperature and then estimate the relative humidity using Equation 12.6.

According to the principle of energy conservation, when the ice is mixed with the tea, the heat lost by the tea is gained by the ice, or $Q_{tea} = Q_{ice}$. The heat gained by the ice is used to melt the ice at 0.0 °C; the remainder of the heat is used to bring the water at 0.0 °C up to the final equilibrium temperature T_{eq}.

SOLUTION

$$Q_{tea} = Q_{ice}$$

$$c_{water} m_{tea} (30.0\ °C - T_{eq}) = m_{ice} L_f + c_{water} m_{ice} (T_{eq} - 0.00\ °C)$$

Solving for T_{eq}, we have

$$T_{eq} = \frac{c_{water} m_{tea} (30.0\ °C) - m_{ice} L_f}{c_{water} (m_{tea} + m_{ice})}$$

$$= \frac{[4186\ J/(kg \cdot C°)]\ (0.300\ kg)(30.0\ °C) - (0.0670\ kg)(33.5 \times 10^4\ J/kg)}{[4186\ J/(kg \cdot C°)]\ (0.300\ kg + 0.0670\ kg)} = 9.91\ °C$$

According to the vapor pressure curve that accompanies problem 71, at a temperature of 9.91 °C, the equilibrium vapor pressure is approximately 1250 Pa. At 30 °C, the equilibrium vapor pressure is approximately 4400 Pa. Therefore, according to Equation 12.6, the percent relative humidity is approximately

$$\text{Percent relative humidity} = \left(\frac{1250\ Pa}{4400\ Pa}\right) \times 100 = \boxed{28\%}$$

76. ***REASONING AND SOLUTION*** At 10 °C the equilibrium vapor pressure is 1250 Pa. At 25 °C, the equilibrium vapor pressure is 3200 Pa. To get the smallest possible value for the relative humidity assume that the air at 10 °C is saturated. That is, take the partial pressure to be 1250 Pa. Then we have,

$$\text{Percent relative humidity} = \left(\frac{\text{Partial pressure}}{\text{Equilibrium vapor pressure}}\right) \times 100 = \left(\frac{1250\ \text{Pa}}{3200\ \text{Pa}}\right) \times 100 = \boxed{39\%}$$

77. ***REASONING AND SOLUTION***

$$\Delta L = \alpha L_0 \Delta T = (2.0 \times 10^{-5}\ C^{o-1})(62\ m)(105\ °C - 23\ °C) = \boxed{0.10\ m} \qquad (12.2)$$

78. ***REASONING AND SOLUTION***

a. To change water at 100.0 °C to steam we have

$$Q = mL_f = (2.00\ \text{kg})(22.6 \times 10^5\ \text{J/kg}) = \boxed{4.52 \times 10^6\ \text{J}}$$

b. For water at 0.0 °C, we have $Q = (cm\Delta T)_{water} + mL_f$

$$Q = [4186\ \text{J/(kg}\cdot\text{C°)}](2.00\ \text{kg})(100.0\ \text{C°}) + 4.52 \times 10^6\ \text{J} = \boxed{5.36 \times 10^6\ \text{J}}$$

79. SSM ***REASONING*** From the conservation of energy, the heat lost by the mercury is equal to the heat gained by the water. As the mercury loses heat, its temperature decreases; as the water gains heat, its temperature rises to its boiling point. Any remaining heat gained by the water will then be used to vaporize the water.

According to Equation 12.4, the heat lost by the mercury is $Q_{\text{mercury}} = (cm\Delta T)_{\text{mercury}}$. The heat required to vaporize the water is, from Equation 12.5, $Q_{\text{vap}} = (m_{\text{vap}}L_{\text{v}})_{\text{water}}$. Thus, the total amount of heat gained by the water is $Q_{\text{water}} = (cm\Delta T)_{\text{water}} + (m_{\text{vap}}L_{\text{v}})_{\text{water}}$.

SOLUTION

$$Q_{\substack{\text{lost by}\\ \text{mercury}}} = Q_{\substack{\text{gained by}\\ \text{water}}}$$

$$(cm\Delta T)_{\text{mercury}} = (cm\Delta T)_{\text{water}} + (m_{\text{vap}}L_{\text{v}})_{\text{water}}$$

where $\Delta T_{\text{mercury}} = (205\ ^\circ\text{C} - 100.0\ ^\circ\text{C})$ and $\Delta T_{\text{water}} = (100.0\ ^\circ\text{C} - 80.0\ ^\circ\text{C})$. Solving for the mass of the water that vaporizes gives

$$m_{\text{vap}} = \frac{c_{\text{mercury}}m_{\text{mercury}}\Delta T_{\text{mercury}} - c_{\text{water}}m_{\text{water}}\Delta T_{\text{water}}}{(L_{\text{v}})_{\text{water}}}$$

$$= \frac{[139\ \text{J/(kg}\cdot\text{C}^\circ)](2.10\ \text{kg})(105\ \text{C}^\circ) - [4186\ \text{J/(kg}\cdot\text{C}^\circ)](0.110\ \text{kg})(20.0\ \text{C}^\circ)}{22.6\times10^5\ \text{J/kg}}$$

$$= \boxed{9.49\times10^{-3}\ \text{kg}}$$

80. ***REASONING*** Since there is no heat lost or gained by the system, the heat lost by the coffee in cooling down must be equal to the heat gained by the ice as it melts plus the heat gained by the melted water as it subsequently heats up. The heat Q lost or gained by a substance is given by Equation 12.4 as $Q = cm\Delta T$, where c is the specific heat capacity, m is the mass, and ΔT is the change in temperature. The heat that is required to change ice at 0 °C into liquid water at 0 °C is given by Equation 12.5 as $Q = m_{\text{ice}}L_{\text{f}}$, where m_{ice} is the mass of ice and L_{f} is the latent heat of fusion for water. Thus, we have that

$$\underbrace{m_{\text{coffee}}c_{\text{coffee}}\Delta T_{\text{coffee}}}_{\text{Heat lost by coffee}} = \underbrace{m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{water}}\Delta T_{\text{water}}}_{\text{Heat gained by ice and liquid water}}$$

The mass of the coffee can be expressed in terms of its density as $m_{\text{coffee}} = \rho_{\text{coffee}}V_{\text{coffee}}$ (Equation 11.1). The change in temperature of the coffee is $\Delta T_{\text{coffee}} = 85\ °\text{C} - T$, where T is the final temperature of the coffee. The change in temperature of the water is $\Delta T_{\text{water}} = T - 0\ °\text{C}$. With these substitutions, the equation above becomes

$$\rho_{\text{coffee}}V_{\text{coffee}}c_{\text{coffee}}\left(85\ °\text{C} - T\right) = m_{\text{ice}}L_{\text{f}} + m_{\text{ice}}c_{\text{water}}\left(T - 0\ °\text{C}\right)$$

Solving this equation for the final temperature gives

$$T = \frac{-m_{\text{ice}}L_{\text{f}} + \rho_{\text{coffee}}V_{\text{coffee}}c_{\text{coffee}}\left(85\ °\text{C}\right)}{\rho_{\text{coffee}}V_{\text{coffee}}c_{\text{coffee}} + m_{\text{ice}}c_{\text{water}}}$$

$$= \frac{-(2)\left(11\times10^{-3}\ \text{kg}\right)\left(3.35\times10^{5}\ \text{J/kg}\right) + \left(1.0\times10^{3}\ \text{kg/m}^3\right)\left(150\times10^{-6}\ \text{m}^3\right)\left[4186\ \text{J/}\left(\text{kg}\cdot\text{C}°\right)\right](85\ °\text{C})}{\left(1.0\times10^{3}\ \text{kg/m}^3\right)\left(150\times10^{-6}\ \text{m}^3\right)\left[4186\ \text{J/}\left(\text{kg}\cdot\text{C}°\right)\right] + (2)\left(11\times10^{-3}\ \text{kg}\right)\left[4186\ \text{J/}\left(\text{kg}\cdot\text{C}°\right)\right]}$$

$$= \boxed{64\ °\text{C}}$$

81. ***REASONING AND SOLUTION*** The relation, $\Delta L = \alpha L_0 \Delta T$, written in terms of the diameter d of the rod, is

$$\Delta T = \frac{\Delta d}{\alpha d_0} = \frac{0.0026\text{ cm}}{\left(12 \times 10^{-6}\ \text{C}^{\circ -1}\right)\left(2.0026\text{ cm}\right)} = \boxed{110\,\text{C}^{\circ}} \tag{12.2}$$

82. ***REASONING AND SOLUTION*** $\Delta d = \alpha\, d_0 \Delta T$ gives

$$\alpha = \frac{\Delta d}{d_0\, \Delta T} = \frac{2.3 \times 10^{-5}\ \text{m}}{(1.8 \times 10^{-2}\ \text{m})(75\,\text{C}^\circ)} = \boxed{1.7 \times 10^{-5}\ (\text{C}^\circ)^{-1}} \tag{12.2}$$

83. **SSM** ***REASONING AND SOLUTION*** Both the gasoline and the tank expand as the temperature increases. According to Equation 12.3, the volume expansion of the gasoline is

$$\Delta V_g = \beta_g V_0 \Delta T = \left[950 \times 10^{-6} \ (\text{C}°)^{-1}\right](20.0 \text{ gal})(18 \text{ C}°) = 0.34 \text{ gal}$$

while the volume of the steel tank expands by an amount

$$\Delta V_s = \beta_s V_0 \Delta T = \left[36 \times 10^{-6} \ (\text{C}°)^{-1}\right](20.0 \text{ gal})(18 \text{ C}°) = 0.013 \text{ gal}$$

The amount of gasoline which spills out is

$$\Delta V_g - \Delta V_s = \boxed{0.33 \text{ gal}}$$

84. ***REASONING*** We assume that no heat is lost through the chest to the outside. Then, energy conservation dictates that the heat gained by the soda is equal to the heat lost by the watermelon in reaching the final temperature T_f. Each quantity of heat is given by Equation 12.4, $Q = cm\Delta T$, where we write the change in temperature ΔT as the higher temperature minus the lower temperature.

SOLUTION Starting with the statement of energy conservation, we have

$$\text{Heat gained by soda} = \text{Heat lost by watermelon}$$

$$(cm\Delta T)_{\text{soda}} = (cm\Delta T)_{\text{watermelon}}$$

$$[3800\ \text{J/(kg}\cdot\text{C°)}](12 \times 0.35\ \text{kg})(T_f - 5.0\ °\text{C}) = [4186\ \text{J/(kg}\cdot\text{C°)}](6.5\ \text{kg})(27\ °\text{C} - T_f)$$

Suppressing units for simplicity and algebraically simplifying, we have

$$1.6 \times 10^4\ T_f - 8.0 \times 10^4 = 7.3 \times 10^5 - 2.7 \times 10^4 T_f$$

Solving for T_f we obtain

$$T_f = \frac{8.1 \times 10^5}{4.3 \times 10^4} = \boxed{19\ °\text{C}}$$

85. ***REASONING AND SOLUTION***

$$\text{Percent relative humidity} = \left(\frac{\text{Partial pressure}}{\text{Equilibrium vapor pressure}}\right) \times 100 = 35\%$$

Using the vaporization curve from problem 71, we see that at 27 °C, the equilibrium vapor pressure is 3800 Pa. The partial pressure is, therefore,

$$\text{Partial pressure} = \frac{(\text{Percent relative humidity})(\text{Equilibrium vapor pressure})}{100}$$

$$= \frac{(35\%)(3800\text{ Pa})}{100} = 1300\text{ Pa}$$

From the diagram we see that the dew point (100% relative humidity at equilibrium temperature) is

$$\boxed{\text{Dew Point} = 10\ ^\circ\text{C}}$$

86. ***REASONING AND SOLUTION*** The space invaders temperature of 58 °I is 58 °I – 25 °I or 33 I° above the Space invaders ice point. Additionally, the space invaders degree is

$$\frac{100\ °\text{C} - 0\ °\text{C}}{156\ °\text{I} - 25\ °\text{I}} = 0.76\ \text{C}°/\text{I}°$$

times smaller than the Celsius degree. Now

$$58\ °\text{I} = (33\ °\text{I})(0.76\ \text{C}°/\text{I}°) = \boxed{25\ °\text{C}}$$

87. SSM WWW ***REASONING*** The system is comprised of the unknown material, the glycerin, and the aluminum calorimeter. From the principle of energy conservation, the heat gained by the unknown material is equal to the heat lost by the glycerin and the calorimeter. The heat gained by the unknown material is used to melt the material and then raise its temperature from the initial value of –25.0 °C to the final equilibrium temperature of $T_{eq} = 20.0\ °C$.

SOLUTION

$$Q_{\substack{\text{gained by}\\ \text{unknown}}} = Q_{\substack{\text{lost by}\\ \text{glycerine}}} + Q_{\substack{\text{lost by}\\ \text{calorimeter}}}$$

$$m_u L_f + c_u m_u \Delta T_u = c_{gl} m_{gl} \Delta T_{gl} + c_{al} m_{al} \Delta T_{al}$$

or

$$(0.10\ \text{kg}) L_f + [160\ \text{J/(kg}\cdot\text{C°)}](0.10\ \text{kg})(45.0\ \text{C°}) = [2410\ \text{J/(kg}\cdot\text{C°)}](0.100\ \text{kg})(7.0\ \text{C°})$$
$$+ [9.0\times10^2\ \text{J/(kg}\cdot\text{C°)}](0.150\ \text{kg})(7.0\ \text{C°})$$

Solving for L_f yields,

$$L_f = \boxed{1.9\times10^4\ \text{J/kg}}$$

88. ***REASONING*** The amount Q of heat required to melt an iceberg at 0 °C is equal to mL_f, where m is its mass and L_f is the latent heat of fusion for water (see Table 12.3). The mass is related to the density ρ and the volume V of the ice by Equation 11.1, $m = \rho V$.

SOLUTION

a. The amount of heat required to melt the iceberg is

$$Q = mL_f = \rho V L_f \tag{12.5}$$

$$= \left(917\ \text{kg/m}^3\right) \underbrace{\left(120 \times 10^3\ \text{m}\right)\left(35 \times 10^3\ \text{m}\right)\left(230\ \text{m}\right)}_{\text{Volume}} \left(3.35 \times 10^5\ \text{J/kg}\right)$$

$$= \boxed{3.0 \times 10^{20}\ \text{J}}$$

b. The number of years it would take to melt the iceberg is equal to the energy required to melt it divided by the energy consumed per year by the U.S.

$$\text{Number of years} = \frac{3.0 \times 10^{20}\ \text{J}}{9.3 \times 10^{19}\ \text{J/y}} = \boxed{3.2\ \text{years}}$$

89. ***REASONING AND SOLUTION*** The coffee and mug absorb the heat produced by the heater. So,

$$Q_h = Q_{coffee} + Q_{mug} = [4186\ \text{J/(kg·C°)}](0.30\ \text{kg})(75\ \text{C°}) + [950\ \text{J/(kg·C°)}](0.25\ \text{kg})(75\ \text{C°})$$

$$= 1.12 \times 10^5\ \text{J}$$

The average power $\overline{P}$ is equal to the heat Q_h divided by the time t

$$\overline{P} = \frac{Q_h}{t} = \frac{1.12 \times 10^5\ \text{J}}{120\ \text{s}} = \boxed{930\ \text{W}} \qquad (6.10b)$$

90. ***REASONING AND SOLUTION***

a. The ruler will try to shrink as the temperature is lowered so a $\boxed{\text{tension}}$ is needed to keep it from shrinking.

b. The change in length of the ruler is given by $\Delta L = \alpha L_0 \Delta T$, Equation 12.2. The stress needed to stretch the ruler this amount is given by Stress = $F/A = Y(\Delta L/L_0)$, Equation 10.17. Substituting for ΔL gives

$$\text{Stress} = Y\alpha\Delta T = (2.0 \times 10^{11}\ \text{N/m}^2)(12 \times 10^{-6}\ \text{C}^{\circ -1})[25\ ^\circ\text{C} - (-15\ ^\circ\text{C})] = \boxed{9.6 \times 10^{7}\ \text{N/m}^2}$$

91. **SSM** ***REASONING AND SOLUTION*** As the rock falls through a distance h, its initial potential energy $m_{\text{rock}}gh$ is converted into kinetic energy. This kinetic energy is then converted into heat when the rock is brought to rest in the pail. If we ignore the heat absorbed by the pail, then

$$m_{\text{rock}}gh = c_{\text{rock}}m_{\text{rock}}\Delta T + c_{\text{water}}m_{\text{water}}\Delta T$$

Solving for ΔT yields

$$\Delta T = \frac{m_{\text{rock}}gh}{c_{\text{rock}}m_{\text{rock}} + c_{\text{water}}m_{\text{water}}}$$

Substituting values yields

$$\Delta T = \frac{(0.20\text{ kg})(9.80\text{ m/s}^2)(15\text{ m})}{[1840\text{ J/(kg}\cdot\text{C°)}](0.20\text{ kg}) + [4186\text{ J/(kg}\cdot\text{C°)}](0.35\text{ kg})} = \boxed{0.016\text{ C°}}$$

92. ***REASONING AND SOLUTION***

a. As the wheel heats up, it will expand. Its radius, and therefore, its moment of inertia, will increase. Since no net external torque acts on the wheel, conservation of angular momentum applies where, according to Equation 9.10, the angular momentum is given by: $L = I\omega$, where I is the moment of inertia and ω is the angular velocity. When the moment of inertia increases at the higher temperature, the angular speed must decrease in order for the angular momentum to remain the same.

Thus, $\boxed{\text{the angular speed of the wheel decreases as the wheel heats up}}$.

b. According to the principle of conservation of angular momentum,

$$\underbrace{I_0\omega_0}_{\substack{\text{Initial angular}\\ \text{momentum, } L_0}} = \underbrace{I_f\omega_f}_{\substack{\text{Final angular}\\ \text{momentum, } L_f}}$$

Solving for ω_f, we have, treating the bicycle wheel as a thin-walled hollow hoop ($I = MR^2$, see Table 9.1)

$$\omega_f = \omega_0\left(\frac{I_0}{I_f}\right) = \omega_0\left(\frac{MR_0^2}{MR_f^2}\right) = \omega_0\left(\frac{R_0}{R_f}\right)^2$$

According to Equation 12.2, $\Delta R = \alpha R_0 \Delta T$, and the final radius of the wheel at the higher temperature is,

$$R_f = R_0 + \Delta R = R_0 + \alpha R_0 \Delta T = R_0(1 + \alpha\Delta T)$$

Therefore, the angular speed of the wheel at the higher temperature is

$$\omega_f = \omega_0\left[\frac{R_0}{R_0(1+\alpha\Delta T)}\right]^2 = \omega_0\left(\frac{1}{1+\alpha\Delta T}\right)^2$$

$$= (18.00\text{ rad/s})\left\{\frac{1}{1+[12\times10^{-6}\ (\text{C}°)^{-1}][300.0\ °\text{C} - (-100.0\ °\text{C})]}\right\}^2 = \boxed{17.83\text{ rad/s}}$$

93. ***REASONING AND SOLUTION*** The steel ball and aluminum plate expand when heat is added to the system. Once the correct amount of heat has been added, the two have the same radius and the ball fits into the plate. Upon heating, the change in length is given by $\Delta L = \alpha L_0 \Delta T$, where $\Delta L = (L_f - L_0)$.

We require that the final lengths of the steel and aluminum be equal, i.e., $(L_f)_{st} = (L_f)_{al}$. Substituting for ΔL and equating the final lengths yields

$$(L_f)_{st} = (L_0)_{st}(1 + \alpha_{st}\Delta T) = (L_f)_{al} = (L_0)_{al}(1 + \alpha_{al}\Delta T)$$

Solving for ΔT:

$$\Delta T = (0.0010)/(\alpha_{al} - 1.0010\ \alpha_{st}) = 91\ C^\circ$$

The amount of heat that must be added is therefore,

$$Q = (cm\Delta T)_{st} + (cm\Delta T)_{al}$$

$$Q = \{[452\ J/(kg{\cdot}C^\circ)](1.5\ kg) + [9.00 \times 10^2\ J/(kg{\cdot}C^\circ)](0.85\ kg)\}(91\ C^\circ) = \boxed{1.3 \times 10^5\ J}$$

94. ***REASONING AND SOLUTION*** The figure below (at the left) shows the forces that act on the middle of the aluminum wire for any value of the angle θ. The figure below (at the right) shows the same forces after they have been resolved into x and y components.

Applying Newton's second law to the vertical forces in figure on the right gives $2T(\sin\theta) - W = 0$. Solving for T gives:

$$T = \frac{W}{2(\sin\theta)} \tag{1}$$

The figures below show how the angle is related to the initial length L_0 of the wire and to the final length L after the temperature drops.

If the distance between the supports does not change, then the distance x is the same in both figures. Thus, at the original temperature,

$$x = L_0 (\cos\theta_0) \tag{2}$$

while at the lower temperature

$$x = L (\cos\theta) \tag{3}$$

Equating the right hand sides of Equations (2) and (3) leads to

$$\cos\theta = \frac{L_0 \cos\theta_0}{L} \tag{4}$$

Now, $L = L_0 + \Delta L$, and from Equation 12.2, it follows that

$$\frac{\Delta L}{L_0} = \alpha\Delta T$$

Thus, Equation (4) becomes

$$\cos\theta = \frac{L_0 \cos\theta_0}{L_0 + \Delta L} = \frac{\cos\theta_0}{1 + (\Delta L / L_0)} = \frac{\cos\theta_0}{1 + \alpha\Delta T}$$

From the figure in the text $\theta_0 = 3.00°$. After the temperature of the wire drops by 20.0 C°, the wire makes an angle θ with the horizontal where

$$\theta = \cos^{-1}\left(\frac{\cos\theta_0}{1+\alpha\Delta T}\right) = \cos^{-1}\left(\frac{\cos 3.00°}{1+\left(23\times10^{-6}\ \text{C}°^{-1}\right)\left(-20.0\ \text{C}°\right)}\right) = 2.446°$$

Using this value for θ in Equation (1) gives

$$\text{T} = \frac{\text{W}}{2\sin\theta} = \frac{85.0\ \text{N}}{2\sin 2.446°} = \boxed{996\ \text{N}}$$

1. $\boxed{\textbf{SSM}}$ ***REASONING*** The heat conducted through the iron poker is given by Equation 13.1, $Q = (kA\,\Delta T)t / L$. If we assume that the poker has a circular cross-section, then its cross-sectional area is $A = \pi r^2$. Table 13.1 gives the thermal conductivity of iron as $79\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)$.

SOLUTION The amount of heat conducted from one end of the poker to the other in 5.0 s is, therefore,

$$Q = \frac{(k\,A\,\Delta T)t}{L} = \frac{\left[79\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)\right]\pi\left(5.0\times10^{-3}\ \text{m}\right)^2(502\ °\text{C} - 26\ °\text{C})(5.0\ \text{s})}{1.2\ \text{m}} = \boxed{12\ \text{J}}$$

2. ***REASONING AND SOLUTION*** The rate at which energy is gained through the refrigerator walls is

$$\frac{Q}{t} = \frac{kA\Delta T}{L} = \frac{[0.030\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)](5.3\ \text{m}^2)(2.0\times10^1\ \text{C}^\circ)}{0.075\ \text{m}} = 42\ \text{J/s}$$

Therefore, the amount of heat per second that must be removed from the unit to keep it cool is $\boxed{42\ \text{J/s}}$.

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3. ***REASONING AND SOLUTION*** The heat Q conducted during a time t through one side of the cubical box with walls of thickness L and cross-sectional area A is given by Equation 13.1:

$$Q = \frac{(kA\,\Delta T)\ t}{L}$$

Since the cube has six faces, the total heat conducted through all six faces is $Q_{\text{total}} = 6Q$, or

$$Q_{\text{total}} = \frac{6(kA\Delta T)\ t}{L}$$

Solving for the thermal conductivity, k, we have

$$k = \frac{Q_{\text{total}}\,L}{6\,A\Delta Tt} = \frac{(3.10\times10^{6}\ \text{J})(3.00\times10^{-2}\ \text{m})}{6(0.350\ \text{m})^{2}\,[21.0\ °\text{C}-(-78.5\ °\text{C})](24\ \text{h})}\left(\frac{1.00\ \text{h}}{3600\ \text{s}}\right) = \boxed{1.47\times10^{-2}\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)}$$

4. ***REASONING AND SOLUTION*** Since 1 kWh of energy costs \$ 0.10, we know that $Q = 10.0 \times 10^3$ W·h of energy can be purchased with \$ 1.00. Using Equation 13.1, we find that the time required is

$$t = \frac{QL}{kA\,\Delta T} = \frac{(10.0 \times 10^3\ \text{W}\cdot\text{h})(0.10\ \text{m})}{[1.1\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)](9.0\ \text{m}^2)(20.0\ \text{C}^\circ - 12.8\ \text{C}^\circ)} = \boxed{14\ \text{h}}$$

5. **SSM** ***REASONING*** The heat transferred in a time t is given by Equation 13.1, $Q = (k\,A\,\Delta T)t\,/\,L$. If the same amount of heat per second is conducted through the two plates, then $(Q/t)_{\text{al}} = (Q/t)_{\text{st}}$. Using Equation 13.1, this becomes

$$\frac{k_{\text{al}}\,A\,\Delta T}{L_{\text{al}}} = \frac{k_{\text{st}}\,A\,\Delta T}{L_{\text{st}}}$$

This expression can be solved for L_{st}.

SOLUTION Solving for L_{st} gives

$$L_{\text{st}} = \frac{k_{\text{st}}}{k_{\text{al}}}L_{\text{al}} = \frac{14\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)}{240\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)}(0.035\ \text{m}) = \boxed{2.0\times10^{-3}\ \text{m}}$$

6. ***REASONING*** To find the total heat conducted, we will apply Equation 13.1 to the steel portion and the iron portion of the rod. In so doing, we use the area of a square for the cross section of the steel. The area of the iron is the area of the circle minus the area of the square. The radius of the circle is one half the length of the diagonal of the square.

SOLUTION In preparation for applying Equation 13.1, we need the area of the steel and the area of the iron. For the steel, the area is simply $A_{\text{Steel}} = L^2$, where L is the length of a side of the square. For the iron, the area is $A_{\text{Iron}} = \pi R^2 - L^2$. To find the radius R, we use the Pythagorean theorem, which indicates that the length D of the diagonal is related to the length of the sides according to $D^2 = L^2 + L^2$. Therefore, the radius of the circle is $R = D/2 = \sqrt{2}L/2$. For the iron, then, the area is

$$A_{\text{Iron}} = \pi R^2 - L^2 = \pi\left(\frac{\sqrt{2}L}{2}\right)^2 - L^2 = \left(\frac{\pi}{2} - 1\right)L^2$$

Taking values for the thermal conductivities of steel and iron from Table 13.1 and applying Equation 13.1, we find

$$Q_{\text{Total}} = Q_{\text{Steel}} + Q_{\text{Iron}}$$

$$= \left[\frac{(kA\Delta T)t}{L}\right]_{\text{Steel}} + \left[\frac{(kA\Delta T)t}{L}\right]_{\text{Iron}} = \left[k_{\text{Steel}}L^2 + k_{\text{Iron}}\left(\frac{\pi}{2} - 1\right)L^2\right]\frac{(\Delta T)t}{L}$$

$$= \left[\left(14\ \frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{C}^\circ}\right)(0.010\ \text{m})^2 + \left(79\ \frac{\text{J}}{\text{s}\cdot\text{m}\cdot\text{C}^\circ}\right)\left(\frac{\pi}{2} - 1\right)(0.010\ \text{m})^2\right]$$

$$\times \frac{(78\ ^\circ\text{C} - 18\ ^\circ\text{C})(120\ \text{s})}{0.50\ \text{m}} = \boxed{85\ \text{J}}$$

7. $\boxed{\textit{SSM}}$ $\boxed{\textit{WWW}}$ ***REASONING AND SOLUTION*** The conductance of an 0.080 mm thick sample of Styrofoam of cross-sectional area A is

$$\frac{k_s A}{L_s} = \frac{\left[0.010\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)\right] A}{0.080\times10^{-3}\ \text{m}} = [125\ \text{J}/(\text{s}\cdot\text{m}^2\cdot\text{C}^\circ)]\, A$$

The conductance of a 3.5 mm thick sample of air of cross-sectional area A is

$$\frac{k_a A}{L_a} = \frac{\left[0.0256\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)\right] A}{3.5\times10^{-3}\ \text{m}} = [7.3\ \text{J}/(\text{s}\cdot\text{m}^2\cdot\text{C}^\circ)]\, A$$

Dividing the conductance of Styrofoam by the conductance of air for samples of the same cross-sectional area A, gives

$$\frac{[125\ \text{J}/(\text{s}\cdot\text{m}^2\cdot\text{C}^\circ)]\, A}{[7.3\ \text{J}/(\text{s}\cdot\text{m}^2\cdot\text{C}^\circ)]\, A} = 17$$

Therefore, the body can adjust the conductance of the tissues beneath the skin by $\boxed{\text{a factor of 17}}$.

8. ***REASONING AND SOLUTION*** The heat lost in each case is given by $Q = (kA\Delta T)t/L$. For the goose down jacket

$$Q_g = \frac{[0.025\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)]A\Delta Tt}{1.5\times10^{-2}\ \text{m}}$$

For the wool jacket

$$Q_w = \frac{[0.040\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)]A\Delta Tt}{5.0\times10^{-3}\ \text{m}}$$

Now

$$Q_w / Q_g = \boxed{4.8}$$

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9. ***REASONING AND SOLUTION*** The amount of heat conducted through the Styrofoam is

$$Q = \frac{k\,A\,\Delta T\,t}{L} = \frac{\left[0.010\ \mathrm{J}/(\mathrm{s}\cdot\mathrm{m}\cdot\mathrm{C}^\circ)\right](0.016\ \mathrm{m}^2)(35\ \mathrm{C}^\circ)(3600\ \mathrm{s})}{2.0\times10^{-3}\ \mathrm{m}} = 1.0\times10^4\ \mathrm{J}$$

The mass of ice that this heat will melt is

$$m = Q/L_f = (1.0\times10^4\ \mathrm{J})/(3.35\times10^5\ \mathrm{J/kg}) = \boxed{3.0\times10^{-2}\ \mathrm{kg}}$$

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10. ***REASONING AND SOLUTION*** Using Equation 13.1, $Q = (kA\,\Delta T)t / L$, we obtain

$$\left(\frac{Q}{At}\right) = \frac{k\Delta T}{L} \tag{1}$$

Before Equation (1) can be applied to the ice-aluminum sandwich, the temperature T at the interface must be determined. We find the temperature at the interface by noting that the heat conducted through the ice must be equal to the heat conducted through the aluminum: $Q_{\text{ice}} = Q_{\text{aluminum}}$. Applying Equation 13.1 to this condition, we have

$$\left(\frac{kA\Delta Tt}{L}\right)_{\text{ice}} = \left(\frac{kA\Delta Tt}{L}\right)_{\text{aluminum}} \tag{2}$$

or

$$\frac{[2.2\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)]A[(-10.0\ °\text{C})\ -\ \text{T}]t}{0.0050\ \text{m}} = \frac{[240\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)]A[T-(-25.0\ °\text{C})]t}{0.0015\ \text{m}}$$

The factors A and t can be eliminated algebraically. Solving for T gives $T = -24.959$ °C for the temperature at the interface.

a. Applying Equation (1) to the ice leads to

$$\left(\frac{Q}{At}\right)_{\text{ice}} = \frac{[2.2\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)][(-10.0\,°\text{C})-(-24.959\,°\text{C})]}{0.0050\ \text{m}} = \boxed{6.58\times10^{3}\ \text{J}/(\text{s}\cdot\text{m}^2)}$$

Since heat is not building up in the materials, the rate of heat transfer per unit area is the same throughout the ice-aluminum combination. Thus, this must be the heat per second per square meter that is conducted through the ice-aluminum "sandwich."

b. Applying Equation (1) to the aluminum in the absence of any ice gives:

$$\left(\frac{Q}{At}\right)_{\text{Al}} = \frac{[240\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)][(-10.0\,°\text{C})-(-25.0\,°\text{C})]}{0.0015\ \text{m}} = \boxed{2.40\times10^{6}\ \text{J}/(\text{s}\cdot\text{m}^2)}$$

11. SSM *REASONING AND SOLUTION*

a. If we ignore the loss of heat through the sides of the rod and assume that heat does not accumulate at any point, then, from energy conservation, the rate at which heat is conducted through the two-rod combination must be the same at all points. In particular, at the interface of the two rods, $Q_a / t = Q_c / t$. Let T represent the temperature at the interface. Using Equation 13.1, we have

$$\frac{k_a A(T_a - T)}{L} = \frac{k_c A(T - T_c)}{L}$$

Solving for T, we find

$$T = \frac{k_a T_a + k_c T_c}{k_a + k_c} = \frac{[240\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)](302\ °\text{C}) + [390\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)](25\ °\text{C})}{240\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°) + 390\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)} = \boxed{130\ °\text{C}}$$

b. Now that the temperature of the interface is known, Equation 13.1 can be used to calculate the amount of heat Q that flows through either section of the unit in a time t. Since the rate of heat flow is the same at all points throughout the unit as discussed above, we need only calculate Q for one of the rods that make up the unit. Using the aluminum rod, and considering the heat flow from its free end to the interface, we have

$$Q = \frac{k_a A(T_a - T)}{L} t = \frac{[240\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)](4.0\times10^{-4}\ \text{m}^2)(302\ °\text{C} - 130\ °\text{C})}{4.0\times10^{-2}\ \text{m}}(2.0\ \text{s}) = \boxed{830\ \text{J}}$$

c. According to Equation 13.1, the temperature T_d at a distance d from the hot end of the aluminum rod is

$$T_a - T_d = \frac{Qd}{k_a A t} = \frac{(Q/t)d}{k_a A}$$

where, from the answer to part (b), we know that $Q/t = (830\ \text{J})/(2.0\ \text{s}) = 415\ \text{J/s}$. Solving for T_d, we have

$$T_d = T_a - \frac{(Q/t)d}{k_a A} = 302\ °\text{C} - \frac{(415\ \text{J/s})(1.5\times10^{-2}\ \text{m})}{[240\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)](4.0\times10^{-4}\ \text{m}^2)} = \boxed{237\ °\text{C}}$$

12. ***REASONING AND SOLUTION*** The rate of heat transfer is the same for all three materials so

$$Q/t = k_p A\Delta T_p/L = k_b A\Delta T_b/L = k_w A\Delta T_w/L$$

Let T_i be the inside temperature, T_1 be the temperature at the plasterboard-brick interface, T_2 be the temperature at the brick-wood interface, and T_o be the outside temperature. Then

$$k_p T_i - k_p T_1 = k_b T_1 - k_b T_2 \qquad (1)$$

and

$$k_b T_1 - k_b T_2 = k_w T_2 - k_w T_o \qquad (2)$$

Solving (1) for T_2 gives

$$T_2 = (k_p + k_b)T_1/k_b - (k_p/k_b)T_i$$

a. Substituting this into (2) and solving for T_1 yields

$$T_1 = \frac{(k_p/k_b)(1 + k_w/k_b)T_i + (k_w/k_b)T_0}{(1 + k_w/k_b)(1 + k_p/k_b) - 1} = \boxed{21\,°C}$$

b. Using this value in (1) yields

$$\boxed{T_2 = 18\ °C}$$

13. ***REASONING AND SOLUTION*** The rate at which heat is transferred from the burner to the water is

$$Q/t = mL_V/t = (0.20\text{ kg})(22.6 \times 10^5\text{ J/kg})/(3.0 \times 10^2\text{ s}) = 1500\text{ W}$$

Between the water and the aluminum-steel interface we have

$$\Delta T = \frac{(Q/t)L}{k_a A} = \frac{(1500\,W)(2.5 \times 10^{-3}\text{ m})}{[240\text{ J}/(\text{s}\cdot\text{m}\cdot\text{C}°)](0.020\text{ m}^2)} = 0.78\text{ C}°$$

The temperature of the aluminum-steel interface is $T = 100.00\text{ C}° + 0.78\text{ C}° = 100.78\text{ C}°$.

Between the aluminum-steel interface and the burner we know

$$\Delta T = \frac{(Q/t)L}{k_s A} = \frac{(1500\,W)(1.2 \times 10^{-3}\text{ m})}{[14\text{ J}/(\text{s}\cdot\text{m}\cdot\text{C}°)](0.020\text{ m}^2)} = 6.4\text{ C}°$$

The temperature of the steel at the burner is $T = 100.78\text{ °C} + 6.4\text{ °C} = \boxed{107.2\text{ °C}}$.

14. ***REASONING AND SOLUTION*** The heat conducted through the layered slab can be found from Equation 13.1:

$$Q = \frac{kA\Delta Tt}{L}$$

We must first find the temperature at the interface between materials 1 and 2. The temperature at the interface is found by noting that since there are no sources or sinks for heat in the slab, $Q_1 = Q_2$. Applying Equation 13.1 to this condition gives

$$\frac{k_1 A\Delta T_1 t}{L_1} = \frac{k_2 A\Delta T_2 t}{L_2} \tag{1}$$

If T represents the temperature at the interface between the two materials, then Equation (1) becomes

$$\frac{k_1}{L_1}(T_H - T) = \frac{k_2}{L_2}(T - T_C)$$

Solving for T gives

$$T = \frac{\frac{k_1}{L_1}T_H + \frac{k_2}{L_2}T_C}{\frac{k_1}{L_1} + \frac{k_2}{L_2}} \tag{2}$$

Applying Equation 13.1 to material 1, we obtain

$$Q = \frac{k_1 A(T_H - T)t}{L_1} \tag{3}$$

Substituting the right hand side of Equation (2) into Equation (3) gives

$$Q = \frac{k_1 At}{L_1}\left[T_H - \left(\frac{\frac{k_1}{L_1}T_H + \frac{k_2}{L_2}T_C}{\frac{k_1}{L_1} + \frac{k_2}{L_2}}\right)\right]$$

$$Q = At\left[\frac{k_1}{L_1}T_H - \left(\frac{\frac{k_1^2}{L_1^2}T_H + \frac{k_1 k_2}{L_1 L_2}T_C}{\frac{k_1}{L_1} + \frac{k_2}{L_2}}\right)\right]$$

$$Q = At\left[\frac{\frac{L_1 k_2 k_1}{L_1}T_H + \frac{L_2 k_1^2}{L_1}T_H - \frac{L_1 L_2 k_1^2}{L_1^2}T_H - k_1 k_2 T_C}{L_1 k_2 + L_2 k_1}\right]$$

$$Q = At\left[\frac{k_1k_2T_H - k_1k_2T_C}{L_1k_2 + L_2k_1}\right] = At(T_H - T_C)\left(\frac{k_1k_2}{L_1k_2 + L_2k_1}\right)$$

$$Q = At(T_H - T_C)\left(\frac{L_1k_2 + L_2k_1}{k_1k_2}\right)^{-1} = At(T_H - T_C)\left(\frac{L_1}{k_1} + \frac{L_2}{k_2}\right)^{-1}$$

Therefore,

$$\boxed{Q = \frac{A(T_H - T_C)t}{\left(\frac{L_1}{k_1} + \frac{L_2}{k_2}\right)}}$$

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15. **SSM** **WWW** ***REASONING*** If the cylindrical rod were made of solid copper, the amount of heat it would conduct in a time t is, according to Equation 13.1, $Q_{\text{copper}} = (k_{\text{copper}} A_2 \Delta T / L)t$. Similarly, the amount of heat conducted by the lead-copper combination is the sum of the heat conducted through the copper portion of the rod and the heat conducted through the lead portion: $Q_{\text{combination}} = \left[k_{\text{copper}}(A_2 - A_1)\Delta T / L + k_{\text{lead}} A_1 \Delta T / L\right] t$. Since the lead-copper combination conducts one-half the amount of heat than does the solid copper rod, $Q_{\text{combination}} = \frac{1}{2} Q_{\text{copper}}$, or

$$\frac{k_{\text{copper}}(A_2 - A_1)\Delta T}{L} + \frac{k_{\text{lead}} A_1 \Delta T}{L} = \frac{1}{2}\frac{k_{\text{copper}} A_2 \Delta T}{L}$$

This expression can be solved for A_1 / A_2, the ratio of the cross-sectional areas. Since the cross-sectional area of a cylinder is circular, $A = \pi r^2$. Thus, once the ratio of the areas is known, the ratio of the radii can be determined.

SOLUTION Solving for the ratio of the areas, we have

$$\frac{A_1}{A_2} = \frac{k_{\text{copper}}}{2(k_{\text{copper}} - k_{\text{lead}})}$$

The cross-sectional areas are circular so that $A_1 / A_2 = (\pi r_1^2)/(\pi r_2^2) = (r_1 / r_2)^2$; therefore,

$$\frac{r_1}{r_2} = \sqrt{\frac{k_{\text{copper}}}{2(k_{\text{copper}} - k_{\text{lead}})}} = \sqrt{\frac{390\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)}{2[390\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ) - 35\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)]}} = \boxed{0.74}$$

16. ***REASONING AND SOLUTION*** We know from Equation 13.2 that

$$A = \frac{Q/t}{e\sigma T^4} = \frac{6.0\times10^1\ \text{W}}{(0.36)\left[5.67\times10^{-8}\ \text{J}/(\text{s}\cdot\text{m}^2\cdot\text{K}^4)\right](3273\ \text{K})^4} = \boxed{2.6\times10^{-5}\ \text{m}^2}$$

17. ***REASONING AND SOLUTION*** For a blackbody $P_b = \sigma T^4 A$, and for the object $P_0 = e\sigma T^4 A$. Division of the above yields $P_0/P_b = e$. Thus,

$$e = \frac{P_0}{P_b} = \frac{30\text{ W}}{90\text{ W}} = \boxed{0.3}$$

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18. ***REASONING AND SOLUTION*** The energy per second emitted by the Earth is, according to Equation 13.2 $P = e\sigma T^4 A$. Since $e = 1$, the energy per second per square meter that the Earth radiates is

$$P/A = \sigma T^4 = [5.67 \times 10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{K}^4)](295\ \text{K})^4 = \boxed{4.29 \times 10^2\ \text{J/(s}\cdot\text{m}^2)}$$

19. SSM WWW ***REASONING AND SOLUTION*** Solving the Stefan-Boltzmann law, Equation 13.2, for the time t, and using the fact that $Q_{\text{blackbody}} = Q_{\text{bulb}}$, we have

$$t_{\text{blackbody}} = \frac{Q_{\text{blackbody}}}{\sigma T^4 A} = \frac{Q_{\text{bulb}}}{\sigma T^4 A} = \frac{P_{\text{bulb}}\, t_{\text{bulb}}}{\sigma T^4 A}$$

where P_{bulb} is the power rating of the light bulb. Therefore,

$$t_{\text{blackbody}} = \frac{(100.0\ \text{J/s})\,(3600\ \text{s})}{\left[5.67\times10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{K}^4)\right](303\ \text{K})^4\ \left[(6\ \text{sides})(0.0100\ \text{m})^2/\text{side}\right]}$$

$$\times\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1\ \text{da}}{24\ \text{h}}\right) = \boxed{14.5\ \text{da}}$$

20. ***REASONING AND SOLUTION*** From the Stefan-Boltzmann law, we know that $T^4 = (Q/t)/(e\sigma A)$ or $T = [(Q/t)/(e\sigma A)]^{1/4}$. Therefore, if we double the energy (2Q), the temperature increases by a factor of

$$T = 2^{1/4} = \boxed{1.19}$$

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21. **SSM** ***REASONING AND SOLUTION*** The net power generated by the stove is given by Equation 13.3, $P_{\text{net}} = e\sigma A(T^4 - T_0^4)$. Solving for T gives

$$T = \left(\frac{P_{\text{net}}}{e\sigma A} + T_0^4 \right)^{1/4}$$

$$= \left\{ \frac{7300 \text{ W}}{(0.900)[5.67 \times 10^{-8} \text{ J/(s} \cdot \text{m}^2 \cdot \text{K}^4)](2.00 \text{ m}^2)} + (302 \text{ K})^4 \right\}^{1/4} = \boxed{532 \text{ K}}$$

22. ***REASONING*** The net rate at which energy is being lost via radiation can not exceed the production rate of 115 J/s, if the body temperature is to remain constant. The net rate at which an object at temperature T radiates energy in a room where the temperature is T_0 is given by Equation 13.3 as $P_{\text{net}} = e\sigma A(T^4 - T_0{}^4)$. P_{net} is the net energy per second radiated. We need only set P_{net} equal to 115 J/s and solve for T_0. We note that the temperatures in this equation must be expressed in Kelvins, not degrees Celsius.

SOLUTION According to Equation 13.3, we have

$$P_{\text{net}} = e\sigma A\left(T^4 - T_0^4\right) \quad \text{or} \quad T_0^4 = T^4 - \frac{P_{\text{net}}}{e\sigma A}$$

Using Equation 12.1 to convert from degrees Celsius to Kelvins, we have $T = 34 + 273 = 307$ K. Using this value, it follows that

$$T_0 = \sqrt[4]{T^4 - \frac{P_{\text{net}}}{e\sigma A}}$$

$$= \sqrt[4]{(307\ \text{K})^4 - \frac{115\ \text{J/s}}{0.700\left[5.67\times10^{-8}\ \text{J}/\left(\text{s}\cdot\text{m}^2\cdot\text{K}^4\right)\right]\left(1.40\ \text{m}^2\right)}} = \boxed{287\ \text{K}\ (14\ ^\circ\text{C})}$$

23. ***REASONING AND SOLUTION*** The heat Q conducted during a time t through a wall of thickness L and cross sectional area A is given by Equation 13.1:

$$Q = \frac{kA\Delta Tt}{L}$$

The radiant energy Q, emitted in a time t by a wall that has a Kelvin temperature T, surface area A, and emissivity e is given by Equation (13.2):

$$Q = e\sigma T^4 At$$

If the amount of radiant energy emitted per second per square meter at 0 °C is the same as the heat lost per second per square meter due to conduction, then

$$\left(\frac{Q}{tA}\right)_{\text{conduction}} = \left(\frac{Q}{tA}\right)_{\text{radiation}}$$

Making use of Equations 13.1 and 13.2, the equation above becomes

$$\frac{k\Delta T}{L} = e\sigma T^4$$

Solving for the emissivity e gives:

$$e = \frac{k\Delta T}{L\sigma T^4} = \frac{[1.1\ \text{J/(s}\cdot\text{m}\cdot K)](293.0\ \text{K} - 273.0\ \text{K})}{(0.10\ \text{m})[5.67\times10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{K}^4)]\ (273.0\ \text{K})^4} = \boxed{0.70}$$

Remark on units: Notice that the units for the thermal conductivity were expressed as J/(s·m·K) even though they are given in Table 13.1 as J/(s·m·C°) . The two units are equivalent since the "size" of a Celsius degree is the same as the "size" of a Kelvin; that is, 1 C° = 1 K. Kelvins were used, rather than Celsius degrees, to ensure consistency of units. However, Kelvins must be used in Equation 13.2 or any equation that is derived from it.

24. ***REASONING*** The heat Q necessary to vaporize a mass m of any substance at its boiling point is $Q = mL_V$ where L_V is the latent heat of vaporization. Therefore, the mass vaporized by an amount of heat Q is $m = Q/L_V$.

For the liquid helium system in question, it continually absorbs heat through radiation. The net power absorbed is given by Equation 13.3, $P_{net} = e\sigma A(T^4 - T_0^4)$ where T_0 is the temperature of the liquid helium, and T is the temperature maintained by the shield. Since the container is a perfect blackbody radiator, $e = 1$. Thus, the rate at which the mass of liquid helium boils away through the venting value is

$$\frac{m}{t} = \frac{(Q/t)}{L_v} = \frac{P_{net}}{L_v} = \frac{e\sigma A(T^4 - T_0^4)}{L_v}$$

This expression can be multiplied by the time t to determine the mass vaporized during that time.

SOLUTION The rate at which liquid helium mass boils away is

$$\frac{m}{t} = \frac{(1)[5.67\times10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{K}^4)]4\pi(0.30\ \text{m})^2[(77\ \text{K})^4 - (4.2\ \text{K})^4]}{2.1\times10^4\ \text{J/kg}} = 1.07\times10^{-4}\ \text{kg/s}$$

The mass of liquid helium that boils away in one hour is, therefore,

$$\left(1.07\times10^{-4}\ \frac{\text{kg}}{\text{s}}\right)(1.0\ \text{h})\left(\frac{3600\ \text{s}}{1.0\ \text{h}}\right) = \boxed{0.39\ \text{kg}}$$

25. **SSM** ***REASONING*** The total radiant power emitted by an object that has a Kelvin temperature T, surface area A, and emissivity e can be found by rearranging Equation 13.2, the Stefan-Boltzmann law: $Q = e\sigma T^4 At$. The emitted power is $P = Q/t = e\sigma T^4 A$. Therefore, when the original cylinder is cut perpendicular to its axis into N smaller cylinders, the ratio of the power radiated by the pieces to that radiated by the original cylinder is

$$\frac{P_{\text{pieces}}}{P_{\text{original}}} = \frac{e\sigma T^4 A_2}{e\sigma T^4 A_1} \tag{1}$$

where A_1 is the surface area of the original cylinder, and A_2 is the sum of the surface areas of all N smaller cylinders. The surface area of the original cylinder is the sum of the surface area of the ends and the surface area of the cylinder body; therefore, if L and r represent the length and cross-sectional radius of the original cylinder, with $L = 10r$,

$$\begin{aligned} A_1 &= (\text{area of ends}) + (\text{area of cylinder body}) \\ &= 2(\pi r^2) + (2\pi r)L = 2(\pi r^2) + (2\pi r)(10r) = 22\pi r^2 \end{aligned}$$

When the original cylinder is cut perpendicular to its axis into N smaller cylinders, the total surface area A_2 is

$$A_2 = N2(\pi r^2) + (2\pi r)L = N2(\pi r^2) + (2\pi r)(10r) = (2N + 20)\pi r^2$$

Substituting the expressions for A_1 and A_2 into Equation (1), we obtain the following expression for the ratio of the power radiated by the N pieces to that radiated by the original cylinder

$$\frac{P_{\text{pieces}}}{P_{\text{original}}} = \frac{e\sigma T^4 A_2}{e\sigma T^4 A_1} = \frac{(2N + 20)\pi r^2}{22\pi r^2} = \frac{N + 10}{11}$$

SOLUTION Since the total radiant power emitted by the N pieces is twice that emitted by the original cylinder, $P_{\text{pieces}} / P_{\text{original}} = 2$, we have $(N + 10)/11 = 2$. Solving this expression for N gives $N = 12$. Therefore, $\boxed{\text{there are 12 smaller cylinders}}$.

26. ***REASONING AND SOLUTION*** For the conduction of heat through the asbestos shell we have $\Delta T = QL/(k_a A_2 t)$. Now Q is the heat absorbed by the inner surface of the shell which is the same as the net radiant energy lost by the sphere since the system is in equilibrium.

The radiant energy lost by the sphere is

$$Q_L = e\sigma T_1^4 A_1 t$$

The radiant energy gained by the sphere is

$$Q_g = e\sigma T_2^4 A_1 t$$

The net loss of energy for the sphere is then

$$Q = e\sigma A_1 (T_1^4 - T_2^4) t$$

The temperature difference between the surfaces of the shell is

$$\Delta T = (e\sigma L/k_a)(r_1/r_2)^2 (T_1^4 - T_2^4) = 42\ C°$$

Therefore,

$$T = 600\ °C - 42\ C° = \boxed{558\ °C}$$

27. SSM ***REASONING AND SOLUTION*** According to Equation 13.1, the heat per second lost is

$$\frac{Q}{t} = \frac{kA\,\Delta T}{L} = \frac{[0.040\ \text{J/(s}\cdot\text{m}\cdot\text{C}^{\circ})]\ (1.6\ \text{m}^2)(25\ \text{C}^{\circ})}{2.0\times10^{-3}\ \text{m}} = \boxed{8.0\times10^{2}\ \text{J/s}}$$

28. ***REASONING AND SOLUTION***

a. The heat lost by the oven is

$$Q = \frac{(kA\Delta T)t}{L} = \frac{[0.045\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}^\circ)](1.6\ \text{m}^2)(160\ ^\circ\text{C} - 50\ ^\circ\text{C})(6.0\ \text{h})\left(\frac{3600\ \text{s}}{1\ \text{h}}\right)}{0.020\ \text{m}}$$

$$= \boxed{8.6 \times 10^6\ \text{J}}$$

b. Now 1 J = 2.78×10^{-7} kWh, so Q = 2.4 kWh. At \$ 0.10 per kWh, the cost is $\boxed{\$\ 0.24}$.

29. ***REASONING AND SOLUTION*** According to Equation 13.2, the radiant energy Q, emitted in a time t by an object that has a Kelvin temperature T, an emissivity e, and a surface area A, is given by $Q = e\sigma T^4 At$. Solving for the temperature T, we have

$$T = \left(\frac{Q/t}{e\sigma A}\right)^{1/4} = \left\{\frac{880\ \text{J/s}}{(0.75)[5.67\times10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{K}^4)](1.0\ \text{m}^2)}\right\}^{1/4} = \boxed{380\ \text{K}}$$

30. ***REASONING AND SOLUTION***

a. The radiant power lost by the body is

$$P_L = e\sigma T^4 A = (0.80)[5.67 \times 10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{K}^4)](307\ \text{K})^4(1.5\ \text{m}^2) = 604\ \text{W}$$

The radiant power gained by the body from the room is

$$P_g = (0.80)[5.67 \times 10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{T}^4)](298\ \text{K})^4(1.5\ \text{m}^2) = 537\ \text{W}$$

The net loss of radiant power is $P = P_L - P_g = \boxed{67\ \text{W}}$

b. The net energy lost by the body is

$$Q = Pt = (67\ \text{W})(3600\ \text{s})\left(\frac{1\ \text{Calorie}}{4186\ \text{J}}\right) = \boxed{58\ \text{Calories}}$$

31. **SSM** ***REASONING AND SOLUTION*** The power radiated per square meter by the car when it has reached a temperature T is given by the Stefan-Boltzmann law, Equation 13.2, $P_{\text{radiated}} / A = e\sigma T^4$, where $P_{\text{radiated}} = Q/t$. Solving for T we have

$$T = \left[\frac{(P_{\text{radiated}} / A)}{e\sigma}\right]^{1/4} = \left\{\frac{560\ \text{W/m}^2}{(1.00)\left[5.67\times10^{-8}\ \text{J/(s}\cdot\text{m}^2\cdot\text{K}^4)\right]}\right\}^{1/4} = \boxed{320\ \text{K}}$$

32. ***REASONING AND SOLUTION*** The temperature difference is given by

$$\Delta T = \frac{L(Q/t)}{kA} = \frac{(0.20\ \text{m})(2.7\ \text{J/s})}{[110\ \text{J/(s}\cdot\text{m}\cdot\text{C}^\circ)](4.0\times10^{-4}\ \text{m}^2)} = 12\ \text{C}^\circ$$

The temperature 0.20 m from the hot end is

$$T = 295\ ^\circ\text{C} - 12\ \text{C}^\circ = \boxed{283\ ^\circ\text{C}}$$

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33. ***REASONING AND SOLUTION*** According to Equation 13.2, for the sphere we have $Q/t = e\sigma A_s T_s^4$, and for the cube $Q/t = e\sigma A_c T_c^4$. Equating and solving we get

$$T_c^4 = (A_s/A_c)T_s^4$$

Now

$$A_s/A_c = (4\pi R^2)/(6L^2)$$

The volume of the sphere and the cube are the same, $(4/3)\,\pi R^3 = L^3$, so $R = \left(\frac{3}{4\pi}\right)^{1/3} L$.

The ratio of the areas is $\frac{A_s}{A_c} = \frac{4\pi R^2}{6L^2} = \frac{4\pi}{6}\left(\frac{3}{4\pi}\right)^{2/3} = 0.806$. The temperature of the cube is, then

$$T_c = \left(\frac{A_s}{A_c}\right)^{1/4} T_s = (0.806)^{1/4}(773\text{ K}) = \boxed{732\text{ K}}$$

34. ***REASONING*** The heat lost per second due to conduction through the glass is given by Equation 13.1 as $Q/t = (kA\Delta T)/L$. In this expression, we have no information for the thermal conductivity k, the cross-sectional area A, or the length L. Nevertheless, we can apply the equation to the initial situation and again to the situation where the outside temperature has fallen. This will allow us to eliminate the unknown variables from the calculation.

SOLUTION Applying Equation 13.1 to the initial situation and to the situation after the outside temperature has fallen, we obtain

$$\left(\frac{Q}{t}\right)_{\text{Initial}} = \frac{kA\left(T_{\text{In}} - T_{\text{Out, initial}}\right)}{L} \quad \text{and} \quad \left(\frac{Q}{t}\right)_{\text{Colder}} = \frac{kA\left(T_{\text{In}} - T_{\text{Out, colder}}\right)}{L}$$

Dividing these two equations to eliminate the common variables gives

$$\frac{(Q/t)_{\text{Colder}}}{(Q/t)_{\text{Initial}}} = \frac{\dfrac{kA\left(T_{\text{In}} - T_{\text{Out, colder}}\right)}{L}}{\dfrac{kA\left(T_{\text{In}} - T_{\text{Out, initial}}\right)}{L}} = \frac{T_{\text{In}} - T_{\text{Out, colder}}}{T_{\text{In}} - T_{\text{Out, initial}}}$$

Remembering that twice as much heat is lost per second when the outside is colder, we find

$$\frac{2(Q/t)_{\text{Initial}}}{(Q/t)_{\text{Initial}}} = 2 = \frac{T_{\text{In}} - T_{\text{Out, colder}}}{T_{\text{In}} - T_{\text{Out, initial}}}$$

Solving for the colder outside temperature gives

$$T_{\text{Out, colder}} = 2T_{\text{Out, initial}} - T_{\text{In}} = 2(5.0\ ^\circ\text{C}) - (25\ ^\circ\text{C}) = \boxed{-15\ ^\circ\text{C}}$$

35. SSM WWW ***REASONING*** The rate at which heat is conducted along either rod is given by Equation 13.1, $Q/t = (kA\,\Delta T)/L$. Since both rods conduct the same amount of heat per second, we have

$$\frac{k_s A_s\,\Delta T}{L_s} = \frac{k_i A_i\,\Delta T}{L_i} \tag{1}$$

Since the same temperature difference is maintained across both rods, we can algebraically cancel the ΔTs. Because both rods have the same mass, $m_S = m_i$; in terms of the densities of silver and iron, the statement about the equality of the masses becomes $\rho_S(L_S A_S) = \rho_i(L_i A_i)$, or

$$\frac{A_s}{A_i} = \frac{\rho_i L_i}{\rho_s L_s} \tag{2}$$

Equations (1) and (2) may be combined to find the ratio of the lengths of the rods. Once the ratio of the lengths is known, Equation (2) can be used to find the ratio of the cross-sectional areas of the rods. If we assume that the rods have circular cross sections, then each has an area of $A = \pi r^2$. Hence, the ratio of the cross-sectional areas can be used to find the ratio of the radii of the rods.

SOLUTION

a. Solving Equation (1) for the ratio of the lengths and substituting the right hand side of Equation (2) for the ratio of the areas, we have

$$\frac{L_s}{L_i} = \frac{k_s A_s}{k_i A_i} = \frac{k_s\left(\rho_i L_i\right)}{k_i\left(\rho_s L_s\right)} \quad \text{or} \quad \left(\frac{L_s}{L_i}\right)^2 = \frac{k_s \rho_i}{k_i \rho_s}$$

Solving for the ratio of the lengths, we have

$$\frac{L_s}{L_i} = \sqrt{\frac{k_s \rho_i}{k_i \rho_s}} = \sqrt{\frac{[420\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)](7860\ \text{kg}/\text{m}^3)}{[79\ \text{J}/(\text{s}\cdot\text{m}\cdot\text{C}°)](10\ 500\ \text{kg}/\text{m}^3)}} = \boxed{2.0}$$

b. From Equation (2) we have

$$\frac{\pi r_s^2}{\pi r_i^2} = \frac{\rho_i L_i}{\rho_s L_s} \quad \text{or} \quad \left(\frac{r_s}{r_i}\right)^2 = \frac{\rho_i L_i}{\rho_s L_s}$$

Solving for the ratio of the radii, we have

$$\frac{r_s}{r_i} = \sqrt{\frac{\rho_i}{\rho_s}\left(\frac{L_i}{L_s}\right)} = \sqrt{\frac{7860\ \text{kg}/\text{m}^3}{10\ 500\ \text{kg}/\text{m}^3}\left(\frac{1}{2.0}\right)} = \boxed{0.61}$$

36. ***REASONING AND SOLUTION*** The heat which must be removed to form a volume, V, of ice is

$$Q = mL_f = \rho V L_f = \rho A h L_f$$

The heat is conducted through the ice to the air, so Q is $Q = kA(\Delta T)t/L$. Thus, we have

$$h = \frac{k\,\Delta T\,t}{\rho L_f L} = \frac{[2.2\ \mathrm{J/(s\cdot m\cdot C^\circ)}](15\ \mathrm{C^\circ})(3.0\times10^2\ \mathrm{s})}{(917\ \mathrm{kg/m^3})(3.35\times10^5\ \mathrm{J/kg})(0.30\ \mathrm{m})} = 1.1\times10^{-4}\ \mathrm{m} = \boxed{0.11\ \mathrm{mm}}$$

1. **SSM** ***REASONING AND SOLUTION*** Since hemoglobin has a molecular mass of 64 500 u, the mass per mole of hemoglobin is 64 500 g/mol. The number of hemoglobin molecules per mol is Avogadro's number, or $6.022 \times 10^{23}\ \text{mol}^{-1}$. Therefore, one molecule of hemoglobin has a mass (in kg) of

$$\left(\frac{64\ 500\ \text{g/mol}}{6.022\times10^{23}\ \text{mol}^{-1}}\right)\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right)=\boxed{1.07\times10^{-22}\ \text{kg}}$$

2. ***REASONING AND SOLUTION***

a. The molecular mass of $C_{55}H_{72}MgN_4O_5$ is

$$55(12.011\text{ u}) + 72(1.00794\text{ u}) + 24.305\text{ u} + 4(14.0067\text{ u}) + 5(15.9994\text{ u}) = \boxed{893.51\text{ u}}$$

b. The mass per mol is 893.51 g/mol. The mass of 3.00 mol is

$$(893.51\text{ g/mol})\ (3.00\text{ mol}) = \boxed{2680\text{ g}}$$

3. ***REASONING AND SOLUTION***

a. The molecular mass of a molecule is the sum of the atomic masses of its atoms. Thus, the molecular mass of aspartame ($C_{14}H_{18}N_2O_5$) is (see the periodic table on the inside of the text's back cover)

$$\text{Molecular mass} = \underbrace{14\ (12.011\ \text{u})}_{\substack{\text{Mass of 14}\\ \text{carbon atoms}}} + \underbrace{18\ (1.00794\ \text{u})}_{\substack{\text{Mass of 18}\\ \text{hydrogen atoms}}}$$

$$+ \underbrace{2\ (14.0067\ \text{u})}_{\substack{\text{Mass of 2}\\ \text{nitrogen atoms}}} + \underbrace{5\ (15.9994)}_{\substack{\text{Mass of 5}\\ \text{oxygen atoms}}} = \boxed{294.307\ \text{u}}$$

b. The mass per mole of aspartame is 294.307 g/mol. The number of aspartame molecules per mole is Avogadro's number, or $6.022 \times 10^{23}\ \text{mol}^{-1}$. Therefore, the mass of one aspartame molecule (in kg) is

$$\left(\frac{294.307\ \text{g/mol}}{6.022\times10^{23}\ \text{mol}^{-1}}\right)\left(\frac{1\ \text{kg}}{1000\ \text{g}}\right) = \boxed{4.887\times10^{-25}\ \text{kg}}$$

4. ***REASONING*** The number of molecules in a known mass of material is the number n of moles of the material times the number N_A of molecules per mole (Avogadro's number). We can find the number of moles by dividing the known mass m by the mass per mole.

SOLUTION Using the periodic table on the inside of the text's back cover, we find that the molecular mass of Tylenol ($C_8H_9NO_2$) is

$$\text{Molecular mass of Tylenol} = \underbrace{8\ (12.011\ \text{u})}_{\text{Mass of 8 carbon atoms}} + \underbrace{9\ (1.00794\ \text{u})}_{\text{Mass of 9 hydrogen atoms}}$$

$$+ \underbrace{14.0067\ \text{u}}_{\text{Mass of nitrogen atom}} + \underbrace{2\ (15.9994)}_{\text{Mass of 2 oxygen atoms}} = 151.165\ \text{u}$$

The molecular mass of Advil ($C_{13}H_{18}O_2$) is

$$\text{Molecular mass of Advil} = \underbrace{13\ (12.011\ \text{u})}_{\text{Mass of 13 carbon atoms}} + \underbrace{18\ (1.00794\ \text{u})}_{\text{Mass of 18 hydrogen atoms}} + \underbrace{2\ (15.9994)}_{\text{Mass of 2 oxygen atoms}} = 206.285\ \text{u}$$

a. Therefore, the number of molecules of pain reliever in the standard dose of Tylenol is

$$\text{Number of molecules} = \text{n}\,\text{N}_\text{A} = \left(\frac{m}{\text{Mass per mole}}\right) N_\text{A}$$

$$= \left(\frac{325 \times 10^{-3}\ \text{g}}{151.165\ \text{g/mol}}\right)\left(6.022 \times 10^{23}\ \text{mol}^{-1}\right) = \boxed{1.29 \times 10^{21}}$$

b. Similarly, the number of molecules of pain reliever in the standard dose of Advil is

$$\text{Number of molecules} = \text{n}\,\text{N}_\text{A} = \left(\frac{m}{\text{Mass per mole}}\right) N_\text{A}$$

$$= \left(\frac{2.00 \times 10^{-1}\ \text{g}}{206.285\ \text{g/mol}}\right)\left(6.022 \times 10^{23}\ \text{mol}^{-1}\right) = \boxed{5.84 \times 10^{20}}$$

5. **SSM** ***REASONING AND SOLUTION*** The number n of moles contained in a sample is equal to the number N of atoms in the sample divided by the number N_A of atoms per mole (Avogadro's number):

$$n = \frac{N}{N_A} = \frac{30.1\times10^{23}}{6.022\times10^{23}\ \text{mol}^{-1}} = 5.00\ \text{mol}$$

Since the sample has a mass of 135 g, the mass per mole is

$$\frac{135\ \text{g}}{5.00\ \text{mol}} = 27.0\ \text{g/mol}$$

The mass per mole (in g/mol) of a substance has the same numerical value as the atomic mass of the substance. Therefore, the atomic mass is 27.0 u. The periodic table of the elements reveals that the unknown element is $\boxed{\text{aluminum}}$.

6. ***REASONING*** The number n of moles of water molecules in the glass is equal to the mass m of water divided by the mass per mole. According to Equation 11.1, the mass of water is equal to its density ρ times its volume V. Thus, we have

$$n = \frac{m}{\text{Mass per mole}} = \frac{\rho V}{\text{Mass per mole}}$$

The volume of the cylindrical glass is $V = \pi r^2 h$, where r is the radius of the cylinder and h is its height. The number of moles of water can be written as

$$n = \frac{\rho V}{\text{Mass per mole}} = \frac{\rho\left(\pi r^2 h\right)}{\text{Mass per mole}}$$

SOLUTION The molecular mass of water (H_2O) is 2(1.00794 u) + (15.9994 u) = 18.0 u. The mass per mole of H_2O is 18.0 g/mol. The density of water (see Table 11.1) is 1.00×10^3 kg/m^3 or 1.00 g/cm^3.

$$n = \frac{\rho\left(\pi r^2 h\right)}{\text{Mass per mole}} = \frac{\left(1.00\ \text{g/cm}^3\right)(\pi)(4.50\ \text{cm})^2(12.0\ \text{cm})}{18.0\ \text{g/mol}} = \boxed{42.4\ \text{mol}}$$

7. **SSM** **WWW** ***REASONING AND SOLUTION*** For a given mass of the liquid phase and the vapor phase, $m_{\text{liquid}} = m_{\text{vapor}}$, or using Equation 11.1, $\rho_{\text{liquid}} V_{\text{liquid}} = \rho_{\text{vapor}} V_{\text{vapor}}$. In this result, ρ and V stand for density and volume, respectively. The ratio of the volumes, is therefore,

$$\frac{V_{\text{vapor}}}{V_{\text{liquid}}} = \frac{\rho_{\text{liquid}}}{\rho_{\text{vapor}}}$$

If we assume that the volume of each phase is filled with N cubes, with one molecule at the center of each cube, then the volume is $V = Nd^3$, where d is the length of one side of a cube. The number N is the same for each phase, since each has the same mass. Since each cube contains one molecule at its center, d is also the distance between neighboring molecules. It follows that, for either phase, $d = \sqrt[3]{V/N}$. Therefore,

$$\frac{d_{\text{vapor}}}{d_{\text{liquid}}} = \left(\frac{V_{\text{vapor}}/N}{V_{\text{liquid}}/N}\right)^{1/3} = \left(\frac{\rho_{\text{liquid}}}{\rho_{\text{vapor}}}\right)^{1/3} = \left(\frac{958\ \text{kg/m}^3}{0.598\ \text{kg/m}^3}\right)^{1/3} = \boxed{11.7}$$

8. ***REASONING AND SOLUTION*** The molecular mass of C_2H_5OH is

$$2(12.011\ \text{u}) + 6(1.00794\ \text{u}) + 15.9994\ \text{u} = 46.069\ \text{u}$$

a. $(46.069\ \text{g/mol})/(6.022 \times 10^{23}/\text{mol}) = 7.65 \times 10^{-23}\ \text{g} = \boxed{7.65 \times 10^{-26}\ \text{kg}}$

b. $2.00 \times 10^{-3}\ \text{m}^3$ of this substance (density = 806 kg/m^3) has a mass $m = \rho V = 1.61$ kg. The number of molecules is, therefore, equal to

$$N = (1.61\ \text{kg})/(7.65 \times 10^{-26}\ \text{kg}) = \boxed{2.11 \times 10^{25}}$$

9. ***REASONING*** We can use the ideal gas law, Equation 14.1 ($PV = nRT$) to find the number of moles of helium in the Goodyear blimp, since the pressure, volume, and temperature are known. Once the number of moles are known, we can find the mass of helium in the blimp.

SOLUTION The number n of moles of helium in the blimp is, according to Equation 14.1,

$$n = \frac{PV}{RT} = \frac{(1.1 \times 10^5 \text{ Pa})(5400 \text{ m}^3)}{[8.31 \text{ J/(mol} \cdot \text{K)}](280 \text{ K})} = 2.55 \times 10^5 \text{ mol}$$

According to the periodic table on the inside of the text's back cover, the atomic mass of helium is 4.002 60 u. Therefore, the mass per mole is 4.002 60 g/mol. The mass m of helium in the blimp is, then,

$$m = \left(2.55 \times 10^5 \text{ mol}\right)\left(4.002\,60 \text{ g/mol}\right)\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) = \boxed{1.0 \times 10^3 \text{ kg}}$$

10. ***REASONING AND SOLUTION*** To find the temperature T_2, use the ideal gas law with n and V constant. Thus, $P_1/T_1 = P_2/T_2$. Then,

$$T_2 = T_1(P_2/P_1) = (284\ \text{K})[(3.01 \times 10^5\ \text{Pa})/(2.81 \times 10^5\ \text{Pa})] = \boxed{304\ \text{K}}$$

11. **SSM** ***REASONING*** According to the ideal gas law (Equation 14.1), $PV = nRT$. Since n, the number of moles, is constant, $n_1R = n_2R$. Thus, according to Equation 14.1, we have

$$\frac{P_1V_1}{T_1} = \frac{P_2V_2}{T_2}$$

SOLUTION Solving for T_2, we have

$$T_2 = \left(\frac{P_2}{P_1}\right)\left(\frac{V_2}{V_1}\right)T_1 = \left(\frac{48.5\,P_1}{P_1}\right)\left[\frac{V_1/16}{V_1}\right](305\text{ K}) = \boxed{925\text{ K}}$$

12. ***REASONING*** The number N of molecules in an ideal gas is given by Equation 14.2 as $N = PV / kT$, where P, V, and T are the absolute pressure, volume, and Kelvin temperature of the gas, and k is Boltzmann's constant. The ratio of the number of air molecules in the two rooms is equal to the ratio of the ideal gas laws for the air in each room.

SOLUTION The number of air molecules in rooms A and B is

$$N_A = \frac{P_A V_A}{kT_A} \quad \text{and} \quad N_B = \frac{P_B V_B}{kT_B} \tag{14.2}$$

Dividing N_A by N_B, and noting that $P_A = P_B$ and $V_A = V_B$, we see that

$$\frac{N_A}{N_B} = \frac{\dfrac{P_A V_A}{kT_A}}{\dfrac{P_B V_B}{kT_B}} = \frac{T_B}{T_A} = \frac{(19 + 273)\text{K}}{(-8.0 + 273)\text{K}} = \boxed{1.10}$$

13. ***REASONING AND SOLUTION*** Assume the ideal gas law holds for O_2. Then

$$n = \frac{PV}{RT} = \frac{\left(1.5 \times 10^{7}\ \text{Pa}\right)\left(2.8 \times 10^{-3}\ \text{m}^{3}\right)}{\left[8.31\ \text{J/(mol} \cdot \text{K)}\right]\left(296\ \text{K}\right)} = 1.71\ \text{mol}$$

The mass of O_2 is (31.9988 g/mol)(1.71 mol) = 550 g = $\boxed{0.550\ \text{kg}}$.

14. ***REASONING AND SOLUTION*** The ideal gas law gives

$$V_2 = (P_1/P_2)(T_2/T_1)\,V_1 = (65.0)[(297\text{ K})/(288\text{ K})](1.00\text{ m}^3) = \boxed{67.0\text{ m}^3}$$

15. **SSM** ***REASONING AND SOLUTION*** According to the ideal gas law (Equation 14.1), the total number of moles of fresh air in the sample is

$$n = \frac{PV}{RT} = \frac{(1.0 \times 10^5 \text{ Pa})(5.0 \times 10^{-4} \text{ m}^3)}{[8.31 \text{ J/(mole} \cdot \text{K)}]\ (310 \text{ K})} = 1.94 \times 10^{-2} \text{ mol}$$

The total number of molecules in the sample is nN_A, where N_A is Avogadro's number. Since the sample contains approximately 21% oxygen, the total number of oxygen molecules in the sample is $(0.21)nN_A$ or

$$(0.21)(1.94 \times 10^{-2} \text{ mol})(6.022 \times 10^{23} \text{ mol}^{-1}) = \boxed{2.5 \times 10^{21}}$$

16. ***REASONING AND SOLUTION*** Assuming constant temperature we have $P_1V_1 = P_2V_2$. The volume of the cylinder is V = Ah where A is the cross-sectional area of the tank and h the height of the air column. Since the cross-sectional area doesn't change, we have $P_1h_1 = P_2h_2$. Thus,

$$h_2 = h_1(P_1/P_2) = (0.80\text{ m})(2.0\text{ atm})/(6.0\text{ atm}) = \boxed{0.27\text{ m}}$$

17. ***REASONING*** According to the ideal gas law (Equation 14.1) $PV = nRT$. We need to put this in terms of the mass density, $\rho = m/V$, of the ideal gas. We can then set $P_{\text{He}} = P_{\text{Ne}}$, and solve the resulting expression for T_{Ne}, the temperature of the neon.

SOLUTION We begin by writing Equation 14.1 in terms of the mass density ρ for an ideal gas. Recall that the number n of moles of a substance is equal to its mass m in grams, divided by its mass per mole M. Therefore, $PV = mRT/M$, and we have $P = (m/V)RT/M = \rho RT/M$, where ρ is the same for each gas. Since the two gases have the same absolute pressures, $P_{\text{He}} = P_{\text{Ne}}$, or

$$\frac{\rho R T_{\text{He}}}{M_{\text{He}}} = \frac{\rho R T_{\text{Ne}}}{M_{\text{Ne}}}$$

The term ρR can be eliminated algebraically from this result. Solving for the temperature of the neon T_{Ne} and using the mass per mole for helium (4.0026 g/mol) and neon (20.179 g/mol), we find

$$T_{\text{Ne}} = T_{\text{He}}\left(\frac{M_{\text{Ne}}}{M_{\text{He}}}\right) = (175\text{ K})\left(\frac{20.179\text{ g/mol}}{4.0026\text{ g/mol}}\right) = \boxed{882\text{ K}}$$

18. ***REASONING*** According to Equation 14.2, $PV = NkT$, where P is the pressure, V is the volume, N is the number of molecules in the sample, k is Boltzmann's constant, and T is the Kelvin temperature. The number of gas molecules per unit volume in the atmosphere is $N/V = P/(kT)$. This can be used to find the desired ratio for the two planets.

SOLUTION We have

$$\frac{(N/V)_{\text{Venus}}}{(N/V)_{\text{Earth}}} = \frac{P_{\text{Venus}}/(kT_{\text{Venus}})}{P_{\text{Earth}}/(kT_{\text{Earth}})} = \left(\frac{P_{\text{Venus}}}{P_{\text{Earth}}}\right)\left(\frac{T_{\text{Earth}}}{T_{\text{Venus}}}\right) = \left(\frac{9.0\times10^{6}\ \text{Pa}}{1.0\times10^{5}\ \text{Pa}}\right)\left(\frac{320\ \text{K}}{740\ \text{K}}\right) = \boxed{39}$$

Thus, we can conclude that the atmosphere of Venus is 39 times "thicker" than that of Earth.

19. **SSM** ***REASONING*** The graph that accompanies problem 71 in Chapter 12 can be used to determine the equilibrium vapor pressure of water in the air when the temperature is 30.0 °C (303 K). Equation 12.6 can then be used to find the partial pressure of water in the air at this temperature. Using this pressure, the ideal gas law can then be used to find the number of moles of water vapor per cubic meter.

SOLUTION According to the graph that accompanies problem 71 in Chapter 12, the equilibrium vapor pressure of water vapor at 30.0 °C is approximately 4250 Pa. According to Equation 12.6,

$$\begin{matrix}\text{Partial}\\ \text{pressure of}\\ \text{water vapor}\end{matrix} = \begin{pmatrix}\text{Percent relative}\\ \text{humidity}\end{pmatrix}\begin{pmatrix}\text{Equilibrium vapor pressure of}\\ \text{water at the existing temperature}\end{pmatrix} \times \frac{1}{100\%}$$

$$= \frac{(55\%)(4250\ \text{Pa})}{100\%} = 2.34 \times 10^3\ \text{Pa}$$

The ideal gas law then gives the number of moles of water vapor per cubic meter of air as

$$\frac{n}{V} = \frac{P}{RT} = \frac{(2.34 \times 10^3\ \text{Pa})}{[8.31\ \text{J/(mol} \cdot \text{K)}](303\ \text{K})} = \boxed{0.93\ \text{mol/m}^3} \qquad (14.1)$$

20. ***REASONING***

a. The volume V of gas in the tank is related to the Kelvin temperature T and absolute pressure P of the gas by Equation 14.1, $V = nRT / P$, where n is the number of moles of gas and R is the universal gas constant.

b. The mass of chlorine gas that has leaked out of the tank is equal to the initial mass (11.0 g) minus the mass remaining in the tank at the later time. The mass at the later time is equal to the mass per mole (in g/mol) of chlorine times the number n of moles remaining in the tank. The number of moles of gas remaining can be found from the ideal gas law, $n = PV / RT$ (Equation 14.1)

SOLUTION

a. The number n of moles of chlorine originally in the tank is equal to the mass of chlorine (11.0 g) divided by the mass per mole of chlorine (70.9 g/mol), so n = (11.0 g)/(70.9 g/mol). The temperature of the gas is 82 °C, which, when converted to the Kelvin temperature scale, is (82 + 273)K. The volume of the tank is

$$V = \frac{nRT}{P} = \frac{\left(\frac{11.0\text{ g}}{70.9\text{ g/mol}}\right)\left[8.31\text{ J/}(\text{mol}\cdot\text{K})\right]\left[(273+82)\text{K}\right]}{5.60\times10^{5}\text{ Pa}} = \boxed{8.17\times10^{-4}\text{ m}^3} \qquad (14.1)$$

b. The mass m_{Cl} of chlorine gas that has leaked out of the tank is m_{Cl} = 11.0 g – m_R, where m_R is the mass of the gas remaining in the tank. The remaining mass, in turn, is equal to the number n_R of moles of gas times the mass per mole (70.9 g/mol) of chlorine. Thus,

$$m_{Cl} = 11.0\text{ g} - m_R = 11.0\text{ g} - n_R\,(70.9\text{ g/mol})$$

The number of remaining moles of gas can be found from the ideal gas law, $n_R = PV / RT$, so that

$$m_{Cl} = 11.0\text{ g} - \left(\frac{PV}{RT}\right)(70.9\text{ g/mol})$$

$$= 11.0\text{ g} - \frac{\left(3.80\times10^{5}\text{ Pa}\right)\left(8.17\times10^{-4}\text{ m}^3\right)}{\left[8.31\text{ J/}(\text{mol}\cdot\text{K})\right]\left[(273+31)\text{K}\right]}(70.9\text{ g/mol}) = \boxed{2.3\text{ g}}$$

21. ***REASONING AND SOLUTION*** Find the number of moles of gas present for each case.

$$n_1 = \frac{PV}{RT_1} = \frac{(1.01 \text{ X } 10^5 \text{ Pa})(60.0 \text{ m}^3)}{R(289 \text{ K})}$$

$$n_2 = \frac{PV}{RT_2} = \frac{(1.01 \text{ X } 10^5 \text{ Pa})(60.0 \text{ m}^3)}{R(302 \text{ K})}$$

The number of moles lost is $n_1 - n_2$. The mass that is lost is, therefore,

$$m_{lost} = (n_1 - n_2)(28.01 \text{ g/mol}) = 3.0 \times 10^3 \text{ g} = \boxed{3.0 \text{ kg}}$$

22. ***REASONING AND SOLUTION*** If the pressure at the surface is P_1 and the pressure at a depth h is P_2, we have that $P_2 = P_1 + \rho gh$. We also know that $P_1V_1 = P_2V_2$. Then,

$$\frac{V_1}{V_2} = \frac{P_2}{P_1} = \frac{P_1 + \rho gh}{P_1} = 1 + \frac{\rho gh}{P_1}$$

Therefore,

$$\frac{V_1}{V_2} = 1 + \frac{(1.000 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(0.200 \text{ m})}{(1.01 \times 10^5 \text{ Pa})} = \boxed{1.02}$$

23. **SSM** **WWW** ***REASONING*** Since the temperature of the confined air is constant, Boyle's law applies, and $P_{\text{surface}}V_{\text{surface}} = P_h V_h$, where P_{surface} and V_{surface} are the pressure and volume of the air in the tank when the tank is at the surface of the water, and P_h and V_h are the pressure and volume of the trapped air after the tank has been lowered a distance h below the surface of the water. Since the tank is completely filled with air at the surface, V_{surface} is equal to the volume V_{tank} of the tank. Therefore, the fraction of the tank's volume that is filled with air when the tank is a distance h below the water's surface is

$$\frac{V_h}{V_{\text{tank}}} = \frac{V_h}{V_{\text{surface}}} = \frac{P_{\text{surface}}}{P_h}$$

We can find the absolute pressure at a depth h using Equation 11.4. Once the absolute pressure is known at a depth h, we can determine the ratio of the pressure at the surface to the pressure at the depth h.

SOLUTION According to Equation 11.4, the trapped air pressure at a depth $h = 40.0\text{ m}$ is

$$P_{\text{h}} = P_{\text{surface}} + \rho g h = (1.01\times10^5\text{ Pa}) + \left[(1.00\times10^3\text{ kg/m}^3)(9.80\text{ m/s}^2)(40.0\text{ m})\right]$$

$$= 4.93\times10^5\text{ Pa}$$

where we have used a value of $\rho = 1.00\times10^3\text{ kg/m}^3$ for the density of water. The desired volume fraction is

$$\frac{V_h}{V_{\text{tank}}} = \frac{P_{\text{surface}}}{P_h} = \frac{1.01\times10^5\text{ Pa}}{4.93\times10^5\text{ Pa}} = \boxed{0.205}$$

24. ***REASONING AND SOLUTION***

a. Since the heat gained by the gas in one tank is equal to the heat lost by the gas in the other tank, $Q_1 = Q_2$, or (letting the subscript 1 correspond to the neon in the left tank, and letting 2 correspond to the neon in the right tank) $cm_1\Delta T_1 = cm_2\Delta T_2$,

$$cm_1(T - T_1) = cm_2(T_2 - T)$$

$$m_1(T - T_1) = m_2(T_2 - T)$$

Solving for T gives

$$T = \frac{m_2T_2 + m_1T_1}{m_2 + m_1} \qquad (1)$$

The masses m_1 and m_2 can be found by first finding the number of moles n_1 and n_2. From the ideal gas law, $PV = nRT$, so

$$n_1 = \frac{P_1V_1}{RT_1} = \frac{(5.0\times10^5\ \text{Pa})(2.0\ \text{m}^3)}{[8.31\ \text{J/(mol}\cdot\text{K)}](220\ \text{K})} = 5.5\times10^2\ \text{mol}$$

This corresponds to a mass $m_1 = (5.5\times10^2\ \text{mol})(20.179\ \text{g/mol}) = 1.1\times10^4\ \text{g} = 1.1\times10^1\ \text{kg}$. Similarly, $n_2 = 2.4 \times 10^2$ mol and $m_2 = 4.9 \times 10^3$ g = 4.9 kg. Substituting these mass values into Equation (1) yields

$$T = \frac{(4.9\ \text{kg})(580\ \text{K}) + (1.1\times10^1\ \text{kg})(220\ \text{K})}{(4.9\ \text{kg}) + (1.1\times10^1\ \text{kg})} = \boxed{3.3\times10^2\ \text{K}}$$

b. From the ideal gas law,

$$P = \frac{nRT}{V} = \frac{[(5.5\times10^2\ \text{mol}) + (2.4\times10^2\ \text{mol})][8.31\ \text{J/(mol}\cdot\text{K)}](3.3\times10^2\ \text{K})}{(2.0\ \text{m}^3 + 5.8\ \text{m}^3)} = \boxed{2.8\times10^5\ \text{Pa}}$$

25. ***REASONING AND SOLUTION*** At the instant just before the balloon lifts off, the buoyant force from the outside air has a magnitude that equals the magnitude of the total weight. According to Archimedes' principle, the buoyant force is the weight of the displaced outside air (density $\rho_0 = 1.29 \times 10^3$ kg/m^3). The mass of the displaced outside air is $\rho_0 V$, where $V = 650$ m^3. The corresponding weight is the mass times the magnitude g of the acceleration due to gravity. Thus, we have

$$\underbrace{(\rho_0 V)g}_{\text{Buoyant force}} = \underbrace{m_{\text{total}} g}_{\substack{\text{Total weight}\\ \text{of balloon}}} \qquad (1)$$

The total mass of the balloon is $m_{\text{total}} = m_{\text{load}} + m_{\text{air}}$, where $m_{\text{load}} = 320$ kg and m_{air} is the mass of the hot air within the balloon. The mass of the hot air can be calculated from the ideal gas law by using it to obtain the number of moles n of air and multiplying n by the mass per mole of air, $M = 29 \times 10^{-3}$ kg/mol:

$$m_{\text{air}} = nM = \left(\frac{PV}{RT}\right)M$$

Thus, the total mass of the balloon is $m_{\text{total}} = m_{\text{load}} + PVM/(RT)$ and Equation (1) becomes

$$\rho_0 V = m_{\text{load}} + \left(\frac{PV}{RT}\right)M$$

Solving for T gives

$$T = \frac{PVM}{(\rho_0 V - m_{\text{load}})R}$$

$$= \frac{(1.01\times10^5\ \text{Pa})(650\ \text{m}^3)(29\times10^{-3}\ \text{kg/mol})}{\left[(1.29\times10^3\ \text{kg/m}^3)(650\ \text{m}^3) - 320\ \text{kg}\right]\left[8.31\ \text{J/(mol}\cdot\text{K)}\right]} = \boxed{440\ \text{K}}$$

26. ***REASONING AND SOLUTION*** The volume of the cylinder is V = AL where A is the cross- sectional area of the piston and L is the length. We know $P_1V_1 = P_2V_2$ so that the new pressure, P_2 can be found. We have

$$P_2 = P_1(V_1/V_2) = P_1A_1L_1/A_2L_2 = P_1L_1/L_2 \quad (\text{since } A_1 = A_2)$$

$$P_2 = (1.01 \times 10^5 \text{ Pa})(L/2L) = 5.05 \times 10^4 \text{ Pa}$$

The force on the piston and spring is, therefore,

$$F = P_2A = (5.05 \times 10^4 \text{ Pa})\pi(0.0500 \text{ m})^2 = 397 \text{ N}$$

The spring constant is k = F/x so

$$k = F/x = (397 \text{ N})/(0.200 \text{ m}) = \boxed{1.98\times10^3 \text{ N/m}}$$

27. $\boxed{\text{SSM}}$ ***REASONING*** According to the ideal gas law (Equation 14.1), $PV = nRT$. Since n, the number of moles of the gas, is constant, $n_1R = n_2R$. Therefore, $P_1V_1 / T_1 = P_2V_2 / T_2$, where $T_1 = 273$ K and T_2 is the temperature we seek. Since the beaker is cylindrical, the volume V of the gas is equal to Ad, where A is the cross-sectional area of the cylindrical volume and d is the height of the region occupied by the gas, as measured from the bottom of the beaker. With this substitution for the volume, the expression obtained from the ideal gas law becomes

$$\frac{P_1 d_1}{T_1} = \frac{P_2 d_2}{T_2} \qquad (1)$$

where the pressures P_1 and P_2 are equal to the sum of the atmospheric pressure and the pressure caused by the mercury in each case. These pressures can be determined using Equation 11.4. Once the pressures are known, Equation (1) can be solved for T_2.

SOLUTION Using Equation 11.4, we obtain the following values for the pressures P_1 and P_2. Note that the initial height of the mercury is $h_1 = (1.520\text{ m})/2 = 0.760\text{ m}$, while the final height of the mercury is $h_2 = (1.520\text{ m})/4 = 0.380\text{ m}$.

$$P_1 = P_0 + \rho g h_1 = (1.01 \times 10^5\text{ Pa}) + \left[(1.36 \times 10^4\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.760\text{ m})\right] = 2.02 \times 10^5\text{ Pa}$$

$$P_2 = P_0 + \rho g h_2 = (1.01 \times 10^5\text{ Pa}) + \left[(1.36 \times 10^4\text{ kg/m}^3)(9.80\text{ m/s}^2)(0.380\text{ m})\right] = 1.52 \times 10^5\text{ Pa}$$

In these pressure calculations, the density of mercury is $\rho = 1.36 \times 10^4\text{ kg/m}^3$. In Equation (1) we note that $d_1 = 0.760\text{ m}$ and $d_2 = 1.14\text{ m}$. Solving Equation (1) for T_2 and substituting values, we obtain

$$T_2 = \left(\frac{P_2 d_2}{P_1 d_1}\right) T_1 = \left[\frac{(1.52 \times 10^5\text{ Pa})(1.14\text{ m})}{(2.02 \times 10^5\text{ Pa})(0.760\text{ m})}\right](273\text{ K}) = \boxed{308\text{ K}}$$

28. ***REASONING AND SOLUTION*** From Equation 14.5 $\left[PV = \frac{2}{3}N(\overline{KE})\right]$, we obtain

$$\overline{KE} = \frac{3PV}{2N} = \frac{3(4.5\times10^{5}\ \text{Pa})(8.5\times10^{-3}\ \text{m}^{3})}{2\left[(2.0\ \text{mol})(6.022\times10^{23}\ \text{mol}^{-1})\right]} = \boxed{4.8\times10^{-21}\ \text{J}}$$

29. ***REASONING AND SOLUTION*** Using $\overline{KE} = (1/2)\, mv_{rms}^2 = (3/2)\, kT$, we can solve for v_{rms}.

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23}\ \text{J/K})(6.0 \times 10^{3}\ \text{K})}{1.67 \times 10^{-27}\ \text{kg}}} = \boxed{1.2 \times 10^{4}\ \text{m/s}} \qquad (14.6)$$

30. ***REASONING*** The average kinetic energy per molecule is proportional to the Kelvin temperature of the carbon dioxide gas. This relation is expressed by Equation 14.6 as $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$, where m is the mass of a carbon dioxide molecule. The mass m is equal to the molecular mass of carbon dioxide (44.0 u), expressed in kilograms.

SOLUTION Solving Equation 14.6 for the temperature of the gas, we have

$$T = \frac{mv_{\text{rms}}^2}{3k} = \frac{(44.0\text{ u})\left(\frac{1.66\times10^{-27}\text{ kg}}{1\text{ u}}\right)(650\text{ m/s})^2}{3\left(1.38\times10^{-23}\text{ J/K}\right)} = \boxed{750\text{ K}}$$

31. **SSM** ***REASONING AND SOLUTION*** Using the expressions for $\overline{v^2}$ and $(\overline{v})^2$ given in the statement of the problem, we obtain:

a. $$\overline{v^2} = \frac{1}{3}(v_1^2 + v_2^2 + v_3^2) = \frac{1}{3}\left[(3.0\text{ m/s})^2 + (7.0\text{ m/s})^2 + (9.0\text{ m/s})^2\right] = \boxed{46.3\text{ m}^2/\text{s}^2}$$

b. $$\left(\overline{v}\right)^2 = \left[\tfrac{1}{3}(v_1 + v_2 + v_3)\right]^2 = \left[\tfrac{1}{3}(3.0\text{ m/s} + 7.0\text{ m/s} + 9.0\text{ m/s})\right]^2 = \boxed{40.1\text{ m}^2/\text{s}^2}$$

$\overline{v^2}$ and $(\overline{v})^2$ are *not* equal, because they are two different physical quantities.

32. ***REASONING*** We know the number n of moles, the volume V, the rms speed v_{rms} of the molecules, and the mass m of each one. We may use Equation 14.5 directly to find the absolute pressure of the gas. First, however, we must determine the number of oxygen molecules in the container from a knowledge of the number of moles present.

SOLUTION The number N of oxygen molecules in the container is equal to the number n of moles times the number of molecules per mole (Avogadro's number): $N = nN_A =$ $(3.00 \text{ mol})(6.022 \times 10^{22} \text{ mol}^{-1})$. The absolute pressure exerted on the wall is

$$P = \left(\frac{N}{3}\right)\left(\frac{mv_{\text{rms}}^2}{V}\right) \tag{14.5}$$

$$= \left[\frac{(3.00 \text{ mol})(6.022 \times 10^{23} \text{ mol}^{-1})}{3}\right]\left[\frac{(5.31 \times 10^{-26} \text{ kg})(483 \text{ m/s})^2}{0.0247 \text{ m}^3}\right] = \boxed{3.02 \times 10^5 \text{ Pa}}$$

33. **SSM** ***REASONING*** The internal energy of the neon at any Kelvin temperature T is given by Equation 14.7, $U = (3/2)nRT$. Therefore, when the temperature of the neon increases from an initial temperature T_i to a final temperature T_f, the internal energy of the neon increases by an amount

$$\Delta U = U_f - U_i = \frac{3}{2} nR\,(T_f - T_i)$$

In order to use this equation, we must first determine n. Since the neon is confined to a tank, the number of moles n is constant, and we can use the information given concerning the initial conditions to determine an expression for the quantity nR. According to the ideal gas law (Equation 14.1),

$$nR = \frac{P_i V_i}{T_i}$$

These two expressions can be combined to obtain an equation in terms of the variables that correspond to the data given in the problem statement.

SOLUTION Combining the two expressions and substituting the given values yields

$$\Delta U = \frac{3}{2}\left(\frac{P_i V_i}{T_i}\right)(T_f - T_i)$$

$$= \frac{3}{2}\left[\frac{(1.01\times 10^5\ \text{Pa})(680\ \text{m}^3)}{(293.2\ \text{K})}\right](294.3\ \text{K} - 293.2\ \text{K}) = \boxed{3.9\times 10^5\ \text{J}}$$

34. ***REASONING*** The behavior of the molecules is described by Equation 14.5: $PV = \frac{2}{3}N(\frac{1}{2}mv_{\text{rms}}^2)$. Since the pressure and volume of the gas are kept constant, while the number of molecules is doubled, we can write $P_1V_2 = P_2V_2$, where the subscript 1 refers to the initial condition, and the subscript 2 refers to the conditions after the number of molecules is doubled. Thus,

$$\tfrac{2}{3}N_1\left[\tfrac{1}{2}m(v_{\text{rms}})_1^2\right] = \tfrac{2}{3}N_2\left[\tfrac{1}{2}m(v_{\text{rms}})_2^2\right] \quad \text{or} \quad N_1(v_{\text{rms}})_1^2 = N_2(v_{\text{rms}})_2^2$$

The last expression can be solved for $(v_{\text{rms}})_2$, the final translational rms speed.

SOLUTION Since the number of molecules is doubled, $N_2 = 2N_1$. Solving the last expression above for $(v_{\text{rms}})_2$, we find

$$(v_{\text{rms}})_2 = (v_{\text{rms}})_1\sqrt{\frac{N_1}{N_2}} = (463\text{ m/s})\sqrt{\frac{N_1}{2N_1}} = \frac{463\text{ m/s}}{\sqrt{2}} = \boxed{327\text{ m/s}}$$

35. ***REASONING AND SOLUTION*** Since the argon atom has a kinetic energy equal to its average translational kinetic energy,

$$\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kT \qquad (1)$$

Using conservation of mechanical energy, we have $E_{\text{earth's surface}} = E_{\text{highest altitude}}$, or $\frac{1}{2}mv_{rms}^2 = mgh$, where the gravitational potential energy near the earth's surface has been chosen to be zero. Using Equation (1), this statement of energy conservation becomes $\frac{3}{2}kT = mgh$. Solving for h gives

$$h = \frac{3kT}{2mg} \qquad (2)$$

In order to use Equation (2), we must find the mass of the argon atom:

$$m = \frac{39.948 \text{ g/mol}}{6.022 \times 10^{23} \text{ mol}^{-1}} = 6.634 \times 10^{-23} \text{ g} = 6.634 \times 10^{-26} \text{ kg}$$

Substituting into Equation (2) gives

$$h = \frac{3(1.38 \times 10^{-23} \text{ J/K})(295 \text{ K})}{2(6.634 \times 10^{-26} \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{9400 \text{ m}}$$

36. ***REASONING AND SOLUTION*** Since we are treating the air as an ideal gas, PV = nRT, so that $U = \frac{5}{2}nRT = \frac{5}{2}PV = \frac{5}{2}(7.7\times10^{6}\ \text{Pa})(5.6\times10^{5}\ \text{m}^{3}) = 1.1\times10^{13}\ \text{J}$. The number of joules of energy consumed per day by one house is

$$30.0\ \text{kW}\cdot\text{h} = \left(30.0\times10^{3}\ \frac{\text{J}\cdot\text{h}}{\text{s}}\right)\left(\frac{3600\ \text{s}}{1\ \text{h}}\right) = 1.08\times10^{8}\ \text{J}$$

The number of homes that could be served for one day by 1.1×10^{13} J of energy is

$$\left(1.1\times10^{13}\ \text{J}\right)\left(\frac{1\ \text{home}}{1.08\times10^{8}\ \text{J}}\right) = \boxed{1.0\times10^{5}\ \text{homes}}$$

37. **SSM** **WWW** ***REASONING AND SOLUTION*** The average force exerted by one electron on the screen is, from the impulse-momentum theorem (Equation 7.4), $\overline{F} = \Delta p / \Delta t = m\Delta v / \Delta t$. Therefore, in a time Δt, N electrons exert an average force $\overline{F} = Nm\Delta v / \Delta t = (N / \Delta t) m\Delta v$. Since the pressure on the screen is the average force per unit area (Equation 10.19), we have

$$P = \frac{\overline{F}}{A} = \frac{(N/\Delta t)m\Delta v}{A}$$

$$= \frac{(6.2 \times 10^{16} \text{ electrons/s})(9.11 \times 10^{-31} \text{ kg})(8.4 \times 10^{7} \text{ m/s} - 0 \text{ m/s})}{1.2 \times 10^{-7} \text{ m}^2} = \boxed{4.0 \times 10^{1} \text{ Pa}}$$

38. ***REASONING*** When perspiration absorbs heat from the body, the perspiration vaporizes. The amount Q of heat required to vaporize a mass $m_{\text{perspiration}}$ of perspiration is given by Equation 12.5 as $Q = m_{\text{perspiration}}L_{\text{v}}$, where L_{v} is the latent heat of vaporization for water at body temperature. The average energy $\bar{E}$ given to a single water molecule is equal to the heat Q divided by the number N of water molecules.

SOLUTION Since $\bar{E} = Q/N$ and $Q = m_{\text{perspiration}}L_{\text{v}}$, we have

$$\bar{E} = \frac{Q}{N} = \frac{m_{\text{perspiration}}L_{\text{v}}}{N}$$

But the mass of perspiration is equal to the mass $m_{\text{H}_2\text{O molecule}}$ of a single water molecule times the number N of water molecules. The mass of a single water molecule is equal to its molecular mass (18.0 u), converted into kilograms. The average energy given to a single water molecule is

$$\bar{E} = \frac{m_{\text{perspiration}}L_{\text{v}}}{N} = \frac{m_{\text{H}_2\text{O molecule}}N\,L_{\text{v}}}{N}$$

$$\bar{E} = m_{\text{H}_2\text{O molecule}}L_{\text{v}} = (18.0\text{ u})\left(\frac{1.66\times10^{-27}\text{ kg}}{1\text{ u}}\right)\left(2.42\times10^{6}\text{ J/kg}\right) = \boxed{7.23\times10^{-20}\text{ J}}$$

39. ***REASONING AND SOLUTION*** According to Fick's law, we have

$$t = \frac{mL}{DA\Delta C} = \frac{\left(8.0\times10^{-13}\ \text{kg}\right)\left(0.015\ \text{m}\right)}{\left(5.0\times10^{-10}\ \text{m}^2/\text{s}\right)\left(7.0\times10^{-4}\ \text{m}^2\right)\left(3.0\times10^{-3}\ \text{kg/m}^3\right)} = \boxed{11\ \text{s}}$$

40. ***REASONING AND SOLUTION*** According to Fick's law, $m = DA(\Delta C)t/L$, where $m/t = 6.00 \times 10^{-14}$ kg/s and the cross-sectional area $A = \pi r^2 = 6.16 \times 10^{-4}$ m^2. Therefore,

$$\frac{\Delta C}{L} = \frac{m/t}{DA} = \frac{6.00 \times 10^{-14}\ \text{kg/s}}{\left(10.6 \times 10^{-10}\ \text{m}^2/\text{s}\right)\left(6.16 \times 10^{-4}\ \text{m}^2\right)} = \boxed{0.0919\ \text{kg/m}^4}$$

41. **SSM** ***REASONING AND SOLUTION*** According to Fick's law of diffusion (Equation 14.8), the mass of ethanol that diffuses through the cylinder in one hour (3600 s) is

$$m = \frac{(DA\,\Delta C)t}{L} = \frac{(12.4\times10^{-10}\ \mathrm{m^2/s})(4.00\times10^{-4}\ \mathrm{m^2})(1.50\ \mathrm{kg/m^3})(3600\ \mathrm{s})}{(0.0200\ \mathrm{m})} = \boxed{1.34\times10^{-7}\ \mathrm{kg}}$$

42. ***REASONING AND SOLUTION*** Fick's law of diffusion gives

$$v = \frac{L}{t} = \frac{D\,A\,\Delta C}{m} = \frac{\left(4.2\times10^{-5}\ \text{m}^2/\text{s}\right)\left(4.0\times10^{-4}\ \text{m}^2\right)\left(3.5\times10^{-2}\ \text{kg/m}^3\right)}{8.4\times10^{-8}\ \text{kg}} = \boxed{7.0\times10^{-3}\ \text{m/s}}$$

43. ***REASONING AND SOLUTION***

a. As stated, the time required for the first solute molecule to traverse a channel of length L is $t = L^2/(2D)$. Therefore, for water vapor in air at 293 K, where the diffusion constant is $D = 2.4 \times 10^{-5}\ \text{m}^2/\text{s}$, the time t required for the first water molecule to travel $L = 0.010\ \text{m}$ is

$$t = \frac{L^2}{2D} = \frac{(0.010\ \text{m})^2}{2(2.4 \times 10^{-5}\ \text{m}^2/\text{s})} = \boxed{2.1\ \text{s}}$$

b. If a water molecule were traveling at the translational rms speed v_{rms} for water, the time t it would take to travel the distance $L = 0.010\ \text{m}$ would be given by $t = L/v_{\text{rms}}$, where, according to Equation 14.6 ($\overline{\text{KE}} = \frac{1}{2}mv_{\text{rms}}^2$), $v_{\text{rms}} = \sqrt{2(\overline{\text{KE}})/m}$. Before we can use the last expression for the translation rms speed v_{rms}, we must determine the mass m of a water molecule and the average translational kinetic energy $\overline{\text{KE}}$.

Using the periodic table on the inside of the text's back cover, we find that the molecular mass of a water molecule is

$$\underbrace{2(1.00794\ \text{u})}_{\substack{\text{Mass of two}\\ \text{hydrogen atoms}}} + \underbrace{15.9994\ \text{u}}_{\substack{\text{Mass of one}\\ \text{oxygen atom}}} = 18.0153\ \text{u}$$

The mass of a single molecule is

$$m = \frac{18.0153 \times 10^{-3}\ \text{kg/mol}}{6.022 \times 10^{23}\ \text{mol}^{-1}} = 2.99 \times 10^{-26}\ \text{kg}$$

The average translational kinetic energy of water molecules at 293 K is, according to Equation 14.6,

$$\overline{\text{KE}} = \tfrac{3}{2}kT = \tfrac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})(293\ \text{K}) = 6.07 \times 10^{-21}\ \text{J}$$

Therefore the translational rms speed of water molecules is

$$v_{\text{rms}} = \sqrt{\frac{2(\overline{\text{KE}})}{m}} = \sqrt{\frac{2(6.07 \times 10^{-21}\ \text{J})}{2.99 \times 10^{-26}\ \text{kg}}} = 637\ \text{m/s}$$

Thus, the time t required for a water molecule to travel the distance $L = 0.010\ \text{m}$ at this speed is

$$t = \frac{L}{v_{\text{rms}}} = \frac{0.010\ \text{m}}{637\ \text{m/s}} = \boxed{1.6 \times 10^{-5}\ \text{s}}$$

c. In part (a), when a water molecule diffuses through air, it makes millions of collisions each second with air molecules. The speed and direction changes abruptly as a result of each collision. Between collisions, the water molecules move in a straight line at constant speed. Although a water molecule does move very quickly between collisions, it wanders only very slowly in a zigzag path from one end of the channel to the other. In contrast, a water molecule traveling unobstructed at its translational rms speed [as in part (b)], will

have a larger displacement over a much shorter time. Therefore, the answer to part (a) is much longer than the answer to part (b).

44. ***REASONING AND SOLUTION***

a. The average concentration is $C_{av} = (1/2)(C_1 + C_2) = (1/2)C_2 = m/V = m/(AL)$, so that $C_2 = 2m/(AL)$. Fick's law then becomes $m = DAC_2t/L = DA(2m/AL)t/L = 2Dmt/L^2$. Solving for t yields

$$\boxed{t = L^2/(2D)}$$

b. Substituting into this expression yields

$$t = (2.5 \times 10^{-2}\ \text{m})^2/[2(1.0 \times 10^{-5}\ \text{m}^2/\text{s})] = \boxed{31\ \text{s}}$$

45. ***REASONING AND SOLUTION*** Equation 14.8, gives Fick's law of diffusion: $m = \frac{DA\Delta Ct}{L}$.

Solving for the time t gives

$$t = \frac{mL}{DA\Delta C} \qquad (1)$$

The time required for the water to evaporate is equal to the time it takes for 2.0 grams of water vapor to traverse the tube and can be calculated from Equation (1) above. Since air in the tube is completely dry at the right end, the concentration of water vapor is zero, $C_1 = 0\ \text{kg/m}^3$, and $\Delta C = C_2 - C_1 = C_2$. The concentration at the left end of the tube, C_2, is equal to the density of the water vapor above the water. This can be found from the ideal gas law:

$$PV = nRT \qquad \Rightarrow \qquad P = \frac{\rho RT}{M}$$

where M is the molecular mass of water vapor. From the figure that accompanies problem 71 in Chapter 12, the equilibrium vapor pressure of water at 20 °C is 2.4×10^3 Pa.

$$\therefore C_2 = \rho = \frac{PM}{RT} = \frac{(2.4 \times 10^3\ \text{Pa})(.0180152\ \text{kg/mol})}{[8.31\ \text{J/(mol}\cdot\text{K)}](293\ \text{K})} = 1.8 \times 10^{-2}\ \text{kg/m}^3$$

Substituting values into Equation (1) gives

$$t = \frac{(2.0 \times 10^{-3}\ \text{kg})(0.15\ \text{m})}{(2.4 \times 10^{-5}\ \text{m}^2/\text{s})(3.0 \times 10^{-4}\ \text{m}^2)(1.8 \times 10^{-2}\ \text{kg/m}^3)} = \boxed{2.3 \times 10^6\ \text{s}}$$

This is about 27 days!

46. ***REASONING AND SOLUTION*** PV = nRT, where T = (27 + 273)K. Therefore, $n = PV/RT = 9.6 \times 10^{-11}$ mol, and the number of molecules is

$$N = nN_A = (9.6 \times 10^{-11}\ \text{mol})(6.022 \times 10^{23}\ \text{mol}^{-1}) = \boxed{5.8 \times 10^{13}}$$

47. **SSM** ***REASONING AND SOLUTION*** The number of moles of air that is pumped into the tire is $\Delta n = n_f - n_i$. According to the ideal gas law (Equation 14.1), $n = PV/(RT)$; therefore,

$$\Delta n = n_f - n_i = \frac{P_f V_f}{RT_f} - \frac{P_i V_i}{RT_i} = \frac{V}{RT}\left(P_f - P_i\right)$$

Substitution of the data given in the problem leads to

$$\Delta n = \frac{4.1 \times 10^{-4}\ \text{m}^3}{\left[8.31\ \text{J/(mol}\cdot\text{K)}\right](296\ \text{K})}\left[\left(6.2 \times 10^5\ \text{Pa}\right) - \left(4.8 \times 10^5\ \text{Pa}\right)\right] = \boxed{2.3 \times 10^{-2}\ \text{mol}}$$

48. ***REASONING*** Since the absolute pressure, volume, and temperature are known, we may use the ideal gas law in the form of Equation 14.1 to find the number of moles of gas. When the volume and temperature are raised, the new pressure can also be determined by using the ideal gas law.

SOLUTION

a. The number of moles of gas is

$$n = \frac{PV}{RT} = \frac{\left(1.72 \times 10^5 \text{ Pa}\right)\left(2.81 \text{ m}^3\right)}{\left[8.31 \text{ J/}\left(\text{mol} \cdot \text{K}\right)\right]\left[\left(273.15 + 15.5\right) \text{ K}\right]} = \boxed{201 \text{ mol}} \qquad (14.1)$$

b. When the volume is raised to 4.16 m^3 and the temperature raised to 28.2 °C, the pressure of the gas is

$$P = \frac{nRT}{V} = \frac{\left(201 \text{ mol}\right)\left[8.31 \text{ J/}\left(\text{mol} \cdot \text{K}\right)\right]\left[\left(273.15 + 28.2\right) \text{ K}\right]}{\left(4.16 \text{ m}^3\right)} = \boxed{1.21 \times 10^5 \text{ Pa}} \qquad (14.1)$$

49. **SSM** **WWW** ***REASONING*** Since mass is conserved, the mass flow rate is the same at all points, as described by the equation of continuity (Equation 11.8). Therefore, the mass flow rate at which CCl_4 enters the tube is the same as that at point A. The concentration difference of CCl_4 between point A and the left end of the tube, ΔC, can be calculated by using Fick's law of diffusion (Equation 14.8). The concentration of CCl_4 at point A can be found from $C_A = C_{\text{left end}} - \Delta C$.

SOLUTION

a. As discussed above in the reasoning, the mass flow rate of CCl_4 as it passes point A is the same as the mass flow rate at which CCl_4 enters the left end of the tube; therefore, the mass flow rate of CCl_4 at point A is $\boxed{5.00 \times 10^{-13}\ \text{kg/s}}$.

b. Solving Fick's law for ΔC, we obtain

$$\Delta C = \frac{mL}{DAt} = \frac{(m/t)L}{DA}$$

$$= \frac{(5.00\times10^{-13}\ \text{kg/s})(5.00\times10^{-3}\ \text{m})}{(20.0\times10^{-10}\ \text{m}^2/\text{s})(3.00\times10^{-4}\ \text{m}^2)} = 4.2\times10^{-3}\ \text{kg/m}^3$$

Then,

$$C_A = C_{\text{left end}} - \Delta C = (1.00\times10^{-2}\ \text{kg/m}^3) - (4.2\times10^{-3}\ \text{kg/m}^3) = \boxed{5.8\times10^{-3}\ \text{kg/m}^3}$$

50. ***REASONING***

According to Equation 14.6, the average kinetic energy per particle in an ideal gas is directly proportional to the Kelvin temperature of the gas, $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$.

SOLUTION Solving this relation for the mass gives

$$m = \frac{3kT}{v_{\text{rms}}^2} = \frac{3\left(1.38\times10^{-23}\ \text{J/K}\right)\left(0+273\right)\text{K}}{\left(1300\ \text{m/s}\right)^2} = 6.69\times10^{-27}\ \text{kg}$$

The mass in atomic mass units is

$$\left(6.69\times10^{-27}\ \text{kg}\right)\left(\frac{1\ \text{u}}{1.66\times10^{-27}\ \text{kg}}\right) = \boxed{4.0\ \text{u}}$$

Consulting the periodic table, we see that the element whose atomic mass is 4.0 u is $\boxed{\text{helium}}$.

51. ***REASONING AND SOLUTION*** To find the rms-speed of the CO_2 we need to first find the temperature. We can do this since we know the rms-speed of the H_2O molecules. Using $(1/2)\,mv_{rms}^2 = (3/2)kT$, we can solve for the temperature to get $T = mv_{rms}^2/(3k)$ where the mass of an H_2O molecule is $(18.015\text{ g/mol})/(6.022 \times 10^{23}\text{ mol}^{-1}) = 2.99 \times 10^{-23}$ g.

Thus, $T = (2.99 \times 10^{-26}\text{ kg})(648\text{ m/s})^2/[3(1.38 \times 10^{-23}\text{ J/K})] = 303$ K. The molecular mass of CO_2 is 44.01 u, hence the mass of a CO_2 molecule is 7.31×10^{-26} kg.

The rms-speed for CO_2 is

$$v_{rms} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23}\text{ J/K})(303\text{ K})}{7.31 \times 10^{-26}\text{ kg}}} = \boxed{414\text{ m/s}}$$

52. ***REASONING AND SOLUTION*** From the drawing in the text we can see that the volume of the cylinder is $V = Ah$, where A is the cross-sectional area of the piston. Assuming that pressure is constant, $V_1/T_1 = V_2/T_2$ or $A_1h_1/T_1 = A_2h_2/T_2$. Since $A_1 = A_2$, we have

$$h_2 = h_1(T_2/T_1) = (0.120\text{ m})[(318\text{ K})/(273\text{ K})] = \boxed{0.140\text{ m}}$$

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53. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The number of moles of water in the drop is $n = m/M$, where m is the mass of the drop and M is the mass per mole of water. The mass of a drop of volume V and density ρ can be obtained from Equation 11.1 as $m = \rho V$. Thus, the number of molecules in the drop is

$$N = N_A n = \frac{N_A \rho V}{M}$$

where $N_A = 6.022 \times 10^{23}\ \text{mol}^{-1}$ is Avogadro's number. Since the drop is spherical, its volume is given by $V = \frac{4}{3}\pi r^3$, where r is the radius of the drop. Substituting the values for the data given in the problem, using a density of $\rho = 1.00 \times 10^3\ \text{kg/m}^3$ for water, and using $M = 18.015 \times 10^{-3}\ \text{kg/mol}$ for water yields

$$N = \frac{\left(6.022\times10^{23}\ \text{mol}^{-1}\right)\left(1.00\times10^{3}\ \dfrac{\text{kg}}{\text{m}^3}\right)\frac{4}{3}\pi\left(9.00\times10^{-4}\text{m}\right)^3}{18.015\times10^{-3}\ \dfrac{\text{kg}}{\text{mol}}} = \boxed{1.02\times10^{20}}$$

54. ***REASONING AND SOLUTION*** We know from the ideal gas law that $PV = nRT$ and $U = (3/2)nRT = (3/2)PV = 9300\ \text{J}$. A 0.25-hp engine provides (0.25 hp)(746 W)/(1 hp) = 187 J/s. In order to equal the internal energy, therefore, the engine must run for a time of

$$(9300\ \text{J})/(187\ \text{J/s}) = \boxed{5.0 \times 10^{1}\ \text{s}}$$

55. **SSM** ***REASONING AND SOLUTION*** Since the density of aluminum is $2700\ \text{kg/m}^3$, the number of atoms of aluminum per cubic meter is, using the given data,

$$N/V = \left(\frac{2700 \times 10^3\ \text{g/m}^3}{26.9815\ \text{g/mol}}\right)\left(6.022 \times 10^{23}\ \text{mol}^{-1}\right) = 6.0 \times 10^{28}\ \text{atoms/m}^3$$

Assuming that the volume of the solid contains many small cubes, with one atom at the center of each, then there are $(6.0 \times 10^{28})^{1/3}$ atoms/m $= 3.9 \times 10^9$ atoms/m along each edge of a 1.00-m^3 cube. Therefore, the spacing between the centers of neighboring atoms is

$$d = \frac{1}{3.9 \times 10^9\ \text{/m}} = \boxed{2.6 \times 10^{-10}\ \text{m}}$$

56. ***REASONING AND SOLUTION***

a. Assuming that the direction of travel of the bullets is positive, the average change in momentum per second is

$$\Delta p/\Delta t = m\Delta v/\Delta t = (200)(0.0050\text{ kg})[(0\text{ m/s}) - 1200\text{ m/s})]/(10.0\text{ s}) = \boxed{-120\text{ N}}$$

b. The average force exerted on the bullets is $\overline{F} = \Delta p/\Delta t$. According to Newton's third law, the average force exerted on the wall is $-\overline{F} = \boxed{120\text{ N}}$.

c. The pressure P is the magnitude of the force on the wall per unit area, so

$$P = (120\text{ N})/(3.0 \times 10^{-4}\text{ m}^2) = \boxed{4.0 \times 10^5\text{ Pa}}$$

57. ***REASONING AND SOLUTION*** We need to determine the amount of He inside the balloon. Begin by using Archimedes' principle; the balloon is being buoyed up by a force equal to the weight of the air displaced. The buoyant force, F_b, therefore, is equal to

$$F_b = mg = \rho\, Vg = (1.19\ \text{kg/m}^3)(4/3)\pi\,(1.50\ \text{m})^3(9.80\ \text{m/s}^2) = 164.9\ \text{N}$$

Since the balloon has a mass of 3.00 kg (weight = 29.4 N), the He inside the balloon weighs 164.9 N – 29.4 N = 135.5 N. Hence, the mass of the helium present in the balloon is m = 13.8 kg. Now we can determine the number of moles of He present in the balloon:

$$n = m/M = (13.8\ \text{kg})/(4.0026 \times 10^{-3}\ \text{kg/mol}) = 3450\ \text{mol}$$

Using the ideal gas law to find the pressure, we have

$$P = nRT/V = (3450\ \text{mol})[8.31\ \text{J/(mol·K)}](305\ \text{K})/[\,(4/3)\pi\,(1.50\ \text{m})^3] = \boxed{6.19 \times 10^5\ \text{Pa}}$$

1. SSM ***REASONING*** Since the change in the internal energy and the heat released in the process are given, the first law of thermodynamics (Equation 15.1) can be used to find the work done. Since we are told how much work is required to make the car go one mile, we can determine how far the car can travel. When the gasoline burns, its internal energy decreases and heat flows into the surroundings; therefore, both ΔU and Q are negative.

SOLUTION According to the first law of thermodynamics, the work that is done when one gallon of gasoline is burned in the engine is

$$W = Q - \Delta U = -1.00 \times 10^8 \text{ J} - (-1.19 \times 10^8 \text{ J}) = 0.19 \times 10^8 \text{ J}$$

Since 6.0×10^5 J of work is required to make the car go one mile, the car can travel

$$0.19 \times 10^8 \text{ J} \left(\frac{1 \text{ mile}}{6.0 \times 10^5 \text{ J}} \right) = \boxed{32 \text{ miles}}$$

2. ***REASONING*** Since the student does work, W is positive, according to our convention. Since his internal energy decreases, the change ΔU in the internal energy is negative. The first law of thermodynamics will allow us to determine the heat Q.

SOLUTION

a. The work is $\boxed{W = +1.6 \times 10^4 \text{ J}}$.

b. The change in internal energy is $\boxed{\Delta U = -4.2 \times 10^4 \text{ J}}$.

c. Applying the first law of thermodynamics from Equation 15.1, we find that

$$Q = \Delta U + W = \left(-4.2 \times 10^4 \text{ J}\right) + \left(1.6 \times 10^4 \text{ J}\right) = \boxed{-2.6 \times 10^4 \text{ J}}$$

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3. ***REASONING AND SOLUTION*** The change in the internal energy of the gas can be determined from Equation 14.7 as

$$\Delta U = \tfrac{3}{2} nR\Delta T = \tfrac{3}{2}(1.00\,\text{mol})[8.32\,\text{J/(mol}\cdot\text{K)}](550\,\text{K} - 350\,\text{K}) = 2500\,\text{J}$$

a. The heat can be found from the first law of thermodynamics:

$$Q = \Delta U + W = 2500\,\text{J} + (-6200\,\text{J}) = \boxed{-3700\,\text{J}}$$

b. Since Q is negative, $\boxed{\text{heat flows out of the gas}}$.

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4. ***REASONING*** Energy in the form of work leaves the system, while energy in the form of heat enters. More energy leaves than enters, so we expect the internal energy of the system to decrease, that is, we expect the change ΔU in the internal energy to be negative. The first law of thermodynamics will confirm our expectation. As far as the environment is concerned, we note that when the system loses energy, the environment gains it, and when the system gains energy the environment loses it. Therefore, the change in the internal energy of the environment must be opposite to that of the system.

SOLUTION

a. The system gains heat so Q is positive, according to our convention. The system does work, so W is also positive, according to our convention. Applying the first law of thermodynamics from Equation 15.1, we find for the system that

$$\Delta U = Q - W = (77\ \text{J}) - (164\ \text{J}) = \boxed{-87\ \text{J}}$$

As expected, this value is negative, indicating a decrease.

b. The change in the internal energy of the environment is opposite to that of the system, so that $\boxed{\Delta U_{\text{environment}} = +87\ \text{J}}$.

5. SSM ***REASONING AND SOLUTION*** In going from the initial state to the intermediate state, the internal energy of the system changes according to Equation 15.1:

$$\Delta U_1 = U_f - U_i = Q - W = +165 \text{ J} - (+312 \text{ J}) = -147 \text{ J}$$

Since internal energy is a function of state, the change in the internal energy in going from the initial state to the intermediate state and back to the initial state is zero. That is, $\Delta U_{\text{total}} = \Delta U_1 + \Delta U_2 = 0$. Therefore, in returning to the initial state, the change in the internal energy is $\Delta U_2 = -\Delta U_1 = +147 \text{ J}$.

a. Thus, the work involved is

$$W = Q - \Delta U_2 = -114 \text{ J} - (+147 \text{ J}) = \boxed{-261 \text{ J}}$$

b. Since the work is negative, $\boxed{\text{work is done on the system}}$.

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6. ***REASONING AND SOLUTION***

a. For the weight lifter

$$\Delta U = Q - W$$

$$= -mL_V - W = -(0.150\text{ kg})(2.42 \times 10^6\text{ J/kg}) - 1.40 \times 10^5\text{ J} = \boxed{-5.03 \times 10^5\text{ J}}$$

b. Since 1 nutritional calorie = 4186 J, the number of nutritional calories is

$$\left(5.03 \times 10^5\text{ J}\right)\left(\frac{1\text{ Calorie}}{4186\text{ J}}\right) = \boxed{1.20 \times 10^2\text{ nutritional calories}}$$

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7. ***REASONING*** The work done during an isobaric process is the product of the pressure P and the change in volume, $V_f - V_i$, where V_f and V_i are the final and initial volumes, respectively (see Equation 15.2).

SOLUTION

a. The work done when the volume is compressed is

$$W = P(V_f - V_i) = (1.5 \times 10^5 \text{ Pa})(2.0 \times 10^{-3} \text{ m}^3 - 7.0 \times 10^{-3} \text{ m}^3) = \boxed{-7.5 \times 10^2 \text{ J}}$$

b. The work done when the volume expands is

$$W = P(V_f - V_i) = (1.5 \times 10^5 \text{ Pa})(8.0 \times 10^{-3} \text{ m}^3 - 2.0 \times 10^{-3} \text{ m}^3) = \boxed{9.0 \times 10^2 \text{ J}}$$

8. ***REASONING*** We will apply the first law of thermodynamics as given in Equation 15.1 ($\Delta U = Q - W$) to the overall process. First, however, we add the changes in the internal energy to obtain the overall change ΔU and add the work values to get the overall work W.

SOLUTION In both steps the internal energy increases, so overall we have $\Delta U = 228\text{ J} + 115\text{ J} = +343\text{ J}$. In both steps the work is negative according to our convention, since it is done on the system. Overall, then, we have $W = -166\text{ J} - 177\text{ J} = -343\text{ J}$. Using the first law of thermodynamics from Equation 15.1, we find

$$\Delta U = Q - W \quad \text{or} \quad Q = \Delta U + W = (+343\text{ J}) + (-343\text{ J}) = \boxed{0\text{ J}}$$

Since the heat is zero, the overall process is $\boxed{\text{adiabatic}}$.

9. SSM ***REASONING*** The work done in the process is equal to the "area" under the curved line between A and B in the drawing. From the graph, we find that there are about 78 "squares" under the curve. Each square has an "area" of

$$(2.0\times10^{4}\ \text{Pa})(2.0\times10^{-3}\ \text{m}^{3})=4.0\times10^{1}\ \text{J}$$

SOLUTION

a. The work done in the process has a magnitude of

$$W=(78)(4.0\times10^{1}\ \text{J})=\boxed{3100\ \text{J}}$$

b. The final volume is smaller than the initial volume, so the gas is compressed. Therefore, work is done on the gas so the work is $\boxed{\text{negative}}$.

10. ***REASONING AND SOLUTION***

a. Starting at point A, the work done during the first (vertical) straight-line segment is

$$W_1 = P_1\Delta V_1 = P_1(0\ \text{m}^3) = 0\ \text{J}$$

For the second (horizontal) straight-line segment, the work is

$$W_2 = P_2\Delta V_2 = 10(1.0 \times 10^4\ \text{Pa})6(2.0 \times 10^{-3}\ \text{m}^3) = 1200\ \text{J}$$

For the third (vertical) straight-line segment the work is

$$W_3 = P_3\Delta V_3 = P_3(0\ \text{m}^3) = 0\ \text{J}$$

For the fourth (horizontal) straight-line segment the work is

$$W_4 = P_4\Delta V_4 = 15(1.0 \times 10^4\ \text{Pa})6(2.0 \times 10^{-3}\ \text{m}^3) = 1800\ \text{J}$$

The total work done is

$$W = W_1 + W_2 + W_3 + W_4 = \boxed{+3.0 \times 10^3\ \text{J}}$$

b. Since the total work is positive, work is done $\boxed{\text{by the system}}$.

11. ***REASONING AND SOLUTION*** The work done by the expanding gas is

$$W = Q - \Delta U = 2050\ \text{J} - 1730\ \text{J} = 320\ \text{J}$$

The work, according to Equation 6.1, is also the magnitude F of the force exerted on the piston times the magnitude s of its displacement. But the force is equal to the weight mg of the block and piston, so that the work is $W = Fs = mgs$. Thus, we have

$$s = \frac{W}{mg} = \frac{320\ \text{J}}{(135\ \text{kg})(9.80\ \text{m/s}^2)} = \boxed{0.24\ \text{m}}$$

---

12. ***REASONING*** For segment AB, there is no work, since the volume is constant. For segment BC the process is isobaric and Equation 15.2 applies. For segment CA, the work can be obtained as the area under the line CA in the graph.

SOLUTION

a. For segment *AB*, the process is isochoric, that is, the volume is constant. For a process in which the volume is constant, no work is done, so $\boxed{W = 0\ \text{J}}$.

b. For segment BC, the process is isobaric, that is, the pressure is constant. Here, the volume is increasing, so the gas is expanding against the outside environment. As a result, the gas does work, which is positive according to our convention. Using Equation 15.2 and the data in the drawing, we obtain

$$W = P\left(V_{\text{f}} - V_{\text{i}}\right)$$

$$= \left(7.0 \times 10^{5}\ \text{Pa}\right)\left[\left(5.0 \times 10^{-3}\ \text{m}^{3}\right) - \left(2.0 \times 10^{-3}\ \text{m}^{3}\right)\right] = \boxed{+2.1 \times 10^{3}\ \text{J}}$$

c. For segment CA, the volume of the gas is decreasing, so the gas is being compressed and work is being done on it. Therefore, the work is negative, according to our convention. The magnitude of the work is the area under the segment CA. We estimate that this area is 15 of the squares in the graphical grid. The area of each square is

$$(1.0 \times 10^{5}\ \text{Pa})(1.0 \times 10^{-3}\ \text{m}^{3}) = 1.0 \times 10^{2}\ \text{J}$$

The work, then, is

$$\text{W} = -\ 15\ (1.0 \times 10^{2}\ \text{J}) = \boxed{-1.5 \times 10^{3}\ \text{J}}$$

13. **SSM** ***REASONING*** According to Equation 15.2, $W = P\Delta V$, the average pressure $\overline{P}$ of the expanding gas is equal to $\overline{P} = W / \Delta V$, where the work W done by the gas on the bullet can be found from the work-energy theorem (Equation 6.3). Assuming that the barrel of the gun is cylindrical with radius r, the volume of the barrel is equal to its length L multiplied by the area (πr^2) of its cross section. Thus, the change in volume of the expanding gas is $\Delta V = L\pi r^2$.

SOLUTION The work done by the gas on the bullet is given by Equation 6.3 as

$$W = \tfrac{1}{2}m(v_{\text{final}}^2 - v_{\text{initial}}^2) = \tfrac{1}{2}(2.6\times10^{-3}\ \text{kg})[(370\ \text{m/s})^2 - 0] = 180\ \text{J}$$

The average pressure of the expanding gas is, therefore,

$$\overline{P} = \frac{W}{\Delta V} = \frac{180\ \text{J}}{(0.61\ \text{m})\pi(2.8\times10^{-3}\ \text{m})^2} = \boxed{1.2\times10^7\ \text{Pa}}$$

14. ***REASONING AND SOLUTION*** The rod's volume increases by an amount $\Delta V = \beta V_0 \Delta T$, according to Equation 12.3. The work done by the expanding aluminum is, from Equation 15.2,

$$W = P\Delta V = P\beta V_0 \Delta T = (1.01 \times 10^5 \text{ Pa})(69 \times 10^{-6}/\text{C}^\circ)(1.4 \times 10^{-3} \text{ m}^3)(3.0 \times 10^2 \text{ C}^\circ) = \boxed{2.9 \text{ J}}$$

15. SSM WWW ***REASONING AND SOLUTION*** The first law of thermodynamics states that $\Delta U = Q - W$. The work W involved in an isobaric process is, according to Equation 15.2, $W = P\Delta V$. Combining these two expressions leads to $\Delta U = Q - P\Delta V$. Solving for Q gives

$$Q = \Delta U + P\Delta V \tag{1}$$

Since this is an expansion, $\Delta V > 0$, so $P\Delta V > 0$. From the ideal gas law, $PV = nRT$, we have $P\Delta V = nR\Delta T$. Since $P\Delta V > 0$, it follows that $nR\Delta T > 0$. The internal energy of an ideal gas is directly proportional to its Kelvin temperature T. Therefore, since $nR\Delta T > 0$, it follows that $\Delta U > 0$. Since both terms on the right hand side of Equation (1) are positive, the left hand side of Equation (1) must also be positive. Thus, Q is positive. By the convention described in the text, this means that

$$\boxed{\text{heat can only flow into an ideal gas during an isobaric expansion}}$$

16. ***REASONING AND SOLUTION*** According to the first law of thermodynamics, the change in internal energy is $\Delta U = Q - W$. The work can be obtained from the area under the graph. There are sixty squares of area under the graph, so the positive work of expansion is

$$W = 60\left(1.0 \times 10^4 \text{ Pa}\right)\left(2.0 \times 10^{-3} \text{ m}^3\right) = 1200 \text{ J}$$

Since $Q = 2700$ J, the change in internal energy is

$$\Delta U = Q - W = 2700 \text{ J} - 1200 \text{ J} = \boxed{1500 \text{ J}}$$

17. ***REASONING AND SOLUTION*** Since the pan is open, the process takes place at constant (atmospheric) pressure P_0. The work involved in an isobaric process is given by Equation 15.2: $W = P_0\Delta V$. The change in volume of the liquid as it is heated is given according to Equation 12.3 as $\Delta V = \beta V_0 \Delta T$, where β is the coefficient of volume expansion. Table 12.1 gives $\beta = 207 \times 10^{-6}\ (C°)^{-1}$ for water. The heat absorbed by the water is given by Equation 12.4 as $Q = cm\Delta T$, where $c = 4186\ J/(kg \cdot C°)$ is the specific heat capacity of liquid water according to Table 12.2. Therefore,

$$\frac{W}{Q} = \frac{P_0 \Delta V}{cm\Delta T} = \frac{P_0 \beta V_0 \Delta T}{cm\Delta T} = \frac{P_0 \beta}{c(m/V_0)} = \frac{P_0 \beta}{c\rho}$$

where $\rho = 1.00 \times 10^3\ kg/m^3$ is the density of the water. Thus, we find

$$\frac{W}{Q} = \frac{P_0 \beta}{c\rho} = \frac{(1.01 \times 10^5\ Pa)(207 \times 10^{-6}\ C°^{-1})}{[4186\ J/(kg \cdot C°)](1.00 \times 10^3\ kg/m^3)} = \boxed{4.99 \times 10^{-6}}$$

18. ***REASONING AND SOLUTION*** For an isothermal process $Q = W = nRT \ln (V_f/V_i)$, where $V_f/V_i = 4.00$. Thus,

$$W = (3.00\ \text{mol})\left[8.31\ \text{J}/(\text{mol}\cdot\text{K})\right](373\ \text{K})\ln(4.00) = \boxed{1.29 \times 10^4\ \text{J}}$$

19. SSM ***REASONING*** According to the first law of thermodynamics (Equation 15.1), $\Delta U = Q - W$. For a monatomic ideal gas (Equation 14.7), $U = \frac{3}{2}nRT$. Therefore, for the process in question, the change in the internal energy is $\Delta U = \frac{3}{2}nR\Delta T$. Combining the last expression for ΔU with Equation 15.1 yields

$$\tfrac{3}{2}nR\Delta T = Q - W$$

This expression can be solved for ΔT.

SOLUTION

a. The heat is $Q = +1200\text{ J}$, since it is absorbed by the system. The work is $W = +2500\text{ J}$, since it is done *by* the system. Solving the above expression for ΔT and substituting the values for the data given in the problem statement, we have

$$\Delta T = \frac{Q - W}{\frac{3}{2}nR} = \frac{1200\text{ J} - 2500\text{ J}}{\frac{3}{2}(0.50\text{ mol})[8.31\text{ J}/(\text{mol}\cdot\text{K})]} = \boxed{-2.1\times10^{2}\text{ K}}$$

b. Since $\Delta T = T_{\text{final}} - T_{\text{initial}}$ is negative, T_{initial} must be greater than T_{final}; this change represents a $\boxed{\text{decrease}}$ in temperature.

Alternatively, one could deduce that the temperature decreases from the following physical argument. Since the system loses more energy in doing work than it gains in the form of heat, the internal energy of the system decreases. Since the internal energy of an ideal gas depends only on the temperature, a decrease in the internal energy must correspond to a decrease in the temperature.

20. ***REASONING AND SOLUTION*** As Section 14.1 discusses, the number of moles n is given by the mass m divided by the mass per mole:

$$n = \frac{m}{\text{Mass per mole}} = \frac{6.0\ \text{g}}{4.0\ \text{g/mol}} = 1.5\ \text{mol}$$

For an isothermal process we have (see Equation 15.3)

$$\ln\left(\frac{V_f}{V_i}\right) = \frac{W}{nRT} = \frac{9600\ \text{J}}{(1.5\ \text{mol})\left[8.31\ \text{J}/(\text{mol}\cdot\text{K})\right](370\ \text{K})} = 2.08$$

Therefore, $V_f/V_i = e^{2.08} = \boxed{8.0}$.

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21. ***REASONING AND SOLUTION*** For an adiabatic process with an ideal gas, $P_f V_f^{\gamma} = P_i V_i^{\gamma}$, as indicated by Equation 15.5. Therefore,

$$P_f / P_i = \left(V_i / V_f\right)^{\gamma} = (2)^{5/3} = \boxed{3.17}$$

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22. ***REASONING*** An adiabatic process is one for which no heat enters or leaves the system, so $Q = 0$ J. The work is given as $W = +610$ J, where the plus sign denotes that the gas does work, according to our convention. Knowing the heat and the work, we can use the first law of thermodynamics to find the change ΔU in internal energy as $\Delta U = Q - W$ (Equation 15.1). Knowing the change in the internal energy, we can find the change in the temperature by recalling that the internal energy of a monatomic ideal gas is $U = \frac{3}{2}nRT$, according to Equation 14.7. As a result, it follows that $\Delta U = \frac{3}{2}nR\Delta T$.

SOLUTION Using the first law from Equation 15.5 and the change in internal energy from Equation 14.7, we have

$$\Delta U = Q - W \quad \text{or} \quad \tfrac{3}{2}nR\Delta T = Q - W$$

Therefore, we find

$$\Delta T = \frac{2(Q-W)}{3nR} = \frac{2[(0\text{ J})-(610\text{ J})]}{3(0.50\text{ mol})[8.31\text{ J}/(\text{mol}\cdot\text{K})]} = \boxed{-98\text{ K}}$$

The change in temperature is a decrease.

23. **SSM** ***REASONING*** When the expansion is isothermal, the work done can be calculated from Equation (15.3): $W = nRT\ln\left(V_f / V_i\right)$. When the expansion is adiabatic, the work done can be calculated from Equation 15.4: $W = \frac{3}{2}nR(T_i - T_f)$.

Since the gas does the same amount of work whether it expands adiabatically or isothermally, we can equate the right hand sides of these two equations. We also note that since the initial temperature is the same for both cases, the temperature T in the isothermal expansion is the same as the initial temperature T_i for the adiabatic expansion. We then have

$$nRT_i \ln\left(\frac{V_f}{V_i}\right) = \frac{3}{2}nR(T_i - T_f)$$

or

$$\ln\left(\frac{V_f}{V_i}\right) = \frac{\frac{3}{2}(T_i - T_f)}{T_i}$$

SOLUTION Solving for the ratio of the volumes gives

$$\frac{V_f}{V_i} = e^{\frac{3}{2}(T_i - T_f)/T_i} = e^{\frac{3}{2}(405\text{ K} - 245\text{ K})/(405\text{ K})} = \boxed{1.81}$$

24. ***REASONING AND SOLUTION***

Step A → B

The internal energy of a monatomic ideal gas is U = (3/2)nRT. Thus, the change is

$$\Delta U = \tfrac{3}{2}nR\,\Delta T = \tfrac{3}{2}(1.00\text{ mol})\left[8.31\text{ J}/(\text{mol}\cdot\text{K})\right](800.0\text{ K} - 400.0\text{ K}) = \boxed{4990\text{ J}}$$

The work for this constant pressure step is W = PΔV. But the ideal gas law applies, so

$$W = P\Delta V = nR\,\Delta T = (1.00\text{ mol})\left[8.31\text{ J}/(\text{mol}\cdot\text{K})\right](800.0\text{ K} - 400.0\text{ K}) = \boxed{3320\text{ J}}$$

The first law of thermodynamics indicates that the heat is

$$\begin{aligned} Q &= \Delta U + W = \tfrac{3}{2}nR\,\Delta T + nR\,\Delta T \\ &= \tfrac{5}{2}(1.00\text{ mol})\left[8.31\text{ J}/(\text{mol}\cdot\text{K})\right](800.0\text{ K} - 400.0\text{ K}) = \boxed{8310\text{ J}} \end{aligned}$$

Step B → C

The internal energy of a monatomic ideal gas is U = (3/2)nRT. Thus, the change is

$$\Delta U = \tfrac{3}{2}nR\,\Delta T = \tfrac{3}{2}(1.00\text{ mol})\left[8.31\text{ J}/(\text{mol}\cdot\text{K})\right](400.0\text{ K} - 800.0\text{ K}) = \boxed{-4990\text{ J}}$$

The volume is constant in this step, so the work done by the gas is $\boxed{W = 0}$.

The first law of thermodynamics indicates that the heat is

$$Q = \Delta U + W = \Delta U = \boxed{-4990\text{ J}}$$

Step C → D

The internal energy of a monatomic ideal gas is U = (3/2)nRT. Thus, the change is

$$\Delta U = \tfrac{3}{2}nR\,\Delta T = \tfrac{3}{2}(1.00\text{ mol})\left[8.31\text{ J}/(\text{mol}\cdot\text{K})\right](200.0\text{ K} - 400.0\text{ K}) = \boxed{-2490\text{ J}}$$

The work for this constant pressure step is W = PΔV. But the ideal gas law applies, so

$$W = P\Delta V = nR\,\Delta T = (1.00\text{ mol})\left[8.31\text{ J}/(\text{mol}\cdot\text{K})\right](200.0\text{ K} - 400.0\text{ K}) = \boxed{-1660\text{ J}}$$

The first law of thermodynamics indicates that the heat is

$$\begin{aligned} Q &= \Delta U + W = \tfrac{3}{2}nR\,\Delta T + nR\,\Delta T \\ &= \tfrac{5}{2}(1.00\text{ mol})\left[8.31\text{ J}/(\text{mol}\cdot\text{K})\right](200.0\text{ K} - 400.0\text{ K}) = \boxed{-4150\text{ J}} \end{aligned}$$

Step D → A

The internal energy of a monatomic ideal gas is U = (3/2)nRT. Thus, the change is

$$\Delta U = \tfrac{3}{2} nR\,\Delta T = \tfrac{3}{2}(1.00\ \text{mol})\left[8.31\ \text{J}/(\text{mol}\cdot\text{K})\right](400.0\ \text{K} - 200.0\ \text{K}) = \boxed{2490\ \text{J}}$$

The volume is constant in this step, so the work done by the gas is $\boxed{W = 0}$

The first law of thermodynamics indicates that the heat is

$$Q = \Delta U + W = \Delta U = \boxed{2490\ \text{J}}$$

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25. ***REASONING AND SOLUTION*** Equation 15.5 indicates that $P_f V_f^{\gamma} = P_i V_i^{\gamma}$, while the ideal gas law indicates that P = nRT/V for both initial and final conditions. Therefore, we have

$$\left(\frac{nRT_i}{V_i}\right)V_i^{\gamma} = \left(\frac{nRT_f}{V_f}\right)V_f^{\gamma} \quad \text{or} \quad T_i V_i^{\gamma-1} = T_f V_f^{\gamma-1}$$

Rearranging this result and using the initial and final temperatures (in Kelvins) gives

$$\frac{V_i}{V_f} = \left(\frac{T_f}{T_i}\right)^{1/(\gamma-1)} = \left(\frac{954\ \text{K}}{3.00 \times 10^2\ \text{K}}\right)^{1/\left(\frac{7}{5}-1\right)} = \boxed{18.0}$$

---

26. ***REASONING*** For an ideal gas, we know that pressure, volume, and kelvin temperature are related by the ideal gas law $PV = nRT$ (Equation 14.1), where n is the number of moles and R is the universal gas constant. In addition, for an adiabatic process, we know that $P_iV_i^{\gamma} = P_fV_f^{\gamma}$ (Equation 15.5). Our information is in terms of temperatures and pressures, so we will solve the ideal gas law for the volume V and substitute into the adiabatic equation to obtain a relationship containing only pressures and temperatures. From this relationship, we will obtain the desired pressure ratio.

SOLUTION From the ideal gas law, we have $V = nRT/P$. Substitution into Equation 15.5 gives

$$P_iV_i^{\gamma} = P_fV_f^{\gamma} \quad \text{or} \quad P_i\left(\frac{nRT_i}{P_i}\right)^{\gamma} = P_f\left(\frac{nRT_f}{P_f}\right)^{\gamma}$$

Simplifying, this result becomes

$$P_i^{1-\gamma}T_i^{\gamma} = P_f^{1-\gamma}T_f^{\gamma} \quad \text{or} \quad \left(\frac{P_i}{P_f}\right)^{1-\gamma} = \left(\frac{T_f}{T_i}\right)^{\gamma}$$

We find, then, that

$$\frac{P_i}{P_f} = \left(\frac{T_f}{T_i}\right)^{\gamma/(1-\gamma)} = \left(\frac{1}{2}\right)^{(5/3)/[1-(5/3)]} = \left(\frac{1}{2}\right)^{-5/2} = \boxed{5.66}$$

27. ***REASONING AND SOLUTION***

a. Since the curved line between A and C is an isotherm, the initial and final temperatures are the same. Since the internal energy of an ideal monatomic gas is $U = (3/2)nRT$, the initial and final energies are also the same, and the change in the internal energy is $\Delta U = 0$. The first law of thermodynamics, then, indicates that for the process A→B→C, we have

$$\Delta U = 0 = Q - W \quad \text{or} \quad Q = W$$

The heat is equal to the work. Determining the work from the area beneath the straight line segments AB and BC, we find that

$$Q = W = -\left(4.00 \times 10^5 \text{ Pa}\right)\left(0.400 \text{ m}^3 - 0.200 \text{ m}^3\right) = \boxed{-8.00 \times 10^4 \text{ J}}$$

b. The minus sign is included because the gas is compressed, so that work is done on the gas. Since the answer for Q is negative, we conclude that $\boxed{\text{heat flows out of the gas}}$.

28. ***REASONING AND SOLUTION*** Let the left be side 1 and the right be side 2. Since the partition moves to the right, side 1 does work on side 2, so that the work values involved satisfy the relation $W_1 = -W_2$. Using Equation 15.4 for each work value, we find that

$$\frac{3}{2}nR\left(T_{1i} - T_{1f}\right) = -\frac{3}{2}nR\left(T_{2i} - T_{2f}\right) \qquad \text{or}$$

$$T_{1f} + T_{2f} = T_{1i} + T_{2i} = 525\text{ K} + 275\text{ K} = 8.00 \times 10^2\text{ K}$$

We now seek a second equation for the two unknowns T_{1f} and T_{2f}. Equation 15.5 for an adiabatic process indicates that $P_{1i}V_{1i}^{\gamma} = P_{1f}V_{1f}^{\gamma}$ and $P_{2i}V_{2i}^{\gamma} = P_{2f}V_{2f}^{\gamma}$. Dividing these two equations and using the facts that $V_{1i} = V_{2i}$ and $P_{1f} = P_{2f}$, gives

$$\frac{P_{1i}V_{1i}^{\gamma}}{P_{2i}V_{2i}^{\gamma}} = \frac{P_{1f}V_{1f}^{\gamma}}{P_{2f}V_{2f}^{\gamma}} \qquad \text{or} \qquad \frac{P_{1i}}{P_{2i}} = \left(\frac{V_{1f}}{V_{2f}}\right)^{\gamma}$$

Using the ideal gas law, we find that

$$\frac{P_{1i}}{P_{2i}} = \left(\frac{V_{1f}}{V_{2f}}\right)^{\gamma} \qquad \text{becomes} \qquad \frac{nRT_{1i}/V_{1i}}{nRT_{2i}/V_{2i}} = \left(\frac{nRT_{1f}/P_{1f}}{nRT_{2f}/P_{2f}}\right)^{\gamma}$$

Since $V_{1i} = V_{2i}$ and $P_{1f} = P_{2f}$, the result above reduces to

$$\frac{T_{1i}}{T_{2i}} = \left(\frac{T_{1f}}{T_{2f}}\right)^{\gamma} \qquad \text{or} \qquad \frac{T_{1f}}{T_{2f}} = \left(\frac{T_{1i}}{T_{2i}}\right)^{1/\gamma} = \left(\frac{525\text{ K}}{275\text{ K}}\right)^{1/\gamma} = 1.474$$

Using this expression for the ratio of the final temperatures in $T_{1f} + T_{2f} = 8.00 \times 10^2\text{ K}$, we find that

a. $\boxed{T_{1f} = 477\text{ K}}$ and b. $\boxed{T_{2f} = 323\text{ K}}$

29. SSM ***REASONING AND SOLUTION***

a. The final temperature of the adiabatic process is given by solving Equation 15.4 for T_f.

$$T_f = T_i - \frac{W}{\frac{3}{2}nR} = 393\text{ K} - \frac{825\text{ J}}{\frac{3}{2}(1.00\text{ mol})[8.31\text{ J}/(\text{mol}\cdot\text{K})]} = \boxed{327\text{ K}}$$

b. According to Equation 15.5 for the adiabatic expansion of an ideal gas, $P_iV_i^\gamma = P_fV_f^\gamma$. Therefore,

$$V_f^\gamma = V_i^\gamma\left(\frac{P_i}{P_f}\right)$$

From the ideal gas law, $PV = nRT$; therefore, the ratio of the pressures is given by

$$\frac{P_i}{P_f} = \left(\frac{T_i}{T_f}\right)\left(\frac{V_f}{V_i}\right)$$

Combining the previous two equations gives

$$V_f^\gamma = V_i^\gamma\left(\frac{T_i}{T_f}\right)\left(\frac{V_f}{V_i}\right)$$

Solving for V_f we obtain

$$\frac{V_f^\gamma}{V_f} = \left(\frac{T_i}{T_f}\right)\left(\frac{V_i^\gamma}{V_i}\right) \qquad \text{or} \qquad V_f^{(\gamma-1)} = \left(\frac{T_i}{T_f}\right)V_i^{(\gamma-1)}$$

$$V_f = \left[\left(\frac{T_i}{T_f}\right)V_i^{(\gamma-1)}\right]^{1/(\gamma-1)} = V_i\left(\frac{T_i}{T_f}\right)^{1/(\gamma-1)}$$

Therefore,

$$V_f = V_i\left(\frac{T_i}{T_f}\right)^{1/(\gamma-1)} = (0.100\text{ m}^3)\left(\frac{393\text{ K}}{327\text{ K}}\right)^{1/(2/3)} = (0.100\text{ m}^3)\left(\frac{393\text{ K}}{327\text{ K}}\right)^{3/2} = \boxed{0.132\text{ m}^3}$$

30. ***REASONING AND SOLUTION*** The heat required for an isobaric process is

$$Q = C_p\, n\, \Delta T = \left(\tfrac{5}{2} R\right) n\, \Delta T$$

$$= \tfrac{5}{2}\left[8.31\ \mathrm{J/(mol \cdot K)}\right]\left(\frac{8.0\ \mathrm{g}}{39.9\ \mathrm{g/mol}}\right)(75\ \mathrm{K}) = \boxed{310\ \mathrm{J}}$$

31. SSM ***REASONING AND SOLUTION*** The amount of heat required to change the temperature of the gas is given by Equation 15.6, where C_P is given by Equation 15.7.

$$Q = C_P n\Delta T = \tfrac{5}{2} Rn\Delta T = \tfrac{5}{2}[8.31 \text{ J/(mol}\cdot\text{K)}]\ (1.5 \text{ mol})\ (77 \text{ K}) = \boxed{2400 \text{ J}}$$

32. ***REASONING*** The heat added is given by Equation 15.6 as $Q = Cn\Delta T$, where C is the molar specific heat capacity, n is the number of moles, and ΔT is the change in temperature. Using the subscripts "P" and "V", we will apply this equation for constant pressure and constant volume conditions, respectively. We do not have a value for n. However, it will not be necessary, if we take advantage of the fact that $C_P/C_V = 5/3$ for a monatomic ideal gas, as indicated in Equation 15.9.

SOLUTION Applying Equation 15.6, we obtain

$$Q_P = C_P n(\Delta T)_P \quad \text{and} \quad Q_V = C_V n(\Delta T)_V$$

Dividing the constant pressure by the constant volume equation gives

$$\frac{Q_P}{Q_V} = \frac{C_P n(\Delta T)_P}{C_V n(\Delta T)_V} = \frac{C_P(\Delta T)_P}{C_V(\Delta T)_V}$$

Using Equation 15.9 ($C_P/C_V = 5/3$) and the fact that $Q_P/Q_V = 2$, we find

$$2 = \frac{5(\Delta T)_P}{3(\Delta T)_V} \quad \text{or} \quad (\Delta T)_P = \frac{6}{5}(\Delta T)_V = \frac{6}{5}(35\text{ K}) = \boxed{42\text{ K}}$$

33. SSM ***REASONING AND SOLUTION*** According to the first law of thermodynamics (Equation 15.1), $\Delta U = U_f - U_i = Q - W$. Since the internal energy of this gas is doubled by the addition of heat, the initial and final internal energies are U and $2U$, respectively. Therefore,

$$\Delta U = U_f - U_i = 2U - U = U$$

Equation 15.1 for this situation then becomes $U = Q - W$. Solving for Q gives

$$Q = U + W \tag{1}$$

The initial internal energy of the gas can be calculated from Equation 14.7:

$$U = \frac{3}{2}nRT = \frac{3}{2}(2.5\text{ mol})[8.31\text{ J/(mol}\cdot\text{K)}](350\text{ K}) = 1.1\times10^4\text{ J}$$

a. If the process is carried out isochorically (i.e., at constant volume), then $W = 0$, and the heat required to double the internal energy is

$$Q = U + W = U + 0 = \boxed{1.1\times10^4\text{ J}}$$

b. If the process is carried out isobarically (i.e., at constant pressure), then $W = P\Delta V$, and Equation (1) above becomes

$$Q = U + W = U + P\Delta V \tag{2}$$

From the ideal gas law, $PV = nRT$, we have that $P\Delta V = nR\Delta T$, and Equation (2) becomes

$$Q = U + nR\Delta T \tag{3}$$

The internal energy of an ideal gas is directly proportional to its Kelvin temperature. Since the internal energy of the gas is doubled, the final Kelvin temperature will be twice the initial Kelvin temperature, or $\Delta T = 350$ K. Substituting values into Equation (3) gives

$$Q = 1.1\times10^4\text{ J} + (2.5\text{ mol})[8.31\text{ J/(mol}\cdot\text{K)}](350\text{ K}) = \boxed{1.8\times10^4\text{ J}}$$

34. ***REASONING AND SOLUTION***

a. The amount of heat needed to raise the temperature of the gas at constant volume is given by Equations 15.6 and 15.8, $Q = n\,C_V\,\Delta T$. Solving for ΔT yields

$$\Delta T = \frac{Q}{nC_v} = \frac{5.24 \times 10^3\ \text{J}}{(3.00\ \text{mol})\left(\frac{3}{2}R\right)} = \boxed{1.40 \times 10^2\ \text{K}}$$

b. The change in the internal energy of the gas is given by the first law of thermodynamics with $W = 0$, since the gas is heated at constant volume:

$$\Delta U = Q - W = 5.24 \times 10^3\ \text{J} - 0 = \boxed{5.24 \times 10^3\ \text{J}}$$

c. The change in pressure can be obtained from the ideal gas law,

$$\Delta P = \frac{nR\Delta T}{V} = \frac{(3.00\ \text{mol})R(1.40 \times 10^2\ \text{K})}{1.50\ \text{m}^3} = \boxed{2.33 \times 10^3\ \text{Pa}}$$

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35. ***REASONING AND SOLUTION*** The heat added at constant pressure is $Q = C_p n\Delta T = (5R/2)\, n\Delta T$. The work done during the process is $W = P\Delta V$. The ideal gas law requires that $\Delta V = nR\Delta T/P$, so $W = nR\Delta T$. The required ratio is then

$$Q/W = \boxed{5/2}$$

36. ***REASONING*** The heat added is given by Equation 15.6 as $Q = C_V n\Delta T$, where C_V is the molar specific heat capacity at constant volume, n is the number of moles, and ΔT is the change in temperature. But the heat is supplied by the heater at a rate of ten watts, or ten joules per second, so $Q = (10.0\text{ W})t$, where t is the on-time for the heater. In addition, we know that the ideal gas law applies: $PV = nRT$ (Equation 14.1). Since the volume is constant while the temperature changes by an amount ΔT, the amount by which the pressure changes is ΔP. This change in pressure is given by the ideal gas law in the form $(\Delta P)V = nR(\Delta T)$.

SOLUTION Using Equation 15.6 and the expression $Q = (10.0\text{ W})t$ for the heat delivered by the heater, we have

$$Q = C_V n\Delta T \quad \text{or} \quad (10.0\text{ W})t = C_V n\Delta T \quad \text{or} \quad t = \frac{C_V n\Delta T}{10.0\text{ W}}$$

Using the ideal gas law in the form $(\Delta P)V = nR(\Delta T)$, we can express the change in temperature as $\Delta T = (\Delta P)V/nR$. With this substitution for ΔT, the expression for the time becomes

$$t = \frac{C_V n(\Delta P)V}{(10.0\text{ W})nR}$$

According to Equation 15.8, $C_V = \frac{3}{2}R$ for a monatomic ideal gas, so we find

$$t = \frac{3Rn(\Delta P)V}{2(10.0\text{ W})nR} = \frac{3(\Delta P)V}{2(10.0\text{ W})} = \frac{3(5.0\times10^4\text{ Pa})(1.00\times10^{-3}\text{ m}^3)}{2(10.0\text{ W})} = \boxed{7.5\text{ s}}$$

37. SSM ***REASONING*** According to Equations 15.6 and 15.7, the heat supplied to a monatomic ideal gas at constant pressure is $Q = C_P n\,\Delta T$, with $C_P = \frac{5}{2}R$. Thus, $Q = \frac{5}{2}nR\Delta T$. The percentage of this heat used to increase the internal energy by an amount ΔU is

$$\text{Percentage} = \left(\frac{\Delta U}{Q}\right) \times 100\ \% = \left(\frac{\Delta U}{\frac{5}{2}nR\,\Delta T}\right) \times 100\ \% \qquad (1)$$

But according to the first law of thermodynamics, $\Delta U = Q - W$. The work W is $W = P\Delta V$, and for an ideal gas $P\Delta V = nR\Delta T$. Therefore, the work W becomes $W = P\Delta V = nR\Delta T$ and the change in the internal energy is $\Delta U = Q - W = \frac{5}{2}nR\Delta T - nR\Delta T = \frac{3}{2}nR\Delta T$. Combining this expression for ΔU with Equation (1) above yields a numerical value for the percentage of heat being supplied to the gas that is used to increase its internal energy.

SOLUTION

a. The percentage is

$$\text{Percentage} = \left(\frac{\Delta U}{\frac{5}{2}nR\Delta T}\right) \times 100\ \% = \left(\frac{\frac{3}{2}nR\,\Delta T}{\frac{5}{2}nR\,\Delta T}\right) \times 100\ \% = \left(\frac{3}{5}\right) \times 100\ \% = \boxed{60.0\ \%}$$

b. The remainder of the heat, or $\boxed{40.0\%}$, is used for the work of expansion.

38. ***REASONING*** The change in the internal energy of the gas can be found using the first law of thermodynamics, since the heat added to the gas is known and the work can be calculated by using Equation 15.2, $W = P\,\Delta V$. The molar specific heat capacity at constant pressure can be evaluated by using Equation 15.6 and the ideal gas law.

SOLUTION

a. The change in the internal energy is

$$\Delta U = Q - W = Q - P\Delta V$$

$$= 31.4\text{ J} - \left(1.40 \times 10^4\text{ Pa}\right)\left(8.00 \times 10^{-4}\text{ m}^3 - 3.00 \times 10^{-4}\text{ m}^3\right) = \boxed{24.4\text{ J}}$$

b. According to Equation 15.6, the molar specific heat capacity at constant pressure is $C_p = Q/(n\,\Delta T)$. The term $n\,\Delta T$ can be expressed in terms of the pressure and change in volume by using the ideal gas law:

$$P\,\Delta V = n\,R\,\Delta T \quad \text{or} \quad n\,\Delta T = P\,\Delta V/R$$

Substituting this relation for $n\,\Delta T$ into $C_p = Q/(n\,\Delta T)$, we obtain

$$C_p = \frac{Q}{\dfrac{P\Delta V}{R}} = \frac{31.4\text{ J}}{\dfrac{\left(1.40 \times 10^4\text{ Pa}\right)\left(5.00 \times 10^{-4}\text{ m}^3\right)}{R}} = \boxed{37.3\text{ J/(mol}\cdot\text{K)}}$$

39. ***REASONING AND SOLUTION*** Let P, V, and T represent the initial values of pressure, volume, and temperature. The first process is isochoric, so

$$Q_1 = C_V n\,\Delta T_1 = (3R/2)n\,\Delta T_1$$

The ideal gas law for this process gives $\Delta T_1 = 2PV/(nR)$, so $Q_1 = 3PV$.

The second process is isobaric, so

$$Q_2 = C_P n\,\Delta T_2 = (5R/2)n\,\Delta T_2$$

The ideal gas law for this process gives $\Delta T_2 = 3PV/nR$, so $Q_2 = (15/2)\,PV$. The total heat is $Q = Q_1 + Q_2 = (21/2)\,PV$.

But at conditions of standard temperature and pressure (see Section 14.2), $P = 1.01 \times 10^5$ Pa and $V = 22.4$ liters $= 22.4 \times 10^{-3}\ \text{m}^3$, so

$$Q = \tfrac{21}{2}PV = \tfrac{21}{2}\left(1.01 \times 10^5\ \text{Pa}\right)\left(22.4 \times 10^{-3}\ \text{m}^3\right) = \boxed{2.38 \times 10^4\ \text{J}}$$

40. ***REASONING*** According to Equation 15.11, the efficiency of a heat engine is $e = W/Q_H$, where W is the work and Q_H is the input heat. Thus, the work is $W = eQ_H$. We can apply this result before and after the tune-up to compute the extra work produced.

SOLUTION Using Equation 15.11, we find the work before and after the tune-up as follows:

$$W_{\text{Before}} = e_{\text{Before}} Q_H \quad \text{and} \quad W_{\text{After}} = e_{\text{After}} Q_H$$

Subtracting the "before" equation from the "after" equation gives

$$W_{\text{After}} - W_{\text{Before}} = e_{\text{After}} Q_H - e_{\text{Before}} Q_H = \left(e_{\text{After}} - e_{\text{Before}}\right) Q_H$$

$$= 0.050(1300\ \text{J}) = \boxed{65\ \text{J}}$$

41. SSM ***REASONING AND SOLUTION*** The efficiency of a heat engine is defined by Equation 15.11 as $e = W / Q_H$, where W is the work done and Q_H is the heat input. The principle of energy conservation requires that $Q_H = W + Q_C$, where Q_C is the heat rejected to the cold reservoir (Equation 15.12). Combining Equations 15.11 and 15.12 gives

$$e = \frac{W}{W + Q_C} = \frac{16\ 600\ \text{J}}{16\ 600\ \text{J} + 9700\ \text{J}} = \boxed{0.631}$$

42. ***REASONING AND SOLUTION*** We know that $Q_H = W + Q_C$, so that

$$W = Q_H - Q_C = 2.41 \times 10^4 \text{ J} - 5.86 \times 10^3 \text{ J} = \boxed{1.82 \times 10^4 \text{ J}}$$

43. ***REASONING AND SOLUTION***

a. The efficiency is $e = W/Q_H$, so that

$$Q_H = W/e = (5500\ \text{J})/(0.64) = \boxed{8600\ \text{J}}$$

b. The rejected heat is

$$Q_C = Q_H - W = 8600\ \text{J} - 5500\ \text{J} = \boxed{3100\ \text{J}}$$

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44. ***REASONING AND SOLUTION*** We are given that $Q_C = 3W$, so that $Q_H = Q_C + W = 4W$. Thus, the efficiency of the engine is

$$e = W/Q_H = W/4W = 1/4 = \boxed{0.25}$$

45. SSM WWW ***REASONING*** The efficiency of either engine is given by Equation 15.13, $e = 1 - (Q_C / Q_H)$. Since engine A receives three times more input heat, produces five times more work, and rejects two times more heat than engine B, it follows that $Q_{HA} = 3Q_{HB}$, $W_A = 5W_B$, and $Q_{CA} = 2Q_{CB}$. As required by the principle of energy conservation for engine A (Equation 15.12),

$$\underbrace{Q_{HA}}_{3Q_{HB}} = \underbrace{Q_{CA}}_{2Q_{CB}} + \underbrace{W_A}_{5W_B}$$

Thus,

$$3Q_{HB} = 2Q_{CB} + 5W_B \tag{1}$$

Since engine B also obeys the principle of energy conservation (Equation 15.12),

$$Q_{HB} = Q_{CB} + W_B \tag{2}$$

Substituting Q_{HB} from Equation (2) into Equation (1) yields

$$3(Q_{CB} + W_B) = 2Q_{CB} + 5W_B$$

Solving for W_B gives

$$W_B = \frac{1}{2}Q_{CB}$$

Therefore, Equation (2) predicts for engine B that

$$Q_{HB} = Q_{CB} + W_B = \frac{3}{2}Q_{CB}$$

SOLUTION

a. Substituting $Q_{CA} = 2Q_{CB}$ and $Q_{HA} = 3Q_{HB}$ into Equation 15.13 for engine A, we have

$$e_A = 1 - \frac{Q_{CA}}{Q_{HA}} = 1 - \frac{2Q_{CB}}{3Q_{HB}} = 1 - \frac{2Q_{CB}}{3(\frac{3}{2}Q_{CB})} = 1 - \frac{4}{9} = \boxed{\frac{5}{9}}$$

b. Substituting $Q_{HB} = \frac{3}{2}Q_{CB}$ into Equation 15.13 for engine B, we have

$$e_B = 1 - \frac{Q_{CB}}{Q_{HB}} = 1 - \frac{Q_{CB}}{\frac{3}{2}Q_{CB}} = 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$$

46. ***REASONING AND SOLUTION*** The maximum efficiency is given by

$$e_{max} = 1 - (T_C/T_H) = 1 - [(620\text{ K})/(950\text{ K})] = 0.35$$

Since the engine operates at three-fifths maximum efficiency, its efficiency is

$$e = 0.60\ e_{max} = (0.60)(0.35) = \boxed{0.21}$$

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47. ***REASONING AND SOLUTION*** The efficiency is given by

$$e = 1 - (T_C/T_H) = 1 - [(200\text{ K})/(500\text{ K})] = 0.6 = W/Q_H$$

The work is

$$W = e\,Q_H = (0.6)(5000\text{ J}) = \boxed{3000\text{ J}}$$

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48. ***REASONING*** Suppose that the plant used a Carnot engine to produce its work with the given hot reservoir (temperature = 520 K). We can calculate the cold-reservoir temperature necessary for the Carnot engine to have an efficiency of 33% directly from Equation 15.15. For a cold-reservoir temperature higher than this value, even a Carnot engine would have an efficiency less than 33%. Therefore, the cold-reservoir temperature calculated from Equation 15.15 is the value we seek.

SOLUTION Using Equation 15.15, we find

$$\text{Efficiency} = 1 - \frac{T_{\text{C}}}{T_{\text{H}}}$$

$$T_{\text{C}} = (1 - \text{Efficiency})T_{\text{H}} = (1 - 0.33)(521\ \text{K}) = \boxed{350\ \text{K}}$$

49. SSM ***REASONING*** The efficiency e of a Carnot engine is given by Equation 15.15, $e = 1 - (T_C / T_H)$, where, according to Equation 15.14, $(Q_C / Q_H) = (T_C / T_H)$. Since the efficiency is given along with T_C and Q_C, Equation 15.15 can be used to calculate T_H. Once T_H is known, the ratio T_C / T_H is thus known, and Equation 15.14 can be used to calculate Q_H.

SOLUTION

a. Solving Equation 15.15 for T_H gives

$$T_H = \frac{T_C}{1-e} = \frac{378\text{ K}}{1-0.700} = \boxed{1260\text{ K}}$$

b. Solving Equation 15.14 for Q_H gives

$$Q_H = Q_C\left(\frac{T_H}{T_C}\right) = (5230\text{ J})\left(\frac{1260\text{ K}}{378\text{ K}}\right) = \boxed{1.74\times10^4\text{ J}}$$

50. ***REASONING*** For a Carnot engine, we know that ratio of the heat Q_C discarded to the cold reservoir to the heat Q_H leaving the hot reservoir is equal to the ratio of the reservoir temperatures or $Q_C/Q_H = T_C/T_H$ (Equation 15.14). However, we are given W/Q_C, the ratio of the work to the discarded heat, not Q_C/Q_H. But we can use the first law of thermodynamics (the conservation of energy) to express W in terms of the heats: $W = Q_H - Q_C$ (Equation 15.12). In this way, we can relate the fact that $W/Q_C = 3.00$ to the temperature ratio and ultimately to the temperature that we seek.

SOLUTION Using the facts that $W/Q_C = 3.00$ and $W = Q_H - Q_C$ (Equation 15.12), we have

$$\frac{W}{Q_C} = \frac{Q_H - Q_C}{Q_C} = 3.00 \quad \text{or} \quad \frac{Q_H}{Q_C} - 1 = 3.00$$

This means that

$$\frac{Q_H}{Q_C} = 4.00 \quad \text{or} \quad \frac{Q_C}{Q_H} = 0.250$$

According to Equation 15.14, we now have

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H} = 0.250 \quad \text{or} \quad T_H = \frac{T_C}{0.250} = \frac{292\ \text{K}}{0.250} = \boxed{1170\ \text{K}}$$

51. ***REASONING AND SOLUTION*** In order to find out how many kilograms of ice in the tub are melted, we must determine Q_C, the amount of exhaust heat delivered to the cold reservoir. Since the hot reservoir consists of boiling water ($T_H = 373.0$ K) and the cold reservoir consists of ice and water ($T_C = 273.0$ K), the efficiency of this engine is

$$e = 1 - \frac{T_C}{T_H} = 1 - \frac{(273.0\text{ K})}{(373.0\text{ K})} = 0.2681$$

The work done by the engine is

$$W = eQ_H = (0.2681)(6800\text{ J}) = 1823\text{ J}$$

Therefore, the amount of heat delivered to the cold reservoir is

$$Q_C = Q_H - W = 6800\text{ J} - 1823\text{ J} = 4977\text{ J}$$

Using the definition of the latent heat of fusion (melting) L_f, we find that the amount of ice that melts is

$$m = \frac{Q_C}{L_f} = \frac{4977\text{ J}}{33.5\times10^4\text{ J/kg}} = \boxed{0.015\text{ kg}}$$

52. ***REASONING AND SOLUTION*** From Equation 15.14 we know that the input heat Q_H and the exhaust heat Q_C of a Carnot engine are related to the Kelvin temperatures of the hot and cold reservoirs according to $Q_H / Q_C = T_H / T_C$. We also know that $Q_C = Q_H - W$, according to Equation 15.12.

a. Therefore, we find T_1 as follows:

$$\frac{Q_H}{Q_C} = \frac{5550\ \text{J}}{5550\ \text{J} - 1750\ \text{J}} = \frac{T_1}{503\ \text{K}} \qquad \text{or} \qquad \boxed{T_1 = 735\ \text{K}}$$

b. Similarly, we find T_2 as follows:

$$\frac{Q_H}{Q_C} = \frac{5550\ \text{J} - 1750\ \text{J}}{5550\ \text{J} - 1750\ \text{J} - 1750\ \text{J}} = \frac{503\ \text{K}}{T_2} \qquad \text{or} \qquad \boxed{T_2 = 271\ \text{K}}$$

53. SSM ***REASONING*** The maximum efficiency e at which the power plant can operate is given by Equation 15.15, $e = 1 - (T_C / T_H)$. The power output is given; it can be used to find the work output W for a 24 hour period. With the efficiency and W known, Equation 15.11, $e = W / Q_H$, can be used to find Q_H. Q_C can then be found from Equation 15.12, $Q_H = W + Q_C$.

SOLUTION

a. The maximum efficiency is

$$e = 1 - \frac{T_C}{T_H} = 1 - \frac{323\text{ K}}{505\text{ K}} = \boxed{0.360}$$

b. Since the power output of the power plant is P = 84 000 kW, the required heat input Q_H for a 24 hour period is

$$Q_H = \frac{W}{e} = \frac{Pt}{e} = \frac{(8.4 \times 10^7\text{ J/s})(24\text{ h})}{0.360}\left(\frac{3600\text{ s}}{1\text{ h}}\right) = 2.02 \times 10^{13}\text{ J}$$

Therefore, solving Equation 15.12 for Q_C, we have

$$Q_C = Q_H - W = 2.02 \times 10^{13}\text{ J} - 7.3 \times 10^{12}\text{ J} = \boxed{1.3 \times 10^{13}\text{ J}}$$

54. ***REASONING AND SOLUTION*** The amount of work delivered by the engines can be determined from Equation 15.12, $Q_H = W + Q_C$. Solving for W for each engine gives:

$$W_1 = (Q_H)_1 - (Q_C)_1 \qquad \text{and} \qquad W_2 = (Q_H)_2 - (Q_C)_2$$

The total work delivered by the two engines is

$$W = W_1 + W_2 = [(Q_H)_1 - (Q_C)_1] + [(Q_H)_2 - (Q_C)_2]$$

But we know that $(Q_H)_2 = (Q_C)_1$, so that

$$W = [(Q_H)_1 - (Q_C)_1] + [(Q_C)_1 - (Q_C)_2] = (Q_H)_1 - (Q_C)_2 \qquad (1)$$

Since these are Carnot engines,

$$\frac{(Q_C)_1}{(Q_H)_1} = \frac{(T_C)_1}{(T_H)_1} \qquad \Rightarrow \qquad (Q_C)_1 = (Q_H)_1 \frac{(T_C)_1}{(T_H)_1} = (4800\text{ J})\frac{670\text{ K}}{890\text{ K}} = 3.61 \times 10^3\text{ J}$$

Similarly, noting that $(Q_H)_2 = (Q_C)_1$ and that $(T_H)_2 = (T_C)_1$, we have

$$(Q_C)_2 = (Q_H)_2 \frac{(T_C)_2}{(T_H)_2} = (Q_C)_1 \frac{(T_C)_2}{(T_C)_1} = (3.61 \times 10^3\text{ J})\frac{420\text{ K}}{670\text{ K}} = 2.26 \times 10^3\text{ J}$$

Substituting into Equation (1) gives

$$W = 4800\text{ J} - 2.26 \times 10^3\text{ J} = \boxed{2.5 \times 10^3\text{ J}}$$

55. SSM ***REASONING AND SOLUTION*** The efficiency e of the power plant is three-fourths its Carnot efficiency so, according to Equation 15.15,

$$e = 0.75\left(1 - \frac{T_C}{T_H}\right) = 0.75\left(1 - \frac{40\text{ K} + 273\text{ K}}{285\text{ K} + 273\text{ K}}\right) = 0.33$$

The power output of the plant is 1.2×10^9 watts. According to Equation 15.11, $e = W / Q_H = (\text{Power} \cdot t) / Q_H$. Therefore, at 33% efficiency, the heat input per unit time is

$$\frac{Q_H}{t} = \frac{\text{Power}}{e} = 3.6 \times 10^9 \text{ J/s}$$

From the principle of conservation of energy, the heat output per unit time must be

$$\frac{Q_C}{t} = \frac{Q_H}{t} - \text{Power} = 2.4 \times 10^9 \text{ J/s}$$

The rejected heat is carried away by the flowing water and, according to Equation 12.4, $Q_C = cm\Delta T$. Therefore,

$$\frac{Q_C}{t} = \frac{cm\Delta T}{t} \quad \text{or} \quad \frac{Q_C}{t} = c(m/t)\Delta T$$

Solving the last equation for ΔT, we have

$$\Delta T = \frac{Q_C}{tc(m/t)} = \frac{(Q_C/t)}{c(m/t)} = \frac{2.4 \times 10^9 \text{ J/s}}{[4186 \text{ J/(kg} \cdot \text{C°)}](1.0 \times 10^5 \text{ kg/s})} = \boxed{5.7\text{ C°}}$$

56. ***REASONING AND SOLUTION*** The engine performs work to lift the block where,

$$W = (1/2)mv^2 + mgh = (1/2)(15.0\text{ kg})(8.50\text{ m/s})^2 + (15.0\text{ kg})(9.80\text{ m/s}^2)(5.00\text{ m}) = 1280\text{ J}$$

The efficiency is $e = 1 - (T_C/T_H) = W/Q_H$, so that

$$Q_H = \frac{W}{1 - \dfrac{T_C}{T_H}} = \frac{1280\text{ J}}{1 - \dfrac{395\text{ K}}{845\text{ K}}} = \boxed{2.40 \times 10^3\text{ J}}$$

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57. ***REASONING AND SOLUTION*** We know that

$$Q_C = Q_H - W = 14\,200\text{ J} - 800\text{ J} = 13\,400\text{ J}$$

Therefore,

$$T_C = T_H(Q_C/Q_H) = (301\text{ K})[(13\,400\text{ J})/(14\,200\text{ J})] = \boxed{284\text{ K}}$$

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58. ***REASONING*** For any refrigerator, the first law of thermodynamics (Equation 15.12) indicates that $W = Q_H - Q_C$. In this expression, we know that $W = 2500$ J and wish to find Q_C. To do so, we need information about Q_H. But the refrigerator is a Carnot device, so we know in addition that $Q_C/Q_H = T_C/T_H$ (Equation 15.14). With this additional equation, we can solve for Q_H and substitute into the first law, obtaining in the process an equation that contains only Q_C as an unknown.

SOLUTION From Equation 15.14 we have

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H} \quad \text{or} \quad Q_H = Q_C\left(\frac{T_H}{T_C}\right)$$

Substituting this expression for Q_H into the first law of thermodynamics gives

$$W = Q_H - Q_C = Q_C\left(\frac{T_H}{T_C}\right) - Q_C$$

Solving for Q_C, we find

$$Q_C = \frac{W}{\frac{T_H}{T_C} - 1} = \frac{2500\ \text{J}}{\frac{299\ \text{K}}{277\ \text{K}} - 1} = \boxed{3.1 \times 10^4\ \text{J}}$$

59. SSM WWW ***REASONING AND SOLUTION*** Equation 15.14 holds for a Carnot air conditioner as well as a Carnot engine. Therefore, solving Equation 15.14 for Q_C, we have

$$Q_C = Q_H \left(\frac{T_C}{T_H} \right) = (6.12 \times 10^5 \text{ J}) \left(\frac{299 \text{ K}}{312 \text{ K}} \right) = \boxed{5.86 \times 10^5 \text{ J}}$$

60. ***REASONING AND SOLUTION*** The coefficient of performance is $CP = Q_C/W$, so that

$$W = Q_C/CP = (4100\ \text{J})/(4.6) = \boxed{890\ \text{J}}$$

61. ***REASONING AND SOLUTION***

a. The work is

$$W = Q_H - Q_C = 3140\ \text{J} - 2090\ \text{J} = \boxed{1050\ \text{J}}$$

b. The coefficient of performance (CP) is

$$CP = Q_H/W = (3140\ \text{J})/(1050\ \text{J}) = \boxed{2.99}$$

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62. ***REASONING*** The coefficient of performance of an air conditioner is Q_C/W, according to Equation 15.16, where Q_C is the heat removed from the house and W is the work required for the removal. In addition, we know that the first law of thermodynamics (energy conservation) applies, so that $W = Q_H - Q_C$, according to Equation 15.12. In this equation Q_H is the heat discarded outside. While we have no direct information about the heats Q_C and Q_H, we do know that the air conditioner is a Carnot device. This means that Equation 15.14 applies: $Q_C/Q_H = T_C/T_H$. Thus, the given temperatures will allow us to calculate the coefficient of performance.

SOLUTION Using Equation 15.16 for the definition of the coefficient of performance and Equation 15.12 for the fact that $W = Q_H - Q_C$, we have

$$\text{Coefficient of performance} = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C} = \frac{Q_C / Q_H}{1 - Q_C / Q_H}$$

Equation 15.14 applies, so that $Q_C/Q_H = T_C/T_H$. With this substitution, we find

$$\text{Coefficient of performance} = \frac{Q_C / Q_H}{1 - Q_C / Q_H} = \frac{T_C / T_H}{1 - T_C / T_H}$$

$$= \frac{T_C}{T_H - T_C} = \frac{297\ \text{K}}{(311\ \text{K}) - (297\ \text{K})} = \boxed{21}$$

63. ***REASONING AND SOLUTION*** The efficiency of the engine is $e = W/Q_H$. The coefficient of performance is $CP = Q_H/W = 1/e$, so

$$CP = \frac{1}{e} = \frac{1}{0.7} = \boxed{1.4}$$

64. ***REASONING AND SOLUTION*** The amount of heat removed from the ice Q_C is

$$Q_C = mL_f = (2.0\text{ kg})(33.5 \times 10^4\text{ J/kg}) = 6.7 \times 10^5\text{ J}$$

The amount of heat leaving the refrigerator Q_H is therefore,

$$Q_H = Q_C(T_H/T_C) = (6.7 \times 10^5\text{ J})(300\text{ K})/(258\text{ K}) = 7.8 \times 10^5\text{ J}$$

The amount of work done by the refrigerator is therefore,

$$W = Q_H - Q_C = 1.1 \times 10^5\text{ J}$$

At \$0.10 per kWh (or \$0.10 per 3.6×10^6 J), the cost is

$$\frac{\$0.10}{3.6 \times 10^6\text{ J}}\left(1.1 \times 10^5\text{ J}\right) = \boxed{\$3.0 \times 10^{-3} = 0.30\text{ cents}}$$

65. SSM WWW ***REASONING*** Let the coefficient of performance be represented by the variable η. Then according to Equation 15.16, $\eta = Q_C / W$. From the statement of energy conservation for a Carnot refrigerator (Equation 15.12), $W = Q_H - Q_C$. Combining Equations 15.16 and 15.12 leads to

$$\eta = \frac{Q_C}{Q_H - Q_C} = \frac{Q_C / Q_C}{(Q_H - Q_C)/Q_C} = \frac{1}{(Q_H / Q_C) - 1}$$

Replacing the ratio of the heats with the ratio of the Kelvin temperatures, according to Equation 15.14, leads to

$$\eta = \frac{1}{(T_H / T_C) - 1} \tag{1}$$

The heat Q_C that must be removed from the refrigerator when the water is cooled can be calculated using Equation 12.4, $Q_C = cm\Delta T$; therefore,

$$W = \frac{Q_C}{\eta} = \frac{cm\Delta T}{\eta} \tag{2}$$

SOLUTION

a. Substituting values into Equation (1) gives

$$\eta = \frac{1}{\frac{T_H}{T_C} - 1} = \frac{1}{\frac{(20.0 + 273.15)\text{ K}}{(6.0 + 273.15)\text{ K}} - 1} = \boxed{2.0 \times 10^1}$$

b. Substituting values into Equation (2) gives

$$W = \frac{cm\Delta T}{\eta} = \frac{[4186\text{ J/(kg}\cdot\text{C°)}](5.00\text{ kg})(14.0\text{ C°})}{2.0 \times 10^1} = \boxed{1.5 \times 10^4\text{ J}}$$

66. ***REASONING AND SOLUTION*** Let CP denote the coefficient of performance. By definition (Equation 15.16), CP $= Q_C/W$, so that

$$W = \frac{Q_C}{CP} = \frac{7.6 \times 10^4 \text{ J}}{2.0} = 3.8 \times 10^4 \text{ J}$$

Thus, the amount of heat that is pumped out of the back of the air conditioner is

$$Q_H = W + Q_C = 3.8 \times 10^4 \text{ J} + 7.6 \times 10^4 \text{ J} = 1.14 \times 10^5 \text{ J}$$

The temperature rise in the room can be found as follows:

$$Q_H = C_v n \Delta T$$

Solving for ΔT gives

$$\Delta T = \frac{Q_H}{C_v n} = \frac{Q_H}{\frac{5}{2} Rn} = \frac{1.14 \times 10^5 \text{ J}}{\frac{5}{2}[8.31 \text{ J/(mol} \cdot \text{K)}](3800 \text{ mol})} = \boxed{1.4 \text{ K}}$$

67. SSM ***REASONING*** The efficiency of the Carnot engine is, according to Equation 15.15,

$$e = 1 - \frac{T_C}{T_H} = 1 - \frac{842\text{ K}}{1684\text{ K}} = \frac{1}{2}$$

Therefore, the work delivered by the engine is, according to Equation 15.11,

$$W = e\,Q_H = \frac{1}{2}Q_H$$

The heat pump removes an amount of heat Q_H from the cold reservoir. Thus, the amount of heat Q' delivered to the hot reservoir of the heat pump is

$$Q' = Q_H + W = Q_H + \frac{1}{2}Q_H = \frac{3}{2}Q_H$$

Therefore, $Q'/Q_H = 3/2$. According to Equation 15.14, $Q'/Q_H = T'/T_C$, so $T'/T_C = 3/2$.

SOLUTION Solving for T' gives

$$T' = \frac{3}{2}T_C = \frac{3}{2}(842\text{ K}) = \boxed{1.26 \times 10^3\text{ K}}$$

68. ***REASONING AND SOLUTION*** The change in entropy is $\Delta S = Q/T$, where

$$Q = mL_S = (4.00\text{ kg})(5.77 \times 10^5\text{ J/kg}) = 2.31 \times 10^6\text{ J}$$

Thus,

$$\Delta S = Q/T = (2.31 \times 10^6\text{ J})/(194.7\text{ K}) = \boxed{1.19 \times 10^4\text{ J/K}}$$

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69. ***REASONING*** The change in entropy of the universe is equal to the change in entropy of the inside of the house plus the change in entropy of the outside of the house.

SOLUTION The change in entropy is given by $\Delta S = Q/T$, as expressed by Equation 15.18. Thus, the change in entropy of the universe is

$$\Delta S_{\text{universe}} = \left(\frac{Q}{T}\right)_{\text{inside}} + \left(\frac{Q}{T}\right)_{\text{outside}}$$

$$= \frac{-1.30 \times 10^5 \text{ J}}{293 \text{ K}} + \frac{1.30 \times 10^5 \text{ J}}{258 \text{ K}} = \boxed{6.0 \times 10^1 \text{ J/K}}$$

70. ***REASONING AND SOLUTION*** Equation 15.19 gives the unavailable work as

$$W_{unavailable} = T_0 \Delta S \tag{1}$$

where $T_0 = 248\ \text{K}$. We also know that $W_{unavailable} = 0.300\ Q$. Furthermore, we can apply Equation 15.18 to the heat lost from the 394-K reservoir and the heat gained by the reservoir at temperature T, with the result that

$$\Delta S = \frac{-Q}{394\ \text{K}} + \frac{Q}{T}$$

With these substitutions for T_0, $W_{unavailable}$, and ΔS, Equation (1) becomes

$$0.300\ Q = (248\ \text{K})\left(\frac{-Q}{394\ \text{K}} + \frac{Q}{T}\right) \qquad \text{or} \qquad \boxed{T = 267\ \text{K}}$$

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71. SSM ***REASONING AND SOLUTION***

a. Since the energy that becomes unavailable for doing work is zero for the process, we have from Equation 15.19, $W_{\text{unavailable}} = T_0 \Delta S_{\text{universe}} = 0$. Therefore, $\Delta S_{\text{universe}} = 0$ and according to the discussion in Section 15.11, the process is $\boxed{\text{reversible}}$.

b. Since the process is reversible, we have (see Section 15.11)

$$\Delta S_{\text{universe}} = \Delta S_{\text{system}} + \Delta S_{\text{surroundings}} = 0$$

Therefore,

$$\Delta S_{\text{surroundings}} = -\Delta S_{\text{system}} = \boxed{-125 \text{ J/K}}$$

72. ***REASONING AND SOLUTION***

a. We know that the hot and cold waters exchange equal amounts of heat, i.e.,

$\Delta Q_{hw} = \Delta Q_{cw}$, so that $(mc\Delta T)_{hw} = (mc\Delta T)_{cw}$, or

$$(1.00\text{ kg})[4186\text{ J/(kg}\cdot\text{C}^\circ)](373\text{ K} - T_f) = (2.00\text{ kg})[4186\text{ J/(kg}\cdot\text{C}^\circ)](T_f - 283\text{ K})$$

Solving for T_f, we obtain $\boxed{T_f = 313\text{K}}$.

b. Since $\Delta S = mc\ln(T_f/T_i)$:

$$\Delta S_{hw} = m_{hw}c\ln[(313\text{ K})/(373\text{ K})] = -734\text{ J/K}$$

$$\Delta S_{cw} = m_{cw}c\ln[(313\text{ K})/(283\text{ K})] = +844\text{ J/K}$$

Therefore,

$$\Delta S_{universe} = \Delta S_{hw} + \Delta S_{cw} = \boxed{1.10 \times 10^2\text{ J/K}}$$

c. The energy unavailable for doing work is, therefore,

$$W_{unavailable} = T_0\,\Delta S_{universe} = (273\text{ K})(1.10 \times 10^2\text{ J/K}) = \boxed{3.00 \times 10^4\text{ J}}$$

73. ***REASONING AND SOLUTION***

a. We begin by finding the final temperature of the water/ice mixture. We have,

$$(cm\Delta T)_w = (mL_f)_{ice} + (mc\Delta T)_{iw}$$

Substituting numbers,

$$(5.00\text{ kg})[4186\text{ J/(kg}\cdot\text{C°)}](80.0 - T_f)$$

$$= (2.00\text{ kg})(33.5 \times 10^4\text{ J/kg}) + (2.00\text{ kg})[4186\text{ J/(kg}\cdot\text{C°)}](T_f)$$

Solving for the final temperature yields $T_f = 34.3°C = 307.3$ K.

Now determine the total change in entropy $\Delta S_{universe}$ for the universe from the sum of the following three contributions:

$$\Delta S_w = mc\ln(T_f/T_i) = (5.00\text{ kg})[4186\text{ J/(kg}\cdot\text{C°)}]\ln[(307.3\text{ K})/(353\text{ K})] = -2900\text{ J/K}$$

$$\Delta S_{iw} = (2.00\text{ kg})[4186\text{ J/(kg}\cdot\text{C°)}]\ln[(307.3\text{ K})/(273\text{ K})] = +991\text{ J/K}$$

$$\Delta S_{ice} = Q/T = (2.00\text{ kg})(33.5 \times 10^4\text{ J/kg})/(273\text{ K}) = +2450\text{ J/K}$$

$$\Delta S_{universe} = \Delta S_w + \Delta S_{iw} + \Delta S_{ice} = \boxed{+541\text{ J/K}}$$

b. The entropy of the universe $\boxed{\text{increases}}$, because the mixing process is irreversible.

74. ***REASONING*** Since the gas is expanding adiabatically, the work done is given by Equation 15.4 as $W = \frac{3}{2}nR\left(T_i - T_f\right)$. Once the work is known, we can use the first law of thermodynamics to find the change in the internal energy of the gas.

SOLUTION

a. The work done by the expanding gas is

$$W = \tfrac{3}{2}nR\left(T_i - T_f\right) = \tfrac{3}{2}(5.0\text{ mol})\left[8.31\text{ J}/(\text{mol}\cdot\text{K})\right](370\text{ K} - 290\text{ K}) = \boxed{+5.0\times10^3\text{ J}}$$

b. Since the process is adiabatic, Q = 0, and the change in the internal energy is

$$\Delta U = Q - W = 0 - 5.0\times10^3\text{ J} = \boxed{-5.0\times10^3\text{ J}}$$

75. SSM *REASONING AND SOLUTION*

a. Since the temperature of the gas is kept constant at all times, the process is isothermal; therefore, the internal energy of an ideal gas does not change and $\boxed{\Delta U = 0}$.

b. From the first law of thermodynamics (Equation 15.1), $\Delta U = Q - W$. But $\Delta U = 0$, so that $Q = W$. Since work is done on the gas, the work is negative, and $\boxed{\text{Q} = -6.1 \times 10^3 \text{ J}}$.

c. The work done in an isothermal compression is given by Equation 15.3:

$$W = nRT \ln\left(\frac{V_\text{f}}{V_\text{i}}\right)$$

Therefore, the temperature of the gas is

$$T = \frac{W}{nR \ln\left(V_\text{f} / V_\text{i}\right)} = \frac{-6.1 \times 10^3 \text{ J}}{(3.0 \text{ mol})[8.31 \text{ J}/(\text{mol}\cdot\text{K})] \ \ln\left[(2.5 \times 10^{-2} \text{ m}^3)/(5.5 \times 10^{-2} \text{ m}^3)\right]} = \boxed{310 \text{ K}}$$

76. ***REASONING AND SOLUTION*** For engine A: $Q_C = 0.72Q_H$, which means that the efficiency of engine A is

$$e_A = 1 - (Q_C/Q_H)_A = 1 - 0.72 = 0.28$$

Since engine B has twice this efficiency, we have $e_B = 1 - (Q_C/Q_H)_B = 0.56$. Solving gives

$$(Q_C/Q_H)_B = 1 - 0.56 = 0.44$$

So engine B discards $\boxed{44\%}$ of its input heat.

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77. ***REASONING AND SOLUTION*** The change in the internal energy of the material is

$$\Delta U = Q = cm\Delta T = [1100\ \text{J/(kg}\cdot\text{C°)}](2.0\ \text{kg})(6.0\ \text{C°}) = \boxed{13\ 000\ \text{J}}$$

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78. ***REASONING AND SOLUTION*** The heat delivered to the kitchen is Q_H, where

$$Q_H = Q_C(T_H/T_C) = (3.00 \times 10^4 \text{ J})(298 \text{ K})/(276 \text{ K}) = \boxed{3.24 \times 10^4 \text{ J}}$$

79. SSM ***REASONING AND SOLUTION*** The change in entropy ΔS of a system for a process in which heat Q enters or leaves the system reversibly at a constant temperature T is given by Equation 15.18, $\Delta S = (Q/T)_R$. For a phase change, $Q = mL$, where L is the latent heat (see Section 12.8).

a. If we imagine a reversible process in which 3.00 kg of ice melts into water at 273 K, the change in entropy of the water molecules is

$$\Delta S = \left(\frac{Q}{T}\right)_R = \left(\frac{mL_f}{T}\right)_R = \frac{(3.00\text{ kg})(3.35\times10^5\text{ J/kg})}{273\text{ K}} = \boxed{3.68\times10^3\text{ J/K}}$$

b. Similarly, if we imagine a reversible process in which 3.00 kg of water changes into steam at 373 K, the change in entropy of the water molecules is

$$\Delta S = \left(\frac{Q}{T}\right)_R = \left(\frac{mL_v}{T}\right)_R = \frac{(3.00\text{ kg})(2.26\times10^6\text{ J/kg})}{373\text{ K}} = \boxed{1.82\times10^4\text{ J/K}}$$

c. Since the change in entropy is greater for the vaporization process than for the fusion process, the $\boxed{\text{vaporization process creates more disorder}}$ in the collection of water molecules.

80. ***REASONING AND SOLUTION*** The amount of heat removed at constant volume is

$$Q = nC_V \Delta T = (2.5 \text{ mol})\left(\tfrac{3}{2}R\right)(35 \text{ K}) = \boxed{1100 \text{ J}}$$

81. ***REASONING AND SOLUTION*** We know that the efficiency is given by $e = 1 - (T_C/T_H)$ $= 1 - [(265\text{ K})/(298\text{ K})] = 0.111$. The coefficient of performance is

$$CP = \frac{Q_H}{W} = \frac{1}{e} = \frac{1}{0.111} = \boxed{9.03}$$

82. ***REASONING AND SOLUTION*** The efficiency of the engine is $e = 1 - (T_C/T_H)$ so

(i) Increase T_H by 40 K; $e = 1 - [(350\text{ K})/(690\text{ K})] = 0.493$

(ii) Decrease T_C by 40 K; $e = 1 - [(310\text{ K})/(650\text{ K})] = 0.523$

The greatest improvement is made by $\boxed{\text{lowering}}$ the temperature of the cold reservoir.

83. SSM ***REASONING*** The work done in an isobaric process is given by Equation 15.2, $W = P\Delta V$; therefore, the pressure is equal to $P = W/\Delta V$. In order to use this expression, we must first determine a numerical value for the work done; this can be calculated using the first law of thermodynamics (Equation 15.1), $\Delta U = Q - W$.

SOLUTION Solving Equation 15.1 for the work W, we find

$$W = Q - \Delta U = 1500\text{ J} - (+4500\text{ J}) = -3.0 \times 10^3\text{ J}$$

Therefore, the pressure is

$$P = \frac{W}{\Delta V} = \frac{-3.0 \times 10^3\text{ J}}{-0.010\text{ m}^3} = \boxed{3.0 \times 10^5\text{ Pa}}$$

The change in volume ΔV, which is the final volume minus the initial volume, is negative because the final volume is 0.010 m^3 *less* than the initial volume.

84. ***REASONING AND SOLUTION***

a. The work is the area under the path ACB. There are 48 "squares" under the path, so that

$$W = -48\left(2.0 \times 10^4 \text{ Pa}\right)\left(2.0 \times 10^{-3} \text{ m}^3\right) = -1900 \text{ J}$$

The minus sign is included because the gas is compressed, so that work is done on it. Since there is no temperature change between A and B (the line AB is an isotherm) and the gas is ideal, $\Delta U = 0$, so

$$Q = \Delta U + W = W = \boxed{-1900 \text{ J}}$$

b. The negative answer for W means that heat flows $\boxed{\text{out}}$ of the gas.

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85. ***REASONING AND SOLUTION*** The amount of heat removed when the ice freezes is

$$Q_C = cm\Delta T + mL_f$$

$$= [4186\ \text{J/(kg·C°)}](1.50\ \text{kg})(20.0\ \text{C°}) + (1.50\ \text{kg})(33.5 \times 10^4\ \text{J/kg}) = 6.28 \times 10^5\ \text{J}$$

Since the coefficient of performance is $CP = Q_C/W$, the work done by the refrigerator is

$$W = Q_C/CP = (6.28 \times 10^5\ \text{J})/3.00 = 2.09 \times 10^5\ \text{J}$$

The heat delivered to the kitchen is

$$Q_H = Q_C + W = 8.37 \times 10^5\ \text{J}$$

The space heater has a power output P of

$$P = Q_H/t = 3.00 \times 10^3\ \text{J/s}$$

Therefore,

$$t = Q_H/P = (8.37 \times 10^5\ \text{J})/(3.00 \times 10^3\ \text{J/s}) = \boxed{279\ \text{s}}$$

86. ***REASONING AND SOLUTION***

a. From the first law of thermodynamics, the heat is $Q = \Delta U + W$. According to Equation 14.7, the change in internal energy is $\Delta U = (3/2)nR\Delta T$. But for an ideal gas, $nR\Delta T = P\Delta V$, so that $\Delta U = (3/2)P\Delta V$. In addition, the work is $W = P\Delta V$ for a constant-pressure process. The heat that flows is

$$Q = \Delta U + W = \tfrac{3}{2}P\Delta V + P\Delta V = \tfrac{5}{2}P\Delta V$$

$$= \tfrac{5}{2}\left(2.6 \times 10^5 \text{ Pa}\right)\left(6.2 \times 10^{-3} \text{ m}^3\right) = \boxed{4.0 \times 10^3 \text{ J}}$$

b. Since the heat Q is positive, $\boxed{\text{heat flows into the gas}}$.

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87. **SSM** ***REASONING*** According to Equation 15.5, $P_i V_i^{\gamma} = P_f V_f^{\gamma}$. The ideal gas law states that $V = nRT / P$ for both the initial and final conditions. Thus, we have

$$P_i \left(\frac{nRT_i}{P_i} \right)^{\gamma} = P_f \left(\frac{nRT_f}{P_f} \right)^{\gamma} \quad \text{or} \quad \frac{P_f}{P_i} = \left(\frac{T_i}{T_f} \right)^{\gamma/(1-\gamma)}$$

Since the ratio of the temperatures is known, the last expression can be solved for the final pressure P_f.

SOLUTION Since $T_i / T_f = 1/2$, and $\gamma = 5/3$, we find that

$$P_f = P_i \left(\frac{T_i}{T_f} \right)^{\gamma/(1-\gamma)} = \left(1.50 \times 10^5 \text{ Pa}\right) \left(\frac{1}{2} \right)^{(5/3)/[1-(5/3)]} = \boxed{8.49 \times 10^5 \text{ Pa}}$$

88. ***REASONING AND SOLUTION*** The change in volume is $\Delta V = -sA$, where s is the distance through which the piston drops and A is the piston area. The minus sign is included because the volume decreases. Thus,

$$s = \frac{-\Delta V}{A}$$

The ideal gas law states that $\Delta V = nR\Delta T/P$. But $Q = C_p n\,\Delta T = \frac{5}{2} Rn\,\Delta T$. Thus, $\Delta T = Q/\left(\frac{5}{2} Rn\right)$. Using these expressions for ΔV and ΔT, we find that

$$s = \frac{-nR\,\Delta T/P}{A} = \frac{-nR\left[Q/\left(\frac{5}{2} Rn\right)\right]}{PA} = \frac{-Q}{\frac{5}{2} PA}$$

$$= \frac{-(-2093\text{ J})}{\frac{5}{2}\left(1.01\times10^{5}\text{ Pa}\right)\left(3.14\times10^{-2}\text{ m}^{2}\right)} = \boxed{0.264\text{ m}}$$

89. ***REASONING AND SOLUTION*** The total heat generated by the students is

$$Q = (200)(130\ \text{W})(3000\ \text{s}) = 7.8 \times 10^7\ \text{J}$$

For the isochoric process.

$$Q = C_V n\Delta T = (5R/2)n\ \Delta T$$

The number of moles of air in the room is found from the ideal gas law to be

$$n = \frac{PV}{RT} = \frac{\left(1.01 \times 10^5\ \text{Pa}\right)\left(1200\ \text{m}^3\right)}{\left[8.31\ \text{J}/\left(\text{mol}\cdot\text{K}\right)\right]\left(294\ \text{K}\right)} = 5.0 \times 10^4\ \text{mol}$$

Now

$$\Delta T = \frac{Q}{\frac{5}{2}Rn} = \frac{7.8 \times 10^7\ \text{J}}{\frac{5}{2}\left[8.31\ \text{J}/\left(\text{mol}\cdot\text{K}\right)\right]\left(5.0 \times 10^4\ \text{mol}\right)} = \boxed{75\ \text{K}}$$

90. ***REASONING AND SOLUTION*** The temperature of the gasoline engine input is $T_1 = 904$ K, the exhaust temperature is $T_2 = 412$ K, and the air temperature is $T_3 = 300$ K. The efficiency of the engine/exhaust is

$$e_1 = 1 - (T_2/T_1) = 0.544$$

The efficiency of the second engine is

$$e_2 = 1 - (T_3/T_2) = 0.272$$

The work done by each segment is $W_1 = (e_1)(Q_{H1})$ and $W_2 = (e_2)(Q_{H2}) = (e_2)(Q_{C1})$ since

$$Q_{H2} = Q_{C1}$$

Now examine $(W_1 + W_2)/W_1$ to find the ratio of the total work produced by both engines to that produced by the first engine alone.

$$(W_1 + W_2)/W_1 = (e_1Q_{H1} + e_2Q_{C1})/(e_1Q_{H1}) = 1 + (e_2/e_1)(Q_{C1}/Q_{H1})$$

But, $e_1 = 1 - (Q_{C1}/Q_{H1})$, so that $(Q_{C1}/Q_{H1}) = 1 - e_1$. Therefore,

$$\frac{W_1 + W_2}{W_1} = 1 + \frac{e_2}{e_1}(1 - e_1)$$

$$= 1 + \frac{e_2}{e_1} - e_2 = 1 + 0.500 - 0.272 = \boxed{1.23}$$

91. **SSM** ***REASONING AND SOLUTION*** We wish to find an expression for the overall efficiency e in terms of the efficiencies e_1 and e_2. From the problem statement, the overall efficiency of the two-engine device is

$$e = \frac{W_1 + W_2}{Q_H} \tag{1}$$

where Q_H is the input heat to engine 1. The efficiency of a heat engine is defined by Equation 15.11, $e = W / Q_H$, so we can write

$$W_1 = e_1 Q_H \tag{2}$$

and

$$W_2 = e_2 Q_{H2}$$

Since the heat rejected by engine 1 is used as input heat for the second engine, $Q_{H2} = Q_{C1}$, and the expression above for W_2 can be written as

$$W_2 = e_2 Q_{C1} \tag{3}$$

According to Equation 15.12, we have $Q_{C1} = Q_H - W_1$, so that Equation (3) becomes

$$W_2 = e_2 (Q_H - W_1) \tag{4}$$

Substituting Equations (2) and (4) into Equation (1) gives

$$e = \frac{e_1 Q_H + e_2 (Q_H - W_1)}{Q_H} = \frac{e_1 Q_H + e_2 (Q_H - e_1 Q_H)}{Q_H}$$

Algebraically canceling the Q_H's in the right hand side of the last expression gives the desired result:

$$\boxed{e = e_1 + e_2 - e_1 e_2}$$

92. ***REASONING AND SOLUTION*** The three-step process is shown on the PV diagram at the right. From the first law of thermodynamics,

$$Q = \Delta U + W \qquad (1)$$

However, the ideal gas is back in its initial state at the end of the three-step process, so that $\Delta U = 0$ overall. With this value for ΔU, Equation (1) becomes $Q = W$, and we conclude that

$$Q = \underbrace{W_{1\to 2}}_{\text{isothermal}} + \underbrace{W_{2\to 3}}_{\text{isobaric}} + \underbrace{W_{3\to 1}}_{\text{isochoric}}$$

Using Equations 15.3 for the isothermal work and Equation 15.2 for the isobaric work, and remembering that there is no work done in an isochoric process, we find that

$$Q = nRT\ln\left(\frac{3V_0}{V_0}\right) + P\left(V_0 - 3V_0\right) = nRT\ln 3 - 2PV_0$$

In this result, T = 438 K, P is the pressure for step $2 \to 3$, and $2PV_0 = 2(P3V_0)/3 = 2nRT/3$. In addition, we know that n = 1 mol. Therefore,

$$Q = nRT\ln 3 - 2PV_0 = nRT\ln 3 - \tfrac{2}{3}nRT = nRT\left(\ln 3 - \tfrac{2}{3}\right)$$

$$= (1\text{ mol})\left[8.31\text{ J}(\text{mol}\cdot\text{K})\right](438\text{ K})\left(\ln 3 - \tfrac{2}{3}\right) = \boxed{1.57\times 10^3\text{ J}}$$

Since this answer is positive, $\boxed{\text{heat is absorbed}}$ by the gas.

1. **SSM** ***REASONING AND SOLUTION*** The frequency f (in hertz) is the number of wave cycles that pass an observer and is the reciprocal of the period T (in seconds). The period T is the time required for one complete cycle of the wave to pass a fixed point. From the data given in the problem, $T = (\text{time for } n \text{ cycles})/n = (120 \text{ s})/10 = 12 \text{ s}$. Therefore,

$$f = \frac{1}{T} = \frac{1}{12 \text{ s}} = \boxed{0.083 \text{ Hz}}$$

2. ***REASONING AND SOLUTION*** The "wavelength" of the magnetized regions is

$$\lambda = v/f = (0.048 \text{ m/s})/(15\,000 \text{ Hz}) = \boxed{3.2 \times 10^{-6} \text{ m}}$$

3. **SSM** ***REASONING*** When the end of the Slinky is moved up and down continuously, a transverse wave is produced. The distance between two adjacent crests on the wave, is, by definition, one wavelength. The wavelength is related to the speed and frequency of a periodic wave by Equation 16.1, $\lambda = v/f$. In order to use Equation 16.1, we must first determine the frequency of the wave. The wave on the Slinky will have the same frequency as the simple harmonic motion of the hand. According to the data given in the problem statement, the frequency is $f = (2.00 \text{ cycles})/(1 \text{ s}) = 2.00 \text{ Hz}$.

SOLUTION Substituting the values for λ and f, we find that the distance between crests is

$$\lambda = \frac{v}{f} = \frac{0.50 \text{ m/s}}{2.00 \text{ Hz}} = \boxed{0.25 \text{ m}}$$

4. ***REASONING AND SOLUTION***

a. Since 15 boxcars pass by in 12.0 s, the boxcars pass by with a frequency of

$$f = \frac{15}{12.0\ \text{s}} = \boxed{1.25\ \text{Hz}}$$

b. Since the length of a boxcar corresponds to the wavelength λ of a wave, we have, from Equation 16.1, that

$$v = f\lambda = (1.25\ \text{Hz})(14.0\ \text{m}) = \boxed{17.5\ \text{m/s}}$$

5. ***REASONING AND SOLUTION*** From Equation 16.1, we have $\lambda = v/f$. But $v = x/t$, so we find

$$\lambda = \frac{v}{f} = \frac{x}{tf} = \frac{2.5\text{ m}}{(1.7\text{ s})(3.0\text{ Hz})} = \boxed{0.49\text{ m}}$$

6. ***REASONING*** The speed of a Tsunamis is equal to the distance x it travels divided by the time t it takes for the wave to travel that distance. The frequency f of the wave is equal to its speed divided by the wavelength λ, $f = v/\lambda$ (Equation 16.1). The period T of the wave is related to its frequency by Equation 10.5, $T = 1/f$.

SOLUTION

a. The speed of the wave is (in m/s)

$$v = \frac{x}{t} = \frac{3700 \times 10^3 \text{ m}}{5.3 \text{ h}}\left(\frac{1 \text{ h}}{3600 \text{ s}}\right) = \boxed{190 \text{ m/s}}$$

b. The frequency of the wave is

$$f = \frac{v}{\lambda} = \frac{190 \text{ m/s}}{750 \times 10^3 \text{ m}} = \boxed{2.5 \times 10^{-4} \text{ Hz}} \qquad (16.1)$$

c. The period of any wave is the reciprocal of its frequency:

$$T = \frac{1}{f} = \frac{1}{2.5 \times 10^{-4} \text{ Hz}} = \boxed{4.0 \times 10^3 \text{ s}} \qquad (10.5)$$

7. **SSM** ***REASONING*** As the transverse wave propagates, the colored dot moves up and down in simple harmonic motion with a frequency of 5.0 Hz. The amplitude (1.3 cm) is the magnitude of the maximum displacement of the dot from its equilibrium position.

SOLUTION The period T of the simple harmonic motion of the dot is $T = 1/f = 1/(5.0\text{ Hz}) = 0.20\text{ s}$. In one period the dot travels through one complete cycle of its motion, and covers a vertical distance of $4 \times (1.3\text{ cm}) = 5.2\text{ cm}$. Therefore, in 3.0 s the dot will have traveled a *total vertical distance* of

$$\left(\frac{3.0\text{ s}}{0.20\text{ s}}\right)(5.2\text{ cm}) = \boxed{78\text{ cm}}$$

8. ***REASONING AND SOLUTION*** The period of the waves is T = 7.0 s/3 = 2.3 s. The frequency is then f = 1/T = 0.43 Hz. The wavelength is λ = 4.0 m. The speed is

$$v = \lambda f = (4.0\text{ m})(0.43\text{ Hz}) = \boxed{1.7\text{ m/s}}$$

9. ***REASONING*** During each cycle of the wave, a particle of the string moves through a total distance that equals $4A$, where A is the amplitude of the wave. The number of wave cycles per second is the frequency f of the wave. Therefore, the distance moved per second by a string particle is $4Af$. The time to move through a total distance L, then, is $t = L/(4Af)$. According to Equation 16.1, however, we have that $f = v/\lambda$, so that

$$t = \frac{L}{4Af} = \frac{L}{4A(v/\lambda)}$$

SOLUTION Using the result obtained above, we find that

$$t = \frac{L\lambda}{4Av} = \frac{(1.0 \times 10^3 \text{ m})(0.18 \text{ m})}{4(2.0 \times 10^{-3} \text{ m})(450 \text{ m/s})} = \boxed{5.0 \times 10^1 \text{ s}}$$

10. ***REASONING AND SOLUTION*** The maximum acceleration of the dot occurs at the extreme positions and is

$$a_{\max} = (2\pi f)^2 A = \left[2\pi(4.0\text{ Hz})\right]^2 \left(5.4\times10^{-3}\text{ m}\right) = \boxed{3.4\text{ m/s}^2} \tag{10.10}$$

11. ***REASONING AND SOLUTION*** When traveling with the waves, the skier "sees" the waves to be traveling with a velocity of $v_w - v_s$ and with a period of $T_1 = 0.600$ s. The distance between crests is (in meters)

$$\lambda = (v_s - v_w)T_1 = 7.20 - 0.600v_w \qquad (1)$$

Similarly, for the skier traveling opposite the waves

$$\lambda = (v_s + v_w)T_2 = 6.00 + 0.500v_w \qquad (2)$$

a. Subtracting Equation (2) from Equation(1) and solving for v_w gives $v_w = \boxed{1.1 \text{ m/s}}$.

b. Substituting the value for v_w into Equation (1) gives $\lambda = \boxed{6.55 \text{ m}}$.

12. ***REASONING AND SOLUTION*** The speed of the wave is

$$v = \sqrt{\frac{F}{m/L}} = f\lambda \tag{16.2}$$

Solving for the mass m of the string gives

$$m = \frac{FL}{f^2\lambda^2} = \frac{(2.3\text{ N})(0.75\text{ m})}{(150\text{ Hz})^2(0.40\text{ m})^2} = \boxed{4.8\times10^{-4}\text{ kg}}$$

13. **SSM** ***REASONING*** The tension F in the violin string can be found by solving Equation 16.2 for F to obtain $F = mv^2 / L$, where v is the speed of waves on the string and can be found from Equation 16.1 as $v = f\lambda$.

SOLUTION Combining Equations 16.2 and 16.1 and using the given data, we obtain

$$F = \frac{mv^2}{L} = (m/L)f^2\lambda^2 = (7.8 \times 10^{-4}\ \text{kg/m})(440\ \text{Hz})^2(65 \times 10^{-2}\ \text{m})^2 = \boxed{64\ \text{N}}$$

14. ***REASONING AND SOLUTION*** Initially, the speed of the wave is $v_o = \lambda f_o$. After the adjustment the speed of the wave is $v = 2\lambda f_o$. Dividing, we obtain $v/v_o = 2$. Now for each case

$$F_o = (m/L)v_o^2 \quad \text{and} \quad F = (m/L)v^2$$

Division yields

$$F/F_o = v^2/v_o^2 = 4$$

Hence,

$$F = 4F_o = 4(58\text{ N}) = \boxed{230\text{ N}}$$

15. **SSM** **WWW** ***REASONING*** According to Equation 16.2, the linear density of the string is given by $(m/L) = F/v^2$, where the speed v of waves on the middle C string is given by Equation 16.1, $v = \lambda f = \lambda / T$.

SOLUTION Combining Equations 16.2 and 16.1 and using the given data, we obtain

$$m/L = \frac{F}{v^2} = \frac{FT^2}{\lambda^2} = \frac{(944\text{ N})(3.82\times10^{-3}\text{ s})^2}{(1.26\text{ m})^2} = \boxed{8.68\times10^{-3}\text{ kg/m}}$$

16. ***REASONING*** Each pulse travels a distance that is given by vt, where v is the wave speed and t is the travel time up to the point when they pass each other. The sum of the distances traveled by each pulse must equal the 50.0-m length of the wire, since each pulse starts out from opposite ends of the wires.

SOLUTION Using v_A and v_B to denote the speeds on either wire, we have

$$v_A t + v_B t = 50.0 \text{ m}$$

Solving for the time t and using Equation 16.2 $\left(v = \sqrt{\dfrac{F}{m/L}} \right)$, we find

$$t = \frac{50.0 \text{ m}}{v_A + v_B} = \frac{50.0 \text{ m}}{\sqrt{\dfrac{F_A}{m/L}} + \sqrt{\dfrac{F_B}{m/L}}} = \frac{50.0 \text{ m}}{\sqrt{\dfrac{6.00 \times 10^2 \text{ N}}{0.020 \text{ kg/m}}} + \sqrt{\dfrac{3.00 \times 10^2 \text{ N}}{0.020 \text{ kg/m}}}} = \boxed{0.17 \text{ s}}$$

17. ***REASONING AND SOLUTION*** The hanging mass imparts to the wire a tension F = Mg so that g = F/M. For the pulse on the wire, Equation 16.2 indicates that

$$F = (m/L)v^2$$

which leads to

$$g = (m/L)v^2/M$$

The speed of the wave on the wire is calculated from the experiment to be v = L/t. Now

$$g = \frac{(m/L)L^2}{Mt^2} = \frac{(3.1 \times 10^{-4}\ \text{kg/m})(1.5\ \text{m})^2}{(0.085\ \text{kg})(0.083\ \text{s})^2} = \boxed{1.2\ \text{m/s}^2}$$

18. ***REASONING*** A particle of the string is moving in simple harmonic motion. The maximum speed of the particle is given by Equation 10.8 as $v_{max} = A\omega$, where A is the amplitude of the wave and ω is the angular frequency. The angular frequency is related to the frequency f by Equation 10.6, $\omega = 2\pi f$, so the maximum speed can be written as $v_{max} = 2\pi f A$. The speed v of a wave on a string is related to the frequency f and wavelength λ by Equation 16.1, $v = f\lambda$. The ratio of the maximum particle speed to the speed of the wave is

$$\frac{v_{max}}{v} = \frac{2\pi f A}{f\lambda} = \frac{2\pi A}{\lambda}$$

The equation can be used to find the wavelength of the wave.

SOLUTION Solving the equation above for the wavelength, we have

$$\lambda = \frac{2\pi A}{\left(\frac{v_{max}}{v}\right)} = \frac{2\pi(4.5\text{ cm})}{3.1} = \boxed{9.1\text{ cm}}$$

19. **SSM** ***REASONING*** Newton's second law can be used to analyze the motion of the blocks using the methods introduced and developed in Chapter 4. In this way we can determine an expression that relates the magnitude of the pulling force **P** to the magnitude of the tension **F** in the wire. Equation 16.2 [$v = \sqrt{F/(m/L)}$] can then be used to find the tension in the wire.

SOLUTION The following figures show a schematic of the situation described in the problem and the free-body diagrams for each block, where $m_1 = 42.0$ kg and $m_2 = 19.0$ kg.

Newton's second law for block 1 is, taking forces that point to the right as positive, $F = m_1 a$, or $a = F/m_1$. For block 2, we obtain $P - F = m_2 a$. Using the expression for a obtained from the equation for block 1, we have

$$P - F = F\left(\frac{m_2}{m_1}\right) \qquad \text{or} \qquad P = F\left(\frac{m_2}{m_1}\right) + F = F\left(\frac{m_2}{m_1} + 1\right)$$

According to Equation 16.2, $F = v^2(m/L)$, where m/L is the mass per unit length of the wire. Combining this expression for F with the expression for P, we have

$$P = v^2(m/L)\left(\frac{m_2}{m_1} + 1\right) = (352 \text{ m/s})^2(8.50 \times 10^{-4} \text{ kg/m})\left(\frac{19.0 \text{ kg}}{42.0 \text{ kg}} + 1\right) = \boxed{153 \text{ N}}$$

20. ***REASONING AND SOLUTION*** Initially, the tension in the wire is

$$F_o = (m/L)v^2 = (7.0 \times 10^{-3}\ \text{kg/m})(46\ \text{m/s})^2$$

As the temperature is lowered, the wire will attempt to shrink by an amount $\Delta L = \alpha L \Delta T$. Since the wire cannot shrink, a stress will develop.

$$\text{stress} = Y\Delta L/L = Y\alpha\Delta T$$

This stress corresponds to an additional tension:

$$F' = (\text{stress})A = YA\alpha\Delta T = (1.1 \times 10^{11}\ \text{Pa})(1.1 \times 10^{-6}\ \text{m}^2)(17 \times 10^{-6}/\text{C}^\circ)(14\ \text{C}^\circ)$$

The total tension in the wire at the lower temperature is now $F = F_0 + F'$, so that the new speed of the waves on the wire is

$$v = \sqrt{\frac{F_0 + F'}{m/L}} = \boxed{79\ \text{m/s}}$$

21. ***REASONING AND SOLUTION*** If the string has length L, the time required for a wave on the string to travel from the center of the circle to the ball is

$$t = \frac{L}{v_{wave}} \tag{1}$$

The speed of the wave is given by text Equation 16.2

$$v_{wave} = \sqrt{\frac{F}{m_{string}/L}} \tag{2}$$

The tension F in the string acts as a centripetal force on the ball, so that

$$F = m_{ball}\omega^2 r = m_{ball}\omega^2 L \tag{3}$$

Eliminating the tension F from Equations (2) and (3) above yields

$$v_{wave} = \sqrt{\frac{m_{ball}\omega^2 L}{m_{string}/L}} = \sqrt{\frac{m_{ball}\omega^2 L^2}{m_{string}}} = L\sqrt{\frac{m_{ball}\omega^2}{m_{string}}}$$

Substituting this expression for v_{wave} into Equation (1) gives

$$t = \frac{L}{L\sqrt{\frac{m_{ball}\omega^2}{m_{string}}}} = \sqrt{\frac{m_{string}}{m_{ball}\omega^2}} = \sqrt{\frac{0.0230\text{ kg}}{(15.0\text{ kg})(12.0\text{ rad/s})^2}} = \boxed{3.26 \times 10^{-3}\text{ s}}$$

22. ***REASONING AND SOLUTION*** We know that $v = 7.1$ m/s, $A = 0.15$ m, $\lambda = 0.28$ m. For a wave traveling in the +x direction, Equation 16.3 indicates that $y = A \sin(2\pi ft - 2\pi x/\lambda)$. Now $f = v/\lambda$, so

$$2\pi f = 2\pi v/\lambda = 2\pi(7.1 \text{ m/s})/(0.28 \text{ m}) = 160 \text{ rad/s}$$

Similarly, we find that

$$2\pi/\lambda = 2\pi/(0.28 \text{ m}) = 22 \text{ rad/m}$$

Therefore,

$$\boxed{y = 0.15 \sin(160\, t - 22x)}$$

23. **SSM** ***REASONING*** The mathematical form for the displacement of a wave traveling in the $-x$ direction is given by Equation 16.4: $y = A\sin\left(2\pi f t + \frac{2\pi x}{\lambda}\right)$. Using Equation 16.1 and the fact that $f = 1/T$, we obtain the following numerical values for f and λ: $f = 1/T = 1.3\text{ Hz}$, and $\lambda = v/f = 9.2\text{ m}$. Omitting units and substituting these values for f and λ into Equation 16.4 gives

$$\boxed{y = (0.37\text{ m})\sin\left(2.6\pi t + 0.22\pi x\right)}$$

24. ***REASONING AND SOLUTION*** We find from the upper graph that $\lambda = 0.060\text{ m} - 0.020\text{ m} = 0.040\text{ m}$ and $A = 0.010\text{ m}$. From the lower graph we find that $T = 0.30\text{ s} - 0.10\text{ s} = 0.20\text{ s}$. Then, $f = 1/(0.20\text{ s}) = 5.0\text{ Hz}$. Substituting these into Equation 16.3 we get

$$y = A\sin\left(2\pi f t - \frac{2\pi x}{\lambda}\right) \qquad \text{and} \qquad \boxed{y = (0.010\text{ m})\sin(10\pi t - 50\pi x)}$$

25. ***REASONING AND SOLUTION***

a. Comparing with Equations 16.3 and 16.4, we see that the wave travels in the $\boxed{\text{+x direction}}$

b. The displacement at x = 13 m and t = 38 s is

$$y = (0.26\text{ m})\sin\left[\pi(38) - (3.7)\pi(13)\right] = \boxed{-0.080\text{ m}}$$

26. ***REASONING*** The speed of a wave on the string is given by Equation 16.2 as $v = \sqrt{\dfrac{F}{m/L}}$, where F is the tension in the string and m/L is the mass per unit length (or linear density) of the string. The wavelength λ is the speed of the wave divided by its frequency f (Equation 16.1).

SOLUTION

a. The speed of the wave on the string is

$$v = \sqrt{\frac{F}{(m/L)}} = \sqrt{\frac{15\text{ N}}{0.85\text{ kg/m}}} = \boxed{4.2\text{ m/s}}$$

b. The wavelength is

$$\lambda = \frac{v}{f} = \frac{4.2\text{ m/s}}{12\text{ Hz}} = \boxed{0.35\text{ m}}$$

c. The amplitude of the wave is $A = 3.6\text{ cm} = 3.6 \times 10^{-2}$ m. Since the wave is moving along the $-x$ direction, the mathematical expression for the wave is given by Equation 16.4 as

$$y = A\sin\left(2\pi f t + \frac{2\pi x}{\lambda}\right)$$

Substituting in the numbers for A, f, and λ, we have

$$y = A\sin\left(2\pi f t + \frac{2\pi x}{\lambda}\right) = \left(3.6\times10^{-2}\text{ m}\right)\sin\left[2\pi(12\text{ Hz})t + \frac{2\pi x}{0.35\text{ m}}\right]$$

$$= \boxed{\left(3.6\times10^{-2}\text{ m}\right)\sin\left[\left(75\text{ s}^{-1}\right)t + \left(18\text{ m}^{-1}\right)x\right]}$$

27. **SSM** ***REASONING*** According to Equation 16.2, the tension F in the string is given by $F = v^2(m/L)$. Since $v = \lambda f$ from Equation 16.1, the expression for F can be written

$$F = (\lambda f)^2 \left(\frac{m}{L}\right) \tag{1}$$

where the quantity m/L is the linear density of the string. In order to use Equation (1), we must first obtain values for f and λ; these quantities can be found by analyzing the expression for the displacement of a string particle.

SOLUTION The displacement is given by $y = (0.021\text{ m})\sin(25t - 2.0x)$. Inspection of this equation and comparison with Equation 16.3, $y = A\sin\left(2\pi f t - \frac{2\pi x}{\lambda}\right)$, gives

$$2\pi f = 25\text{ rad/s} \quad \text{or} \quad f = \frac{25}{2\pi}\text{ Hz}$$

and

$$\frac{2\pi}{\lambda} = 2.0\text{ m}^{-1} \quad \text{or} \quad \lambda = \frac{2\pi}{2.0}\text{ m}$$

Substituting these values f and λ into Equation (1) gives

$$F = (\lambda f)^2\left(\frac{m}{L}\right) = \left[\left(\frac{2\pi}{2.0}\text{ m}\right)\left(\frac{25\text{ Hz}}{2\pi}\right)\right]^2 (1.6\times10^{-2}\text{ kg/m}) = \boxed{2.5\text{ N}}$$

28. ***REASONING*** Using either Equation 16.3 or 16.4 with $x = 0$ m, we obtain

$$y = A\sin(2\pi f t)$$

Applying this equation with $f = 175$ Hz, we can find the times at which $y = 0.10$ m.

SOLUTION Using the given values for y and A, we obtain

$$0.10\ \text{m} = (0.20\ \text{m})\sin(2\pi f t) \quad \text{or} \quad 2\pi f t = \sin^{-1}\left(\frac{0.10\ \text{m}}{0.20\ \text{m}}\right) = \sin^{-1}(0.50)$$

The smallest two angles for which the sine function is 0.50 are 30° and 150°. However, the values for $2\pi ft$ must be expressed in radians. Thus, we find that the smallest two angles for which the sine function is 0.50 are expressed in radians as follows:

$$(30°)\left(\frac{2\pi}{360°}\right) \quad \text{and} \quad (150°)\left(\frac{2\pi}{360°}\right)$$

The times corresponding to these angles can be obtained as follows:

$$2\pi f t_1 = (30°)\left(\frac{2\pi}{360°}\right) \qquad 2\pi f t_2 = (150°)\left(\frac{2\pi}{360°}\right)$$

$$t_1 = \frac{30°}{f(360°)} \qquad t_2 = \frac{150°}{f(360°)}$$

Subtracting and using $f = 175$ Hz gives

$$t_2 - t_1 = \frac{150°}{f(360°)} - \frac{30°}{f(360°)} = \frac{(150° - 30°)}{(175\ \text{Hz})(360°)} = \boxed{1.9 \times 10^{-3}\ \text{s}}$$

29. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The speed of sound in an ideal gas is given by Equation 16.5, $v = \sqrt{\gamma kT/m}$. The ratio of the speed of sound v_2 in the container after the temperature is raised to the speed v_1 before the temperature change is

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

Thus, the new speed is

$$v_2 = v_1\sqrt{\frac{T_2}{T_1}} = (1220\text{ m/s})\sqrt{\frac{405\text{ K}}{201\text{ K}}} = \boxed{1730\text{ m/s}}$$

30. ***REASONING*** Since the sonar pulse travels to the object and back, the total distance it travels is $2x$, where x is the distance from the submarine to the object. The total distance is equal to the product of the speed v_{seawater} of the sonar wave in seawater and the time t for the pulse to reach the object and return, $2x = v_{\text{seawater}}t$. We can use this relation to find the distance to the object.

SOLUTION Solving the equation above for x, and using the value of v_{seawater} from Table 16.1, we have

$$x = \frac{v_{\text{seawater}}t}{2} = \frac{(1522\ \text{m/s})(1.30\ \text{s})}{2} = \boxed{989\ \text{m}}$$

31. ***REASONING AND SOLUTION***

$$\lambda = v/f = (343 \text{ m/s})/(4185.6 \text{ Hz}) = \boxed{8.19 \times 10^{-2} \text{ m}} \tag{16.1}$$

32. ***REASONING*** A rail can be approximated as a long slender bar, so the speed of sound in the rail is given by Equation 16.7. With this equation and the given data for Young's modulus and the density of steel, we can determine the speed of sound in the rail. Then, we will be able to compare this speed to the speed of sound in air at 20 °C, which is 343 m/s.

SOLUTION The speed of sound in the rail is

$$v = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{2.0 \times 10^{11}\ \text{N/m}^2}{7860\ \text{kg/m}^3}} = 5.0 \times 10^3\ \text{m/s}$$

This speed is greater than the speed of sound in air at 20 °C by a factor of

$$\frac{v_{\text{rail}}}{v_{\text{air}}} = \frac{5.0 \times 10^3\ \text{m/s}}{343\ \text{m/s}} = \boxed{15}$$

33. **SSM** ***REASONING*** If we treat the sample of argon atoms like an ideal monatomic gas ($\gamma = 1.67$) at 298 K, Equation 14.6 $\left(\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT\right)$ can be solved for the root-mean-square speed v_{rms} of the argon atoms. The speed of sound in argon can be found from Equation 16.5: $v = \sqrt{\gamma\, kT/m}$.

SOLUTION We first find the mass of an argon atom. Since the molecular mass of argon is 39.9 u, argon has a mass per mole of 39.9×10^{-3} kg/mol. Thus, the mass of a single argon atom is

$$\text{m} = \frac{39.9\times10^{-3}\ \text{kg/mol}}{6.022\times10^{23}\ \text{mol}^{-1}} = 6.63\times10^{-26}\ \text{kg}$$

a. Solving Equation 14.6 for v_{rms} and substituting the data given in the problem statement, we find

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38\times10^{-23}\ \text{J/K})(298\ \text{K})}{6.63\times10^{-26}\ \text{kg}}} = \boxed{431\ \text{m/s}}$$

b. The speed of sound in argon is, according to Equation 16.5,

$$v = \sqrt{\frac{\gamma\, kT}{m}} = \sqrt{\frac{(1.67)(1.38\times10^{-23}\ \text{J/K})(298\ \text{K})}{6.63\times10^{-26}\ \text{kg}}} = \boxed{322\ \text{m/s}}$$

34. ***REASONING AND SOLUTION*** The sound wave will travel at constant speed through the water (speed = v_{water}) and the copper block (speed = v_{copper}). Thus, the times it takes for the sound wave to travel downward through the water a distance d_{water} and downward through the copper a distance d_{copper} are

$$\Delta t_{water} = \frac{d_{water}}{v_{water}} \qquad \text{and} \qquad \Delta t_{copper} = \frac{d_{copper}}{v_{copper}}$$

The time required for the incident sound wave to reach the bottom of the copper block just before reflection is equal to the time required for the reflected sound wave to reach the surface of the water. Using values for the speed of sound in water and copper from Table 16.1 in the text, we find that the total time interval between when the sound enters and leaves the water is, then,

$$\Delta t_{TOTAL} = 2(\Delta t_{water} + \Delta t_{copper}) = 2\left[\left(\frac{d_{water}}{v_{water}}\right) + \left(\frac{d_{copper}}{v_{copper}}\right)\right]$$

$$\Delta t_{TOTAL} = 2\left[\left(\frac{0.45\text{ m}}{1482\text{ m/s}}\right) + \left(\frac{0.15\text{ m}}{5010\text{ m/s}}\right)\right] = \boxed{6.7 \times 10^{-4}\text{ s}}$$

35. ***REASONING AND SOLUTION***

a. In order to determine the order of arrival of the three waves, we need to know the speeds of each wave. The speeds for air, water and steel are

$$v_a = 343 \text{ m/s}, v_w = 1482 \text{ m/s}, v_s = 5040 \text{ m/s}$$

The order of arrival is $\boxed{\text{steel wave first, water wave second, air wave third}}$.

b. Calculate the length of time each wave takes to travel 125 m.

$$t_s = (125 \text{ m})/(5040 \text{ m/s}) = 0.025 \text{ s}$$
$$t_w = (125 \text{ m})/(1482 \text{ m/s}) = 0.084 \text{ s}$$
$$t_a = (125 \text{ m})/(343 \text{ m/s}) = 0.364 \text{ s}$$

Therefore, the delay times are

$$\Delta t_{12} = t_w - t_s = 0.084 \text{ s} - 0.025 \text{ s} = \boxed{0.059 \text{ s}}$$
$$\Delta t_{13} = t_a - t_s = 0.364 \text{ s} - 0.025 \text{ s} = \boxed{0.339 \text{ s}}$$

36. ***REASONING AND SOLUTION*** The wheel must rotate at a frequency of $f = (2200\ \text{Hz})/20 = 110\ \text{Hz}$. The angular speed ω of the wheel is

$$\omega = 2\pi f = 2\pi(110\ \text{Hz}) = \boxed{690\ \text{rad/s}}$$

37. ***REASONING AND SOLUTION*** The speed of sound in an ideal gas is given by text Equation 16.5

$$v = \sqrt{\frac{\gamma kT}{m}}$$

where m is the mass of a single gas particle (atom or molecule). Solving for T gives

$$T = \frac{mv^2}{\gamma k} \qquad (1)$$

The mass of a single helium atom is

$$\frac{4.003 \text{ g/mol}}{6.022\times10^{23} \text{ mol}^{-1}}\left(\frac{1 \text{ kg}}{1000 \text{ g}}\right) = 6.650\times10^{-27} \text{ kg}$$

The speed of sound in oxygen at 0 °C is 316 m/s. Since helium is a monatomic gas, $\gamma = 1.67$. Then, substituting into Equation (1) gives

$$T = \frac{(6.65\times10^{-27} \text{ kg})(316 \text{ m/s})^2}{(1.67)(1.38\times10^{-23} \text{ J/K})} = \boxed{28.8 \text{ K}}$$

38. ***REASONING AND SOLUTION*** The lightening bolt and the thunder are generated at the same point at the same instant. If the speed of light were 961 m/s, while the speed of sound remains at 343 m/s, the time required for light to travel 1.0 mile (1.6×10^3 m) would be

$$(1.6 \times 10^3 \text{ m})/(961 \text{ m/s}) = 1.7 \text{ s}$$

The time required for the thunder to travel this distance is still 4.7 s. Thus, the flash reaches the observer after 1.7 s, and the thunder reaches the observer 4.7 s – 1.7 s = 3.0 s later. The time interval between seeing the flash and hearing the thunder is approximately 3 s for each mile between the observer and the lightening bolt. Therefore, $\boxed{X = 3}$.

39. $\boxed{\text{SSM}}$ ***REASONING*** Equation 16.7 relates the Young's modulus Y, the mass density ρ, and the speed of sound v in a long slender solid bar. According to Equation 16.7, the Young's modulus is given by $Y = \rho v^2$. The data given in the problem can be used to compute values for both ρ and v.

SOLUTION Using the values of the data given in the problem statement, we find that the speed of sound in the bar is

$$v = \frac{L}{t} = \frac{0.83 \text{ m}}{1.9 \times 10^{-4} \text{ s}} = 4.4 \times 10^3 \text{ m/s}$$

where L is the length of the rod and t is the time required for the wave to travel the length of the rod. The mass density of the bar is, from Equation 11.1, $\rho = m / V = m/(LA)$ where m, and A are, respectively, the mass and the cross-sectional area of the rod. The density of the rod is, therefore,

$$\rho = \frac{m}{LA} = \frac{2.1 \text{ kg}}{(0.83 \text{ m})(1.3 \times 10^{-4} \text{ m}^2)} = 1.9 \times 10^4 \text{ kg/m}^3$$

Using these values, we find that the bulk modulus for the rod is

$$Y = \rho v^2 = (1.9 \times 10^4 \text{ kg/m}^3)(4.4 \times 10^3 \text{ m/s})^2 = 3.7 \times 10^{11} \text{ N/m}^2$$

Comparing this value to those given in Table 10.1, we conclude that the bar is most likely made of $\boxed{\text{tungsten}}$.

40. ***REASONING*** Let v_P represent the speed of the primary wave and v_S the speed of the secondary wave. The travel times for the primary and secondary waves are t_P and t_S, respectively. If x is the distance from the earthquake to the seismograph, then $t_P = x/v_P$ and $t_S = x/v_S$. The difference in the arrival times is

$$t_S - t_P = \frac{x}{v_S} - \frac{x}{v_P} = x\left(\frac{1}{v_S} - \frac{1}{v_P}\right)$$

We can use this equation to find the distance from the earthquake to the seismograph.

SOLUTION Solving the equation above for x gives

$$x = \frac{t_S - t_P}{\frac{1}{v_S} - \frac{1}{v_P}} = \frac{78\text{ s}}{\frac{1}{4.5\times10^3\text{ m/s}} - \frac{1}{8.0\times10^3\text{ m/s}}} = \boxed{8.0\times10^5\text{ m}}$$

41. **SSM** ***REASONING*** The sound will spread out uniformly in all directions. For the purposes of counting the echoes, we will consider only the sound that travels in a straight line parallel to the ground and reflects from the vertical walls of the cliff.

Let the distance between the hunter and the closer cliff be x_1 and the distance from the hunter to the further cliff be x_2.

The first echo arrives at the location of the hunter after traveling a total distance $2x_1$ in a time t_1, so that, if v_s is the speed of sound, $t_1 = 2x_1 / v_s$. Similarly, the second echo arrives after reflection from the far wall and in an amount of time t_2 after the firing of the gun. The quantity t_2 is related to the distance x_2 and the speed of sound v_s according to $t_2 = 2x_2 / v_s$.

The time difference between the first and second echo is, therefore,

$$\Delta t = t_2 - t_1 = \frac{2}{v_s}(x_2 - x_1) \tag{1}$$

The third echo arrives in a time t_3 after the second echo. It arises from the sound of the second echo that is reflected from the closer cliff wall. Thus, $t_3 = 2x_1 / v_s$, or, solving for x_1, we have

$$x_1 = \frac{v_s t_3}{2} \tag{2}$$

Combining Equations (1) and (2), we obtain

$$\Delta t = t_2 - t_1 = \frac{2}{v_s}\left(x_2 - \frac{v_s t_3}{2}\right)$$

Solving for x_2, we have

$$x_2 = \frac{v_s}{2}(\Delta t + t_3) \tag{3}$$

The distance between the cliffs can be found from $d = x_1 + x_2$, where x_1 and x_2 can be determined from Equations (2) and (3), respectively.

SOLUTION According to Equation (2), the distance x_1 is

$$x_1 = \frac{(343\text{ m/s})(1.1\text{ s})}{2} = 190\text{ m}$$

According to Equation (3), the distance x_2 is

$$x_2 = \frac{(343\text{ m/s})}{2}(1.6\text{ s} + 1.1\text{ s}) = 460\text{ m}$$

Therefore, the distance between the cliffs is

$$d = x_1 + x_2 = 190 \text{ m} + 460 \text{ m} = \boxed{650 \text{ m}}$$

42. ***REASONING AND SOLUTION*** If y_a is the one-way distance the sound travels in air ($y_a = 10.0$ m) and y_w is the distance the sound travels in water, then the total travel time, t, is given by

$$t = 2y_a / v_a + 2y_w / v_w$$

where the speeds can be obtained from Table 16.1. We can, therefore, solve for the depth of the lake.

$$y_w = \left[t - 2\left(\frac{y_a}{v_a}\right)\right]\left(\frac{v_w}{2}\right) = \left[(0.140\text{ s}) - 2\left(\frac{10.0\text{ m}}{343\text{ m/s}}\right)\left(\frac{1482\text{ m/s}}{2}\right)\right] = \boxed{61\text{ m}}$$

43. ***REASONING AND SOLUTION*** The speed of sound in an ideal gas is given by text Equation 16.5:

$$v = \sqrt{\frac{\gamma kT}{m}}$$

where $\gamma = c_p/c_v$, k is Boltzmann's constant, T is the Kelvin temperature of the gas, and m is the mass of a single gas molecule. If m_{TOTAL} is the mass of the gas sample and N is the total number of molecules in the sample, then the above equation can be written as

$$v = \sqrt{\frac{\gamma kT}{(m_{\text{TOTAL}}/N)}} = \sqrt{\frac{\gamma N\,k\,T}{m_{\text{TOTAL}}}} \qquad (1)$$

For an ideal gas, PV = NkT, so that Equation (1) becomes,

$$v = \sqrt{\frac{\gamma PV}{m_{\text{TOTAL}}}} = \sqrt{\frac{(1.67)(3.5\times10^5\ \text{Pa})(2.5\ \text{m}^3)}{2.3\ \text{kg}}} = \boxed{8.0\times10^2\ \text{m/s}}$$

44. ***REASONING AND SOLUTION*** Since the frequency ($f = v/\lambda$) is the same in each gas, we have

$$v_{Kr}/\lambda_{Kr} = v_{Ne}/\lambda_{Ne}$$

then

$$\lambda_{Ne} = \lambda_{Kr}(v_{Ne}/v_{Kr})$$

We know that

$$v = \sqrt{\frac{\gamma kT}{m}}$$

The ratio of the speeds is

$$\frac{v_{Ne}}{v_{Kr}} = \sqrt{\frac{m_{Kr}}{m_{Ne}}}$$

We need to calculate the mass of each type of atom in the gas:

$$m_{Kr} = (83.8 \times 10^{-3}\ \text{kg/mol})/(6.022 \times 10^{23}/\text{mol}) = 1.39 \times 10^{-25}\ \text{kg}$$
$$m_{Ne} = (20.2 \times 10^{-3}\ \text{kg/mol})/(6.022 \times 10^{23}/\text{mol}) = 3.35 \times 10^{-26}\ \text{kg}$$

So we have,

$$\lambda_{Ne} = \lambda_{Kr}\sqrt{\frac{m_{Kr}}{m_{Ne}}} = (1.25\ \text{m})\sqrt{\frac{1.39 \times 10^{-25}\ \text{kg}}{3.35 \times 10^{-26}\ \text{kg}}} = \boxed{2.55\ \text{m}}$$

45. ***REASONING AND SOLUTION*** The speed of sound in an ideal gas is

$$v = \sqrt{\frac{\gamma kT}{m}}$$

so that the mass of a gas molecule is

$$m = \gamma kT/v^2 = (1.67)(1.38 \times 10^{-23}\ \text{J/K})(3.00 \times 10^2\ \text{K})/(363\ \text{m/s})^2$$

$$m = 5.25 \times 10^{-23}\ \text{g}$$

We now need to determine what fraction of the gas is argon and what fraction is neon. First find the mass of each molecule.

$$m_{ar} = (39.9\ \text{g/mol})/(6.022 \times 10^{23}/\text{mol}) = 6.63 \times 10^{-23}\ \text{g}$$

$$m_{ne} = (20.2\ \text{g/mol})/(6.022 \times 10^{23}/\text{mol}) = 3.35 \times 10^{-23}\ \text{g}$$

Let q be the fraction of gas that is in the form of argon and p be the fraction that is neon. We know that $q + p = 1$. Also, $qm_{ar} + pm_{ne} = m$. Substituting $p = 1 - q$ in this equation gives

$$qm_{ar} + (1 - q)m_{ne} = m$$

and we can now solve for the fraction

$$q = (m - m_{ne})/(m_{ar} - m_{ne})$$

Suppressing the units and canceling the factor of 10^{-23}, we find

$$q = (5.25 - 3.35)/(6.63 - 3.35) = 0.57 = \boxed{57\%\ \text{argon}}$$

and

$$p = 1 - q = 1 - 0.57 = 0.43 = \boxed{43\%\ \text{neon}}$$

46. ***REASONING AND SOLUTION*** The speed of sound in fresh water is

$$v_{\text{fresh}} = \sqrt{\frac{B_{\text{ad}}}{\rho}} = \sqrt{\frac{2.20\times10^{9}\ \text{Pa}}{1.00\times10^{3}\ \text{kg/m}^{3}}} = 1.48\times10^{3}\ \text{m/s}$$

The speed of sound in salt water is

$$v_{\text{salt}} = \sqrt{\frac{B_{\text{ad}}}{\rho}} = \sqrt{\frac{2.37\times10^{9}\ \text{Pa}}{1025\ \text{kg/m}^{3}}} = 1.52\times10^{3}\ \text{m/s}$$

$$\frac{1.52\times10^{3}\ \text{m/s}}{1.48\times10^{3}\ \text{m/s}} = 1.03$$

Thus, the sonar unit erroneously calculates that the objects are 1.03 times the distance measured in salt water:

$$\text{d}_{\text{actual}} = (1.03)(10.0\ \text{m}) = \boxed{10.3\ \text{m}}$$

47. **SSM** **WWW** ***REASONING*** We must determine the time t for the warning to travel the vertical distance $h = 10.0$ m from the prankster to the ears of the man when he is just under the window. The desired distance above the man's ears is the distance that the balloon would travel in this time and can be found with the aid of the equations of kinematics.

SOLUTION Since sound travels with constant speed v_s, the distance h and the time t are related by $h = v_s t$. Therefore, the time t required for the warning to reach the ground is

$$t = \frac{h}{v_s} = \frac{10.0 \text{ m}}{343 \text{ m/s}} = 0.0292 \text{ s}$$

We now proceed to find the distance that the balloon travels in this time. To this end, we must find the balloon's final speed v_y, after falling from rest for 10.0 m. Since the balloon is dropped from rest, we use Equation 3.6*b* ($v_y^2 = v_{0y}^2 + 2a_y y$) with $v_{0y} = 0$m/s:

$$v_y = \sqrt{v_{0y}^2 + 2a_y y} = \sqrt{(0 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(10.0 \text{ m})} = 14.0 \text{ m/s}$$

Using this result, we can find the balloon's speed 0.0292 seconds before it hits the man by solving Equation 3.3b ($v_y = v_{0y} + a_y t$) for v_{0y}:

$$v_{0y} = v_y - a_y t = \left[(14.0 \text{ m/s}) - (9.80 \text{ m/s}^2)(0.0292 \text{ s})\right] = 13.7 \text{ m/s}$$

Finally, we can find the desired distance y above the man's head from Equation 3.5b:

$$y = v_{0y}t + \frac{1}{2}a_y t^2 = (13.7 \text{ m/s})(0.0292 \text{ s}) + \frac{1}{2}(9.80 \text{ m/s}^2)(0.0292 \text{ s})^2 = \boxed{0.404 \text{ m}}$$

48. ***REASONING AND SOLUTION*** We have $I = P/A$. Therefore,

$$P = IA = (3.2 \times 10^{-6}\ \text{W/m}^2)(2.1 \times 10^{-3}\ \text{m}^2) = \boxed{6.7 \times 10^{-9}\ \text{W}} \tag{16.8}$$

49. **SSM** **WWW** ***REASONING AND SOLUTION*** Since the sound radiates uniformly in all directions, at a distance r from the source, the energy of the sound wave is distributed over the area of a sphere of radius r. Therefore, according to Equation 16.9 [$I = P/(4\pi r^2)$] with $r = 3.8$ m, the power radiated from the source is

$$P = 4\pi I r^2 = 4\pi(3.6 \times 10^{-2}\ \text{W/m}^2)(3.8\ \text{m})^2 = \boxed{6.5\ \text{W}}$$

50. ***REASONING AND SOLUTION*** The energy carried by the sound into the ear is

$$\text{Energy} = IAt = (3.2 \times 10^{-5}\ \text{W/m}^2)(2.1 \times 10^{-3}\ \text{m}^2)(3600\ \text{s}) = \boxed{2.4 \times 10^{-4}\ \text{J}}$$

51. ***REASONING AND SOLUTION*** Since the sound spreads out uniformly in all directions, the intensity is uniform over any sphere centered on the source. From text Equation 16.9,

$$I = \frac{P}{4\pi r^2}$$

Then,

$$\frac{I_1}{I_2} = \frac{P/(4\pi r_1^2)}{P/(4\pi r_2^2)} = \frac{r_2^2}{r_1^2}$$

Solving for r_2, we obtain

$$r_2 = r_1\sqrt{\frac{I_1}{I_2}} = (120\ \text{m})\sqrt{\frac{(2.0\times10^{-6}\ \text{W/m}^2)}{(0.80\times10^{-6}\ \text{W/m}^2)}} = \boxed{190\ \text{m}}$$

52. ***REASONING AND SOLUTION*** The intensity of the "direct" sound is given by text Equation 16.9,

$$I_{DIRECT} = \frac{P}{4\pi r^2}$$

The total intensity at the point in question is

$$I_{TOTAL} = I_{DIRECT} + I_{REFLECTED}$$

$$= \left[\frac{1.1\times10^{-3}\ \text{W}}{4\pi(3.0\ \text{m})^2}\right] + 4.4\times10^{-6}\ \text{W/m}^2 = \boxed{1.4\times10^{-5}\ \text{W/m}^2}$$

53. **SSM** ***REASONING AND SOLUTION*** According to Equation 16.8, the power radiated by the speaker is $P = IA = I\pi r^2$, where r is the radius of the circular opening. Thus, the radiated power is

$$P = (17.5\ \text{W/m}^2)(\pi)(0.0950\ \text{m})^2 = 0.496\ \text{W}$$

As a percentage of the electrical power, this is

$$\frac{0.496\ \text{W}}{25.0\ \text{W}} \times 100\ \% = \boxed{1.98\ \%}$$

54. ***REASONING*** According to Equation 16.8, the intensity I of a sound wave is equal to the sound power P divided by the area A through which the power passes; $I = P/A$. The area is that of a circle, so $A = \pi r^2$, where r is the radius. The power, on the other hand, is the energy E per unit time, $P = E/t$, according to Equation 6.10b. Thus, we have

$$I = \frac{P}{A} = \frac{\frac{E}{t}}{\pi r^2}$$

All the variables in this equation are known except for the time.

SOLUTION Solving the expression above for the time yields

$$t = \frac{E}{I\pi r^2} = \frac{4800\text{ J}}{\left(5.9\times10^3\text{ W/m}^2\right)\pi\left(1.8\times10^{-2}\text{ m}\right)^2} = \boxed{8.0\times10^2\text{ s}}$$

55. ***REASONING AND SOLUTION*** The energy delivered to the lasagna in 7.0 minutes (420 s) is

$$Q = cm\Delta T = [3400\ \text{J/(kg·C°)}](0.25\ \text{kg})(80.0\ \text{C°}) = 6.8 \times 10^4\ \text{J} \qquad (12.4)$$

The power delivered to the lasagna is

$$P = \frac{\text{Energy}}{\text{time}} = \frac{6.8 \times 10^4\ \text{J}}{420\ \text{s}} = 1.6 \times 10^2\ \text{W} \qquad (6.10b)$$

Thus, the intensity of the microwaves in the oven is

$$I = \frac{P}{A} = \frac{1.6 \times 10^2\ \text{W}}{1.6 \times 10^{-2}\ \text{m}^2} = \boxed{1.0 \times 10^4\ \text{W/m}^2} \qquad (16.8)$$

56. ***REASONING AND SOLUTION*** The power radiated by the helicopter is

$$P = IA = I\,(4\pi r^2)$$

At the overhead position, the intensity is

$$I_1 = P/\left(4\pi r_1^2\right)$$

At some distance away, the new intensity is

$$I_2 = P/\left(4\pi r_2^2\right)$$

Since $I_1 = 4I_2$, we can write

$$P/\left(4\pi r_1^2\right) = P/\left(4\pi r_2^2\right)$$

This result indicates that $r_2 = 2r_1$. The helicopter must therefore travel a distance d where

$$r_1^2 + d^2 = r_2^2 = 4r_1^2$$

So,

$$d = \sqrt{3}\, r_1 = \sqrt{3}\,(1450\text{ m}) = \boxed{2510\text{ m}}$$

57. **SSM** ***REASONING*** Since the sound is emitted from the rocket uniformly in all directions, the energy carried by the sound wave spreads out uniformly over concentric spheres of increasing radii r_1, r_2, r_3, . . . as the wave propagates. Let r_1 represent the radius when the measured intensity at the ground is I (position 1 for the rocket) and r_2 represent the radius when the measured intensity at the ground is $\frac{1}{3}I$ (position 2 of the rocket). The time required for the energy to spread out over the sphere of radius r_1 is $t_1 = r_1 / v_s$, where v_s is the speed of sound. Similarly, the time required for the energy to spread out over a sphere of radius r_2 is $t_2 = r_2 / v_s$. As the sound wave emitted at position 1 spreads out uniformly, the rocket continues to accelerate upward to position 2 with acceleration a_y for a time t_{12}. Therefore, the time that elapses between the two intensity measurements is

$$t = t_2 - t_1 + t_{12} \tag{1}$$

From Equation 3.3*b*, the time t_{12} is

$$t_{12} = \frac{(v_2 - v_1)}{a_y} \tag{2}$$

where v_1 and v_2 are the speeds of the rocket at positions 1 and 2, respectively. Combining Equations (1) and (2), we obtain

$$t = t_2 - t_1 + \frac{(v_2 - v_1)}{a_y} \tag{3}$$

The respective values of v_1 and v_2 can be found from Equation 3.6b ($v_y^2 = v_{0y}^2 + 2a_y y$) with $v_{0y} = 0$ m/s, and $y = r$. Once values for v_1 and v_2 are known, Equation (3) can be solved directly.

SOLUTION From the data given in the problem statement, $r_1 = 562$ m. We can find r_2 using the following reasoning: from the definition of intensity, $I_1 = P/(4\pi r_1^2)$ and $I_2 = P/(4\pi r_2^2)$. Since $I_1 = 3I_2$, we have

$$P/(4\pi r_1^2) = 3P/(4\pi r_2^2) \qquad \text{or} \qquad r_2 = r_1\sqrt{3} = 973 \text{ m}$$

The times t_1 and t_2 are

$$t_1 = \frac{r_1}{v_s} = \frac{562 \text{ m}}{343 \text{ m/s}} = 1.64 \text{ s}$$

and

$$t_2 = \frac{r_2}{v_s} = \frac{973 \text{ m}}{343 \text{ m/s}} = 2.84 \text{ s}$$

Then, taking up as the positive direction, we have from Equation 3.6b,

$$v_1 = \sqrt{2a_y r_1} = \sqrt{2(58.0 \text{ m/s}^2)(562 \text{ m})} = 255 \text{ m/s}$$

and

$$v_2 = \sqrt{2a_y r_2} = \sqrt{2(58.0\ \text{m/s}^2)(973\ \text{m})} = 336\ \text{m/s}$$

Substituting these values into Equation (3), we have

$$t = 2.84\ \text{s} - 1.64\ \text{s} + \frac{(336\ \text{m/s} - 255\ \text{m/s})}{58.0\ \text{m/s}^2} = \boxed{2.6\ \text{s}}$$

58. ***REASONING AND SOLUTION*** We know $\beta_2 - \beta_1 =$ (10 dB) log (I_2/I_1), so that the ratio of the intensities can be written as

$$I_2/I_1 = 10^{(\beta_2 - \beta_1)/(10\text{ dB})} = 10^{(61\text{ dB} - 23\text{ dB})/(10\text{ dB})} = \boxed{6300}$$

59. ***REASONING*** Knowing that the threshold of hearing corresponds to an intensity of $I_0 = 1.00 \times 10^{-12}$ W/m^2, we can solve Equation 16.10 directly for the desired intensity.

SOLUTION Using Equation 16.10, we find

$$\beta = 115 \text{ dB} = (10 \text{ dB}) \log\left(\frac{\text{I}}{\text{I}_0}\right) \qquad \text{or} \qquad \frac{I}{I_0} = 10^{(115 \text{ dB})/(10 \text{ dB})} = 10^{11.5}$$

Solving for I gives

$$I = I_0 10^{11.5} = \left(1.00 \times 10^{-12} \text{ W/m}^2\right) 10^{11.5} = \boxed{0.316 \text{ W/m}^2}$$

60. ***REASONING*** The sound intensity level outside the room is

$$\beta_{\text{outside}} = (10\text{ dB})\log\left(\frac{I_{\text{outside}}}{I_0}\right) \tag{16.10}$$

where I_0 is the threshold of hearing. Solving for the intensity I_{outside} gives

$$I_{\text{outside}} = I_0 10^{\frac{\beta_{\text{outside}}}{10\text{ dB}}}$$

These two relations will allow us to find the sound intensity outside the room.

SOLUTION From the problem statement we know that $\beta_{\text{outside}} = \beta_{\text{inside}} + 44.0$ dB. We can evaluate β_{inside} by applying Equation 16.10 to the inside of the room:

$$\beta_{\text{inside}} = (10\text{ dB})\log\left(\frac{I_{\text{inside}}}{I_0}\right) = (10\text{ dB})\log\left(\frac{1.20\times10^{-10}\text{ W/m}^2}{1.00\times10^{-12}\text{ W/m}^2}\right) = 20.8\text{ dB}$$

Thus, the sound intensity level outside the room is $\beta_{\text{outside}} = 20.8$ dB + 44.0 dB = 64.8 dB. The sound intensity outside the room is

$$I_{\text{outside}} = I_0 10^{\frac{\beta_{\text{outside}}}{10\text{ dB}}} = \left(1.00\times10^{-12}\text{ W/m}^2\right)10^{\frac{64.8\text{ dB}}{10\text{ dB}}} = \boxed{3.02\times10^{-6}\text{ W/m}^2}$$

61. **SSM** ***REASONING AND SOLUTION*** The intensity level β in decibels (dB) is related to the sound intensity I according to Equation 16.10:

$$\beta = (10\text{ dB})\log\left(\frac{I}{I_0}\right)$$

where the quantity I_0 is the reference intensity. Therefore, we have

$$\beta_2 - \beta_1 = (10\text{ dB})\log\left(\frac{I_2}{I_0}\right) - (10\text{ dB})\log\left(\frac{I_1}{I_0}\right) = (10\text{ dB})\log\left(\frac{I_2/I_0}{I_1/I_0}\right) = (10\text{ dB})\log\left(\frac{I_2}{I_1}\right)$$

Solving for the ratio I_2/I_1, we find

$$30.0\text{ dB} = (10\text{ dB})\log\left(\frac{I_2}{I_1}\right) \qquad or \qquad \frac{I_2}{I_1} = 10^{3.0} = 1000$$

Thus, we conclude that the sound intensity $\boxed{\text{increases by a factor of 1000}}$.

62. ***REASONING AND SOLUTION***

a. $\beta = (10\text{ dB}) \log (P_A/P_B) = (10\text{ dB})\log [(250\text{ W})/(45\text{ W}) = \boxed{7.4\text{ dB}}$

b. $\boxed{\text{No}}$, A will not be twice as loud as B since it requires an increase of 10 dB to double the loudness.

63. **SSM** ***REASONING*** According to Equation 16.10, the sound intensity level β in decibels (dB) is related to the sound intensity I according to $\beta = (10\text{ dB})\log\ (I/I_0)$, where the quantity I_0 is the reference intensity. Since the sound is emitted uniformly in all directions, the intensity, or power per unit area, is given by $I = P/(4\pi r^2)$. Thus, the sound intensity at position 1 can be written as $I_1 = P/(4\pi r_1^2)$, while the sound intensity at position 2 can be written as $I_2 = P/(4\pi r_2^2)$. Therefore, the difference in the sound intensity level β_{21} between the two positions is

$$\beta_{21} = \beta_2 - \beta_1 = (10\text{ dB})\log\left(\frac{I_2}{I_0}\right) - (10\text{ dB})\log\left(\frac{I_1}{I_0}\right) = (10\text{ dB})\log\left(\frac{I_2/I_0}{I_1/I_0}\right) = (10\text{ dB})\ \log\left(\frac{I_2}{I_1}\right)$$

$$\beta_{21} = (10\text{ dB})\log\left[\frac{P/(4\pi r_2^2)}{P/(4\pi r_1^2)}\right] = (10\text{ dB})\log\left(\frac{r_1^2}{r_2^2}\right) = (10\text{ dB})\log\left(\frac{r_1}{r_2}\right)^2$$

$$= (20\text{ dB})\log\left(\frac{r_1}{r_2}\right) = (20\text{ dB})\log\left(\frac{r_1}{2r_1}\right) = (20\text{ dB})\log(1/2) = \boxed{-6.0\text{ dB}}$$

The negative sign indicates that the sound intensity level decreases.

64. ***REASONING AND SOLUTION***

From the information given in problem 62 we have

$$\beta = (10 \text{ dB}) \log (P/P_0)$$

where $P_0 = 1.00$ W. Solving for P in the above equation yields

$$P = P_0 10^{\beta/(10 \text{ dB})} = (1.00 \text{ W})10^{1.75} = \boxed{56.2 \text{ W}}$$

65. ***REASONING AND SOLUTION*** First, we will determine the intensity of the sound coming through the window. Since the sound intensity level at the threshold of hearing is 0 dB, we have

$$\beta - \beta_0 = (10\text{ dB})\log\left(\frac{I}{I_0}\right) - 0\text{ dB}$$

Solving for I, we obtain

$$I = I_0\ 10^{(\beta-\beta_0)/(10\text{ dB})} = (1.00\times10^{-12}\text{ W/m}^2)\,10^{(95\text{ dB})/(10\text{ dB})} = 3.2\times10^{-3}\text{ W/m}^2$$

Then, since

$$\mathrm{I} = \frac{\mathrm{P}}{\mathrm{A}} = \frac{\mathrm{Energy/t}}{\mathrm{A}}$$

We have

$$\text{Energy} = \text{IAt} = (3.2\times10^{-3}\text{ W/m}^2)[(1.1\text{ m})(0.75\text{ m})](3600\text{ s}) = \boxed{9.5\text{ J}}$$

66. ***REASONING*** If I_1 is the sound intensity produced by a single person, then NI_1 is the sound intensity generated by N people. The sound intensity level generated by N people is given by Equation 16.10 as

$$\beta_N = (10\text{ dB})\log\left(\frac{NI_1}{I_0}\right)$$

where I_0 is the threshold of hearing. Solving this equation for N yields

$$N = \left(\frac{I_0}{I_1}\right)10^{\frac{\beta_N}{10\text{ dB}}} \qquad (1)$$

We also know that the sound intensity level for one person is

$$\beta_1 = (10\text{ dB})\log\left(\frac{I_1}{I_0}\right) \quad \text{or} \quad I_1 = I_0\,10^{\frac{\beta_1}{10\text{ dB}}} \qquad (2)$$

Equations (1) and (2) are all that we need in order to find the number of people at the football game.

SOLUTION Substituting the expression for I_1 from Equation (2) into Equation (1) gives the desired result.

$$N = \frac{I_0\,10^{\frac{\beta_N}{10\text{ dB}}}}{I_0\,10^{\frac{\beta_1}{10\text{ dB}}}} = \frac{10^{\frac{109\text{ dB}}{10\text{ dB}}}}{10^{\frac{60.0\text{ dB}}{10\text{ dB}}}} = \boxed{79\,400}$$

67. ***REASONING*** The sound intensity level heard by the gardener increases by 10.0 dB because distance between him and the radio decreases. The intensity of the sound is greater when the radio is closer than when it is further away. Since the unit is emitting sound uniformly (neglecting any reflections), the intensity is inversely proportional to the square of the distance from the radio, according to Equation 16.9. Combining this equation with Equation 16.10, which relates the intensity level in decibels to the intensity I, we can use the 10.0-dB change in the intensity level to find the final vertical position of the radio. Once that position is known, we can then use kinematics to determine the fall time.

SOLUTION Let the sound intensity levels at the initial and final positions of the radio be β_i and β_f, respectively. Using Equation 16.10 for each, we have

$$\beta_f - \beta_i = (10\text{ dB})\log\left(\frac{I_f}{I_0}\right) - (10\text{ dB})\log\left(\frac{I_i}{I_0}\right) = (10\text{ dB})\log\left(\frac{I_f/I_0}{I_i/I_0}\right) = (10\text{ dB})\log\left(\frac{I_f}{I_i}\right) \qquad (1)$$

Since the radiation is uniform, Equation 16.9 can be used to substitute for the intensities I_i and I_f, so that Equation (1) becomes

$$\beta_f - \beta_i = (10\text{ dB})\log\left(\frac{I_f}{I_i}\right) = (10\text{ dB})\log\left[\frac{P/(4\pi h_f)^2}{P/(4\pi h_i)^2}\right] = (10\text{ dB})\log\left(\frac{h_i}{h_f}\right)^2 = (20\text{ dB})\log\left(\frac{h_i}{h_f}\right) \qquad (2)$$

Since $\beta_f - \beta_i = 10.0$ dB, Equation (2) gives

$$\beta_f - \beta_i = 10.0\text{ dB} = (20\text{ dB})\log\left(\frac{h_i}{h_f}\right) \qquad \text{or} \qquad \frac{h_i}{h_f} = 10^{(10.0\text{ dB})/(20\text{ dB})} = 10^{0.500} = 3.16$$

We can now determine the final position of the radio as follows:

$$\frac{h_i}{h_f} = \frac{5.1\text{ m}}{h_f} = 3.16 \qquad \text{or} \qquad h_f = \frac{5.1\text{ m}}{3.16} = 1.61\text{ m}$$

It follows, then, that the radio falls through a distance of 5.1 m – 1.61 m = 3.49 m. Taking upward as the positive direction and noting that the radio falls from rest ($v_0 = 0$ m/s), we can solve Equation 2.8 $\left(y = v_0 t + \frac{1}{2}at^2\right)$ from the equations of kinematics for the fall time t:

$$t = \sqrt{\frac{2y}{a}} = \sqrt{\frac{2(-3.49\text{ m})}{-9.80\text{ m/s}^2}} = \boxed{0.84\text{ s}}$$

68. ***REASONING AND SOLUTION*** The intensity level at each point is given by

$$I = \frac{P}{4\pi r^2}$$

Therefore,

$$\frac{I_1}{I_2} = \left(\frac{r_2}{r_1}\right)^2$$

Since the two intensity levels differ by 2.00 dB, the intensity ratio is

$$\frac{I_1}{I_2} = 10^{0.200} = 1.58$$

Thus,

$$\left(\frac{r_2}{r_1}\right)^2 = 1.58$$

We also know that $r_2 - r_1 = 1.00$ m. We can then solve the two equations simultaneously by substituting, i.e., $r_2 = r_1\sqrt{1.58}$ gives

$$r_1\sqrt{1.58} - r_1 = 1.00 \text{ m}$$

so that

$$r_1 = (1.00 \text{ m})/[\sqrt{1.58} - 1] = \boxed{3.9 \text{ m}}$$

and

$$r_2 = 1.00 \text{ m} + r_1 = \boxed{4.9 \text{ m}}$$

69. **SSM** ***REASONING AND SOLUTION*** The sound intensity level β in decibels (dB) is related to the sound intensity I according to Equation 16.10, $\beta = (10\text{ dB})\log\ (I/I_0)$, where the quantity I_0 is the reference intensity. According to the problem statement, when the sound intensity level triples, the sound intensity also triples; therefore,

$$3\beta = (10\text{ dB})\log\left(\frac{3I}{I_0}\right)$$

Then,

$$3\beta - \beta = (10\text{ dB})\log\left(\frac{3I}{I_0}\right) - (10\text{ dB})\log\left(\frac{I}{I_0}\right) = (10\text{ dB})\log\left(\frac{3I/I_0}{I/I_0}\right)$$

Thus, $2\beta = (10\text{ dB})\log 3$ and

$$\beta = (5\text{ dB})\log 3 = \boxed{2.39\text{ dB}}$$

70. ***REASONING*** In this situation, the observer is moving toward a stationary source of waves. The frequency f_o detected by the moving observer is related to the frequency f_s emitted by the source by

$$f_o = f_s\left(1 + \frac{v_o}{v}\right) \tag{16.13}$$

where v_o is the speed of the observer and v is the speed of the waves. The frequency f_s is related to the speed v of the waves and their wavelength λ by Equation 16.1, $f_s = v/\lambda$. Substituting this value for f_s into Equation 16.13 gives

$$f_o = \left(\frac{v}{\lambda}\right)\left(1 + \frac{v_o}{v}\right)$$

SOLUTION The frequency of the waves, as detected by the moving observer, is

$$f_o = \left(\frac{v}{\lambda}\right)\left(1 + \frac{v_o}{v}\right) = \left(\frac{6.70\text{ m/s}}{13.4\text{ m}}\right)\left(1 + \frac{4.20\text{ m/s}}{6.70\text{ m/s}}\right) = \boxed{0.813\text{ Hz}}$$

71. ***REASONING*** The observed frequency changes because of the Doppler effect. As you drive toward the parked car (a stationary source of sound), the Doppler effect is that given by Equation 16.13. As you drive away from the parked car, Equation 16.14 applies.

SOLUTION Equations 16.13 and 16.14 give the observed frequency f_o in each case:

$$\underbrace{f_{\text{o, toward}} = f_s\left(1+v_o/v\right)}_{\text{Driving toward parked car}} \qquad \text{and} \qquad \underbrace{f_{\text{o, away}} = f_s\left(1-v_o/v\right)}_{\text{Driving away from parked car}}$$

Subtracting the equation on the right from the one on the left gives the change in the observed frequency:

$$f_{\text{o, toward}} - f_{\text{o, away}} = 2f_s v_o/v$$

Solving for the observer's speed (which is your speed), we obtain

$$v_o = \frac{v\left(f_{\text{o, toward}} - f_{\text{o, away}}\right)}{2f_s} = \frac{(343\text{ m/s})(95\text{ Hz})}{2(960\text{ Hz})} = \boxed{17\text{ m/s}}$$

72. ***REASONING AND SOLUTION*** The frequency of the sound reaching your ears is

$$f_o = f_s\left(\frac{1}{1-v_s/v}\right) = (955\text{ Hz})\left[\frac{1}{1-(18\text{ m/s})/(343\text{ m/s})}\right] = 1010\text{ Hz} \qquad (16.11)$$

This corresponds to a wavelength of

$$\lambda = \frac{v}{f_o} = \frac{343\text{ m/s}}{1010\text{ Hz}} = \boxed{0.340\text{ m}} \qquad (16.1)$$

73. **SSM** ***REASONING*** Since you detect a frequency that is smaller than that emitted by the car when the car is stationary, the car must be moving away from you. Therefore, according to Equation 16.12, the frequency f_o heard by a stationary observer from a source moving away from the observer is given by

$$f_o = f_s \left(\frac{1}{1 + \frac{v_s}{v}} \right)$$

where f_s is the frequency emitted from the source when it is stationary with respect to the observer, v is the speed of sound, and v_s is the speed of the moving source. This expression can be solved for v_s.

SOLUTION We proceed to solve for v_s and substitute the data given in the problem statement. Rearrangement gives

$$\frac{v_s}{v} = \frac{f_s}{f_o} - 1$$

Solving for v_s and noting that $f_o / f_s = 0.86$ yields

$$v_s = v\left(\frac{f_s}{f_o} - 1 \right) = (343 \text{ m/s}) \left(\frac{1}{0.86} - 1 \right) = \boxed{56 \text{ m/s}}$$

74. ***REASONING***

a. The frequency must be lowered to 20 000 Hz or less if the person is to hear the sound. If the car and loudspeaker move away from the person, the wavelength of the sound increases. The increase in wavelength gives rise to a decrease in frequency. Thus, the car must move away from the person.

b. Since the car is moving away from the stationary person, the frequency f_o heard by the observer (the person) is related to the frequency f_s emitted by the source (the car) by

$$f_o = f_s \left(\frac{1}{1 + \frac{v_s}{v}} \right) \tag{16.12}$$

where v_s is the speed of the source and v is the speed of sound in air. This equation can be solved for the speed of the source.

SOLUTION

b. Solving Equation 16.12 for v_s, and recognizing that the maximum frequency that can be heard by a normal young person is f_o = 20 000 Hz, we have

$$v_s = v\left(\frac{f_s}{f_o} - 1\right) = (343\text{ m/s})\left(\frac{20\,510\text{ Hz}}{20\,000\text{ Hz}} - 1\right) = \boxed{8.7\text{ m/s}}$$

75. **SSM** ***REASONING AND SOLUTION*** In this situation, both the source and the observer are moving through the air. Therefore, the frequency of sound heard by the crew is given by Equation 16.15 using the plus signs in both the numerator and the denominator.

$$f_o = f_s \left(\frac{1 + \dfrac{v_o}{v}}{1 + \dfrac{v_s}{v}} \right) = (1550 \text{ Hz}) \left(\frac{1 + \dfrac{13.0 \text{ m/s}}{343 \text{ m/s}}}{1 + \dfrac{67.0 \text{ m/s}}{343 \text{ m/s}}} \right) = \boxed{1350 \text{ Hz}}$$

76. ***REASONING*** The source of the sound is stationary, while the observer (the microphone) is moving toward the source. Therefore, the Doppler effect is given by Equation 16.13:

$$f_o = f_s\left(1+\frac{v_o}{v}\right) \qquad \text{or} \qquad f_o - f_s = f_s\left(\frac{v_o}{v}\right)$$

Data is given for $f_o - f_s$, which is the amount by which the observed and Doppler-shifted frequency exceeds the emitted frequency. Thus, we will apply the expression for $f_o - f_s$ to the sound waves in air and in the liquid.

SOLUTION Using v_A and v_L to denote the speed of sound in air and in the liquid, respectively, we have

$$f_{o,A} - f_s = 83.0\text{ Hz} = f_s\left(\frac{v_o}{v_A}\right) \qquad \text{and} \qquad f_{o,L} - f_s = 23.0\text{ Hz} = f_s\left(\frac{v_o}{v_L}\right)$$

Dividing the equation on the left by the equation on the right and solving for v_L, we obtain

$$\frac{83.0\text{ Hz}}{23.0\text{ Hz}} = \frac{f_s\left(\dfrac{v_o}{v_A}\right)}{f_s\left(\dfrac{v_o}{v_L}\right)} = \frac{v_L}{v_A}$$

$$v_L = v_A\left(\frac{83.0\text{ Hz}}{23.0\text{ Hz}}\right) = (343\text{ m/s})\left(\frac{83.0\text{ Hz}}{23.0\text{ Hz}}\right) = \boxed{1240\text{ m/s}}$$

77. ***REASONING AND SOLUTION*** The speed of the Bungee jumper, v_s, after she has fallen a distance y is given by

$$v_s^2 = v_0^2 + 2ay$$

Since she falls from rest, $v_0 = 0$ m/s, and

$$v_s = \sqrt{2ay} = \sqrt{2(-9.80\ \text{m/s}^2)(-11.0\ \text{m})} = 14.7\ \text{m/s}$$

Then, from Equation 16.11

$$f_o = f_s\left(\frac{1}{1 - v_s / v}\right) = (589\ \text{Hz})\left[\frac{1}{1-(14.7\ \text{m/s})/(343\ \text{m/s})}\right] = \boxed{615\ \text{Hz}}.$$

78. ***REASONING***

a. Since the two submarines are approaching each other head on, the frequency f_o detected by the observer (sub B) is related to the frequency f_s emitted by the source (sub A) by

$$f_o = f_s \left(\frac{1 + \frac{v_o}{v}}{1 - \frac{v_s}{v}} \right) \tag{16.15}$$

where v_o and v_s are the speed of the observer and source, respectively, and v is the speed of the underwater sound

b. The sound reflected from submarine B has the same frequency that it detects, namely, f_o. Now sub B becomes the source of sound and sub A is the observer. We can still use Equation 16.15 to find the frequency detected by sub A.

SOLUTION

a. The frequency f_o detected by sub B is

$$f_o = f_s \left(\frac{1 + \frac{v_o}{v}}{1 - \frac{v_s}{v}} \right) = (1550\text{ Hz}) \left(\frac{1 + \frac{8\text{ m/s}}{1522\text{ m/s}}}{1 - \frac{12\text{ m/s}}{1522\text{ m/s}}} \right) = \boxed{1570\text{ Hz}}$$

b. The sound reflected from submarine B has the same frequency that it detects, namely, 1570 Hz. Now sub B is the source of sound whose frequency is f_s = 1570 Hz. The speed of sub B is v_s = 8 m/s. The frequency detected by sub A (whose speed is v_o = 12 m/s) is

$$f_o = f_s \left(\frac{1 + \frac{v_o}{v}}{1 - \frac{v_s}{v}} \right) = (1570\text{ Hz}) \left(\frac{1 + \frac{12\text{ m/s}}{1522\text{ m/s}}}{1 - \frac{8\text{ m/s}}{1522\text{ m/s}}} \right) = \boxed{1590\text{ Hz}}$$

79. **SSM** ***REASONING*** We can use the Doppler shift formula to determine the speed of the motorcycle. Once this speed is known, the equations of kinematics (specifically, Equation 2.9) can be used to determine the distance covered by the motorcycle.

SOLUTION As the motorcycle accelerates, the frequency of the siren, as heard by the driver, decreases When the frequency of the siren heard by the driver is 90.0% of the value it has when the motorcycle is stationary, we have $f_o / f_s = 0.900$. Since the driver (the observer) is moving away from the source, we can use Equation 16.14 { $f_o = f_s[1-(v_o / v)]$ } to obtain the speed of the driver, and hence the speed of the motorcycle;

$$\frac{f_o}{f_s} = 1 - \frac{v_o}{v} = 0.900$$

Solving for v_o, we have

$$v_o = 0.100\ v = 0.100\ (343\ \text{m/s}) = 34.3\ \text{m/s}$$

According to Equation 2.9 ($v^2 = v_0^2 + 2ax$), since the motorcycle starts from rest so that $v_0 = 0$ m/s, the distance traveled by the motorcycle is

$$x = \frac{v^2}{2a} = \frac{(34.3\ \text{m/s})^2}{2(2.81\ \text{m/s}^2)} = \boxed{209\ \text{m}}$$

80. ***REASONING AND SOLUTION*** The maximum observed frequency is f_o^{max}, and the minimum observed frequency is f_o^{min}. We are given that $f_o^{max} - f_o^{min} = 2.1$ Hz, where

$$f_o^{max} = f_s [1 + (v_O/v)] \tag{16.13}$$

and

$$f_o^{min} = f_s [1 - (v_O/v)] \tag{16.14}$$

We have

$$f_o^{max} - f_o^{min} = f_s [1 + (v_O/v)] - f_s [1 - (v_O/v)] = 2f_s (v_O/v)$$

We can now solve for the maximum speed of the microphone, v_O:

$$v_O = (2.1 \text{ Hz})v/(2f_s) = (2.1 \text{ Hz})(343 \text{ m/s})/[2(440 \text{ Hz})] = 0.82 \text{ m/s}$$

Using $v_{max} = v_O = A\omega$, Equation 10.6, where A is the amplitude of the simple harmonic motion and ω is the angular frequency, $\omega = 2\pi/T = 2\pi/(2.0 \text{ s}) = 3.1$ rad/s, we have

$$A = v_O/\omega = (0.82 \text{ m/s})/(3.1 \text{ rad/s}) = \boxed{0.26 \text{ m}}$$

81. ***REASONING AND SOLUTION*** The intensity level in dB is $\beta = (10\text{ dB}) \log (I/I_0)$, Equation 16.10, so

$$I/I_0 = 10^{\beta/(10\text{ dB})} = 10^{1.4} = \boxed{25}$$

82. ***REASONING*** Knowing the wavelength λ, we can obtain the frequency f from Equation 16.1 ($v = f\lambda$), since the speed of sound v in seawater is given. Then, knowing the frequency, we can obtain the period T from Equation 10.5 ($f = 1/T$).

SOLUTION From Equations 16.1 and 10.5, we have

$$v = f\lambda = \left(\frac{1}{T}\right)\lambda \qquad \text{or} \qquad T = \frac{\lambda}{v} = \frac{0.025\ \text{m}}{1470\ \text{m/s}} = \boxed{1.7 \times 10^{-5}\ \text{s}}$$

83. **SSM** ***REASONING AND SOLUTION*** The speed of sound in a liquid is given by Equation 16.6, $v = \sqrt{B_{\text{ad}} / \rho}$, where B_{ad} is the adiabatic bulk modulus and ρ is the density of the liquid. Solving for B_{ad}, we obtain $B_{\text{ad}} = v^2 \rho$. The ratio of the adiabatic bulk modulus of fresh water to that of ethanol at 20°C is, therefore,

$$\frac{(B_{\text{ad}})_{\text{water}}}{(B_{\text{ad}})_{\text{ethanol}}} = \frac{v_{\text{water}}^2 \rho_{\text{water}}}{v_{\text{ethanol}}^2 \rho_{\text{ethanol}}} = \frac{(1482 \text{ m/s})^2 (998 \text{ kg/m}^3)}{(1162 \text{ m/s})^2 (789 \text{ kg/m}^3)} = \boxed{2.06}$$

84. ***REASONING AND SOLUTION*** The speed of a wave on a string is proportional to the square root of the tension, Equation 16.2. If the tension increases by a factor of four then the speed will increase by a factor of two. The new speed will then be $\boxed{600 \text{ m/s}}$.

85. ***REASONING AND SOLUTION*** The Doppler-shifted frequency is

$$f_o = f_s\left(\frac{1}{1 - v_s / v}\right) = (784\text{ Hz})\left[\frac{1}{1-(0.83\text{ m/s})/(343\text{ m/s})}\right] = \boxed{786\text{ Hz}} \qquad (16.11)$$

86. ***REASONING AND SOLUTION*** The period of the wave is the same as the period of the person, so T = 5.00 s.

a. $f = 1/T = \boxed{0.200 \text{ Hz}}$ (10.5)

b. $v = \lambda f = (20.0 \text{ m})(0.200 \text{ Hz}) = \boxed{4.00 \text{ m/s}}$ (16.1)

87. **SSM** ***REASONING AND SOLUTION*** The sound intensity level β in decibels (dB) is related to the sound intensity I according to Equation 16.10, $\beta = (10\text{ dB})\log\ (I/I_0)$, where I_0 is the reference intensity. If β_1 and β_2 represent two different sound level intensities, then,

$$\beta_2 - \beta_1 = (10\text{ dB})\log\left(\frac{I_2}{I_0}\right) - (10\text{ dB})\log\left(\frac{I_1}{I_0}\right) = (10\text{ dB})\log\left(\frac{I_2/I_0}{I_1/I_0}\right) = (10\text{ dB})\log\left(\frac{I_2}{I_1}\right)$$

Therefore,

$$\frac{I_2}{I_1} = 10^{(\beta_2-\beta_1)/(10\text{ dB})}$$

When the difference in sound intensity levels is $\beta_2 - \beta_1 = 1.0\text{ dB}$, the ratio of the sound intensities is

$$\frac{I_2}{I_1} = 10^{(1\text{ dB})/(10\text{ dB})} = \boxed{1.3}$$

88. ***REASONING AND SOLUTION*** The intensity of the sound falls off according to the square of the distance. The intensity at the new location will be

$$I_{78} = \left(\frac{22\text{ m}}{78\text{ m}}\right)^2 \left(3.0 \times 10^{-4}\text{ W/m}^2\right) = \boxed{2.4 \times 10^{-5}\text{ W/m}^2}$$

89. ***REASONING AND SOLUTION***

a. The travel time is

$$t = x/v = (2.70\ \text{m})/(343\ \text{m/s}) = \boxed{7.87 \times 10^{-3}\ \text{s}}$$

b. The wavelength is

$$\lambda = v/f = (343\ \text{m/s})/(523\ \text{Hz}) = 0.656\ \text{m} \tag{16.1}$$

The number of wavelengths in 2.70 m is, therefore,

$$\text{Number of wavelengths} = (2.70\ \text{m})/(0.656\ \text{m}) = \boxed{4.12}$$

90. ***REASONING AND SOLUTION***

a. Comparing with Equation 16.4, we find $A = \boxed{0.45\text{ m}}$. $2f = 8.0$ Hz, so $f = \boxed{4.0\text{ Hz}}$. $2/\lambda = 1.0\text{ m}^{-1}$, so $\lambda = \boxed{2.0\text{ m}}$. Using Equation 16.1, we find

$$v = f\lambda = (2.0\text{ m})(4.0\text{ Hz}) = \boxed{8.0\text{ m/s}}$$

b. Equation 16.4 applies to a wave traveling in the $\boxed{-x\text{ direction}}$.

91. **SSM** ***REASONING AND SOLUTION: METHOD 1*** Since the police car is matching the speed of the speeder, there is no relative motion between the police car and the listener (the speeder). Therefore, the frequency that the speeder hears when the siren is turned on is the same as if the police car and the listener are stationary. Thus, the speeder hears a frequency of $\boxed{860 \text{ Hz}}$.

REASONING AND SOLUTION: METHOD 2 In this situation, both the source (the siren) and the observer (the speeder) are moving through the air. Therefore, the frequency of the siren heard by the speeder is given by Equation 16.15 using the minus signs in both the numerator and the denominator.

$$f_o = f_s \left(\frac{1 - \frac{v_o}{v}}{1 - \frac{v_s}{v}} \right) = (860 \text{ Hz}) \left(\frac{1 - \frac{38 \text{ m/s}}{343 \text{ m/s}}}{1 - \frac{38 \text{ m/s}}{343 \text{ m/s}}} \right) = \boxed{860 \text{ Hz}}$$

92. ***REASONING AND SOLUTION*** Since $f = v/\lambda$ and the frequency of the sound remains unchanged in water, we have

$$v_a/\lambda_a = v_w/\lambda_w$$

or

$$\lambda_w = \lambda_a(v_w/v_a) = (2.74\text{ m})[(1482\text{ m/s})/(343\text{ m/s})] = \boxed{11.8\text{ m}}$$

93. ***REASONING AND SOLUTION*** The emitted power is 2.0×10^5 J/s = 2.0×10^5 W. The intensity is, therefore,

$$I = \frac{P}{A} = \frac{P}{4\pi r^2} = \frac{2.0 \times 10^5 \text{ W}}{4\pi(85 \text{ m})^2} = 2.2 \text{ W/m}^2 \tag{16.9}$$

Thus, the sound intensity level is

$$\beta = (10 \text{ dB}) \log\left(\frac{I}{I_0}\right) = (10 \text{ dB}) \log\left(\frac{2.2 \text{ W/m}^2}{1.00 \times 10^{-12} \text{ W/m}^2}\right) = \boxed{120 \text{ dB}} \tag{16.10}$$

94. ***REASONING AND SOLUTION*** Let β_2 and I_2 denote the intensity level and the intensity, respectively, when two rifles are shot. Let β_1 and I_1 denote the intensity level and intensity, respectively, when one rifle is shot. Then

$$\beta_2 = (10\text{ dB})\log\left(\frac{I_2}{I_0}\right) = (10\text{ dB})\log\left(\frac{2I_1}{I_0}\right)$$

where I_1 is the intensity of a single rifle. Therefore,

$$\frac{2I_1}{I_0} = 10^{\beta_2/(10\text{ dB})}$$

$$I_1 = \left(\frac{I_0}{2}\right)10^{\beta_2/(10\text{ dB})} = \left(\frac{1.00\times10^{-12}\text{ W/m}^2}{2}\right)10^{8.00} = 5.00\times10^{-5}\text{ W/m}^2$$

$$\therefore \beta_1 = (10\text{ dB})\log\left(\frac{I_1}{I_0}\right) = (10\text{ dB})\log\left(\frac{5.00\times10^{-5}\text{ W/m}^2}{1.00\times10^{-12}\text{ W/m}^2}\right) = \boxed{77.0\text{ dB}}$$

95. **SSM** **WWW** ***REASONING*** Using the procedures developed in Chapter 4 for using Newton's second law to analyze the motion of bodies and neglecting the weight of the wire relative to the tension in the wire lead to the following equations of motion for the two blocks:

$$\sum F_x = F - m_1 g\,(\sin\ 30.0°) = 0 \tag{1}$$

$$\sum F_y = F - m_2 g = 0 \tag{2}$$

where F is the tension in the wire. In Equation (1) we have taken the direction of the $+x$ axis for block 1 to be parallel to and up the incline. In Equation (2) we have taken the direction of the $+y$ axis to be upward for block 2. This set of equations consists of two equations in three unknowns, m_1, m_2, and F. Thus, a third equation is needed in order to solve for any of the unknowns. A useful third equation can be obtained by solving Equation 16.2 for F:

$$F = (m/L)v^2 \tag{3}$$

Combining Equation (3) with Equations (1) and (2) leads to

$$(m/L)v^2 - m_1 g\ \sin\ 30.0° = 0 \tag{4}$$

$$(m/L)v^2 - m_2 g = 0 \tag{5}$$

Equations (4) and (5) can be solved directly for the masses m_1 and m_2.

SOLUTION Substituting values into Equation (4), we obtain

$$m_1 = \frac{(m/L)v^2}{g\ \sin 30.0°} = \frac{(0.0250\ \text{kg/m})(75.0\ \text{m/s})^2}{(9.80\ \text{m/s}^2)\ \sin 30.0°} = \boxed{28.7\ \text{kg}}$$

Similarly, substituting values into Equation (5), we obtain

$$m_2 = \frac{(m/L)v^2}{g} = \frac{(0.0250\ \text{kg/m})(75.0\ \text{m/s})^2}{(9.80\ \text{m/s}^2)} = \boxed{14.3\ \text{kg}}$$

96. ***REASONING AND SOLUTION*** First find the speed of the record at a distance of 0.100 m from the center:

$$v = r\omega = (0.100\ \text{m})(3.49\ \text{rad/s}) = 0.349\ \text{m/s} \tag{8.9}$$

The wavelength is, then,

$$\lambda = v/f = (0.349\ \text{m/s})/(5.00 \times 10^3\ \text{Hz}) = \boxed{6.98 \times 10^{-5}\ \text{m}} \tag{16.1}$$

97. ***REASONING AND SOLUTION*** The speed of the wave on the cable is

$$v = \sqrt{\frac{F}{(m/L)}} = \sqrt{\frac{F}{\rho A}} = \sqrt{\frac{1.00 \times 10^4 \text{ N}}{(7860 \text{ kg/m}^3)(2.83 \times 10^{-3} \text{ m}^2)}} = \boxed{21.2 \text{ m/s}} \qquad (16.2)$$

98. ***REASONING AND SOLUTION***

a. According to Equation 16.2, the speed of the wave is

$$v = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{315\ N}{6.50 \times 10^{-3}\ \text{kg/m}}} = \boxed{2.20 \times 10^{2}\ \text{m/s}}$$

b. According to Equations 10.6 and 10.8, the maximum speed of the point on the wire is

$$v_{\max} = (2\pi f)A = 2\pi(585\ \text{Hz})\left(2.50 \times 10^{-3}\ \text{m}\right) = \boxed{9.19\ \text{m/s}}$$

99. $\boxed{\text{SSM}}$ ***REASONING*** We must first find the intensities that correspond to the given sound intensity levels (in decibels). The total intensity is the sum of the two intensities. Once the total intensity is known, Equation 16.10 can be used to find the total sound intensity level in decibels.

SOLUTION Since, according to Equation 16.10, $\beta = (10\text{ dB})\log(I/I_0)$, where I_0 is the reference intensity corresponding to the threshold of hearing ($I_0 = 1.00\times10^{-12}\text{ W/m}^2$), it follows that $I = I_0\ 10^{\beta/(10\text{ dB})}$. Therefore, if $\beta_1 = 75.0$ dB and $\beta_2 = 72.0$ dB at the point in question, the corresponding intensities are

$$I_1 = I_0\ 10^{\beta_1/(10\text{ dB})} = (1.00\times10^{-12}\text{ W/m}^2)\ 10^{(75.0\text{ dB})/(10\text{ dB})} = 3.16\times10^{-5}\text{ W/m}^2$$

$$I_2 = I_0\ 10^{\beta_2/(10\text{ dB})} = (1.00\times10^{-12}\text{ W/m}^2)\ 10^{(72.0\text{ dB})/(10\text{ dB})} = 1.58\times10^{-5}\text{ W/m}^2$$

Therefore, the total intensity I_{total} at the point in question is

$$I_{\text{total}} = I_1 + I_2 = (3.16\times10^{-5}\text{ W/m}^2) + (1.58\times10^{-5}\text{ W/m}^2) = 4.74\times10^{-5}\text{ W/m}^2$$

and the corresponding intensity level β_{total} is

$$\beta_{\text{total}} = (10\text{ dB})\log\left(\frac{I_{\text{total}}}{I_0}\right) = (10\text{ dB})\log\left(\frac{4.74\times10^{-5}\text{ W/m}^2}{1.00\times10^{-12}\text{ W/m}^2}\right) = \boxed{76.8\text{ dB}}$$

100. ***REASONING AND SOLUTION*** We can write

$$\beta_2 - \beta_1 = (10\text{ dB}) \log (I_2/I_1)$$

so

$$I_2 / I_1 = 10^{(\beta_2 - \beta_1)/(10\text{ dB})} = 10^{(78\text{ dB} - 65\text{ dB})/(10\text{ dB})} = 20$$

Since the 78 dB level is 20 times more intense than the 65 dB level, this means that the number of people speaking at the 65 dB level needed to reach the 78 dB level is $\boxed{20}$.

101. ***REASONING AND SOLUTION*** The sound emitted by the plane at A reaches the person after a time t. This time, t, required for the sound wave at A to reach the person is the same as the time required for the plane to fly from A to B.

The figure at the right shows the relevant geometry. During the time t, the plane travels the distance x, while the sound wave travels the distance d. The sound wave travels with constant speed. The plane has a speed of v_0 at A and a speed v at B and travels with constant acceleration. Thus,

$$d = v_{sound}\, t$$

and

$$x = \tfrac{1}{2}\left(v + v_0\right)t$$

From the drawing, we have

$$\sin\theta = \frac{x}{d} = \frac{\frac{1}{2}\left(v + v_0\right)t}{v_{sound}\,t} = \frac{v + v_0}{2v_{sound}}$$

Solving for v gives

$$v = 2v_{sound}\sin\theta - v_0 = 2\left(343\text{ m/s}\right)\sin 36.0° - 164\text{ m/s} = \boxed{239\text{ m/s}}$$

102. ***REASONING AND SOLUTION***

a. The distance traveled is x = vt = (343 m/s)(0.580 s) = 199 m. The one-way distance is half this value or $\boxed{99.5\text{ m}}$.

b. Since the speed is proportional to the square-root of the absolute temperature, i.e.,

$$\frac{v_2}{v_1} = \sqrt{\frac{T_2}{T_1}}$$

or

$$v_2 = v_1\sqrt{\frac{T_2}{T_1}} = (343\text{ m/s})\sqrt{\frac{298\text{ K}}{293\text{ K}}} = 346\text{ m/s}.$$

The distance computed by the theodolite at the higher temperature would be (1/2) vt_2, where v = 343 m/s and t_2 = (199 m)/(346 m/s). Then

$$(1/2)(343\text{ m/s})(199\text{ m})/(346\text{ m/s}) = 98.6\text{ m}$$

The percent error is then

$$\%\text{ error} = [(99.5\text{ m} - 98.6\text{ m})/(99.5\text{ m})] \times 100\% = \boxed{0.9\ \%}$$

1. SSM ***REASONING AND SOLUTION*** According to the principle of linear superposition, when two or more waves are present simultaneously at the same place, the resultant wave is the sum of the individual waves. Therefore, the shape of the string at the indicated times looks like the following:

2. ***REASONING AND SOLUTION*** The shape of the string looks like

3. ***REASONING AND SOLUTION*** In a time of $t = 1$ s, the pulse on the left has moved to the right a distance of 1 cm, while the pulse on the right has moved to the left a distance of 1 cm. Adding the shapes of these two pulses when $t = 1$ s reveals that the height of the resultant pulse is

a. $\boxed{2 \text{ cm}}$ at $x = 3$ cm.

b. $\boxed{1 \text{ cm}}$ at $x = 4$ cm.

4. ***REASONING AND SOLUTION*** As the LYM section of the tube is pulled out, the length of the path on that side increases by $2(0.020\text{ m}) = 0.040\text{ m}$. This additional path difference between the LYM and LXM waves must correspond to $\lambda/2$, since the sound at the microphone goes from a maximum to a minimum. Hence, $\lambda/2 = 0.040\text{ m}$, or $\lambda = 0.080\text{ m}$. Then

$$v = \lambda f = (0.080\text{ m})(12\,000\text{ Hz}) = \boxed{960\text{ m/s}}$$

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5. SSM ***REASONING*** When constructive interference occurs again at point C, the path length difference is two wavelengths, or $\Delta s = 2\lambda = 3.20\text{ m}$. Therefore, we can write the expression for the path length difference as

$$s_{AC} - s_{BC} = \sqrt{s_{AB}^2 + s_{BC}^2} - s_{BC} = 3.20\text{ m}$$

This expression can be solved for s_{AB}.

SOLUTION Solving for s_{AB}, we find that

$$s_{AB} = \sqrt{(3.20\text{ m} + 2.40\text{ m})^2 - (2.40\text{ m})^2} = \boxed{5.06\text{ m}}$$

6. ***REASONING*** For destructive interference to occur, the difference in travel distances for the sound waves must be an integer number of half wavelengths. For larger and larger distances between speaker *B* and the observer at *C*, the difference in travel distances becomes smaller and smaller. Thus, the largest possible distance between speaker *B* and the observer at *C* occurs when the difference in travel distances is just one half wavelength.

SOLUTION Since the triangle *ABC* in Figure 17.7 is a right triangle, we can apply the Pythagorean theorem to obtain the distance d_{AC} as $\sqrt{(5.00\text{ m})^2 + d_{BC}^2}$. Therefore, the difference in travel distances is

$$\sqrt{(5.00\text{ m})^2 + d_{BC}^2} - d_{BC} = \frac{\lambda}{2} = \frac{v}{2f}$$

where we have used Equation 16.1 to express the wavelength λ as $\lambda = v/f$. Solving for the distance d_{BC} gives

$$\sqrt{(5.00\text{ m})^2 + d_{BC}^2} = d_{BC} + \frac{v}{2f} \quad \text{or} \quad (5.00\text{ m})^2 + d_{BC}^2 = \left(d_{BC} + \frac{v}{2f}\right)^2$$

$$(5.00\text{ m})^2 + d_{BC}^2 = d_{BC}^2 + \frac{d_{BC}v}{f} + \frac{v^2}{4f^2} \quad \text{or} \quad (5.00\text{ m})^2 = \frac{d_{BC}v}{f} + \frac{v^2}{4f^2}$$

$$d_{BC} = \frac{(5.00\text{ m})^2 - \dfrac{v^2}{4f^2}}{\dfrac{v}{f}} = \frac{(5.00\text{ m})^2 - \dfrac{(343\text{ m/s})^2}{4(125\text{ Hz})^2}}{\dfrac{343\text{ m/s}}{125\text{ Hz}}} = \boxed{8.42\text{ m}}$$

7. WWW WWW ***REASONING*** The geometry of the positions of the loudspeakers and the listener is shown in the following drawing.

The listener at C will hear either a loud sound or no sound, depending upon whether the interference occurring at C is constructive or destructive. If the listener hears no sound, destructive interference occurs, so

$$d_2 - d_1 = \frac{n\lambda}{2} \qquad n = 1, 3, 5, \ldots \tag{1}$$

SOLUTION Since $v = \lambda f$, according to Equation 16.1, the wavelength of the tone is

$$\lambda = \frac{\text{v}}{f} = \frac{343 \text{ m/s}}{68.6 \text{ Hz}} = 5.00 \text{ m}$$

Speaker B will be closest to Speaker A when $n = 1$ in Equation (1) above, so

$$d_2 = \frac{n\lambda}{2} + d_1 = \frac{5.00 \text{ m}}{2} + 1.00 \text{ m} = 3.50 \text{ m}$$

From the figure above we have that,

$$x_1 = (1.00 \text{ m}) \cos 60.0° = 0.500 \text{ m}$$

$$y = (1.00 \text{ m}) \sin 60.0° = 0.866 \text{ m}$$

Then

$$x_2^2 + y^2 = d_2^2 = (3.50 \text{ m})^2 \qquad \text{or} \qquad x_2 = \sqrt{(3.50 \text{ m})^2 - (0.866 \text{ m})^2} = 3.39 \text{ m}$$

Therefore, the closest that speaker A can be to speaker B so that the listener hears no sound is $x_1 + x_2 = 0.500 \text{ m} + 3.39 \text{ m} = \boxed{3.89 \text{ m}}$.

8. ***REASONING AND SOLUTION*** Since the triangle ABC is a right triangle, the distance AC is given by the Pythagorean theorem as $\sqrt{(3.20\text{ m})^2 + (2.40\text{ m})^2} = 4.00\text{ m}$. The distance BC is given as 2.40 m. Thus, we find that the difference in the travel distances for the waves is 4.00 m – 2.40 m = 1.60 m. Since the speakers vibrate out of phase, destructive interference will occur if this difference in distance is an *integer* number of wavelengths. (See Conceptual Example 2 in the text.)

The smallest frequency that will produce destructive interference corresponds to the longest wavelength that satisfies this condition. The longest wavelength that will result in destructive interference is $\lambda = 1.60$ m. Using the relation $v = \lambda f$, we find that

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{1.60\text{ m}} = \boxed{214\text{ Hz}}$$

9. ***REASONING AND SOLUTION*** Since $v = \lambda f$, the wavelength of the tone is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{73.0 \text{ Hz}} = 4.70 \text{ m}$$

The figure below shows the line between the two speakers and the distances in question.

Constructive interference will occur when the difference in the distances traveled by the two sound waves in reaching point P is an integer number of wavelengths. That is, when

$$(L - x) - x = n\lambda$$

where n is an integer (or zero). Solving for x gives

$$x = \frac{L - n\lambda}{2} \qquad (1)$$

When $n = 0$, $x = L/2 = (7.80 \text{ m})/2 = \boxed{3.90 \text{ m}}$. This corresponds to the point halfway between the two speakers. Clearly in this case, each wave has traveled the same distance and therefore, they will arrive in phase.

When $n = 1$,

$$x = \frac{(7.80 \text{ m}) - (4.70 \text{ m})}{2} = 1.55 \text{ m}$$

Thus, there is a point of constructive interference $\boxed{\text{1.55 m from speaker A}}$. The points of constructive interference will occur symmetrically about the center point at L/2, so there is also a point of constructive interference 1.55 m from speaker B, that is at the point 7.80 m – 1.55 m = $\boxed{\text{6.25 m from speaker A}}$.

When $n > 1$, the values of x obtained from Equation (1) will be negative. These values correspond to positions of constructive interference that lie to the left of A or to the right of C. They do not lie on the line between the speakers.

10. ***REASONING*** In both cases, Equation 17.1 ($\sin\theta = \lambda/D$) offers a direct solution, since the width D of the doorway is given and we can use Equation 16.1 ($\lambda = v/f$) to determine the wavelength λ in terms of the speed v and frequency f.

SOLUTION Using Equations 17.1 and 16.1, we find

$$\sin\theta = \frac{\lambda}{\mathrm{D}} = \frac{v}{Df}$$

Using this result we have

(a) $$\theta = \sin^{-1}\left(\frac{v}{Df}\right) = \sin^{-1}\left[\frac{343\ \mathrm{m/s}}{(0.77\ \mathrm{m})(5.0\times10^{3}\ \mathrm{Hz})}\right] = \boxed{5.1°}$$

(b) $$\theta = \sin^{-1}\left(\frac{v}{Df}\right) = \sin^{-1}\left[\frac{343\ \mathrm{m/s}}{(0.77\ \mathrm{m})(5.0\times10^{2}\ \mathrm{Hz})}\right] = \boxed{63°}$$

11. SSM ***REASONING*** The diffraction angle for the first minimum for a circular opening is given by Equation 17.2: $\sin\theta = 1.22\lambda / D$, where D is the diameter of the opening.

SOLUTION

a. Using Equation 16.1, we must first find the wavelength of the 2.0-kHz tone:

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{2.0 \times 10^3 \text{ Hz}} = 0.17 \text{ m}$$

The diffraction angle for a 2.0-kHz tone is, therefore,

$$\theta = \sin^{-1}\left(1.22 \times \frac{0.17 \text{ m}}{0.30 \text{ m}}\right) = \boxed{44°}$$

b. The wavelength of a 6.0-kHz tone is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{6.0 \times 10^3 \text{ Hz}} = 0.057 \text{ m}$$

Therefore, if we wish to generate a 6.0-kHz tone whose diffraction angle is as wide as that for the 2.0-kHz tone in part (a), we will need a speaker of diameter D, where

$$D = \frac{1.22\ \lambda}{\sin\theta} = \frac{(1.22)(0.057 \text{ m})}{\sin 44°} = \boxed{0.10 \text{ m}}$$

12. ***REASONING*** The angle θ for the first intensity minimum of a diffraction horn is given by Equation 17. 1 as $\sin\theta = \lambda_{DH}/D$, where λ_{DH} is the wavelength of the sound produced by the diffraction horn and D is the width of the horn. Likewise, the angle θ for the first intensity minimum of a circular speaker is given by Equation 17. 2 as $\sin\theta = 1.22\lambda_{CS}/D$, where λ_{CS} is the wavelength of the sound produced by the circular speaker and D is its diameter. These two relations can be used to find the ratio of the wavelengths.

SOLUTION The ratio of the two wavelengths is

$$\frac{\lambda_{DH}}{\lambda_{CS}} = \frac{D\sin\theta}{\dfrac{D\sin\theta}{1.22}} = \boxed{1.22}$$

13. ***REASONING AND SOLUTION*** The condition for the first diffraction minimum for a single slit is given by Equation 17.1:

$$\sin\theta = \frac{\lambda}{D} \qquad \text{or} \qquad \theta = \sin^{-1}\left(\frac{\lambda}{D}\right)$$

Since $v = \lambda f$, the expression for θ becomes

$$\theta = \sin^{-1}\left(\frac{v}{fD}\right) = \sin^{-1}\left[\frac{343\ \text{m/s}}{(8100\ \text{Hz})(0.060\ \text{m})}\right] = 45°$$

Thus, at the frequency 8100 Hz, the person sitting at this angle off to the side of the diffraction horn hears no sound. If the person moves to an angle $\theta/2$, the frequency that interferes destructively (leading again to no sound) is found as follows:

$$\sin\theta = \frac{\lambda}{D} = \frac{v}{fD} \quad \text{or} \quad f = \frac{v}{D\sin\theta} = \left\{\frac{343\ \text{m/s}}{(0.060\ \text{m})\left[\sin\left(45°/2\right)\right]}\right\} = \boxed{1.5\times10^{4}\ \text{Hz}}$$

14. ***REASONING AND SOLUTION*** At 0 °C the speed of sound is air is given as 331 m/s in Table 16.1 in the text. This corresponds to a wavelength of

$$\lambda_1 = v/f = (331 \text{ m/s})/(3.00 \times 10^3 \text{ Hz}) = 0.1103 \text{ m}$$

The diffraction angle is given by Equation 17.2 as

$$\theta_1 = \sin^{-1}\left(\frac{1.22\,\lambda}{\text{D}}\right) = \sin^{-1}\left[\frac{1.22(0.1103 \text{ m})}{0.175 \text{ m}}\right] = 50.3°$$

For an ideal gas, the speed of sound is proportional to the square root of the Kelvin temperature, according to Equation 16.5. Therefore, the speed of sound at 29 °C is

$$v = (331 \text{ m/s})\sqrt{\frac{302 \text{ K}}{273 \text{ K}}} = 348 \text{ m/s}$$

The wavelength at this temperature is $\lambda_2 = (348 \text{ m/s})/(3.00 \times 10^3 \text{ Hz}) = 0.116$ m. This gives a diffraction angle of $\theta_2 = 54.0°$. The change in the diffraction angle is thus

$$\Delta\theta = 54.0° - 50.3° = \boxed{3.7°}$$

15. ***REASONING AND SOLUTION***

The figure at the right shows the geometry of the situation.

The tone will not be heard at seats located at the first diffraction minimum. This occurs when

$$\sin\ \theta = \frac{\lambda}{D} = \frac{v}{fD}$$

That is, the angle θ is given by

$$\theta = \sin^{-1}\left(\frac{v}{fD}\right) = \sin^{-1}\left[\frac{343\ \text{m/s}}{(1.0\times10^{4}\ \text{Hz})(0.075\ \text{m})}\right] = 27.2°$$

From the figure at the right, we see that

$$\tan 27.2° = \frac{x}{8.7\ \text{m}} \quad \Rightarrow \quad x = (8.7\ \text{m})(\tan 27.2°) = 4.47\ \text{m}$$

Thus, seats at which the tone cannot be heard are a distance x on either side of the center seat C. Thus, the distance between the two seats is

$$2x = 2(4.47\ \text{m}) = \boxed{8.9\ \text{m}}$$

16. ***REASONING AND SOLUTION*** The experiment without the putty shows that the tuning fork frequency is either 490 Hz or 494 Hz. The effect of the putty is to decrease the frequency of the tuning fork and since the beat frequency goes down, the original tuning fork frequency must have been higher than 492 Hz. Hence, its frequency is

$$\boxed{494 \text{ Hz}}$$

17. SSM ***REASONING AND SOLUTION*** Two ultrasonic sound waves combine and form a beat frequency that is in the range of human hearing. The frequency of one of the ultrasonic waves is 70 kHz. The beat frequency is the difference between the two sound frequencies. The smallest possible value for the ultrasonic frequency can be found by subtracting the upper limit of human hearing from the value of 70 kHz. The largest possible value for the ultrasonic frequency can be determined by adding the upper limit of human hearing to the value of 70 kHz. We know that the frequency range of human hearing is from 20 Hz to 20 kHz.

a. The smallest possible frequency of the other ultrasonic wave is

$$f = 70\text{ kHz} - 20\text{ kHz} = \boxed{50\text{ kHz}}$$

which results in a beat frequency of $70\text{ kHz} - 50\text{ kHz} = 20\text{ kHz}$.

b. The largest possible frequency for the other wave is

$$f = 70\text{ kHz} + 20\text{ kHz} = \boxed{90\text{ kHz}}$$

which results in a beat frequency of $90\text{ kHz} - 70\text{ kHz} = 20\text{ kHz}$.

18. ***REASONING*** When two frequencies are sounded simultaneously, the beat frequency produced is the difference between the two. Thus, knowing the beat frequency between the tuning fork and one flute tone tells us only the difference between the known frequency and the tuning-fork frequency. It does not tell us whether the tuning-fork frequency is greater or smaller than the known frequency. However, two different beat frequencies and two flute frequencies are given. Consideration of both beat frequencies will enable us to find the tuning-fork frequency.

SOLUTION The fact that a 1-Hz beat frequency is heard when the tuning fork is sounded along with the 262-Hz tone implies that the tuning-fork frequency is either 263 Hz or 261 Hz. We can eliminate one of these values by considering the fact that a 3-Hz beat frequency is heard when the tuning fork is sounded along with the 266-Hz tone. This implies that the tuning-fork frequency is either 269 Hz or 263 Hz. Thus, the tuning-fork frequency must be $\boxed{263\text{ Hz}}$.

19. SSM ***REASONING*** The beat frequency of two sound waves is the difference between the two sound frequencies. From the graphs, we see that the period of the wave in the upper figure is 0.020 s, so its frequency is $f_1 = 1/T_1 = 1/(0.020\text{ s}) = 5.0\times10^1\text{ Hz}$. The frequency of the wave in the lower figure is $f_2 = 1/(0.024\text{ s}) = 4.2\times10^1\text{ Hz}$.

SOLUTION The beat frequency of the two sound waves is

$$f_{\text{beat}} = f_1 - f_2 = 5.0\times10^1\text{ Hz} - 4.2\times10^1\text{ Hz} = \boxed{8\text{ Hz}}$$

20. ***REASONING AND SOLUTION*** The first case requires that the frequency be either

$$440\text{ Hz} - 5\text{ Hz} = 435\text{ Hz} \quad \text{or} \quad 440\text{ Hz} + 5\text{ Hz} = 445\text{ Hz}$$

The second case requires that the frequency be either

$$436\text{ Hz} - 9\text{ Hz} = 427\text{ Hz} \quad \text{or} \quad 436\text{ Hz} + 9\text{ Hz} = 445\text{ Hz}$$

The frequency of the tuning fork is $\boxed{445\text{ Hz}}$.

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21. ***REASONING*** The beat frequency is the difference between the frequency of the sound wave traveling in seawater and that generated by the 440.0-Hz tuning fork. We can use Equation 16.1, $v = f\lambda$, to find the frequency of the sound wave, provided we can determine the speed v of sound in seawater. This can be accomplished by using Equation 16.6.

SOLUTION The speed of sound in a liquid is given by Equation 16.6 as $v = \sqrt{B_{\text{ad}} / \rho}$, where B_{ad} is the adiabatic bulk modulus and ρ is the density of the liquid. The frequency of the sound traveling in sea water is

$$f = \frac{v}{\lambda} = \frac{\sqrt{\dfrac{B_{\text{ad}}}{\rho}}}{\lambda} = \frac{\sqrt{\dfrac{2.31 \times 10^9 \text{ Pa}}{1025 \text{ kg/m}^3}}}{3.35 \text{ m}} = 448 \text{ Hz}$$

The beat frequency is

$$448 \text{ Hz} - 440.0 \text{ Hz} = \boxed{8 \text{ Hz}}$$

22. ***REASONING AND SOLUTION***

a. We know that $f_x - f_y = 8$ Hz. Also, f_x is either 395 Hz or 389 Hz and f_y is either 397 Hz or 387 Hz. The fact that $f_x > f_y$ requires that

$$\boxed{f_y = 387 \text{ Hz} \quad \text{and, therefore,} \quad f_x = 395 \text{ Hz}}$$

b. Now we know that $f_y - f_x = 8$ Hz and f_x is either 395 Hz or 389 Hz and f_y is either 397 Hz or 387 Hz. The fact that $f_x < f_y$ requires that

$$\boxed{f_y = 397 \text{ Hz} \quad \text{and, therefore,} \quad f_x = 389 \text{ Hz}}$$

23. **SSM** ***REASONING*** For standing waves on a string that is clamped at both ends, Equations 17.3 and 16.2 indicate that the standing wave frequencies are

$$f_n = n\left(\frac{v}{2L}\right) \qquad \text{where} \qquad v = \sqrt{\frac{F}{m/L}}$$

Combining these two expressions, we have, with $n = 1$ for the fundamental frequency,

$$f_1 = \frac{1}{2L}\sqrt{\frac{F}{m/L}}$$

This expression can be used to find the ratio of the two fundamental frequencies.

SOLUTION The ratio of the two fundamental frequencies is

$$\frac{f_{\text{old}}}{f_{\text{new}}} = \frac{\frac{1}{2L}\sqrt{\frac{F_{\text{old}}}{m/L}}}{\frac{1}{2L}\sqrt{\frac{F_{\text{new}}}{m/L}}} = \sqrt{\frac{F_{\text{old}}}{F_{\text{new}}}}$$

Since $F_{\text{new}} = 4F_{\text{old}}$, we have

$$f_{\text{new}} = f_{\text{old}}\sqrt{\frac{F_{\text{new}}}{F_{\text{old}}}} = f_{\text{old}}\sqrt{\frac{4\,F_{\text{old}}}{F_{\text{old}}}} = f_{\text{old}}\sqrt{4} = (55.0\text{ Hz})\,(2) = \boxed{1.10\times10^{2}\text{ Hz}}$$

24. ***REASONING AND SOLUTION*** Since the wavelength is twice the distance between two successive nodes, we can use Equation 16.1 and see that

$$v = \lambda f = (2L)f = 2(0.30\text{ m})(4.0\text{ Hz}) = \boxed{2.4\text{ m/s}}$$

25. ***REASONING AND SOLUTION*** Assuming that fretting the string does NOT change the tension, the speed of waves on the string will be the same in both cases. The speed of the waves is given $v = \lambda f = 2Lf$. Applied to the first case (unfretted), this relation gives $v = 2L_1f_1$. Applied to the second case, it gives $v = 2L_2f_2$.

Equating the above equations and rearranging yields

$$L_2 = L_1(f_1/f_2) = (0.62\text{ m})(196\text{ Hz})/(262\text{ Hz}) = \boxed{0.46\text{ m}}$$

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26. ***REASONING*** The time it takes for a wave to travel the length L of the string is $t = L/v$, where v is the speed of the wave. The speed can be obtained since the fundamental frequency is known, and Equation 17.3 (with $n = 1$) gives the fundamental frequency as $f_1 = v/(2L)$. The length is not needed, since it can be eliminated algebraically between this expression and the expression for the time.

SOLUTION Solving Equation 17.3 for the speed gives $v = 2Lf_1$. With this result for the speed, the time for a wave to travel the length of the string is

$$t = \frac{L}{v} = \frac{L}{2Lf_1} = \frac{1}{2f_1} = \frac{1}{2(256\text{ Hz})} = \boxed{1.95 \times 10^{-3}\text{ s}}$$

27. SSM ***REASONING*** According to Equation 17.3, the fundamental (n = 1) frequency of a string fixed at both ends is related to the wave speed v by $f_1 = v/2L$, where L is the length of the string. Thus, the speed of the wave is $v = 2Lf_1$. Combining this with Equation 16.2, $v = \sqrt{F/(m/L)}$, we have, after some rearranging,

$$\frac{F}{L^2} = 4f_1^2(m/L)$$

Since the strings have the same tension and the same lengths between their fixed ends, we have

$$f_{1\mathrm{E}}^2(m/L)_{\mathrm{E}} = f_{1\mathrm{G}}^2(m/L)_{\mathrm{G}}$$

where the symbols "E" and "G" represent the E and G strings on the violin. This equation can be solved for the linear density of the G string.

SOLUTION The linear density of the string is

$$(m/L)_{\mathrm{G}} = \frac{f_{1\mathrm{E}}^2}{f_{1\mathrm{G}}^2}(m/L)_{\mathrm{E}} = \left(\frac{f_{1\mathrm{E}}}{f_{1\mathrm{G}}}\right)^2(m/L)_{\mathrm{E}}$$

$$= \left(\frac{659.3\ \mathrm{Hz}}{196.0\ \mathrm{Hz}}\right)^2(3.47\times10^{-4}\ \mathrm{kg/m}) = \boxed{3.93\times10^{-3}\ \mathrm{kg/m}}$$

28. ***REASONING*** Equation 17.3 (with $n = 1$) gives the fundamental frequency as $f_1 = v/(2L)$, where L is the length of the wire and v is the speed of the waves traveling on the string. The speed is given by Equation 16.2 as $v = \sqrt{\dfrac{F}{m/L}}$, where F is the tension and m is the mass of the wire.

SOLUTION Using Equations 17.3 and 16.2, we obtain

$$f_1 = \frac{v}{2L} = \frac{\sqrt{\dfrac{F}{m/L}}}{2L} = \frac{1}{2}\sqrt{\frac{F}{mL}} = \frac{1}{2}\sqrt{\frac{160\text{ N}}{(6.0\times10^{-3}\text{ kg})(0.41\text{ m})}} = \boxed{130\text{ Hz}}$$

29. ***REASONING*** For a vibrating string that is fixed at both ends, there are an integer number of half-wavelengths between the ends. Each half-wavelength gives rise to one loop in the standing wave. From this information, we can determine the wavelength. Once the wavelength is known, the speed of sound follows from $v = f\lambda$, since the frequency is given in the statement of the problem.

SOLUTION

a. In the distance of 2.50 m there are five loops, so there are five half-wavelengths. Thus, the wavelength of the wave is

$$5\left(\frac{\lambda}{2}\right) = 2.50\text{ m} \qquad \text{or} \qquad \boxed{\lambda = 1.00\text{ m}}$$

b. The speed of the wave is

$$v = f\lambda = (85.0\text{ Hz})(1.00\text{ m}) = \boxed{85.0\text{ m/s}}$$

c. The fundamental frequency arises when there is only one loop in the standing wave. The wavelength is $(1)(\lambda/2) = 2.50$ m, or $\lambda = 5.00$ m. The frequency of the wave is

$$f = \frac{v}{\lambda} = \frac{85.0\text{ m/s}}{5.00\text{ m}} = \boxed{17.0\text{ Hz}}$$

30. ***REASONING*** Each string has a node at each end, so the frequency of vibration is given by Equation 17.3 as $f_n = nv/(2L)$, where $n = 1, 2, 3, \ldots$ The speed v of the wave can be determined from Equation 16.2 as $v = \sqrt{F/(m/L)}$. We will use these two relations to find the lowest frequency that permits standing waves in both strings with a node at the junction.

SOLUTION
Since the frequency of the left string is equal to the frequency of the right string, we can write

$$\frac{n_{\text{left}}\sqrt{\dfrac{F}{(m/L)_{\text{left}}}}}{2\text{L}_{\text{left}}} = \frac{n_{\text{right}}\sqrt{\dfrac{F}{(m/L)_{\text{right}}}}}{2\text{L}_{\text{right}}}$$

Substituting in the data given in the problem yields

$$\frac{n_{\text{left}}\sqrt{\dfrac{190.0\text{ N}}{6.00\times10^{-2}\text{ kg/m}}}}{2(3.75\text{ m})} = \frac{n_{\text{right}}\sqrt{\dfrac{190.0\text{ N}}{1.50\times10^{-2}\text{ kg/m}}}}{2(1.25\text{ m})}$$

This expression gives $n_{\text{left}} = 6n_{\text{right}}$. Letting $n_{\text{left}} = 6$ and $n_{\text{right}} = 1$, the frequency of the left string (which is also equal to the frequency of the right string) is

$$f_6 = \frac{(6)\sqrt{\dfrac{190.0\text{ N}}{6.00\times10^{-2}\text{ kg/m}}}}{2(3.75\text{ m})} = \boxed{45.0\text{ Hz}}$$

31. $\boxed{\textit{SSM}}$ ***REASONING*** We can find the extra length that the D-tuner adds to the E-string by calculating the length of the D-string and then subtracting from it the length of the E string. For standing waves on a string that is fixed at both ends, Equation 17.3 gives the frequencies as $f_n = n(v/2L)$. The ratio of the fundamental frequency of the D-string to that of the E-string is

$$\frac{f_{\mathrm{D}}}{f_{\mathrm{E}}} = \frac{v/(2L_{\mathrm{D}})}{v/(2L_{\mathrm{E}})} = \frac{L_{\mathrm{E}}}{L_{\mathrm{D}}}$$

This expression can be solved for the length L_{D} of the D-string in terms of quantities given in the problem statement.

SOLUTION The length of the D-string is

$$L_{\mathrm{D}} = L_{\mathrm{E}}\left(\frac{f_{\mathrm{E}}}{f_{\mathrm{D}}}\right) = (0.628\ \mathrm{m})\left(\frac{41.2\ \mathrm{Hz}}{36.7\ \mathrm{Hz}}\right) = 0.705\ \mathrm{m}$$

The length of the E-string is extended by the D-tuner by an amount

$$L_{\mathrm{D}} - L_{\mathrm{E}} = 0.705\ \mathrm{m} - 0.628\ \mathrm{m} = \boxed{0.077\ \mathrm{m}}$$

32. ***REASONING AND SOLUTION*** We are given $f_j = \sqrt[12]{2}\, f_{j-1}$.

a. The length of the unfretted string is $L_0 = v/(2f_0)$ and the length of the string when it is pushed against fret 1 is $L_1 = v/(2f_1)$. The distance between the frets is

$$L_0 - L_1 = \frac{v}{2f_0} - \frac{v}{2f_1} = \left(\frac{v}{2f_0}\right)\left(1 - \frac{f_0}{f_1}\right) = \left(\frac{v}{2f_0}\right)\left(1 - \frac{1}{\sqrt[12]{2}}\right)$$

$$= L_0\left(1 - \frac{1}{\sqrt[12]{2}}\right) = (0.628\text{ m})(0.0561) = \boxed{0.0352\text{ m}}$$

b. The frequencies corresponding to the sixth and seventh frets are $f_6 = \left(\sqrt[12]{2}\right)^6 f_0$ and $f_7 = \left(\sqrt[12]{2}\right)^7 f_0$. The distance between fret 6 and fret 7 is

$$L_6 - L_7 = \frac{v}{2f_6} - \frac{v}{2f_7} = \frac{v}{2\left(\sqrt[12]{2}\right)^6 f_0} - \frac{v}{2\left(\sqrt[12]{2}\right)^7 f_0} = \left(\frac{v}{2f_0}\right)\left[\frac{1}{\left(\sqrt[12]{2}\right)^6} - \frac{1}{\left(\sqrt[12]{2}\right)^7}\right]$$

$$= L_0\left[\frac{1}{\left(\sqrt[12]{2}\right)^6} - \frac{1}{\left(\sqrt[12]{2}\right)^7}\right] = (0.628\text{ m})(0.0397) = \boxed{0.0249\text{ m}}$$

33. **SSM** **WWW** ***REASONING*** The natural frequencies of the cord are, according to Equation 17.3, $f_n = nv/(2L)$, where $n = 1, 2, 3, ...$ The speed v of the waves on the cord is, according to Equation 16.2, $v = \sqrt{F/(m/L)}$, where F is the tension in the cord. Combining these two expressions, we have

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{F}{m/L}} \qquad \text{or} \qquad \left(\frac{f_n 2L}{n}\right)^2 = \frac{F}{m/L}$$

Applying Newton's second law of motion, $\Sigma F = ma$, to the forces that act on the block and are parallel to the incline gives

$$F - Mg\sin\theta = Ma = 0 \qquad \text{or} \qquad F = Mg\sin\theta$$

where $Mg\sin\theta$ is the component of the block's weight that is parallel to the incline. Substituting this value for the tension into the equation above gives

$$\left(\frac{f_n 2L}{n}\right)^2 = \frac{Mg\sin\theta}{m/L}$$

This expression can be solved for the angle θ and evaluated at the various harmonics. The answer can be chosen from the resulting choices.

SOLUTION Solving this result for $\sin\theta$ shows that

$$\sin\theta = \frac{(m/L)}{Mg}\left(\frac{f_n 2L}{n}\right)^2 = \frac{1.20\times10^{-2}\ \text{kg/m}}{(15.0\ \text{kg})(9.80\ \text{m/s}^2)}\left[\frac{(165\ \text{Hz})2(0.600\ \text{m})}{n}\right]^2 = \frac{3.20}{n^2}$$

Thus, we have

$$\theta = \sin^{-1}\left(\frac{3.20}{n^2}\right)$$

Evaluating this for the harmonics corresponding to the range of n from $n = 2$ to $n = 4$, we have

$$\theta = \sin^{-1}\left(\frac{3.20}{2^2}\right) = 53.1° \text{ for } n = 2$$

$$\theta = \sin^{-1}\left(\frac{3.20}{3^2}\right) = 20.8° \text{ for } n = 3$$

$$\theta = \sin^{-1}\left(\frac{3.20}{4^2}\right) = 11.5° \text{ for } n = 4$$

The angles between 15.0° and 90.0° are $\boxed{\theta = 20.8°}$ and $\boxed{\theta = 53.1°}$.

34. ***REASONING*** Equation 17.5 (with $n = 1$) gives the fundamental frequency as $f_1 = v/(4L)$, where L is the length of the auditory canal and v is the speed of sound.

SOLUTION Using Equation 17.5, we obtain

$$f_1 = \frac{v}{4L} = \frac{343 \text{ m/s}}{4(0.029 \text{ m})} = \boxed{3.0 \times 10^3 \text{ Hz}}$$

35. ***REASONING AND SOLUTION*** The length L of a tube open at both ends can be found from Equation 17.4, $L = nv/(2f_n)$. For the fundamental frequency, $n = 1$, to the ratio of the length of the piccolo to that of the flute is

$$\frac{L_{\text{piccolo}}}{L_{\text{flute}}} = \frac{\dfrac{(1)v}{2(f_1)_{\text{piccolo}}}}{\dfrac{(1)v}{2(f_1)_{\text{flute}}}} = \frac{(f_1)_{\text{flute}}}{(f_1)_{\text{piccolo}}} = \frac{261.6\text{ Hz}}{587.3\text{ Hz}} = \boxed{0.445}$$

36. ***REASONING AND SOLUTION*** We know that $L = v/(2f)$. For 20.0 Hz

$$L = (343 \text{ m/s})/[2(20.0 \text{ Hz})] = \boxed{8.6 \text{ m}}$$

For 20.0 kHz

$$L = (343 \text{ m/s})/[2(20.0 \times 10^3 \text{ Hz})] = \boxed{8.6 \times 10^{-3} \text{ m}}$$

37. **SSM** ***REASONING AND SOLUTION*** The distance between one node and an adjacent antinode is $\lambda/4$. Thus, we must first determine the wavelength of the standing wave. A tube open at only one end can develop standing waves only at the odd harmonic frequencies. Thus, for a tube of length L producing sound at the third harmonic ($n = 3$), $L = 3(\lambda/4)$. Therefore, the wavelength of the standing wave is

$$\lambda = \tfrac{4}{3}L = \tfrac{4}{3}(1.5\text{ m}) = 2.0\text{ m}$$

and the distance between one node and the adjacent antinode is $\lambda/4 = \boxed{0.50\text{ m}}$.

38. ***REASONING*** The fundamental (n = 1) frequency for a tube open at both ends is (see Equation 17.4) $f_1 = v/(2L)$. The fundamental frequency of a tube open at only one end is given by Equation 17.5 as $f_1 = v/(4L)$. Since we know the fundamental frequency of the tube open at only one end, we can use these relations to determine the other fundamental frequency.

SOLUTION

a. The ratio of the fundamental frequency of a tube open at both ends to that of a tube open at only one end is

$$\frac{(f_1)_{\text{open at both ends}}}{(f_1)_{\text{open at only one end}}} = \frac{\dfrac{v}{2L}}{\dfrac{v}{4L}} = 2$$

Thus,

$$(f_1)_{\text{open at both ends}} = 2(f_1)_{\text{open at only one end}} = 2(130.8\text{ Hz}) = \boxed{261.6\text{ Hz}}$$

b. The length L of the tube can be found from the relation $f_1 = v/(2L)$:

$$L = \frac{v}{2f_1} = \frac{343\text{ m/s}}{2(261.6\text{ Hz})} = \boxed{0.656\text{ m}}$$

39. $\boxed{\textit{SSM}}$ ***REASONING AND SOLUTION*** Since both tubes have the same length, the wavelength of the fundamental frequency is the same for both gases. From Equation 16.1 we have $f_{\mathrm{H}} = v_{\mathrm{H}} / \lambda$ and $f_{\mathrm{N}} = v_{\mathrm{N}} / \lambda$, where the subscript "H" refers to helium and "N" refers to neon. Dividing the first equation by the second gives

$$f_{\mathrm{H}} / f_{\mathrm{N}} = v_{\mathrm{H}} / v_{\mathrm{N}}$$

We know from Equation 16.5 that the speed of sound in a monatomic ideal gas is given by $v = \sqrt{\gamma kT / m}$, where $\gamma = 1.40$ and m is the mass of one atom of the gas. Therefore,

$$\frac{v_{\mathrm{H}}}{v_{\mathrm{N}}} = \sqrt{\frac{m_{\mathrm{N}}}{m_{\mathrm{H}}}}$$

Substituting the relation above into $f_{\mathrm{H}}/f_{\mathrm{N}} = v_{\mathrm{H}}/v_{\mathrm{N}}$, and using the fact that the atomic mass of neon is $m_{\mathrm{N}} = 20.179$ u and that of helium is $m_{\mathrm{H}} = 4.0026$ u, yields

$$f_{\mathrm{H}} = f_{\mathrm{N}} \sqrt{\frac{m_{\mathrm{N}}}{m_{\mathrm{H}}}} = (268\ \mathrm{Hz}) \sqrt{\frac{20.179\ \mathrm{u}}{4.0026\ \mathrm{u}}} = \boxed{602\ \mathrm{Hz}}$$

40. ***REASONING AND SOLUTION*** According to Equation 16.5, the molecular mass is given by $m = 1.40\ kT/v^2$. The speed of waves in the tube is $v = \lambda f = 2Lf$.

Thus,

$$m = \frac{1.40\ kT}{4L^2 f^2} = \frac{(1.40)(1.38 \times 10^{-23}\ \text{J/K})(293\ \text{K})}{4(0.248\ \text{m})^2(294\ \text{Hz})^2} = \boxed{2.66 \times 10^{-25}\ \text{kg}}$$

41. ***REASONING AND SOLUTION*** The distance between the nodes of the standing wave is $L = \lambda/2 = v/(2f)$. The man travels this distance in a time

$$t = 1/(3.0\ \text{Hz}) = 0.33\ \text{s}$$

His speed is then

$$v_{\text{listener}} = L/t = v/(2ft) = (343\ \text{m/s})/[2(0.33\ \text{s})(440\ \text{Hz})] = \boxed{1.2\ \text{m/s}}$$

42. ***REASONING*** The well is open at the top and closed at the bottom, so it can be approximated as a column of air that is open at only one end. According to Equation 17.5, the natural frequencies for such an air column are

$$f_{\mathrm{n}} = n\left(\frac{v}{4L}\right) \quad \text{where} \quad n = 1, 3, 5, \ldots$$

The depth L of the well can be calculated from the speed of sound, v = 343 m/s, and a knowledge of the natural frequencies f_{n}.

SOLUTION We know that two of the natural frequencies are 42 and 70.0 Hz. The ratio of these two frequencies is

$$\frac{70.0\ \text{Hz}}{42\ \text{Hz}} = \frac{5}{3}$$

Therefore, the value of n for each frequency is n = 3 for the 42-Hz sound, and n = 5 for the 70.0-Hz sound. Using n = 3, for example, the depth of the well is

$$L = \frac{nv}{4f_3} = \frac{3(343\ \text{m/s})}{4(42\ \text{Hz})} = \boxed{6.1\ \text{m}}$$

43. SSM WWW ***REASONING*** According to Equation 11.4, the absolute pressure at the bottom of the mercury is $P = P_{\text{atm}} + \rho g h$, where the height h of the mercury column is the original length L_0 of the air column minus the shortened length L. Hence,

$$P = P_{\text{atm}} + \rho g(L_0 - L)$$

SOLUTION From Equation 17.5, the fundamental (n = 1) frequency f_1 of the shortened tube is $f_1 = 1(v/4L)$, where L is the length of the air column in the tube. Likewise, the frequency f_3 of the third (n = 3) harmonic in the original tube is $f_3 = 3(v/4L_0)$, where L_0 is the length of the air column in the original tube. Since $f_1 = f_3$, we have that

$$1\left(\frac{v}{4L}\right) = 3\left(\frac{v}{4L_0}\right) \quad \textit{or} \quad L = \tfrac{1}{3}L_0$$

The pressure at the bottom of the mercury is

$$P = P_{\text{atm}} + \rho g\left(\tfrac{2}{3}L_0\right)$$

$$= 1.01\times10^5\ \text{Pa} + (13\ 600\ \text{kg/m}^3)(9.80\ \text{m/s}^2)\left(\tfrac{2}{3}\times0.75\ \text{m}\right) = \boxed{1.68\times10^5\ \text{Pa}}$$

44. ***REASONING AND SOLUTION*** The original tube has a fundamental given by $f = v/(4L)$, so that its length is $L = v/(4f)$. The cut tube that has one end closed has a length of $L_c = v/(4f_c)$, while the cut tube that has both ends open has a length $L_o = v/(2f_o)$.

We know that $L = L_c + L_o$. Substituting the expressions for the lengths and solving for f gives

$$f = \frac{f_o f_c}{2f_c + f_o} = \frac{(425\text{ Hz})(675\text{ Hz})}{2(675\text{ Hz}) + 425\text{ Hz}} = \boxed{162\text{ Hz}}$$

45. ***REASONING AND SOLUTION*** We know that at the fundamental frequency $\lambda = 2L = 2(0.10\text{ m}) = 0.20\text{ m}$. Therefore,

$$v = f\lambda = (440\text{ Hz})(0.20\text{ m}) = \boxed{88\text{ m/s}}$$

46. ***REASONING AND SOLUTION***

a. For a string fixed at both ends the fundamental frequency is $f_1 = v/(2L)$ so $f_n = nf_1$.

$$\boxed{f_2 = 800\text{ Hz}, \quad f_3 = 1200\text{ Hz}, \quad f_4 = 1600\text{ Hz}}$$

b. For a pipe with both ends open the fundamental frequency is $f_1 = v/(2L)$ so $f_n = nf_1$.

$$\boxed{f_2 = 800\text{ Hz}, \quad f_3 = 1200\text{ Hz}, \quad f_4 = 1600\text{ Hz}}$$

c. For a pipe open at one end only the fundamental frequency is $f_1 = v/(4L)$ so $f_n = nf_1$ with n odd.

$$\boxed{f_3 = 1200\text{ Hz}, \quad f_5 = 2000\text{ Hz}, \quad f_7 = 2800\text{ Hz}}$$

47. SSM ***REASONING*** The tones from the two speakers will produce destructive interference with the smallest frequency when the path length difference at C is one-half of a wavelength. From Figure 17.7, we see that the path length difference is $\Delta s = s_{AC} - s_{BC}$. From Example 1, we know that $s_{AC} = 4.00\text{ m}$, and from Figure 17.7, $s_{BC} = 2.40\text{ m}$. Therefore, the path length difference is $\Delta s = 4.00\text{ m} - 2.40\text{ m} = 1.60\text{ m}$.

SOLUTION Thus, destructive interference will occur when

$$\frac{\lambda}{2} = 1.60\text{ m} \qquad \text{or} \qquad \lambda = 3.20\text{ m}$$

This corresponds to a frequency of

$$f = \frac{v}{\lambda} = \frac{343\text{ m/s}}{3.20\text{ m}} = \boxed{107\text{ Hz}}$$

48. ***REASONING AND SOLUTION*** The beat frequency is $f_{beat} = 529\text{ Hz} - 524\text{ Hz} = 5\text{ Hz}$. The period of the beats is then

$$T_{beat} = 1/f_{beat} = 1/(5\text{ Hz}) = \boxed{0.2\text{ s}}$$

49. ***REASONING AND SOLUTION*** We know that $f_A = v/(2L_A)$ and $f_B = v/(4L_B)$. Since the frequencies are equal $2L_A = 4L_B$. Thus,

$$L_B = (1/2)\, L_A = (1/2)(0.60\text{ m}) = \boxed{0.30\text{ m}}$$

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50. ***REASONING AND SOLUTION*** The shape of the string looks like

51. $\boxed{\textbf{SSM}}$ ***REASONING*** The fundamental frequency f_1 is given by Equation 17.3 with $n = 1$: $f_1 = v/(2L)$. Since values for f_1 and L are given in the problem statement, we can use this expression to find the speed of the waves on the cello string. Once the speed is known, the tension F in the cello string can be found by using Equation 16.2, $v = \sqrt{F/(m/L)}$.

SOLUTION Combining Equations 17.3 and 16.2 yields

$$2Lf_1 = \sqrt{\frac{F}{m/L}}$$

Solving for F, we find that the tension in the cello string is

$$F = 4L^2 f_1^2 (m/L) = 4(0.800\text{ m})^2 (65.4\text{ Hz})^2 (1.56\times10^{-2}\text{ kg/m}) = \boxed{171\text{ N}}$$

52. ***REASONING AND SOLUTION***

a. From the graph we see that the wavelength of the wave is $\boxed{8.0\text{ cm}}$.

b. A standing wave is comprised of two identical waves traveling in opposite directions. If one of the waves has a velocity of +12.0 cm/s, then the other wave must have a velocity of $\boxed{-12.0\text{ cm/s}}$

c. The frequency of each wave is found from Equation 16.1:

$$f = \frac{v}{\lambda} = \frac{12.0\text{ m/s}}{8.0\text{ cm}} = \boxed{1.5\text{ Hz}}$$

d. A "dot" attached to the string vibrates up and down in simple harmonic motion. Equation 10.8 gives the maximum speed of an object executing simple harmonic motion as

$$v_{\max} = A\omega = A(2\pi f) = (3.0\text{ cm})2\pi(1.5\text{ Hz}) = \boxed{28.3\text{ cm/s}}$$

53. ***REASONING AND SOLUTION*** Since the tube is closed at both ends, a node must appear at each end in order for a standing wave to exist in the tube. Hence, $L = \lambda/2$ for the shortest wave, $L = \lambda$ for the next , $L = 3\lambda/2$ for the next and so on up to the nth. Thus, $L = n\lambda/2$. So $\lambda = 2L/n$. The frequency is always given by $f = v/\lambda$, so

$$\boxed{f_n = nv/(2L), \quad \text{where } n = 1, 2, 3, \ldots}$$

54. ***REASONING AND SOLUTION*** The speed of the speakers is

$$v_S = 2\pi r/t = 2\pi(9.01\text{ m})/(20.0\text{ s}) = 2.83\text{ m/s}$$

The sound that an observer hears coming from the right speaker is Doppler shifted to a new frequency given by Equation 16.11 as

$$f_{OR} = \frac{f_s}{1 - v_s / v} = \frac{100.0\text{ Hz}}{1 - \left[(2.83\text{ m/s})/(343.00\text{ m/s})\right]} = 100.83\text{ Hz}$$

The sound that an observer hears coming from the left speaker is shifted to a new frequency given by Equation 16.12 as

$$f_{OL} = \frac{f_s}{1 + v_s / v} = \frac{100.0\text{ Hz}}{1 + \left[(2.83\text{ m/s})/(343.00\text{ m/s})\right]} = 99.18\text{ Hz}$$

The beat frequency heard by the observer is then

$$100.83\text{ Hz} - 99.18\text{ Hz} = \boxed{1.7\text{ Hz}}$$

55. SSM WWW ***REASONING*** The beat frequency produced when the piano and the other instrument sound the note (three octaves higher than middle C) is $f_{\text{beat}} = f - f_0$, where f is the frequency of the piano and f_0 is the frequency of the other instrument ($f_0 = 2093$ Hz). We can find f by considering the temperature effects and the mechanical effects that occur when the temperature drops from 25.0 °C to 20.0 °C.

SOLUTION The fundamental frequency f_0 of the wire at 25.0 °C is related to the tension F_0 in the wire by

$$f_0 = \frac{v}{2L_0} = \frac{\sqrt{F_0/(m/L)}}{2L_0} \tag{1}$$

where Equations 17.3 and 16.2 have been combined.

The amount ΔL by which the piano wire attempts to contract is (see Equation 12.2) $\Delta L = \alpha L_0 \Delta T$, where α is the coefficient of linear expansion of the wire, L_0 is its length at 25.0 °C, and ΔT is the amount by which the temperature drops. Since the wire is prevented from contracting, there must be a stretching force exerted at each end of the wire. According to Equation 10.17, the magnitude of this force is

$$\Delta F = Y\left(\frac{\Delta L}{L_0}\right)A$$

where Y is the Young's modulus of the wire, and A is its cross-sectional area. Combining this relation with Equation 12.2, we have

$$\Delta F = Y\left(\frac{\alpha L_0 \Delta T}{L_0}\right)A = \alpha(\Delta T)YA$$

Thus, the frequency f at the lower temperature is

$$f = \frac{v}{2L_0} = \frac{\sqrt{(F_0 + \Delta F)/(m/L)}}{2L_0} = \frac{\sqrt{\left[F_0 + \alpha(\Delta T)YA\right]/(m/L)}}{2L_0} \tag{2}$$

Using Equations (1) and (2), we find that the frequency f is

$$f = f_0 \frac{\sqrt{\left[F_0 + \alpha(\Delta T)YA\right]/(m/L)}}{\sqrt{F_0/(m/L)}} = f_0\sqrt{\frac{F_0 + \alpha(\Delta T)YA}{F_0}}$$

$$f = (2093\text{ Hz})\sqrt{\frac{818.0\text{ N} + (12\times10^{-6}/\text{C}°)(5.0\text{ C}°)(2.0\times10^{11}\text{ N/m}^2)(7.85\times10^{-7}\text{ m}^2)}{818.0\text{ N}}}$$

$$= 2105\text{ Hz}$$

Therefore, the beat frequency is $2105\text{ Hz} - 2093\text{ Hz} = \boxed{12\text{ Hz}}$.

CHAPTER 1 | *INTRODUCTION AND MATHEMATICAL CONCEPTS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION***

a. The SI unit for x is m. The SI units for the quantity vt are

$$\frac{\text{m}}{\text{s}} \div (\text{s}) = \text{m}\left(\frac{\text{m}}{\text{s}}\right)(\text{s}) = \text{m}$$

Therefore, the units on the left hand side of the equation are consistent with the units on the right hand side.

b. As described in part *a*, the SI units for the quantities x and vt are both m. The SI units for the quantity $\frac{1}{2}at^2$ are

$$\left(\frac{\text{m}}{\text{s}^2}\right)(\text{s}^2) = \text{m}$$

Therefore, the units on the left hand side of the equation are consistent with the units on the right hand side.

c. The SI unit for v is m/s. The SI unit for the quantity at is

$$\left(\frac{\text{m}}{\text{s}^2}\right)(\text{s}) = \frac{\text{m}}{\text{s}}$$

Therefore, the units on the left hand side of the equation are consistent with the units on the right hand side.

d. As described in part *c*, the SI units of the quantities v and at are both m/s. The SI unit of the quantity $\frac{1}{2}at^3$ is

$$\left(\frac{\text{m}}{\text{s}^2}\right)(\text{s}^3) = \text{m}\times\text{s}$$

Thus, the units on the left hand side are *not* consistent with the units on the right hand side. In fact, the right hand side is not a valid operation because it is not possible to add physical quantities that have different units.

e. The SI unit for the quantity v^3 is m^3/s^3. The SI unit for the quantity $2ax^3$ is

$$\left(\frac{\mathrm{m}}{\mathrm{s}^2}\right)(\mathrm{m}^2) = \frac{\mathrm{m}^3}{\mathrm{s}^2}$$

Therefore, the units on the left hand side of the equation are *not* consistent with the units on the right hand side.

f. The SI unit for the quantity t is s. The SI unit for the quantity $\sqrt{\frac{2x}{a}}$ is

$$\sqrt{\frac{\mathrm{m}}{(\mathrm{m}/\mathrm{s}^2)}} = \sqrt{\mathrm{m}\left(\frac{\mathrm{s}^2}{\mathrm{m}}\right)} = \sqrt{\mathrm{s}^2} = \mathrm{s}$$

Therefore, the units on the left hand side of the equation are consistent with the units on the right hand side.

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2. ***REASONING AND SOLUTION*** The quantity tan q is dimensionless and has no units. The units of the ratio x/v are

$$\frac{\mathrm{m}}{(\mathrm{m}/\mathrm{s})} = \mathrm{m}\left(\frac{\mathrm{s}}{\mathrm{m}}\right) = \mathrm{s}$$

Thus, the units on the left side of the equation are not consistent with those on the right side, and the equation tan $q = x/v$ is not a possible relationship between the variables x, v, and q.

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3. ***REASONING AND SOLUTION*** It is not always possible to add two numbers that have the same dimensions. In order to add any two physical quantities they must be expressed in the same *units*. Consider the two lengths: 1.00 m and 1.00 cm. Both quantities are lengths and, therefore, have the dimension [L]. Since the units are different, however, these two numbers cannot be added.

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4. ***REASONING AND SOLUTION***

a. The dimension of a physical quantity describes the physical *nature* of the quantity and the *kind* of unit that is used to express the quantity. It is possible for two quantities to have the same dimensions but different units. All lengths, for example, have the dimension [L]. However, a length may be expressed in any length unit, such as kilometers, meters, centimeters, millimeters, inches, feet or yards.

As another illustration, the quantities 100 g and 1.5 kg are masses and have the dimensions [M]; however, they have different units.

b. All quantities with the same units must have the same dimensions. For example, all quantities expressed in kilograms have the dimension [M]; all quantities expressed in meters have the dimensions [L].

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5. ***REASONING AND SOLUTION*** For the equation to be valid, the dimensions of the left hand side of the equation must be the same as the dimensions on the right hand side. Since the quantity c has no dimensions, it does not contribute to the dimensions of the right hand side, regardless of the value of n. Therefore, the value of n cannot be determined from dimensional analysis.

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6. ***REASONING AND SOLUTION*** The following table shows the value of sin q, cos q, the ratio (sin q)/(cos q) and tan q.

q	sin q	cos q	(sin q)/(cos q)	tan q
30.0	0.500	0.866	0.577	0.577
39.0	0.629	0.777	0.810	0.810
53.0	0.799	0.602	1.33	1.33
60.0	0.866	0.500	1.73	1.73

From the definitions given in Equations 1.1-1.3, we have

$$\frac{\sin\ q}{\cos\ q} = \frac{h_o\ /\ h}{h_a\ /\ h} = \frac{h_o}{h_a} = \tan\ q$$

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7. ***REASONING AND SOLUTION***

a. The graph below shows sin q plotted on the vertical axis and q on the horizontal axis, with q in 15° increments from q = 0 to q = 720°.

b. The graph below shows cos q plotted on the vertical axis and q on the horizontal axis, with q in 15° increments from $q = 0$ to $q = 720°$.

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8. ***REASONING AND SOLUTION*** A vector quantity has both magnitude and direction. The number of people attending a football game, the number of days in a month, and the number of pages in a book can all be completely specified by giving a magnitude only. Hence, none of these quantities can be considered a vector.

---

9. ***REASONING AND SOLUTION*** For two vectors to be equal, they must be equal in magnitude and have the same direction. Only vectors **A**, **B**, and **D** have the same magnitude. These three vectors are shown below:

Clearly, only vectors **A** and **D** have the same magnitude and point in the same direction; hence, **A** and **D** are equal.

---

10. ***REASONING AND SOLUTION*** For two vectors to be equal, they must be equal in magnitude and have the same direction. Thus, two vectors with the same magnitude are not necessarily equal. They must also point in the same direction.

---

11. ***REASONING AND SOLUTION*** One can arrive back at the starting point after making eight consecutive displacements that add to zero only if one traverses three of the edges on one face and three edges on the opposite face (six displacements; the remaining two displacements occur in going from one opposite face to the other). For any particular starting point, there are four independent ways to traverse three edges on opposite faces. These are illustrated in the figures below. In (A) and (B), one traverses three consecutive edges before moving to the opposite face; in (C) and (D), one traverses two consecutive edges, moves to the opposite face, traverses three consecutive edges, then moves back to the original face and traverses the third edge on that face ending at the starting point.

For any particular starting point, one can initially move in any one of three possible directions. Thus, there are a total of 4 (independent ways) 3 (possible directions) or 12 ways to arrive back at the starting point that involve eight displacement vectors.

---

12. ***REASONING AND SOLUTION***
(a) Any vector has a zero component in the direction that is perpendicular to the direction of that vector. Thus, it *is* possible for one component of a vector to be zero, while the vector itself is not zero.

(b) In order for a vector to be zero, its components in all mutually perpendicular directions must also be zero. Therefore, it is *not* possible for a vector to be zero, while one of its components is not zero.

---

13. ***REASONING AND SOLUTION*** If two or more vectors have a resultant of zero, then, when they are arranged in a tail-to-head fashion the head of the last vector must touch the tail of the first vector. In order to satisfy this requirement with two vectors, the vectors must be equal in magnitude and opposite in direction. Therefore, it is *not* possible to add two *perpendicular* vectors so that the vector sum is zero.

---

14. ***REASONING AND SOLUTION*** Three or more vectors with unequal magnitudes may add together so that their vector sum is zero. To do so, they must be oriented so that, when they are placed in a tail-to-head fashion, the head of the last vector touches the tail of the first vector, as in the drawing at the right.

$\mathbf{A} + \mathbf{B} + \mathbf{C} = \mathbf{0}$

---

15. ***REASONING AND SOLUTION*** The vector equation $\mathbf{A} + \mathbf{B} = 0$, implies that $\mathbf{A} = -\mathbf{B}$.

a. The magnitude of **A** must be equal to the magnitude of **B**.

b. The vectors **A** and **B** must point in opposite directions, as indicated by the minus sign in $\mathbf{A} = -\mathbf{B}$.

The same conclusions can be reached from a geometric argument. If $\mathbf{A} + \mathbf{B} = 0$, then, when **A** and **B** are placed tail-to-head, the head of **B** must touch the tail of **A**. Thus, the vectors **A** and **B** must have the same length (i.e., the same magnitude) and point in opposite directions.

---

16. ***REASONING AND SOLUTION*** The equation $\mathbf{A} + \mathbf{B} = \mathbf{C}$ tells us that the vector **C** is the resultant of the vectors **A** and **B**. The magnitudes of the vectors **A**, **B**, and **C** are related by $A^2 + B^2 = C^2$. This has the same form as the Pythagorean theorem that relates the length of the two sides of a right triangle and the length of the hypotenuse. Thus, the vectors **A** and **B** must be at right angles (or perpendicular) to each other.

---

17. ***REASONING AND SOLUTION*** The equation $\mathbf{A} + \mathbf{B} = \mathbf{C}$ tells us that the vector **C** is the resultant of the vectors **A** and **B**. The magnitudes of the vectors **A**, **B**, and **C** are related by $A + B = C$. In other words, the length of the vector **C** is equal to the combined lengths of vectors **A** and **B**. Therefore, the vectors **A** and **B** must point in the same direction.

---

18. ***REASONING AND SOLUTION*** If the magnitude of a vector is doubled, we may conclude that the magnitude of each component of the vector has also doubled.

This is best explained using the arbitrary vector, **A**, shown at the right. The angle q, that specifies the direction of **A**, is given by

$$q = \tan^{1}\left(\frac{A_y}{A_x}\right)$$

Suppose that the magnitude of **A** is doubled. The vector **A** is now twice as long, but its direction (and therefore the angle q) remains unchanged. In order for the angle q to remain the same after the magnitude of **A** is doubled, both $\mathbf{A}_x$ and $\mathbf{A}_y$ must double, so that the ratio A_y / A_x remains the same.

---

19. ***REASONING AND SOLUTION***

a. As the angle increases from 0° to 90°, the x component of the vector decreases in magnitude while the y component increases in magnitude. When the vector has rotated through an angle of 90°, the x component has zero magnitude and the magnitude of the y component is equal to the magnitude of the original vector (see figure below).

b. As the angle increases from 90° to 180°, the y component of the vector decreases in magnitude while the x component increases in magnitude. When the vector has rotated through an angle of 180°, the y component has zero magnitude and the magnitude of the x component is equal to the magnitude of the original vector.

c. As the angle increases from 180° to 270°, the x component of the vector decreases in magnitude while the y component increases in magnitude. When the vector has rotated through an angle of 270°, the x component has zero magnitude and the magnitude of the y component is equal to the magnitude of the original vector.

d. As the angle increases from 270° to 360°, the y component of the vector decreases in magnitude while the x component increases in magnitude. When the vector has rotated through an angle of 360°, the y component has zero magnitude and the magnitude of the x component is equal to the magnitude of the original vector.

---

20. ***REASONING AND SOLUTION*** In general, a vector that has a component of zero along the x axis of a certain axes system does *not* have a component of zero along the x axis of another (rotated) axes system. Consider the vector **A** shown below.

Figure 1

Figure 2

Relative to the x, y axes shown in Figure 1, the vector **A** has a component of zero along the x axis. Figure 2 shows the same vector along with a new axes system x', y'. The primed axes system is rotated at an arbitrary angle q counterclockwise relative to the unprimed x, y axes system. Clearly the component of **A** along the x' axis is non zero for *most* arbitrary values of the (rotation) angle q. The exceptions occur when $q = 0$, 180° and 360°.

CHAPTER 2 | *KINEMATICS IN ONE DIMENSION*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The displacement of the honeybee for the trip is *not* the same as the distance traveled by the honeybee. As stated in the question, the distance traveled by the honeybee is 2 km. The displacement for the trip is the shortest distance between the initial and final positions of the honeybee. Since the honeybee returns to the hive, its initial position and final position are the same; therefore, the displacement of the honeybee is zero.

---

2. ***REASONING AND SOLUTION*** The buses *do not* have equal velocities. Velocity is a vector, with both magnitude and direction. In order for two vectors to be equal, they must have the same magnitude and the same direction. The direction of the velocity of each bus points in the direction of motion of the bus. Thus, the directions of the velocities of the buses are different. Therefore, the velocities are not equal, even though the speeds are the same.

---

3. ***REASONING AND SOLUTION*** The average speed of a vehicle is defined as the total distance covered by the vehicle divided by the time required for the vehicle to cover the distance. Both distance and time are scalar quantities. Since the average speed is the ratio of two scalar quantities, it is a scalar quantity.

---

4. ***REASONING AND SOLUTION*** Consider the four traffic lights **1**, **2**, **3** and **4** shown below. Let the distance between lights **1** and **2** be x_{12}, the distance between lights **2** and **3** be x_{23}, and the distance between lights **3** and **4** be x_{34}.

The lights can be timed so that if a car travels with a constant speed v, red lights can be avoided in the following way. Suppose that at time $t = 0$ s, light **1** turns green while the rest are red. Light **2** must then turn green in a time t_{12}, where $t_{12} = x_{12}/v$. Light **3** must turn green in a time t_{23} after light **2** turns green, where $t_{23} = x_{23}/v$. Likewise, light **4** must turn green in a time t_{34} after light **3** turns green, where $t_{34} = x_{34}/v$. Note that the timing of traffic lights is more complicated than indicated here when groups of cars are stopped at light **1.**

Then the acceleration of the cars, the reaction time of the drivers, and other factors must be considered.

---

5. ***REASONING AND SOLUTION*** The velocity of the car is a vector quantity with both magnitude and direction The speed of the car is a scalar quantity and has nothing to do with direction. It is possible for a car to drive around a track at constant speed. As the car drives around the track, however, the car must change direction. Therefore, the direction of the velocity *changes*, and the velocity cannot be constant. The *incorrect* statement is (a).

---

6. ***REASONING AND SOLUTION*** Answers will vary. One example is a ball that is thrown straight up in the air. When the ball is at its highest point, its velocity is momentarily zero. Since the ball is close to the surface of the earth, its acceleration is nearly constant and is equal to the acceleration due to gravity. Thus, the velocity of the ball is momentarily zero, but since the ball is accelerating, the velocity is changing. An instant later, the velocity of the ball is nonzero as the ball begins to fall. Another example is a swimmer in a race, reversing directions at the end of the pool.

---

7. ***REASONING AND SOLUTION*** The acceleration of an object is the rate at which its velocity is changing. No information can be gained concerning the acceleration of an object if all that is known is the velocity of the object at a single instant. No conclusion can be reached concerning the accelerations of the two vehicles, so the car does not necessarily have a greater acceleration.

---

8. ***REASONING AND SOLUTION*** It *is* possible for the instantaneous velocity at any point during a trip to have a negative value, even though the average velocity for the entire trip has a positive value. The average velocity for the trip is the displacement for the trip divided by the elapsed time. It depends only on the initial and final positions, and the time required for the trip. The average velocity contains no information concerning the actual path taken by the object. Let us assume that the object is constrained to move in a straight line with directions designated as positive or negative. The direction of the average velocity is the same as the direction of the displacement, while the direction of the instantaneous velocity is the same as the instantaneous direction of motion. The average velocity will be positive if the displacement vector points in the positive direction. At any point in the trip, the object could temporarily reverse direction and move in the negative direction. While the object is moving in the negative direction, its instantaneous velocity is negative. As long as the overall displacement is positive, the average velocity for the trip is positive.

---

9. ***REASONING AND SOLUTION*** Acceleration is the rate of change of velocity. The average velocity for an object over a time interval t is given by $\bar{\mathbf{v}} = \mathbf{x} / t$, where $\mathbf{x}$ is the displacement of the object during the time interval.

A runner runs half the remaining distance to the finish line every ten seconds. In each successive ten-second interval, the distance covered by the runner, and hence the runner's

displacement, becomes smaller by a factor of 2. Thus, during each successive ten-second interval, the ratio $\mathbf{x} / t$ becomes smaller by a factor of 2. The acceleration, however, is the *change* in velocity per unit time. Assume that the distance to the finish line is 100 m and consider the change in the average velocity between the first and second 10 s intervals; it is $v_{1,2}$ = (1/2)(50 m)/(10 s) – (1/2)(100 m)/(10 s) = – 2.5 m/s. Now consider the change in average velocity between the second and third 10 s intervals; it is $v_{2,3}$ = (1/2)(25 m)/(10 s) – (1/2)(50 m)/(10 s) = – 1.25 m/s. Thus, as time passes, the magnitude of the velocity changes by a decreasing amount from one 10 s interval to the next. As a result, the acceleration of the runner does not have a constant magnitude.

---

10. ***REASONING AND SOLUTION*** An object moving with a constant acceleration will slow down if the acceleration vector points in the opposite direction to the velocity vector; however, if the acceleration remains constant, the object will never come to a permanent halt. As time increases, the magnitude of the velocity will get smaller and smaller. At some time, the velocity will be instantaneously zero. If the acceleration is constant, however, the velocity vector will continue to change at the same rate. An instant after the velocity is zero, the magnitude of the velocity will begin increasing in the same direction as the acceleration. As time increases, the velocity of the object will then increase in the same direction as the acceleration. In other words, if the acceleration truly remains constant, the object will slow down, stop *for an instant*, reverse direction and then speed up.

---

11. ***REASONING AND SOLUTION*** Other than being horizontal, the motion of an experimental vehicle that slows down, comes to a momentary halt, reverses direction and then speeds up with a constant acceleration of 9.80 m/s^2, is identical to that of a ball that is thrown straight upward near the surface of the earth, comes to a halt, and falls back down. In both cases, the acceleration is constant in magnitude and direction and the objects begin their motion with the acceleration and velocity vectors pointing in opposite directions.

---

12. ***REASONING AND SOLUTION*** The first ball has an initial speed of zero, since it is dropped from rest. It picks up speed on the way down, striking the ground at a speed v_f. The second ball has a motion that is the reverse of that of the first ball. The second ball starts out with a speed v_f and loses speed on the way up. By symmetry, the second ball will come to a halt at the top of the building. Thus, in approaching the crossing point, the second ball travels faster than the first ball. Correspondingly, the second ball must travel farther on its way to the crossing point than the first ball does. Thus, the crossing point must be located in the upper half of the building.

---

13. ***REASONING AND SOLUTION*** Two objects are thrown vertically upward, first one, and then, a bit later, the other. The time required for either ball to reach its maximum height can be found from Equation 2.4: $v = v_0 + at$. At the maximum height, v = 0 m/s; solving for t yields $t = v_0 / a$, where a is the acceleration due to gravity. Clearly, the time required to reach the maximum height depends on the initial speed with which the object was thrown.

Since the second object is launched later, its initial speed must be less than the initial speed of the first object in order that both objects reach their maximum heights at the same instant. The maximum height that each object attains can be found from Equation 2.9: $v^2 = v_0^2 + 2ay$. At the maximum height, v = 0 m/s; solving for y gives $y = v_0^2 / (2a)$, where a is the acceleration due to gravity. Since the second object has a smaller initial speed v_0, it will also attain a smaller maximum height. Thus, it is *not* possible for both objects to reach the same maximum height at the same instant.

---

14. ***REASONING AND SOLUTION*** The magnitude of the muzzle velocity of the bullet can be found (to a very good approximation) by solving Equation 2.9, $v^2 = v_0^2 + 2ax$, with v_0 = 0 m/s; that is

$$v = \sqrt{2ax}$$

where a is the acceleration of the bullet and x is the distance traveled by the bullet before it leaves the barrel of the gun (i.e., the length of the barrel).

Since the muzzle velocity of the rifle with the shorter barrel is greater than the muzzle velocity of the rifle with the longer barrel, the product ax must be greater for the bullet in the rifle with the shorter barrel. But x is smaller for the rifle with the shorter barrel, thus the *acceleration of the bullet must be larger in the rifle with the shorter barrel.*

CHAPTER 3 | *KINEMATICS IN TWO DIMENSIONS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** In addition to the x and y axes, the z axis would be required to completely describe motion in three dimensions. Motion along the z axis can be described in terms of the kinematic variables z, a_z, v_z, v_{0z}, and t. In analogy with the equations of kinematics for the x and y components, the following equations would be necessary to describe motion along the z axis:

$$v_z = v_{0z} + a_z t \qquad z = v_{0z} t + \tfrac{1}{2} a_z t^2$$

$$z = \tfrac{1}{2}(v_{0z} + v_z)t \qquad v_z^2 = v_{0z}^2 + 2a_z z$$

---

2. ***REASONING AND SOLUTION*** An object thrown upward at an angle q will follow the trajectory shown below. Its acceleration is that due to gravity, and, therefore, always points downward. The acceleration is denoted by a_y in the figure. In general, the velocity of the object has two components, v_x and v_y. Since $a_x = 0$, v_x always equals its initial value. The y component of the velocity, v_y, decreases as the object rises, drops to zero when the object is at its highest point, and then increases in magnitude as the object falls downward.

a. Since $v_y = 0$ when the object is at its highest point, the velocity of the object points only in the x direction. As suggested in the figure below, the acceleration will be perpendicular to the velocity when the object is at its highest point and $v_y = 0$.

b. In order for the velocity and acceleration to be parallel, the x component of the velocity would have to drop to zero. However, v_x always remains equal to its initial value; therefore, the velocity and the acceleration can never be parallel.

---

3. ***REASONING AND SOLUTION*** As long as air resistance is negligible, the acceleration of a projectile is constant and equal to the acceleration due to gravity. The acceleration of the projectile, therefore, is the same at every point in its trajectory, and can *never* be zero.

---

4. ***REASONING AND SOLUTION*** If a baseball were pitched on the moon, it would still fall downwards as it travels toward the batter. However the acceleration due to gravity on the moon is roughly 6 times less than that on earth. Thus, in the time it takes to reach the batter, the ball will not fall as far vertically on the moon as it does on earth. Therefore, the pitcher's mound on the moon would be at a lower height than it is on earth.

---

5. ***REASONING AND SOLUTION*** The figure below shows the ball's trajectory. The velocity of the ball (along with its x and y components) are indicated at three positions. As long as air resistance is neglected, we know that $a_x = 0$ and a_y is the acceleration due to gravity. Since $a_x = 0$, the x component of the velocity remains the same and is given by v_{0x}. The initial y component of the velocity is v_{0y} and decreases as the ball approaches the highest point, where $v_y = 0$. The magnitude of the y component of the velocity then increases as the ball falls downward. Just before the ball strikes the ground, the y velocity component is equal in magnitude to v_{0y}. The speed is the magnitude of the velocity.

a. Since $v_y = 0$ when the ball is at the highest point in the trajectory, the speed is a minimum there.

b. Similarly, since v_y is a maximum at the initial and final positions of the motion, the speed is a maximum at these positions.

---

6. ***REASONING AND SOLUTION*** If there were a wind blowing parallel to the ground and toward the kicker in Figure 3.12, the acceleration component in the horizontal direction would not be zero. The horizontal component of the velocity would decrease as time goes on. The flight time of the ball, however, depends only on the vertical component of the initial velocity. Thus, the flight time of the ball would not be affected by the wind. The horizontal range of the football will be shortened, but the flight time will remain the same.

---

7. ***REASONING AND SOLUTION*** The wrench will hit at the same place on the deck of the ship regardless of whether the sailboat is at rest or moving with a constant velocity. If the sailboat is at rest, the wrench will fall straight down hitting the deck at some point *P*. If the sailboat is moving with a constant velocity, the motion of the wrench will be two dimensional. However, the horizontal component of the velocity of the wrench will be the same as the velocity of the sailboat. Therefore, the wrench will always remain above the same point *P* as it is falling. This situation is analogous to the falling care package which is described in Example 2 of the text.

---

8. ***REASONING AND SOLUTION*** The two bullets differ only in their horizontal motion. One bullet has $v_x = 0$, while the other bullet has $v_x = v_{0x}$. The time of flight, however, is determined only by the vertical motion, and both bullets have the same initial vertical velocity component ($v_{0y} = 0$). Both bullets, therefore, reach the ground at the same time.

---

9. ***REASONING AND SOLUTION*** Since the launch speed of projectile A is twice that of B, it follows that

$$(v_{0x})_{\mathrm{A}} = 2(v_{0x})_{\mathrm{B}} \quad \text{and} \quad (v_{0y})_{\mathrm{A}} = 2(v_{0y})_{\mathrm{B}}$$

As seen from the result of Example 5, the maximum height attained by either projectile is directly proportional to the square of v_{0y}^2; therefore, the ratio of the maximum heights is $H_{\mathrm{A}} / H_{\mathrm{B}} = (2)^2 = 4$. Example 7 in the text shows that the range of either projectile is directly proportional to the product of v_{0x} and t. Example 6 shows that t is proportional to v_{0y}; thus,

$$\frac{R_{\mathrm{A}}}{R_{\mathrm{B}}} = \frac{(v_{0x})_{\mathrm{A}}\, t_{\mathrm{A}}}{(v_{0x})_{\mathrm{B}}\, t_{\mathrm{B}}} = \frac{(v_{0x})_{\mathrm{A}}\,(v_{0y})_{\mathrm{A}}}{(v_{0x})_{\mathrm{B}}\,(v_{0y})_{\mathrm{B}}} = \frac{\left[2(v_{0x})_{\mathrm{B}}\right]\left[2(v_{0y})_{\mathrm{B}}\right]}{(v_{0x})_{\mathrm{B}}\,(v_{0y})_{\mathrm{B}}} = 4$$

---

10. ***REASONING AND SOLUTION***

a. The displacement is greater for the stone that is thrown horizontally, because it has the same vertical component as the dropped stone and, in addition, has a horizontal component.

b. The impact speed is greater for the stone that is thrown horizontally. The reason is that it has the same vertical velocity component as the dropped stone but, in addition, also has a horizontal component that equals the throwing velocity.

c. The time of flight is the same in each case, because the vertical part of the motion for each stone is the same. That is, each stone has an initial vertical velocity component of zero and falls through the same height.

---

11. ***REASONING AND SOLUTION*** A leopard springs upward at a 45° angle and then falls back to the ground. If we neglect air resistance, the leopard will follow the usual parabolic trajectory of projectile motion. If we let v_0 denote the initial speed of the leopard, then, at any time *t*, the *x* and *y* components of the velocity of the leopard are given by

$$v_x = v_0 \cos 45° = \tfrac{\sqrt{2}}{2} v_0$$

$$v_y = v_0 \sin 45° + a_y t = \tfrac{\sqrt{2}}{2} v_0 + a_y t$$

The speed at any time *t* is then given by

$$v = \sqrt{v_x^2 + v_y^2}$$

As the leopard rises, its speed decreases reaching a minimum at the highest point in the trajectory. Then, as the leopard falls back to the ground, its speed increases. To determine the minimum speed of the leopard, we note that when the leopard is at the highest point in its trajectory, $v_y = 0$. Therefore, the minimum speed of the leopard is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{\left(\tfrac{\sqrt{2}}{2} v_0\right)^2 + 0} = 0.707 v_0$$

In other words, as the leopard moves upward, its speed decreases from its initial value of v_0 to its minimum value of $0.707\, v_0$ at the leopard's highest point. We can conclude, therefore, that the leopard never has a speed that is one-half of its initial value.

---

12. ***REASONING AND SOLUTION*** A football quarterback throws a pass on the run and then keeps running without changing his velocity. If the quarterback throws the ball *straight upward*, and keeps running without changing his velocity, the quarterback can catch the ball. The initial horizontal velocity component of the ball will be equal to the (horizontal) velocity of the quarterback. If air resistance is neglected, the ball is not accelerated in the

horizontal direction; therefore, the ball retains its initial horizontal velocity component. As a result, the ball remains directly above the quarterback at all times during the flight. Thus, the ball will fall into the hands of the quarterback. This situation is analogous to the bullet in Conceptual Example 4, which remains directly above the gun barrel during its flight.

---

13. ***REASONING AND SOLUTION*** Since the plastic bottle moves with the current, the passenger is estimating the velocity of the boat relative to the water. Therefore, the passenger cannot conclude that the boat is moving at 5 m/s with respect to the shore.

---

14. ***REASONING AND SOLUTION*** The plane flies at the same speed with respect to the ground during the entire flight. Therefore, even though the earth is rotating, the speed of the plane relative to the earth is identical in both cases, the distances are the same, and, therefore, the two flight times are identical.

---

15. ***REASONING AND SOLUTION***

a. Since the child is at rest with respect to the floor of the RV and with respect to the cannon, there is no relative motion between the child, the floor of the RV and the cannon. Therefore, the range of the marble toward the front is the same as the range toward the rear from the point of view of the child.

b. The velocity of the marble relative to an observer standing still on the ground is $\mathbf{v}_{\mathrm{MG}} = \mathbf{v}_{\mathrm{MV}} + \mathbf{v}_{\mathrm{VG}}$, where $\mathbf{v}_{\mathrm{MV}}$ is the velocity of the marble relative to the vehicle and $\mathbf{v}_{\mathrm{VG}}$ is the velocity of the vehicle relative to the ground. Suppose that the RV moves in the positive direction; then the quantity $\mathbf{v}_{\mathrm{VG}}$ is positive. When the marble is shot toward the front of the RV, the x component of $\mathbf{v}_{\mathrm{MV}}$ will also be a positive quantity. Alternatively, when the marble is shot toward the back of the RV, the x component of $\mathbf{v}_{\mathrm{MV}}$ is a negative quantity. Thus, the magnitude of the x component of $\mathbf{v}_{\mathrm{MG}}$ is smaller when the marble is shot toward the rear than when it is shot toward the front. The time that the marble is in the air is determined by the vertical motion which is the same in both cases. Since the magnitude of the x component of $\mathbf{v}_{\mathrm{MG}}$ is smaller when the marble is shot toward the rear, it will cover less distance in the time it takes to hit the floor of the RV compared to when the marble is shot toward the front. Therefore, from the point of view of an observer on the ground, the range of the marble is greater when the marble is shot toward the front.

---

16. ***REASONING AND SOLUTION*** The time required for any given swimmer to cross the river is equal to the width of the river divided by the magnitude of the component of the velocity that is parallel to the width of the river. All three swimmers can swim equally fast relative to the water; however, all three swim at different angles relative to the current. Since swimmer A heads straight across the width of the river, swimmer A will have the largest velocity component parallel to the width of the river; therefore, swimmer A crosses the river in the least time.

CHAPTER 4 | *FORCES AND NEWTON'S LAWS OF MOTION*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** When the car comes to a sudden halt, the upper part of the body continues forward (as predicted by Newton's first law) if the force exerted by the lower back muscles is not great enough to give the upper body the same deceleration as the car. The lower portion of the body is held in place by the force of friction exerted by the car seat and the floor.

   When the car rapidly accelerates, the upper part of the body tries to remain at a constant velocity (again as predicted by Newton's first law). If the force provided by the lower back muscles is not great enough to give the upper body the same acceleration as the car, the upper body appears to be pressed backward against the seat as the car moves forward.

---

2. ***REASONING AND SOLUTION*** When the birdfeeder is hanging freely and no one is pulling on the dangling (lower) cord, there is a tension in the cord between the birdfeeder and the tree limb (the upper cord), because the upper cord supports the weight of the birdfeeder. When the lower cord is pulled down with a slow continuous pull, the tension in both cords increases slowly. Since the upper cord has a larger tension to begin with, it always has the greater tension as the lower cord is pulled. Thus, the upper cord snaps first.

   On the other hand, when the child gives the lower cord a sudden, downward pull, the tension in the lower cord increases suddenly. However, the tension in the upper cord does not increase as suddenly. The reason is that the birdfeeder has a large mass, so it accelerates very slowly. Thus, the upper cord is stretched slowly and, consequently, the tension in the upper cord rises slowly. Since the tension rises much faster in the lower cord, it is the first to snap.

---

3. ***REASONING AND SOLUTION*** If the net external force acting on an object is zero, it is possible for the object to be traveling with a nonzero velocity. According to Newton's second law, $\mathbf{F} = m\mathbf{a}$, if the net external force $\mathbf{F}$ is zero, the acceleration $\mathbf{a}$ is also zero. If the acceleration is zero, the velocity must be constant, both in magnitude and in direction. Thus, an object can move with a constant nonzero velocity when the net external force is zero.

---

4. ***REASONING AND SOLUTION*** According to Newton's second law, a net force is required to give an object a non-zero acceleration.

   a. If an object is moving with a constant acceleration of 9.80 m/s^2, we can conclude that there is a net force on the object.

b. If an object moves with a constant velocity of 9.80 m/s, its acceleration is zero; therefore, we can conclude that the net force acting on the object is zero.

---

5. ***REASONING AND SOLUTION*** An object will not necessarily accelerate when two or more forces are applied to the object simultaneously. The applied forces may cancel so the net force is zero; in such a case, the object will not accelerate. The resultant of all the forces that act on the object must be nonzero in order for the object to accelerate.

---

6. ***REASONING AND SOLUTION***
Since the father and the daughter are standing on ice skates, there is virtually no friction between their bodies and the ground. We can assume, therefore, that the only horizontal force that acts on the daughter is due to the father, and similarly, the only horizontal force that acts on the father is due to the daughter.

a. According to Newton's third law, when they push off against each other, the force exerted on the father by the daughter must be equal in magnitude and opposite in direction to the force exerted on the daughter by the father. In other words, both the father and the daughter experience pushing forces of equal magnitude.

b. According to Newton's second law, $\mathbf{F} = m\mathbf{a}$. Therefore, $\mathbf{a} = \mathbf{F}/m$. The magnitude of the net force on the father is the same as the magnitude of the net force on the daughter, so we can conclude that, since the daughter has the smaller mass, she will acquire the larger acceleration.

---

7. ***REASONING AND SOLUTION***

a. The force of the gymnast on the trampoline causes the elastic surface of the trampoline to deform.

b. The reaction force exerted by the trampoline on the gymnast causes the gymnast to decelerate and come to a momentary stop.

---

8. ***REASONING AND SOLUTION*** The two oppositely directed pushing forces predicted by Newton's third law act on different bodies; one force acts on the crate, while the other acts on the person who is pushing on the crate. Since the two forces act on different objects, they cannot cancel each other. Whether or not the crate moves depends on the net force that acts on the crate. If the crate does not move under the action of a single pushing force, the only reasonable conclusion is that there must be another force *acting on the crate* that cancels the pushing force. The other force is the force of static friction.

---

9. ***REASONING AND SOLUTION*** The magnitude of the gravitational force between any two of the particles is given by Newton's law of universal gravitation: $F = Gm_1 m_2 / r^2$ where m_1 and m_2 are the masses of the particles and r is the distance between them. Since the particles have equal masses, we can arrange the particles so that each one experiences a net gravitational force that has the same magnitude if we arrange the particles so that the distance between any two of the particles is the same. Therefore, the particles should be placed at the corners of an equilateral triangle with all three sides of equal length.

---

10. ***REASONING AND SOLUTION*** The mass of an object is a quantitative measure of its inertia. The mass of an object is an intrinsic property of the object and is independent of the location of the object. The weight of an object is the gravitational force exerted on the object by the earth. The gravitational force depends on the distance between the object and the center of the earth. Therefore, when an object is moved from sea level to the top of a mountain, its weight will change, while the mass of the object remains constant.

---

11. ***REASONING AND SOLUTION*** The weight of the ball always acts downward. The force of air resistance will always act in the direction that is opposite to the direction of motion of the ball. The net force on the ball is the resultant of the weight and the force of air resistance.

a. As the ball moves upward, the force of air resistance acts downward. Since air resistance and the weight of the ball act in the same direction in this case, the net force on the ball will be greater in magnitude than the weight of the ball.

b. As the ball falls downward, the force of air resistance is upward. Since air resistance and the weight of the ball act in opposite directions, the net force that acts on the ball will be smaller in magnitude than the weight of the ball.

Note that in both cases, the net force points downward since the object accelerates downward in both cases.

---

12. ***REASONING AND SOLUTION*** From Equation 4.5, $W = mg$, we know that the weight of an object is directly proportional to its mass. The proportionality constant on a given planet is g, the magnitude of the acceleration due to gravity on that planet.

If object A weighs twice as much as object B at the same spot on the earth, then the mass of object A is twice as much as the mass of object B. The mass of any object is independent of the object's location in the universe. Since the weight of an object is directly proportional to its mass, object A will weigh twice as much as object B at the same spot on any planet. While the values of the weights will differ from planet to planet, as the value of g varies from planet to planet, the ratio of the weights (which equals the ratio of their masses) will be the same on any planet.

---

13. ***REASONING AND SOLUTION*** Equations 4.4 and 4.5 may be combined to give the acceleration due to gravity at a distance r from the center of the earth: $g = GM_E / r^2$. Since this expression depends on r, the acceleration of a freely falling object does depend on its location. The acceleration due to gravity will be greater at Death Valley, California (where r is smaller) than it is on the top of Mt. Everest (where r is greater). Since r is measured from the center of the earth, however, these values will be very close.

---

14. ***REASONING AND SOLUTION*** Assuming that the accelerating mechanism remains attached to the rocket, the acceleration will be greater when the rocket is fired horizontally. The accelerating mechanism provides an acceleration that points in the initial direction of motion of the rocket. The net acceleration is the resultant of the accelerating mechanism and the acceleration due to gravity. When the rocket is fired horizontally, these accelerations will be at right angles to each other. When the rocket is fired straight up, these accelerations will be in opposite directions. The magnitude of the resultant will be greater when these two accelerations are at right angles rather than when they are in opposition. Therefore, the acceleration will be greater when the rocket is fired horizontally.

If we assume that the accelerating mechanism is not attached to the rocket, then once the rocket is fired, the only force on the rocket is that due to gravity, and the rocket has the acceleration due to gravity regardless of its orientation. In this case, the acceleration of the rocket is the same regardless of whether it is fired straight up or fired horizontally.

---

15. ***REASONING AND SOLUTION*** If the elevator were at rest, or moving with a constant velocity, the scale would read the true weight of $mg = 98$ N. When the elevator is accelerating, the scale reading will differ from 98 N and will display the apparent weight, F_N, which is given by Equation 4.6: $F_N = mg + ma$ where a, the acceleration of the elevator, is positive when the elevator accelerates upward and negative when the elevator accelerates downward.

a. When the apparent weight is $F_N = 75$ N, the apparent weight is less than the true weight ($mg = 98$ N) so a must be negative. The elevator is accelerating downward.

b. When the apparent weight is $F_N = 120$ N, the apparent weight is greater than the true weight ($mg = 98$ N) so a must be positive. The elevator is accelerating upward.

---

16. ***REASONING AND SOLUTION*** The apparent weight will differ from the true weight only in an accelerating elevator. When the scale in an elevator reads the true weight, the only conclusion that can be made is that the elevator has zero acceleration. Therefore, one cannot conclude whether the elevator is moving with any constant velocity upward, any constant velocity downward, or whether the elevator is at rest, since each of these conditions involves zero acceleration.

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17. ***REASONING AND SOLUTION*** If the elevator were at rest, or moving with a constant velocity, the scale will read the true weight mg. When the elevator is accelerating, the scale reading will differ from the true weight and will register the apparent weight, F_N, which is given by Equation 4.6: $F_N = mg + ma$ where a, the acceleration of the elevator, is positive when the elevator accelerates upward and negative when the elevator accelerates downward.

Since the scale registers 600 N when the elevator is moving with constant velocity, we know that the true weight is 600 N.

a. The elevator is moving upward. When the elevator slows down, its acceleration vector points downward. The term ma will be negative; therefore, the scale reading will be less than 600 N.

b. When the elevator is stopped, the scale will register the true weight; therefore, the scale reads 600 N.

c. The elevator is moving downward and speeding up. The acceleration vector points downward, and the term ma is negative. Therefore, the scale registers a value that is less than 600 N.

---

18. ***REASONING AND SOLUTION*** Since the sled moves with constant velocity, the force of kinetic friction is present. The magnitude of this force is given by $\mu_k F_N$, where μ_k is the coefficient of kinetic friction and F_N is the magnitude of the normal force that acts on the sled. Furthermore, the horizontal component of the applied force must be equal in magnitude to the force of kinetic friction, since there is no acceleration.

When the person pulls on the sled, the vertical component of the pulling force tends to decrease the magnitude of the normal force relative to that when the sled is not being pulled or pushed. On the other hand, when the person pushes on the sled, the vertical component of the pushing force tends to increase the normal force relative to that when the sled is not being pulled or pushed. Therefore, when the sled is pulled, the magnitude of the force of kinetic friction, and therefore the magnitude of the applied force, is less than when the sled is pushed.

---

19. ***REASONING AND SOLUTION*** We know that $\mu_s = 2.0\,\mu_k$ for a crate in contact with a cement floor. The maximum force of static friction is $f_s^{MAX} = \mu_s F_N$ while the force of kinetic friction is $f_k = \mu_k F_N$. As long as the crate is on the cement floor, we can conclude that the magnitude of the maximum static frictional force acting on the crate will always be twice the magnitude of the kinetic frictional force on the moving crate, once the crate has begun moving. However, the force of static friction may not have its maximum value. Thus, the magnitude of the static frictional force is not always twice the magnitude of the kinetic frictional force.

---

20. ***REASONING AND SOLUTION*** A box rests on the floor of a stationary elevator. Because of static friction, a force is required to start the box sliding across the floor of the elevator. The magnitude of this force is given by $f_s^{MAX} = m_s F_N$, where F_N is the magnitude of the normal force exerted on the box by the floor of the elevator. When the elevator is stationary, the magnitude of the normal force exerted on the box is *mg* where *m* is the mass of the crate and *g* is the magnitude of the acceleration due to gravity. When the crate accelerates upward, the floor of the elevator will push against the box to accelerate it upward; therefore, when the crate accelerates upward, the magnitude of the normal force will be greater than *mg*. When the elevator accelerates downward, the normal force between the box and the floor of the elevator will be less than when the elevator is stationary; therefore, the magnitude of the normal force will be less than *mg*. Thus, the magnitude of the force required to start the box sliding across the floor of the elevator are ranked as follows in ascending order: (c) elevator accelerating downward; (a) elevator stationary; (b) elevator accelerating upward.

---

21. ***REASONING AND SOLUTION*** When the rope is tied to a tree and pulled by the ten people, the tension in the rope is twice as great as it was when it was used in a tug-of-war with five people on each team. Therefore, the rope is more likely to break when it is tied to the tree and pulled by ten people.

    Note that when the rope is tied to the tree and pulled by ten people, the situation is equivalent to a tug-of-war with *ten* people on each team.

---

22. ***REASONING AND SOLUTION*** An object is in equilibrium when its acceleration is zero. When a stone is thrown from the top of a cliff, its acceleration is the acceleration due to gravity; therefore, the stone is not in equilibrium.

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23. ***REASONING AND SOLUTION*** An object is in equilibrium when its acceleration is zero.

    a. If a single nonzero force acts on an object, the object will accelerate according to Newton's second law. The object is not in equilibrium.

    b. If two forces that point in mutually perpendicular directions act on an object, the object will experience a net force. By Newton's second law, the object will, therefore, have a nonzero acceleration. The object is not in equilibrium.

    c. If two forces that point in directions that are not perpendicular act on the object, the object may or may not be in equilibrium, depending on how the forces are oriented. In general, the resultant of two such forces is nonzero, the object will accelerate, and it is not in equilibrium. In the special case where the two forces point in opposite directions and have the same magnitude, the net force is zero, the object has zero acceleration, and the object is, therefore, in equilibrium.

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24. ***REASONING AND SOLUTION*** A circus performer hangs from a stationary rope. Since there is no acceleration, the tension in the rope must be equal in magnitude to the weight of the performer. She then begins to climb upward by pulling herself up, hand-over-hand. Whether the tension in the rope is greater than or equal to the tension when she hangs stationary depends on whether or not she accelerates as she moves upward. When she moves upward at constant velocity, the tension in the rope will be the same. When she accelerates upward, the rope must support the net upward force in addition to her weight; therefore, in this case, the tension in the rope will be greater than when she hangs stationary.

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25. ***REASONING AND SOLUTION*** If a sky diver with an open parachute approaches the ground with a constant velocity, the acceleration of the sky diver is zero, and the sky diver is, therefore, in equilibrium. The two forces responsible for the equilibrium are the weight of the sky diver and the force exerted on the sky diver by the parachute. These forces must be equal in magnitude and opposite in direction so that their resultant is zero.

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26. ***REASONING AND SOLUTION*** There are three forces that act on the ring as shown in the figure below. The weight of the block, which acts downward, and two forces of tension that act along the rope away from the ring. Since the ring is at rest, the net force on the ring is zero. The weight of the block is balanced by the vertical components of the tension in the rope. Clearly, the rope can never be made horizontal, for then there would be no vertical components of the tension forces to balance the weight of the block.

---

27. ***REASONING AND SOLUTION*** A freight train is accelerating on a level track. The tension in the coupling between the engine and the first car depends on the total mass being pulled by the engine. Therefore, other things being equal, if some of the cargo in the last car were transferred to any one of the other cars, the tension in the coupling between the engine and the first car would remain the same. This is because the transfer does not change the total mass being pulled by the engine.

CHAPTER 5 | *DYNAMICS OF UNIFORM CIRCULAR MOTION*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The car will accelerate if its velocity changes in magnitude, in direction, or both. If a car is traveling at a constant speed of 35 m/s, it can be accelerating if its direction of motion is changing.

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2. ***REASONING AND SOLUTION*** Consider two people, one on the earth's surface at the equator, and the other at the north pole. If we combine Equations 5.1 and 5.2, we see that the centripetal acceleration of an object moving in a circle of radius r with period T can be written as $a_c = (4p^2 r)/T^2$. The earth rotates about an axis that passes approximately through the north pole and is perpendicular to the plane of the equator. Since both people are moving on the earth's surface, they have the same period T. The person at the equator moves in a larger circle so that r is larger for the person at the equator. Therefore, the person at the equator has a larger centripetal acceleration than the person at the north pole.

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3. ***REASONING AND SOLUTION*** The equations of kinematics (Equations 3.3 - 3.6) cannot be applied to uniform circular motion because an object in uniform circular motion does not have a constant acceleration. While the acceleration vector is constant in magnitude $\left(a = v^2/r\right)$, its direction changes constantly -- it always points toward the center of the circle. As the object moves around the circle the direction of the acceleration must constantly change. Because of this changing direction, the condition of constant acceleration that is required by Equations 3.3 – 3.6 is violated.

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4. ***REASONING AND SOLUTION*** Acceleration is the rate of change of velocity. In order to have an acceleration, the velocity vector must change either in magnitude or direction, or both. Therefore, if the velocity of the object is constant, the acceleration must be zero. On the other hand, if the speed of the object is constant, the object could be accelerating if the direction of the velocity is changing.

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5. ***REASONING AND SOLUTION*** When the car is moving at constant speed along the straight segments (i.e., *AB* and *DE*) , the acceleration is zero. Along the curved segments, the magnitude of the acceleration is given by v^2/r. Since the speed of the car is constant, the magnitude of the acceleration is largest where the radius r is smallest. Ranked from smallest to largest the magnitudes of the accelerations in each of the four sections are: *AB* or *DE, CD, BC*.

---

6. ***REASONING AND SOLUTION*** From Example 7, the maximum safe speed with which a car can round an unbanked horizontal curve of radius r is given by $v = \sqrt{m_s gr}$. Since the acceleration due to gravity on the moon is roughly one sixth that on earth, the safe speed for the same curve on the moon would be less than that on earth. In other words, other things being equal, it would be more difficult to drive at high speed around an unbanked curve on the moon as compared to driving around the same curve on the earth.

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7. ***REASONING AND SOLUTION*** A bug lands on a windshield wiper. The wipers are turned on. Since the wipers move along the arc of a circle, the bug will experience a centripetal acceleration, and hence, a centripetal force must be present. The magnitude of the centripetal force is given by $F_c = mv^2 / r$. In order for the bug to remain at rest on the wiper blade, the force of static friction between the bug and the wiper blade must contribute in a major way to the centripetal force. Without the centripetal force, the bug will be dislodged. When the wipers are turned on at a higher setting, v is larger, and the centripetal force required to keep the bug moving along the arc of the circle is larger than if the wipers are turned on the low setting. Since the high setting requires a larger centripetal force to keep the bug on the wiper, the bug is more likely to be dislodged at that setting than at the low setting.

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8. ***REASONING AND SOLUTION*** From Example 7, the maximum safe speed with which a car can round an unbanked curve of radius r is given by $v = \sqrt{m_s gr}$. This expression is independent of the mass (and therefore the weight) of the car. Thus, the chance of a light car safely rounding an unbanked curve on an icy road is the same as that for a heavier car (assuming that all other factors are the same).

---

9. ***REASONING AND SOLUTION*** Since the speed and radius of the circle are constant, the centripetal acceleration is constant. As the water leaks out, however, the mass of the object undergoing the uniform circular motion decreases. Centripetal force is mass times the centripetal acceleration, so that the centripetal force applied to the container must be decreasing. It is the tension in the rope that provides the centripetal force. You are holding the free end of the rope and pulling on it in order to create the tension. Therefore, you must be reducing your pull as the water leaks out. In turn, according to Newton's third law, the rope must be pulling back on your hand with a force of decreasing magnitude, and you feel this pull weakening as time passes.

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10. ***REASONING AND SOLUTION*** As the propeller rotates faster, the centripetal acceleration of the parts of the propeller increase. As the centripetal acceleration increases, the centripetal force required to cause the various parts of the propeller to rotate in the circle also increases. When the necessary centripetal force exceeds the mechanical forces that hold the propeller together, the propeller will come apart.

---

11. ***REASONING AND SOLUTION*** The centripetal force on the penny is given by $F_c = mv^2 / r$, where $v = 2pr / T$ and T is the constant period of the turntable. Therefore, the centripetal force on the penny is given by $F_c = 4p^2 mr / T^2$. Clearly, the penny will require the largest centripetal force to remain in place when located at the largest value of r; that is, at the edge of the turntable.

---

12. ***REASONING AND SOLUTION*** A model airplane on a guideline can fly in a circle because the tension in the guideline provides the horizontal centripetal force necessary to pull the plane into a horizontal circle. A real airplane has no such horizontal forces. The air on the wings on a real plane exerts an upward lifting force that is perpendicular to the wings. The plane must bank so that a component of the lifting force can be oriented horizontally, thereby providing the required centripetal force to cause the plane to fly in a circle.

---

13. ***REASONING AND SOLUTION***

a. Referring to Figure 5.10 in the text, we can see that the centripetal force on the plane is $L \sin q = mv^2 / r$, where L is the magnitude of the lifting force. In addition, the vertical component of the lifting force must balance the weight of the plane, so that $L \cos q = mg$. Dividing these two equations reveals that $\tan q = v^2 / (rg)$.

b. The banking condition for a car traveling at speed v around a curve of radius r, banked at angle q is $\tan q = v^2 / (rg)$, according to Equation 5.4 in the text.

c. The speed v of a satellite in a circular orbit of radius r about the earth is given by $v = \sqrt{GM_E / r}$, according to Equation 5.5 in the text.

d. The minimum speed required for a loop-the-loop trick around a loop of radius r is $v = \sqrt{rg}$, according to the discussion in Section 5.7 of the text. According to Equations 4.4 and 4.5, $g = GM_E / r^2$. Thus, any expression that depends on g also depends on M_E and would be affected by a change in the earth's mass. Such is the case for each of the four situations discussed above.

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14. ***REASONING AND SOLUTION*** When the string is whirled in a horizontal circle, the tension in the string, F_T, provides the centripetal force which causes the stone to move in a circle. Since the speed of the stone is constant, $mv^2 / r = F_T$ and the tension in the string is constant.

When the string is whirled in a vertical circle, the tension in the string and the weight of the stone both contribute to the centripetal force, depending on where the stone is on the circle. Now, however, the tension increases and decreases as the stone traverses the vertical circle. When the stone is at the lowest point in its swing, the tension in the string pulls the stone upward, while the weight of the stone acts downward. Therefore, the centripetal force

is $mv^2 / r = F_T \quad mg$. Solving for the tension shows that $F_T = mv^2 / r + mg$. This tension is larger than in the horizontal case. Therefore, the string has a greater chance of breaking when the stone is whirled in a vertical circle.

---

15. ***REASONING AND SOLUTION*** A fighter pilot pulls out of a dive on a vertical circle and begins to climb upward. As the pilot moves along the circle, all parts of his body, including the blood in his head, must experience a centripetal force in order to remain on the circle. The blood, however, is not rigidly attached to the body and does not experience the requisite centripetal force until it flows out of the head, away from the circle's center, and collects in the lower body parts, which ultimately push on it enough to keep it on the circular path.

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16. ***REASONING AND SOLUTION*** When a car moves around an unbanked horizontal curve, the centripetal force that keeps the car on the road so that it can negotiate the curve comes from the force of static friction. If car A cannot negotiate the curve, then the force of static friction between the treads of car A's tires and the road is not great enough to provide the centripetal force. The coefficient of static friction between the tires and the road are less for car A than for car B, since car B can negotiate the turn.

CHAPTER 6 | *WORK AND ENERGY*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The work done by $\mathbf{F}_1$ in moving the box through a displacement **s** is $W_1 = (F_1 \cos 0°)s = F_1 s$. The work done by $\mathbf{F}_2$ is $W_2 = (F_2 \cos q)s$. From the drawing, we see that $F_1 > F_2 \cos q$; therefore, the force $\mathbf{F}_1$ does more work.

---

2. ***REASONING AND SOLUTION*** The force **P** acts along the displacement; therefore, it does positive work. Both the normal force $\mathbf{F}_N$ and the weight $m\mathbf{g}$ are perpendicular to the displacement; therefore, they do zero work. The kinetic frictional force $\mathbf{f}_k$ acts opposite to the direction of the displacement; therefore, it does negative work.

---

3. ***REASONING AND SOLUTION*** Work can be positive or negative. Work is positive when the force has a component in the same direction as the direction of the displacement, or equivalently, when the angle q between the force **F** and the displacement **s** is less than 90°. The work is negative when the force has a component in the direction *opposite* to the displacement; that is, when q is greater than 90°.

   Since the force does positive work on a particle that has a displacement pointing in the $+x$ direction, the force must have an x component that points in the $+x$ direction. Furthermore, since the same force does negative work on a particle that has a displacement pointing in the $+y$ direction, the force has a y component that points in the *negative* y direction.

   When the x and y components of this force are sketched head to tail, as in the figure at the right, we see that the force must point in the fourth quadrant.

---

4. ***REASONING AND SOLUTION*** The sailboat moves at constant velocity, and, therefore, has zero acceleration. From Newton's second law, we know that the net external force on the sailboat must be zero.

   a. There is no work done on the sailboat by a zero net external force.

   b. Work is done by the individual forces that act on the boat; namely the wind that propels the boat forward and the water that resists the motion of the boat. Since the wind propels the boat forward, it does positive work on the boat. Since the force of the water is a resistive force, it acts opposite to the displacement of the boat, and, therefore, it does

negative work. Since the total work done on the boat is zero, each force must do an equal amount of work with one quantity being positive, and the other being negative.

Note: The answer to part (a) could have been deduced from the work-energy theorem as well. Since the velocity of the boat is constant, the kinetic energy of the boat does not change and the total work done on the boat is zero.

---

5. ***REASONING AND SOLUTION*** The speed of the ball decreases; therefore, the ball is subjected to an external resistive force. A resistive force always points opposite to the direction of the displacement of the ball. Therefore, the external force does negative work.

Using the work-energy theorem, we see that the change in the kinetic energy of the ball is negative; therefore, the total work done on the ball is negative. We can conclude, therefore, that the net force did negative work on the ball.

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6. ***REASONING AND SOLUTION*** The kinetic energy of an object of mass m moving with speed v is given by $\text{KE} = \frac{1}{2}mv^2$. The kinetic energy depends on both the mass and the speed of the object. The mass of an automobile is significantly greater than the mass of a motorcycle. Therefore, even if an automobile is moving slowly, it is possible that the product $\frac{1}{2}mv^2$ is greater for the car than it is for the faster-moving motorcycle.

---

7. ***REASONING AND SOLUTION*** A net external force acts on a particle. This net force is not zero. From Newton's second law, we can conclude that the net external force causes the particle to accelerate. Since the particle experiences an acceleration, its velocity must change. The change in velocity, however, may occur as a change in magnitude only, a change in direction only, or a change in both magnitude and direction.

a. This is sufficient information to conclude that the velocity of the particle changes; however, there is not sufficient information to determine exactly how the velocity changes.

b. There is not sufficient information to determine if the kinetic energy of the particle changes. In terms of the work-energy theorem, the kinetic energy will change if the net external force does work on the particle. But without knowing the direction of the net external force with respect to the particle's displacement, we cannot know if work is done.

c. There is not sufficient information to determine if the speed of the particle changes. Kinetic energy is $\frac{1}{2}mv^2$, so the speed v will change if the kinetic energy changes. But, as explained in part (b), there is insufficient information to determine whether the kinetic energy changes.

---

8. ***REASONING AND SOLUTION*** The speed of a particle doubles and then doubles again, because a net external force acts on it. Let m and v_0 represent the mass and initial speed of the particle, respectively. During the first doubling, the change in the kinetic energy of the particle is

$$\mathrm{KE_f} \quad \mathrm{KE_0} = \tfrac{1}{2}m\left(2v_0\right)^2 \quad \tfrac{1}{2}mv_0^2 = \tfrac{3}{2}mv_0^2$$

During the second doubling, the change in the kinetic energy of the particle is

$$\mathrm{KE_f} \quad \mathrm{KE_0} = \tfrac{1}{2}m\left(4v_0\right)^2 \quad \tfrac{1}{2}m\left(2v_0\right)^2 = 6mv_0^2$$

From the work-energy theorem, we know that a change in kinetic energy is equal to the work done by the net external force. Therefore, more work is done by the net force during the second doubling.

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9. ***REASONING AND SOLUTION*** A shopping bag hangs straight down from the hand of a person who walks across a horizontal floor at constant velocity.

a. The force exerted on the shopping bag by the hand is perpendicular to the direction of motion; hence, the work done by the hand is zero.

b. If the person then rides up an escalator at constant velocity, a component of the force exerted by the hand will point in the direction of the displacement and do a positive amount of work.

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10. ***REASONING AND SOLUTION*** The change in the gravitational potential energy of the astronaut is given by PE = mgH, where H is the height of the ladder. The value of g is greater on earth than it is on the moon; therefore, the gravitational potential energy of the astronaut changes by a greater amount on earth.

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11. ***REASONING AND SOLUTION*** A net external nonconservative force does positive work on a particle, and both its kinetic and potential energies change.

a. The change in the total mechanical energy of the particle is given by $E = \mathrm{KE} + \mathrm{PE}$. From Equation 6.7b, $W_{\mathrm{nc}} = \mathrm{KE} + \mathrm{PE}$; in other words, the work done on the particle by the net external nonconservative force is equal to the change in the particle's total mechanical energy. Since the force does positive work on the particle, we can conclude from Equation 6.7b that the particle's total mechanical energy must increase.

b. We do not have sufficient information to reach any conclusions on the individual changes in the kinetic and potential energies. Either KE or PE could be positive or negative,

while the sum $KE + PE$ is positive. If either the change in the kinetic energy or the change in the potential energy is negative, the only conclusion that can be reached is that the magnitude of the negative quantity must be smaller than the magnitude of the positive quantity.

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12. ***REASONING AND SOLUTION*** If the total mechanical energy of an object is conserved, then the sum of the kinetic energy and the potential energy must be constant.

    a. If the kinetic energy decreases, the gravitational potential energy must increase by the same amount that the kinetic energy decreases.

    b. If the potential energy decreases, the kinetic energy must increase by the same amount that the potential energy decreases.

    c. If the kinetic energy does not change, the potential energy cannot change either.

---

13. ***REASONING AND SOLUTION*** Yes, the speed of the Magnum XL-200 would be 0 m/s at the top of the second hill. As the roller coaster descends the first hill, its total mechanical energy remains constant at all times. Its gravitational potential energy at the top of the hill is converted entirely into kinetic energy at the bottom. When the roller coaster ascends the second hill, its total mechanical energy also remains constant, and is equal to what it had during the descent of the first hill. As the roller coaster moves up the hill, kinetic energy is converted into gravitational potential energy. By the time it reaches the top of the second hill, all of its kinetic energy has been converted into gravitational potential energy, and so its speed is 0 m/s at the top.

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14. ***REASONING AND SOLUTION*** As the person moves downward from the top of the Ferris wheel, his displacement points downward. Since the person's weight also points downward, the work done by gravity is positive. As the person moves upward from the bottom, his displacement points upward. Since the weight still points downward, the work done by gravity is negative. The magnitude of the work done in each half cycle is the same; therefore, the net work done in one revolution is zero.

---

15. ***REASONING AND SOLUTION*** Car A turns off its engine and coasts up the hill. Car B keeps its engine running and drives up the hill at constant speed. If air resistance and friction are negligible, then only the motion of car A is an example of the principle of conservation of mechanical energy. During the motion of car A, there are no nonconservative forces present. As car A coasts up the hill it loses kinetic energy and gains an equal amount of gravitational potential energy. The motion of car B is not described by the principle of conservation of mechanical energy, because the driving force that propels car B up the hill is a nonconservative force.

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16. ***REASONING AND SOLUTION*** A trapeze artist, starting from rest, swings downward on the bar, lets go at the bottom of the swing, and falls freely to the net. An assistant, standing on the same platform as the trapeze artist, jumps from rest straight downward.

a. The work done by gravity, on either person, is $W = mgh$, where m is the mass of the person, and h is the magnitude of the vertical component of the person's displacement. The value of h is the same for both the trapeze artist and the assistant; however, the value of m is, in general, different for the trapeze artist and the assistant. Therefore, gravity does the most work on the more massive person.

b. Both the trapeze artist and the assistant strike the net with *the same speed.* Here's why. They both start out at rest on the platform above the net. As the trapeze artist swings downward, before letting go of the bar, she moves along the arc of a circle. The work done by the tension in the trapeze cords is zero because it points perpendicular to the circular path of motion. Thus, if air resistance is ignored, the work done by the nonconservative forces is zero, $W_{\text{nc}} = 0$ J. The total mechanical energy of each person is conserved, regardless of the path taken to the net. Since both the trapeze artist and the assistant had the same initial total mechanical energy (all potential), they must have the same total mechanical energy (all kinetic) when they reach the net. That is, for either person, $mgh = (1/2)mv^2$. Solving for v gives $v = \sqrt{2gh}$, independent of the mass of the person. The value of h is the same for both the trapeze artist and the assistant; therefore, they strike the net with the same speed.

---

17. ***REASONING AND SOLUTION*** Since each plane has the same speed, the kinetic energy of each fuel tank will be the same at the instant of release. Since each plane is at the same height above the ground, each fuel tank must fall through the same vertical displacement. Therefore, the work done by gravity on each fuel tank is the same. From the work-energy theorem, each fuel tank will gain the same amount of kinetic energy during the fall. Therefore, each fuel tank will hit the ground with the same speed.

---

18. ***REASONING AND SOLUTION*** Average power $\overline{P}$ is the average rate at which work W is done: $\overline{P} = W/t$. It is not correct to conclude that one engine is doing twice the work of another just because it is generating twice the power. If one engine generates twice the power as the other, it may be generating the same amount of work in one half of the time.

CHAPTER 7 | *IMPULSE AND MOMENTUM*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The linear momentum **p** of an object is the product of its mass and its velocity. Since the automobiles are identical, they have the same mass; however, although the automobiles have the same speed, they have different velocities. One automobile is traveling east, while the other one is traveling west. Therefore, the automobiles do not have the same momentum. Note that both momenta have the same magnitude, however, one car has a momentum that points east, while the other car has a momentum that points west.

---

2. ***REASONING AND SOLUTION*** Since linear momentum is a vector quantity, the total linear momentum of any system is the resultant of the linear momenta of the constituents. The people who are standing around have zero momentum. Those who move randomly carry momentum randomly in all directions. Since there is such a large number of people, there is, on average, just as much linear momentum in any one direction as in any other. On average, the resultant of this random distribution is zero. Therefore, the approximate linear momentum of the Times Square system is zero.

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3. ***REASONING AND SOLUTION***
   a. Yes. Momentum is a vector, and the two objects have the same momentum. This means that the direction of each object's momentum is the same. Momentum is mass times velocity, and the direction of the momentum is the same as the direction of the velocity. Thus, the velocity directions must be the same.

   b. No. Momentum is mass times velocity. The fact that the objects have the same momentum means that the product of the mass and the magnitude of the velocity is the same for each. Thus, the magnitude of the velocity of one object can be smaller, for example, as long as the mass of that object is proportionally greater to keep the product of mass and velocity unchanged.

---

4. ***REASONING AND SOLUTION***
   a. If a single object has kinetic energy, it must have a velocity; therefore, it must have linear momentum as well.

   b. In a system of two or more objects, the individual objects could have linear momenta that cancel each other. In this case, the linear momentum of the system would be zero. The kinetic energies of the objects, however, are scalar quantities that are always positive; thus, the total kinetic energy of the system of objects would necessarily be nonzero. Therefore, it is possible for a system of two or more objects to have a total kinetic energy that is not zero but a total momentum that is zero.

---

5. ***REASONING AND SOLUTION*** The impulse-momentum theorem, Equation 7.4, states that $\overline{\mathbf{F}}\ t = m\mathbf{v}_f \quad m\mathbf{v}_0$.

a. If an airplane is flying horizontally with a constant momentum during a time t, then from Equation 7.4, $\overline{\mathbf{F}}\ t = 0$. There is no net impulse $\overline{\mathbf{F}}\ t$ on the plane during this time interval.

b. The fact that the net impulse on the plane is zero indicates that the impulses of the two horizontal forces must cancel each other. That is, the impulse of the thrust is equal in magnitude and opposite in direction to the impulse of the resistive force.

---

6. ***REASONING AND SOLUTION*** The severity of the collision is determined by the amount of momentum transferred by the colliding object. If the child is moving twice as fast as the adult, and the mass of the child and the bicycle is one half that of the adult, the magnitude of the linear momenta (mass x speed) of the child and the adult are the same. Therefore, it does not matter whether one is struck by the fast-moving child or the slow-moving adult.

---

7. ***REASONING AND SOLUTION*** An object slides along the surface of the earth and slows down because of kinetic friction.

a. If the object and the earth are considered to be part of the system, then the force of kinetic friction arises between components of the system. Therefore, the force of kinetic friction is an internal force, not an external force.

b. A force can only change the linear momentum of a system if the force is an external force. Since the force of friction is an internal force, it cannot change the total linear momentum of the two-body system. In fact, the total linear momentum of the two-body system remains the same. The linear momentum lost by the object is equal to the linear momentum gained by the earth.

---

8. ***REASONING AND SOLUTION*** The impulse-momentum theorem, Equation 7.4, states that $\overline{\mathbf{F}}\ t = m\mathbf{v}_f \quad m\mathbf{v}_0$. Assuming that the golf ball is at rest when it is struck with the club, $\overline{\mathbf{F}}\ t = m\mathbf{v}_f$.

During a good "follow-through" when driving a golf ball, the club is in contact with the ball for the longest possible time. From the impulse-momentum theorem, it is clear that when the contact time t is a maximum, the final linear momentum $m\mathbf{v}_f$ of the ball is a maximum. In other words, during a good "follow through" the maximum amount of momentum is transferred to the ball. Therefore, the ball will travel through a larger horizontal distance.

---

9. ***REASONING AND SOLUTION***

a. Since the water leaves each nozzle with a speed that is greater than the speed inside the arm, the quantity $m\mathbf{v}_f \quad m\mathbf{v}_0$ is positive. From the impulse-momentum theorem, $\overline{\mathbf{F}} \quad t = m\mathbf{v}_f \quad m\mathbf{v}_0$, and we can deduce that there is a net positive or outward impulse. Therefore, a net outward force is exerted on the water.

b. From Newton's third law, the water must exert a net force that is equal in magnitude, but negative and directed toward the nozzle. The nozzle and the arm, in turn, move. Since each arm is free to rotate about the vertical axis, the arm will whirl.

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10. ***REASONING AND SOLUTION*** To throw the villain forward, Superman must exert a force on him. For a system comprised of Superman and the villain, this force is an internal force. Internal forces cannot change the total linear momentum of the two-person system. Therefore, when Superman throws the villain forward, the villain gains forward momentum, and Superman must gain an equal amount of momentum in the backward direction. Therefore, Superman could not remain stationary after throwing the villain forward.

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11. ***REASONING AND SOLUTION*** Since the satellite explodes in outer space far from any other body, it may be considered to be an isolated system. The forces of the explosion are forces that are internal to the system. Therefore, after the explosion, the total linear momentum of all the pieces must be equal to the linear momentum of the satellite before it exploded.

---

12. ***REASONING AND SOLUTION*** Let the system be defined by the jetliner (and all the other passengers, luggage, etc.) and the walking passenger. When the passenger walks toward the front of the plane, the linear momentum of this passenger increases. Any forces exerted by this passenger on the jetliner are internal forces and cannot change the total momentum of the system. Therefore, from momentum conservation, the forward momentum of the jetliner will decrease by the same amount as the momentum gained by the passenger. This amount is small compared to the forward momentum of the jetliner and goes unnoticed.

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13. ***REASONING AND SOLUTION***

a. No. The person overhead jumps straight down and, therefore, applies only a vertical force to the boat. Since friction and air resistance are negligible, no horizontal force is applied to the boat. According to the impulse-momentum theorem, this means that the horizontal momentum of the boat cannot change.

b. The speed of the boat decreases. Momentum is mass times velocity. The only way for the horizontal momentum of the boat to remain unchanged when the mass increases due to the presence of the jumper is for the magnitude of the boat's velocity (that is, the speed) to decrease.

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14. ***REASONING AND SOLUTION*** Let the system be comprised of the asteroid, the catapult and the supply of stones. When the catapult is used to "throw" chunks of stones into space, the force exerted by the catapult is an internal force; therefore, the total momentum of the system must remain the same. When rocks are thrown in one direction, they carry linear momentum in that direction. From the conservation of momentum, the asteroid must carry an equal amount of momentum in the opposite direction. It will, therefore, move in that direction. Such a device could be used as a propulsion system to move the asteroid closer to earth.

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15. ***REASONING AND SOLUTION*** For a system comprised of the three balls, there is no net external force. The forces that occur when the three balls collide are internal forces. Therefore, the total linear momentum of the system is conserved. Note, however, that the momentum of each ball is not conserved. The momentum of any given ball will change as it interacts with the other balls; the momentum of each ball will change in such a way as to conserve the momentum of the system. It is the momentum of the system of balls, not the momentum of an individual ball, that is conserved.

---

16. ***REASONING AND SOLUTION*** An elastic collision is one in which the total kinetic energy of the system after the collision is equal to the total kinetic energy before the collision. The kinetic energy of the individual objects in the system will, in general, change during a collision. They will change so that the total kinetic energy of the system remains the same before and after the collision. Even in an elastic collision, however, the kinetic energy of each object is not necessarily the same before and after the collision.

---

17. ***REASONING AND SOLUTION*** Example 7 concerns the elastic collision between two balls: one initially moving (ball 1), and one initially at rest (ball 2). The results of that example show that the final speeds of the balls are given by

$$v_{\mathrm{f1}} = \left(\frac{m_1 \quad m_2}{m_1 + m_2} \right) v_{01} \tag{7.8a}$$

and

$$v_{\mathrm{f2}} = \left(\frac{2m_1}{m_1 + m_2} \right) v_{01} \tag{7.8b}$$

If the two objects have equal mass, $m_1 = m_2$, and Equation 7.8a indicates that $v_{\mathrm{f1}} = 0$, and Equation 7.8b indicates that $v_{\mathrm{f2}} = v_{01}$. In other words, the first object is stopped completely and the second object takes off with the velocity the first object originally had.

---

18. ***REASONING AND SOLUTION*** Many objects have a point, a line, or a plane of symmetry. If the mass of the system is uniformly distributed, the center of mass of such an object lies at that point, on that line, or in that plane. The point of symmetry of a doughnut is at the geometric center of the hole. Thus, the center of mass of a doughnut is at the center of the hole.

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19. ***REASONING AND SOLUTION*** Since more of the mass of the bat is located near the heavier end of the bat, the center of mass of the bat will be located nearer the heavier end.

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20. ***REASONING AND SOLUTION*** A sunbather is lying on a floating, stationary raft. She then gets up and walks to one end of the raft. The sunbather and the raft are considered as an isolated system.

a. As the sunbather walks to one end of the raft, she exerts a force on the raft; however, the force is internal to the isolated system. Since there are no external forces acting on the system, the linear momentum of the system cannot change. Since the linear momentum of the system is initially zero, it must remain zero. Therefore, the velocity of the center of mass of the system must be zero.

b. The sunbather has linear momentum as she walks to one end of the raft. Since the linear momentum of the isolated system must remain zero, the raft must acquire a linear momentum that is equal in magnitude and opposite in direction to that of the sunbather. From the definition of linear momentum, $\mathbf{p} = m\mathbf{v}$, we know that the direction of the linear momentum of an object is the same as the direction of the velocity of the object. Thus, the raft acquires a velocity that is opposite to the direction of motion of the sunbather.

CHAPTER 8 | *ROTATIONAL KINEMATICS*

CONCEPTUAL QUESTIONS

1. ***REASONING AND SOLUTION*** The figures below show two axes in the plane of the paper and located so that the points B and C move in circular paths having the same radii (radius = r).

2. ***REASONING AND SOLUTION*** When a pair of scissors is used to cut a string, each blade of the scissors does not have the same angular velocity at a given instant during the cut. The angular speed of each blade is the same; however, each blade rotates in the opposite direction. Therefore, it is correct to conclude that the blades have opposite angular velocities at any instant during the cut.

3. ***REASONING AND SOLUTION*** Just before the clock is unplugged, the second hand is rotating so that its angular velocity is clockwise. The second hand moves with a constant angular velocity, so the angular acceleration is zero. When the plug is pulled, the second hand will continue to rotate with its clockwise angular velocity, but it will slow down. Therefore, the angular acceleration must be opposite to the angular velocity, or counterclockwise.

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4. ***REASONING AND SOLUTION*** The tangential speed, v_T, of a point on the earth's surface is related to the earth's angular speed w according to $v_T = rw$, Equation 8.9, where r is the perpendicular distance from the point to the earth's rotation axis. At the equator, r is equal to the earth's radius. As one moves away from the equator toward the north or south geographic pole, the distance r becomes smaller. Since the earth's rotation axis passes through the geographic poles, r is effectively zero at those locations. Therefore, your tangential speed would be a minimum if you stood as close as possible to either the north or south geographic pole.

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5. ***REASONING AND SOLUTION***

a. A thin rod rotates at a constant angular speed about an axis of rotation that is perpendicular to the rod at its center. As the rod rotates, each point at a distance r from the center on one half of the rod has the same tangential speed as the point at a distance r from the center on the other half of the rod. This is true for all values of r for $0 < r \quad (L/2)$ where L is the length of the rod.

b. If the rod rotates about an axis that is perpendicular to the rod at one end, no two points are the same distance from the axis of rotation. Therefore, no two points on the rod have the same tangential speed.

---

6. ***REASONING AND SOLUTION*** The wheels are rotating with a constant angular velocity.

a. Since the angular velocity is constant, each wheel has zero angular acceleration, = 0 rad/s. Since the tangential acceleration a_T is related to the angular acceleration through Equation 8.10, $a_T = ra$, every point on the rim has zero tangential acceleration.

b. Since the particles on the rim of the wheels are moving along a circular path, they must have a centripetal acceleration. This can be supported by Equation 8.11, $a_c = rw^2$, where a_c is the magnitude of the centripetal acceleration. Since w is nonzero, a_c is nonzero.

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7. ***REASONING AND SOLUTION*** The wheel and the two points under consideration are shown in the figure at the right.

a. Each point must undergo the same angular displacement in the same time interval. Therefore, at any given instant, the angular velocity of both points is the same.

b. Each point must increase its angular velocity at the same rate; therefore, each point has the same angular acceleration.

c. The tangential speed is given by $v_T = rw$. Since both points have the same angular speed w, but point **1** is further from the center than point **2**, point **1** has the larger tangential speed.

d. The tangential acceleration is given by $a_T = ra$. Since both points have the same angular acceleration a, but point **1** is further from the center than point **2**, point **1** has the larger tangential acceleration.

e. The centripetal acceleration is given by $a_c = rw^2$. Since both points have the same angular speed w, but point **1** is further from the center than point **2**, point **1** has the larger centripetal acceleration.

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8. ***REASONING AND SOLUTION*** For a building that is located on the earth's equator, the points in the building rotate about the center of the earth. The tangential speed of any point is given by $v_T = rw$, where, in this case, r is measured from the center of the earth. The top floor of the building has a larger value of r than the other floors; therefore, the top floor has the greater tangential speed.

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9. ***REASONING AND SOLUTION*** The centripetal acceleration of any point in the space station is given by $a_c = rw^2$ (Equation 8.11),where r is the distance from the point to the axis of rotation of the space station. If the centripetal acceleration is adjusted to be g at a given value of r (such as the astronaut's feet), the centripetal acceleration will be different at other values of r. Therefore, if the adjustment is made so that the centripetal acceleration at the astronaut's feet equals g, then the centripetal acceleration will not equal g at the astronaut's head.

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10. ***REASONING AND SOLUTION*** The tangential speed for any point on the tip of a clock hand is given by $v_T = rw$. The angular speeds of the second hand, the minute hand, and the hour hand differ greatly, with the second hand having the largest angular speed and the hour hand having the smallest. If one desired to create a clock in which the tips of the second hand, the minute hand, and the hour hand moved with the same tangential speed, the lengths

of the arms would also have to differ greatly, with the second hand having the smallest arm and the hour hand having the largest. Such a clock would not be very practical.

---

11. ***REASONING AND SOLUTION*** Any point on a rotating object possesses a centripetal acceleration that is directed radially toward the axis of rotation. This also applies to a tire on a moving car and is true regardless of whether the car has a constant linear velocity or whether it is accelerating.

---

12. ***REASONING AND SOLUTION*** The bicycle wheel has an angular acceleration. The arrows are perpendicular to the radius of the wheel. The magnitude of the arrows increases with increasing distance from the center in accordance with $v_T = rw$, or $a_T = r\alpha$. The arrows in the picture could represent either the tangential velocity or the tangential acceleration. The arrows are not directed radially inward; therefore, they cannot represent the centripetal acceleration.

---

13. ***REASONING AND SOLUTION*** The speedometer of a truck uses a device that measures the angular speed of the tires. The angular speed is related to the linear speed of the truck by $v = rw$.

Suppose two trucks are traveling side-by-side along a highway at the same linear speed v, and one truck has larger wheels than the other. The angular speed of the larger-diameter wheels is less than that of the smaller-diameter wheels. Since the speedometer uses a device that measures the angular speed of the tires, the speedometer reading on the truck with the larger wheels will indicate a smaller linear speed than that on the truck with the smaller wheels.

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14. ***REASONING AND SOLUTION*** When a fan is shut off, the fan blades gradually slow down until they eventually stop. The angular acceleration is never really constant because it gradually decreases, becoming zero at the instant the blades stop rotating. In solving problems, it is sometimes convenient to approximate such motion as having constant angular acceleration. This is a fairly good approximation for a short time interval just after the switch is turned off.

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15. ***REASONING AND SOLUTION*** Three examples that involve rotation about an axis that is not fixed:

(1) The motion of a Frisbee. The axis of the rotating Frisbee moves through the air with the Frisbee.
(2) The motion of the earth in its orbit. The rotation axis of the earth changes position as the earth revolves around the sun.
(3) The motion of a twirling baton that has been thrown into the air. The rotation axis constantly changes location as the baton rises and falls.

CHAPTER 9 | *ROTATIONAL DYNAMICS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The magnitude of the torque produced by a force **F** is given by $t = F\ell$, where ℓ is the lever arm. When a long pipe is slipped over a wrench handle, the length of the wrench handle is effectively increased. The same force can be applied with a larger lever arm. Therefore, a greater torque can be used to remove the "frozen" nut.

---

2. ***REASONING AND SOLUTION*** The magnitude of the torque produced by a force **F** is given by $t = F\ell$ where ℓ is the lever arm.

   a. A large force can be used to produce a small torque by choosing a small lever arm. Then the product $F\ell$ is small even though the magnitude of the force, F, is large. If the line of action passes through the axis of rotation, $\ell = 0$, and the torque is zero regardless of how large F is.

   b. A small force may be used to produce a large torque by choosing a large lever arm ℓ. Then, the product $F\ell$ can be large even though the magnitude of the force F is small.

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3. ***REASONING AND SOLUTION*** In a cassette deck, the magnetic tape applies a torque to the supply reel. The magnitude of the torque is equal to the product of the tension F_T in the tape, assumed to be constant, and the lever arm ℓ. As shown below, the lever arm is the radius of the tape remaining on the supply reel. Figure A shows the reel and tape at an earlier time; figure B shows the tape and the reel at a later time.

Figure A

Figure B

As the reel becomes empty, the lever arm ℓ gets smaller, as suggested in the figures above. Therefore, the torque exerted on the reel by the tape becomes smaller.

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4. ***REASONING AND SOLUTION*** A flat rectangular sheet of plywood is fixed so that it can rotate about an axis perpendicular to the sheet through one corner, as shown in the figure. In order to produce the largest possible torque with the force **F** (acting in the plane of the paper), the force **F** must be applied with the largest possible lever arm. This can be attained by orienting the force **F** so that it is perpendicular to the diagonal that passes through the rotation axis as shown below. The lever arm is equal to the length of the diagonal of the plywood.

---

5. ***REASONING AND SOLUTION*** Work and torque are both the product of force and distance. Work and torque are distinctly different physical quantities, as is evident by considering the distances in the definitions. Work is defined by $W = (F \cos q)s$, according to Equation 6.1, where F is the magnitude of the force, q is the angle between the force and the displacement, and s is the magnitude of the displacement. The magnitude of the torque is defined as the magnitude of the force times the lever arm, according to Equation 9.1. In the definition of work, the "distance" is the magnitude of the displacement over which the force acts. In the definition of torque, the distance is the lever arm, a "static" distance. The lever arm is not the same physical quantity as the displacement. Therefore, work and torque are different quantities.

---

6. ***REASONING AND SOLUTION*** A person stands on a train, both feet together, facing a window. The front of the train is to the person's left. When the train starts to move forward, a force of static friction is applied to the person's feet. This force tends to produce a torque about the center of gravity of the person, causing the person to fall backward. If the right foot is slid out toward the rear of the train, the normal force of the floor on the right foot produces a counter torque about the center of gravity of the person. The resultant of these two torques is zero, and the person can maintain his balance.

---

7. ***REASONING AND SOLUTION*** If we assume that the branch is fairly uniform, then in the spring, just when the fruit begins to grow, the center of gravity of the branch is near the center of the branch. As the fruit grows, the mass at the outer end of the branch increases. Therefore, as the fruit grows, the center of gravity of the fruit-growing branch moves along the branch toward the fruit.

---

8. ***REASONING AND SOLUTION*** For a rigid body in equilibrium, there can be no net torque acting on the object *with respect to any axis*. Consider an axis perpendicular to the plane of the paper at the right end of the rod. The lines of action of both $\mathbf{F}_1$ and $\mathbf{F}_3$ pass through this axis. Therefore, no torque is created by either of these forces. However, the force $\mathbf{F}_2$ does create a torque with respect to this axis. As a result, there is a net torque acting on the object with respect to at least one axis, and the three forces shown in the drawing cannot keep the object in equilibrium.

---

9. ***REASONING AND SOLUTION*** The forces that keep the *right section* of the ladder in equilibrium are shown in the free-body diagram at the right.

They are: the weight of the section, $m\mathbf{g}$, the normal force exerted on the ladder by the floor, $\mathbf{F}_N$, the tension in the crossbar, $\mathbf{F}_T$, and the reaction force due to the left side of the ladder, $\mathbf{F}_R$.

---

10. ***REASONING AND SOLUTION***
Treating the wine rack and the bottle as a rigid body, the two external forces that keep it in equilibrium are its weight, $m\mathbf{g}$, located at the center of gravity, and the normal force, $\mathbf{F}_N$, exerted on the base by the table.

Note that the weight acts at the center of gravity (CG), which must be located exactly above the point where the normal force acts, as is the case in Figure 9.10*c*.

---

11. ***REASONING AND SOLUTION*** The torque required to give the triangular sheet an angular acceleration a is given by the rotational analog of Newton's second law for a rigid body: $t = Ia$. The greatest net torque will be required about the axis for which the moment of inertia I is the greatest. The greatest value for I occurs when the greatest portion of the mass of the sheet is far from the axis. This will be for axis B. Therefore, for axis B, the greatest net external torque is required to bring the rectangular plate up to 10.0 rad/s in 10.0 s, starting from rest.

---

12. ***REASONING AND SOLUTION*** The object has an angular velocity and an angular acceleration due to torques that are present.

a. If additional torques are applied so as to make the net torque suddenly equal to zero, the object will continue to rotate with constant angular velocity. The angular velocity will remain constant at the value that it had at the instant that the net torque became zero.

b. If all the torques are suddenly removed, the effect is the same as when the net torque is zero as in part (a) above. That is, the object will continue to rotate with constant angular velocity. The angular velocity will remain constant at the value that it had at the instant that the torques are removed.

---

13. ***REASONING AND SOLUTION*** The space probe is initially moving with a constant translational velocity and zero angular velocity through outer space.

a. When the two engines are fired, each generates a thrust T in opposite directions; hence, the net force on the space probe is zero. Since the net force on the probe is zero, there is no translational acceleration and the translational velocity of the probe remains the same.

b. The thrust of each engine produces a torque about the center of the space probe. If R is the radius of the probe, the thrust of each engine produces a torque of magnitude TR, each in the same direction, so that the net torque on the probe has magnitude $2TR$. Since there is a net torque on the probe, there will be an angular acceleration. The angular velocity of the probe will increase.

---

14. ***REASONING AND SOLUTION*** When you are lying down with your hands behind your head, your moment of inertia I about your stomach is greater than it is when your hands are on your stomach. Sit-ups are then more difficult because, with a greater I, you must exert a greater torque about your stomach to give your upper torso the same angular acceleration, according to Equation 9.7.

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15. ***REASONING AND SOLUTION*** The translational kinetic energy of a rigid body depends only on the mass and the speed of the body. It does not depend on how the mass is distributed. Therefore, for purposes of computing the body's translational kinetic energy, the mass of a rigid body can be considered as concentrated at its center of mass.

Unlike the translational kinetic energy, the moment of inertia of a body *does depend* on the location and orientation of the axis relative to the particles that make up the rigid body. Therefore, for purposes of computing the body's moment of inertia, the mass of a rigid body cannot be considered as concentrated at its center of mass. The distance of any individual mass particle from the axis is important.

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16. ***REASONING AND SOLUTION*** A thin sheet of plastic is uniform and has the shape of an equilateral triangle, as shown in the figure at the right. The axes A and B are two rotation axes that are perpendicular to the plane of the triangle.

If the triangle rotates about each axis with the same angular speed w, the rotational kinetic energy of the sheet is given by $\text{KE} = \frac{1}{2} Iw^2$. The rotational kinetic energy will be greater if the rotation occurs about the axis for which the triangle has the larger moment of inertia I. The value for I will be greater when more of the mass is located farther away from the axis, as is the case for axis B. Therefore, the triangle has the greater rotational kinetic energy when it rotates about axis B.

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17. ***REASONING AND SOLUTION*** With feet flat on the floor and body straight, you can lean over until your center of gravity is above your toes. If you lean any farther, it is no longer possible to balance the torque due to your weight with a torque created by the upward reaction force that pushing downward with your toes produces. Bob's center of gravity must be closer to the ground. The reason is that, with the center of gravity closer to the ground, a greater amount of leaning can occur before it moves beyond his toes.

---

18. ***REASONING AND SOLUTION*** Let each object have mass M and radius R. The objects are initially at rest. In rolling down the incline, the objects lose gravitational potential energy Mgh, where h is the height of the incline. The objects gain kinetic energy equal to $(1/2)Iw^2 + (1/2)Mv^2$, where I is the moment of inertia about the center of the object, v is the linear speed of the center of mass at the bottom of the incline, and w is the angular speed about the center of mass at the bottom. From conservation of mechanical energy

$$Mgh = \tfrac{1}{2} Iw^2 + \tfrac{1}{2} Mv^2$$

Since the objects roll down the incline, $w = v/R$. For the hoop, $I = MR^2$, hence,

$$Mgh = \tfrac{1}{2}(MR^2)\frac{v^2}{R^2} + \tfrac{1}{2} Mv^2$$

Solving for v gives $v = \sqrt{gh}$ for the hoop. Repeating the calculation for the solid cylinder, the spherical shell, and the solid sphere gives

$v = \sqrt{gh}$	(hoop)
$v = 1.15\sqrt{gh}$	(solid cylinder)
$v = 1.10\sqrt{gh}$	(spherical shell)
$v = 1.20\sqrt{gh}$	(solid sphere)

The solid sphere is faster at the bottom than any of the other objects. The solid cylinder is faster at the bottom than the spherical shell or the hoop. The spherical shell is faster at the bottom than the hoop. Thus, the solid sphere reaches the bottom first, followed by the solid cylinder, the spherical shell, and the hoop, in that order.

---

19. ***REASONING AND SOLUTION*** A woman sits on the spinning seat of a piano stool with her arms folded. Let the woman and the piano stool comprise the system. When the woman extends her arms, the torque that she applies is an internal torque. Since there is no net external torque acting on the system, the angular momentum of the woman and the piano stool, Iw, must remain constant.

    a. With her arms extended, the woman's moment of inertia about the rotation axis, I, is greater than it was when her arms were folded. Since the product of Iw must remain constant and I increases, w must decrease. Therefore, the woman's angular velocity decreases.

    b. As discussed above, since there is no net external torque acting on the system, the angular momentum of the system remains the same.

---

20. ***REASONING AND SOLUTION*** Consider the earth to be an isolated system. Note that the earth rotates about an axis that passes through the North and South poles and is perpendicular to the plane of the equator. If the ice cap at the South Pole melted and the water were uniformly distributed over the earth's oceans, the mass at the South Pole would be uniformly distributed and, on average, be farther from the earth's rotational axis. The moment of inertia of the earth would, therefore, increase. Since the earth is an isolated system, any torques involved in the redistribution of the water would be internal torques; therefore, the angular momentum of the earth must remain the same. If the moment of inertia increases, and the angular momentum is to remain constant, the angular velocity of the earth must decrease.

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21. ***REASONING AND SOLUTION*** Note that the earth rotates about an axis that passes through the North and South poles and is perpendicular to the plane of the equator. When rivers like the Mississippi carry sediment toward the equator, they redistribute the mass from a more uniform distribution to a distribution with more mass concentrated around the equator. This increases the moment of inertia of the earth. If the earth is considered to be an isolated system, then any torques involved in the redistribution of mass are internal torques. Therefore, the angular momentum of the earth must remain constant. The moment of inertia increases, and the angular momentum must remain the same; therefore, the angular velocity must decrease.

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22. ***REASONING AND SOLUTION*** Let the system be the rotating cloud of interstellar gas. The gravitational force that pulls the particles together is internal to the system; hence, the torque resulting from the gravitational force is an internal torque. Since there is no net external torque acting on the collapsing cloud, the angular momentum, Iw, of the cloud must remain constant. Since the cloud is shrinking, its moment of inertia I decreases. Since the product Iw remains constant, the angular velocity w must increase as I decreases. Therefore, the angular velocity of the formed star would be greater than the angular velocity of the original gas cloud.

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23. ***REASONING AND SOLUTION*** A person is hanging motionless from a vertical rope over a swimming pool. She lets go of the rope and drops straight down. Consider the woman to be the system. When she lets go of the rope her angular momentum is zero. It is possible for the woman to curl into a ball; however, since her angular momentum is zero, she cannot change her angular velocity by exerting any internal torques. In order to begin spinning, an external torque must be applied to the woman. Therefore, it is not possible for the woman to curl into a ball and start spinning.

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24. ***REASONING AND SOLUTION*** The box that creates the greatest torque is the one that appears shaded in the drawing at the right.

All boxes give rise to the same force $m\mathbf{g}$. Each box produces a torque about the axle. The box that produces the greatest torque is the one for which the lever arm ℓ is the greatest. By inspection of the drawing, the lever arm relative to the axle is greatest for the shaded box.

CHAPTER 10 | *SIMPLE HARMONIC MOTION AND ELASTICITY*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A horizontal spring is attached to an immovable wall. Two people pull on the spring. They then detach it from the wall and pull on opposite ends of the horizontal spring. They pull just as hard in each case.

   Suppose that each person pulls with a force **P**. When the spring is attached to the wall, both people pull on the same end, so that they pull with a total force 2**P**. The wall exerts a force –2**P** at the other end of the spring. The tension in the spring has magnitude 2*P*. When the two people pull on opposite ends of the horizontal spring, one person exerts a force **P** at one end of the spring, and the other person exerts a force –**P** at the other end of the spring. The tension in the spring has magnitude *P*.

   The tension in the spring is greater when it is attached to the immovable wall; therefore, the spring will stretch more in this case.

---

2. ***REASONING AND SOLUTION*** Identical springs are attached to a box in two different ways, as shown in the drawing in the text. Initially, the springs are unstrained. The box is then pulled to the right and released. The displacement of the box is the same in both cases.

   For the box on the left, each spring is stretched and its displacement is $+\ x$, where the plus sign indicates a displacement to the right. According to Equation 10.2, the restoring force exerted by each spring on the box is $F = \ k(+\ x)$. Thus, the net force due to both springs is $F = \ 2k(+\ x)$.

   For the box on the right, one spring is stretched and the other is compressed. However, the displacement of each spring is still $+\ x$. The net force exerted on the box is $F = \ k(+\ x) \quad k(+\ x) = \ 2k(+\ x)$.

   Thus, both boxes experience the same net force.

---

3. ***REASONING AND SOLUTION*** Figures 10.11 and 10.14 show the velocity and the acceleration, respectively, of the shadow of a ball that undergoes uniform circular motion. The shadow undergoes simple harmonic motion.

   a. The velocity of the shadow is given by Equation 10.7: $v = \ Aw \sin q$. The velocity of the shadow will be zero when $q = 0$ or p rad. From Figure 10.11, we see that the velocity equals zero each time the shadow reaches the right and left endpoints of its motion (that is, when the ball crosses the *x* axis).

   b. The acceleration of the shadow is given by Equation 10.9: $a = \ Aw^2 \cos q$. The acceleration of the shadow will be zero when q is $p/2$ or $3p/2$. From Figure 10.14, we see that the acceleration equals zero each time that the shadow passes through the point $x = 0$ m (that is, when the ball crosses the *y* axis).

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4. ***REASONING AND SOLUTION*** Simple harmonic motion is the oscillatory motion that occurs when a restoring force of the form of Equation 10.2, $F = kx$, acts on an object. The force changes continually as the displacement x changes.

A steel ball is dropped onto a concrete floor. Over and over again, it rebounds to its original height. During the time when the ball is in the air, either falling down or rebounding up, the only force acting on the ball is its weight, which is constant. Thus, the motion of the bouncing ball is not simple harmonic motion.

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5. ***REASONING AND SOLUTION*** The vibrational frequency f of an object in simple harmonic motion is given by (see Example 6): $f = [1/(2p)]\sqrt{k/m}$. The shock absorbers of a car are "springs" that obey this relation. As the number of passengers in a car increases, the total mass supported by any one of the shock absorbers increases. Therefore, the vibrational frequency of the shock absorber, f, must decrease.

---

6. ***REASONING AND SOLUTION*** A block is attached to a horizontal spring and slides back and forth in simple harmonic motion on a frictionless horizontal surface. A second identical block is suddenly attached to the first block when the first block is at one extreme end of the oscillation cycle.

a. Since the attachment is made at one extreme end of the oscillation cycle, where the velocity is zero, the extreme end of the oscillation cycle will remain at the same point; in other words, the amplitude remains the same.

b. The angular frequency of an object of mass m in simple harmonic motion at the end of a spring of force constant k is given by Equation 10.11: $w = \sqrt{k/m}$. Since the mass m is doubled while the force constant k remains the same, the angular frequency decreases by a factor of $\sqrt{2}$. The vibrational frequency f is related to w by $f = w/(2p)$; the vibrational frequency f will also decrease by a factor of $\sqrt{2}$.

c. The maximum speed of oscillation is given by Equation 10.8: $v_{\max} = Aw$. Since the amplitude, A, remains the same and the angular frequency, w, decreases by a factor of $\sqrt{2}$, the maximum speed of oscillation also decreases by a factor of $\sqrt{2}$.

---

7. ***REASONING AND SOLUTION*** The time required for a particle in simple harmonic motion to travel through one complete cycle (the period) is independent of the amplitude of the motion, even though at larger amplitudes the particle travels further. This is possible because, at larger amplitudes, the maximum speed of the particle is greater. Thus, even though the particle must cover larger distances at larger amplitudes, it does so with greater speeds.

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8. ***REASONING AND SOLUTION*** The back-and-forth motion of the saw blade is simple harmonic motion for the following reason. The shadow of the ball in Figure 10.9 has a displacement x that moves in simple harmonic motion (see Equation 10.3). The horizontal back-and-forth motion of the saw blade is identical to the displacement x of the ball's shadow, since the saw blade is driven by a pin mounted on a rotating disk. Therefore, the motion of the saw blade is simple harmonic motion.

---

9. ***REASONING AND SOLUTION*** The elastic potential energy that a spring has by virtue of being stretched or compressed is given by Equation 10.13: $\text{PE}_{\text{elastic}} = (1/2)kx^2$, where x is the amount by which the spring is stretched or compressed relative to its unstrained length. The amount of stretch or compression appears squared, so that the elastic potential energy is positive and independent of the sign of x. Therefore, the amount of elastic potential energy stored in a spring when it is compressed by one centimeter is the same as when it is stretched by the same amount.

---

10. ***REASONING AND SOLUTION*** We can deduce from Equation 10.16 and Example 10 that, for small angles, the period, T, of a simple pendulum is given by $T = 2p\sqrt{L/g}$ where L is the length of the pendulum.

If a grandfather clock is running slowly, then its period is too long, so one needs to decrease the period to make the clock keep the correct time. Since $T = 2p\sqrt{L/g}$, we, therefore, need to decrease the length of the pendulum.

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11. ***REASONING AND SOLUTION*** From Equations 10.5 and 10.11, we can deduce that the period of the simple harmonic motion of an ideal spring is given by $T = 2p\sqrt{m/k}$, where m is the mass at the end of the ideal spring and k is the spring constant. We can deduce from Equations 10.5 and 10.16 that, for small angles, the period, T, of a simple pendulum is given by $T = 2p\sqrt{L/g}$ where L is the length of the pendulum.

In principle, the motion of a simple pendulum and an object on an ideal spring can both be used to provide the period of a clock. However, it is clear from the expressions for the period given above that the period of the mass-spring system depends only on the mass and the spring constant, while the period of the pendulum depends on the acceleration due to gravity. Therefore, a pendulum clock is likely to become more inaccurate when it is carried to the top of a high mountain where the value of g will be smaller than it is at sea level.

---

12. ***REASONING AND SOLUTION*** We can deduce from Equations 10.5 and 10.16 that, for small angles, the period, T, of a simple pendulum is given by $T = 2p\sqrt{L/g}$ where L is the length of the pendulum. This can be solved for the acceleration due to gravity to yield: $g = 4p^2 L/T^2$.

If you were held prisoner in a room and had only a watch and a pair of shoes with shoelaces of known length, you could determine whether this room is on earth or on the moon in the following way: You could use one of the shoelaces and one of the shoes to make a pendulum. You could then set the pendulum into oscillation and use the watch to

measure the period of the pendulum. The acceleration due to gravity could then be calculated from the expression above. If the value is close to 9.80 m/s^2, then it can be concluded that the room is on earth. If the value is close to 1.6 m/s^2, then it can be concluded that the room is on the moon.

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13. ***REASONING AND SOLUTION*** The playground swing may be treated, to a good approximation, as a simple pendulum. The period of a simple pendulum is given by $T = 2p\sqrt{L/g}$. This expression for the period depends only on the length of the pendulum and the acceleration due to gravity; for angles less than 10° the period is independent of the amplitude of the motion. Therefore, if one person is pulled back 4° from the vertical while another person is pulled back 8° from the vertical, they will both have the same period. If they are released simultaneously, they will both come back to the starting points at the same time.

---

14. ***REASONING AND SOLUTION*** A car travels over a road that contains a series of equally spaced bumps. If the distance between successive bumps is d and the car travels at a constant horizontal velocity **v**, then the car will encounter a bump at regular intervals separated by a time $T = d/v$. The frequency at which the car encounters the bumps is $f = 1/T = v/d$. If the oscillation frequency of the suspension system of the car is v/d, then the bumps will "drive" the suspension system into large amplitude oscillations. That is, the bumps will drive the suspension system at resonance.

---

15. ***REASONING AND SOLUTION*** The amount of force F needed to stretch a rod is given by Equation 10.17: $F = Y(DL/L_0)A$, where A is the cross-sectional area of the rod, L_0 is the original length, L is the change in length, and Y is Young's modulus of the material.

Since the cylinders are made of the same material, they have the same Young's modulus. However, the cross-sectional area of the material in the hollow cylinder is smaller than that of the solid cylinder since most of its cross section is empty. When identical forces of magnitude F are applied to the right end of each cylinder, each stretches by an amount $L = (FL_0)/(YA)$. Since the hollow cylinder has a smaller cross-sectional area of material compared to the solid cylinder, the hollow cylinder will stretch the most.

---

16. ***REASONING AND SOLUTION*** Young's modulus for steel is thousands of times greater than that for rubber. This means that, all other factors being equal, the force needed to stretch a steel rod by a given amount must be thousands of times greater than the force needed to stretch a rubber rod by the same amount. It does not mean that steel stretches much more easily than rubber.

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17. ***REASONING AND SOLUTION*** A trash compactor crushes empty aluminum cans, thereby reducing the total volume of the cans by 75%. The value given in Table 10.3 for the bulk modulus of aluminum cannot be used to calculate the change in pressure generated in the trash compactor. The value of the bulk modulus in Table 10.3 pertains to solid

aluminum objects. Most of the 75% reduction in the total volume of the cans arises from collapsing the cans and forcing air out of them.

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18. ***REASONING AND SOLUTION*** Pressure is defined in terms of the magnitude of the force per unit area, where the force is applied perpendicular to the surface.

Both sides of Equation 10.18, $F = S(DX / L_0)A$ can be divided by the area A to give F/A on the left side. This F/A term cannot be called a pressure, such as the pressure that appears in Equation 10.19, because the force F in the "F/A" term is a shear force that is parallel to the surface, not perpendicular to it.

---

19. ***REASONING AND SOLUTION*** When the block rests on the ground, the ground exerts an upward normal force on the block. Regardless of which face the block is resting on, the magnitude of the normal force is equal to the weight of the block. The magnitude of the force per unit area is the stress.

a. Face B has the smallest area; therefore, the stress, F/A, is largest when the block rests on face B.

b. Face C has the largest area; therefore, the stress, F/A, is smallest when the block rests on face C.

CHAPTER 11 | *FLUIDS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A pile of empty aluminum cans has a volume of 1.0 m^3. This volume includes the inner volume of each can which contains air. It also includes the air in the spaces between the cans. Therefore, most of the volume is occupied by air, not by aluminum. The mass of the pile of cans is *not* $r_{Al}V = (2700\ \text{kg/m}^3)(1.0\ \text{m}^3) = 2700\ \text{kg}$, because the volume of aluminum in the pile of cans is not 1.0 m^3.

---

2. ***REASONING AND SOLUTION*** Initially, the net force on the plunger is zero. The pressure in the bottle is equal to atmospheric pressure; therefore, the magnitude of the force per unit area pushing outward on the plunger is equal to the magnitude of the force per unit area that pushes inward on the plunger. After the first pull-and-release cycle, the force per unit area pushing outward on the plunger is less than the force per unit area pushing inward, because there is less pressure inside the bottle. Therefore, a larger external force is required to pull the plunger. On the 15th pull, the force per unit area pushing outward is very close to zero. The net force on the plunger is inward and due mostly to atmospheric pressure. Therefore, on the 15th pull, a much greater force will be required than on previous pulls.

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3. ***REASONING AND SOLUTION*** A person could not balance her entire weight on the pointed end of a single nail, because it would penetrate her skin. According to Equation 11.3, the pressure exerted by the nail is $P = F/A$ where F represents the weight of the person, and A is the area of the tip of the nail. Since the tip of the nail has a very small radius, its area is very small; therefore, the pressure that the nail exerts on the person is large. The reason she can safely lie on a "bed of nails" is that the effective area of the nails is very large if the nails are closely spaced. Thus, the weight of the person F is distributed over all the nails so that the pressure exerted by any one nail is small.

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4. ***REASONING AND SOLUTION*** As you climb a mountain, your ears "pop" because of the change in atmospheric pressure.

   a. As you climb up, the outside pressure becomes lower than the pressure in your inner ear. The outward force per unit area on your eardrum is greater than the inward force per unit area; therefore, your eardrum moves outward.

   b. As you climb down, the outside pressure becomes greater than the pressure in your inner ear. The outward force per unit area on your eardrum is less than the inward force per unit area on your eardrum; therefore, your eardrum moves inward.

---

5. ***REASONING AND SOLUTION*** The bottle of juice is sealed under a partial vacuum. Therefore, when the seal is intact, the button remains depressed, because the pressure inside the bottle is less than the atmospheric pressure outside of the bottle. The force per unit area pushing up on the button from the inside is significantly smaller than the force per unit area pushing down on the outside. When the seal is broken, air rushes inside the bottle. The force pushing up on the button increases and offsets the force pushing down. The natural springiness of the material from which the lid is made can then make the button "pop up."

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6. ***REASONING AND SOLUTION*** Even though the tube is thin, pouring liquid into it increases the pressure uniformly throughout the vessel by an amount rgh, where h is the height of the water in the tube. Apparently, before the tube is full, the outward force exerted on the walls of the container due to the fluid pressure is greater than the container can withstand. Therefore, the container bursts.

---

7. ***REASONING AND SOLUTION*** A closed tank is completely filled with water. A valve is opened at the bottom of the tank and the water begins to flow out. The water at the valve is pushed out by the force that results from the pressure exerted by the column of water above the valve. This force must be larger than the opposing inward force caused by atmospheric pressure at the valve. When these two forces are equal, the water will cease to exit at the valve. Therefore, when the water stops flowing, there will still be a noticeable amount of water in the tank.

   On the other hand, if the tank were open to the atmosphere, rather than closed, the water at the valve would experience an additional outward force due to atmospheric pressure on the top of the water column and the tank would empty completely.

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8. ***REASONING AND SOLUTION*** When you drink through a straw, you draw the air out of the straw, and the external air pressure leads to the unbalanced force that pushes the liquid up into the straw. This action requires the presence of an atmosphere. The moon has no atmosphere, so you could not use a straw to sip a drink on the moon.

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9. ***REASONING AND SOLUTION*** The *gauge pressure* of a fluid enclosed in a container is the amount by which the container pressure *exceeds* atmospheric pressure. The actual pressure of the fluid, taking atmospheric pressure into account, is called the absolute pressure. On earth, at sea level, the atmosphere exerts a pressure of $1.0 \quad 10^5 \text{ Pa}$ on every surface with which it is in contact.

   An exam question asks, "Why does the cork fly out with a loud 'pop' when a bottle of champagne is opened?" To answer this question, a student replies, "Because the gas pressure in the bottle is about $0.3 \quad 10^5 \text{ Pa}$." The cork flies out because the absolute pressure within the bottle exceeds the atmospheric pressure on the outside. Assuming that the student recognizes this fact, the student must be referring to a gauge pressure of $0.3 \quad 10^5 \text{ Pa}$. Then, the absolute pressure within the bottle would be $1.0 \quad 10^5 \text{ Pa}$ plus $0.3 \quad 10^5 \text{ Pa}$ and would indeed exceed atmospheric pressure.

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10. ***REASONING AND SOLUTION*** A scuba diver is below the surface of the water when a storm approaches, dropping the air pressure above the water. According to Equation 11.4, the absolute pressure at the location of the scuba diver is $P_{\text{diver}} = P_0 + rgh$ where P_0 is atmospheric pressure, r is the density of the water, and h is the depth of the diver below the surface. Since the storm causes the atmospheric pressure to drop, the absolute water pressure at the location of the diver will also drop. If the diver were wearing a sensitive gauge designed to measure the *absolute pressure*, it would register the drop in pressure.

---

11. ***REASONING AND SOLUTION*** According to Archimedes' principle, the water will apply a buoyant force to the steel beam; the magnitude of the buoyant force will be equal to the weight of the water displaced by the beam. Therefore, the buoyant force experienced by the beam is given by $(r_{\text{water}} V_{\text{beam}})g$ where r_{water} is the density of the water and V_{beam} is the volume of the beam. If we neglect any change in water density with depth, then, the quantity $r_{\text{water}} V_{\text{beam}}$ has the same value regardless of whether the beam is suspended vertically or horizontally under the water. Therefore, both the vertically suspended and the horizontally suspended beam experience the same buoyant force.

---

12. ***REASONING AND SOLUTION*** Archimedes' principle states that any fluid applies a buoyant force to an object that is partially or completely immersed in it; the magnitude of the buoyant force equals the weight of the fluid that the object displaces.

A glass beaker, filled to the brim with water, is resting on a scale. A block is placed in the water, causing some of it to spill over. The water that spills is wiped away, and the beaker is still filled to the brim.

a. According to Table 11.1, the density of wood is 550 kg/m^3, while the density of water at 4 °C is $1.000 \quad 10^3$ kg/m^3. Since the density of the wood is less than the density of water, the block will float with part of its volume above the surface of the water. According to Archimedes' principle, the buoyant force exerted on the bottom surface of the block must be equal in magnitude to the weight of water that is displaced by the block. However, since the block floats on the surface of the water, the buoyant force must be equal in magnitude and opposite in direction to the weight of the block, as suggested in the free body diagram at the right. Hence, the weight of the block *must be equal to* the weight of the water displaced by the block. The weight of the contents of the beaker remains the same. Therefore, the initial and final scale readings are the same if the block is made of wood.

b. According to Table 11.1, the density of iron is 7860 kg/m^3, while the density of water at 4 °C is $1.000 \quad 10^3$ kg/m^3. Since the density of iron is greater than the density of water, the iron block will sink to the bottom of the beaker. Let V represent the volume of water that spills over. According to the definition of mass density (Equation 11.1) the mass of water

that spills over is equal to $m_{\text{water}} = r_{\text{water}}V$. The beaker is still filled to the brim, but the block now occupies the volume V where the water used to be. According to Equation 11.1, the mass that now occupies the volume V is given by $m_{\text{block}} = r_{\text{iron}}V$. Since $r_{\text{iron}} > r_{\text{water}}$, the mass of the block is greater than the mass of the water that spilled over. The weight of the contents of the beaker increases. Therefore, the final reading on the scale will be greater than the initial reading.

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13. ***REASONING AND SOLUTION*** According to Archimedes' principle, any fluid applies a buoyant force to an object that is partially or completely immersed in it; the magnitude of the buoyant force equals the weight of the fluid that the object displaces. Therefore, the magnitude of the buoyant force exerted on an object immersed in water is given by $F_{\text{B}} = r_{\text{water}}Vg$, where r_{water} is the density of water, V is the volume displaced by the immersed object, and g is the magnitude of the acceleration due to gravity. If the acceleration due to gravity on a distant planet is less than it is on earth, then, other factors remaining the same, the buoyant force will be less on the planet than it is on earth. However, the weight of the object will also be less than the weight of the object on earth.

When an object floats in water, the upward buoyant force exerted by the water must be equal in magnitude and opposite in direction to the weight of the object, as shown at the right. Hence, $F_{\text{B}} = m_{\text{object}}g$. It follows that $r_{\text{water}}Vg = m_{\text{object}}g$. Notice that the acceleration due to gravity, g, appears on both sides of this equation. Algebraically canceling the g's we have $r_{\text{water}}V = m_{\text{object}}$. Therefore, the object will float so that it displaces a volume of water V, where $V = m_{\text{object}} / r_{\text{water}}$. This result is independent of g. It is the same on earth as it is on the distant planet. Therefore, it would be no more difficult to float in water on this planet than it would be on earth.

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14. ***REASONING AND SOLUTION*** An ice cube is placed in a glass, and the glass is filled to the brim with water. When the ice cube melts, and the temperature of the water returns to its initial value, the water level remains the same.

According to Table 11.1, the density of ice is less than that of liquid water; therefore, the ice will float in the glass with some of the ice above the level of the water. It will displace a volume of water V_{water} that is less than the volume of the ice cube, V_{ice}. According to Archimedes' principle, the water will apply a buoyant force to the ice cube; the magnitude of this force will be equal to the weight of the water that is displaced by the ice cube. Since the ice cube floats, the buoyant force must be equal in magnitude and opposite in direction to the weight of the ice cube. That is, $m_{\text{ice}}g = m_{\text{water}}g$; thus, the mass of the ice cube must be equal to the mass of the water that is displaced by the ice cube. When the ice cube melts, and the water has returned to its initial temperature, the mass of the water that originated from the ice cube occupies the volume V_{water}. In other words, the water that originated from the ice cube will occupy a volume that is exactly equal to the volume that was displaced by the ice cube. Therefore, the water level will remain the same.

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15. ***REASONING AND SOLUTION*** As a person dives toward the bottom of a swimming pool, the pressure increases noticeably. The buoyant force, however, remains the same. According to Archimedes' principle, the water applies a buoyant force to the swimmer, the magnitude of which is equal to the weight of the water displaced by the swimmer. In other words, the buoyant force experienced by the swimmer is given by $(\rho_{\text{water}} V_{\text{swimmer}})g$, where ρ_{water} is the density of the water and V_{swimmer} is the volume of water displaced by the swimmer. If we neglect any change in water density with depth, then the quantity $\rho_{\text{water}} V_{\text{swimmer}}$ remains the same as the swimmer approaches the bottom of the pool.

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16. ***REASONING AND SOLUTION*** According to Archimedes' principle, the magnitude of the buoyant force exerted on an object immersed in water is equal to the weight of the water displaced by the object.

As discussed in Section 5.6 of the text, objects in orbiting satellites are in uniform circular motion and "fall" with the same acceleration toward the center of the orbit. The apparent weight of an object in an orbiting satellite is, therefore, zero. Thus, the apparent weight of the water displaced by the swimmer is zero and the swimmer will not experience a buoyant force. This assumes, as stated, that there is no artificial gravity.

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17. ***REASONING AND SOLUTION*** When the gate is not in use, the hollow, triangular-shaped gate is filled with seawater and rests on the ocean floor. As shown in the figure, the gate is anchored to the ocean floor by a hinge mechanism, about which it can rotate. When a dangerously high tide is expected, water is pumped out of the gate, and the structure rotates into a position where it can protect against flooding.

When the gate is filled with sea water, its average density (including the gate material) is greater than that of seawater, and the gate sinks to the ocean floor. When water is pumped from the gate, the density of the "gate system" decreases. Therefore, the mass, and hence the weight, of the "gate system" decreases. When the buoyant force exerted on the bottom of the "gate system" by the sea water exceeds the weight of the "gate system," the gate begins to rise. Since the left end of the gate is anchored to the hinge mechanism, the gate will rotate about the hinge as its right end rises.

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18. ***REASONING AND SOLUTION*** In steady flow, the velocity **v** of a fluid particle at any point is constant in time. On the other hand, a fluid accelerates when it moves into a region of smaller cross-sectional area, as shown in the figure below.

a. A fluid particle at X with speed v_X must be accelerated to the right in order to acquire the greater speed v_Y at Y. From Newton's second law, this acceleration can arise only from a net force that acts in the direction XY. If there are no other external forces acting on the fluid, this force must arise from the change in pressure within the fluid. The pressure at X must be greater than the pressure at Y.

b. The definition of steady flow makes no reference as to how the velocity of a fluid particle varies from point to point as the fluid flows. It simply states that the velocity of a fluid particle *at any particular point* is constant in time. Therefore, the condition of steady flow does not rule out the acceleration discussed in part (a).

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19. ***REASONING AND SOLUTION*** Consider a stream of water that falls from a faucet. According to the equation of continuity, the mass flow rate rAv must be the same at every point along the stream (r = water density, A = cross-sectional area of the stream, v = speed of stream). As the water falls, it is accelerated due to gravity; therefore, the speed of the water increases as it falls. Since the density of water is uniform throughout the stream, when v increases, the cross-sectional area A of the stream must decrease in order to maintain a constant mass flow rate. Therefore, the cross-sectional area of the stream becomes smaller as the water falls.

If the water is shot upward, as it is in a fountain, the velocity of the stream is upward, while the acceleration due to gravity is directed downward. Therefore, the speed of the stream decreases as the stream rises. Since v decreases, the cross-sectional area A must increase in order to maintain a constant mass flow rate rAv. Therefore, the cross-sectional area of the stream becomes larger as the water is shot upward.

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20. ***REASONING AND SOLUTION*** Suppose you are driving your car alongside a moving truck. You will notice that your car will be pulled toward the truck as it passes.

Consider the air at two points. One point O is on the open side of your car, that is, the side that is not adjacent to the truck. The other point B is between the car and the truck. Bernoulli's principle, as applied to a constant elevation (Equation 11.12), indicates that where the air speed is smaller, the air pressure is greater. Since the car is pulled toward the truck, the pressure at point O must be greater than at point B. Therefore, the air at point O is not moving very fast, while the air at point B is rushing more rapidly around the curved surfaces of the front of the vehicles and through the narrow space between them.

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21. ***REASONING AND SOLUTION*** Bernoulli's equation applies to the steady flow of a nonviscous, incompressible fluid. Turbulent flow is an extreme type of unsteady flow. It occurs when there are sharp obstacles or bends in the path of a fast-moving fluid. Bernoulli's equation does not apply to turbulent flow. When water cascades down a rock-strewn spillway, the water exhibits turbulent flow. The velocity at any particular point changes erratically from one instant to another. Therefore, Bernoulli's equation could not be used to describe water flowing down the rock-strewn spillway.

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22. ***REASONING AND SOLUTION*** Two sheets of paper are held by adjacent corners, so that they hang downward. They are oriented so that they are parallel and slightly separated by a gap through which the floor can be seen. When air is blown strongly down through the gap (see the drawing at the right), the sheets come closer together.

In accord with Bernoulli's equation, the moving air between the sheets has a lower pressure than the stationary air on either side of the sheets. The greater pressure on either side of the sheets generates an inward force on the outer surface of the sheets, and the sheets are pushed together.

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23. ***REASONING AND SOLUTION*** The figure below shows a baseball, ***as viewed from the side***, moving to the right with no spin. Since the air flows with the same speed above and below the ball, the pressure is the same above and below the ball. There is no net force to cause the ball to curve in any particular direction (except for gravity which results in the usual parabolic trajectory).

Without spin

If the ball is given a spin that is counterclockwise when viewed from the side, as shown below, the air close to the surface of the ball is dragged with the ball. In accord with Bernoulli's equation, the air on the top half of the ball is "speeded up" (lower pressure), while that on the lower half of the ball is slowed down (higher pressure).

Because of the pressure difference, a deflection force is generated that is directed from the higher pressure side of the ball to the lower pressure side of the ball. Therefore, the ball curves upward on its way to the plate.

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24. ***REASONING AND SOLUTION*** You are traveling on a train with your window open. As the train approaches its rather high operating speed, your ears "pop."

When the train is stationary, the air inside and outside the train is stationary. The pressures inside and outside, therefore, are equal. When the train moves at a high speed, the air rushes rapidly by the window. According to Bernoulli's principle as expressed in Equation 11.12, the faster moving air outside has a lower pressure than the nearly stationary air inside. Under this condition some of the inside air is pushed out the window. As a result, the pressure within the train drops. However, the pressure of the air in your inner ear is still at the higher level that it was when the train was stationary. Thus, your ears "pop" outward.

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25. ***REASONING AND SOLUTION*** A passenger is smoking a cigarette in the back seat of a moving car. To remove the smoke, the driver opens a window just a bit. Relative to the car, the air in the car is stationary, while the air outside the car is moving. According to Bernoulli's principle, as expressed in Equation 11.12, the moving air has a lower pressure than the stationary air. Hence, the higher-pressure air inside the car will flow through the opening to the lower-pressure region on the outside. Thus, the smoke will be drawn from the higher-pressure region in the car to the lower-pressure region outside.

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26. ***REASONING AND SOLUTION*** As discussed in the text, the dynamic lift on airplane wings occurs because the air travels faster over the curved upper surface of the wing than it does over the flatter lower surface (see Figure 11.34). According to Bernoulli's equation, the pressure above the wing is lower (higher air speed), while the pressure below the wing is higher (lower air speed). The net upward pressure on the wing causes an upward force that lifts the plane.

The airport in Phoenix, Arizona, has occasionally been closed to large planes because of unusually low air density. According to Equation 11.12, the net upward pressure on the wing of a plane is given by $P_{\text{bottom}} \quad P_{\text{top}} = (1/2)\,r_{\text{air}}\,(v_{\text{top}}^2 \quad v_{\text{bottom}}^2)$ where P_{bottom} and P_{top} are the air pressures on the bottom and top surfaces of the wing, while v_{top} and v_{bottom} are the air speeds at the top and bottom wing surfaces. When the air density is low, the quantity $(v_{\text{top}}^2 \quad v_{\text{bottom}}^2)$ must be greater than it is under normal conditions to provide the same minimum upward force to lift the plane. The speeds, v_{top} and v_{bottom}, depend on the speed of the plane relative to the ground. If the runway is not exceptionally long, the plane may not have sufficient distance over which to increase its speed so that the quantity $(v_{\text{top}}^2 \quad v_{\text{bottom}}^2)$ is large enough for take-off.

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27. ***REASONING AND SOLUTION*** To change the oil in a car, you remove a plug beneath the engine and let the old oil run out. Your car has been sitting in the garage on a cold day. Before changing the oil, it is advisable to run the engine for a while.

As discussed in the text, the viscosity of liquids usually decreases as the temperature is increased. Since the car was sitting in a cold garage, the viscosity of the oil will be higher than it is at room temperature. Therefore, if the plug is removed, the oil will run out, but at a speed which is small compared to its speed of flow at room temperature. If the engine is run for a while, the oil will heat up and its viscosity will decrease. The oil will then flow more easily, and it can be drained from the car in a much shorter time than if it were at the ambient temperature of the garage.

CHAPTER 12 | *TEMPERATURE AND HEAT*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The length L_0 of the tape rule changes by an amount L when its temperature changes by an amount T, where L is given by $L = aL_0 T$ and a is the coefficient of linear expansion. When the length of the tape rule changes, the location of the graduations changes and makes the rule inaccurate. For year-round outdoor use, the range of T will be large; therefore, to minimize the thermal expansion of the rule we would want the rule to be made from a material with a small coefficient of linear expansion. Since the coefficient of linear expansion of steel is less than that of aluminum, we would choose a steel tape rule for highest accuracy.

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2. ***REASONING AND SOLUTION*** The distance between the two fine lines engraved near the ends of the bar is unique only at a specified temperature. When the temperature of the bar is raised, the length of the bar increases, and the distance between the engraved lines increases. Since the bar was an international standard for length, it was essential that the distance between the lines always remained the same; therefore, it was important that the bar be kept at a constant temperature.

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3. ***REASONING AND SOLUTION*** The plate is made of aluminum; the spherical ball is made of brass. The coefficient of linear expansion of aluminum is greater than the coefficient of linear expansion of brass. Therefore, if the plate and the ball are heated, both will expand; however, the diameter of the hole in the aluminum plate will expand more than the diameter of the brass ball. In order to prevent the ball from falling through the hole, the plate and the ball must be cooled. Both the diameter of the hole in the plate and the diameter of the ball will contract. The diameter of the hole will decrease more than the diameter of the ball, thereby preventing the ball from falling through the hole.

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4. ***REASONING AND SOLUTION*** Reinforced concrete is concrete that is reinforced with embedded steel rods. For this arrangement to be practical, it is essential that the coefficient of linear expansion of concrete is the same as that of steel. When the reinforced concrete is heated or cooled, both the concrete and the steel must expand or contract by the same amount. If the coefficients of linear expansion were not the same, the concrete and steel would expand and contract at different rates when heated or cooled. If the steel expanded or contracted more than the concrete, it would place internal stresses on the concrete, thereby causing it to weaken and possibly crack.

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5. ***REASONING AND SOLUTION*** A rod is hung from an aluminum frame such that there is a small gap between the rod and the floor. The rod and the frame are heated uniformly; both the frame and the rod will expand. The linear expansion of the vertical portion of the frame and the linear expansion of the rod can be determined from Equation 12.2: $L = aL_0 \quad T$.

a. If the rod is made of aluminum, the fractional increase in length L / L_0 is the same for the vertical portion of the frame and the rod. Since the rod is initially shorter than the vertical portion of the frame, the rod will never be as long as the vertical portion of the frame. Therefore, the rod will never touch the floor.

b. If the rod is made of lead, the heated rod could touch the floor. The coefficient of linear expansion of lead is greater than that of aluminum. Therefore, the fractional change in the length of the rod will be greater than the fractional change in the length of the vertical portion of the frame. If the temperature is raised high enough, the length of the rod could expand enough to fill the gap and touch the floor.

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6. ***REASONING AND SOLUTION*** A simple pendulum is made using a long thin metal wire. From Equations 10.6 and 10.16, we know that the period of the pendulum is proportional to $\sqrt{L / g}$, where L is the length of the wire. When the temperature drops, the length of the wire decreases; therefore, the period of the pendulum decreases.

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7. ***REASONING AND SOLUTION*** A bimetallic strip is made from two thin strips of metals that have different coefficients of linear expansion; they are bonded together. When the strip is heated, the metal with the larger value of a expands more than the metal with the smaller a. Since the two metals are bonded together, the strip bends into an arc.

According to Equation 12.2, an object of length L_0 and coefficient of linear expansion a experiences a change in length L when the temperature changes by an mount T, where $L = aL_0 \quad T$. Thus, for two metal strips of the same length, the strip with the larger value of a will experience a greater change in length for the same temperature change.

According to Table 12.1, the relative difference in the values of a for aluminum and steel is greater than for copper and silver. Therefore, if two bimetallic strips have identical shapes, and one is made from aluminum and steel, and the other is made from copper and silver, the aluminum-steel strip will bend more for a given change in temperature.

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8. ***REASONING AND SOLUTION*** A hot steel ring fits snugly over a cold brass cylinder. The temperatures of the ring and the cylinder are, respectively, above and below room temperature. As the assembly equilibrates to room temperature, the diameter of the steel ring contracts while the diameter of the brass cylinder expands. The ring and the cylinder, therefore, fit much more snugly than they did initially. It becomes nearly impossible to pull the ring off the cylinder once the assembly has equilibrated at room temperature.

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9. ***REASONING AND SOLUTION*** The coefficient of volume expansion of Pyrex glass is only about 39 percent that of common glass. Therefore, the fractional change in volume, V / V_0, of a Pyrex baking dish is significantly less than that of common glass when both objects are taken from a refrigerator and placed in a hot oven. The Pyrex dish, therefore, is less likely to crack from thermal stress.

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10. ***REASONING AND SOLUTION*** An ordinary mercury-in-glass thermometer works on the following principle. When the thermometer is placed in contact with the object whose temperature is to be measured, the thermometer comes into thermal equilibrium with the object. As thermal equilibrium is reached, the glass tube and the enclosed mercury expand or contract, depending on whether the thermometer heats up or cools down. The coefficient of volume expansion of mercury is about 7 times greater than that of ordinary glass. Thus, the expansion or contraction of the mercury column relative to the scale on the glass is evident when one reads the thermometer. If the coefficients of volume expansion of mercury and glass were both the same, both would expand or contract by the same amount. The reading on the thermometer would never change.

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11. ***REASONING AND SOLUTION*** According to Archimedes' principle, any fluid applies a buoyant force to an object that is partially or completely immersed in it; the magnitude of the buoyant force equals the weight of the fluid that the object displaces. Therefore, the magnitude of the buoyant force exerted on an object immersed in water is given by $F_B = r_{water} V g$, where r_{water} is the density of water, and V is the volume displaced by the immersed object.

As shown in Figure 12.20, the density of cold water (above 4 °C) is greater than the density of warm water (above 4 °C). Therefore, cold water provides a greater buoyant force than warm water.

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12. ***REASONING AND SOLUTION*** When the bulb of a mercury-in-glass thermometer is inserted into boiling water, the glass is heated first. Therefore, the glass envelope expands. The cavity that contains the mercury increases in volume, and the mercury level drops slightly. Then, the mercury begins to be heated, and its volume expands. Since the coefficient of volume expansion for mercury is much greater than that of the glass, the mercury expands to a greater extent than the glass cavity, and the mercury level then begins to rise.

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13. ***REASONING AND SOLUTION*** When an amount of heat Q is added to an object of mass m, its temperature will increase. According to Equation 12.4, the amount by which the temperature will increase is given by $T = Q / cm$, where c is the specific heat capacity of the object.

Two different objects are supplied with equal amounts of heat. The objects could have the same specific heat capacities, but they could have different masses. The objects could have the same masses, but they could have different specific heat capacities. Or, the objects

could have both different masses and different specific heat capacities. In any of these cases, the temperature changes of the objects would not be the same.

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14. ***REASONING AND SOLUTION*** Two objects are made from the same material, but have different masses and temperatures. When the two objects are placed in contact, they will come into thermal equilibrium and reach a common temperature. The heat gained by the cooler object must be equal to the heat lost by the warmer object, so that $cm_1 \quad T_1 = cm_2 \quad T_2$, where one object has mass m_1 and the other object has mass m_2. Since the objects are made of the same material, they have the same specific heat capacities so that $m_1 \quad T_1 = m_2 \quad T_2$ or $T_2 = \left(m_1 / m_2\right) \quad T_1$. Suppose that m_1 is greater than m_2. Then, T_2 is greater than T_1, so the less massive object must undergo the greater temperature change.

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15. ***REASONING AND SOLUTION*** Two identical mugs contain hot coffee from the same pot. One mug is full, while the other is only one-quarter full. According to Equation 12.4, the amount of heat that must be removed from either cup to bring it to room temperature is $Q = cm \quad T$, where m is the mass of coffee in the cup, c is the specific heat capacity of the coffee and T is the temperature difference between the initial temperature of the coffee and room temperature. Since the mug that is full contains a larger mass m of coffee, more heat Q must be removed from the full mug than from the mug that is only one-quarter full. Therefore, the mug that is full remains warmer longer.

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16. ***REASONING AND SOLUTION*** As heat flows from the higher-temperature air to the lower-temperature water bottle, it passes through the wet sock. Most of the heat causes the water in the sock to evaporate, and only a relatively small amount of heat reaches the water bottle. Thus, as long as the sock remains wet, the water bottle remains cool.

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17. ***REASONING AND SOLUTION*** An alcohol rub has a cooling effect on a sick patient for the following reason. In order for the alcohol to evaporate, heat must be supplied to it. The evaporating alcohol removes heat from the skin of the sick patient. The evaporating alcohol, therefore, helps to lower the body temperature of the patient.

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18. ***REASONING AND SOLUTION*** Your head feels colder under an air-conditioning vent when your hair is wet than when it is dry. When your hair is wet, the water evaporates. In order to evaporate, heat must be supplied to the water. Therefore, heat is removed from your head as the water evaporates, thereby causing a cooling effect on your head.

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19. ***REASONING AND SOLUTION*** Suppose that the latent heat of vaporization of H_2O were one-tenth of its actual value.

a. Other factors being equal, it would take only one-tenth the amount of heat to "boil away" a pot of water on the stove. Assuming that the stove supplies heat at a constant rate, it would, therefore, take a shorter time for a pot of water to boil away.

b. The evaporative cooling mechanism of the human body would be less effective. When a given amount of water evaporates from the skin, it would now require only one-tenth as much heat. Therefore, only one-tenth as much heat would be removed from the skin surface for a given amount of perspiration.

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20. ***REASONING AND SOLUTION*** Orchard owners sometimes spray a film of water over fruit blossoms to protect them when a hard freeze is expected. The thin film of water will freeze before the water in the tissues of the blossoms. When the thin film of water freezes at 0 °C, each kilogram of water releases an amount of heat equal to the latent heat of fusion. Some of this heat will warm and, therefore, protect the blossoms from reaching the damaging temperature of – 4 °C.

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21. ***REASONING AND SOLUTION*** When a liquid is heated, its equilibrium vapor pressure increases as the temperature increases, as shown in Figure 12.31 for water. When the vapor pressure of the liquid is equal to the atmospheric pressure above the liquid, the liquid begins to boil. Therefore, at a higher altitude, where the atmospheric pressure is less than at lower altitudes, boiling is more easily achieved. At a higher altitude, the vapor pressure does not have to be raised as much to reach the boiling point, and the temperature required to boil a liquid at the higher altitude is less than that at a lower altitude.

A camping stove is used to boil water on a mountain. Since water will boil at a lower temperature at higher altitudes, it does not necessarily follow that the same stove can boil water at lower altitudes. If the stove cannot provide sufficient heat Q to raise the temperature of the water to the temperature at which the vapor pressure of the liquid is equal to the atmospheric pressure at the lower altitude, the water will not boil.

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22. ***REASONING AND SOLUTION*** If a bowl of water is placed in a closed container and water vapor is pumped away rapidly enough, the liquid water turns to ice. As water vapor is pumped away, the pressure in the jar is reduced. The liquid attempts to reestablish the equilibrium vapor pressure, so more water molecules enter the gaseous state. As the water vaporizes, heat must be supplied in the form of the latent heat of vaporization. Because of the heat loss, the remaining liquid is cooled. If this heat loss occurs rapidly enough, the surroundings cannot replenish the heat loss, and, when sufficient heat has been removed, the remaining liquid turns to ice.

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23. ***REASONING AND SOLUTION*** If medical instruments were sterilized in an open container of water, the water would be subjected to atmospheric pressure. Therefore, the highest possible temperature that could be attained is 100 °C. In order to raise the temperature above 100 °C, we would have to raise the pressure above the liquid, as suggested by Figure 12.30. An autoclave heats medical instruments in water under high pressure. We can see from Figure 12.31 that at pressures above atmospheric pressure, the boiling point of water is greater than 100 °C. Therefore, by sterilizing medical instruments in an autoclave, it is possible to attain temperatures much greater than 100 °C.

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24. ***REASONING AND SOLUTION*** A bottle of carbonated soda is left outside in subfreezing temperatures. The temperature of the soda is "subfreezing," but the soda remains in the liquid state. Because the bottle is sealed, the pressure above the soda remains too high for the soda to freeze. When the soda is brought inside and opened, the pressure drops to atmospheric pressure. Because the temperature of the soda is below the freezing point at atmospheric pressure, the soda freezes.

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25. ***REASONING AND SOLUTION*** A bowl of water is covered tightly and allowed to sit at a constant temperature of 23 °C for a long time. Since the bowl is tightly covered, none of the water vapor can escape, and since the bowl is allowed to sit for a long time at a constant temperature, the vapor above the liquid attains equilibrium with the liquid. The partial pressure of the water vapor will equal the equilibrium vapor pressure of water at 23 °C, and the relative humidity will be 100%.

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26. ***REASONING AND SOLUTION*** Dew forms on Tuesday night and not on Monday night, even though Monday night is the cooler night. Since dew did not form on Monday night, we can conclude that the temperature on Monday night was above the dew point. The partial pressure of water in the air must have been greater on Tuesday night; then, the dew point would occur at a higher temperature. As long as the temperature on Tuesday is below the dew point, dew will form.

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27. ***REASONING AND SOLUTION*** Two rooms in a house have the same temperature. One of the rooms contains an indoor swimming pool. The partial pressure of water vapor in the air will be greater in the room that contains the indoor pool. Therefore, the dew point will be higher in the room with the pool. On a cold day, the windows of one of the rooms are "steamed up." Water will condense on the window when the temperature of the air next to the glass falls below the dew point. Since only one of the rooms has "steamed" windows, and the dew point is higher for the room with the pool, then the outside temperature must be less than the dew point of the room with the pool, but greater than the dew point for the room without the pool. Therefore, the windows are "steamed up" in the room with the pool.

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28. ***REASONING AND SOLUTION*** A jar is half filled with boiling water. The lid is then screwed on the jar. The temperature of the jar and the lid will be close to the boiling point of water. As the jar is cooled to room temperature, the lid and the jar contract. Since the coefficient of linear expansion of the metal jar lid is greater than the coefficient of linear expansion of the glass jar, the lid will contract more than the jar. Therefore, the lid will fit more snugly when the jar has cooled down to room temperature and be difficult to remove.

In addition, as the temperature of the water vapor inside the jar drops, the pressure inside becomes less than the outside pressure. Thus, there is a net force pushing the lid onto the jar, making the lid harder to turn.

CHAPTER 13 | *THE TRANSFER OF HEAT*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** Convection is the process in which heat is carried from one place to another by the bulk movement of the medium. In liquids and gases, the molecules are free to move; hence, convection occurs as a result of bulk molecular motion. In solids, however, the molecules are generally bound to specific locations (lattice sites). While the molecules in a solid can vibrate about their equilibrium locations, they are not free to move from place to place within the solid. Therefore, convection does not generally occur in solids.

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2. ***REASONING AND SOLUTION*** A heavy drape, hung close to a cold window, reduces heat loss through the window by interfering with the process of convection. Without the drape, convection currents bring the warm air of the room into contact with the cold window. With the drape, convection currents are less prominent, and less room air is circulated directly past the cold surface of the window.

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3. ***REASONING AND SOLUTION*** Forced convection plays the principal role in the wind chill factor. The wind mixes the cold ambient air with the warm layer of air that immediately surrounds the exposed portions of your body. The forced convection removes heat from your exposed body surfaces, thereby making you feel colder than you would otherwise feel if there were no wind.

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4. ***REASONING AND SOLUTION*** A road surface is exposed to the air on its upper surface and to the earth on its lower surface. Even when the air temperature is at the freezing point, the road surface may be above this temperature as heat flows through the road from the earth. In order for a road to freeze, sufficient heat must be lost from the earth by conduction through the road surface. The temperature of the earth under the road must be reduced at least to the freezing point. A bridge is exposed to the air on both its upper and lower surfaces. It will, therefore, lose heat from both surfaces and reach thermal equilibrium with the air much more quickly than an ordinary roadbed. It is reasonable, then, that the bridge surface will usually freeze before the road surface.

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5. ***REASONING AND SOLUTION*** A piece of Styrofoam and a piece of wood are sandwiched together to form a layered slab. The two pieces have the same thickness and cross-sectional area. The exposed surfaces have constant temperatures. The temperature of the exposed Styrofoam surface is greater than the temperature of the exposed wood surface. The rate of heat flow through either layer can be determined from Equation 13.1: $Q/t = kA \quad T/L$, where k is the thermal conductivity of the layer, A and L are the cross-sectional area and thickness of the layer, respectively, and T is the temperature difference between the ends of the layer. Since heat is not trapped within the sandwich, the rate at

which heat flows through the sandwich, Q/t, must be uniform throughout both layers. Therefore, $(kA\ \ T/L)_{\text{Styrofoam}} = (kA\ \ T/L)_{\text{wood}}$. Since both layers have the same cross-sectional area and thickness, A and L are the same for both layers. Therefore, $k_{\text{Styrofoam}}\ \ T_{\text{Styrofoam}} = k_{\text{wood}}\ \ T_{\text{wood}}$. From Table 13.1, we see that the thermal conductivity of Styrofoam is less than the thermal conductivity of wood; therefore, the temperature difference between the two ends of the wood layer must be smaller than the temperature difference between the two ends of the Styrofoam layer. From this, we can conclude that the temperature at the Styrofoam-wood interface must be closer to the lower temperature of the exposed wood surface.

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6. ***REASONING AND SOLUTION*** When heat is transferred from place to place inside the human body by the flow of blood, the main method of heat transfer is forced convection, similar to that illustrated for the radiator fluid in Figure 13.7. The heart is analogous to the water pump in the figure.

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7. ***REASONING AND SOLUTION*** Some animals have hair, the strands of which are hollow, air-filled tubes. Other animals have hair that is composed of solid, tubular strands. For animals that live in very cold climates, hair that is composed of hollow air-filled tubes would be advantageous for survival. Since air has a small thermal conductivity, hair shafts composed of hollow air-filled tubes would reduce the loss of body heat by conduction. Since hair shafts are small, no appreciable convection would occur within them. Thus, the hollow air-filled structure of the hair shaft inhibits the loss of heat by conduction.

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8. ***REASONING AND SOLUTION*** A poker used in a fireplace is held at one end, while the other end is in the fire. Such pokers are made of iron rather than copper because the thermal conductivity of iron is roughly smaller by a factor of five than the thermal conductivity of copper. Therefore, the transfer of heat along the poker by conduction is considerably reduced by using iron. Hence, one end of the poker can be placed in the fire, and the other end will remain cool enough to be comfortably handled.

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9. ***REASONING AND SOLUTION*** Snow, with air trapped within it, is a thermal insulator, because air has a relatively low thermal conductivity and the small, dead-air spaces inhibit heat transfer by convection. Therefore, a lack of snow allows the ground to freeze at depths greater than normal.

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10. ***REASONING AND SOLUTION*** Table 13.1 indicates that the thermal conductivity of steel is 14 $\text{J/(s} \times \text{m} \times \text{C°)}$, while that of concrete is 1.1 $\text{J/(s} \times \text{m} \times \text{C°)}$. According to Equation 13.1, $Q = kA\ \ Tt/L$, this implies that heat will flow more readily through a volume of steel than it will through an identically shaped volume of concrete. Therefore, while steel reinforcement bars can enhance the structural stability of concrete walls, they degrade the insulating value of the concrete.

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11. ***REASONING AND SOLUTION*** A potato will bake faster if a nail is driven into it before it is placed in the oven. Since the nail is metal, we can assume that the thermal conductivity of the nail is greater than the thermal conductivity of the potato. The nail conducts more heat from the oven to the interior of the potato than does the flesh of the potato, thereby causing the potato to bake faster.

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12. ***REASONING AND SOLUTION*** Several days after a snowstorm, the roof on a house is uniformly covered with snow. On a neighboring house, the snow on the roof has completely melted. Since one of the houses still has snow on the roof, it is reasonable to conclude that the ambient temperature is still below the freezing point of water. Since the snow has melted from the roof of the neighboring house, we can conclude that the heat required to melt the snow must have come through the attic and the roof by conduction. Hence, the house which has the uniform layer of snow on the roof is probably better insulated. The better the insulation, the smaller is the amount of heat conducted through the roof to melt the snow.

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13. ***REASONING AND SOLUTION*** One car has a metal body, while another car has a plastic body. On a cold winter day, these cars are parked side by side. The metal car feels colder to the touch of your bare hand even though both cars are at the same temperature. This is because your fingers are sensitive to the rate at which heat is transferred to or from them, rather than to the temperature itself. The metal car feels colder than the plastic car at the same temperature, because heat flows from your bare hand into the metal car more readily than it flows into the plastic car. The flow occurs into the metal more readily, because the thermal conductivity of the metal is greater than that of the plastic.

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14. ***REASONING AND SOLUTION*** Many high-quality pots have copper bases and polished stainless steel sides. Since copper has a high thermal conductivity, heat can readily enter the bottom of the pot by means of conduction. Since the temperature of the pot is greater than the temperature of its environment, the pot will lose heat by means of radiation. Polished stainless steel has a low emissivity; that is, it is a poor emitter of radiant energy. Hence, by making the sides of the pot polished stainless steel, the amount of heat that would be lost by radiation is minimized. This design is optimal. If the pot were constructed entirely of copper, the bottom would efficiently conduct heat into the pan; however, heat would also be conducted efficiently into the sides of the pot, raising their temperature and increasing the loss from the sides via radiation. If, on the other hand, the pot were constructed entirely of stainless steel, the loss of heat through radiant energy would be minimized; however, since stainless steel has a low thermal conductivity, heat would not efficiently enter the bottom of the pot through conduction.

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15. ***REASONING AND SOLUTION*** The radiant energy Q emitted in a time t by an object that has a Kelvin temperature T, a surface area A, and an emissivity e, is given by Equation 13.2, $Q = e s T^4 A t$, where s is the Stefan-Boltzmann constant.

We now consider two objects that have the same size and shape. Object A has an emissivity of 0.3, and object B has an emissivity of 0.6. Since each object radiates the same power, $e_A s T_A^4 A_A = e_B s T_B^4 A_B$. The Stefan-Boltzmann constant is a universal constant, and since the objects have the same size and shape, $A_A = A_B$; therefore, $e_A T_A^4 = e_B T_B^4$, or $T_A / T_B = \sqrt[4]{e_B / e_A} = \sqrt[4]{2}$. Hence, the Kelvin temperature of A is $\sqrt[4]{2}$ or 1.19 times the Kelvin temperature of B, not twice the temperature of B.

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16. ***REASONING AND SOLUTION*** A concave mirror can be used to start a fire by directing sunlight onto a small spot on a piece of paper. The mirror, being a good reflector, is a poor absorber; therefore it will not absorb much of the sun's radiant energy. Paper is a better absorber than the mirror, so the paper absorbs the radiant energy. Therefore, the mirror does not get as hot as the paper. Eventually, the paper gets hot enough to burn.

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17. ***REASONING AND SOLUTION*** Two strips of material, A and B, are identical except that they have emissivities of 0.4 and 0.7, respectively. The strips are heated to the same temperature and have a bright glow. The emissivity is the ratio of the energy that an object actually radiates to the energy that the object would radiate if it were a perfect emitter. The strip with the higher emissivity will radiate more energy per second than the strip with the lower emissivity, other things being equal. Therefore, strip B will have the brighter glow.

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18. ***REASONING AND SOLUTION*** The thermal conductivity of the bottom of the pot is greater than the thermal conductivity of air; therefore, the portion of the heating element beneath the pot loses heat by conduction through the bottom of the pot. The exposed portion of the heating element loses some heat through convection, but the convective process is not as efficient as the conductive process through the bottom of the pot. The exposed portion of the heating element will, therefore, lose less heat and be at a higher temperature than the portion of the heating element beneath the pot. Thus, the exposed portion glows cherry red.

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19. ***REASONING AND SOLUTION*** If we consider a glove and a mitten, each of the same "size" and made of the same material, we can deduce that the mitten has less surface area A exposed to the cold winter air. Thus, according to Equation 13.1, $Q = kA \quad Tt / L$, we can conclude that the mitten will conduct less heat per unit time from the hand to the winter air. Therefore, to keep your hands as warm as possible during skiing, you should wear mittens as opposed to gloves.

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20. ***REASONING AND SOLUTION*** Two identical hot cups of cocoa are sitting on a kitchen table. One has a metal spoon in it and one does not. After five minutes, the cocoa with the metal spoon in it will be cooler. The metal spoon conducts heat from the cocoa to the handle of the spoon. Convection currents in the air and radiation then remove the heat from the spoon handle. The conduction-convection-radiation process removes heat from the cocoa, thereby cooling it faster than the cocoa that does not have a spoon in it.

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21. ***REASONING AND SOLUTION*** The radiant energy Q emitted in a time t by an object that has a Kelvin temperature T, a surface area A, and an emissivity e, is given by Equation 13.2: $Q = e s T^4 A t$, where s is the Stefan-Boltzmann constant.

a. A hot solid cube will cool more rapidly if it is cut in half, rather than if it is left intact. Since the cube is warmer than its environment, it will lose heat primarily through radiation. Convection currents will also remove some heat from the surface of the cube. When the cube has been cut in half, the surface area of the solid has been increased. If the length of one edge of the original cube is L, then cutting the cube in half increases the surface area from $6L^2$ to $8L^2$. From Equation 13.2, the amount of heat Q radiated in a time t is proportional to the surface area of the cube; therefore, the cube will radiate more rapidly and cool more rapidly if it is cut in half.

b. One pound of spaghetti noodles has a larger effective surface area than one pound of lasagna noodles. Imagine cutting many spaghetti noodles from one large lasagna noodle, in a way similar to what was done to the cube in part (a). Since the effective surface area of the spaghetti noodles is greater than that of the lasagna, heat will be radiated from the surface of the spaghetti noodles more effectively than heat will be radiated from the surface of the lasagna noodles. Therefore, the spaghetti noodles will cool more rapidly from the same initial temperature than the lasagna noodles.

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22. ***REASONING AND SOLUTION*** The black asphalt is a better absorber than the cement; the black asphalt will absorb more of the sun's radiant energy than the cement. Since the sun has been shining all day, the asphalt will be at a higher temperature than the cement. The temperature of the asphalt is apparently above the freezing point of water, while the temperature of the cement playground is below the freezing point of water. Therefore, when snow hits the asphalt, it melts immediately, while the snow collects on the cement.

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23. ***REASONING AND SOLUTION*** The radiant energy Q emitted in a time t by an object that has a Kelvin temperature T, a surface area A, and an emissivity e, is given by Equation 13.2: $Q = e s T^4 A t$, where s is the Stefan-Boltzmann constant.

If you are stranded in the mountains in bitter cold weather, you could minimize energy losses from your body by curling up into the tightest possible ball. In doing so, you minimize your effective surface area. Therefore, A in Equation 13.2 is made smaller, and you would radiate less heat.

CHAPTER 14 | *THE IDEAL GAS LAW AND KINETIC THEORY*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION***
a. Avogadro's number N_A is the number of particles per mole of substance. Therefore, one mole of hydrogen gas (H_2) and one mole of oxygen gas (O_2) contain the same number (Avogadro's number) of molecules.

b. One mole of a substance has a mass in grams that is equal to the atomic or molecular mass of the substance. The molecular mass of oxygen is greater than the molecular mass of hydrogen. Therefore, one mole of oxygen has more mass than one mole of hydrogen.

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2. ***REASONING AND SOLUTION*** Substances A and B have the same mass densities. Therefore, the mass per unit volume of substance A is equal to that of substance B.

a. The mass of one mole of a substance depends on the molecular mass of the substance. In general, the molecular masses of substances A and B will differ, and one mole of each substance will not occupy the same volume; therefore, even though substances A and B have the same mass density, one mole of substance A will not have the same mass as substance B.

b. Since the mass per unit volume of substance A is the same as the mass per unit volume of substance B, 1 m^3 of substance A has the same mass as 1 m^3 of substance B.

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3. ***REASONING AND SOLUTION*** A tightly sealed house has a large ceiling fan that blows air out of the house and into the attic. The fan is turned on, and the owners forget to open any windows or doors. As the fan transports air molecules from the house into the attic, the number of air molecules in the house decreases. Since the house is tightly sealed, the volume of the house remains constant. If the temperature of the air inside the house remains constant, then from the ideal gas law, $PV = nRT$, the pressure in the house must decrease. The air pressure in the attic, however, increases. The fan must now blow air from a lower pressure region to a higher pressure region. Thus, it becomes harder for the fan to do its job.

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4. ***REASONING AND SOLUTION*** According to the ideal gas law (Equation 14.1), $PV = nRT$. Therefore, the pressure of a gas confined to a fixed volume is directly proportional to the Kelvin temperature. Therefore, when the temperature is increased, the pressure increases proportionally.

The gas above the liquid in a can of hair spray is at a relatively high pressure. If the temperature of the can is increased to a sufficiently high temperature, the pressure of the confined gas could increase to a value larger than the walls of the can will sustain. The can

would then explode. Therefore, the label on hair spray cans usually contains the warning "Do not store at high temperatures."

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5. ***REASONING AND SOLUTION*** When an electric furnace in a tightly sealed house is turned on for a while, the temperature of the air in the house increases. Since the house is tightly sealed, both the volume of the air and the number of air molecules remain constant. If we assume that air behaves like an ideal gas, then, from the ideal gas law, $PV = nRT$, the pressure in the house must increase.

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6. ***REASONING AND SOLUTION*** At the sea coast, there is a cave that can be entered only by swimming beneath the water through a submerged passage and emerging into a pocket of air within the cave. Since the cave is not vented to the external atmosphere, the air in the cave contains a fixed number of moles n. As the tide comes in, the water level in the cave rises, thereby decreasing the volume of the air above the water. According to the ideal gas law (Equation 14.1), $PV = nRT$. The pressure of the air in the cave is, therefore, inversely proportional to the air volume. Since the volume of the air decreases, the pressure increases. Thus, if you are in the cave when the tide comes in, your ears will "pop" inward in a manner that is analogous to what happens when you climb down a mountain.

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7. ***REASONING AND SOLUTION*** Atmospheric pressure decreases with increasing altitude. Helium filled weather balloons are underinflated when they are launched. As the balloon rises, the pressure exerted on the outside of it decreases. The number of helium molecules in the balloon is fixed. If we assume that the temperature of the atmosphere remains constant, then from the ideal gas law, $PV = nRT$, we see that the volume of the helium in the balloon will increase, thereby further inflating the balloon. If the balloon is fully inflated when it is launched from earth, it would burst when it reaches an altitude where the expanded volume of the helium is greater than the maximum volume of the balloon.

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8. ***REASONING AND SOLUTION*** A slippery cork is being pressed into a very full bottle of wine. When released, the cork slowly slides back out. If some of the wine is removed from the bottle before the cork is inserted, the cork does not slide out. When the bottle is very full, the volume of air in the bottle above the wine is relatively small. Therefore, pushing the cork in reduces the volume of that air by an appreciable fraction. As a result, the pressure of the air increases appreciably and becomes large enough to push the slippery cork back out of the bottle. If some of the wine is removed, the volume of air above the wine is much larger to begin with, and pushing the cork in reduces the volume of that air by a much smaller fraction. Consequently, the pressure of the air increases by a much smaller amount and does not become large enough to push the cork back out.

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9. ***REASONING AND SOLUTION*** Packing material consists of "bubbles" of air trapped between bonded layers of plastic. The packing material exerts normal forces on the packed object at each place where the bubbles make contact with the object. The motion of the

packed object is thereby restricted. The magnitude of the normal force that any given bubble can exert depends on the pressure of the air trapped inside the bubble. The magnitude of the normal force is equal to the pressure of the air times the area of contact between the bubble and the object. Since the air is trapped in the bubbles, the number of air molecules and the volume of the gas are fixed. The pressure exerted by the bubbles, then, depends on the temperature according to the ideal gas law: $PV = nRT$. On colder days, T is smaller than on warmer days; therefore, on colder days, the pressure P of the air in the bubbles is less than on warmer days. When the pressure is smaller, the magnitude of the normal force that each bubble can exert on a packed object is smaller. Therefore, the packing material offers less protection on cold days.

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10. ***REASONING AND SOLUTION*** The kinetic theory of gases assumes that a gas molecule rebounds with the same speed after colliding with the wall of a container. From the impulse-momentum theorem, Equation 7.4, we know that $\mathbf{F}\ t = m(\mathbf{v}_f\ \ \mathbf{v}_0)$, where the magnitude of $\mathbf{F}$ is the magnitude of the force on the wall. This implies that in a time interval t, a gas molecule of mass m, moving with velocity $\mathbf{v}$ before the collision and $-\mathbf{v}$ after the collision, exerts a force of magnitude $F = 2mv/t$ on the walls of the container, since the magnitude of $\mathbf{v}_f\ \ \mathbf{v}_0$ is $2v$. If the speed of the gas molecule after the collision is less than that before the collision, the force exerted on the wall of the container will be less than $F = 2mv/t$, since the magnitude of $\mathbf{v}_f\ \ \mathbf{v}_0$ is then less than $2v$. Therefore, the pressure of the gas will be less than that predicted by kinetic theory.

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11. ***REASONING AND SOLUTION*** The relationship between the average kinetic energy per particle in an ideal gas and the Kelvin temperature T of the gas is given by Equation 14.6: $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$.

If the translational speed of each molecule in an ideal gas is tripled, then the root-mean-square speed for the gas is also tripled. From Equation 14.6, the Kelvin temperature is proportional to the square of the root-mean-square speed. Therefore, the Kelvin temperature will increase by a factor of 9.

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12. ***REASONING AND SOLUTION*** If the temperature of an ideal gas is doubled from 50 to 100 °C, the average translational kinetic energy per particle does not double. Equation 14.6, $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$, relates the average kinetic energy per particle to the Kelvin temperature of the gas, not the Celsius temperature. If the Celsius temperature increases from 50 to 100 °C, the Kelvin temperature increases from 323.15 to 373.15 K. This represents a fractional increase of 1.15; therefore, the average translational kinetic energy per particle increases only by a factor of 1.15.

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13. ***REASONING AND SOLUTION*** According to Equation 14.7, the internal energy of a sample consisting of n moles of a monatomic ideal gas at Kelvin temperature T is $U = \frac{3}{2}nRT$; therefore, the internal energy of such an ideal gas depends only on the Kelvin

temperature. If the pressure and volume of this sample is changed *isothermally*, the internal energy of the ideal gas will remain the same. Physically, this means that the experimenter would have to change the pressure and volume in such a way, that the product PV remains the same. This can be verified from the ideal gas law (Equation 14.1), $PV = nRT$; if the values of P and V are varied so that the product PV remains constant, then T will remain constant and, from Equation 14.7, the internal energy of the gas remains the same.

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14. ***REASONING AND SOLUTION*** The atoms in a container of helium have the same translational rms speed as the molecules in a container of argon. Equation 14.6 relates the average translational kinetic energy per particle in an ideal gas to the Kelvin temperature: $\frac{1}{2}mv_{\text{rms}}^2 = \frac{3}{2}kT$. Since the mass of an argon atom is greater than the mass of a helium atom, then, from Equation 14.6, the average translational kinetic energy per atom of the argon atoms is greater than the average translational kinetic energy per atom of the helium atoms. Therefore, the temperature of the argon atoms is greater than that of the helium atoms.

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15. ***REASONING AND SOLUTION*** When an object is placed in a warm environment, its temperature increases for two reasons: (1) the object gains electromagnetic radiation from objects in the warm environment, and (2) the object gains heat from air molecules that collide with the object.

Suppose that an astronaut were placed in the ionosphere, where the temperature of the ionized gas is about 1000 K, and the density of the gas is on the order of 10^{11} molecules / m^3. Since the density is extremely low, the distances between gas molecules are very large. Even though the temperature of the gas is large, there is an insufficient number of molecules to radiate sufficient energy to heat the astronaut. The number of molecules that collide with the astronaut per unit time is very small; therefore, even though the average kinetic energy of the gas molecules is large, the amount of heat transferred by the small number of collisions per unit time is low. Most of the surface area of the astronaut is in contact with empty space. The tissues of the astronaut would radiate more electromagnetic radiation than they absorb. Therefore, the astronaut would not burn up; in fact, if the astronaut remained in the environment very long, he would freeze to death.

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16. ***REASONING AND SOLUTION*** Fick's law of diffusion relates the mass m of solute that diffuses in a time t through a solvent contained in a channel of length L and cross-sectional area A: $m = (DA \quad C)t / L$, where C is the concentration difference between the ends of the channel and D is the diffusion constant.

In the lungs, oxygen in very small sacs (alveoli) diffuses into the blood. The walls of the alveoli are thin, so the oxygen diffuses over a small distance L. Since the number of alveoli is large, the effective area A across which diffusion occurs is very large. From Fick's law, we see that the mass of oxygen that diffuses per unit time is directly proportional to the effective cross-sectional area A and inversely proportional to the diffusion distance L. Since A is large and L is small, the mass of oxygen per second that diffuses into the blood is large.

CHAPTER 15 | *THERMODYNAMICS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The plunger of a bicycle tire pump is pushed down rapidly with the end of the pump sealed so that no air escapes. Since the compression occurs rapidly, there is no time for heat to flow into or out of the system. Therefore, to a very good approximation, the process may be treated as an adiabatic compression that is described by Equation 15.4:

$$W = (3/2)nR(T_i \quad T_f)$$

The person who pushes the plunger down does work on the system, therefore W is negative. It follows that the term $(T_i \quad T_f)$ must also be negative. Thus, the final temperature T_f must be greater than the initial temperature T_i. This increase in temperature is evidenced by the fact that the pump becomes warm to the touch.

Alternate Explanation:
Since the compression occurs rapidly, there is no time for heat to flow into or out of the system. Therefore, to a very good approximation, the process may be treated as an adiabatic compression. According to the first law of thermodynamics, the change in the internal energy is $U = Q \quad W = \quad W$, since $Q = 0$ for adiabatic processes. Since work is done on the system, W is negative; therefore the change in the internal energy, U, is positive. The work done by the person pushing the plunger is manifested as an increase in the internal energy of the air in the pump. The internal energy of an ideal gas is proportional to the Kelvin temperature. Since the internal energy of the gas increases, the temperature of the air in the pump must also increase. This increase in temperature is evidenced by the fact that the pump becomes warm to the touch.

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2. ***REASONING AND SOLUTION*** The work done in an isobaric process is given by Equation 15.2: $W = P(V_f \quad V_i)$. According to the first law of thermodynamics, the change in the internal energy is $U = Q \quad W = Q \quad P(V_f \quad V_i)$.

One hundred joules of heat is added to a gas, and the gas expands at constant pressure (isobarically). Since the gas expands, the final volume will be greater than the initial volume. Therefore, the term $P(V_f \quad V_i)$ will be positive. Since $Q = +100$ J, and the term $P(V_f \quad V_i)$ is positive, the change in the internal energy must be less than 100 J. It is not possible that the internal energy increases by 200 J.

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3. ***REASONING AND SOLUTION*** The internal energy of an ideal gas is proportional to its Kelvin temperature (see Equation 14.7). In an isothermal process the temperature remains constant; therefore, the internal energy of an ideal gas remains constant throughout an isothermal process. Thus, if a gas is compressed isothermally and its internal energy increases, the gas is not an ideal gas.

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4. ***REASONING AND SOLUTION*** According to the first law of thermodynamics (Equation 15.1), the change in the internal energy is $U = Q - W$. The process is isochoric, which means that the volume is constant. Consequently, no work is done, so $W = 0$. The process is also adiabatic, which means that no heat enters or leaves the system, so $Q = 0$. According to the first law, then, $U = Q - W = 0$. There is no change in the internal energy, and the internal energy of the material at the end of the process is the same as it was at the beginning.

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5. ***REASONING AND SOLUTION***

a. It is possible for the temperature of a substance to rise without heat flowing into the substance. Consider, for example, the adiabatic compression of an ideal gas. Since the process is an adiabatic process, $Q = 0$. The work done by the external agent increases the internal energy of the gas. Since the internal energy of an ideal gas is proportional to the Kelvin temperature, the temperature of the gas must increase.

b. The temperature of a substance does not necessarily have to change because heat flows into or out of it. Consider, for example, the isothermal expansion of an ideal gas. Since the internal energy of an ideal gas is proportional to the Kelvin temperature, the internal energy, U, remains constant during an isothermal process. The first law of thermodynamics gives $U = Q \quad W = 0$, or $Q = W$. The heat that is added to the gas during the isothermal expansion is used by the gas to perform the work involved in the expansion. The temperature of the gas remains unchanged. Similarly, in an isothermal compression, the work done on the gas as the gas is compressed causes heat to flow out of the gas while the temperature of the gas remains constant.

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6. ***REASONING AND SOLUTION*** The text drawing shows a pressure-volume graph in which a gas undergoes a two-step process from A to B and from B to C.

From A to B: The volume V of the gas increases at constant pressure P. According to the ideal gas law (Equation 14.1), $PV = nRT$, the temperature T of the gas must increase. According to Equation 14.7, $U = (3/2)nRT$, if T increases, then U, the change in the internal energy, must be positive. Since the volume increases at constant pressure (V increases), we know from Equation 15.2, $W = P \quad V$, that the work done is positive. The first law of thermodynamics (Equation 15.1) states that $U = Q \quad W$; since U and W are both positive, Q must also be positive.

From B to C The pressure P of the gas increases at constant volume V. According to the ideal gas law (Equation 14.1), $PV = nRT$, the temperature T of the gas must increase. According to Equation 14.7, $U = (3/2)nRT$, if T increases, then U, the change in the internal energy, must be positive. Since the process occurs isochorically ($V = 0$), and according to Equation 15.2, $W = P \quad V$, the work done is zero. The first law of thermodynamics (Equation 15.1) states that $U = Q \quad W$; since $W = 0$, Q is also positive since U is positive.

These results are summarized in the table below:

	U	Q	W
$A \quad B$	+	+	+
$B \quad C$	+	+	0

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7. ***REASONING AND SOLUTION*** Since the process is an adiabatic process, $Q = 0$. Since the gas expands into chamber B under zero external pressure, the work done by the gas is $W = P \quad V = 0$. According to the first law of thermodynamics, the change in the internal energy is, therefore, zero: $U = Q \quad W = 0$. The internal energy of an ideal gas is proportional to the Kelvin temperature of the gas (Equation 14.7). Since the change in the internal energy of the gas is zero, the temperature change of the gas is zero. The final temperature of the gas is the same as the initial temperature of the gas.

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8. ***REASONING AND SOLUTION*** A material contracts when it is heated. To determine the molar specific heat capacities, we first calculate the heat Q needed to raise the temperature of the material by an amount T. From the first law of thermodynamics, $Q = DU + W$. When the heating occurs at constant pressure, the work done is given by Equation 15.2: $W = P \quad V = P(V_f \quad V_i)$. When the volume is constant, $V = 0$. Therefore, we have:

$$Q_P = \quad U + P(V_f \quad V_i) \qquad \text{and} \qquad Q_V = \quad U$$

Equation 15.6 indicates that the molar heat capacities will be given by $C = Q/(n \quad T)$. Therefore

$$C_P = \frac{U + P(V_f \quad V_i)}{n \quad T} \qquad \text{and} \qquad C_V = \frac{U}{n \quad T}$$

Since the material contracts when it is heated, V_f is less than V_i. Therefore the term $P(V_f \quad V_i)$ is negative. Hence, the numerator of C_P is smaller than the numerator of C_V. Therefore, C_V is larger than C_P.

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9. ***REASONING AND SOLUTION*** When a solid melts at constant pressure, the volume of the resulting liquid does not differ much from the volume of the solid. According to the first law of thermodynamics, $U = Q \quad W = Q \quad P(V_f \quad V_i) \quad Q$. Hence, the heat that must be added to melt the solid is used primarily to increase the internal energy of the molecules. The internal energy of the liquid has increased by an amount $Q = mL_f$ compared to that of the solid, where m is the mass of the material and L_f is the latent heat of fusion.

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10. ***REASONING AND SOLUTION*** According to Equation 14.7, the Kelvin temperature T of the gas is related to its internal energy U by $U = (3/2)nRT$. The change in the internal energy is given by the first law of thermodynamics (Equation 15.1), $U = Q \quad W$.

It is desired to heat a gas so that its temperature will be as high as possible. If the process occurs at constant pressure, so that the volume of the gas increases, work is done by the gas. The available heat is used to do work *and* to increase the internal energy of the gas. On the other hand, if the process is carried out at constant volume, the work done is zero, and all of the heat increases the internal energy of the gas. From Equation 14.7, the internal energy is directly proportional to the Kelvin temperature of the gas. Since the internal energy increases by a greater amount when the process occurs at constant volume, the temperature increase is greatest under conditions of constant volume. Therefore, if it is desired to heat a gas so that its temperature will be as high as possible, you should heat it under conditions of constant volume.

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11. ***REASONING AND SOLUTION*** A hypothetical device takes 10 000 J of heat from a hot reservoir and 5000 J of heat from a cold reservoir and produces 15 000 J of work.

a. According to the first law of thermodynamics, $U = Q \quad W$. This is a statement of energy conservation. The hypothetical device does not violate energy conservation. It does not create or destroy energy It converts one form of energy (15 000 J of heat) into another form of energy (15 000 J of work) with no gain or loss.

b. This hypothetical device does violate the second law of thermodynamics. It converts all of its input heat (15 000 J) into work (15 000 J). Therefore, the efficiency of this device is 1.0 or 100 %. But Equation 15.15 is a consequence of the second law of thermodynamics and sets the limits of the maximum possible efficiency of any heat device. Since T_C is greater than 0 K, the ratio T_C / T_H must be positive. Furthermore, since $T_C < T_H$, the ratio T_C / T_H must be less than one. Therefore, Equation 15.15 implies that the efficiency of any device must be less than 1 or 100%. Since the efficiency of the hypothetical device is equal to 100%, it violates the second law of thermodynamics.

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12. ***REASONING AND SOLUTION*** According to the second law of thermodynamics, heat flows spontaneously from a substance at a higher temperature to a substance at a lower temperature and does not flow spontaneously in the reverse direction. Therefore, according to the second law of thermodynamics, work must be done to remove heat from a substance at a lower temperature and deposit it in a substance at a higher temperature. In other words, the second law requires that energy in the form of work must be supplied to an air conditioner in order for it to remove heat from a cool space and deposit the heat in a warm space. An advertisement for an automobile that claimed the same gas mileage with and without the air conditioner would be suspect. Since the car would use more energy with the air conditioner on, the car would use more gasoline. Therefore, the mileage should be less with the air conditioner running.

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13. ***REASONING AND SOLUTION*** Carnot's principle states that the most efficient engine operating between two temperatures is a reversible engine. This means that a *reversible* engine operating between the temperatures of 600 and 400 K must be more efficient than an *irreversible* engine operating between the *same two temperatures*. No comparison can be made with an irreversible engine operating between two temperatures that are different than 600 and 400 K.

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14. ***REASONING AND SOLUTION*** The efficiency of a Carnot engine is given by Equation 15.15: $\text{efficiency} = 1 \quad (T_C / T_H)$. Three reversible engines A, B, and C, use the same cold reservoir for their exhaust heats. They use different hot reservoirs with the following temperatures: (A) 1000 K; (B) 1100 K; and (C) 900 K. We can rank these engines in order of increasing efficiency according to the following considerations. The ratio T_C / T_H is inversely proportional to the value of T_H. The ratio T_C / T_H will be smallest for engine B; therefore, the quantity $1 - (T_C / T_H)$ will be largest for engine B. Thus, engine B has the largest efficiency. Similarly, the ratio T_C / T_H will be largest for engine C; therefore, the quantity $1 - (T_C / T_H)$ will be smallest for engine C. Thus, engine C has the smallest efficiency. Hence, the engines are, in order of increasing efficiency: engine C, engine A, and engine B.

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15. ***REASONING AND SOLUTION*** The efficiency of a Carnot engine is given by Equation 15.15: $\text{efficiency} = 1 \quad (T_C / T_H)$.

a. Lowering the Kelvin temperature of the cold reservoir by a factor of four makes the ratio T_C / T_H one-fourth as great.

b. Raising the Kelvin temperature of the hot reservoir by a factor of four makes the ratio T_C / T_H one-fourth as great.

c. Cutting the Kelvin temperature of the cold reservoir in half and doubling the Kelvin temperature of the hot reservoir makes the ratio T_C / T_H one-fourth as great.

Therefore, all three possible improvements have the same effect on the efficiency of a Carnot engine.

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16. ***REASONING AND SOLUTION*** A refrigerator is kept in a garage that is not heated in the cold winter or air-conditioned in the hot summer. In order to make ice cubes, the refrigerator uses electrical energy to provide the work to remove heat from the interior of the freezer and deposit the heat outside of the refrigerator. In the summer, the "hot" reservoir will be at a higher temperature than it is in the winter. Therefore, more work will be required to remove heat from the interior of the freezer in the summer, and the refrigerator will use more electrical energy. Hence, it will cost more for the refrigerator to make a kilogram of ice cubes in the summer.

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17. ***REASONING AND SOLUTION*** The coefficient of performance of a heat pump is given by Equation 15.17: coefficient of performance = Q_H / W. From the conservation of energy, $Q_H = W + Q_C$. Thus, the ratio Q_H / W can be written $1 + (Q_C / W)$. The job of a heat pump is to remove heat from a cold reservoir, and deliver it to a hot reservoir; therefore, the ratio Q_C / W must be nonzero and positive. Hence, the coefficient of performance, $1 + (Q_C / W)$, must always be greater than one.

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18. ***REASONING AND SOLUTION*** In a refrigerator, the interior of the unit is the cold reservoir, while the warmer exterior of the room is the hot reservoir. An air conditioner is like a refrigerator, except that the room being cooled is the cold reservoir, and the outdoor environment is the hot reservoir. Therefore, an air conditioner cools the inside of the house, while a refrigerator warms the interior of the house.

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19. ***REASONING AND SOLUTION*** Heat pumps *can* deliver more energy into your house than they consume in operating. A heat pump consumes an amount of energy *W,* which it uses to make heat Q_C flow from the cold outdoors into the warm house. The amount of energy the heat pump delivers to the house is $Q_H = W + Q_C$. This is greater than the energy, *W*, consumed by the heat pump.

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20. ***REASONING AND SOLUTION*** A refrigerator is advertised as being easier to "live with" during the summer, because it puts into your kitchen only the heat that it removes from the food. The advertisement is describing a refrigerator in which heat is removed from the interior of the refrigerator and deposited outside the refrigerator without requiring any work. Since no work is required, the flow must be spontaneous. This violates the second law of thermodynamics, which states that heat spontaneously flows from a higher-temperature substance to a lower-temperature substance, and does not flow spontaneously in the reverse direction. Heat can be made to flow from a cold reservoir to a hot reservoir, but only when work is done. Both the heat and the work are deposited in the hot reservoir.

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21. ***REASONING AND SOLUTION*** On a summer day, a window air conditioner cycles on and off, according to how the temperature within the room changes. When the unit is on it will be depositing heat, along with the work required to remove the heat, to the outside. Therefore, the outside of the unit will be hotter when the unit is on. Hence, you would be more likely to fry an egg on the outside part of the unit when the unit is on.

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22. ***REASONING AND SOLUTION*** The second law of thermodynamics states that the total entropy of the universe does not change when a reversible process occurs ($S_{\text{universe}} = 0$) and increases when an irreversible process occurs ($S_{\text{universe}} > 0$).

An event happens somewhere in the universe and, as a result, the entropy of an object changes by –5 J/K. If the event is a reversible process, then the entropy change for the rest of the universe must be +5 J/K; this results in a total entropy change of zero for the universe. If the process is irreversible, the only possible choice for the change in the entropy of the rest of the universe is +10 J/K; this results in a total entropy change of +5 J/K for the universe. The choices –5 J/K and 0 J/K are not possible choices for the entropy change of the rest of the universe, because they imply that the total entropy change would be negative. This would violate the second law of thermodynamics.

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23. ***REASONING AND SOLUTION*** When water freezes from a less-ordered liquid to a more-ordered solid, its entropy decreases. This decrease in entropy does not violate the second law of thermodynamics, because it is a decrease for only one part of the universe. In terms of entropy, the second law indicates that the total change in entropy for the entire universe must be either zero (reversible process) or greater than zero (irreversible process). In the case of freezing water, heat must be removed from the water and deposited in the environment. The entropy of the environment increases as a result. If the freezing occurs reversibly, the increase in entropy of the environment will exactly match the decrease in entropy of the water, with the result that $S_{\text{universe}} = S_{\text{water}} + S_{\text{environment}} = 0$. If the freezing occurs irreversibly, then the increase in entropy of the environment will exceed the decrease in entropy of the water, with the result that $S_{\text{universe}} = S_{\text{water}} + S_{\text{environment}} > 0$.

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24. ***REASONING AND SOLUTION*** Since we can interpret the increase of entropy as an increase in disorder, the more disordered system will have the greater entropy.

a. The popcorn that results from the kernels is more disorderly than a handful of popcorn kernels; therefore, the popcorn that results from the kernels has the greater entropy.

b. A salad has more disorder after it has been tossed; therefore, the tossed salad has the greater entropy.

c. A messy apartment is more disorderly than a neat apartment; therefore, a messy apartment has the greater entropy.

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25. ***REASONING AND SOLUTION*** A glass of water contains a teaspoon of dissolved sugar. After a while, the water evaporates, leaving behind sugar crystals. The entropy of the sugar crystals is less than the entropy of the dissolved sugar, because the sugar crystals are in a more ordered state. However, think about the water. The entropy of the gaseous water vapor is greater than the entropy of the liquid water, because the molecules in the vapor are in a less ordered state. Since the increase in the entropy of the water is greater than the decrease in entropy of the sugar, the net change in entropy of the universe is positive. The process, therefore, does not violate the entropy version of the second law of thermodynamics.

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26. ***REASONING AND SOLUTION*** Since we can interpret the increase of entropy as an increase in disorder, the more disordered state will have the greater entropy. The finished building is the most ordered state; therefore it has the smallest entropy. The burned-out shell of a building is the most disordered state; therefore it has the largest amount of entropy.

The states can be ranked in order of decreasing entropy (largest first) as follows: (3) the burned-out shell of a building, (1) the unused building material, and (2) the building.

CHAPTER 16 | *WAVES AND SOUND*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** As Figure 16.4 shows, in a water wave, the wave motion of the water includes both transverse and longitudinal components. The water at the surface moves on nearly circular paths. When the wave passes beneath a fishing float, the float will simultaneously bob up and down, as well as move back and forth horizontally. Thus, the float will move in a nearly circular path in the vertical plane. It is not really correct, therefore, to say that the float bobs straight "up and down."

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2. ***REASONING AND SOLUTION*** "Domino Toppling" is an event that consists of lining up an incredible number of dominoes and then letting them topple, one after another. As the dominoes topple, their displacements contain both vertical and horizontal components. Therefore, the disturbance that propagates along the line of dominoes has both longitudinal (horizontal) and transverse (vertical) components.

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3. ***REASONING AND SOLUTION*** A longitudinal wave moves along a Slinky at a speed of 5 m/s. We *cannot* conclude that one coil of the Slinky moves through a distance of 5 m in one second. The quantity 5 m/s is the longitudinal wave speed, v_{speed}; it specifies how fast the *disturbance* travels along the spring. The wave speed depends on the properties of the spring. Like the transverse wave speed, the longitudinal wave speed depends upon the tension F in the spring and its linear mass density m/L. As long as the tension and the linear mass density remain the same, the disturbance will travel along the spring at constant speed.

   The particles in the Slinky oscillate longitudinally in simple harmonic motion with the same amplitude and frequency as the source. As with all particles in simple harmonic motion, the particle speed is not constant. The particle speed is a maximum as the particle passes through its equilibrium position and reaches zero when the particle has reached its maximum displacement from the equilibrium position. The particle speed depends upon the amplitude and frequency of the particle's motion. Thus, the particle speed, and therefore the longitudinal speed of a single coil, depends upon the properties of the source that causes the disturbance.

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4. ***REASONING AND SOLUTION*** A wave moves on a string with constant velocity. It is *not* correct to conclude that the particles of the string always have zero acceleration. As Conceptual Example 3 discusses, it is important to distinguish between the speed of the waves on the string, v_{wave}, and the speed of the particles in the string, v_{particle}. The wave speed v_{wave} is determined by the properties of the string; namely, the tension in the string and the linear mass density of the string. These properties determine the speed with which the disturbance travels along the string. The wave speed will remain constant as long as these properties remain unchanged.

The particles in the string oscillate transversely in simple harmonic motion with the same amplitude and frequency as the source of the disturbance. Like all particles in simple harmonic motion, the acceleration of the particles continually changes. It is zero when the particles pass through their equilibrium positions and is a maximum when the particles are at their maximum displacements from their equilibrium positions.

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5. ***REASONING AND SOLUTION*** A wire is strung tightly between two immovable posts. The speed of a transverse wave on the wire is given by Equation 16.2: $v_{\text{wave}} = \sqrt{F/(m/L)}$. The wire will expand because of the increase in temperature. Since the length of the wire increases slightly, it will sag, and the tension in the wire will decrease. From Equation 16.2, we see that the speed of the wave is directly proportional to the square root of the tension in the wire. If we ignore any change in the mass per unit length of the wire, then we can conclude that, when the temperature is increased, the speed of waves on the wire will decrease.

---

6. ***REASONING AND SOLUTION*** A rope of mass m is hanging down from the ceiling. Nothing is attached to the loose end of the rope. A transverse wave is traveling up the rope.

The tension in the rope is not constant. The lower portion of the rope pulls down on the higher portions of the rope. If we imagine that the rope is divided into small segments, we see that the segments near the top of the rope are being pulled down by more weight than the segments near the bottom. Therefore, the tension in the rope increases as we move up the rope. The speed of a transverse wave on the rope is given by Equation 16.2: $v_{\text{wave}} = \sqrt{F/(m/L)}$. From Equation 16.2 we see that, as the tension F in the rope increases, the speed of the wave increases. Therefore, as the transverse wave travels up the rope, the speed of the wave increases.

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7. ***REASONING AND SOLUTION*** One end of each of two identical strings is attached to a wall. Each string is being pulled tightly by someone at the other end. A transverse pulse is sent traveling along one of the strings. A bit later, an identical pulse is sent traveling along the other string. In order for the second pulse to catch up with the first pulse, the speed of the pulse in the second string must be increased. The speed of the transverse pulse on the second string is given by Equation 16.2: $v_{\text{wave}} = \sqrt{F/(m/L)}$. This equation indicates that we can increase the speed of the second pulse by increasing the tension F in the string. Thus, the second string must be pulled more tightly.

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8. ***REASONING AND SOLUTION*** In Section 4.10 the concept of a "massless" rope is discussed. For a truly massless rope, the linear density of the rope, m/L, is zero. From Equation 16.2, $v_{\text{wave}} = \sqrt{F/(m/L)}$, the wave speed would be infinite if (m/L) were zero. Therefore, if the rope were really massless, the speed of transverse waves on the rope would be infinite, and a transverse wave would be instantaneously transmitted from one end of the

rope to the other. It would not take any time for a transverse wave to travel the length of a massless rope.

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9. ***REASONING AND SOLUTION*** As the disturbance moves outward when a sound wave is produced, it compresses the air directly in front of it. This compression causes the air pressure to rise slightly, resulting in a condensation that travels outward. The condensation is followed by a region of decreased pressure, called a rarefaction. Both the condensation and rarefaction travel away from the speaker at the speed of sound. As the condensations and rarefactions of the sound wave move away from the disturbance, the individual air molecules are not carried with the wave. Each molecule executes simple harmonic motion about a fixed equilibrium position. As the wave passes by, all the particles in the region of the disturbance participate in this motion. There are no particles that are always at rest.

---

10. ***REASONING AND SOLUTION*** Assuming that we can treat air as an ideal gas, then the speed of sound in air is given by Equation 16.5, $v = \sqrt{g\ kT / m}$, where g is the ratio of the specific heats c_P / c_V, k is Boltzmann's constant, T is the Kelvin temperature, and m is the mass of a molecule of the gas.

We can see from Equation 16.5 that the speed of sound in air is proportional to the square root of the Kelvin temperature of the gas. Therefore, on a hot day, the speed of sound in air is greater than it is on a cold day. Hence, we would expect an echo to return to us more quickly on a hot day as compared to a cold day, other things being equal.

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11. ***REASONING AND SOLUTION*** A loudspeaker produces a sound wave. The sound wave travels from air into water. As indicated in Table 16.1, the speed of sound in water is approximately four times greater than it is in air. We are told in the hint that the frequency of the sound wave does not change as the sound enters the water. The relationship between the frequency f, the wavelength l, and the speed v of a wave is given by Equation 16.1: $v = f\ l$. Since the wave speed increases and the frequency remains the same as the sound enters the water, the wavelength of the sound must increase.

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12. ***REASONING AND SOLUTION*** A person is making JELL-O. It starts out as a liquid and then sets into a gel. In general, sound travels slowest in gases, faster in liquids, and fastest in solids. When the JELL-O sets into a gel, it is more "solid;" therefore, the speed of sound should be larger in the set JELL-O than when the JELL-O is in the liquid state.

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13. ***REASONING AND SOLUTION*** Animals that rely on an acute sense of hearing for survival often have relatively large external ear parts. Sound intensity is defined as the sound power P that passes perpendicularly through a surface divided by the area A of that surface: $I = P / A$. For low intensity sounds, the power per unit area is small. Relatively large outer ears have a greater area than smaller outer ears. Hence, large outer ears intercept and direct more sound power into the auditory system than smaller outer ears do.

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14. ***REASONING AND SOLUTION*** A source is emitting sound uniformly in all directions. According to Equation 16.9, $I = P/(4\pi r^2)$, the intensity of such a source varies as $1/r^2$. Thus, the intensity I at a point in space depends on the distance of that point from the source. A flat surface faces the source. As suggested in the following figure, the distance between the source and the flat sheet varies, in general, from point to point on the sheet. The figure indicates that, as we move up the screen, the distance between the source and the screen increases. Therefore, the sound intensity is not the same at all points on the screen.

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15. ***REASONING AND SOLUTION*** Two people talk simultaneously and each creates an intensity level of 65 dB at a certain point. The total intensity level at this point does *not* equal 130 dB. Intensity levels are defined in terms of logarithms. According to Equation 16.10, the sound intensity level in decibels is given by $\beta = (10\text{ dB})\log(I/I_0)$, where I_0 is the threshold of human hearing. We can show, using Equation 16.10, that an intensity level of 65 dB corresponds to a sound intensity of $3.2 \quad 10^{-6}\ \text{W/m}^2$. If two people simultaneously talk and each creates an intensity of $3.2 \quad 10^{-6}\ \text{W/m}^2$ at a certain point, then the total *intensity* at that point is $6.4 \quad 10^{-6}\ \text{W/m}^2$. However, since the *intensity level* is defined in terms of logarithms, the intensity level at that point is (from Equation 16.10) 68 dB. While the second person doubles the intensity at the point in question, he only increases the intensity level (or the loudness) by 3 dB.

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16. ***REASONING AND SOLUTION*** Two cars, one behind the other, are traveling in the same direction at the same speed. The velocity of either car relative to the other one is zero. Since there is no relative motion between the cars, either driver will hear the other's horn at exactly the same frequency that would be heard if both cars were at rest.

---

17. ***REASONING AND SOLUTION*** According to Table 16.1, the speed of sound in air at 20 °C is v = 343 m/s, while its value in water at the same temperature is v = 1482 m/s. These values influence the Doppler effect, because of which an observer hears a frequency f_o that is different from the frequency f_s that is emitted by the source of sound. For purposes of this question, we assume that f_s = 1000 Hz and that the speed at which the source moves is v_s = 25 m/s. Our conclusions, however, will be valid for any values of f_s and v_s .

a. The Doppler-shifted frequency when the source approaches the observer is given by Equation 16.11 as $f_o = f_s \big/ 1 \quad (v_s / v)$. Applying this equation for air and water, we find

Air
$$f_o = \frac{1000 \text{ Hz}}{1 \quad (25 \text{ m/s})/(343 \text{ m/s})} = 1079 \text{ Hz}$$

Water
$$f_o = \frac{1000 \text{ Hz}}{1 \quad (25 \text{ m/s})/(1482 \text{ m/s})} = 1017 \text{ Hz}$$

The change in frequency due to the Doppler effect in air is greater.

b. The Doppler-shifted frequency when the source moves away from the observer is given by Equation 16.12 as $f_o = f_s \big/ 1 + (v_s / v)$. Applying this equation for air and water, we find

Air
$$f_o = \frac{1000 \text{ Hz}}{1 + (25 \text{ m/s})/(343 \text{ m/s})} = 932 \text{ Hz}$$

Water
$$f_o = \frac{1000 \text{ Hz}}{1 + (25 \text{ m/s})/(1482 \text{ m/s})} = 983 \text{ Hz}$$

The change in frequency due to the Doppler effect in air is greater.

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18. ***REASONING AND SOLUTION*** A music fan at a swimming pool is listening to a radio on a diving platform. The radio is playing a constant frequency tone when this fellow, clutching his radio, jumps.

a. The frequency heard by a person left behind on the platform is given by Equation 16.12: $f_o = f_s \big/ 1 + (v_s / v)$. Since the denominator, $1 + (v_s / v)$, is necessarily greater than one, the fraction in curly brackets is necessarily less than one. Therefore, f_o is less than f_s, and a person left behind on the platform will hear a frequency that is smaller than the frequency produced by the radio. As the radio falls, it accelerates, so its speed increases; as v_s increases, the quantity $1 + (v_s / v)$ also increases, and the quantity in the curly brackets decreases, so that f_o decreases with time. Therefore, the observed frequency is not constant and decreases during the fall.

b. The frequency heard by a person down below floating on a rubber raft is given by Equation 16.11: $f_o = f_s \big/ 1 \quad (v_s / v)$. We can assume that during the fall $v_s < v$ at all times so that the quantity v_s / v is less than one, and the denominator, $1 \quad (v_s / v)$, is necessarily less than one. Therefore, the fraction in curly brackets is necessarily greater than one, and f_o is greater than f_s. A person down below floating on a rubber raft will hear a

frequency that is greater than the frequency produced by the radio. As the radio falls, v_S increases as discussed in part (a), so that the quantity $1 \quad (v_s / v)$ decreases. The quantity in the curly brackets increases, so that f_o increases with time. Therefore, the observed frequency is not constant and increases during the fall.

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19. ***REASONING AND SOLUTION*** When a car is at rest, its horn emits a frequency of 600 Hz. A person standing in the middle of the street hears the horn with a frequency of 580 Hz. As shown in Figure 16.29, when a vehicle is moving, a stationary observer in front of the vehicle will hear sound of a shorter wavelength, while an observer behind the vehicle will hear sound of a longer wavelength. We can deduce, therefore, from Equation 16.1, $v = f\ l$, that the person standing in front of the vehicle will hear sound of a higher frequency than when the car is at rest, while a person standing behind the vehicle will hear sound of a lower frequency than when the car is at rest. Since the person standing in the middle of the street hears a frequency of 580 Hz, which is less than the stationary frequency of 600 Hz, the person is behind the car. Therefore, the person need not jump out of the way.

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20. ***REASONING AND SOLUTION*** A car is speeding toward a large wall and sounds the horn. The sound of the horn in front of the car will have a shorter wavelength and a higher frequency than the wavelength and frequency of the sound of the horn when the car is at rest. This Doppler shift occurs because the source is moving. The echo is produced when this higher-frequency sound is reflected from the wall. The wall now acts as a stationary source of this higher-frequency sound. The driver is an observer who is moving toward a stationary source. Since the driver is moving toward the echo, he will hear a frequency that is greater than the sound heard by a stationary observer. Thus, the echo heard by the driver is Doppler-shifted twice. The sound is shifted once to a higher frequency because the source of the sound is moving. Then the sound is Doppler shifted to a higher frequency again because the driver is moving relative to the stationary source (of the echo).

CHAPTER 17 *THE PRINCIPLE OF LINEAR SUPERPOSITION AND INTERFERENCE PHENOMENA*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The principle of linear superposition states that when two or more waves are present simultaneously at the same place, the resultant wave is the sum of the individual waves. This principle does *not* imply that two sound waves, passing through the same place at the same time, always create a louder sound than either wave alone. The resultant wave pattern depends on the relative phases of the two sound waves when they meet. If two sound waves arrive at the same place at the same time, and they are exactly in phase, then the two waves will interfere constructively and create a louder sound than either wave alone. On the other hand, if two waves arrive at the same place at the same time, and they are exactly out of phase, destructive interference will occur; the net effect is a mutual cancellation of the sound. If the two sound waves have the same amplitude and frequency, they will completely cancel each other and no sound will be heard.

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2. ***REASONING AND SOLUTION*** If you are sitting at the overlap point between the two speakers in Figure 17.4, the two sound waves reaching you are exactly out of phase. You hear no sound because of destructive interference. If one of the speakers is suddenly shut off, then only one sound wave will reach your ears. Since there is no other sound wave to interfere with this sound wave, you will hear the sound from the single speaker.

---

3. ***REASONING AND SOLUTION*** Consider the situation in Figure 17.3. If you walk along a line that is perpendicular to the line between the speakers and passes through the overlap point, you will always be equidistant from both speakers. Therefore the sound waves along this line always overlap exactly in phase. Hence, you will always hear the same loudness; you will not observe the loudness to change from loud to faint to loud.

   On the other hand, if you walk along a line that passes through the overlap point and is parallel to the line between the speakers, your distance from the two speakers will vary such that the difference in path lengths traveled by the two waves will vary. At certain points the path length difference between the two sound waves will be an integer number of wavelengths; constructive interference will occur at these points, and the sound intensity will be a maximum. At other points, the path length difference between the two sound waves will be an odd number of half-wavelengths [$(1/2)l$, $(3/2)l$, $(5/2)l$, etc.]; destructive interference will occur and the sound intensity will be a minimum. In between the points of constructive and destructive interference, the waves will be out of phase by varying degrees. Therefore, as you walk along this line, you will observe the sound intensity to alternate between faint and loud.

---

4. ***REASONING AND SOLUTION*** If the width of the speakers is D, then sound of wavelength l will diffract more readily if the ratio l / D is large. Since longer wavelengths correspond to lower frequencies, we see that lower-frequency sounds diffract more readily from a given speaker than higher-frequency sounds. Since the frequencies of the sounds of the female vocalists are higher than the frequencies of the sounds of the rhythmic bass, the sounds of the female vocalists will not diffract to the same extent as the sounds of the rhythmic bass. Thus, diffraction allows the bass tones to penetrate the regions to either side of the stage, but it does not permit the same help to the sounds of the female vocalists.

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5. ***REASONING AND SOLUTION*** When a wave encounters an obstacle or the edges of an opening, it bends or diffracts around them. The extent of the diffraction depends on the ratio of the wavelength l to the size D of the obstacle or opening. If the ratio l / D is small, little diffraction occurs. As the ratio l / D is made larger, the wave diffracts to a greater extent. In Example 1 in Section 16.2, it is shown that the wavelength of AM radio waves is 244 m, while the wavelength of FM radio waves is 3.26 m. For a given obstacle, the ratio l / D will be greater for AM radio waves; therefore AM radio waves will diffract more readily around a given obstacle than FM waves.

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6. ***REASONING AND SOLUTION*** A tuning fork has a frequency of 440 Hz. The string of a violin and this tuning fork, when sounded together, produce a beat frequency of 1 Hz. This beat frequency is the difference between the frequency of the tuning fork and the frequency of the violin string. A violin string of frequency 439 Hz, as well as a violin string of 441 Hz, will produce a beat frequency of 1 Hz, when sounded together with the 440 Hz tuning fork. We conclude that, from these two pieces of information alone, it is *not* possible to distinguish between these two possibilities. Therefore, it is *not* possible to determine the frequency of the violin string.

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7. ***REASONING AND SOLUTION*** Tuning forks vibrate at 438 Hz and 440 Hz. When sounded together, a listener hears a beat frequency of 2 Hz, the difference in frequency between the two forks. The forks are then vibrated underwater with the listener also underwater. The forks vibrate at 438 and 440 Hz, just as they do in air. The speed of sound, however, is four times faster in water than in air. From the relation $v = l f$, we see that the wavelength of the sound in water is four times longer than it is in air. Since the speed of the waves is four times greater than it is in air, the regions of constructive and destructive interference move past the ears of the listener four times more rapidly than they do in air. However, the regions of condensation and rarefaction in water are separated by four times the distance than they are in air. Therefore, at a given point in space, the amplitude variation of the resultant wave occurs at the same rate that it does in air. Consequently, the listener hears a beat frequency of 2 Hz, just as he does in air.

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8. ***REASONING AND SOLUTION*** The frequency of a guitar string is given by Equation 17.3, $f_n = n\left[v/(2L)\right]$, where v is the speed of the wave on the string, L is the distance between the two fixed ends of the guitar string, and n = 1, 2, 3, 4, . . . The speed v

of the wave is given by Equation 16.2, $v = \sqrt{F/(m/L)}$, where F is the tension in the guitar string, and m/L is the mass per unit length of the string.

If the tension in a guitar string is doubled, then, according to Equation 16.2, the speed of the wave on the string will increase by a factor of $\sqrt{2}$. According to Equation 17.3, the frequency of oscillation is directly proportional to the speed of the wave. Therefore, the frequency of oscillation will *not* double; rather, it will *increase by a factor of* $\sqrt{2}$.

---

9. ***REASONING AND SOLUTION*** A string is attached to a wall and vibrates back and forth as in Figure 17.18. The vibration frequency and length of the string are fixed. The tension in the string is changed. From Equation 16.2, $v = \sqrt{F/(m/L)}$, we see that increasing the tension F results in increasing the speed v of the waves on the string. From the relationship $v = l f$, we see that, since the frequency remains fixed, an increase in the wave speed results in an increase in the wavelength of the wave.

It is observed that at certain values of the tension, a standing wave pattern develops. Since the two ends of the string are "fixed," the ends of the string are nodes; thus, the length L of the string must contain an integer number of half-wavelengths. Therefore, standing waves will occur at the wavelengths $l_n = 2L/n$, where n = 1, 2, 3, 4, . . . The largest possible wavelength that will result in a standing wave pattern occurs when n = 1, and the wavelength is equal to twice the length of the string. If the tension is increased beyond the value for which l = 2L, the string cannot sustain a standing wave pattern.

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10. ***REASONING AND SOLUTION*** A string is being vibrated back and forth as in Figure 17.18*a*. The tension in the string is decreased by a factor of four, with the frequency and the length of the string remaining the same. From Equation 16.2, $v = \sqrt{F/(m/L)}$, we see that decreasing the tension F by a factor of four results in decreasing the wave speed v by a factor of $\sqrt{4}$ or 2. From the relationship $v = l f$, we see that, when the frequency f is fixed, decreasing the wave speed by a factor of 2 results in decreasing the wavelength l by a factor of 2. Therefore, when the tension in the string is decreased by a factor of 4, the wavelength of the resulting standing wave pattern will decrease by a factor of 2. In Figure 17.18*a*, the wavelength of the standing wave is equal to twice the length of the string. Since decreasing the tension reduces the wavelength by a factor of 2, the new wavelength is equal to the length of the string. With this new wavelength, the pattern shown below results.

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11. ***REASONING AND SOLUTION*** a. Since the bottom end of the rope is free to move, there is an antinode at the bottom.

b. Since the rope has mass, it has weight. The upper part of the rope supports more of the weight than the lower part does. Thus, the tension in the rope is greater near the top than near the bottom. The distance between successive nodes is one-half the wavelength of the waves traveling up and down the rope. But the wavelength is equal to the speed of the waves divided by the frequency, according to Equation 16.1. In turn, the speed is proportional to the square root of the tension, according to Equation 16.2. As a result, the wave speed is greater near the top than near the bottom and so is the wavelength. Since the separation between successive nodes is one-half the wavelength, it is also greater near the top than near the bottom.

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12. ***REASONING AND SOLUTION*** A relatively large sound can be created by blowing across the top of an empty soda bottle because standing waves develop in the bottle. The soda bottle behaves like a tube open only at one end. The frequency of standing waves in the bottle are, therefore, given by Equation 17.5: $f_n = nv/(4L)$, where v is the speed of sound in air, L is the length of the air column in the bottle, and n = 1, 3, 5, . . . When the bottle is partially filled with water, the length of the air column L is decreased. From Equation 17.5, the frequency of the standing wave in the bottle is increased. Therefore, the pitch of the sound becomes higher.

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13. ***REASONING AND SOLUTION*** Consider the tube at the bottom of Figure 17.23. We suppose that the air in the tube is replaced with a gas in which the speed of sound is twice what it is in air. If the frequency of the tuning fork remains unchanged, we see from the relation $v = l\,f$ that the wavelength of the sound will be twice as long in the gas as it is in air. The resulting pattern is shown in the drawing at the right.

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14. ***REASONING AND SOLUTION*** When a concert performer generates a 2093-Hz tone, the wavelength of the sound is

$$l = \frac{v}{f} = \frac{343 \text{ m/s}}{2093 \text{ Hz}} = 0.164 \text{ m or } 16.4 \text{ cm}.$$

If there is excessive reflection of the sound that the performer generates, a large amplitude standing wave can result. The distance between an antinode (maximum loudness) and the next adjacent node (zero loudness) on a standing wave is one-quarter of a wavelength. For a standing sound wave with a frequency of 2093 Hz, the distance between an antinode and the adjacent node is (16.4/4) cm or 4.1 cm. It would be possible, therefore, for a listener to move a distance of only 4.1 cm and hear the loudness of the tone change from loud to faint.

---

15. ***REASONING AND SOLUTION*** The speed of sound v in an ideal gas is given by Equation 16.5, $v = \sqrt{g\,kT/m}$, where $g = C_P / C_V$ is the ratio of the specific heat capacities, k is Boltzmann's constant, T is the Kelvin temperature, and m is the mass of a molecule of the gas. Thus, the speed of sound in an ideal gas is directly proportional to the square root of the Kelvin temperature T.

If the instrument is open at both ends, then the frequency of sound produced by the instrument is given by Equation 17.4, $f_n = n\left[v/(2L)\right]$, where v is the speed of sound, L is the distance between the two open ends of the instrument, and $n = 1, 2, 3, 4, \ldots$ Similarly, if the instrument is open at only one end and closed at the other end, the frequency of sound produced by the instrument is given by Equation 17.5, $f_n = n\left[v/(4L)\right]$, where v is the speed of sound, L is the distance between the open and closed ends of the instrument, and $n = 1, 3, 5, 7, \ldots$

When a wind instrument is brought inside from the cold outdoors, its frequency will change because the temperature inside is higher than the temperature outside. If we can treat air as an ideal gas, then according to Equation 16.5, the speed of sound will increase because T increases. According to both Equations 17.4 and 17.5, the frequency produced by a wind instrument is directly proportional to v, the speed of sound; therefore, since the speed of sound increases with increasing temperature, the frequency of the sound produced by the instrument will also increase.

---

16. ***REASONING AND SOLUTION*** Tones produced by a typical orchestra are complex sound waves, and most have fundamental frequencies less than 5000 Hz. Since these tones are complex sound waves, each one consists of a mixture of the fundamental and the higher harmonic frequencies. The higher harmonics are integer multiples of the fundamental frequency. For example, in a tone with a fundamental frequency of 5000 Hz, the fourth harmonic will have a frequency of 20 000 Hz. Since most orchestra tones have fundamental frequencies that are less than 5000 Hz, a high-quality stereo system must be able to reproduce accurately all frequencies up to 20 000 Hz in order to include at least the fourth harmonic of all tones.

INSTRUCTOR'S SOLUTIONS MANUAL

to accompany

Physics Fifth Edition

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Volume 2

Chapters 18 32

PREFACE for VOLUME 2

This volume contains the complete solutions to the problems and conceptual questions for chapters 18 through 32. These chapters include electricity and magnetism, light and optics, and modern physics. The solutions for chapters 1 through 17 [mechanics (including fluids), thermal physics, and wave motion] are contained in Volume 1.

The manual is organized so that the solutions to the problems are in the front part of the book (part A), while those for the conceptual questions are in the back (part B).

An electronic version of this manual is also available on the Instructor Resource CD. The files are available in two formats: Microsoft Word, and PDF files of each individual solution.

There are two types of icons at the beginning of some problems. The SSM and WWW icons indicate, respectively, that the solution is also available in the Student Solutions Manual and on the World Wide Web at http://www.wiley.com/college/cutnell.

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1. **SSM** ***REASONING AND SOLUTION*** The charge on a single electron is -1.60×10^{-19} C. In order to give a neutral silver dollar a charge of +2.4 μC, we must remove an amount of negative charge equal to –2.4 μC. This corresponds to

$$\left(-2.4 \times 10^{-6}\ \text{C}\right)\left(\frac{1\ \text{electron}}{-1.60 \times 10^{-19}\ \text{C}}\right) = \boxed{1.5 \times 10^{13}\ \text{electrons}}$$

2. ***REASONING AND SOLUTION*** The total charge to be removed is – 5.0 μC. The number of electrons corresponding to this charge is

$$N = (-5.0 \times 10^{-6}\ \text{C})/(-1.60 \times 10^{-19}\text{C}) = \boxed{3.1 \times 10^{13}}$$

3. ***REASONING AND SOLUTION*** The total charge of the electrons is

$$q = N(-e) = (6.0 \times 10^{13})(-1.60 \times 10^{-19}\ \text{C})$$

$$q = -9.6 \times 10^{-6}\ \text{C} = -9.6\ \mu\text{C}$$

The net charge on the sphere is, therefore,

$$q_{net} = +8.0\ \mu\text{C} - 9.6\ \mu\text{C} = \boxed{-1.6\ \mu\text{C}}$$

---

4. ***REASONING AND SOLUTION*** Object A is negative because of an *excess* of electrons, while object B is positive because of a *deficiency* of electrons. The mass difference between the charged objects arises because the mass of A is greater due to the additional electrons, while the mass of object B is less due to the loss of electrons. Since $q = Ne$, where e is the magnitude of the charge on one electron, the number of excess electrons on object A is

$$N = \frac{q}{e} = \frac{3.0 \times 10^{-6}\ \text{C}}{1.60 \times 10^{-19}\ \text{C}} = 1.9 \times 10^{13}$$

This corresponds to an increase in mass given by

$$\Delta m = \left(1.9 \times 10^{13}\ \text{electrons}\right)\left(\frac{9.11 \times 10^{-31}\ \text{kg}}{1\ \text{electron}}\right) = 1.7 \times 10^{-17}\ \text{kg}$$

Since both objects carry charge of the same magnitude, N is also equal to the number of electrons lost by object B. Hence, the mass of B is reduced by the amount Δm. If M is the mass of either object when they are electrically neutral, then the mass of each charged object is:

$$M_A = M + \Delta m \qquad \text{while} \qquad M_B = M - \Delta m$$

The mass difference between the charged objects is, therefore,

$$\Delta M = M_A - M_B = (M + \Delta m) - (M - \Delta m)$$

$$= 2\Delta m = 2(1.7 \times 10^{-17}\ \text{kg}) = \boxed{3.4 \times 10^{-17}\ \text{kg}}$$

From the discussion above, $\boxed{\text{object A has the larger mass}}$.

5. **SSM** ***REASONING*** Identical conducting spheres equalize their charge upon touching. When spheres A and B touch, an amount of charge $+q$, flows from A and instantaneously neutralizes the $-q$ charge on B leaving B momentarily neutral. Then, the remaining amount of charge, equal to $+4q$, is equally split between A and B, leaving A and B each with equal amounts of charge $+2q$. Sphere C is initially neutral, so when A and C touch, the $+2q$ on A splits equally to give $+q$ on A and $+q$ on C. When B and C touch, the $+2q$ on B and the $+q$ on C combine to give a total charge of $+3q$, which is then equally divided between the spheres B and C; thus, B and C are each left with an amount of charge $+1.5q$.

SOLUTION Taking note of the initial values given in the problem statement, and summarizing the final results determined in the *Reasoning* above, we conclude the following:

a. Sphere C ends up with an amount of charge equal to $\boxed{+1.5q}$.

b. The charges on the three spheres before they were touched, are, according to the problem statement, $+5q$ on sphere A, $-q$ on sphere B, and zero charge on sphere C. Thus, the total charge on the spheres is $+5q - q + 0 = \boxed{+4q}$.

c. The charges on the spheres after they are touched are $+q$ on sphere A, $+1.5q$ on sphere B, and $+1.5q$ on sphere C. Thus, the total charge on the spheres is $+q + 1.5q + 1.5q = \boxed{+4q}$.

6. ***REASONING***

a. The number N of electrons is 10 times the number of water molecules in 1 liter of water. The number of water molecules is equal to the number n of moles of water molecules times Avogadro's number N_A: $N = 10\,n\,N_A$.

b. The net charge of all the electrons is equal to the number of electrons times the change on one electron.

SOLUTION

a. The number N of water molecules is equal to $10\ n\ N_A$, where n is the number of moles of water molecules and N_A is Avogadro's number. The number of moles is equal to the mass m of 1 liter of water divided by the mass per mole of water. The mass of water is equal to its density ρ times the volume, as expressed by Equation 11.1. Thus, the number of electrons is

$$N = 10\ n\ N_A = 10\left(\frac{m}{18.0\ \text{g/mol}}\right)N_A = 10\left(\frac{\rho\ V}{18.0\ \text{g/mol}}\right)N_A$$

$$= 10\left[\frac{\left(1000\ \text{kg/m}^3\right)\left(1.00\times10^{-3}\ \text{m}^3\right)\left(\frac{1000\ \text{g}}{1\ \text{kg}}\right)}{18.0\ \text{g/mol}}\right]\left(6.022\times10^{23}\ \text{mol}^{-1}\right)$$

$$= \boxed{3.35\times10^{26}\ \text{electrons}}$$

b. The net charge Q of all the electrons is equal to the number of electrons times the change on one electron: $Q = \left(3.35\times10^{26}\right)\left(-1.60\times10^{-19}\ \text{C}\right) = \boxed{-5.36\times10^{7}\ \text{C}}$.

7. **SSM** **WWW** ***REASONING*** Initially, the two spheres are neutral. Since negative charge is removed from the sphere which loses electrons, it then carries a net positive charge. Furthermore, the neutral sphere to which the electrons are added is then negatively charged. Once the charge is transferred, there exists an electrostatic force on each of the two spheres, the magnitude of which is given by Coulomb's law (Equation 18.1), $F = kq_1q_2/r^2$.

SOLUTION

a. Since each electron carries a charge of -1.60×10^{-19} C, the amount of negative charge removed from the first sphere is

$$\left(3.0\times10^{13}\ \text{electrons}\right)\left(\frac{1.60\times10^{-19}\ \text{C}}{1\ \text{electron}}\right) = 4.8\times10^{-6}\ \text{C}$$

Thus, the first sphere carries a charge $+4.8 \times 10^{-6}$ C, while the second sphere carries a charge -4.8×10^{-6} C. The magnitude of the electrostatic force that acts on each sphere is, therefore,

$$F = \frac{kq_1q_2}{r^2} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(4.8\times10^{-6}\ \text{C})^2}{(0.50\ \text{m})^2} = \boxed{0.83\ \text{N}}$$

b. Since the spheres carry charges of opposite sign, the force is $\boxed{\text{attractive}}$.

8. ***REASONING AND SOLUTION*** The electric force exerted on each proton can be obtained from Coulomb's law

$$F = \frac{k\,q_1 q_2}{r^2} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)\left(1.60\times10^{-19}\ \mathrm{C}\right)}{\left(3.0\times10^{-15}\,\mathrm{m}\right)^2} = \boxed{26\ \mathrm{N}}$$

9. ***REASONING AND SOLUTION*** The electrostatic forces decreases with the square of the distance separating the charges. If this distance is increased by a factor of 5 then the force will decrease by a factor of 25. The new force is, then,

$$F = \frac{3.5\text{ N}}{25} = \boxed{0.14\text{ N}}$$

10. ***REASONING*** The magnitude of the electrostatic force that acts on particle 1 is given by Coulomb's law as $F = k\,q_1\,q_2 / r^2$. This equation can be used to find the magnitude q_2 of the charge.

SOLUTION Solving Coulomb's law for the magnitude q_2 of the charge gives

$$q_2 = \frac{F\,r^2}{k\,q_1} = \frac{(3.4\ \text{N})(0.26\ \text{m})^2}{\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.5\times10^{-6}\ \text{C}\right)} = \boxed{7.3\times10^{-6}\ \text{C}} \tag{18.1}$$

Since q_1 is positive and experiences an attractive force, the charge q_2 must be $\boxed{\text{negative}}$.

11. **SSM** ***REASONING AND SOLUTION*** The net electrostatic force on charge 3 at $x = +3.0$ m is the vector sum of the forces on charge 3 due to the other two charges, 1 and 2. According to Coulomb's law (Equation 18.1), the magnitude of the force on charge 3 due to charge 1 is

Figure 1

$$F_{13} = \frac{kq_1 q_3}{r_{13}^2}$$

where the distance between charges 1 and 3 is r_{13}.

According to the Pythagorean theorem, $r_{13}^2 = x^2 + y^2$. Therefore,

$$F_{13} = \frac{(8.99 \times 10^9 \ \text{N} \cdot \text{m}^2/\text{C}^2)(18 \times 10^{-6}\ \text{C})(45 \times 10^{-6}\ \text{C})}{(3.0\ \text{m})^2 + (3.0\ \text{m})^2} = 0.405\ \text{N}$$

Charges 1 and 3 are equidistant from the origin, so that $\theta = 45°$ (see Figure 1). Since charges 1 and 3 are both positive, the force on charge 3 due to charge 1 is repulsive and along the line that connects them, as shown in Figure 2. The components of F_{13} are:

$$F_{13x} = F_{13} \cos 45° = 0.286\ \text{N} \qquad \text{and} \qquad F_{13y} = -F_{13} \sin 45° = -0.286\ \text{N}$$

The second force on charge 3 is the attractive force (opposite signs) due to its interaction with charge 2 located at the origin. The magnitude of the force on charge 3 due to charge 2 is, according to Coulomb's law ,

Figure 2

$$F_{23} = \frac{kq_2 q_3}{r_{23}^2} = \frac{kq_2 q_3}{x^2}$$

$$= \frac{(8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2)(12 \times 10^{-6}\ \text{C})(45 \times 10^{-6}\ \text{C})}{(3.0\ \text{m})^2}$$

$$= 0.539\ \text{N}$$

Since charges 2 and 3 have opposite signs, they attract each other, and charge 3 experiences a force to the left as shown in Figure 2. Taking up and to the right as the positive directions, we have

$$F_{3x} = F_{13x} + F_{23x} = +0.286\ \text{N} - 0.539\ \text{N} = -0.253\ \text{N}$$

$$F_{3y} = F_{13y} = -0.286\ \text{N}$$

Using the Pythagorean theorem, we find the magnitude of F_3 to be

$$F_3 = \sqrt{F_{3x}^2 + F_{3y}^2} = \sqrt{(-0.253\ \text{N})^2 + (-0.286\ \text{N})^2} = \boxed{0.38\ \text{N}}$$

Figure 3

The direction of F_3 relative to the $-x$ axis is specified by the angle ϕ, where

$$\phi = \tan^{-1}\left(\frac{0.286\ \text{N}}{0.253\ \text{N}}\right) = \boxed{49^\circ \text{ below the } -x \text{ axis}}$$

12. ***REASONING AND SOLUTION*** Calculate the magnitude of each force acting on the center charge. Using Coulomb's law, we can write

$$F_{43} = \frac{k\,q_4 q_3}{r_{43}^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00\times10^{-6}\ \text{C}\right)\left(3.00\times10^{-6}\ \text{C}\right)}{\left(0.100\ \text{m}\right)^2}$$

$$= 10.8\ \text{N}\ \ \text{(toward the south)}$$

$$F_{53} = \frac{k\,q_5 q_3}{r_{53}^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.00\times10^{-6}\ \text{C}\right)\left(3.00\times10^{-6}\ \text{C}\right)}{\left(0.100\ \text{m}\right)^2}$$

$$= 13.5\ \text{N}\ \ \text{(toward the east)}$$

Adding F_{43} and F_{53} vectorially, we have

$$F = \sqrt{F_{43}^2 + F_{53}^2} = \sqrt{\left(10.8\ \text{N}\right)^2 + \left(13.5\ \text{N}\right)^2} = \boxed{17.3\ \text{N}}$$

$$\theta = \tan^{-1}\left(\frac{F_{43}}{F_{53}}\right) = \tan^{-1}\left(\frac{10.8\ \text{N}}{13.5\ \text{N}}\right) = \boxed{38.7^\circ\ \text{S of E}}$$

13. ***REASONING AND SOLUTION*** Place the –9.0 μC charge at the apex of the triangle (north), the +8.0 μC charge at the lower left corner (west), and the +2.0 μC charge at the lower right (east). Find the forces on the +2.0 μC charge due to the –9.0 μC and +8.0 μC charges.

$$F_{92} = \frac{kq_9q_2}{r_{92}^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(9.0\times10^{-6}\ \text{C})(2.0\times10^{-6}\ \text{C})}{(0.15\ \text{m})^2}$$

$$F_{92} = 7.2\ \text{N, at } 60.0°\ \text{N of W}$$

The components are

$$F_{92x} = F_{92}\cos 60.0° = 3.6\ \text{N (due west)}$$

$$F_{92y} = F_{92}\sin 60.0° = 6.2\ \text{N (due north)}$$

Similarly,

$$F_{82} = \frac{kq_8q_2}{r_{82}^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.0\times10^{-6}\ \text{C})(2.0\times10^{-6}\ \text{C})}{(0.15\ \text{m})^2}$$

$$F_{92} = 6.4\ \text{N, directed due east}$$

The resultant force on the + 2.0 μC charge is obtained from

$$F_2 = \sqrt{F_x^2 + F_y^2} = \sqrt{(6.4\ \text{N} - 3.6\ \text{N})^2 + (6.2\ \text{N})^2} = \boxed{6.8\ \text{N}}$$

14. ***REASONING***

a. The magnitude of the electrostatic force that acts on each sphere is given by Coulomb's law as $F = k\,q_1\,q_2 / r^2$, where q_1 and q_2 are the magnitudes of the charges, and r is the distance between the centers of the spheres.

b. When the spheres are brought into contact, the net charge after contact and separation must be equal to the net charge before contact. Since the spheres are identical, the charge on each after being separated is one-half the net charge. Coulomb's law can be applied again to determine the magnitude of the electrostatic force that each sphere experiences.

SOLUTION

a. The magnitude of the force that each sphere experiences is given by Coulomb's law as:

$$F = \frac{k\,q_1\,q_2}{r^2} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(20.0\times10^{-6}\ \mathrm{C}\right)\left(50.0\times10^{-6}\ \mathrm{C}\right)}{\left(2.50\times10^{-2}\ \mathrm{m}\right)^2} = \boxed{1.44\times10^{4}\ \mathrm{N}}$$

Because the charges have opposite signs, the force is $\boxed{\text{attractive}}$.

b. The net charge on the spheres is $-20.0\ \mu\mathrm{C} + 50.0\ \mu\mathrm{C} = +30.0\ \mu\mathrm{C}$. When the spheres are brought into contact, the net charge after contact and separation must be equal to the net charge before contact, or $+30.0\ \mu\mathrm{C}$. Since the spheres are identical, the charge on each after being separated is one-half the net charge, so $q_1 = q_2 = +15.0\ \mu\mathrm{C}$. The electrostatic force that acts on each sphere is now

$$F = \frac{k\,q_1\,q_2}{r^2} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(15.0\times10^{-6}\ \mathrm{C}\right)\left(15.0\times10^{-6}\ \mathrm{C}\right)}{\left(2.50\times10^{-2}\ \mathrm{m}\right)^2} = \boxed{3.24\times10^{3}\ \mathrm{N}}$$

Since the charges now have the same signs, the force is $\boxed{\text{repulsive}}$.

15. SSM WWW ***REASONING*** Each particle will experience an electric force due to the presence of the other charge. According to Coulomb's law (Equation 18.1), the magnitude of the force felt by each particle can be calculated from $F = kq_1q_2/r^2$, where q_1 and q_2 are the respective charges on particles 1 and 2 and r is the distance between them. According to Newton's second law, the magnitude of the force experienced by each particle is given by $F = ma$, where a is the acceleration of the particle.

SOLUTION

a. Since the two particles have identical positive charges, $q_1 = q_2 = q$, and we have, using the data for particle 1,

$$\frac{kq^2}{r^2} = m_1a_1$$

Solving for q, we find that

$$q = \sqrt{\frac{m_1a_1r^2}{k}} = \sqrt{\frac{(6.00\times10^{-6}\ \text{kg})\,(4.60\times10^{3}\ \text{m/s}^2)\,(2.60\times10^{-2}\ \text{m})^2}{8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{4.56\times10^{-8}\ \text{C}}$$

b. Since each particle experiences a force of the same magnitude (From Newton's third law), we can write $F_1 = F_2$, or $m_1a_1 = m_2a_2$. Solving this expression for the mass m_2 of particle 2, we have

$$m_2 = \frac{m_1a_1}{a_2} = \frac{(6.00\times10^{-6}\ \text{kg})(4.60\times10^{3}\ \text{m/s}^2)}{8.50\times10^{3}\ \text{m/s}^2} = \boxed{3.25\times10^{-6}\ \text{kg}}$$

16. ***REASONING*** The unknown charge q must be positive. To see why, consider the unknown charge at the upper right corner in the drawing at the right (the unknown charge at the lower left corner could also be used). Three forces act on this charge: (1) F_O is the repulsive force due to the other unknown charge on the opposite corner, (2) F_B is the attractive force due to the negative charge at the lower right corner, and (3) F_L is the attractive force due to the negative charge at the upper left corner. These three forces add to give a net force of zero. The unknown charge can not be negative, because then F_B and F_L would have directions opposite to those shown in the shown in the drawing, and the sum of F_O, F_B, and F_L could not be zero. We note that the magnitudes of F_B and F_L are equal according to Coulomb's law (Equation 18.1), since the sides of the square have equal lengths and the charge magnitudes are q and 0.70 μC in each case. We also note that the directions of F_B and F_L are perpendicular. Thus, the resultant of F_B and F_L is given by the Pythagorean theorem and points along the diagonal of the square, directly opposite to the direction of F_O. Since the vector sum of F_O, F_B, and F_L is zero, the magnitude of the resultant of F_B and F_L must equal the magnitude of F_O, and it is with this fact in mind that we begin our solution.

SOLUTION Using the Pythagorean to express the magnitude of the resultant of F_B and F_L, which is equal to F_O, we have

$$\sqrt{F_B^2 + F_L^2} = F_O \qquad (1)$$

Coulomb's law indicates that

$$F_B = F_L = \frac{k\left(0.70 \times 10^{-6}\ \text{C}\right)q}{d^2} \qquad \text{and} \qquad F_O = \frac{kq^2}{\left(\sqrt{2}\ d\right)^2} = \frac{kq^2}{2d^2}$$

where we have used d for the length of a side of the square and the fact that the diagonal of the square has a length of $\sqrt{d^2 + d^2} = \sqrt{2}\ d$. Substituting these expressions for F_O, F_B, and F_L into Equation (1), we find

$$\sqrt{\left[\frac{k\left(0.70 \times 10^{-6}\ \text{C}\right)q}{d^2}\right]^2 + \left[\frac{k\left(0.70 \times 10^{-6}\ \text{C}\right)q}{d^2}\right]^2} = \sqrt{2\left[\frac{k\left(0.70 \times 10^{-6}\ \text{C}\right)q}{d^2}\right]^2} = \frac{kq^2}{2d^2}$$

Simplifying this result shows that

$$\frac{\sqrt{2}\ k\left(0.70 \times 10^{-6}\ \text{C}\right)q}{d^2} = \frac{kq^2}{2d^2} \qquad \text{or} \qquad q = 2\sqrt{2}\left(0.70 \times 10^{-6}\ \text{C}\right) = \boxed{+2.0\ \mu\text{C}}$$

17. ***REASONING AND SOLUTION*** The new force acting on each of the charges A and B is

$$F = kq_A q_B / r^2$$

where

$$q_A + q_B = 0$$

since the net charge on the system is zero. We want the net force to equal 45.0 N, and can write

$$F = kq_A{}^2 / r^2$$

so

$$q_A = r\sqrt{\frac{F}{k}} = (0.0200\ \text{m})\sqrt{\frac{45.0\ \text{N}}{8.99 \times 10^9\ \text{N} \cdot \text{m}^2/\text{C}^2}}$$

$$= 1.41 \times 10^{-6}\ \text{C}$$

Therefore, we need to remove –0.41 μC of charge from A and give it to B. The number of electrons will then be

$$N = q/(-e) = (-0.41 \times 10^{-6}\ \text{C})/(-1.60 \times 10^{-19}\ \text{C}) = \boxed{2.6 \times 10^{12}}.$$

18. ***REASONING***

a. There are two electrostatic forces that act on q_1; that due to q_2 and that due to q_3. The magnitudes of these forces can be found by using Coulomb's law. The magnitude and direction of the net force that acts on q_1 can be determined by using the method of vector components.

b. According to Newton's second law, Equation 4.2b, the acceleration of q_1 is equal to the net force divided by its mass. However, there is only one force acting on it, so this force is the net force.

SOLUTION

a. The magnitude F_{12} of the force exerted on q_1 by q_2 is given by Coulomb's law, Equation 18.1, where the distance is specified in the drawing:

$$F_{12} = \frac{k\,q_1 q_2}{r_{12}^2} = \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(8.00\times10^{-6}\ \mathrm{C}\right)\left(5.00\times10^{-6}\ \mathrm{C}\right)}{\left(1.30\ \mathrm{m}\right)^2} = 0.213\ \mathrm{N}$$

Since the magnitudes of the charges and the distances are the same, the magnitude of $\mathbf{F}_{13}$ is the same as the magnitude of $\mathbf{F}_{12}$, or F_{13} = 0.213 N. From the drawing it can be seen that the x-components of the two forces cancel, so we need only to calculate the y components of the forces.

Force	y component
$\mathbf{F}_{12}$	$+F_{12}\sin 23.0° = +(0.213\ \mathrm{N})\sin 23.0° = +0.0832\ \mathrm{N}$
$\mathbf{F}_{13}$	$+F_{13}\sin 23.0° = +(0.213\ \mathrm{N})\sin 23.0° = +0.0832\ \mathrm{N}$
$\mathbf{F}$	$F_y = +0.166\ \mathrm{N}$

Thus, the net force is $\boxed{\mathbf{F} = +0.166\ \mathrm{N}\ \text{(directed along the } +y \text{ axis)}}$.

b. According to Newton's second law, Equation 4.2b, the acceleration of q_1 is equal to the net force divided by its mass. However, there is only one force acting on it, so this force is the net force:

$$\mathbf{a} = \frac{\mathbf{F}}{m} = \frac{+0.166\ \mathrm{N}}{1.50\times10^{-3}\ \mathrm{kg}} = \boxed{+111\ \mathrm{m/s^2}}$$

where the plus sign indicates that $\boxed{\text{the acceleration is along the +y axis}}$.

19. **SSM** ***REASONING AND SOLUTION*** Before the spheres have been charged, they exert no forces on each other. After the spheres are charged, each sphere experiences a repulsive force F due to the charge on the other sphere, according to Coulomb's law (Equation 18.1). Therefore, since each sphere has the same charge, the magnitude F of this force is

$$F = \frac{kq_1q_2}{r^2} = \frac{(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2)(1.60 \times 10^{-6} \text{ C})^2}{(0.100 \text{ m})^2} = 2.30 \text{ N}$$

The repulsive force on each sphere compresses the spring to which it is attached. The magnitude of this repulsive force is related to the amount of compression by Equation 10.1: $F = kx$. Therefore, solving for k, we find

$$k = \frac{F}{x} = \frac{2.30 \text{ N}}{0.0250 \text{ m}} = \boxed{92.0 \text{ N/m}}$$

20. ***REASONING AND SOLUTION*** The magnitude of the force exerted on each charge is given by Coulomb's law, $F = kq_1q_2/r^2$. In this case we also have the constraint that

$$q_1 + q_2 = q = 9.00\ \mu C \qquad \text{or} \qquad q_1 = q - q_2$$

Using this in Coulomb's law, we can eliminate q_1 and solve for q_2, i.e.,

$$F = k(q - q_2)q_2/r^2$$

gives

$$q_2^{\ 2} - qq_2 + Fr^2/k = 0$$

where $F = 8.00 \times 10^{-3}$ N, and $r = 3.00$ m. Solving the above quadratic equation for q_2 we obtain;

$$q_2 = \boxed{1.00 \times 10^{-6}\ C} \quad \text{and} \quad q_1 = \boxed{8.00 \times 10^{-6}\ C}$$

21. ***REASONING*** This is a problem that deals with motion in a circle of radius r. As Chapter 5 discusses, a centripetal force acts on the plane to keep it on its circular path. The centripetal force F_c is the name given to the net force that acts on the plane in the radial direction and points toward the center of the circle. When there are no electric charges present, only the tension in the guideline supplies this force, and it has a value T_{max} at the moment the line breaks. However, when there is a charge of +q on the plane and a charge of –q on the guideline at the center of the circle, there are two contributions to the centripetal force. One is the electrostatic force of attraction between the charges and is given by Coulomb's law (Equation 18.1) as $F = kq^2/r^2$. The other is the tension T_{max}, which is characteristic of the rope and has the same value as when no charges are present. Whether or not charges are present, the centripetal force is equal to the mass m times the centripetal acceleration, according to Newton's second law and stated in Equation 5.3, $F_c = mv^2/r$. In this expression v is the speed of the plane. Since we are given information about the plane's kinetic energy, we will use the definition of kinetic energy, which is KE = $mv^2/2$, according to Equation 6.2.

SOLUTION From the definition of kinetic energy, we see that $mv^2 = 2(\text{KE})$, so that Equation 5.3 for the centripetal force becomes

$$F_c = \frac{mv^2}{r} = \frac{2(\text{KE})}{r}$$

Applying this result to the situations with and without the charges, we get

$$\underbrace{T_{max} + \frac{kq^2}{r^2}}_{\text{Centripetal force}} = \frac{2(\text{KE})_{\text{charged}}}{r} \qquad (1) \qquad\qquad \underbrace{T_{max}}_{\text{Centripetal force}} = \frac{2(\text{KE})_{\text{uncharged}}}{r} \qquad (2)$$

Subtracting Equation (2) from Equation (1) eliminates T_{max} and gives

$$\frac{kq^2}{r^2} = \frac{2\left[(\text{KE})_{\text{charged}} - (\text{KE})_{\text{uncharged}}\right]}{r}$$

Solving for q gives

$$q = \sqrt{\frac{2r\left[(\text{KE})_{\text{charged}} - (\text{KE})_{\text{uncharged}}\right]}{k}} = \sqrt{\frac{2(3.0\text{ m})(51.8\text{ J} - 50.0\text{ J})}{8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = \boxed{3.5\times10^{-5}\text{ C}}$$

22. ***REASONING AND SOLUTION*** Assume that before the objects are touched that the left object has a negative charge of magnitude q_1 and the right object has a charge q_2. The force between them then has a magnitude of

$$F = kq_1q_2/r^2$$

After touching the charge on each object is the same and of magnitude $(q_2 - q_1)/2$. The magnitude of the force between the objects is now

$$F = k[(q_2 - q_1)/2]^2/r^2$$

Equating the equations and rearranging gives

$$q_2^{\ 2} - 6q_1q_2 + q_1^{\ 2} = 0$$

The solutions to this quadratic equation are

$$q_1 = 5.58\ \mu C,\ \ q_2 = 0.957\ \mu C \quad \text{and} \quad q_1 = 0.957\ \mu C,\ \ q_2 = 5.58\ \mu C$$

The charge, q_1, was assumed negative, so the possible solutions are

$\boxed{-5.58\ \mu C \text{ on left},\ \ +0.957\ \mu C \text{ on right}}$ and $\boxed{-0.957\ \mu C \text{ on left},\ \ +5.58\ \mu C \text{ on right}}$

23. **SSM** ***REASONING*** The charged insulator experiences an electric force due to the presence of the charged sphere shown in the drawing in the text. The forces acting on the insulator are the downward force of gravity (i.e., its weight, $W = mg$), the electrostatic force $F = kq_1q_2/r^2$ (see Coulomb's law, Equation 18.1) pulling to the right, and the tension T in the wire pulling up and to the left at an angle θ with respect to the vertical as shown in the drawing in the problem statement. We can analyze the forces to determine the desired quantities θ and T.

SOLUTION.

a. We can see from the diagram that

$$T_x = F \qquad \text{which gives} \qquad T\sin\theta = kq_1q_2/r^2$$

and

$$T_y = W \qquad \text{which gives} \qquad T\cos\theta = mg$$

Dividing the first equation by the second yields $(T\sin\theta)/(T\cos\theta) = \tan\theta = kq_1q_2/(mgr^2)$.

Solving for θ, we find that

$$\theta = \tan^{-1}\left(\frac{kq_1q_2}{mgr^2}\right)$$

$$= \tan^{-1}\left[\frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(0.600\times10^{-6}\ \text{C})(0.900\times10^{-6}\ \text{C})}{(8.00\times10^{-2}\ \text{kg})(9.80\ \text{m/s}^2)(0.150\ \text{m})^2}\right] = \boxed{15.4°}$$

b. Since $T\cos\theta = mg$, the tension can be obtained as follows:

$$T = \frac{mg}{\cos\theta} = \frac{(8.00\times10^{-2}\ \text{kg})\ (9.80\ \text{m/s}^2)}{\cos 15.4°} = \boxed{0.813\ \text{N}}$$

24. ***REASONING AND SOLUTION***

a. To find the charge on each ball we first need to determine the electric force acting on each ball. This can be done by noting that each thread makes an angle of 18° with respect to the vertical.

$$F_e = mg \tan 18° = (8.0 \times 10^{-4}\ \text{kg})(9.80\ \text{m/s}^2) \tan 18° = 2.547 \times 10^{-3}\ \text{N}$$

We also know that

$$F_e = kq_1q_2/r^2 = kq^2/r^2$$

where r = 2(0.25 m) sin 18° = 0.1545 m. Now

$$q = r\sqrt{\frac{F_e}{k}} = (0.1545\ \text{m})\sqrt{\frac{2.547 \times 10^{-3}\ \text{N}}{8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{8.2 \times 10^{-8}\ \text{C}}$$

b. The tension is due to the combination of the weight of the ball and the electric force, the two being perpendicular to one another. The tension is therefore,

$$T = \sqrt{(mg)^2 + F_e^2}$$

$$= \sqrt{\left[\left(8.0 \times 10^{-4}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\right]^2 + \left(2.547 \times 10^{-3}\ \text{N}\right)^2} = \boxed{8.2 \times 10^{-3}\ \text{N}}$$

---

25. ***SOLUTION*** Knowing the electric field at a spot allows us to calculate the force that acts on a charge placed at that spot, without knowing the nature of the object producing the field. This is possible because the electric field is defined as $\mathbf{E} = \mathbf{F}/q_0$, according to Equation 18.2. This equation can be solved directly for the force **F**, if the field **E** and charge q_0 are known.

SOLUTION Using Equation 18.2, we find that the force has a magnitude of

$$F = Eq = (260\ 000\ \text{N/C})\left(7.0 \times 10^{-6}\ \text{C}\right) = 1.8\ \text{N}$$

If the charge were positive, the direction of the force would be due west, the same as the direction of the field. But the charge is negative, so the force points in the opposite direction or due east. Thus, the force on the charge is $\boxed{\text{1.8 N due east}}$.

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26. ***REASONING AND SOLUTION*** The electric field lines must originate on the positive charges and terminate on the negative charges. They cannot cross one another. Furthermore, the number of field lines beginning or ending on any charge must be proportional to the magnitude of the charge. If 10 electric field lines leave the +5q charge, then six lines must originate from the +3q charge, and eight lines must end on each –4q charge. The drawing shows the electric field lines that meet these criteria.

27. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The force **F** exerted on a charge q_0 placed in an electric field **E** can be determined from Equation 18.2, the definition of electric field ($\mathbf{E} = \mathbf{F}/q_0$). Writing this in terms of magnitudes, and taking due east as the positive direction, we have, solving for F,

$$F = q_0 E = (+3.0 \times 10^{-5}\ \text{C})(+15\ 000\ \text{N/C}) = \boxed{+0.45\ \text{N}}$$

where the plus sign indicates that the force on the charge is directed $\boxed{\text{due east}}$.

28. ***REASONING***

a. The drawing shows the two point charges q_1 and q_2. Point A is located at $x = 0$ cm, and point B is at $x = +6.0$ cm.

Since q_1 is positive, the electric field points away from it. At point A, the electric field E_1 points to the left, in the $-x$ direction. Since q_2 is negative, the electric field points toward it. At point A, the electric field E_2 points to the right, in the $+x$ direction. The net electric field is $E = -E_1 + E_2$. We can use Equation 18.3, $E = kq/r^2$, to find the magnitude of the electric field due to each point charge.

b. The drawing shows the electric field produced by the charges q_1 and q_2 at point B, which is located at $x = +6.0$ cm.

Since q_1 is positive, the electric field points away from it. At point B, the electric field points to the right, in the $+x$ direction. Since q_2 is negative, the electric field points toward it. At point B, the electric field points to the right, in the $+x$ direction. The net electric field is $E = +E_1 + E_2$.

SOLUTION

a. The net electric field at the origin (point A) is $E = -E_1 + E_2$:

$$E = -E_1 + E_2 = \frac{-k\,q_1}{r_1^2} + \frac{k\,q_2}{r_2^2}$$

$$= \frac{-\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(8.5\times10^{-6}\ \mathrm{C}\right)}{\left(3.0\times10^{-2}\ \mathrm{m}\right)^2} + \frac{\left(8.99\times10^9\ \mathrm{N\cdot m^2/C^2}\right)\left(21\times10^{-6}\ \mathrm{C}\right)}{\left(9.0\times10^{-2}\ \mathrm{m}\right)^2}$$

$$= \boxed{-6.2\times10^7\ \mathrm{N/C}}$$

The minus sign tells us that the net electric field points along the $-x$ axis.

b. The net electric field at $x = +6.0$ cm (point B) is $E = E_1 + E_2$:

$$E = E_1 + E_2 = \frac{k\,q_1}{r_1^2} + \frac{k\,q_2}{r_2^2}$$

$$= \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(8.5\times10^{-6}\ \mathrm{C}\right)}{\left(3.0\times10^{-2}\ \mathrm{m}\right)^2} + \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(21\times10^{-6}\ \mathrm{C}\right)}{\left(3.0\times10^{-2}\ \mathrm{m}\right)^2}$$

$$= \boxed{+2.9\times10^{8}\ \mathrm{N/C}}$$

The plus sign tells us that the net electric field points along the $+x$ axis.

29. ***REASONING AND SOLUTION***

a. In order for the field to be zero, the point cannot be between the two charges. Instead, it must be located on the line between the two charges on the side of the positive charge and away from the negative charge. If x is the distance from the positive charge to the point in question, then the negative charge is at a distance (3.0 m + x) meters from this point. For the field to be zero here we have

$$kq_-/(3.0\text{ m} + x)^2 = kq_+/x^2$$

Substituting and solving for x yields (the units have been suppressed for convenience),

$$(12 \times 10^{-6})x^2 - 6.0(4.0 \times 10^{-6})x - 9.0(4.0 \times 10^{-6}) = 0$$

which we can solve for x using the quadratic equation. We obtain $\boxed{x = 3.0\text{ m}}$.

b. Since the field is zero at this point, the force acting on a charge at that point would be $\boxed{0\text{ N}}$.

30. ***REASONING AND SOLUTION*** The electric field due to a point charge is

$$E = kq/r^2 \qquad \text{so that} \qquad kq = Er^2$$

Since the charge is the same at either location, we can write

$$E_1 r_1^{\ 2} = E_2 r_2^{\ 2}$$

Solving for E_2 we obtain

$$E_2 = E_1(r_1^{\ 2}/r_2^{\ 2}) = (160\ \text{N/C})[(0.15\ \text{m})^2/(0.45\ \text{m})^2] = \boxed{18\ \text{N/C}}$$

31. **SSM** ***REASONING*** Before the 3.0-μC point charge q is introduced into the region, the region contains a uniform electric field $\mathbf{E}$ of magnitude 1.6×10^4 N/C. After the 3.0-μC charge is introduced into the region, the net electric field changes. In addition to the uniform electric field $\mathbf{E}$, the region will also contain the electric field $\mathbf{E}_q$ due to the point charge q. The field at any point in the region is the vector sum of $\mathbf{E}$ and $\mathbf{E}_q$. The field $\mathbf{E}_q$ is radial as discussed in the text, and its magnitude at any distance r from the charge q is given by Equation 18.3, $E_q = kq/r^2$. There will be one point P in the region where the net electric field $\mathbf{E}_{\text{net}}$ is zero. This point is located where the field $\mathbf{E}$ has the same magnitude and points in the direction opposite to the field $\mathbf{E}_q$. We will use this reasoning to find the distance r_0 from the charge q to the point P.

SOLUTION Let us assume that the field $\mathbf{E}$ points to the right and that the charge q is *negative* (the problem is done the same way if q is positive, although then the relative positions of P and q will be reversed). Since q is negative, its electric field is radially *inward* (i.e., toward q); therefore, in order for the field $\mathbf{E}_q$ to point in the opposite direction to $\mathbf{E}$, the charge q will have to be to the left of the point P where $\mathbf{E}_{\text{net}}$ is zero, as shown in the following drawing.

From Equation 18.3, $E_q = kq/r_0^2$, and solving for the distance r_0, we have $r_0 = \sqrt{kq/E_q}$. Since the magnitude E_q must be equal to the magnitude of E at the point P, we have

$$r_0 = \sqrt{\frac{kq}{E}} = \sqrt{\frac{(8.99 \times 10^9 \ \text{Nm}^2/\text{C}^2)\,(3.0 \times 10^{-6} \ \text{C})}{1.6 \times 10^4 \ \text{N/C}}} = \boxed{1.3 \ \text{m}}$$

32. ***REASONING AND SOLUTION*** The electric field is defined by Equation 18.2: $\mathbf{E} = \mathbf{F}/q_0$. Thus, the magnitude of the force exerted on a charge q in an electric field of magnitude E is given by

$$F = qE \tag{1}$$

The magnitude of the electric field can be determined from its *x* and *y* components by using the Pythagorean theorem:

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{(6.00 \times 10^3 \text{ N/C})^2 + (8.00 \times 10^3 \text{ N/C})^2} = 1.00 \times 10^4 \text{ N/C}$$

a. From Equation (1) above, the magnitude of the force on the charge is

$$F = (7.50 \times 10^{-6} \text{ C})(1.00 \times 10^4 \text{ N/C}) = \boxed{7.5 \times 10^{-2} \text{ N}}$$

b. From the defining equation for the electric field, it follows that the direction of the force on a charge is the same as the direction of the field, provided that the charge is positive. Thus, the angle that the force makes with the *x* axis is given by

$$\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{8.00 \times 10^3 \text{ N/C}}{6.00 \times 10^3 \text{ N/C}}\right) = \boxed{53.1°}$$

33. ***REASONING AND SOLUTION*** The electric field between the plates of a parallel plate capacitor is given by

$$E = \frac{q}{\varepsilon_0 A} \tag{18.4}$$

Solving for the area of the plate, A, yields

$$A = \frac{q}{\varepsilon_0 E} = \frac{0.15 \times 10^{-6}\ \text{C}}{\left[8.85 \times 10^{-12}\ \text{C}^2/\left(\text{N} \cdot \text{m}^2\right)\right]\left(2.4 \times 10^5\ \text{N/C}\right)} = \boxed{7.1 \times 10^{-2}\ \text{m}^2}$$

34. ***REASONING*** The two charges lying on the x axis produce no net electric field at the coordinate origin. This is because they have identical charges, are located the same distance from the origin, and produce electric fields that point in opposite directions. The electric field produced by q_3 at the origin points away from the charge, or along the $-y$ direction. The electric field produced by q_4 at the origin points toward the charge, or along the $+y$ direction. The net electric field is, then, $E = -E_3 + E_4$, where E_3 and E_4 can be determined by using Equation 18.3.

SOLUTION The net electric field at the origin is

$$E = -E_3 + E_4 = \frac{-k\,q_3}{r_3^2} + \frac{k\,q_4}{r_4^2}$$

$$= \frac{-\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(3.0\times10^{-6}\ \mathrm{C}\right)}{\left(5.0\times10^{-2}\ \mathrm{m}\right)^2} + \frac{\left(8.99\times10^{9}\ \mathrm{N\cdot m^2/C^2}\right)\left(8.0\times10^{-6}\ \mathrm{C}\right)}{\left(7.0\times10^{-2}\ \mathrm{m}\right)^2}$$

$$= \boxed{+3.9\times10^{6}\ \mathrm{N/C}}$$

The plus sign indicates that $\boxed{\text{the net electric field points along the +y direction}}$.

35. **SSM** ***REASONING*** Since the charged droplet (charge = q) is suspended motionless in the electric field **E**, the net force on the droplet must be zero. There are two forces that act on the droplet, the force of gravity $\mathbf{W} = m\mathbf{g}$, and the electric force $\mathbf{F} = q\mathbf{E}$ due to the electric field. Since the net force on the droplet is zero, we conclude that $mg = qE$. We can use this reasoning to determine the sign and the magnitude of the charge on the droplet.

SOLUTION

a. Since the net force on the droplet is zero, and the weight W points downward, the electric force F = qE must point upward. Since the electric field points upward, the excess charge on the droplet must be $\boxed{\text{positive}}$ in order for the force F to point upward.

b. The excess charge on the droplet is, using the expression $mg = qE$,

$$q = \frac{mg}{E} = \frac{(3.50 \times 10^{-9}\ \text{kg})(9.80\ \text{m/s}^2)}{8480\ \text{N/C}} = 4.04 \times 10^{-12}\ \text{C}$$

The charge on a proton is 1.60×10^{-19} C, so the excess number of protons is

$$\left(4.04 \times 10^{-12}\ \text{C}\right)\left(\frac{1\ \text{proton}}{1.60 \times 10^{-19}\ \text{C}}\right) = \boxed{2.53 \times 10^{7}\ \text{protons}}$$

36. ***REASONING AND SOLUTION*** The figure at the right shows the configuration given in text Figure 18.22*b*. The electric field at the center of the rectangle is the resultant of the electric fields at the center due to each of the four charges. As discussed in Conceptual Example 11, the magnitudes of the electric field at the center due to each of the four charges are equal. However, the fields produced by the charges in corners 1 and 3 are in opposite directions. Since they have the same magnitudes, they combine to give zero resultant.

Figure 1

The fields produced by the charges in corners 2 and 4 point in the same direction (toward corner 2).

Thus, $E_C = E_{C2} + E_{C4}$, where E_C is the magnitude of the electric field at the center of the rectangle, and E_{C2} and E_{C4} are the magnitudes of the electric field at the center due to the charges in corners 2 and 4 respectively. Since both E_{C2} and E_{C4} have the same magnitude, we have $E_C = 2\,E_{C2}$.

The distance r, from any of the charges to the center of the rectangle, can be found using the Pythagorean theorem:

$$d = \sqrt{(3.00\text{ cm})^2 + (5.00\text{ cm})^2} = 5.83\text{ cm}$$

$$\text{Therefore, } r = \frac{d}{2} = 2.92\text{ cm} = 2.92 \times 10^{-2}\text{ m}$$

Figure 2

The electric field at the center has a magnitude of

$$E_C = 2E_{C2} = \frac{2kq_2}{r^2} = \frac{2(8.99 \times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(8.60 \times 10^{-12}\text{ C})}{(2.92 \times 10^{-2}\text{ m})^2} = \boxed{1.81 \times 10^2\text{ N/C}}$$

The figure at the right shows the configuration given in text Figure 18.22*c*. All four charges contribute a non-zero component to the electric field at the center of the rectangle. As discussed in Conceptual Example 11, the contribution from the charges in corners 2 and 4 point toward corner 2 and the contribution from the charges in corners 1 and 3 point toward corner 1.

Figure 3

Notice also, the magnitudes of E_{24} and E_{13} are equal, and, from the first part of this problem, we can know that

$$E_{24} = E_{13} = 1.81 \times 10^2\text{ N/C}$$

The electric field at the center of the rectangle is the vector sum of $\mathbf{E_{24}}$ and $\mathbf{E_{13}}$. The x components of $\mathbf{E_{24}}$ and $\mathbf{E_{13}}$ are equal in magnitude and opposite in direction; hence

$$(E_{13})_x - (E_{24})_x = 0$$

Therefore,

$$E_C = (E_{13})_y + (E_{24})_y = 2(E_{13})_y = 2(E_{13})\sin\theta$$

From Figure 2 above,

$$\sin\ \theta = \frac{5.00\text{ cm}}{d} = \frac{5.00\text{ cm}}{5.83\text{ cm}} = 0.858$$

and

$$E_C = 2(E_{13})\sin\theta = 2(1.81\times10^2\ \text{N/C})(0.858) = \boxed{3.11\times10^2\ \text{N/C}}$$

37. ***REASONING AND SOLUTION*** The average force $\overline{\mathbf{F}}$ on the proton can be determined from the impulse-momentum theorem (Equation 7.4):

$$\overline{\mathbf{F}}\Delta t = \Delta(m\mathbf{v})$$

$$\overline{F} = \frac{mv - mv_0}{\Delta t} = \frac{(5.0 \times 10^{-23}\ \text{kg} \cdot \text{m/s}) - (1.5 \times 10^{-23}\ \text{kg} \cdot \text{m/s})}{6.3 \times 10^{-6}\ \text{s}} = 5.6 \times 10^{-18}\ \text{N}$$

From the definition of electric field: $\mathbf{E} = \dfrac{\overline{\mathbf{F}}}{q_0}$, we have

$$E = \frac{\overline{F}}{q} = \frac{5.6 \times 10^{-18}\ \text{N}}{1.60 \times 10^{-19}\ \text{C}} = \boxed{35\ \text{N/C}}$$

38. ***REASONING AND SOLUTION*** From kinematics, $v_y^2 = v_{0y}^2 + 2a_y y$. Since the electron starts from rest, $v_{0y} = 0$ m/s. The acceleration of the proton is given by

$$a_y = \frac{F}{m} = \frac{eE}{m}$$

where e and m are the electron's charge magnitude and mass, respectively, and E is the magnitude of the electric field. The magnitude of the electric field between the plates of a parallel plate capacitor is $E = \sigma/\varepsilon_0$, where σ is the magnitude of the charge per unit area on each plate. Thus, $a_y = e\sigma/(m\varepsilon_0)$. Combining this expression for a with the kinematics equation we have

$$v_y^2 = 2\left(\frac{e\sigma}{m\varepsilon_0}\right)y$$

Solving for v_y gives

$$v_y = \sqrt{\frac{2e\sigma y}{m\varepsilon_0}} = \sqrt{\frac{2(1.60\times10^{-19}\ \text{C})(1.8\times10^{-7}\ \text{C/m}^2)(1.5\times10^{-2}\ \text{m})}{(9.11\times10^{-31}\ \text{kg})[8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)]}} = \boxed{1.0\times10^7\ \text{m/s}}$$

39. **SSM** **WWW** ***REASONING*** The figure shows the arrangement of the three charges. Let $\mathbf{E}_q$ represent the electric field at the empty corner due to the $-q$ charge. Furthermore, let $\mathbf{E}_1$ and $\mathbf{E}_2$ be the electric fields at the empty corner due to charges $+q_1$ and $+q_2$, respectively.

According to the Pythagorean theorem, the distance from the charge $-q$ to the empty corner along the diagonal is given by $\sqrt{(2d)^2 + d^2} = \sqrt{5d^2} = d\sqrt{5}$. The magnitude of each electric field is given by Equation 18.3, $E = kq/r^2$. Thus, the magnitudes of each of the electric fields at the empty corner are given as follows: $E_q = kq/r^2 = kq/\left(d\sqrt{5}\right)^2 = kq/(5d^2)$, since the length of the diagonal is $d\sqrt{5}$; $E_1 = kq_1/(4d^2)$; and $E_2 = kq_2/d^2$. The angle θ that the diagonal makes with the horizontal is $\theta = \tan^{-1}(d/2d) = 26.57°$. Since the net electric field E_{net} at the empty corner is zero, the horizontal component of the net field must be zero, and we have

$$E_1 - E_q \cos 26.57° = 0 \qquad \text{or} \qquad kq_1/\left(4d^2\right) - kq\left(\cos 26.57°\right)/\left(5d^2\right) = 0$$

Similarly, the vertical component of the net field must be zero, and we have

$$E_2 - E_q \sin 26.57° = 0 \qquad \text{or} \qquad kq_2/d^2 - kq\left(\sin 26.57°\right)/\left(5d^2\right) = 0$$

These last two expressions can be solved for the charges q_1 and q_2.

SOLUTION Solving the last two expressions for the charges q_1 and q_2, we have

$$q_1 = \tfrac{4}{5} q \cos 26.57° = \boxed{0.716\, q}$$

$$q_2 = \tfrac{1}{5} q \sin 26.57° = \boxed{0.0895\, q}$$

40. ***REASONING*** The magnitude E of the electric field is defined as the magnitude F of the electric force exerted on a small test charge divided by the charge: $E = F/q$. According to Newton's second law, Equation 4.2, the net force acting on an object is equal to its mass m times its acceleration a. Since there is only one force acting on the object, it is the net force. Thus, the magnitude of the electric field can be written as

$$E = \frac{F}{q} = \frac{ma}{q}$$

The acceleration is related to the initial and final velocities, v_0 and v, and the time t through Equation 2.4, as $a = \dfrac{v - v_0}{t}$. Substituting this expression for a into the one above for E gives

$$E = \frac{ma}{q} = \frac{m\left(\dfrac{v - v_0}{t}\right)}{q} = \frac{m(v - v_0)}{qt}$$

SOLUTION The magnitude E of the electric field is

$$E = \frac{m(v - v_0)}{qt} = \frac{(9.0 \times 10^{-5}\ \text{kg})(2.0 \times 10^{3}\ \text{m/s} - 0\ \text{m/s})}{(7.5 \times 10^{-6}\ \text{C})(0.96\ \text{s})} = \boxed{2.5 \times 10^{4}\ \text{N/C}}$$

41. ***REASONING AND SOLUTION*** The net field at the center of the circle is E_n, and since E_1 and E_2 are perpendicular to one another, we see that

$$E_1 = E_n \cos 20.0° \quad \text{and} \quad E_2 = E_n \sin 20.0°$$

Solving for E_n in each equation and equating the two expressions gives

$$E_1/(\cos 20.0°) = E_2/(\sin 20.0°)$$

Using $E = kq/r^2$ we have

$$q_1/(\cos 20.0°) = q_2/(\sin 20.0°)$$

so

$$(q_2/q_1) = (\sin 20.0°/\cos 20.0°) = \boxed{0.364}$$

42. ***REASONING AND SOLUTION*** From two-dimensional kinematics, taking the entry point as the origin, we have

$$x = v_{0x}t + \frac{1}{2}a_x t^2 \tag{1}$$

$$y = v_{0y}t + \frac{1}{2}a_y t^2 \tag{2}$$

There is no acceleration in the x-direction, so $a_x = 0\ \text{m/s}^2$. Initially, the electron travels in the +x direction, so $v_{0y} = 0$ m/s. Solving Equation (1) for t and substituting into Equation (2) gives:

$$y = \frac{1}{2}a_y\left(\frac{x}{v_{0x}}\right)^2 \tag{3}$$

From Newton's second law and the definition of electric field we have

$$a_y = \frac{F}{m} = \frac{eE}{m}$$

Thus, Equation (3) becomes

$$y = \frac{1}{2}\left(\frac{eE}{m}\right)\left(\frac{x}{v_{0x}}\right)^2$$

where e is the magnitude of the electron's charge and m is its mass. Solving for E gives

$$E = \frac{2my}{e}\left(\frac{v_{0x}}{x}\right)^2 = \frac{2(9.11\times10^{-31}\ \text{kg})(1.50\times10^{-3}\ \text{m})}{(1.60\times10^{-19}\ \text{C})}\left(\frac{7.00\times10^{6}\ \text{m/s}}{2.00\times10^{-2}\ \text{m}}\right)^2 = \boxed{2.09\times10^{3}\ \text{N/C}}$$

43. **SSM** ***REASONING*** The net electric field at point P in Figure 1 is the vector sum of the fields $\mathbf{E}_+$ and $\mathbf{E}_-$, which are due, respectively, to the charges $+q$ and $-q$. These fields are shown in Figure 2.

Figure 1

Figure 2

According to Equation 18.3, the magnitudes of the fields E_+ and E_- are the same, since the triangle is an isosceles triangle with equal sides of length ℓ. Therefore, $E_+ = E_- = kq/\ell^2$. The vertical components of these two fields cancel, while the horizontal components reinforce, leading to a total field at point P that is horizontal and has a magnitude of

$$E_P = E_+ \cos\alpha + E_- \cos\alpha = 2\left(\frac{kq}{\ell^2}\right)\cos\alpha$$

At point M in Figure 1, both $\mathbf{E}_+$ and $\mathbf{E}_-$ are horizontal and point to the right. Again using Equation 18.3, we find

$$E_M = E_+ + E_- = \frac{kq}{\ell^2} + \frac{kq}{\ell^2} = \frac{2kq}{\ell^2}$$

Since $E_M/E_P = 9.0$, we have

$$\frac{E_M}{E_P} = \frac{2kq/d^2}{2kq(\cos\alpha)/\ell^2} = \frac{1}{(\cos\alpha)d^2/\ell^2} = 9.0$$

But from Figure 1, we can see that $d/\ell = \cos\alpha$. Thus, it follows that

$$\frac{1}{\cos^3\alpha} = 9.0 \qquad \text{or} \qquad \cos\alpha = \sqrt[3]{1/9.0} = 0.48$$

The value for α is, then, $\alpha = \cos^{-1}(0.48) = \boxed{61°}$.

44. ***REASONING AND SOLUTION*** The electric field due to a parallel plate capacitor is given by $E = \sigma/\varepsilon_o$ (Equation 18.4).

a. The induced charge density σ is, therefore,

$$\sigma = \varepsilon_o E = [8.85 \times 10^{-12}\ C^2/(N \cdot m^2)](480\ N/C) = \boxed{4.2 \times 10^{-9}\ C/m^2}$$

b. The area of one face of the circular coin is

$$A = \pi r^2 = \pi (0.019\ m)^2$$

The total charge is

$$q = \sigma A = \boxed{4.8 \times 10^{-12}\ C}$$

45. ***REASONING AND SOLUTION*** Since the thread makes an angle of 30.0° with the vertical, it can be seen that the electric force on the ball, F_e, and the gravitational force, mg, are related by

$$F_e = mg \tan 30.0°$$

The force F_e is due to the charged ball being in the electric field of the parallel plate capacitor. That is,

$$F_e = Eq$$

where q is the ball's charge and E is the field due to the plates, given by

$$E = Q/(\varepsilon_o A) \qquad (18.4)$$

Combining equations gives

$$mg \tan 30.0° = qQ/(\varepsilon_o A)$$

Solving for Q yields

$$Q = \frac{\varepsilon_0 A mg \tan 30.0°}{q}$$

$$= \frac{\left[8.85 \times 10^{-12}\ C^2/\left(N \cdot m^2\right)\right]\left(0.0150\ m^2\right)\left(6.50 \times 10^{-3}\ kg\right)\left(9.80\ m/s^2\right) \tan 30.0°}{0.150 \times 10^{-6}\ C}$$

$$= \boxed{3.25 \times 10^{-8}\ C}$$

46. ***REASONING AND SOLUTION*** Gauss' Law is given by text Equation 18.7: $\Phi_E = \dfrac{Q}{\varepsilon_0}$, where Q is the *net charge* enclosed by the Gaussian surface.

a. $$\Phi_E = \frac{3.5 \times 10^{-6}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)} = \boxed{4.0 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}}$$

b. $$\Phi_E = \frac{-2.3 \times 10^{-6}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)} = \boxed{-2.6 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}}$$

c. $$\Phi_E = \frac{(3.5 \times 10^{-6}\ \text{C}) + (-2.3 \times 10^{-6}\ \text{C})}{8.85 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)} = \boxed{1.4 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}}$$

47. **SSM** ***REASONING*** As discussed in Section 18.9, the electric flux Φ_E through a surface is equal to the component of the electric field that is normal to the surface multiplied by the area of the surface, $\Phi_E = E_\perp A$, where $E_\perp$ is the component of **E** that is normal to the surface of area A. We can use this expression and the figure in the text to determine the flux through the two surfaces.

SOLUTION

a. The flux through surface 1 is

$$(\Phi_E)_1 = (E\cos 35°)A_1 = (250\ \text{N/C})(\cos 35°)(1.7\ \text{m}^2) = \boxed{350\ \text{N}\cdot\text{m}^2/\text{C}}$$

b. Similarly, the flux through surface 2 is

$$(\Phi_E)_2 = (E\cos 55°)A_2 = (250\ \text{N/C})(\cos 55°)(3.2\ \text{m}^2) = \boxed{460\ \text{N}\cdot\text{m}^2/\text{C}}$$

48. ***REASONING AND SOLUTION*** The maximum possible flux occurs when the electric field is parallel to the normal of the rectangular surface (that is, when the angle between the direction of the field and the direction of the normal is zero). Then

$$\Phi_E = (E \cos \phi)A = (580 \text{ N/C})(\cos 0°)(0.16 \text{ m})(0.38 \text{ m}) = \boxed{35 \text{ N} \cdot \text{m}^2 / \text{C}}$$

49. ***REASONING AND SOLUTION***

a. In all three cases, the net charge enclosed by the surface is the same, because the net charge enclosed by each surface is the same; therefore, by Gauss' Law, the electric flux through the surfaces described in parts (a) through (c) is the same:

$$\Phi_E = \frac{Q}{\varepsilon_0} = \frac{2.0 \times 10^{-6}\ \text{C}}{8.85 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)} = \boxed{2.3 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}}$$

b. $\Phi_E = \boxed{2.3 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}}$

c. $\Phi_E = \boxed{2.3 \times 10^5\ \text{N} \cdot \text{m}^2/\text{C}}$

50. ***REASONING AND SOLUTION*** Since the electric field is uniform, its magnitude and direction are the same at each point on the wall. The angle ϕ between the electric field and the normal to the wall is 35°. Therefore, the electric flux is

$$\Phi_E = (E \cos \phi)\, A = (150 \text{ N/C})(\cos 35°)[(5.9 \text{ m})(2.5 \text{ m})] = \boxed{1.8 \times 10^3 \text{ N} \cdot \text{m}^2/\text{C}}$$

51. **SSM** ***REASONING*** The electric flux through each face of the cube is given by $\Phi_E = (E\cos\phi)A$ (see Section 18.9) where E is the magnitude of the electric field at the face, A is the area of the face, and ϕ is the angle between the electric field and the outward normal of that face. We can use this expression to calculate the electric flux Φ_E through each of the six faces of the cube.

SOLUTION

a. On the bottom face of the cube, the outward normal points parallel to the $-y$ axis, in the opposite direction to the electric field, and $\phi = 180°$. Therefore,

$$(\Phi_E)_{\text{bottom}} = (1500\ \text{N/C})(\cos 180°)(0.20\ \text{m})^2 = \boxed{-6.0\times10^1\ \text{N}\cdot\text{m}^2/\text{C}}$$

On the top face of the cube, the outward normal points parallel to the $+y$ axis, and $\phi = 0.0°$. The electric flux is, therefore,

$$(\Phi_E)_{\text{top}} = (1500\ \text{N/C})(\cos 0.0°)(0.20\ \text{m})^2 = \boxed{+6.0\times10^1\ \text{N}\cdot\text{m}^2/\text{C}}$$

On each of the other four faces, the outward normals are perpendicular to the direction of the electric field, so $\phi = 90°$. So for each of the four side faces,

$$(F_E)_{\text{sides}} = (1500\ \text{N/C})(\cos 90°)(0.20\ \text{m})^2 = \boxed{0\ \text{N}\cdot\text{m}^2/\text{C}}$$

b. The total flux through the cube is

$$(\Phi_E)_{\text{total}} = (\Phi_E)_{\text{top}} + (\Phi_E)_{\text{bottom}} + (\Phi_E)_{\text{side 1}} + (\Phi_E)_{\text{side 2}} + (\Phi_E)_{\text{side 3}} + (\Phi_E)_{\text{side 4}}$$

Therefore,

$$(\Phi_E)_{\text{total}} = (+6.0\times10^1\ \text{N}\cdot\text{m}^2/\text{C}) + (-6.0\times10^1\ \text{N}\cdot\text{m}^2/\text{C}) + 0 + 0 + 0 + 0 = \boxed{0\ \text{N}\cdot\text{m}^2/\text{C}}$$

52. ***REASONING AND SOLUTION*** Since both charge distributions are uniformly spread over concentric spherical shells, the electric field possesses spherical symmetry. Gauss' law can be used to determine the magnitude of the electric field, provided we choose spherical Gaussian surfaces (concentric with the spherical shells) to evaluate the electric flux. To find the magnitude of the electric field at any distance r from the center of the spherical shells, we construct a spherical Gaussian surface of radius r. The electric flux through this Gaussian surface is

$$\Phi_E = \Sigma(E \cos \phi)\Delta A$$

Because the charge distributions have spherical symmetry, we expect the electric field to be directed radially. That is, the electric field is everywhere perpendicular to the Gaussian surface. Thus, for any surface element, ϕ will be 0 or 180°. Furthermore, since the charge distribution possesses spherical symmetry, we expect the electric field to be uniform in magnitude over any sphere concentric with the shells. Thus, E is constant over any Gaussian surface concentric with the shells. Then, $(E \cos \phi)$ can be factored out of the summation.

$$\Phi_E = \Sigma(E \cos \phi)\Delta A = (E \cos \phi)\Sigma\Delta A$$

where $\Sigma\Delta A$ is the sum of the area elements that make up the Gaussian surface. This sum must equal the surface area of the Gaussian surface or

$$\Phi_E = (E \cos \phi)\Sigma\Delta A = (E \cos \phi)(4\pi r^2)$$

where r is the radius of the Gaussian surface. From Gauss' law this becomes

$$(E \cos \phi)(4\pi r^2) = \frac{Q}{\varepsilon_0} \qquad (1)$$

where Q is the net charge enclosed by the Gaussian surface.

a. **r = 0.20 m**

The Gaussian surface encloses both shells. The net charge enclosed is

$$(+5.1 \times 10^{-6}\ \text{C}) + (-1.6 \times 10^{-6}\ \text{C}) = +3.5 \times 10^{-6}\ \text{C}$$

Since the net charge is positive, E will be radially outward for all points on the Gaussian surface, and $\phi = 0.0°$ for all elements on the Gaussian surface.

Solving Equation (1) for E gives

$$E = \frac{Q}{\varepsilon_0(4\pi r^2)} = \frac{3.5 \times 10^{-6}\ \text{C}}{[8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)][4\pi(0.20\ \text{m})^2]} = \boxed{7.9 \times 10^5\ \text{N/C}}$$

The direction of **E** is $\boxed{\text{radially outward}}$, because the net charge within the Gaussian surface is positive.

b. **r = 0.10 m**

The Gaussian surface encloses only the inner shell. The net charge enclosed is

$$Q = -1.6 \times 10^{-6}\ \text{C}$$

Since the net charge is negative, E will be radially inward for all points on the Gaussian surface, and ϕ = 180° for all elements on the Gaussian surface.

Solving Equation (1) for E gives

$$E = -\frac{Q}{\varepsilon_0 (4\pi r^2)} = -\frac{(-1.6 \times 10^{-6}\ \text{C})}{[8.85 \times 10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)][4\pi(0.10\ \text{m})^2]} = \boxed{1.4 \times 10^6\ \text{N/C}}$$

The direction of **E** is $\boxed{\text{radially inward}}$, because the net charge within the Gaussian surface is negative.

c. **r = 0.025 m**

The net charge enclosed by the Gaussian surface is zero. This implies that the net electric flux is zero, so the electric field is either a constant or zero everywhere within the Gaussian surface. However, an electric field does not exist within the Gaussian surface, because there are only negative charges on the shell of radius r_1, so electric field lines cannot originate from any place on this shell. Thus, $\boxed{E = 0\ \text{N/C}}$ in this region.

53. ***REASONING*** Because the charge is distributed uniformly along the straight wire, the electric field is directed radially outward, as the end view of the wire given below illustrates.

And because of symmetry, the magnitude of the electric field is the same at all points equidistant from the wire. In this situation we will use a Gaussian surface that is a cylinder concentric with the wire. The drawing shows that this cylinder is composed of three parts, the two flat ends (1 and 3) and the curved wall (2). We will evaluate the electric flux for this three-part surface and then set it equal to Q/ε_0 (Gauss' law) to find the magnitude of the electric field.

SOLUTION Surfaces 1 and 3 – the flat ends of the cylinder – are parallel to the electric field, so $\cos\phi = \cos 90° = 0$. Thus, there is no flux through these two surfaces: $\Phi_1 = \Phi_3 = 0\ \mathrm{N\cdot m^2/C}$.

Surface 2 – the curved wall – is everywhere perpendicular to the electric field **E**, so $\cos\phi = \cos 0° = 1$. Furthermore, the magnitude E of the electric field is the same for all points on this surface, so it can be factored outside the summation in Equation 18.6:

$$\Phi_2 = \Sigma\left(E\cos 0°\right)\Delta A = E\Sigma A$$

The area ΣA of this surface is just the circumference $2\pi r$ of the cylinder times its length L: $\Sigma A = (2\pi r)L$. The electric flux through the entire cylinder is, then,

$$\Phi_E = \Phi_1 + \Phi_2 + \Phi_3 = 0 + E\left(2\pi rL\right) + 0 = E\left(2\pi rL\right)$$

Following Gauss' law, we set Φ_E equal to Q/ε_0, where Q is the net charge inside the Gaussian cylinder: $E(2\pi rL) = Q/\varepsilon_0$. The ratio Q/L is the charge per unit length of the wire and is known as the linear charge density λ: $\lambda = Q/L$. Solving for E, we find that

$$\boxed{E = \frac{Q/L}{2\pi\varepsilon_0 r} = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

54. ***REASONING AND SOLUTION*** The electric field lines must originate on the positive charges and terminate on the negative charge. They cannot cross one another. Furthermore, the number of field lines beginning or terminating on any charge must be proportional to the magnitude of the charge. Thus, for every field line that leaves the charge +q, two field lines must leave the charge +2q. These three lines must terminate on the -3q charge. If the sketch is to have six field lines, two of them must originate on +q, and four of them must originate on the charge +2q.

55. **SSM** ***REASONING AND SOLUTION*** The magnitude of the force of attraction between the charges is given by Coulomb's law (Equation 18.1): $F = kq_1q_2/r^2$, where q_1 and q_2 are the magnitudes of the charges and r is the separation of the charges. Let F_A and F_B represent the magnitudes of the forces between the charges when the separations are r_A and $r_B = r_A/9$, respectively. Then

$$\frac{F_B}{F_A} = \frac{kq_1q_2/r_B^2}{kq_1q_2/r_A^2} = \left(\frac{r_A}{r_B}\right)^2 = \left(\frac{r_A}{r_A/9}\right)^2 = (9)^2 = 81$$

Therefore, we can conclude that $F_B = 81F_A = (81)(1.5\text{ N}) = \boxed{120\text{ N}}$.

56. ***REASONING AND SOLUTION*** The +2q of charge initially on the sphere lies entirely on the outer surface. When the +q charge is placed inside of the sphere, then a $\boxed{-q}$ charge will still be induced on the interior of the sphere. An additional +q will appear on the outer surface, giving a net charge of $\boxed{+3q}$.

57. ***REASONING*** The drawing at the right shows the set-up. The force on the $+q$ charge at the origin due to the other $+q$ charge is given by Coulomb's law (Equation 18.1), as is the force due to the $+2q$ charge. These two forces point to the left, since each is repulsive. The sum of the two is twice the force on the $+q$ charge at the origin due to the other $+q$ charge alone.

SOLUTION Applying Coulomb's law, we have

$$\underbrace{\frac{k(q)(q)}{(0.50\ \text{m})^2}}_{\substack{\text{Force due to } +q \\ \text{charge at } x=+0.50\ \text{m}}} + \underbrace{\frac{k(2q)(q)}{(d)^2}}_{\substack{\text{Force due to } +2q \\ \text{charge at } x=+d}} = \underbrace{2\frac{k(q)(q)}{(0.50\ \text{m})^2}}_{\substack{\text{Twice the force due to} \\ +q \text{ charge at } x=+0.50\ \text{m}}}$$

Rearranging this result and solving for d give

$$\frac{k(2q)(q)}{(d)^2} = \frac{k(q)(q)}{(0.50\ \text{m})^2} \qquad \text{or} \qquad d^2 = 2(0.50\ \text{m})^2 \qquad \text{or} \qquad d = \pm 0.71\ \text{m}$$

We reject the negative root, because a negative value for d would locate the $+2q$ charge to the left of the origin. Then, the two forces acting on the charge at the origin would have different directions, contrary to the statement of the problem. Therefore, the $+2q$ charge is located at a position of $\boxed{x = +0.71\ \text{m}}$.

58. ***REASONING AND SOLUTION***

a. The electrostatic force acting on q_2 is the vector sum of the forces due to charges q_1 and q_3, i.e., $\mathbf{F_2 = F_{12} + F_{32}}$. We can write for the magnitude of each force,

$$F_{12} = \frac{kq_1q_2}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(25\times10^{-6}\ \text{C})(11\times10^{-6}\ \text{C})}{(2.0\ \text{m})^2} = 0.62\ \text{N}$$

The direction of the force is to the right (+x direction).

$$F_{32} = \frac{kq_3q_2}{r^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(45\times10^{-6}\ \text{C})(11\times10^{-6}\ \text{C})}{(1.5\ \text{m})^2} = 2.0\ \text{N}$$

The direction of the force is to the left (–x direction).

Since F_{32} is larger than F_{12}, the net force is $\boxed{\text{to the left (–x direction)}}$ and has a magnitude of

$$F_2 = 2.0\ \text{N} - 0.62\ \text{N} = \boxed{1.4\ \text{N}}$$

b. If q_2 were negative, the net force would still be 1.4 N, but would be in the +x direction instead. This is because F_{12} would now be to the left (since q_1 attracts q_2), with the same magnitude, and F_{32} would act to the right. The resulting force $F_2 = \boxed{1.4\ \text{N}}$ would now be $\boxed{\text{to the right (+x direction)}}$.

59. **SSM** **WWW** ***REASONING*** Two forces act on the charged ball (charge q); they are the downward force of gravity $m\mathbf{g}$ and the electric force $\mathbf{F}$ due to the presence of the charge q in the electric field $\mathbf{E}$. In order for the ball to float, these two forces must be equal in magnitude and opposite in direction, so that the net force on the ball is zero (Newton's second law). Therefore, $\mathbf{F}$ must point upward, which we will take as the positive direction. According to Equation 18.2, $\mathbf{F} = q\mathbf{E}$. Since the charge q is negative, the electric field $\mathbf{E}$ must point downward, as the product $q\mathbf{E}$ in the expression $\mathbf{F} = q\mathbf{E}$ must be positive, since the force $\mathbf{F}$ points upward. The magnitudes of the two forces must be equal, so that $mg = qE$. This expression can be solved for E.

SOLUTION The magnitude of the electric field $\mathbf{E}$ is

$$E = \frac{mg}{q} = \frac{(0.012\ \text{kg})(9.80\ \text{m/s}^2)}{18 \times 10^{-6}\ \text{C}} = \boxed{6.5 \times 10^3\ \text{N/C}}$$

As discussed in the reasoning, this electric field points $\boxed{\text{downward}}$.

60. ***REASONING*** The drawing at the right shows the set-up. Here, the electric field **E** points along the $+y$ axis and applies a force of $+\mathbf{F}$ to the $+q$ charge and a force of $-\mathbf{F}$ to the $-q$ charge, where $q = 8.0\ \mu\text{C}$ denotes the magnitude of each charge. Each force has the same magnitude of $F = Eq$, according to Equation 18.2. The torque is measured as discussed in Section 9.1. According to Equation 9.1, the torque produced by each force has a magnitude given by the magnitude of the force times the lever arm, which is the perpendicular distance between the point of application of the force and the axis of rotation. In the drawing the z axis is the axis of rotation and is midway between the ends of the rod. Thus, the lever arm for each force is half the length L of the rod or $L/2$, and the magnitude of the torque produced by each force is $\tau = (Eq)(L/2)$.

SOLUTION The $+\mathbf{F}$ and the $-\mathbf{F}$ force each cause the rod to rotate in the same sense about the z axis. Therefore, the torques from these forces reinforce one another. Using $\tau = (Eq)(L/2)$ for each torque, we find that the magnitude of the net torque is

$$\tau = Eq\left(\frac{L}{2}\right) + Eq\left(\frac{L}{2}\right) = EqL = \left(5.0 \times 10^{3}\ \text{N/C}\right)\left(8.0 \times 10^{-6}\ \text{C}\right)\left(4.0\ \text{m}\right) = \boxed{0.16\ \text{N}\cdot\text{m}}$$

61. **SSM** ***REASONING AND SOLUTION***

a. Since the gravitational force between the spheres is one of attraction and the electrostatic force must balance it, the electric force must be one of repulsion. Therefore, the charges must have $\boxed{\text{the same algebraic signs, both positive or both negative}}$.

b. There are two forces that act on each sphere; they are the gravitational attraction F_G of one sphere for the other, and the repulsive electric force F_E of one sphere on the other. From the problem statement, we know that these two forces balance each other, so that $F_G = F_E$. The magnitude of F_G is given by Newton's law of gravitation (Equation 4.3: $F_G = Gm_1m_2/r^2$), while the magnitude of F_E is given by Coulomb's law (Equation 18.1: $F_E = kq_1q_2/r^2$). Therefore, we have $Gm_1m_2/r^2 = kq_1q_2/r^2$, and since the spheres have the same mass and carry charges of the same magnitude, $Gm^2/r^2 = kq^2/r^2$. Solving for q, we find

$$q = m\sqrt{\frac{G}{k}} = (2.0 \times 10^{-6}\ \text{kg})\sqrt{\frac{6.67 \times 10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2}{8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{1.7 \times 10^{-16}\ \text{C}}$$

62. ***REASONING AND SOLUTION*** The force on q_1 due to q_2 is given by Coulomb's law:

$$F_{12} = \frac{kq_1q_2}{r_{12}^2} \tag{1}$$

The force on q_1 due to the electric field of the capacitor is given by

$$F_{1C} = q_1E_C = q_1\left(\frac{\sigma}{\varepsilon_0}\right) \tag{2}$$

Equating the right hand sides of Equations (1) and (2) above gives

$$\frac{kq_1q_2}{r_{12}^2} = q_1\left(\frac{\sigma}{\varepsilon_0}\right)$$

Solving for r_{12} gives

$$r_{12} = \sqrt{\frac{\varepsilon_0 kq_2}{\sigma}}$$

$$= \sqrt{\frac{[8.85\times10^{-12}\ \text{C}^2/(\text{N}\cdot\text{m}^2)](8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-6}\ \text{C})}{(1.30\times10^{-4}\ \text{C/m}^2)}} = \boxed{5.53\times10^{-2}\ \text{m}}$$

63. ***REASONING AND SOLUTION*** We need to find the electric force, and hence the electric field, at the point in question. The net force is

$$\mathbf{F} = m\mathbf{a} = (2.0 \times 10^{-3}\ \text{kg})(3.5 \times 10^{3}\ \text{m/s}^2) = 7.0\ \text{N}\ \ (+x\ \text{direction})$$

The electric field is

$$E = F/q = (7.0\ \text{N})/(25 \times 10^{-6}\ \text{C}) = \boxed{2.8 \times 10^{5}\ \text{N/C}}$$

Since the charge is negative, the field is directed opposite to the direction of the force. Therefore, the electric field is along the $\boxed{-x\ \text{direction}}$.

64. ***REASONING AND SOLUTION***

a. Since the spring is stretched, the electric force must be a repulsion. Therefore, the charges must be $\boxed{\text{the same polarity (both positive or both negative)}}$.

b. The force needed to stretch the spring is $F = k_{\text{spring}}\, x$, which is provided by the electric force given by Coulomb's law.

$$\frac{kq^2}{r^2} = k_{\text{spring}}\, x$$

$$q = \sqrt{\frac{k_{\text{spring}}\, x r^2}{k}} = \sqrt{\frac{(220\ \text{N/m})(0.020\ \text{m})(0.34\ \text{m})^2}{8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2}} = \boxed{7.5 \times 10^{-6}\ \text{C}}$$

65. **SSM** ***REASONING*** The charge at A experiences a force $\mathbf{F_A}$ given by $\mathbf{F_A} = q_A\mathbf{E_A}$, where $\mathbf{E_A}$ is the net electric field at A. The net electric field $\mathbf{E_A}$ is the vector sum of the fields due to the presence of the other two charges. In order for the net force on the charge at corner A to point along the vertical, the fourth charge that is placed at the empty corner, must produce an electric field $\mathbf{E_q}$ at A with an x-component that is equal in magnitude and opposite in direction to the x-component of $\mathbf{E_A}$. The x-component of $\mathbf{E_A}$ is due solely to the +3.0 μC charge in the lower right corner. According to Equation 18.3, then, $E_{Ax} = kq_{\text{right}}/(16d^2)$, where q_{right} is the charge in the lower right corner. Since q_{right} is positive, E_{Ax} points to the left. The x component of the field $\mathbf{E_q}$ must, therefore, be equal to $E_{qx} = kq_{\text{right}}/(16d^2)$ and point to the right. We will now use this expression to solve for q.

SOLUTION According to Equation 18.3, the electric field $\mathbf{E_q}$ has a magnitude that is given by

$$E_q = kq/r^2 = kq/(d^2 + 16d^2) = kq/(17d^2)$$

Furthermore, this field must point along the diagonal from the charge q_A toward the empty corner, as shown in the following drawing. Therefore, the charge placed at the empty corner must be negative.

The angle θ is given by $\theta = \tan^{-1}[d/(4d)] = \tan^{-1}(0.250) = 14.0°$. Using the arguments developed in the reasoning, we have $E_{qx} = kq_{\text{right}}/(16d^2)$, or

$$E_{qx} = E_q \cos\theta = \frac{kq}{17d^2}\cos\theta = \frac{kq_{\text{right}}}{16d^2}$$

Solving for the charge magnitude q, we obtain

$$q = \frac{q_{\text{right}}}{\cos\theta}\left(\frac{17}{16}\right) = \left(\frac{3.0\times10^{-6}\ \text{C}}{\cos 14.0°}\right)\left(\frac{17}{16}\right) = 3.3\times10^{-6}\ \text{C}$$

Thus, the charge at the empty corner is $\boxed{-3.3\times10^{-6}\ \text{C}}$.

66. ***REASONING*** The proton (charge = $+e = 1.60 \times 10^{-19}$ C and mass = $m = 1.67 \times 10^{-27}$ kg) moves in the direction of the electric field because of the force that the field applies to the proton. This force does work and thereby changes the proton's kinetic energy. According to the work-energy theorem (Equation 6.3), the work done causes the proton's kinetic energy to change. Kinetic energy is $mv^2/2$, where v is the speed. The work-energy theorem will involve the final speed of the proton and we will use it to obtain that speed.

SOLUTION According to Equation 6.3, the work-energy theorem is $W = \mathrm{KE_f} - \mathrm{KE_0}$, where W is the work done by the net external force that acts on the proton and $\mathrm{KE_f}$ and $\mathrm{KE_0}$ are, respectively, the final and initial kinetic energies. The force applied by the electric field **E** is the only force acting, so it is the net force and, according to Equation 18.2, has a magnitude of $F = Ee$. The direction of the force is in the same direction as the electric field, since the proton has a positive charge. Since the motion is in the direction of the field and the force, the work done by the force is given by Equation 6.1 as $W = Fs = Ees$, where s is the distance traveled. Thus, the work-energy theorem can be stated as follows:

$$W = \mathrm{KE_f} - \mathrm{KE_0} \qquad \text{or} \qquad Ees = \frac{1}{2}mv_\mathrm{f}^2 - \frac{1}{2}mv_0^2$$

Solving this result for the final speed v_f, we find

$$v_\mathrm{f} = \sqrt{\frac{2Ees}{m} + v_0^2}$$

$$= \sqrt{\frac{2\left(2.3 \times 10^3\ \mathrm{N/C}\right)\left(1.60 \times 10^{-19}\ \mathrm{C}\right)\left(2.0 \times 10^{-3}\ \mathrm{m}\right)}{1.67 \times 10^{-27}\ \mathrm{kg}} + \left(2.5 \times 10^4\ \mathrm{m/s}\right)^2}$$

$$= \boxed{3.9 \times 10^4\ \mathrm{m/s}}$$

67. ***REASONING AND SOLUTION*** In order for the net force on any charge to be directed inward toward the center of the square, the charges must be placed with alternate + and – signs on each successive corner. The force on any charge due to an adjacent charge is

$$F = \frac{kq^2}{r^2} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right)\left(2.0 \times 10^{-6} \text{ C}\right)^2}{\left(0.30 \text{ m}\right)^2} = 0.40 \text{ N}$$

The forces due to two adjacent charges are perpendicular to one another and produce a resultant force

$$F' = \sqrt{2F^2} = \sqrt{2(0.40 \text{ N})^2}$$

$$F' = 0.57 \text{ N}.$$

The force due to the diagonal charge is

$$F'' = kq^2/(2r^2) = 0.20 \text{ N}$$

since the diagonal distance is $r\sqrt{2}$. The force F'' is directed opposite to F' (since the diagonal charges are of the same sign). Therefore, the net force acting on any of the charges is directed inward and has a magnitude

$$F_{net} = F' - F'' = 0.57 \text{ N} - 0.20 \text{ N} = \boxed{0.37 \text{ N}}$$

1. **SSM** ***REASONING AND SOLUTION*** Combining Equations 19.1 and 19.3, we have

$$W_{AB} = \text{EPE}_A - \text{EPE}_B = q_0(V_A - V_B) = (+1.6\times10^{-19}\ \text{C})(0.070\ \text{V}) = \boxed{1.1\times10^{-20}\ \text{J}}$$

2. ***REASONING*** Equation 19.1 indicates that the work done by the electric force as the particle moves from point A to point B is $W_{AB} = \text{EPE}_A - \text{EPE}_B$. For motion through a distance s along the line of action of a constant force of magnitude F, the work is given by Equation 6.1 as either $+Fs$ (if the force and the displacement have the same direction) or $-Fs$ (if the force and the displacement have opposite directions). Here, $\text{EPE}_A - \text{EPE}_B$ is given to be positive, so we can conclude that the work is $W_{AB} = +Fs$ and that the force points in the direction of the motion from point A to point B. The electric field is given by Equation 18.2 as $\mathbf{E} = \mathbf{F}/q_0$, where q_0 is the charge.

SOLUTION a. Using Equation 19.1 and the fact that $W_{AB} = +Fs$, we find

$$W_{AB} = +Fs = \text{EPE}_A - \text{EPE}_B$$

$$F = \frac{\text{EPE}_A - \text{EPE}_B}{s} = \frac{9.0 \times 10^{-4}\ \text{J}}{0.20\ \text{m}} = \boxed{4.5 \times 10^{-3}\ \text{N}}$$

As discussed in the reasoning, the direction of the force is $\boxed{\text{from } A \text{ toward } B}$.

b. From Equation 18.2, we find that the electric field has a magnitude of

$$E = \frac{F}{q_0} = \frac{4.5 \times 10^{-3}\ \text{N}}{1.5 \times 10^{-6}\ \text{C}} = \boxed{3.0 \times 10^{3}\ \text{N/C}}$$

The direction is the same as that of the force on the positive charge, namely $\boxed{\text{from } A \text{ toward } B}$.

3. ***REASONING*** The number N of electrons that jump from your hand (point A) to the door knob (point B) is equal to the total charge q that jumps divided by the charge $-e$ of one electron: $N = q/(-e)$, where $e = 1.6 \times 10^{-19}$ C. We can determine q by using Equation 19.4, which relates the work W_{AB} done by the electric force to the difference in electric potentials, $V_B - V_A$, and the charge. The difference in potentials is given as $V_B - V_A = 2.0 \times 10^4$ V.

SOLUTION The number of electrons that jumps from your hand to the door knob is

$$N = \frac{q}{-e} = \frac{\dfrac{-W_{AB}}{V_B - V_A}}{-e} = \frac{\dfrac{-1.5 \times 10^{-7}\ \text{J}}{2.0 \times 10^4\ \text{V}}}{-1.6 \times 10^{-19}\ \text{C}} = \boxed{4.7 \times 10^7}$$

4. ***REASONING AND SOLUTION***

a. According to Equation 19.4, the work done by the electric force as the electron goes from point A (the cathode) to point B (the anode) is

$$W_{AB} = -q(V_B - V_A) = -(-1.6 \times 10^{-19}\ \text{C})(+125\,000\ \text{V}) = \boxed{+2.00 \times 10^{-14}\ \text{J}}$$

b. The only force that acts on the electron is the conservative electric force. Therefore, the total energy of the electron is conserved as it moves from point A to point B:

$$\underbrace{\tfrac{1}{2}mv_A^2 + \text{EPE}_A}_{\substack{\text{Total energy at}\\ \text{point } A}} = \underbrace{\tfrac{1}{2}mv_B^2 + \text{EPE}_B}_{\substack{\text{Total energy at}\\ \text{point } B}}$$

Since the electron starts from rest, $v_A = 0$. The electric potential V is related to the electric potential energy EPE by $V = \text{EPE}/q$ (see Equation 19.3). With these changes, the equation above gives the kinetic energy of the electron at point B (the anode) to be

$$\tfrac{1}{2}mv_B^2 = -\text{EPE}_B + \text{EPE}_A$$

$$= -q\left(V_B - V_A\right) = -\left(-1.60 \times 10^{-19}\ \text{C}\right)\left(125\,000\ \text{V}\right) = \boxed{2.00 \times 10^{-14}\ \text{J}}$$

5. SSM ***REASONING*** The only force acting on the moving electron is the conservative electric force. Therefore, the total energy of the electron (the sum of the kinetic energy KE and the electric potential energy EPE) remains constant throughout the trajectory of the electron. Let the subscripts A and B refer to the initial and final positions, respectively, of the electron. Then,

$$\tfrac{1}{2}mv_A^2 + \text{EPE}_A = \tfrac{1}{2}mv_B^2 + \text{EPE}_B$$

Solving for v_B gives

$$v_B = \sqrt{v_A^2 - \frac{2}{m}(\text{EPE}_B - \text{EPE}_A)}$$

Since the electron starts from rest, $v_A = 0$. The difference in potential energies is related to the difference in potentials by Equation 19.4, $\text{EPE}_B - \text{EPE}_A = q(V_B - V_A)$.

SOLUTION The speed v_B of the electron just before it reaches the screen is

$$v_B = \sqrt{-\frac{2q}{m}(V_B - V_A)} = \sqrt{-\frac{2(-1.6\times10^{-19}\ \text{C})}{9.11\times10^{-31}\ \text{kg}}(25\ 000\ \text{V})} = \boxed{9.4\times10^{7}\ \text{m/s}}$$

6. ***REASONING AND SOLUTION***

a. The only force that acts on the particle is the conservative electric force. Therefore, the total energy of the particle is conserved as it moves from point A to point B:

$$\underbrace{\tfrac{1}{2}mv_A^2 + \text{EPE}_A}_{\substack{\text{Total energy at}\\ \text{point } A}} = \underbrace{\tfrac{1}{2}mv_B^2 + \text{EPE}_B}_{\substack{\text{Total energy at}\\ \text{point } B}}$$

Since the particle starts from rest, $v_A = 0$. The electric potential V is related to the electric potential energy EPE by $V = \text{EPE}/q$ (see Equation 19.3). With these changes, we can solve the equation above for the potential difference $V_B - V_A$:

$$V_B - V_A = \frac{1}{q}\left(0 - \tfrac{1}{2}mv_B^2\right) = \frac{1}{-1.5\times10^{-6}\ \text{C}}\left[0 - \tfrac{1}{2}\left(2.5\times10^{-6}\ \text{kg}\right)\left(42\ \text{m/s}\right)^2\right] = \boxed{1500\ \text{V}}$$

b. $\boxed{\text{Point } B}$ is at the higher potential, because a negative charge accelerates from a lower potential to a higher potential.

7. ***REASONING AND SOLUTION*** The power rating P is defined as the work W_{AB} done by the battery divided by the time t,

$$P = \frac{W_{AB}}{t}$$

The work done by the electric force as the charge moves from point A (the positive terminal), through the electric motor, and to point B (the negative terminal) is

$$W_{AB} = q\left(V_A - V_B\right) = \left(1300\text{ C}\right)\left(320\text{ V}\right) = 4.2 \times 10^5\text{ J} \tag{19.4}$$

The power rating is

$$P = \frac{W_{AB}}{t} = \frac{4.2 \times 10^5\text{ J}}{8.0\text{ s}} = 5.2 \times 10^4\text{ W}$$

Since 746 W = 1 hp, the minimum horsepower rating of the car is

$$\left(5.20 \times 10^4\text{ W}\right)\frac{1\text{ hp}}{746\text{ W}} = \boxed{7.0 \times 10^1\text{ hp}}$$

8. ***REASONING*** According to Equation 19.4, the energy available is $\Delta(\text{EPE}) = q_0 \Delta V$, where q_0 is the charge and ΔV is the potential difference between the battery terminals, that is, 12 V. The energy (heat) need to boil m kilograms of water is given by Equation 12.5 as $Q = mL_\text{v}$, where $L_\text{v} = 22.6 \times 10^5$ J/kg is the latent heat of vaporization of water (see Table 12.3).

SOLUTION Using Equations 19.4 and 12.5, we obtain

$$\Delta(\text{EPE}) = mL_\text{v} = q_0 \Delta V$$

$$m = \frac{q_0 \Delta V}{L_\text{v}} = \frac{(7.5 \times 10^5 \text{ C})(12 \text{ V})}{22.6 \times 10^5 \text{ J/kg}} = \boxed{4.0 \text{ kg}}$$

9. SSM WWW ***REASONING*** The only force acting on the moving charge is the conservative electric force. Therefore, the total energy of the charge remains constant. Applying the principle of conservation of energy between locations A and B, we obtain

$$\tfrac{1}{2}mv_A^2 + \text{EPE}_A = \tfrac{1}{2}mv_B^2 + \text{EPE}_B$$

Since the charged particle starts from rest, $v_A = 0$. The difference in potential energies is related to the difference in potentials by Equation 19.4, $\text{EPE}_B - \text{EPE}_A = q(V_B - V_A)$. Thus, we have

$$q(V_A - V_B) = \tfrac{1}{2}mv_B^2 \qquad (1)$$

Similarly, applying the conservation of energy between locations C and B gives

$$q(V_C - V_B) = \tfrac{1}{2}m(2v_B)^2 \qquad (2)$$

Dividing Equation (1) by Equation (2) yields

$$\frac{V_A - V_B}{V_C - V_B} = \frac{1}{4}$$

This expression can be solved for V_B.

SOLUTION Solving for V_B, we find that

$$V_B = \frac{4V_A - V_C}{3} = \frac{4(452\text{ V}) - 791\text{ V}}{3} = \boxed{339\text{ V}}$$

10. ***REASONING*** The gravitational and electric forces are conservative forces, so the total energy of the particle remains constant as it moves from point A to point B. Recall from Equation 6.5 that the gravitational potential energy (GPE) of a particle of mass m is GPE = mgh, where h is the height of the particle above the earth's surface. The conservation of energy is written as

$$\tfrac{1}{2}mv_A^2 + mgh_A + \text{EPE}_A = \tfrac{1}{2}mv_B^2 + mgh_B + \text{EPE}_B \qquad (1)$$

We will use this equation several times to determine the initial speed v_A of the negatively charged particle.

SOLUTION

When the negatively charged particle is thrown upward, it attains a maximum height of h. For this particle we have:

$v_A = ?$	$v_B = 0$ (at maximum height)
$\text{EPE}_A = (-q)\,V_A$	$\text{EPE}_B = (-q)V_B$
$h_A = 0$ (ground level)	$h_B = h$ (the maximum height)

Solving the conservation of energy equation, Equation (1), for v_A and substituting in the data above gives

$$v_A = \sqrt{\frac{2}{m}\left[mgh + (-q)(V_B - V_A)\right]} \qquad (2)$$

Equation (2) cannot be solved as it stands because the height h and the potential difference $(V_B - V_A)$ are not known. We now make use of the fact that a positively charged particle, when thrown straight upward with an initial speed of 30.0 m/s, also reaches the maximum height h. For this particle we have:

$v_A = 30.0$ m/s	$v_B = 0$ (at maximum height)
$\text{EPE}_A = (+q)\,V_A$	$\text{EPE}_B = (+q)V_B$
$h_A = 0$ (ground level)	$h_B = h$

Solving the conservation of energy equation, Equation (1), for the potential difference $(V_B - V_A)$ and substituting in the data above gives

$$V_B - V_A = \frac{1}{+q}\left[\tfrac{1}{2}m(30.0\text{ m/s})^2 - mgh\right] \qquad (3)$$

Substituting Equation (3) into Equation (2) gives, after some algebraic simplifications,

$$v_A = \sqrt{4gh - (30.0\text{ m/s})^2} \qquad (4)$$

Equation (4) cannot be solved because the height h is still unknown. We now make use of the fact that the uncharged particle, when thrown straight upward with an initial speed of 25.0 m/s, also reaches the maximum height h. For this particle we have:

$v_A = 25.0$ m/s	$v_B = 0$ (at maximum height)
$\text{EPE}_A = qV_A = 0$ (since $q = 0$)	$\text{EPE}_B = qV_B = 0$ (since $q = 0$)
$h_A = 0$ (ground level)	$h_A = h$

Solving Equation (1) with this data for the maximum height h yields

$$h = \frac{(25.0\ \text{m/s})^2}{2g} = \frac{(25.0\ \text{m/s})^2}{2(9.80\ \text{m/s}^2)} = 31.9\ \text{m}$$

Substituting $h = 31.9$ m into Equation (4) gives $v_A = \boxed{18.7\ \text{m/s}}$.

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11. **SSM** ***REASONING AND SOLUTION*** The electric potential V at a distance r from a point charge q is given by Equation 19.6, $V = kq/r$. Solving this expression for q, we find that

$$q = \frac{rV}{k} = \frac{(0.25\text{ m})(+130\text{ V})}{9.0\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{+3.6\times10^{-9}\ \text{C}}$$

12. ***REASONING*** The potential of each charge q at a distance r away is given by Equation 19.6 as $V = kq/r$. By applying this expression to each charge, we will be able to find the desired ratio, because the distances are given for each charge.

SOLUTION According to Equation 19.6, the potentials of each charge are

$$V_A = \frac{kq_A}{r_A} \quad \text{and} \quad V_B = \frac{kq_B}{r_B}$$

Since we know that $V_A = V_B$, it follows that

$$\frac{kq_A}{r_A} = \frac{kq_B}{r_B} \quad \text{or} \quad \frac{q_B}{q_A} = \frac{r_B}{r_A} = \frac{0.43\text{ m}}{0.18\text{ m}} = \boxed{2.4}$$

13. ***REASONING*** The potential V at a distance r from a proton is $V = k(+e)/r$ (see Equation 19.6), where $+e$ is the charge of the proton. When an electron ($q = -e$) is placed at a distance r from the proton, the electric potential energy is EPE $= -eV$, as per Equation 19.3.

SOLUTION The difference in the electric potential energies when the electron and proton are separated by $r_{\text{final}} = 5.29 \times 10^{-11}$ m and when they are very far apart ($r_{\text{initial}} = \infty$) is

$$\text{EPE}_{\text{final}} - \text{EPE}_{\text{initial}} = \frac{(-e)ke}{r_{\text{final}}} - \frac{(-e)ke}{r_{\text{initial}}}$$

$$= -(8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\left(1.60 \times 10^{-19}\ \text{C}\right)^2$$

$$\times\left(\frac{1}{5.29 \times 10^{-11}\ \text{m}} - \frac{1}{\infty}\right) = \boxed{-4.35 \times 10^{-18}\ \text{J}}$$

14. ***REASONING AND SOLUTION*** The potential at point A is

$$V_A = \frac{kq}{r_A} = \frac{\left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2\right)\left(-3.00 \times 10^{-8} \text{ C}\right)}{2.00 \text{ m}} = -135 \text{ V}$$

Similarly, the potential at point B is $V_B = kq/r_B = -89.9$ V

The difference in the potentials is

$$V_B - V_A = \boxed{+45 \text{ V}}$$

$$\boxed{\text{Point } B \text{ is at the higher potential.}}$$

15. **SSM** **WWW** ***REASONING*** Initially, suppose that one charge is at C and the other charge is held fixed at B. The charge at C is then moved to position A. According to Equation 19.4, the work W_{CA} done by the electric force as the charge moves from C to A is $W_{CA} = q(V_C - V_A)$, where, from Equation 19.6, $V_C = kq/d$ and $V_A = kq/r$. From the figure at the right we see that $d = \sqrt{r^2 + r^2} = \sqrt{2}r$. Therefore, we find that

$$W_{CA} = q\left(\frac{kq}{\sqrt{2}r} - \frac{kq}{r}\right) = \frac{kq^2}{r}\left(\frac{1}{\sqrt{2}} - 1\right)$$

SOLUTION Substituting values, we obtain

$$W_{CA} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(3.0\times10^{-6}\ \text{C})^2}{0.500\ \text{m}}\left(\frac{1}{\sqrt{2}} - 1\right) = \boxed{-4.7\times10^{-2}\ \text{J}}$$

16. ***REASONING AND SOLUTION*** The initial and final electric potential energies of the configuration are, respectively,

$$\text{EPE}_A = \frac{kq_1q_2}{r_A} \qquad \text{and} \qquad \text{EPE}_B = \frac{kq_1q_2}{r_B}$$

Since $\text{EPE}_B = 2\ \text{EPE}_A$, we have

$$\frac{kq_1q_2}{r_B} = 2\frac{kq_1q_2}{r_A}$$

or

$$r_B = \frac{r_A}{2} = \frac{0.74\ \text{m}}{2} = \boxed{0.37\ \text{m}}.$$

17. ***REASONING AND SOLUTION*** Let s be the length of the side of the square and Q be the value of the unknown charge. The potential at either of the vacant corners is

$$V = 0 = \frac{k(9q)}{s} + \frac{k(-8q)}{s} + \frac{kQ}{s/\sqrt{2}}$$

so

$$\boxed{Q = \frac{-q}{\sqrt{2}}}$$

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18. ***REASONING AND SOLUTION*** Let the first spot where the potential is zero be a distance x to the left of the negative charge. Then,

$$\frac{k(2q)}{d-x} = \frac{kq}{x} \qquad or \qquad x = \boxed{\frac{d}{3}}$$

Let the second spot where the potential is zero be a distance x to the right of the negative charge. Then,

$$\frac{k(2q)}{d+x} = \frac{kq}{x} \qquad or \qquad x = \boxed{d}$$

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19. **SSM** ***REASONING*** The only force acting on the moving charge is the conservative electric force. Therefore, the sum of the kinetic energy KE and the electric potential energy EPE is the same at points A and B:

$$\tfrac{1}{2}mv_A^2 + \text{EPE}_A = \tfrac{1}{2}mv_B^2 + \text{EPE}_B$$

Since the particle comes to rest at B, $v_B = 0$. Combining Equations 19.3 and 19.6, we have

$$\text{EPE}_A = qV_A = q\left(\frac{kq_1}{d}\right)$$

and

$$\text{EPE}_B = qV_B = q\left(\frac{kq_1}{r}\right)$$

where d is the initial distance between the fixed charge and the moving charged particle, and r is the distance between the charged particles after the moving charge has stopped. Therefore, the expression for the conservation of energy becomes

$$\tfrac{1}{2}mv_A^2 + \frac{kqq_1}{d} = \frac{kqq_1}{r}$$

This expression can be solved for r. Once r is known, the distance that the charged particle moves can be determined.

SOLUTION Solving the expression above for r gives

$$r = \frac{kqq_1}{\tfrac{1}{2}mv_A^2 + \dfrac{kqq_1}{d}}$$

$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-8.00\times10^{-6}\ \text{C})(-3.00\times10^{-6}\ \text{C})}{\tfrac{1}{2}(7.20\times10^{-3}\ \text{kg})(65.0\ \text{m/s})^2 + \dfrac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(-8.00\times10^{-6}\ \text{C})(-3.00\times10^{-6}\ \text{C})}{0.0450\ \text{m}}}$$

$$= 0.0108\ \text{m}$$

Therefore, the charge moves a distance of $0.0450\ \text{m} - 0.0108\ \text{m} = \boxed{0.0342\ \text{m}}$.

20. ***REASONING*** It will not matter in what order the group is assembled. For convenience, we will assemble the group from one end of the line to the other. The potential energy of each charge added to the group will be determined and the four values added together to get the total. At each step, the electric potential energy of an added charge q_0 is given by Equation 19.3 as EPE = $q_0 V_{\text{Total}}$, where V_{Total} is the potential at the point where the added charge is placed. The potential V_{Total} will be determined by adding together the contributions from the charges previously put in position, each according to Equation 19.6 ($V = kq/r$).

SOLUTION The first charge added to the group has no electric potential energy, since the spot where it goes has a total potential of $V_{\text{Total}} = 0$ J, there being no charges in the vicinity to create it:

$$\text{EPE}_1 = q_0 V_{\text{Total}} = \left(2.0 \times 10^{-6}\ \text{C}\right)\left(0\ \text{V}\right) = 0\ \text{J}$$

The second charge experiences a total potential that is created by the first charge:

$$V_{\text{Total}} = \frac{kq}{r} = \frac{\left(8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(+2.0 \times 10^{-6}\ \text{C}\right)}{0.40\ \text{m}} = 4.5 \times 10^{4}\ \text{V}$$

$$\text{EPE}_2 = q_0 V_{\text{Total}} = \left(2.0 \times 10^{-6}\ \text{C}\right)\left(4.5 \times 10^{4}\ \text{V}\right) = 0.090\ \text{J}$$

The third charge experiences a total potential that is created by the first and the second charges:

$$V_{\text{Total}} = \frac{\left(8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(+2.0 \times 10^{-6}\ \text{C}\right)}{0.80\ \text{m}} + \frac{\left(8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(+2.0 \times 10^{-6}\ \text{C}\right)}{0.40\ \text{m}} = 6.7 \times 10^{4}\ \text{V}$$

$$\text{EPE}_3 = q_0 V_{\text{Total}} = \left(2.0 \times 10^{-6}\ \text{C}\right)\left(6.7 \times 10^{4}\ \text{V}\right) = 0.13\ \text{J}$$

The fourth charge experiences a total potential that is created by the first, second, and third charges:

$$V_{\text{Total}} = \frac{\left(8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(+2.0 \times 10^{-6}\ \text{C}\right)}{1.2\ \text{m}} + \frac{\left(8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(+2.0 \times 10^{-6}\ \text{C}\right)}{0.80\ \text{m}} + \frac{\left(8.99 \times 10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(+2.0 \times 10^{-6}\ \text{C}\right)}{0.40\ \text{m}} = 8.2 \times 10^{4}\ \text{V}$$

$$\text{EPE}_4 = q_0 V_{\text{Total}} = \left(2.0 \times 10^{-6}\ \text{C}\right)\left(8.2 \times 10^{4}\ \text{V}\right) = 0.16\ \text{J}$$

The total electric potential energy of the group is

$$EPE_{Total} = EPE_1 + EPE_2 + EPE_3 + EPE_4$$

$$= 0\ J + 0.090\ J + 0.13\ J + 0.16\ J = \boxed{0.38\ J}$$

21. ***REASONING*** The only force acting on each proton is the conservative electric force. Therefore, the total energy (kinetic energy plus electric potential energy) is conserved at all points along the motion. For two points, A and B, the conservation of energy is

$$\underbrace{\tfrac{1}{2}mv_A^2 + \tfrac{1}{2}mv_A^2}_{\text{Initial kinetic energy of the two protons}} + \underbrace{\text{EPE}_A}_{\text{Initial electric potential energy}} = \underbrace{\tfrac{1}{2}mv_B^2 + \tfrac{1}{2}mv_B^2}_{\text{Final kinetic energy of the two protons}} + \underbrace{\text{EPE}_B}_{\text{Final electric potential energy}}$$

The electric potential energy of two protons (charge = $+e$) that are separated by a distance r is EPE = ke^2/r. By substituting this expression for EPE into the conservation of energy, we will be able to determine the distance of closest approach.

SOLUTION When the protons are very far apart ($r_A = \infty$), so that $\text{EPE}_A = 0$. At the distance r_B of closest approach, the speed of each proton is momentarily zero ($v_B = 0$). With these substitutions, the conservation of energy equation reduces to

$$\tfrac{1}{2}mv_A^2 + \tfrac{1}{2}mv_A^2 = \frac{ke^2}{r_B}$$

Solving for r_B, the distance of closest approach is

$$r_B = \frac{ke^2}{mv_A^2} = \frac{\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.6\times10^{-19}\ \text{C}\right)^2}{\left(1.67\times10^{-27}\ \text{kg}\right)\left(1.5\times10^6\ \text{m/s}\right)^2} = \boxed{6.1\times10^{-14}\ \text{m}}$$

22. ***REASONING AND SOLUTION*** The figure at the right shows two identical charges, q, fixed to diagonally opposite corners of a square. The potential at corner A is caused by the presence of the two charges. It is given by

$$(V_A)_0 = \frac{kq}{r} + \frac{kq}{r} = \frac{2kq}{r}$$

Since both charges are the same distance from corner B, this is equal to the potential at corner B as well. If a third charge, q_3, is placed at the center of the square, the potential at corner A (as well as corner B) becomes

$$(V_A)_f = \frac{kq}{r} + \frac{kq}{r} + \frac{kq_3}{(d/2)}$$

We can find d by applying the Pythagorean theorem to the geometry in the figure above:

$$d = \sqrt{r^2 + r^2} = \sqrt{2}r \qquad \text{so that} \qquad \frac{d}{2} = \frac{r}{\sqrt{2}}$$

The expression for $(V_A)_f$ becomes

$$(V_A)_f = \frac{kq}{r} + \frac{kq}{r} + \frac{kq_3\sqrt{2}}{r}.$$

From the problem statement, we know that the addition of q_3 causes the potential at A and B to change sign without changing magnitude.

In other words, $(V_A)_f = -(V_A)_0$, or

$$\frac{kq}{r} + \frac{kq}{r} + \frac{kq_3\sqrt{2}}{r} = -\frac{2kq}{r}$$

Solving for q_3 gives

$$q_3 = -(2\sqrt{2})q = -(2\sqrt{2})(1.7 \times 10^{-6}\ \text{C}) = \boxed{-4.8 \times 10^{-6}\ \text{C}}$$

23. SSM ***REASONING*** Initially, the three charges are infinitely far apart. We will proceed as in Example 8 by adding charges to the triangle, one at a time, and determining the electric potential energy at each step. According to Equation 19.3, the electric potential energy EPE is the product of the charge q and the electric potential V at the spot where the charge is placed, EPE = qV. The total electric potential energy of the group is the sum of the energies of each step in assembling the group.

SOLUTION Let the corners of the triangle be numbered clockwise as 1, 2 and 3, starting with the top corner. When the first charge (q_1 = 8.00 μC) is placed at a corner 1, the charge has no electric potential energy, $\text{EPE}_1 = 0$. This is because the electric potential V_1 produced by the other two charges at corner 1 is zero, since they are infinitely far away.

Once the 8.00-μC charge is in place, the electric potential V_2 that it creates at corner 2 is

$$V_2 = \frac{kq_1}{r_{21}}$$

where r_{21} = 5.00 m is the distance between corners 1 and 2, and q_1 = 8.00 μC. When the 20.0-μC charge is placed at corner 2, its electric potential energy EPE_2 is

$$\text{EPE}_2 = q_2 V_2 = q_2\left(\frac{kq_1}{r_{21}}\right)$$

$$= \left(20.0\times10^{-6}\ \text{C}\right)\left[\frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.00\times10^{-6}\ \text{C}\right)}{5.00\ \text{m}}\right] = 0.288\ \text{J}$$

The electric potential V_3 at the remaining empty corner is the sum of the potentials due to the two charges that are already in place on corners 1 and 2:

$$V_3 = \frac{kq_1}{r_{31}} + \frac{kq_2}{r_{32}}$$

where q_1 = 8.00 μC, r_{31} = 3.00 m, q_2 = 20.0 μC, and r_{32} = 4.00 m. When the third charge (q_3 = −15.0 μC) is placed at corner 3, its electric potential energy EPE_3 is

$$\text{EPE}_3 = q_3 V_3 = q_3\left(\frac{kq_1}{r_{31}} + \frac{kq_2}{r_{32}}\right) = q_3 k\left(\frac{q_1}{r_{31}} + \frac{q_2}{r_{32}}\right)$$

$$= \left(-15.0\times10^{-6}\ \text{C}\right)\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{8.00\times10^{-6}\ \text{C}}{3.00\ \text{m}} + \frac{20.0\times10^{-6}\ \text{C}}{4.00\ \text{m}}\right) = -1.034\ \text{J}$$

The electric potential energy of the entire array is given by

$$\text{EPE} = \text{EPE}_1 + \text{EPE}_2 + \text{EPE}_3 = 0 + 0.288\ \text{J} + (-1.034\ \text{J}) = \boxed{-0.746\ \text{J}}$$

24. ***REASONING AND SOLUTION***

a. Let d be the distance between the charges. The potential at the point $x_1 = 4.00$ cm to the left of the negative charge is

$$V = 0 = \frac{kq_1}{d - x_1} - \frac{kq_2}{x_1}$$

which gives

$$\frac{q_1}{q_2} = \frac{d}{x_1} - 1 \qquad (1)$$

Similarly, at the point $x_2 = 7.00$ cm to the right of the negative charge we have

$$V = 0 = \frac{kq_1}{x_2 + d} - \frac{kq_2}{x_2}$$

which gives

$$\frac{q_1}{q_2} = \frac{d}{x_2} + 1 \qquad (2)$$

Equating Equations (1) and (2) and solving for d gives $d = \boxed{0.187 \text{ m}}$.

b. Using the above value for d in Equation (1) yields $\frac{q_1}{q_2} = \boxed{3.67}$.

25. ***REASONING AND SOLUTION*** The electrical potential energy of the group of charges is

$$\text{EPE} = kq_1q_2/d + kq_1q_3/(2d) + kq_2q_3/d = 0$$

so

$$q_1q_2 + (1/2)q_1q_3 + q_2q_3 = 0$$

a. If $q_1 = q_2 = q$, then

$$q + (1/2)q_3 + q_3 = 0 \qquad \text{or} \qquad \boxed{q_3 = -\tfrac{2}{3}q}$$

b. If $q_1 = q$ and $q_2 = -q$ then

$$-q + (1/2)q_3 - q_3 = 0 \qquad \text{or} \qquad \boxed{q_3 = -2q}$$

26. ***REASONING*** The only force acting on each particle is the conservative electric force. Therefore, the total energy (kinetic energy plus electric potential energy) is conserved as the particles move apart. In addition, the net external force acting on the system of two particles is zero (the electric force that each particle exerts on the other is an internal force). Thus, the total linear momentum of the system is also conserved. We will use the conservation of energy and the conservation of linear momentum to find the final speed of each particle.

SOLUTION For two points, *A* and *B*, along the motion, the conservation of energy is

$$\underbrace{\tfrac{1}{2}mv_{1,A}^2 + \tfrac{1}{2}mv_{2,A}^2}_{\text{Initial kinetic energy of the two particles}} + \underbrace{\frac{kq_1q_2}{r_A}}_{\text{Initial electric potential energy}} = \underbrace{\tfrac{1}{2}mv_{1,B}^2 + \tfrac{1}{2}mv_{2,B}^2}_{\text{Final kinetic energy of the two particles}} + \underbrace{\frac{kq_1q_2}{r_B}}_{\text{Final electric potential energy}}$$

Setting $v_{1,A} = v_{2,A} = 0$ since the particles are initially at rest, and letting $r_B = \frac{1}{2}r_A$, the conservation of energy equation becomes

$$\tfrac{1}{2}mv_{1,B}^2 + \tfrac{1}{2}mv_{2,B}^2 = -\frac{kq_1q_2}{r_A} \qquad (1)$$

This equation cannot be solved for $v_{1,B}$ because the final speed $v_{2,B}$ of the second particle is not known. To find this speed, we will use the conservation of linear momentum:

$$\underbrace{mv_{1,A} + mv_{2,A}}_{\text{Initial linear momentum}} = \underbrace{mv_{1,B} + mv_{2,B}}_{\text{Final linear momentum}}$$

Setting $v_{1,A} = v_{2,A} = 0$ and solving for $v_{2,B}$ gives $v_{2,B} = -v_{1,B}$. Substituting this result into Equation (1) and solving for $v_{1,B}$ yields

$$v_{1,B} = \sqrt{\frac{-kq_1q_2}{mr_A}}$$

$$= \sqrt{\frac{-\left(9.0\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(+5.0\times10^{-6}\ \text{C}\right)\left(-5.0\times10^{-6}\ \text{C}\right)}{\left(6.0\times10^{-3}\ \text{kg}\right)\left(0.80\ \text{m}\right)}} = \boxed{6.8\ \text{m/s}}$$

This is also the speed of $v_{2,B}$.

27. **SSM** ***REASONING AND SOLUTION*** Since all points on the equipotential surface are the same distance r from the point charge, the potential is given by Equation 19.6,

$$V = \frac{kq}{r} = \frac{(8.99 \times 10^9 \ \text{N} \cdot \text{m}^2 / \text{C}^2)(3.0 \times 10^{-7} \ \text{C})}{0.15 \ \text{m}} = \boxed{18 \ 000 \ \text{V}}$$

28. ***REASONING*** The potential of a point charge is given by Equation 19.6 ($V = kq/r$). This expression can be solved directly for the charge q.

SOLUTION Using Equation 19.6, we find

$$V = \frac{kq}{r} \quad \text{or} \quad q = \frac{rV}{k} = \frac{(0.18\ \text{m})(+36\ \text{V})}{8.99 \times 10^{9}\ \text{N} \cdot \text{m}^2 / \text{C}^2} = \boxed{+7.2 \times 10^{-10}\ \text{C}}$$

29. ***REASONING*** The magnitude E of the electric field is given by Equation 19.7 (without the minus sign) as $E = \frac{\Delta V}{\Delta s}$, where ΔV is the potential difference between the two metal conductors of the spark plug, and Δs is the distance between the two conductors. We can use this relation to find ΔV.

SOLUTION The potential difference between the conductors is

$$\Delta V = E\Delta s = \left(4.7 \times 10^{7}\ \text{V/m}\right)\left(0.75 \times 10^{-3}\ \text{m}\right) = \boxed{3.5 \times 10^{4}\ \text{V}}$$

30. ***REASONING AND SOLUTION*** From Equation 19.7 we know that $E = -\dfrac{\Delta V}{\Delta s}$, where ΔV is the potential difference between the two surfaces of the membrane, and Δs is the distance between them. If A is a point on the positive surface and B is a point on the negative surface, then $\Delta V = V_A - V_B = 0.070$ V. The electric field between the surfaces is

$$E = -\frac{\Delta V}{\Delta s} = -\frac{V_B - V_A}{\Delta s} = \frac{V_A - V_B}{\Delta s} = \frac{0.070\ \text{V}}{8.0 \times 10^{-9}\ \text{m}} = \boxed{8.8 \times 10^{6}\ \text{V/m}}$$

31. **SSM** ***REASONING AND SOLUTION*** Equation 19.7 gives the result directly:

$$E = -\frac{\Delta V}{\Delta s} = -\frac{V_B - V_A}{\Delta s} = -\frac{28\text{ V} - 95\text{ V}}{0.016\text{ m}} = \boxed{4.2 \times 10^3\text{ V/m}}$$

The electric field points from high potential to low potential. Thus, it points from $\boxed{A \text{ to } B}$.

32. ***REASONING AND SOLUTION*** Let E_1 represent the magnitude of the electric field on the first equipotential surface, and E_2 represent the magnitude of the electric field when it has shrunk to one-half of its initial value. Then

$$E_1 = \frac{kq}{r_1^{\,2}} \qquad \text{and} \qquad E_2 = \frac{kq}{r_2^{\,2}}$$

Since $E_2 = \frac{1}{2}E_1$, we have

$$\frac{kq}{r_2^{\,2}} = \frac{1}{2}\frac{kq}{r_1^{\,2}} \qquad \text{so that} \qquad \frac{r_2}{r_1} = \sqrt{2}$$

Thus, the electric field shrinks to one-half its original value at a distance of $r_2 = \sqrt{2}\ r_1$.

The potential difference between the first equipotential surface and the locus of points where the electric field has dropped to one-half of its original magnitude is

$$\Delta V = \frac{kq}{r_2} - \frac{kq}{r_1} = \frac{kq}{r_1}\left(\frac{1}{\sqrt{2}} - 1\right)$$

$$= \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/C^2)(2.00\times10^{-6}\ \text{C})}{1.60\ \text{m}}\left(\frac{1}{\sqrt{2}} - 1\right) = -3290\ \text{V}$$

Since the potential difference between two successive surfaces is 1.00×10^3 V, we have

$$\frac{\begin{array}{c}\text{Magnitude of}\\ \Delta V \text{ from } r_1 \text{ to } r_2\end{array}}{\begin{array}{c}\Delta V \text{ between successive}\\ \text{equipotential surfaces}\end{array}} = \frac{3290\ \text{V}}{1.00\times10^{3}\ \text{V}} = 3.29$$

Therefore, the number of equipotential surfaces crossed in going radially outward from r_1 to r_2 is $\boxed{3}$.

---

33. ***REASONING AND SOLUTION*** The drawing shows the electric field and the three points, A, B, and C, around the point P, which we take as the origin. We choose the upward direction as being positive. Thus, $E = -3.0 \times 10^3$ V/m, since the electric field points straight down.

a. The electric potential at point A can be determined from Equation 19.7 as

$$\underbrace{E}_{-3.0\times 10^3\ \mathrm{V/m}} = -\frac{\Delta V}{\Delta s} = -\frac{V_P - V_A}{\Delta s} = -\frac{135\ \mathrm{V} - V_A}{0 - 8.0\times 10^{-3}\ \mathrm{m}}$$

so that $\boxed{V_A = 159\ \mathrm{V}}$.

b. The electric potential at point B is found in a similar manner:

$$\underbrace{E}_{-3.0\times 10^3\ \mathrm{V/m}} = -\frac{\Delta V}{\Delta s} = -\frac{V_B - V_P}{\Delta s} = -\frac{V_B - 135\ \mathrm{V}}{-3.3\times 10^{-3}\ \mathrm{m} - 0}$$

so that $\boxed{V_B = 125\ \mathrm{V}}$

c. $\Delta V = 0$, since the path is perpendicular to the electric field, and no work is done in moving a charge along such a path: $\boxed{V_C = 135\ \mathrm{V}}$

34. ***REASONING AND SOLUTION*** Let point A be on the x-axis where the potential is 515 V. Let point B be on the x-axis where the potential is 495 V. From Equation 19.7, the electric field is

$$E = -\frac{\Delta V}{\Delta s} = -\frac{V_B - V_A}{\Delta s} = -\frac{495\text{ V} - 515\text{ V}}{-2\left(6.0 \times 10^{-3}\text{ m}\right)} = -1.7 \times 10^3\text{ V/m}$$

The magnitude of the electric field is $\boxed{1.7 \times 10^3\text{ V/m}}$. Since the electric field is negative, it points to the $\boxed{\text{left}}$, from the high toward the low potential.

35. **SSM** ***REASONING*** The total energy of the particle on the equipotential surface A is $E_A = \frac{1}{2}mv_A^2 + qV_A$. Similarly, the total energy of the particle when it reaches equipotential surface B is $E_B = \frac{1}{2}mv_B^2 + qV_B$. According to Equation 6.8, the work W done by the outside force is equal to the final total energy E_B minus the initial total energy E_A, $W = E_B - E_A$.

SOLUTION The work done by the outside force in moving the particle from A to B is,

$$W = E_B - E_A = \left(\tfrac{1}{2}mv_B^2 + qV_B\right) - \left(\tfrac{1}{2}mv_A^2 + qV_A\right) = \tfrac{1}{2}m\left(v_B^2 - v_A^2\right) + q(V_B - V_A)$$

$$= \tfrac{1}{2}(5.00\times10^{-2}\ \text{kg})\left[(3.00\ \text{m/s})^2 - (2.00\ \text{m/s})^2\right]$$

$$+ (4.00\times10^{-5}\ \text{C})\left[7850\ \text{V} - 5650\ \text{V}\right] = \boxed{0.213\ \text{J}}$$

36. ***REASONING AND SOLUTION*** The capacitance, voltage, and charge are related by

$$V = \frac{q}{C} = \frac{7.2 \times 10^{-5}\ \text{C}}{6.0 \times 10^{-6}\ \text{F}} = \boxed{12\ \text{V}} \tag{19.8}$$

37. **SSM** ***REASONING*** According to Equation 19.11, the energy stored in a capacitor with capacitance C and potential V across its plates is $\text{Energy} = \frac{1}{2}CV^2$.

SOLUTION Therefore, solving Equation 19.11 for V, we have

$$V = \sqrt{\frac{2(\text{Energy})}{C}} = \sqrt{\frac{2(73\text{ J})}{120\times10^{-6}\text{ F}}} = \boxed{1.1\times10^{3}\text{ V}}$$

38. ***REASONING AND SOLUTION*** The capacitance is $C = q_0/V_0 = q/V$. The new charge q is, therefore,

$$q = \frac{q_0 V}{V_0} = \frac{\left(5.3 \times 10^{-5}\ \text{C}\right)\left(9.0\ \text{V}\right)}{6.0\ \text{V}} = \boxed{8.0 \times 10^{-5}\ \text{C}}$$

39. SSM ***REASONING*** The charge that resides on the outer surface of the cell membrane is $q = CV$, according to Equation 19.8. Before we can use this expression, however, we must first determine the capacitance of the membrane. If we assume that the cell membrane behaves like a parallel plate capacitor filled with a dielectric, Equation 19.10 ($C = \kappa \varepsilon_0 A / d$) applies as well.

SOLUTION The capacitance of the cell membrane is

$$C = \frac{\kappa \varepsilon_0 A}{d} = \frac{(5.0)(8.85 \times 10^{-12}\ \text{F/m})(5.0 \times 10^{-9}\ \text{m}^2)}{1.0 \times 10^{-8}\ \text{m}} = 2.2 \times 10^{-11}\ \text{F}$$

a. The charge on the outer surface of the membrane is, therefore,

$$q = CV = (2.2 \times 10^{-11}\ \text{F})(60.0 \times 10^{-3}\ \text{V}) = \boxed{1.3 \times 10^{-12}\ \text{C}}$$

b. If the charge in part (a) is due to K^+ ions with charge $+e$ ($e = 1.6 \times 10^{-19}$ C), the number of ions present on the outer surface of the membrane is

$$\text{Number of } K^+ \text{ ions} = \frac{1.3 \times 10^{-12}\ \text{C}}{1.6 \times 10^{-19}\ \text{C}} = \boxed{8.1 \times 10^{6}}$$

40. ***REASONING AND SOLUTION*** The total charge transferred is given by

$$q = CV = (2.5 \times 10^{-8}\ \text{F})(450\ \text{V})$$

The number of electrons transferred is, then,

$$\text{Number of electrons} = \frac{q}{e} = \frac{(2.5 \times 10^{-8}\ \text{F})(450\ \text{V})}{1.60 \times 10^{-19}\ \text{C}} = \boxed{7.0 \times 10^{13}}$$

41. ***REASONING AND SOLUTION***

a. We know that

$$\text{Energy} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}(750 \times 10^{-6}\ \text{F})(330\ \text{V})^2 = \boxed{41\ \text{J}}$$

b. The power developed by the flash is

$$P = \frac{\text{Energy}}{\text{Time}} = \frac{41\ \text{J}}{5.0 \times 10^{-3}\ \text{s}} = \boxed{8200\ \text{W}}$$

42. ***REASONING*** Equation 19.11 (Energy = $CV^2/2$) gives the energy stored by a capacitor. Applying this expression to each capacitor will allow us to determine the voltage ratio, since the two energies are given.

SOLUTION Applying Equation19.11 to each capacitor gives

$$(\text{Energy})_{\text{A}} = \tfrac{1}{2}CV_{\text{A}}^2 \quad \text{and} \quad (\text{Energy})_{\text{B}} = \tfrac{1}{2}CV_{\text{B}}^2$$

Dividing the A-equation by the B-equation, we find

$$\frac{(\text{Energy})_{\text{A}}}{(\text{Energy})_{\text{B}}} = \frac{\tfrac{1}{2}CV_{\text{A}}^2}{\tfrac{1}{2}CV_{\text{B}}^2} \quad \text{or} \quad \frac{(\text{Energy})_{\text{A}}}{(\text{Energy})_{\text{B}}} = \frac{V_{\text{A}}^2}{V_{\text{B}}^2}$$

$$V_{\text{A}} = V_{\text{B}}\sqrt{\frac{(\text{Energy})_{\text{A}}}{(\text{Energy})_{\text{B}}}} = (12\ \text{V})\sqrt{\frac{310\ \text{J}}{34\ \text{J}}} = \boxed{36\ \text{V}}$$

43. SSM ***REASONING*** According to Equation 19.11, the energy stored in a capacitor with a capacitance C and potential V across its plates is $\text{Energy} = \frac{1}{2}CV^2$. Once we determine how much energy is required to operate a 75-W light bulb for one minute, we can then use the expression for the energy to solve for V.

SOLUTION The energy stored in the capacitor, which is equal to the energy required to operate a 75-W bulb for one minute (= 60 s), is

$$\text{Energy} = Pt = (75\ \text{W})(60\ \text{s}) = 4500\ \text{J}$$

Therefore, solving Equation 19.11 for V, we have

$$V = \sqrt{\frac{2(\text{Energy})}{C}} = \sqrt{\frac{2(4500\ \text{J})}{3.3\ \text{F}}} = \boxed{52\ \text{V}}$$

44. ***REASONING AND SOLUTION*** The electric energy stored in the region between the metal spheres is

$$\text{Energy} = \tfrac{1}{2}\kappa\varepsilon_0 E^2(\text{Volume}) \tag{19.12}$$

However, the magnitude E of the electric field is related to the potential difference ΔV between the spheres and the distance Δs between them via Equation 19.7, $E = \Delta V/\Delta s$. Thus,

$$\text{Energy} = \tfrac{1}{2}\kappa\varepsilon_0\left(\frac{\Delta V}{\Delta s}\right)^2(\text{Volume})$$

$$= \tfrac{1}{2}(2.1)\left(8.85\times10^{-12}\ \frac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\left(\frac{3.0\ \text{V}}{0.0020\ \text{m}}\right)^2\underbrace{\left[4\pi(0.1500\ \text{m})^2(0.0020\ \text{m})\right]}_{\text{Volume}}$$

$$= \boxed{1.2\times10^{-8}\ \text{J}}$$

45. ***REASONING AND SOLUTION*** From Equations 19.9 and 18.4, we have

$$A = \frac{q}{\kappa \varepsilon_0 E} = \frac{1.7 \times 10^{-7}\ \text{C}}{(3.5)\left(8.85 \times 10^{-12}\ \dfrac{\text{C}^2}{\text{N} \cdot \text{m}^2}\right)(1.4 \times 10^7\ \text{N/C})} = 3.9 \times 10^{-4}\ \text{m}^2$$

Since $A = \pi r^2$, the radius of the plates is

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{3.9 \times 10^{-4}\ \text{m}^2}{\pi}} = \boxed{1.1 \times 10^{-2}\ \text{m}}$$

46. ***REASONING AND SOLUTION*** The charge on the empty capacitor is $q_0 = C_0V_0$. With the dielectric in place, the charge remains the same. However, the new capacitance is $C = \kappa C_0$ and the new voltage is V. Thus,

$$q_0 = CV = \kappa C_0\, V = C_0V_0$$

Solving for the new voltage yields

$$V = V_0/\kappa = (12.0\text{ V})/\ 2.8 = \ 4.3\text{ V}$$

The potential difference is 12.0 – 4.3 = $\boxed{7.7\text{ V}}$. The change in potential is a $\boxed{\text{decrease}}$.

47. SSM WWW ***REASONING*** If we assume that the motion of the proton and the electron is horizontal in the $+x$ direction, the motion of the proton is determined by Equation 2.8, $x = v_0 t + \frac{1}{2} a_p t^2$, where x is the distance traveled by the proton, v_0 is its initial speed, and a_p is its acceleration. If the distance between the capacitor places is d, then this relation becomes $\frac{1}{2} d = v_0 t + \frac{1}{2} a_p t^2$, or

$$d = 2v_0 t + a_p t^2 \tag{1}$$

We can solve Equation (1) for the initial speed v_0 of the proton, but, first, we must determine the time t and the acceleration a_p of the proton . Since the proton strikes the negative plate at the same instant the electron strikes the positive plate, we can use the motion of the electron to determine the time t.

For the electron, $\frac{1}{2} d = \frac{1}{2} a_e t^2$, where we have taken into account the fact that the electron is released from rest. Solving this expression for t we have $t = \sqrt{d / a_e}$. Substituting this expression into Equation (1), we have

$$d = 2v_0 \sqrt{\frac{d}{a_e}} + \left(\frac{a_p}{a_e} \right) d \tag{2}$$

The accelerations can be found by noting that the magnitudes of the forces on the electron and proton are equal, since these particles have the same magnitude of charge. The force on the electron is $F = eE = eV / d$, and the acceleration of the electron is, therefore,

$$a_e = \frac{F}{m_e} = \frac{eV}{m_e d} \tag{3}$$

Newton's second law requires that $m_e a_e = m_p a_p$, so that

$$\frac{a_p}{a_e} = \frac{m_e}{m_p} \tag{4}$$

Combining Equations (2), (3) and (4) leads to the following expression for v_0, the initial speed of the proton:

$$v_0 = \frac{1}{2} \left(1 - \frac{m_e}{m_p} \right) \sqrt{\frac{eV}{m_e}}$$

SOLUTION Substituting values into the expression above, we find

$$v_0 = \frac{1}{2} \left(1 - \frac{9.11 \times 10^{-31}\ \text{kg}}{1.67 \times 10^{-27}\ \text{kg}} \right) \sqrt{\frac{(1.6 \times 10^{-19}\ \text{C})(175\ \text{V})}{9.11 \times 10^{-31}\ \text{kg}}} = \boxed{2.77 \times 10^6\ \text{m/s}}$$

48. ***REASONING AND SOLUTION*** The electron and proton experience forces of the same magnitude in the same electric field, since they have the same magnitude of charge. Newton's second law gives

$$m_p a_p = m_e a_e$$

where a_p and a_e are the accelerations of the proton and the electron. Kinematics gives

$$a_p = 2x_p/t^2 \qquad \text{and} \qquad a_e = 2x_e/t^2$$

so

$$x_p = \frac{m_e}{m_p} x_e = \frac{9.11 \times 10^{-31}\ \text{kg}}{1.67 \times 10^{-27}\ \text{kg}}(0.0400\ \text{m}) = \boxed{2.18 \times 10^{-5}\ \text{m}}$$

49. ***REASONING AND SOLUTION*** First, we need to find the magnitude of the charge. Since $V = kq/r$, we have

$$q = \frac{Vr}{k} = \frac{(164\ \text{V})(0.20\ \text{m})}{9.0 \times 10^9\ \text{N} \cdot \text{m}^2 / \text{C}^2} = 3.6 \times 10^{-9}\ \text{C}$$

Thus, at a distance of 0.80 m the potential would be

$$V = \frac{kq}{r} = \frac{(9.0 \times 10^9\ \text{N} \cdot \text{m}^2 / \text{C}^2)(3.6 \times 10^{-9}\ \text{C})}{0.80\ \text{m}} = \boxed{41\ \text{V}}$$

50. ***REASONING AND SOLUTION***

a. The change in the electric potential energy is

$$\text{EPE}_A - \text{EPE}_B = W_{AB} = \boxed{5.80 \times 10^{-3}\ \text{J}}$$

b. The potential difference between the points is

$$V_A - V_B = \frac{\text{EPE}_A - \text{EPE}_B}{q} = \frac{5.80 \times 10^{-3}\ \text{J}}{1.80 \times 10^{-4}\ \text{C}} = \boxed{32.2\ \text{V}}$$

c. $\boxed{\text{Point } A}$ has the higher potential.

51. **SSM** ***REASONING AND SOLUTION*** Equation 19.10 gives the capacitance for a parallel plate capacitor filled with a dielectric of constant κ: $C = \kappa \varepsilon_0 A / d$. Solving for κ, we have

$$\kappa = \frac{Cd}{\varepsilon_0 A} = \frac{(7.0 \times 10^{-6}\ \text{F})(1.0 \times 10^{-5}\ \text{m})}{(8.85 \times 10^{-12}\ \text{F/m})(1.5\ \text{m}^2)} = \boxed{5.3}$$

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52. ***REASONING AND SOLUTION*** The only force that acts on the α-particle is the conservative electric force. Therefore, the total energy of the α-particle is conserved as it moves from point A to point B:

$$\underbrace{\tfrac{1}{2}mv_A^2 + \text{EPE}_A}_{\substack{\text{Total energy at}\\ \text{point } A}} = \underbrace{\tfrac{1}{2}mv_B^2 + \text{EPE}_B}_{\substack{\text{Total energy at}\\ \text{point } B}}$$

Since the α-particle starts from rest, $v_A = 0$. The electric potential V is related to the electric potential energy EPE by $V = \text{EPE}/q$ (see Equation 19.3). With these changes, the equation above gives the kinetic energy of the α-particle at point B to be

$$\tfrac{1}{2}mv_B^2 = \text{EPE}_A - \text{EPE}_B = q\left(V_A - V_B\right)$$

Since an α-particle contains two protons, its charge is $q = 2e = 3.2 \times 10^{-19}$ C. Thus, the kinetic energy (in electron-volts) is

$$\tfrac{1}{2}mv_B^2 = q\left(V_A - V_B\right) = \left(3.2 \times 10^{-19}\ \text{C}\right)\left[+250\ \text{V} - \left(-150\ \text{V}\right)\right]$$

$$= 1.28 \times 10^{-16}\ \text{J}\left(\frac{1.0\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}}\right) = \boxed{8.0 \times 10^2\ \text{eV}}$$

53. ***REASONING AND SOLUTION*** The capacitance is given by

$$C = \frac{k\varepsilon_0 A}{d} = \frac{5\left(8.85 \times 10^{-12}\ \text{F/m}\right)\left(5 \times 10^{-6}\ \text{m}^2\right)}{1 \times 10^{-8}\ \text{m}} = \boxed{2 \times 10^{-8}\ \text{F}}$$

54. ***REASONING AND SOLUTION*** The initial and final separations of the charges are the same. Since the electric potential energy depends only on the separation of the charges, there is no change in the electric potential energy, and, hence,

$$\boxed{\text{no work is done}}.$$

55. SSM WWW ***REASONING AND SOLUTION*** As described in the problem statement, the charges jump between your hand and a doorknob. If we assume that the electric field is uniform, Equation 19.7 applies, and we have

$$E = -\frac{\Delta V}{\Delta s} = -\frac{V_{\text{knob}} - V_{\text{hand}}}{\Delta s}$$

Therefore, solving for the potential difference between your hand and the doorknob, we have

$$V_{\text{knob}} - V_{\text{hand}} = -E\,\Delta s = -(-3.0 \times 10^{6}\ \text{N/C})(3.0 \times 10^{-3}\ \text{m}) = \boxed{+9.0 \times 10^{3}\ \text{V}}$$

56. ***REASONING*** The electric field E that exists between two points in space is, according to Equation 19.7, proportional to the electric potential difference ΔV between the points divided by the distance Δx between them: $E = -\Delta V/\Delta x$.

SOLUTION

a. The electric field in the region from A to B is

$$E = -\frac{\Delta V}{\Delta x} = -\frac{5.0\text{ V} - 5.0\text{ V}}{0.20\text{ m} - 0\text{ m}} = \boxed{0\text{ V/m}}$$

b. The electric field in the region from B to C is

$$E = -\frac{\Delta V}{\Delta x} = -\frac{3.0\text{ V} - 5.0\text{ V}}{0.40\text{ m} - 0.20\text{ m}} = \boxed{1.0 \times 10^{1}\text{ V/m}}$$

c. The electric field in the region from C to D is

$$E = -\frac{\Delta V}{\Delta x} = -\frac{1.0\text{ V} - 3.0\text{ V}}{0.80\text{ m} - 0.40\text{ m}} = \boxed{5.0\text{ V/m}}$$

57. ***REASONING AND SOLUTION*** The potential difference is constant so $V_0 = q_0/C_0 = q/C = q/(\kappa C_0)$. Thus,

$$q = kq_0 = kC_0V_0 = (4.0)(2.7 \times 10^{-6}\ \text{F})(12\ \text{V}) = 1.3 \times 10^{-4}\ \text{C}$$

The original charge is $q_0 = C_0V_0 = 3.2 \times 10^{-5}$ C. The surface charge Q is, therefore,

$$Q = q - q_0 = \boxed{1.0 \times 10^{-4}\ \text{C}}$$

58. ***REASONING*** The only force acting on each particle is the conservative electric force. Therefore, the total energy (kinetic energy plus electric potential energy) is conserved as the particles move apart. In addition, the net external force acting on the system of two particles is zero (the electric force that each particle exerts on the other is an internal force). Thus, the total linear momentum of the system is also conserved. We will use the conservation of energy and the conservation of linear momentum to find the initial separation of the particles.

SOLUTION For two points, A and B, along the motion, the conservation of energy is

$$\underbrace{\tfrac{1}{2}m_1v_{1,A}^2+\tfrac{1}{2}m_2v_{2,A}^2}_{\text{Initial kinetic energy of the two particles}}+\underbrace{\frac{kq_1q_2}{r_A}}_{\text{Initial electric potential energy}}=\underbrace{\tfrac{1}{2}m_1v_{1,B}^2+\tfrac{1}{2}m_2v_{2,B}^2}_{\text{Final kinetic energy of the two particles}}+\underbrace{\frac{kq_1q_2}{r_B}}_{\text{Final electric potential energy}}$$

Solving this equation for $1/r_A$ and setting $v_{1,A} = v_{2,A} = 0$ since the particles are initially at rest, we obtain

$$\frac{1}{r_A}=\frac{1}{r_B}+\frac{1}{kq_1q_2}\left(\tfrac{1}{2}m_1v_{1,B}^2+\tfrac{1}{2}m_2v_{2,B}^2\right) \qquad (1)$$

This equation cannot be solved for the initial separation r_A, because the final speed $v_{2,B}$ of the second particle is not known. To find this speed, we will use the conservation of linear momentum:

$$\underbrace{m_1v_{1,A}+m_2v_{2,A}}_{\text{Initial linear momentum}}=\underbrace{m_1v_{1,B}+m_2v_{2,B}}_{\text{Final linear momentum}}$$

Setting $v_{1,A} = v_{2,A} = 0$ and solving for $v_{2,B}$ gives

$$v_{2,B}=-\frac{m_1}{m_2}v_{1,B}=-\frac{3.00\times10^{-3}\text{ kg}}{6.00\times10^{-3}\text{ kg}}(125\text{ m/s})=-62.5\text{ m/s}$$

Substituting this value for $v_{2,B}$ into Equation (1) yields

$$\frac{1}{r_A}=\frac{1}{0.100\text{ m}}+\frac{1}{\left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(8.00\times10^{-6}\text{ C}\right)^2}$$
$$\times\left[\tfrac{1}{2}\left(3.00\times10^{-3}\text{ kg}\right)\left(125\text{ m/s}\right)^2+\tfrac{1}{2}\left(6.00\times10^{-3}\text{ kg}\right)\left(-62.5\text{ m/s}\right)^2\right]$$

$$\boxed{r_A=1.41\times10^{-2}\text{ m}}$$

59. **SSM** ***REASONING*** According to Equation 19.10, the capacitance of a parallel plate capacitor filled with a dielectric is $C = \kappa\varepsilon_0 A/d$, where κ is the dielectric constant, A is the area of one plate, and d is the distance between the plates.

From the definition of capacitance (Equation 19.8), $q = CV$. Thus, using Equation 19.10, we see that the charge q on a parallel plate capacitor that contains a dielectric is given by $q = (\kappa\varepsilon_0 A/d)V$. Since each dielectric occupies one-half of the volume between the plates, the area of each plate in contact with each material is $A/2$. Thus,

$$q_1 = \frac{\kappa_1\varepsilon_0(A/2)}{d}V = \frac{\kappa_1\varepsilon_0 A}{2d}V \qquad \text{and} \qquad q_2 = \frac{\kappa_2\varepsilon_0(A/2)}{d}V = \frac{\kappa_2\varepsilon_0 A}{2d}V$$

According to the problem statement, the total charge stored by the capacitor is

$$q_1 + q_2 = CV \tag{1}$$

where q_1 and q_2 are the charges on the plates in contact with dielectrics 1 and 2, respectively. Using the expressions for q_1 and q_2 above, Equation (1) becomes

$$CV = \frac{\kappa_1\varepsilon_0 A}{2d}V + \frac{\kappa_2\varepsilon_0 A}{2d}V = \frac{\kappa_1\varepsilon_0 A + \kappa_2\varepsilon_0 A}{2d}V = \frac{(\kappa_1+\kappa_2)\varepsilon_0 A}{2d}V$$

This expression can be solved for C.

SOLUTION Solving for C, we obtain $\boxed{C = \dfrac{\varepsilon_0 A(\kappa_1+\kappa_2)}{2d}}$

60. ***REASONING AND SOLUTION*** The information about the electric field requires that

$$kq_2/(1.00\text{ m})^2 = kq_1/(4.00\text{ m})^2 \qquad \text{so} \qquad q_2 = (1/16.0)\, q_1$$

Let x be the distance of the zero-potential point from the negative charge. Then

$$kq_1/(d + x) = kq_2/x$$

if the point is to the right of q_2 and

$$kq_1/(d - x) = kq_2/x$$

if the point is to the left of q_2. Solving for x gives

$$\boxed{x = 0.200\text{ m to the right of the negative charge}}$$

$$\boxed{x = 0.176\text{ m to the left of the negative charge}}$$

1. **SSM** ***REASONING*** Since current is defined as charge per unit time, the current used by the portable compact disc player is equal to the charge provided by the battery pack (180 C) divided by the time in which the charge is delivered (2.0 h).

SOLUTION The amount of current that the player uses in operation is determined from Equation 20.1:

$$I = \frac{\Delta q}{\Delta t} = \frac{180\ \text{C}}{2.0\ \text{h}} \underbrace{\left(\frac{1.0\ \text{h}}{3600\ \text{s}}\right)}_{\substack{\text{Converts hours} \\ \text{to seconds}}} = \boxed{0.025\ \text{A}}$$

2. ***REASONING*** The current I is defined in Equation 20.1 as the amount of charge Δq per unit of time Δt that flows in a wire. Therefore, the amount of charge is the product of the current and the time interval. The number of electrons is equal to the charge that flows divided by the magnitude of the charge on an electron.

SOLUTION

a. The amount of charge that flows is

$$\Delta q = I\,\Delta t = (18\text{ A})\left(2.0\times10^{-3}\text{ s}\right) = \boxed{3.6\times10^{-2}\text{ C}}$$

b. The number of electrons N is equal to the amount of charge divided by e, the magnitude of the charge on an electron.

$$N = \frac{\Delta q}{e} = \frac{3.6\times10^{-2}\text{ C}}{1.60\times10^{-19}\text{ C}} = \boxed{2.3\times10^{17}}$$

3. ***REASONING AND SOLUTION*** We know that $V = IR$. Therefore,

$$I = \frac{V}{R} = \frac{120\ \text{V}}{580\ \Omega} = \boxed{0.21\ \text{A}}$$

4. ***REASONING AND SOLUTION*** Ohm's law gives

$$I = \frac{V}{R} = \frac{120\ \text{V}}{14\ \Omega} = \boxed{8.6\ \text{A}}$$

5. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** Ohm's law (Equation 20.2, $V = IR$) gives the result directly

$$I = \frac{V}{R} = \frac{240\text{ V}}{11\ \Omega} = \boxed{22\text{ A}}$$

6. ***REASONING AND SOLUTION*** First determine the total charge delivered to the battery using Equation 20.1:

$$\Delta q = I\Delta t = (6.0\ \text{A})(5.0\ \text{h})[(3600\ \text{s})/(1\ \text{h})] = 1.1 \times 10^5\ \text{C}$$

To find the energy delivered to the battery, multiply this charge by the energy per unit charge (i.e., the voltage) to get

$$\text{Energy} = (\Delta q)V = (1.1 \times 10^5\ \text{C})(12\ \text{V}) = \boxed{1.3 \times 10^6\ \text{J}}$$

7. ***REASONING*** As discussed in Section 20.1, the voltage gives the energy per unit charge. Thus, we can determine the energy delivered to the toaster by multiplying the voltage V by the charge Δq that flows during a time Δt of one minute. The charge can be obtained by solving Equation 20.1, $I = (\Delta q)/(\Delta t)$, since the current I can be obtained from Ohm's law.

SOLUTION Remembering that voltage is energy per unit charge, we have

$$\text{Energy} = V\,\Delta q$$

Solving Equation 20.1 for Δq gives $\Delta q = I\,\Delta t$, which can be substituted in the previous result to give

$$\text{Energy} = V\,\Delta q = VI\,\Delta t$$

According to Ohm's law (Equation 20.2), the current is $I = V/R$, which can be substituted in the energy expression to show that

$$\text{Energy} = VI\,\Delta t = V\left(\frac{V}{R}\right)\Delta t = \frac{V^2\,\Delta t}{R} = \frac{(120\text{ V})^2\,(60\text{ s})}{14\ \Omega} = \boxed{6.2\times10^4\text{ J}}$$

8. ***REASONING AND SOLUTION***

a. The total charge that can be delivered is

$$\Delta q = (220\ \text{A}\cdot\text{h})[3600\ \text{s}/(1\ \text{h})] = \boxed{7.9 \times 10^5\ \text{C}}$$

b. The maximum current is

$$I = (220\text{A}\cdot\text{h})/[(38\ \text{min})(1\ \text{hr})/(60\ \text{min})] = \boxed{350\ \text{A}}$$

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9. **SSM** ***REASONING*** The number N of protons that strike the target is equal to the amount of electric charge Δq striking the target divided by the charge e of a proton, $N = (\Delta q)/e$. From Equation 20.1, the amount of charge is equal to the product of the current I and the time Δt. We can combine these two relations to find the number of protons that strike the target in 15 seconds.

The heat Q that must be supplied to change the temperature of the aluminum sample of mass m by an amount ΔT is given by Equation 12.4 as $Q = cm\Delta T$, where c is the specific heat capacity of aluminum. The heat is provided by the kinetic energy of the protons and is equal to the number of protons that strike the target times the kinetic energy per proton. Using this reasoning, we can find the change in temperature of the block for the 15 second-time interval.

SOLUTION

a. The number N of protons that strike the target is

$$N = \frac{\Delta q}{e} = \frac{I\Delta t}{e} = \frac{(0.50 \times 10^{-6}\ \text{A})(15\ \text{s})}{1.6 \times 10^{-19}\ \text{C}} = \boxed{4.7 \times 10^{13}}$$

b. The amount of heat Q provided by the kinetic energy of the protons is

$$Q = (4.7 \times 10^{13}\ \text{protons})(4.9 \times 10^{-12}\ \text{J/proton}) = 230\ \text{J}$$

Since $Q = cm\Delta T$ and since Table 12.2 gives the specific heat of aluminum as $c = 9.00 \times 10^2$ J/(kg·C°), the change in temperature of the block is

$$\Delta T = \frac{Q}{cm} = \frac{230\ \text{J}}{\left(9.00 \times 10^2\ \dfrac{\text{J}}{\text{kg} \cdot \text{C}^\circ}\right)(15 \times 10^{-3}\ \text{kg})} = \boxed{17\ \text{C}^\circ}$$

10. ***REASONING AND SOLUTION*** Using Equation 20.3 and the resistivity from Table 20.1, we find

$$R = \frac{\rho L}{A} = \frac{\left(2.82 \times 10^{-8}\ \Omega \cdot \text{m}\right)\left(10.0 \times 10^{3}\ \text{m}\right)}{4.9 \times 10^{-4}\ \text{m}^{2}} = \boxed{0.58\ \Omega}$$

11. **SSM** **WWW** ***REASONING*** The resistance of a metal wire of length L, cross-sectional area A and resistivity ρ is given by Equation 20.3: $R = \rho L / A$. Solving for A, we have $A = \rho L / R$. We can use this expression to find the ratio of the cross-sectional area of the aluminum wire to that of the copper wire.

SOLUTION Forming the ratio of the areas and using resistivity values from Table 20.1, we have

$$\frac{A_{\text{aluminum}}}{A_{\text{copper}}} = \frac{\rho_{\text{aluminum}} L / R}{\rho_{\text{copper}} L / R} = \frac{\rho_{\text{aluminum}}}{\rho_{\text{copper}}} = \frac{2.82 \times 10^{-8}\ \Omega \cdot \text{m}}{1.72 \times 10^{-8}\ \Omega \cdot \text{m}} = \boxed{1.64}$$

12. ***REASONING AND SOLUTION*** Solving Equation 20.5 for α yields

$$\alpha = \frac{\left(R/R_0\right)-1}{T-T_0} = \frac{\left[\left(43.7\ \Omega\right)/\left(38.0\ \Omega\right)\right]-1}{55\ ^\circ\text{C} - 25\ ^\circ\text{C}} = \boxed{0.0050\ \left(\text{C}^\circ\right)^{-1}}$$

13. ***REASONING AND SOLUTION*** The resistance of the cable is

$$R = \frac{V}{I} = \frac{\rho L}{A}$$

Since $A = \pi r^2$, the radius of the cable is

$$r = \sqrt{\frac{\rho L I}{\pi V}} = \sqrt{\frac{\left(1.72 \times 10^{-8}\ \Omega \cdot \text{m}\right)\left(0.24\ \text{m}\right)\left(1200\ \text{A}\right)}{\pi\left(1.6 \times 10^{-2}\ \text{V}\right)}} = \boxed{9.9 \times 10^{-3}\ \text{m}}$$

14. ***REASONING*** The materials in Table 20.1 are listed according to their resistivities, so we need to find the resistivity of this material. The resistivity depends on the resistance R, the length L, and the cross-sectional A of the wire (see Equation 20.3). We know the length of the wire, and the cross-sectional area can be found from the radius r as $A = \pi r^2$. The resistance depends on the voltage V and current I through the relation $R = V / I$, as expressed by Equation 20.2.

SOLUTION

The resistivity is

$$\rho = \frac{R\,A}{L} = \frac{R\left(\pi r^2\right)}{L} \tag{20.3}$$

We also know that $R = V / I$, so the resistivity becomes

$$\rho = \frac{\left(\frac{V}{I}\right)\left(\pi r^2\right)}{L} = \frac{V \pi r^2}{I\,L} = \frac{(0.0320\ \text{V})\pi\left(1.03 \times 10^{-3}\ \text{m}\right)^2}{(1.35\ \text{A})(2.80\ \text{m})} = 2.82 \times 10^{-8}\ \Omega \cdot \text{m}$$

An inspection of Table 20.1 shows that the material that has this resistivity is $\boxed{\text{aluminum}}$.

15. ***REASONING AND SOLUTION*** First, we find α from the known values of T, T_0, R, and R_0. We have from Equation 20.5

$$\alpha = \frac{\dfrac{R}{R_0} - 1}{T - T_0} = \frac{\dfrac{47.6\ \Omega}{35.0\ \Omega} - 1}{100.0\ °\text{C} - 20.0\ °\text{C}} = 0.0045\ (\text{C}°)^{-1}$$

We know that $R = R_0[1 + \alpha(T - T_0)]$, and using $R = 37.8\ \Omega$ we find

$$T = T_0 + \frac{\dfrac{R}{R_0} - 1}{\alpha} = 20.0\ °\text{C} + \frac{\dfrac{37.8\ \Omega}{35.0\ \Omega} - 1}{0.0045\ (\text{C}°)^{-1}} = \boxed{37.8\ °\text{C}}$$

16. ***REASONING*** The resistance R of the wire depends on its length L, so we can use Equation 20.3 to express the length in terms of the resistance:

$$L = \frac{RA}{\rho}$$

where A is the cross-sectional area of the wire and ρ is the resistivity of tungsten. The cross-sectional area can be expressed in terms of the radius r of the wire as $A = \pi r^2$. The resistance depends on the voltage V and current I through the relation $R = V/I$, Equation 20.2. Thus, the length of the wire can be expressed as

$$L = \frac{RA}{\rho} = \frac{\left(\frac{V}{I}\right)\left(\pi r^2\right)}{\rho}$$

According to Equation 20.4, the resistivity ρ at the temperature T depends on the resistivity ρ_0 at the temperature T_0 through the relation

$$\rho = \rho_0\left[1 + \alpha\left(T - T_0\right)\right]$$

where α is the temperature coefficient of resistivity. Substituting this expression for ρ into the expression for the length of the wire, we have

$$L = \frac{\left(\frac{V}{I}\right)\left(\pi r^2\right)}{\rho_0\left[1 + \alpha\left(T - T_0\right)\right]}$$

SOLUTION The length of the wire is

$$L = \frac{\left(\frac{V}{I}\right)\left(\pi r^2\right)}{\rho_0\left[1 + \alpha\left(T - T_0\right)\right]}$$

$$= \frac{\left(\frac{120\text{ V}}{1.5\text{ A}}\right)\pi\left(0.075 \times 10^{-3}\text{ m}\right)^2}{\left(5.6 \times 10^{-8}\ \Omega\cdot\text{m}\right)\left\{1 + \left[4.5 \times 10^{-3}\left(\text{C}^\circ\right)^{-1}\right]\left(1320\ ^\circ\text{C} - 20.0\ ^\circ\text{C}\right)\right\}} = \boxed{3.7\text{ m}}$$

The resistivity at 20.0 °C, $\rho_0 = 5.6 \times 10^{-8}\ \Omega\cdot\text{m}$, was obtained from Table 20.1.

17. $\boxed{\text{SSM}}$ $\boxed{\text{WWW}}$ ***REASONING*** The resistance of a metal wire of length L, cross-sectional area A and resistivity ρ is given by Equation 20.3: $R = \rho L / A$. The volume V_2 of the new wire will be the same as the original volume V_1 of the wire, where volume is the product of length and cross-sectional area. Thus, $V_1 = V_2$ or $A_1 L_1 = A_2 L_2$. Since the new wire is three times longer than the first wire, we can write

$$A_1 L_1 = A_2 L_2 = A_2 (3L_1) \quad \text{or} \quad A_2 = A_1 / 3$$

We can form the ratio of the resistances, use this expression for the area A_2, and find the new resistance.

SOLUTION The resistance of the new wire is determined as follows:

$$\frac{R_2}{R_1} = \frac{\rho L_2 / A_2}{\rho L_1 / A_1} = \frac{L_2 A_1}{L_1 A_2} = \frac{(3L_1) A_1}{L_1 (A_1 / 3)} = 9$$

Solving for R_2, we find that

$$R_2 = 9R_1 = 9(21.0\ \Omega) = \boxed{189\ \Omega}$$

18. ***REASONING AND SOLUTION*** Suppose that when the initial temperature of the wire is T_0 the resistance is R_0, and when the temperature rises to T the resistance is R. The relation between temperature and resistance is given by Equation 20.5 as $R = R_0[1 + \alpha (T - T_0)]$, where α is the temperature coefficient of resistivity. The initial and final resistances are related to the voltage and current as $R_0 = V/I_0$ and $R = V/I$, where the voltage V across the wire is the same in both cases. Substituting these values for R_0 and R into Equation 20.5 and solving for T, we arrive at

$$T = T_0 + \frac{\left(\dfrac{I_0}{I} - 1\right)}{\alpha} = 20\,°\text{C} + \frac{\left(\dfrac{1.50\ \text{A}}{1.30\ \text{A}} - 1\right)}{4.5 \times 10^{-4}\ (\text{C}°)^{-1}} = \boxed{360\ °\text{C}}$$

19. **SSM** **WWW** ***REASONING*** We will ignore any changes in length due to thermal expansion. Although the resistance of each section changes with temperature, the total resistance of the composite does not change with temperature. Therefore,

$$\underbrace{\left(R_{\text{tungsten}}\right)_0 + \left(R_{\text{carbon}}\right)_0}_{\text{At room temperature}} = \underbrace{R_{\text{tungsten}} + R_{\text{carbon}}}_{\text{At temperature } T}$$

From Equation 20.5, we know that the temperature dependence of the resistance for a wire of resistance R_0 at temperature T_0 is given by $R = R_0[1 + \alpha(T - T_0)]$, where α is the temperature coefficient of resistivity. Thus,

$$\left(R_{\text{tungsten}}\right)_0 + \left(R_{\text{carbon}}\right)_0 = \left(R_{\text{tungsten}}\right)_0 (1 + \alpha_{\text{tungsten}} \Delta T) + \left(R_{\text{carbon}}\right)_0 (1 + \alpha_{\text{carbon}} \Delta T)$$

Since ΔT is the same for each wire, this simplifies to

$$\left(R_{\text{tungsten}}\right)_0 \alpha_{\text{tungsten}} = -\left(R_{\text{carbon}}\right)_0 \alpha_{\text{carbon}} \qquad (1)$$

This expression can be used to find the ratio of the resistances. Once this ratio is known, we can find the ratio of the lengths of the sections with the aid of Equation 20.3 ($L = RA/\rho$).

SOLUTION From Equation (1), the ratio of the resistances of the two sections of the wire is

$$\frac{\left(R_{\text{tungsten}}\right)_0}{\left(R_{\text{carbon}}\right)_0} = -\frac{\alpha_{\text{carbon}}}{\alpha_{\text{tungsten}}} = -\frac{-0.0005\ [(\text{C}^\circ)^{-1}]}{0.0045\ [(\text{C}^\circ)^{-1}]} = \frac{1}{9}$$

Thus, using Equation 20.3, we find the ratio of the tungsten and carbon lengths to be

$$\frac{L_{\text{tungsten}}}{L_{\text{carbon}}} = \frac{\left(R_0 A/\rho\right)_{\text{tungsten}}}{\left(R_0 A/\rho\right)_{\text{carbon}}} = \frac{\left(R_{\text{tungsten}}\right)_0}{\left(R_{\text{carbon}}\right)_0}\left(\frac{\rho_{\text{carbon}}}{\rho_{\text{tungsten}}}\right) = \left(\frac{1}{9}\right)\left(\frac{3.5\times10^{-5}\,\Omega\cdot\text{m}}{5.6\times10^{-8}\,\Omega\cdot\text{m}}\right) = \boxed{70}$$

where we have used resistivity values from Table 20.1 and the fact that the two sections have the same cross-sectional areas.

20. ***REASONING AND SOLUTION*** The mass m of the aluminum wire is equal to the density d of aluminum times the volume of the wire. The wire is cylindrical, so its volume is equal to the cross-sectional area A times the length L; $m = dAL$.

The cross-sectional area of the wire is related to its resistance R and length L by Equation 20.3; $R = \rho L/A$, where ρ is the resistivity of aluminum. Therefore, the mass of the aluminum wire can be written as

$$m = dAL = d\left(\frac{\rho L}{R}\right)L$$

The resistance R is given by Ohm's law as $R = V/I$, so the mass of the wire becomes

$$m = d\left(\frac{\rho L}{R}\right)L = \frac{d\rho L^2 I}{V}$$

$$m = \frac{(2700\ \text{kg/m}^3)(2.82 \times 10^{-8}\ \Omega \cdot \text{m})(175\ \text{m})^2\ (125\ \text{A})}{0.300\ \text{V}} = \boxed{9.7 \times 10^2\ \text{kg}}$$

21. ***REASONING AND SOLUTION*** According to Equation 20.6, the power delivered to the battery to charge it is

$$P = IV = (19.0\text{ A})(12.0\text{ V}) = \boxed{228\text{ W}}$$

22. ***REASONING AND SOLUTION***

a. According to Equation 20.6c, the resistance is

$$R = V^2/P = (12\ \text{V})^2/(33\ \text{W}) = \boxed{4.4\ \Omega}$$

b. According to Equation 20.6a, the current is

$$I = P/V = (33\ \text{W})/(12\ \text{V}) = \boxed{2.8\ \text{A}}$$

23. **SSM** ***REASONING*** According to Equation 6.10b, the energy used is Energy $= Pt$, where P is the power and t is the time. According to Equation 20.6a, the power is $P = IV$, where I is the current and V is the voltage. Thus, Energy $= IVt$, and we apply this result first to the drier and then to the computer.

SOLUTION The energy used by the drier is

$$\text{Energy} = Pt = IVt = (16\text{ A})(240\text{ V})(45\text{ min})\underbrace{\left(\frac{60\text{ s}}{1.00\text{ min}}\right)}_{\substack{\text{Converts minutes}\\ \text{to seconds}}} = 1.04\times10^{7}\text{ J}$$

For the computer, we have

$$\text{Energy} = 1.04\times10^{7}\text{ J} = IVt = (2.7\text{ A})(120\text{ V})t$$

Solving for t we find

$$t = \frac{1.04\times10^{7}\text{ J}}{(2.7\text{ A})(120\text{ V})} = 3.21\times10^{4}\text{ s} = \left(3.21\times10^{4}\text{ s}\right)\left(\frac{1.00\text{ h}}{3600\text{ s}}\right) = \boxed{8.9\text{ h}}$$

24. ***REASONING*** The total cost of keeping all the TVs turned on is equal to the number of TVs times the cost to keep each one on. The cost for one TV is equal to the energy it consumes times the cost per unit of energy ($0.10 per kW·h). The energy that a single set uses is, according to Equation 6.10b, the power it consumes times the time of use.

SOLUTION The total cost is

$$\text{Total cost} = (110 \text{ million sets})(\text{Cost per set})$$

$$= (110 \text{ million sets})\left[\text{Energy (in kW} \cdot \text{h) used per set}\right]\left(\frac{\$0.10}{1 \text{ kW} \cdot \text{h}}\right)$$

The energy (in kW·h) used per set is the product of the power and the time, where the power is expressed in kilowatts and the time is in hours:

$$\text{Energy used per set} = Pt = (75 \text{ W})\left(\frac{1 \text{ kW}}{1000 \text{ W}}\right)(6.0 \text{ h}) \qquad (6.10b)$$

The total cost of operating the TV sets is

$$\text{Total cost} = (110 \text{ million sets})\left[(75 \text{ W})\left(\frac{1 \text{ kW}}{1000 \text{ W}}\right)(6.0 \text{ h})\right]\left(\frac{\$0.10}{1 \text{ kW} \cdot \text{h}}\right) = \boxed{\$5.0 \times 10^6}$$

25. ***REASONING AND SOLUTION*** According to Equation 20.6a, we know $P = IV$, so that

$$I = P/V = (140\text{ W})/(120\text{ V}) = \boxed{1.2\text{ A}}$$

26. ***REASONING AND SOLUTION*** The power delivered is P = VI so

a. $$P_{bd} = VI_{bd} = (120\text{ V})(11\text{ A}) = \boxed{1300\text{ W}}$$

b. $$P_{vc} = VI_{vc} = (120\text{ V})(4.0\text{ A}) = \boxed{480\text{ W}}$$

c. The energy is E = Pt so,

$$E_{bd}/E_{vc} = (P_{bd}t_{bd})/(P_{vc}t_{vc}) = (1300\text{ W})(15\text{ min})/[(480\text{ W})(30.0\text{ min})] = \boxed{1.4}$$

27. **SSM** ***REASONING AND SOLUTION*** As a function of temperature, the resistance of the wire is given by Equation 20.5: $R = R_0\left[1+\alpha(T-T_0)\right]$, where α is the temperature coefficient of resistivity. From Equation 20.6c, we have $P = V^2/R$. Combining these two equations, we have

$$P = \frac{V^2}{R_0\left[1+\alpha\left(T-T_0\right)\right]} = \frac{P_0}{1+\alpha\left(T-T_0\right)}$$

where $P_0 = V^2/R_0$, since the voltage is constant. But $P = \frac{1}{2}P_0$, so we find

$$\frac{P_0}{2} = \frac{P_0}{1+\alpha\left(T-T_0\right)} \qquad \text{or} \qquad 2 = 1+\alpha\left(T-T_0\right)$$

Solving for T, we find

$$T = \frac{1}{\alpha} + T_0 = \frac{1}{0.0045\ (\text{C}°)^{-1}} + 28° = \boxed{250\ °\text{C}}$$

28. ***REASONING AND SOLUTION*** We know that the resistance of the wire can be obtained from

$$P = V^2/R \qquad \text{or} \qquad R = V^2/P$$

We also know that $R = \rho L/A$. Solving for the length, noting that $A = \pi r^2$, and using $\rho = 100 \times 10^{-8}\ \Omega\cdot\text{m}$ from Table 20.1, we find

$$L = \frac{RA}{\rho} = \frac{\left(V^2/P\right)\left(\pi r^2\right)}{\rho} = \frac{V^2 \pi r^2}{\rho P} = \frac{(120\ \text{V})^2\, \pi \left(6.5 \times 10^{-4}\ \text{m}\right)^2}{\left(100 \times 10^{-8}\ \Omega\cdot\text{m}\right)\left(4.00 \times 10^2\ \text{W}\right)} = \boxed{50\ \text{m}}$$

29. ***REASONING AND SOLUTION*** The electrical energy is converted to heat, so we have

$$Q = mc\Delta T = mc(T - T_0)$$

This energy is also given by

$$Q = Pt = I^2R_{av}t = I^2[1/2\ (R + R_0)]t$$

where we are considering the average resistance R_{av} over the one minute interval. Also

$$R = R_0[1 + \alpha(T - T_0)]$$

We have

$$Q = I^2[1/2\ (R + R_0)]t = I^2\{1/2\ R_0[2 + \alpha(T - T_0)]\}t = mc(T - T_0)$$

Solving for T in the above expression yields

$$T = T_0 + \frac{\dfrac{R_0 I^2 t}{mc}}{1 - \dfrac{\alpha R_0 I^2 t}{2mc}}$$

Using $R_0 = 12\ \Omega$, $I = 0.10$ A, $t = 60.0$ s, $m = 1.3 \times 10^{-3}$ kg, $c = 452$ J/(kg·C°) from Table 12.2, $\alpha = 0.0050\ (C°)^{-1}$, $T_0 = 20.0$ °C, we obtain $\boxed{T = 33\ °C}$.

30. ***REASONING*** The average power is given by Equation 20.15c as $\overline{P} = V_{\text{rms}}^2 / R$. In this expression the rms voltage V_{rms} appears. However, we seek the peak voltage V_0. The relation between the two types of voltage is given by Equation 20.13 as $V_{\text{rms}} = V_0 / \sqrt{2}$, so we can obtain the peak voltage by using Equation 20.13 to substitute into Equation 20.15c.

SOLUTION Substituting V_{rms} from Equation 20.13 into Equation 20.15c gives

$$\overline{P} = \frac{V_{\text{rms}}^2}{R} = \frac{\left(V_0 / \sqrt{2}\right)^2}{R} = \frac{V_0^2}{2R}$$

Solving for the peak voltage V_0 gives

$$V_0 = \sqrt{2R\overline{P}} = \sqrt{2(4.0\ \Omega)(55\ \text{W})} = \boxed{21\ \text{V}}$$

31. ***REASONING AND SOLUTION*** The expression relating the peak current, I_0, to the rms-current, I_{rms}, is

$$I_{rms} = I_0/\sqrt{2} = (2.50\text{ A})/\sqrt{2} = \boxed{1.77\text{ A}}$$

32. ***REASONING***

a. We can obtain the frequency of the alternating current by comparing this specific expression for the current with the more general one in Equation 20.8.

b. The resistance of the light bulb is, according to Equation 20.14, equal to the rms-voltage divided by the rms-current. The rms-voltage is given, and we can obtain the rms-current by dividing the peak current by $\sqrt{2}$, as expressed by Equation 20.12.

c. The average power is given by Equation 20.15a as the product of the rms-current and the rms-voltage.

SOLUTION

a. By comparing $I = (0.707\text{ A})\sin\left[(314\text{ Hz})\,t\right]$ with the general expression (see Equation 20.8) for the current in an ac circuit, $I = I_0 \sin 2\pi f t$, we see that

$$2\pi f t = (314\text{ Hz})\,t \qquad \text{or} \qquad f = \frac{314\text{ Hz}}{2\pi} = \boxed{50.0\text{ Hz}}$$

b. The resistance is equal to $V_{\text{rms}}/I_{\text{rms}}$, where the rms-current is related to the peak current I_0 by $I_{\text{rms}} = I_0/\sqrt{2}$. Thus, the resistance of the light bulb is

$$R = \frac{V_{\text{rms}}}{I_{\text{rms}}} = \frac{V_{\text{rms}}}{\dfrac{I_0}{\sqrt{2}}} = \frac{\sqrt{2}\,(120.0\text{ V})}{0.707\text{ A}} = \boxed{2.40\times10^{2}\ \Omega} \qquad (20.14)$$

c. The average power is the product of the rms-current and rms-voltage:

$$\bar{P} = I_{\text{rms}}V_{\text{rms}} = \left(\frac{I_0}{\sqrt{2}}\right)V_{\text{rms}} = \left(\frac{0.707\text{ A}}{\sqrt{2}}\right)(120.0\text{ V}) = \boxed{60.0\text{ W}} \qquad (20.15a)$$

33. **SSM** ***REASONING AND SOLUTION***

a. According to Equation 20.15c, the average power consumed by the iron is

$$\overline{P} = \frac{V_{\text{rms}}^2}{R} = \frac{(120\text{ V})^2}{16\ \Omega} = \boxed{9.0 \times 10^2\text{ W}}$$

b. The peak power is $P_{\text{peak}} = 2\overline{P} = \boxed{1.8 \times 10^3\text{ W}}$.

34. ***REASONING AND SOLUTION*** The power P dissipated in the extension cord is $P = I^2R$ (Equation 20.6b). The resistance R is related to the length L of the wire and its cross-sectional area A by Equation 20.3, $R = \rho L/A$, where ρ is the resistivity of copper. The cross-sectional area of the wire can be expressed as

$$A = \frac{\rho L}{R} = \frac{\rho I^2}{\left(\frac{P}{L}\right)}$$

where the ratio P/L is the power per unit length of copper wire that the heater produces. The wire is cylindrical, so its cross-sectional area is $A = \pi r^2$. Thus, the smallest radius of wire that can be used is

$$r = I\sqrt{\frac{\rho}{\pi\left(\frac{P}{L}\right)}} = (18\text{ A})\sqrt{\frac{1.72\times10^{-8}\ \Omega\cdot\text{m}}{\pi(1.0\text{ W/m})}} = \boxed{1.3\times10^{-3}\text{ m}}$$

Note that we have used 1.0 W/m as the power per unit length, rather than 2.0 W/m. This is because an extension cord is composed of two copper wires. If the maximum power per unit length that the extension cord itself can produce is 2.0 W/m, then each wire can produce only a maximum of 1.0 W/m.

35. ***REASONING AND SOLUTION*** The total amount of energy needed is

$$E = Pt = I^2Rt = (25\ \text{A})^2(5.3\ \Omega)(31\ \text{d})[(9\ \text{h})/(1\ \text{d})][(3600\ \text{s})/(1\ \text{h})] = 3.3 \times 10^9\ \text{J}$$

To find the cost, then,

$$\text{Cost} = [\$0.10/(1\ \text{kWh})](3.3 \times 10^9\ \text{J})(1\ \text{kWh})/(3.6 \times 10^6\ \text{J}) = \boxed{\$\ 92}$$

36. ***REASONING AND SOLUTION***

a. According to Equation 20.15c,

$$P = V^2_{rms}/R = (240\ V)^2/(29\ \bullet) = \boxed{2.0 \times 10^3\ W}$$

b. Assuming (3/4) of the energy is converted to heat,

$$Q = (3/4)Pt = mc\Delta T$$

Solving for t yields

$$t = (4/3)mc(T_f - T_0)/P = (4/3)(1.9\ kg)[4186\ J/(kg{\cdot}C°)](85\ C°)/(2.0 \times 10^3\ W) = \boxed{450\ s}$$

37. $\boxed{\text{SSM}}$ ***REASONING*** According to Equation 6.10b, the energy supplied to the water in the form of heat is $Q = Pt$, where P is the power and t is the time. According to Equation 20.6c, the power is $P = V^2/R$, so that $Q = (V^2/R)t$, where V is the voltage and R is the resistance. But for a mass m of water with a specific heat capacity c, the heat Q is related to the change in temperature ΔT by $Q = mc\Delta T$ according to Equation 12.4. Our solution uses these two expressions for Q.

SOLUTION Combining the two expressions for Q, we have $Q = mc\Delta T = V^2t/R$. Solving for t, we have

$$t = \frac{Rcm\Delta T}{V^2} \tag{1}$$

The mass of the water can be determined from its volume V_{water}. From Equation 11.1, we have $m = \rho_{\text{water}} V_{\text{water}}$, where $\rho_{\text{water}} = 1.000 \times 10^3 \text{ kg/m}^3$ is the mass density of water (see Table 11.1). Therefore, Equation (1) becomes

$$t = \frac{Rc\rho_{\text{water}} V_{\text{water}} \Delta T}{V^2} \tag{2}$$

The value (see Table 12.2) for the specific heat of water is 4186 J/(kg•C°). The volume of water is

$$V_{\text{water}} = (52 \text{ gal})\left(\frac{3.79 \times 10^{-3} \text{ m}^3}{1.00 \text{ gal}}\right) = 0.197 \text{ m}^3$$

Substituting values into Equation (2), we obtain

$$t = \frac{(3.0\ \Omega)[4186 \text{ J/(kg} \cdot \text{C°)}](1.000 \times 10^3 \text{ kg/m}^3)(0.197 \text{ m}^3)(53\ ^\circ\text{C} - 11\ ^\circ\text{C})}{(120 \text{ V})^2} = 7220 \text{ s}$$

$$= (7220 \text{ s})\left(\frac{1.00 \text{ h}}{3600 \text{ s}}\right) = \boxed{2.0 \text{ h}}$$

38. ***REASONING AND SOLUTION*** The energy Q_1 that is released when the water cools from an initial temperature T to a final temperature of 0.0 °C is given by Equation 12.4 as $Q_1 = cm(T - 0.0\ °C)$. The energy Q_2 released when the water turns into ice at 0.0 °C is $Q_2 = mL_f$, where L_f is the latent heat of fusion for water. Since power P is energy divided by time, the power produced is

$$P = \frac{Q_1 + Q_2}{t} = \frac{cm(T - 0.0\ °C) + mL_f}{t}$$

The power produced by an electric heater is, according to Equation 20.6a, P = IV. Substituting this expression for P into the equation above and solving for the current I, we get

$$I = \frac{cm(T - 0.0\ °C) + mL_f}{tV}$$

$$I = \frac{(4186\ \text{J/kg}\cdot\text{C}°)(660\ \text{kg})(10.0\ \text{C}°) + (660\ \text{kg})(33.5 \times 10^4\ \text{J/kg})}{(9.0\ \text{h})\left(\frac{3600\ \text{s}}{\text{h}}\right)(240\ \text{V})} = \boxed{32\ \text{A}}$$

39. $\boxed{\text{SSM}}$ ***REASONING*** The equivalent series resistance R_S is the sum of the resistances of the three resistors. The potential difference V can be determined from Ohm's law as $V = IR_S$.

SOLUTION

a. The equivalent resistance is

$$R_S = 25\ \Omega + 45\ \Omega + 75\ \Omega = \boxed{145\ \Omega}$$

b. The potential difference across the three resistors is

$$V = IR_S = (0.51\ \text{A})(145\ \Omega) = \boxed{74\ \text{V}}$$

40. ***REASONING*** Since the two resistors are connected in series, they are equivalent to a single equivalent resistance that is the sum of the two resistances, according to Equation 20.16. Ohm's law (Equation 20.2) can be applied with this equivalent resistance to give the battery voltage.

SOLUTION According to Ohm's law, we find

$$V = IR_s = I\left(R_1 + R_2\right) = \left(0.12\ \text{A}\right)\left(47\ \Omega + 28\ \Omega\right) = \boxed{9.0\ \text{V}}$$

41. ***REASONING AND SOLUTION*** The equivalent resistance of the circuit is

$$R_s = R_1 + R_2 = 36.0\ \Omega + 18.0\ \Omega = 54.0\ \Omega$$

Ohm's law for the circuit gives $I = V/R_s = (15.0\ \text{V})/(54.0\ \Omega) = 0.278\ \text{A}$

a. Ohm's law for R_1 gives $V_1 = (0.278\ \text{A})(36.0\ \Omega) = \boxed{10.0\ \text{V}}$

b. Ohm's law for R_2 gives $V_2 = (0.278\ \text{A})(18.0\ \Omega) = \boxed{5.00\ \text{V}}$

42. ***REASONING*** According to Equation 20.2, the resistance R of the resistor is equal to the voltage V_R across it divided by the current I, or $R = V_R / I$. Since the resistor, the lamp, and the voltage source are in series, the voltage across the resistor is $V_R = 120.0\ \text{V} - V_L$, where V_L is the voltage across the lamp. Thus, the resistance is

$$R = \frac{120.0\ \text{V} - V_L}{I}$$

Since V_L is known, we need only determine the current in the circuit. Since we know the voltage V_L across the lamp and the power P dissipated by it, we can use Equation 20.6a to find the current: $I = P / V_L$. The resistance can be written as

$$R = \frac{120.0\ \text{V} - V_L}{P / V_L}$$

SOLUTION Substituting the known values for V_L and P into the equation above, the resistance is

$$R = \frac{120.0\ \text{V} - 25\ \text{V}}{(60.0\ \text{W})/(25\ \text{V})} = \boxed{4.0 \times 10^1\ \Omega}$$

43. **SSM** ***REASONING*** Using Ohm's law (Equation 20.2) we can write an expression for the voltage across the original circuit as $V = I_0 R_0$. When the additional resistor R is inserted in series, assuming that the battery remains the same, the voltage across the new combination is given by $V = I(R + R_0)$. Since V is the same in both cases, we can write $I_0 R_0 = I(R + R_0)$. This expression can be solved for R_0.

SOLUTION Solving for R_0, we have $I_0 R_0 - IR_0 = IR$ or $R_0(I_0 - I) = IR$; therefore,

$$R_0 = \frac{IR}{I_0 - I} = \frac{(12.0\text{ A})(8.00\ \Omega)}{15.0\text{ A} - 12.0\text{ A}} = \boxed{32\ \Omega}$$

44. ***REASONING AND SOLUTION***

a. The equivalent resistance of the circuit is

$$R_s = 9.0\ \Omega + 5.0\ \Omega + 1.0\ \Omega = 15.0\ \Omega$$

The current through each of the resistors is from Ohm's law

$$I = (24\ \text{V})/(15.0\ \Omega) = \boxed{1.6\ \text{A}}$$

b. The voltage drop across the 9.0-Ω resistor is $V_1 = (1.6\ \text{A})(9.0\ \Omega) = \boxed{14\ \text{V}}$

The drop across the 5.0 Ω resistor is $V_2 = (1.6\ \text{A})(5.0\ \Omega) = \boxed{8.0\ \text{V}}$

The drop across the 1.0 Ω resistor is $V_3 = (1.6\ \text{A})(1.0\ \Omega) = \boxed{1.6\ \text{V}}$

c. The power dissipated in the 9.0 Ω resistor is

$$P_1 = I^2R_1 = (1.6\ \text{A})^2(9.0\ \Omega) = \boxed{23\ \text{W}}$$

Similarly, for the 5.0 Ω resistor, $P_2 = (1.6\ \text{A})^2(5.0\ \Omega) = \boxed{13\ \text{W}}$

and for the 1.0 Ω resistor $P_3 = (1.6\ \text{A})^2(1.0\ \Omega) = \boxed{2.6\ \text{W}}$

45. ***REASONING AND SOLUTION*** The 5.0 Ω resistor can tolerate a current of

$$I = \sqrt{\frac{P_1}{R_1}} = \sqrt{\frac{20.0\ \text{W}}{5.0\ \Omega}} = 2.0\ \text{A}$$

Similarly, it is found that the 30.0 Ω and 15.0 Ω resistors can tolerate currents of 0.577 A and 0.816 A, respectively. The maximum current in the circuit can, therefore, only be 0.577 A.

a. Ohm's law for the circuit gives

$$V = IR_S = (0.577\ \text{A})(50.0\ \Omega) = \boxed{28.9\ \text{V}}$$

b. The battery must deliver

$$P = IV = (0.577\ \text{A})(28.9\ \text{V}) = \boxed{16.7\ \text{W}}$$

46. ***REASONING*** The answer is not 340 W + 240 W = 580 W. The reason is that each heater contributes resistance to the circuit when they are connected in series across the battery. For a series connection, the resistances add together to give the equivalent total resistance, according to Equation 20.16. Thus, the total resistance is greater than the resistance of either heater. The greater resistance means that the current from the battery is less than when either heater is present by itself. Since the power for each heater is $P = I^2R$, according to Equation 20.6b, the smaller current means that the power delivered to an individual heater is less when both are connected than when that heater is connected alone. We approach this problem by remembering that the total power delivered to the series combination of the heaters is the power delivered to the equivalent series resistance.

SOLUTION Let the resistances of the two heaters be R_1 and R_2. Correspondingly, the powers delivered to the heaters when each is connected alone to the battery are P_1 and P_2. For the series connection, the equivalent total resistance is $R_1 + R_2$, according to Equation 20.16. Using Equation 20.6c, we can write the total power delivered to this equivalent resistance as

$$P = \frac{V^2}{R_1 + R_2} \tag{1}$$

But according to Equation 20.6c, as applied to the situations when each heater is connected by itself to the battery, we have

$$P_1 = \frac{V^2}{R_1} \qquad \text{or} \qquad R_1 = \frac{V^2}{P_1} \tag{2}$$

$$P_2 = \frac{V^2}{R_2} \qquad \text{or} \qquad R_2 = \frac{V^2}{P_2} \tag{3}$$

Substituting Equations (2) and (3) into Equation (1) gives

$$P = \frac{V^2}{\dfrac{V^2}{P_1} + \dfrac{V^2}{P_2}} = \frac{1}{\dfrac{1}{P_1} + \dfrac{1}{P_2}} = \frac{P_1 P_2}{P_1 + P_2} = \frac{(340\ \text{W})(240\ \text{W})}{340\ \text{W} + 240\ \text{W}} = \boxed{140\ \text{W}}$$

47. ***REASONING*** Ohm's law provides the basis for our solution. We will use it to express the current from the battery when both resistors are connected and when only one resistor at a time is connected. When both resistors are connected, we will use Ohm's law with the series equivalent resistance, which is $R_1 + R_2$, according to Equation 20.16. The problem statement gives values for amounts by which the current increases when one or the other resistor is removed. Thus, we will focus attention on the difference between the currents given by Ohm's law.

SOLUTION When R_2 is removed, leaving only R_1 connected, the current increases by 0.20 A. In this case, using Ohm's law to express the currents, we have

$$\underbrace{\frac{V}{R_1}}_{\substack{\text{Current given by}\\ \text{Ohm's law when}\\ \text{only } R_1 \text{ is present}}} - \underbrace{\frac{V}{R_1 + R_2}}_{\substack{\text{Current given by}\\ \text{Ohm's law when}\\ R_1 \text{ and } R_2 \text{ are present}}} = \frac{VR_2}{R_1\left(R_1 + R_2\right)} = 0.20\text{ A} \qquad (1)$$

When R_1 is removed, leaving only R_2 connected, the current increases by 0.10 A. In this case, using Ohm's law to express the currents, we have

$$\underbrace{\frac{V}{R_2}}_{\substack{\text{Current given by}\\ \text{Ohm's law when}\\ \text{only } R_2 \text{ is present}}} - \underbrace{\frac{V}{R_1 + R_2}}_{\substack{\text{Current given by}\\ \text{Ohm's law when}\\ R_1 \text{ and } R_2 \text{ are present}}} = \frac{VR_1}{R_2\left(R_1 + R_2\right)} = 0.10\text{ A} \qquad (2)$$

Multiplying Equation (1) and Equation (2), we obtain

$$\left[\frac{V R_2}{R_1\left(R_1 + R_2\right)}\right]\left[\frac{V R_1}{R_2\left(R_1 + R_2\right)}\right] = \left(0.20\text{ A}\right)\left(0.10\text{ A}\right)$$

Simplifying this result algebraically shows that

$$\frac{V^2}{\left(R_1 + R_2\right)^2} = \left(0.20\text{ A}\right)\left(0.10\text{ A}\right) \quad \text{or} \quad \frac{V}{R_1 + R_2} = \sqrt{\left(0.20\text{ A}\right)\left(0.10\text{ A}\right)} = 0.14\text{ A} \qquad (3)$$

a. Using the result for $V/(R_1 + R_2)$ from Equation (3) to substitute into Equation (1) gives

$$\frac{V}{R_1} - 0.14\text{ A} = 0.20\text{ A} \quad \text{or} \quad R_1 = \frac{V}{0.20\text{ A} + 0.14\text{ A}} = \frac{12\text{ V}}{0.20\text{ A} + 0.14\text{ A}} = \boxed{35\,\Omega}$$

b. Using the result for $V/(R_1 + R_2)$ from Equation (3) to substitute into Equation (2) gives

$$\frac{V}{R_2} - 0.14\text{ A} = 0.10\text{ A} \quad \text{or} \quad R_2 = \frac{V}{0.10\text{ A} + 0.14\text{ A}} = \frac{12\text{ V}}{0.10\text{ A} + 0.14\text{ A}} = \boxed{5.0 \times 10^1\,\Omega}$$

48. ***REASONING AND SOLUTION*** The rule for combining parallel resistors is

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

which gives

$$\frac{1}{R_2} = \frac{1}{R_P} - \frac{1}{R_1} = \frac{1}{115\,\Omega} - \frac{1}{155\,\Omega} \qquad \text{or} \qquad \boxed{R_2 = 446\,\Omega}$$

49. **SSM** ***REASONING AND SOLUTION*** Since the circuit elements are in parallel, the equivalent resistance can be obtained directly from Equation 20.17:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{16\ \Omega} + \frac{1}{8.0\ \Omega} \qquad \text{or} \qquad \boxed{R_p = 5.3\ \Omega}$$

50. ***REASONING AND SOLUTION*** The power P dissipated in a resistance R is given by Equation 20.6c as $P = V^2/R$. The resistance R_{50} of the 50.0-W filament is

$$R_{50} = \frac{V^2}{P} = \frac{(120.0\text{ V})^2}{50.0\text{ W}} = \boxed{288\,\Omega}$$

The resistance R_{100} of the 100.0-W filament is

$$R_{100} = \frac{V^2}{P} = \frac{(120.0\text{ V})^2}{100.0\text{ W}} = \boxed{144\,\Omega}$$

51. $\boxed{\text{SSM}}$ ***REASONING*** Since the resistors are connected in parallel, the voltage across each one is the same and can be calculated from Ohm's Law (Equation 20.2: $V = IR$). Once the voltage across each resistor is known, Ohm's law can again be used to find the current in the second resistor. The total power consumed by the parallel combination can be found calculating the power consumed by each resistor from Equation 20.6b: $P = I^2R$. Then, the total power consumed is the sum of the power consumed by each resistor.

SOLUTION Using data for the second resistor, the voltage across the resistors is equal to

$$V = IR = (3.00\ \text{A})(64.0\ \Omega) = 192\ \Omega$$

a. The current through the 42.0-Ω resistor is

$$I = \frac{V}{R} = \frac{192\ \text{V}}{42.0\ \Omega} = \boxed{4.57\ \text{A}}$$

b. The power consumed by the 42.0-Ω resistor is

$$P = I^2R = (4.57\ \text{A})^2(42.0\ \Omega) = 877\ \text{W}$$

while the power consumed by the 64.0-Ω resistor is

$$P = I^2R = (3.00\ \text{A})^2(64.0\ \Omega) = 576\ \text{W}$$

Therefore the total power consumed by the two resistors is $877\ \text{W} + 576\ \text{W} = \boxed{1450\ \text{W}}$.

52. ***REASONING AND SOLUTION*** Each piece has a resistance of 1/3 R. Then

$$1/R_p = 1/(1/3\ R) + 1/(1/3\ R) + 1/(1/3\ R) = 9/R \qquad \text{or} \qquad R_p = \boxed{R/9}$$

53. ***REASONING*** The total power is given by Equation 20.15c as $\overline{P} = V_{\text{rms}}^2 / R_{\text{p}}$, where R_{p} is the equivalent parallel resistance of the heater and the lamp. Since the total power and the rms voltage are known, we can use this expression to obtain the equivalent parallel resistance. This equivalent resistance is related to the individual resistances of the heater and the lamp via Equation 20.17, which is $R_{\text{p}}^{-1} = R_{\text{heater}}^{-1} + R_{\text{lamp}}^{-1}$. Since R_{heater} is given, R_{lamp} can be found once R_{p} is known.

SOLUTION According to Equation 20.15c, the equivalent parallel resistance is

$$R_{\text{p}} = \frac{V_{\text{rms}}^2}{\overline{P}}$$

Using this result in Equation 20.17 gives

$$\frac{1}{R_{\text{p}}} = \frac{1}{V_{\text{rms}}^2 / \overline{P}} = \frac{1}{R_{\text{heater}}} + \frac{1}{R_{\text{lamp}}}$$

Rearranging this expression shows that

$$\frac{1}{R_{\text{lamp}}} = \frac{\overline{P}}{V_{\text{rms}}^2} - \frac{1}{R_{\text{heater}}} = \frac{84\ \text{W}}{(120\ \text{V})^2} - \frac{1}{6.0 \times 10^2\ \Omega} = 4.2 \times 10^{-3}\ \Omega^{-1}$$

Therefore,

$$R_{\text{lamp}} = \frac{1}{4.2 \times 10^{-3}\ \Omega^{-1}} = \boxed{240\ \Omega}$$

54. ***REASONING*** The electric heater and the toaster are connected in parallel with the voltage source, so each receives the same voltage as the source. The rms-current through the toaster is given by Equation 20.14, $I_{\text{rms}} = V_{\text{rms}}/R$, where V_{rms} is the rms-voltage across the toaster and R is its resistance. The total power supplied to the heater and toaster is given by Equation 20.15c as $\overline{P} = V_{\text{rms}}^2 / R_{\text{P}}$, where R_{P} is the equivalent resistance of the parallel circuit.

SOLUTION

a. The voltage across the heater is the same as that of the generator, $V_{\text{rms}} = \boxed{120.0\text{ V}}$.

b. The current through the toaster is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R_{\text{toaster}}} = \frac{120.0\text{ V}}{17.0\ \Omega} = \boxed{7.06\text{ A}} \qquad (20.14)$$

c. The average power supplied to the heater and toaster is $\overline{P} = V_{\text{rms}}^2 / R_{\text{P}}$. The equivalent resistance can be obtained from Equation 20.17:

$$\frac{1}{R_{\text{P}}} = \frac{1}{R_{\text{heater}}} + \frac{1}{R_{\text{toaster}}}$$

The average power becomes

$$\overline{P} = \frac{V_{\text{rms}}^2}{R_{\text{P}}} = V_{\text{rms}}^2\left(\frac{1}{R_{\text{heater}}} + \frac{1}{R_{\text{toaster}}}\right) = (120.0\text{ V})^2\left(\frac{1}{9.60\ \Omega} + \frac{1}{17.0\ \Omega}\right) = \boxed{2350\text{ W}}$$

55. $\boxed{\text{SSM}}$ ***REASONING*** The equivalent resistance of the three devices in parallel is R_p, and we can find the value of R_p by using our knowledge of the total power consumption of the circuit; the value of R_p can be found from Equation 20.6c, $P = V^2 / R_p$. Ohm's law (Equation 20.2, $V = IR$) can then be used to find the current through the circuit.

SOLUTION

a. The total power used by the circuit is P = 1650 W + 1090 W +1250 W = 3990 W. The equivalent resistance of the circuit is

$$R_p = \frac{V^2}{P} = \frac{(120\text{ V})^2}{3990\text{ W}} = \boxed{3.6\ \Omega}$$

b. The total current through the circuit is

$$I = \frac{V}{R_p} = \frac{120\text{ V}}{3.6\ \Omega} = \boxed{33\text{ A}}$$

This current is larger than the rating of the circuit breaker; therefore, the $\boxed{\text{breaker will open}}$.

56. ***REASONING AND SOLUTION*** The resistance of the parallel combination is given by

$$1/R_p = 1/R + 1/(2.00\ \Omega)$$

and the series combination has a resistance of $R_s = R + 2.00\ \Omega$. Ohm's law gives for each case

$$I_p = V/R_p = V/R + V/(2.00\ \Omega)$$

and

$$I_s = V/R_s = V/(R + 2.00\ \Omega)$$

We know that $I_p = 5I_s$. Using this with the above equations and suppressing units yields the quadratic equation

$$R^2 - 6.0\ R + 4.00 = 0$$

The two solutions of this equation are $\boxed{5.24\ \bullet}$ and $\boxed{0.76\ \bullet}$.

57. ***REASONING*** Since the defogger wires are connected in parallel, the total resistance of all thirteen wires can be obtained from Equation 20.17:

$$\frac{1}{R_p} = \frac{13}{R} \qquad \text{or} \qquad R_p = \frac{R}{13}$$

where R is the individual resistance of one of the wires. The heat required to melt the ice is given by $Q = mL_f$, where m is the mass of the ice and L_f is the latent heat of fusion of the ice (see Section 12.8). Therefore, using Equation 20.6c, we can see that the power or energy dissipated per unit time in the wires and used to melt the ice is

$$P = \frac{V^2}{R_p} = \underbrace{\frac{mL_f}{t}}_{\text{Energy/time}} \qquad \text{or} \qquad \frac{V^2}{R/13} = \frac{mL_f}{t}$$

According to Equation 20.3, $R = \rho L / A$, where the length of the wire is L, its cross-sectional area is A and its resistivity is ρ; therefore, the last expression can be written

$$\frac{13V^2}{R} = \frac{13V^2}{\rho L / A} = \frac{mL_f}{t}$$

This expression can be solved for the area A.

SOLUTION Solving the above expression for A, and substituting given data, and obtaining the latent heat of fusion of water from Table 12.3, we find that

$$A = \frac{\rho L m L_f}{13V^2 t}$$

$$= \frac{(88.0 \times 10^{-8}\ \Omega \cdot \text{m})(1.30\ \text{m})(2.10 \times 10^{-2}\ \text{kg})(33.5 \times 10^{4}\ \text{J/kg})}{13(12.0\ \text{V})^2\ (120\ \text{s})} = \boxed{3.58 \times 10^{-8}\ \text{m}^2}$$

58. ***REASONING*** To find the current, we will use Ohm's law, together with the proper equivalent resistance. The coffee maker and frying pan are in series, so their equivalent resistance is given by Equation 20.16 as $R_{\text{coffee}} + R_{\text{pan}}$. This total resistance is in parallel with the resistance of the bread maker, so the equivalent resistance of the parallel combination can be obtained from Equation 20.17 as $R_{\text{p}}^{-1} = (R_{\text{coffee}} + R_{\text{pan}})^{-1} + R_{\text{bread}}^{-1}$.

SOLUTION Using Ohm's law and the expression developed above for R_{p}^{-1}, we find

$$I = \frac{V}{R_{\text{p}}} = V\left(\frac{1}{R_{\text{coffee}} + R_{\text{pan}}} + \frac{1}{R_{\text{bread}}}\right) = (120\ \text{V})\left(\frac{1}{14\ \Omega + 16\ \Omega} + \frac{1}{23\ \Omega}\right) = \boxed{9.2\ \text{A}}$$

59. $\boxed{\text{SSM}}$ ***REASONING*** We will analyze the combination in parts. The 65 and 85-Ω resistors are in parallel. This parallel combination is in series with the 63-Ω resistor.

SOLUTION The equivalent resistance for the parallel combination of the 65 and 85-Ω resistors can be determined as follows, using Equation 20.17:

$$\frac{1}{R_{\text{p}}} = \frac{1}{65\,\Omega} + \frac{1}{85\,\Omega} = 0.027\,\Omega^{-1} \qquad \text{or} \qquad R_{\text{p}} = \frac{1}{0.027\,\Omega^{-1}} = 37\,\Omega$$

This equivalent resistance is in series with the 63-Ω resistance, the two giving the following total resistance between points *A* and *B*, according to Equation 20.16:

$$R_{\text{s}} = R_{AB} = 63\,\Omega + 37\,\Omega = \boxed{1.0 \times 10^{2}\ \Omega}$$

60. ***REASONING AND SOLUTION*** The 6.00 Ω, 5.00 Ω and 3.00 Ω resistors are in series with an equivalent resistance of $R_s = 14.0\ \Omega$.

This equivalent resistor is in parallel with the 8.00-Ω resistor, so

$$1/R_p = 1/(14.0\ \Omega) + 1/(8.00\ \Omega) \qquad \text{or} \qquad R_p = 5.09\ \Omega$$

This new equivalent resistor is in series with the 4.00-Ω resistor, so $R_s' = 9.09\ \Omega$.

R_s' is in parallel with the 10.0-Ω resistor, so

$$1/R_p' = 1/(9.09\ \Omega) + 1/(10.0\ \Omega) \qquad \text{or} \qquad R_p' = 4.76\ \Omega$$

Finally, R_p' is in series with the 2.00-Ω , so the total equivalent resistance is $\boxed{6.76\ \Omega}$.

61. $\boxed{\text{SSM}}$ ***REASONING*** When two or more resistors are in series, the equivalent resistance is given by Equation 20.16: $R_s = R_1 + R_2 + R_3 + \ldots$. Likewise, when resistors are in parallel, the expression to be solved to find the equivalent resistance is given by Equation 20.17: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$. We will successively apply these to the individual resistors in the figure in the text beginning with the resistors on the right side of the figure.

SOLUTION Since the 4.0-Ω and the 6.0-Ω resistors are in series, the equivalent resistance of the combination of those two resistors is 10.0 Ω. The 9.0-Ω and 8.0-Ω resistors are in parallel; their equivalent resistance is 4.24 Ω. The equivalent resistances of the parallel combination (9.0 Ω and 8.0 Ω) and the series combination (4.0 Ω and the 6.0 Ω) are in parallel; therefore, their equivalent resistance is 2.98 Ω. The 2.98-Ω combination is in series with the 3.0-Ω resistor, so that equivalent resistance is 5.98 Ω. Finally, the 5.98-Ω combination and the 20.0-Ω resistor are in parallel, so the equivalent resistance between the points A and B is $\boxed{4.6\ \Omega}$.

62. ***REASONING*** The circuit diagram is shown at the right. We can find the current in the 120.0-Ω resistor by using Ohm's law, provided that we can obtain a value for V_{AB}, the voltage between points A and B in the diagram. To find V_{AB}, we will also apply Ohm's law, this time by multiplying the current from the battery times R_{AB}, the equivalent parallel resistance between A and B. The current from the battery can be obtained by applying Ohm's law once again, now to the entire circuit and using the total equivalent resistance of the series combination of the 20.0-Ω resistor and R_{AB}. Once the current in the 120.0-Ω resistor is found, the power dissipated in it can be obtained from Equation 20.6b, $P = I^2R$.

SOLUTION

a. According to Ohm's law, the current in the 120.0-Ω resistor is $I_{120} = V_{AB}/(120.0\ \Omega)$. To find V_{AB}, we note that the equivalent parallel resistance between points A and B can be obtained from Equation 20.17 as follows:

$$\frac{1}{R_{AB}} = \frac{1}{60.0\ \Omega} + \frac{1}{120.0\ \Omega} \qquad \text{or} \qquad R_{AB} = 40.0\ \Omega$$

This resistance of 40.0 Ω is in series with the 20.0-Ω resistance, so that, according to Equation 20.16, the total equivalent resistance connected across the battery is 40.0 Ω + 20.0 Ω = 60.0 Ω. Applying Ohm's law to the entire circuit, we can see that the current from the battery is

$$I = \frac{15.0\ \text{V}}{60.0\ \Omega} = 0.250\ \text{A}$$

Again applying Ohm's law, this time to the resistance R_{AB}, we find that

$$V_{AB} = (0.250\ \text{A})R_{AB} = (0.250\ \text{A})(40.0\ \Omega) = 10.0\ \text{V}$$

Finally, we can see that the current in the 120.0-Ω resistor is

$$I_{120} = \frac{V_{AB}}{120\ \Omega} = \frac{10.0\ \text{V}}{120\ \Omega} = \boxed{8.33 \times 10^{-2}\ \text{A}}$$

b. The power dissipated in the 120.0-Ω resistor is given by Equation 20.6b as

$$P = I_{120}^2 R = \left(8.33 \times 10^{-2}\ \text{A}\right)^2 (120.0\ \Omega) = \boxed{0.833\ \text{W}}$$

63. ***REASONING AND SOLUTION*** The resistors in the small network have an equivalent resistance

$$\frac{1}{R_p} = \frac{1}{2.0\ \Omega + 1.0\ \Omega} + \frac{1}{5.0\ \Omega + 1.0\ \Omega} \qquad \text{or} \qquad R_p = 2.0\ \Omega$$

This resistance is in series with the 4.0-Ω resistor so the equivalent resistance of the circuit is R = 6.0 Ω. Therefore, Ohm's law gives the total current in the circuit to be

$$I = V/R = (12\ \text{V})/(6.0\ \Omega) = 2.0\ \text{A}$$

This current, upon entering the parallel branch, will split in the ratios of 3:9 and 6:9, with the largest current entering the smallest resistance path. The 5.0-Ω resistor then has a current of

$$I = (3/9)(2.0\ \text{A})$$

The power dissipated in this resistor is

$$P = [(3/9)(2.0\ \text{A})]^2(5.0\ \Omega) = \boxed{2.2\ \text{W}}$$

64. ***REASONING*** The power P delivered to the circuit is, according to Equation 20.6c, $P = V^2 / R_{12345}$, where V is the voltage of the battery and R_{12345} is the equivalent resistance of the five-resistor circuit. The voltage and power are known, so that the equivalent resistance can be calculated. We will use our knowledge of resistors wired in series and parallel to evaluate R_{12345} in terms of the resistance R of each resistor. In this manner we will find the value for R.

SOLUTION First we note that all the resistors are equal, so $R_1 = R_2 = R_3 = R_4 = R_5 = R$. We can find the equivalent resistance R_{12345} as follows. The resistors R_3 and R_4 are in series, so the equivalent resistance R_{34} of these two is $R_{34} = R_3 + R_4 = 2R$. The resistors R_2, R_{34}, and R_5 are in parallel, and the reciprocal of the equivalent resistance R_{2345} is

$$\frac{1}{R_{2345}} = \frac{1}{R_2} + \frac{1}{R_{34}} + \frac{1}{R_5} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{R} = \frac{5}{2R}$$

so $R_{2345} = 2R/5$. The resistor R_1 is in series with R_{2345}, and the equivalent resistance of this combination is the equivalent resistance of the circuit. Thus, we have

$$R_{12345} = R_1 + R_{2345} = R + \frac{2R}{5} = \frac{7R}{5}$$

The power delivered to the circuit is

$$P = \frac{V^2}{R_{12345}} = \frac{V^2}{\left(\frac{7R}{5}\right)}$$

Solving for the resistance R, we find that

$$R = \frac{5V^2}{7P} = \frac{5(45\text{ V})^2}{7(58\text{ W})} = \boxed{25\ \Omega}$$

65. **SSM** **WWW** ***REASONING*** Since we know that the current in the 8.00-Ω resistor is 0.500 A, we can use Ohm's law ($V = IR$) to find the voltage across the 8.00-Ω resistor. The 8.00-Ω resistor and the 16.0-Ω resistor are in parallel; therefore, the voltages across them are equal. Thus, we can also use Ohm's law to find the current through the 16.0-Ω resistor. The currents that flow through the 8.00-Ω and the 16.0-Ω resistors combine to give the total current that flows through the 20.0-Ω resistor. Similar reasoning can be used to find the current through the 9.00-Ω resistor.

SOLUTION

a. The voltage across the 8.00-Ω resistor is $V_8 = (0.500\text{ A})(8.00\ \Omega) = 4.00\text{ V}$. Since this is also the voltage that is across the 16.0-Ω resistor, we find that the current through the 16.0-Ω resistor is $I_{16} = (4.00\text{ V})/(16.0\ \Omega) = 0.250\text{ A}$. Therefore, the total current that flows through the 20.0-Ω resistor is

$$I_{20} = 0.500\text{ A} + 0.250\text{ A} = \boxed{0.750\text{ A}}$$

b. The 8.00-Ω and the 16.0-Ω resistors are in parallel, so their equivalent resistance can be obtained from Equation 20.17, $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$, and is equal to 5.33 Ω. Therefore, the equivalent resistance of the upper branch of the circuit is $R_{\text{upper}} = 5.33\ \Omega + 20.0\ \Omega = 25.3\ \Omega$, since the 5.33-Ω resistance is in series with the 20.0-Ω resistance. Using Ohm's law, we find that the voltage across the upper branch must be $V = (0.750\text{ A})(25.3\ \Omega) = 19.0\text{ V}$. Since the lower branch is in parallel with the upper branch, the voltage across both branches must be the same. Therefore, the current through the 9.00-Ω resistor is, from Ohm's law,

$$I_9 = \frac{V_{\text{lower}}}{R_9} = \frac{19.0\text{ V}}{9.00\ \Omega} = \boxed{2.11\text{ A}}$$

66. ***REASONING AND SOLUTION*** The equivalent resistance of the initial configuration is given by

$$1/R_p = 3/R \qquad \text{or} \qquad R_p = R/3$$

The parallel part of the final configuration has a resistance of $R_p' = R/2$, so the total equivalent resistance is

$$R_s = R/2 + R = 3R/2$$

Now $R_s = R_p + 700\ \Omega$, so $3R/2 = R/3 + 700\ \Omega$, which gives $R = \boxed{600\ \Omega}$.

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67. ***REASONING AND SOLUTION*** The terminal voltage of the battery is

$$V_T = \text{Emf} - \text{Ir}$$

where

$$I = \text{Emf}/(R + r)$$

so

$$r/(R + r) = 1 - V_T/\text{Emf} = 1 - (8.30\ \text{V})/(9.00\ \text{V}) = 0.078$$

Solving for the internal resistance gives $r = 0.078\ (R + r)$, or

$$r = 0.078\ R/(1 - 0.078) = 0.078(1.4\ \Omega)/(1 - 0.078) = \boxed{0.12\ \Omega}$$

68. ***REASONING AND SOLUTION*** Ohm's law gives

$$r = V/I = (1.5\ \text{V})/(28\ \text{A}) = \boxed{0.054\ \Omega}$$

69. **SSM** ***REASONING*** The terminal voltage of the battery is given by $V_{\text{terminal}} = \text{Emf} - Ir$, where r is the internal resistance of the battery. Since the terminal voltage is observed to be one-half of the emf of the battery, we have $V_{\text{terminal}} = \text{Emf}/2$ and $I = \text{Emf}/(2r)$. From Ohm's law, the equivalent resistance of the circuit is $R = \text{emf}/I = 2r$. We can also find the equivalent resistance of the circuit by considering that the identical bulbs are in parallel across the battery terminals, so that the equivalent resistance of the N bulbs is found from

$$\frac{1}{R_{\text{p}}} = \frac{N}{R_{\text{bulb}}} \qquad \text{or} \qquad R_{\text{p}} = \frac{R_{\text{bulb}}}{N}$$

This equivalent resistance is in series with the battery, so we find that the equivalent resistance of the circuit is

$$R = 2r = \frac{R_{\text{bulb}}}{N} + r$$

This expression can be solved for N.

SOLUTION Solving the above expression for N, we have

$$N = \frac{R_{\text{bulb}}}{2r - r} = \frac{R_{\text{bulb}}}{r} = \frac{15\ \Omega}{0.50\ \Omega} = \boxed{30}$$

70. ***REASONING*** According to the discussion in Section 20.9, the emf of a battery is equal to its terminal voltage V plus the voltage V_r across the internal resistance; Emf $= V + V_r$. According to Equation 20.6a, however, the voltage across the internal resistance is related to the current I and the power P dissipated by the internal resistance as $V_r = P/I$. Thus, the emf of the battery can be expressed as

$$\text{Emf} = V + \frac{P}{I}$$

SOLUTION Using the result found above, the emf of the battery is

$$\text{Emf} = V + \frac{P}{I} = 23.4\text{ V} + \frac{34.0\text{ W}}{55.0\text{ A}} = \boxed{24.0\text{ V}}$$

71. ***REASONING AND SOLUTION*** The equivalent resistance of the circuit is

$$R = 0.15\ \Omega + 1.50\ \Omega = 1.65\ \Omega$$

The current in the circuit is then

$$I = (12.0\ \text{V})/(1.65\ \Omega) = 7.27\ \text{A}$$

The terminal voltage is

$$V_T = 12.0\ \text{V} - (7.27\ \text{A})(0.15\ \Omega) = \boxed{10.9\ \text{V}}$$

72. ***REASONING AND SOLUTION***

a. In the first case the parallel resistance of the 75.0 Ω and the 45.0 Ω resistors have an equivalent resistance that can be calculated using Equation 20.17:

$$\frac{1}{R_p} = \frac{1}{75.0\ \Omega} + \frac{1}{45.0\ \Omega} \qquad \text{or} \qquad R_p = 28.1\ \Omega$$

Ohm's law, Emf = IR gives Emf = (0.294 A)(28.1 Ω + r), or

$$\text{Emf} = 8.26\ \text{V} + (0.294\ \text{A})r \tag{1}$$

In the second case, Emf = (0.116 A)(75.0 Ω + r), or

$$\text{Emf} = 8.70\ \text{V} + (0.116\ \text{A})r \tag{2}$$

Multiplying Equation (1) by 0.116 A, Equation (2) by 0.294 A, and subtracting yields

Emf = $\boxed{8.99\ \text{V}}$.

b. Substituting this result into Equation (1) and solving for r gives r = $\boxed{2.5\ \Omega}$.

73. $\boxed{\text{SSM}}$ ***REASONING*** The current I can be found by using Kirchhoff's loop rule. Once the current is known, the voltage between points A and B can be determined.

SOLUTION

a. We assume that the current is directed clockwise around the circuit. Starting at the upper-left corner and going clockwise around the circuit, we set the potential drops equal to the potential rises:

$$\underbrace{(5.0\ \Omega)I + (27\ \Omega)I + 10.0\ \text{V} + (12\ \Omega)I + (8.0\ \Omega)I}_{\text{Potential drops}} = \underbrace{30.0\ \text{V}}_{\text{Potential rises}}$$

Solving for the current gives $\boxed{I = 0.38\ \text{A}}$.

b. The voltage between points A and B is

$$V_{AB} = 30.0\ \text{V} - (0.38\ \text{A})(27\ \Omega) = \boxed{2.0 \times 10^{1}\ \text{V}}$$

c. $\boxed{\text{Point B}}$ is at the higher potential.

74. ***REASONING AND SOLUTION***

a. The voltage between A and B is just the voltage across the resistor. Ohm's law gives

$$V_{AB} = IR = (2.0\ \text{A})(6.0\ \Omega) = \boxed{12\ \text{V}}$$

b. The voltage between A and C is the sum of the drop across the resistor and the rise across the battery:

$$V_{AC} = -12\ \text{V} + 36\ \text{V} = \boxed{24\ \text{V}}$$

75. ***REASONING AND SOLUTION*** Apply the loop rule to get (with the units suppressed)

$$9.0\ \text{V} = \text{I}(0.015\ \Omega) + \text{I}(0.015\ \Omega) + 8.0\ \text{V}$$

Solving this equation for the current gives $\text{I} = \boxed{33\ \text{A}}$.

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76. ***REASONING*** First, we draw a current I_1 in the 6.00-Ω resistor. We can express I_1 in terms of the other currents in the circuit, I and 3.00 A, by applying the junction rule to junction *f;* the sum of the currents into the junction must equal the sum of the currents out of the junction.

$$\underbrace{I}_{\substack{\text{Current into}\\ \text{junction } f}} = \underbrace{I_1 + 3.00\text{ A}}_{\substack{\text{Current out of}\\ \text{junction } f}} \qquad \text{or} \qquad I_1 = I - 3.00\text{ A}$$

In order to obtain values for I and V we apply the loop rule to the top and bottom loops of the circuit.

SOLUTION Applying the loop rule to the top loop (*abcf*) we have

$$\underbrace{(3.00\text{ A})(4.00\,\Omega) + (3.00\text{ A})(8.00\,\Omega)}_{\text{Potential drops}} = \underbrace{24.0\text{ V} + (I - 3.00\text{ A})(6.00\,\Omega)}_{\text{Potential rises}}$$

This equation can be solved directly for the current; $I = \boxed{5.00\text{ A}}$.

Applying the loop rule to the bottom loop (*fcde*) we have

$$\underbrace{(I - 3.00\text{ A})(6.00\,\Omega) + 24.0\text{ V} + I\,(2.00\,\Omega)}_{\text{Potential drops}} = \underbrace{V}_{\text{Potential rises}}$$

Substituting $I = 5.00$ A into this equation and solving for V gives $V = \boxed{46.0\text{ V}}$.

77. **SSM** ***REASONING*** We begin by labeling the currents in the three resistors. The drawing below shows the directions chosen for these currents. The directions are arbitrary, and if any of them is incorrect, then the analysis will show that the corresponding value for the current is negative.

We then mark the resistors with the plus and minus signs that serve as an aid in identifying the potential drops and rises for the loop rule, recalling that conventional current is always directed from a higher potential (+) toward a lower potential (–). Thus, given the directions chosen for I_1, I_2, and I_3, the plus and minus signs *must* be those shown in the drawing. We can now use Kirchhoff's rules to find the voltage across the 5.0-Ω resistor.

SOLUTION Applying the loop rule to the left loop (and suppressing units for onvenience) gives

$$5.0I_1 + 10.0I_3 + 2.0 = 10.0 \tag{1}$$

Similarly, for the right loop,

$$10.0I_2 + 10.0I_3 + 2.0 = 15.0 \tag{2}$$

If we apply the junction rule to the upper junction, we obtain

$$I_1 + I_2 = I_3 \tag{3}$$

Subtracting Equation (2) from Equation (1) gives

$$5.0I_1 - 10.0I_2 = -5.0 \tag{4}$$

We now multiply Equation (3) by 10 and add the result to Equation (2); the result is

$$10.0I_1 + 20.0I_2 = 13.0 \tag{5}$$

If we then multiply Equation (4) by 2 and add the result to Equation (5), we obtain $20.0I_1 = 3.0$, or solving for I_1, we obtain $I_1 = 0.15\text{ A}$. The fact that I_1 is positive means that the current in the drawing has the correct direction. The voltage across the 5.0-Ω resistor can be found from Ohm's law:

$$V = (0.15\text{ A})(5.0\ \Omega) = \boxed{0.75\text{ V}}$$

Current flows from the higher potential to the lower potential, and the current through the 5.0-Ω flows from left to right, so the $\boxed{\text{left end of the resistor}}$ is at the higher potential.

78. ***REASONING AND SOLUTION***

Let the current through the 20.0 Ω be I_1 and flow to the right.

Let the current through the 10.0 Ω be I_2 and flow up.

Let the current through the 5.0 Ω be I_3 and flow to the right.

Applying the loop rule to the left loop gives

$$20.0\ I_1 - 10.0\ I_2 = 0$$

and to the right loop

$$10.0\ I_2 + 5.0\ I_3 = 30.0$$

The junction rule applied to the upper junction gives

$$I_1 + I_2 - I_3 = 0$$

A simultaneous solution of the above gives $I_2 = \boxed{1.71\ \text{A}}$.

Since the answer is positive, the current flows in the assumed direction. That is, $\boxed{\text{from the bottom to the top}}$.

79. ***REASONING*** To find the voltage between points B and D, we will find the current in the 60.0-Ω resistor and then use Ohm's law to find the voltage as $V = IR$. To find the current we will use Kirchhoff's laws, the set-up for which is shown in the circuit diagram at the right. In this diagram we have marked the current in each resistor. It is I_{60} that we seek. Note that we have marked each resistor with plus and minus signs, to denote which end of the resistor is at the higher and which end is at the lower potential. Given our choice for the current directions, the plus and minus signs <u>must</u> be those shown, and they will help us apply Kirchhoff's loop rule correctly. If our value for I_{60} turns out to be negative, it will mean that the actual direction for this current is opposite to that in the diagram.

SOLUTION Applying the junction rule to junctions B and D gives

$$\underbrace{I_{40} = I_{60} + I_{70}}_{\text{At junction } B} \qquad (1) \qquad\qquad \underbrace{I_{60} + I_{20} = I_{50}}_{\text{At junction } D} \qquad (2)$$

Applying the loop rule to loops ABD, BCD, and ADC (including the battery) gives

$$\underbrace{I_{40}(40.0\ \Omega) + I_{60}(60.0\ \Omega)}_{\text{Potential drops, loop } ABD} = \underbrace{I_{20}(20.0\ \Omega)}_{\substack{\text{Potential rises,}\\ \text{loop } ABD}} \quad \text{or} \quad I_{40}(2.00) + I_{60}(3.00) = I_{20} \qquad (3)$$

$$\underbrace{I_{70}(70.0\ \Omega)}_{\substack{\text{Potential drops,}\\ \text{loop } BCD}} = \underbrace{I_{50}(50.0\ \Omega) + I_{60}(60.0\ \Omega)}_{\text{Potential rises, loop } BCD} \quad \text{or} \quad I_{70}(7.00) = I_{50}(5.00) + I_{60}(6.00) \qquad (4)$$

$$\underbrace{I_{20}(20.0\ \Omega) + I_{50}(50.0\ \Omega)}_{\text{Potential drops, loop } ADC} = \underbrace{20.0\ \text{V}}_{\substack{\text{Potential rises,}\\ \text{loop } ADC}} \quad \text{or} \quad I_{20}(2.00) + I_{50}(5.00) = 2.00 \qquad (5)$$

Equations (1)– (5) are five equations in five unknowns and must be solved simultaneously. Remember that it is I_{60} we seek, so our approach will be to eliminate the other four unknowns. Substituting I_{50} from Equation (2) into Equation (5) gives

$$I_{20}(7.00) + I_{60}(5.00) = 2.00 \qquad (6)$$

Substituting I_{40} from Equation (1) into Equation (3) gives

$$I_{60}(5.00) + I_{70}(2.00) = I_{20} \qquad (7)$$

Solving Equation (5) for I_{50} and substituting the result into Equation (4) gives

$$I_{70}(7.00) = 2.00 - I_{20}(2.00) + I_{60}(6.00) \qquad (8)$$

Solving Equation (8) for I_{70} and substituting the result into Equation (7) gives

$$I_{60}(47.0) - I_{20}(11.0) = -4.00 \tag{9}$$

Solving Equation (6) for I_{20} and substituting the result into Equation (9) gives

$$I_{60}(47.0) - \left[\frac{2.00 - I_{60}(5.00)}{7.00}\right](11.0) = -4.00 \qquad \text{or} \qquad I_{60} = -0.0156\ \text{A}$$

Since this result is negative, the current in the 60.0-Ω resistor is opposite to that shown in the diagram given in the reasoning step, that is, from point *D* up toward point *B*. Thus, point *D* is at a higher potential than point *B*, because conventional current is always directed from the high toward the low potential. Using Ohm's law, we find that the voltage between *D* and *B* is

$$V = I_{60}R = (0.0156\ \text{A})(60.0\ \Omega) = \boxed{0.94\ \text{V, with point } D \text{ at the higher potential}}$$

80. ***REASONING*** Since only 0.100 mA out of the available 60.0 mA is needed to cause a full-scale deflection of the galvanometer, the shunt resistor must allow the excess current of 59.9 mA to detour around the meter coil, as the drawing at the right indicates. The value for the shunt resistance can be obtained by recognizing that the 50.0-Ω coil resistance and the shunt resistance are in parallel, both being connected between points *A* and *B* in the drawing. Thus, the voltage across each resistance is the same.

SOLUTION Expressing voltage as the product of current and resistance, we find that

$$\underbrace{\left(59.9\times10^{-3}\ \text{A}\right)(R)}_{\text{Voltage across shunt resistance}} = \underbrace{\left(0.100\times10^{-3}\ \text{A}\right)(50.0\ \Omega)}_{\text{Voltage across coil resistance}}$$

$$R = \frac{\left(0.100\times10^{-3}\ \text{A}\right)(50.0\ \Omega)}{59.9\times10^{-3}\ \text{A}} = \boxed{0.0835\ \Omega}$$

81. **SSM** ***REASONING AND SOLUTION*** According to Ohm's law, the voltage across the series combination of the galvanometer and the resistor, under full-scale conditions is $V = I(R + R_g)$ where R_g is the resistance of the galvanometer. Therefore, solving for R, we have

$$R = \frac{V}{I} - R_g = \frac{30.0\ \text{V}}{8.30 \times 10^{-3}\ \text{A}} - 180\ \Omega = \boxed{3.43 \times 10^3\ \Omega}$$

82. ***REASONING*** As discussed in Section 20.11, some of the current (6.20 mA) goes directly through the galvanometer and the remainder I goes through the shunt resistor. Since the resistance of the coil R_C and the shunt resistor R are in parallel, the voltage across each is the same. We will use this fact to determine how much current goes through the shunt resistor. This value, plus the 6.20 mA that goes through the galvanometer, is the maximum current that this ammeter can read.

SOLUTION The voltage across the coil resistance is equal to the voltage across the shunt resistor, so

$$\underbrace{\left(6.20\times10^{-3}\ \text{A}\right)\left(20.0\ \Omega\right)}_{\substack{\text{Voltage across}\\ \text{coil resistance}}}=\underbrace{\left(I\right)\left(24.8\times10^{-3}\ \Omega\right)}_{\substack{\text{Voltage across}\\ \text{shunt resistor}}}$$

So $I = 5.00$ A. The maximum current is 5.00 A + 6.20 mA = $\boxed{5.01\ \text{A}}$.

83. ***REASONING AND SOLUTION*** The maximum voltage which can appear across the voltmeter is

$$V = IR_{\text{v}} = (180 \times 10^{-6}\ \text{A})(140\ 000\ \Omega) = \boxed{25\ \text{V}}$$

84. ***REASONING AND SOLUTION*** For the 20.0 V scale

$$V_1 = I(R_1 + R_c)$$

For the 30.0 V scale

$$V_2 = I(R_2 + R_c)$$

Subtracting and rearranging yields

$$I = \frac{V_2 - V_1}{R_2 - R_1} = \frac{30.0\ \text{V} - 20.0\ \text{V}}{2930\ \Omega - 1680\ \Omega} = \boxed{8.00 \times 10^{-3}\ \text{A}}$$

Substituting this value into either of the equations for V_1 or V_2 gives $R_c = \boxed{820\ \Omega}$.

85. **SSM** ***REASONING AND SOLUTION***

a. According to Ohm's law (Equation 20.2, $V = IR$) the current in the circuit is

$$I = \frac{V}{R+R} = \frac{V}{2R}$$

The voltage across either resistor is IR, so that we find

$$IR = \left(\frac{V}{2R}\right)R = \frac{V}{2} = \frac{60.0\text{ V}}{2} = \boxed{30.0\text{ V}}$$

b. The voltmeter's resistance is $R_v = V/I = (60.0\text{ V})/(5.00\times10^{-3}\text{ A}) = 12.0\times10^{3}\ \Omega$, and this resistance is in parallel with the resistance $R = 1550\ \Omega$. The equivalent resistance of this parallel combination can be obtained as follows:

$$\frac{1}{R_p} = \frac{1}{R_v} + \frac{1}{R} \qquad \text{or} \qquad R_p = \frac{RR_v}{R+R_v} = \frac{(1550\ \Omega)(12.0\times10^{3}\ \Omega)}{1550\ \Omega + 12.0\times10^{3}\ \Omega} = 1370\ \Omega$$

The voltage registered by the voltmeter is IR_p, where I is the current supplied by the battery to the series combination of the other 1550-Ω resistor and R_p. According to Ohm's law

$$I = \frac{60.0\text{ V}}{1550\ \Omega + 1370\ \Omega} = 0.0205\text{ A}$$

Thus, the voltage registered by the voltmeter is

$$IR_p = (0.0205\text{ A})(1370\ \Omega) = \boxed{28.1\text{ V}}$$

86. ***REASONING AND SOLUTION***

a. Add the parallel capacitances to get

$$C_p = 4.0\ \mu\text{F} + 8.0\ \mu\text{F} = \boxed{12.0\ \mu\text{F}}$$

b. The total charge stored on the two capacitors is

$$Q = C_p V = \left(12.0 \times 10^{-6}\ \text{F}\right)\left(25\ \text{V}\right) = \boxed{3.0 \times 10^{-4}\ \text{C}}$$

87. ***REASONING*** Our approach to this problem is to deal with the arrangement in parts. We will combine separately those parts that involve a series connection and those that involve a parallel connection.

SOLUTION The 24, 12, and 8.0-μF capacitors are in series. Using Equation 20.19, we can find the equivalent capacitance for the three capacitors:

$$\frac{1}{C_s} = \frac{1}{24\ \mu\text{F}} + \frac{1}{12\ \mu\text{F}} + \frac{1}{8.0\ \mu\text{F}} \qquad \text{or} \qquad C_s = 4.0\ \mu\text{F}$$

This 4.0-μF capacitance is in parallel with the 4.0-μF capacitance already shown in the text diagram. Using Equation 20.18, we find that the equivalent capacitance for the parallel group is

$$C_p = 4.0\ \mu\text{F} + 4.0\ \mu\text{F} = 8.0\ \mu\text{F}$$

This 8.0-μF capacitance is between the 5.0 and the 6.0-μF capacitances and in series with them. Equation 20.19 can be used, then, to determine the equivalent capacitance between *A* and *B* in the text diagram:

$$\frac{1}{C_s} = \frac{1}{5.0\ \mu\text{F}} + \frac{1}{8.0\ \mu\text{F}} + \frac{1}{6.0\ \mu\text{F}} \qquad \text{or} \qquad C_s = \boxed{2.0\ \mu\text{F}}$$

88. ***REASONING AND SOLUTION*** The equivalent capacitance of the circuit is

$$1/C = 1/(4.0\ \mu F) + 1/(6.0\ \mu F) + 1/(12.0\ \mu F) \qquad \text{or} \qquad C = 2.0\ \mu F$$

The total charge provided by the battery is, then,

$$Q = CV = (2.0 \times 10^{-6}\ F)(50.0\ V) = 1.0 \times 10^{-4}\ C$$

This charge appears on each capacitor in a series circuit, so the voltage across the 4.0-μF capacitor is

$$V_1 = Q/C_1 = (1.0 \times 10^{-4}\ C)/(4.0 \times 10^{-6}\ F) = \boxed{25\ V}$$

89. **SSM** ***REASONING*** The magnitude q of the charge on each plate of a capacitor is related to the voltage V across the capacitor plates and the capacitance C by Equation 19.8: $q = CV$. When two or more capacitors are in series, the equivalent capacitance of the combination can be obtained from Equation 20.19: $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \cdots$. Equation 20.18 gives the equivalent capacitance for two or more capacitors in parallel: $C_p = C_1 + C_2 + C_3 + \ldots$.

In this problem two of the capacitors are in series, while the third capacitor is in parallel with the series combination. We proceed by first finding the equivalent capacitance of the circuit, and then we can use Equation 19.8 to find the total charge that the battery delivers to all the capacitors.

SOLUTION The equivalent capacitance of the series combination can be obtained as follows:

$$\frac{1}{C_s} = \frac{1}{3.0\ \mu\text{F}} + \frac{1}{4.0\ \mu\text{F}} = 0.583\ (\mu\text{F})^{-1} \qquad \text{or} \qquad C_s = \frac{1}{0.583\ (\mu\text{F})^{-1}} = 1.72\ \mu\text{F}$$

The equivalent capacitance of the entire circuit is

$$C_{\text{equivalent}} = C_s = 1.72\ \mu\text{F} + 10.0\ \mu\text{F} = 11.72\ \mu\text{F}$$

Therefore the total charge delivered by the battery is

$$q = C_{\text{equivalent}} V = (11.72 \times 10^{-6}\ \text{F})(40.0\ \text{V}) = \boxed{4.69 \times 10^{-4}\ \text{C}}$$

90. ***REASONING AND SOLUTION*** Let C_o be the capacitance of an empty capacitor. Then the capacitances are

$$C_1 = 3.00\ C_o \quad \text{and} \quad C_2 = 5.00\ C_o$$

The series capacitance of the two is

$$1/C_s = 1/(2.50\ C_o) + 1/(4.00\ C_o) \qquad \text{or} \qquad C_s = 1.54\ C_o$$

Now

$$\kappa C_o = 1.54\ C_o \qquad \text{or} \qquad \kappa = \boxed{1.54}$$

91. ***REASONING AND SOLUTION*** The charges stored on capacitors in series are equal and equal to the charge separated by the battery. The total energy stored in the capacitors is

$$\text{Energy} = Q^2/(2C_1) + Q^2/(2C_2)$$

$$\text{Energy} = (1/C_1 + 1/C_2)Q^2/2$$

According to Equation 20.19, the quantity in the parentheses is just the reciprocal of the equivalent capacitance C of the circuit, so

$$\boxed{\text{Energy} = Q^2/(2C)}$$

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92. ***REASONING AND SOLUTION*** The 7.00 and 3.00-μF capacitors are in parallel. According to Equation 20.18, the equivalent capacitance of the two is 7.00 μF + 3.00 μF = 10.0 μF. This 10.0-μF capacitance is in series with the 5.00-μF capacitance. According to Equation 20.19, the equivalent capacitance of the complete arrangement can be obtained as follows:

$$\frac{1}{C} = \frac{1}{10.0\ \mu\text{F}} + \frac{1}{5.00\ \mu\text{F}} = 0.300\ (\mu\text{F})^{-1} \qquad \text{or} \qquad C = \frac{1}{0.300\ (\mu\text{F})^{-1}} = 3.33\ \mu\text{F}$$

The battery separates an amount of charge

$$Q = CV = (3.33 \times 10^{-6}\ \text{F})(30.0\ \text{V}) = 99.9 \times 10^{-6}\ \text{C}$$

This amount of charge resides on the 5.00 μF capacitor, so its voltage is

$$V_5 = (99.9 \times 10^{-6}\ \text{C})/(5.00 \times 10^{-6}\ \text{F}) = 20.0\ \text{V}$$

The loop rule gives the voltage across the 3.00 μF capacitor to be

$$V_3 = 30.0\ \text{V} - 20.0\ \text{V} = 10.0\ \text{V}$$

This is also the voltage across the 7.00 μF capacitor, since it is in parallel, so $V_7 = \boxed{10.0\ \text{V}}$.

93. **SSM** ***REASONING*** When two or more capacitors are in series, the equivalent capacitance of the combination can be obtained from Equation 20.19, $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$. Equation 20.18 gives the equivalent capacitance for two or more capacitors in parallel: $C_p = C_1 + C_2 + C_3 + \ldots$. The energy stored in a capacitor is given by $\frac{1}{2}CV^2$, according to Equation 19.11. Thus, the energy stored in the series combination is $\frac{1}{2}C_sV_s^2$, where

$$\frac{1}{C_s} = \frac{1}{7.0\ \mu\text{F}} + \frac{1}{3.0\ \mu\text{F}} = 0.476\ (\mu\text{F})^{-1} \qquad \text{or} \qquad C_s = \frac{1}{0.476\ (\mu\text{F})^{-1}} = 2.10\ \mu\text{F}$$

Similarly, the energy stored in the parallel combination is $\frac{1}{2}C_pV_p^2$ where

$$C_p = 7.0\ \mu\text{F} + 3.0\ \mu\text{F} = 10.0\ \mu\text{F}$$

The voltage required to charge the parallel combination of the two capacitors to the same total energy as the series combination can be found by equating the two energy expressions and solving for V_p.

SOLUTION Equating the two expressions for the energy, we have

$$\tfrac{1}{2}C_sV_s^2 = \tfrac{1}{2}C_pV_p^2$$

Solving for V_p, we obtain the result

$$V_p = V_s\sqrt{\frac{C_s}{C_p}} = (24\ \text{V})\sqrt{\frac{2.10\ \mu\text{F}}{10.0\ \mu\text{F}}} = \boxed{11\text{V}}$$

94. ***REASONING AND SOLUTION*** Charge is conserved during the re-equilibrium. Therefore, using q_0 and q_f to denote the initial and final charges, respectively, we have

$$q_{10} + q_{20} = 18.0\ \mu C = q_{1f} + q_{2f} \qquad (1)$$

After equilibrium has been established the capacitors will have equal voltages across them, since they are connected in parallel. Thus, $V_f = q_{1f}/C_1 = q_{2f}/C_2$, which leads to

$$q_{1f} = q_{2f}(C_1/C_2) = q_{2f}\,(2.00\ \mu F)/(8.00\ \mu F) = 0.250\ q_{2f}$$

Substituting this result into Equation (1) gives

$$18.0\ \mu C = 0.250\ q_{2f} + q_{2f} \qquad \text{or} \qquad q_{2f} = 14.4\ \mu C$$

Hence,

$$V_f = q_{2f}/C_2 = (14.4 \times 10^{-6}\ C)/(8.00 \times 10^{-6}\ F) = \boxed{1.80\ V}$$

95. **SSM** ***REASONING*** The charge q on a discharging capacitor in a RC circuit is given by Equation 20.22: $q = q_0 e^{-t/RC}$, where q_0 is the original charge at time $t = 0$ s. Once t (time for one pulse) and the ratio q/q_0 are known, this expression can be solved for C.

SOLUTION Since the pacemaker delivers 81 pulses per minute, the time for one pulse is $[1 \text{ min}/(81 \text{ pulses})] \times [60.0 \text{ s}/(1.00 \text{ min})] = 0.74 \text{ s}$. Since one pulse is delivered every time the fully-charged capacitor loses 63.2% of its original charge, the charge remaining is 36.8% of the original charge. Thus, we have $q = (0.368)q_0$, or $q/q_0 = 0.368$.

From Equation 20.22, we have

$$\frac{q}{q_0} = e^{-t/RC}$$

Taking the natural logarithm of both sides, we have,

$$\ln\left(\frac{q}{q_0}\right) = -\frac{t}{RC}$$

Solving for C, we find

$$C = \frac{-t}{R\ln(q/q_0)} = \frac{-(0.74 \text{ s})}{(1.8 \times 10^6\ \Omega)\ln(0.368)} = \boxed{4.1 \times 10^{-7} \text{ F}}$$

96. ***REASONING AND SOLUTION*** According to Equation 20.21, we find that

$$R = \frac{\tau}{C} = \frac{3.0\,\text{s}}{750 \times 10^{-6}\ \text{F}} = \boxed{4.0 \times 10^{3}\ \Omega}$$

97. ***REASONING*** The time constant of an RC circuit is given by Equation 20.21 as $\tau = RC$, where R is the resistance and C is the capacitance in the circuit. The two resistors are wired in parallel, so we can obtain the equivalent resistance by using Equation 20.17. The two capacitors are also wired in parallel, and their equivalent capacitance is given by Equation 20.18. The time constant is the product of the equivalent resistance and equivalent capacitance.

SOLUTION The equivalent resistance of the two resistors in parallel is

$$\frac{1}{R_P} = \frac{1}{2.0\ \text{k}\Omega} + \frac{1}{4.0\ \text{k}\Omega} = \frac{3}{4.0\ \text{k}\Omega} \quad \text{or} \quad R_P = 1.3\ \text{k}\Omega \tag{20.17}$$

The equivalent capacitance is

$$C_P = 3.0\ \mu\text{F} + 6.0\ \mu\text{F} = 9.0\ \mu\text{F} \tag{20.18}$$

The time constant for the charge to build up is

$$\tau = R_P\, C_P = \left(1.3 \times 10^3\ \Omega\right)\left(9.0 \times 10^{-6}\ \text{F}\right) = \boxed{1.2 \times 10^{-2}\ \text{s}}$$

98. ***REASONING AND SOLUTION*** In drawing (a) the equivalent capacitance is of the series capacitors is $C_s = C/2$. These are in parallel with a capacitance, C, so that the equivalent capacitance is

$$C_A = C/2 + C = (3/2)\,C$$

The time constant is $\tau_A = (3/2)\,C\,R$ so that

$$C = (2/3)\,(\tau_A/R) \qquad (1)$$

In drawing (b) the equivalent capacitance is $C_B = (2/3)C$. Therefore, with the aid of Equation (1), we have

$$\tau_B = RC_B = (2/3)\,C\,R = (4/9)\,\tau_A = (4/9)(0.34\text{ s}) = \boxed{0.15\text{ s}}$$

99. ***REASONING AND SOLUTION*** For charging a capacitor, Equations 20.20 and 20.21 indicate that

$$q = q_0(1 - e^{-t/\tau})$$

or

$$t/\tau = -\ln(1 - q/q_0) = -\ln(0.0010) = \boxed{6.9}$$

100. ***REASONING AND SOLUTION*** According to Equation 20.6c, the power dissipated by the iron is

$$P = \frac{V^2}{R} = \frac{(120\ \text{V})^2}{24\ \Omega} = \boxed{6.0 \times 10^2\ \text{W}}$$

101. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The equivalent capacitance of the combination can be obtained immediately from Equation 20.19, $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} \ldots$.

$$\frac{1}{C_s} = \frac{1}{3.0\ \mu\text{F}} + \frac{1}{7.0\ \mu\text{F}} + \frac{1}{9.0\ \mu\text{F}} = 0.587\ (\mu\text{F})^{-1} \qquad \text{or} \qquad C_s = \frac{1}{0.587\ (\mu\text{F})^{-1}} = \boxed{1.7\ \mu\text{F}}$$

102. ***REASONING AND SOLUTION*** The definition of current in Equation 20.1 gives

$$I = \frac{\Delta q}{\Delta t} = \frac{35\ \text{C}}{1.00 \times 10^{-3}\ \text{s}} = \boxed{3.5 \times 10^{4}\ \text{A}}$$

103. **SSM** ***REASONING AND SOLUTION*** Ohm's law (Equation 20.2), $V = IR$, gives the result directly:

$$R = \frac{V}{I} = \frac{9.0\ \text{V}}{0.11\ \text{A}} = \boxed{82\ \Omega}$$

104. ***REASONING AND SOLUTION*** Label the currents with the resistor values. Take I_3 to the right, I_2 to the left and I_1 to the right. Applying the loop rule to the top loop (suppressing the units) gives

$$I_1 + 2.0\ I_2 = 1.0 \qquad (1)$$

and to the bottom loop gives

$$2.0\ I_2 + 3.0\ I_3 = 5.0 \qquad (2)$$

Applying the junction rule to the left junction gives

$$I_2 = I_1 + I_3 \qquad (3)$$

Solving Equations (1) , (2) and (3) simultaneously, we find $I_2 = \boxed{0.73\ \text{A}}$.

The positive sign shows that the assumed direction is correct. That is, to the $\boxed{\text{left}}$.

105. ***REASONING AND SOLUTION*** From Equation 20.5 we have that $R = R_0[1 + \alpha (T - T_0)]$. Solving for T gives

$$T = T_0 + \frac{\left(R / R_0\right) - 1}{\alpha} = 20.0\ ^\circ\text{C} + \frac{\left[\left(99.6\ \Omega\right)/\left(125\ \Omega\right)\right] - 1}{3.72 \times 10^{-3}\ (\text{C}^\circ)^{-1}} = \boxed{-34.6\ ^\circ\text{C}}$$

106. ***REASONING*** The magnitude q of the charge on one plate of a capacitor is given by Equation 19.8 as $q = CV_1$, where $C = 9.0\ \mu\text{F}$ and V_1 is the voltage across the capacitor. Since the capacitor and the resistor R_1 are in parallel, the voltage across the capacitor is equal to the voltage across R_1. From Equation 20.2 we know that the voltage across the 4.0-Ω resistor is given by $V_1 = IR_1$, where I is the current in the circuit. Thus, the charge can be expressed as

$$q = CV_1 = C\left(IR_1\right)$$

The current is equal to the battery voltage V divided by the equivalent resistance of the two resistors in series, so that

$$I = \frac{V}{R_S} = \frac{V}{R_1 + R_2}$$

Substituting this result for I into the equation for q yields

$$q = C\left(IR_1\right) = C\left(\frac{V}{R_1 + R_2}\right)R_1$$

SOLUTION The magnitude of the charge on one of the plates is

$$q = C\left(\frac{V}{R_1 + R_2}\right)R_1 = \left(9.0 \times 10^{-6}\ \text{F}\right)\left(\frac{12\ \text{V}}{4.0\ \Omega + 2.0\ \Omega}\right)\left(4.0\ \Omega\right) = \boxed{7.2 \times 10^{-5}\ \text{C}}$$

107. ***REASONING AND SOLUTION*** The voltages across the galvanometer and shunt resistances are equal since they are in parallel, so

$$I_g R_g = I_s R_s \qquad \text{or} \qquad R_s = (I_g/I_s)R_g$$

where $I_g = 0.150 \times 10^{-3}$ A and $I_s = 4.00 \times 10^{-3}$ A $- 0.150 \times 10^{-3}$ A. Then

$$R_s = (0.150 \times 10^{-3}\ \text{A})(12.0\ \Omega)/(3.85 \times 10^{-3}\ \text{A}) = 0.468\ \Omega$$

The equivalent parallel resistance can be obtained as follows:

$$\frac{1}{R_p} = \frac{1}{0.468\ \Omega} + \frac{1}{12.0\ \Omega} \qquad \text{or} \qquad R_p = \boxed{0.450\ \Omega}$$

108. ***REASONING AND SOLUTION*** The voltage V_{Cu} between the ends of the copper rod is given by Ohm's law as $V_{cu} = IR_{Cu,}$ where R_{Cu} is the resistance of the copper rod. The current I in the circuit is equal to the voltage V of the battery that is connected across the free ends of the copper-iron rod divided by the equivalent resistance of the rod. The copper and iron rods are joined end-to-end, so the same current passes through each. Thus, they are connected in series, so the equivalent resistance R_S is $R_S = R_{Cu} + R_{Fe}$. Thus, the current is

$$I = \frac{V}{R_S} = \frac{V}{R_{Cu} + R_{Fe}}$$

The voltage across the copper rod is

$$V_{Cu} = IR_{Cu} = \frac{V}{R_{Cu} + R_{Fe}} R_{Cu}$$

The resistance of the copper and iron rods is given by $R_{Cu} = \rho_{Cu}L/A$ and $R_{Fe} = \rho_{Fe}L/A$, where the length L and cross-sectional area A is the same for both rods. Substituting these expressions for the resistances into the equation above and using resistivities from Table 20.1 yield

$$V_{Cu} = \left(\frac{V}{\rho_{Cu} + \rho_{Fe}}\right)\rho_{Cu}$$

$$V_{Cu} = \left(\frac{12\text{ V}}{1.72 \times 10^{-8}\ \Omega \cdot \text{m} + 9.7 \times 10^{-8}\ \Omega \cdot \text{m}}\right)(1.72 \times 10^{-8}\ \Omega \cdot \text{m}) = \boxed{1.8\text{ V}}$$

109. **SSM** ***REASONING*** When two or more resistors are in series, the equivalent resistance is given by Equation 20.16: $R_s = R_1 + R_2 + R_3 + \ldots$. When resistors are in parallel, the expression to be solved to find the equivalent resistance is Equation 20.17: $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$. We will use these relations to determine the eight different values of resistance that can be obtained by connecting together the three resistors: 1.00, 2.00, and 3.00 Ω.

SOLUTION When all the resistors are connected in series, the equivalent resistance is the sum of all three resistors and the equivalent resistance is $\boxed{6.00\ \Omega}$. When all three are in parallel, we have from Equation 20.17, the equivalent resistance is $\boxed{0.545\ \Omega}$.

We can also connect two of the resistors in parallel and connect the parallel combination in series with the third resistor. When the 1.00 and 2.00-Ω resistors are connected in parallel and the 3.00-Ω resistor is connected in series with the parallel combination, the equivalent resistance is $\boxed{3.67\ \Omega}$. When the 1.00 and 3.00-Ω resistors are connected in parallel and the 2.00-Ω resistor is connected in series with the parallel combination, the equivalent resistance is $\boxed{2.75\ \Omega}$. When the 2.00 and 3.00-Ω resistors are connected in parallel and the 1.00-Ω resistor is connected in series with the parallel combination, the equivalent resistance is $\boxed{2.20\ \Omega}$.

We can also connect two of the resistors in series and put the third resistor in parallel with the series combination. When the 1.00 and 2.00-Ω resistors are connected in series and the 3.00-Ω resistor is connected in parallel with the series combination, the equivalent resistance is $\boxed{1.50\ \Omega}$. When the 1.00 and 3.00-Ω resistors are connected in series and the 2.00-Ω resistor is connected in parallel with the series combination, the equivalent resistance is $\boxed{1.33\ \Omega}$. Finally, when the 2.00 and 3.00-Ω resistors are connected in series and the 1.00-Ω resistor is connected in parallel with the series combination, the equivalent resistance is $\boxed{0.833\ \Omega}$.

110. ***REASONING AND SOLUTION*** The aluminum and copper portions may be viewed a being connected in parallel since the same voltage appears across them. Using a and b to denote the inner and outer radii, respectively, we have for the equivalent resistance that

$$\frac{1}{R_p} = \frac{1}{R_{Al}} + \frac{1}{R_{Cu}} = \frac{A_{Cu}}{\rho_{Cu}L} + \frac{A_{Al}}{\rho_{Al}L}$$

$$\frac{1}{R_p} = \frac{\pi a^2}{\rho_{Cu}L} + \frac{\pi(b^2 - a^2)}{\rho_{Al}L}$$

Using resistivity values from Table 20.1, we find that $R_p = \boxed{0.00116\ \Omega}$.

111. $\boxed{\text{SSM}}$ ***REASONING*** The resistance of one of the wires in the extension cord is given by

Equation 20.3: $R = \rho L / A$, where the resistivity of copper is $\rho = 1.72 \times 10^{-8}\ \Omega \cdot \text{m}$, according to Table 20.1. Since the two wires in the cord are in series with each other, their total resistance is $R_{\text{cord}} = R_{\text{wire 1}} + R_{\text{wire 2}} = 2\rho L / A$. Once we find the equivalent resistance of the entire circuit (extension cord + trimmer), Ohm's law can be used to find the voltage applied to the trimmer.

SOLUTION

a. The resistance of the extension cord is

$$R_{\text{cord}} = \frac{2\rho L}{A} = \frac{2(1.72 \times 10^{-8}\ \Omega \cdot \text{m})(46\ \text{m})}{1.3 \times 10^{-6}\ \text{m}^2} = \boxed{1.2\ \Omega}$$

b. The total resistance of the circuit (cord + trimmer) is, since the two are in series,

$$R_{\text{s}} = 1.2\ \Omega + 15.0\ \Omega = 16.2\ \Omega$$

Therefore from Ohm's law (Equation 20.2: $V = IR$), the current in the circuit is

$$I = \frac{V}{R_{\text{s}}} = \frac{120\ \text{V}}{16.2\ \Omega} = 7.4\ \text{A}$$

Finally, the voltage applied to the trimmer alone is (again using Ohm's law),

$$V_{\text{trimmer}} = (7.4\ \text{A})(15.0\ \Omega) = \boxed{110\ \text{V}}$$

112. ***REASONING AND SOLUTION*** The power dissipated in the resistor if the battery has no internal resistance is

$$P_0 = I_0^2 R$$

where $I_0 = V/R$, so

$$P_0 = V^2/R \qquad (1)$$

The power dissipated if the battery has an internal resistance is $P = I^2R$, where $I = V/(R + r)$ and $P = 0.900\ P_0$. Thus,

$$0.900\ P_0 = V^2R/(R + r)^2 \qquad (2)$$

Division of Equation (2) by Equation (1) gives

$$R^2/(R + r)^2 = 0.900 \qquad \text{or} \qquad R/(R + r) = 0.949 \qquad \text{or} \qquad r/R = \boxed{0.054}$$

113. ***REASONING AND SOLUTION*** Using Equation 20.5, we can equate resistances

$$\underbrace{R_{0i}\left[1+\alpha_i\left(T-T_0\right)\right]}_{\substack{\text{Resistance}\\ \text{for iron wire}}}=\underbrace{R_{0g}\left[1+\alpha_g\left(T-T_0\right)\right]}_{\substack{\text{Resistance}\\ \text{for gold wire}}}$$

We can solve this equation for T to obtain

$$T=T_0+\frac{R_{0i}-R_{0g}}{R_{0g}\alpha_g-R_{0i}\alpha_i}$$

$$=20.0\ ^\circ\text{C}+\frac{5.90\ \Omega-6.70\ \Omega}{(6.70\ \Omega)\left[0.0034\ (\text{C}^\circ)^{-1}\right]-(5.90\ \Omega)\left[0.0050\ (\text{C}^\circ)^{-1}\right]}=\boxed{140\ ^\circ\text{C}}$$

114. ***REASONING AND SOLUTION*** Each resistor can tolerate a current of no more than

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{0.25\ \text{W}}{47\ \Omega}} = 0.073\ \text{A}$$

Ohm's law applied to a series circuit containing N such resistors gives $V = IR_S = INR$, so

$$N = \frac{V}{IR} = \frac{9.0\ \text{V}}{(0.073\ \text{A})(47\ \Omega)} = 2.6$$

Only $\boxed{\text{three}}$ resistors can be used.

115. $\boxed{\text{SSM}}$ ***REASONING*** The foil effectively converts the capacitor into two capacitors in series. Equation 19.10 gives the expression for the capacitance of a capacitor of plate area A and plate separation d (no dielectric): $C_0 = \varepsilon_0 A / d$. We can use this expression to determine the capacitance of the individual capacitors created by the presence of the foil. Then using the fact that the "two capacitors" are in series, we can use Equation 20.19 to find the equivalent capacitance of the system.

SOLUTION Since the foil is placed one-third of the way from one plate of the original capacitor to the other, we have $d_1 = (2/3)d$, and $d_2 = (1/3)d$. Then

$$C_1 = \frac{\varepsilon_0 A}{(2/3)d} = \frac{3\varepsilon_0 A}{2d}$$

and

$$C_2 = \frac{\varepsilon_0 A}{(1/3)d} = \frac{3\varepsilon_0 A}{d}$$

Since these two capacitors are effectively in series, it follows that

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3\varepsilon_0 A/(2d)} + \frac{1}{3\varepsilon_0 A/d} = \frac{3d}{3\varepsilon_0 A} = \frac{d}{\varepsilon_0 A}$$

But $C_0 = \varepsilon_0 A / d$, so that $d/(\varepsilon_0 A) = 1/C_0$, and we have

$$\frac{1}{C_s} = \frac{1}{C_0} \qquad \text{or} \qquad \boxed{C_s = C_0}$$

116. ***REASONING AND SOLUTION*** The resistance of the thermistor decreases by 15% relative to its normal value of 37.0 °C. That is,

$$\Delta R/R_0 = (R - R_0)/R_0 = -0.15$$

According to Equation 20.5, we have

$$R = R_0[1 + \alpha(T - T_0)] \quad \text{or} \quad (R - R_0) = \alpha R_0(T - T_0) \quad \text{or} \quad (R - R_0)/R_0 = \alpha(T - T_0) = -0.15$$

Rearranging this result gives

$$T = T_0 + (-0.15)/\alpha = 37.0\ °\text{C} + (-0.15)/(-0.060/\text{C}°) = \boxed{39.5\ °\text{C}}$$

1. **SSM** ***REASONING AND SOLUTION*** According to Equation 21.1, the magnitude B of the magnetic field is

$$B = \frac{F}{q_0 v \sin\theta} = \frac{8.7\times10^{-3}\ \text{N}}{(12\times10^{-6}\ \text{C})(9.0\times10^{6}\ \text{m/s})\sin 90.0^\circ} = \boxed{8.1\times10^{-5}\ \text{T}}$$

2. ***REASONING AND SOLUTION*** The magnitude of the force can be determined using Equation 21.1, $F = qvB \sin \theta$, where θ is the angle between the velocity and the magnetic field. The direction of the force is determined by using Right-Hand Rule No. 1.

a. $F = qvB \sin 30.0° = (8.4 \times 10^{-6}\ \text{C})(45\ \text{m/s})(0.30\ \text{T}) \sin 30.0° = \boxed{5.7 \times 10^{-5}\ \text{N}}$,

directed $\boxed{\text{into the paper}}$.

b. $F = qvB \sin 90.0° = (8.4 \times 10^{-6}\ \text{C})(45\ \text{m/s})(0.30\ \text{T}) \sin 90.0° = \boxed{1.1 \times 10^{-4}\ \text{N}}$,

directed $\boxed{\text{into the paper}}$.

c. $F = qvB \sin 150° = (8.4 \times 10^{-6}\ \text{C})(45\ \text{m/s})(0.30\ \text{T}) \sin 150° = \boxed{5.7 \times 10^{-5}\ \text{N}}$,

directed $\boxed{\text{into the paper}}$.

3. ***REASONING AND SOLUTION***

a. The magnitude of the magnetic force is given by $F = qvB \sin\theta$. We have, therefore,

$$B = \frac{F}{qv\sin\theta} = \frac{7.31\times10^{-3}\ \text{N}}{\left(25.0\times10^{-6}\ \text{C}\right)\left(4.50\times10^{3}\ \text{m/s}\right)\sin 90.0^\circ} = \boxed{6.50\times10^{-2}\ \text{T}}$$

b. For the second charge,

$$v_2 = \frac{F}{qB\sin\theta} = \frac{1.90\times10^{-3}\ \text{N}}{\left(5.00\times10^{-6}\ \text{C}\right)\left(6.50\times10^{-2}\ \text{T}\right)\sin 40.0^\circ} = \boxed{9.10\times10^{3}\ \text{m/s}}$$

4. ***REASONING*** According to Equation 21.1, the magnetic force has a magnitude of $F = qvB \sin\theta$. The field B and the directional angle θ are the same for each particle. Particle 1, however, travels faster than particle 2. By itself, a faster speed v would lead to a greater force magnitude F. But the force on each particle is the same. Therefore, particle 1 must have a smaller charge to counteract the effect of its greater speed.

SOLUTION Applying Equation 21.1 to each particle, we have

$$\underbrace{F = q_1 v_1 B \sin\theta}_{\text{Particle 1}} \quad \text{and} \quad \underbrace{F = q_2 v_2 B \sin\theta}_{\text{Particle 2}}$$

Dividing the equation for particle 1 by the equation for particle 2 and remembering that $v_1 = 3v_2$ gives

$$\frac{F}{F} = \frac{q_1 v_1 B \sin\theta}{q_2 v_2 B \sin\theta} \quad \text{or} \quad 1 = \frac{q_1 v_1}{q_2 v_2} \quad \text{or} \quad \frac{q_1}{q_2} = \frac{v_2}{v_1} = \frac{v_2}{3v_2} = \boxed{\tfrac{1}{3}}$$

5. SSM ***REASONING*** According to Equation 21.1, the magnitude of the magnetic force on a moving charge is $F = q_0 vB \sin\theta$. Since the magnetic field points due north and the proton moves eastward, $\theta = 90.0°$. Furthermore, since the magnetic force on the moving proton balances its weight, we have $mg = q_0 vB \sin\theta$, where m is the mass of the proton. This expression can be solved for the speed v.

SOLUTION Solving for the speed v, we have

$$v = \frac{mg}{q_0 B \sin\theta} = \frac{(1.67\times10^{-27}\ \text{kg})(9.80\ \text{m/s}^2)}{(1.6\times10^{-19}\ \text{C})(2.5\times10^{-5}\ \text{T})\sin 90.0°} = \boxed{4.1\times10^{-3}\ \text{m/s}}$$

6. ***REASONING*** According to Equation 21.1, the magnetic force has a magnitude of $F = qvB \sin\theta$, where q is the magnitude of the charge, B is the magnitude of the magnetic field, v is the speed, and θ is the angle of the velocity with respect to the field. As θ increases from 0° to 90°, the force increases. Therefore, the angle we seek must lie between 25° and 90°.

SOLUTION Letting $\theta_1 = 25°$ and θ_2 be the desired angle, we can apply Equation 21.1 to both situations as follows:

$$\underbrace{F = qvB\sin\theta_1}_{\text{Situation 1}} \quad \text{and} \quad \underbrace{2F = qvB\sin\theta_2}_{\text{Situation 2}}$$

Dividing the equation for situation 2 by the equation for situation 1 gives

$$\frac{2F}{F} = \frac{qvB\sin\theta_2}{qvB\sin\theta_1} \quad \text{or} \quad \sin\theta_2 = 2\sin\theta_1 = 2\sin 25° = 0.85$$

$$\theta_2 = \sin^{-1}(0.85) = \boxed{58°}$$

7. ***REASONING*** The angle θ between the electron's velocity and the magnetic field can be found from Equation 21.1,

$$\sin\theta = \frac{F}{qvB}$$

According to Newton's second law, the magnitude F of the force is equal to the product of the electron's mass m and the magnitude a of its acceleration, $F = ma$.

SOLUTION The angle θ is

$$\theta = \sin^{-1}\left(\frac{ma}{qvB}\right) = \sin^{-1}\left[\frac{(9.11\times10^{-31}\ \text{kg})(3.50\times10^{14}\ \text{m/s}^2)}{(1.60\times10^{-19}\ \text{C})(6.80\times10^{6}\ \text{m/s})(8.70\times10^{-4}\ \text{T})}\right] = \boxed{19.7°}$$

8. ***REASONING AND SOLUTION*** Equation 21.1 gives the magnitude F of the maximum force on the particle as $F = q_0 vB \sin 90.0° = q_0 vB$, where B is the magnitude of the net magnetic field. Since the two field components B_x and B_y are perpendicular, the Pythagorean theorem indicates that $B = \sqrt{B_x^2 + B_y^2}$. Therefore, we find that $B = \sqrt{B_x^2 + B_y^2} = \dfrac{F}{q_0 v}$. Squaring this result and solving for the y component of the magnetic field gives

$$B_y = \sqrt{\frac{F^2}{(q_0 v)^2} - B_x^2} = \sqrt{\frac{(0.455\ \text{N})^2}{\left[(6.50 \times 10^{-5}\ C)(2.00 \times 10^4\ \text{m/s})\right]^2} - (0.200\ \text{T})^2} = \boxed{0.287\ \text{T}}$$

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9. SSM WWW ***REASONING*** The direction in which the electrons are deflected can be determined using Right-Hand Rule No. 1 and reversing the direction of the force (RHR-1 applies to positive charges, and electrons are negatively charged).

Each electron experiences an acceleration a given by Newton's second law of motion, $a = F/m$, where F is the net force and m is the mass of the electron. The only force acting on the electron is the magnetic force, $F = q_0 vB \sin\theta$, so it is the net force. The speed v of the electron is related to its kinetic energy KE by the relation $\text{KE} = \frac{1}{2}mv^2$. Thus, we have enough information to find the acceleration.

SOLUTION

a. According to RHR-1, if you extend your right hand so that your fingers point along the direction of the magnetic field **B** and your thumb points in the direction of the velocity **v** of a *positive* charge, your palm will face in the direction of the force **F** on the positive charge.

For the electron in question, the fingers of the right hand should be oriented downward (direction of **B**) with the thumb pointing to the east (direction of **v**). The palm of the right hand points due north (the direction of **F** on a positive charge). Since the electron is negatively charged, it will be deflected $\boxed{\text{due south}}$.

b. The acceleration of an electron is given by Newton's second law, where the net force is the magnetic force. Thus,

$$a = \frac{F}{m} = \frac{q_0 vB \sin\theta}{m}$$

Since the kinetic energy is $\text{KE} = \frac{1}{2}mv^2$, the speed of the electron is $v = \sqrt{2(\text{KE})/m}$. Thus, the acceleration of the electron is

$$a = \frac{q_0 vB \sin\theta}{m} = \frac{q_0 \sqrt{\dfrac{2(\text{KE})}{m}}\, B \sin\theta}{m}$$

$$= \frac{(1.60\times10^{-19}\ \text{C})\sqrt{\dfrac{2(2.40\times10^{-15}\ \text{J})}{9.11\times10^{-31}\ \text{kg}}}\,(2.00\times10^{-5}\ \text{T})\sin 90.0^\circ}{9.11\times10^{-31}\ \text{kg}} = \boxed{2.55\times10^{14}\ \text{m/s}^2}$$

10. ***REASONING AND SOLUTION***

b. The magnitude of the magnetic field is $B = mv/qr$ (see Equation 21.2). Therefore,

$$B = \frac{(9.11 \times 10^{-31}\ \text{kg})(6.0 \times 10^{6}\ \text{m/s})}{(1.60 \times 10^{-19}\ \text{C})(1.3 \times 10^{-3}\ \text{m})} = \boxed{2.6 \times 10^{-2}\ \text{T}}$$

c. From Newton's second law, the electron's acceleration is $a = F/m$, where the force can be obtained using Equation 21.1:

$$F = qvB \sin\theta$$

$$= (1.60 \times 10^{-19}\ \text{C})(6.0 \times 10^{6}\ \text{m/s})(2.6 \times 10^{-2}\ \text{T}) \sin 90.0^\circ = 2.5 \times 10^{-14}\ \text{N}$$

The acceleration is, therefore,

$$a = \frac{F}{m} = \frac{2.5 \times 10^{-14}\ \text{N}}{9.11 \times 10^{-31}\ \text{kg}} = \boxed{2.7 \times 10^{16}\ \text{m/s}^2}$$

11. ***REASONING AND SOLUTION*** The charge can be found from Equation 21.2 as

$$q = \frac{mv}{Br} = \frac{\left(6.6 \times 10^{-27}\ \text{kg}\right)\left(4.4 \times 10^{5}\ \text{m/s}\right)}{\left(0.75\ \text{T}\right)\left(0.012\ \text{m}\right)} = 3.2 \times 10^{-19}\ \text{C}$$

Since $e = 1.6 \times 10^{-19}$ C, we see that the charge of the ionized helium is $\boxed{+2e}$.

12. ***REASONING*** The magnetic field applies the maximum magnetic force to the moving charge, because the motion is perpendicular to the field. This force is perpendicular to both the field and the velocity. The electric field applies an electric force to the charge that is in the same direction as the field, since the charge is positive. These two forces are shown in the drawing, and they are perpendicular to one another. Therefore, the magnitude of the net field can be obtained using the Pythagorean theorem.

SOLUTION According to Equation 21.1, the magnetic force has a magnitude of $F_{\text{magnetic}} = qvB \sin\theta$, where q is the magnitude of the charge, B is the magnitude of the magnetic field, v is the speed, and $\theta = 90°$ is the angle of the velocity with respect to the field. Thus, $F_{\text{magnetic}} = qvB$. According to Equation 18.2, the electric force has a magnitude of $F_{\text{electric}} = qE$. Using the Pythagorean theorem, we find the magnitude of the net force to be

$$F = \sqrt{F_{\text{magnetic}}^2 + F_{\text{electric}}^2} = \sqrt{(qvB)^2 + (qE)^2} = q\sqrt{(vB)^2 + E^2}$$

$$= (1.8 \times 10^{-6}\ \text{C})\sqrt{\left[(3.1 \times 10^{6}\ \text{m/s})(1.2 \times 10^{-3}\ \text{T})\right]^2 + (4.6 \times 10^{3}\ \text{N/C})^2} = \boxed{1.1 \times 10^{-2}\ \text{N}}$$

13. SSM ***REASONING AND SOLUTION***

a. The speed of a proton can be found from Equation 21.2 ($r = mv/qB$),

$$v = \frac{qBr}{m} = \frac{(1.6\times10^{-19}\ \text{C})(0.30\ \text{T})(0.25\ \text{m})}{1.67\times10^{-27}\ \text{kg}} = \boxed{7.2\times10^{6}\ \text{m/s}}$$

b. The magnitude F_c of the centripetal force is given by Equation 5.3,

$$F_c = \frac{mv^2}{r} = \frac{(1.67\times10^{-27}\ \text{kg})(7.2\times10^{6}\ \text{m/s})^2}{0.25\ \text{m}} = \boxed{3.5\times10^{-13}\ \text{N}}$$

14. ***REASONING AND SOLUTION*** The radius of the circular path of a charged particle in a magnetic field is given by $r = mv/qB$.

a. For an electron

$$r = \frac{mv}{qB} = \frac{(9.11 \times 10^{-31}\ \text{kg})(9.0 \times 10^{6}\ \text{m/s})}{(1.6 \times 10^{-19}\ \text{C})(1.2 \times 10^{-7}\ \text{T})} = \boxed{4.3 \times 10^{2}\ \text{m}}$$

b. For a proton, only the mass changes in the calculation above. Using $m = 1.67 \times 10^{-27}$ kg, we obtain $r = \boxed{7.8 \times 10^{5}\ \text{m}}$.

15. SSM WWW ***REASONING AND SOLUTION*** In one revolution, the particle moves once around the circumference of the circle. Therefore, it travels a distance of $d = 2\pi r$, where r is the radius of the circle. Since the particle moves at constant speed, $v = d/t$, and the time required for one revolution is $t = d/v$. According to Equation 21.2, $r = mv/qB$, so $v = qBr/m$. Thus, the time required for the particle to complete one revolution is

$$t = \frac{d}{v} = \frac{2\pi r}{qBr/m} = \frac{2\pi}{B(q/m)} = \frac{2\pi}{(0.72\ \text{T})(5.7\times10^{8}\ \text{C/kg})} = \boxed{1.5\times10^{-8}\ \text{s}}$$

16. ***REASONING AND SOLUTION*** The magnitudes of the magnetic and electric forces are equal. Therefore, $F_B = F_E$, or $qvB = qE$. Solving for v yields,

$$v = \boxed{E/B}$$

17. ***REASONING AND SOLUTION*** The radius of curvature for a particle in a mass spectrometer is given by (see Section 21.4)

$$r = \sqrt{\frac{2mV}{qB^2}} = \sqrt{\frac{2\left(3.27 \times 10^{-25}\ \text{kg}\right)\left(1.00 \times 10^{3}\ \text{V}\right)}{\left(3.2 \times 10^{-19}\ \text{C}\right)\left(0.500\ \text{T}\right)^2}} = \boxed{0.0904\ \text{m}}$$

18. ***REASONING*** The radius of the circular path is given by Equation 21.2 as $r = mv/(qB)$, where m is the mass of the species, v is the speed, q is the magnitude of the charge, and B is the magnitude of the magnetic field. To use this expression, we must know something about the speed. Information about the speed can be obtained by applying the conservation of energy principle. The electric potential energy lost as a charged particle "falls" from a higher to a lower electric potential is gained by the particle as kinetic energy.

SOLUTION For an electric potential difference V and a charge q, the electric potential energy lost is qV, according to Equation 19.4. The kinetic energy gained is $\frac{1}{2}mv^2$. Thus, energy conservation dictates that

$$qV = \tfrac{1}{2}mv^2 \quad \text{or} \quad v = \sqrt{\frac{2qV}{m}}$$

Substituting this result into Equation 21.2 for the radius gives

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2qV}{m}} = \frac{1}{B}\sqrt{\frac{2mV}{q}}$$

Using e to denote the magnitude of the charge on an electron, we note that the charge for species X^+ is $+e$, while the charge for species X^{2+} is $+2e$. With this in mind, we find for the ratio of the radii that

$$\frac{r_1}{r_2} = \frac{\frac{1}{B}\sqrt{\frac{2mV}{e}}}{\frac{1}{B}\sqrt{\frac{2mV}{2e}}} = \sqrt{2} = \boxed{1.41}$$

19. SSM ***REASONING*** From the discussion in Section 21.3, we know that when a charged particle moves perpendicular to a magnetic field, the trajectory of the particle is a circle. The drawing shows a particle moving in the plane of the paper (the magnetic field is perpendicular to the paper). If the particle is moving initially through the coordinate origin and to the right (along the $+x$ axis), the subsequent circular path of the particle will intersect the y axis at the greatest possible value, which is equal to twice the radius r of the circle.

SOLUTION

a. From the drawing above, it can be seen that the largest value of y is equal to the diameter ($2r$) of the circle. When the particle passes through the coordinate origin its velocity must be parallel to the $+x$ axis. Thus, the angle is $\boxed{\theta = 0°}$.

b. The maximum value of y is twice the radius r of the circle. According to Equation 21.2, the radius of the circular path is $r = mv/qB$. The maximum value y_{max} is, therefore,

$$y_{max} = 2r = 2\left(\frac{mv}{qB}\right) = 2\left[\frac{(3.8\times10^{-8}\ \text{kg})(44\ \text{m/s})}{(7.3\times10^{-6}\ \text{C})(1.6\ \text{T})}\right] = \boxed{0.29\ \text{m}}$$

20. ***REASONING AND SOLUTION*** The magnitudes of the magnetic and electric forces must be equal. Therefore,

$$F_B = F_E \quad \text{or} \quad qvB = qE$$

This relation can be solved to give the speed of the particle, $v = E/B$. We also know that when the electric field is turned off, the particle travels in a circular path of radius $r = mv/qB$. Substituting $v = E/B$ into this equation and solving for q/m gives

$$\frac{q}{m} = \frac{E}{rB^2} = \frac{3.80 \times 10^3 \text{ N/C}}{(4.30 \times 10^{-2} \text{ m})(0.360 \text{ T})^2} = \boxed{6.8 \times 10^5 \text{ C/kg}}$$

21. ***REASONING AND SOLUTION*** The proton will miss the opposite plate if the distance between the plates is equal to the radius of the circular orbit of the proton. Therefore,

$$B = \frac{mv}{qr} = \frac{\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(2.2 \times 10^{6}\ \text{m/s}\right)}{\left(1.6 \times 10^{-19}\ \text{C}\right)\left(0.18\ \text{m}\right)} = \boxed{0.13\ \text{T}}$$

22. ***REASONING AND SOLUTION*** Equation 21.2 gives the speed as $v = rqB/m$, or $mv^2 = r^2q^2B^2/m$. Since the kinetic energy is $\text{KE} = \frac{1}{2}mv^2$, it follows that $\text{KE} = \dfrac{r^2q^2B^2}{2m}$. Recognizing that q, B, and m are the same for both particles, we find that

$$\frac{\text{KE}_3}{\text{KE}_1} = \frac{\dfrac{r_3^2q^2B^2}{2m}}{\dfrac{r_1^2q^2B^2}{2m}} = \left(\frac{r_3}{r_1}\right)^2 = (22)^2 = \boxed{484}$$

23. **SSM** ***REASONING AND SOLUTION*** According to Right-Hand Rule No. 1, the magnetic force on the positively charged particle is toward the bottom of the page in the drawing in the text. If the presence of the electric field is to double the magnitude of the net force on the charge, the electric field must also be $\boxed{\text{directed toward the bottom of the page}}$.

Note that this results in the electric field being perpendicular to the magnetic field, even though the electric force and the magnetic force are in the same direction.

Furthermore, if the magnitude of the net force on the particle is twice the magnetic force, the electric force must be equal in magnitude to the magnetic force. In other words, combining Equations 18.2 and 21.1, we find $qE = qvB\sin\theta$, with $\sin\theta = \sin\ 90.0° = 1.0$. Then, solving for E

$$E = vB\sin\theta = (270\ \text{m/s})(0.52\ \text{T})(1.0) = \boxed{140\ \text{V/m}}$$

24. ***REASONING AND SOLUTION*** The angular speed is $\omega = v/r = qB/m$. Therefore,

$$\omega = \frac{qB}{m} = \frac{\left(3.2 \times 10^{-19}\ \text{C}\right)\left(0.0210\ \text{T}\right)}{6.64 \times 10^{-27}\ \text{kg}} = \boxed{1.01 \times 10^{6}\ \text{rad/s}}$$

25. ***REASONING AND SOLUTION*** The drawings show the two circular paths leading to the target T when the proton is projected from the origin O. In each case, the center of the circle is at C. Since the target is located at $x = -0.10$ m and $y = -0.10$ m, the radius of each circle is $r = 0.10$ m. The speed with which the proton is projected can be obtained from Equation 21.2, if we remember that the charge and mass of a proton are $q = 1.6 \times 10^{-19}$ C and $m = 1.67 \times 10^{-27}$ kg, respectively:

$$v = \frac{rqB}{m} = \frac{(0.10\text{ m})(1.6 \times 10^{-19}\text{ C})(0.010\text{ T})}{1.67 \times 10^{-27}\text{ kg}} = \boxed{9.6 \times 10^{4}\text{ m/s}}$$

26. ***REASONING*** According to Equation 21.3, the magnetic force has a magnitude of $F = ILB \sin \theta$, where I is the current, B is the magnitude of the magnetic field, L is the length of the wire, and $\theta = 90°$ is the angle of the wire with respect to the field.

SOLUTION Using Equation 21.3, we find that

$$L = \frac{F}{IB \sin\theta} = \frac{7.1 \times 10^{-5}\ \text{N}}{(0.66\ \text{A})(4.7 \times 10^{-5}\ \text{T})\sin 58°} = \boxed{2.7\ \text{m}}$$

27. **SSM** ***REASONING AND SOLUTION*** The magnetic force exerted on the power line is given by Equation 21.3,

$$F = ILB\sin\theta = (1400\ \text{A})(120\ \text{m})(5.0\times10^{-5}\ \text{T})(\sin 75°) = \boxed{8.1\ \text{N}}$$

28. ***REASONING AND SOLUTION*** The magnitude F of the force on a current I is given by Equation 21.3 as $F = ILB \sin\theta$, where L is the length of the wire and θ is the angle between the wire and a magnetic field that has a magnitude B. Applying this equation in the two situations described in this problem and recognizing that L, B, and θ are the same in each, we find that

$$\frac{F_1}{F_2} = \frac{I_1 LB \sin\theta}{I_2 LB \sin\theta} = \frac{I_1}{I_2} \quad \text{so} \quad \frac{0.030\text{ N}}{0.047\text{ N}} = \frac{2.7\text{ A}}{I_2} \quad \text{or} \quad I_2 = (2.7\text{ A})\left(\frac{0.047\text{ N}}{0.030\text{ N}}\right) = \boxed{4.2\text{ A}}$$

29. ***REASONING AND SOLUTION*** The force on each side can be found from $F = ILB \sin \theta$. For the top side, $\theta = 90.0°$, so

$$F = (12\text{ A})(0.32\text{ m})(0.25\text{ T}) \sin 90.0° = \boxed{0.96\text{ N}}$$

The force on the bottom side ($\theta = 90.0°$) is the same as that on the top side, $F = \boxed{0.96\text{ N}}$.

For each of the other two sides $\theta = 0°$, so that the force is $F = \boxed{0\text{ N}}$.

30. ***REASONING*** The magnitude of the magnetic force exerted on a long straight wire is given by Equation 21. 3 as $F = ILB \sin \theta$. The direction of the magnetic force is predicted by Right-Hand Rule No. 1. The net force on the triangular loop is the vector sum of the forces on the three sides.

SOLUTION

a. The direction of the current in side AB is opposite to the direction of the magnetic field, so the angle θ between them is $\theta = 180°$. The magnitude of the magnetic force is

$$F_{AB} = ILB \sin \theta = ILB \sin 180° = \boxed{0\text{ N}}$$

For the side BC, the angle is $\theta = 55.0°$, and the length of the side is $L = \dfrac{2.00\text{ m}}{\cos 55.0°} = 3.49\text{ m}$. The magnetic force is

$$F_{BC} = ILB \sin \theta = (4.70\text{ A})(3.49\text{ m})(1.80\text{ T}) \sin 55.0° = \boxed{24.2\text{ N}}$$

An application of Right-Hand No. 1 shows that the magnetic force on side BC is directed $\boxed{\text{perpendicularly out of the paper}}$, toward the reader.

For the side AC, the angle is $\theta = 90.0°$, and the length of the side is $L = (2.00\text{ m}) \tan 55.0° = 2.86\text{ m}$. The magnetic force is

$$F_{AC} = ILB \sin \theta = (4.70\text{ A})(2.86\text{ m})(1.80\text{ T}) \sin 90.0° = \boxed{24.2\text{ N}}$$

An application of Right-Hand No. 1 shows that the magnetic force on side AC is directed $\boxed{\text{perpendicularly into the paper}}$, away from the reader.

b. The net force is the vector sum of the forces on the three sides. Taking the positive direction as being out of the paper, the net force is

$$\sum F = 0\text{ N} + 24.2\text{ N} + (-24.2\text{ N}) = \boxed{0\text{ N}}$$

31. **SSM** ***REASONING AND SOLUTION*** The force on a current-carrying wire is given by Equation 21.3: $F = ILB\sin\theta$. Solving for the angle θ, we find that the angle between the wire and the magnetic field is

$$\theta = \sin^{-1}\left(\frac{F}{ILB}\right) = \sin^{-1}\left[\frac{5.46\text{ N}}{(21.0\text{ A})(0.655\text{ m})(0.470\text{ T})}\right] = \boxed{57.6°}$$

32. ***REASONING*** A maximum magnetic force is exerted on the wire by the field components that are perpendicular to the wire, and no magnetic force is exerted by field components that are parallel to the wire. Thus, the wire experiences a force only from the x- and y-components of the field. The z-component of the field may be ignored, since it is parallel to the wire. We can use the Pythagorean theorem to find the net field in the x, y plane. This net field, then, is perpendicular to the wire and makes an angle of $\theta = 90°$ with respect to the wire. Equation 21.3 can be used to calculate the magnitude of the magnetic force that this net field applies to the wire.

SOLUTION According to Equation 21.3, the magnetic force has a magnitude of $F = ILB \sin\theta$, where I is the current, B is the magnitude of the magnetic field, L is the length of the wire, and θ is the angle of the wire with respect to the field. Using the Pythagorean theorem, we find that the net field in the x, y plane is

$$B = \sqrt{B_x^2 + B_y^2}$$

Using this field in Equation 21.3, we calculate the magnitude of the magnetic force to be

$$F = ILB \sin\theta = IL\sqrt{B_x^2 + B_y^2}\ \sin\theta$$

$$= (4.3\ \text{A})(0.25\ \text{m})\sqrt{(0.10\ \text{T})^2 + (0.15\ \text{T})^2}\ \sin 90° = \boxed{0.19\ \text{N}}$$

33. SSM *REASONING AND SOLUTION*

a. From Right-Hand Rule No. 1, if we extend the right hand so that the fingers point in the direction of the magnetic field, and the thumb points in the direction of the current, the palm of the hand faces the direction of the magnetic force on the current.

The springs will stretch when the magnetic force exerted on the copper rod is downward, toward the bottom of the page. Therefore, if you extend your right hand with your fingers pointing out of the page and the palm of your hand facing the bottom of the page, your thumb points left-to-right along the copper rod. Thus, the current flows $\boxed{\text{left-to-right}}$ in the copper rod.

b. The downward magnetic force exerted on the copper rod is, according to Equation 21.3

$$F = ILB\sin\theta = (12\text{ A})(0.85\text{ m})(0.16\text{ T})\sin 90.0° = 1.6\text{ N}$$

From Hooke's law, we know that the net force exerted on the rod by the two springs is $F = kx + kx = 2kx$, where k is the spring constant. Solving for x, we find that

$$x = \frac{F}{2k} = \frac{1.6\text{ N}}{2(75\text{ N/m})} = \boxed{1.1\times 10^{-2}\text{ m}}$$

34. ***REASONING AND SOLUTION*** Since the rod rotates neither clockwise nor counterclockwise about the axis at P, the torques relative to that axis must balance to zero. There are two torques that must be considered, that due to the magnetic force and that due to the weight of the rod. We consider both of these to act at the rod's center of gravity, which is at the center of the rod (length = L), because the rod is uniform. According to RHR-1, the magnetic force acts perpendicular to the rod and is directed up and to the left in the drawing. Equation 21.3 gives the magnitude F of the magnetic force as $F = ILB \sin 90.0°$, since the current is perpendicular to the field. Therefore, the magnetic torque is counterclockwise. The weight is mg and acts downward, producing a clockwise torque. Thus, we have for the torques:

$$\tau_{\text{magnetic}} = \underbrace{(ILB)}_{\text{force}}\underbrace{(L/2)}_{\text{lever arm}} \qquad \text{and} \qquad \tau_{\text{weight}} = \underbrace{(mg)}_{\text{force}}\underbrace{\left[(L/2)\cos\theta\right]}_{\text{lever arm}}$$

Equating these torques, we find that

$$\underbrace{(ILB)(L/2)}_{\text{magnetic torque}} = \underbrace{(mg)\left[(L/2)\cos\theta\right]}_{\text{weight torque}} \qquad \text{or} \qquad \cos\theta = \frac{ILB}{mg}$$

$$\theta = \cos^{-1}\left[\frac{(3.8\text{ A})(0.40\text{ m})(0.31\text{ T})}{(0.080\text{ kg})(9.80\text{ m/s}^2)}\right] = \boxed{53°}$$

35. ***REASONING*** The drawing below shows a side view of the conducting rails and the aluminum rod. Three forces act on the rod: (1) its weight mg, (2) the magnetic force F, and the normal force F_N. An application of Right-Hand Rule No. 1 shows that the magnetic force is directed to the left, as shown in the drawing. Since the rod slides down the rails at a constant velocity, its acceleration is zero. If we choose the x-axis to be along the rails, Newton's second law states that the net force along the x-direction is zero: $\Sigma F_x = ma_x = 0$. Using the components of F and mg that are along the x-axis, Newton's second law becomes

$$\underbrace{-F\cos 30.0° + mg\sin 30.0°}_{\Sigma F_x} = 0$$

The magnetic force is given by Equation 21.3 as $F = ILB \sin\theta$, where $\theta = 90.0°$ is the angle between the magnetic field and the current. We can use these two relations to find the current in the rod.

SOLUTION Substituting the expression $F = ILB \sin 90.0°$ into Newton's second law and solving for the current I, we obtain

$$I = \frac{mg\sin 30.0°}{(LB\sin 90.0°)\cos 30.0°} = \frac{(0.20\text{ kg})(9.80\text{ m/s}^2)\sin 30.0°}{[(1.6\text{ m})(0.050\text{ T})\sin 90.0°]\cos 30.0°} = \boxed{14\text{ A}}$$

36. ***REASONING AND SOLUTION*** The maximum torque occurs when $\phi = 90.0°$ so that $\tau = NIAB$. For a square loop, $A = (L/4)^2 = (0.50\text{ m}/4)^2 = 1.6 \times 10^{-2}\text{ m}^2$. So,

$$\tau = NIAB = (1)(12\,A)(1.6 \times 10^{-2}\text{ m}^2)(0.12\text{ T}) = \boxed{0.023\text{ N}\cdot\text{m}}$$

37. SSM ***REASONING AND SOLUTION***

a. The magnetic moment of the coil is

$$\textit{Magnetic moment} = NIA = (50)(15\text{ A})\left[\pi(0.10\text{ m})^2\right] = \boxed{24\text{ A}\cdot\text{m}^2}$$

where we have used the fact that the area of the circular loop is $A = \pi r^2 = \pi(0.10\text{ m})^2$.

b. According to Equation 21.4, the torque is the product of the magnetic moment NIA and $B\sin\phi$. However, the maximum torque occurs when $\phi = 90.0°$, so we have

$$\tau = (\textit{Magnetic moment})(B\sin 90.0°) = (24\text{ A}\cdot\text{m}^2)(0.20\text{ T}) = \boxed{4.8\text{ N}\cdot\text{m}}$$

38. ***REASONING*** According to Equation 21.4, the maximum torque is $\tau_{max} = NIAB$, where N is the number of turns in the coil, I is the current, $A = \pi r^2$ is the area of the circular coil, and B is the magnitude of the magnetic field. We can apply the maximum-torque expression to each coil, noting that τ_{max}, N, and I are the same for each.

SOLUTION Applying Equation 21.4 to each coil, we have

$$\underbrace{\tau_{max} = NI\pi r_1^2 B_1}_{\text{Coil 1}} \quad \text{and} \quad \underbrace{\tau_{max} = NI\pi r_2^2 B_2}_{\text{Coil 2}}$$

Dividing the expression for coil 2 by the expression for coil 1 gives

$$\frac{\tau_{max}}{\tau_{max}} = \frac{NI\pi r_2^2 B_2}{NI\pi r_1^2 B_1} \quad \text{or} \quad 1 = \frac{r_2^2 B_2}{r_1^2 B_1}$$

Solving for r_2, we obtain

$$r_2 = r_1\sqrt{\frac{B_1}{B_2}} = (5.0\text{ cm})\sqrt{\frac{0.18\text{ T}}{0.42\text{ T}}} = \boxed{3.3\text{ cm}}$$

39. ***REASONING*** The magnetic moment of the current-carrying triangle is the product of the current I and the area A of the triangle. The magnitude τ of the torque exerted on the triangle by the magnetic field is equal to the product of the magnetic moment, the magnitude B of the magnetic field, and the sine of the angle ϕ between the magnetic field and the normal to the plane of the triangle, $\tau = (\textit{Magnetic moment})\, B \sin\phi$.

SOLUTION

a. The area of a triangle is equal to one-half the base times the height,

$$A = \tfrac{1}{2}(2.00\text{ m})\left[(2.00\text{ m})\tan 55.0°\right] = 2.86\text{ m}^2$$

The magnetic moment is

$$\textit{Magnetic moment} = IA = (4.70\text{ A})(2.86\text{ m}^2) = \boxed{13.4\text{ A}\cdot\text{m}^2}$$

b. The magnitude of the net torque exerted on the triangle is

$$\tau = (\textit{Magnetic moment})\, B \sin\phi \qquad (21.4)$$

$$= (13.4\text{ A}\cdot\text{m}^2)(1.80\text{ T})\sin 90.0° = \boxed{24.1\text{ N}\cdot\text{m}}$$

40. ***REASONING AND SOLUTION*** The torque is given by $\tau = NIAB \sin\phi$.

a. The maximum torque occurs when $\phi = 90.0°$ ($\sin\phi = 1$). In this case we want the torque to be 80.0% of the maximum value, so

$$NIAB \sin\phi = 0.800\left(NIAB \sin 90.0°\right) \qquad \text{so that} \qquad \phi = \sin^{-1}(0.800) = \boxed{53.1°}$$

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41. SSM WWW ***REASONING*** The torque on the loop is given by Equation 21.4, $\tau = NIAB\sin\phi$. From the drawing in the text, we see that the angle ϕ between the normal to the plane of the loop and the magnetic field is $90° - 35° = 55°$. The area of the loop is $0.70\ \text{m} \times 0.50\ \text{m} = 0.35\ \text{m}^2$.

SOLUTION

a. The magnitude of the net torque exerted on the loop is

$$\tau = NIAB\sin\phi = (75)(4.4\ \text{A})(0.35\ \text{m}^2)(1.8\ \text{T})\ \sin 55° = \boxed{170\ \text{N}\cdot\text{m}}$$

b. As discussed in the text, when a current-carrying loop is placed in a magnetic field, the loop tends to rotate such that its normal becomes aligned with the magnetic field. The normal to the loop makes an angle of 55° with respect to the magnetic field. Since this angle decreases as the loop rotates, the $\boxed{35°\ \text{angle increases}}$.

42. ***REASONING*** According to Equation 21.4, the maximum torque is $\tau_{max} = NIAB$, where N is the number of turns in the coil, I is the current, $A = \pi r^2$ is the area of the circular coil, and B is the magnitude of the magnetic field. Since the coil contains only one turn, the length L of the wire is the circumference of the circle, so that $L = 2\pi r$ or $r = L/(2\pi)$. Since N, I, and B are known we can solve for L.

SOLUTION According to Equation 21.4 and the fact that $r = L/(2\pi)$, we have

$$\tau_{max} = NI\pi r^2 B = NI\pi\left(\frac{L}{2\pi}\right)^2 B$$

Solving this result for L gives

$$L = \sqrt{\frac{4\pi\tau_{max}}{NIB}} = \sqrt{\frac{4\pi\left(8.4\times10^{-4}\ \text{N}\cdot\text{m}\right)}{(1)(3.7\ \text{A})(0.75\ \text{T})}} = \boxed{0.062\ \text{m}}$$

43. ***REASONING AND SOLUTION*** In Figure 21.22a the magnetic torque is a maximum. Equation 21.4 gives this maximum torque as $\tau_{max} = NIAB$. The torque from the brake balances this magnetic torque. The brake torque is $\tau_{brake} = F_{brake}r$, where F_{brake} is the brake force, and r is the radius of the shaft and also the lever arm. The maximum value for the brake force available from static friction is $F_{brake} = \mu_s F_N$, where F_N is the normal force pressing the brake shoe against the shaft. The brake torque, then, is $\tau_{brake} = \mu_s F_N r$. Therefore, we can obtain the magnitude of the minimum normal force as follows:

$$\tau_{brake} = \tau_{max} \qquad \text{or} \qquad \mu_s F_N r = NIAB$$

$$F_N = \frac{NIAB}{\mu_s r} = \frac{(380)(0.16\ A)(2.5 \times 10^{-3}\ \text{m}^2)(0.12\ \text{T})}{(0.70)(0.010\ \text{m})} = \boxed{2.6\ \text{N}}$$

44. ***REASONING AND SOLUTION*** According to Equation 21.4, the maximum torque for a single turn is $\tau_{max} = IAB$. When the length L of the wire is used to make the square, each side of the square has a length $L/4$. The area of the square is $A_{square} = (L/4)^2$. For the rectangle, since two sides have a length d, while the other two sides have a length $2d$, it follows that $L = 6d$, or $d = L/6$. The area is $A_{rectangle} = 2d^2 = 2(L/6)^2$. Using Equation 21.4 for the square and the rectangle, we obtain

$$\frac{\tau_{square}}{\tau_{rectangle}} = \frac{IA_{square}B}{IA_{rectangle}B} = \frac{A_{square}}{A_{rectangle}} = \frac{(L/4)^2}{2(L/6)^2} = \boxed{1.13}$$

45. SSM ***REASONING*** The magnetic moment of the orbiting electron can be found from the expression $Magnetic\ moment = NIA$. For this situation, $N = 1$. Thus, we need to find the current and the area for the orbiting charge.

SOLUTION The current for the orbiting charge is, by definition (see Equation 20.1), $I = \Delta q / \Delta t$, where Δq is the amount of charge that passes a given point during a time interval Δt. Since the charge ($\Delta q = e$) passes by a given point once per revolution, we can find the current by dividing the total orbiting charge by the period T of revolution.

$$I = \frac{\Delta q}{T} = \frac{\Delta q}{2\pi r / v} = \frac{(1.6 \times 10^{-19}\ \text{C})(2.2 \times 10^{6}\ \text{m/s})}{2\pi(5.3 \times 10^{-11}\ \text{m})} = 1.06 \times 10^{-3}\ \text{A}$$

The area of the orbiting charge is

$$A = \pi r^2 = \pi(5.3 \times 10^{-11}\ \text{m})^2 = 8.82 \times 10^{-21}\ \text{m}^2$$

Therefore, the magnetic moment is

$$Magnetic\ moment = NIA = (1)(1.06 \times 10^{-3}\ \text{A})(8.82 \times 10^{-21}\ \text{m}^2) = \boxed{9.3 \times 10^{-24}\ \text{A} \cdot \text{m}^2}$$

46. ***REASONING AND SOLUTION*** The magnitude B of the magnetic field at a distance r from a long straight wire carrying a current I is $B = \mu_0 I/(2\pi r)$. Thus, the distance is

$$r = \frac{\mu_0 I}{2\pi B} = \frac{\left(4\pi \times 10^{-7}\ \mathrm{T\cdot m/A}\right)\left(48\ \mathrm{A}\right)}{2\pi\left(8.0 \times 10^{-5}\ \mathrm{T}\right)} = \boxed{0.12\ \mathrm{m}} \tag{21.5}$$

47. ***REASONING AND SOLUTION*** The current associated with the lightning bolt is

$$I = \frac{\Delta q}{\Delta t} = \frac{15\text{ C}}{1.5 \times 10^{-3}\text{ s}} = 1.0 \times 10^{4}\text{ A}$$

The magnetic field near this current is given by

$$B = \frac{\mu_o I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\text{ T}\cdot\text{m/A}\right)\left(1.0 \times 10^{4}\text{ A}\right)}{2\pi\left(25\text{ m}\right)} = \boxed{8.0 \times 10^{-5}\text{ T}}$$

48. ***REASONING AND SOLUTION*** The magnetic field at the center of a current loop of radius R is given by $B = \mu_0 I/(2R)$, so that

$$R = \frac{\mu_0 I}{2B} = \frac{\left(4\pi \times 10^{-7}\ \mathrm{T \cdot m/A}\right)\left(12\ \mathrm{A}\right)}{2\left(1.8 \times 10^{-4}\ \mathrm{T}\right)} = \boxed{4.2 \times 10^{-2}\ \mathrm{m}}$$

49. **SSM** ***REASONING AND SOLUTION*** The magnetic field inside a long solenoid is given by Equation 21.7:

$$B = \mu_0 nI = (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})\left(\frac{1400\ \text{turns}}{0.65\ \text{m}}\right)(4.7\ \text{A}) = \boxed{1.3 \times 10^{-2}\ \text{T}}$$

50. ***REASONING AND SOLUTION***

a. In Figure 21.28a the magnetic field that exists at the location of each wire points upward. Since the current in each wire is the same, the fields at the locations of the wires also have the same magnitudes. Therefore, a single external field that points $\boxed{\text{downward}}$ will cancel the mutual repulsion of the wires, if this external field has a magnitude that equals that of the field produced by either wire.

b. Equation 21.5 gives the magnitude of the field produced by a long straight wire. The external field must have this magnitude:

$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(25\ \text{A}\right)}{2\pi\left(0.016\ \text{m}\right)} = \boxed{3.1 \times 10^{-4}\ \text{T}}$$

51. SSM ***REASONING*** The two rods attract each other because they each carry a current I in the same direction. The bottom rod floats because it is in equilibrium. The two forces that act on the bottom rod are the downward force of gravity $m\mathbf{g}$ and the upward magnetic force of attraction to the upper rod. If the two rods are a distance s apart, the magnetic field generated by the top rod at the location of the bottom rod is (see Equation 21.5) $B = \mu_0 I/(2\pi s)$. According to Equation 21.3, the magnetic force exerted on the bottom rod is $F = ILB\sin\theta = \mu_0 I^2 L\sin\theta/(2\pi s)$, where θ is the angle between the magnetic field at the location of the bottom rod and the direction of the current in the bottom rod. Since the rods are parallel, the magnetic field is perpendicular to the direction of the current (RHR-2), and $\theta = 90.0°$, so that $\sin\theta = 1.0$.

SOLUTION Taking upward as the positive direction, the net force on the bottom rod is

$$\frac{\mu_0 I^2 L\sin\theta}{2\pi s} - mg = 0$$

Solving for I, we find

$$I = \sqrt{\frac{2\pi mgs}{\mu_0 L}} = \sqrt{\frac{2\pi(0.073\text{ kg})(9.80\text{ m/s}^2)(8.2\times10^{-3}\text{ m})}{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(0.85\text{ m})}} = \boxed{190\text{ A}}$$

52. ***REASONING*** The magnitude of the magnetic field at the center of a circular loop of current is given by Equation 21.6 as $B = N\mu_0 I/(2R)$, where N is the number of turns, μ_0 is the permeability of free space, I is the current, and R is the radius of the loop. The field is perpendicular to the plane of the loop. Magnetic fields are vectors, and here we have two fields, each perpendicular to the plane of the loop producing it. Therefore, the two field vectors are perpendicular, and we must add them as vectors to get the net field. Since they are perpendicular, we can use the Pythagorean theorem to calculate the magnitude of the net field.

SOLUTION Using Equation 21.6 and the Pythagorean theorem, we find that the magnitude of the net magnetic field at the common center of the two loops is

$$B_{\text{net}} = \sqrt{\left(\frac{N\mu_0 I}{2R}\right)^2 + \left(\frac{N\mu_0 I}{2R}\right)^2} = \sqrt{2}\left(\frac{N\mu_0 I}{2R}\right)$$

$$= \frac{\sqrt{2}(1)\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1.7\ \text{A})}{2(0.040\ \text{m})} = \boxed{3.8 \times 10^{-5}\ \text{T}}$$

53. ***REASONING AND SOLUTION*** Let the current in the left-hand wire be labeled I_1 and that in the right-hand wire I_2.

a. At point A: B_1 is *up* and B_2 is *down*, so we subtract them to get the net field. We have

$$B_1 = \mu_0 I_1/(2\pi d_1) = \mu_0(8.0\text{ A})/[2\pi(0.030\text{ m})]$$
$$B_2 = \mu_0 I_2/(2\pi d_2) = \mu_0(8.0\text{ A})/[2\pi(0.150\text{ m})]$$

So the net field at point A is

$$B_A = B_1 - B_2 = \boxed{4.3 \times 10^{-5}\text{ T}}$$

b. At point B: B_1 and B_2 are both *down* so we add the two. We have

$$B_1 = \mu_0(8.0\text{ A})/[2\pi(0.060\text{ m})]$$
$$B_2 = \mu_0(8.0\text{ A})/[2\pi(0.060\text{ m})]$$

So the net field at point B is

$$B_B = B_1 + B_2 = \boxed{5.3 \times 10^{-5}\text{ T}}$$

54. ***REASONING AND SOLUTION*** The net force on the wire loop is a sum of the forces on each segment of the loop. The forces on the two segments perpendicular to the long straight wire cancel each other out. The net force on the loop is therefore the sum of the forces on the parallel segments (near and far). These are

$$F_n = \mu_o I_1 I_2 L/(2\pi d_n) = \mu_o(12\text{ A})(25\text{ A})(0.50\text{ m})/[2\pi(0.11\text{ m})] = 2.7 \times 10^{-4}\text{ N}$$

$$F_f = \mu_o I_1 I_2 L/(2\pi d_f) = \mu_o(12\text{ A})(25\text{ A})(0.50\text{ m})/[2\pi(0.26\text{ m})] = 1.2 \times 10^{-4}\text{ N}$$

Note: F_n is a force of attraction, while F_f is a repulsive one. The magnitude of the net force is, therefore,

$$F = F_n - F_f = 2.7 \times 10^{-4}\text{ N} - 1.2 \times 10^{-4}\text{ N} = \boxed{1.5 \times 10^{-4}\text{ N}}$$

55. ***REASONING AND SOLUTION*** The magnitude B_i of the magnetic field due to the inner coil is

$$B_i = \mu_0 I_i N_i/(2r_i)$$

The magnetic field B_0 due to the outer coil must cancel that due to the inner coil. We have

$$B_0 = \mu_0 I_0 N_0/(2r_0)$$

and this must equal B_i. Therefore,

$$\mu_0 I_i N_i/(2r_i) = \mu_0 I_0 N_0/(2r_0)$$

so that

$$I_0 = I_i \left(\frac{N_i}{N_0}\right)\left(\frac{r_0}{r_i}\right) = (6.0\text{ A})\left(\frac{120\text{ turns}}{150\text{ turns}}\right)\left(\frac{0.017\text{ m}}{0.012\text{ m}}\right) = \boxed{6.8\text{ A}}$$

In order for the two fields to be equal in magnitude but opposite in direction, the current in the outer coil must flow in the $\boxed{\text{opposite direction}}$ as the inner coil current.

56. ***REASONING*** Two wires that are parallel and carry current in the same direction exert attractive magnetic forces on one another, as Section 21.7 discusses. This attraction between the wires causes the spring to compress. When compressed, the spring exerts an elastic restoring force on each wire, as Section 10.1 discusses. For each wire, this restoring force acts to push the wires apart and balances the magnetic force, thus keeping the separation between the wires from decreasing to zero. Equation 10.2 (without the minus sign) gives the magnitude of the restoring force as F = kx, where k is the spring constant and x is the magnitude of the displacement of the spring from its unstrained length. By setting the magnitude of the magnetic force equal to the magnitude of the restoring force, we will be able to find the separation between the rods when the current is present.

SOLUTION According to Equation 21.3, the magnetic force has a magnitude of $F = ILB \sin\theta$, where I is the current, B is the magnitude of the magnetic field, L is the length of the wire, and θ is the angle of the wire with respect to the field. Using RHR-2 reveals that the magnetic field produced by either wire is perpendicular to the other wire, so that $\theta = 90°$ in Equation 21.3, which becomes $F = ILB$. According to Equation 21.5 the magnitude of the magnetic field produced by a long straight wire is $B = \mu_0 I/(2\pi r)$. Substituting this expression into Equation 21.3 gives the magnitude of the magnetic force as

$$F = IL\left(\frac{\mu_0 I}{2\pi r}\right) = \frac{\mu_0 I^2 L}{2\pi r}$$

Equating this expression to the magnitude of the restoring force from the spring gives

$$\frac{\mu_0 I^2 L}{2\pi r} = kx$$

Solving for the separation r, we find

$$r = \frac{\mu_0 I^2 L}{2\pi kx} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(950\ \text{A}\right)^2\left(0.50\ \text{m}\right)}{2\pi\left(150\ \text{N/m}\right)\left(0.020\ \text{m}\right)} = \boxed{0.030\ \text{m}}$$

57. SSM ***REASONING*** According to Equation 21.6 the magnetic field at the center of a circular, current-carrying loop of N turns and radius r is $B = N\mu_0 I/(2r)$. The number of turns N in the coil can be found by dividing the total length L of the wire by the circumference after it has been wound into a circle. The current in the wire can be found by using Ohm's law, $I = V/R$.

SOLUTION The number of turns in the wire is

$$N = \frac{L}{2\pi r}$$

The current in the wire is

$$I = \frac{V}{R} = \frac{12.0\ \text{V}}{(5.90\times10^{-3}\ \Omega/\text{m})L} = \frac{2.03\times10^{3}}{L}\ \text{A}$$

Therefore, the magnetic field at the center of the coil is

$$B = N\left(\frac{\mu_0 I}{2r}\right) = \left(\frac{L}{2\pi r}\right)\left(\frac{\mu_0 I}{2r}\right) = \frac{\mu_0 L I}{4\pi r^2}$$

$$= \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)L\left(\dfrac{2.03\times10^{3}}{L}\ \text{A}\right)}{4\pi(0.140\ \text{m})^2} = \boxed{1.04\times10^{-2}\ \text{T}}$$

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58. ***REASONING AND SOLUTION*** The forces acting on each wire are the magnetic force F, the gravitational force mg, and the tension T in the strings. Each string makes an angle of 7.5° with respect to the vertical. From the drawing at the right we can relate the magnetic force to the gravitational force. Since the wire is in equilibrium, Newton's second law requires that $\Sigma F_x = 0$ and $\Sigma F_y = 0$. These equations become

$$\underbrace{-T\sin 7.5° + F}_{\Sigma F_x} = 0 \quad \text{and} \quad \underbrace{T\cos 7.5° - mg}_{\Sigma F_y} = 0$$

Solving the first equation for T, and then substituting the result into the second equation gives (after some simplification)

$$\tan 7.5° = \frac{F}{mg} \qquad (1)$$

The magnetic force F exerted on one wire by the other is $F = \dfrac{\mu_0 I^2 L}{2\pi d}$, where d is the distance between the wires [$d/2$ = (1.2 m) sin 7.5°, so that $d = 0.31$ m], I is the current (which is the same for each wire), and L is the length of each wire. Substituting this relation for F into Equation (1) and then solving for the current, gives

$$I = \sqrt{\left(\frac{m}{L}\right) g \tan 7.5° \left(\frac{2\pi d}{\mu_0}\right)}$$

$$= \sqrt{(0.050\ \text{kg/m})(9.80\ \text{m/s}^2)\tan 7.5° \left[\frac{2\pi(0.31\ \text{m})}{\mu_0}\right]} = \boxed{320\ \text{A}}$$

59. ***REASONING AND SOLUTION*** The currents in wires 1 and 2 produce the magnetic fields B_1 and B_2 at the empty corner, as shown in the drawing. The directions of these fields can be obtained using RHR-2. Since there are equal currents in wires 1 and 2 and since these wires are each the same distance r from the empty corner, B_1 and B_2 have equal magnitudes. Using Equation 21.5, we can write the field magnitude as $B_1 = B_2 = \mu_0 I/(2\pi r)$. Since B_1 and B_2 are perpendicular, it follows from the Pythagorean theorem that they combine to produce a net magnetic field that has the direction shown in the following drawing and has a magnitude B_{1+2} given by

$$B_{1+2} = \sqrt{B_1^2 + B_2^2} = \sqrt{\left(\frac{\mu_0 I}{2\pi r}\right)^2 + \left(\frac{\mu_0 I}{2\pi r}\right)^2} = \frac{\sqrt{2}\,\mu_0 I}{2\pi r}$$

The current in wire 3 produces a field B_3 at the empty corner. Since B_3 and B_{1+2} combine to give a zero net field, B_3 must have a direction opposite to that of B_{1+2}. Thus, B_3 must point upward and to the left, and RHR-2 indicates that

the current in wire 3 must be directed out of the plane of the paper

Moreover, the magnitudes of B_3 and B_{1+2} must be the same. Recognizing that wire 3 is a distance of $d = \sqrt{r^2 + r^2} = \sqrt{2}\,r$ from the empty corner, we have

$$B_3 = B_{1+2} \qquad \text{or} \qquad \frac{\mu_0 I_3}{2\pi\left(\sqrt{2}\,r\right)} = \frac{\sqrt{2}\,\mu_0 I}{2\pi r} \qquad \text{so that} \qquad \boxed{\frac{I_3}{I} = 2}$$

60. ***REASONING AND SOLUTION*** Ampere's law in the form of Equation 21.8 indicates that $\Sigma B_{\parallel}\Delta\ell = \mu_0 I$. Since the magnetic field is everywhere perpendicular to the plane of the paper, it is everywhere perpendicular to the circular path and has no component $B_{\parallel}$ that is parallel to the circular path. Therefore, Ampere's law reduces to $\Sigma B_{\parallel}\Delta\ell = 0 = \mu_0 I$, so that $\boxed{\text{the net current passing through the circular surface is zero}}$.

61. **SSM** ***REASONING*** Since the two wires are next to each other, the net magnetic field is everywhere parallel to $\Delta\ell$ in Figure 21.40. Moreover, the net magnetic field **B** has the same magnitude B at each point along the circular path, because each point is at the same distance from the wires. Thus, in Ampere's law (Equation 21.8), $B_{\parallel} = B$, $I = I_1 + I_2$, and we have

$$\Sigma B_{\parallel} \Delta\ell = B(\Sigma\Delta\ell) = \mu_0 (I_1 + I_2)$$

But $\Sigma\Delta\ell$ is just the circumference ($2\pi r$) of the circle, so Ampere's law becomes

$$B(2\pi r) = \mu_0 (I_1 + I_2)$$

This expression can be solved for B.

SOLUTION

a. When the currents are in the same direction, we find that

$$B = \frac{\mu_0 (I_1 + I_2)}{2\pi r} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(28\ \text{A} + 12\ \text{A})}{2\pi(0.72\ \text{m})} = \boxed{1.1 \times 10^{-5}\ \text{T}}$$

b. When the currents have opposite directions, a similar calculation shows that

$$B = \frac{\mu_0 (I_1 - I_2)}{2\pi r} = \frac{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(28\ \text{A} - 12\ \text{A})}{2\pi(0.72\ \text{m})} = \boxed{4.4 \times 10^{-6}\ \text{T}}$$

62. ***REASONING AND SOLUTION***

a. An end-on view of the copper cylinder is a circle, as the drawing shows. The dots around the circle represent the current coming out of the paper toward you. The larger dashed circle of radius r is the closed path used in Ampere's law and is centered on the axis of the cylinder. Equation 21.8 gives Ampere's law as $\Sigma B_{\parallel}\Delta\ell = \mu_0 I$. Because of the symmetry of the arrangement in the drawing, we have $B_{\parallel} = B$ for all $\Delta\ell$ on the circular path, so that Ampere's law becomes

$$\Sigma B_{\parallel}\Delta\ell = B(\Sigma\Delta\ell) = \mu_0 I$$

In this result, I is the net current through the circular surface bounded by the dashed path. In other words, it is the current I in the copper tube. Furthermore, $\Sigma\Delta\ell$ is the circumference of the circle, so we find that

$$B(\Sigma\Delta\ell) = B(2\pi r) = \mu_0 I \quad \text{or} \quad \boxed{B = \frac{\mu_0 I}{2\pi r}}$$

b. The setup here is similar to that in part a, except that the smaller dashed circle of radius r is now the closed path used in Ampere's law (see the drawing at the right). With this change, the derivation then proceeds exactly as in part a. Now, however, there is no current through the circular surface bounded by the dashed path, because all of the current is outside the path. Therefore, $I = 0$, and $B = \dfrac{\mu_0 I}{2\pi r} = \boxed{0\ \text{T}}$.

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63. ***REASONING AND SOLUTION*** The drawing shows an end-on view of the solid cylinder. The dots represent the current in the cylinder coming out of the paper toward you. The dashed circle of radius r is the closed path used in Ampere's law and is centered on the axis of the cylinder. Equation 21.8 gives Ampere's law as $\Sigma B_{\parallel}\Delta\ell = \mu_0 I$. Because of the symmetry of the arrangement in the drawing, we have $B_{\parallel} = B$ for all $\Delta\ell$ on the circular path, so that Ampere's law becomes

$$\Sigma B_{\parallel}\Delta\ell = B\left(\Sigma\Delta\ell\right) = \mu_0 I$$

In this result, $\Sigma\Delta\ell = 2\pi r$, the circumference of the circle. The current I is the part of the total current that comes through the area πr^2 bounded by the dashed path. We can calculate this current by using the current per unit cross-sectional area of the solid cylinder. This current per unit area is called the current density. The current I is the current density times the area πr^2:

$$I = \underbrace{\left(\frac{I_0}{\pi R^2}\right)}_{\substack{\text{current}\\ \text{density}}}\left(\pi r^2\right) = \frac{I_0 r^2}{R^2}$$

Thus, Ampere's law becomes

$$B\left(\Sigma\Delta\ell\right) = \mu_0 I \qquad \text{or} \qquad B\left(2\pi r\right) = \mu_0\left(\frac{I_0 r^2}{R^2}\right) \qquad \text{or} \qquad B = \boxed{\frac{\mu_0 I_0 r}{2\pi R^2}}$$

64. ***REASONING AND SOLUTION*** The speed of the electron can be determined using $eV = (1/2)mv^2$ so that

$$v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\left(1.60 \times 10^{-19}\ \text{C}\right)\left(19\,000\ \text{V}\right)}{9.11 \times 10^{-31}\ \text{kg}}} = 8.17 \times 10^{7}\ \text{m/s}$$

The magnetic force is given by

$$F = qvB \sin\theta = (1.60 \times 10^{-19}\ \text{C})(8.17 \times 10^{7}\ \text{m/s})(0.28\ \text{T}) \sin 90.0^{\circ} = \boxed{3.7 \times 10^{-12}\ \text{N}}$$

65. SSM ***REASONING*** The coil carries a current and experiences a torque when it is placed in an external magnetic field. Thus, when the coil is placed in the magnetic field due to the solenoid, it will experience a torque given by Equation 21.4: $\tau = NIAB\sin\phi$, where N is the number of turns in the coil, A is the area of the coil, B is the magnetic field inside the solenoid, and ϕ is the angle between the normal to the plane of the coil and the magnetic field. The magnetic field in the solenoid can be found from Equation 21.7: $B = \mu_0 nI$, where n is the number of turns per unit length of the solenoid and I is the current.

SOLUTION The magnetic field inside the solenoid is

$$B = \mu_0 nI = (4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1400\ \text{turns/m})(3.5\ \text{A}) = 6.2 \times 10^{-3}\ \text{T}$$

The torque exerted on the coil is

$$\tau = NIAB\sin\phi = (50)(0.50\ \text{A})(1.2 \times 10^{-3}\ \text{m}^2)(6.2 \times 10^{-3}\ \text{T})(\sin 90.0°) = \boxed{1.9 \times 10^{-4}\ \text{N}\cdot\text{m}}$$

66. ***REASONING AND SOLUTION*** Use $B = F/(qv \sin \theta)$. For both the electron and the proton the magnitude of B is the same,

$$B = (8.0 \times 10^{-14}\ \text{N})/[(1.60 \times 10^{-19}\ \text{C})(4.5 \times 10^{6}\ \text{m/s}) \sin 90.0°] = \boxed{0.11\ \text{T}}$$

a. For a proton, the direction of the magnetic field would be $\boxed{\text{upward}}$, away from the surface of the earth, perpendicular to both v and F.

b. For an electron, which has a negative charge, the magnetic field would be directed $\boxed{\text{downward}}$, toward the surface of the earth, perpendicular to both v and F.

67. ***REASONING AND SOLUTION*** The torque is given by

$$\tau = (\textit{Magnetic Moment})\, B \sin\phi = (1.4 \times 10^{-26}\ \text{A}\cdot\text{m}^2)(0.65\ \text{T}) \sin 64° = \boxed{8.2 \times 10^{-27}\ \text{N}\cdot\text{m}}$$

68. ***REASONING AND SOLUTION*** We require the magnetic field of the wire to be equal in magnitude but opposite in direction to the external magnetic field. We know that $B = \mu_0 I/(2\pi r)$, so we can solve for r to get

$$r = \frac{\mu_0 I}{2\pi B} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(305\ \text{A}\right)}{2\pi\left(7.00 \times 10^{-3}\ \text{T}\right)} = \boxed{8.71 \times 10^{-3}\ \text{m}}$$

69. SSM ***REASONING AND SOLUTION*** According to Equation 21.1, $B = \dfrac{F}{q_0 (v \sin\theta)}$.

Solving this for the angle θ, we find

$$\theta = \sin^{-1}\left(\frac{F}{q_0 v B}\right) = \sin^{-1}\left[\frac{2.30 \times 10^{-7}\ \text{N}}{(1.70 \times 10^{-5}\ \text{C})(2.80 \times 10^{2}\ \text{m/s})(5.00 \times 10^{-5}\ \text{T})}\right] = \boxed{75.1° \text{ and } 105°}$$

70. ***REASONING AND SOLUTION*** The direction of the magnetic force on the current is given by Right-Hand Rule No. 1. Using this rule, it can be shown that the current is directed $\boxed{\text{east-to-west}}$. The magnitude of the force is given by $F = ILB \sin\theta$. Solving for I we get

$$I = \frac{F}{LB\sin\theta} = \frac{0.058\ \text{N}}{(46\ \text{m})(3.2 \times 10^{-5}\ \text{T})\sin 90.0^\circ} = \boxed{39\ \text{A}}$$

71. ***REASONING AND SOLUTION*** The radius of the circular path of the ion in a mass spectrometer can be obtained using the relation $r = \sqrt{\frac{2mV}{eB^2}}$ (see Section 21.4). For the deuteron, we have

$$r = \sqrt{\frac{2mV}{eB^2}} = \sqrt{\frac{2\left(3.34 \times 10^{-27}\ \text{kg}\right)\left(2.00 \times 10^{3}\ \text{V}\right)}{\left(1.60 \times 10^{-19}\ \text{C}\right)\left(0.600\ \text{T}\right)^2}} = \boxed{0.0152\ \text{m}}$$

72. ***REASONING AND SOLUTION*** The magnetic field due to the circular loop alone is $B_1 = \frac{\mu_0 I_1}{2R}$. The field due to the straight wire is $B_2 = \frac{\mu_0 I_2}{2\pi H}$. These two fields cancel at the center of the loop, so that their magnitudes must be equal:

$$\frac{\mu_0 I_1}{2R} = \frac{\mu_0 (6.6\, I_1)}{2\pi H} \qquad \text{or} \qquad \boxed{H = 2.1\, R}$$

73. SSM WWW ***REASONING*** The particle travels in a semicircular path of radius r, where r is given by Equation 21.2 ($r = mv/qB$). The time spent by the particle in the magnetic field is given by $t = s/v$, where s is the distance traveled by the particle and v is its speed. The distance s is equal to one-half the circumference of a circle ($s = \pi r$).

SOLUTION We find that

$$t = \frac{s}{v} = \frac{\pi r}{v} = \frac{\pi}{v}\left(\frac{mv}{qB}\right) = \frac{\pi m}{qB} = \frac{\pi(6.0\times10^{-8}\ \text{kg})}{(7.2\times10^{-6}\ \text{C})(3.0\ \text{T})} = \boxed{8.7\times10^{-3}\ \text{s}}$$

74. ***REASONING AND SOLUTION*** Using Right-Hand Rule No. 2, the magnetic fields due to wires A and B will cancel out on the side of wire A away from wire B. If x is the distance (in meters) from this point to wire A, the distance to wire B is $(1.0 + x)$ meters. At this point then,

$$B_A = B_B \quad \text{so that} \quad \mu_0 I_A/(2\pi x) = \mu_0 I_B/[2\pi(1.0 + x)]$$

Using $I_A = I_B/3$ and simplifying, we obtain

$$(I_B/3)/x = I_B/(1.0 + x), \text{ which reduces to } \quad 1.0 + x = 3x$$

Solving this equation yields $x = \boxed{0.50 \text{ m}}$, on the side of wire A that is away from wire B.

75. ***REASONING*** The positive plate has a charge q and is moving downward with a speed v at right angles to a magnetic field of magnitude B. The magnitude F of the magnetic force exerted on the positive plate is $F = qvB \sin 90.0°$. The charge on the positive plate is related to the magnitude E of the electric field that exists between the plates by (see Equation 18.4) $q = \varepsilon_0 AE$, where A is the area of the positive plate. Substituting this expression for q into the expression for the magnetic force gives the answer in terms of known quantities.

SOLUTION

$$F = (\varepsilon_0 AE)vB$$

$$= \left[8.85 \times 10^{-12}\ \mathrm{C^2/(N \cdot m^2)}\right](7.5 \times 10^{-4}\ \mathrm{m^2})(170\ \mathrm{N/C})(32\ \mathrm{m/s})(3.6\ \mathrm{T}) = \boxed{1.3 \times 10^{-10}\ \mathrm{N}}$$

An application of Right-Hand Rule No. 1 shows that the magnetic force is perpendicular to the plane of the page and $\boxed{\text{directed out of the page}}$, toward the reader.

76. ***REASONING AND SOLUTION*** Find the magnitude of the magnetic field due to each wire, with the left-hand wire being #1 and the right-hand wire being #2. We have, for the magnitudes of B_1 and B_2

$$B_1 = B_2 = \mu_0 I/(2\pi r) = \mu_0 (85.0 \text{ A})/[2\pi(0.150 \text{ m})] = 1.13 \times 10^{-4} \text{ T}$$

The direction of B_1 is 30.0° below the horizontal to the right. The direction of B_2 is 30.0° below the horizontal to the left. The components of B_1 and B_2 are

$$B_{1x} = +B_1 \cos 30.0° = +9.79 \times 10^{-5} \text{ T}$$
$$B_{1y} = -B_1 \sin 30.0° = -5.65 \times 10^{-5} \text{ T}$$
$$B_{2x} = -B_2 \cos 30.0° = -9.79 \times 10^{-5} \text{ T}$$
$$B_{2y} = -B_2 \sin 30.0° = -5.65 \times 10^{-5} \text{ T}$$

The components of the magnetic field at P, then, are

$$B_x = B_{1x} + B_{2x} = 0, \quad \text{and} \quad B_y = B_{1y} + B_{2y} = -2(5.65 \times 10^{-5} \text{ T}) = -1.13 \times 10^{-4} \text{ T}$$

The net field is

$$B = \sqrt{B_x^2 + B_y^2} = \boxed{1.13 \times 10^{-4} \text{ T, down (toward the bottom of the page)}}$$

77. SSM ***REASONING*** The magnetic moment of the rotating charge can be found from the expression $Magnetic\ moment = NIA$. For this situation, $N = 1$. Thus, we need to find the current and the area for the rotating charge. This can be done by resorting to first principles.

SOLUTION The current for the rotating charge is, by definition (see Equation 20.1), $I = \Delta q / \Delta t$, where Δq is the amount of charge that passes by a given point during a time interval Δt. Since the charge passes by once per revolution, we can find the current by dividing the total rotating charge by the period T of revolution.

$$I = \frac{\Delta q}{T} = \frac{\Delta q}{2\pi / \omega} = \frac{\omega\, \Delta q}{2\pi} = \frac{(150\ \text{rad/s})(4.0 \times 10^{-6}\ \text{C})}{2\pi} = 9.5 \times 10^{-5}\ \text{A}$$

The area of the rotating charge is $A = \pi r^2 = \pi (0.20\ \text{m})^2 = 0.13\ \text{m}^2$

Therefore, the magnetic moment is

$$Magnetic\ moment = NIA = (1)(9.5 \times 10^{-5}\ \text{A})(0.13\ \text{m}^2) = \boxed{1.2 \times 10^{-5}\ \text{A} \cdot \text{m}^2}$$

1. **SSM** ***REASONING AND SOLUTION*** According to Equation 22.1, the motional emf when the velocity **v**, the magnetic field **B**, and the length **L** are mutually perpendicular is given by $\text{Emf} = vBL$. Therefore in order for a spark to jump across the gap, the rod would have to be moving in the magnetic field with speed

$$v = \frac{\text{Emf}}{BL} = \frac{940\text{ V}}{(4.8\text{ T})(1.3\text{ m})} = \boxed{150\text{ m/s}}$$

2. ***REASONING AND SOLUTION***

a. The right-hand rule shows that positive charge would be forced to the $\boxed{\text{driver's side}}$.

b. According to Equation 22.1, the width of the car is

$$L = \frac{\text{Emf}}{vB} = \frac{2.4 \times 10^{-3}\ \text{V}}{(25\ \text{m/s})\left(4.8 \times 10^{-5}\ \text{T}\right)} = \boxed{2.0\ \text{m}}$$

3. ***REASONING AND SOLUTION*** Using Equation 22.1, we find

$$\text{Emf} = vBL = (220\ \text{m/s})(5.0 \times 10^{-6}\ \text{T})(59\ \text{m}) = \boxed{0.065\ \text{V}}$$

4. ***REASONING AND SOLUTION*** The motional emf generated by a conductor moving perpendicular to a magnetic field is given by Equation 22.1 as Emf = vBL, where v and L are the speed and length, respectively, of the conductor, and B is the magnitude of the magnetic field. The emf would have been

$$\text{Emf} = vBL = \left(7.6 \times 10^{3}\ \text{m/s}\right)\left(5.1 \times 10^{-5}\ \text{T}\right)\left(2.0 \times 10^{4}\ \text{m}\right) = \boxed{7800\ \text{V}}$$

5. **WWW** ***REASONING AND SOLUTION*** For each of the three rods in the drawing, we have the following:

***Rod A*:** The motional emf is **zero**, because the velocity of the rod is parallel to the direction of the magnetic field, and the charges do not experience a magnetic force.

***Rod B*:** The motional emf is, according to Equation 22.1,

$$\text{Emf} = vBL = (2.7\ \text{m/s})(0.45\ \text{T})(1.3\ \text{m}) = \boxed{1.6\ \text{V}}$$

The positive end of Rod 2 is **end 2**.

***Rod C*:** The motional emf is **zero**, because the magnetic force F on each charge is directed perpendicular to the length of the rod. For the ends of the rod to become charged, the magnetic force must be directed parallel to the length of the rod.

6. ***REASONING AND SOLUTION*** Ohm's law requires $I = \text{Emf}/R$, where $\text{Emf} = vBL$ and $R = \rho L/A$. Hence,

$$I = \frac{BvA}{\rho} = \frac{(0.050\ \text{T})(2.0\ \text{m/s})(3.1\times10^{-6}\ \text{m}^2)}{9.7\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{3.2\ \text{A}}$$

7. **SSM** ***REASONING*** The moving rod acts as a battery does in supplying an emf or voltage in the circuit. The speed of the rod is related to the emf by Equation 22.1, $\text{Emf} = vBL$. From Equation 20.6c, we have $P = (\text{Emf})^2 / R$, where P is the power consumed by the circuit and R is the resistance in the circuit.

SOLUTION

a. Solving Equation 20.6c for the emf, we have

$$\text{Emf} = \sqrt{PR} = \sqrt{(15\text{ W})(6.0\ \Omega)} = 9.5\text{ V}$$

The speed of the rod is

$$v = \frac{\text{Emf}}{LB} = \frac{9.5\text{ V}}{(2.4\text{ T})(1.2\text{ m})} = \boxed{3.3\text{ m/s}}$$

b. The force exerted on the rod by the magnetic field is given by Equation 21.3 with $\theta = 90°$, and $I = \text{Emf}/R$. Therefore,

$$F = ILB\ (\sin\ 90°) = \left(\frac{\text{Emf}}{R}\right)LB = \left(\frac{9.5\text{ V}}{6.0\ \Omega}\right)(1.2\text{ m})(2.4\text{ T}) = \boxed{4.6\text{ N}}$$

8. ***REASONING*** Once the switch is closed, there is a current in the rod. The magnetic field applies a force to this current and accelerates the rod to the right. As the rod begins to move, however, a motional emf appears between the ends of the rod. This motional emf depends on the speed of the rod and increases as the speed increases. Equally important is the fact that the motional emf opposes the emf of the battery. The net emf causing the current in the rod is the algebraic sum of the two emf contributions. Thus, as the speed of the rod increases and the motional emf increases, the net emf decreases. As the net emf decreases, the current in the rod decreases and so does the force that field applies to the current. Eventually, the speed reaches the point when the motional emf has the same magnitude as the battery emf, and the net emf becomes zero. At this point, there is no longer a net force acting on the rod and the speed remains constant from this point onward, according to Newton's second law. This maximum speed can be determined by using Equation 22.1 for the motional emf, with a value of the motional emf that equals the battery emf.

SOLUTION Using Equation 22.1 (Emf = vBL) and a value of 3.0 V for the emf, we find that the maximum speed of the rod is

$$v = \frac{\text{Emf}}{BL} = \frac{3.0\ \text{V}}{(0.60\ \text{T})(0.20\ \text{m})} = \boxed{25\ \text{m/s}}$$

9. ***REASONING AND SOLUTION***

a. Newton's second law gives the magnetic retarding force to be

$$F = mg = IBL$$

Now the current, I, is

$$I = (\text{Emf})/R = vBL/R$$

so

$$m = \frac{v(BL)^2}{Rg} = \frac{(4.0\ \text{m/s})(0.50\ \text{T})^2(1.3\ \text{m})^2}{(0.75\ \Omega)(9.80\ \text{m/s}^2)} = \boxed{0.23\ \text{kg}}$$

b. The change in height in a time Δt is $\Delta h = -v\Delta t$. The change in gravitational potential energy is

$$\Delta\text{PE} = mg\Delta h = -mgv\Delta t = -(0.23\ \text{kg})(9.80\ \text{m/s}^2)(4.0\ \text{m/s})(0.20\ \text{s}) = \boxed{-1.8\ \text{J}}$$

c. The energy dissipated in the resistor is the amount by which the gravitational potential energy decreases or $\boxed{1.8\ \text{J}}$.

10. ***REASONING AND SOLUTION***

a. The magnetic flux is $\Phi = BA \cos \phi$. Since $\phi = 0.0°$, we have

$$\Phi = (0.35\ \text{T})(0.0160\ \text{m}^2) \cos 0.0° = \boxed{5.6 \times 10^{-3}\ \text{Wb}}$$

b. Since $\phi = 90°$, we have

$$\Phi = (0.35\ \text{T})(0.0160\ \text{m}^2) \cos 90° = \boxed{0\ \text{Wb}}$$

11. $\boxed{\text{SSM}}$ ***REASONING*** The general expression for the magnetic flux through an area A is given by Equation 22.2: $\Phi = BA\cos\phi$ where B is the magnitude of the magnetic field and ϕ is the angle of inclination of the magnetic field **B** with respect to the normal to the area.

The magnetic flux through the door is a maximum when the magnetic field lines are perpendicular to the door and $\phi_1 = 0.0°$ so that $\Phi_1 = \Phi_{max} = BA(\cos 0.0°) = BA$.

SOLUTION When the door rotates through an angle ϕ_2, the magnetic flux that passes through the door decreases from its maximum value to one-third of its maximum value. Therefore $\Phi_2 = \frac{1}{3}\Phi_{max}$, and we have

$$\Phi_2 = BA\cos\phi_2 = \tfrac{1}{3}BA \qquad \text{or} \qquad \cos\phi_2 = \tfrac{1}{3} \qquad \text{or} \qquad \phi_2 = \cos^{-1}\left(\tfrac{1}{3}\right) = \boxed{70.5°}$$

12. ***REASONING*** The magnetic flux is defined in Equation 22.2 as $\Phi = BA\cos\phi$, where B is the magnitude of the magnetic field, A is the area of the loop, and ϕ is the angle between the normal to the surface of the loop and the magnetic field. The change $\Delta\Phi$ in flux is the final flux Φ minus the initial flux Φ_0; $\Delta\Phi = \Phi - \Phi_0$.

SOLUTION Let L_1 and L_2 be the lengths of the left and bottom edges of the loop. The initial area is the area of this rectangular loop plus the area of the semicircle: $A_0 = L_1 \times L_2 + \frac{1}{2}\pi r^2$. The initial magnetic flux is $\Phi_0 = B\left(L_1 \times L_2 + \frac{1}{2}\pi r^2\right)\cos 0°$. The final area is the area of the rectangular loop minus the area of the semicircle: $A_0 = L_1 \times L_2 - \frac{1}{2}\pi r^2$. The final magnetic flux is $\Phi = B\left(L_1 \times L_2 - \frac{1}{2}\pi r^2\right)\cos 0°$. The change in flux is

$$\Delta\Phi = \Phi - \Phi_0 = B\left(L_1 \times L_2 - \tfrac{1}{2}\pi r^2\right)\cos 0° - B\left(L_1 \times L_2 + \tfrac{1}{2}\pi r^2\right)\cos 0°$$

$$= -B\pi r^2 \cos 0° = -(0.75\ \text{T})\pi(0.20\ \text{m})^2 \cos 0° = \boxed{-9.4 \times 10^{-2}\ \text{Wb}}$$

Note that the change in flux does not depend on L_1 or L_2.

13. ***REASONING AND SOLUTION*** When computing the magnetic flux, we multiply the component of the magnetic field that is parallel to the normal to the surface by the area of the surface.

a. For the surface that lies in the xy plane, the normal is parallel to the z axis, so

$$\Phi = B_z A = (0.30\ \text{T})(2.0 \times 10^{-2}\ \text{m})^2 = \boxed{1.2 \times 10^{-4}\ \text{Wb}}$$

b. For the surface that lies in the xz plane, the normal is parallel to the y axis, so

$$\Phi = B_y A = (0.80\ \text{T})(2.0 \times 10^{-2}\ \text{m})^2 = \boxed{3.2 \times 10^{-4}\ \text{Wb}}$$

c. For the surface that lies in the yz plane, the normal is parallel to the x axis, so

$$\Phi = B_x A = (0.50\ \text{T})(2.0 \times 10^{-2}\ \text{m})^2 = \boxed{2.0 \times 10^{-4}\ \text{Wb}}$$

14. ***REASONING AND SOLUTION***

a. According to Equation 22.2, we have $\Phi = BA \cos \phi$. If the wall faces north, then

$$\Phi = B_H A \cos 0.0° + B_V A \cos 90° = B_H A = (2.6 \times 10^{-5}\ \text{T})(28\ \text{m}^2) = \boxed{7.3 \times 10^{-4}\ \text{Wb}}$$

b. If the wall faces east, then

$$\Phi = B_H A \cos 90° + B_V A \cos 90° = \boxed{0\ \text{Wb}}$$

c. The normal to the floor is vertical, so

$$\Phi = B_H A \cos 90° + B_V A \cos 0.0° = B_V A = (4.2 \times 10^{-5}\ \text{T})(112\ \text{m}^2) = \boxed{4.7 \times 10^{-3}\ \text{Wb}}$$

15. $\boxed{\text{SSM}}$ $\boxed{\text{WWW}}$ ***REASONING*** The general expression for the magnetic flux through an area A is given by Equation 22.2: $\Phi = BA\cos\phi$, where B is the magnitude of the magnetic field and ϕ is the angle of inclination of the magnetic field **B** with respect to the normal to the surface.

SOLUTION Since the magnetic field **B** is parallel to the surface for the triangular ends and the bottom surface, the flux through each of these three surfaces is $\boxed{0\text{ Wb}}$.

The flux through the 1.2 m by 0.30 m face is

$$\Phi = (0.25\text{ T})(1.2\text{ m})(0.30\text{ m})\cos 0.0° = \boxed{0.090\text{ Wb}}$$

For the 1.2 m by 0.50 m side, the area makes an angle ϕ with the magnetic field **B**, where

$$\phi = 90° - \tan^{-1}\left(\frac{0.30\text{ m}}{0.40\text{ m}}\right) = 53°$$

Therefore,

$$\Phi = (0.25\text{ T})(1.2\text{ m})(0.50\text{ m})\cos 53° = \boxed{0.090\text{ Wb}}$$

16. ***REASONING AND SOLUTION*** The change in flux $\Delta\Phi$ is given by $\Delta\Phi = B\,\Delta A \cos\phi$, where ΔA is the area of the loop that leaves the region of the magnetic field in a time Δt. This area is the product of the width of the rectangle (0.080 m) and the length $v\,\Delta t$ of the side that leaves the magnetic field, $\Delta A = (0.080\text{ m})\,v\,\Delta t$.

$$\Delta\Phi = B\,\Delta A \cos\phi = (2.4\text{ T})\,(0.080\text{ m})(0.020\text{ m/s})(2.0\text{ s})\cos 0.0° = \boxed{7.7 \times 10^{-3}\text{ Wb}}$$

17. ***REASONING AND SOLUTION***

a. The change in flux through the loop is $\Delta\Phi = (\Delta B)A \cos \phi$, so the magnitude for the emf given by Faraday's law is

$$\text{Emf} = N[(\Delta B)/(\Delta t)]A \cos \phi$$

$$\text{Emf} = 300[(0.40\ \text{T})/(0.80\ \text{s})](5.0 \times 10^{-3}\ \text{m}^2) \cos 30.0° = \boxed{0.65\ \text{V}}$$

b. The current in the resistor is, according to Ohm's law,

$$I = (\text{Emf})/R = (0.65\ \text{V})/(6.0\ \Omega) = \boxed{0.11\ \text{A}}$$

18. ***REASONING AND SOLUTION*** The average emf induced in the circular wire is given by

$$\text{Emf} = -B\left(\frac{A - A_0}{t - t_0}\right)\cos\phi$$

The change in the area is equal to the final area of the circle ($A = 0\ \text{m}^2$) minus the initial area ($A_0 = \pi r^2$).

$$\text{Emf} = -B\left(\frac{0 - \pi r^2}{t - t_0}\right)\cos\phi$$

$$\text{Emf} = -(0.55\ \text{T})\left[\frac{0 - \pi\left(2.0 \times 10^{-2}\ \text{m}\right)^2}{0.25\ \text{s} - 0}\right]\cos 0.0° = \boxed{2.8 \times 10^{-3}\ \text{V}}$$

19. $\boxed{\text{SSM}}$ ***REASONING*** According to Equation 22.3, the average emf induced in a coil of N loops is $\text{Emf} = -N\Delta\Phi/\Delta t$.

SOLUTION For the circular coil in question, the flux through a single turn changes by

$$\Delta\Phi = BA\cos 45° - BA\cos 90° = BA\cos 45°$$

during the interval of $\Delta t = 0.010\ \text{s}$. Therefore, for N turns, Faraday's law gives the magnitude of the emf (without the minus sign) as

$$\text{Emf} = \frac{NBA\cos 45°}{\Delta t}$$

Since the loops are circular, the area A of each loop is equal to πr^2. Solving for B, we have

$$B = \frac{(\text{Emf})\Delta t}{N\pi r^2\cos 45°} = \frac{(0.065\ \text{V})(0.010\ \text{s})}{(950)\pi(0.060\ \text{m})^2\cos 45°} = \boxed{8.6\times10^{-5}\ \text{T}}$$

20. ***REASONING*** According to Faraday's law as given in Equation 22.3 (without the minus sign), the magnitude of the emf is Emf = $\Delta\Phi/\Delta t$ for a single turn ($N = 1$). Since the normal is parallel to the magnetic field, the angle ϕ between the normal and the field is $\phi = 0°$ when calculating the flux Φ from Equation 22.2: $\Phi = BA \cos 0° = BA$. We will use this expression for the flux in Faraday's law.

SOLUTION Representing the flux as $\Phi = BA$, we find that Faraday's law (without the minus sign) becomes

$$\text{Emf} = \frac{\Delta\Phi}{\Delta t} = \frac{\Delta(BA)}{\Delta t} = \frac{B\,\Delta A}{\Delta t}$$

In this result we have used the fact that the field magnitude B is constant. Rearranging this equation gives

$$\frac{\Delta A}{\Delta t} = \frac{\text{Emf}}{B} = \frac{2.6\text{ V}}{1.7\text{ T}} = \boxed{1.5\text{ m}^2/\text{s}}$$

21. ***REASONING AND SOLUTION*** Faraday's law gives (using only the magnitude of the induced emf)

$$\Delta t = BA \cos 0.0°/(\text{Emf}) = (1.5\ \text{T})(0.032\ \text{m}^2)/(0.010\ \text{V}) = \boxed{4.8\ \text{s}}$$

22. ***REASONING*** We will use Faraday's law of electromagnetic induction, Equation 22.3, to find the emf induced in the loop. Once this value has been determined, we can employ Equation 22.3 again to find the rate at which the area changes.

SOLUTION

a. The magnitude of the induced emf is given by Equation 22.3 without the minus sign:

$$\text{Emf} = N\left(\frac{\Phi - \Phi_0}{t - t_0}\right) = N\left(\frac{BA\cos\phi - B_0 A\cos\phi}{t - t_0}\right) = NA\cos\phi\left(\frac{B - B_0}{t - t_0}\right)$$

$$= (1)(0.35\text{ m} \times 0.55\text{ m})\cos 65°\left(\frac{2.1\text{ T} - 0\text{ T}}{0.45\text{ s} - 0\text{ s}}\right) = \boxed{0.38\text{ V}}$$

b. When the magnetic field is constant and the area is changing in time, Faraday's law can be written as (again, without the minus sign)

$$\text{Emf} = N\left(\frac{\Phi - \Phi_0}{t - t_0}\right) = N\left(\frac{BA\cos\phi - BA_0\cos\phi}{t - t_0}\right)$$

$$= NB\cos\phi\left(\frac{A - A_0}{t - t_0}\right) = NB\cos\phi\left(\frac{\Delta A}{\Delta t}\right)$$

Solving this equation for $\Delta A/\Delta t$ and substituting in the value of 0.38 V for the emf, we find that

$$\frac{\Delta A}{\Delta t} = \frac{\text{Emf}}{NB\cos\phi} = \frac{0.38\text{ V}}{(1)(2.1\text{ T})\cos 65°} = \boxed{0.43\text{ m}^2/\text{s}}$$

23. ***REASONING AND SOLUTION*** The rod may be viewed as sweeping out an area whose initial value is 0.0 m^2 and final value is $\pi r^2/4$. The flux change due to this change in area is

$$\Delta\Phi = B\pi r^2/4$$

Faraday's law gives the magnitude of the emf to be

$$\text{Emf} = \Delta\Phi/\Delta t = (1/4)(0.16\text{ T})(\pi)(1.5\text{ m})^2/(0.66\text{ s}) = \boxed{0.43\text{ V}}$$

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24. ***REASONING*** The energy dissipated in the resistance is given by Equation 6.10b as the power dissipated multiplied by the time, $E = Pt$. The power, according to Equation 20.6c, is the square of the induced emf divided by the resistance, $P = (\text{Emf})^2/R$. The induced emf can be determined from Faraday's law of electromagnetic induction, Equation 22.3.

SOLUTION Expressing the energy consumed as $E = Pt$, and substituting in $P = (\text{Emf})^2/R$, we find

$$E = Pt = \frac{(\text{Emf})^2 t}{R}$$

The induced emf is given by Faraday's law as $\text{Emf} = -N(\Delta\Phi/\Delta t)$, and the resistance R is equal to the resistance per unit length ($3.3 \times 10^{-2}\ \Omega/\text{m}$) times the length of the circumference of the loop, $2\pi r$. Thus, the energy consumed is

$$E = \frac{\left(-N\dfrac{\Delta\Phi}{\Delta t}\right)^2 t}{\left(3.3\times10^{-2}\ \Omega/\text{m}\right)2\pi r} = \frac{\left[-N\left(\dfrac{\Phi-\Phi_0}{t-t_0}\right)\right]^2 t}{\left(3.3\times10^{-2}\ \Omega/\text{m}\right)2\pi r}$$

$$= \frac{\left[-N\left(\dfrac{BA\cos\phi - B_0A\cos\phi}{t-t_0}\right)\right]^2 t}{\left(3.3\times10^{-2}\ \Omega/\text{m}\right)2\pi r} = \frac{\left[-NA\cos\phi\left(\dfrac{B-B_0}{t-t_0}\right)\right]^2 t}{\left(3.3\times10^{-2}\ \Omega/\text{m}\right)2\pi r}$$

$$= \frac{\left[-(1)\pi(0.12\ \text{m})^2(\cos 0°)\left(\dfrac{0.60\ \text{T} - 0\ \text{T}}{0.45\ \text{s} - 0\ \text{s}}\right)\right]^2 (0.45\ \text{s})}{\left(3.3\times10^{-2}\ \Omega/\text{m}\right)2\pi(0.12\ \text{m})} = \boxed{6.6\times10^{-2}\ \text{J}}$$

25. **SSM** **WWW** ***REASONING AND SOLUTION*** According to Ohm's law, the emf developed by the coil is Emf = IR, which, when combined with the definition of electric current becomes Emf = $(\Delta q)R/(\Delta t)$. The magnitude of the emf is also given by Equation 22.3 (Faraday's law) without the minus sign:

$$\text{Emf} = \text{N}\frac{\Delta\Phi}{\Delta t} = \text{N}\left(\frac{BA\cos 0° - 0}{\Delta t}\right) = \frac{NBA}{\Delta t}$$

Combining the two expressions for the emf, we have

$$\frac{(\Delta q)R}{\Delta t} = \frac{NBA}{\Delta t} \qquad \text{or} \qquad B = \frac{(\Delta q)R}{NA} = \frac{(8.87\times10^{-3}\ \text{C})(45.0\ \Omega)}{(1850)(4.70\times10^{-4}\ \text{m}^2)} = \boxed{0.459\ \text{T}}$$

26. ***REASONING AND SOLUTION*** In time, Δt, the flux change through the loop ABC is $\Delta\Phi = B\Delta A$, where

$$\Delta A = [\omega\Delta t/(2\pi)](\pi R^2)$$

Faraday's law gives the magnitude of the induced emf to be

$$\text{Emf} = \Delta\Phi/\Delta t = B\omega R^2/2 = (3.8 \times 10^{-3}\ \text{T})(15\ \text{rad/s})(0.50\ \text{m})^2/2 = 7.1 \times 10^{-3}\ \text{V}$$

The current through the resistor is

$$I = (\text{Emf})/R = (7.1 \times 10^{-3}\ \text{V})/(3.0\ \Omega) = \boxed{2.4 \times 10^{-3}\ \text{A}}$$

27. ***REASONING AND SOLUTION*** Consider one revolution of either rod. The magnitude of the emf induced across the rod is

$$\text{Emf} = B\Delta A/\Delta t = B(\pi L^2)/\Delta t$$

The angular speed of the rods is $\omega = 2\pi/\Delta t$, so Emf = $BL^2\omega/2$. The rod tips have opposite polarity since they are rotating in opposite directions. Hence, the difference in potentials of the tips is

$$\Delta V = BL^2\,\omega$$

so

$$\omega = \frac{\Delta V}{BL^2} = \frac{4.5 \times 10^3\ \text{V}}{(4.7\ \text{T})(0.68\ \text{m})^2} = \boxed{2100\ \text{rad/s}}$$

28. ***REASONING AND SOLUTION***

a. When the triangle is crossing the + y axis, the magnetic flux through it begins to increase. The induced field must be into the paper by Lenz's law. The induced current must, therefore, be $\boxed{\text{CW}}$.

b. When the triangle crosses the –x axis, the flux through it is not changing at all. No induced field will be produced and there will be $\boxed{\text{NO induced current}}$.

c. Upon crossing the –y axis the coil will experience a decrease in flux so the induced field will be out of the paper. The induced current must then be $\boxed{\text{CCW}}$.

d. As the triangle crosses the +x axis, the flux through it is not changing at all. No induced field will be produced and there will be $\boxed{\text{NO induced current}}$.

29. **SSM** ***REASONING AND SOLUTION*** If the north and south poles of the magnet are interchanged, the currents in the ammeter would simply be reversed in direction.

Therefore, in Figure 22.1*b*, the current will flow $\boxed{\text{right to left}}$ through the ammeter.

Similarly, in Figure 22.1*c*, the current will flow $\boxed{\text{left to right}}$ through the ammeter.

30. ***REASONING AND SOLUTION*** According to Lenz's law, the induced emf has a polarity that leads to an induced current whose direction is such that the induced magnetic field opposes the original flux change. As the loop rotates through one-half a revolution, the area through which the magnetic field passes decreases. The magnetic flux, being proportional to the area, also decreases. The induced magnetic field must oppose this decrease in flux, so the induced magnetic field must strengthen the original magnetic field. Thus, the induced magnetic field points in the same direction as the original magnetic field, or into the plane of the page. According to RHR-2, the induced current flows clockwise around the loop (and right-to-left through the resistor) in order that its magnetic field be directed into the page. Since conventional current flows through a resistor from higher potential toward lower potential, $\boxed{\text{the right end of the resistor must be the positive end}}$.

31. **SSM** ***REASONING*** In solving this problem, we apply Lenz's law, which essentially says that the change in magnetic flux must be opposed by the induced magnetic field.

SOLUTION

a. The magnetic field due to the wire in the vicinity of position 1 is directed out of the paper. The coil is moving closer to the wire into a region of higher magnetic field, so the flux through the coil is increasing. Lenz's law demands that the induced field counteract this increase. The direction of the induced field, therefore, must be into the paper. The current in the coil must be clockwise.

b. At position 2 the magnetic field is directed into the paper and is decreasing as the coil moves away from the wire. The induced magnetic field, therefore, must be directed into the paper, so the current in the coil must be clockwise.

32. ***REASONING AND SOLUTION***

a. When the magnet is above the ring its magnetic field points down through the ring and is increasing as the magnet falls. The induced magnetic field attempts to reduce the increasing field and points up. The induced current in the ring is as shown in the drawing at the right, and then the ring looks like a magnet with its north pole at the top, repelling the north pole of the falling magnet and retarding its motion.

When the magnet is below the ring, its magnetic field still points down through the ring but is decreasing as the magnet falls. The induced magnetic field attempts to bolster the decreasing field and points down. The induced current in the ring is as shown in the drawing, and the ring then looks like a magnet with its north pole at the bottom, attracting the south pole of the falling magnet and retarding its motion.

b. The motion of the magnet is unaffected, since no induced current can flow in the cut ring. No induced current means that no induced magnetic field can be produced to repel or attract the falling magnet.

33. ***REASONING*** The current I in the straight wire produces a circular pattern of magnetic field lines around the wire. The magnetic field at any point is tangent to one of these circular field lines. Thus, the field points perpendicular to the plane of the table. Furthermore, according to Right-Hand Rule No. 2, the field is directed down into the table surface in region 1 above the wire and is directed up out of the table surface in region 2 below the wire (see the drawing at the right). To deduce the direction of any induced current in the circular loop, we consider Faraday's law and the change that occurs in the magnetic flux through the loop due to the field of the straight wire.

SOLUTION As the current I increases, the magnitude of the field that it produces also increases. However, the directions of the fields in regions 1 and 2 do not change and remain as discussed in the reasoning. Since the fields in these two regions always have opposite directions and equal magnitudes at any given radial distance from the straight wire, the flux through the regions add up to give zero for any value of the current. With the flux remaining constant as time passes, Faraday's law indicates that there is no induced emf in the coil. Since there is no induced emf in the coil, there is $\boxed{\text{no induced current}}$.

34. ***REASONING AND SOLUTION***

a. **Location I**

As the loop swings downward, the normal to the loop makes a smaller angle with the applied field. Hence, the flux through the loop is increasing. The induced magnetic field must point generally to the left to counteract this increase. The induced current flows

$$\boxed{x \rightarrow y \rightarrow z}$$

Location II

The angle between the normal to the loop and the applied field is now increasing, so the flux through the loop is decreasing. The induced field must now be generally to the right, and the current flows

$$\boxed{z \rightarrow y \rightarrow x}$$

b. **Location I**

The argument is the same as for location II in part a.

$$\boxed{z \rightarrow y \rightarrow x}$$

Location II

The argument is the same as for location I in part a.

$$\boxed{x \rightarrow y \rightarrow z}$$

35. **SSM** **WWW** ***REASONING AND SOLUTION***

a. From the drawing, we see that the period of the generator (the time for one full cycle) is 0.42 s; therefore, the frequency of the generator is

$$f = \frac{1}{T} = \frac{1}{0.42\text{ s}} = \boxed{2.4\text{ Hz}}$$

b. The angular speed of the generator is related to its frequency by $\omega = 2\pi f$, so the angular speed is

$$\omega = 2\pi(2.4\text{ Hz}) = \boxed{15\text{ rad/s}}$$

c. The magnitude of the maximum emf induced in a rotating planar coil is given by $(\text{Emf})_0 = NAB\omega$ (see Equation 22.4). The magnitude of the magnetic field can be found by solving this expression for B and noting from the drawing that $(\text{Emf})_0 = 28$ V:

$$B = \frac{(\text{Emf})_0}{NA\omega} = \frac{28\text{ V}}{(150)(0.020\text{ m}^2)(15\text{ rad/s})} = \boxed{0.62\text{ T}}$$

36. ***REASONING AND SOLUTION*** If the generators have the same peak emf, number of turns and angular frequency, then $NB_1A_1 \sin \omega t = NB_2A_2 \sin \omega t$, so that $B_1A_1 = B_2A_2$. Thus,

$$B_1 = (0.10\ \text{T})(0.045\ \text{m}^2)/(0.015\ \text{m}^2) = \boxed{0.30\ \text{T}}$$

37. ***REASONING AND SOLUTION*** The maximum emf produced by the generator is $NAB\omega$, so

$$B = \frac{5500\text{ V}}{150\left(0.85\text{ m}^2\right)2\pi\left(60.0\text{ Hz}\right)} = \boxed{0.11\text{ T}}$$

38. ***REASONING AND SOLUTION*** Using Equation 22.5 to take the back emf into account, we find

$$R = \frac{V - \text{Emf}}{I} = \frac{(120.0\ \text{V}) - (72.0\ \text{V})}{3.0\ \text{A}} = \boxed{16\ \Omega}$$

39. **SSM** ***REASONING*** We can use the information given in the problem statement to determine the area of the coil A. Since it is square, the length of one side is $\ell = \sqrt{A}$.

SOLUTION According to Equation 22.4, the maximum emf induced in the coil is $(\text{Emf})_0 = NAB\omega$. Therefore, the length of one side of the coil is

$$\ell = \sqrt{A} = \sqrt{\frac{(\text{Emf})_0}{NB\omega}} = \sqrt{\frac{75.0\text{ V}}{(248)(0.170\text{ T})(79.1\text{ rad/s})}} = \boxed{0.150\text{ m}}$$

40. ***REASONING*** When the motor is running at normal speed, the current is the net emf divided by the resistance R of the armature wire. The net emf is the applied voltage V minus the back emf developed by the rotating coil. We can use this relation to find the back emf. When the motor is just turned on, there is no back emf, so the current is just the applied voltage divided by the resistance of the wire. To limit the starting current to 15.0 A, a resistor R_1 is placed in series with the resistance of the wire, so the equivalent resistance is $R + R_1$. The current to the motor is equal to the applied voltage divided by the equivalent resistance.

SOLUTION

a. According to Equation 22.5, the back emf generated by the motor is

$$\text{Emf} = V - IR = 120.0\text{ V} - (7.00\text{ A})(0.720\ \Omega) = \boxed{115\text{ V}}$$

b. When the motor has been just turned on, the back emf is zero, so the current is

$$I = \frac{V - \text{Emf}}{R} = \frac{120.0\text{ V} - 0\text{ V}}{0.720\ \Omega} = \boxed{167\text{ A}}$$

c. When a resistance R_1 is placed in series with the resistance R of the wire, the equivalent resistance is $R_1 + R$. The current to the motor is

$$I = \frac{V - \text{Emf}}{R_1 + R}$$

Solving this expression for R_1 gives

$$R_1 = \frac{V - \text{Emf}}{I} - R = \frac{120.0\text{ V} - 0\text{ V}}{15.0\text{ A}} - 0.720\ \Omega = \boxed{7.28\ \Omega}$$

41. **SSM** ***REASONING AND SOLUTION*** When the motor just begins to turn the fan blade, there is no back emf. Therefore, Ohm's law indicates that $120.0\text{ V} = I_0R$, where I_0 is the current in the motor and R is the resistance of the motor coils. Solving for I_0 gives

$$I_0 = \frac{120.0\text{ V}}{R} \tag{1}$$

At normal operating speed, there is a back emf, and Equation 22.5 gives the current in the motor as

$$I = \frac{120.0\text{ V} - \text{Emf}}{R} \tag{2}$$

But we know that $I = 0.150I_0$. Therefore, using Equations (1) and (2), we find that

$$\frac{120.0\text{ V} - \text{Emf}}{R} = 0.150\left(\frac{120.0\text{ V}}{R}\right)$$

Eliminating R algebraically, we have

$$120.0\text{ V} - \text{Emf} = 0.150(120.0\text{ V}) \qquad \text{or} \qquad \text{Emf} = 0.850(120.0\text{ V}) = \boxed{102\text{ V}}$$

42. ***REASONING AND SOLUTION*** The peak emf is NABω, where N is the length of the wire divided by the circumference of a turn.

$$\text{Peak emf} = [L/(2\pi\, r)](\pi\, r^2)\omega B$$

$$\text{Peak emf} = Lr\omega B/2 = (5.7\text{ m})(0.14\text{ m})(25\text{ rad/s})(0.20\text{ T})/2 = \boxed{2.0\text{ V}}$$

43. ***REASONING AND SOLUTION***

a. On startup, the back emf of the generator is zero. Then,

$$R = V/I = (117\text{ V})/(12.2\text{ A}) = \boxed{9.59\ \Omega}$$

b. At normal speed

$$\text{Emf} = V - IR = 117\text{ V} - (2.30\text{ A})(9.59\ \Omega) = \boxed{95\text{ V}}$$

c. The back emf of the motor is proportional to the rotational speed, so at 1/3 the normal speed, the back emf is

$$(1/3)(95\text{ V}) = 32\text{ V}$$

The voltage applied to the resistor is then $V = 117\text{ V} - 32\text{ V} = 85\text{ V}$, so the current is

$$I = V/R = (85\text{ V})/(9.59\ \Omega) = \boxed{8.9\text{ A}}$$

44. ***REASONING AND SOLUTION*** Using Equation 22.7, we find

$$(\text{Emf})_2 = -M\frac{\Delta I_1}{\Delta t} \qquad \text{or} \qquad M = \frac{-(\text{Emf})_2\,\Delta t}{\Delta I_1} = \frac{-(0.12\text{ V})(0.14\text{ s})}{1.6\text{ A} - 3.4\text{ A}} = \boxed{9.3\times10^{-3}\text{ H}}$$

45. **SSM** ***REASONING AND SOLUTION*** From the results of Example 13, the self-inductance L of a long solenoid is given by $L = \mu_0 n^2 A\ell$. Solving for the number of turns n per unit length gives

$$n = \sqrt{\frac{L}{\mu_0 A\ell}} = \sqrt{\frac{1.4 \times 10^{-3}\ \text{H}}{(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1.2 \times 10^{-3}\ \text{m}^2)(0.052\ \text{m})}} = 4.2 \times 10^3\ \text{turns/m}$$

Therefore, the total number of turns N is the product of n and the length ℓ of the solenoid:

$$N = n\ell = (4.2 \times 10^3\ \text{turns/m})(0.052\ \text{m}) = \boxed{220\ \text{turns}}$$

46. ***REASONING*** When the current through an inductor changes, the induced emf is given by Equation 22.9 as

$$\text{Emf} = -\text{L}\frac{\Delta I}{\Delta t}$$

where L is the inductance, ΔI is the change in the current, and Δt is the time interval during which the current changes. For each interval, we can determine ΔI and Δt from the graph.

SOLUTION

a. $$\text{Emf} = -L\frac{\Delta I}{\Delta t} = -\left(3.2\times10^{-3}\ \text{H}\right)\left(\frac{4.0\ \text{A} - 0\ \text{A}}{2.0\times10^{-3}\ \text{s} - 0\ \text{s}}\right) = \boxed{-6.4\ \text{V}}$$

b. $$\text{Emf} = -L\frac{\Delta I}{\Delta t} = -\left(3.2\times10^{-3}\ \text{H}\right)\left(\frac{4.0\ \text{A} - 4.0\ \text{A}}{5.0\times10^{-3}\ \text{s} - 2.0\times10^{-3}\ \text{s}}\right) = \boxed{0\ \text{V}}$$

c. $$\text{Emf} = -L\frac{\Delta I}{\Delta t} = -\left(3.2\times10^{-3}\ \text{H}\right)\left(\frac{0\ \text{A} - 4.0\ \text{A}}{9.0\times10^{-3}\ \text{s} - 5.0\times10^{-3}\ \text{s}}\right) = \boxed{+3.2\ \text{V}}$$

47. ***REASONING AND SOLUTION*** The induced emf in the secondary coil is proportional to the mutual inductance. If the primary coil is assumed to be unaffected by the metal, that is $\Delta I_1/\Delta t$ is the same for both cases, then

$$\text{New emf} = 3(0.46\ \text{V}) = \boxed{1.4\ \text{V}}$$

48. ***REASONING AND SOLUTION*** The magnitude of the emf induced in the secondary coil is

$$(\text{Emf})_2 = M\Delta I_1/\Delta t = (2.5 \times 10^{-3}\ \text{H})(3.0\ \text{A})/(0.040\ \text{s}) = 0.188\ \text{V}$$

This emf drives a current in the secondary coil according to Ohm's law:

$$I_2 = (\text{Emf})_2/R_2 = (0.188\ \text{V})/(2.0\ \Omega) = \boxed{0.094\ \text{A}}$$

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49. **SSM** ***REASONING*** The energy density is given by Equation 22.11 as

$$\text{Energy density} = \frac{\text{Energy}}{\text{Volume}} = \frac{1}{2\mu_0} B^2$$

The energy stored is the energy density times the volume.

SOLUTION The volume is the area A times the height h. Therefore, the energy stored is

$$\text{Energy} = \frac{B^2 Ah}{2\mu_0} = \frac{(7.0 \times 10^{-5}\ \text{T})^2 (5.0 \times 10^8\ \text{m}^2)(1500\ \text{m})}{2(4\pi \times 10^{-7}\ \text{T} \cdot \text{m/A})} = \boxed{1.5 \times 10^9\ \text{J}}$$

50. ***REASONING AND SOLUTION*** The energy density in an electric field is Energy density = $\varepsilon_o E^2/2$, and the energy density in a magnetic field is Energy density = $B^2/(2\mu_o)$. Equating and rearranging yields

$$E = B\sqrt{\frac{1}{\varepsilon_0 \mu_0}} = Bc = (12\text{ T})(3.0 \times 10^8\text{ m/s}) = \boxed{3.6 \times 10^9\text{ N/C}}$$

51. ***REASONING AND SOLUTION*** The mutual inductance is $M = N_2\Phi_2/I_1$. The flux through the 125 turn coil is $\Phi_2 = B_1A_2 = \mu_0 n_1 I_1 A_2$, so

$$M = N_2\mu_0 n_1 A_2 = (125)(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})(1750/\text{m})(\pi)(0.0180\ \text{m})^2 = \boxed{2.80 \times 10^{-4}\ \text{H}}$$

52. ***REASONING AND SOLUTION*** If the rectangle is made $\Delta I = I_f$ wide, then the top of the rectangle will intersect the line at LI_f. The work is one-half the area of the rectangle, so

$$\boxed{W = \tfrac{1}{2}\left(LI_f\right)I_f = \tfrac{1}{2}LI_f^2}$$

53. **SSM** ***REASONING*** According to Ohm's law, the average current I induced in the coil is given by $I = (\text{Emf})/R$, where R is the resistance of the coil. To find the induced emf, we use Faraday's law of electromagnetic induction

SOLUTION The magnitude of the induced emf can be found from Faraday's law of electromagnetic induction and is given by Equation 22.3 without the minus sign:

$$\text{Emf} = N\frac{\Delta\Phi}{\Delta t} = N\frac{\Delta\left(BA\cos 0°\right)}{\Delta t} = NA\frac{\Delta B}{\Delta t}$$

We have used the fact that the field within a long solenoid is perpendicular to the cross-sectional area A of the solenoid and makes an angle of 0° with respect to the normal to the area. The field is given by Equation 21.7 as $B = \mu_0 n I$, so the change ΔB in the field is $\Delta B = \mu_0 n \Delta I$, where ΔI is the change in the current. The induced current is, then,

$$I = \frac{NA\dfrac{\Delta B}{\Delta t}}{R} = \frac{NA\dfrac{\mu_0 n\,\Delta I}{\Delta t}}{R}$$

$$= \frac{(10)(6.0\times10^{-4}\ \text{m}^2)\dfrac{(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A})(400\ \text{turns/m})(0.40\ \text{A})}{(0.050\ \text{s})}}{1.5\ \Omega} = \boxed{1.6\times10^{-5}\ \text{A}}$$

54. ***REASONING AND SOLUTION***

a. The transformer is a $\boxed{\text{step-up}}$ transformer.

b. The turns ratio is

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{12\ 000\ \text{V}}{220\ \text{V}} = \boxed{55:1}$$

55. **SSM** ***REASONING AND SOLUTION*** Since the secondary voltage is less than the primary voltage, we can conclude that the transformer used in the doorbell described in the problem is a $\boxed{\text{step-down transformer}}$. The turns ratio is given by the transformer equation, Equation 22.12:

$$\frac{N_s}{N_p} = \frac{V_s}{V_p} = \frac{10.0\text{ V}}{120\text{ V}} = \frac{1}{12}$$

so the turns ratio is $\boxed{1:12}$.

56. ***REASONING AND SOLUTION*** The step down transformer reduces the voltage by a factor of 13.

$$V_2 = (120\ \text{V})/13 = \boxed{9.2\ \text{V}}$$

57. ***REASONING AND SOLUTION*** The power consumed by the air filter is $P_s = I_s V_s$. But according to the transformer equation, we have $V_s = (N_s/N_p)V_p$. Therefore,

$$P_s = I_s V_p (N_s/N_p) = (1.5 \times 10^{-3}\ \text{A})(120\ \text{V})(43) = \boxed{7.7\ \text{W}}$$

58. ***REASONING AND SOLUTION*** According to the transformer equation (Equation 22.12), we have

$$N_s = (V_s/V_p)N_p = [(4320\ \text{V})/(120.0\ \text{V})](21) = \boxed{756}$$

59. **SSM** ***REASONING*** The power lost in heating the wires is given by Equation 20.6b: $P = I^2R$. Before we can use this equation, however, we must determine the total resistance R of the wire and the current that flows through the wire.

SOLUTION

a. The total resistance of one of the wires is equal to the resistance per unit length multiplied by the length of the wire. Thus, we have

$$(5.0\times10^{-2}\ \Omega/\text{km})(7.0\ \text{km}) = 0.35\ \Omega$$

and the total resistance of the transmission line is twice this value or 0.70 Ω. According to Equation 20.6a ($P = IV$), the current flowing into the town is

$$I = \frac{P}{V} = \frac{1.2\times10^6\ \text{W}}{1200\ \text{V}} = 1.0\times10^3\ \text{A}$$

Thus, the power lost in heating the wire is

$$P = I^2R = \left(1.0\times10^3\ \text{A}\right)^2(0.70\ \Omega) = \boxed{7.0\times10^5\ \text{W}}$$

b. According to the transformer equation (Equation 22.12), the stepped-up voltage is

$$V_s = V_p\left(\frac{N_s}{N_p}\right) = (1200\ \text{V})\left(\frac{100}{1}\right) = 1.2\times10^5\ \text{V}$$

According to Equation 20.6a ($P = IV$), the current in the wires is

$$I = \frac{P}{V} = \frac{1.2\times10^6\ \text{W}}{1.2\times10^5\ \text{V}} = 1.0\times10^1\ \text{A}$$

The power lost in heating the wires is now

$$P = I^2R = \left(1.0\times10^1\ \text{A}\right)^2(0.70\ \Omega) = \boxed{7.0\times10^1\ \text{W}}$$

60. ***REASONING*** The starting point here is to realize that the power used to operate the tube comes from the secondary coil and, therefore, is $P_s = I_s V_s$ according to Equation 20.15a. The current I_s in the secondary is given, as is the power P_s. We seek the turns ratio, and the voltage V_s across the secondary is related to the turns ratio by the transformer equation (Equation 22.12).

SOLUTION Using Equation 20.15a for the power delivered by the secondary, we have $P_s = I_s V_s$. According to the transformer equation, the voltage V_s across the secondary is

$$V_s = \left(\frac{N_s}{N_p}\right) V_p$$

Substituting this result into the power expression, we find

$$P_s = I_s V_s = I_s \left(\frac{N_s}{N_p}\right) V_p$$

Rearranging this result shows that the turns ratio is

$$\frac{N_s}{N_p} = \frac{P_s}{I_s V_p} = \frac{95\ \text{W}}{\left(5.3 \times 10^{-3}\ \text{A}\right)\left(120\ \text{V}\right)} = \boxed{150}$$

61. ***REASONING AND SOLUTION*** Ohm's law written for the secondary is $V_s = I_s R_2$.

We know that

$$V_s = (N_s/N_p)V_p \qquad \text{and} \qquad I_s = (N_p/N_s)I_p$$

Substituting these expressions for V_s and I_s into $V_s = I_s R_2$ and recognizing that $R_1 = V_P/I_P$, we find that

$$\boxed{R_1 = \left(\frac{N_p}{N_s}\right)^2 R_2}$$

62. ***REASONING AND SOLUTION*** Faraday's law gives for the magnitude of the change in the magnetic field

$$\Delta B = \frac{(\text{Emf})\Delta t}{NA} = \frac{(1.5\text{ V})(0.050\text{ s})}{(100)(0.040\text{ m})(0.060\text{ m})} = \boxed{0.31\text{ T}}$$

63. ***REASONING AND SOLUTION*** An rms voltage of 120 V corresponds to a maximum emf of $(120\text{ V})\sqrt{2}$ or 170 V. The maximum voltage produced by the generator is $NAB\omega$, so

$$N = \frac{170\text{ V}}{\left(0.016\text{ m}^2\right)\left(7.0\times10^{-5}\text{ T}\right)2\pi\left(60.0\text{ Hz}\right)} = \boxed{4.0\times10^{5}}$$

64. ***REASONING*** Using Equation 22.3 (Faraday's law) without the minus sign and recognizing that $N = 1$, we can write the magnitudes of the emf for parts *a* and *b* as follows:

$$(\text{Emf})_a = \left(\frac{\Delta\Phi}{\Delta t}\right)_a \quad (1) \qquad \text{and} \qquad (\text{Emf})_b = \left(\frac{\Delta\Phi}{\Delta t}\right)_b \quad (2)$$

To solve this problem, we need to consider the change in flux $\Delta\Phi$ and the time interval Δt for both parts of the drawing.

SOLUTION The change in flux is the same for both parts of the drawing and is given by

$$(\Delta\Phi)_a = (\Delta\Phi)_b = \Phi_{\text{inside}} - \Phi_{\text{outside}} = \Phi_{\text{inside}} = BA \quad (3)$$

In Equation (3) we have used the fact that initially the coil is outside the field region, so that $\Phi_{\text{outside}} = 0$ Wb for both cases. Moreover, the field is perpendicular to the plane of the coil and has the same magnitude B over the entire area A of the coil, once it has completely entered the field region. Thus, $\Phi_{\text{inside}} = BA$ in both cases, according to Equation 22.2.

The time interval required for the coil to entire the field region completely can be expressed as the distance the coil travels divided by the speed at which it is pushed. In part *a* of the drawing the distance traveled is W, while in part *b* it is L. Thus, we have

$$(\Delta t)_a = W/v \quad (4) \qquad\qquad (\Delta t)_b = L/v \quad (5)$$

Substituting Equations (3), (4), and (5) into Equations (1) and (2), we find

$$(\text{Emf})_a = \left(\frac{\Delta\Phi}{\Delta t}\right)_a = \frac{BA}{W/v} \quad (6) \qquad\qquad (\text{Emf})_b = \left(\frac{\Delta\Phi}{\Delta t}\right)_b = \frac{BA}{L/v} \quad (7)$$

Dividing Equation (6) by Equation (7) gives

$$\frac{(\text{Emf})_a}{(\text{Emf})_b} = \frac{\dfrac{BA}{W/v}}{\dfrac{BA}{L/v}} = \frac{L}{W} = 3.0 \qquad \text{or} \qquad (\text{Emf})_b = \frac{(\text{Emf})_a}{3.0} = \frac{0.15\text{ V}}{3.0} = \boxed{0.050\text{ V}}$$

65. **SSM** ***REASONING AND SOLUTION*** According to Faraday's law, only the relative motion between the coil and magnet is significant.. It makes no difference if the coil is held fixed and the magnet is moved or the magnet is held fixed and the coil is moved. The results are the same. Therefore,

a. The current will flow from **left to right** as shown in Figure 22.1*b*.

b. The current will flow from **right to left** as shown in Figure 22.1*c*.

66. ***REASONING AND SOLUTION*** The resistance of the primary is

$$R_p = \rho L_p/A \tag{1}$$

The resistance of the secondary is

$$R_s = \rho L_s/A \tag{2}$$

In writing Equations (1) and (2) we have assumed that both coils are made of the same wire, so that the resistivity ρ and the cross-sectional area A is the same for each. Division of the equations gives

$$R_s/R_p = L_s/L_p = (14\ \Omega)/(56\ \Omega) = 1/4$$

The lengths of the wires are proportional to the number of turns so

$$N_s/N_p = L_s/L_p = \boxed{1/4}$$

67. **SSM** ***REASONING AND SOLUTION*** The emf induced in the coil of an ac generator is given by Equation 22.4 as

$$\text{Emf} = NAB\omega \sin\omega t = (500)(1.2\times10^{-2}\ \text{m}^2)(0.13\ \text{T})(34\ \text{rad/s})\sin 27° = \boxed{12\ \text{V}}$$

68. ***REASONING AND SOLUTION*** The energy stored in a capacitor is given by Equation 19.11 as $\frac{1}{2}CV^2$. The energy stored in an inductor is given by Equation 22.10 as $\frac{1}{2}LI^2$. Setting these two equations equal to each other and solving for the current I, we get

$$I = \sqrt{\frac{C}{L}}\,V = \sqrt{\frac{3.0 \times 10^{-6}\ \text{F}}{5.0 \times 10^{-3}\ \text{H}}}\,(35\ \text{V}) = \boxed{0.86\ \text{A}}$$

69. ***REASONING AND SOLUTION*** Faraday's law can be used if we can find the flux change through the triangle as the bar slides.

$$\Delta\Phi = B\Delta A = B(bh/2 - b_0h_0/2)$$

where the b's and h's represent the bases and heights of the triangle. At time zero, the height and base of the triangle are both zero. At time, $\Delta t = t = 5.0$ s, $h = b \tan 15°$ and $b = vt$. Now

$$\Delta\Phi = (1/2)\, B(vt)^2 \tan 15°$$

Faraday's law gives the magnitude of the emf to be

$$\text{Emf} = \Delta\Phi/\Delta t = (1/2)\,(Bv^2t) \tan 15°$$

$$\text{Emf} = (1/2)\,(0.42\text{ T})(0.40\text{ m/s})^2(5.0\text{ s}) \tan 15° = \boxed{0.045\text{ V}}$$

70. ***REASONING AND SOLUTION*** If the applied magnetic field is decreasing in time, then the flux through the circuit is decreasing. Lenz's law requires that an induced magnetic field be produced which attempts to counteract this decrease; hence its direction is out of the paper. The sense of the induced current in the circuit must be CCW. Therefore, the lower plate of the capacitor is positive while the upper plate is negative. The electric field between the plates of the capacitor points from positive to negative so the electric field points $\boxed{\text{upward}}$.

71. **SSM** ***REASONING*** According to Equation 22.3, the average emf induced in a single loop is $\text{emf} = -\Delta\Phi/\Delta t$. Since the magnitude of the magnetic field is changing, the area of the loop remains constant, and the direction of the field is parallel to the normal to the loop, the change in flux through the loop is given by $\Delta\Phi = (\Delta B)A$. Thus the magnitude of the induced emf in the loop is given by $\text{Emf} = (\Delta B)A/\Delta t$.

Similarly, when the area of the loop is changed and the field B has a given value, we find the magnitude of the induced emf to be $\text{Emf} = B\Delta A/\Delta t$.

SOLUTION

a. The magnitude of the induced emf when the field changes in magnitude is

$$\text{Emf} = \frac{(\Delta B)A}{\Delta t} = (0.20\ \text{T/s})(0.018\ \text{m}^2) = \boxed{3.6\times10^{-3}\ \text{V}}$$

b. At a particular value of B (when B is changing), the rate at which the area must change can be obtained from

$$\text{Emf} = \frac{B\Delta A}{\Delta t} \quad \text{or} \quad \frac{\Delta A}{\Delta t} = \frac{\text{Emf}}{B} = \frac{3.6\times10^{-3}\ \text{V}}{1.8\ \text{T}} = \boxed{2.0\times10^{-3}\ \text{m}^2/\text{s}}$$

In order for the induced emf to be zero, the magnitude of the magnetic field and the area of the loop must change in such a way that the flux remains constant. Since the magnitude of the magnetic field is increasing, the area of the loop must decrease, if the flux is to remain constant. Therefore, $\boxed{\text{the area of the loop must be shrunk}}$.

72. ***REASONING*** From the results of Example 13, the self-inductance L of a solenoid is given by

$$L = \mu_0 n^2 A \ell \qquad (1)$$

where ℓ is the length of the solenoid. A toroid is a solenoid that is bent to form a circle of radius R. The length ℓ of the toroid is the circumference of the circle, $\ell = 2\pi R$. Substituting this value for ℓ into the equation above, we can obtain and expression for the self-inductance of the toroid.

SOLUTION Substituting $\ell = 2\pi R$ into Equation (1) gives

$$L = 2\pi\mu_0 n^2 AR$$

$$= 2\pi\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(2400\ \text{m}^{-1}\right)^2\left(1.0\times 10^{-6}\ \text{m}^2\right)\left(0.050\ \text{m}\right) = \boxed{2.3\times 10^{-6}\ \text{H}}$$

73. ***REASONING AND SOLUTION***

a. Using Equation 22.5, we find

$$I = (V - \text{Emf})/R = (120.0\ \text{V} - 108\ \text{V})/(15.0\ \Omega) = \boxed{0.80\ \text{A}}$$

b. If the armature is not rotating, then the back emf is zero and

$$I = V/R = (120.0\ \text{V})/(15.0\ \Omega) = \boxed{8.00\ \text{A}}$$

c. Since the back emf is proportional to the angular speed of the armature, the half speed emf is Emf = (108 V)/2 = 54.0 V. The current is then

$$I = (120.0\ \text{V} - 54.0\ \text{V})/(15.0\ \Omega) = \boxed{4.40\ \text{A}}$$

74. ***REASONING AND SOLUTION*** The mutual inductance is

$$M = \frac{N_2 \Phi_2}{I_1}$$

The flux through loop 2 is

$$\Phi_2 = B_1 A_2 = \left(\frac{\mu_0 N_1 I_1}{2 R_1} \right) \left(\pi R_2^2 \right)$$

Then

$$M = \frac{N_2 \Phi_2}{I_1} = \frac{N_2}{I_1} \left(\frac{\mu_0 N_1 I_1}{2 R_1} \right) \left(\pi R_2^2 \right) = \boxed{\frac{\mu_0 \pi N_1 N_2 R_2^2}{2 R_1}}$$

1. **SSM** ***REASONING AND SOLUTION*** The capacitive reactance is given by Equation 23.2: $X_C = \dfrac{1}{2\pi f C}$. Solving for the frequency f, we find that

$$f = \frac{1}{2\pi X_C C} = \frac{1}{2\pi(168\ \Omega)(7.50\times10^{-6}\ \text{F})} = \boxed{126\ \text{Hz}}$$

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2. ***REASONING AND SOLUTION*** The rms voltage can be calculated using $V = IX_C$, where the capacitive reactance X_C can be found using

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(3.4 \times 10^3 \text{ Hz})(0.86 \times 10^{-6} \text{ F})} = \boxed{54\ \Omega}$$

The voltage is, therefore,

$$V = IX_C = (35 \times 10^{-3} \text{ A})(54\ \Omega) = \boxed{1.9 \text{ V}}$$

3. ***REASONING*** We will first find the capacitance C using the relation $C = 1/(2\pi f X_C)$, where f is the frequency and X_C is the capacitive reactance. The value of the capacitive reactance at a frequency of 510 Hz can then be determined by using this value of the capacitance.

SOLUTION The capacitance is

$$C = \frac{1}{2\pi f X_C} = \frac{1}{2\pi(170\text{ Hz})(36\,\Omega)} = 2.6\times10^{-5}\text{ F} \tag{23.2}$$

When the frequency is 510 Hz, the capacitive reactance is

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(510\text{ Hz})(2.6\times10^{-5}\text{ F})} = \boxed{12\,\Omega}$$

4. ***REASONING*** The rms current in a capacitor is $I_{rms} = V_{rms}/X_C$, according to Equation 23.1. The capacitive reactance is $X_C = 1/(2\pi f C)$, according to Equation 23.2. For the first capacitor, we use $C = C_1$ in these expressions. For the two capacitors in parallel, we use $C = C_P$, where C_P is the equivalent capacitance from Equation 20.18 ($C_P = C_1 + C_2$). Taking the difference between the currents and using the given data, we can obtain the desired value for C_2. The capacitance C_1 is unknown, but it will be eliminated algebraically from the calculation.

SOLUTION Using Equations 23.1 and 23.2, we find that the current in a capacitor is

$$I_{rms} = \frac{V_{rms}}{X_C} = \frac{V_{rms}}{1/(2\pi f C)} = V_{rms}\, 2\pi f C$$

Applying this result to the first capacitor and the parallel combination of the two capacitors, we obtain

$$\underbrace{I_1 = V_{rms}\, 2\pi f C_1}_{\text{Single capacitor}} \quad \text{and} \quad \underbrace{I_{Combination} = V_{rms}\, 2\pi f (C_1 + C_2)}_{\text{Parallel combination}}$$

Subtracting I_1 from $I_{Combination}$, reveals that

$$I_{Combination} - I_1 = V_{rms}\, 2\pi f (C_1 + C_2) - V_{rms}\, 2\pi f C_1 = V_{rms}\, 2\pi f C_2$$

Solving for C_2 gives

$$C_2 = \frac{I_{Combination} - I_1}{V_{rms}\, 2\pi f} = \frac{0.18\ \text{A}}{(24\ \text{V})2\pi(440\ \text{Hz})} = \boxed{2.7 \times 10^{-6}\ \text{F}}$$

5. SSM ***REASONING AND SOLUTION***

a. The equivalent capacitance C_s of two capacitors in series is given by Equation 20.19 as

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{3.00 \times 10^{-6}\ \text{F}} + \frac{1}{6.00 \times 10^{-6}\ \text{F}} \qquad \text{or} \qquad \boxed{C_s = 2.00\ \mu\text{F}}$$

b. The current in the circuit can be found by solving Equation 23.1, $V_{\text{rms}} = I_{\text{rms}} X_C$, for I_{rms}. However, we must first find the capacitive reactance X_C. From Equation 23.2, we have

$$X_C = \frac{1}{2\pi f C_s} = \frac{1}{2\pi(510\ \text{Hz})(2.00 \times 10^{-6}\ \text{F})} = 156\ \Omega$$

The current in the circuit is given by

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_C} = \frac{120\ \text{V}}{156\ \Omega} = \boxed{0.77\ \text{A}}$$

6. ***REASONING*** The capacitance C is related to the capacitive reactance X_C and the frequency f via Equation 23.2 as $C = 1/(2\pi f X_C)$. The capacitive reactance, in turn is related to the rms-voltage V_{rms} and the rms-current I_{rms} by $X_C = V_{rms}/I_{rms}$ (see Equation 23.1). Thus, the capacitance can be written as $C = I_{rms}/(2\pi f V_{rms})$. The magnitude of the maximum charge q on one plate of the capacitor is, from Equation 19.8, the product of the capacitance C and the peak voltage V.

SOLUTION

a. Recall that the rms-voltage V_{rms} is related to the peak voltage V by $V_{rms} = \frac{V}{\sqrt{2}}$. The capacitance is, then,

$$C = \frac{I_{rms}}{2\pi f V_{rms}} = \frac{3.0\text{ A}}{2\pi(750\text{ Hz})\left(\frac{140\text{ V}}{\sqrt{2}}\right)} = \boxed{6.4\times10^{-6}\text{ F}}$$

b. The maximum charge on one plate of the capacitor is

$$q = CV = (6.4\times10^{-6}\text{ F})(140\text{ V}) = \boxed{9.0\times10^{-4}\text{ C}}$$

7. ***REASONING AND SOLUTION*** Equations 23.1 and 23.2 indicate that the rms current in a capacitor is $I = V / X_C$, where V is the rms voltage and $X_C = 1/(2\pi f C)$. Therefore, the current is $I = V 2\pi f C$. For a single capacitor $C = C_1$, and we have

$$I = V 2\pi f C_1$$

For two capacitors in series, Equation 20.19 indicates that the equivalent capacitance can be obtained from $C^{-1} = C_1^{-1} + C_2^{-1}$, which can be solved to show that $C = C_1 C_2 / (C_1 + C_2)$. The total series current is, then,

$$I_{\text{series}} = V 2\pi f C = V 2\pi f \left(\frac{C_1 C_2}{C_1 + C_2} \right)$$

The series current is one-third of the current I. It follows, therefore, that

$$\frac{I_{\text{series}}}{I} = \frac{V 2\pi f \left(\frac{C_1 C_2}{C_1 + C_2} \right)}{V 2\pi f C_1} = \frac{C_2}{C_1 + C_2} = \frac{1}{3} \quad \text{or} \quad \frac{C_1}{C_2} = 2$$

For two capacitors in parallel, Equation 20.18 indicates that the equivalent capacitance is $C = C_1 + C_2$. The total current in this case is

$$I_{\text{parallel}} = V 2\pi f C = V 2\pi f (C_1 + C_2)$$

The ratio of I_{parallel} to the current I in the single capacitor is

$$\frac{I_{\text{parallel}}}{I} = \frac{V 2\pi f (C_1 + C_2)}{V 2\pi f C_1} = \frac{C_1 + C_2}{C_1} = 1 + \frac{C_2}{C_1} = 1 + \frac{1}{2} = \boxed{\frac{3}{2}}$$

8. ***REASONING AND SOLUTION*** We know that $V = IX_L = I(2\pi fL)$, so the frequency is

$$f = \frac{V}{2\pi IL} = \frac{2.1\text{ V}}{2\pi(0.023\text{ A})(0.047\text{ H})} = \boxed{310\text{ Hz}}$$

9. SSM ***REASONING*** The individual reactances are given by Equations 23.2 and 23.4, respectively,

[Capacitive reactance] $$X_C = \frac{1}{2\pi f C}$$

[Inductive reactance] $$X_L = 2\pi f L$$

When the reactances are equal, we have $X_C = X_L$, from which we find

$$\frac{1}{2\pi f C} = 2\pi f L \qquad \text{or} \qquad 4\pi^2 f^2 LC = 1$$

The last expression may be solved for the frequency f.

SOLUTION Solving for f with $L = 52 \times 10^{-3}$ H and $C = 76 \times 10^{-6}$ F, we obtain

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(52\times10^{-3}\ \text{H})(76\times10^{-6}\ \text{F})}} = \boxed{8.0\times10^{1}\ \text{Hz}}$$

10. ***REASONING AND SOLUTION*** The reactance of an inductor is given by Equation 23.4 as $X_L = 2\pi f L$. At the frequencies $f_1 = 1350\text{ Hz}$ and $f_2 = 450\text{ Hz}$, we can use this equation as follows:

$$\frac{X_{L2}}{X_{L1}} = \frac{2\pi f_2 L}{2\pi f_1 L} = \frac{f_2}{f_1} \qquad \text{or} \qquad X_{L2} = X_{L1}\left(\frac{f_2}{f_1}\right) = (480\,\Omega)\left(\frac{450\text{ Hz}}{1350\text{ Hz}}\right) = \boxed{160\,\Omega}$$

11. SSM WWW ***REASONING*** The rms current can be calculated from Equation 23.3, $I_{\text{rms}} = V_{\text{rms}} / X_{\text{L}}$, provided that the inductive reactance is obtained first. Then the peak value of the current I_0 supplied by the generator can be calculated from the rms current I_{rms} by using Equation 20.12, $I_0 = \sqrt{2}\, I_{\text{rms}}$.

SOLUTION At the frequency of $f = 620\ \text{Hz}$, we find, using Equations 23.4 and 23.3, that

$$X_{\text{L}} = 2\pi f L = 2\pi(620\ \text{Hz})(8.2 \times 10^{-3}\ \text{H}) = 32\ \Omega$$

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_{\text{L}}} = \frac{10.0\ \text{V}}{32\ \Omega} = 0.31\ \text{A}$$

Therefore, from Equation 20.12, we find that the *peak value* I_0 of the current supplied by the generator must be

$$I_0 = \sqrt{2}\, I_{\text{rms}} = \sqrt{2}\,(0.31\ \text{A}) = \boxed{0.44\ \text{A}}$$

12. ***REASONING*** The rms current in an inductor is $I_{\text{rms}} = V_{\text{rms}}/X_{\text{L}}$, according to Equation 23.3. The inductive reactance is $X_{\text{L}} = 2\pi f L$, according to Equation 23.4. Applying these expressions to both generators will allow us to obtain the desired current.

SOLUTION Using Equations 23.3 and 23.4, we find that the current in an inductor is

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{X_{\text{L}}} = \frac{V_{\text{rms}}}{2\pi f L}$$

Applying this result to the two generators gives

$$\underbrace{I_1 = \frac{V_{\text{rms}}}{2\pi f_1 L}}_{\text{Generator 1}} \quad \text{and} \quad \underbrace{I_2 = \frac{V_{\text{rms}}}{2\pi f_2 L}}_{\text{Generator 2}}$$

Dividing the equation for generator 2 by the equation for generator 1, we obtain

$$\frac{I_2}{I_1} = \frac{\dfrac{V_{\text{rms}}}{2\pi f_2 L}}{\dfrac{V_{\text{rms}}}{2\pi f_1 L}} = \frac{f_1}{f_2} \quad \text{or} \quad I_2 = I_1 \frac{f_1}{f_2} = (0.30\ \text{A})\frac{1.5\ \text{kHz}}{6.0\ \text{kHz}} = \boxed{0.075\ \text{A}}$$

13. ***REASONING AND SOLUTION*** The inductance of the solenoid is given by (see Section 22.8)

$$L = \mu_0 N^2 A/l = (4\pi \times 10^{-7}\ \text{T·m/A})(135)^2\ (3.1 \times 10^{-5}\ \text{m}^2)/(0.025\ \text{m}) = 2.8 \times 10^{-5}\ \text{H}$$

Thus,

$$X_L = 2\pi f L = 2\pi(18 \times 10^3\ \text{Hz})(2.8 \times 10^{-5}\ \text{H}) = 3.2\ \Omega$$

The rms voltage is

$$V = IX_L = (0.036\ \text{A})(3.2\ \Omega) = 0.12\ \text{V}$$

The *peak* voltage V_0 can be obtained from the rms-voltage V as

$$V_0 = V\sqrt{2} = (0.12\ \text{V})\sqrt{2} = \boxed{0.17\ \text{V}}$$

14. ***REASONING AND SOLUTION*** Equations 23.3 and 23.4 indicate that the rms current in a single inductance L_1 is $I_1 = V/X_{L1}$, where V is the rms voltage and $X_{L1} = 2\pi f L_1$. Therefore, the current is $I_1 = V/(2\pi f L_1)$. Similarly, the current in the second inductor connected across the terminals of the generator is $I_2 = V/(2\pi f L_2)$. The total current delivered by the generator is the sum of these two values:

$$I_{\text{total}} = I_1 + I_2 = \frac{V}{2\pi f L_1} + \frac{V}{2\pi f L_2}$$

But this same total current is delivered to the single inductance L, so it also follows that $I_{\text{total}} = V/(2\pi f L)$. Equating the two expressions for I_{total} shows that

$$\frac{V}{2\pi f L} = \frac{V}{2\pi f L_1} + \frac{V}{2\pi f L_2} \qquad \text{or} \qquad \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$$

Using this result, we determine the value of L as follows:

$$\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2} \qquad \text{or} \qquad L = \frac{L_1 L_2}{L_1 + L_2} = \frac{(0.030\ \text{H})(0.060\ \text{H})}{0.030\ \text{H} + 0.060\ \text{H}} = \boxed{0.020\ \text{H}}$$

15. **SSM** ***REASONING*** The voltage supplied by the generator can be found from Equation 23.6, $V_{\text{rms}} = I_{\text{rms}} Z$. The value of I_{rms} is given in the problem statement, so we must obtain the impedance of the circuit.

SOLUTION The impedance of the circuit is, according to Equation 23.7,

$$Z = \sqrt{R^2 + (X_{\text{L}} - X_{\text{C}})^2} = \sqrt{(275\ \Omega)^2 + (648\ \Omega - 415\ \Omega)^2} = 3.60 \times 10^2\ \Omega$$

The rms voltage of the generator is

$$V_{\text{rms}} = I_{\text{rms}} Z = (0.233\ \text{A})(3.60 \times 10^2\ \Omega) = \boxed{83.9\ \text{V}}$$

16. ***REASONING AND SOLUTION***

a. At a frequency of 609 Hz:

$$X_L = 2\pi fL = 2\pi(609\text{ Hz})(0.0310\text{ H}) = 119\ \Omega$$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(609\text{ Hz})(3.30\times10^{-6}\text{ F})} = 79.2\ \Omega$$

so,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(106\ \Omega)^2 + (119\ \Omega - 79.2\ \Omega)^2} = \boxed{113\ \Omega}$$

b. The phase angle between the current and the voltage is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{119\ \Omega - 79.2\ \Omega}{106\ \Omega}\right) = \boxed{+21°}$$

17. ***REASONING AND SOLUTION*** The power factor is $\cos\phi$, where ϕ can be obtained from Equation 23.8 $\left[\tan\phi = \left(X_L - X_C\right)/R\right]$. To obtain ϕ, we need values for the capacitive and inductive reactances, X_C and X_L, which are obtained from Equations 23.2 and 23.4, respectively:

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(2550\text{ Hz})(2.00\times10^{-6}\text{ F})} = 31.2\ \Omega$$

$$X_L = 2\pi f L = 2\pi(2550\text{ Hz})(4.00\times10^{-3}\text{ H}) = 64.1\ \Omega$$

Using these values and the given value for R, we can obtain the power factor:

$$\tan\phi = \frac{X_L - X_C}{R} \qquad \text{or} \qquad \phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{64.1\ \Omega - 31.2\ \Omega}{47.0\ \Omega}\right) = 35.0°$$

Thus, the power factor is $\cos\phi = \cos 35.0° = \boxed{0.819}$.

18. ***REASONING AND SOLUTION*** In order to find the voltage and phase angle we need to know the impedance of the circuit. We know that

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi\left(4.80\times10^3\ \text{Hz}\right)\left(0.250\times10^{-6}\ \text{F}\right)} = 133\ \Omega$$

The impedance of the circuit can be found using

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{\left(232\ \Omega\right)^2 + \left(0 - 133\ \Omega\right)^2} = 267\ \Omega$$

a. The voltage of the generator is

$$V = IZ = (0.0400\ \text{A})(267\ \Omega) = \boxed{10.7\ \text{V}}$$

b. The phase angle is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 133\ \Omega}{232\ \Omega}\right) = \boxed{-29.8°}$$

19. **SSM** ***REASONING*** We can use the equations for a series RCL circuit to solve this problem provided that we set $X_C = 0$ since there is no capacitor in the circuit. The current in the circuit can be found from Equation 23.6, $V_{rms} = I_{rms} Z$, once the impedance of the circuit has been obtained. Equation 23.8, $\tan\phi = (X_L - X_C)/R$, can then be used (with $X_C = 0$) to find the phase angle between the current and the voltage.

SOLUTION The inductive reactance is (Equation 23.4)

$$X_L = 2\pi f L = 2\pi(106\text{ Hz})(0.200\text{ H}) = 133\,\Omega$$

The impedance of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + X_L^2} = \sqrt{(215\,\Omega)^2 + (133\,\Omega)^2} = 253\,\Omega$$

a. The current through each circuit element is, using Equation 23.6,

$$I_{rms} = \frac{V_{rms}}{Z} = \frac{234\text{ V}}{253\,\Omega} = \boxed{0.925\text{ A}}$$

b. The phase angle between the current and the voltage is, according to Equation 23.8 (with $X_C = 0$),

$$\tan\phi = \frac{X_L - X_C}{R} = \frac{X_L}{R} = \frac{133\,\Omega}{215\,\Omega} = 0.619$$

$$\phi = \tan^{-1}(0.619) = \boxed{31.8°}$$

20. ***REASONING*** The average power dissipated is that dissipated in the resistor and is $\overline{P} = I_{\text{rms}}^2 R$, according to Equation 20.15b. We are given the current I_{rms} but need to find the resistance R. Since the inductive reactance X_{L} is known, we can find the resistance from the impedance, which is $Z = \sqrt{R^2 + X_{\text{L}}^2}$, according to Equation 23.7. Since the voltage and the current are known, we can obtain the impedance from Equation 23.6 as $Z = V_{\text{rms}}/I_{\text{rms}}$.

SOLUTION From Equation 23.7, we can determine the resistance as $R = \sqrt{Z^2 - X_{\text{L}}^2}$. With this expression for the resistance, Equation 20.15b for the power becomes

$$\overline{P} = I_{\text{rms}}^2 R = I_{\text{rms}}^2 \sqrt{Z^2 - X_{\text{L}}^2}$$

Using Equation 23.6 to express the impedance, we obtain the following value for the dissipated power

$$\overline{P} = I_{\text{rms}}^2 \sqrt{Z^2 - X_{\text{L}}^2} = I_{\text{rms}}^2 \sqrt{\left(\frac{V_{\text{rms}}}{I_{\text{rms}}}\right)^2 - X_{\text{L}}^2}$$

$$= (1.75\ \text{A})^2 \sqrt{\left(\frac{115\ \text{V}}{1.75\ \text{A}}\right)^2 - (52.0\ \Omega)^2} = \boxed{123\ \text{W}}$$

21. SSM WWW ***REASONING*** We can use the equations for a series RCL circuit to solve this problem, provided that we set $X_L = 0$ since there is no inductance in the circuit. Thus, according to Equations 23.6 and 23.7, the current in the circuit is $I_{rms} = V_{rms} / \sqrt{R^2 + X_C^2}$. When the frequency f is very large, the capacitive reactance is zero, or $X_C = 0$, in which case the current becomes $I_{rms}(\text{large } f) = V_{rms} / R$. When the current I_{rms} in the circuit is one-half the value of $I_{rms}(\text{large } f)$ that exists when the frequency is very large, we have

$$\frac{I_{rms}}{I_{rms}(\text{large } f)} = \frac{1}{2}$$

We can use these expressions to write the ratio above in terms of the resistance and the capacitive reactance. Once the capacitive reactance is known, the frequency can be determined.

SOLUTION The ratio of the currents is

$$\frac{I_{rms}}{I_{rms}(\text{large } f)} = \frac{V_{rms} / \sqrt{R^2 + X_C^2}}{V_{rms} / R} = \frac{R}{\sqrt{R^2 + X_C^2}} = \frac{1}{2} \qquad \text{or} \qquad \frac{R^2}{R^2 + X_C^2} = \frac{1}{4}$$

Taking the reciprocal of this result gives

$$\frac{R^2 + X_C^2}{R^2} = 4 \qquad \text{or} \qquad 1 + \frac{X_C^2}{R^2} = 4$$

Therefore,

$$\frac{X_C}{R} = \sqrt{3}$$

According to Equation 23.2, $X_C = 1/(2\pi f C)$, so it follows that

$$\frac{X_C}{R} = \frac{1/(2\pi f C)}{R} = \sqrt{3}$$

Thus,

$$f = \frac{1}{2\pi RC\sqrt{3}} = \frac{1}{2\pi(85\,\Omega)(4.0 \times 10^{-6}\ \text{F})\sqrt{3}} = \boxed{270\ \text{Hz}}$$

22. ***REASONING AND SOLUTION***

a. At very large frequencies the impedance of the inductor becomes much larger than that of the capacitor or the resistances of the two resistors. Therefore, current flows only through the top branch of the circuit, i.e., through the capacitor and 290 Ω resistor. For a very large frequency, $X_C = 0$, so $Z = R = 290\ \Omega$. Therefore,

$$I = V/Z = (75\ \text{V})/(290\ \Omega) = \boxed{0.26\ \text{A}}$$

b. At very small frequencies the impedance of the capacitor becomes much larger than that of the inductor or the resistances of the two resistors. Therefore, current flows only through the middle branch of the circuit, i.e., through the inductor and 710 Ω resistor. At very small frequencies, $X_L = 0$, so $Z = R = 710\ \Omega$. Therefore,

$$I = V/Z = (75\ \text{V})/(710\ \Omega) = \boxed{0.11\ \text{A}}$$

23. ***REASONING*** The instantaneous value of the generator voltage is given by $V(t) = V_0 \sin 2\pi ft$, where V_0 is the peak voltage and f is the frequency. We will see that the inductive reactance is greater than the capacitive reactance, $X_L > X_C$, so that the current in the circuit *lags* the voltage by $\pi/2$ radians, or 90°. Thus, the current in the circuit obeys the relation $I(t) = I_0 \sin(2\pi ft - \pi/2)$, where I_0 is the peak current.

SOLUTION

a. The instantaneous value of the voltage at a time of 1.5×10^{-4} s is

$$V(t) = V_0 \sin 2\pi f t = (64.0\text{ V})\sin\left[2\pi(1.00\times10^3\text{ Hz})(1.50\times10^{-4}\text{ s})\right] = \boxed{51.8\text{ V}}$$

Note: When evaluating the sine function in the expression above, be sure to set your calculator to the *radian* mode.

b. The inductive and capacitive reactances are

$$X_L = 2\pi f L = 2\pi(1.00\times10^3\text{ Hz})(4.30\times10^{-3}\text{ H}) = 27.02\,\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(1.00\times10^3\text{ Hz})(8.80\times10^{-6}\text{ F})} = 18.08\,\Omega$$

Since X_L is greater than X_C, the current lags the voltage by $\pi/2$ radians. Thus, the instantaneous current in the circuit is $I(t) = I_0 \sin(2\pi ft - \pi/2)$, where $I_0 = V_0/Z$. The impedance Z of the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{0 + (27.0\,\Omega - 18.1\,\Omega)^2} = 8.94\,\Omega$$

The instantaneous current is

$$I = \frac{V_0}{Z}\sin\left(2\pi f t - \tfrac{1}{2}\pi\right) = \frac{64.0\text{ V}}{8.94\,\Omega}\sin\left[2\pi(1.00\times10^3\text{ Hz})(1.50\times10^{-4}\text{ s}) - \tfrac{1}{2}\pi\right] = \boxed{-4.21\text{ A}}$$

24. ***REASONING AND SOLUTION*** At very low frequencies the capacitors behave as if they were cut out of the circuit, while the inductors behave as if they were replaced with wires that have zero resistance. The circuit behaves as shown in drawing A, and the current delivered by the generator in the limit of very low frequency is $I_{\text{low frequency}} = V / R_1$.

A very high frequencies the capacitors behave as if they were replaced with wires that have zero resistance, while the inductors behave as if they were cut out of the circuit. The circuit now behaves as in drawing B, and the two resistors are in series. The current at very high frequencies is $I_{\text{high frequency}} = V / \left(R_1 + R_2\right)$.

The ratio of the currents is known, so that we can obtain the ratio of the resistances:

$$\frac{I_{\text{low frequency}}}{I_{\text{high frequency}}} = \frac{V / R_1}{V / \left(R_1 + R_2\right)} = \frac{R_1 + R_2}{R_1} = 1 + \frac{R_2}{R_1} = 4 \qquad \text{or} \qquad \boxed{\frac{R_2}{R_1} = 3}$$

A: Low frequency

B: High frequency

25. ***REASONING AND SOLUTION*** With only the resistor in the circuit, the power dissipated is $P_1 = V_0^2/R = 1.000$ W. Therefore, $V_0^2 = 1.000\ R$. When the capacitor is added in series with the resistor, the power dissipated is given by $P_2 = I_2 V_0 \cos\phi = 0.500$ W, where $\cos\phi$ is the power factor, with $\cos\phi = R/Z_2$, and $I_2 = V_0/Z_2$. Substituting yields,

$$P_2 = V_0^2 R/Z_2^2 = R^2/(R^2 + X_C^2) = 0.500 \text{ W}$$

Solving for X_C gives, $X_C = R$. When the inductor is added in series with the resistor, we have $P_3 = V_0 I_3 \cos\phi = 0.250$ W, where $I_3 = V_0/Z_3$ and $\cos\phi = R/Z_3$. Thus,

$$P_3 = R^2/(R^2 + X_L^2) = 0.250 \text{ W}$$

Solving for X_L, we find that $X_L = R\sqrt{3}$. Finally, when both the inductor and capacitor are added in series with the resistor we have

$$P_4 = \frac{V_0^2 R}{R^2 + \left(X_L - X_C\right)^2} = \frac{R^2}{R^2 + \left(R\sqrt{3} - R\right)^2} = \boxed{0.651 \text{ W}}$$

26. ***REASONING AND SOLUTION*** The capacitance can be obtained from Equation 23.10:

$$C = \frac{1}{4\pi^2 L f_0^2} = \frac{1}{4\pi^2 \left(15 \times 10^{-6}\ \text{H}\right)\left(740 \times 10^3\ \text{Hz}\right)^2} = \boxed{3.1 \times 10^{-9}\ \text{F}}$$

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27. **SSM** ***REASONING*** The current in an RCL circuit is given by Equation 23.6, $I_{\text{rms}} = V_{\text{rms}} / Z$, where the impedance Z of the circuit is given by Equation 23.7 as $Z = \sqrt{R^2 + (X_{\text{L}} - X_{\text{C}})^2}$. The current is a maximum when the impedance is a minimum for a given generator voltage. The minimum impedance occurs when the frequency is f_0, corresponding to the condition that $X_{\text{L}} = X_{\text{C}}$, or $2\pi f_0 L = 1/(2\pi f_0 C)$. Solving for the frequency f_0, called the resonant frequency, we find that

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

Note that the resonant frequency depends on the inductance and the capacitance, but does not depend on the resistance.

SOLUTION

a. The frequency at which the current is a maximum is

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(17.0\times10^{-3}\ \text{H})(12.0\times10^{-6}\ \text{F})}} = \boxed{352\ \text{Hz}}$$

b. The maximum value of the current occurs when $f = f_0$. This occurs when $X_{\text{L}} = X_{\text{C}}$, so that $Z = R$. Therefore, according to Equation 23.6, we have

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{V_{\text{rms}}}{R} = \frac{155\ \text{V}}{10.0\ \Omega} = \boxed{15.5\ \text{A}}$$

28. ***REASONING AND SOLUTION*** At resonance, $P = IV$. Therefore,

$$V = P/I = (65.0\ \text{W})/(0.530\ \text{A}) = \boxed{123\ \text{V}}$$

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29. **SSM** ***REASONING AND SOLUTION*** The resonant frequency is given by Equation 23.10 as

$$f_0 = \frac{1}{2\pi\sqrt{LC}} = 9.3 \text{ kHz}$$

If the inductance and capacitance of the circuit are each tripled, the new resonant frequency is

$$f_0' = \frac{1}{2\pi\sqrt{(3L)(3C)}} = \frac{1}{2\pi\sqrt{9LC}} = \frac{1}{3}\left(\frac{1}{2\pi\sqrt{LC}}\right) = \tfrac{1}{3}f_0 = \tfrac{1}{3}(9.3 \text{ kHz}) = \boxed{3.1 \text{ kHz}}$$

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30. ***REASONING*** The resonant frequency is given by Equation 23.10 as $f_0 = 1/\left(2\pi\sqrt{LC}\right)$, where L is the inductance and C is the capacitance. In this problem the capacitance remains unchanged, and with this in mind, we will apply Equation 23.10 twice, once for each value of the inductance. The unknown capacitance can then be eliminated from the resulting equations.

SOLUTION Applying Equation 23.10 for each value of the inductance gives

$$f_{01} = \frac{1}{2\pi\sqrt{L_1 C}} \quad \text{and} \quad f_{02} = \frac{1}{2\pi\sqrt{L_2 C}}$$

Dividing the expression for f_{02} by the expression for f_{01}, we obtain

$$\frac{f_{02}}{f_{01}} = \frac{1/\left(2\pi\sqrt{L_2 C}\right)}{1/\left(2\pi\sqrt{L_1 C}\right)} = \sqrt{\frac{L_1}{L_2}}$$

Solving for f_{02} shows that

$$f_{02} = f_{01}\sqrt{\frac{L_1}{L_2}} = (1.3\text{ kHz})\sqrt{\frac{7.0\text{ mH}}{1.5\text{ mH}}} = \boxed{2.8\text{ kHz}}$$

31. ***REASONING*** Since the resonant frequency f_0 is known, we may use Equation 23.10, $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$ to find the inductance L, provided the capacitance C can be determined. The capacitance can be found by using the definitions of capacitive and inductive reactances.

SOLUTION

a. Solving Equation 23.10 for the inductance, we have

$$L = \frac{1}{4\pi^2 f_0^2 C} \tag{1}$$

where f_0 is the resonant frequency. From Equations 23. 2 and 23.4, the capacitive and inductive reactances are

$$X_C = \frac{1}{2\pi f C} \qquad \text{and} \qquad X_L = 2\pi f L$$

where f is any frequency. Solving the first of these equations for f, substituting the result into the second equation, and solving for C yields $C = \dfrac{L}{X_L X_C}$. Substituting this result into Equation (1) above and solving for L gives

$$L = \frac{1}{2\pi f_0}\sqrt{X_L X_C} = \frac{1}{2\pi(1500\text{ Hz})}\sqrt{(30.0\,\Omega)(5.0\,\Omega)} = \boxed{1.3\times10^{-3}\text{ H}}$$

b. The capacitance is

$$C = \frac{L}{X_L X_C} = \frac{1.3\times10^{-3}\text{ H}}{(30.0\,\Omega)(5.0\,\Omega)} = \boxed{8.7\times10^{-6}\text{ F}}$$

32. ***REASONING AND SOLUTION*** The resonant frequency of the circuit is 225 Hz. Solving for LC at resonance, we get $LC = 1/(4\pi^2 f_0^{\,2})$. When the circuit is *not* at resonance, $X_L/X_C = 5.36$. We have

$$\frac{X_L}{X_C} = \frac{2\pi f L}{\dfrac{1}{2\pi f C}} = 4\pi^2 f^2 LC = 4\pi^2 f^2\left(\frac{1}{4\pi^2 f_0^2}\right) = \frac{f^2}{f_0^2} = 5.36$$

Solving for the frequency f yields,

$$f = f_0\sqrt{5.36} = (225\text{ Hz})\sqrt{5.36} = \boxed{521\text{ Hz}}$$

33. **SSM** ***REASONING*** At the resonant frequency f_0, we have $C = 1/(4\pi^2 f_0^2 L)$. We want to determine some series combination of capacitors whose equivalent capacitance C_s' is such that $f_0' = 3f_0$. Thus,

$$C_s' = \frac{1}{4\pi^2 f_0'^2 L} = \frac{1}{4\pi^2 (3f_0)^2 L} = \frac{1}{9}\left(\frac{1}{4\pi^2 f_0^2 L}\right) = \frac{1}{9}C$$

The equivalent capacitance of a series combination of capacitors is $1/C_s' = 1/C_1 + 1/C_2 + \ldots$ If we require that all the capacitors have the same capacitance C, the equivalent capacitance is

$$\frac{1}{C_s'} = \frac{1}{C} + \frac{1}{C} + \cdots = \frac{n}{C}$$

where n is the total number of identical capacitors. Using the result above, we find that

$$\frac{1}{C_s'} = \frac{1}{\frac{1}{9}C} = \frac{n}{C} \qquad \text{or} \qquad n = 9$$

Therefore, the number of *additional* capacitors that must be inserted in series in the circuit so that the frequency triples is $n' = n - 1 = \boxed{8}$.

34. ***REASONING AND SOLUTION*** At resonance, the power dissipated by the circuit is $P = IV$. At a *non-resonant* frequency, the power dissipated is

$$P' = I'V\cos\phi = \tfrac{1}{2}P = \tfrac{1}{2}IV$$

so that

$$I'\cos\phi = \tfrac{1}{2}I \qquad \text{and} \qquad \cos\phi = \frac{\mathrm{I}}{2\mathrm{I}'} = \frac{R}{Z'}$$

We also know that

$$I = \frac{V}{R} \qquad \text{and} \qquad \mathrm{I}' = \frac{\mathrm{V}}{Z'} \qquad \text{so that} \qquad \frac{\mathrm{I}}{\mathrm{I}'} = \frac{Z'}{R}$$

Combining these results yields

$$\underbrace{\frac{1}{2}\left(\frac{I}{I'}\right)}_{\frac{R}{Z}} = \frac{1}{2}\left(\frac{Z'}{R}\right) \qquad \text{so that} \qquad Z' = R\sqrt{2}$$

Finally, the power factor, $\cos\phi$, is

$$\cos\phi = \frac{R}{Z'} = \frac{R}{R\sqrt{2}} = \boxed{0.707}$$

35. ***REASONING AND SOLUTION***

a. We are given that when $f = 2f_0$ that $Z = 2R = \sqrt{R^2 + \left(X_L - X_C\right)^2}$. We also know by rearranging the resonant frequency equation that

$$L = 1/(4\pi^2 f_0^2 C) \quad \text{and} \quad C = 1/(4\pi^2 f_0^2 L)$$

Squaring the equation for Z we get

$$Z^2 = 4R^2 = R^2 + (X_L - X_L)^2 \quad \text{so that} \quad X_L - X_C = R\sqrt{3}$$

Now,

$$X_L = 2\pi f L = 4\pi f_0 L \quad \text{and} \quad X_C = \frac{1}{2\pi f C} = \frac{1}{4\pi f_0 C}$$

so that

$$X_L - X_C = 4\pi f_0 L - \frac{1}{4\pi f_0 C} = 4\pi f_0 L - \frac{1}{4\pi f_0\left(\dfrac{1}{4\pi^2 f_0^2 L}\right)} = 3\pi f_0 L$$

Combining yields, $\sqrt{3}\,\pi f_0 L = R$. Now look at X_L/R.

$$\frac{X_L}{R} = \frac{4\pi f_0 L}{\sqrt{3}\pi f_0 L} = \boxed{\frac{4}{\sqrt{3}}}$$

b. A similar treatment yields X_C/R:

$$\frac{X_C}{R} = \frac{\dfrac{1}{4\pi f_0 C}}{\dfrac{3}{4\sqrt{3}\pi f_0 C}} = \boxed{\frac{1}{\sqrt{3}}}$$

36. ***REASONING*** From Figure 23.11 we see that $V_0^2 = \left(V_L - V_C\right)^2 + V_R^2$. Since $V_L = 0$ ($L = 0$), and we know V_0 and V_R, we can use this equation to find V_C.

SOLUTION Solving the equation above for V_C gives

$$V_C = \sqrt{V_0^2 - V_R^2} = \sqrt{(45\text{ V})^2 - (24\text{ V})^2} = \boxed{38\text{ V}}$$

37. **SSM** ***REASONING*** The voltage across the capacitor reaches its maximum instantaneous value when the generator voltage reaches its maximum instantaneous value. The maximum value of the capacitor voltage first occurs one-fourth of the way, or one-quarter of a period, through a complete cycle (see the voltage curve in Figure 23.4).

SOLUTION The period of the generator is $T = 1/f = 1/(5.00\text{ Hz}) = 0.200\text{ s}$. Therefore, the least amount of time that passes before the instantaneous voltage across the capacitor reaches its maximum value is $\frac{1}{4}T = \frac{1}{4}(0.200\text{ s}) = \boxed{5.00 \times 10^{-2}\text{ s}}$.

38. ***REASONING AND SOLUTION*** At very high frequencies the capacitors behave as if they were replaced with wires that have zero resistance, while the inductors behave as if they were cut out of the circuit. The drawings below show the circuits under this condition. Circuit I behaves as if the two resistors are in parallel, and the equivalent resistance can be obtained from $R_P^{-1} = R^{-1} + R^{-1}$ as $R_P = R/2$. Circuit II behaves as if the two resistors are in series, and the equivalent resistance is $R_S = R + R = 2R$. In either case, the current is the voltage divided by the resistance. Therefore, the ratio of the currents in the two circuits is

$$\frac{I_{\text{circuit I}}}{I_{\text{circuit II}}} = \frac{V/(R/2)}{V/(2R)} = \boxed{4}$$

Circuit I
High frequency

Circuit II
High frequency

39. ***REASONING AND SOLUTION***

a. Equation 20.14 gives

$$I = \frac{V}{R} = \frac{120\text{ V}}{240\,\Omega} = \boxed{0.50\text{ A}}$$

b. Since there is no inductor, Equations 23.6 and 23.7 apply with $X_L = 0$. Therefore, the current is $I = V/Z = V/\sqrt{R^2 + X_C^2}$. Using Equation 23.2 for X_C, we find

$$I = \frac{V}{\sqrt{R^2 + \left(\frac{1}{2\pi f C}\right)^2}} = \frac{120\text{ V}}{\sqrt{(240\,\Omega)^2 + \left[\frac{1}{2\pi(60.0\text{ Hz})(10.0\times10^{-6}\text{ F})}\right]^2}} = \boxed{0.34\text{ A}}$$

c. When an inductor is present, Equations 23.6 and 23.7 give the current as $I = V/Z = V/\sqrt{R^2 + (X_L - X_C)^2}$. This expression reduces to $I = V/R$ when $X_L = X_C$. Using Equations 23.2 and 23.4 for the reactances, we find that

$$2\pi f L = \frac{1}{2\pi f C} \quad \text{or} \quad L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 (60.0\text{ Hz})^2 (10.0\times10^{-6}\text{ F})} = \boxed{0.704\text{ H}}$$

$\boxed{\text{Yes,}}$ it is possible to return the current to the value calculated in part (a).

40. ***REASONING AND SOLUTION*** We know that

$$V = IX_L = I(2\pi fL) = (0.20\text{ A})(2\pi)(750\text{ Hz})(0.080\text{ H}) = \boxed{75\text{ V}}$$

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41. SSM WWW ***REASONING*** Since the capacitor and the inductor are connected in parallel, the voltage across each of these elements is the same or $V_L = V_C$. Using Equations 23.3 and 23.1, respectively, this becomes $I_{rms} X_L = I_{rms} X_C$. Since the currents in the inductor and capacitor are equal, this relation simplifies to $X_L = X_C$. Therefore, we can find the value of the inductance by equating the expressions (Equations 23.4 and 23.2) for the inductive reactance and the capacitive reactance, and solving for *L*.

SOLUTION Since $X_L = X_C$, we have

$$2\pi f L = \frac{1}{2\pi f C}$$

Therefore, the value of the inductance is

$$L = \frac{1}{4\pi^2 f^2 C} = \frac{1}{4\pi^2 (60.0\text{ Hz})^2 (40.0\times 10^{-6}\text{ F})} = 0.176\text{ H} = \boxed{176\text{ mH}}$$

42. ***REASONING AND SOLUTION*** We begin by calculating the impedance of the circuit using $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$. We have

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(1350\text{ Hz})\left(4.10 \times 10^{-6}\text{ F}\right)} = 28.8\,\Omega$$

$$X_L = 2\pi f L = 2\pi(1350\text{ Hz})(5.30 \times 10^{-3}\text{ H}) = 45.0\,\Omega$$

$$Z = \sqrt{(16.0\,\Omega)^2 + (45.0\,\Omega - 28.8\,\Omega)^2} = 22.8\,\Omega$$

The current is therefore,

$$I = V/Z = (15.0\text{ V})/(22.8\,\Omega) = 0.658\text{ A}$$

Since the circuit elements are in series, the current through each element is the same. The voltage across each element is

$$V_R = IR = (0.658\text{A})(16.0\,\Omega) = \boxed{10.5\text{ V}}$$

$$V_C = IX_C = (0.658\text{ A})(28.8\,\Omega) = \boxed{19.0\text{ V}}$$

$$V_L = IX_L = (0.658\text{ A})(45.0\,\Omega) = \boxed{29.6\text{ V}}$$

43. SSM WWW ***REASONING*** Since we know the values of the resonant frequency of the circuit, the capacitance, and the generator voltage, we can find the value of the inductance from Equation 23.10, the expression for the resonant frequency. The resistance can be found from energy considerations at resonance; the power factor is given by $\cos\phi$, where the phase angle ϕ is given by Equation 23.8, $\tan\phi = (X_L - X_C)/R$.

SOLUTION

a. Solving Equation 23.10 for the inductance *L*, we find that

$$L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2 (1.30\times10^3\ \text{Hz})^2 (5.10\times10^{-6}\ \text{F})} = \boxed{2.94\times10^{-3}\ \text{H}}$$

b. At resonance, $f = f_0$, and the current is a maximum. This occurs when $X_L = X_C$, so that $Z = R$. Thus, the average power $\overline{P}$ provided by the generator is $\overline{P} = V_{rms}^2/R$, and solving for *R* we find

$$R = \frac{V_{rms}^2}{\overline{P}} = \frac{(11.0\ \text{V})^2}{25.0\ \text{W}} = \boxed{4.84\ \Omega}$$

c. When the generator frequency is 2.31 kHz, the individual reactances are

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(2.31\times10^3\ \text{Hz})(5.10\times10^{-6}\ \text{F})} = 13.5\ \Omega$$

$$X_L = 2\pi f L = 2\pi(2.31\times10^3\ \text{Hz})(2.94\times10^{-3}\ \text{H}) = 42.7\ \Omega$$

The phase angle ϕ is, from Equation 23.8,

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{42.7\ \Omega - 13.5\ \Omega}{4.84\ \Omega}\right) = 80.6°$$

The power factor is then given by

$$\cos\phi = \cos\ 80.6° = \boxed{0.163}$$

44. ***REASONING AND SOLUTION*** With the inductor hooked up to the battery we have $V = I_0R$. When the inductor is hooked up to the generator we have $V = (I_0/4)Z$. Since the voltage is the same in each case, we can equate these two expressions: $I_0R = (I_0/4)Z$. Therefore,

$$R = \frac{1}{4}Z = \frac{1}{4}\sqrt{R^2 + X_L^2}$$

Squaring gives

$$R^2 = \frac{1}{16}\left(R^2 + X_L^2\right) \quad \text{or} \quad R = \frac{X_L}{\sqrt{15}}$$

Thus,

$$R = \frac{2\pi f L}{\sqrt{15}} = \frac{2\pi(1200\text{ Hz})(4.5\times10^{-3}\text{ H})}{\sqrt{15}} = \boxed{8.8\,\Omega}$$

45. ***REASONING AND SOLUTION*** The current in an RCL-circuit is

$$I = \frac{V}{\sqrt{R^2 + \left(X_L - X_C\right)^2}}$$

Rearranging terms

$$(X_L - X_C)^2 = (V/I)^2 - R^2$$

Using Equations 23.2 and 23.4 for X_C and X_L, respectively, we obtain

$$2\pi f L - \frac{1}{2\pi f C} = \sqrt{\left(\frac{V}{I}\right)^2 - R^2} = \sqrt{\left(\frac{26.0\text{ V}}{0.141\text{ A}}\right)^2 - \left(108\,\Omega\right)^2} = 149\,\Omega$$

Multiplying by f leads to

$$2\pi f^2 L - (149\ \Omega)f - 1/(2\pi C) = 0$$

or

$$2\pi f^2(5.42 \times 10^{-3}\text{ H}) - (149\ \Omega)f - 1/[(2\pi(0.200 \times 10^{-6}\text{ F})] = 0$$

We can solve this quadratic equation for the frequencies. We obtain

$$f_1 = \boxed{3.11 \times 10^3\text{ Hz}} \quad \text{and} \quad f_2 = \boxed{7.50 \times 10^3\text{ Hz}}$$

1. **SSM** ***REASONING AND SOLUTION*** This is a standard exercise in units conversion. We first determine the number of meters in one light-year. The distance that light travels in one year is

$$d = ct = (3.00 \times 10^8 \text{ m/s})(1.00 \text{ year})\left(\frac{365.25 \text{ days}}{1 \text{ year}}\right)\left(\frac{24 \text{ hours}}{1.0 \text{ day}}\right)\left(\frac{3600 \text{ s}}{1 \text{ hour}}\right) = 9.47\times10^{15} \text{ m}$$

Thus, 1 light year $= 9.47 \times 10^{15}$ m. Then, the distance to Alpha Centauri is

$$(4.3 \text{ light - years})\left(\frac{9.47 \times 10^{15} \text{ m}}{1 \text{ light - year}}\right) = \boxed{4.1 \times 10^{16} \text{ m}}$$

2. ***REASONING AND SOLUTION*** The time is the distance divided by the speed of the waves.

a. $t = s/c = (3.85 \times 10^8 \text{ m})/(3.00 \times 10^8 \text{ m/s}) = \boxed{1.28 \text{ s}}$

b. A calculation similar to that in part a yields $\boxed{t = 1.9 \times 10^2 \text{ s}}$.

3. ***REASONING AND SOLUTION*** Let $f_1 = 88.0 \times 10^6$ Hz then

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}}$$

Let $f_2 = 108.0 \times 10^6$ Hz then

$$f_2 = \frac{1}{2\pi\sqrt{LC_2}}$$

Division and rearrangement of the above expressions give

$$C_2 = (f_1/f_2)^2 C_1 = [(88.0 \times 10^6 \text{ Hz})/(108.0 \times 10^6 \text{ Hz})]^2 (33.0 \times 10^{-12} \text{ F}) = \boxed{2.19 \times 10^{-11} \text{ F}}$$

4. ***REASONING*** The electromagnetic wave will be picked up by the radio when the resonant frequency f_0 of the circuit in Figure 24.5 is equal to the frequency of the broadcast wave, or f_0 = 1400 kHz. This frequency, in turn, is related to the capacitance C and inductance L of the circuit through Equation 23.10, $f_0 = 1/\left(2\pi\sqrt{LC}\right)$. Since C is known, we can use this relation to find the inductance.

SOLUTION Solving the relation $f_0 = 1/\left(2\pi\sqrt{LC}\right)$ for the inductance L, we find that

$$L = \frac{1}{4\pi^2 f_0^2 C} = \frac{1}{4\pi^2\left(1400\times10^3\ \text{Hz}\right)^2\left(8.4\times10^{-11}\ \text{F}\right)} = \boxed{1.5\times10^{-4}\ \text{H}}$$

5. **SSM** ***REASONING*** The equation that represents the wave mathematically is $y = A\sin(2\pi ft - 2\pi x/\lambda)$. In this expression the amplitude is $A = 156$ N/C. The wavelength λ can be calculated using Equation 16.1, and we obtain

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50 \times 10^8 \text{ Hz}} = 2.00 \text{ m}$$

SOLUTION

a. For $t = 0$ s, the wave expression becomes

$$y = A\sin(2\pi ft - 2\pi x/\lambda) = 156\sin\left[2\pi f(0) - \frac{2\pi x}{2.00}\right] = -156\sin\left(\frac{2\pi x}{2.00}\right) = -156\sin(\pi x)$$

In this result, the units are suppressed for convenience. The following table gives the values of the electric field obtained using this version of the wave expression with the given values of the position x. The term πx is in radians when x is in meters, and conversion from radians to degrees is accomplished using the fact that 2π rad = 360°.

x	$y = -156 \sin(\pi x)$
0 m	$-156 \sin(0) = -156 \sin(0°) = 0$
0.50 m	$-156 \sin(0.50\,\pi) = -156 \sin(90°) = -156$
1.00 m	$-156 \sin(1.00\,\pi) = -156 \sin(180°) = 0$
1.50 m	$-156 \sin(1.50\,\pi) = -156 \sin(270°) = +156$
2.00 m	$-156 \sin(2.00\,\pi) = -156 \sin(360°) = 0$

These values for the electric field are plotted in the graph shown at the right.

b. For $t = T/4$, we use the fact that $f = 1/T$, and the wave expression becomes

$$y = A\sin(2\pi ft - 2\pi x/\lambda) = 156\sin\left[2\pi\left(\frac{1}{T}\right)\left(\frac{T}{4}\right) - \frac{2\pi x}{2.00}\right] = 156\sin\left(\frac{\pi}{2} - \pi x\right) = 156\cos(\pi x)$$

In this result, the units are suppressed for convenience. The following table gives the values of the electric field obtained using this version of the wave expression with the given values of the position x. The term πx is in radians when x is in meters, and conversion from radians to degrees is accomplished using the fact that 2π rad = 360°.

x	$y = 156 \cos(\pi x)$

0 m	$156 \cos(0) = 156 \cos(0°) = +156$
0.50 m	$156 \cos(0.50\,\pi) = 156 \cos(90°) = 0$
1.00 m	$156 \cos(1.00\,\pi) = 156 \cos(180°) = -156$
1.50 m	$156 \cos(1.50\,\pi) = 156 \cos(270°) = 0$
2.00 m	$156 \cos(2.00\,\pi) = 156 \cos(360°) = +156$

These values for the electric field are plotted in the graph shown at the right.

6. ***REASONING AND SOLUTION*** The average flux change through the coil in one fourth of the wave period is, according to Faraday's law, $\Delta\Phi = NAB = NB_0A$. The magnitude of the average emf is then emf $= \Delta\Phi/\Delta t = NB_0A/\Delta t$. Now $\Delta t = T/4 = 1/(4\ f)$, so

$$\text{Emf} = 4Nf\ B_0A = 4(450)(1.2 \times 10^6\ \text{Hz})(2.0 \times 10^{-13}\ \text{T})(\pi)(0.25\ \text{m})^2 = \boxed{8.5 \times 10^{-5}\ \text{V}}$$

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7. ***REASONING AND SOLUTION*** Using Equation 16.1, we obtain

$$\lambda = c/f = (2.9979 \times 10^8 \text{ m/s})/(26.965 \times 10^6 \text{ Hz}) = \boxed{11.118 \text{ m}}$$

8. ***REASONING AND SOLUTION*** The longest FM radio wavelength is

$$\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(88.0 \times 10^6 \text{ Hz}) = \boxed{3.41 \text{ m}}$$

The shortest wavelength is

$$\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(108.0 \times 10^6 \text{ Hz}) = \boxed{2.78 \text{ m}}$$

9. **SSM** **WWW** ***REASONING AND SOLUTION*** According to Equation 16.1, the wavelength is $\lambda = c/f$. Since the rods are adjusted so that each one has a length of $\lambda/4$, the length of each rod is

$$L = \frac{\lambda}{4} = \frac{(c/f)}{4} = \frac{(3.00 \times 10^8 \text{ m/s})/(60.0 \times 10^6 \text{ Hz})}{4} = \boxed{1.25 \text{ m}}$$

10. ***REASONING AND SOLUTION*** According to Equation 16.1, the wavelength of these waves is $\lambda = c/f$. Therefore,

$$\frac{\lambda_{\text{MRI}}}{\lambda_{\text{PET}}} = \frac{c/f_{\text{MRI}}}{c/f_{\text{PET}}} = \frac{f_{\text{PET}}}{f_{\text{MRI}}} = \frac{1.23\times10^{20}\ \text{Hz}}{6.38\times10^{7}\ \text{Hz}} = \boxed{1.93\times10^{12}}$$

11. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The number of wavelengths that can fit across the width W of your thumb is W/λ. From Equation 16.1, we know that $\lambda = c/f$, so

$$\text{No. of wavelengths} = \frac{W}{\lambda} = \frac{Wf}{c} = \frac{(2.0 \times 10^{-2}\ \text{m})(5.5 \times 10^{14}\ \text{Hz})}{3.0 \times 10^{8}\ \text{m/s}} = \boxed{3.7 \times 10^{4}}$$

12. ***REASONING AND SOLUTION*** The VHF wavelength is

$$\lambda = c/f = (3.00 \times 10^8 \text{ m/s})/(63.0 \times 10^6 \text{ Hz}) = 4.76 \text{ m}$$

The UHF wavelength is

$$\lambda = (3.00 \times 10^8 \text{ m/s})/(527 \times 10^6 \text{ Hz}) = 0.569 \text{ m}$$

The ratio of the wavelengths is (4.76 m)/(0.569 m) = $\boxed{8.37}$

13. ***REASONING AND SOLUTION*** The wavelength of the microwaves is $\lambda = 4L$, so the frequency is

$$f = c/\lambda = c/(4L) = (3.00 \times 10^8 \text{ m/s})/(4 \times 0.50 \times 10^{-2} \text{ m}) = \boxed{1.5 \times 10^{10} \text{ Hz}}$$

14. ***REASONING AND SOLUTION*** The distance traveled by each wave is $s = ct$. The difference in distance traveled is then

$$\Delta s = c\Delta t = (3.00 \times 10^8 \text{ m/s})(5.0 \times 10^{-7} \text{ s}) = \boxed{150 \text{ m}}$$

15. **SSM** ***REASONING*** We proceed by first finding the time t for sound waves to travel between the astronauts. Since this is the same time it takes for the electromagnetic waves to travel to earth, the distance between earth and the spaceship is $d_{\text{earth-ship}} = ct$.

SOLUTION The time it takes for sound waves to travel at 343 m/s through the air between the astronauts is

$$t = \frac{d_{\text{astronaut}}}{v_{\text{sound}}} = \frac{1.5 \text{ m}}{343 \text{ m/s}} = 4.4 \times 10^{-3} \text{ s}$$

Therefore, the distance between the earth and the spaceship is

$$d_{\text{earth-ship}} = ct = (3.0 \times 10^{8} \text{ m/s})(4.4 \times 10^{-3} \text{ s}) = \boxed{1.3 \times 10^{6} \text{ m}}$$

16. ***REASONING AND SOLUTION*** The time it takes for light from Sirius to reach earth is the distance s divided by the speed of light,

$$t = \frac{s}{c} = \frac{8.3 \times 10^{16}\ \text{m}}{3.0 \times 10^{8}\ \text{m/s}}\left(\frac{1\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1\ \text{d}}{24\ \text{h}}\right)\left(\frac{1\ \text{y}}{365.25\ \text{d}}\right) = \boxed{8.8\ \text{y}}$$

17. ***REASONING*** We see from Figure 24.11, that the rotating mirror is eight-sided. Therefore, the minimum angular speed is that angular speed at which one side of the mirror rotates one-eighth of a revolution during the time it takes for the light to make the round-trip between Mt. San Antonio and Mt. Wilson. The angular speed of the mirror can be determined by dividing the angular displacement of one-eighth of a revolution by the round-trip time, where the round-trip time is the round-trip distance divided by the speed of light.

SOLUTION The round-trip travel time is

$$t = \frac{2(35\times10^{3}\ \text{m})}{3.00\times10^{8}\ \text{m/s}} = 2.3\times10^{-4}\ \text{s}$$

Therefore, the minimum angular speed for the rotating mirror is

$$\omega = \frac{\frac{1}{8}\ \text{revolution}}{2.3\times10^{-4}\ \text{s}} = \boxed{540\ \text{rev/s}}$$

18. ***REASONING AND SOLUTION*** The time t that it takes for the telephone call to go from one city to the other is equal to the distance s traveled by the electromagnetic wave divided by the speed of light, $t = s/c$. The distance $s/2$ from one city to the satellite is given by the Pythagorean theorem as (see the drawing)

$$\frac{s}{2} = \sqrt{(3.6 \times 10^7 \text{ m})^2 + \left(\frac{3.5}{2} \times 10^6 \text{ m}\right)^2} = 3.6 \times 10^7 \text{ m}$$

The time for the wave to travel from one city up to the satellite and back to the other is

$$t = \frac{s}{c} = \frac{2(3.6 \times 10^7 \text{ m})}{3.0 \times 10^8 \text{ m/s}} = \boxed{0.24 \text{ s}}$$

19. **SSM** ***REASONING AND SOLUTION*** If d is the distance between the mirrors, the time required for the echo to be heard is $t = 2d / v_{\text{sound}}$. In this time, the light will travel a total distance D where $D = ct = c(2d / v_{\text{sound}})$. Therefore, the number of times that the flash travels the round-trip distance between the mirrors in the time t is

$$N = \frac{D}{2d} = \frac{2cd / v_{\text{sound}}}{2d} = \frac{c}{v_{\text{sound}}} = \frac{3.00 \times 10^{8} \text{ m/s}}{343 \text{ m/s}} = \boxed{8.75 \times 10^{5}}$$

20. ***REASONING AND SOLUTION*** The time required for the sound to travel from the TV to the viewer is $t_1 = s_1/v$. The time required for the EM waves to travel to the viewer's set is $t = s/c$. The time required for the sound to travel from the celebrity to the reporter is $t_2 = s_2/v$. Now $t_1 + t = t_2$, so

$$s = (s_2 - s_1)(c/v) = (4.1\text{ m} - 2.3\text{ m})(3.00 \times 10^8\text{ m/s})/(343\text{ m/s}) = \boxed{1.6 \times 10^6\text{ m}}$$

21. ***REASONING AND SOLUTION*** Using Equations 24.5b and 24.3, we find the following results.

a. $$E_{\text{rms}} = \sqrt{\frac{\overline{S}}{c\varepsilon_0}} = \sqrt{\frac{1.23\times10^{9}\ \text{W/m}^2}{\left(3.00\times10^{8}\ \text{m/s}\right)\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right]}} = \boxed{6.81\times10^{5}\ \text{N/C}}$$

b. $$B_{\text{rms}} = E_{\text{rms}}/c = \boxed{2.27\times10^{-3}\ \text{T}}$$

22. ***REASONING AND SOLUTION*** According to Equation 24.3, we obtain

$$B = E/c = (1470 \text{ N/C})/(3.00 \times 10^8 \text{ m/s}) = \boxed{4.90 \times 10^{-6} \text{ T}}$$

23. **SSM** ***REASONING*** The rms value E_{rms} of the electric field is related to the average energy density $\bar{u}$ of the microwave radiation according to Equation 24.2b: $\bar{u} = \varepsilon_0 E_{\text{rms}}^2$.

SOLUTION Solving for E_{rms} gives

$$E_{\text{rms}} = \sqrt{\frac{\bar{u}}{\varepsilon_0}} = \sqrt{\frac{4 \times 10^{-14}\ \text{J/m}^3}{8.85 \times 10^{-12}\ \text{C}^2/(\text{N} \cdot \text{m}^2)}} = \boxed{0.07\ \text{N/C}}$$

24. ***REASONING AND SOLUTION*** The intensity S of a wave is the power passing perpendicularly through a surface divided by the area A of the surface. But power is the total energy U per unit time t, so the intensity can be written as

$$S = \frac{\text{Total energy}}{\text{Time} \cdot \text{Area}} = \frac{U}{tA}$$

Equation 24.5c relates the intensity S of the electromagnetic wave to the magnitude B of its magnetic field; namely $S = (c/\mu_0)B^2$. Combining these two results, we have

$$\frac{U}{tA} = \frac{c}{\mu_0} B^2$$

If the rms value for the magnetic field is used, the energy becomes the average energy $\bar{U}$. Thus, the average energy that this wave carries through the window in a 45 s phone call is

$$\bar{U} = \frac{c}{\mu_0} B_{\text{rms}}^2 tA = \left(\frac{3.0 \times 10^8 \text{ m/s}}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} \right) (1.5 \times 10^{-10} \text{ T})^2 (45 \text{ s})(0.20 \text{ m}^2) = \boxed{4.8 \times 10^{-5} \text{ J}}$$

25. **SSM** ***REASONING AND SOLUTION***

a. According to Equation 24.5b, the average intensity is $\bar{S} = c\varepsilon_0 E_{\text{rms}}^2$. In addition, the average intensity is the average power $\bar{P}$ divided by the area A. Therefore,

$$E_{\text{rms}} = \sqrt{\frac{\bar{S}}{c\varepsilon_0}} = \sqrt{\frac{\bar{P}}{c\varepsilon_0 A}}$$

$$= \sqrt{\frac{1.20 \times 10^4 \text{ W}}{(3.00 \times 10^8 \text{ m/s})[8.85 \times 10^{-12} \text{C}^2/(\text{N} \cdot \text{m}^2)](135 \text{ m}^2)}} = \boxed{183 \text{ N/C}}$$

b. Then, from Equation 24.3 ($E_{\text{rms}} = cB_{\text{rms}}$), we have

$$B_{\text{rms}} = \frac{E_{\text{rms}}}{c} = \frac{183 \text{ N/C}}{3.00 \times 10^8 \text{ m/s}} = \boxed{6.10 \times 10^{-7} \text{ T}}$$

26. ***REASONING AND SOLUTION*** The energy is equal to the power P multiplied by the time t. The power, on the other hand, is equal to product of the the intensity S of the wave and the area A through which the wave passes.

$$\text{Energy} = P\,t = (SA)t = (1390\ \text{W/m}^2)(25\ \text{m} \times 45\ \text{m})(3600\ \text{s}) = \boxed{5.6 \times 10^9\ \text{J}}$$

27. ***REASONING*** The electromagnetic solar power that strikes an area $A_\perp$ oriented perpendicular to the direction in which the sunlight is radiated is $P = SA_\perp$, where S is the intensity of the sunlight. In the problem, the solar panels are not oriented perpendicular to the direction of the sunlight, and sunlight strikes the panels at an angle θ with respect to the normal. We wish to find the solar power that impinges on the solar panels when $\theta = 35°$, given that the incident power is 3200 W when $\theta = 55°$.

SOLUTION When the angle that the sunlight makes with the normal to the solar panel is θ, the power that strikes the solar panel is given by $P = SA\cos\theta$, where the area perpendicular to the sunlight is $A_\perp = A\cos\theta$ (see the drawing). Therefore we can write

$$\frac{P_2}{P_1} = \frac{SA\cos\theta_2}{SA\cos\theta_1}$$

where the intensity S of the sunlight that reaches the panel, as well as the area A, are the same in both cases. Therefore, we have

$$\frac{P_2}{P_1} = \frac{\cos\theta_2}{\cos\theta_1}$$

Solving for P_2, we find that the when $\theta_2 = 35°$, the solar power impinging on the panel is

$$P_2 = P_1\left(\frac{\cos\theta_2}{\cos\theta_1}\right) = (3200\text{ W})\left(\frac{\cos 35°}{\cos 55°}\right) = \boxed{4600\text{ W}}$$

28. ***REASONING*** According to Equation 24.5b, the average intensity $\bar{S}$ of the infrared radiation is related to the rms value of the electric field E_{rms} by $\bar{S} = c\varepsilon_0 E_{\text{rms}}^2$. According to Equation 16.8, the average power $\bar{P}$ is equal to the average intensity times the area A to which the power is being delivered, the area being that of a circle or $A = \pi r^2$. Thus, $\bar{P} = \bar{S}A = \bar{S}\left(\pi r^2\right)$. The average power is given by Equation 6.10b as the energy Q absorbed by the leg divided by the time t, so that $t = Q/\bar{P}$. The energy absorbed by the leg is related to the rise in temperature ΔT by Equation 12.4 as $Q = cm\,\Delta T$, where c is the specific heat capacity and m is the mass.

SOLUTION

a. The average intensity of the infrared radiation is

$$\bar{S} = c\varepsilon_0 E_{\text{rms}}^2 \qquad (24.5b)$$

$$= \left(3.0\times10^8\ \text{m/s}\right)\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right]\left(2800\ \text{N/C}\right)^2 = \boxed{2.1\times10^4\ \text{W/m}^2}$$

b. The average power delivered to the leg is

$$\bar{P} = \bar{S}A = \bar{S}\left(\pi r^2\right) = \left(2.1\times10^4\ \text{W/m}^2\right)\pi\left(4.0\times10^{-2}\ \text{m}\right)^2 = \boxed{1.1\times10^2\ \text{W}} \qquad (16.8)$$

c. Combining the relations $t = Q/\bar{P}$ and $Q = cm\,\Delta T$, the time required to raise the temperature by 2.0 C° is

$$t = \frac{Q}{\bar{P}} = \frac{cm\,\Delta T}{\bar{P}} = \frac{\left[3500\ \text{J}/\left(\text{kg}\cdot\text{C}^\circ\right)\left(0.28\ \text{kg}\right)\right]\left(2.0\ \text{C}^\circ\right)}{1.1\times10^2\ \text{W}} = \boxed{18\ \text{s}}$$

29. **SSM** **WWW** ***REASONING AND SOLUTION*** The sun radiates sunlight (electromagnetic waves) uniformly in all directions, so the intensity at a distance r from the sun is given by Equation 16.9 as $S = P/(4\pi r^2)$, where P is the power radiated by the sun. The power that strikes an area $A_\perp$ oriented perpendicular to the direction in which the sunlight is radiated is $P' = SA_\perp$, according to Equation 16.8. The 0.75-m^2 patch of flat land on the equator at point Q is not perpendicular to the direction of the sunlight, however.

The figure at the right shows that

$$A_\perp = (0.75\ \text{m}^2)\ \cos\ 27°$$

Therefore, the power striking the patch of land is

$$P' = SA_\perp = \left(\frac{P}{4\pi r^2}\right)(0.75\ \text{m}^2)\ \cos 27°$$

$$= \left[\frac{3.9\times 10^{26}\ \text{W}}{4\pi(1.5\times 10^{11}\ \text{m})^2}\right](0.75\ \text{m}^2)\ \cos 27° = \boxed{920\ \text{W}}$$

30. ***REASONING AND SOLUTION***

a. The maximum electrical force on the charge is $F = qE_0$. The rms electric field generated by the wave is

$$E_{\text{rms}} = \sqrt{\frac{\overline{S}}{c\varepsilon_0}} = \sqrt{\frac{1390\ \text{W/m}^2}{\left(3.00\times10^8\ \text{m/s}\right)\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right]}} = 724\ \text{N/C}$$

The maximum electric field is

$$E_0 = \sqrt{2}\ E_{\text{rms}} = \sqrt{2}\ (724\ \text{N/C}) = 1020\ \text{N/C}$$

The force is then

$$F = (2.6\times10^{-8}\ \text{C})(1020\ \text{N/C}) = \boxed{2.7\times10^{-5}\ \text{N}}$$

b. The maximum magnetic force on the charge is $F = qvB_0$. The rms magnetic field is $B_{\text{rms}} = E_{\text{rms}}/c$, and the maximum magnetic field is

$$B_0 = \sqrt{2}\ B_{\text{rms}} = \sqrt{2}\ E_{\text{rms}}/c = \sqrt{2}\ (724\ \text{N/C})/(3.00\times10^8\ \text{m/s}) = 3.41\times10^{-6}\ \text{T}$$

so

$$F = (2.6\times10^{-8}\ \text{C})(3.7\times10^4\ \text{m/s})(3.41\times10^{-6}\ \text{T}) = \boxed{3.3\times10^{-9}\ \text{N}}$$

31. ***REASONING*** The Doppler effect for electromagnetic radiation is given by Equation 24.6;

$$f_o = f_s\left(1 \pm \frac{v_{rel}}{c}\right) \qquad \text{if } v_{rel} << c$$

where f_o is the observed frequency, f_s is the frequency emitted by the source, and v_{rel} is the speed of the source relative to the observer. As discussed in the text, the plus sign applies when the source and the observer are moving toward one another, while the minus sign applies when they are moving apart. According to Equation 16.1, the wavelength of these waves is $\lambda = c/f$. Therefore, the Doppler shift can be written in terms of wavelengths:

$$\frac{1}{\lambda_o} = \frac{1}{\lambda_s}\left(1 \pm \frac{v_{rel}}{c}\right) \qquad \text{if } v_{rel} << c$$

SOLUTION

a. The wavelength λ_o of the light observed on earth is greater than the wavelength λ_s of the light when it is emitted from the distant galaxy (the source). Therefore, the frequency of the light observed on earth is less than the frequency of the light when it is emitted from the distant galaxy. Thus, the quantity in the brackets in Equation 24.6 must be less than one. It is equal to $\left[1-(v_{rel}/c)\right]$, and since the minus sign applies, we can conclude that

$$\boxed{\text{the galaxy must be receding from the earth}}\ .$$

b. We can find the speed of the galaxy relative to the earth by solving the wavelength version of Equation 24.6 for v_{rel}:

$$v_{rel} = c\left(1 - \frac{\lambda_s}{\lambda_o}\right) = (3.0\times10^8\ \text{m/s})\left(1 - \frac{500.7\ \text{nm}}{503.7\ \text{nm}}\right) = \boxed{1.8\times10^6\ \text{m/s}}$$

32. ***REASONING*** Using the Doppler effect, we will find the relative speed between the speeding car and the police car. Since we know the speed of the police car relative to the ground, we can determine the speed of the car relative to the ground, once the relative speed v_{rel} is found.

There are two Doppler frequency changes in this situation. First, the speeder's car observes the wave frequency coming from the radar gun to have a frequency f_{o} that is different from the emitted frequency f_{s}. The second Doppler shift occurs after the wave reflects from the speeder's car and returns to the police car.

The Doppler frequency for electromagnetic radiation is given by Equation 24.6, $f_{\text{o}} = f_{\text{s}}[1 \pm (v_{\text{rel}}/c)]$, where v_{rel} is the relative speed between the source and the observer of the radiation, and the plus sign applies when the source and the observer are moving toward one another, while the minus sign applies when they are moving apart. Since the distance between the police car and the speeder's car is increasing, they are moving apart, and according to Equation 24.6, the first Doppler frequency change is given by $f_{\text{o}} - f_{\text{s}} = -f_{\text{s}}(v_{\text{rel}}/c)$. After the wave reflects from the speeder's car it returns to the police car where it is observed to have a frequency f_{o}' that is different from its frequency f_{o} at the instant of reflection. Equation 24.6 may again be used, this time to determine the second Doppler frequency shift: $f_{\text{o}}' - f_{\text{o}} = -f_{\text{o}}(v_{\text{rel}}/c)$. We can use these two equations for the frequency shifts to determine an expression for the total Doppler change in frequency. Adding the two equations, we have

$$(f_{\text{o}}' - f_{\text{o}}) + (f_{\text{o}} - f_{\text{s}}) = -f_{\text{o}}\left(\frac{v_{\text{rel}}}{c}\right) - f_{\text{s}}\left(\frac{v_{\text{rel}}}{c}\right)$$

$$f_{\text{o}}' - f_{\text{s}} = -\left[f_{\text{o}}\left(\frac{v_{\text{rel}}}{c}\right) + f_{\text{s}}\left(\frac{v_{\text{rel}}}{c}\right)\right] \approx -2f_{\text{s}}\left(\frac{v_{\text{rel}}}{c}\right)$$

where we have assumed that f_{s} and f_{o} differ only by a negligibly small amount, so that $f_{\text{o}} \approx f_{\text{s}}$. Rearranging, we have

$$f_{\text{s}} - f_{\text{o}}' \approx 2f_{\text{s}}\left(\frac{v_{\text{rel}}}{c}\right)$$

SOLUTION Solving for the relative speed v_{rel} gives

$$v_{\text{rel}} \approx \left(\frac{f_{\text{s}} - f_{\text{o}}'}{2f_{\text{s}}}\right)c = \left[\frac{320\text{ Hz}}{2(7.0\times10^{9}\text{ Hz})}\right](3.0\times10^{8}\text{ m/s}) = 6.9\text{ m/s}$$

The relative speed v_{rel} is related to the speeds of the vehicles with respect to the ground by $v_{\text{rel}} = v_{\text{speeder}} - v_{\text{police}}$. Therefore, the speeder's speed with respect to the ground is

$$v_{\text{speeder}} = v_{\text{rel}} + v_{\text{police}} = 6.9\text{ m/s} + 25\text{ m/s} = \boxed{32\text{ m/s}}$$

33. **SSM** ***REASONING*** The Doppler effect for electromagnetic radiation is given by Equation 24.6, $f_o = f_s\left(1 \pm v_{rel}/c\right)$, where f_o is the observed frequency, f_s is the frequency emitted by the source, and v_{rel} is the speed of the source relative to the observer. As discussed in the text, the plus sign applies when the source and the observer are moving toward one another, while the minus sign applies when they are moving apart.

SOLUTION

a. At location *A*, the galaxy is moving away from the earth with a relative speed of

$$v_{rel} = (1.6 \times 10^6 \text{ m/s}) - (0.4 \times 10^6 \text{ m/s}) = 1.2 \times 10^6 \text{ m/s}$$

Therefore, the minus sign in Equation 24.6 applies and the observed frequency for the light from region *A* is

$$f_o = f_s\left(1 - \frac{v_{rel}}{c}\right) = (6.200 \times 10^{14} \text{ Hz})\left(1 - \frac{1.2 \times 10^6 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}}\right) = \boxed{6.175 \times 10^{14} \text{ Hz}}$$

b. Similarly, at location *B*, the galaxy is moving away from the earth with a relative speed of

$$v_{rel} = (1.6 \times 10^6 \text{ m/s}) + (0.4 \times 10^6 \text{ m/s}) = 2.0 \times 10^6 \text{ m/s}$$

The observed frequency for the light from region *B* is

$$f_o = f_s\left(1 - \frac{v_{rel}}{c}\right) = (6.200 \times 10^{14} \text{ Hz})\left(1 - \frac{2.0 \times 10^6 \text{ m/s}}{3.0 \times 10^8 \text{ m/s}}\right) = \boxed{6.159 \times 10^{14} \text{ Hz}}$$

34. ***REASONING AND SOLUTION***

a. The polarizer reduces the intensity of the light by a factor of two or to $\boxed{0.55\ \text{W/m}^2}$.

b. The intensity of the light leaving the analyzer is given by Malus' law.

$$S = (0.55\ \text{W/m}^2)\cos^2 75° = \boxed{3.7 \times 10^{-2}\ \text{W/m}^2}$$

35. $\boxed{\textit{SSM}}$ ***REASONING AND SOLUTION*** The average intensity of light leaving the polarizing material is given by Malus' Law (Equation 24.7). Therefore, using Malus' law, we obtain the following results.

a. $$\frac{\bar{S}}{\bar{S}_0} = \cos^2 25° = \boxed{0.82}$$

b. $$\frac{\bar{S}}{\bar{S}_0} = \cos^2 65° = \boxed{0.18}$$

36. ***REASONING*** Since the incident beam is unpolarized, the intensity of the light transmitted by the first sheet of polarizing material is one-half the intensity of the incident beam. The beams striking the second and third sheets of polarizing material are polarized, so the average intensity $\overline{S}$ of the light transmitted by each sheet is given by Malus' law, $\overline{S} = \overline{S}_0 \cos^2\theta$, where $\overline{S}_0$ is the average intensity of the light incident on each sheet.

SOLUTION The average intensity $\overline{S}_1$ of the light leaving the first sheet is one-half the intensity of the incident beam, so $\overline{S}_1 = \frac{1}{2}\left(1260.0\ \text{W/m}^2\right) = 630.0\ \text{W/m}^2$. The intensity $\overline{S}_2$ of the light leaving the second sheet of polarizing material is given by Malus' law, Equation 24.7, $\overline{S}_2 = \overline{S}_1 \cos^2\theta$, where θ is the angle between the polarization of the incident beam and the transmission axis of the second sheet:

$$\overline{S}_2 = \left(630.0\ \text{W/m}^2\right)\cos^2\left(55.0° - 19.0°\right) = 412\ \text{W/m}^2$$

The intensity $\overline{S}_3$ of the light leaving the third sheet of polarizing material is $\overline{S}_3 = \overline{S}_2 \cos^2\theta$, where θ is the angle between the polarization of the incident beam and the transmission axis of the third sheet:

$$\overline{S}_3 = \left(412\ \text{W/m}^2\right)\cos^2\left(100.0° - 55.0°\right) = \boxed{206\ \text{W/m}^2}$$

37. ***REASONING AND SOLUTION*** Malus' law gives

$$S/S_o = \cos^2 \theta = \cos^2 38° = 0.62$$

so that $\boxed{62\ \%}$ of the light is transmitted.

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38. ***REASONING AND SOLUTION*** If the intensity of the unpolarized light is I_0, the intensity of the polarized light leaving the polarizer is $\frac{1}{2}I_0$. By Malus' law, the intensity of the light leaving the insert is $\frac{1}{2}I_0 \cos^2\theta$. From the results of Conceptual Example 8, the intensity of light leaving the analyzer is $\frac{1}{2}I_0 \cos^2\theta \sin^2\theta$. Thus, the intensity I of light that reaches the photocell is

$$I = \tfrac{1}{2}I_o \cos^2\theta \sin^2\theta = \tfrac{1}{2}(150\ \text{W/m}^2)\cos^2 30.0° \sin^2 30.0° = \boxed{14\ \text{W/m}^2}$$

39. **SSM** **WWW** ***REASONING*** The average intensity of light leaving each analyzer is given by Malus' Law (Equation 24.7). Thus, intensity of the light transmitted through the first analyzer is

$$\bar{S}_1 = \bar{S}_0 \cos^2 27°$$

Similarly, the intensity of the light transmitted through the second analyzer is

$$\bar{S}_2 = \bar{S}_1 \cos^2 27° = \bar{S}_0 \cos^4 27°$$

And the intensity of the light transmitted through the third analyzer is

$$\bar{S}_3 = \bar{S}_2 \cos^2 27° = \bar{S}_0 \cos^6 27°$$

If we generalize for the Nth analyzer, we deduce that

$$\bar{S}_N = \bar{S}_{N-1} \cos^2 27° = \bar{S}_0 \cos^{2N} 27°$$

Since we want the light reaching the photocell to have an intensity that is reduced by a least a factor of one hundred relative to the first analyzer, we want $\bar{S}_N / \bar{S}_0 = 0.010$. Therefore, we need to find N such that $\cos^{2N} 27° = 0.010$. This expression can be solved for N.

SOLUTION Taking the common logarithm of both sides of the last expression gives

$$2N \log(\cos 27°) = \log 0.010 \quad \text{or} \quad N = \frac{\log 0.010}{2 \log(\cos 27°)} = \boxed{20}$$

40. ***REASONING*** The polarizer, the insert and the analyzer in the set-up in Figure 24.22*a* all reduce the intensity of the light that reaches the photocell. The polarizer reduces the intensity by a factor of one-half, as described in Section 24.6 of the text; if the average intensity of the incident unpolarized light is $\bar{I}$, the average intensity of the polarized light that leaves the polarizer and strikes the insert is $\bar{S}_0 = \bar{I}/2$. According to Malus' law (see Equation 24.7), the average intensity $\bar{S}_{\text{insert}}$ of the light leaving the insert is $\bar{S}_{\text{insert}} = \bar{S}_0 \cos^2\theta$, where θ is the relative angle between the transmission axes of the polarizer and the insert. The intensity of the light is further reduced as the polarized light passes through the analyzer. Malus' law can be used in succession at each piece of polarizing material to determine the intensity that reaches the photocell, both with and without the presence of the analyzer.

SOLUTION When the analyzer is present, the average intensity reaching the photocell is equal to the average intensity that leaves the analyzer. The average intensity leaving the analyzer is, according to Malus' law, $\bar{S}_{\text{insert}} \cos^2\phi$ where $\bar{S}_{\text{insert}}$ is the average intensity that leaves the insert and reaches the analyzer, and ϕ is the relative angle between the transmission axes of the analyzer and insert. From Figure 24.22*a* we see that $\phi = 90° - \theta$. The average intensity of the light leaving the insert is $\bar{S}_{\text{insert}} = \bar{S}_0 \cos^2\theta$, according to Malus' law. Therefore, when the analyzer is present as shown in Figure 24.22*b,* the average intensity leaving the analyzer and reaching the photocell is

$$\bar{S}_{\text{photocell}} = \bar{S}_{\text{insert}} \cos^2\phi = \left(\bar{S}_0 \cos^2\theta\right)\cos^2\left(90° - \theta\right)$$

This expression can be solved for $\bar{S}_0$ to determine the average intensity leaving the polarizer:

$$\bar{S}_0 = \frac{\bar{S}_{\text{photocell}}}{(\cos^2\theta)\cos^2(90° - \theta)} = \frac{110\ \text{W/m}^2}{(\cos^2 23°)\cos^2(90° - 23°)} = 850\ \text{W/m}^2$$

If the analyzer were removed from the setup, everything else remaining the same, the intensity reaching the photocell would be equal to the intensity that leaves the insert. Therefore, if the analyzer were removed, the intensity reaching the photocell would be

$$\bar{S}_{\text{photocell}} = \bar{S}_{\text{insert}} = \bar{S}_0 \cos^2\theta = (850\ \text{W/m}^2)\cos^2 23° = \boxed{720\ \text{W/m}^2}$$

41. ***REASONING AND SOLUTION*** Using Equation 16.1, we find

$$f = c/\lambda = (3.00 \times 10^8 \text{ m/s})/(2.1 \times 10^{-9} \text{ m}) = \boxed{1.4 \times 10^{17} \text{ Hz}}$$

42. ***REASONING AND SOLUTION*** According to Equation 16.8, we have

$$S = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{1.2 \times 10^{-3}\ \text{W}}{\pi \left(1.0 \times 10^{-3}\ \text{m}\right)^2} = \boxed{3.8 \times 10^2\ \text{W/m}^2}$$

43. **SSM** **WWW** ***REASONING AND SOLUTION*** The actual time t and round-trip distance s are related by $s = ct$. An uncertainty of Δt in the time introduces an uncertainty in the round trip distance Δs, so that $s + \Delta s = c(t + \Delta t)$. Thus, subtraction of these two expression leads to

$$\Delta s = c\Delta t = (3.0 \times 10^8 \text{ m/s})(0.10 \times 10^{-9} \text{ s}) = 0.030 \text{ m}$$

This is the error in the round-trip distance. Therefore, the error in the earth-moon distance is half this distance or $\boxed{0.015 \text{ m}}$.

44. ***REASONING AND SOLUTION*** The average electromagnetic energy contained in a volume is the product of the average energy density $\bar{u}$ and the volume V. However, the average energy density is related to the average intensity $\bar{S}$ of the electromagnetic wave by Equation 24.4 as $\bar{u} = \bar{S}/c$. Therefore,

$$\text{Average energy} = \bar{u}V = \left(\frac{\bar{S}}{c}\right)V = \left(\frac{1.0\times10^{3}\ \text{W/m}^{2}}{3.0\times10^{8}\ \text{m/s}}\right)(5.5\ \text{m}^{3}) = \boxed{1.8\times10^{-5}\ \text{J}}$$

45. ***REASONING AND SOLUTION*** The sun radiates sunlight uniformly in all directions. The intensity S, or power per unit area, of the sunlight at a distance r from the sun is given by Equation 16.9 as $S = P/(4\pi r^2)$, where P is the power radiated by the sun. The intensity of the sunlight at the surface of Mars and Earth, respectively, is

$$S_{\text{Mars}} = \frac{P}{4\pi r_{\text{Mars}}^2} \quad \text{and} \quad S_{\text{Earth}} = \frac{P}{4\pi r_{\text{Earth}}^2}$$

Solving the equation on the right for P and then substituting it into the equation on the left yield, after canceling the factors of 4π,

$$S_{\text{Mars}} = S_{\text{Earth}}\left(\frac{r_{\text{Earth}}}{r_{\text{Mars}}}\right)^2 = (1390\ \text{W/m}^2)\left(\frac{1.50\times10^{11}\ \text{m}}{2.28\times10^{11}\ \text{m}}\right)^2 = \boxed{602\ \text{W/m}^2}$$

46. ***REASONING AND SOLUTION***

a. The frequency of the wave is

$$f = c/\lambda = (3.00 \times 10^8 \text{ m/s})/(274 \text{ m}) = \boxed{1.09 \times 10^6 \text{ Hz}}$$

b. It is an $\boxed{\text{AM radio wave}}$.

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47. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** The average intensity of light leaving each polarizer is given by Malus' Law (Equation 24.7): $\bar{S} = \bar{S}_0 \cos^2 \theta$. Solving for the angle θ and noting that $\bar{S} = 0.10\ \bar{S}_0$, we have

$$\theta = \cos^{-1} \sqrt{\frac{0.100\ \bar{S}_0}{\bar{S}_0}} = \boxed{71.6°}$$

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48. ***REASONING*** There are two Doppler frequency changes in the emitted wave in this case. First, the speeder's car observes the wave frequency coming from the radar gun to have a frequency f_o that is different from the emitted (source) frequency f_s. The wave then reflects and returns to the police car, where it is observed to have a frequency f_o' that is different than its frequency f_o at the instant of reflection. Although the police car is now moving, the relative motion of the two vehicles is one of approach. In Example 6, it is shown that, when the source and the observer of the radar are approaching each other, the magnitude of the difference between frequency of the emitted wave and the wave that returns to the police car after reflecting from the speeder's car is

$$f_o' - f_s \approx 2f_s\left(\frac{v_{rel}}{c}\right)$$

where v_{rel} is the relative speed between the speeding car and the police car.

SOLUTION Since the police car is moving to the right at 27 m/s, while the speeder is coming from behind at 39 m/s, the relative speed v_{rel} is 39 m/s – 27 m/s = 12 m/s.

a. The total Doppler change in frequency is, therefore,

$$f_o' - f_s = 2(8.0\times10^9\ \text{Hz})\left(\frac{12\ \text{m/s}}{3.0\times10^8\ \text{m/s}}\right) = \boxed{640\ \text{Hz}}$$

b. Since the two cars are approaching each other, the plus sign in Equation 24.6 applies, the quantity in brackets in Equation 24.6 will be greater than one, and the Doppler shifted frequency will be greater than the source frequency.

Therefore, $\boxed{\text{the wave that returns to the police car has the greater frequency}}$.

49. ***REASONING*** According to Equation 16.3, the displacement y of a wave that travels in the $+x$ direction and has amplitude A, frequency f, and wavelength λ is given by

$$y = A \sin\left(2\pi f t - \frac{2\pi x}{\lambda}\right)$$

This equation, with $y = E$, applies to the traveling electromagnetic wave in the problem, which is represented mathematically as

$$E = E_0 \sin\left[\left(1.5\times10^{10}\ \text{s}^{-1}\right)t - \left(5.0\times10^{1}\ \text{m}^{-1}\right)x\right]$$

As E_0 is the maximum field strength, it represents the amplitude A of the wave. We can find the frequency and wavelength of this electromagnetic wave by comparing the mathematical form of the electric field with Equation 16.3.

SOLUTION

a. By inspection, we see that $2\pi f = 1.5\times10^{10}\ \text{s}^{-1}$. Therefore, the frequency of the wave is

$$f = \frac{1.5\times10^{10}\ \text{s}^{-1}}{2\pi} = \boxed{2.4\times10^{9}\ \text{Hz}}$$

b. As shown in Figure 17.18, the separation between adjacent nodes in any standing wave is one-half of a wavelength. By inspection of the mathematical form of the electric field and comparison with Equation 16.3, we infer that $2\pi/\lambda = 5.0\times10^{1}\ \text{m}^{-1}$. Therefore,

$$\lambda = \frac{2\pi}{5.0\times10^{1}\ \text{m}^{-1}} = 0.126\ \text{m}$$

Therefore, the nodes in the standing waves formed by this electromagnetic wave are separated by $\lambda/2 = \boxed{0.063\ \text{m}}$.

50. ***REASONING*** The rms value E_{rms} of the electric field is related to the average intensity $\bar{S}$ of the light by Equation 24.5b, $\bar{S} = c\varepsilon_0 E_{\text{rms}}^2$. The average intensity $\bar{S}$ of the light transmitted by the polarizer is related to the incident intensity $\bar{S}_0$ by Malus' law, $\bar{S} = \bar{S}_0 \cos^2\theta$, where θ is the angle between the transmission axis and the direction of polarization. These two relations will allow us to determine the rms value of the electric field.

SOLUTION Combining the two equations given above and solving for the rms value of the electric field, we have

$$E_{\text{rms}} = \sqrt{\frac{\bar{S}_0 \cos^2\theta}{c\varepsilon_0}} = \sqrt{\frac{\left(15\ \text{W/m}^2\right)\cos^2 25^\circ}{\left(3.0\times10^8\ \text{m/s}\right)\left[8.85\times10^{-12}\ \text{C}^2/\left(\text{N}\cdot\text{m}^2\right)\right]}} = \boxed{68\ \text{N/C}}$$

51. **SSM** ***REASONING AND SOLUTION*** Since the sun emits radiation uniformly in all directions, at a distance r from the sun's center, the energy spreads out over a sphere of surface area $4\pi r^2$. Therefore, according to Equation 16.9, $S = P/(4\pi r^2)$, the total power radiated by the sun is

$$P = S(4\pi r^2) = (1390\ \text{W/m}^2)(4\pi)(1.50\times10^{11}\ \text{m})^2 = \boxed{3.93\times10^{26}\ \text{W}}$$

52. ***REASONING AND SOLUTION*** The intensity of the laser light is $S = P/A = cu$, where u is the energy density of the light. The energy in a section of length L of the cylindrical beam is $U = uAL$ or

$$U = PL/c = (0.750\ \text{W})(2.50\ \text{m})/(3.00 \times 10^8\ \text{m/s}) = \boxed{6.25 \times 10^{-9}\ \text{J}}$$

53. ***REASONING AND SOLUTION*** The light energy spreads out uniformly over the surface of a sphere centered on the source. The average intensity $\bar{S}$ of the light at the point r = 8.00 m away from the source is then

$$\bar{S} = \frac{\bar{P}}{A} = \frac{\bar{P}}{4\pi r^2} = \frac{60.0\ \text{W}}{4\pi(8.00\ \text{m})^2} = 7.46 \times 10^{-2}\ \text{W/m}^2$$

a. The rms value of the electric field at the point is

$$E_{rms} = \sqrt{\frac{\bar{S}}{c\varepsilon_0}} = \sqrt{\frac{7.46 \times 10^{-2}\ \text{W/m}^2}{(3.00 \times 10^8\ \text{m/s})(8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2)}} = \boxed{5.30\ \text{N/C}}$$

b. Then

$$B_{rms} = E_{rms}/c = \boxed{1.77 \times 10^{-8}\ \text{T}}$$

54. ***REASONING AND SOLUTION*** The polarizer will transmit a maximum intensity of $(1/2)S_u + S_p$, when its axis is parallel to the polarization direction of the polarized component of the incident light. Then the light intensity at the photocell is

$$S_{max} = [(1/2)\, S_u + S_p] \cos^2 \theta \tag{1}$$

The polarizer transmits minimum light intensity of $(1/2)\, S_u$ when its axis is perpendicular to the polarization direction of the polarized incident light, so

$$S_{min} = (1/2)\, S_u \cos^2 \theta \tag{2}$$

Solving Equation (2) for S_u gives

$$S_u = 2S_{min}/\cos^2 \theta \tag{3}$$

Using Equation (3) in Equation (1) and solving give

$$S_p = (S_{max} - S_{min})/\cos^2 \theta \tag{4}$$

Using Equations (3) and (4), we find that the percent polarization is

$$\frac{100\, S_p}{S_p + S_u} = \frac{\dfrac{100\,(S_{max} - S_{min})}{\cos^2 \theta}}{\dfrac{S_{max} - S_{min} + 2S_{min}}{\cos^2 \theta}}$$

$$\frac{100\, S_p}{S_p + S_u} = \frac{100\,(S_{max} - S_{min})}{S_{max} + S_{min}}$$

1. SSM ***REASONING*** The geometry is shown below. According to the law of reflection, the incident ray, the reflected ray, and the normal to the surface all lie in the same plane, and the angle of reflection θ_r equals the angle of incidence θ_i. We can use the law of reflection and the properties of triangles to determine the angle θ at which the ray leaves M_2.

SOLUTION From the law of reflection, we know that $\phi = 65°$. We see from the figure that $\phi + \alpha = 90°$, or $\alpha = 90° - \phi = 90° - 65° = 25°$. From the figure and the fact that the sum of the interior angles in any triangle is 180°, we have $\alpha + \beta + 120° = 180°$. Solving for β, we find that $\beta = 180° - (120° + 25°) = 35°$. Therefore, since $\beta + \gamma = 90°$, we find that the angle γ is given by $\gamma = 90° - \beta = 90° - 35° = 55°$. Since γ is the angle of incidence of the ray on mirror M_2, we know from the law of reflection that $\boxed{\theta = 55°}$.

2. ***REASONING AND SOLUTION*** The two arrows, A and B are located in front of a plane mirror, and a person at point P is viewing the image of each arrow. As discussed in Conceptual Example 1, light emanating from the arrow is reflected from the mirror and is reflected toward the observer at P. In order for the observer to see the arrow in its entirety, both rays, the one from the top of the arrow and the one from the bottom of the arrow, must pass through the point P.

According to the law of reflection, all rays will be reflected so that the angle of reflection is equal to the angle of incidence. The ray from the top of arrow A strikes the mirror and reflects so that it passes through point P. Likewise, the ray from the bottom of the arrow´ is reflected such that it too passes through point P. Therefore, the observer at P $\boxed{\text{sees the arrow at } A \text{ in its entirety}}$.

Similar reasoning shows that the ray from the top of arrow B passes through point P. However, as the drawing shows, the ray from the bottom of the arrow does not pass through P. This conclusion is true no matter where the bottom ray strikes the mirror. The observer $\boxed{\text{does } \textit{not} \text{ see the arrow at } B \text{ in its entirety}}$.

3. ***REASONING AND SOLUTION*** As seen in the diagram, the angle of incidence θ is found from $\tan\ \theta = (0.90\ \text{m})/(3.6\ \text{m}) = 0.25$. Thus, $\theta = \tan^{-1}(0.25) = \boxed{14°}$.

4. ***REASONING AND SOLUTION*** Referring to Figure 25.9b and Conceptual Example 2, we find the following locations for the three images:

Image 1:	$x = -2.0$ m,	$y = +1.0$ m
Image 2:	$x = +2.0$ m,	$y = -1.0$ m
Image 3:	$x = +2.0$ m,	$y = +1.0$ m

5. SSM WWW ***REASONING AND SOLUTION*** The drawing at the right shows a ray diagram in which the reflected rays have been projected behind the mirror. We can see by inspection of this drawing that, after the rays reflect from the plane mirror, the angle α between them is still $\boxed{10°}$.

6. ***REASONING*** The drawing shows the situation described. The law of reflection indicates that the angle of incidence θ_i is equal to the angle of reflection θ_r.

SOLUTION For the incident and reflected light, we have

$$\underbrace{\tan\theta_i = \frac{d}{3.0\text{ cm}}}_{\text{Incident light}} \quad \text{and} \quad \underbrace{\tan\theta_r = \frac{\ell}{6.0\text{ cm}}}_{\text{Reflected light}}$$

But $\theta_i = \theta_r$, according to the law of reflection, so that $\tan\theta_i = \tan\theta_r$, and we have

$$\frac{d}{3.0\text{ cm}} = \frac{\ell}{6.0\text{ cm}} \quad \text{or} \quad \ell = 2d$$

From the drawing we can see that $d + \ell = 9.0$ cm, and using the fact that $\ell = 2.0d$, we obtain

$$d + \ell = d + 2.0d = 9.0\text{ cm} \quad \text{or} \quad d = 3.0\text{ cm}$$

Therefore, the laser should be aimed at the point at $\boxed{x = +3.0\text{ cm}}$.

7. ***REASONING AND SOLUTION***

a. After the mirror has been rotated, the new angle of incidence is $\theta_i = 45° + 15° = 60°$. The angle of reflection, then, is also equal to 60°. The reflected ray which was originally 90° (45° + 45°) from the original angle of incidence, is now 120° (60° + 60°) from the incident ray's direction. Therefore, the reflected ray has been rotated through

$$\beta = 120° - 90° = \boxed{30°}$$

b. The angle through which the reflected ray is rotated depends only on the angle through which the mirror is rotated, and is independent of the angle of incidence. Therefore, $\boxed{\beta' = 30°}$.

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8. ***REASONING AND SOLUTION*** Denote the walls as side A (left-hand mirror opposite the target), side B (top wall), and side C (where the target is). The directions in the which the laser will be fired will be the angles measured counterclockwise with respect to the +x axis (i.e., the line from *P* to the bottom of wall C). We have

$\boxed{\theta_1 = 11.3°}$, follows a path from C to A to the target.

$\boxed{\theta_2 = 45.0°}$, follows a path directly to the target.

$\boxed{\theta_3 = 71.6°}$, follows a path from B to the target.

$\boxed{\theta_4 = 135°}$, follows a path from A to B to the target.

$\boxed{\theta_5 = 162°}$, follows a path from A to the target.

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9. **SSM** ***REASONING*** The time of travel is proportional to the total distance for each path. Therefore, using the distances identified in the drawing, we have

$$\frac{t_{\text{reflected}}}{t_{\text{direct}}} = \frac{d_{LM} + d_{MP}}{d_{LP}} \qquad (1)$$

We know that the angle of incidence is equal to the angle of reflection. We also know that the lamp L is a distance $2s$ from the mirror, while the person P is a distance s from the mirror.

Therefore, it follows that if $d_{MP} = d$, then $d_{LM} = 2d$. We may use the law of cosines (see Appendix E) to express the distance d_{LP} as

$$d_{LP} = \sqrt{(2d)^2 + d^2 - 2(2d)(d)\cos 2(30.0°)}$$

SOLUTION Substituting the expressions for d_{MP}, d_{LM}, and d_{LP} into Equation (1), we find that the ratio of the travel times is

$$\frac{t_{\text{reflected}}}{t_{\text{direct}}} = \frac{2d + d}{\sqrt{(2d)^2 + d^2 - 2(2d)(d)\cos 2(30.0°)}} = \frac{3}{\sqrt{5 - 4\cos 60.0°}} = \boxed{1.73}$$

10. ***REASONING AND SOLUTION*** The ray diagram is shown in the figure (Note: $f = 5.0$ cm and $d_o = 15.0$ cm).

a. The ray diagram indicates that the $\boxed{\text{image distance is 7.5 cm}}$ in front of the mirror.

b. $\boxed{\text{The image height is 1.0 cm}}$, and the image is inverted relative to the object.

11. ***REASONING AND SOLUTION*** An accurately drawn ray diagram, which will look similar, but not identical, to that in Figure 25.20a, reveals that

a. the $\boxed{\text{image distance is } 3.0 \times 10^1 \text{ cm behind the mirror}}$, and

b. the $\boxed{\text{image height is } 5.0 \text{ cm}}$, and the image is upright relative to the object.

12. ***REASONING AND SOLUTION***

a. A ray diagram, which will look similar, but not identical, to that in Figure 25.22a, reveals that the $\boxed{\text{image distance is 20.0 cm behind the mirror, or } d_i = -20.0\,\text{cm}}$.

b. The ray diagram also shows that $\boxed{\text{the image height is 6.0 cm}}$, and the image is upright relative to the object.

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13. SSM ***REASONING AND SOLUTION*** The ray diagram is shown below. (Note: $f = -50.0$ cm and $d_o = 25.0$ cm)

a. The ray diagram indicates that the image is 16.7 cm behind the mirror.

b. The ray diagram indicates that the image height is 6.67 cm.

14. ***REASONING AND SOLUTION*** An accurately drawn ray diagram, which will look similar to that in Figure 25.19a, reveals that

a. the $\boxed{\text{image distance is 23.6 cm}}$ in front of the mirror, and

b. the $\boxed{\text{image height is 2.14 cm}}$, and the image is inverted relative to the object.

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15. ***REASONING AND SOLUTION*** A plane mirror faces a concave mirror ($f = 8.0$ cm). The following is a ray diagram of an object placed 10.0 cm in front of the plane mirror.

The ray diagram shows the light that is first reflected from the plane mirror and then the concave mirror. The scale is shown in the figure. For the reflection from the plane mirror, as discussed in the text, the image is upright, the same size as the object, and is located as far behind the mirror as the object is in front of it. When the reflected ray reaches the concave mirror, the ray that is initially parallel to the principal axis passes through the focal point F after reflection from the concave mirror. The ray that passes directly through the focal point emerges parallel to the principal axis after reflection from the concave surface. The point of intersection of these two rays locates the position of the image. By inspection, we see that the image is located at $\boxed{10.9 \text{ cm}}$ from the concave mirror.

16. ***REASONING AND SOLUTION*** We know that $d_o = 8.0$ cm and that the image is real and enlarged. A concave mirror produces a real, enlarged image when the object is located between the focal point and the center of curvature, as in Figure 25.19*a*. As a result, the image is also inverted, so that $m = -d_i/d_o = -4$, according to the magnification equation. Thus,

$$d_i = 4d_o = 4(8.0 \text{ cm}) = \boxed{+32 \text{ cm}}$$

17. SSM ***REASONING*** This problem can be solved using the mirror equation, Equation 25.3.

SOLUTION Using the mirror equation with $d_i = +97\ \text{cm}$ and $f = 42\ \text{cm}$, we find

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{42\ \text{cm}} - \frac{1}{97\ \text{cm}} \qquad \text{or} \qquad \boxed{d_o = +74\ \text{cm}}$$

18. ***REASONING AND SOLUTION***

a. The focal length is found from

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{27\text{ cm}} + \frac{1}{-65\text{ cm}} \quad \text{so} \quad \boxed{f = +46\text{ cm}}$$

b. The magnification is

$$m = \frac{-d_i}{d_o} = \frac{-(-65\text{ cm})}{27\text{ cm}} = \boxed{2.4}$$

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19. ***REASONING AND SOLUTION***

a. The image distance is

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{32.0\text{ cm}} - \frac{1}{38.0\text{ cm}} \quad \text{so} \quad \boxed{d_i = 2.0 \times 10^2\text{ cm}}$$

b. The image height is

$$h_i = -h_o \frac{d_i}{d_o} = -(1.20\text{ cm})\left(\frac{2.0 \times 10^2\text{ cm}}{38.0\text{ cm}}\right) = \boxed{-6.3\text{ cm}}$$

c. Since h_i is negative, the image is inverted. Therefore, to make the picture on the wall appear normal, the slide must be oriented $\boxed{\text{upside down}}$.

20. ***REASONING*** The mirror equation relates the object and image distances to the focal length. Thus, we can apply the mirror equation once with the given object and image distances to determine the focal length. Then, we can use the mirror equation again with the focal length and the second object distance to determine the unknown image distance.

SOLUTION According to the mirror equation, we have

$$\underbrace{\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{f}}_{\text{First position of object}} \quad \text{and} \quad \underbrace{\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f}}_{\text{Second position of object}}$$

Since the focal length is the same in both cases, it follows that

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{d_{o2}}+\frac{1}{d_{i2}}$$

$$\frac{1}{d_{i2}}=\frac{1}{d_{o1}}+\frac{1}{d_{i1}}-\frac{1}{d_{o2}}=\frac{1}{25\text{ cm}}+\frac{1}{(-17\text{ cm})}-\frac{1}{19\text{ cm}}=-0.071\text{ cm}^{-1}$$

$$d_{i2}=-14\text{ cm}$$

The negative value for d_{i2} indicates that the image is located $\boxed{14\text{ cm}}$ behind the mirror.

21. SSM ***REASONING*** We have seen that a convex mirror always forms a *virtual image* as shown in Figure 25.22*a* of the text, where the image is *upright* and *smaller* than the object. These characteristics should bear out in the results of our calculations.

SOLUTION The radius of curvature of the convex mirror is 68 cm. Therefore, the focal length is, from Equation 25.2, $f = -(1/2)R = -34\text{ cm}$. Since the image is virtual, we know that $d_i = -22\text{ cm}$.

a. With $d_i = -22\text{ cm}$ and $f = -34\text{ cm}$, the mirror equation gives

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{-34\text{ cm}} - \frac{1}{-22\text{ cm}} \qquad \text{or} \qquad \boxed{d_o = +62\text{ cm}}$$

b. According to the magnification equation, the magnification is

$$m = -\frac{d_i}{d_o} = -\frac{-22\text{ cm}}{62\text{ cm}} = \boxed{+0.35}$$

c. Since the magnification m is positive, the image is $\boxed{\text{upright}}$.

d. Since the magnification m is less than one, the image is $\boxed{\text{smaller}}$ than the object.

22. ***REASONING AND SOLUTION*** We can find the focal length from the mirror equation, provided we have values for both the image and object distances. The image in a convex mirror is upright, so the magnification is a positive quantity. Since the image is half the size of the object, the magnification equation indicates that $m = -d_i / d_o = \frac{1}{2}$, or $d_i = -\frac{1}{2} d_o$. Therefore, the image distance is $d_i = -\frac{1}{2}(13\text{ cm}) = -6.5\text{ cm}$. Using this result in the mirror equation, we find that

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad \text{or} \qquad \frac{1}{13\text{ cm}} + \frac{1}{-6.5\text{ cm}} = \frac{1}{f} \qquad \text{or} \qquad \boxed{f = -13\text{ cm}}$$

23. SSM WWW ***REASONING*** This problem can be solved by using the mirror equation, Equation 25.3, and the magnification equation, Equation 25.4.

SOLUTION

a. Using the mirror equation with $d_i = d_o$ and $f = R/2$, we have

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{R/2} - \frac{1}{d_o} \qquad \text{or} \qquad \frac{2}{d_o} = \frac{2}{R}$$

Therefore, we find that $\boxed{d_o = R}$.

b. According to the magnification equation, the magnification is

$$m = -\frac{d_i}{d_o} = -\frac{d_o}{d_o} = \boxed{-1}$$

c. Since the magnification m is negative, the image is $\boxed{\text{inverted}}$.

24. ***REASONING AND SOLUTION***

a. Since the image of the tooth is enlarged, it cannot be a plane mirror. Convex mirrors will produce smaller images in all cases. Therefore, the mirror is $\boxed{\text{concave}}$.

b. The focal length is

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{2.0\text{ cm}} + \frac{1}{-5.6\text{ cm}} \quad \text{so} \quad \boxed{f = +3.1\text{ cm}}$$

c. The magnification is

$$m = -\frac{d_i}{d_o} = -\frac{-5.6\text{ cm}}{2.0\text{ cm}} = \boxed{+2.8}$$

d. Since m is positive, the image is $\boxed{\text{upright}}$ relative to the object.

25. ***REASONING*** Since the size of the image is one-third that of the object, we know from Equation 25.4 that

$$\underbrace{m}_{1/3} = -\frac{d_i}{d_o} \qquad \text{so that} \qquad d_i = -\tfrac{1}{3}d_o$$

We will substitute this relation into the mirror equation to find the ratio d_o/f.

SOLUTION Substituting $d_i = -\frac{1}{3}d_o$ into the mirror equation gives

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad \text{or} \qquad \frac{1}{d_o} + \frac{1}{-\frac{1}{3}d_0} = \frac{1}{f}$$

Solving this last equation for the ratio d_o/f yields $d_o/f = \boxed{-2}$.

26. ***REASONING*** The mirror equation relates the object and image distances to the focal length. The magnification equation relates the magnification to the object and image distances. The problem neither gives nor asks for information about the image distance. Therefore, we can solve the magnification equation for the image distance and substitute the result into the mirror equation to obtain an expression relating the object distance, the magnification, and the focal length. This expression can be applied to both mirrors A and B to obtain the ratio of the focal lengths.

SOLUTION The magnification equation gives the magnification as $m = -d_i/d_o$. Solving for d_i, we obtain $d_i = -md_o$. Substituting this result into the mirror equation, we obtain

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{d_o}+\frac{1}{(-md_o)}=\frac{1}{f} \quad \text{or} \quad \frac{1}{d_o}\left(1-\frac{1}{m}\right)=\frac{1}{f} \quad \text{or} \quad f=\frac{d_o m}{m-1}$$

Applying this result for the focal length f to each mirror gives

$$f_A=\frac{d_o m_A}{m_A-1} \quad \text{and} \quad f_B=\frac{d_o m_B}{m_B-1}$$

Dividing the expression for f_A by the expression for f_B, we find

$$\frac{f_A}{f_B}=\frac{d_0 m_A/(m_A-1)}{d_0 m_B/(m_B-1)}=\frac{m_A(m_B-1)}{m_B(m_A-1)}=\frac{4.0(2.0-1)}{2.0(4.0-1)}=\boxed{0.67}$$

27. SSM WWW ***REASONING AND SOLUTION*** The magnification equation, Equation 25.4, indicates that $d_i = -md_o$. Substituting this result into the mirror equation, we find that

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{-md_o} = \frac{1}{f}$$

Solving for the object distance d_o, we find

$$d_o = \frac{(m-1)f}{m}$$

After the object is moved to its new position, its object distance becomes d_o' and its magnification becomes $m' = 2m$. Therefore, the amount by which the object is moved is

$$d_o - d_o' = \frac{(m-1)f}{m} - \frac{(m'-1)f}{m'} = \frac{(m-1)f}{m} - \frac{(2m-1)f}{2m}$$

$$= -\frac{f}{2m} = -\frac{-24.0\text{ cm}}{2(0.150)} = \boxed{+80.0\text{ cm}}$$

The positive answer means that the initial object distance is larger than the final object distance, so that $\boxed{\text{the object is moved toward the mirror}}$.

28. ***REASONING AND SOLUTION*** We know that $d_o - d_i = 45.0$ cm. We also have $1/d_o + 1/d_i = 1/f$. Solving the first equation for d_i and substituting the result into the second equation yields,

$$\frac{1}{d_o} + \frac{1}{d_o - 45.0\text{ cm}} = \frac{1}{30.0\text{ cm}}$$

Cross multiplying gives $d_o^2 - 105\ d_o + 1350 = 0$, which we can solve using the quadratic equation to yield two roots, $d_o = (105 \pm 75)/2$.

a. When the object lies beyond the center of curvature we have

$$d_{o+} = (1.80 \times 10^2\text{ cm})/2 = \boxed{+9.0 \times 10^1\text{ cm}} \quad \text{and} \quad d_{i+} = \boxed{+45\text{ cm}}$$

b. When the object lies within the focal point

$$d_{o-} = (3.0 \times 10^1\text{ cm})/2 = \boxed{+15\text{ cm}}, \quad \text{and } d_{i-} = \boxed{-3.0 \times 10^1\text{ cm}}$$

29. ***REASONING AND SOLUTION***

a. From the mirror equation we have

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad \text{so that} \qquad d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}}$$

Since f is negative for a convex mirror, and since d_o is always positive, the quantity $(1/f - 1/d_o)$ must always be negative. Hence, d_i will always be negative, i.e., the image is $\boxed{\text{virtual}}$.

b. We have

$$d_i = \frac{1}{\frac{1}{f} - \frac{1}{d_o}} = \frac{f d_o}{d_o - f}$$

Since $m = -d_i/d_o$, we have that

$$m = \frac{-f}{d_o - f}$$

This expression shows that the magnitude of m is less than 1 for the following reason. Because f is always negative for a convex mirror and d_o is always positive, the denominator is always greater than the numerator. So the image is always smaller than the object. In addition, $m = -d_i/d_o > 0$, since d_i is always negative and d_o is positive. Thus, the image is always $\boxed{\text{upright and smaller}}$.

30. ***REASONING AND SOLUTION***

a. Since the car is "very distant," we can assume that it is infinitely far from the mirror. Therefore, when paraxial rays leave the car and travel parallel to the principal axis, they appear to come from the focal point after reflecting from the mirror. Thus, the focal length is $f = -12$ cm (negative, because the mirror is a convex mirror), and the radius of curvature is $R = -2f = \boxed{24 \text{ cm}}$.

b. The ray diagram is similar to that shown in Figure 25.16.

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31. SSM WWW ***REASONING*** When paraxial light rays that are parallel to the principal axis strike a convex mirror, the reflected rays diverge after being reflected, and appear to originate from the focal point F behind the mirror (see Figure 25.16). We can treat the sun as being infinitely far from the mirror, so it is reasonable to treat the incident rays as paraxial rays that are parallel to the principal axis.

SOLUTION

a. Since the sun is infinitely far from the mirror and its image is a virtual image that lies *behind* the mirror, we can conclude that the mirror is a $\boxed{\text{convex mirror}}$.

b. With $d_i = -12.0\text{ cm}$ and $d_o = \infty$, the mirror equation (Equation 25.3) gives

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{d_i} = \frac{1}{d_i}$$

Therefore, the focal length f lies 12.0 cm behind the mirror (this is consistent with the reasoning above that states that, after being reflected, the rays appear to originate from the focal point behind the mirror). In other words, $f = -12.0\text{ cm}$. Then, according to Equation 25.2, $f = -\frac{1}{2}R$, and the radius of curvature is

$$R = -2f = -2(-12.0\text{ cm}) = \boxed{24.0\text{ cm}}$$

32. ***REASONING AND SOLUTION***

a. The height of the shortest mirror would be one-half the height of the person. Therefore,

$$h = H/2 = (1.70 \text{ m} + 0.12 \text{ m})/2 = \boxed{0.91 \text{ m}}$$

b. The bottom edge of the mirror should be above the floor by

$$h' = (1.70 \text{ m})/2 = \boxed{0.85 \text{ m}}$$

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33. ***REASONING AND SOLUTION*** From the mirror equation, we have that

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{17\text{ cm}} - \frac{1}{38\text{ cm}} \quad \text{so} \quad \boxed{d_i = +31\text{ cm}}$$

34. ***REASONING AND SOLUTION***

a. The image distance is

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-2.0\text{ m}} - \frac{1}{15\text{ m}} \quad \text{so} \quad \boxed{d_i = -1.8\text{ m}}$$

b. Since the image distance is negative, the image is $\boxed{\text{virtual}}$.

c. The height of the object is

$$h_i = -h_o \frac{d_i}{d_o} = -(1.6\text{ m})\frac{-1.8\text{ m}}{15\text{ m}} = \boxed{0.19\text{ cm}}$$

35. SSM ***REASONING*** This problem can be solved using the mirror equation, Equation 25.3.

SOLUTION Using the mirror equation with $d_i = +26\ \text{cm}$ and $f = 12\ \text{cm}$, we find

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = \frac{1}{12\ \text{cm}} - \frac{1}{26\ \text{cm}} \qquad \text{or} \qquad \boxed{d_o = +22\ \text{cm}}$$

36. ***REASONING*** The drawing shows two plane mirrors that intersect at an angle of 50°. An incident light ray reflects from one mirror and then the other. The various angles are labeled θ, α, β, γ, ε and ϕ.

We wish to find the numerical value of the angle θ. We will do this using both the law of reflection, which states that when light is reflected from a surface, the angle of incidence is equal to the angle of reflection, and the fact that the sum of the interior angles of a triangle is 180°.

SOLUTION Since the normal to the surface for the incident ray forms a 90° angle with the horizontal mirror, we know that $\gamma = 90° - \alpha$. Then, using the same reasoning, $\varepsilon = 90° - \beta$. Finally, from the diagram, we see that $\theta + \phi = 180°$, or, solving for θ, we find that $\theta = 180° - \phi$.

We now proceed to reduce this expression to angles and expressions of known values. From the figure, $2\alpha + 2\beta + \phi = 180°$, so that $\phi = 180° - 2(\alpha + \beta)$. Therefore, we find that $\theta = 180° - \phi = 180° - 180° + 2(\alpha + \beta)$, or

$$\theta = 2(\alpha + \beta) \tag{1}$$

We now proceed to find the quantity $\alpha + \beta$. For the triangle formed by the intersection of the mirrors and the first reflected ray, $180° = 50° + \gamma + \varepsilon$. Since $\gamma = 90° - \alpha$ and $\varepsilon = 90° - \beta$ as shown above, this becomes

$$180° = 50° + (90° - \alpha) + (90° - \beta) = 50° + 180° - (\alpha + \beta)$$

or

$$\alpha + \beta = 50°$$

Substituting into Equation (1),

$$\theta = 2(\alpha + \beta) = 2 \times 50° = \boxed{100°}$$

37. ***REASONING AND SOLUTION*** To obtain the focal length, we will use the mirror equation. To do so, it is necessary to obtain values for the object and image distances. Since the image distance for virtual image 1 in Figure 25.21b is a negative quantity, $-d_i$ will be a positive quantity. Therefore, the distance between the readout device and image 1 is $d_o - d_i = 2.00\text{ m}$. Equation 25.4 gives the magnification m as

$$m = -d_i / d_o = 4.00 \qquad \text{so that} \qquad d_i = -4.00\, d_o$$

As a result

$$d_o - d_i = d_o - \left(-4.00\, d_o\right) = 2.00\text{ m} \qquad \text{or} \qquad d_o = 0.400\text{ m}$$

The image distance is

$$d_i = -4.00\, d_o = -4.00\left(0.400\text{ m}\right) = -1.60\text{ m}$$

We can now obtain the focal length from the mirror equation:

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{0.400\text{ m}} + \frac{1}{-1.60\text{ m}} = \frac{1}{f} \qquad \text{or} \qquad \boxed{f = 0.533\text{ m}}$$

38. ***REASONING*** The image in the plane mirror is located 15.0 cm behind the mirror. This is because the candle is located 15.0 cm in front of the mirror, and plane mirrors always produce images that are the same distance behind the mirror as the object is in front of it. We know that the image moves 7.0 cm farther away from the mirror, when the plane mirror replaces the convex mirror. Therefore, the image in the convex mirror must have been 15.0 cm – 7.0 cm = 8.0 cm behind the mirror. This means that the image distance for the convex mirror is –8.0 cm. Knowing both the object and image distances, we can use the mirror equation to calculate the focal length.

SOLUTION According to the mirror equation, we have

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{15.0\text{ cm}} + \frac{1}{(-8.0\text{ cm})} = -0.058\text{ cm}^{-1} \quad \text{or} \quad \boxed{f = -17\text{ cm}}$$

39. **SSM** ***REASONING*** We need to know the focal length of the mirror and can obtain it from the mirror equation, Equation 25.3, as applied to the first object:

$$\frac{1}{d_{o1}}+\frac{1}{d_{i1}}=\frac{1}{14.0\text{ cm}}+\frac{1}{-7.00\text{ cm}}=\frac{1}{f} \qquad \text{or} \qquad f=-14.0\text{ cm}$$

According to the magnification equation, Equation 25.4, the image height h_i is related to the object height h_o as follows: $h_i = mh_o = \left(-d_i / d_o\right)h_o$.

SOLUTION Applying this result to each object, we find that $h_{i2} = h_{i1}$, or

$$\left(\frac{-d_{i2}}{d_{o2}}\right)h_{o2}=\left(\frac{-d_{i1}}{d_{o1}}\right)h_{o1}$$

Therefore,

$$d_{i2}=d_{o2}\left(\frac{d_{i1}}{d_{o1}}\right)\left(\frac{h_{o1}}{h_{o2}}\right)$$

Using the fact that $h_{o2} = 2h_{o1}$, we have

$$d_{i2}=d_{o2}\left(\frac{d_{i1}}{d_{o1}}\right)\left(\frac{h_{o1}}{h_{o2}}\right)=d_{o2}\left(\frac{-7.00\text{ cm}}{14.0\text{ cm}}\right)\left(\frac{h_{o1}}{2h_{o1}}\right)=-0.250\,d_{o2}$$

Using this result in the mirror equation, as applied to the second object, we find that

$$\frac{1}{d_{o2}}+\frac{1}{d_{i2}}=\frac{1}{f}$$

or

$$\frac{1}{d_{o2}}+\frac{1}{-0.250\,d_{o2}}=\frac{1}{-14.0\text{ cm}}$$

Therefore,

$$\boxed{d_{o2}=+42.0\text{ cm}}$$

40. ***REASONING AND SOLUTION*** The magnification is $m = -d_i / d_o = +1/4$, so that $d_i = -(1/4)d_o$. Thus,

$$\frac{1}{f} = \frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{d_o} + \frac{1}{\left(-\frac{1}{4}d_o\right)} = \frac{-3}{d_o}$$

Therefore, $f = -(1/3)d_o$, for the convex side. Since the radius of curvature is the same in each case, the focal length of the concave side is $f' = +\frac{1}{3}d_o'$. So,

$$\frac{1}{d_o'} + \frac{1}{d_i'} = \frac{1}{f'} = \frac{1}{\frac{1}{3}d_o'} \qquad \text{so that} \qquad \frac{1}{d_i'} = \frac{2}{d_o'}$$

The magnification of the concave side is, therefore, $m' = -d_i'/d_o' = \boxed{-1/2}$.

41. ***REASONING AND SOLUTION*** We can see from the diagram that tan $\theta = x/(L/2)$. We also see that tan $\theta = (L - x)/L$.
Equating,

$$\frac{x}{\frac{1}{2}L} = \frac{L-x}{L} \qquad \text{or} \qquad x = \tfrac{1}{3}L$$

Therefore,

$$\theta = \tan^{-1}\left(\frac{\frac{1}{3}L}{\frac{1}{2}L}\right) = \boxed{33.7°}$$

1. **SSM** ***REASONING AND SOLUTION*** The speed of light in benzene v is related to the speed of light in vacuum c by the index of refraction n. The index of refraction is defined by Equation 26.1 ($n = c/v$). According to Table 26.1, the index of refraction of benzene is 1.501. Therefore, solving for v, we have

$$v = \frac{c}{n} = \frac{3.00 \times 10^8 \text{ m/s}}{1.501} = \boxed{2.00 \times 10^8 \text{ m/s}}$$

2. ***REASONING AND SOLUTION*** Using Equation 26.1 for diamond and for ice, we have

$$v_d = c/n_d \qquad \text{and} \qquad v_i = c/n_i$$

Dividing the above equations reveals that

$$v_d/v_i = n_i/n_d = 1.309/2.419 = \boxed{0.5411}$$

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3. ***REASONING*** The wavelength λ is related to the frequency f and speed v of the light in a material by Equation 16.1 ($\lambda = v/f$). The speed of the light in each material can be expressed using Equation 26.1 ($v = c/n$) and the refractive indices n given in Table 26.1. With these two equations, we can obtain the desired ratio.

SOLUTION Using Equations 16.1 and 26.1, we find

$$\lambda = \frac{v}{f} = \frac{c/n}{f} = \frac{c}{fn}$$

Using this result and recognizing that the frequency f and the speed c of light in a vacuum do not depend on the material, we obtain the ratio of the wavelengths as follows:

$$\frac{\lambda_{\text{alcohol}}}{\lambda_{\text{disulfide}}} = \frac{\left(\dfrac{c}{fn}\right)_{\text{alcohol}}}{\left(\dfrac{c}{fn}\right)_{\text{disulfide}}} = \frac{\dfrac{c}{f}\left(\dfrac{1}{n}\right)_{\text{alcohol}}}{\dfrac{c}{f}\left(\dfrac{1}{n}\right)_{\text{disulfide}}} = \frac{n_{\text{disulfide}}}{n_{\text{alcohol}}} = \frac{1.632}{1.362} = \boxed{1.198}$$

4. ***REASONING*** We can identify the substance in Table 26.1 if we can determine its index of refraction. The index of refraction n is equal to the speed of light c in a vacuum divided by the speed of light v in the substance, or $n = c/v$. According to Equation 16.1, however, the speed of light is related to its wavelength λ and frequency f via $v = f\lambda$. Combining these two equations by eliminating the speed v yields $n = c/(f\lambda)$.

SOLUTION The index of refraction of the substance is

$$n = \frac{c}{f\lambda} = \frac{2.998 \times 10^{8}\ \text{m/s}}{\left(5.403 \times 10^{14}\ \text{Hz}\right)\left(340.0 \times 10^{-9}\ \text{m}\right)} = 1.632$$

An examination of Table 26.1 shows that the substance is $\boxed{\text{carbon disulfide}}$.

5. **SSM** **WWW** ***REASONING*** Since the light will travel in glass at a constant speed v, the time it takes to pass perpendicularly through the glass is given by $t = d/v$, where d is the thickness of the glass. The speed v is related to the vacuum value c by Equation 26.1: $n = c/v$.

SOLUTION Substituting for v from Equation 26.1 and substituting values, we obtain

$$t = \frac{d}{v} = \frac{nd}{c} = \frac{(1.5)(4.0\times10^{-3}\ \text{m})}{3.00\times10^{8}\ \text{m/s}} = \boxed{2.0\times10^{-11}\ \text{s}}$$

6. ***REASONING AND SOLUTION*** If $n_B = c/v$, then $n_A = c/(1.25\ v)$, so

$$n_A/n_B = 1/1.25 = \boxed{0.800}$$

7. ***REASONING AND SOLUTION*** The speed of light in the vacuum is $c = (3.50\text{ km})/t$. The speed of light in the liquid is $v = (2.50\text{ km})/t$, so

$$n = c/v = (3.50)/(2.50) = \boxed{1.40}$$

8. ***REASONING*** Distance traveled is the speed times the travel time. Assuming that t is the time it takes for the light to travel through the two sheets, it would travel a distance of ct in a vacuum, where its speed is c. Thus, to find the desired distance, we need to determine the travel time t. This time is the sum of the travel times in each sheet. The travel time in each sheet is determined by the thickness of the sheet and the speed of the light in the material. The speed in the material is less than the speed in a vacuum and depends on the refractive index of the material.

SOLUTION In the ice of thickness d_i, the speed of light is v_i, and the travel time is $t_i = d_i/v_i$. Similarly, the travel time in the quartz sheet is $t_q = d_q/v_q$. Therefore, the desired distance ct is

$$ct = c\left(t_i + t_q\right) = c\left(\frac{d_i}{v_i} + \frac{d_q}{v_q}\right) = d_i\frac{c}{v_i} + d_q\frac{c}{v_q}$$

Since Equation 26.1 gives the refractive index as $n = c/v$ and since Table 26.1 gives the indices of refraction for ice and quartz as $n_i = 1.309$ and $n_q = 1.544$, the result just obtained can be written as follows:

$$ct = d_i\frac{c}{v_i} + d_q\frac{c}{v_q} = d_i n_i + d_q n_q = (2.0\text{ cm})(1.309) + (1.1\text{ cm})(1.544) = \boxed{4.3\text{ cm}}$$

9. $\boxed{\text{SSM}}$ ***REASONING*** We begin by using Snell's law (Equation 26.2: $n_1 \sin\theta_1 = n_2 \sin\theta_2$) to find the index of refraction of the material. Then we will use Equation 26.1, the definition of the index of refraction ($n = c/v$) to find the speed of light in the material.

SOLUTION From Snell's law, the index of refraction of the material is

$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2} = \frac{(1.000)\sin 63.0°}{\sin 47.0°} = 1.22$$

Then, from Equation 26.1, we find that the speed of light v in the material is

$$v = \frac{c}{n_2} = \frac{3.00 \times 10^8 \text{ m/s}}{1.22} = \boxed{2.46 \times 10^8 \text{ m/s}}$$

10. ***REASONING AND SOLUTION*** Applying Snell's law at the gas-solid interface gives the angle of refraction θ_2 to be

$$(1.00)\sin 35.0° = (1.55)\sin\theta_2 \qquad \text{or} \qquad \theta_2 = 21.7°$$

Since the refractive index of the liquid is the same as that of the solid, light is not refracted when it enters the liquid. Therefore, the light enters the liquid at an angle of $\boxed{21.7°}$.

11. **SSM** ***REASONING*** We will use the geometry of the situation to determine the angle of incidence. Once the angle of incidence is known, we can use Snell's law to find the index of refraction of the unknown liquid. The speed of light v in the liquid can then be determined.

SOLUTION From the drawing in the text, we see that the angle of incidence at the liquid-air interface is

$$\theta_1 = \tan^{-1}\left(\frac{5.00 \text{ cm}}{6.00 \text{ cm}}\right) = 39.8°$$

The drawing also shows that the angle of refraction is 90.0°. Thus, according to Snell's law (Equation 26.2: $n_1 \sin\theta_1 = n_2 \sin\theta_2$), the index of refraction of the unknown liquid is

$$n_1 = \frac{n_2 \sin\theta_2}{\sin\theta_1} = \frac{(1.000)(\sin 90.0°)}{\sin 39.8°} = 1.56$$

From Equation 26.1 ($n = c/v$), we find that the speed of light in the unknown liquid is

$$v = \frac{c}{n_1} = \frac{3.00 \times 10^8 \text{ m/s}}{1.56} = \boxed{1.92 \times 10^8 \text{ m/s}}$$

12. ***REASONING AND SOLUTION*** The actual height d of the diving board above the water can be obtained by using Equation 26.3. As usual, n_1 is the index of refraction of the medium (air) associated with the incident ray, and n_2 is that of the medium (water) associated with the refracted ray. Taking the refractive index of water from Table 26.1, we find

$$d = d'\left(\frac{n_1}{n_2}\right) = (4.0\text{ m})\left(\frac{1.00}{1.33}\right) = \boxed{3.0\text{ m}}$$

13. ***REASONING AND SOLUTION*** Snell's law gives

$$(1.45)\sin 65.0° = n_2 \sin 53.0°$$

Solving, we find that the index of refraction is $n_2 = \boxed{1.64}$.

14. ***REASONING*** When the incident light is in a vacuum, Snell's law, Equation 26.2, can be used to express the relation between the angle of incidence (35.0°), the (unknown) index of refraction n_2 of the glass and the (unknown) angle θ_2 of refraction for the light entering the slab: $(1.00)\sin 35.0° = n_2 \sin \theta_2$ When the incident light is in the liquid, we can again use Snell's law to express the relation between the index of refraction n_1 of the liquid, the angle of incidence (20.3°), the index of refraction n_2 of the glass, and the (unknown) angle of refraction θ_2: $n_1 \sin 20.3° = n_2 \sin \theta_2$. By equating these two equations, we can determine the index of refraction of the liquid.

SOLUTION Setting the two equations above equal to each other and solving for the index of refraction of the liquid gives

$$n_1 \sin 20.3° = (1.00)\sin 35.0° \quad \text{and} \quad n_1 = \frac{(1.00)\sin 35.0°}{\sin 20.3°} = \boxed{1.65}$$

15. **SSM** **WWW** The drawing at the right shows the geometry of the situation using the same notation as that in Figure 26.7. In addition to the text's notation, let t represent the thickness of the pane, let L represent the length of the ray in the pane, let x (shown twice in the figure) equal the displacement of the ray, and let the difference in angles $\theta_1 - \theta_2$ be given by ϕ.

We wish to find the amount x by which the emergent ray is displaced relative to the incident ray. This can be done by applying Snell's law at each interface, and then making use of the geometric and trigonometric relations in the drawing.

SOLUTION If we apply Snell's law (see Equation 26.2) to the bottom interface we obtain $n_1 \sin\theta_1 = n_2 \sin\theta_2$. Similarly, if we apply Snell's law at the top interface where the ray emerges, we have $n_2 \sin\theta_2 = n_3 \sin\theta_3 = n_1 \sin\theta_3$. Comparing this with Snell's law at the bottom face, we see that $n_1 \sin\theta_1 = n_1 \sin\theta_3$, from which we can conclude that $\theta_3 = \theta_1$. Therefore, the emerging ray is parallel to the incident ray.

From the geometry of the ray and thickness of the pane, we see that $L\cos\theta_2 = t$, from which it follows that $L = t/\cos\theta_2$. Furthermore, we see that $x = L\sin\phi = L\sin(\theta_1 - \theta_2)$. Substituting for L, we find

$$x = L\sin(\theta_1 - \theta_2) = \frac{t\sin(\theta_1 - \theta_2)}{\cos\theta_2}$$

Before we can use this expression to determine a numerical value for x, we must find the value of θ_2. Solving the expression for Snell's law at the bottom interface for θ_2, we have

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2} = \frac{(1.000)(\sin 30.0°)}{1.52} = 0.329 \qquad \text{or} \qquad \theta_2 = \sin^{-1} 0.329 = 19.2°$$

Therefore, the amount by which the emergent ray is displaced relative to the incident ray is

$$x = \frac{t\sin(\theta_1 - \theta_2)}{\cos\theta_2} = \frac{(6.00\text{ mm})\sin(30.0° - 19.2°)}{\cos 19.2°} = \boxed{1.19\text{ mm}}$$

16. ***REASONING AND SOLUTION***

a. Snell's law applied at the air-ice interface gives

$$\sin\theta_2 = (n_1/n_2)\sin\theta_1 = (1.000/1.309)\sin 45° = 0.54$$

$$\theta_2 = \sin^{-1}(0.54) = \boxed{33°}$$

b. This angle becomes the angle of incidence at the ice-water interface, so

$$\sin\theta_2 = (1.309/1.333)\sin 33° = 0.53 \qquad \text{or} \qquad \theta_2 = \sin^{-1}(0.53) = \boxed{32°}$$

17. ***REASONING AND SOLUTION*** The horizontal distance of the chest from the normal is found from Figure 26.5*a* to be $x = d \tan \theta_1$ and $x = d' \tan \theta_2$, where θ_1 is the angle from the dashed normal to the solid rays and θ_2 is the angle from the dashed normal to the dashed rays. Hence,

$$d' = (\tan \theta_1/\tan \theta_2)d$$

Snell's law applied at the interface gives

$$n_1 \sin \theta_1 = n_2 \sin \theta_2$$

For small angles, $\sin \theta_1 \approx \tan \theta_1$ and $\sin \theta_2 \approx \tan \theta_2$, so

$$\tan \theta_1/ \tan \theta_2 \approx \sin \theta_1/\sin \theta_2 = (n_2/n_1)$$

Now $d' = d (\tan \theta_1/\tan \theta_2)$. Therefore,

$$\boxed{d' \approx d\left(\frac{n_2}{n_1}\right)}$$

18. ***REASONING*** Following the discussion in Conceptual Example 4, we have the drawing at the right to use as a guide. In this drawing the symbol d refers to depths in the water, while the symbol h refers to heights in the air above the water. Moreover, symbols with a prime denote apparent distances, and unprimed symbols denote actual distances. We will use Equation 26.3 to relate apparent distances to actual distances. In so doing, we will use the fact that the refractive index of air is essentially $n_{\text{air}} = 1$ and denote the refractive index of water by $n_{\text{w}} = 1.333$ (see Table 26.1).

SOLUTION To the fish, the man appears to be a distance above the air-water interface that is given by Equation 26.3 as $h' = h\left(n_{\text{w}}/1\right)$. Thus, measured above the eyes of the fish, the man appears to be located at a distance of

$$h' + d = h\left(\frac{n_{\text{w}}}{1}\right) + d \tag{1}$$

To the man, the fish appears to be a distance below the air-water interface that is given by Equation 26.3 as $d' = d\left(1/n_{\text{w}}\right)$. Thus, measured below the man's eyes, the fish appears to be located at a distance of

$$h + d' = h + d\left(\frac{1}{n_{\text{w}}}\right) \tag{2}$$

Dividing Equation (1) by Equation (2) and using the fact that $h = d$, we find

$$\frac{h' + d}{h + d'} = \frac{h\left(\frac{n_{\text{w}}}{1}\right) + d}{h + d\left(\frac{1}{n_{\text{w}}}\right)} = \frac{n_{\text{w}} + 1}{1 + \frac{1}{n_{\text{w}}}} = n_{\text{w}} \tag{3}$$

In Equation (3), $h' + d$ is the distance we seek, and $h + d'$ is given as 2.0 m. Thus, we find

$$h' + d = n_{\text{w}}\left(h + d'\right) = \left(1.333\right)\left(2.0\text{ m}\right) = \boxed{2.7\text{ m}}$$

19. ***REASONING AND SOLUTION*** Using Equation 26.3, we have for the block in air

$$d_a' = (1/n_p)d$$

and for the block in water

$$d_w' = (n_w/n_p)d$$

Therefore, using the refractive index for water given in Table 26.1, we find

$$d_w' = n_w d_a' = (1.333)(1.6\text{ cm}) = \boxed{2.1\text{ cm}}$$

20. ***REASONING AND SOLUTION*** The light rays coming from the bottom of the beaker are refracted at two interfaces, the water-oil interface and the oil-air interface. When the rays enter the oil from the water, they appear to have originated from an apparent depth d′ below the water-oil interface. This apparent depth is given by Equation 26.3 as

$$d' = d\left(\frac{n_{\text{oil}}}{n_{\text{water}}}\right) = (15.0\text{ cm})\left(\frac{1.48}{1.33}\right) = 16.7\text{ cm}$$

When the rays reach the top of the oil, a distance of 15.0 cm above the water, they can be regarded as having originated from a depth of 15.0 cm + 16.7 cm = 31.7 cm below the oil-air interface. When the rays enter the air, they are refracted again and appear to have come from an apparent depth d″ below the oil-air interface. This apparent depth is given by Equation 26.3 as

$$d'' = (31.7\text{ cm})\left(\frac{n_{\text{air}}}{n_{\text{oil}}}\right) = (31.7\text{ cm})\left(\frac{1.00}{1.48}\right) = \boxed{21.4\text{ cm}}$$

21. **SSM** ***REASONING*** The drawing at the right shows the situation. As discussed in the text, when the observer is directly above, the apparent depth d' of the object is related to the actual depth by Equation 26.3:

$$d' = d\left(\frac{n_2}{n_1}\right)$$

In this case, we must apply Equation 26.3 twice; once for the rays in the glass, and once again for the rays in the water.

SOLUTION We refer to the drawing for our notation and begin at the logo. To an observer *in the water* directly above the logo, the apparent depth of the logo is $d'_g = d_g\left(n_w / n_g\right)$. When viewed directly from above in air, the logo's apparent depth is, $d'_w = \left(d_w + d'_g\right)\left(n_{air} / n_w\right)$, where we have used the fact that when viewed from air, the logo's actual depth appears to be $d_w + d'_g$. Substituting the expression for d'_g into the expression for d'_w, we obtain

$$d'_w = (d_w + d'_g)\left(\frac{n_{air}}{n_w}\right) = d_w\left(\frac{n_{air}}{n_w}\right) + d_g\left(\frac{n_w}{n_g}\right)\left(\frac{n_{air}}{n_w}\right) = d_w\left(\frac{n_{air}}{n_w}\right) + d_g\left(\frac{n_{air}}{n_g}\right)$$

$$= (1.50\text{ cm})\left(\frac{1.000}{1.333}\right) + (3.20\text{ cm})\left(\frac{1.000}{1.52}\right) = \boxed{3.23\text{ cm}}$$

22. ***REASONING AND SOLUTION***

a. The apparent "depth" of the fish is given by

$$d' = (1.000/1.333)(15.0 \text{ cm}) = \boxed{11.3 \text{ cm}}$$

b. The image of the fish is located 15.0 cm behind the mirror. The apparent "depth" of the image is then

$$d' = (1.000/1.333)(45.0 \text{ cm}) = \boxed{33.8 \text{ cm}}$$

c. Since the apparent "depth" is inversely proportional to the index of refraction of the liquid, a $\boxed{\text{larger}}$ index of refraction may cause the image to appear in front of the mirror.

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23. **SSM** ***REASONING AND SOLUTION*** According to Equation 26.4, the critical angle is related to the refractive indices n_1 and n_2 by $\sin\theta_c = n_2 / n_1$, where $n_1 > n_2$. Solving for n_1, we find

$$n_1 = \frac{n_2}{\sin\theta_c} = \frac{1.000}{\sin 40.5°} = \boxed{1.54}$$

24. ***REASONING AND SOLUTION*** Using Equation 26.4 and taking the refractive index for carbon disulfide from Table 26.1, we obtain

$$\theta_c = \sin^{-1}(1.000/1.632) = \boxed{37.79°}$$

25. ***REASONING AND SOLUTION***

a. The index of refraction n_2 of the liquid must match that of the glass, or $n_2 = \boxed{1.50}$.

b. When none of the light is transmitted into the liquid, the angle of incidence must be equal to or greater than the critical angle. According to Equation 26.4, the critical angle θ_c is given by $\sin \theta_c = n_2/n_1$, where n_2 is the index of refraction of the liquid and n_1 is that of the glass. Therefore,

$$n_2 = n_1 \sin \theta_c = (1.50) \sin 58.0° = \boxed{1.27}$$

If n_2 were larger than 1.27, the critical angle would also be larger, and light would be transmitted from the glass into the liquid. Thus, $n_2 = 1.27$ represents the largest index of refraction of the liquid such that none of the light is transmitted into the liquid.

26. ***REASONING AND SOLUTION*** Only the light which has an angle of incidence less than or equal θ_c can escape. This light leaves the source in a cone whose apex angle is $2\theta_c$. The radius of this cone at the surface of the water (n = 1.333, see Table 26.1) is $R = d \tan \theta_c$. Now

$$\theta_c = \sin^{-1} (1.000/1.333) = 48.6°$$

so

$$R = (2.2 \text{ m}) \tan 48.6° = \boxed{2.5 \text{ m}}$$

27. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** If a person's eyes are very close to the surface of the water, a light ray coming from the shark will be seen even when it is refracted through an angle of 90.0° as it enters the air. In this situation, the ray strikes the water-air interface at the critical angle. The critical angle θ_c is given by Equation 26.4 as

$$\theta_c = \sin^{-1}\left(\frac{1.00}{1.333}\right) = 48.6°$$

where we have used $n = 1.333$ for the refractive index of water (see Table 26.1). The horizontal distance x of the shark from the boat is related to the depth (4.5 m) of the shark and the critical angle by trigonometry:

$$x = (4.5\text{ m})\tan 48.6° = \boxed{5.1\text{ m}}$$

If the shark is farther than 5.1 m from the boat, a light ray from the shark will strike the water-air interface at an angle that is greater than the critical angle. The ray will be totally reflected back into the water, and the person will not see the shark.

28. ***REASONING AND SOLUTION***

a. Using Equation 26.4 and the refractive index for crown glass given in Table 26.1, we find that the critical angle for a crown glass-air interface is

$$\theta_c = \sin^{-1}(1.00/1.523) = 41.0°$$

The light will be totally reflected at point A since the incident angle of 60.0° is greater than θ_c. The incident angle at point B, however, is 30.0° and smaller than θ_c. The light will exit first at $\boxed{\text{point B}}$.

b. The critical angle for crown glass-water is

$$\theta_c = \sin^{-1}(1.333/1.523) = 61.1°$$

The incident angle at point A is less than this, so the light will first exit at $\boxed{\text{point A}}$.

29. ***REASONING*** Total internal reflection will occur at point P provided that the angle α in the drawing at the right exceeds the critical angle. This angle is determined by the angle θ_2 at which the light rays enter the quartz slab. We can determine θ_2 by using Snell's law of refraction and the incident angle, which is given as $\theta_1 = 34°$.

SOLUTION Using n for the refractive index of the fluid that surrounds the crystalline quartz slab and n_q for the refractive index of quartz and applying Snell's law give

$$n \sin\theta_1 = n_q \sin\theta_2 \qquad \text{or} \qquad \sin\theta_2 = \frac{n}{n_q}\sin\theta_1 \tag{1}$$

But when α equals the critical angle, we have from Equation 26.4 that

$$\sin\alpha = \sin\theta_c = \frac{n}{n_q} \tag{2}$$

According to the geometry in the drawing above, $\alpha = 90° - \theta_2$. As a result, Equation (2) becomes

$$\sin\left(90° - \theta_2\right) = \cos\theta_2 = \frac{n}{n_q} \tag{3}$$

Squaring Equation (3), using the fact that $\sin^2\theta_2 + \cos^2\theta_2 = 1$, and substituting from Equation (1), we obtain

$$\cos^2\theta_2 = 1 - \sin^2\theta_2 = 1 - \frac{n^2}{n_q^2}\sin^2\theta_1 = \frac{n^2}{n_q^2} \tag{4}$$

Solving Equation (4) for n and using the value given in Table 26.1 for the refractive index of crystalline quartz, we find

$$n = \frac{n_q}{\sqrt{1+\sin^2\theta_1}} = \frac{1.544}{\sqrt{1+\sin^2 34°}} = \boxed{1.35}$$

30. ***REASONING*** When the light enters the optical fiber, the angle of incidence θ_1 is related to the angle of refraction θ_2 by Snell's law, Equation 26.2, as $n_{\text{air}} \sin \theta_1 = n_{\text{flint}} \sin \theta_2$. As the drawing shows, the ray of light in the flint glass strikes the crown glass at an angle of incidence of $90° - \theta_2$. According to the problem statement, this angle is the critical angle for the interface between the two glasses.

According to Equation 26.4, the critical angle is related to the indices of refraction by

$$\sin \underbrace{\theta_c}_{90° - \theta_2} = \frac{n_{\text{crown}}}{n_{\text{flint}}}$$

This relation and Snell's law can be used to find the angle of incidence.

SOLUTION The critical angle is

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{crown}}}{n_{\text{flint}}}\right) = \sin^{-1}\left(\frac{1.523}{1.667}\right) = 66.01°$$

Since $\theta_c = 90° - \theta_2$, it follows that $\theta_2 = 90° - 66.01° = 23.99°$. Using Snell's law, $n_{\text{air}} \sin \theta_1 = n_{\text{flint}} \sin \theta_2$, we find that the angle of incidence is

$$\theta_1 = \sin^{-1}\left(\frac{n_{\text{flint}} \sin \theta_2}{n_{\text{air}}}\right) = \sin^{-1}\left(\frac{1.667 \sin 23.99°}{1.000}\right) = \boxed{42.67°}$$

31. **SSM** ***REASONING*** Since the light reflected from the coffee table is completely polarized parallel to the surface of the glass, the angle of incidence must be the Brewster angle ($\theta_B = 56.7°$) for the air-glass interface. We can use Brewster's law (Equation 26.5: $\tan \theta_B = n_2 / n_1$) to find the index of refraction n_2 of the glass.

SOLUTION Solving Brewster's law for n_2, we find that the refractive index of the glass is

$$n_2 = n_1 \tan\theta_B = (1.000)(\tan 56.7°) = \boxed{1.52}$$

32. ***REASONING*** Using the value given for the critical angle in Equation 26.4 ($\sin\theta_c = n_2/n_1$), we can obtain the ratio of the refractive indices. Then, using this ratio in Equation 26.5 (Brewster's law), we can obtain Brewster's angle θ_B.

SOLUTION From Equation 26.4, with $n_2 = n_{air} = 1$ and $n_2 = n_{liquid}$, we have

$$\sin\theta_c = \sin 39° = \frac{1}{n_{liquid}} \tag{1}$$

According to Brewster's law,

$$\tan\theta_B = \frac{n_2}{n_1} = \frac{1}{n_{liquid}} \tag{2}$$

Substituting Equation (2) into Equation (1), we find

$$\tan\theta_B = \frac{1}{n_{liquid}} = \sin 39° = 0.63 \qquad \text{or} \qquad \theta_B = \tan^{-1}(0.63) = \boxed{32°}$$

33. ***REASONING AND SOLUTION***

a. Using Brewster's law (Equation 26.5) and taking values for the refractive indices from Table 26.1, we find

$$\theta_B = \tan^{-1}(1.333/1.000) = \boxed{53.12°}$$

b. Similarly, we find

$$\theta_B = \tan^{-1}(1.309/1.000) = \boxed{52.62°}$$

34. ***REASONING AND SOLUTION*** The incident angle must equal Brewster's angle for the light to be 100% polarized. Using the refractive index of carbon tetrachloride given in Table26.1, we have

$$\theta_B = \tan^{-1}(1.461/1.000) = 55.61°$$

Snell's law applied at the interface gives

$$\sin \theta_2 = (1.000/1.461) \sin 55.61° \qquad \text{or} \qquad \theta_2 = \boxed{34.39°}$$

35. $\boxed{\text{SSM}}$ $\boxed{\text{WWW}}$ ***REASONING*** Brewster's law (Equation 26.5: $\tan\theta_B = n_2/n_1$) relates the angle of incidence θ_B at which the reflected ray is completely polarized parallel to the surface to the indices of refraction n_1 and n_2 of the two media forming the interface. We can use Brewster's law for light incident from above to find the ratio of the refractive indices n_2/n_1. This ratio can then be used to find the Brewster angle for light incident from below on the same interface.

SOLUTION The index of refraction for the medium in which the incident ray occurs is designated by n_1. For the light striking from above $n_2/n_1 = \tan\theta_B = \tan 65.0° = 2.14$. The same equation can be used when the light strikes from below if the indices of refraction are interchanged

$$\theta_B = \tan^{-1}\left(\frac{n_1}{n_2}\right) = \tan^{-1}\left(\frac{1}{n_2/n_1}\right) = \tan^{-1}\left(\frac{1}{2.14}\right) = \boxed{25.0°}$$

36. ***REASONING*** When light is incident at the Brewster angle, we know that the angle between the refracted ray and the reflected ray is 90°. This relation will allow us to determine the Brewster angle. By applying Snell's law to the incident and refracted rays, we can find the index of refraction of the glass.

SOLUTION The drawing shows the incident, reflected, and refracted rays.

a. We see from the drawing that $\theta_B + 90° + \theta_2 = 180°$, so that $\theta_B = 90° - 29.9° = \boxed{60.1°}$.

b. Applying Snell's law at the vacuum/glass interface gives

$$n_{\text{vacuum}} \sin \theta_B = n_{\text{glass}} \sin \theta_2$$

$$n_{\text{glass}} = \left[\frac{(1.00)\sin 60.1°}{\sin 29.9°} \right] = \boxed{1.74}$$

37. ***REASONING AND SOLUTION*** From Snell's law we have

$$\sin\theta_B = \left(\frac{n_2 \sin\theta_2}{n_1}\right)$$

But from Brewster's law, Equation 26.5, $n_2/n_1 = \tan\theta_B$. Substituting this expression for n_2/n_1 into Snell's law, we see that

$$\sin\theta_B = \tan\theta_B \sin\theta_2 = \left(\frac{\sin\theta_B}{\cos\theta_B}\right)\sin\theta_2$$

This result shows that $\cos\theta_B = \sin\theta_2$. Since $\sin\theta_2 = \cos(90° - \theta_2)$, we have that $\cos\theta_B = \sin\theta_2 = \cos(90° - \theta_2)$. Thus, $\theta_B = 90° - \theta_2$, so $\theta_B + \theta_2 = 90°$, and

$$\boxed{\text{the reflected and refracted rays are perpendicular}}\ .$$

38. ***REASONING AND SOLUTION*** Snell's law for the yellow light is

$$(1.000)\sin 45.0° = n_y \sin \theta_2$$

and for the blue light is

$$(1.000)\sin \theta_1 = n_b \sin \theta_2$$

Dividing these equations gives

$$\sin\theta_1 = \frac{n_b}{n_y}\sin 45.0° = \left(\frac{1.684}{2.417}\right)\sin 45.0° \qquad \text{or} \qquad \theta_1 = \boxed{29.5°}$$

39. $\boxed{\text{SSM}}$ ***REASONING*** Because the refractive index of the glass depends on the wavelength (i.e., the color) of the light, the rays corresponding to different colors are bent by different amounts in the glass. We can use Snell's law (Equation 26.2: $n_1 \sin\theta_1 = n_2 \sin\theta_2$) to find the angle of refraction for the violet ray and the red ray. The angle between these rays can be found by the subtraction of the two angles of refraction.

SOLUTION In Table 26.2 the index of refraction for violet light in crown glass is 1.538, while that for red light is 1.520. According to Snell's law, then, the sine of the angle of refraction for the violet ray in the glass is $\sin\theta_2 = (1.000/1.538)\ \sin 45.00° = 0.4598$, so that

$$\theta_2 = \sin^{-1}(0.4598) = 27.37°$$

Similarly, for the red ray, $\sin\theta_2 = (1.000/1.520)\ \sin 45.00° = 0.4652$, from which it follows that

$$\theta_2 = \sin^{-1}(0.4652) = 27.72°$$

Therefore, the angle between the violet ray and the red ray in the glass is

$$27.72° - 27.37° = \boxed{0.35°}$$

40. ***REASONING*** The angle of each refracted ray in the crown glass can be obtained from Snell's law (Equation 26.2) as $n_{\text{diamond}} \sin \theta_1 = n_{\text{crown glass}} \sin \theta_2$, where θ_1 is the angle of incidence and θ_2 is the angle of refraction.

SOLUTION The angles of refraction for the red and blue rays are:

Blue ray

$$\theta_2 = \sin^{-1}\left(\frac{n_{\text{diamond}} \sin \theta_1}{n_{\text{crown glass}}}\right) = \sin^{-1}\left[\frac{(2.444)\sin 35.00°}{1.531}\right] = 66.29°$$

Red ray

$$\theta_2 = \sin^{-1}\left(\frac{n_{\text{diamond}} \sin \theta_1}{n_{\text{crown glass}}}\right) = \sin^{-1}\left[\frac{2.410 \sin 35.00°}{1.520}\right] = 65.43°$$

The angle between the blue and red rays is

$$\theta_{\text{blue}} - \theta_{\text{red}} = 66.29° - 65.43° = \boxed{0.86°}$$

41. ***REASONING AND SOLUTION*** Both the red and violet rays strike the prism at 0° and will have 0° angles of refraction. Both rays will then have 25.0° angle of incidence at the glass-air interface. Snell's law gives

Red ray $\sin \theta_R = (1.662/1.000) \sin 25.0°$ or $\theta_R = \boxed{44.6°}$

Violet ray $\sin \theta_V = (1.698/1.000) \sin 25.0°$ or $\theta_V = \boxed{45.9°}$

42. ***REASONING AND SOLUTION*** From geometry, the angle of incidence at the left face of the prism is $\theta_1 = 27.0°$. The angle of refraction θ_2 of the light entering the prism is given by Snell's law, $n_1 \sin \theta_1 = n_2 \sin \theta_2$. Thus,

$$\theta_2 = \sin^{-1}\left(\frac{1.48 \sin 27.0°}{1.31}\right) = 30.86°$$

Geometry can be used again to show that the angle of incidence θ_3 at the right face of the prism is $\theta_3 = 23.14°$. The angle of refraction θ_4 of the light entering the liquid is given by Snell's law

$$\theta_4 = \sin^{-1}\left(\frac{1.31 \sin 23.14°}{1.48}\right) = \boxed{20.4°}$$

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43. **SSM** ***REASONING*** We can use Snell's law (Equation 26.2: $n_1 \sin\theta_1 = n_2 \sin\theta_2$) at each face of the prism. At the first interface where the ray enters the prism, $n_1 = 1.000$ for air and $n_2 = n_g$ for glass. Thus, Snell's law gives

$$(1)\sin 60.0° = n_g \sin\theta_2 \qquad \text{or} \qquad \sin\theta_2 = \frac{\sin 60.0°}{n_g} \tag{1}$$

We will represent the angles of incidence and refraction at the second interface as θ_1' and θ_2', respectively. Since the triangle is an equilateral triangle, the angle of incidence at the second interface, where the ray emerges back into air, is $\theta_1' = 60.0° - \theta_2$. Therefore, at the second interface, where $n_1 = n_g$ and $n_2 = 1.000$, Snell's law becomes

$$n_g \sin\ (60.0° - \theta_2) = (1)\sin\theta_2' \tag{2}$$

We can now use Equations (1) and (2) to determine the angles of refraction θ_2' at which the red and violet rays emerge into the air from the prism.

SOLUTION

Red Ray The index of refraction of flint glass at the wavelength of red light is $n_g = 1.662$. Therefore, using Equation (1), we can find the angle of refraction for the red ray as it enters the prism:

$$\sin\theta_2 = \frac{\sin 60.0°}{1.662} = 0.521 \qquad \text{or} \qquad \theta_2 = \sin^{-1} 0.521 = 31.4°$$

Substituting this value for θ_2 into Equation (2), we can find the angle of refraction at which the red ray emerges from the prism:

$$\sin\theta_2' = 1.662 \sin(60.0° - 31.4°) = 0.796 \qquad \text{or} \qquad \theta_2' = \sin^{-1} 0.796 = \boxed{52.7°}$$

Violet Ray For violet light, the index of refraction for glass is $n_g = 1.698$. Again using Equation (1), we find

$$\sin\theta_2 = \frac{\sin 60.0°}{1.698} = 0.510 \qquad \text{or} \qquad \theta_2 = \sin^{-1} 0.510 = 30.7°$$

Using Equation (2), we find

$$\sin\theta_2' = 1.698 \sin(60.0° - 30.7°) = 0.831 \qquad \text{or} \qquad \theta_2' = \sin^{-1} 0.831 = \boxed{56.2°}$$

44. ***REASONING AND SOLUTION*** Using the thin lens equation, we find

$$1/f = 1/d_o + 1/d_i = 1/(13\text{ cm}) + 1/(-5.0\text{ cm}) \qquad \text{or} \qquad f = \boxed{-8.1\text{ cm}}$$

45. **SSM** ***REASONING AND SOLUTION*** Equation 26.6 gives the thin-lens equation which relates the object and image distances d_o and d_i, respectively, to the focal length f of the lens: $(1/d_o)+(1/d_i)=(1/f)$.

The optical arrangement is similar to that in Figure 26.26. The problem statement gives values for the focal length ($f = 50.0\text{ mm}$) and the maximum lens-to-film distance ($d_i = 275\text{ mm}$). Therefore, the maximum distance that the object can be located in front of the lens is

$$\frac{1}{d_o}=\frac{1}{f}-\frac{1}{d_i}=\frac{1}{50.0\text{ mm}}-\frac{1}{275\text{ mm}} \quad \text{or} \quad \boxed{d_o = 61.1\text{ mm}}$$

46. ***REASONING AND SOLUTION*** The image distance for the first case is

$$1/d_i = 1/f - 1/d_o = 1/(200.0\text{ mm}) - 1/(3.5 \times 10^3\text{ mm}) \qquad \text{or} \qquad d_i = 212\text{ mm}$$

and, similarly, for the second case it is $d_i = 201$ mm. Thus, the lens must be capable of moving through a distance of $212\text{ mm} - 201\text{ mm} = 11\text{ mm}$ or $\boxed{0.011\text{ m}}$.

47. ***REASONING AND SOLUTION***

a. According to the thin-lens equation, we have

$$1/d_i = 1/f - 1/d_o = 1/(-25\text{ cm}) - 1/(38\text{ cm}) \qquad \text{or} \qquad d_i = \boxed{-15\text{ cm}}$$

b. The image is $\boxed{\text{virtual}}$ since the image distance is negative.

48. ***REASONING*** The height h_i of the person's image on the film is related to his actual height h_o, the image distance d_i, and the object distance d_0 by the magnification equation, Equation 26.7. All these variables are known, except the image distance. Since the object distance and the focal length are also known, we can use the thin lens equation, Equation 26.6, to find the image distance.

SOLUTION The image height is

$$h_i = h_o\left(-\frac{d_i}{d_o}\right)$$

The image distance d_i can be obtained from the thin-lens equation

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{85.0\times10^{-3}\ \text{m}} - \frac{1}{16.0\ \text{m}} = 11.7\ \text{m}^{-1} \quad \text{or} \quad d_i = 0.0855\ \text{m}$$

The height of the image on the film is

$$h_i = h_o\left(-\frac{d_i}{d_o}\right) = (145\ \text{cm})\left(-\frac{0.0855\ \text{m}}{16.0\ \text{m}}\right) = \boxed{-0.775\ \text{cm}}$$

The minus sign tells us that the image is $\boxed{\text{inverted}}$ relative to the object.

49. **SSM** ***REASONING*** The ray diagram is constructed by drawing the paths of two rays from a point on the object. For convenience, we will choose the top of the object. The ray that is parallel to the principal axis will be refracted by the lens so that it passes through the focal point on the right of the lens. The ray that passes through the center of the lens passes through undeflected. The image is formed at the intersection of these two rays. In this case, the rays do not intersect on the right of the lens. However, if they are extended backwards they intersect on the left of the lens, locating a virtual, upright, and enlarged image.

SOLUTION

a. The ray-diagram, drawn to scale, is shown below.

From the diagram, we see that the image distance is $\boxed{d_i = -75 \text{ cm}}$ and the magnification is $\boxed{+2.5}$. The negative image distance indicates that the image is virtual. The positive magnification indicates that the image is larger than the object.

b. From the thin-lens equation [Equation 26.6: $(1/d_o) + (1/d_i) = (1/f)$], we obtain

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{50.0 \text{ m}} - \frac{1}{30.0 \text{ cm}} \quad \text{or} \quad \boxed{d_i = -75.0 \text{ cm}}$$

The magnification equation (Equation 26.7) gives the magnification to be

$$m = -\frac{d_i}{d_o} = -\frac{-75.0 \text{ cm}}{30.0 \text{ cm}} = \boxed{+2.50}$$

50. ***REASONING*** The distance from the lens to the screen, the image distance, can be obtained directly from the thin-lens equation, Equation 26.6, since the object distance and focal length are known. The width and height of the image on the screen can be determined by using Equation 26.7, the magnification equation.

SOLUTION

a. The distance d_i to the screen is

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{105.00\text{ mm}} - \frac{1}{108.00\text{ mm}} = 2.646 \times 10^{-4}\text{ mm}^{-1}$$

so that $d_i = 3.78 \times 10^3\text{ mm} = \boxed{3.78\text{ m}}$.

b. According to the magnification equation, the width and height of the image on the screen are

Width

$$h_i = h_o\left(-\frac{d_i}{d_o}\right) = (24.0\text{ mm})\left(-\frac{3.78 \times 10^3\text{ mm}}{108\text{ mm}}\right) = -8.40 \times 10^2\text{ mm}$$

The width is $\boxed{8.40 \times 10^2\text{ mm}}$.

Height

$$h_i = h_o\left(-\frac{d_i}{d_o}\right) = (36.0\text{ mm})\left(-\frac{3.78 \times 10^3\text{ mm}}{108\text{ mm}}\right) = -1.26 \times 10^3\text{ mm}$$

The height is $\boxed{1.26 \times 10^3\text{ mm}}$.

51. ***REASONING AND SOLUTION*** Solving the thin-lens equation for the image distance yields

$$d_i = \frac{fd_o}{d_o - f}$$

The magnification is

$$m = -\frac{d_i}{d_o} = -\frac{f}{d_0 - f}$$

a. For f = 35.0 mm and d_o = 8.00 m, the magnification is m = – 4.39 × 10^{-3}, and we find

$$h_i = (-4.39 \times 10^{-3})(1.80\text{ m}) = \boxed{-7.90 \times 10^{-3}\text{ m}}$$

b. For f = 150.0 mm and d_o = 8.00 m, the magnification is m = – 1.91 × 10^{-2} and we find

$$h_i = (-1.91 \times 10^{-2})(1.80\text{ m}) = \boxed{-3.44 \times 10^{-2}\text{ m}}$$

52. ***REASONING*** A diverging lens always produces a virtual image, so that the image distance d_i is negative. Moreover, the object distance d_o is positive. Therefore, the distance between the object and the image is $d_o + d_i = 6.0$ cm, rather than $d_o - d_i = 6.0$ cm. The equation $d_o + d_i = 6.0$ cm and the thin-lens equation constitute two equations in two unknowns, and we will solve them simultaneously to obtain values for d_i and d_o.

SOLUTION

a. Solving the equation $d_o + d_i = 6.0$ cm for d_o, substituting into the thin-lens equation, and suppressing the units give

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f} \qquad \text{or} \qquad \frac{1}{6.0 - d_i} + \frac{1}{d_i} = \frac{1}{-3.0} \tag{1}$$

Grouping the terms on the left of Equation (1) over a common denominator, we have

$$\frac{d_i + 6.0 - d_i}{d_i\left(6.0 - d_i\right)} = \frac{6.0}{d_i\left(6.0 - d_i\right)} = \frac{1}{-3.0} \tag{2}$$

Cross-multiplying and rearranging in Equation (2) gives

$$d_i\left(6.0 - d_i\right) = -18 \qquad \text{or} \qquad d_i^2 - 6.0d_i - 18 = 0 \tag{3}$$

Using the quadratic formula to solve Equation (3), we obtain

$$d_i = \frac{-(-6.0) \pm \sqrt{(-6.0)^2 - 4(1.00)(-18)}}{2(1.00)} = \boxed{-2.2 \text{ cm}}$$

We have discarded the positive root, because we know that d_i must be negative for the virtual image.

b. Using the fact that $d_o + d_i = 6.0$ cm, we find that the object distance is

$$d_o = 6.0 \text{ cm} - d_i = (6.0 \text{ cm}) - (-2.2 \text{ cm}) = \boxed{8.2 \text{ cm}}$$

53. **SSM** ***REASONING*** The magnification equation (Equation 26.7) relates the object and image distances d_o and d_i, respectively, to the relative size of the of the image and object: $m = -(d_i / d_o)$. We consider two cases: in case 1, the object is placed 18 cm in front of a diverging lens. The magnification for this case is given by m_1. In case 2, the object is moved so that the magnification m_2 is reduced by a factor of 2 compared to that in case 1. In other words, we have $m_2 = \frac{1}{2} m_1$. Using Equation 26.7, we can write this as

$$-\frac{d_{i2}}{d_{o2}} = -\frac{1}{2}\left(\frac{d_{i1}}{d_{o1}}\right) \tag{1}$$

This expression can be solved for d_{o2}. First, however, we must find a numerical value for d_{i1}, and we must eliminate the variable d_{i2}.

SOLUTION

The image distance for case 1 can be found from the thin-lens equation [Equation 26.6: $(1/d_o) + (1/d_i) = (1/f)$]. The problem statement gives the focal length as $f = -12$ cm. Since the object is 18 cm in front of the diverging lens, $d_{o1} = 18$ cm. Solving for d_{i1}, we find

$$\frac{1}{d_{i1}} = \frac{1}{f} - \frac{1}{d_{o1}} = \frac{1}{-12\text{ cm}} - \frac{1}{18\text{ cm}} \qquad \text{or} \qquad d_{i1} = -7.2\text{ cm}$$

where the minus sign indicates that the image is virtual. Solving Equation (1) for d_{o2}, we have

$$d_{o2} = 2d_{i2}\left(\frac{d_{o1}}{d_{i1}}\right) \tag{2}$$

To eliminate d_{i2} from this result, we note that the thin-lens equation applied to case 2 gives

$$\frac{1}{d_{i2}} = \frac{1}{f} - \frac{1}{d_{o2}} = \frac{d_{o2} - f}{f d_{o2}} \qquad \text{or} \qquad d_{i2} = \frac{f d_{o2}}{d_{o2} - f}$$

Substituting this expression for d_{i2} into Equation (2), we have

$$d_{o2} = \left(\frac{2 f d_{o2}}{d_{o2} - f}\right)\left(\frac{d_{o1}}{d_{i1}}\right) \qquad \text{or} \qquad d_{o2} - f = 2f\left(\frac{d_{o1}}{d_{i1}}\right)$$

Solving for d_{o2}, we find

$$d_{o2} = f\left[2\left(\frac{d_{o1}}{d_{i1}}\right) + 1\right] = (-12\text{cm})\left[2\left(\frac{18\text{ cm}}{-7.2\text{ cm}}\right) + 1\right] = \boxed{48\text{ cm}}$$

54. ***REASONING AND SOLUTION*** The focal length of the lens can be obtained from the thin-lens equation as follows:

$$1/f = 1/(4.00\text{ m}) + 1/(0.210\text{ m}) \qquad \text{or} \qquad f = 0.200\text{ m}$$

The same equation applied to the projector gives

$$1/d_o = 1/(0.200\text{ m}) - 1/(0.500\text{ m}) \qquad \text{or} \qquad d_o = \boxed{0.333\text{ m}}$$

55. SSM WWW ***REASONING*** The optical arrangement is similar to that in Figure 26.26. We begin with the thin-lens equation, [Equation 26.6: $(1/d_o)+(1/d_i)=(1/f)$]. Since the distance between the moon and the camera is so large, the object distance d_o is essentially infinite, and $1/d_o = 1/\infty = 0$. Therefore the thin-lens equation becomes $1/d_i = 1/f$ or $d_i = f$. The diameter of the moon's imagine on the slide film is equal to the image height h_i, as given by the magnification equation (Equation 26.7: $h_i / h_o = -d_i / d_o$).

When the slide is projected onto a screen, the situation is similar to that in Figure 26.27. In this case, the thin-lens and magnification equations can be used in their usual forms.

SOLUTION

a. Solving the magnification equation for h_i gives

$$h_i = -h_o \frac{d_i}{d_o} = (-3.48\times10^6\ \text{m})\left(\frac{50.0\times10^{-3}\ \text{m}}{3.85\times10^8\ \text{m}}\right) = -4.52\times10^{-4}\ \text{m}$$

The diameter of the moon's image on the slide film is, therefore, $\boxed{4.52\times10^{-4}\ \text{m}}$.

b. From the magnification equation, $h_i = -h_o\left(d_i / d_o\right)$. We need to find the ratio d_i / d_o. Beginning with the thin-lens equation, we have

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{f} \quad \text{or} \quad \frac{1}{d_o}=\frac{1}{f}-\frac{1}{d_i} \quad \text{which leads to} \quad \frac{d_i}{d_o}=\frac{d_i}{f}-\frac{d_i}{d_i}=\frac{d_i}{f}-1$$

Therefore,

$$h_i = -h_o\left(\frac{d_i}{f}-1\right) = -(4.52\times10^{-4}\ \text{m})\left(\frac{15.0\ \text{m}}{110.0\times10^{-3}\ \text{m}}-1\right) = -6.12\times10^{-2}\ \text{m}$$

The diameter of the image on the screen is $\boxed{6.12\times10^{-2}\ \text{m}}$.

56. ***REASONING AND SOLUTION*** The thin-lens equation for the first situation is

$$1/d_o + 1/d_i = 1/f$$

The magnification equation can be solved for the image distance and used in the thin-lens equation, so that

$$1/d_o(1 - 1/m) = 1/f$$

In the second situation where f' = 50.0 mm

$$1/d_o'(1 - 1/m) = 1/f'$$

Dividing the equations, keeping in mind that the image should be the same size in both cases, that is, m is the same, we get

$$d_o'/d_o = f'/f = (50.0 \text{ mm})/(300.0 \text{ mm}) = 1/6.00$$

or

$$d_o' = (72 \text{ m})/(6.00) = \boxed{12 \text{ m}}$$

57. ***REASONING AND SOLUTION*** Let d represent the distance between the object and the screen. Then, $d_o + d_i = d$. Using this expression in the thin-lens equation gives

$$1/d_o + 1/(d - d_o) = 1/f \qquad \text{or} \qquad d_o^2 - dd_o + df = 0$$

With d = 125 cm and f = 25.0 cm, the quadratic formula yields solutions of

$$d_o = \boxed{+35 \text{ cm}} \text{ and } d_o = \boxed{+90.5 \text{ cm}}$$

58. ***REASONING AND SOLUTION*** The thin-lens equation written for the first situation gives (suppressing the units)

$$\frac{1}{20.0}+\frac{1}{d_{\mathrm{i}}}=\frac{1}{f} \tag{1}$$

The thin-lens equation written for the second situation gives

$$\frac{1}{16.0}+\frac{1}{d_{\mathrm{i}}+2.70}=\frac{1}{f} \tag{2}$$

Since the right hand sides of Equations (1) and (2) are the same, we have

$$\frac{1}{16.0}+\frac{1}{d_{\mathrm{i}}+2.70}=\frac{1}{20.0}+\frac{1}{d_{\mathrm{i}}} \qquad \text{or} \qquad \frac{1}{16.0}-\frac{1}{20.0}=\frac{1}{d_{\mathrm{i}}}-\frac{1}{d_{\mathrm{i}}+2.70}$$

Combining terms over common denominators gives

$$\frac{20.0-16.0}{(16.0)(20.0)}=\frac{d_{\mathrm{i}}+2.70-d_{\mathrm{i}}}{d_{\mathrm{i}}\left(d_{\mathrm{i}}+2.70\right)} \qquad \text{or} \qquad \frac{1}{80.0}=\frac{2.70}{d_{\mathrm{i}}\left(d_{\mathrm{i}}+2.70\right)}$$

Cross multiplying and rearranging terms gives

$$d_{\mathrm{i}}^{2}+2.70d_{\mathrm{i}}-216=0$$

Using the quadratic formula, we find

$$d_{\mathrm{i}}=\frac{-2.70\pm\sqrt{(2.70)^{2}-4(1.00)(-216)}}{2(1.00)}=+13.4\text{ cm}$$

where we have chosen the positive root since the lens produces a real image. Substituting this value for the image distance into Equation (1), we find

$$\frac{1}{20.0\text{ cm}}+\frac{1}{13.4\text{ cm}}=\frac{1}{f} \qquad \text{or} \qquad f=\boxed{8.0\text{ cm}}$$

59. $\boxed{\text{SSM}}$ ***REASONING*** The problem can be solved using the thin-lens equation [Equation 26.6: $(1/d_o)+(1/d_i)=(1/f)$] twice in succession. We begin by using the thin lens-equation to find the location of the image produced by the converging lens; this image becomes the object for the diverging lens.

SOLUTION

a. The image distance for the converging lens is determined as follows:

$$\frac{1}{d_{i1}}=\frac{1}{f}-\frac{1}{d_{o1}}=\frac{1}{12.0\text{ cm}}-\frac{1}{36.0\text{ cm}} \quad \text{or} \quad d_{i1}=18.0\text{ cm}$$

This image acts as the object for the diverging lens. Therefore,

$$\frac{1}{d_{i2}}=\frac{1}{f}-\frac{1}{d_{o2}}=\frac{1}{-6.00\text{ cm}}-\frac{1}{(30.0\text{ cm}-18.0\text{ cm})} \quad \text{or} \quad d_{i2}=-4.00\text{ cm}$$

Thus, the final image is located $\boxed{\text{4.00 cm to the left of the diverging lens}}$.

b. The magnification equation (Equation 26.7: $h_i/h_o=-d_i/d_o$) gives

$$\underbrace{m_c=-\frac{d_{i1}}{d_{o1}}=-\frac{18.0\text{ cm}}{36.0\text{ cm}}=-0.500}_{\text{Converging lens}} \qquad \underbrace{m_d=-\frac{d_{i2}}{d_{o2}}=-\frac{-4.00\text{ cm}}{12.0\text{ cm}}=0.333}_{\text{Diverging lens}}$$

Therefore, the overall magnification is given by the product $m_c m_d=\boxed{-0.167}$.

c. Since the final image distance is negative, we can conclude that the image is $\boxed{\text{virtual}}$.

d. Since the overall magnification of the image is negative, the image is $\boxed{\text{inverted}}$.

e. The magnitude of the overall magnification is less than one; therefore, the final image is $\boxed{\text{smaller}}$.

60. ***REASONING*** We will consider one lens at a time, using the thin-lens equation for each. The key to the solution is the fact that the image formed by the first lens serves as the object for the second lens.

SOLUTION Using the thin-lens equation, we find the image distance for the first lens:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-8.0\text{ cm}} - \frac{1}{4.0\text{ cm}} \qquad \text{or} \qquad d_i = -2.7\text{ cm}$$

The negative value for d_i indicates that the image is virtual and located 2.7 cm to the left of the first lens. The lenses are 16 cm apart, so this image is located 2.7 cm + 16 cm = 18.7 cm from the second lens. Since this image serves as the object for the second lens, we can locate the image formed by the second lens with the aid of the thin-lens equation, with d_o = 18.7 cm:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{-8.0\text{ cm}} - \frac{1}{18.7\text{ cm}} \qquad \text{or} \qquad d_i = \boxed{-5.6\text{ cm}}$$

61. ***REASONING AND SOLUTION*** For the first image, the image distance is given by the thin-lens equation:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{0.080\text{ m}} - \frac{1}{0.040\text{ m}} \qquad \text{or} \qquad d_i = -0.080\text{ m}$$

The object distance to the second lens is 0.080 m + 0.120 m = 0.200 m. For the final image, the image distance is

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{0.080\text{ m}} - \frac{1}{(0.200\text{ m})}$$

$$d_i = \boxed{0.13\text{ m to the right of the second lens}}$$

62. ***REASONING*** This is a two-lens problem, and so the image produced by the first lens acts as the object for the second lens. Since we know the final image distance and the focal point of the second (diverging) lens, we can determine the object distance for this lens by using the thin-lens equation. This object is the image produced by the first (converging) lens. By employing the thin-lens equation again, we can calculate how far the object is from the converging lens.

SOLUTION The focal length of the second lens is $f_2 = -28.0$ cm, and the image distance is $d_{i2} = -20.7$ cm. The minus sign arises because the image falls to the left of the lens, which, by convention, is a negative distance. According to the thin-lens equation, the object distance d_{o2} for the second lens is

$$\frac{1}{d_{o2}} = \frac{1}{f_2} - \frac{1}{d_{i2}} = \frac{1}{-28.0\text{ cm}} - \frac{1}{-20.7\text{ cm}} = 0.0126\text{ cm}^{-1} \quad \text{or} \quad d_{o2} = 79.4\text{ cm}$$

Since d_{o2} is positive, the object lies 79.4 cm to the left of the second lens. However, the first lens is 56.0 cm to the left of the second lens, so the separation between this object and the first lens is 79.4 cm – 56.0 cm = 23.4 cm. This object is the image produced by the first lens. However, the image distance is $d_{i1} = -23.4$ cm, since the image falls to the left of the first lens and, by convention, is a negative distance. Using the thin-lens equation, we find that the object distance d_{o1} for the first lens is

$$\frac{1}{d_{o1}} = \frac{1}{f_1} - \frac{1}{d_{i1}} = \frac{1}{24.0\text{ cm}} - \frac{1}{-23.4\text{ cm}} = 0.0844\text{ cm}^{-1} \quad \text{or} \quad d_{o1} = \boxed{11.8\text{ cm}}$$

63. ***REASONING AND SOLUTION***

a. The image distance for the diverging lens is

$$1/d_i = 1/f - 1/d_o = 1/(-8.00\text{ cm}) - 1/(12.0\text{ cm}) \qquad \text{or} \qquad d_i = -4.80\text{ cm}$$

This image serves as the object for the converging lens, so its focal length is

$$1/f' = 1/d_i' + 1/d_o' = 1/(-37.0\text{ cm}) + 1/(12.8\text{ cm}) \qquad \text{or} \qquad f' = \boxed{19.6\text{ cm}}$$

b. The magnification of the diverging lens is

$$m = -(-4.80\text{ cm}/12.0\text{ cm}) = +0.400$$

and the magnification of the converging lens is

$$m' = -(-37.0\text{ cm}/12.8\text{ cm}) = +2.89$$

The overall magnification is, then,

$$mm' = (0.400)(2.89) = 1.16$$

and the final image height is

$$h_i' = mm'\,h_o = (1.16)(0.75\text{ cm}) = \boxed{0.87\text{ cm}}$$

64. ***REASONING AND SOLUTION*** Let d be the separation of the lenses. The first lens forms its image at

$$1/d_i = 1/f - 1/d_o = 1/(16.0\text{ cm}) - 1/(20.0\text{ cm}) \qquad \text{or} \qquad d_i = 80.0\text{ cm}$$

This image serves as the object for the second lens, so the object distance for the second lens is d – 80.0 cm. According to the thin-lens equation, the reciprocal of the image distance for the second lens is

$$1/d_i' = 1/f - 1/(d - 80.0\text{ cm}) \qquad (1)$$

The magnification of the first lens is

$$m = -(80.0\text{ cm})/(20.0\text{ cm}) = -4.00$$

Since the overall magnification of the combination must be +1.000, the magnification of the second lens must be –0.250. Therefore, applying the magnification equation to the second lens, we have

$$d_i' = 0.250\,(d - 80.0\text{ cm}) \qquad (2)$$

Substituting Equation (2) into Equation (1) and rearranging yields d = $\boxed{160.0\text{ cm}}$.

65. $\boxed{\text{SSM}}$ ***REASONING*** We begin by using the thin-lens equation [Equation 26.6: $(1/d_o)+(1/d_i)=(1/f)$] to locate the image produced by the lens. This image is then treated as the object for the mirror.

SOLUTION

a. The image distance from the diverging lens can be determined as follows:

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{-8.00\text{ cm}}-\frac{1}{20.0\text{ cm}} \qquad \text{or} \qquad d_i=-5.71\text{ cm}$$

The image produced by the lens is 5.71 cm to the left of the lens. The distance between this image and the concave mirror is 5.71 cm + 30.0 cm = 35.7 cm. The mirror equation [Equation 25.3: $(1/d_o)+(1/d_i)=(1/f)$] gives the image distance from the mirror:

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{12.0\text{ cm}}-\frac{1}{35.7\text{ cm}} \qquad \text{or} \qquad \boxed{d_i=18.1\text{ cm}}$$

b. The image is $\boxed{\text{real}}$, because d_i is a positive number, indicating that the final image lies to the left of the concave mirror.

c. The image is $\boxed{\text{inverted}}$, because a diverging lens always produces an upright image, and the concave mirror produces an inverted image when the object distance is greater than the focal length of the mirror.

66. ***REASONING AND SOLUTION***

a. The image distance for the first lens is

$$1/d_i = 1/f_1 - 1/d_o = 1/(9.00\text{ cm}) - 1/(12.0\text{ cm}) \qquad \text{or} \qquad d_i = 36\text{ cm}$$

Since the lenses are separated by 18.0 cm, the value of $d_i = 36$ cm places the image 18 cm *to the right* of the second lens. This image serves as the object for the second lens. This is a case in which the object, being to the right of the lens, has a negative object distance, as indicated in the Reasoning Strategy given in Section 26.8. The image distance for the second lens is

$$1/d_i' = 1/f_2 - 1/d_o' = 1/(6.00\text{ cm}) - 1/(-18\text{ cm}) \qquad \text{or} \qquad d_i' = +4.50\text{ cm}$$

The positive sign indicates that the final image lies $\boxed{\text{4.50 cm to the right of the second lens}}$ **.**

b. The magnification of the first lens is

$$m = -(36\text{ cm})/(12\text{ cm}) = -3.0$$

and the magnification of the second lens is

$$m' = -(4.50)/(-18.0) = +0.25$$

The overall magnification is then $mm' = \boxed{-0.75}$.

c. The final image is $\boxed{\text{real}}$ since its distance is positive.

d. The final image is $\boxed{\text{inverted}}$ since the overall magnification is negative.

e. The final image is $\boxed{\text{smaller}}$ than the object since the magnitude of the overall magnification is less than one.

67. ***REASONING*** We will apply the thin-lens equation to solve this problem. In doing so, we must be careful to take into account the fact that the lenses of the glasses are worn at a distance of 2.0 or 3.0 cm from her eyes.

SOLUTION

a. The object distance is 25.0 cm – 2.0 cm, since it is measured relative to the lenses, which are worn 2.0 cm from the eyes. As discussed in the text, the lenses form a virtual image located at the near point. The image distance must be negative for a virtual image, but the value is not –48.0 cm, because the glasses are worn 2.0 cm from the eyes. Instead, the image distance is –48.0 cm + 2.0 cm. Using the thin-lens equation, we can find the focal length as follows:

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{25.0\text{ cm}-2.0\text{ cm}}+\frac{1}{-48.0\text{ cm}+2.0\text{ cm}}=\frac{1}{f} \qquad \text{or} \qquad f=\boxed{46.0\text{ cm}}$$

b. Similarly, we find

$$\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{25.0\text{ cm}-3.0\text{ cm}}+\frac{1}{-48.0\text{ cm}+3.0\text{ cm}}=\frac{1}{f} \qquad \text{or} \qquad f=\boxed{43.0\text{ cm}}$$

68. ***REASONING AND SOLUTION***

a. The woman sees distant objects clearly, so she must be $\boxed{\text{hyperopic}}$.

b. The thin-lens equation gives the refractive power to be

$$1/f = 1/d_o + 1/d_i = 1/(25\text{ cm} - 2.0\text{ cm}) + 1/(-65\text{ cm} + 2.0\text{ cm}) = 0.028\text{ cm}^{-1} = \boxed{2.8\text{ diopters}}\ .$$

69. $\boxed{\text{SSM}}$ ***REASONING*** The far point is 5.0 m from the right eye, and 6.5 m from the left eye. For an object infinitely far away ($d_o = \infty$), the image distances for the corrective lenses are then –5.0 m for the right eye and –6.5 m for the left eye, the negative sign indicating that the images are virtual images formed to the left of the lenses. The thin-lens equation [Equation 26.6: $(1/d_o) + (1/d_i) = (1/f)$] can be used to find the focal length. Then, Equation 26.8 can be used to find the refractive power for the lens for each eye.

SOLUTION Since the object distance d_o is essentially infinite, $1/d_o = 1/\infty = 0$, and the thin-lens equation becomes $1/d_i = 1/f$, or $d_i = f$. Therefore, for the right eye, $f = -5.0\text{ m}$, and the refractive power is (see Equation 26.8)

[Right eye]

$$\begin{matrix}\text{Refractive power}\\ \text{(in diopters)}\end{matrix} = \frac{1}{f} = \frac{1}{(-5.0\text{ m})} = \boxed{-0.20\text{ diopters}}$$

Similarly, for the left eye, $f = -6.5\text{ m}$, and the refractive power is

[Left eye]

$$\begin{matrix}\text{Refractive power}\\ \text{(in diopters)}\end{matrix} = \frac{1}{f} = \frac{1}{(-6.5\text{ m})} = \boxed{-0.15\text{ diopters}}$$

70. ***REASONING*** The thin-lens equation, Equation 26.6, can be used to find the distance from the blackboard to her eyes (the object distance). The distance from her eye lens to the retina is the image distance, and the focal length of her lens is the reciprocal of the refractive power (see Equation 26.8). The magnification equation, Equation 26.7, can be used to find the height of the image on her retina.

SOLUTION

a. The thin-lens equation can be used to find the object distance d_o. However, we note from Equation 26.8 that $1/f = 57.50\ \text{m}^{-1}$ and $d_i = 0.01750$ m, so that

$$\frac{1}{d_o} = \frac{1}{f} - \frac{1}{d_i} = 57.50\ \text{m}^{-1} - \frac{1}{0.01750\ \text{m}} = 0.36\ \text{m}^{-1} \quad \text{or} \quad d_i = \boxed{2.8\ \text{m}}$$

b. The magnification equation can be used to find the height h_i of the image on the retina

$$h_i = h_o\left(-\frac{d_i}{d_o}\right) = (5.00\ \text{cm})\left(-\frac{0.01750\ \text{m}}{2.8\ \text{m}}\right) = \boxed{-3.1\times10^{-2}\ \text{cm}} \qquad (26.7)$$

71. ***REASONING*** Nearsightedness is corrected using diverging lenses to form a virtual image at the far point of the eye, as Section 26.10 discusses. The far point is given as 5.2 m, so we know that the image distance for the contact lenses is $d_i = -5.2$ m. The minus sign indicates that the image is virtual. The thin-lens equation can be used to determine the focal length.

SOLUTION According to the thin-lens equation, we have

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{12.0\text{ cm}} + \frac{1}{-5.2\text{ cm}} = \frac{1}{f} \qquad \text{or} \qquad f = \boxed{-9.2\text{ cm}}$$

72. ***REASONING AND SOLUTION*** The magnification is $h_i/h_o = -d_i/d_o$, so $h_o/d_o = -h_i/d_i$. Therefore, the size of the image on the retina, h_i, is

$$h_i = h_o(-d_i/d_o) = (2.0\text{ mm})(-1.7\text{ cm})/(25\text{ cm}) = \boxed{-0.14\text{ mm}}\ .$$

73. ***REASONING AND SOLUTION*** If the far point is 3.62 m from the eyes, the focal length of the lens is –3.62 m , or – 362 cm. The near point of 25 cm then represents the virtual image distance, i.e., $d_i = -25$ cm. Therefore, using the thin-lens equation, we find

$$1/d_o = 1/f - 1/d_i = [1/(-362\text{ cm})] - [1/(-25\text{ cm})] \qquad \text{or} \qquad d_o = \boxed{26.9\text{ cm}}$$

74. ***REASONING AND SOLUTION*** We need to determine the focal lengths for Bill's glasses and for Anne's glasses. Using the thin-lens equation we have

[Bill] $1/f_B = 1/d_o + 1/d_i = 1/(23.0\text{ cm}) + 1/(-123\text{ cm})$ or $f_B = 28.3\text{ cm}$

[Anne] $1/f_A = 1/d_o + 1/d_i = 1/(23.0\text{ cm}) + 1/(-73.0\text{ cm})$ or $f_A = 33.6\text{ cm}$

Now find d_o' for Bill and Anne when they switch glasses.

a. Anne:

$$1/d_o' = 1/f_B - 1/d_i = 1/(28.3\text{ cm}) - 1/(-73\text{ cm}) \quad \text{or} \quad d_o' = 20.4\text{ cm}$$

Relative to the eyes, this becomes 20.4 cm + 2.00 cm = $\boxed{22.4\text{ cm}}$.

b. Bill:

$$1/d_o' = 1/f_A - 1/d_i = 1/(33.6\text{ cm}) - 1/(-123\text{ cm}) \quad \text{or} \quad d_o' = 26.4\text{ cm}$$

Relative to the eyes, this becomes 26.4 cm + 2.00 cm = $\boxed{28.4\text{ cm}}$.

75. ***REASONING AND SOLUTION*** The far point of 6.0 m tells us that the focal length of the lens is f = –6.0 m.

a. The image distance can be found using

$$1/d_i = 1/f - 1/d_o = 1/(-6.0\text{ m}) - 1/(18.0\text{ m}) \qquad \text{or} \qquad d_i = \boxed{-4.5\text{ m}}$$

b. The image size as obtained from the magnification is

$$h_i = h_o(-d_i/d_o) = (2.0\text{ m})[-(-4.5\text{ m})/(18.0\text{ m})] = \boxed{0.50\text{ m}}$$

76. ***REASONING AND SOLUTION***

a. The angular size of an object is $\theta \bullet h_o/d_o$, where h_o is the height of the object and d_o is the distance to the object. For the spectator watching the game live, we find

$$\theta \approx h_o/d_o = (1.9\text{ m})/(75\text{ m}) = \boxed{0.025\text{ rad}}$$

b. Similarly for the TV viewer, we find

$$\theta \approx h_o/d_o = (0.12\text{ m})/(3.0\text{ m}) = \boxed{0.040\text{ rad}}$$

c. Since the angular size of the player on the TV is greater than the angular size seen by the spectator, $\boxed{\text{the player looks larger on television}}$.

77. **SSM** ***REASONING*** The angular size of a distant object in radians is approximately equal to the diameter of the object divided by the distance from the eye. We will use this definition to calculate the angular size of the quarter, and then, calculate the angular size of the sun; we can then form the ratio $\theta_{\text{quarter}} / \theta_{\text{sun}}$.

SOLUTION The angular sizes are

$$\theta_{\text{quarter}} \approx \frac{2.4 \text{ cm}}{70.0 \text{ cm}} = 0.034 \text{ rad} \qquad \text{and} \qquad \theta_{\text{sun}} \approx \frac{1.39 \times 10^{9} \text{ m}}{1.50 \times 10^{11} \text{ m}} = 0.0093 \text{ rad}$$

Therefore, the ratio of the angular sizes is

$$\frac{\theta_{\text{quarter}}}{\theta_{\text{sun}}} = \frac{0.034 \text{ rad}}{0.0093 \text{ rad}} = \boxed{3.7}$$

78. ***REASONING*** The angular magnification M of a magnifying glass is given by Equation 26.10 as

$$M = \frac{\theta'}{\theta} \approx \left(\frac{1}{f} - \frac{1}{d_i}\right) N$$

where $\theta' = 0.0380$ rad is the angular size of the final image produced by the magnifying glass, $\theta = 0.0150$ rad is the reference angular size of the object seen at the near point without the magnifying glass, and N is the near point of the eye. The largest possible angular magnification occurs when the image is at the near point of the eye, or $d_i = -N$, where the minus sign denotes that the image lies on the left side of the lens (the same side as the object). This equation can be solved to find the focal length of the magnifying glass.

SOLUTION Letting $d_i = -N$, and solving Equation 26.10 for the focal length f gives

$$f = \frac{N}{\frac{\theta'}{\theta} - 1} = \frac{21.0 \text{ cm}}{\frac{0.0380 \text{ rad}}{0.0150 \text{ rad}} - 1} = \boxed{13.7 \text{ cm}}$$

79. ***REASONING AND SOLUTION*** With the magnifying glass held so that the image is at the near point of the eye, we have from Equation 26.10 that $M = (N/f) + 1$. Solving for f,

$$f = N/(M - 1) = (0.30 \text{ m})/(3.4 - 1) = \boxed{0.13 \text{ m}}$$

80. ***REASONING AND SOLUTION*** Using Equation 26.10, we have

$$M = \left(\frac{1}{f} - \frac{1}{d_{\mathrm{i}}}\right) N$$

Solving for the image distance gives

$$d_{\mathrm{i}} = \frac{fN}{N - fM} = \frac{(22\ \mathrm{cm})(36\ \mathrm{cm})}{(36\ \mathrm{cm}) - (22\ \mathrm{cm})(2.5)} = \boxed{-42\ \mathrm{cm}}$$

81. $\boxed{\text{SSM}}$ ***REASONING*** The angular magnification of a magnifying glass is given by Equation 26.10: $M \approx \left[(1/f)-(1/d_i)\right]N$, where N is the distance from the eye to the near-point. For maximum magnification, the closest to the eye that the image can be is at the near point, with $d_i = -N$ (where the minus sign indicates that the image lies to the left of the lens and is virtual). In this case, Equation 26.10 becomes $M_{max} \approx N/f + 1$. At minimum magnification, the image is as far from the eye as it can be ($d_i = -\infty$); this occurs when the object is placed at the focal point of the lens. In this case, Equation 26.10 simplifies to $M_{min} \approx N/f$.

Since the woman observes that for clear vision, the maximum angular magnification is 1.25 times larger than the minimum angular magnification, we have $M_{max} = 1.25 M_{min}$. This equation can be written in terms of N and f using the above expressions, and then solved for f.

SOLUTION We have

$$\frac{N}{f} + 1 = 1.25\,\frac{N}{f}$$

Solving for f, we find that

$$f = 0.25\,N = (0.25)(25\text{ cm}) = \boxed{6.3\text{ cm}}$$

82. ***REASONING AND SOLUTION*** The information given allows us to determine the near point for this farsighted person. With $f = 45.4$ cm and $d_o = 25.0$ cm, we find from the thin-lens equation that

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_0} = \frac{1}{45.4\ \text{cm}} - \frac{1}{25.0\ \text{cm}} \qquad \text{or} \qquad d_i = -55.6\ \text{cm}$$

Therefore, this person's near point, N, is 55.6 cm. We now need to find the focal length of the magnifying glass based on the near point for a normal eye, i.e., $M = N/f + 1$, where $N = 25.0$ cm. Thus,

$$f = N/(M - 1) = (25.0\ \text{cm})/(7.50 - 1) = 3.85\ \text{cm}$$

We can now determine the maximum angular magnification for the farsighted person

$$M = N/f + 1 = (55.6\ \text{cm})/(3.85\ \text{cm}) + 1 = \boxed{15.4}$$

83. ***REASONING AND SOLUTION*** According to Equation 26.11, the angular magnification of the microscope is

$$M \approx -\frac{(L - f_e)N}{f_o f_e} = -\frac{(14.0\text{ cm} - 2.5\text{ cm})(25.0\text{ cm})}{(0.50\text{ cm})(2.5\text{ cm})} = -2.3 \times 10^2$$

Now the new angle is

$$\theta' = M\theta = \left(-2.3 \times 10^2\right)\left(2.1 \times 10^{-5}\text{ rad}\right) = -4.8 \times 10^{-3}\text{ rad}$$

The magnitude of the angle is $\boxed{4.8 \times 10^{-3}\text{ rad}}$.

84. ***REASONING AND SOLUTION*** Using Equation 26.11, we find

$$f_e = \frac{LN}{N - M f_o} = \frac{(18\text{ cm})(25\text{ cm})}{(25\text{ cm}) - (-83)(1.5\text{ cm})} = \boxed{3.0\text{ cm}}$$

85. **SSM** ***REASONING*** The angular magnification of a compound microscope is given by Equation 26.11:

$$M \approx -\frac{(L - f_e)N}{f_o f_e}$$

where f_o is the focal length of the objective, f_e is the focal length of the eyepiece, and L is the separation between the two lenses. This expression can be solved for f_o, the focal length of the objective.

SOLUTION Solving for f_o, we find that the focal length of the objective is

$$f_o = -\frac{(L - f_e)N}{f_e M} = -\frac{(16.0\text{ cm} - 1.4\text{ cm})(25\text{ cm})}{(1.4\text{ cm})(-320)} = \boxed{0.81\text{ cm}}$$

86. ***REASONING AND SOLUTION*** The angular magnification of the microscope when using the 100 diopter objective is

$$M_1 \approx -\frac{(L - f_e)N}{f_{o1} f_e}$$

and when using the 300 diopter objective it is

$$M_2 \approx -\frac{(L - f_e)N}{f_{o2} f_e}$$

Division of the equations results in

$$M_2/M_1 = f_{o1}/f_{o2} = (300 \text{ diopters})/(100 \text{ diopters}) = 3$$

Since the angular magnification of the 300 diopter objective is three times greater than that of the 100 diopter objective the angle will be $\boxed{9 \times 10^{-3} \text{ rad}}$

87. ***REASONING AND SOLUTION*** First, we find the focal length of the magnifying glass. The maximum angular magnification of the magnifying glass is $M = N/f + 1$.

Rearranging this expression gives

$$f = N/(M - 1) = (25.0 \text{ cm})/(12.0 - 1) = 2.27 \text{ cm}$$

This lens is now used as the eyepiece for a microscope. Rearranging Equation 26.11, we can find the focal length of the objective of this microscope as follows:

$$f_o = -(L - f_e)N/(f_e M) = -(23.0 \text{ cm} - 2.27 \text{ cm})(25.0 \text{ cm})/[(2.27 \text{ cm})(-525)] = \boxed{0.435 \text{ cm}}$$

88. ***REASONING*** The angular magnification of a compound microscope is given by Equation 26.11. All the necessary data are given in the statement of the problem, so the angular magnification can be calculated directly. In order to find how far the object is from the objective, examine Figure 26.33*a*. The object distance d_{o1} for the first lens is related to its focal length f_1 and image distance d_{i1} by the thin-lens equation (Equation 26.6). From the drawing we see that the image distance is approximately equal to the distance L between the lenses minus the focal length f_e of the eyepiece, or $d_{i1} \approx L - f_e$. Thus, the object distance is given by

$$\frac{1}{d_{o1}} = \frac{1}{f_o} - \frac{1}{d_{i1}} \approx \frac{1}{f_o} - \frac{1}{L - f_e}$$

The magnification due to the objective is given by Equation 26.7 as $m_{\text{objective}} = -d_{i1}/d_{o1}$. Since both d_{i1} and d_{o1} are now known, the magnification can be evaluated.

SOLUTION

a. According to Equation 26.11, the angular magnification of the compound microscope is

$$M \approx -\frac{(L - f_e)N}{f_o f_e} = -\frac{(26.0\text{ cm} - 6.50\text{ cm})(35.0\text{ cm})}{(3.50\text{ cm})(6.50\text{ cm})} = \boxed{30.0}$$

b. Using the thin-lens equation, we can determine the object distance from the objective as follows:

$$\frac{1}{d_{o1}} = \frac{1}{f_o} - \frac{1}{L - f_e} = \frac{1}{3.50\text{ cm}} - \frac{1}{26.0\text{ cm} - 6.50\text{ cm}} = 0.234\text{ cm}^{-1}$$

or $d_{o1} = \boxed{4.27\text{ cm}}$

c. The magnification m of the objective is given by Equation 26.7 as

$$m_{\text{objective}} = -\frac{d_{i1}}{d_{o1}} = -\frac{26.0\text{ cm} - 6.50\text{ cm}}{4.27\text{ cm}} = \boxed{-4.57}$$

89. **SSM** ***REASONING AND SOLUTION*** The angular magnification of an astronomical telescope, is given by Equation 26.12 as $M \approx -f_o / f_e$. Solving for the focal length of the eyepiece, we find

$$f_e \approx -\frac{f_o}{M} = -\frac{48.0 \text{ cm}}{(-184)} = \boxed{0.261 \text{ cm}}$$

90. ***REASONING AND SOLUTION***

a. From Equation 26.12, we have $M = -f_o/f_e$, so that

$$f_o = -Mf_e = -(-155)(5.00 \times 10^{-3}\ \text{m}) = \boxed{0.775\ \text{m}}$$

b. The length of the telescope is the sum of the focal lengths of the objective and eyepiece, so

$$L = f_o + f_e = 0.775\ \text{m} + 0.005\ \text{m}\ \boxed{0.780\ \text{m}}$$

91. ***REASONING*** Knowing the angles subtended at the unaided eye and with the telescope will allow us to determine the angular magnification of the telescope. Then, since the angular magnification is related to the focal lengths of the eyepiece and the objective, we will use the known focal length of the eyepiece to determine the focal length of the objective.

SOLUTION From Equation 26.12, we have

$$M = \frac{\theta'}{\theta} = -\frac{f_{\text{o}}}{f_{\text{e}}}$$

where θ is the angle subtended by the unaided eye and θ' is the angle subtended when the telescope is used. We note that θ' is negative, since the telescope produces an inverted image. Thus, using Equation 26.12, we find

$$f_{\text{o}} = -\frac{f_{\text{e}}\theta'}{\theta} = -\frac{(0.032\text{ m})\left(-2.8\times10^{-3}\text{ rad}\right)}{8.0\times10^{-5}\text{ rad}} = \boxed{1.1\text{ m}}$$

92. ***REASONING AND SOLUTION*** First we calculate the focal lengths of the eyepiece and objective lenses using Equation 26.8: Refractive power (RP) = 1/f.

[Objective] $f_o = 1/(RP) = 1/(1.25 \text{ diopters}) = 0.800 \text{ m}$

[Eyepiece] $f_e = 1/(RP) = 1/(250 \text{ diopters}) = 4.0 \times 10^{-3} \text{ m}$

Using Equation 26.12, we find that the angular magnification is

$$M = -f_o/f_e = -(0.800 \text{ m})/(4.0 \times 10^{-3} \text{ m}) = \boxed{-2.0 \times 10^{2}}$$

93. **SSM** ***REASONING AND SOLUTION***

a. The lens with the largest focal length should be used for the objective of the telescope. Since the refractive power is the reciprocal of the focal length (in meters), the lens with the smallest refractive power is chosen as the objective, namely, the $\boxed{\text{1.3 - diopter lens}}$.

b. According to Equation 26.8, the refractive power is related to the focal length f by Refractive power (in diopters) $= 1/[\text{f(in meters)}]$. Since we know the refractive powers of the two lenses, we can solve Equation 26.8 for the focal lengths of the objective and the eyepiece. We find that $f_o = 1/(1.3 \text{ diopters}) = 0.77 \text{ m}$. Similarly, for the eyepiece, $f_e = 1/(11 \text{ diopters}) = 0.091 \text{ m}$. Therefore, the distance between the lenses should be

$$L \approx f_o + f_e = 0.77 \text{ m} + 0.091 \text{ m} = \boxed{0.86 \text{ m}}$$

c. The angular magnification of the telescope is given by Equation 26.12 as

$$M \approx -\frac{f_o}{f_e} = -\frac{0.77 \text{ m}}{0.091 \text{ m}} = \boxed{-8.5}$$

94. ***REASONING*** The angular magnification M of an astronomical telescope is given by Equation 26.12 as $M \approx -f_o/f_e$, where f_o and f_e are the focal lengths of the objective and eyepiece, respectively. We are given a value for f_e, but not for f_o. However, an astronomical telescope is used to view objects that are very distant. Therefore, the image produced by the objective is located very near to its focal length (see Figure 26.42*a*). For the greatest possible magnification, the focal point of the eyepiece is also placed very close to the first image, so that $f_o \approx L - f_e$, where L is the distance between the lenses.

SOLUTION The angular magnification is

$$M \approx -\frac{f_o}{f_e} \approx -\frac{L - f_e}{f_e} = -\frac{1.25\text{ m} - 0.050\text{ m}}{0.050\text{ m}} = \boxed{-24}$$

95. ***REASONING AND SOLUTION***

a. The magnification is

$$M = -f_o/f_e = -(19.4\text{ m})/(0.100\text{ m}) = \boxed{-194}$$

b. The angular size of the crater is

$$\theta = h_o/d_o = (1500\text{ m})/(3.77 \times 10^8\text{ m}) = 4.0 \times 10^{-6}\text{ rad}$$

The angular magnification is, $M = \theta'/\theta$, so that

$$\theta' = M\theta = (-194)(4.0 \times 10^{-6}\text{ rad}) = -7.8 \times 10^{-4}\text{ rad}$$

Since $h_i' = \theta' f_e$, we have

$$h_i' = \theta' f_e = (-7.8 \times 10^{-4}\text{ rad})(0.100\text{ m}) = \boxed{-7.8 \times 10^{-5}\text{ m}}$$

c. The apparent distance is shorter by a factor of 194, so

$$\text{Apparent Distance} = (3.77 \times 10^8\text{ m})/(194) = \boxed{1.94 \times 10^6\text{ m}}$$

96. ***REASONING AND SOLUTION*** Use the thin–lens equation to find the first image distance:

$$1/d_i = 1/f_o - 1/d_o = 1/(1.500\text{ m}) - 1/(114.00\text{ m}) \qquad \text{or} \qquad d_i = 1.520\text{ m}$$

The magnification is

$$M = -d_i/d_o = -(1.520\text{ m})/(114.00\text{ m}) = -0.01333$$

Now use this "first image" as the object for the second lens,

$$1/d_i' = 1/f_e - 1/d_o' = 1/(0.070\text{ m}) - 1/(0.050\text{ m}) \qquad \text{or} \qquad d_i' = -0.18\text{ m}$$

The magnification in the second case is

$$M' = -d_i'/d_o' = +3.6$$

The total linear magnification is therefore,

$$M_l = M \times M' = (-0.01333)(+3.6) = -0.048$$

However, we need the angular magnification, so

$$\theta' = -h_i'/d_i' \text{ and } \theta = h_o/(d_o + f_0 + f_e)$$

where we'll use $h_o = 1$ m, $h_i' = -0.048$ m, $d_o + f_0 + f_e = (114.00\text{ m} + 1.500\text{ m} + 0.070\text{ m})$ $= 115.57$ m, and $d_i' = -0.18$ m. Therefore,

$$\theta' = -(-0.048\text{ m})/(-0.18\text{ m}) = -0.27\text{ rad}$$

and

$$\theta = (1.0\text{ m})/(115.57\text{ m}) = 8.65 \times 10^{-3}\text{ rad}$$

So that,

$$M_{ang} = \theta'/\theta = (-0.27\text{ rad})/(8.65 \times 10^{-3}\text{ rad}) = \boxed{-31}$$

97. **SSM** ***REASONING*** The ray diagram is constructed by drawing the paths of two rays from a point on the object. For convenience, we choose the top of the object. The ray that is parallel to the principal axis will be refracted by the lens and pass through the focal point on the right side. The ray that passes through the center of the lens passes through undeflected. The image is formed at the intersection of these two rays on the right side of the lens.

SOLUTION The following ray diagram (to scale) shows that $\boxed{d_i = 18 \text{ cm}}$ and reveals a real, inverted, and enlarged image.

98. ***REASONING AND SOLUTION*** The angular magnification of a compound microscope is negative, and Equation 26.9 gives $M = \theta'/\theta$, so that

$$\theta' = M\,\theta = (-160)(4.0 \times 10^{-3}\ \text{rad}) = -0.64\ \text{rad}$$

The magnitude of this angular size is $\boxed{0.64\ \text{rad}}$.

99. ***REASONING AND SOLUTION*** The critical angle for the glass-liquid interface must be larger than 65.0° if the light is to penetrate into the liquid. Using Equation 26.4, we have

$$n_L/n > \sin 65.0° \qquad \text{or} \qquad n_L > n \sin 65.0° = (1.60) \sin 65.0° = \boxed{1.45}$$

100. ***REASONING AND SOLUTION*** The angle of incidence is found from the drawing to be

$$\theta_1 = \tan^{-1}\left(\frac{8.0\text{ m}}{2.5\text{ m}}\right) = 73°$$

Snell's law gives the angle of refraction to be

$$\sin\theta_2 = (n_1/n_2)\sin\theta_1 = (1.000/1.333)\sin 73° = 0.72 \qquad \text{or} \qquad \theta_2 = 46°$$

The distance d is found from the drawing to be

$$d = 8.0\text{ m} + (4.0\text{ m})\tan\theta_2 = \boxed{12.1\text{ m}}$$

101. **SSM** ***REASONING AND SOLUTION*** Since the far point is 220 cm, we know that the image distance is $d_i = -220$ cm, when the object is infinitely far from the lens ($d_o = \infty$). Thus, the thin-lens equation (Equation 26.6) becomes

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{\infty} + \frac{1}{(-220\text{ cm})} = \frac{1}{(-220\text{ cm})} = \frac{1}{f} \qquad \text{or} \qquad \boxed{f = -220\,\text{cm}}$$

102. ***REASONING AND SOLUTION*** Using Equation 26.3, we find

$$d = (n_1/n_2)\, d' = (1.546/1.000)(2.5 \text{ cm}) = \boxed{3.9 \text{ cm}}$$

103. ***REASONING*** The height of the mountain's image is given by the magnification equation as $h_i = -h_o d_i / d_o$. To use this expression, however, we will need to know the image distance d_i, which can be determined using the thin-lens equation. Knowing the image distance, we can apply the expression for the image height directly to calculate the desired ratio.

SOLUTION According to the thin-lens equation, we have

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} \tag{1}$$

For both pictures, the object distance d_o is very large compared to the focal length f. Therefore, $1/d_o$ is negligible compared to $1/f$, and the thin-lens equation indicates that $d_i = f$. As a result, the magnification equation indicates that the image height is given by

$$h_i = -\frac{h_o d_i}{d_o} = -\frac{h_o f}{d_o} \tag{2}$$

Applying Equation (2) for the two pictures and noting that in each case the object height h_o and the focal length f are the same, we find

$$\frac{(h_i)_{5\text{ km}}}{(h_i)_{14\text{ km}}} = \frac{\left(-\dfrac{h_o f}{d_o}\right)_{5\text{ km}}}{\left(-\dfrac{h_o f}{d_o}\right)_{14\text{ km}}} = \frac{(d_o)_{14\text{ km}}}{(d_o)_{5\text{ km}}} = \frac{14\text{ km}}{5.0\text{ km}} = \boxed{2.8}$$

104. ***REASONING AND SOLUTION***

a. Using the thin-lens equation, we obtain

$$1/d_i = 1/f - 1/d_o = 1/(-32\text{ cm}) - 1/(+19\text{ cm}) \qquad \text{or} \qquad d_i = \boxed{-12\text{ cm}}$$

b. Using the magnification equation, we find

$$m = -(d_i/d_o) = -(-12\text{ cm})/(19\text{ cm}) = \boxed{+0.63}$$

c. The image is $\boxed{\text{virtual}}$ since d_i is negative.

d. The image is $\boxed{\text{upright}}$ since m is +.

e. The image is $\boxed{\text{reduced}}$ in size since $m < 1$.

105. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION***

a. We know from the law of reflection (Section 25.2), that the angle of reflection is equal to the angle of incidence, so the reflected ray is reflected at $\boxed{43°}$.

b. Snell's law of refraction (Equation 26.2: $n_1 \sin\theta_1 = n_2 \sin\theta_2$ can be used to find the angle of refraction. Table 26.1 indicates that the index of refraction of water is 1.333. Solving for θ_2 and substituting values, we find that

$$\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2} = \frac{(1.000)(\sin 43°)}{1.333} = 0.51 \qquad \text{or} \qquad \theta_2 = \sin^{-1} 0.51 = \boxed{31°}$$

106. ***REASONING AND SOLUTION*** Using Equation 26.10, with $(1/f) - (1/d_i) = 1/d_o$, we have

$$M = N/d_o = (72\text{ cm})/(4.0\text{ cm}) = \boxed{18}$$

107. $\boxed{\text{SSM}}$ ***REASONING AND SOLUTION*** An optometrist prescribes contact lenses that have a focal length of 55.0 cm.

a. The focal length is positive (+55.0 cm); therefore, we can conclude that the lenses are $\boxed{\text{converging}}$.

b. As discussed in the text (see Section 26.10), farsightedness is corrected by converging lenses. Therefore, the person who wears these lens is $\boxed{\text{farsighted}}$.

c. If the lenses are designed so that objects no closer than 35.0 cm can be seen clearly, we have $d_o = 35.0\text{ cm}$. The thin-lens equation (Equation 26.6) gives the image distance:

$$\frac{1}{d_i} = \frac{1}{f} - \frac{1}{d_o} = \frac{1}{55.0\text{ cm}} - \frac{1}{35.0\text{ cm}} \qquad \text{or} \qquad d_i = -96.3\text{ cm}$$

Thus, the near point is located $\boxed{96.3\text{ cm}}$ from the eyes.

108. ***REASONING AND SOLUTION*** Using Snell's law, we have

$$n_A \sin 72° = n_B \sin 56° \qquad \text{or} \qquad \frac{n_A}{n_B} = \frac{\sin 56°}{\sin 72°} = \boxed{0.87}$$

109. ***REASONING AND SOLUTION***

a. The sun is so far from the lens that the incident rays are nearly parallel to the principal axis, so the image distance d_i is nearly equal to the focal length of the lens. The magnification of the lens is

$$m = -\frac{d_i}{d_o} = -\frac{10.0 \times 10^{-2}\ \text{m}}{1.50 \times 10^{11}\ \text{m}} = -6.67 \times 10^{-13}$$

The image height h_i is

$$h_i = mh_o = (-6.67 \times 10^{-13})(1.39 \times 10^{9}\ \text{m}) = -9.27 \times 10^{-4}\ \text{m}$$

The diameter of the sun's image on the paper is the magnitude of h_i, or 9.27×10^{-4} m. The area A of the image is

$$A = \tfrac{1}{4}\pi(9.27 \times 10^{-4}\ \text{m})^2 = \boxed{6.74 \times 10^{-7}\ \text{m}^2}$$

b. The intensity I of the light wave is the power P that strikes the paper perpendicularly divided by the illuminated area A (see Equation 16.8)

$$I = \frac{P}{A} = \frac{0.530\ \text{W}}{6.74 \times 10^{-7}\ \text{m}^2} = \boxed{7.86 \times 10^{5}\ \text{W/m}^2}$$

110. ***REASONING AND SOLUTION*** We note that the object is placed 20.0 cm from the lens. Since the focal point of the lens is $f = -20.0$ cm, the object is situated at the focal point. In the scale drawing that follows, we locate the image using the two rays labeled a and b, which originate at the top of the object.

a. Measuring according to the scale used in the drawing, we find that the image is located 10.0 cm to the left of the lens. The lens is a diverging lens and forms a virtual image, so the image distance is $d_i = \boxed{-10.0 \text{ cm}}$.

b. Measuring the heights of the image and the object in the drawing, we find that the magnification is m = $\boxed{+0.500}$.

111. **SSM** ***REASONING AND SOLUTION***

a. A real image must be projected on the drum; therefore, the lens in the copier must be a $\boxed{\text{converging lens}}$.

b. If the document and its copy have the same size, but are inverted with respect to one another, the magnification equation (Equation 26.7) indicates that $m = -d_i / d_o = -1$. Therefore, $d_i / d_o = 1$ or $d_i = d_o$. Then, the thin-lens equation (Equation 26.6) gives

$$\frac{1}{d_i} + \frac{1}{d_o} = \frac{1}{f} = \frac{2}{d_o} \qquad \text{or} \qquad d_o = d_i = 2f$$

Therefore the document is located at a distance $\boxed{2f}$ from the lens.

c. Furthermore, the image is located at a distance of $\boxed{2f}$ from the lens.

112. ***REASONING AND SOLUTION***

a. With the image at the near point of the eye, the angular magnification is M = N/f +1. Find the focal length for M = 6.0, using

$$f = N/(M - 1) = (25\text{ cm})/(6.0 - 1) = 5.0\text{ cm} = 0.050\text{ m}$$

The refractive power of this lens is, therefore,

$$\text{Refractive power} = 1/f = 1/(0.050\text{ m}) = \boxed{2.0 \times 10^1\text{ diopters}}$$

b. When the image of the stamp is 45 cm from the eye,

$$M = (1/f - 1/d_i)N = [1/(0.050\text{ m}) - 1/(-0.45\text{ m})](0.25\text{ m}) = \boxed{5.6}$$

113. **SSM** ***REASONING*** A contact lens is placed directly on the eye. Therefore, the object distance, which is the distance from the book to the lens, is 25.0 cm. The near point can be determined from the thin-lens equation [Equation 26.6: $(1/d_o)+(1/d_i)=(1/f)$].

SOLUTION

a. Using the thin-lens equation, we have

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{65.0\text{ cm}}-\frac{1}{25.0\text{ cm}} \qquad \text{or} \qquad d_i=-40.6\text{ cm}$$

In other words, at age 40, the man's near point is 40.6 cm. Similarly, when the man is 45, we have

$$\frac{1}{d_i}=\frac{1}{f}-\frac{1}{d_o}=\frac{1}{65.0\text{ cm}}-\frac{1}{29.0\text{ cm}} \qquad \text{or} \qquad d_i=-52.4\text{ cm}$$

and his near point is 52.4 cm. Thus, the man's near point has changed by $52.4\text{ cm}-40.6\text{ cm}=\boxed{11.8\text{ cm}}$.

b. With $d_o=25.0\text{ cm}$ and $d_i=-52.4\text{ cm}$, the focal length of the lens is found as follows:

$$\frac{1}{f}=\frac{1}{d_o}+\frac{1}{d_i}=\frac{1}{25.0\text{ cm}}+\frac{1}{(-52.4\text{ cm})} \qquad \text{or} \qquad \boxed{f=47.8\text{ cm}}$$

114. ***REASONING*** To find the distance through which the object must be moved, we must obtain the object distances for the two situations described in the problem. To do this, we combine the thin-lens equation and the magnification equation, since data for the magnification is given.

SOLUTION

a. Since the magnification is positive, the image is upright, and the object must be located within the focal point of the lens, as in Figure 26.28. When the magnification is negative and has a magnitude greater than one, the object must be located between the focal point and the point that is at a distance of twice the focal length from the lens, as in Figure 26.27. Therefore, the object should be moved $\boxed{\text{away from the lens}}$.

b. According to the thin-lens equation, we have

$$\frac{1}{d_i}+\frac{1}{d_o}=\frac{1}{f} \qquad \text{or} \qquad d_i=\frac{d_o f}{d_o-f} \tag{1}$$

According to the magnification equation, with d_i expressed as in Equation (2), we have

$$m=-\frac{d_i}{d_o}=-\frac{1}{d_o}\left(\frac{d_o f}{d_o-f}\right)=\frac{f}{f-d_o} \qquad \text{or} \qquad d_o=\frac{f(m-1)}{m} \tag{2}$$

Applying Equation (2) to the two cases described in the problem, we have

$$(d_o)_{+m}=\frac{f(m-1)}{m}=\frac{f(+4.0-1)}{+4.0}=\frac{3.0f}{4.0} \tag{3}$$

$$(d_o)_{-m}=\frac{f(m-1)}{m}=\frac{f(-4.0-1)}{-4.0}=\frac{5.0f}{4.0} \tag{4}$$

Subtracting Equation (3) from Equation (4), we find that the object must be moved away from the lens by an additional distance of

$$(d_o)_{-m}-(d_o)_{+m}=\frac{5.0f}{4.0}-\frac{3.0f}{4.0}=\frac{2.0f}{4.0}=\frac{0.30\text{ m}}{2.0}=\boxed{0.15\text{ m}}$$

115. $\boxed{\text{SSM}}$ ***REASONING*** The angular magnification of a refracting telescope is 32 8000 times larger when you look through the correct end of the telescope than when you look through the wrong end. We wish to find the angular magnification, $M = -f_o / f_e$ (see Equation 26.12) of the telescope. Thus, we proceed by finding the ratio of the focal lengths of the objective and the eyepiece and using Equation 26.12 to find *M*.

SOLUTION When you look through the correct end of the telescope, the angular magnification of the telescope is $M_c = -f_o / f_e$. If you look through the wrong end, the roles of the objective and eyepiece lenses are interchanged, so that the angular magnification would be $M_w = -f_e / f_o$. Therefore,

$$\frac{M_c}{M_w} = \frac{-f_o / f_e}{-f_e / f_o} = \left(\frac{f_o}{f_e}\right)^2 = 32\ 800 \qquad \text{or} \qquad \frac{f_o}{f_e} = \pm\sqrt{32\ 800} = \pm 181$$

The angular magnification of the telescope is negative, so we choose the positive root and obtain $M = -f_o / f_e = -(+181) = \boxed{-181}$.

116. ***REASONING AND SOLUTION*** From the drawing we see that $d_o = x + f$ and $d_i = x' + f$. Substituting these two expressions into the thin-lens equation, we obtain

$$\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{x+f} + \frac{1}{x'+f} = \frac{1}{f}$$

Combining the terms on the left over a common denominator gives

$$\frac{x+f+x'+f}{(x+f)(x'+f)} = \frac{x+x'+2f}{(x+f)(x'+f)} = \frac{1}{f}$$

Cross-multiplying shows that

$$f(x+x'+2f) = (x+f)(x'+f)$$

Expanding and simplifying this result, we obtain

$$fx + fx' + 2f^2 = xx' + fx' + xf + f^2 \qquad \text{or} \qquad \boxed{xx' = f^2}$$

117. ***REASONING AND SOLUTION*** If the near point is 79.0 cm, then $d_i = -79.0$ cm, and $d_o = 25.0$ cm. Using the thin-lens equation, we find that the focal length of the correcting lens is

$$f = d_o d_i/(d_o + d_i) = (25.0\text{ cm})(-79.0\text{ cm})/(25.0\text{ cm} - 79.0\text{ cm}) = +36.6\text{ cm}$$

a. The distance d_o' to the poster can be obtained as follows:

$$1/d_o' = 1/f - 1/d_i' = 1/(36.6\text{ cm}) - 1/(-217\text{ cm}) \qquad \text{or} \qquad d_o' = \boxed{31.3\text{ cm}}$$

b. The image size is

$$h_i = h_o(-d_i'/d_o') = (0.350\text{ m})[-(-217\text{ cm})/(31.3\text{ cm})] = \boxed{2.43\text{ m}}$$

1. **SSM** ***REASONING AND SOLUTION*** To decide whether constructive or destructive interference occurs, we need to determine the wavelength of the wave. For electromagnetic waves, Equation 16.1 can be written $f\lambda = c$, so that

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m}}{536 \times 10^3 \text{ Hz}} = 5.60 \times 10^2 \text{ m}$$

Since the two wave sources are in phase, constructive interference occurs when the path difference is an integer number of wavelengths, and destructive interference occurs when the path difference is an odd number of half wavelengths. We find that the path difference is 8.12 km – 7.00 km = 1.12 km. The number of wavelengths in this path difference is $\left(1.12 \times 10^3\ m\right) / \left(5.60 \times 10^2\ m\right) = 2.00$. Therefore, $\boxed{\text{constructive interference occurs}}$.

2. ***REASONING AND SOLUTION***

a. We know that $\theta = \sin^{-1}(m\lambda/d)$. Also, $m = 1$ for the first order bright fringe, so

$$\theta = \sin^{-1}\left[\frac{(1)(630\times10^{-9}\text{ m})}{5.3\times10^{-5}\text{ m}}\right] = \boxed{0.68°}$$

b. For the second-order bright fringe, $m = 2$ so $\theta = \boxed{1.4°}$.

c. For the third-order bright fringe, $m = 3$ so $\theta = \boxed{2.0°}$.

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3. ***REASONING AND SOLUTION*** For destructive interference, $(m + 1/2)\lambda = d \sin \theta$. For the second dark fringe, $m = 1$ so

$$\frac{d}{\lambda} = \frac{m + \frac{1}{2}}{\sin\theta} = \frac{1 + \frac{1}{2}}{\sin 5.4°} = \boxed{16}$$

4. ***REASONING*** Let ℓ_1 and ℓ_2 be the distances from source 1 and source 2, respectively. For constructive interference the condition is $\ell_2 - \ell_1 = m\lambda$, where λ is the wavelength and $m = 0, 1, 2, 3, \ldots$.For destructive interference the condition is $\ell_2 - \ell_1 = (m + \frac{1}{2})\lambda$, where $m = 0, 1, 2, 3, \ldots$.The fact that the wavelength is greater than the separation between the sources will allow us to decide which type of interference we have and to locate the two places in question.

SOLUTION a. Since the places we seek lie on the line between the two sources and since the separation between the sources is 4.00 m, we know that $\ell_2 - \ell_1$ cannot have a value greater than 4.00 m. Therefore, with a wavelength of 5.00 m, the only way constructive interference can occur is at a place where $m = 0$. But this would mean that $\ell_2 = \ell_1$, and there is only one place where this is true, namely, at the midpoint between the sources. But we know that there are two places. As a result, we conclude that

$\boxed{\text{destructive interference is occurring}}$.

b. For destructive interference, the possible values for the integer m are $m = 0, 1, 2, 3, \ldots$, and we must decide which to use. The condition for destructive interference is $\ell_2 - \ell_1 = (m + \frac{1}{2})\lambda$. Since $\lambda = 5.00$ m, all values of m greater than or equal to one result in $\ell_2 - \ell_1$ being greater than 4.00 m, which we already know is not possible. Therefore, we conclude that $m = 0$, and the condition for destructive interference becomes $\ell_2 - \ell_1 = \frac{1}{2}\lambda$. We also know that $\ell_2 + \ell_1 = 4.00$ m, since the separation between the sources is 4.00 m. In other words, we have

$$\ell_2 - \ell_1 = \tfrac{1}{2}\lambda = \tfrac{1}{2}(5.00\text{ m}) = 2.50\text{ m} \quad \text{and} \quad \ell_2 + \ell_1 = 4.00\text{ m}$$

Adding these two equations gives

$$2\ell_2 = 6.50\text{ m} \quad \text{or} \quad \ell_2 = 3.25\text{ m}$$

Since $\ell_2 + \ell_1 = 4.00$m, it follows that

$$\ell_1 = 4.00\text{ m} - \ell_2 = 4.00\text{ m} - 3.25\text{ m} = 0.75\text{ m}$$

These results indicate that the two places are $\boxed{\text{3.25 m and 0.75 m}}$ from one of the speakers.

5. **SSM** ***REASONING AND SOLUTION*** The angular position θ of the bright fringes of a double slit are given by Equation 27.1 as $\sin\theta = m\lambda / d$, with the order of the fringe specified by $m = 0, 1, 2, 3, \ldots$. Solving for λ, we have

$$\lambda = \frac{d \sin\theta}{m} = \frac{(3.8 \times 10^{-5}\ \text{m}) \sin 2.0°}{2} = 6.6 \times 10^{-7}\ \text{m} = \boxed{660\ \text{nm}}$$

6. ***REASONING AND SOLUTION*** The angular position of a third order fringe is given by $\theta = \sin^{-1} 3\lambda/d$, and the position of the fringe on the screen is $y = L \tan \theta$.

For the third-order red fringe,

$$\theta = \sin^{-1}\left[\frac{3\left(665 \times 10^{-9}\ \text{m}\right)}{0.158 \times 10^{-3}\ \text{m}}\right] = 0.724°$$

and

$$y = (2.24\ \text{m}) \tan 0.724° = 2.83 \times 10^{-2}\ \text{m}$$

For the third-order green fringe, $\theta = 0.615°$ and $y = 2.40 \times 10^{-2}$ m.

The distance between the fringes is 2.83×10^{-2} m $- 2.40 \times 10^{-2}$ m = $\boxed{4.3 \times 10^{-3}\ \text{m}}$.

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7. ***REASONING AND SOLUTION*** From Example 1 we have that $y = L \tan \theta$. Since for small angles, $\tan \theta \approx \sin \theta = \lambda/d$, $y = (L/d)\lambda$. The first case gives

$$L/d = y/\lambda = (2.40 \times 10^{-2}\ \text{m})/(475 \times 10^{-9}\ \text{m}) = 5.05 \times 10^{4}$$

Then the fringe separation in the second case is

$$y = (5.05 \times 10^{4})(611 \times 10^{-9}\ \text{m}) = \boxed{0.0309\ \text{m}}$$

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8. ***REASONING AND SOLUTION*** The first-order orange fringes occur farther out from the center than do the first-order blue fringes. Therefore, the screen must be moved $\boxed{\text{toward the slits}}$ so that the orange fringes will appear on the screen. The distance between the screen and the slits is L, and the amount by which the screen must be moved toward the slits is $L_{\text{blue}} - L_{\text{orange}}$. We know that $L_{\text{blue}} = 0.500\text{ m}$, and must, therefore, determine L_{orange}. We begin with Equation 27.1, as it applies to first-order fringes, that is, $\sin\theta = \lambda/d$. Furthermore, as discussed in Example 1 in the text, $\tan\theta = y/L$, where y is the distance from the center of the screen to a fringe. Since the angle θ locating the fringes is small, $\sin\theta \approx \tan\theta$, and we have that

$$\frac{\lambda}{d} \approx \frac{y}{L} \qquad \text{or} \qquad L = \frac{yd}{\lambda}$$

Writing this result for L for both colors and dividing the two equations gives

$$\frac{L_{\text{blue}}}{L_{\text{orange}}} = \frac{yd/\lambda_{\text{blue}}}{yd/\lambda_{\text{orange}}} = \frac{\lambda_{\text{orange}}}{\lambda_{\text{blue}}}$$

where we have recognized that y is one-half the screen width (the same for both colors), and that the slit separation d is the same for both colors. Using the result above, we find that

$$L_{\text{blue}} - L_{\text{orange}} = L_{\text{blue}} - \left(\frac{\lambda_{\text{blue}}}{\lambda_{\text{orange}}}\right)L_{\text{blue}} = L_{\text{blue}}\left(1 - \frac{\lambda_{\text{blue}}}{\lambda_{\text{orange}}}\right)$$

$$= (0.500\text{ m})\left[1 - \frac{471\text{ nm}}{611\text{ nm}}\right] = \boxed{0.115\text{ m}}$$

9. SSM WWW ***REASONING*** The light that travels through the plastic has a different path length than the light that passes through the unobstructed slit. Since the center of the screen now appears dark, rather than bright, destructive interference, rather than constructive interference occurs there. This means that the difference between the number of wavelengths in the plastic sheet and that in a comparable thickness of air is $\frac{1}{2}$.

SOLUTION The wavelength of the light in the plastic sheet is given by Equation 27.3 as

$$\lambda_{\text{plastic}} = \frac{\lambda_{\text{vacuum}}}{n} = \frac{586 \times 10^{-9}\ \text{m}}{1.60} = \boxed{366 \times 10^{-9}\ \text{m}}$$

The number of wavelengths contained in a plastic sheet of thickness t is

$$N_{\text{plastic}} = \frac{t}{\lambda_{\text{plastic}}} = \frac{t}{366 \times 10^{-9}\ \text{m}}$$

The number of wavelengths contained in an equal thickness of air is

$$N_{\text{air}} = \frac{t}{\lambda_{\text{air}}} = \frac{t}{586 \times 10^{-9}\ \text{m}}$$

where we have used the fact that $\lambda_{\text{air}} \approx \lambda_{\text{vacuum}}$. Destructive interference occurs when the difference, $N_{\text{plastic}} - N_{\text{air}}$, in the number of wavelengths is $\frac{1}{2}$:

$$N_{\text{plastic}} - N_{\text{air}} = \frac{1}{2}$$

$$\frac{t}{366 \times 10^{-9}\ \text{m}} - \frac{t}{586 \times 10^{-9}\ \text{m}} = \frac{1}{2}$$

Solving this equation for t yields $t = 487 \times 10^{-9}\ \text{m} = \boxed{487\ \text{nm}}$.

10. ***REASONING AND SOLUTION*** A phase shift equivalent to one-half a wavelength occurs at the top air-plastic interface, so the condition for destructive interference is $2t = m\lambda_{\text{film}}$. Since $\lambda_{\text{film}} = \lambda_{\text{vacuum}}/n$, the thickness of the film can be written as

$$t = \frac{m}{2}\left(\frac{\lambda_{\text{vacuum}}}{n}\right)$$

For $m = 1$:

$$t = \frac{1}{2}\left(\frac{589\text{ nm}}{1.61}\right) = \boxed{183\text{ nm}}$$

For $m = 2$:

$$t = \frac{2}{2}\left(\frac{589\text{ nm}}{1.61}\right) = \boxed{366\text{ nm}}$$

11. SSM ***REASONING*** To solve this problem, we must express the condition for destructive interference in terms of the film thickness t and the wavelength λ_{film} of the light as it passes through the magnesium fluoride coating. We must also take into account any phase changes that occur upon reflection.

SOLUTION Since the coating is intended to be nonreflective, its thickness must be chosen so that destructive interference occurs between waves 1 and 2 in the drawing. For destructive interference, the combined phase difference between the two waves must be an odd integer number of half wavelengths. The phase change for wave 1 is equivalent to one-half of a wavelength, since this light travels from a smaller refractive index ($n_{\text{air}} = 1.00$) toward a larger refractive index ($n_{\text{film}} = 1.38$).

Similarly, there is a phase change when wave 2 reflects from the right surface of the film, since this light also travels from a smaller refractive index ($n_{\text{film}} = 1.38$) toward a larger one ($n_{\text{lens}} = 1.52$). Therefore, a phase change of one-half wavelength occurs at both boundaries, so the net phase change between waves 1 and 2 due to reflection is zero. Since wave 2 travels back and forth through the film and, and since the light is assumed to be at nearly normal incidence, the extra distance traveled by wave 2 compared to wave 1 is twice the film thickness, or $2t$. Thus, in this case, the minimum condition for destructive interference is

$$2t = \tfrac{1}{2}\lambda_{\text{film}}$$

The wavelength of light in the coating is

$$\lambda_{\text{film}} = \frac{\lambda_{\text{vacuum}}}{n} = \frac{565\text{ nm}}{1.38} = 409\text{ nm} \qquad (27.3)$$

Solving the above expression for t, we find that the minimum thickness that the coating can have is

$$t = \tfrac{1}{4}\lambda_{\text{film}} = \tfrac{1}{4}(409\text{ nm}) = \boxed{102\text{ nm}}$$

12. ***REASONING AND SOLUTION*** The condition for destructive interference is $2t = \lambda_{\text{film}}$, so $2t = \lambda_{\text{vacuum}}/n$.

a. For removal of violet light, which makes the film look yellow,

$$t = \frac{1}{2}\left(\frac{410\text{ nm}}{1.40}\right) = \boxed{150\text{ nm}}$$

b. For removal of yellow light, which makes the film look violet,

$$t = \frac{1}{2}\left(\frac{580\text{ nm}}{1.40}\right) = \boxed{210\text{ nm}}$$

13. ***REASONING AND SOLUTION*** The only change from Example 5 is $\lambda_{\text{film}} = \lambda_{\text{vacuum}}/n =$ (552 nm)/1.33 = 415 nm. Since $t = \frac{1}{2}\left(m + \frac{1}{2}\right)\lambda_{\text{film}}$,

$$m = \frac{2t}{\lambda_{\text{film}}} - \frac{1}{2} = \frac{2\left(4.10 \times 10^{-5}\ \text{m}\right)}{415 \times 10^{-9}\ \text{m}} - \frac{1}{2} = 197$$

Since the first bright fringe occurs when $m = 0$, the number of bright fringes is $m + 1 = \boxed{198}$.

14. ***REASONING*** In air the index of refraction is nearly $n = 1$, while in the film it is greater than one. A phase change occurs whenever light travels through a material with a smaller refractive index toward a material with a larger refractive index and reflects from the boundary between the two. The phase change is equivalent to $\frac{1}{2}\lambda_{\text{film}}$, where λ_{film} is the wavelength in the film. This phase change occurs at the top surface of the film, where the light first strikes it. However, no phase change occurs when light that has penetrated the film reflects back upward from the bottom surface. This is because a phase change does not occur when light travels through a material with a larger refractive index toward a material with a smaller refractive index and reflects from the boundary between the two. Thus, to evaluate destructive interference correctly, we must consider a net phase change of $\frac{1}{2}\lambda_{\text{film}}$ due to reflection as well as the extra distance traveled by the light within the film.

SOLUTION Figure 27.42 shows the soap film and the two rays of light that represent the interfering light waves. At nearly perpendicular incidence, ray 2 travels a distance of $2t$ further than ray 1, where t is the thickness of the film. In addition, the net phase change for the two rays is $\frac{1}{2}\lambda_{\text{film}}$, as discussed in the reasoning section. We must combine this amount with the extra travel distance to determine the condition for destructive interference. For destructive interference, the combined total must be an odd-integer number of half-wavelengths in the film:

$$\underbrace{2t}_{\substack{\text{Extra distance}\\ \text{traveled by}\\ \text{ray 2}}} + \underbrace{\tfrac{1}{2}\lambda_{\text{film}}}_{\substack{\text{Half wavelength}\\ \text{net phase change}\\ \text{due to reflection}}} = \underbrace{\tfrac{1}{2}\lambda_{\text{film}}, \tfrac{3}{2}\lambda_{\text{film}}, \tfrac{5}{2}\lambda_{\text{film}}, \ldots}_{\substack{\text{Condition for}\\ \text{destructive}\\ \text{interference}}}$$

Subtracting the term $\frac{1}{2}\lambda_{\text{film}}$ from the left side of this equation and from each term on the right side, we obtain

$$2t = 0, \lambda_{\text{film}}, 2\lambda_{\text{film}}, \ldots$$

The minimum nonzero thickness is $t = \lambda_{\text{film}}/2$. But the wavelength in the film is related to the vacuum-wavelength according to Equation 27.3: $\lambda_{\text{film}} = \lambda_{\text{vacuum}}/n$. Thus, the minimum nonzero thickness is $t = \lambda_{\text{vacuum}}/(2n)$. Applying this result to both regions of the film allows us to obtain the desired ratio:

$$\frac{t_{\text{magenta}}}{t_{\text{yellow}}} = \frac{\lambda_{\text{vacuum, green}}/(2n)}{\lambda_{\text{vacuum, blue}}/(2n)} = \frac{\lambda_{\text{vacuum, green}}}{\lambda_{\text{vacuum, blue}}} = \frac{555\text{ nm}}{469\text{ nm}} = \boxed{1.18}$$

15. SSM WWW ***REASONING*** To solve this problem, we must express the condition for constructive interference in terms of the film thickness t and the wavelength λ_{film} of the light in the soap film. We must also take into account any phase changes that occur upon reflection.

SOLUTION For the reflection at the top film surface, the light travels from air, where the refractive index is smaller ($n = 1.00$), toward the film, where the refractive index is larger ($n = 1.33$). Associated with this reflection there is a phase change that is equivalent to one-half of a wavelength. For the reflection at the bottom film surface, the light travels from the film, where the refractive index is larger ($n = 1.33$), toward air, where the refractive index is smaller ($n = 1.00$). Associated with this reflection, there is no phase change. As a result of these two reflections, there is a net phase change that is equivalent to one-half of a wavelength. To obtain the condition for constructive interference, this net phase change must be added to the phase change that arises because of the film thickness t, which is traversed twice by the light that penetrates it. For constructive interference we find that

$$2t + \tfrac{1}{2}\lambda_{\text{film}} = \lambda_{\text{film}}, 2\lambda_{\text{film}}, 3\lambda_{\text{film}}, \ldots$$

or

$$2t = \left(m + \tfrac{1}{2}\right)\lambda_{\text{film}}, \qquad \text{where } m = 0, 1, 2, \ldots$$

Equation 27.3 indicates that $\lambda_{\text{film}} = \lambda_{\text{vacuum}} / n$. Using this expression and the fact that $m = 0$ for the minimum thickness t, we find that the condition for constructive interference becomes

$$2t = \left(m + \tfrac{1}{2}\right)\lambda_{\text{film}} = \left(0 + \tfrac{1}{2}\right)\left(\frac{\lambda_{\text{vacuum}}}{n}\right)$$

or

$$t = \frac{\lambda_{\text{vacuum}}}{4n} = \frac{611\text{ nm}}{4(1.33)} = \boxed{115\text{ nm}}$$

16. ***REASONING*** The number m of bright fringes in an air wedge is discussed in Example 5, where it is shown that m is related to the thickness t of the film and the wavelength λ_{film} of the light within the film by (see Figure 27.13)

$$\underbrace{2t}_{\substack{\text{Extra distance}\\ \text{traveled by}\\ \text{wave 2}}} + \underbrace{\tfrac{1}{2}\lambda_{\text{film}}}_{\substack{\text{Half-wavelength}\\ \text{shift due to reflection}\\ \text{of wave 1}}} = m\lambda_{\text{film}} \quad \text{where} \quad m = 1, 2, 3, \ldots$$

For a given thickness, we can solve this equation for the number of bright fringes.

SOLUTION Since the light is traveling in a film of air, $\lambda_{\text{film}} = \lambda_{\text{vacuum}} = 550$ nm. The number m of bright fringes is

$$m = \frac{2t}{\lambda_{\text{vacuum}}} + \frac{1}{2} = \frac{2\left(1.37 \times 10^{-5}\ \text{m}\right)}{550 \times 10^{-9}\ \text{m}} + \frac{1}{2} = 50.3$$

Thus, there are $\boxed{\text{50 bright fringes}}$.

17. ***REASONING AND SOLUTION*** The condition for destructive interference is $2t = \lambda_{\text{film}} = \lambda_{\text{vacuum}}/n$, so that

$$nt = \lambda_{\text{vacuum}}/2 = (660\text{ nm})/2 = 330\text{ nm}$$

The condition for constructive interference is $2t = (m + 1/2)\lambda_{\text{film}} = (m + 1/2)(\lambda_{\text{vacuum}}/n)$, so

$$\lambda_{\text{vacuum}} = \frac{2nt}{m+\frac{1}{2}} = \frac{2(330\text{ nm})}{m+\frac{1}{2}}$$

For $m = 0$, $\lambda_{\text{vacuum}} = 1320$ nm, for $m = 1$, $\lambda_{\text{vacuum}} = 440$ nm, and for $m = 2$, $\lambda_{\text{vacuum}} = 264$ nm.

The only visible wavelength that will give constructive interference is $\boxed{440\text{ nm}}$.

18. ***REASONING AND SOLUTION***

a. The condition for constructive interference is

$$2t = m\lambda_{\text{film}} = m(\lambda_{\text{vacuum}}/n_{\text{water}})$$

For $\lambda_{\text{vacuum}} = 432$ nm,

$$t = \frac{m}{2}\left(\frac{432\text{ nm}}{1.33}\right)$$

For $\lambda_{\text{vacuum}} = 648$ nm,

$$t = \frac{m'}{2}\left(\frac{648\text{ nm}}{1.33}\right)$$

Equating the two expressions for t, we find that $m/m' = 1.50$. For minimum thickness, this means that $m = 3$ and $m' = 2$. Then

$$t = \frac{m}{2}\left(\frac{432\text{ mn}}{1.33}\right) = \frac{3}{2}\left(\frac{432\text{ nm}}{1.33}\right) = \boxed{487\text{ nm}}$$

b. The condition for destructive interference is $2t = (m + 1/2)\,(\lambda_{\text{vacuum}}/n_{\text{water}})$. Thus,

$$\lambda_{\text{vacuum}} = \frac{2n_{\text{water}}t}{m+\frac{1}{2}} = \frac{2(1.33)(487\times10^{-9}\text{ m})}{m+\frac{1}{2}}$$

The wavelength lying within the specified range is for $m = 2$, so that

$$\lambda_{\text{vacuum}} = \boxed{518\text{ nm}}$$

19. SSM ***REASONING*** This problem can be solved by using Equation 27.4 for the value of the angle θ when $m = 1$ (first dark fringe).

SOLUTION

a. When the slit width is $W = 1.8 \times 10^{-4}$ m and $\lambda = 675 \text{ nm} = 675 \times 10^{-9}$ m, we find, according to Equation 27.4,

$$\theta = \sin^{-1}\left(m\frac{\lambda}{W}\right) = \sin^{-1}\left[(1)\frac{675 \times 10^{-9} \text{ m}}{1.8 \times 10^{-4} \text{ m}}\right] = \boxed{0.21°}$$

b. Similarly, when the slit width is $W = 1.8 \times 10^{-6}$ m and $\lambda = 675 \times 10^{-9}$ m, we find

$$\theta = \sin^{-1}\left[(1)\frac{675 \times 10^{-9} \text{ m}}{1.8 \times 10^{-6} \text{ m}}\right] = \boxed{22°}$$

20. ***REASONING AND SOLUTION***

a. We know that $\theta = \sin^{-1}(\lambda/W)$ for the first ($m = 1$) dark fringe. Therefore,

$$\theta = \sin^{-1}\left(\frac{660 \times 10^{-9}\ \text{m}}{0.91\ \text{m}}\right) = \boxed{4.2 \times 10^{-5}\ \text{degrees}}$$

b. The wavelength of the sound wave is $\lambda = v/f = (343\ \text{m/s})/(440\ \text{Hz}) = 0.78\ \text{m}$. For the sound wave,

$$\theta = \sin^{-1}\left(\frac{0.78\ \text{m}}{0.91\ \text{m}}\right) = \boxed{59°}$$

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21. ***REASONING AND SOLUTION*** We have $\theta = \sin^{-1}(2\lambda/W)$ for the second dark fringe.

a. So

$$\theta = \sin^{-1}\left[\frac{2\left(430\times10^{-9}\text{ m}\right)}{2.1\times10^{-6}\text{ m}}\right] = \boxed{24°}$$

b. And

$$\theta = \sin^{-1}\left[\frac{2\left(660\times10^{-9}\text{ m}\right)}{2.1\times10^{-6}\text{ m}}\right] = \boxed{39°}$$

22. ***REASONING AND SOLUTION*** For the first-order dark fringe in a single-slit diffraction pattern, Equation 27.4 with $m = 1$ indicates that $\sin\theta = \lambda/W$, or $W = \lambda/\sin\theta$. Using this result, we find that

$$\frac{W_A}{W_B} = \frac{\lambda/\sin\theta_A}{\lambda/\sin\theta_B} = \frac{\sin\theta_B}{\sin\theta_A} = \frac{\sin 56°}{\sin 34°} = \boxed{1.5}$$

23. SSM WWW ***REASONING*** According to Equation 27.4, the angles at which the dark fringes occur are given by $\sin\theta = m\lambda / W$, where W is the slit width. In Figure 27.24, we see from the trigonometry of the situation that $\tan\theta = y / L$. Therefore, the latter expression can be used to determine the angle θ, and Equation 27.4 can be used to find the wavelength λ.

SOLUTION The angle θ is given by

$$\theta = \tan^{-1}\left(\frac{y}{\mathrm{L}}\right) = \tan^{-1}\left(\frac{3.5\times10^{-3}\ \mathrm{m}}{4.0\ \mathrm{m}}\right) = 0.050°$$

The wavelength of the light is

$$\lambda = W\sin\theta = \left(5.6\times10^{-4}\ \mathrm{m}\right)\sin 0.050° = 4.9\times10^{-7}\ \mathrm{m} = \boxed{490\ \mathrm{nm}}$$

24. ***REASONING AND SOLUTION*** The width of the bright fringe that is next to the central bright fringe is $y_2 - y_1$, where y_1 is the distance from the center of the central bright fringe to the first-order dark fringe, and y_2 is the analogous distance for the second-order dark fringe. To find the fringe width, we follow the approach outlined in Example 6 in the text. According to Equation 27.4, the dark fringes for single slit diffraction are located by $\sin\theta = m\lambda/W$. For first-order fringes, $m = 1$, and $\sin\theta_1 = \lambda / W$. For second-order fringes, $m = 2$, and $\sin\theta_2 = 2\lambda / W$. Therefore, we find that

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{W}\right) = \sin^{-1}\left(\frac{480 \times 10^{-9}\ \text{m}}{2.0 \times 10^{-5}\ \text{m}}\right) = 1.4°$$

$$\theta_2 = \sin^{-1}\left(\frac{2\lambda}{W}\right) = \sin^{-1}\left[\frac{2\left(480 \times 10^{-9}\ \text{m}\right)}{2.0 \times 10^{-5}\ \text{m}}\right] = 2.8°$$

From Example 6 we know that the distance y is given by $y = L \tan\theta$, where L is the distance between the slit and the screen. Therefore, it follows that

$$y_2 - y_1 = L\left(\tan\theta_2 - \tan\theta_1\right) = (0.50\ \text{m})\left(\tan 2.8° - \tan 1.4°\right) = \boxed{0.012\ \text{m}}$$

25. **SSM** ***REASONING AND SOLUTION*** Using Equation 27.4 for the first-order dark fringes ($m = 1$) and referring to Figure 27.24 in the text, we see that

$$\sin\theta = (1)\frac{\lambda}{W} = \frac{y}{\sqrt{L^2 + y^2}}$$

Since the distance L between the slit and the screen equals the width $2y$ of the central bright fringe, this equation becomes

$$\frac{\lambda}{W} = \frac{y}{\sqrt{(2y)^2 + y^2}} = \frac{1}{\sqrt{5}} = \boxed{0.447}$$

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26. ***REASONING*** The width of the central bright fringe is defined by the location of the first dark fringe on either side of it. According to Equation 27.4, the angle θ locating the first dark fringe can be obtained from $\sin\theta = \lambda/W$, where λ is the wavelength of the light and W is the width of the slit. According to the drawing, $\tan\theta = y/L$, where y is half the width of the central bright fringe and L is the distance between the slit and the screen.

SOLUTION Since the angle θ is small, we can use the fact that $\sin\theta \approx \tan\theta$. Since $\sin\theta = \lambda/W$ and $\tan\theta = y/L$, we have

$$\frac{\lambda}{W} = \frac{y}{L} \quad \text{or} \quad W = \frac{\lambda L}{y}$$

Applying this result to both slits gives

$$\frac{W_2}{W_1} = \frac{\lambda L / y_2}{\lambda L / y_1} = \frac{y_1}{y_2}$$

$$W_2 = W_1 \frac{y_1}{y_2} = \left(3.2 \times 10^{-5}\ \text{m}\right) \frac{\frac{1}{2}\left(1.2\ \text{cm}\right)}{\frac{1}{2}\left(1.9\ \text{cm}\right)} = \boxed{2.0 \times 10^{-5}\ \text{m}}$$

27. ***REASONING AND SOLUTION*** It is given that $2y = 450\ W$ and $L = 18\ 000\ W$. We know $\lambda/W = \sin\theta$. Now $\sin\theta \approx \tan\theta = y/L$, so

$$\frac{\lambda}{W} = \frac{y}{L} = \frac{225\,W}{18\,000\,W} = \boxed{0.013}$$

28. ***REASONING AND SOLUTION*** The minimum angular separation of the cars must be $\theta_{min} = 1.22\ \lambda/D$, and the separation of the cars is $y = L\theta_{min} = 1.22\ L\lambda/D$.

a. For red light, $\lambda = 665$ nm, and

$$y = (1.22)(8690\text{ m})(665 \times 10^{-9}\text{ m})/(2.00 \times 10^{-3}\text{ m}) = \boxed{3.53\text{ m}}$$

b. For violet light, $\lambda = 405$ nm, and

$$y = (1.22)(8690\text{ m})(405 \times 10^{-9}\text{ m})/(2.00 \times 10^{-3}\text{ m}) = \boxed{2.15\text{ m}}$$

29. ***REASONING AND SOLUTION***

a. The minimum angular separation of the points must be $\theta_{\text{min}} = 1.22\ \lambda/D$, where $\lambda = 550\ \text{nm}$. Thus,

$$\theta_{\text{min}} = \frac{1.22\lambda}{D} = \frac{1.22\left(550 \times 10^{-9}\ \text{m}\right)}{2.0 \times 10^{-3}\ \text{m}} = 3.4 \times 10^{-4}\ \text{rad}$$

The distance at which the points can be resolved is

$$L = \frac{y}{\theta_{\text{min}}} = \frac{0.0738\ \text{m}}{3.4 \times 10^{-4}\ \text{rad}} = \boxed{220\ \text{m}}$$

b. $\boxed{\text{No}}$

30. ***REASONING*** The drawing shows the two asteroids that are a distance r from the Hubble Space Telescope and separated by a distance s. If s is much smaller than r, the angle θ (measured in radians) is given by $\theta = s/r$. The smallest angle that can just be resolved by the telescope is given by Equation 27.6 as

$$\theta_{\text{min}} = 1.22\frac{\lambda}{D}$$

where λ is the wavelength of the light and D is the diameter of the telescope's aperture. These two relations can be used to find the separation distance s.

SOLUTION The distance between the asteroids is

$$s = r\theta = r\left(\frac{1.22\lambda}{D}\right) = \left(2.0\times10^{10}\text{ m}\right)\left[\frac{1.22\left(550\times10^{-9}\text{ m}\right)}{2.4\text{ m}}\right] = \boxed{5.6\times10^{3}\text{ m}}$$

31. **SSM** ***REASONING AND SOLUTION*** According to Rayleigh's criterion (Equation 27.6), the minimum angular separation of the two objects is

$$\theta_{\min}\,(\text{in rad}) \approx 1.22\frac{\lambda}{D} = 1.22\left(\frac{565\times10^{-9}\ \text{m}}{1.02\ \text{m}}\right) = 6.76\times10^{-7}\ \text{rad}$$

Therefore, from the figure above, the separation y of the two objects is

$$y = L\theta_{\min} = (3.75\times10^{4}\ \text{m})(6.76\times10^{-7}\ \text{rad}) = \boxed{0.0254\ \text{m}}$$

32. ***REASONING*** At the aperture the star and its planet must subtend an angle at least as large as that given by the Rayleigh criterion, which is $\theta_{\min} \approx 1.22\,\lambda/D$, where λ is the wavelength of the light and D is the diameter of the aperture. The angle $\theta_{\min}$ is given by this criterion in radians. We can obtain the angle subtended at the telescope aperture by using the separation s between the planet and the star and the distance L of the star from the earth. According to Equation 8.1, the angle in radians is $\theta_{\min} = s/L$.

SOLUTION Using the Rayleigh criterion and Equation 8.1, we have

$$\frac{s}{L} \approx 1.22\frac{\lambda}{D} \quad \text{or} \quad D \approx \frac{1.22\,\lambda\,L}{s} = \frac{1.22\left(550\times10^{-9}\ \text{m}\right)\left(4.2\times10^{17}\ \text{m}\right)}{1.2\times10^{11}\ \text{m}} = \boxed{2.3\ \text{m}}$$

33. ***REASONING AND SOLUTION***

a. Equation 27.6$\left(\theta_{\min} = 1.22\,\lambda / D\right)$ gives the minimum angle $\theta_{\min}$ that two point objects can subtend at an aperture of diameter D and still be resolved. The angle must be measured in radians. For a dot separation s and a distance L between the painting and the eye, Equation 8.1 gives the angle in radians as $\theta_{\min} = s / L$. Therefore, we find that

$$\frac{1.22\,\lambda}{D} = \frac{s}{L} \qquad \text{or} \qquad L = \frac{s\,D}{1.22\,\lambda}$$

With λ = 550 nm and a pupil diameter of D = 2.5 mm, the distance L is

$$L = \frac{s\,D}{1.22\,\lambda} = \frac{\left(1.5 \times 10^{-3}\ \text{m}\right)\left(2.5 \times 10^{-3}\ \text{m}\right)}{\left(1.22\right)\left(550 \times 10^{-9}\ \text{m}\right)} = \boxed{5.6\ \text{m}}$$

b. The calculation here is similar to that in part a, except that n = 1.00 and D = 25 mm for the camera. Therefore, the distance L for the camera is

$$L = \frac{s\,D}{1.22\,\lambda} = \frac{\left(1.5 \times 10^{-3}\ \text{m}\right)\left(25 \times 10^{-3}\ \text{m}\right)}{\left(1.22\right)\left(550 \times 10^{-9}\ \text{m}\right)} = \boxed{56\ \text{m}}$$

34. ***REASONING AND SOLUTION*** The minimum angular separation of the mice is $\theta_{min} = 1.22\ \lambda/D$, where $\lambda = 550$ nm. Thus,

$$\theta_{min} = 1.22\ \lambda/D = 1.22(550 \times 10^{-9}\ \text{m})/(6.0 \times 10^{-3}\ \text{m}) = 1.12 \times 10^{-4}\ \text{rad}$$

The distance at which the mice can just be resolved as separate objects is

$$y = \frac{0.010\ \text{m}}{\tan\theta_{min}} = \frac{0.010\ \text{m}}{1.12 \times 10^{-4}} = 89\ \text{m}$$

So the eagle moves a distance of $d = 176\ \text{m} - 89\ \text{m} = 87\ \text{m}$ at a speed of 17 m/s. Therefore, the amount of time which passes is

$$t = \frac{d}{v} = \frac{87\ \text{m}}{17\ \text{m/s}} = \boxed{5.1\ \text{s}}$$

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35. SSM WWW ***REASONING*** When light of wavelength λ passes through a circular opening of diameter D, a circular diffraction pattern results, and Equation 27.5 [$\sin\theta = 1.22(\lambda / D)$] locates the first circular dark fringe relative to the central bright spot. Therefore, Equation 27.5 can be used to find the angular separation θ of the central bright spot and the first dark fringe. The geometry of the situation can be used to find the diameter of the central bright spot on the moon.

SOLUTION From Equation 27.5,

$$\theta = \sin^{-1}\left(\frac{1.22\lambda}{D}\right) = \sin^{-1}\left[\frac{1.22(694.3\times10^{-9}\ \text{m})}{0.20\ \text{m}}\right] = 4.2\times10^{-6}\ \text{rad}$$

From the figure below, we see that $2\theta = \dfrac{d}{R}$.

Therefore, the diameter of the spot on the moon is

$$d = 2\theta R = 2(4.2\times10^{-6}\ \text{rad})(3.77\times10^{8}\ \text{m}) = \boxed{3.2\times10^{3}\ \text{m}}$$

36. ***REASONING AND SOLUTION***

a. Equation 27.6$\left(\theta_{\min} = 1.22\,\lambda / D\right)$ gives the minimum angle $\theta_{\min}$ that two point objects can subtend at an aperture of diameter D and still be resolved. The angle must be measured in radians. For a separation s between the two circles and a distance L between the concentric arrangement and the camera, Equation 8.1 gives the angle in radians as $\theta_{\min} = s / L$. Therefore, we find that

$$\frac{1.22\,\lambda}{D} = \frac{s}{L} \qquad \text{or} \qquad L = \frac{s\,D}{1.22\,\lambda}$$

Since s = 0.040 m – 0.010 m = 0.030 m, we calculate that

$$L = \frac{s\,D}{1.22\,\lambda} = \frac{(0.030\text{ m})\left(12.5\times10^{-3}\text{ m}\right)}{(1.22)\left(555\times10^{-9}\text{ m}\right)} = \boxed{550\text{ m}}$$

b. The calculation here is similar to that in part a, except that the separation s is between one side of a diameter of the small circle and the other side, or s = 0.020 m:

$$L = \frac{s\,D}{1.22\,\lambda} = \frac{(0.020\text{ m})\left(12.5\times10^{-3}\text{ m}\right)}{(1.22)\left(555\times10^{-9}\text{ m}\right)} = \boxed{370\text{ m}}$$

37. **SSM** ***REASONING AND SOLUTION*** According to Equation 27.7, the angles that correspond to the first-order ($m = 1$) maximum for the two wavelengths in question are:

a. for $\lambda = 660 \text{ nm} = 660 \times 10^{-9} \text{ m}$,

$$\theta = \sin^{-1}\left(m\frac{\lambda}{d}\right) = \sin^{-1}\left[(1)\left(\frac{660 \times 10^{-9} \text{ m}}{1.1 \times 10^{-6} \text{ m}}\right)\right] = \boxed{37°}$$

b. for $\lambda = 410 \text{ nm} = 410 \times 10^{-9} \text{ m}$,

$$\theta = \sin^{-1}\left(m\frac{\lambda}{d}\right) = \sin^{-1}\left[(1)\left(\frac{410 \times 10^{-9} \text{ m}}{1.1 \times 10^{-6} \text{ m}}\right)\right] = \boxed{22°}$$

38. ***REASONING*** For a diffraction grating the angle θ that locates a bright fringe can be found using Equation 27.7, $\sin\theta = m\lambda/d$, where λ is the wavelength, d is the separation between the grating slits, and the order m is $m = 0, 1, 2, 3, \ldots$. By applying this relation to both cases, we will be able to determine the unknown wavelength, because the order and the slit separation are the same for both.

SOLUTION We will use λ_1 and λ_2 to denote the known and unknown wavelengths, respectively. The corresponding angles are θ_1 and θ_2. Applying Equation 27.7 to the two cases, we have

$$\sin\theta_1 = \frac{m\lambda_1}{d} \quad \text{and} \quad \sin\theta_2 = \frac{m\lambda_2}{d}$$

Dividing the expression for case 2 by the expression for case 1 gives

$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{m\lambda_2/d}{m\lambda_1/d} = \frac{\lambda_2}{\lambda_1}$$

Solving for λ_2, we find

$$\lambda_2 = \lambda_1 \frac{\sin\theta_2}{\sin\theta_1} = (420\text{ nm})\frac{\sin 41°}{\sin 26°} = \boxed{630\text{ nm}}$$

39. ***REASONING AND SOLUTION*** We know that

$$\tan\theta = y/L = (8.94 \times 10^{-2}\ \text{m})/(0.625\ \text{m}) \quad \text{so} \quad \theta = 8.14°$$

Therefore,

$$\lambda = d \sin\theta = (4.17 \times 10^{-6}\ \text{m}) \sin 8.14° = \boxed{5.90 \times 10^{-7}\ \text{m}}$$

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40. ***REASONING AND SOLUTION*** We have

$$d = \frac{3\lambda}{\sin\theta} = \frac{3(621 \times 10^{-9}\ \text{m})}{\sin 18.0°} = 6.03 \times 10^{-6}\ \text{m}$$

The number of lines per centimeter is

$$1/d = 1/(6.03 \times 10^{-4}\ \text{cm}) = \boxed{1660\ \text{lines/cm}}$$

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41. SSM ***REASONING AND SOLUTION*** The geometry of the situation is shown below.

From the geometry, we have

$$\tan\theta = \frac{y}{L} = \frac{0.60\text{ mm}}{3.0\text{ mm}} = 0.20 \qquad \text{or} \qquad \theta = 11.3°$$

Then, solving Equation 27.7 with $m = 1$ for the separation d between the slits, we have

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)\left(780\times10^{-9}\text{ m}\right)}{\sin 11.3°} = \boxed{4.0\times10^{-6}\text{ m}}$$

42. ***REASONING AND SOLUTION*** For the first order, $\lambda/d = \sin 16.0° = 0.2756$. For the third order

$$\theta = \sin^{-1}\left(3\frac{\lambda}{d}\right) = \sin^{-1}\left[3(0.2756)\right] = \boxed{55.8°}$$

43. SSM ***REASONING*** The angle θ that locates the first-order maximum produced by a grating with 3300 lines/cm is given by Equation 27.7, $\sin\theta = m\lambda / d$, with the order of the fringes given by $m = 0, 1, 2, 3, \ldots$ Any two of the diffraction patterns will overlap when their angular positions are the same.

SOLUTION Since the grating has 3300 lines/cm, we have

$$d = \frac{1}{3300 \text{ lines/cm}} = 3.0 \times 10^{-4} \text{ cm} = 3.0 \times 10^{-6} \text{ m}$$

a. In first order, $m = 1$; therefore, for violet light,

$$\theta = \sin^{-1}\left(m\frac{\lambda}{d}\right) = \sin^{-1}\left[(1)\left(\frac{410 \times 10^{-9} \text{ m}}{3.0 \times 10^{-6} \text{ m}}\right)\right] = \boxed{7.9°}$$

Similarly for red light,

$$\theta = \sin^{-1}\left(m\frac{\lambda}{d}\right) = \sin^{-1}\left[(1)\left(\frac{660 \times 10^{-9} \text{ m}}{3.0 \times 10^{-6} \text{ m}}\right)\right] = \boxed{13°}$$

b. Repeating the calculation for the second order maximum ($m = 2$), we find that

($m = 2$)	
for violet	$\theta = 16°$
for red	$\theta = 26°$

c. Repeating the calculation for the third order maximum ($m = 3$), we find that

($m = 3$)	
for violet	$\theta = 24°$
for red	$\theta = 41°$

d. Comparisons of the values for θ calculated in parts (a), (b) and (c) show that the $\boxed{\text{second and third orders overlap}}$.

44. ***REASONING*** The maximum number of bright fringes that can be seen on either side of the central maximum occurs when the angle is $\theta = 90.0°$. Equation 27.7 gives the relationship between the angle and the wavelength as

$$\underbrace{\sin 90.0°}_{=1} = m\frac{\lambda}{d}$$

Because there are only three bright fringes, $m = 3$. We can use the equation above to find the separation d between the slits, and then the number of lines per centimeter for the grating.

SOLUTION The separation between the slits is

$$d = \frac{m\lambda}{\sin\theta} = \frac{3\left(510 \times 10^{-9}\ \text{m}\right)}{\sin 90.0°} = 1.53 \times 10^{-6}\ \text{m}$$

The number of lines per centimeter is

$$\frac{1}{d} = \frac{1}{1.53 \times 10^{-4}\ \text{cm}} = \boxed{6540\ \text{lines/cm}}$$

45. ***REASONING AND SOLUTION***

a. The angular positions of the specified orders are equal, so $\lambda/d_A = 2\lambda/d_B$, or

$$\frac{d_B}{d_A} = \boxed{2}$$

b. Similarly, we have for the m_A order of grating A and the m_B order of grating B that $m_A\lambda/d_A = m_B\lambda/d_B$, so $m_A = m_B/2$.

The next highest orders which overlap are

$$m_B = \boxed{4},\ m_A = \boxed{2} \qquad \text{and} \qquad m_B = \boxed{6},\ m_A = \boxed{3}$$

46. ***REASONING AND SOLUTION*** The interference minima are given by Equation 27.2 with $m = 0$, since we are asked for the smallest angle:

$$\theta = \sin^{-1}[\lambda/(2d)], \quad \text{where} \quad \lambda = v/f = (343 \text{ m/s})/(80.0 \text{ Hz}) = 4.29 \text{ m}$$

Now

$$\theta = \sin^{-1}[(4.29 \text{ m})/(14.0 \text{ m})] = \boxed{17.8°}$$

47. **SSM** ***REASONING*** The slit separation d is given by Equation 27.1 with $m = 1$; namely $d = \lambda / \sin\theta$. As shown in Example 1 in the text, the angle θ is given by $\theta = \tan^{-1}(y / L)$.

SOLUTION The angle θ is

$$\theta = \tan^{-1}\left(\frac{0.037\ \text{m}}{4.5\ \text{m}}\right) = 0.47°$$

Therefore, the slit width d is

$$d = \frac{\lambda}{\sin\theta} = \frac{490 \times 10^{-9}\ \text{m}}{\sin 0.47°} = \boxed{6.0 \times 10^{-5}\ \text{m}}$$

48. ***REASONING AND SOLUTION*** The position of the first minimum is $y = L \tan \theta$ (see Example 6), so the width of the central maximum is $2y = 2L \tan \theta$. We know that $\theta = \sin^{-1}(\lambda/W)$. For $\lambda = 635$ nm and $W = 4.30 \times 10^{-5}$ m, $\theta = 0.846°$ and

$$2y = 2(1.32 \text{ m})(\tan 0.846°) = \boxed{0.0390 \text{ m}}$$

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49. ***REASONING AND SOLUTION*** This problem is analogous to Example 3 in the text, where the condition for destructive interference is obtained as $2t = m\lambda_{\text{film}}$. For the minimum non-zero thickness, $m = 1$. In addition, the wavelength in the film is given by Equation 27.3: $\lambda_{\text{film}} = \lambda_{\text{vacuum}} / n$. Therefore, green light is removed when the minimum non-zero thickness is

$$t = \frac{m}{2}\lambda_{\text{film}} = \frac{1}{2}\left(\frac{\lambda_{\text{vacuum}}}{n}\right) = \frac{1}{2}\left(\frac{551\text{ nm}}{1.33}\right) = \boxed{207\text{ nm}}$$

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50. ***REASONING AND SOLUTION*** The position of the third dark fringe (m = 2) is given by $\sin\theta = \left(m + \frac{1}{2}\right)\frac{\lambda}{d} = \left(2 + \frac{1}{2}\right)\frac{\lambda}{d}$. The position of the fourth bright fringe (m = 4) is given by $\sin\theta = m\frac{\lambda'}{d} = 4\frac{\lambda'}{d}$. Equating these two expressions and solving for λ' yields

$$\lambda' = \frac{\left(2 + \frac{1}{2}\right)\lambda}{4} = \frac{\left(2 + \frac{1}{2}\right)(645\text{ nm})}{4} = \boxed{403\text{ nm}}$$

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51. **SSM** ***REASONING*** According to Rayleigh's criterion, the two taillights must be separated by a distance s sufficient to subtend an angle $\theta_{\text{min}} \approx 1.22\lambda / D$ at the pupil of the observer's eye. Recalling that this angle must be expressed in radians, we relate θ_{min} to the distances s and L.

SOLUTION The wavelength λ is 660 nm. Therefore, we have from Equation 27.6

$$\theta_{\text{min}} \approx 1.22\frac{\lambda}{D} = 1.22\left(\frac{660\times10^{-9}\ \text{m}}{7.0\times10^{-3}\ \text{m}}\right) = 1.2\times10^{-4}\ \text{rad}$$

According to Equation 8.1, the distance L between the observer and the taillights is

$$L = \frac{s}{\theta_{\text{min}}} = \frac{1.2\ \text{m}}{1.2\times10^{-4}\ \text{rad}} = \boxed{1.0\times10^{4}\ \text{m}}$$

52. ***REASONING AND SOLUTION*** Equation 27.7 gives the angles that define the positions of the principal maxima of a diffraction grating in air as $\sin\theta = m\lambda_{air}/d$. But, the wavelength in air is related to that in a vacuum by $\lambda_{air} = \lambda_{vacuum}/n_{air} = \lambda_{vacuum}/1$. Thus, we have

$$\sin 33° = \frac{m\lambda_{vacuum}}{d} \tag{1}$$

When the diffraction grating is immersed in water, the analogous relation is

$$\sin\theta_{water} = \frac{m\lambda_{water}}{d} = \frac{m\left(\dfrac{\lambda_{vacuum}}{n_{water}}\right)}{d} = \frac{m\left(\dfrac{\lambda_{vacuum}}{1.33}\right)}{d} \tag{2}$$

Dividing Equation (2) by Equation (1) and algebraically simplifying, we get

$$\frac{\sin\theta_{water}}{\sin 33°} = \frac{1}{1.33}$$

Thus,

$$\sin\theta_{water} = \frac{\sin 33°}{1.33} \quad \text{or} \quad \boxed{\theta_{water} = 24°}$$

53. ***REASONING AND SOLUTION***

a. The minimum angular separation of the cells must be $\theta_{min} = 1.22\ \lambda/D$. The separation of the cells is

$$y = L\theta_{min} = 1.22\ L\lambda/D$$

Now $D = f$, the focal length of the lens. Also, $L = f$, so

$$\boxed{y = 1.22\lambda}$$

b. The wavelength must be $\boxed{\text{shorter}}$ to resolve cells that are closer together.

54. ***REASONING AND SOLUTION***

The position of a particular wavelength on the screen is

$$y = L \tan \theta \qquad \text{where} \qquad \theta = \sin^{-1}(m\lambda/d)$$

a. In the first order ($m = 1$), $\theta = \sin^{-1}(\lambda/d)$. For $\lambda = 410$ nm,

$$\theta = \sin^{-1}(410 \times 10^{-9}\ \text{m})/(2.2 \times 10^{-6}\ \text{m}) = 11°$$

and

$$y = (3.2\ \text{m}) \tan 11° = 0.62\ \text{m}$$

For $\lambda = 660$ nm, we find similarly that $\theta = 17°$ and $y = 0.98$ m.

The width of the spectrum is 0.98 m – 0.62 m = $\boxed{0.36\ \text{m}}$

b. In the second order ($m = 2$), $\theta = \sin^{-1}(2\lambda/d)$. For $\lambda = 410$ nm, we find that $\theta = 22°$ and $y = 1.3$ m.

For $\lambda = 660$ nm, we find that $\theta = 37°$ and $y = 2.4$ m.

The width of the spectrum is 2.4 m – 1.3 m = $\boxed{1.1\ \text{m}}$

55. **SSM** ***REASONING*** For a diffraction grating, the angular position θ of a principal maximum on the screen is given by Equation 27.7 as $\sin\theta = m\lambda / d$ with $m = 0, 1, 2, 3, \ldots$

SOLUTION When the fourth-order principal maximum of light A exactly overlaps the third-order principal maximum of light B, we have

$$\sin\theta_A = \sin\theta_B$$

$$\frac{4\lambda_A}{d} = \frac{3\lambda_B}{d} \qquad \text{or} \qquad \frac{\lambda_A}{\lambda_B} = \boxed{\frac{3}{4}}$$

56. ***REASONING AND SOLUTION***

a. The condition for constructive interference is $2t = \frac{1}{2}\lambda_{\text{film}} = \frac{1}{2}\left(\frac{\lambda_{\text{vacuum}}}{n_{\text{gasoline}}}\right)$. Therefore,

$$t = \frac{\lambda_{\text{vacuum}}}{4n_{\text{gasoline}}} = \frac{580 \times 10^{-9}\ \text{m}}{4(1.40)} = \boxed{1.0 \times 10^{-7}\ \text{m}}$$

b. Now, the condition for constructive interference is $2t = \lambda_{\text{film}} = \lambda_{\text{vacuum}}/n_{\text{gasoline}}$, so

$$t = \frac{\lambda_{\text{vacuum}}}{2n_{\text{gasoline}}} = \frac{580 \times 10^{-9}\ \text{m}}{2(1.40)} = \boxed{2.1 \times 10^{-7}\ \text{m}}$$

57. ***REASONING AND SOLUTION*** The last maximum formed by the grating corresponds to $\theta = 90.0°$, so that

$$m = \frac{d}{\lambda}\sin 90.0° = \frac{d}{\lambda} = \frac{1.78 \times 10^{-6}\ \text{m}}{471 \times 10^{-9}\ \text{m}} = 3.78$$

where we have used $d = \dfrac{1}{5620\ \text{lines/cm}} = 1.78 \times 10^{-4}\ \text{cm} = 1.78 \times 10^{-6}\ \text{m}$.

Thus, the last maximum formed by the grating is $m = 3$. This maximum lies at

$$\theta = \sin^{-1}\left(m\frac{\lambda}{d}\right) = \sin^{-1}\left[3\left(\frac{1}{3.78}\right)\right] = 52.5°$$

The distance from the center of the screen to the $m = 3$ maximum is

$$y = (0.750\ \text{m})\tan 52.5° = 0.977\ \text{m}$$

The screen must have a width of $2y = \boxed{1.95\ \text{m}}$

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58. ***REASONING AND SOLUTION*** The condition for destructive interference is $2t = m\lambda$. The thickness t of the "air wedge" at the 100th dark fringe is related to the radius R of the fringe and the radius of curvature r of the lens in the following way.

If s is the length of the circular arc along the lens from the center to the fringe, then the angle subtended by this arc is $\theta = s/r$. Since r is large, the arc is almost straight and is the hypotenuse of a right triangle in the air wedge of sides t and R, and angle $(1/2)\theta$ opposite t.

In the right triangle

$$(1/2)\theta \approx \tan (1/2)\theta = t/R \quad \text{so} \quad R = 2tr/s$$

Now if $(1/2)\theta$ is very small, then $s \approx R$, so

$$R^2 = 2tr = (100)\lambda r = (100)(654 \times 10^{-9}\ \text{m})(10.0\ \text{m})$$

so

$$\boxed{R = 0.0256\ \text{m}}$$

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1. **SSM** ***REASONINIG*** Since the "police car" is moving relative to the earth observer, the earth observer measures a greater time interval Δt between flashes. Since both the proper time Δt_0 (as observed by the officer) and the dilated time Δt (as observed by the person on earth) are known, the speed of the "police car" relative to the observer can be determined from the time dilation relation, Equation 28.1.

SOLUTION According to Equation 28.1, the dilated time interval between flashes is $\Delta t = \Delta t_0 / \sqrt{1-(v^2/c^2)}$, where Δt_0 is the proper time. Solving for the speed v, we find

$$v = c\sqrt{1-\left(\frac{\Delta t_0}{\Delta t}\right)^2} = (3.0\times10^8 \text{ m/s})\sqrt{1-\left(\frac{1.5 \text{ s}}{2.5 \text{ s}}\right)^2} = \boxed{2.4\times10^8 \text{ m/s}}$$

2. ***REASONINIG*** The total time for the trip is one year. This time is the proper time interval Δt_o, because it is measured by an observer (the astronaut) who is at rest relative to the beginning and ending events (the times when the trip started and ended) and who sees them at the same location in spacecraft. On the other hand, the astronaut measures the clocks on earth to run at the dilated time interval Δt, which is the time interval of one hundred years**.** The relation between the two time intervals is given by Equation 28.1, which can be used to find the speed of the spacecraft.

SOLUTION The dilated time interval Δt is related to the proper time interval Δt_o by $\Delta t = \Delta t_o / \sqrt{1-\left(v^2/c^2\right)}$. Solving this equation for the speed v of the spacecraft yields

$$v = c\sqrt{1-\left(\frac{\Delta t_o}{\Delta t}\right)^2} = c\sqrt{1-\left(\frac{1\text{ yr}}{100\text{ yr}}\right)^2} = \boxed{0.999\,95c} \tag{28.1}$$

3. **SSM** ***REASONINIG AND SOLUTION*** The period Δt_0 of the rotating radar antenna is given by Equation 10.4 as

$$\Delta t_0 = \frac{2\pi}{\omega_0} = \frac{2\pi}{0.25 \text{ rad/s}} = 25 \text{ s}$$

This period, or time interval, is a proper time interval, because an earth-based observer measures the beginning and ending of each revolution at the same place. An observer moving relative to the earth measures a dilated time interval given by Equation 28.1 as

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{25 \text{ s}}{\sqrt{1-\frac{(0.80\,c)^2}{c^2}}} = 42 \text{ s}$$

The angular speed ω of the rotating antenna as measured by the moving observer is, therefore,

$$\omega = \frac{2\pi}{\Delta t} = \frac{2\pi}{42 \text{ s}} = \boxed{0.15 \text{ rad/s}}$$

4. ***REASONINIG AND SOLUTION*** The time interval for 8.0 breaths in your rest frame is $\Delta t_0 = 1.0$ min. The same time interval as measured by someone on the earth is given by Equation 28.1:

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2/c^2}} = \frac{1.0 \text{ min}}{\sqrt{1-(0.975c/c)^2}} = \frac{1.0 \text{ min}}{\sqrt{1-(0.975)^2}} = 4.5 \text{ min}$$

Therefore, your breathing rate as measured by an earth-based observer is

$$\frac{8.0 \text{ breaths}}{4.5 \text{ min}} = \boxed{1.8 \text{ breaths per min}}$$

5. ***REASONINIG AND SOLUTION*** According to Equations 10.4 and 10.11, the period Δt_0 of oscillation on earth is

$$\Delta t_0 = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{5.00\text{ kg}}{49.3\text{ N/m}}} = 2.00\text{ s}$$

Equation 28.1 can be used to find the dilated period Δt measured by the moving observer

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{2.00\text{ s}}{\sqrt{1-\left(\frac{2.80\times10^8\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2}} = \boxed{5.57\text{ s}}$$

6. ***REASONINIG AND SOLUTION*** Using Equation 28.1 with

$$\sqrt{1-\left(\frac{v}{c}\right)^2} \approx 1-\tfrac{1}{2}\left(\frac{v}{c}\right)^2$$

we have

$$\Delta t_0 = \Delta t\left[1-\tfrac{1}{2}\left(\frac{v}{c}\right)^2\right]$$

The difference between Δt and Δt_0 is

$$\text{Time difference} = \Delta t - \Delta t_0 = \Delta t\left(\tfrac{1}{2}\right)\left(\frac{v}{c}\right)^2$$

$$\text{Time difference} = (15\text{ days})\left(\frac{8.64\times10^4\text{ s}}{1\text{ day}}\right)\left(\tfrac{1}{2}\right)\left(\frac{7800\text{ m/s}}{3.00\times10^8\text{ m/s}}\right)^2 = \boxed{4.4\times10^{-4}\text{ s}}$$

7. ***REASONINIG AND SOLUTION*** The proper time is the time it takes for the bacteria to double its number, i.e., $\Delta t_0 = 24.0$ hours. For the earth based sample to grow to 256 bacteria, it would take 8 days ($2^n = 256$ or $n = 8$). The "doubling time" for the space culture would be

$$\Delta t = \frac{\Delta t_0}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{24.0\text{ h}}{\sqrt{1-\left(\frac{0.866\,c}{c}\right)^2}} = 48.0\text{ h} \quad \text{or} \quad 2\text{ days}$$

In eight earth days, the space bacteria would undergo $n' = \left(\frac{1}{2}\right)8 = 4$ "doublings". The number of space bacteria is

$$\text{Number of Space Bacteria} = 2^{n'} = 2^4 = \boxed{16}$$

8. ***REASONINIG AND SOLUTION*** The proper length is $L_0 = 4.2 \times 10^6$ m, so the length measured by the UFO voyagers traveling at $v = 0.70c$ is, using Equation 28.2,

$$L = L_0\sqrt{1-\left(\frac{v}{c}\right)^2} = \left(4.2\times10^6\ \text{m}\right)\sqrt{1-\left(\frac{0.70\,c}{c}\right)^2} = \boxed{3.0\times10^6\ \text{m}}$$

9. **SSM** ***REASONINIG AND SOLUTION*** The length L_0 that the person measures for the UFO when it lands is the proper length, since the UFO is at rest with respect to the person. Therefore, from Equation 28.2 we have

$$L_0 = \frac{L}{\sqrt{1-\frac{v^2}{c^2}}} = \frac{230\text{ m}}{\sqrt{1-\frac{(0.90\,c)^2}{c^2}}} = \boxed{530\text{ m}}$$

10. ***REASONINIG*** The distance between earth and the center of the galaxy is the proper length L_o, because it is the distance measured by an observer who is at rest relative to the earth and the center of the galaxy. A person on board the spaceship is moving with respect to them and measures a contracted length L that is related to the proper length by Equation 28.2 as $L = L_o\sqrt{1-\left(v^2/c^2\right)}$. The contracted distance is also equal to the product of the spaceship's speed v the time interval measured by a person on board the spaceship. This time interval is the proper time interval Δt_0 because the person on board the spaceship measures the beginning and ending events (the times when the trip starts and ends) at the same location relative to a coordinate system fixed to the spaceship. Thus, the contracted distance is also $L = v\Delta t_0$. By setting the two expressions for L equal to each other, we can find the how long the trip will take according to a clock on board the spaceship.

SOLUTION Setting $L = L_o\sqrt{1-\left(v^2/c^2\right)}$ equal to $L = v\Delta t_o$ and solving for the proper time interval Δt_o gives

$$\Delta t_o = \frac{L_o}{v}\sqrt{1-\left(v^2/c^2\right)}$$

$$= \frac{(23\,000\text{ ly})\left(\dfrac{9.47\times10^{15}\text{ m}}{1\text{ ly}}\right)}{0.9990\left(3.00\times10^8\text{ m/s}\right)}\sqrt{1-\left[\frac{(0.9990c)^2}{c^2}\right]}\left(\frac{1\text{ yr}}{3.16\times10^7\text{ s}}\right) = \boxed{1.0\times10^3\text{ yr}}$$

11. **SSM** **WWW** ***REASONINIG AND SOLUTION*** The diameter D of the planet, as measured by a moving spacecraft, is given in terms of the proper diameter D_0 by Equation 28.2. Taking the ratio of the diameter D_A of the planet measured by spaceship A to the diameter D_B measured by spaceship B, we find

$$\frac{D_A}{D_B} = \frac{D_0\sqrt{1-\frac{v_A^{\ 2}}{c^2}}}{D_0\sqrt{1-\frac{v_B^{\ 2}}{c^2}}} = \frac{\sqrt{1-\frac{(0.60\,c)^2}{c^2}}}{\sqrt{1-\frac{(0.80\,c)^2}{c^2}}} = \boxed{1.3}$$

12. ***REASONINIG AND SOLUTION*** Since the spacecraft is moving parallel to the length of the space pad, only the length, but not the width, of the pad is contracted. Taking $L = 1800$ m in Equation 28.2 we can solve for the proper length, i.e.,

$$L_0 = \frac{L}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{1800 \text{ m}}{\sqrt{1-\left(\frac{0.85\,c}{c}\right)^2}} = 3400 \text{ m}$$

The dimensions of the landing pad, according to the engineer who built it, are

$$\text{Dimensions} = \boxed{3400 \text{ m} \times 1500 \text{ m}}$$

13. ***REASONINIG AND SOLUTION*** Since the proper length is the same in either case, we have $(L_0)_A = (L_0)_B$, or using Equation 28.2,

$$L_A \Big/ \sqrt{1 - v_A^2 / c^2} = L_B \Big/ \sqrt{1 - v_B^2 / c^2}$$

Therefore,

$$L_B = L_A \sqrt{\frac{1 - v_B^2 / c^2}{1 - v_A^2 / c^2}} = L_A \sqrt{\frac{1 - (0.80c / c)^2}{1 - (0.60c / c)^2}}$$

or

$$L_B = (8.0 \text{ light years}) \sqrt{\frac{1 - (0.80)^2}{1 - (0.60)^2}} = \boxed{6.0 \text{ light years}}$$

14. ***REASONINIG AND SOLUTION*** The side x will have its length contracted as viewed from the rocket. Taking the proper length to be $L_0 = x$, we can find the dilated length $L = x'$ from Equation 28.2:

$$x' = x\sqrt{1-\left(\frac{v}{c}\right)^2} = x\sqrt{1-\left(\frac{0.730\,c}{c}\right)^2} = 0.683\,x$$

The new angle, can be found using tan $\theta' = y/x' = y/(0.683\,x)$ = tan 30.0°/(0.683),

$$\boxed{\theta' = 40.2°}$$

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15. **SSM** **WWW** ***REASONINIG*** Only the sides of the rectangle that lie in the direction of motion will experience length contraction. In order to make the rectangle look like a square, each side must have a length of $L = 2.0$ m. Thus, we move along the long side, taking the proper length to be $L_0 = 3.0$ m. We can solve for the speed using Equation 28.2. Then, with this speed, we can use the relation for length contraction to find L for the short side as we move along it.

SOLUTION From Equation 28.2, $L = L_0\sqrt{1-(v^2/c^2)}$, we find that

$$v = c\sqrt{1-\left(\frac{L}{L_0}\right)^2} = c\sqrt{1-\left(\frac{2.0\text{ m}}{3.0\text{ m}}\right)^2} = 0.75\,c$$

Moving at this speed along the short side, we take $L_0 = 2.0$ m and find L:

$$L = L_0\sqrt{1-\left(\frac{v}{c}\right)^2} = (2.0\text{ m})\sqrt{1-\left(\frac{0.75\,c}{c}\right)^2} = 1.3\text{ m}$$

The observed dimensions of the rectangle are, therefore, $\boxed{3.0\text{ m} \times 1.3\text{ m}}$, since the long side is not contracted due to motion along the short side.

16. ***REASONINIG AND SOLUTION***

a. The proper distance is L_0 = 12.0 light years (12.0 ly). First we determine the distance traveled as viewed by each twin:

[Twin A] $$L_A = L_0\sqrt{1-\left(\frac{v_A}{c}\right)^2} = (12.0\text{ ly})\sqrt{1-\left(\frac{0.900c}{c}\right)^2} = 5.23\text{ ly}$$

[Twin B] $$L_B = L_0\sqrt{1-\left(\frac{v_B}{c}\right)^2} = (12.0\text{ ly})\sqrt{1-\left(\frac{0.500c}{c}\right)^2} = 10.4\text{ ly}$$

Then, we determine the time to reach the planet for each twin:

[Twin A] $t_A = L_A/v_A$ = (5.23 ly)/(0.900c) = 5.81 years

[Twin B] $t_B = L_B/v_B$ = (10.4 ly)/(0.500c) = 20.8 years

Next, we determine the age of each twin when each arrives at the planet:

[Twin A] 19.0 years + 5.81 years = 24.8 years

[Twin B] 19.0 years + 20.8 years = 39.8 years

However, twin A has to wait for twin B to arrive. As seen from earth,

$$\Delta t_A = (12.0\text{ ly})/(0.900c) = 13.3\text{ y}$$
$$\Delta t_B = (12.0\text{ ly})/(0.500c) = 24.0\text{ y}$$

Therefore, twin A must wait another 24.0 y – 13.3 y = 10.7 y for twin B to arrive. When twin B arrives, twin A has an age of 24.8 y + 10.7 y = 35.5 y. The difference between their ages when they meet is, then,

$$\text{Age difference} = 39.8\text{ y} - 35.5\text{ y} = \boxed{4.3\text{ y}}$$

b. Thus, $\boxed{\text{twin B, traveling at } 0.500\,c,}$ is older.

17. ***REASONINIG AND SOLUTION*** The magnitude of the nonrelativistic momentum of a particle is given by Equation 7.2 as $p_0 = mv$. The magnitude of relativistic momentum of a particle is given by Equation 28.3:

$$p = \frac{mv}{\sqrt{1 - v^2/c^2}} = \frac{p_0}{\sqrt{1 - v^2/c^2}}$$

When the magnitude of the relativistic momentum of a particle is three times the magnitude of its nonrelativistic momentum, we have

$$3p_0 = \frac{p_0}{\sqrt{1 - v^2/c^2}} \qquad \text{or} \qquad \sqrt{1 - v^2/c^2} = \tfrac{1}{3}$$

Squaring and rearranging the last expression gives

$$1 - \frac{v^2}{c^2} = \tfrac{1}{9} \qquad \text{or} \qquad \frac{v^2}{c^2} = 1 - \tfrac{1}{9} = \tfrac{8}{9}$$

Taking the square root, we find

$$v = \sqrt{\tfrac{8}{9}}\, c = 0.943c = \boxed{2.83 \times 10^8 \text{ m/s}}$$

18. ***REASONINIG AND SOLUTION*** According to Equation 28.3, the magnitude of the momentum is

$$p = \frac{mv}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{(1550\text{ kg})(15.0\text{ m/s})}{\sqrt{1-\left(\frac{15.0\text{ m/s}}{25.0\text{ m/s}}\right)^2}} = \boxed{2.91\times10^4\text{ kg}\cdot\text{m/s}}$$

19. $\boxed{\text{SSM}}$ ***REASONINIG AND SOLUTION*** The magnitude of the relativistic momentum p of the rocket is given by Equation 28.3, $p = mv/\sqrt{1-(v^2/c^2)}$. Solving this expression for the speed v of the rocket gives

$$v = \frac{p}{\sqrt{m^2 + \frac{p^2}{c^2}}} = \frac{3.15\times10^{13}\ \text{kg}\cdot\text{m/s}}{\sqrt{(1.40\times10^5\ \text{kg})^2 + \frac{(3.15\times10^{13}\ \text{kg}\cdot\text{m/s})^2}{(3.00\times10^8\ \text{m/s})^2}}} = \boxed{1.80\times10^8\ \text{m/s}}$$

20. ***REASONINIG*** The height of the woman as measured by the observer is given by Equation 28.2 as $h = h_0\sqrt{1-(v/c)^2}$, where h_0 is her proper height. In order to use this equation, we must determine the speed v of the woman relative to the observer. We are given the magnitude of her relativistic momentum, so we can determine v from p.

SOLUTION According to Equation 28.3 $p = mv/\sqrt{1-v^2/c^2}$, so $mv = p\sqrt{1-v^2/c^2}$
Squaring both sides, we have

$$m^2v^2 = p^2(1-v^2/c^2) \qquad \text{or} \qquad m^2v^2 + p^2\frac{v^2}{c^2} = p^2$$

$$v^2\left(m^2 + \frac{p^2}{c^2}\right) = p^2 \qquad \text{or} \qquad v^2 = \frac{p^2}{m^2 + \dfrac{p^2}{c^2}}$$

Solving for v and substituting values, we have

$$v = \frac{p}{\sqrt{m^2 + \dfrac{p^2}{c^2}}} = \frac{2.0\times10^{10}\ \text{kg}\cdot\text{m/s}}{\sqrt{(55\ \text{kg})^2 + \left(\dfrac{2.0\times10^{10}\ \text{kg}\cdot\text{m/s}}{3.00\times10^8\ \text{m/s}}\right)^2}} = 2.3\times10^8\ \text{m/s}$$

Then, the height that the observer measures for the woman is

$$h = h_0\sqrt{1-\left(\frac{v}{c}\right)^2} = (1.6\ \text{m})\sqrt{1-\left(\frac{2.3\times10^8\ \text{m/s}}{3.0\times10^8\ \text{m/s}}\right)^2} = \boxed{1.0\ \text{m}}$$

21. ***REASONINIG AND SOLUTION*** The total momentum of the man/woman system is conserved, since no net external force acts on the system. Therefore, the final total momentum $p_m + p_w$ must equal the initial total momentum, which is zero. As a result, $p_m = -p_w$ where Equation 28.3 must be used for the momenta p_m and p_w. Thus, we find

$$\frac{m_m v_m}{\sqrt{1-(v_m/c)^2}} = -\frac{m_w v_w}{\sqrt{1-(v_w/c)^2}}$$

We know that m_m = 88 kg, m_w = 54 kg, and v_w = +2.5 m/s. Remember that c has the hypothetical value of 3.0 m/s. Solving for v_m, we find $\boxed{v_m = -2.0 \text{ m/s}}$.

22. ***REASONINIG AND SOLUTION*** The mass equivalent is given by $E_0 = \text{KE} = mc^2$ or

$$m = \frac{\text{KE}}{c^2} = \frac{7.8 \times 10^{-13}\ \text{J}}{\left(3.00 \times 10^{8}\ \text{m/s}\right)^2} = \boxed{8.7 \times 10^{-30}\ \text{kg}} \tag{28.5}$$

23. **SSM** ***REASONINIG*** According to the work-energy theorem, Equation 6.3, the work that must be done on the electron to accelerate it from rest to a speed of $0.990c$ is equal to the kinetic energy of the electron when it is moving at $0.990c$.

SOLUTION Using Equation 28.6, we find that

$$\mathrm{KE} = mc^2\left(\frac{1}{\sqrt{1-(v^2/c^2)}}-1\right)$$

$$= (9.11\times10^{-31}\ \mathrm{kg})(3.00\times10^{8}\ \mathrm{m/s})^2\left(\frac{1}{\sqrt{1-(0.990\,c)^2/c^2}}-1\right) = \boxed{5.0\times10^{-13}\ \mathrm{J}}$$

24. ***REASONINIG*** The mass m of the aspirin is related to its rest energy E_0 by Equation 28.5, $E_0 = mc^2$. Since it requires 1.1×10^8 J to operate the car for twenty miles, we can calculate the number of miles that the car can go on the energy that is equivalent to the mass of one tablet.

SOLUTION The number N of miles the car can go on one aspirin tablet is

$$N = \frac{E_0}{\left(1.1\times10^8\ \text{J}\right)/\left(20.0\ \text{mi}\right)} = \frac{mc^2}{\left(1.1\times10^8\ \text{J}\right)/\left(20.0\ \text{mi}\right)}$$

$$= \frac{\left(325\times10^{-3}\ \text{kg}\right)\left(3.0\times10^8\ \text{m/s}\right)^2}{\left(1.1\times10^8\ \text{J}\right)/\left(20.0\ \text{mi}\right)} = \boxed{5.3\times10^9\ \text{mi}}$$

25. ***REASONINIG AND SOLUTION*** The energy E_0 produced in one year is the product of the power P generated and the time t, $E_0 = Pt$. This energy is equivalent to an amount of mass m given by Equation 28.5 as $E_0 = mc^2$. Thus, we have that

$$mc^2 = Pt \quad \text{or} \quad m = \frac{Pt}{c^2}$$

The mass of nuclear fuel consumed in one year (3.15×10^7 s) is

$$m = \frac{Pt}{c^2} = \frac{\left(3.0 \times 10^9 \text{ W}\right)\left(3.15 \times 10^7 \text{ s}\right)}{\left(3.00 \times 10^8 \text{ m/s}\right)^2} = \boxed{1.1 \text{ kg}}$$

26. ***REASONINIG AND SOLUTION***

a. The energy Q needed to heat the water is

$$Q = cm\,\Delta T = [4186\ \text{J/(kg·C°)}](4.0\ \text{kg})(60.0\ °\text{C} - 20.0\ °\text{C}) = \boxed{6.7\times10^{5}\,\text{J}} \qquad (12.4)$$

b. Using $\Delta E_0 = (\Delta m)c^2$, we have

$$\Delta m = \Delta E_0/c^2 = (6.7 \times 10^5\ \text{J})/(3.00 \times 10^8\ \text{m/s})^2 = \boxed{7.4\times10^{-12}\ \text{kg}} \qquad (28.5)$$

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27. $\boxed{\text{SSM}}$ ***REASONINIG AND SOLUTION***

a. In Section 28.6 it is shown that when the speed of a particle is $0.01c$ (or less), the relativistic kinetic energy becomes nearly equal to the nonrelativistic kinetic energy. Since the speed of the particle here is $0.001c$, the ratio of the relativistic kinetic energy to the nonrelativistic kinetic energy is $\boxed{1.0}$.

b. Taking the ratio of the relativistic kinetic energy, Equation 28.6, to the nonrelativistic kinetic energy, $\frac{1}{2}mv^2$, we find that

$$\frac{mc^2\left(\dfrac{1}{\sqrt{1-(v^2/c^2)}}-1\right)}{\frac{1}{2}mv^2}=2\left(\frac{c}{v}\right)^2\left(\frac{1}{\sqrt{1-(v^2/c^2)}}-1\right)$$

$$=2\left(\frac{c}{0.970\,c}\right)^2\left(\frac{1}{\sqrt{1-(0.970\,c)^2/c^2}}-1\right)=\boxed{6.6}$$

28. ***REASONINIG AND SOLUTION***

a. The kinetic energy is provided by the work done to accelerate the electron. We have

$$\text{KE} = W = qV = (1.6 \times 10^{-19}\ \text{C})(2.40 \times 10^{7}\ \text{V}) = \boxed{3.8\times10^{-12}\ \text{J}} \tag{19.4}$$

b. We need to calculate the speed of the electron. From Equation 28.6 we find that

$$v = c\sqrt{1 - \frac{1}{\left(\frac{\text{KE}}{mc^2} + 1\right)^2}} = \boxed{0.9998c}$$

29. ***REASONINIG*** The relativistic momentum is given by Equation 28.3 as $p = mv/\sqrt{1-v^2/c^2}$, while the total energy of an object is given by Equation 28.4 as $E = mc^2/\sqrt{1-v^2/c^2}$. We wish to use these expressions to show that $E^2 = p^2c^2 + (mc^2)^2$.

SOLUTION We begin by squaring both sides of Equation 28.4 to obtain

$$E^2 = \frac{(mc^2)^2}{1-v^2/c^2} = (mc^2)^2 \frac{1}{1-v^2/c^2}$$

We then add zero to the numerator of the right hand side in the form $0 = -v^2/c^2 + v^2/c^2$. We next regroup terms and simplify algebraically:

$$E^2 = (mc^2)^2\left(\frac{1-v^2/c^2+v^2/c^2}{1-v^2/c^2}\right) = (mc^2)^2\left(\frac{1-v^2/c^2}{1-v^2/c^2}\right) + \left[\frac{(mc^2)^2(v^2/c^2)}{1-v^2/c^2}\right]$$

$$E^2 = (mc^2)^2 + \left(\frac{m^2c^2v^2}{1-v^2/c^2}\right) = (mc^2)^2 + \frac{(mv)^2}{1-v^2/c^2}c^2$$

Rearranging and substituting $p = mv/\sqrt{1-v^2/c^2}$ from Equation 28.3 show that this is equal to

$$E^2 = \left(\frac{mv}{\sqrt{1-v^2/c^2}}\right)^2 c^2 + (mc^2)^2 = p^2c^2 + (mc^2)^2$$

Therefore, we have shown that $\boxed{E^2 = p^2c^2 + (mc^2)^2}$.

30. ***REASONINIG*** Let's define the following relative velocities:

v_{AB} = velocity of galaxy **A** relative to galaxy **B**
v_{AE} = velocity of galaxy **A** relative to **E**arth
v_{EB} = velocity of **E**arth relative to galaxy **B**

These three velocities are related by the velocity-addition formula, Equation 28.7:

$$v_{AB} = \frac{v_{AE} + v_{EB}}{1 + \frac{v_{AE}v_{EB}}{c^2}}$$

Let's assume that galaxy A is moving to the right, along the $+x$ direction, so its velocity relative to earth is $v_{AE} = +0.75c$. Galaxy B is moving along the $-x$ axis with a speed of $0.55c$ relative to earth, so its velocity is $v_{BE} = -0.55c$. According to the velocity-addition formula, we need to know v_{EB}, not v_{BE}. However, they are related by $v_{EB} = -v_{BE}$, so that $v_{EB} = -(-0.55c) = +0.55c$.

SOLUTION The velocity of galaxy A relative to galaxy B is

$$v_{AB} = \frac{v_{AE} + v_{EB}}{1 + \frac{v_{AE}v_{EB}}{c^2}} = \frac{+0.75c + 0.55c}{1 + \frac{(+0.75c)(+0.55c)}{c^2}} = +0.920c$$

The speed of galaxy A relative to galaxy B is $\boxed{0.920c}$.

31. **SSM** ***REASONINIG*** Let's define the following relative velocities, assuming that the spaceship and exploration vehicle are moving in the positive direction.

v_{ES} = velocity of **E**xploration vehicle relative to the **S**paceship.
v_{EO} = velocity of **E**xploration vehicle relative to an **O**bserver on earth = $+0.70c$
v_{SO} = velocity of **S**paceship relative to an **O**bserver on earth = $+0.50c$

The velocity v_{ES} can be determined from the velocity-addition formula, Equation 28.7:

$$v_{ES} = \frac{v_{EO} + v_{OS}}{1 + \dfrac{v_{EO}v_{OS}}{c^2}}$$

The velocity v_{OS} of the observer on earth relative to the spaceship is not given. However, we know that v_{OS} is the negative of v_{SO}, so $v_{OS} = -v_{SO} = -(+0.50c) = -0.50c$.

SOLUTION The velocity of the exploration vehicle relative to the spaceship is

$$v_{ES} = \frac{v_{EO} + v_{OS}}{1 + \dfrac{v_{EO}v_{OS}}{c^2}} = \frac{+0.70c + (-0.50c)}{1 + \dfrac{(+0.70c)(-0.50c)}{c^2}} = \boxed{+0.31c}$$

32. ***REASONINIG AND SOLUTION*** Let's define the following relative velocities, assuming that the direction away from the earth is the positive direction:

v_{IE} = velocity of **I**ons relative to **E**arth

v_{IS} = velocity of **I**ons relative to the **S**paceship = $-0.80c$. This velocity is negative because the spaceship is moving away from the earth (in the positive direction), and the ions are emitted from the engine in an opposite direction (the negative direction) relative to the spaceship.

v_{SE} = velocity of **S**paceship relative to **E**arth = $+0.70c$

These velocities are related by the velocity-addition formula, Equation 28.7:

$$v_{IE} = \frac{v_{IS} + v_{SE}}{1 + \frac{v_{IS}v_{SE}}{c^2}} = \frac{-0.80c + 0.70c}{1 + \frac{(-0.80c)(+0.70c)}{c^2}} = \boxed{-0.23c}$$

33. **SSM** **WWW** ***REASONINIG*** Since the crew is initially at rest relative to the escape pod, the length of 45 m is the proper length L_0 of the pod. The length of the escape pod as determined by an observer on earth can be obtained from the relation for length contraction given by Equation 28.2, $L = L_0\sqrt{1-\left(v_{PE}^2/c^2\right)}$. The quantity v_{PE} is the speed of the escape pod relative to the earth, which can be found from the velocity-addition formula, Equation 28.7. The following are the relative velocities, assuming that the direction away from the earth is the positive direction:

v_{PE} = velocity of the escape **P**od relative to **E**arth.

v_{PR} = velocity of escape **P**od relative to the **R**ocket = $-0.55c$. This velocity is negative because the rocket is moving away from the earth (in the positive direction), and the escape pod is moving in an opposite direction (the negative direction) relative to the rocket.

v_{RE} = velocity of **R**ocket relative to **E**arth = $+0.75c$

These velocities are related by the velocity-addition formula, Equation 28.7.

SOLUTION The relative velocity of the escape pod relative to the earth is

$$v_{PE} = \frac{v_{PR}+v_{RE}}{1+\dfrac{v_{PR}v_{RE}}{c^2}} = \frac{-0.55c+0.75c}{1+\dfrac{(-0.55c)(+0.75c)}{c^2}} = +0.34c$$

The speed of the pod relative to the earth is $0.34c$. The length of the pod as determined by an observer on earth is

$$L = L_0\sqrt{1-\frac{v_{PE}^2}{c^2}} = (45\text{ m})\sqrt{1-\frac{(0.34c)^2}{c^2}} = \boxed{42\text{ m}}$$

34. ***REASONINIG*** We define the following relative velocities, assuming that the rocket approaching the earth from the right is traveling in the positive direction:

v_{RL} = velocity of the **R**ight rocket relative to the **L**eft rocket
v_{RE} = velocity of the **R**ight rocket relative to the person on **E**arth = $+0.75c$
v_{LE} = velocity of the **L**eft rocket relative to the person on **E**arth = $-0.65c$

The velocity v_{RL} can be found from the velocity-addition formula, Equation 28.7:

$$v_{RL} = \frac{v_{RE} + v_{EL}}{1 + \frac{v_{RE}v_{EL}}{c^2}}$$

The velocity v_{RE} is given, but v_{EL}, the velocity of the earth relative to the left rocket, is not. However, we know that v_{EL} is the negative of v_{LE}, so $v_{EL} = -v_{LE} = -(-0.65c) = +0.65c$.

SOLUTION The velocity of the right rocket relative to the left rocket is

$$v_{RL} = \frac{v_{RE} + v_{EL}}{1 + \frac{v_{RE}v_{EL}}{c^2}} = \frac{+0.75c + 0.65c}{1 + \frac{(+0.75c)(+0.65c)}{c^2}} = +0.94c$$

The relative speed is $\boxed{0.94c}$.

35. ***REASONINIG AND SOLUTION***

a. The following relative velocities are pertinent to this problem. It is assumed that particle 1 is moving in the positive direction, and, therefore, particle 2 is moving in the negative direction.

$v_{P_1P_2}$ = velocity of particle 1 (**P$_1$**) relative to particle 2 (**P$_2$**)

v_{P_1L} = velocity of particle 1 (**P$_1$**) relative to an observer in the **L**aboratory

$= +2.10 \times 10^8$ m/s

v_{P_2L} = velocity of particle 2 (**P$_2$**) relative to an observer in the **L**aboratory

$= -2.10 \times 10^8$ m/s

The velocity $v_{P_1P_2}$ can be obtained from the velocity-addition formula, Equation 28.7:

$$v_{P_1P_2} = \frac{v_{P_1L} + v_{LP_2}}{1 + \dfrac{v_{P_1L}v_{LP_2}}{c^2}}$$

The velocity v_{P_1L} is given, but v_{LP_2}, the velocity of the laboratory observer relative to particle 2, is not. However, we know that v_{LP_2} is the negative of v_{P_2L}, so $v_{LP_2} = -v_{P_2L}$ $= -\left(-2.10 \times 10^8 \text{ m/s}\right) = +2.10 \times 10^8 \text{ m/s}$.

SOLUTION The velocity of the right rocket relative to the left rocket is

$$v_{P_1P_2} = \frac{v_{P_1L} + v_{LP_2}}{1 + \dfrac{v_{P_1L}v_{LP_2}}{c^2}} = \frac{+2.10 \times 10^8 \text{ m/s} + 2.10 \times 10^8 \text{ m/s}}{1 + \dfrac{\left(+2.10 \times 10^8 \text{ m/s}\right)\left(+2.10 \times 10^8 \text{ m/s}\right)}{c^2}} = \boxed{+2.82 \times 10^8 \text{ m/s}}$$

b. The relativistic momentum is given by Equation 28.3, where the speed is that determined in part a, that is, 0.940*c*. Therefore,

$$p = \frac{mv_{P_1P_2}}{\sqrt{1 - \left(\dfrac{v_{P_1P_2}}{c}\right)^2}} = \frac{\left(2.16 \times 10^{-25} \text{ kg}\right)\left(+2.82 \times 10^8 \text{ m/s}\right)}{\sqrt{1 - \left(\dfrac{2.82 \times 10^8 \text{ m/s}}{3.00 \times 10^8 \text{ m/s}}\right)^2}} = \boxed{1.8 \times 10^{-16} \text{ kg} \cdot \text{m/s}}$$

36. ***REASONINIG AND SOLUTION*** The total energy for each particle is given by

$$E = \frac{mc^2}{\sqrt{1-\left(\frac{v}{c}\right)^2}} = \frac{\left(9.11\times10^{-31}\text{ kg}\right)\left(3.00\times10^{8}\text{ m/s}\right)^2}{\sqrt{1-\left(\frac{0.20\,c}{c}\right)^2}} = 8.4\times10^{-14}\text{ J} \tag{28.4}$$

The annihilation energy is twice this value, so $E' = 2E = \boxed{1.7\times10^{-13}\text{J}}$.

37. **SSM** **WWW** ***REASONINIG*** All standard meter sticks at rest have a length of 1.00 m for observers who are at rest with respect to them. Thus, 1.00 m is the proper length L_0 of the meter stick. When the meter stick moves with speed v relative to an earth-observer, its length $L = 0.500$ m will be a contracted length. Since both L_0 and L are known, v can be found directly from Equation 28.2, $L = L_0\sqrt{1-\left(v^2/c^2\right)}$.

SOLUTION Solving Equation 28.2 for v, we find that

$$v = c\sqrt{1-\left(\frac{L}{L_0}\right)^2} = (3.00\times10^8\ \text{m/s})\sqrt{1-\left(\frac{0.500\ \text{m}}{1.00\ \text{m}}\right)^2} = \boxed{2.60\times10^8\ \text{m/s}}$$

38. ***REASONINIG*** The expression for time dilation is, according to Equation 28.1,

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - v^2 / c^2}}$$

For a given event, it relates the proper time interval Δt_0 to the time interval Δt that would be measured by an observer moving at a speed v relative to the frame of reference in which the event takes place.

We must consider two situations; in the first situation, the Klingon spacecraft has a speed of $0.75c$ with respect to the earth. In the second situation, the craft has a speed of $0.94c$ relative to the earth. We will refer to these two situations as A and B, respectively.

Since the proper time interval always has the same value, $(\Delta t_0)_A = (\Delta t_0)_B$. We can express both sides of this expression using Equation 28.1. The result can be solved for Δt_B.

SOLUTION Use of Equation 28.1 gives

$$\Delta t_A \sqrt{1 - v_A^2 / c^2} = \Delta t_B \sqrt{1 - v_B^2 / c^2}$$

$$\Delta t_B = \Delta t_A \frac{\sqrt{1 - v_A^2 / c^2}}{\sqrt{1 - v_B^2 / c^2}} = \Delta t_A \sqrt{\frac{1 - (v_A / c)^2}{1 - (v_B / c)^2}} = (37.0 \text{ h}) \sqrt{\frac{1 - (0.75c/c)^2}{1 - (0.94c/c)^2}} = \boxed{72 \text{ h}}$$

39. ***REASONINIG AND SOLUTION***

a. According to Equation 7.2, the magnitude of the classical momentum of the spaceship is

$$p_0 = mv = (2.0\times10^7\ \text{kg})(0.85)(3.00\times10^8\ \text{m/s}) = \boxed{5.1\times10^{15}\ \text{kg}\cdot\text{m/s}}$$

b. The magnitude of the relativistic momentum of the ship is given by Equation 28.3:

$$p = \frac{mv}{\sqrt{1-v^2/c^2}} = \frac{p_0}{\sqrt{(1-(0.85c/c)^2}} = \frac{5.1\times10^{15}\,\text{kg}\cdot\text{m/s}}{\sqrt{(1-(0.85)^2}} = \boxed{9.7\times10^{15}\,\text{kg}\cdot\text{m/s}}$$

40. ***REASONINIG AND SOLUTION***

a. According to the second postulate of special relativity, all observers measure the speed of light to be c, regardless of their velocities relative to each other. Therefore, the aliens aboard the hostile spacecraft see the photons of the laser approach $\boxed{\text{at the speed of light, } c}$.

b. To find the velocity of the ions relative to the aliens, we define the relative velocities as follows. (The direction of the ions and the intergalactic cruiser is taken to be the positive direction.)

v_{IS} = velocity of the **I**ons relative to the alien **S**pacecraft

v_{IC} = velocity of the **I**ons relative to the intergalactic **C**ruiser = $+0.950c$

v_{CS} = velocity of the intergalactic **C**ruiser relative to the alien **S**pacecraft = $+0.800c$

These velocities are related by the velocity-addition formula, Equation 28.7. The velocity of the ions relative to the alien spacecraft is:

$$v_{IS} = \frac{v_{IC} + v_{CS}}{1 + \dfrac{v_{IC}v_{CS}}{c^2}} = \frac{+0.950c + 0.800c}{1 + \dfrac{(+0.950c)(+0.800c)}{c^2}} = \boxed{+0.994c}$$

c. The aliens see the laser light (photons) moving with respect to the cruiser at a speed

$$U = 1.000c - 0.800c = \boxed{0.200c}$$

d. The aliens see the ions moving away from the cruiser at speed

$$U' = 0.994c - 0.800c = \boxed{0.194c}$$

41. **SSM** ***REASONINIG*** The total linear momentum of the system is conserved, since no net external force acts on the system. Therefore, the final total momentum $p_1 + p_2$ of the two fragments must equal the initial total momentum, which is zero since the particle is initially at rest. As a result, $p_1 = -p_2$, where Equation 28.3 must be used for the magnitudes of the momenta p_1 and p_2. Thus, we find

$$\frac{m_1 v_1}{\sqrt{1-(v_1^2/c^2)}} = \frac{-m_2 v_2}{\sqrt{1-(v_2^2/c^2)}}$$

SOLUTION Letting fragment 2 be the more-massive fragment, we have that $v_1 = 0.800c$, $m_1 = 1.67\times10^{-27}$ kg, and $m_2 = 5.01\times10^{-27}$ kg. Squaring both sides of the above equation, rearranging terms, substituting the known values for v_1, m_1, and m_2, we find that

$$\frac{v_2^2}{1-(v_2^2/c^2)} = \frac{m_1^2 v_1^2}{m_2^2\left[1-(v_1^2/c^2)\right]} = \frac{(1.67\times10^{-27}\text{ kg})^2(0.800\,c)^2}{(5.01\times10^{-27}\text{ kg})^2\left[1-(0.800\,c/c)^2\right]} = 0.1975\,c^2$$

Therefore,

$$v_2^2 = 0.1975\,c^2\left[1-(v_2^2/c^2)\right] = 0.1975\,c^2 - 0.1975\,v_2^2$$

Solving for v_2 gives

$$v_2 = \pm\sqrt{\frac{0.1975\,c^2}{1.1975}} = \pm0.406\,c$$

We reject the positive root, since then both fragments would be moving in the same direction after the break-up and the system would have a non-zero momentum. According to the principle of conservation of momentum, the total momentum after the break-up must be zero, just as it was before the break-up. The momentum of the system will be zero only if the velocity v_2 is opposite to the velocity v_1. Hence, we chose the negative root and $\boxed{v_2 = -0.406\,c}$.

42. ***REASONINIG AND SOLUTION*** Any change in the energy of a system is equivalent to a change in the mass of the system, according to $\Delta E_0 = (\Delta m)c^2$ (see Section 28.6). Therefore, the change in mass of a system when its energy changes by an amount ΔE_0 is given by

$$\Delta m = \frac{\Delta E_0}{c^2} \tag{1}$$

We want to know how close two stationary electrons have to be positioned so that their total mass is twice what it is when the electrons are very far apart. Let $m_{\text{total}} = 2m_{\text{e}}$ represent the total mass of the system when the electrons are very far apart. Then, when the electrons have been brought together so that their total mass becomes $2m_{\text{total}}$, we have

$$m_{\text{total}} + \Delta m = 2m_{\text{total}} \tag{2}$$

When the two electrons are very far apart, their electric potential energy is zero. When they are brought together so that their separation is *r*, their electric potential energy is

$$\text{EPE} = \frac{ke^2}{r}$$

where *e* is the charge on the electron (see Equations 19.3 and 19.6). Therefore, the change in energy of the system is

$$\Delta E_0 = \text{EPE} - 0 = \frac{ke^2}{r} \tag{3}$$

Combining Equations (1) and (2), we have

$$m_{\text{total}} + \frac{\Delta E_0}{c^2} = 2m_{\text{total}}$$

Substituting the right hand side of Equation (3) for ΔE_0, we have

$$m_{\text{total}} + \frac{1}{c^2}\left(\frac{ke^2}{r}\right) = 2m_{\text{total}}$$

Further simplification and solving for *r* leads to

$$\frac{1}{c^2}\left(\frac{ke^2}{r}\right) = m_{\text{total}}$$

$$r = \frac{ke^2}{m_{\text{total}}c^2} = \frac{ke^2}{2m_{\text{e}}c^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{2(9.11\times10^{-31}\ \text{kg})(3.00\times10^8\ \text{m/s})^2} = \boxed{1.40\times10^{-15}\ \text{m}}$$

43. ***REASONINIG AND SOLUTION*** The length L_M of the man's rocket as measured by the woman is related to the proper length L_{0M} of the man's rocket by the length-contraction formula

$$L_M = L_{0M}\sqrt{1-\frac{v^2}{c^2}}$$

However, the problem states that L_M is also equal to L_{0W}, the proper length of the woman's rocket. Setting the expression above for L_M equal to L_{0W} and solving for L_{0M}, we obtain

$$L_{0M} = \frac{L_{0W}}{\sqrt{1-\frac{v^2}{c^2}}}$$

The length L_W of the woman's rocket as measured by the man is related to the proper length L_{0W} of the woman's rocket by the length-contraction formula

$$L_W = L_{0W}\sqrt{1-\frac{v^2}{c^2}}$$

Dividing this expression for L_W into the one directly above it for L_{0M} gives

$$\frac{L_{0M}}{L_W} = \frac{\frac{L_{0W}}{\sqrt{1-(v/c)^2}}}{L_{0W}\sqrt{1-(v/c)^2}} = \frac{1}{1-(v/c)^2} = \frac{1}{1-(0.940c)^2/c^2} = \boxed{8.59}$$

1. **SSM** ***REASONING*** The energy of the photon is related to its frequency by Equation 29.2, $E = hf$. Equation 16.1, $v = f\lambda$, relates the frequency and the wavelength for any wave.

SOLUTION Combining Equations 29.2 and 16.1, and noting that the speed of a photon is c, the speed of light in a vacuum, we have

$$\lambda = \frac{c}{f} = \frac{c}{(E/h)} = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^{8}\ \text{m/s})}{6.4\times10^{-19}\ \text{J}} = 3.1\times10^{-7}\ \text{m} = \boxed{310\ \text{nm}}$$

2. ***REASONING AND SOLUTION*** The frequency of a photon is $f = E/h$. For $E = 3.3 \times 10^{-16}$ J, $f = 5.0 \times 10^{17}$ Hz, which is in the $\boxed{\text{X-ray}}$ region of the spectrum. For $E = 1.3 \times 10^{-20}$ J, $f = 2.0 \times 10^{13}$ Hz, which is in the $\boxed{\text{infrared}}$ region.

3. ***REASONING AND SOLUTION*** The energy of a single photon is

$$E = hf = (6.63 \times 10^{-34}\ \text{J·s})(98.1 \times 10^{6}\ \text{Hz}) = 6.50 \times 10^{-26}\ \text{J}$$

The number of photons emitted per second is

$$\frac{\text{Power radiated}}{\text{Energy per photon}} = \frac{5.0 \times 10^{4}\ \text{W}}{6.50 \times 10^{-26}\ \text{J}} = \boxed{7.7 \times 10^{29}\ \text{photons/s}}$$

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4. ***REASONING*** The photons of this wave must carry at least enough energy to equal the work function. Then the electrons are ejected with zero kinetic energy. Since the energy of a photon is $E = hf$ according to Equation 29.2, where f is the frequency of the wave, we have that $W_0 = hf$. Equation 16.1 relates the frequency to the wavelength λ according to $f = c/\lambda$, where c is the speed of light. Thus, it follows that $W_0 = hc/\lambda$.

SOLUTION Using Equations 29.2 and 16.1, we find that

$$W_0 = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{485\times10^{-9}\ \text{m}} = 4.10\times10^{-19}\ \text{J}$$

Since 1 eV = 1.60×10^{-19} J, it follows that

$$W_0 = \left(4.10\times10^{-19}\ \text{J}\right)\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.56\ \text{eV}}$$

5. **SSM** ***REASONING*** According to Equation 29.3, the work function W_0 is related to the photon energy hf and the maximum kinetic energy $\mathrm{KE}_{\max}$ by $W_0 = hf - \mathrm{KE}_{\max}$. This expression can be used to find the work function of the metal.

SOLUTION $\mathrm{KE}_{\max}$ is 6.1 eV. The photon energy (in eV) is, according to Equation 29.2,

$$hf = \left(6.63 \times 10^{-34}\ \mathrm{J \cdot s}\right)\left(3.00 \times 10^{15}\ \mathrm{Hz}\right)\left(\frac{1\ \mathrm{eV}}{1.60 \times 10^{-19}\ \mathrm{J}}\right) = 12.4\ \mathrm{eV}$$

The work function is, therefore,

$$W_0 = hf - \mathrm{KE}_{\max} = 12.4\ \mathrm{eV} - 6.1\ \mathrm{ev} = \boxed{6.3\ \mathrm{eV}}$$

6. ***REASONING AND SOLUTION*** In the first case, the energy of the incident photon is given by Equation 29.3 as

$$hf = \mathrm{KE}_{\max} + W_0 = 0.68 \text{ eV} + 2.75 \text{ eV} = 3.43 \text{ eV}$$

In the second case, a rearrangement of Equation 29.3 yields

$$\mathrm{KE}_{\max} = hf - W_0 = 3.43 \text{ eV} - 2.17 \text{ eV} = \boxed{1.26 \text{ eV}}$$

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7. ***REASONING AND SOLUTION*** The energy of an FM photon is given by

$$E_{\text{FM}} = hf = (6.63 \times 10^{-34}\ \text{J·s})(91.9 \times 10^{6}\ \text{Hz}) = 6.09 \times 10^{-26}\ \text{J}$$

The energy of an AM photon is

$$E_{\text{AM}} = (6.63 \times 10^{-34}\ \text{J·s})(665 \times 10^{3}\ \text{Hz}) = 4.41 \times 10^{-28}\ \text{J}$$

The number of AM photons needed to equal the energy of one FM photon is

$$E_{\text{FM}}/E_{\text{AM}} = \boxed{138}$$

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8. ***REASONING AND SOLUTION*** The work function of the material is

$$W_0 = \frac{hc}{\lambda} - \mathrm{KE}_{\max} = \frac{hc}{\lambda} - \tfrac{1}{2}mv_{\max}^2$$

$$= \frac{(6.63\times10^{-34}\ \mathrm{J\cdot s})(3.00\times10^{8}\ \mathrm{m/s})}{238\times10^{-9}\ \mathrm{m}} - \tfrac{1}{2}(9.11\times10^{-31}\ \mathrm{kg})(3.75\times10^{5}\ \mathrm{m/s})^2$$

$$= \left(7.72\times10^{-19}\ \mathrm{J}\right)\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right) = 4.82\ \mathrm{eV}$$

The metal with this work function is $\boxed{\text{gold}}$.

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9. **SSM** ***REASONING AND SOLUTION*** The number of photons per second, N, entering the owl's eye is $N = SA/E$, where S is the intensity of the beam, A is the area of the owl's pupil, and E is the energy of a single photon. Assuming that the owl's pupil is circular, $A = \pi r^2 = \pi\left(\frac{1}{2}d\right)^2$, where d is the diameter of the owl's pupil. Combining Equations 29.2 and 16.1, we have $E = hf = hc/\lambda$. Therefore,

$$N = \frac{SA\lambda}{hc} = \frac{(5.0\times10^{-13}\ \text{W/m}^2)\pi\left[\frac{1}{2}\left(8.5\times10^{-3}\ \text{m}\right)\right]^2(510\times10^{-9}\ \text{m})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.0\times10^{8}\ \text{m/s})} = \boxed{73\ \text{photons/s}}$$

10. ***REASONING AND SOLUTION*** The work function of the material (using $\lambda = 196$ nm) is found from

$$W_0 = hf = hc/\lambda = 1.01 \times 10^{-18}\ \text{J}$$

The maximum kinetic energy of the ejected electron is (using $\lambda = 141$ nm)

$$\text{KE}_{\text{max}} = hf - W_0 = hc/\lambda - W_0 = 3.96 \times 10^{-19}\ \text{J}$$

The speed of the electron is then

$$v = \sqrt{\frac{2\left(\text{KE}_{\text{max}}\right)}{m}} = \sqrt{\frac{2\left(3.96 \times 10^{-19}\ \text{J}\right)}{9.11 \times 10^{-31}\ \text{kg}}} = \boxed{9.32 \times 10^{5}\ \text{m/s}}$$

11. SSM WWW ***REASONING*** The heat required to melt the ice is given by $Q = mL_f$, where m is the mass of the ice and L_f is the latent heat of fusion for water (see Section 12.8). Since, according to Equation 29.2, each photon carries an energy of $E = hf$, the energy content of N photons is $E_{\text{Total}} = Nhf$. According to Equation 16.1, $f = c/\lambda$, so we have

$$E_{\text{Total}} = \frac{Nhc}{\lambda}$$

If we assume that all of the photon energy is used to melt the ice, then, $E_{\text{Total}} = Q$, so that

$$\underbrace{\frac{Nhc}{\lambda}}_{E_{\text{Total}}} = \underbrace{mL_f}_{Q}$$

This expression may be solved for N to determine the required number of photons.

SOLUTION

a. We find that

$$N = \frac{mL_f\lambda}{hc} = \frac{(2.0\text{ kg})(33.5\times10^4\text{ J/kg})(620\times10^{-9}\text{ m})}{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^8\text{ m/s})} = \boxed{2.1\times10^{24}\text{ photons}}$$

b. The number N' of molecules in 2.0-kg of water is

$$N' = (2.0\text{ kg})\left(\frac{1\text{ mol}}{18\times10^{-3}\text{ kg}}\right)\left(\frac{6.022\times10^{23}\text{ molecules}}{1\text{ mol}}\right) = 6.7\times10^{25}\text{ molecules}$$

Therefore, on average, the number of water molecules that one photon converts from the ice phase to the liquid phase is

$$\frac{N'}{N} = \frac{6.7\times10^{25}\text{ molecules}}{2.1\times10^{24}\text{ photons}} = \boxed{32\text{ molecules/photon}}$$

12. ***REASONING AND SOLUTION***

a. According to Equation 24.5b, the electric field can be found from $E = \sqrt{S/(\varepsilon_0 c)}$. The intensity S of the beam is

$$S = \frac{\text{Energy per unit time}}{A} = \frac{Nhf}{A} = \frac{Nh}{A}\left(\frac{c}{\lambda}\right)$$

$$= \frac{\left(1.30 \times 10^{18}\ \text{photons/s}\right)\left(6.63 \times 10^{-34}\ \text{J}\cdot\text{s}\right)}{\pi\left(1.00 \times 10^{-3}\ \text{m}\right)^2}\left(\frac{3.00 \times 10^{8}\ \text{m/s}}{514.5 \times 10^{-9}\ \text{m}}\right)$$

$$= \boxed{1.60 \times 10^{5}\ \text{W/m}^2}$$

where N is the number of photons per second emitted. Then,

$$E = \sqrt{S/\left(\varepsilon_0 c\right)} = \boxed{7760\ \text{N/C}}$$

b. According to Equation 24.3, the average magnetic field is

$$B = E/c = \boxed{2.59 \times 10^{-5}\ \text{T}}$$

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13. ***REASONING AND SOLUTION*** We know that

$$p = \frac{h}{\lambda} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{0.13\ \text{m}} = \boxed{5.1 \times 10^{-33}\ \text{kg} \cdot \text{m/s}}$$

14. ***REASONING AND SOLUTION*** The momentum of the photon is $p = h/\lambda$ and that of the electron is $p = mv$. Equating and solving for the wavelength of the photon,

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{\left(9.11 \times 10^{-31}\ \text{kg}\right)\left(2.0 \times 10^{5}\ \text{m/s}\right)} = \boxed{3.6 \times 10^{-9}\ \text{m}}$$

15. SSM *REASONING AND SOLUTION*

a. According to Equation 29.6, the magnitude of the momentum of the incident photon is

$$p = \frac{h}{\lambda} = \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{0.3120 \times 10^{-9}\ \text{m}} = \boxed{2.124 \times 10^{-24}\ \text{kg} \cdot \text{m/s}}$$

b. The wavelength of the scattered photon is, from Equation 29.7,

$$\lambda' = \lambda + \frac{h}{mc}(1 - \cos\theta)$$

where θ is the scattering angle. Combining this expression with Equation 29.6, we find that the magnitude of the momentum of the scattered photon is

$$p' = \frac{h}{\lambda'} = \frac{h}{\lambda + \left(\frac{h}{mc}\right)(1 - \cos\theta)}$$

$$= \frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{0.3120 \times 10^{-9}\ \text{m} + \left[\frac{6.626 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.109 \times 10^{-31}\ \text{kg})(2.998 \times 10^{8}\ \text{m/s})}\right](1 - \cos 135.0°)}$$

$$= \boxed{2.096 \times 10^{-24}\ \text{kg} \cdot \text{m/s}}$$

16. ***REASONING*** The change in the wavelength in Compton's experiment is given by Equation 29.7 as $\lambda' - \lambda = \frac{h}{mc}(1-\cos\theta)$. In this problem, we do not know the value of the incident wavelength λ. However, we do know that it is the same for the two given values of the wavelength λ' of the scattered photons. Thus, we can apply Equation 29.7 for each of the given values of λ' and eliminate the incident wavelength from our calculation.

SOLUTION Applying Equation 29.7 for each of the given wavelength values, we have

$$\lambda_1' - \lambda = \frac{h}{mc}(1-\cos\theta_1) \quad \text{and} \quad \lambda_2' - \lambda = \frac{h}{mc}(1-\cos\theta_2)$$

Subtracting the first equation from the second gives

$$\lambda_2' - \lambda - (\lambda_1' - \lambda) = \frac{h}{mc}(1-\cos\theta_2) - \frac{h}{mc}(1-\cos\theta_1)$$

$$\lambda_2' - \lambda_1' = \frac{h}{mc}(-\cos\theta_2 + \cos\theta_1)$$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})}{(9.11\times10^{-31}\ \text{kg})(3.00\times10^{8}\ \text{m/s})}(-\cos 70.0° + \cos 30.0°)$$

$$= \boxed{1.27\times10^{-12}\ \text{m}}$$

17. ***REASONING AND SOLUTION*** Since there are no external forces that act on the system, conservation of linear momentum applies. Since the photon is scattered at $\theta = 180°$, the collision is "head-on" and all motion occurs along the horizontal direction, which we will take as the x axis. For an initially stationary electron, the conservation of linear momentum states that:

$$\underbrace{p_0}_{\substack{\text{Momentum}\\\text{of incident}\\\text{photon}}} = \underbrace{p_f}_{\substack{\text{Momentum}\\\text{of scattered}\\\text{photon}}} + \underbrace{p_{\text{electron}}}_{\substack{\text{Momentum}\\\text{of recoil}\\\text{electron}}}$$

If we take the right to be the positive direction, the associated scalar equation is

$$p_0 = -p_f + p_{\text{electron}} \qquad \text{or} \qquad p_{\text{electron}} = p_0 + p_f$$

Using Equation 29.6, we can write the expression for the momentum gained by the electron as

$$p_{\text{electron}} = \frac{h}{\lambda_0} + \frac{h}{\lambda_f} = h\left(\frac{1}{\lambda_0} + \frac{1}{\lambda_f}\right)$$

Substituting numerical values, we have

$$p_{\text{electron}} = (6.626 \times 10^{-34}\ \text{J}\cdot\text{s})\left(\frac{1}{0.2800 \times 10^{-9}\ \text{m}} + \frac{1}{0.2849 \times 10^{-9}\ \text{m}}\right)$$

$$= \boxed{4.692 \times 10^{-24}\ \text{kg}\cdot\text{m/s}}$$

18. ***REASONING AND SOLUTION***

a. We have

$$\lambda = \lambda' - (h/mc)(1 - \cos 163°) = \boxed{0.1819 \text{ nm}}$$

b. For the incident photon

$$E = hf = hc/\lambda = \boxed{1.092 \times 10^{-15} \text{ J}}$$

c. For the scattered photon

$$E' = hf' = hc/\lambda' = \boxed{1.064 \times 10^{-15} \text{ J}}$$

d. The kinetic energy of the recoil electron is, therefore,

$$\text{KE} = E - E' = \boxed{2.8 \times 10^{-17} \text{ J}}$$

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19. SSM WWW ***REASONING*** The change in wavelength that occurs during Compton scattering is given by Equation 29.7:

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) \qquad \text{or} \qquad (\lambda' - \lambda)_{\max} = \frac{h}{mc}(1 - \cos 180°) = \frac{2h}{mc}$$

$(\lambda' - \lambda)_{\max}$ is the maximum change in the wavelength, and to calculate it we need a value for the mass m of a nitrogen molecule. This value can be obtained from the mass per mole M of nitrogen (N_2) and Avogadro's number N_A, according to $m = M / N_A$ (see Section 14.1).

SOLUTION Using a value of $M = 0.028\ \text{kg/mol}$, we obtain the following result for the maximum change in the wavelength:

$$(\lambda' - \lambda)_{\max} = \frac{2h}{mc} = \frac{2h}{\left(\frac{M}{N_A}\right)c} = \frac{2\left(6.63 \times 10^{-34}\ \text{J}\cdot\text{s}\right)}{\left(\frac{0.028\ \text{kg/mol}}{6.02 \times 10^{23}\ \text{mol}^{-1}}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}$$

$$= \boxed{9.50 \times 10^{-17}\ \text{m}}$$

20. ***REASONING*** Since the net external force acting on the system (the photon and the electron) is zero, the conservation of linear momentum applies. In addition, there are no nonconservative forces, so the conservation of total energy applies as well. Since the photon scatters at an angle of $\theta = 180.0°$ in Figure 29.10, the collision is "head-on." Thus, the motion takes place entirely along the horizontal direction, which we will take as the x axis, with the right as being the positive direction.

The conservation of linear momentum gives rise to Equation 29.7, which relates the difference $\lambda' - \lambda$ between the scattered and incident X-ray photon wavelengths to the scattering angle θ of the electron as

$$\lambda' - \lambda = \frac{h}{mc}(1 - \cos\theta) = \frac{h}{mc}(1 - \cos 180.0°) = \frac{2h}{mc} \tag{1}$$

The conservation of total energy is written as

$$\underbrace{\frac{hc}{\lambda}}_{\substack{\text{Energy}\\\text{of incident}\\\text{photon}}} + \underbrace{0}_{\substack{\text{Initial}\\\text{kinetic}\\\text{energy}\\\text{of electron}}} = \underbrace{\frac{hc}{\lambda'}}_{\substack{\text{Energy}\\\text{of scattered}\\\text{photon}}} + \underbrace{\tfrac{1}{2}mv^2}_{\substack{\text{Final}\\\text{kinetic}\\\text{energy}\\\text{of electron}}} \tag{2}$$

Equations (1) and (2) will permit us to find the wavelength λ of the incident X-ray photon.

SOLUTION Solving Equation (1) for λ' and substituting the result into Equation (2) gives

$$\frac{hc}{\lambda} = \frac{hc}{\dfrac{2h}{mc} + \lambda} + \tfrac{1}{2}mv^2$$

Algebraically rearranging this result, we obtain a quadratic equation for λ:

$$\lambda^2 + \underbrace{\left(\frac{2h}{mc}\right)}_{4.85\times10^{-12}\ \text{m}} \lambda - \underbrace{\frac{2h^2}{m\left(\tfrac{1}{2}mv^2\right)}}_{9.70\times10^{-20}\ \text{m}^2} = 0$$

where we have used $h = 6.63 \times 10^{-34}$ J·s, $m = 9.11 \times 10^{-31}$ kg, $c = 3.00 \times 10^8$ m/s, and $v = 4.67 \times 10^6$ m/s. Solving this quadratic equation for λ, we obtain

$$\boxed{\lambda = 3.09 \times 10^{-10}\ \text{m}}$$

21. ***REASONING AND SOLUTION*** We know that $\lambda = h/mv$. Solving for the mass yields

$$m = \frac{h}{\lambda v} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{\left(8.4 \times 10^{-14}\ \text{m}\right)\left(1.2 \times 10^{6}\ \text{m/s}\right)} = \boxed{6.6 \times 10^{-27}\ \text{kg}}$$

22. ***REASONING AND SOLUTION*** The de Broglie wavelength λ is given by Equation 29.8 as $\lambda = h/p$, where p is the magnitude of the momentum of the particle. The magnitude of the momentum is $p = mv$, where m is the mass and v is the speed of the particle. Using this expression in Equation 29.8, we find that

$$\lambda = \frac{h}{mv} \quad \text{or} \quad v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(0.282 \times 10^{-9}\ \text{m}\right)} = \boxed{1.41 \times 10^{3}\ \text{m/s}}$$

23. SSM ***REASONING*** In order for the person to diffract to the same extent as the sound wave, the de Broglie wavelength of the person must be equal to the wavelength of the sound wave.

SOLUTION

a. Therefore,

$$\lambda_{\text{sound}} = \lambda_{\text{person}}$$

$$\lambda_{\text{sound}} = \frac{h}{m_{\text{person}} v_{\text{person}}}$$

Solving for v_{person}, and using the relation $\lambda_{\text{sound}} = v_{\text{sound}} / f_{\text{sound}}$ (Equation 16.1), we have

$$v_{\text{person}} = \frac{h}{m_{\text{person}} (v_{\text{sound}} / f_{\text{sound}})} = \frac{h f_{\text{sound}}}{m_{\text{person}} v_{\text{sound}}}$$

$$= \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})(128\ \text{Hz})}{(55.0\ \text{kg})(343\ \text{m/s})} = \boxed{4.50 \times 10^{-36}\ \text{m/s}}$$

b. At the speed calculated in part (a), the time required for the person to move a distance of one meter is

$$t = \frac{x}{v} = \frac{1.0\ \text{m}}{4.50 \times 10^{-36}\ \text{m/s}} \underbrace{\left(\frac{1.0\ \text{h}}{3600\ \text{s}}\right)\left(\frac{1\ \text{day}}{24.0\ \text{h}}\right)\left(\frac{1\ \text{year}}{365.25\ \text{days}}\right)}_{\text{Factors to convert seconds to years}} = \boxed{7.05 \times 10^{27}\ \text{years}}$$

24. ***REASONING AND SOLUTION***

a. We know $E = hc/\lambda$ for a photon. The energy of the photon is

$$E = 5.0\,\text{eV}\left(\frac{1.60\times10^{-19}\ \text{J}}{1\,\text{eV}}\right) = 8.0\times10^{-19}\ \text{J}$$

The wavelength is

$$\lambda = \frac{hc}{E} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{8.0\times10^{-19}\ \text{J}} = \boxed{2.5\times10^{-7}\ \text{m}}$$

b. The speed of the 5.0-eV electron is

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2\left(8.0\times10^{-19}\ \text{J}\right)}{9.11\times10^{-31}\ \text{kg}}} = 1.3\times10^{6}\ \text{m/s}$$

The de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.3\times10^{6}\ \text{m/s}\right)} = \boxed{5.6\times10^{-10}\ \text{m}}$$

25. ***REASONING AND SOLUTION*** The average kinetic energy of a helium atom is

$$\text{KE} = (3/2)kT = (3/2)(1.38 \times 10^{-23}\ \text{J/K})(293\ \text{K}) = 6.07 \times 10^{-21}\ \text{J}$$

The speed of the atom corresponding to the average kinetic energy is

$$v = \sqrt{\frac{2(\text{KE})}{m}} = \sqrt{\frac{2\left(6.07 \times 10^{-21}\ \text{J}\right)}{6.65 \times 10^{-27}\ \text{kg}}} = 1.35 \times 10^{3}\ \text{m/s}$$

The deBroglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{\left(6.65 \times 10^{-27}\ \text{kg}\right)\left(1.35 \times 10^{3}\ \text{m/s}\right)} = \boxed{7.38 \times 10^{-11}\ \text{m}}$$

26. ***REASONING AND SOLUTION*** The de Broglie wavelength of the neutrons is (see Equation 29.8)

$$\lambda = \frac{h}{p} = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{(1.67 \times 10^{-27}\ \text{kg})(2.80 \times 10^{3}\ \text{m/s})} = 1.42 \times 10^{-10}\ \text{m}$$

According to Rayleigh's criterion, the minimum angle θ_{min} between the two objects, such that they are just resolved using these neutrons, is

$$\theta_{\text{min}} = 1.22\frac{\lambda}{D} = 1.22\left(\frac{1.42 \times 10^{-10}\ \text{m}}{0.100 \times 10^{-3}\ \text{m}}\right) = \boxed{1.73 \times 10^{-6}\ \text{rad}}$$

27. SSM ***REASONING AND SOLUTION*** The de Broglie wavelength λ of the woman is given by Equation 29.8 as $\lambda = h/p$, where p is the magnitude of her momentum. The magnitude of the momentum is $p = mv$, where m is the woman's mass and v is her speed. According to Equation 3.6b of the equations of kinematics, the speed v is given by $v = \sqrt{2a_y y}$, since the woman jumps from rest. In this expression, $a_y = -9.80\ \text{m/s}^2$ and $y = -9.5\ \text{m}$. With these considerations we find that

$$\lambda = \frac{h}{mv} = \frac{h}{m\sqrt{2a_y y}} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{(41\ \text{kg})\sqrt{2(-9.80\ \text{m/s}^2)(-9.5\ \text{m})}} = \boxed{1.2 \times 10^{-36}\ \text{m}}$$

28. ***REASONING*** The de Broglie wavelength is given by $\lambda = h/p$ according to Equation 29.8, where h is Planck's constant and p is the magnitude of the momentum. Thus, to evaluate the wavelength, we need information about the momentum. While we do not have values for the momentum of particle B before the collision or for the object that moves away after the collision, we do know that the total momentum is conserved. We know this, because it is given that no external forces act on the particles. As Section 7.2 discusses, the total momentum is conserved in such circumstances. This fact will allow us to solve the problem.

SOLUTION The de Broglie wavelength of the object that moves off after the collision is given by Equation 29.8 as $\lambda_{\text{after}} = h/p_{\text{after}}$. Since momentum is mass times velocity, the magnitude of the momentum that the object has after the collision is $p_{\text{after}} = (m_A + m_B)v$, where v is the common speed of the two particles. We can evaluate this momentum be using the law of conservation of momentum, which indicates that the total momentum after the collision is the same as it is before the collision. Before the collision only particle B is moving, so that the total momentum at that time has a value of just $m_B v_{0B}$, where v_{0B} is the initial velocity of particle B. Momentum conservation, then, dictates that

$$\underbrace{(m_A + m_B)v}_{\substack{\text{Total momentum}\\ \text{after collision}}} = \underbrace{m_B v_{0B}}_{\substack{\text{Total momentum}\\ \text{before collision}}}$$

Using this result, we find that the desired de Broglie wavelength is

$$\lambda_{\text{after}} = \frac{h}{p_{\text{after}}} = \frac{h}{(m_A + m_B)v} = \frac{h}{m_B v_{0B}}$$

But the term on the far right is just the given de Broglie wavelength of the incident particle B. Therefore, we conclude that $\boxed{\lambda_{\text{after}} = 2.0 \times 10^{-34}\ \text{m}}$.

29. ***REASONING*** The only force acting on the moving charge is the conservative electric force. Therefore, the total energy of the electron at all points in its trajectory as it is accelerated through the potential difference V remains the same. In other words, if A and B are the end points of the trajectory, then

$$\underbrace{\mathrm{KE_A} + \mathrm{EPE_A}}_{\text{Total energy at A}} = \underbrace{\mathrm{KE_B} + \mathrm{EPE_B}}_{\text{Total energy at B}}$$

Since the electron starts from rest $\mathrm{KE_A} = 0$, and we have

$$\mathrm{EPE_A} - \mathrm{EPE_B} = \tfrac{1}{2}mv_{\mathrm{B}}^2$$

According to Equation 19.4,

$$\mathrm{EPE_B} - \mathrm{EPE_A} = q(V_{\mathrm{B}} - V_{\mathrm{A}}) = qV = -eV$$

Combining these results, we find

$$\tfrac{1}{2}mv_{\mathrm{B}}^2 = eV$$

Solving for V, we have

$$V = \frac{mv_{\mathrm{B}}^2}{2e}$$

We can express the speed v_{B} of the electron in terms of its de Broglie wavelength:

$$\lambda = \frac{h}{p} = \frac{h}{mv_{\mathrm{B}}} \qquad \text{or} \qquad v_{\mathrm{B}} = \frac{h}{m\lambda}$$

The expression for V becomes

$$V = \frac{m}{2e}\left(\frac{h}{m\lambda}\right)^2 = \frac{1}{2me}\left(\frac{h}{\lambda}\right)^2$$

SOLUTION Given the data in the problem statement and the fact that the mass and the magnitude of the charge of the electron are, respectively, $m = 9.11 \times 10^{-31}$ kg and $e = 1.6 \times 10^{-19}$ C, we find that the potential difference V is

$$V = \frac{1}{2(9.11 \times 10^{-31}\ \mathrm{kg})(1.6 \times 10^{-19}\ \mathrm{C})}\left(\frac{6.63 \times 10^{-34}\ \mathrm{J \cdot s}}{1.0 \times 10^{-11}\ \mathrm{m}}\right)^2 = \boxed{1.5 \times 10^4\ \mathrm{V}}$$

30. ***REASONING AND SOLUTION*** The energy of the photon is $E = hf = hc/\lambda_{\text{photon}}$, while the kinetic energy of the particle is KE = $(1/2)mv^2 = h^2/(2m\lambda^2)$. Equating the two energies and rearranging the result gives $\lambda_{\text{photon}}/\lambda = (2mc/h)\lambda$. Now the speed of the particle is $v = 0.050c$, so $\lambda = h/(0.050\ mc)$, and

$$\lambda_{\text{photon}}/\lambda = 2/0.050 = \boxed{4.0 \times 10^1}$$

31. SSM WWW ***REASONING AND SOLUTION*** According to the uncertainty principle, the minimum uncertainty in the momentum can be determined from $\Delta p_y \Delta y = h/(2\pi)$. Since $p_y = mv_y$, it follows that $\Delta p_y = m\Delta v_y$. Thus, the minimum uncertainty in the velocity of the oxygen molecule is given by

$$\Delta v_y = \frac{h}{2\pi m \Delta y} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{2\pi\left(5.3 \times 10^{-26}\ \text{kg}\right)\left(0.25 \times 10^{-3}\ \text{m}\right)} = \boxed{8.0 \times 10^{-6}\ \text{m/s}}$$

32. ***REASONING*** We know that the object is somewhere on the line. Therefore, the uncertainty in the object's position is $\Delta y = 2.5$ m. The minimum uncertainty in the object's momentum is Δp_y and is specified by the Heisenberg uncertainty principle (Equation 29.10) in the form $(\Delta p_y)(\Delta y) = h/(2\pi)$. Since momentum is mass m times velocity v, the uncertainty in the velocity Δv is related to the uncertainty in the momentum by $\Delta v = (\Delta p_y)/m$.

SOLUTION

a. Using the uncertainty principle, we find the minimum uncertainty in the momentum as follows:

$$\left(\Delta p_y\right)\left(\Delta y\right) = \frac{h}{2\pi}$$

$$\Delta p_y = \frac{h}{2\pi\,\Delta y} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi\left(2.5\ \text{m}\right)} = \boxed{4.2\times10^{-35}\ \text{kg}\cdot\text{m/s}}$$

b. For a golf ball this uncertainty in momentum corresponds to an uncertainty in velocity that is given by

$$\Delta v_y = \frac{\Delta p_y}{m} = \frac{4.2\times10^{-35}\ \text{kg}\cdot\text{m/s}}{0.045\ \text{kg}} = \boxed{9.3\times10^{-34}\ \text{m/s}}$$

c. For an electron this uncertainty in momentum corresponds to an uncertainty in velocity that is given by

$$\Delta v_y = \frac{\Delta p_y}{m} = \frac{4.2\times10^{-35}\ \text{kg}\cdot\text{m/s}}{9.11\times10^{-31}\ \text{kg}} = \boxed{4.6\times10^{-5}\ \text{m/s}}$$

33. ***REASONING AND SOLUTION*** The minimum uncertainty Δp in the momentum of the proton is given by the Heisenberg uncertainty principle (Equation 29.10) as $\Delta p = h/(2\pi\Delta d)$ where Δd is the uncertainty in position; Therefore,

$$\Delta p = \frac{h}{2\pi\Delta d} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi(5.5\times10^{-15}\ \text{m})} = \boxed{1.9\times10^{-20}\ \text{kg}\cdot\text{m/s}}$$

34. ***REASONING AND SOLUTION*** We know from Equation 29.8 that

$$h = \lambda p = \lambda mv = (0.90\ \text{m})(82\ \text{kg})(0.50\ \text{m/s}) = \boxed{37\ \text{J}\cdot\text{s}}$$

35. SSM WWW ***REASONING AND SOLUTION*** If we assume that the number of particles that strikes the screen outside the bright fringe is negligible, the particles that pass through the slit will hit the screen somewhere in the central bright fringe. The angular width of the central bright fringe is equal to 2θ, where θ is shown in Figure 29.15. From this figure we see that

$$\theta = \tan^{-1}\left(\frac{\Delta p_y}{p_x}\right)$$

According to Equation 29.10, the minimum uncertainty in Δp_y occurs when

$$(\Delta p_y)(\Delta y) = \frac{h}{2\pi}$$

Since the electron can pass anywhere through the width W of the slit, the uncertainty in the y position of the electron is $\Delta y = W$. Thus, the minimum uncertainty can be written as

$$(\Delta p_y)W = \frac{h}{2\pi} \qquad \text{or} \qquad \Delta p_y = \frac{h}{2\pi W}$$

Using the fact that $p_x = h/\lambda$, we find

$$\theta = \tan^{-1}\left(\frac{h}{p_x 2\pi W}\right) = \tan^{-1}\left[\frac{h}{(h/\lambda)2\pi W}\right] = \tan^{-1}\left(\frac{\lambda}{2\pi W}\right)$$

$$= \tan^{-1}\left[\frac{633\times10^{-9}\ \text{m}}{2\pi(0.200\times10^{-3}\ \text{m})}\right] = 2.89\times10^{-2}\ \text{degrees}$$

Therefore, the minimum range of angles over which the particles spread is $\boxed{-0.0289° \le \theta \le +0.0289°}$.

36. ***REASONING AND SOLUTION*** The uncertainty in the position is $\Delta y = \lambda = h/mv$. Now, since $\Delta p \Delta y = m\Delta v \Delta y = h/2\pi$, we have $m\Delta v \Delta y = m\Delta v(h/mv) = h\Delta v/v = h/2\pi$. Solving for Δv yields

$$\Delta v = v/2\pi = (4.5 \times 10^5 \text{ m/s})/2\pi = \boxed{7.2 \times 10^4 \text{ m/s}}$$

37. ***REASONING AND SOLUTION*** The de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{\left(1.3 \times 10^{-4}\ \text{kg}\right)\left(0.020\ \text{m/s}\right)} = \boxed{2.6 \times 10^{-28}\ \text{m}}$$

38. ***REASONING*** The energy of a photon of frequency f is, according to Equation 29.2, $E = hf$, where h is Planck's constant. Since the frequency and wavelength are related by $f = c/\lambda$ (see Equation 16.1), the energy of a photon can be written in terms of the wavelength as $E = hc/\lambda$. These expressions can be solved for both the wavelength and the frequency.

SOLUTION

a. The wavelength of the photon is

$$\lambda = \frac{hc}{E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{1.22\times10^{-18}\ \text{J}} = \boxed{1.63\times10^{-7}\ \text{m}}$$

b. Using the answer from part (a), we find that the frequency of the photon is

$$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{1.63\times10^{-7}\ \text{m}} = \boxed{1.84\times10^{15}\ \text{Hz}}$$

Alternatively, we could use Equation 29.2 directly to obtain the frequency:

$$f = \frac{E}{h} = \frac{1.22\times10^{-18}\ \text{J}}{6.63\times10^{-34}\ \text{J}\cdot\text{s}} = \boxed{1.84\times10^{15}\ \text{Hz}}$$

c. The wavelength and frequency values shown in Figure 24.9 indicate that this photon corresponds to electromagnetic radiation in the $\boxed{\text{ultraviolet region}}$ of the electromagnetic spectrum.

39. SSM ***REASONING AND SOLUTION*** The de Broglie wavelength λ is given by Equation 29.8 as $\lambda = h/p$, where p is the magnitude of the momentum of the particle. The magnitude of the momentum is $p = mv$, where m is the mass and v is the speed of the particle. Using this expression in Equation 29.8, we find that $\lambda = h/(mv)$, or

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}\ \text{J}\cdot\text{s}}{\left(1.67 \times 10^{-27}\ \text{kg}\right)\left(1.30 \times 10^{-14}\ \text{m}\right)} = 3.05 \times 10^{7}\ \text{m/s}$$

The kinetic energy of the proton is

$$\text{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}(1.67 \times 10^{-27}\ \text{kg})(3.05 \times 10^{7}\ \text{m/s})^2 = \boxed{7.77 \times 10^{-13}\ \text{J}}$$

40. ***REASONING AND SOLUTION*** The de Broglie wavelength is given by Equation 29.8 as $\lambda = h/p$. Since momentum is mass times velocity, or $p = mv$, it follows that

$$\frac{\lambda_{\text{electron}}}{\lambda_{\text{proton}}} = \frac{\dfrac{h}{m_{\text{electron}}v}}{\dfrac{h}{m_{\text{proton}}v}} = \frac{m_{\text{proton}}}{m_{\text{electron}}} = \frac{1.67 \times 10^{-27}\ \text{kg}}{9.11 \times 10^{-31}\ \text{kg}} = \boxed{1830}$$

41. ***REASONING AND SOLUTION*** Equation 29.6 gives the magnitude p of the photon momentum as $p = h/\lambda$, where λ is the wavelength. Using this expression for both the red and the violet light, we find that

$$\frac{p_{\text{violet}}}{p_{\text{red}}} = \frac{h/\lambda_{\text{violet}}}{h/\lambda_{\text{red}}} = \frac{\lambda_{\text{red}}}{\lambda_{\text{violet}}} = \frac{730\text{ nm}}{380\text{ nm}} = \boxed{1.9}$$

42. ***REASONING AND SOLUTION*** Equation 29.3 gives

$$\mathrm{KE}_{\max} = hf - W_o = \frac{hc}{\lambda} - W_o$$

$$= \frac{\left(6.63\times10^{-34}\ \mathrm{J\cdot s}\right)\left(3.00\times10^{8}\ \mathrm{m/s}\right)}{215\times10^{-9}\ \mathrm{m}} - \left(3.68\ \mathrm{eV}\right)\left(\frac{1.60\times10^{-19}\ \mathrm{J}}{1\ \mathrm{eV}}\right) = 3.36\times10^{-19}\ \mathrm{J}$$

Converting to electron volts

$$\mathrm{KE}_{\max} = \left(3.36\times10^{-19}\ \mathrm{J}\right)\left(\frac{1\ \mathrm{eV}}{1.60\times10^{-19}\ \mathrm{J}}\right) = \boxed{2.10\ \mathrm{eV}}$$

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43. SSM ***REASONING*** The width of the central bright fringe in the diffraction patterns will be identical when the electrons have the same de Broglie wavelength as the wavelength of the photons in the red light. The de Broglie wavelength of one electron in the beam is given by Equation 29.8, $\lambda_{\text{electron}} = h / p$, where $p = mv$.

SOLUTION Following the reasoning described above, we find

$$\lambda_{\text{red light}} = \lambda_{\text{electron}}$$

$$\lambda_{\text{red light}} = \frac{h}{m_{\text{electron}} v_{\text{electron}}}$$

Solving for the speed of the electron, we have

$$v_{\text{electron}} = \frac{h}{m_{\text{electron}} \lambda_{\text{red light}}} = \frac{6.63 \times 10^{-34}\ \text{J} \cdot \text{s}}{(9.11 \times 10^{-31}\ \text{kg})(661 \times 10^{-9}\ \text{m})} = \boxed{1.10 \times 10^{3}\ \text{m/s}}$$

44. ***REASONING*** We will first calculate the potential energy of the system at each of the two separations, and then find the energy difference for the two configurations. Since the electric potential energy lost by the system is carried off by a photon that is emitted during the process, the energy difference must be equal to the energy of the photon. The wavelength of the photon can then by found using Equation 29.2 with Equation 16.1: $E = hc/\lambda$.

SOLUTION The initial potential energy of the system is (see Equations 19.3 and 19.6)

$$\text{EPE}_1 = eV_1 = e\left(\frac{kq}{r_1}\right)$$

$$= (1.6\times10^{-19}\ \text{C})\left[\frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.30\times10^{-6}\ \text{C})}{0.420\ \text{m}}\right] = 2.84\times10^{-14}\ \text{J}$$

The final potential energy is

$$\text{EPE}_2 = eV_2 = (1.6\times10^{-19}\ \text{C})\left[\frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.30\times10^{-6}\ \text{C})}{1.58\ \text{m}}\right] = 7.56\times10^{-15}\ \text{J}$$

The energy difference, and therefore the energy of the emitted photon, is

$$\Delta E = \text{EPE}_1 - \text{EPE}_2 = 2.84\times10^{-14}\ \text{J} - 7.56\times10^{-15}\ \text{J} = 2.08\times10^{-14}\ \text{J}$$

The wavelength of this photon is

$$\lambda = \frac{hc}{\Delta E} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{2.08\times10^{-14}\ \text{J}} = \boxed{9.56\times10^{-12}\ \text{m}}$$

45. ***REASONING AND SOLUTION*** The momentum of an incident photon is given by $p = h/\lambda$. The total momentum carried by the photons to the surface in a time Δt is

$$P = Nh\Delta t/\lambda, \qquad \text{where} \qquad N = 3.0 \times 10^{18} \text{ photons/s}$$

The impulse momentum principle gives the average force exerted on the wall to be $F = \Delta P/\Delta t$.

a. $\Delta P = 2P$, since the photons reflect from the mirror, so

$$F = 2Nh/\lambda = 2(3.0 \times 10^{18} \text{ photons /s})(6.63 \times 10^{-34} \text{ J·s})/(395 \times 10^{-9} \text{ m}) = \boxed{1.0 \times 10^{-8} \text{ N}}$$

b. $\Delta P = P$, since the photons are totally absorbed by the wall, so $F = Nh/\lambda$ and

$$F = \boxed{5.0 \times 10^{-9} \text{ N}}$$

1. **SSM** ***REASONING*** Assuming that the hydrogen atom is a sphere of radius r_{atom}, its volume V_{atom} is given by $\frac{4}{3}\pi r_{\text{atom}}^3$. Similarly, if the radius of the nucleus is r_{nucleus}, the volume V_{nucleus} is given by $\frac{4}{3}\pi r_{\text{nucleus}}^3$.

SOLUTION

a. According to the given data, the nuclear dimensions are much smaller than the orbital radius of the electron; therefore, we can treat the nucleus as a point about which the electron orbits. The electron is normally at a distance of about 5.3×10^{-11} m from the nucleus, so we can treat the atom as a sphere of radius $r_{\text{atom}} = 5.3 \times 10^{-11}$ m. The volume of the atom is

$$V_{\text{atom}} = \frac{4}{3}\pi r_{\text{atom}}^3 = \frac{4}{3}\pi(5.3 \times 10^{-11}\ \text{m})^3 = \boxed{6.2 \times 10^{-31}\ \text{m}^3}$$

b. Similarly, since the nucleus has a radius of approximately $r_{\text{nucleus}} = 1 \times 10^{-15}$ m, its volume is

$$V_{\text{nucleus}} = \frac{4}{3}\pi r_{\text{nucleus}}^3 = \frac{4}{3}\pi(1 \times 10^{-15}\ \text{m})^3 = \boxed{4 \times 10^{-45}\ \text{m}^3}$$

c. The percentage of the atomic volume occupied by the nucleus is

$$\frac{V_{\text{nucleus}}}{V_{\text{atom}}} \times 100\% = \frac{r_{\text{nucleus}}^3}{r_{\text{atom}}^3} \times 100\% = \left(\frac{1 \times 10^{-15}\ \text{m}}{5.3 \times 10^{-11}\ \text{m}}\right)^3 \times 100\% = \boxed{7 \times 10^{-13}\ \%}$$

2. ***REASONING*** According to the discussion in Conceptual Example 1, the radius of the electron orbit is about 1×10^5 times as great as the radius of the nucleus. In the scale model, the radius of the electron's orbit must also be 1×10^5 times as great as the radius of ball.

SOLUTION Since the ball that represents the nucleus has a radius of 3.2 cm, the distance between the nucleus and the nearest electron in the model must be

$$(3.2\text{ cm})\left(1\times10^5\right)=\left(3.2\times10^5\text{ cm}\right)\left(\frac{1\text{ mi}}{1.61\times10^5\text{ cm}}\right)=\boxed{2\text{ mi}}$$

3. ***REASONING AND SOLUTION*** The density ρ is the mass m per unit volume V or $\rho = m/V$. Since the volume of a sphere is $V = 4\pi r^3/3$, the density can be written as $\rho = 3m/(4\pi r^3)$. We use m_p and m_e to denote the mass of a proton and the mass of an electron, respectively. Considering that the hydrogen nucleus is a single proton with a radius of r_p, it follows that the desired density ratio is

$$\frac{\rho_{\text{nucleus}}}{\rho_{\text{atom}}} = \frac{3m_p/\left(4\pi r_p^3\right)}{3\left(m_p + m_e\right)/\left(4\pi r_{\text{atom}}^3\right)} = \frac{m_p r_{\text{atom}}^3}{\left(m_p + m_e\right) r_p^3}$$

$$= \frac{\left(1.67\times10^{-27}\ \text{kg}\right)\left(5.3\times10^{-11}\ \text{m}\right)^3}{\left(1.67\times10^{-27}\ \text{kg} + 9.11\times10^{-31}\ \text{kg}\right)\left(1.0\times10^{-15}\ \text{m}\right)^3} = \boxed{1.5\times10^{14}}$$

4. ***REASONING AND SOLUTION*** The de Broglie wavelength is given by $\lambda = h/(mv)$. In order to find the speed v, we use the fact that the kinetic energy is given by KE = $mv^2/2$, Equation 6.2. Thus,

$$v = \sqrt{\frac{2\text{KE}}{m}} = \sqrt{\frac{2\left(7.00\times10^{-13}\ \text{J}\right)}{6.64\times10^{-27}\ \text{kg}}} = 1.45\times10^{7}\ \text{m/s}$$

Now

$$\lambda = \frac{h}{mv} = \frac{6.626\times10^{-34}\ \text{J}\cdot\text{s}}{\left(6.64\times10^{-27}\ \text{kg}\right)\left(1.45\times10^{7}\ \text{m/s}\right)} = \boxed{6.88\times10^{-15}\ \text{m}} \tag{29.8}$$

5. **SSM** ***REASONING*** The copper nucleus has a total charge $Ze = 29 \times 1.6 \times 10^{-19}$ C, and radius $r_{\text{nucleus}} = 4.8 \times 10^{-15}$ m. The required work can be found from Equation 19.1: $W_{\text{AB}} = \text{EPE}_{\text{A}} - \text{EPE}_{\text{B}}$. According to Equation 19.3, $\text{EPE} = Vq$, where $q = e$ for a proton. Therefore, $W_{\text{AB}} = V_{\text{A}}e - V_{\text{B}}e$. The proton is initially at infinity, so $V_{\text{A}} = 0$ V and $V_{\text{B}} = kZe/r_{\text{nucleus}}$, according to Equation 19.6. Thus, $W_{\text{AB}} = -kZe^2/r_{\text{nucleus}}$.

SOLUTION The work done by the electric force is, therefore,

$$W_{\text{AB}} = \frac{-kZe^2}{r_{\text{nucleus}}} = \frac{-(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(29)(1.6 \times 10^{-19}\ \text{C})^2}{4.8 \times 10^{-15}\ \text{m}} = -1.39 \times 10^{-12}\ \text{J}$$

$$W_{\text{AB}} = \left(-1.39 \times 10^{-12}\ \text{J}\right)\underbrace{\left(\frac{1.0\ \text{eV}}{1.6 \times 10^{-19}\ \text{J}}\right)}_{\substack{\text{Converts from joules}\\ \text{to electron volts}}} = \boxed{-8.7 \times 10^6\ \text{eV}}$$

6. ***REASONING AND SOLUTION*** The distance of closest approach can be obtained by setting KE = EPE or KE = $(kZe/r)(2e)$, and solving for r to get $r = kZ(2e^2)/(\text{KE})$.

$$r = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(79)2(1.602 \times 10^{-19}\ \text{C})^2/(5.0 \times 10^{-13}\ \text{J}) = \boxed{7.3 \times 10^{-14}\ \text{m}}$$

7. **SSM** **WWW** ***REASONING AND SOLUTION*** The radii for Bohr orbits are given by Equation 30.10:

$$r_n = (5.29 \times 10^{-11}\ \text{m})\frac{n^2}{Z}$$

The Bohr radii for a doubly ionized lithium atom Li^{2+} are given by Equation 30.10 with $Z = 3$; therefore, the radius of the $n = 5$ Bohr orbit in a doubly ionized lithium atom Li^{2+} is

$$r_5 = (5.29 \times 10^{-11}\ \text{m})\frac{5^2}{3} = \boxed{4.41 \times 10^{-10}\ \text{m}}$$

8. ***REASONING AND SOLUTION*** The first excited state occurs for $n = 2$ and has an energy given by Equation 30.13:

$$E_2 = -(13.6\text{ eV})\frac{Z^2}{n^2} = -(13.6\text{ eV})\frac{1^2}{2^2} = -3.40\text{ eV}$$

After absorbing 2.86 eV of energy, the electron has an energy

$$E_n = -3.40\text{ eV} + 2.86\text{ eV} = -0.54\text{ eV}$$

Again using Equation 30.13, we find $E_n = -0.54\text{ eV} = -(13.6\text{ eV})(1^2/n^2)$. Solving for n, we obtain

$$n = \sqrt{\frac{13.6\text{ eV}}{0.54\text{ eV}}} = \boxed{5}$$

9. ***REASONING AND SOLUTION***

a. The longest wavelength in the Pfund series occurs for the transition $n = 6$ to $n = 5$, so that

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right) = \left(1.097 \times 10^7 \text{ m}^{-1}\right)\left(\frac{1}{5^2} - \frac{1}{6^2}\right) \quad \text{or} \quad \boxed{\lambda = 7458 \text{ nm}}$$

b. The shortest wavelength occurs when $1/n^2 = 0$, so that

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right) = \left(1.097 \times 10^7 \text{ m}^{-1}\right)\left(\frac{1}{5^2}\right) \quad \text{or} \quad \boxed{\lambda = 2279 \text{ nm}}$$

c. The lines in the Pfund series occur in the $\boxed{\text{infrared region}}$.

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10. ***REASONING*** The atomic number for helium is $Z = 2$. The ground state is the $n = 1$ state, the first excited state is the $n = 2$ state, and the second excited state is the $n = 3$ state. With $Z = 2$ and $n = 3$, we can use Equation 30.10 to find the radius of the ion.

SOLUTION The radius of the second excited state is

$$r_3 = \left(5.29 \times 10^{-11}\ \text{m}\right)\frac{n^2}{Z} = \left(5.29 \times 10^{-11}\ \text{m}\right)\frac{3^2}{2} = \boxed{2.38 \times 10^{-10}\ \text{m}} \tag{30.10}$$

11. **SSM** ***REASONING*** According to Equation 30.14, the wavelength λ emitted by the hydrogen atom when it makes a transition from the level with n_i to the level with n_f is given by

$$\frac{1}{\lambda} = \frac{2\pi^2 mk^2 e^4}{h^3 c}(Z^2)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \quad \text{with} \quad n_i, n_f = 1,\ 2,\ 3, \ldots \quad \text{and} \quad n_i > n_f$$

where $2\pi^2 mk^2 e^4/(h^3 c) = 1.097 \times 10^7\ \text{m}^{-1}$ and $Z = 1$ for hydrogen. Once the wavelength for the particular transition in question is determined, Equation 29.2 ($E = hf = hc/\lambda$) can be used to find the energy of the emitted photon.

SOLUTION In the Paschen series, $n_f = 3$. Using the above expression with $Z = 1$, $n_i = 7$ and $n_f = 3$, we find that

$$\frac{1}{\lambda} = (1.097 \times 10^7\ \text{m}^{-1})\,(1^2)\left(\frac{1}{3^2} - \frac{1}{7^2}\right) \qquad \text{or} \qquad \lambda = 1.005 \times 10^{-6}\ \text{m}$$

The photon energy is

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^8\ \text{m/s})}{1.005 \times 10^{-6}\ \text{m}} = \boxed{1.98 \times 10^{-19}\ \text{J}}$$

12. ***REASONING*** Since the atom emits two photons as it returns to the ground state, one is emitted when the electron falls from $n = 3$ to $n = 2$, and the other is emitted when it subsequently drops from $n = 2$ to $n = 1$. The wavelengths of the photons emitted during these transitions are given by Equation 30.14 with the appropriate values for the initial and final numbers, n_i and n_f.

SOLUTION The wavelengths of the photons are

n = 3 to n = 2

$$\frac{1}{\lambda} = \left(1.097 \times 10^7 \text{ m}^{-1}\right)(1)^2\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = 1.524 \times 10^6 \text{ m}^{-1} \qquad (30.14)$$

$$\lambda = \boxed{6.56 \times 10^{-7} \text{ m}}$$

n = 2 to n = 1

$$\frac{1}{\lambda} = \left(1.097 \times 10^7 \text{ m}^{-1}\right)(1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = 8.228 \times 10^6 \text{ m}^{-1} \qquad (30.14)$$

$$\lambda = \boxed{1.22 \times 10^{-7} \text{ m}}$$

13. ***REASONING*** The Bohr expression as it applies to any one-electron species of atomic number Z, is given by Equation 30.13: $E_n = -(13.6\text{ eV})(Z^2/n^2)$. For certain values of the quantum number n, this expression predicts equal electron energies for singly ionized helium He^+ ($Z = 2$) and doubly ionized lithium Li^+ ($Z = 3$). As stated in the problem, the quantum number n is different for the equal energy states for each species.

SOLUTION For equal energies, we can write

$$(E_n)_{He} = (E_n)_{Li} \qquad \text{or} \qquad -(13.6\text{ eV})\frac{Z_{He}^2}{n_{He}^2} = -(13.6\text{ eV})\frac{Z_{Li}^2}{n_{Li}^2}$$

Simplifying, this becomes

$$\frac{Z_{He}^2}{n_{He}^2} = \frac{Z_{Li}^2}{n_{Li}^2} \qquad \text{or} \qquad \frac{4}{n_{He}^2} = \frac{9}{n_{Li}^2}$$

Thus,

$$n_{He} = n_{Li}\sqrt{\frac{4}{9}} = n_{Li}\left(\frac{2}{3}\right)$$

Therefore, the value of the helium energy level is equal to the lithium energy level for any value of n_{He} that is two-thirds of n_{Li}. For quantum numbers less than or equal to 9, an equality in energy levels will occur for $n_{He} = 2, 4, 6$ corresponding to $n_{Li} = 3, 6, 9$. The results are summarized in the following table.

n_{He}	n_{Li}	*Energy*
2	3	–13.6 eV
4	6	–3.40 eV
6	9	–1.51 eV

14. ***REASONING AND SOLUTION*** The energy of the hydrogen atom, $Z = 1$, in electron volts is given by Equation 30.13, $E_n = -13.6/n^2$, which requires a value for the quantum number n. A value for n can be obtained from Equation 30.10, $r_n = \left(5.29 \times 10^{-11}\ \text{m}\right) n^2$, which gives the radius of the hydrogen atom in meters.

$$n^2 = \frac{r_n}{5.29 \times 10^{-11}\ \text{m}} = \frac{4.761 \times 10^{-10}\ \text{m}}{5.29 \times 10^{-11}\ \text{m}} = 9 \qquad \text{or} \qquad n = 3$$

$$E_n = -\frac{13.6\ \text{eV}}{n^2} = -\frac{13.6}{3^2} = \boxed{-1.51\ \text{eV}}$$

15. **SSM** ***REASONING AND SOLUTION*** For the Paschen series, $n_f = 3$. The range of wavelengths occurs for values of $n_i = 4$ to $n_i = \infty$. Using Equation 30.14, we find that the shortest wavelength occurs for $n_i = \infty$ and is given by

$$\frac{1}{\lambda} = \left(1.097 \times 10^7 \text{ m}^{-1}\right)(1)^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \left(1.097 \times 10^7 \text{ m}^{-1}\right)\left(\frac{1}{3^2}\right) \quad \text{or} \quad \underbrace{\lambda = 8.204 \times 10^{-7} \text{ m}}_{\substack{\text{Shortest wavelength in} \\ \text{Paschen series}}}$$

The longest wavelength in the Paschen series occurs for $n_i = 4$ and is given by

$$\frac{1}{\lambda} = \left(1.097 \times 10^7 \text{ m}^{-1}\right)\left(\frac{1}{3^2} - \frac{1}{4^2}\right) \quad \text{or} \quad \underbrace{\lambda = 1.875 \times 10^{-6} \text{ m}}_{\substack{\text{Longest wavelength in} \\ \text{Paschen series}}}$$

For the Brackett series, $n_f = 4$. The range of wavelengths occurs for values of $n_i = 5$ to $n_i = \infty$. Using Equation 30.14, we find that the shortest wavelength occurs for $n_i = \infty$ and is given by

$$\frac{1}{\lambda} = \left(1.097 \times 10^7 \text{ m}^{-1}\right)(1)^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \left(1.097 \times 10^7 \text{ m}^{-1}\right)\left(\frac{1}{4^2}\right) \quad \text{or} \quad \underbrace{\lambda = 1.459 \times 10^{-6} \text{ m}}_{\substack{\text{Shortest wavelength in} \\ \text{Brackett series}}}$$

The longest wavelength in the Brackett series occurs for $n_i = 5$ and is given by

$$\frac{1}{\lambda} = \left(1.097 \times 10^7 \text{ m}^{-1}\right)\left(\frac{1}{4^2} - \frac{1}{5^2}\right) \quad \text{or} \quad \underbrace{\lambda = 4.051 \times 10^{-6} \text{ m}}_{\substack{\text{Longest wavelength in} \\ \text{Brackett series}}}$$

Since the longest wavelength in the Paschen series falls within the Brackett series, the wavelengths of the two series overlap.

16. ***REASONING AND SOLUTION***

According to Newton's second law, as expressed in Equation 30.6, $mv^2 = kZe^2/r$, which says that KE = –(1/2) (EPE). Since KE + EPE = E (total energy), we have –(1/2) (EPE) + EPE = E, or EPE = $2E$ = 2(– 4.90 eV) = $\boxed{-9.80 \text{ eV}}$.

Since KE = –(1/2) (EPE), we have KE = – (1/2)(–9.80 eV) = $\boxed{+ 4.90 \text{ eV}}$.

17. **SSM** ***REASONING*** Singly ionized helium, He^+, is a hydrogen-like species with $Z = 2$. The wavelengths of the series of lines produced when the electron makes a transition from higher energy levels into the $n_f = 4$ level are given by Equation 30.14 with $Z = 2$ and $n_f = 4$:

$$\frac{1}{\lambda} = \left(1.097 \times 10^7 \text{ m}^{-1}\right)(2^2)\left(\frac{1}{4^2} - \frac{1}{n_i^2}\right)$$

SOLUTION Solving this expression for n_i gives

$$n_i = \left[\frac{1}{4^2} - \frac{1}{4\lambda(1.097 \times 10^7 \text{ m}^{-1})}\right]^{-1/2}$$

Evaluating this expression at the limits of the range for λ, we find that $n_i = 19.88$ for $\lambda = 380 \text{ nm}$, and $n_i = 5.58$ for $\lambda = 750 \text{ nm}$. Therefore, the values of n_i for energy levels from which the electron makes the transitions that yield wavelengths in the range between 380 nm and 750 nm are $\boxed{6 \le n_i \le 19}$.

18. ***REASONING*** For each species, the wavelengths appearing in the line spectra can be calculated using Equation 30.14:

$$\frac{1}{\lambda} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

The shortest wavelength for a given series of lines occurs when an electron is in an initial energy level with a principal quantum number of $n_i = \infty$. Therefore, $1/n_i^2 = 1/\infty = 0$, and the shortest wavelength is given by

$$\frac{1}{\lambda} = \frac{RZ^2}{n_f^2}$$

In this expression, the value of n_f is the same for the series for Li^{2+} and for Be^{3+}. The term R is the Rydberg constant and is also the same for both ionic species. Thus, rearranging this equation gives

$$\lambda Z^2 = \frac{n_f^2}{R} = \text{constant}$$

which is the basis for our solution.

SOLUTION Since λZ^2 is the same for each species, we have

$$\left(\lambda Z^2\right)_{Li} = \left(\lambda Z^2\right)_{Be} \qquad \text{or} \qquad \lambda_{Be} = \frac{\left(\lambda Z^2\right)_{Li}}{Z_{Be}^2} = \frac{(40.5\text{ nm})(3)^2}{(4)^2} = \boxed{22.8\text{ nm}}$$

19. ***REASONING AND SOLUTION*** We need to use Equation 30.14 to find the spacing between the longest and next-to-the longest wavelengths in the Balmer series. In order to do this, we need to first find these two wavelengths.

Longest:

$$\frac{1}{\lambda_1} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \left(1.097 \times 10^7 \text{ m}^{-1}\right)\left(\frac{1}{2^2} - \frac{1}{3^2}\right) \qquad \text{or} \qquad \lambda_1 = 656.3 \text{ nm}$$

Next-to-longest:

$$\frac{1}{\lambda_2} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \left(1.097 \times 10^7 \text{ m}^{-1}\right)\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \qquad \text{or} \qquad \lambda_2 = 486.2 \text{ nm}$$

Equation 27.7 states that $\sin\theta = m\lambda/d$. Using the small angle approximation, $\sin\theta \approx \tan\theta \approx \theta = y/L$, so that $y/L = m\lambda/d$. The position of the fringe due to the longest wavelength is $y_1 = m\lambda_1 L/d$. For the next-to-longest, $y_2 = m\lambda_2 L/d$. The difference in the positions on the screen is therefore, $y_1 - y_2 = (mL/d)(\lambda_1 - \lambda_2)$ which gives

$$d = (mL)(\lambda_1 - \lambda_2)/(y_1 - y_2)$$

$$d = (1)(0.81 \text{ m})(656.3 \times 10^{-9} \text{ m} - 486.2 \times 10^{-9} \text{ m})/(0.0300 \text{ m})$$

$$d = 4.59 \times 10^{-6} \text{ m} = 4.59 \times 10^{-4} \text{ cm}$$

This spacing between adjacent lines corresponds to a number of lines per centimeter that is

$$\text{Number of lines per cm} = 1/(4.59 \times 10^{-4} \text{ cm}) = \boxed{2180 \text{ lines/cm}}$$

20. ***REASONING AND SOLUTION*** The shortest wavelength, λ_s, occurs when n_i = infinity, so that $1/n_i = 0$. In that case Equation 30.14 becomes

$$\frac{1}{\lambda_s} = \frac{RZ^2}{n_f^2} \qquad \text{or} \qquad RZ^2 = \frac{n_f^2}{\lambda_s}$$

The longest wavelength in the series occurs when $n_i = n_f + 1$.

$$\frac{1}{\lambda_l} = RZ^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) = \left(\frac{n_f^2}{\lambda_s}\right)\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

A little algebra gives n_f as follows:

$$\lambda_l/\lambda_s = (41.02 \times 10^{-9}\ \text{m})/(22.79 \times 10^{-9}\ \text{m}) = (n_f + 1)^2/(2n_f + 1)$$

or

$$n_f^2 - 1.600\ n_f - 0.800 = 0$$

Solving the quadratic equation yields only one positive root, which is $n_f = 2$. Therefore, $1/\lambda_s = (1/22.79 \times 10^{-9}\ \text{m}) = RZ^2/n_f^2 = RZ^2/4$, which gives $Z = 4$. As a result, the next-to-the-longest wavelength is:

$$1/\lambda = R(4)^2(1/2^2 - 1/4^2) \qquad \text{or} \qquad \boxed{\lambda = 30.39\ \text{nm}}$$

21. ***REASONING*** The orbital quantum number ℓ has values of 0, 1, 2, ..., $(n - 1)$, according to the discussion in Section 30.5. Since $\ell = 5$, we can conclude, therefore, that $n \geq 6$. This knowledge about the principal quantum number n can be used with Equation 30.13, $E_n = -(13.6\text{ eV})Z^2/n^2$, to determine the smallest value for the total energy E_n.

SOLUTION The smallest value of E_n (i.e., the most negative) occurs when $n = 6$. Thus, using $Z = 1$ for hydrogen, we find

$$E_n = -(13.6\text{ eV})\frac{Z^2}{n^2} = -(13.6\text{ eV})\frac{1^2}{6^2} = \boxed{-0.378\text{ eV}}$$

22. ***REASONING AND SOLUTION*** The largest value of ℓ for $n = 4$ is $\ell = n - 1 = 3$. The largest value of m_ℓ for $\ell = 3$ is $m_\ell = 3$. Since m_ℓ runs from minus its largest value to plus that value in integer steps, we have

$$m_\ell = \boxed{-3,\ -2,\ -1,\ 0,\ +1,\ +2,\ +3}$$

23. $\boxed{\text{SSM}}$ $\boxed{\text{WWW}}$ ***REASONING*** The values that ℓ can have depend on the value of n, and only the following integers are allowed: $\ell = 0,\ 1,\ 2,\ \ldots (n-1)$. The values that m_ℓ can have depend on the value of ℓ, with only the following positive and negative integers being permitted: $m_\ell = -\ell,\ \ldots -2, -1, 0, +1, +2, \ldots +\ell$.

SOLUTION Thus, when $n = 6$, the possible values of ℓ are 0, 1, 2, 3, 4, 5. Now when $m_\ell = 2$, the possible values of ℓ are 2, 3, 4, 5, ... These two series of integers overlap for the integers 2, 3, 4, and 5. Therefore, the possible values for the orbital quantum number ℓ that this electron could have are $\boxed{\ell = 2,\ 3,\ 4,\ 5}$.

24. ***REASONING***

a. The ground state is the $n = 1$ state, the first excited state is the $n = 2$ state, and the second excited state is the $n = 3$ state. The total energy (in eV) of a hydrogen atom in the $n = 3$ state is given by Equation 30.13.

b. According to quantum mechanics, the magnitude L of the angular momentum is given by Equation 30.15 as $L = \sqrt{\ell(\ell+1)}(h/2\pi)$, where ℓ is the orbital quantum number. The discussion in Section 30.5 indicates that the maximum value that ℓ can have is one less than the principal quantum number, so that $\ell_{\text{max}} = n - 1$.

c. Equation 30.16 gives the z-component L_z of the angular momentum as $L_z = m_\ell(h/2\pi)$, where m_ℓ is the magnetic quantum number. According to the discussion in Section 30.5, the maximum value that m_ℓ can attain is when it is equal to the orbital quantum number, which is ℓ_{max}.

SOLUTION

a. The total energy of the hydrogen atom is given by Equation 30.13. Using $n = 3$, we have

$$E_3 = -\frac{(13.6\text{ eV})(1)^2}{3^2} = \boxed{-1.51\text{ eV}}$$

b. The maximum orbital quantum number is $\ell_{\text{max}} = n - 1 = 3 - 1 = 2$. The maximum angular momentum L_{max} has a magnitude given by Equation 30.15:

$$L_{\text{max}} = \sqrt{\ell_{\text{max}}(\ell_{\text{max}}+1)}\frac{h}{2\pi} = \sqrt{2(2+1)}\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{2\pi} = \boxed{2.58\times10^{-34}\text{ J}\cdot\text{s}}$$

c. The maximum value for the z-component L_z of the angular momentum is $(\text{with } m_\ell = \ell_{\text{max}} = 2)$

$$L_z = m_\ell\frac{h}{2\pi} = (2)\frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{2\pi} = \boxed{2.11\times10^{-34}\text{ J}\cdot\text{s}}$$

25. ***REASONING AND SOLUTION*** The magnitude of the angular momentum in a hydrogen atom is given by Equation 30.15:

$$L = \sqrt{\ell(\ell+1)}\left(\frac{h}{2\pi}\right)$$

Solving for ℓ and using the given value for L to show that $4\pi^2 L^2 / h^2 = 12$, we find that

$$\ell^2 + \ell - 4\pi^2 L^2/h^2 = \ell^2 + \ell - 12 = 0 \qquad \text{or} \qquad \ell = -4 \text{ or } +3$$

Since ℓ must be positive, we take $\ell = 3$ for this case. The z-component of the angular momentum is given by $L_z = m_\ell h/(2\pi)$. We need to determine the possible values of m_ℓ for the value of ℓ found above. We know that m_ℓ can have values $m_\ell = 0, \pm 1, \pm 2, \ldots, \pm \ell$. Therefore, the possible values of m_ℓ are $m_\ell = 3, 2, 1, 0, -1, -2, -3$. With $h = 6.626 \times 10^{-34}$ J·s, the seven possible values for L_z are, therefore,

$$L_z = 3\left(\frac{h}{2\pi}\right) = \boxed{3.16 \times 10^{-34}\ \text{J}\cdot\text{s}} \qquad \text{or} \qquad L_z = -3\left(\frac{h}{2\pi}\right) = \boxed{-3.16 \times 10^{-34}\ \text{J}\cdot\text{s}}$$

$$L_z = 2\left(\frac{h}{2\pi}\right) = \boxed{2.11 \times 10^{-34}\ \text{J}\cdot\text{s}} \qquad \text{or} \qquad L_z = -2\left(\frac{h}{2\pi}\right) = \boxed{-2.11 \times 10^{-34}\ \text{J}\cdot\text{s}}$$

$$L_z = 1\left(\frac{h}{2\pi}\right) = \boxed{1.05 \times 10^{-34}\ \text{J}\cdot\text{s}} \qquad \text{or} \qquad L_z = -1\left(\frac{h}{2\pi}\right) = \boxed{-1.05 \times 10^{-34}\ \text{J}\cdot\text{s}}$$

$$L_z = 0\left(\frac{h}{2\pi}\right) = \boxed{0\ \text{J}\cdot\text{s}}$$

26. ***REASONING AND SOLUTION***

a. For the angular momentum, Bohr's value is given by Equation 30.8, with $n = 1$,

$$L_n = \frac{nh}{2\pi} = \boxed{\frac{h}{2\pi}}$$

According to quantum theory, the angular momentum is given by Equation 30.15. For $n = 1$, $\ell = 0$

$$L = \sqrt{\ell(\ell+1)}\left(\frac{h}{2\pi}\right) = \sqrt{0(0+1)}\left(\frac{h}{2\pi}\right) = \boxed{0\ \text{J}\cdot\text{s}}$$

b. For $n = 3$; Bohr theory gives

$$L_n = \frac{nh}{2\pi} = \boxed{\frac{3h}{2\pi}}$$

while quantum mechanics gives

[$n = 3$, $\ell = 0$] $$L = \sqrt{\ell(\ell+1)}\left(\frac{h}{2\pi}\right) = \sqrt{0(0+1)}\left(\frac{h}{2\pi}\right) = \boxed{0\ \text{J}\cdot\text{s}}$$

[$n = 3$, $\ell = 1$] $$L = \sqrt{\ell(\ell+1)}\left(\frac{h}{2\pi}\right) = \sqrt{1(1+1)}\left(\frac{h}{2\pi}\right) = \boxed{\frac{\sqrt{2}h}{2\pi}}$$

[$n = 3$, $\ell = 2$] $$L = \sqrt{\ell(\ell+1)}\left(\frac{h}{2\pi}\right) = \sqrt{2(2+1)}\left(\frac{h}{2\pi}\right) = \boxed{\frac{\sqrt{6}h}{2\pi}}$$

27. **SSM** **WWW** ***REASONING*** Let θ denote the angle between the angular momentum L and its z-component L_z. We can see from the figure at the right that $L_z = L\cos\theta$. Using Equation 30.16 for L_z and Equation 30.15 for L, we have

$$\cos\theta = \frac{L_z}{L} = \frac{m_\ell h/(2\pi)}{\sqrt{\ell(\ell+1)}\left[h/(2\pi)\right]} = \frac{m_\ell}{\sqrt{\ell(\ell+1)}}$$

The smallest value for θ corresponds to the largest value of cos θ. For a given value of ℓ, the largest value for cos θ corresponds to the largest value for m_ℓ. But the largest possible value for m_ℓ is $m_\ell = \ell$. Therefore, we find that

$$\cos\theta = \frac{\ell}{\sqrt{\ell(\ell+1)}} = \sqrt{\frac{\ell}{\ell+1}}$$

SOLUTION The smallest value for θ corresponds to the largest value for ℓ. When the electron is in the $n = 5$ state, the largest allowed value of ℓ is $\ell = 4$; therefore, we see that

$$\cos\theta = \sqrt{\frac{\ell}{\ell+1}} = \sqrt{\frac{4}{4+1}} \qquad \text{or} \qquad \theta = \cos^{-1}\left(\sqrt{4/5}\right) = \boxed{26.6°}$$

28. ***REASONING AND SOLUTION***

$$\boxed{1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^1}$$

29. ***REASONING AND SOLUTION*** According to Figure 30.17, the energy sublevel with $n = 4$, $\ell = 0$, which corresponds to the notation 4s, is lower in energy than the $n = 3$, $\ell = 2$, (3d) energy sublevel. Thus, the 4s energy sublevel will be filled before the 3d energy sublevel. Therefore, using Figure 30.17 as a guide, we find that the ground state electronic configuration of arsenic ($Z = 33$) is

$$\boxed{1\text{s}^2\ 2\text{s}^2\ 2\text{p}^6\ 3\text{s}^2\ 3\text{p}^6\ 4\text{s}^2\ 3\text{d}^{10}\ 4\text{p}^3}$$

30. ***REASONING***

a. The maximum number of electrons that the $n = 1$, $\ell = 0$ subshell can contain is 2 (see Figure 30.17). If the atom is in its ground state, the third electron must go into the $n = 2$, $\ell = 0$ subshell. For this subshell, the magnetic quantum number can only be zero ($m_\ell = 0$), and the electron can have a spin quantum number of either $m_s = +\frac{1}{2}$ or $-\frac{1}{2}$.

b. If the atom is in its first excited state, the third electron must go into the $n = 2$, $\ell = 1$ subshell. For $\ell = 1$, there are three possibilities for m_ℓ: $m_\ell = +1$, 0, and –1. For each value of m_ℓ, the electron can have a spin quantum number of either $m_s = +\frac{1}{2}$ or $-\frac{1}{2}$.

SOLUTION

a. According to the discussion in the Reasoning section, the third electron can have one of two possibilities for the four quantum numbers: $\boxed{n = 2, \ell = 0, m_\ell = 0, m_s = +\frac{1}{2}}$ and $\boxed{n = 2, \ell = 0, m_\ell = 0, m_s = -\frac{1}{2}}$.

b. When the third electron is in the first excited state, there are six possibilities for the four quantum numbers:

$$\boxed{n = 2, \ell = 1, m_\ell = +1, m_s = +\tfrac{1}{2}},\ \boxed{n = 2, \ell = 1, m_\ell = +1, m_s = -\tfrac{1}{2}},$$

$$\boxed{n = 2, \ell = 1, m_\ell = 0, m_s = +\tfrac{1}{2}},\ \boxed{n = 2, \ell = 1, m_\ell = 0, m_s = -\tfrac{1}{2}},$$

$$\boxed{n = 2, \ell = 1, m_\ell = -1, m_s = +\tfrac{1}{2}},\ \boxed{n = 2, \ell = 1, m_\ell = -1, m_s = -\tfrac{1}{2}}.$$

31. **SSM** ***REASONING*** In the theory of quantum mechanics, there is a selection rule that restricts the initial and final values of the orbital quantum number ℓ. The selection rule states that when an electron makes a transition between energy levels, the value of ℓ may not remain the same or increase or decrease by more than one. In other words, the rule requires that $\Delta\ell = \pm 1$.

SOLUTION

a. For the transition $2\text{s} \rightarrow 1\text{s}$, the electron makes a transition from the 2s state ($n = 2$, $\ell = 0$) to the 1s state ($n = 1$, $\ell = 0$). Since the value of ℓ is the same in both states, $\Delta\ell = 0$, and we can conclude that this energy level transition is $\boxed{\text{not permitted}}$.

b. For the transition $2\text{p} \rightarrow 1\text{s}$, the electron makes a transition from the 2p state ($n = 2$, $\ell = 1$) to the 1s state ($n = 1$, $\ell = 0$). The value of ℓ changes so that $\Delta\ell = 0 - 1 = -1$, and we can conclude that this energy level transition is $\boxed{\text{permitted}}$.

c. For the transition $4\text{p} \rightarrow 2\text{p}$, the electron makes a transition from the 4p state ($n = 4$, $\ell = 1$) to the 2p state ($n = 2$, $\ell = 1$). Since the value of ℓ is the same in both states, $\Delta\ell = 0$, and we can conclude that this energy level transition is $\boxed{\text{not permitted}}$.

d. For the transition $4\text{s} \rightarrow 2\text{p}$, the electron makes a transition from the 4s state ($n = 4$, $\ell = 0$) to the 2p state ($n = 2$, $\ell = 1$). The value of ℓ changes so that $\Delta\ell = 1 - 0 = +1$, and we can conclude that this energy level transition is $\boxed{\text{permitted}}$.

e. For the transition $3\text{d} \rightarrow 3\text{s}$, the electron makes a transition from the 3d state ($n = 3$, $\ell = 2$) to the 3s state ($n = 3$, $\ell = 0$). The value of ℓ changes so that $\Delta\ell = 0 - 2 = -2$, and we can conclude that this energy level transition is $\boxed{\text{not permitted}}$.

32. ***REASONING AND SOLUTION*** Following the order of filling of subshells given in Figure 30.17, the required condition is found for the following electron configuration:

$$1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,4s^2\,3d^{10}\,4p^6\,5s^2$$

In this configuration there are 10 electrons in the 3d subshell. In each of the 1s, 2s, 3s, 4s, and 5s subshells there are 2 electrons, for a total of 10. To identify the atom, we note that the total number of electrons in all of the subshells is 38. According to the periodic table, the atom with Z = 38 is $\boxed{\text{strontium, Sr}}$.

33. **SSM** ***REASONING*** This problem is similar to Example 10 in the text. We use Equation 30.14 with the initial value of n being $n_i = 2$, and the final value being $n_f = 1$. As in Example 10, we use a value of Z that is one less than the atomic number of the atom in question (in this case, a value of $Z = 41$ rather than 42); this accounts approximately for the shielding effect of the single K-shell electron in canceling out the attraction of one nuclear proton.

SOLUTION Using Equation 30.14, we obtain

$$\frac{1}{\lambda} = (1.097 \times 10^7 \text{ m}^{-1})(41)^2 \left(\frac{1}{1^2} - \frac{1}{2^2}\right) \qquad \text{or} \qquad \boxed{\lambda = 7.230 \times 10^{-11} \text{ m}}$$

34. ***REASONING AND SOLUTION*** From Equation 30.14 we have

$$\frac{1}{\lambda} = R(Z-1)^2\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

Solving for Z we obtain

$$Z = \sqrt{\frac{1/(\lambda R)}{\left(1/n_f^2\right) - \left(1/n_i^2\right)}} + 1 = \sqrt{\frac{\left[1/\left(4.5\times10^{-9}\ \text{m}\right)\left(1.097\times10^{7}\ \text{m}^{-1}\right)\right]}{\left(1/1^2\right) - \left(1/2^2\right)}} + 1 = 6.2$$

Therefore, the element is likely to be $\boxed{\text{carbon}}$ (Z = 6).

35. ***REASONING AND SOLUTION*** 25.0% of the incident kinetic energy means that the energy available for producing X-rays is

$$E = (0.250)(1.60 \times 10^{-19}\ \text{C})(45.0 \times 10^{3}\ \text{V}) = 1.80 \times 10^{-15}\ \text{J}$$

The wavelength is then

$$\lambda = hc/E = (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})(3.00 \times 10^{8}\ \text{m/s})/(1.80 \times 10^{-15}\ \text{J}) = \boxed{1.11 \times 10^{-10}\ \text{m}}$$

36. ***REASONING AND SOLUTION*** To knock a K-shell electron out of a copper atom, we take $n_f = 1$ and n_i = infinity. Because of shielding, take $Z = 28$ (rather than $Z = 29$), and use the Bohr model to find $E = hc/\lambda = hcRZ^2/n_f^2$.

$$E = (6.63 \times 10^{-34}\ \text{J s})(3.00 \times 10^{8}\ \text{m/s})(1.097 \times 10^{7}\ \text{m}^{-1})(28)^2 = 1.71 \times 10^{-15}\ \text{J}$$

The minimum potential difference needed is

$$V = E/e = (1.71 \times 10^{-15}\ \text{J})/(1.60 \times 10^{-19}\ \text{C}) = \boxed{10\ 700\text{V}}$$

37. **SSM** ***REASONING*** In the spectrum of X-rays produced by the tube, the cutoff wavelength λ_0 and the voltage V of the tube are related according to Equation 30.17, $V = hc/(e\lambda_0)$. Since the voltage is increased from zero until the K_α X-ray just appears in the spectrum, it follows that $\lambda_0 = \lambda_\alpha$ and $V = hc/(e\lambda_\alpha)$. Using Equation 30.14 for $1/\lambda_\alpha$, we find that

$$V = \frac{hc}{e\lambda_\alpha} = \frac{hcR(Z-1)^2}{e}\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

In this expression we have replaced Z with $Z-1$, in order to account for shielding, as explained in Example 10 in the text.

SOLUTION The desired voltage is, then,

$$V = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)\left(1.097\times10^{7}\ \text{m}^{-1}\right)(47-1)^2}{\left(1.60\times10^{-19}\ \text{C}\right)}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \boxed{21\,600\ \text{V}}$$

38. ***REASONING*** The kinetic energy of each electron is given by Equation 6.2 as $\frac{1}{2}mv^2$. If all of this energy goes into producing an X-ray photon, the photon has the highest possible energy. The energy E of a photon is related to its frequency f by Equation 29.2 as $E = hf$. We can express the frequency in terms of the wavelength λ by using Equation 16.1, $f = c/\lambda$. Combining these relations gives

$$\underbrace{\tfrac{1}{2}mv^2}_{\substack{\text{Kinetic energy}\\ \text{of a photon}}} = hf = \frac{hc}{\lambda}$$

SOLUTION Solving the expression above for the speed of an electron just before it strikes the target, we find that

$$v = \sqrt{\frac{2hc}{m\lambda}} = \sqrt{\frac{2\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.20\times10^{-10}\ \text{m}\right)}} = \boxed{6.03\times10^{7}\ \text{m/s}}$$

39. ***REASONING AND SOLUTION***

$$E = hf = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{193\times10^{-9}\ \text{m}} = \boxed{1.03\times10^{-18}\ \text{J}}$$

40. ***REASONING***

a. Average power is energy per unit time (see Equation 6.10b). Thus, the energy of a pulse is the product of the average power and its time of duration.

b. The number of photons in each pulse is the energy of a pulse divided by the energy of a single photon. According to Equations 29.2 and 16.1, the energy of a photon is equal to hc/λ, where λ is its wavelength.

SOLUTION

a. The energy E of a pulse is the product of the average power $\overline{P}$ and its duration time t:

$$E = \overline{P}t = \left(5.00 \times 10^{-3}\ \text{W}\right)\left(25.0 \times 10^{-3}\ \text{s}\right) = \boxed{1.25 \times 10^{-4}\ \text{J}}$$

b. The number N of photons in each pulse is the energy of a pulse divided by the energy of a single photon:

$$N = \frac{E}{\left(\dfrac{hc}{\lambda}\right)} = \frac{1.25 \times 10^{-4}\ \text{J}}{\dfrac{\left(6.63 \times 10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00 \times 10^{8}\ \text{m/s}\right)}{633 \times 10^{-9}\ \text{m}}} = \boxed{3.98 \times 10^{14}}$$

41. SSM WWW ***REASONING*** The number of photons emitted by the laser will be equal to the total energy carried in the beam divided by the energy per photon.

SOLUTION The total energy carried in the beam is, from the definition of power,

$$E_{\text{total}} = Pt = (1.5\ \text{W})(0.050\ \text{s}) = 0.075\ \text{J}$$

The energy of a single photon is given by Equations 29.2 and 16.1 as

$$E_{\text{photon}} = hf = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})(3.00 \times 10^{8}\ \text{m/s})}{514 \times 10^{-9}\ \text{m}} = 3.87 \times 10^{-19}\ \text{J}$$

where we have used the fact that $514\ \text{nm} = 514 \times 10^{-9}\ \text{m}$. Therefore, the number of photons emitted by the laser is

$$\frac{E_{\text{total}}}{E_{\text{photon}}} = \frac{0.075\ \text{J}}{3.87 \times 10^{-19}\ \text{J/photon}} = \boxed{1.9 \times 10^{17}\ \text{photons}}$$

42. ***REASONING AND SOLUTION*** According to Equations 29.2 and 16.1, $E = hc/\lambda$, the energy of a single photon of the dye laser is

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})(3.00 \times 10^{8}\ \text{m/s})}{585 \times 10^{-9}\ \text{m}} = 3.40 \times 10^{-19}\ \text{J}$$

while the energy of a single photon from the CO_2 laser is

$$E = \frac{hc}{\lambda} = \frac{(6.63 \times 10^{-34}\ \text{J} \cdot \text{s})(3.00 \times 10^{8}\ \text{m/s})}{1.06 \times 10^{-5}\ \text{m}} = 1.88 \times 10^{-20}\ \text{J}$$

The ratio of these two photon energies is

$$\frac{3.40 \times 10^{-19}\ \text{J}}{1.88 \times 10^{-20}\ \text{J}} = 18.1$$

We can only have a whole number of photons, therefore, the minimum number of photons that the carbon dioxide laser must produce to deliver at least as much energy to a target as does a single photon from the dye laser is $\boxed{19}$.

43. ***REASONING AND SOLUTION*** For either laser, the number of photons is given by the energy E produced divided by the energy per photon. Since power P is energy per unit time, the energy produced in a time t is $E = Pt$. For a photon frequency f the energy per photon is hf. But the photon frequency f and wavelength λ are related to the speed of light c by $f = c/\lambda$. Therefore, the energy per photon is hc/λ. The number of photons is, then,

$$\text{Number of photons} = \frac{\text{Energy produced}}{\text{Energy per photon}} = \frac{Pt}{hc/\lambda} = \frac{Pt\lambda}{hc}$$

Since each laser produces the same number of photons, it follows that

$$\left(\frac{Pt\lambda}{hc}\right)_{\text{He/Ne}} = \left(\frac{Pt\lambda}{hc}\right)_{\text{SS}} \qquad \text{or} \qquad t_{\text{He/Ne}} = \frac{(Pt\lambda)_{\text{SS}}}{(P\lambda)_{\text{He/Ne}}}$$

where the "He/Ne" and "SS" refer to the helium/neon and solid state lasers, respectively. The time for the helium/neon laser is, then,

$$t_{\text{He/Ne}} = \frac{(Pt\lambda)_{\text{SS}}}{(P\lambda)_{\text{He/Ne}}} = \frac{\left(1.0\times10^{14}\ \text{W}\right)\left(1.1\times10^{-11}\ \text{s}\right)(1060\ \text{nm})}{\left(1.0\times10^{-3}\ \text{W}\right)(633\ \text{nm})} = 1.8\times10^{6}\ \text{s}$$

This is a time of $\left(1.8\times10^{6}\ \text{s}\right)\left[(1\ \text{day})/\left(8.64\times10^{4}\ \text{s}\right)\right] = \boxed{21\ \text{days}}$.

44. ***REASONING AND SOLUTION*** According to Figure 30.17, the energy sublevel with $n = 4$, $\ell = 0$, which corresponds to the notation $4s$, is lower in energy than the $n = 3$, $\ell = 2$, $(3d)$ energy sublevel. Thus, the $4s$ energy sublevel will be filled before the $3d$ energy sublevel. Therefore, using Figure 30.17 as a guide, we find that the ground state electronic configuration of manganese ($Z = 25$) is

$$\boxed{1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^5}$$

45. **SSM** ***REASONING AND SOLUTION*** In the ground state, the electron in a hydrogen atom is in the $n = 1$ energy level. To ionize such a hydrogen atom, the electron must be taken from the $n = 1$ level to the level for which n is infinity. The electromagnetic radiation needed to accomplish this corresponds to a Lyman line in which the final state n is taken to be infinity. Equation 30.1 gives the reciprocal wavelength for the Lyman series. With $n = \infty$, Equation 30.1 reveals that for the longest wavelength

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right) = (1.097 \times 10^7 \text{ m}^{-1})\left(\frac{1}{1^2} - \frac{1}{\infty^2}\right) \qquad \text{or} \qquad \boxed{\lambda = 91.2 \text{ nm}}$$

46. ***REASONING AND SOLUTION*** According to Equation 30.4, the energy for a K_α X-ray photon is $E_2 - E_1 = hf$. But since the frequency f is related to the wavelength λ and the speed of light c by $f = c/\lambda$, it follows that $E_2 - E_1 = hc/\lambda$. Using the Bohr expression for $1/\lambda$ given in Equation 30.14, we find that the photon energy is given by

$$E_2 - E_1 = hcR(Z-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

In this expression we have replaced Z with $Z - 1$, in order to account for shielding, as explained in Example 10 in the text. The photon energy is, then,

$$E_2 - E_1 = \left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)\left(1.097\times10^{7}\ \text{m}^{-1}\right)(82-1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$$

$$= \boxed{1.07\times10^{-14}\ \text{J}}$$

47. ***REASONING AND SOLUTION*** The table below lists the possible sets of the four quantum numbers that correspond to the electrons in a completely filled $4f$ subshell:

n	ℓ	m_ℓ	m_s
4	3	3	1/2
4	3	3	–1/2
4	3	2	1/2
4	3	2	–1/2
4	3	1	1/2
4	3	1	–1/2
4	3	0	1/2
4	3	0	–1/2
4	3	–1	1/2
4	3	–1	–1/2
4	3	–2	1/2
4	3	–2	–1/2
4	3	–3	1/2
4	3	–3	–1/2

48. ***REASONING AND SOLUTION***

a. The energy difference between the two K_β energy levels is $E = hf = hc/\lambda$.

$$E = (6.63 \times 10^{-34}\ \text{J·s})(3.00 \times 10^{8}\ \text{m/s})/(1.84 \times 10^{-11}\ \text{m}) = \boxed{1.08 \times 10^{-14}\ \text{J}}$$

b. Converting to electron volts, we have

$$(1.08 \times 10^{-14}\ \text{J})(1\ \text{eV})/(1.60 \times 10^{-19}\ \text{J}) = \boxed{6.75 \times 10^{4}\ \text{eV}}$$

49. $\boxed{\text{SSM}}$ ***REASONING*** According to the Bohr model, the energy E_n (in eV) of the electron in an orbit is given by Equation 30.13: $E_n = -13.6\left(Z^2/n^2\right)$. In order to find the principal quantum number of the state in which the electron in a doubly ionized lithium atom Li^{2+} has the same total energy as a ground state electron in a hydrogen atom, we equate the right hand sides of Equation 30.13 for the hydrogen atom and the lithium ion. This gives

$$-(13.6)\left(\frac{Z^2}{n^2}\right)_{\text{H}} = -(13.6)\left(\frac{Z^2}{n^2}\right)_{\text{Li}} \qquad \text{or} \qquad n_{\text{Li}}^2 = \left(\frac{n^2}{Z^2}\right)_{\text{H}} Z_{\text{Li}}^2$$

This expression can be evaluated to find the desired principal quantum number.

SOLUTION For hydrogen, $Z = 1$, and $n = 1$ for the ground state. For lithium Li^{2+}, $Z = 3$. Therefore,

$$n_{\text{Li}}^2 = \left(\frac{n^2}{Z^2}\right)_{\text{H}} Z_{\text{Li}}^2 = \left(\frac{1^2}{1^2}\right)\left(3^2\right) \qquad \text{or} \qquad \boxed{n_{\text{Li}} = 3}$$

50. ***REASONING*** The energy levels and radii of a hydrogenic species of atomic number Z are given by Equations 30.13 and 30.10, respectively: $E_n = -(13.6\text{ eV})(Z^2/n^2)$ and $r_n = (5.29\times10^{-11}\text{ m})(n^2/Z)$. We can use Equation 30.13 to find the value of Z for the unidentified ionized atom and then calculate the radius of the $n = 5$ orbit using Equation 30.10.

SOLUTION Solving Equation 30.13 for atomic number Z of the unknown species, we have

$$Z = \sqrt{\frac{E_n n^2}{-13.6\text{ eV}}} = \sqrt{\frac{(-30.6\text{ eV})(2)^2}{-13.6\text{ eV}}} = 3$$

Therefore, the radius of the $n = 5$ orbit is

$$r_5 = (5.29\times10^{-11}\text{ m})\left(\frac{5^2}{3}\right) = \boxed{4.41\times10^{-10}\text{ m}}$$

51. ***REASONING AND SOLUTION*** We can first determine the maximum magnetic quantum number using Equation 30.16, i.e., $L_z = m_\ell h/(2\pi)$, so that

$$m_\ell = 2\pi L_z/h = 2\pi\,(2.11 \times 10^{-34}\ \text{J·s})/(6.626 \times 10^{-34}\ \text{J·s}) = 2$$

Therefore, we can have $\ell \geq 2$, and $n \geq 3$. From Equation 30.13, we have

[*n* = 3] $E = (-\,13.6\ \text{eV})/3^2 = \boxed{-1.51\ \text{eV}}$

[*n* = 4] $E = (-\,13.6\ \text{eV})/4^2 = \boxed{-0.850\ \text{eV}}$

[*n* = 5] $E = (-\,13.6\ \text{eV})/5^2 = \boxed{-0.544\ \text{eV}}$

52. ***REASONING AND SOLUTION*** From the diagram we see that an integral number of half wavelengths fit into the "box". That is, $n\lambda/2 = L$, where $n = 1, 2, 3, \ldots$ Using the de Broglie equation yields $\lambda = h/(mv)$. Combining these two expressions yields, $nh/(2mv) = L$, and rearranging gives the velocity as $v = nh/(2mL)$. Finally, the kinetic energy can be written

$$\text{KE} = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(\frac{nh}{2mL}\right)^2$$

$$\boxed{\text{KE} = \frac{n^2h^2}{8mL^2} \quad \text{where} \quad n = 1,2,3,\ldots}$$

53. **SSM** ***REASONING*** A wavelength of 410.2 nm is emitted by the hydrogen atoms in a high-voltage discharge tube. This transition lies in the visible region (380–750 nm) of the hydrogen spectrum. Thus, we can conclude that the transition is in the Balmer series and, therefore, that $n_f = 2$. The value of n_i can be found using Equation 30.14, according to which the Balmer series transitions are given by

$$\frac{1}{\lambda} = R\left(1^2\right)\left(\frac{1}{2^2} - \frac{1}{n_i^2}\right) \qquad n = 3, 4, 5, \ldots$$

This expression may be solved for n_i for the energy transition that produces the given wavelength.

SOLUTION Solving for n_i, we find that

$$n_i = \frac{1}{\sqrt{\frac{1}{2^2} - \frac{1}{R\lambda}}} = \frac{1}{\sqrt{\frac{1}{2^2} - \frac{1}{(1.097 \times 10^7\ \text{m}^{-1})(410.2 \times 10^{-9}\ \text{m})}}} = 6$$

Therefore, the initial and final states are identified by $\boxed{n_i = 6 \text{ and } n_f = 2}$.

54. ***REASONING AND SOLUTION***

a. To find an expression for v_n, we use Equation 30.8, $L_n = mv_n r_n = nh/(2\pi)$, and substitute for r_n from Equation 30.9:

$$mv_n\left(\frac{h^2n^2}{4\pi^2 mke^2 Z}\right) = \frac{nh}{2\pi} \qquad \text{or} \qquad \boxed{v_n = \frac{2\pi ke^2 Z}{nh}}$$

b. For the hydrogen atom ($Z = 1$) in the $n = 1$ orbit,

$$v_n = 2\pi(8.99 \times 10^9\ \text{N·m}^2/\text{C}^2)(1.602 \times 10^{-19}\ \text{C})^2/(6.626 \times 10^{-34}\ \text{J·s}) = \boxed{2.19 \times 10^6\ \text{m/s}}$$

c. For the $n = 2$ orbit of hydrogen,

$$v_n = 2\pi k e^2 Z/(2h) = \boxed{1.09 \times 10^6\ \text{m/s}}$$

d. The speeds found in parts (b) and (c) are well below the speed of light, 3.0×10^8 m/s, and $\boxed{\text{are consistent with ignoring relativistic effects}}$.

1. **SSM** ***REASONING*** For an element whose chemical symbol is X, the symbol for the nucleus is ${}^{A}_{Z}\mathrm{X}$ where A represents the number of protons and neutrons (the nucleon number) and Z represents the number of protons (the atomic number) in the nucleus.

SOLUTION

a. The symbol ${}^{195}_{78}\mathrm{X}$ indicates that the nucleus in question contains $Z = 78$ protons, and $N = A - Z = 195 - 78 = \boxed{117 \text{ neutrons}}$. From the periodic table, we see that $Z = 78$ corresponds to $\boxed{\text{platinum, Pt}}$.

b. Similar reasoning indicates that the nucleus in question is $\boxed{\text{sulfur, S}}$, and the nucleus contains $N = A - Z = 32 - 16 = \boxed{16 \text{ neutrons}}$.

c. Similar reasoning indicates that the nucleus in question is $\boxed{\text{copper, Cu}}$, and the nucleus contains $N = A - Z = 63 - 29 = \boxed{34 \text{ neutrons}}$.

d. Similar reasoning indicates that the nucleus in question is $\boxed{\text{boron, B}}$, and the nucleus contains $N = A - Z = 11 - 5 = \boxed{6 \text{ neutrons}}$.

e. Similar reasoning indicates that the nucleus in question is $\boxed{\text{plutonium, Pu}}$, and the nucleus contains $N = A - Z = 239 - 94 = \boxed{145 \text{ neutrons}}$.

2. ***REASONING AND SOLUTION***

a. An element X can be represented by ${}^{A}_{Z}\mathrm{X}$, where A is the nucleon number, and Z is the atomic number (number of protons). In addition, $A = Z + N$, where N is the number of neutrons in the nucleus. Therefore, ${}^{18}_{8}\mathrm{O}$ has $\boxed{\text{8 protons and 10 neutrons}}$.

b. Similarly, ${}^{120}_{50}\mathrm{Sn}$ has $\boxed{\text{50 protons and 70 neutrons}}$.

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3. ***REASONING AND SOLUTION*** Solving Equation 31.2 for A gives

$$A = r^3/(1.2 \times 10^{-15}\ \text{m})^3$$

If r is doubled, then A will increase by a factor of $2^3 = \boxed{8}$

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4. ***REASONING*** For an element whose chemical symbol is X, the symbol for the nucleus is ${}^{A}_{Z}\mathrm{X}$, where A represents the total number of protons and neutrons (the nucleon number) and Z represents the number of protons in the nucleus (the atomic number). The number of neutrons N is related to A and Z by Equation 31.1: $A = Z + N$.

SOLUTION For the nucleus ${}^{208}_{82}\mathrm{Pb}$, we have $Z = 82$ and $A = 208$.

a. The net electrical charge of the nucleus is equal to the total number of protons multiplied by the charge on a single proton. Since the ${}^{208}_{82}\mathrm{Pb}$ nucleus contains 82 protons, the net electrical charge of the ${}^{208}_{82}\mathrm{Pb}$ nucleus is

$$q_{\text{net}} = (82)(+1.60\times10^{-19}\ \mathrm{C}) = \boxed{+1.31\times10^{-17}\ \mathrm{C}}$$

b. The number of neutrons is $N = A - Z = 208 - 82 = \boxed{126}$.

c. By inspection, the number of nucleons is $A = \boxed{208}$.

d. The approximate radius of the nucleus can be found from Equation 31.2, namely

$$r = (1.2\times10^{-15}\ \mathrm{m})\,A^{1/3} = (1.2\times10^{-15}\ \mathrm{m})(208)^{1/3} = \boxed{7.1\times10^{-15}\ \mathrm{m}}$$

e. The nuclear density is the mass per unit volume of the nucleus. The total mass of the nucleus can be found by multiplying the mass m_{nucleon} of a single nucleon by the total number A of nucleons in the nucleus. Treating the nucleus as a sphere of radius r, the nuclear density is

$$\rho = \frac{m_{\text{total}}}{V} = \frac{m_{\text{nucleon}}A}{\frac{4}{3}\pi r^3} = \frac{m_{\text{nucleon}}A}{\frac{4}{3}\pi\left[(1.2\times10^{-15}\ \mathrm{m})\,A^{1/3}\right]^3} = \frac{m_{\text{nucleon}}}{\frac{4}{3}\pi(1.2\times10^{-15}\ \mathrm{m})^3}$$

Therefore,

$$\rho = \frac{1.67\times10^{-27}\ \mathrm{kg}}{\frac{4}{3}\pi(1.2\times10^{-15}\ \mathrm{m})^3} = \boxed{2.3\times10^{17}\ \mathrm{kg/m^3}}$$

5. SSM WWW ***REASONING AND SOLUTION*** The surface area of a sphere is $\text{Area} = 4\pi r^2$. But according to Equation 31.2, the radius of a nucleus in meters is $r = \left(1.2 \times 10^{-15}\ \text{m}\right) A^{1/3}$, where A is the nucleon number. With this expression for r, the surface area becomes $\text{Area} = 4\pi\left(1.2 \times 10^{-15}\ \text{m}\right)^2 A^{2/3}$. The ratio of the largest to the smallest surface area is, then,

$$\frac{\text{Largest area}}{\text{Smallest area}} = \frac{4\pi\left(1.2 \times 10^{-15}\ \text{m}\right)^2 A_{\text{largest}}^{2/3}}{4\pi\left(1.2 \times 10^{-15}\ \text{m}\right)^2 A_{\text{smallest}}^{2/3}} = \frac{209^{2/3}}{1^{2/3}} = \boxed{35.2}$$

6. ***REASONING*** The nucleus is roughly spherical, so its volume is $V = \frac{4}{3}\pi r^3$. The radius r is given by Equation 31.2 as $r \approx (1.2 \times 10^{-15}\ \text{m})A^{1/3}$, where A is the atomic mass number or nucleon number. Therefore, the volume is given by $V = \frac{4}{3}\pi\left(1.2 \times 10^{-15}\ \text{m}\right)^3 A$ and is proportional to A. We can apply this expression to the unknown nucleus and to the nickel nucleus, knowing that the ratio of the two volumes is 2:1. This ratio provides the solution we seek.

SOLUTION Applying the expression for the volume to each nucleus gives

$$\frac{V}{V_{\text{Ni}}} = \frac{\frac{4}{3}\pi\left(1.2 \times 10^{-15}\ \text{m}\right)^3 A}{\frac{4}{3}\pi\left(1.2 \times 10^{-15}\ \text{m}\right)^3 (60)} = \frac{A}{60} = 2 \quad \text{or} \quad A = 120$$

The nucleon number is equal to the number of neutrons N plus the number of protons or the atomic number Z, so $A = N + Z$. Therefore, the atomic number for the unknown nucleus is

$$Z = A - N = 120 - 70 = 50$$

The unknown nucleus, then, is ${}^{120}_{50}\text{X}$. Reference to the periodic table reveals that the element that has an atomic number of $Z = 50$ is tin (Sn). Thus, $\boxed{{}^{A}_{Z}\text{X} = {}^{120}_{50}\text{Sn}}$.

7. ***REASONING AND SOLUTION*** For isotope X, $A_X = Z + N = Z + Z = 2Z$ (equal number of protons and neutrons, $Z = N$). For isotope Y, $A_Y = Z + N' = Z + 2N = Z + 2Z = 3Z$. The radii are related by

$$\frac{r_Y}{r_X} = \left(\frac{A_Y}{A_X}\right)^{1/3} = \left(\frac{3Z}{2Z}\right)^{1/3} = \left(\frac{3}{2}\right)^{1/3} = \boxed{1.14}$$

8. ***REASONING AND SOLUTION***

a. The volume of the nucleus is found using

$$V = (4/3)\pi r^3 = (4/3)\pi(1.2 \times 10^{-15}\ \text{m})^3\ A = (7.2 \times 10^{-45}\ \text{m}^3)\ A$$

The mass of the nucleus is $m = (1.67 \times 10^{-27}\ \text{kg})\ A$, so the density of the nucleus is

$$\rho = m/V = \boxed{2.3 \times 10^{17}\ \text{kg/m}^3}$$

b. The mass is

$$m = \rho V = (2.3 \times 10^{17}\ \text{kg/m}^3)(4/3)\pi(2.3 \times 10^{-3}\ \text{m})^3 = \boxed{1.2 \times 10^{10}\ \text{kg}}$$

c. The number of supertankers would be $N = (1.2 \times 10^{10}\ \text{kg})/(1.5 \times 10^{8}\ \text{kg}) = \boxed{80}$

---

9. SSM WWW ***REASONING*** According to Equation 31.2, the radius of a nucleus in meters is $r = \left(1.2 \times 10^{-15}\ \text{m}\right) A^{1/3}$, where A is the nucleon number. If we treat the neutron star as a uniform sphere, its density (Equation 11.1) can be written as

$$\rho = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi r^3}$$

Solving for the radius r, we obtain,

$$r = \sqrt[3]{\frac{M}{\frac{4}{3}\pi\rho}}$$

This expression can be used to find the radius of a neutron star of mass M and density ρ.

SOLUTION As discussed in Conceptual Example 1, nuclear densities have the same approximate value in all atoms. If we consider a uniform spherical nucleus, then the density of nuclear matter is approximately given by

$$\rho = \frac{M}{V} \approx \frac{A \times (\text{mass of a nucleon})}{\frac{4}{3}\pi r^3} = \frac{A \times (\text{mass of a nucleon})}{\frac{4}{3}\pi\left[(1.2 \times 10^{-15}\ \text{m})\, A^{1/3}\right]^3}$$

$$= \frac{1.67 \times 10^{-27}\ \text{kg}}{\frac{4}{3}\pi(1.2 \times 10^{-15}\ \text{m})^3} = 2.3 \times 10^{17}\ \text{kg/m}^3$$

Substituting values into the expression for r determined above, we have

$$r = \sqrt[3]{\frac{(0.40)(1.99 \times 10^{30}\ \text{kg})}{\frac{4}{3}\pi(2.3 \times 10^{17}\ \text{kg/m}^3)}} = \boxed{9.4 \times 10^3\ \text{m}}$$

10. ***REASONING AND SOLUTION***

a. The total mass of the separated nucleons is

$$m = 27(1.007\ 825\ \text{u}) + 32(1.008\ 665\ \text{u}) = 59.488\ 555\ \text{u}$$

The mass defect is then

$$\Delta m = 59.488\ 555\ \text{u} - 58.933\ 198\ \text{u} = \boxed{0.555\ 357\ \text{u}}$$

b. Expressed in kilograms we have

$$\Delta m = (0.555\ 357\ \text{u})\left(\frac{1.6605 \times 10^{-27}\ \text{kg}}{1\ \text{u}}\right) = \boxed{9.2217 \times 10^{-28}\ \text{kg}}$$

---

11. **SSM** ***REASONING AND SOLUTION*** The symbol ${}^{16}_{8}\mathrm{O}$ indicates that the oxygen nucleus contains $A = 16$ nucleons. According to the binding energy per nucleon curve in Figure 31.6, the corresponding binding energy per nucleon (for $A = 16$) is 8.00 MeV/nucleon. Therefore, the total binding energy of ${}^{16}_{8}\mathrm{O}$ is

$$(16 \text{ nucleons})\left(\frac{8.00 \text{ MeV}}{\text{nucleon}}\right) = \boxed{128 \text{ MeV}}$$

12. ***REASONING AND SOLUTION*** $^{27}_{13}\text{Al}$ contains

$$Z = 13 \text{ protons and } N = A - Z = 27 - 13 = 14 \text{ neutrons}$$

The mass of a hydrogen atom is 1.007 825 u. The mass of an aluminum atom is given as 26.981 539 u. The mass of a neutron is given in Table 31.1 as 1.008 665 u. Thus, the mass defect Δm is

$$\Delta m = 13\ (1.007\ 825\ \text{u}) + 14\ (1.008\ 665\ \text{u}) - 26.981\ 539\ \text{u} = 0.241\ 496\ \text{u}$$

Since 1 u = 931.5 MeV, this mass defect corresponds to a binding energy of

$$(0.241\,496\ \text{u})\left(\frac{931.5\ \text{MeV}}{1\ \text{u}}\right) = \boxed{225.0\ \text{MeV}}$$

13. ***REASONING AND SOLUTION*** The earth revolves around the sun, and the two represent a bound system that has a binding energy of 2.6×10^{33} J. Therefore, to completely separate the earth and the sun so that they are infinitely far apart requires an amount of energy equal to the binding energy, namely $\Delta E_0 = 2.6 \times 10^{33}$ J.

Any change in the energy of a system causes a change in the mass of the system according to $\Delta E_0 = (\Delta m)c^2$ (see Section 28.6). Therefore, the change in mass of the earth-sun system when its energy changes by an amount $\Delta E_0 = 2.6 \times 10^{33}$ J is given by

$$\Delta m = \frac{\Delta E_0}{c^2} = \frac{2.6 \times 10^{33}\ \text{J}}{(3.0 \times 10^8\ \text{m/s})^2} = 2.9 \times 10^{16}\ \text{kg}$$

Therefore, the difference between the mass of the separated system and that of the bound system is $\boxed{2.9 \times 10^{16}\ \text{kg}}$.

14. ***REASONING*** The atomic mass given for ${}^{206}_{82}\text{Pb}$ includes the 82 electrons in the neutral atom. Therefore, when computing the mass defect, we must account for these electrons. We do so by using the atomic mass of 1.007 825 u for the hydrogen atom ${}^{1}_{1}\text{H}$, which also includes the single electron, instead of the atomic mass of a proton. To obtain the binding energy in MeV, we will use the fact that 1 u is equivalent to 931.5 MeV.

SOLUTION

a. Noting that the number of neutrons is 206 – 82 = 124, we can obtain the mass defect Δm as follows:

$$\Delta m = \underbrace{82(1.007\ 825\ \text{u})}_{\substack{\text{82 free hydrogen atoms}\\ \text{(protons plus electrons)}}} + \underbrace{124(1.008\ 665\ \text{u})}_{\text{124 free neutrons}} - \underbrace{205.974\ 440\ \text{u}}_{\substack{\text{Intact lead atom}\\ \text{(including 82 electrons)}}} = \boxed{1.741\ 670\ \text{u}}$$

b. Since 1 u is equivalent to 931.5 MeV, the binding energy is

$$\text{Binding energy} = (1.741\ 670\ \text{u})\left(\frac{931.5\ \text{MeV}}{1\ \text{u}}\right) = \boxed{1622\ \text{MeV}}$$

c. The binding energy per nucleon is

$$\text{Binding energy per nucleon} = \frac{\text{Binding energy}}{\text{Number of nucleons}}$$

$$= \frac{1622\ \text{MeV}}{206} = \boxed{7.87\ \text{MeV}}$$

15. SSM ***REASONING*** Since we know the difference in binding energies for the two isotopes, we can determine the corresponding mass defect. Also knowing that the isotope with the larger binding energy contains one more neutron than the other isotope gives us enough information to calculate the atomic mass difference between the two isotopes.

SOLUTION The mass defect corresponding to a binding energy difference of 5.03 MeV is

$$(5.03\ \text{MeV})\left(\frac{1\ \text{u}}{931.5\ \text{MeV}}\right) = 0.005\,40\ \text{u}$$

Since the isotope with the larger binding energy has one more neutron ($m = 1.008\,665\ \text{u}$) than the other isotope, the difference in atomic mass between the two isotopes is

$$1.008\,665\ \text{u} - 0.005\,40\ \text{u} = \boxed{1.003\,27\ \text{u}}$$

16. ***REASONING AND SOLUTION***

a. The mass defect is

$$\Delta m = 13.005\ 738\ \text{u} + 1.008\ 665\ \text{u} - 14.003\ 074\ \text{u} = 0.011\ 329\ \text{u}$$

The binding energy of the neutron is then

$$(0.011\,329\ \text{u})\left(\frac{931.5\ \text{MeV}}{1\ \text{u}}\right) = \boxed{10.55\ \text{MeV}}$$

b. The mass defect is

$$\Delta m = 13.003\ 355\ \text{u} + 1.007\ 825\ \text{u} - 14.003\ 074\ \text{u} = 0.008\ 106\ \text{u}$$

The binding energy of the proton is then

$$(0.008\,106\ \text{u})\left(\frac{931.5\ \text{MeV}}{1\ \text{u}}\right) = \boxed{7.55\ \text{MeV}}$$

c. The $\boxed{\text{neutron}}$ is more tightly bound, since it has the larger binding energy.

17. ***REASONING AND SOLUTION***

a. The decay process looks like $\boxed{{}^{212}_{84}\text{Po} \rightarrow {}^{208}_{82}\text{Pb} + {}^{4}_{2}\text{He}}$.

b. In this case $\boxed{{}^{232}_{92}\text{U} \rightarrow {}^{228}_{90}\text{Th} + {}^{4}_{2}\text{He}}$

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18. ***REASONING AND SOLUTION***

a. The decay reaction is: ${}^{242}_{94}\text{Pu} \rightarrow {}^{A}_{Z}\text{X} + {}^{4}_{2}\text{He}$. Therefore, $242 = A + 4$, so that $A = 238$. In addition, $94 = Z + 2$, so that $Z = 92$. Thus, the daughter nucleus is $\boxed{{}^{238}_{92}\text{U}}$.

b. The decay reaction is ${}^{24}_{11}\text{Na} \rightarrow {}^{A}_{Z}\text{X} + {}^{0}_{-1}\text{e}$. Therefore, $24 = A$. In addition, $11 = Z - 1$, so that $Z = 12$. Thus the daughter nucleus is $\boxed{{}^{24}_{12}\text{Mg}}$.

c. The decay reaction is ${}^{13}_{7}\text{N} \rightarrow {}^{A}_{Z}\text{X} + {}^{0}_{1}\text{e}$. Therefore, $13 = A$. In addition, $7 = Z + 1$, so that $Z = 6$. Thus, the daughter nucleus is $\boxed{{}^{13}_{6}\text{C}}$

---

19. ***REASONING AND SOLUTION***

a. The decay process looks like $\boxed{{}^{14}_{6}\mathrm{C} \rightarrow {}^{14}_{7}\mathrm{N} + {}^{0}_{-1}\mathrm{e}}$

b. For the next reaction $\boxed{{}^{212}_{82}\mathrm{Pb} \rightarrow {}^{212}_{83}\mathrm{Bi} + {}^{0}_{-1}\mathrm{e}}$

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20. ***REASONING AND SOLUTION***

a. The reaction looks like $\boxed{{}^{14}_{6}\text{C} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e}}$

b. The mass change during the reaction is

$$\Delta m = 14.003\ 241\ \text{u} - 14.003\ 074\ \text{u} = 0.000\ 167\ \text{u}$$

Since 1 u = 931.5 MeV, the energy released during the reaction is

$$E = (0.000\,167\ \text{u})\left(\frac{931.5\ \text{MeV}}{1\ \text{u}}\right) = \boxed{0.156\ \text{MeV}}$$

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21. **SSM** ***REASONING AND SOLUTION*** The general form for β^- decay is

$$\underbrace{{}^{A}_{Z}\mathrm{P}}_{\substack{\text{Parent}\\ \text{nucleus}}} \rightarrow \underbrace{{}^{A}_{Z+1}\mathrm{D}}_{\substack{\text{Daughter}\\ \text{nucleus}}} + \underbrace{{}^{0}_{-1}\mathrm{e}}_{\substack{\beta^- \text{ particle}\\ \text{(electron)}}}$$

Therefore, the β^- decay process for ${}^{35}_{16}\mathrm{S}$ is $\boxed{{}^{35}_{16}\mathrm{S} \rightarrow {}^{35}_{17}\mathrm{Cl} + {}^{0}_{-1}\mathrm{e}}$.

22. ***REASONING AND SOLUTION*** The mass of the products is

$$m = 222.017\ 57\ \text{u} + 4.002\ 60\ \text{u} = 226.\ 020\ 17\ \text{u}$$

The mass defect for the decay is $\Delta m = 226.025\ 40\ \text{u} - 226.020\ 17\ \text{u} = 0.005\ 23\ \text{u}$, which corresponds to an energy of

$$(0.005\ 23\ \text{u})\left(\frac{931.5\ \text{MeV}}{1\ \text{u}}\right) = \boxed{4.87\ \text{MeV}}$$

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23. SSM WWW ***REASONING AND SOLUTION*** The general form for β^+ decay is

$$\underbrace{{}^{A}_{Z}\mathrm{P}}_{\substack{\text{Parent}\\ \text{nucleus}}} \rightarrow \underbrace{{}^{A}_{Z-1}\mathrm{D}}_{\substack{\text{Daughter}\\ \text{nucleus}}} + \underbrace{{}^{0}_{+1}\mathrm{e}}_{\substack{\beta^+ \text{ particle}\\ \text{(positron)}}}$$

a. Therefore, the β^+ decay process for ${}^{18}_{9}\mathrm{F}$ is $\boxed{{}^{18}_{9}\mathrm{F} \rightarrow {}^{18}_{8}\mathrm{O} + {}^{0}_{+1}\mathrm{e}}$.

b. Similarly, the β^+ decay process for ${}^{15}_{8}\mathrm{O}$ is $\boxed{{}^{15}_{8}\mathrm{O} \rightarrow {}^{15}_{7}\mathrm{N} + {}^{0}_{+1}\mathrm{e}}$.

24. ***REASONING*** Electric charge must be conserved, and the number of nucleons must be conserved. Applying these two conservation principles, we will be able to obtain two equations containing the two unknown quantities N_α and N_β.

SOLUTION The overall decay process can be written as follows:

$$^{220}_{86}\text{Rn} \rightarrow {}^{208}_{82}\text{Pb} + N_\alpha\left({}^{4}_{2}\text{He}\right) + N_\beta\left({}^{0}_{-1}\text{e}\right)$$

Since electric charge must be conserved, the 86 protons on the left must equal the total number of protons on the right, the result being

$$86 = 82 + N_\alpha(2) + N_\beta(-1)$$

Since the nucleon number must be conserved also, the 220 nucleons on the left must equal the total number of nucleons on the right, so that

$$220 = 208 + N_\alpha(4) + N_\beta(0)$$

Solving this result for the number of α particles gives $\boxed{N_\alpha = 3}$. Substituting $N_\alpha = 3$ into the conservation-of-charge equation, we find that the number of β^- particles is $\boxed{N_\beta = 2}$.

25. ***REASONING AND SOLUTION*** The decay reaction is ${}^{239}_{94}\text{Pu} \rightarrow {}^{235}_{92}\text{U} + {}^{4}_{2}\text{He}$. The mass defect of the reaction is

$$\Delta m = 239.052\ 16\ \text{u} - 4.002\ 603\ \text{u} - 235.043\ 924\ \text{u} = 0.005\ 63\ \text{u}$$

This corresponds to an energy release of

$$(0.005\ 63\ \text{u})\left(\frac{931.5\ \text{MeV}}{1\ \text{u}}\right) = 5.24\ \text{MeV}$$

This energy is assumed to be kinetic energy of the α particle (mass $m = 6.44 \times 10^{-27}$ kg, see Example 2), so the speed of the α particle is

$$v = \sqrt{\frac{2(\text{KE})}{\text{m}}} = \sqrt{\frac{2\left(5.24 \times 10^{6}\ \text{eV}\right)\left(\frac{1.60 \times 10^{-19}\ \text{J}}{1\ \text{eV}}\right)}{6.64 \times 10^{-27}\ \text{kg}}} = \boxed{1.59 \times 10^{7}\ \text{m/s}}$$

26. ***REASONING AND SOLUTION*** The β decay reaction is

$$^{208}_{81}\text{Tl} \rightarrow {}^{208}_{82}\text{Pb} + {}^{0}_{-1}\text{e}$$

so

$$^{A}_{Z}\text{X} \rightarrow {}^{208}_{82}\text{Pb} + {}^{4}_{2}\text{He}$$

gives

$$\boxed{^{A}_{Z}\text{X} = {}^{212}_{84}\text{Po}}$$

27. **SSM** ***REASONING*** Energy is released during the β decay. To find the energy released, we determine how much the mass has decreased because of the decay and then calculate the equivalent energy. The reaction and masses are shown below:

$$\underbrace{{}^{22}_{11}\text{Na}}_{21.994\,434\text{ u}} \rightarrow \underbrace{{}^{22}_{10}\text{Ne}}_{21.991\,383\text{ u}} + \underbrace{{}^{0}_{+1}\text{e}}_{5.485\,799\times10^{-4}\text{ u}}$$

SOLUTION The decrease in mass is

$$21.994\ 434\text{ u} - \left(21.991\ 383\text{ u} + 5.485\ 799\times10^{-4}\text{ u} + 5.485\ 799\times10^{-4}\text{ u}\right) = 0.001\ 954\text{ u}$$

where the extra electron mass takes into account the fact that the atomic mass for sodium includes the mass of 11 electrons, whereas the atomic mass for neon includes the mass of only 10 electrons.

Since 1 u is equivalent to 931.5 MeV, the released energy is

$$\left(0.001\,954\text{ u}\right)\left(\frac{931.5\text{ MeV}}{1\text{ u}}\right) = \boxed{1.82\text{ MeV}}$$

28. ***REASONING*** As Section 7.2 discusses, the principle of conservation of linear momentum indicates that the total momentum of an isolated system is conserved. Assuming that no external net force acts on the beryllium nucleus, it is isolated and the emission of the gamma ray satisfies this principle. The magnitude of the momentum of the gamma ray photon is h/λ, according to Equation 29.6, where h is Planck's constant and λ is the wavelength. We take the direction of the photon as the positive direction. Then, using Equation 7.2, we write the momentum of the recoiling beryllium nucleus as $-mv_{\text{Be}}$, where m is the mass and v_{Be} is the speed. The conservation principle indicates that the total momentum after the emission must be equal to the total momentum before the emission, so we have

$$\underbrace{-mv_{\text{Be}}}_{\substack{\text{Momentum of nucleus}\\ \text{after emission}}} + \underbrace{\frac{h}{\lambda}}_{\substack{\text{Momentum of gamma}\\ \text{ray photon}}} = \underbrace{0}_{\substack{\text{Total momentum}\\ \text{before emission}}}$$

This expression can be solved for the wavelength.

SOLUTION Solving the conservation-of-momentum expression, we obtain

$$\lambda = \frac{h}{mv_{\text{Be}}}$$

Before we can use this equation, the mass of the beryllium must be converted from atomic mass units to kilograms:

$$m_{\text{Be}} = (7.017\ \text{u})\left(\frac{1.6605\times10^{-27}\ \text{kg}}{1\ \text{u}}\right) = 1.165\times10^{-26}\ \text{kg}$$

The wavelength, then, is

$$\lambda = \frac{h}{mv_{\text{Be}}} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(1.165\times10^{-26}\ \text{kg})(2.19\times10^{4}\ \text{m/s})} = \boxed{2.60\times10^{-12}\ \text{m}}$$

29. ***REASONING AND SOLUTION*** Since one-eighth of the nuclei remain, three half-lives have elapsed. Therefore, $T_{1/2} = (9.0 \text{ days})/3 = \boxed{3.0 \text{ days}}$.

30. ***REASONING AND SOLUTION*** The number of seconds in 14.28 days is $T_{1/2} = 1.23 \times 10^6$ s. The decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{1.26 \times 10^6 \text{ s}} = \boxed{5.62 \times 10^{-7} \text{ s}^{-1}}$$

31. **SSM** ***REASONING AND SOLUTION*** According to Equation 31.5, $N = N_0 e^{-\lambda t}$, the decay constant is

$$\lambda = -\frac{1}{t}\ln\left(\frac{N}{N_0}\right) = -\frac{1}{20\text{ days}}\ln\left(\frac{8.14\times10^{14}}{4.60\times10^{15}}\right) = 0.0866\text{ days}^{-1}$$

The half-life is, from Equation 31.6,

$$T_{1/2} = \frac{0.693}{\lambda} = \frac{0.693}{0.0866\text{ days}^{-1}} = \boxed{8.00\text{ days}}$$

32. ***REASONING AND SOLUTION*** According to Equation 31.5, the fraction of an initial sample remaining after a time t is $N / N_0 = e^{-\lambda t}$, where λ is the decay constant. The decay constant is related to the half-life $T_{1/2}$. According to Equation 31.6, the decay constant is $\lambda = 0.693 / T_{1/2}$. Therefore, the fraction remaining is

$$\frac{N}{N_0} = e^{-0.693\, t / T_{1/2}} = e^{-0.693\left[(30.0\text{ days})/(8.04\text{ days})\right]} = 0.0753$$

This fraction corresponds to a percentage of $\boxed{7.53\ \%}$.

33. ***REASONING AND SOLUTION*** The amount remaining is 0.0100% = 0.000 100. We know $N / N_0 = e^{-0.693\,t/T_{1/2}}$. Therefore, we find

$$t = -\frac{T_{1/2}}{0.693}\ln\left(\frac{N}{N_0}\right) = -\frac{28.5\text{ yr}}{0.693}\ln(0.000\,100) = \boxed{379\text{ yr}}$$

34. ***REASONING AND SOLUTION*** The activity is $A = \lambda N$. The decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{(5.27\text{ yr})\left(\dfrac{3.156\times10^{7}\text{ s}}{1\text{ yr}}\right)} = 4.17\times10^{-9}\text{ s}^{-1}$$

As discussed in Section 14.1, the number N of nuclei is the number of moles of nuclei times Avogadro's number (which is the number of nuclei per mole). Thus,

$$N = \underbrace{\left(\frac{0.50\text{ g}}{59.9\text{ g/mol}}\right)}_{\text{Number of moles}}\underbrace{\left(6.02\times10^{23}\text{ mol}^{-1}\right)}_{\text{Avogadro's number}} = 5.0\times10^{21}$$

Therefore, $A = \lambda N = (4.17\times10^{-9}\text{ s}^{-1})(5.0\times10^{21}) = \boxed{2.1\times10^{13}\text{ Bq}}$.

35. **SSM** ***REASONING*** We can find the decay constant from Equation 31.5, $N = N_0 e^{-\lambda t}$. If we multiply both sides by the decay constant λ, we have

$$\lambda N = \lambda N_0 e^{-\lambda t} \qquad \text{or} \qquad A = A_0 e^{-\lambda t}$$

where A_0 is the initial activity and A is the activity after a time t. Once the decay constant is known, we can use the same expression to determine the activity after a total of six days.

SOLUTION Solving the expression above for the decay constant λ, we have

$$\lambda = -\frac{1}{t}\ln\left(\frac{A}{A_0}\right) = -\frac{1}{2\text{ days}}\ln\left(\frac{285\text{ disintegrations/min}}{398\text{ disintegrations/min}}\right) = 0.167\text{ days}^{-1}$$

Then the activity four days after the second day is

$$A = (285\text{ disintegrations/min})e^{-(0.167\text{ days}^{-1})(4.00\text{ days})} = \boxed{146\text{ disintegrations/min}}$$

36. ***REASONING AND SOLUTION*** The decay constant is

$$\lambda = (0.693)/T_{1/2} = (0.693)/(1.60 \times 10^3 \text{yr}) = 4.33 \times 10^{-4} \text{ yr}^{-1}$$

The mass m of the radium is proportional to the number N of atoms present, so $N/N_0 = m/m_0$. Since

$$N = N_0 e^{-\lambda t}, \quad \text{then} \quad m = m_0 e^{-\lambda t}$$

The mass remaining after fifty years is

$$m = \left(1.000 \times 10^{-9} \text{ kg}\right) e^{-\left(4.33 \times 10^{-4} \text{ yr}^{-1}\right)\left(50.0 \text{ yr}\right)} = 0.979 \times 10^{-9} \text{ kg}$$

The difference in mass, which is the mass of radium that has disappeared, is

$$\Delta m = 1.000 \times 10^{-9} \text{ kg} - 0.979 \times 10^{-9} \text{ kg} = \boxed{2.1 \times 10^{-11} \text{ kg}}$$

37. ***REASONING AND SOLUTION*** We know that $A = \lambda N$. The decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{\left(1.6 \times 10^3 \text{ yr}\right)\left(\dfrac{3.156 \times 10^7 \text{ s}}{1 \text{ yr}}\right)} = 1.372 \times 10^{-11} \text{ s}^{-1}$$

As discussed in Section 14.1, the number N of Radium nuclei is the number of moles of nuclei times Avogadro's number (which is the number of nuclei per mole). Thus,

$$N = \underbrace{\left(\frac{1.00 \text{ g}}{226 \text{ g/mol}}\right)}_{\substack{\text{Number}\\ \text{of moles}}}\underbrace{\left(6.02 \times 10^{23} \text{ mol}^{-1}\right)}_{\substack{\text{Avogadro's}\\ \text{number}}} = 2.664 \times 10^{21}$$

Therefore, $A = \lambda N = (1.372 \times 10^{-11} \text{ s}^{-1})(2.664 \times 10^{21}) = \boxed{3.7 \times 10^{10} \text{ Bq}}$.

38. ***REASONING*** The activity of a radioactive sample is given by the magnitude of Equation 31.4 as λN, where λ is the decay constant and N is the number of radioactive nuclei. The decay constant is related to the half life by Equation 31.6 as $\lambda = 0.693/T_{1/2}$. Thus, the activity can be expressed as $0.693\ N/T_{1/2}$. The number of nuclei can be obtained as discussed in Section 14.1, by multiplying the number of moles by Avogadro's number N_A. The number of moles is the mass m divided by the mass per mole M. Therefore, the number of radioactive nuclei is $N = mN_A/M$, and our expression for the activity becomes

$$\text{Activity} = \frac{0.693\ N}{T_{1/2}} = \frac{0.693\ mN_A}{T_{1/2}\ M}$$

We can apply this result to both krypton and xenon and then set the two activities equal to obtain the mass of krypton.

SOLUTION Setting the activity of the krypton equal to the activity of the xenon, we obtain

$$\frac{0.693\ m_{Kr} N_A}{(T_{1/2})_{Kr}\ M_{Kr}} = \frac{0.693\ m_{Xe} N_A}{(T_{1/2})_{Xe}\ M_{Xe}} \quad \text{or} \quad \frac{m_{Kr}}{(T_{1/2})_{Kr}\ M_{Kr}} = \frac{m_{Xe}}{(T_{1/2})_{Xe}\ M_{Xe}}$$

Solving for m_{Kr} gives

$$m_{Kr} = \frac{m_{Xe}(T_{1/2})_{Kr}\ M_{Kr}}{(T_{1/2})_{Xe}\ M_{Xe}} = \frac{(2.00\ \text{g})(1.840\ \text{s})(91.9\ \text{g/mol})}{(13.6\ \text{s})(139.9\ \text{g/mol})} = \boxed{0.178\ \text{g}}$$

39. **SSM** ***REASONING*** According to Equation 31.5, the number of nuclei remaining after a time t is $N = N_0 e^{-\lambda t}$. Using this expression, we find the ratio N_A / N_B as follows:

$$\frac{N_A}{N_B} = \frac{N_{0A} e^{-\lambda_A t}}{N_{0B} e^{-\lambda_B t}} = e^{-(\lambda_A - \lambda_B)t}$$

where we have used the fact that initially the numbers of the two types of nuclei are equal $(N_{0A} = N_{0B})$. Taking the natural logarithm of both sides of the equation above shows that

$$\ln(N_A / N_B) = -(\lambda_A - \lambda_B)t \qquad \text{or} \qquad \lambda_A - \lambda_B = \frac{-\ln(N_A / N_B)}{t}$$

SOLUTION Since $N_A / N_B = 3.00$ when $t = 3.00$ days, it follows that

$$\lambda_A - \lambda_B = \frac{-\ln(3.00)}{3.00 \text{ days}} = -0.366 \text{ days}^{-1}$$

But we need to find the half-life of species B, so we use Equation 31.6, which indicates that $\lambda = 0.693 / T_{1/2}$. With this expression for λ, the result for $\lambda_A - \lambda_B$ becomes

$$0.693\left(\frac{1}{T_{1/2}^A} - \frac{1}{T_{1/2}^B}\right) = -0.366$$

Since $T_{1/2}^B = 1.50$ days, the result above can be solved to show that $\boxed{T_{1/2}^A = 7.23 \text{ days}}$.

40. ***REASONING AND SOLUTION*** We know $t = -(1/\lambda)\ln(A/A_0)$ where

$$\lambda = 1.21 \times 10^{-4}\ \text{yr}^{-1} \quad \text{and} \quad A/A_0 = 0.21$$

Therefore, $t = -\dfrac{1}{1.21 \times 10^{-4}\ \text{yr}^{-1}} \ln(0.21) = \boxed{13\,000\ \text{yr}}$.

41. **SSM** ***REASONING AND SOLUTION*** The answer can be obtained directly from Equation 31.5, combined with Equation 31.6:

$$\frac{N}{N_0} = e^{-\lambda t} = e^{-(0.693)t/T_{1/2}} = e^{-(0.693)(41\,000\text{ yr})/(5730\text{ yr})} = 0.0070$$

The percent of atoms remaining is $\boxed{0.70\ \%}$.

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42. ***REASONING AND SOLUTION***

a. According to Equations 31.5 and 31.6, the ratio N / N_0 is given by

$$\frac{N}{N_0} = e^{-0.693\,t/T_{1/2}} = e^{-0.693\left[(5.00\text{ yr})/(5730\text{ yr})\right]} = \boxed{0.999}$$

b. Similarly, we find

$$\frac{N}{N_0} = e^{-0.693\,t/T_{1/2}} = e^{-0.693\left[(3600\text{ s})/(122.2\text{ s})\right]} = \boxed{1.36 \times 10^{-9}}$$

c. Similarly, we find

$$\frac{N}{N_0} = e^{-0.693\,t/T_{1/2}} = e^{-0.693\left[(5.00\text{ yr})/(12.33\text{ yr})\right]} = \boxed{0.755}$$

43. ***REASONING AND SOLUTION***

a. We know that

$$t = -\frac{T_{1/2}}{0.693}\ln\left(\frac{A}{A_0}\right) = -\frac{5730\text{ yr}}{0.693}\ln\left(\frac{0.0061\text{ Bq}}{0.23\text{ Bq}}\right) = \boxed{3.0\times10^4\text{ yr}}$$

b. If A_0 is 40% larger, then $A_0 = (1.4)(0.23\text{ Bq}) = 0.32\text{ Bq}$, so that

$$t = -\frac{T_{1/2}}{0.693}\ln\left(\frac{A}{A_0}\right) = -\frac{5730\text{ yr}}{0.693}\ln\left(\frac{0.0061\text{ Bq}}{0.32\text{ Bq}}\right) = \boxed{3.3\times10^4\text{ yr}}$$

44. ***REASONING AND SOLUTION*** The time during which radioactive decay occurred is $t = 1988\text{ yr} - 1200\text{ yr} = 788\text{ yr}$. According to Equations 31.5 and 31.6, the ratio N / N_0 that corresponds to this time is given by

$$\frac{N}{N_0} = e^{-0.693\,t/T_{1/2}} = e^{-0.693\left[(788\text{ yr})/(5730\text{ yr})\right]} = 0.909$$

The percentage of the original ${}^{14}_{6}\text{C}$ nuclei remaining, then, is $\boxed{90.9\%}$.

45. SSM ***REASONING*** According to Equation 31.5, $N = N_0 e^{-\lambda t}$. If we multiply both sides by the decay constant λ, we have

$$\lambda N = \lambda N_0 e^{-\lambda t} \qquad \text{or} \qquad A = A_0 e^{-\lambda t}$$

where A_0 is the initial activity and A is the activity after a time t. The decay constant λ is related to the half-life through Equation 31.6: $\lambda = 0.693 / T_{1/2}$. We can find the age of the fossils by solving for the time t. The maximum error can be found by evaluating the limits of the accuracy as given in the problem statement.

SOLUTION The age of the fossils is

$$t = -\frac{T_{1/2}}{0.693} \ln\left(\frac{A}{A_0}\right) = -\frac{5730 \text{ yr}}{0.693} \ln\left(\frac{0.10 \text{ Bq}}{0.23 \text{ Bq}}\right) = \boxed{6900 \text{ yr}}$$

The maximum error can be found as follows:

When there is an error of +10 %, $A = 0.10 \text{ Bq} + 0.010 \text{ Bq} = 0.11 \text{ Bq}$, and we have

$$t = -\frac{5730 \text{ yr}}{0.693} \ln\left(\frac{0.11 \text{ Bq}}{0.23 \text{ Bq}}\right) = 6100 \text{ yr}$$

Similarly, when there is an error of –10 %, $A = 0.10 \text{ Bq} - 0.010 \text{ Bq} = 0.090 \text{ Bq}$, and we have

$$t = -\frac{5730 \text{ yr}}{0.693} \ln\left(\frac{0.090 \text{ Bq}}{0.23 \text{ Bq}}\right) = 7800 \text{ yr}$$

The maximum error in the age of the fossils is $7800 \text{ yr} - 6900 \text{ yr} = \boxed{900 \text{ yr}}$.

46. ***REASONING*** The number of radioactive nuclei remaining after a time t is given by Equations 31.5 and 31.6 as

$$N = N_0 e^{-0.693\,t/T_{1/2}}$$

where N_0 is the number of radioactive nuclei present at $t = 0$ s and $T_{1/2}$ is the half life for the decay. The activity A is proportional to the number of radioactive nuclei that a sample contains, so it follows that

$$A = A_0 e^{-0.693\,t/T_{1/2}} \tag{1}$$

We know that the activity for the ancient carbon in the sample is 0.011 Bq per gram of carbon, whereas for the fresh carbon it is 0.23 Bq per gram of carbon. Knowing the percentage composition of the contaminated sample, we can determine its activity A by using the given percentages with the known activities:

$$A_{\text{Contaminated}} = 0.980\,A_{\text{Ancient}} + 0.020\,A_{\text{Fresh}} \tag{2}$$

The true age and the apparent age can be obtained by applying Equation (1) to the corresponding activities.

SOLUTION a. Using Equation (1), we find that the true age is

$$A_{\text{Ancient}} = A_0 e^{-0.693\,t/T_{1/2}} \quad \text{or} \quad 0.011\text{ Bq} = (0.23\text{ Bq})e^{-0.693\,t/(5730\text{ yr})}$$

Taking the natural logarithm of both sides of this result gives

$$\ln\left(\frac{0.011\text{ Bq}}{0.23\text{ Bq}}\right) = -\frac{0.693\,t}{5730\text{ yr}}$$

$$t = \frac{-(5730\text{ yr})\ln\left(\frac{0.011\text{ Bq}}{0.23\text{ Bq}}\right)}{0.693} = \boxed{25\,000\text{ yr}}$$

b. Using Equation (2) to find the activity of the contaminated sample, we obtain

$$\begin{aligned} A_{\text{Contaminated}} &= 0.980\,A_{\text{Ancient}} + 0.020\,A_{\text{Fresh}} \\ &= 0.980(0.011\text{ Bq}) + 0.020(0.23\text{ Bq}) = 0.0154\text{ Bq} \end{aligned}$$

Using this activity in Equation (1), we find that the apparent age of the sample is

$$t = \frac{-(5730\text{ yr})\ln\left(\frac{0.0154\text{ Bq}}{0.23\text{ Bq}}\right)}{0.693} = \boxed{22\,000\text{ yr}}$$

47. ***REASONING AND SOLUTION*** For $^{202}_{80}\text{Hg}$ the mass of the separated nucleons is

$$m = 80(1.007\ 825\ \text{u}) + 122(1.008\ 665\ \text{u}) = 203.683\ 130\ \text{u}$$

The mass defect is then $\Delta m = 203.683\ 130\ \text{u} - 201.970\ 617\ \text{u} = 1.712\ 513\ \text{u}$.

This corresponds to a total binding energy of $(1.712\ 513\ \text{u})\left(\dfrac{931.5\ \text{MeV}}{1\ \text{u}}\right) = 1595\ \text{MeV}$ and

a binding energy per nucleon of $\dfrac{1595\ \text{MeV}}{202\ \text{nucleons}} = \boxed{7.90\ \text{MeV per nucleon}}$.

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48. ***REASONING AND SOLUTION*** Equation 31.2 gives for the radius of the nucleus

$$r = (1.2 \times 10^{-15}\ \text{m})A^{1/3} = (1.2 \times 10^{-15}\ \text{m})(48)^{1/3} = \boxed{4.4 \times 10^{-15}\ \text{m}}$$

49. **SSM** ***REASONING AND SOLUTION*** The number of radioactive nuclei that remains in a sample after a time t is given by Equation 31.5, $N = N_0 e^{-\lambda t}$, where λ is the decay constant. From Equation 31.6, we know that the decay constant is related to the half-life by $T_{1/2} = 0.693/\lambda$; therefore, $\lambda = 0.693/T_{1/2}$ and we can write

$$\frac{N}{N_0} = e^{-(0.693/T_{1/2})t} \qquad \text{or} \qquad \frac{t}{T_{1/2}} = -\frac{1}{0.693}\ln\left(\frac{N}{N_0}\right)$$

When the number of radioactive nuclei decreases to one-millionth of the initial number, $N/N_0 = 1.00\times10^{-6}$; therefore, the number of half-lives is

$$\frac{t}{T_{1/2}} = -\frac{1}{0.693}\ln\left(1.00\times10^{-6}\right) = \boxed{19.9}$$

50. ***REASONING AND SOLUTION*** The reaction and the associated masses are as follows:

$$\underbrace{{}^{3}_{1}\mathrm{H}}_{3.016\,050\ \mathrm{u}} \rightarrow \underbrace{{}^{3}_{2}\mathrm{He} + {}^{0}_{-1}\mathrm{e}}_{3.016\,030\ \mathrm{u}}$$

The mass defect Δm for the reaction is

$$\Delta m = 3.016\,050\ \mathrm{u} - 3.016\,030\ \mathrm{u} = 0.000\,020\ \mathrm{u}$$

Since 1 u = 931.5 MeV, this mass defect corresponds to a released energy of

$$(0.000\,020\ \mathrm{u})\left(\frac{931.5\ \mathrm{MeV}}{1\ \mathrm{u}}\right) = \boxed{0.019\ \mathrm{MeV}}$$

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51. ***REASONING*** We can determine the identity of X in each of the decay processes by noting that for each process, the sum of A and Z for the decay products must equal the values of A and Z for the parent nuclei.

SOLUTION

a.

$$^{211}_{82}\text{Pb} \rightarrow {}^{211}_{83}\text{Bi} + \text{X}$$

Using the reasoning discussed above, X must have $A = 0$ and $Z = -1$. Therefore X must be an electron, $^{0}_{-1}\text{e}$; $\boxed{\text{X represents a } \beta^- \text{ particle (electron)}}$.

b.

$$^{11}_{6}\text{C} \rightarrow {}^{11}_{5}\text{B} + \text{X}$$

Similar reasoning suggests that X must have $A = 0$ and $Z = +1$. Therefore X must be a positron; $^{0}_{+1}\text{e}$; $\boxed{\text{X represents a } \beta^+ \text{ particle (positron)}}$.

c.

$$^{231}_{90}\text{Th}^* \rightarrow {}^{231}_{90}\text{Th} + \text{X}$$

Similar reasoning suggests that X must have $A = 0$ and $Z = 0$. Therefore X must be a gamma ray; $\boxed{\text{X represents a } \gamma \text{ ray}}$.

d.

$$^{210}_{84}\text{Po} \rightarrow {}^{206}_{82}\text{Pb} + \text{X}$$

Using the reasoning discussed above, X must have $A = 4$ and $Z = 2$. Therefore X must be a helium nucleus, $^{4}_{2}\text{He}$; $\boxed{\text{X represents an } \alpha \text{ particle (helium nucleus)}}$.

52. ***REASONING*** According to Equation 31.5, the number of nuclei remaining after a time t is $N = N_0 e^{-\lambda t}$. If we multiply both sides of this equation by the decay constant λ, we have $\lambda N = \lambda N_0 e^{-\lambda t}$. Recognizing that λN is the activity A, we have $A = A_0 e^{-\lambda t}$, where A_0 is the activity at time $t = 0$. A_0 can be determined from the fact that we know the mass of the specimen, and that the activity of one gram of carbon in a living organism is 0.23 Bq. The decay constant λ can be determined from the value of 5730 yr for the half-life of ${}^{14}_{6}\text{C}$ using Equation 31.6. With known values for A_0 and λ, the given activity of 1.6 Bq can be used to determine the age t of the specimen.

SOLUTION For ${}^{14}_{6}\text{C}$, the decay constant is

$$\lambda = \frac{0.693}{T_{1/2}} = \frac{0.693}{5730 \text{ yr}} = 1.21 \times 10^{-4} \text{ yr}^{-1}$$

The activity at time $t = 0$ is $A_0 = (9.2 \text{ g})(0.23 \text{ Bq/g}) = 2.1 \text{ Bq}$. Since $A = 1.6 \text{ Bq}$ and $A_0 = 2.1 \text{ Bq}$, the age of the specimen can be determined from

$$A = 1.6 \text{ Bq} = (2.1 \text{ Bq})\, e^{-(1.21 \times 10^{-4} \text{ yr}^{-1})\, t}$$

Taking the natural logarithm of both sides leads to

$$\ln\left(\frac{1.6 \text{ Bq}}{2.1 \text{ Bq}}\right) = -(1.21 \times 10^{-4} \text{ yr}^{-1})\, t$$

Therefore, the age of the specimen is

$$t = \frac{\ln\left(\dfrac{1.6 \text{ Bq}}{2.1 \text{ Bq}}\right)}{-1.21 \times 10^{-4} \text{ yr}^{-1}} = \boxed{2.2 \times 10^{3} \text{ yr}}$$

53. SSM WWW ***REASONING AND SOLUTION*** As shown in Figure 31.18, if the first dynode produces 3 electrons, the second produces 9 electrons (3^2), the third produces 27 electrons (3^3), so the Nth produces 3^N electrons. The number of electrons that leaves the 14th dynode and strikes the 15th dynode is

$$3^{14} = \boxed{4\ 782\ 969 \text{ electrons}}$$

54. ***REASONING AND SOLUTION*** The conservation of linear momentum applied to the reaction gives

$$m_\alpha v_\alpha + m_T v_T = 0 \tag{1}$$

The energy released in the decay is assumed to be kinetic, so

$$(1/2)m_\alpha v_\alpha^{\ 2} + (1/2)m_T v_T^{\ 2} = 4.3 \text{ MeV} \tag{2}$$

Solving Equation (1) for v_α, substituting into Equation (2) and rearranging, gives the kinetic energy of the thorium atom.

$$\tfrac{1}{2}m_T v_T^2 = \frac{4.3 \text{ MeV}}{1 + \dfrac{m_T}{m_\alpha}} = \frac{4.3 \text{ MeV}}{1 + \dfrac{234.0436 \text{ u}}{4.0026 \text{ u}}} = \boxed{0.072 \text{ MeV}}$$

The kinetic energy of the α particle is, then, 4.3 MeV – 0.072 MeV = $\boxed{4.2 \text{ MeV}}$.

55. ***REASONING AND SOLUTION*** Since the isotopes have the same binding energies, they have the same mass defects, so that

$$\underbrace{Zm_{\text{h}} + (A_1 - Z)m_{\text{n}} - M_1}_{\substack{\text{Isotope \#1 mass defect:} \\ A_1 = \text{nucleon number} \\ M_1 = \text{atomic mass}}} = \underbrace{Zm_{\text{h}} + (A_2 - Z)m_{\text{n}} - M_2}_{\substack{\text{Isotope \#2 mass defect:} \\ A_2 = \text{nucleon number} \\ M_2 = \text{atomic mass}}}$$

In this expression, Z is the atomic number of each isotope, m_{h} is the atomic mass of a hydrogen atom, and m_{n} is the mass of a neutron. Simplifying the expression shows that $A_1 m_{\text{n}} - M_1 = A_2 m_{\text{n}} - M_2$. We also know that $A_2 = A_1 + 2$. Substituting this in the equation above and simplifying the result shows that

$$M_2 - M_1 = 2m_{\text{n}} = 2(1.008\,665\text{ u}) = \boxed{2.017\,330\text{ u}}$$

56. ***REASONING AND SOLUTION*** $A = A_0 e^{-\lambda t}$ for each of the substances. Equating and rearranging yields

$$A_{0,\mathrm{g}} e^{-\lambda_{\mathrm{g}} t} = A_{0,\mathrm{i}} e^{-\lambda_{\mathrm{i}} t}$$

But, $A_{0,\mathrm{g}} = 5\,A_{0,\mathrm{i}}$. Substituting this result into the equation above and algebraically rearranging the variables, we find

$$e^{-\left(\lambda_{\mathrm{g}} - \lambda_{\mathrm{i}}\right)t} = \tfrac{1}{5}$$

Taking the natural logarithm of each side of this equation results in

$$t = -\frac{\ln\left(\frac{1}{5}\right)}{\lambda_{\mathrm{g}} - \lambda_{\mathrm{i}}}$$

Now,

$$\lambda_{\mathrm{g}} = \frac{0.693}{T_{1/2}} = \frac{0.693}{2.69\text{ days}} = 0.258\text{ days}^{-1}$$

$$\lambda_{\mathrm{i}} = \frac{0.693}{T_{1/2}} = \frac{0.693}{8.04\text{ days}} = 0.0862\text{ days}^{-1}$$

so,

$$t = -\frac{\ln\left(\frac{1}{5}\right)}{0.258\text{ days}^{-1} - 0.0862\text{ days}^{-1}} = \boxed{9.37\text{ days}}$$

1. **SSM** ***REASONING AND SOLUTION*** The exposure in roentgens is given by Equation 32.1:

$$\text{Exposure (in roentgens)} = \left(\frac{1}{2.58\times10^{-4}}\right)\frac{q}{m}$$

The total charge q in the beam is equal to the number of ions in the beam multiplied by the charge on each ion, namely $e = 1.6\times10^{-19}\ \text{C}$. Therefore, the exposure is

$$\text{Exposure} = \left(\frac{1}{2.58\times10^{-4}}\right)\frac{(1.7\times10^{12})(1.6\times10^{-19}\ \text{C})}{4.0\times10^{-3}\ \text{kg}} = \boxed{0.26\ \text{roentgens}}$$

2. ***REASONING AND SOLUTION*** According to Equation 32.4, the biologically equivalent dose (BED) is the product of the absorbed dose (AD) and the relative biological effectiveness (RBE): BED = AD × RBE. For the protons, BED = (60 rad)(10) = 600 rem. For the α-particles to have the same BED, the absorbed dose is

$$\text{AD} = \text{BED/RBE} = (600\ \text{rem})/(20) = \boxed{30\ \text{rad}}$$

3. ***REASONING AND SOLUTION*** The absorbed dose (AD) is given by Equation 32.2,

$$\text{AD} = \frac{\text{Energy absorbed}}{\text{Mass of absorbing material}}$$

so

$$\text{Energy absorbed} = \text{AD} \times \text{Mass} = (2.5 \times 10^{-5}\ \text{Gy})(65\ \text{kg}) = \boxed{1.6 \times 10^{-3}\ \text{J}}$$

4. ***REASONING AND SOLUTION***

a. The biologically equivalent dose is equal to the product of the absorbed dose and the RBE (see Equation 32.4):

$$\text{Biologically equivalent dose} = (\text{Absorbed dose})(\text{RBE}) = (38\text{ rad})(12) = \boxed{460\text{ rem}}$$

b. According to the discussion at the end of Section 32.1, a person exposed to a whole-body, single dose of 460 rem has $\boxed{\text{a 50\% chance of dying}}$.

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5. **SSM** **WWW** ***REASONING AND SOLUTION*** The energy absorbed is given by the absorbed dose in grays times the mass of the person in kilograms, according to Equation 32.2. To find the absorbed dose from the biologically equivalent dose (BED), we turn to Equation 32.4:

$$\text{Absorbed dose in rad} = \frac{\text{BED}}{\text{RBE}} = \frac{45 \times 10^{-3}\ \text{rem}}{12}$$

Using the fact that 0.01 Gy = 1 rad, we find that

$$\text{Absorbed dose in grays} = \left(\frac{45 \times 10^{-3}}{12}\ \text{rad}\right)\left(\frac{0.01\ \text{Gy}}{1\ \text{rad}}\right)$$

$$\text{Energy} = (\text{Absorbed dose in grays})(\text{Mass}) = \left(\frac{45 \times 10^{-5}}{12}\ \text{Gy}\right)(75\ \text{kg}) = \boxed{2.8 \times 10^{-3}\ \text{J}}$$

6. ***REASONING AND SOLUTION*** According to Equation 32.2, the absorbed dose (AD) is equal to the energy absorbed by the tumor divided by its mass:

$$\text{AD} = \frac{\text{Energy absorbed}}{\text{Mass}} = \frac{(25\ \text{s})\left(1.6\times10^{10}\ \text{s}^{-1}\right)\left(4.0\times10^{6}\ \text{eV}\right)\left(\dfrac{1.60\times10^{-19}\ \text{J}}{1\ \text{eV}}\right)}{0.015\ \text{kg}}$$

$$= 1.7\times10^{1}\ \text{Gy} = 1.7\times10^{3}\ \text{rd}$$

The biologically equivalent dose (BED) is equal to the product of the absorbed dose (AD) and the RBE (see Equation 32.4):

$$\text{BED} = \text{AD}\times\text{RBE} = (1.7\times10^{3}\ \text{rd})(14) = \boxed{2.4\times10^{4}\ \text{rem}} \qquad (32.4)$$

7. **SSM** ***REASONING*** According to Equation 32.2, the absorbed dose is the energy absorbed divided by the mass of absorbing material:

$$\text{Absorbed dose} = \frac{\text{Energy absorbed}}{\text{Mass of absorbing material}}$$

The energy absorbed in this case is the sum of three terms: (1) the heat needed to melt a mass m of ice at 0.0 °C into liquid water at 0.0 °C, which is mL_f, according to the definition of the latent heat of fusion L_f (see Section 12.8); (2) the heat needed to raise the temperature of liquid water by an amount ΔT, which is $cm\Delta T$, where c is the specific heat capacity and ΔT is the change in temperature from 0.0 to 100.0 °C, according to Equation 12.4; (3) the heat needed to vaporize liquid water at 100.0 °C into steam at 100.0 °C, which is mL_v, according to the definition of the latent heat of vaporization L_v (see Section 12.8). Once the energy absorbed is determined, the absorbed dose can be determined using Equation 32.2.

SOLUTION Using the value of $c = 4186$ J/(kg·C°) for liquid water from Table 12.2 and the values of $L_f = 33.5 \times 10^4$ J/kg and $L_v = 22.6 \times 10^5$ J/kg from Table 12.3, we find that

$$\begin{aligned}\text{Absorbed dose in grays} &= \frac{\text{Energy}}{\text{mass}} = \frac{mL_f + cm\Delta T + mL_v}{m} = L_f + c\Delta T + L_v \\ &= 33.5 \times 10^4 \text{ J/kg} + \left[4186 \text{ J/}\left(\text{kg} \cdot \text{C°}\right)\right]\left(100.0 \text{ C°}\right) + 22.6 \times 10^5 \text{ J/kg} \\ &= 3.01 \times 10^6 \text{ J/kg}\end{aligned}$$

Using the fact that 0.01 Gy = 1 rd, we find that

$$\text{Absorbed dose} = \left(3.01 \times 10^6 \text{ Gy}\right)\left(\frac{1 \text{ rd}}{0.01 \text{ Gy}}\right) = \boxed{3.01 \times 10^8 \text{ rd}}$$

8. ***REASONING*** The number of nuclei in the beam is equal to the energy absorbed by the tumor divided by the energy per nucleus (130 MeV). According to Equation 32.2, the energy (in joules) absorbed by the tumor is equal to the absorbed dose (expressed in grays) times the mass of the tumor. The absorbed dose (expressed in rads) is equal to the biologically equivalent dose divided by the RBE of the radiation (see Equation 32.4). We can use these concepts to determine the number of nuclei in the beam.

SOLUTION The number N of nuclei in the beam is equal to the energy E absorbed by the tumor divided by the energy per nucleus. Since the energy absorbed is equal to the absorbed dose (in Gy) times the mass m (see Equation 32.2) we have

$$N = \frac{E}{\text{Energy per nucleus}} = \frac{[\text{Absorbed dose (in Gy)}]m}{\text{Energy per nucleus}}$$

We can express the absorbed dose in terms of rad units, rather than Gy units, by noting that 1 rad = 0.01 Gy. Therefore,

$$\text{Absorbed dose (in Gy)} = \text{Absorbed dose (in rad)}\left(\frac{0.01\text{ Gy}}{1\text{ rad}}\right)$$

The number of nuclei can now be written as

$$N = \frac{E}{\text{Energy per nucleus}} = \frac{[\text{Absorbed dose (in rad)}]\left(\frac{0.01\text{ Gy}}{1\text{ rad}}\right)m}{\text{Energy per nucleus}}$$

We know from Equation 32.4 that the Absorbed dose (in rad) is equal to the Biologically equivalent dose divided by the RBE, so that

$$N = \frac{E}{\text{Energy per nucleus}} = \frac{\left[\frac{\text{Biologically equivalent dose}}{\text{RBE}}\right]\left(\frac{0.01\text{ Gy}}{1\text{ rad}}\right)m}{\text{Energy per nucleus}}$$

$$= \frac{\left[\frac{180\text{ rem}}{16}\right]\left(\frac{0.01\text{ Gy}}{1\text{ rad}}\right)(0.17\text{ kg})}{\left(130\times10^{6}\text{ eV}\right)\left(\frac{1.60\times10^{-19}\text{ J}}{1\text{ eV}}\right)} = \boxed{9.2\times10^{8}}$$

9. ***REASONING AND SOLUTION*** According to Equation 32.2, the energy absorbed is equal to the product of the absorbed dose (AD) and the mass of the tumor:

$$\text{Energy} = \text{AD} \times \text{Mass} = (12\ \text{Gy})(2.0\ \text{kg}) = 24\ \text{J}$$

This energy is carried by ΔN particles in time Δt, so that

$$\text{Energy} = (\Delta N/\Delta t)(850\ \text{s})(0.40 \times 10^{6}\ \text{eV})(1.60 \times 10^{-19}\ \text{J})/(1\ \text{eV})$$

Therefore,

$$\frac{\Delta N}{\Delta t} = \boxed{4.4 \times 10^{11}\ \text{s}^{-1}}$$

10. ***REASONING AND SOLUTION*** The reaction can be written as

$$^{10}_{5}\text{B} + {}^{4}_{2}\text{He} \rightarrow {}^{1}_{1}\text{H} + {}^{13}_{Z}\text{X}$$

Since electric charge is conserved, $5 + 2 = 1 + Z$. Therefore, we have

$$\boxed{Z = 6}\text{, and } \boxed{\text{X} = \text{carbon}}$$

11. **SSM** ***REASONING AND SOLUTION*** When a nitrogen ${}^{14}_{7}\mathrm{N}$ nucleus absorbs a deuterium ${}^{2}_{1}\mathrm{H}$ nucleus during a nuclear reaction, the compound nucleus formed will have $A = 14 + 2 = 16$, and $Z = 7 + 1 = 8$. Reference to the periodic table reveals that the compound nucleus must, therefore, be

$$\boxed{\text{oxygen } {}^{16}_{8}\mathrm{O}}$$

12. ***REASONING AND SOLUTION*** The reaction can be can be written as

$$^{27}_{13}\text{Al} + {}^{4}_{2}\text{He} \rightarrow {}^{A}_{15}\text{P} + {}^{1}_{0}\text{n}$$

We therefore see that

$$27 + 4 = A + 1 \qquad \text{or} \qquad A = \boxed{30}$$

13. ***REASONING AND SOLUTION***

a. Since the total number of nucleons is conserved, we have $43 + 4 = A + 46$ or $A = 1$. Since the total electric charge is conserved, we have $20 + 2 = Z + 21$ or $Z = 1$. The unknown quantity must, therefore, be a $\boxed{\text{proton, } {}^{1}_{1}\text{H}}$.

b. In a fashion similar to that in part a, we have $A = 12 + 1 - 9 = 4$ and $Z = 6 - 4 = 2$. The unknown quantity must, therefore, be an $\boxed{\text{alpha particle, } {}^{4}_{2}\text{He}}$.

c. Similarly, $A = 9 + 1 - 4 = 6$ and $Z = 4 + 1 - 2 = 3$. The unknown quantity must, therefore, be $\boxed{\text{lithium, } {}^{6}_{3}\text{Li}}$.

d. The nucleon number is $A = 17 + 1 - 4 = 14$. The atomic number is $Z = 8 + 1 - 2 = 7$. The unknown quantity must, therefore, be $\boxed{\text{nitrogen, } {}^{14}_{7}\text{N}}$.

e. The nucleon number is $A = 55 + 1 = 56$. The atomic number is $Z = 25$. The unknown quantity must, therefore, be $\boxed{\text{manganese, } {}^{56}_{25}\text{Mn}}$.

14. ***REASONING AND SOLUTION***

a. We note that ${}^{1}_{0}\mathrm{n}$ is a neutron (n) and ${}^{1}_{1}\mathrm{H}$ is a proton (p), so the reaction can be written as $\boxed{{}^{14}_{7}\mathrm{N}(n, p){}^{14}_{6}\mathrm{C}}$.

b. This reaction can be written as $\boxed{{}^{238}_{92}\mathrm{U}(n, \gamma){}^{239}_{92}\mathrm{U}}$.

c. We note that ${}^{2}_{1}\mathrm{H}$ is a deuteron (d), so the reaction can be written as $\boxed{{}^{24}_{12}\mathrm{Mg}(n, d){}^{23}_{11}\mathrm{Na}}$.

15. **SSM** ***REASONING*** Energy is released from this reaction. Consequently, the combined mass of the daughter nucleus ${}^{12}_{6}\text{C}$ and the α particle ${}^{4}_{2}\text{He}$ is less than the combined mass of the parent nucleus ${}^{14}_{7}\text{N}$ and ${}^{2}_{1}\text{H}$. The mass defect is equivalent to the energy released. We proceed by determining the difference in mass in atomic mass units and then use the fact that 1 u is equivalent to 931.5 MeV (see Section 31.3).

SOLUTION The reaction and the atomic masses are as follows:

$$\underbrace{{}^{2}_{1}\text{H}}_{2.014\,102\text{ u}} + \underbrace{{}^{14}_{7}\text{N}}_{14.003\,074\text{ u}} \rightarrow \underbrace{{}^{12}_{6}\text{C}}_{12.000\,000\text{ u}} + \underbrace{{}^{4}_{2}\text{He}}_{4.002\,603\text{ u}}$$

The mass defect Δm for this reaction is

$$\Delta m = 2.014\,102\text{ u} + 14.003\,074\text{ u} - 12.000\,000\text{ u} - 4.002\,603\text{ u} = 0.014\,573\text{ u}$$

Since 1 u = 931.5 MeV, the energy released is

$$(0.014\,573\text{ u})\left(\frac{931.5\text{ MeV}}{1\text{ u}}\right) = \boxed{13.6\text{ MeV}}$$

16. ***REASONING AND SOLUTION*** The reaction can be written as

$$ {}^{A}_{Z}\mathrm{X} + {}^{63}_{29}\mathrm{Cu} \rightarrow {}^{62}_{29}\mathrm{Cu} + {}^{1}_{1}\mathrm{H} + {}^{1}_{0}\mathrm{n} $$

Therefore, $A + 63 = 62 + 1 + 1$, so that we find $A = 1$. In addition, $Z + 29 = 29 + 1$, so that $Z = 1$. Thus, the unknown particle ${}^{A}_{Z}\mathrm{X}$ is a proton ${}^{1}_{1}\mathrm{H}$. Therefore, the compound nucleus has

$$ Z = 29 + 1 = 30 \qquad \text{and} \qquad A = 63 + 1 = 64 $$

The compound nucleus is $\boxed{\text{zinc, } {}^{64}_{30}\mathrm{Zn}}$.

17. ***REASONING AND SOLUTION*** During a fission reaction, neutrons are produced in addition to the other fission products. We can see from the reaction that the missing nucleon number is

$$A = 235 + 1 - 93 - 141 = 2$$

Thus, the nucleons produced are $\boxed{\text{2 neutrons}}$.

18. ***REASONING AND SOLUTION*** To find the number of neutrons we use the fact that the total number of nucleons is conserved. Therefore,

$$235 + 1 = 133 + 99 + A \qquad \text{or} \qquad A = 4$$

Since each neutron has $A = 1$, we conclude that the number of neutrons released is $\boxed{4}$.

19. **SSM** ***REASONING*** The rest energy of the uranium nucleus can be found by taking the atomic mass of the $^{235}_{92}\text{U}$ atom, subtracting the mass of the 92 electrons, and then using the fact that 1 u is equivalent to 931.5 MeV (see Section 31.3). According to Table 31.1, the mass of an electron is $5.485\,799 \times 10^{-4}$ u. Once the rest energy of the uranium nucleus is found, the desired ratio can be calculated.

SOLUTION The mass of $^{235}_{92}\text{U}$ is 235.043 924 u. Therefore, subtracting the mass of the 92 electrons, we have

$$\text{Mass of } ^{235}_{92}\text{U nucleus} = 235.043\,924 \text{ u} - 92(5.485\,799 \times 10^{-4}\text{u}) = 234.993\,455 \text{ u}$$

The energy equivalent of this mass is

$$(234.993\,455 \text{ u})\left(\frac{931.5 \text{ MeV}}{1 \text{ u}}\right) = 2.189 \times 10^{5} \text{ MeV}$$

Therefore, the ratio is

$$\frac{200 \text{ MeV}}{2.189 \times 10^{5} \text{ MeV}} = \boxed{9.0 \times 10^{-4}}$$

20. ***REASONING AND SOLUTION*** In order to find the mass of the two fragments we need to know the equivalent mass of 225.0 MeV of energy. We know that 1 u = 931.5 MeV. Therefore, the mass equivalent of 225.0 MeV is

$$(225.0 \text{ MeV})(1 \text{ u})/(931.5 \text{ MeV}) = 0.2415 \text{ u}$$

By balancing each side of the fission reaction we can find the mass of the fragments, i.e.,

$$m(\text{fragments}) = m(\text{U-235}) + m(\text{neutron}) - m(3 \text{ neutrons}) - m_{\text{equiv}}$$

$$m(\text{fragments}) = 235.043\,924 \text{ u} - 2(1.008\,665 \text{ u}) - 0.2415 \text{ u} = \boxed{232.7851 \text{ u}}$$

21. ***REASONING AND SOLUTION*** The energy of the neutron after the first collision is $(1.5 \times 10^6 \text{ eV})(0.65)$. After the second collision it is $(1.5 \times 10^6 \text{ eV})(0.65)(0.65)$. Thus, after the nth collision it is

$$(1.5 \times 10^6 \text{ eV})(0.65)^n = 0.040 \text{ eV}$$

Solving for n gives

$$n \log(0.65) = \log\left(\frac{0.040 \text{ eV}}{1.5 \times 10^6 \text{ eV}}\right) = -7.57 \qquad \text{or} \qquad n = \frac{-7.57}{\log(0.65)} = 40.4$$

Therefore, 40 collisions will reduce the energy to something slightly greater than 0.040 eV, and to reduce the energy to at least 0.040 eV, $\boxed{41}$ collisions are needed.

22. ***REASONING*** The conservation of nucleon number states that the total number of nucleons (protons plus neutrons) before the reaction occurs must be equal to the total number of nucleons after the reaction. This conservation law will allow us to find the atomic mass number A of the unknown nucleus. The conservation of electric charge states that the net electric charge of the particles before the reaction must be equal to the net charge after the reaction. This conservation law will allow us to find the atomic number Z of the unknown nucleus. With a knowledge of the atomic number, we can use the periodic table to identify the element.

SOLUTION

a. The total number of nucleons before the reaction is 1 + 232 = 233. The total number of nucleons after the reaction is A. Setting these two numbers equal to each other yields $\boxed{A = 233}$. The net electric charge before the reaction is 0 + 90 = 90. The net electric charge after the reaction is Z. Setting these two numbers equal to each other yields $\boxed{Z = 90}$. A check of the periodic table shows that this element is $\boxed{\text{Thorium } ({}^{233}_{90}\text{Th})}$.

b. The ${}^{233}_{90}\text{Th}$ nucleus subsequently undergoes β^- decay $\left({}^{0}_{-1}\text{e}\right)$, as does its daughter. The first reaction is ${}^{233}_{90}\text{Th} \rightarrow {}^{A}_{Z}\text{X} + {}^{0}_{-1}\text{e}$. By employing an analysis similar to that used in part (a), the unknown nucleus is found to be ${}^{233}_{91}\text{Pa}$. This daughter nucleus also undergoes β^- decay according to the reaction ${}^{233}_{91}\text{Pa} \rightarrow {}^{A}_{Z}\text{X} + {}^{0}_{-1}\text{e}$. Using the analysis of part (a) again, we see that the final unknown nucleus is $\boxed{{}^{233}_{92}\text{U}}$.

23. **SSM** ***REASONING*** We first determine the energy released by 1.0 kg of $^{235}_{92}\text{U}$. Using the data given in the problem statement, we can then determine the number of kilograms of coal that must be burned to produce the same energy.

SOLUTION The energy equivalent of one atomic mass unit is given in the text (see Section 31.3) as

$$1\ \text{u} = 1.4924 \times 10^{-10}\ \text{J} = 931.5\ \text{MeV}$$

Therefore, the energy released in the fission of 1.0 kg of $^{235}_{92}\text{U}$ is

$$(1.0\ \text{kg of}\ ^{235}_{92}\text{U})\left(\frac{1.0 \times 10^{3}\ \text{g/kg}}{235\ \text{g/mol}}\right)\left(\frac{6.022 \times 10^{23}\ \text{nuclei}}{1.0\ \text{mol}}\right)$$

$$\times \left(\frac{2.0 \times 10^{2}\ \text{MeV}}{\text{nuclei}}\right)\left(\frac{1.4924 \times 10^{-10}\ \text{J}}{931.5\ \text{MeV}}\right) = 8.2 \times 10^{13}\ \text{J}$$

When 1.0 kg of coal is burned, about 3.0×10^{7} J is released; therefore the number of kilograms of coal that must be burned to produce an energy of 8.2×10^{13} J is

$$m_{\text{coal}} = \left(8.2 \times 10^{13}\ \text{J}\right)\left(\frac{1.0\ \text{kg}}{3.0 \times 10^{7}\ \text{J}}\right) = \boxed{2.7 \times 10^{6}\ \text{kg}}$$

24. ***REASONING AND SOLUTION*** In one kilogram of U-235 there are

$$(6.02 \times 10^{23}/\text{mole})(1000\text{ g})/(235\text{ g/mole}) = 2.56 \times 10^{24}\text{ nuclei}$$

Each nucleus produces 2.0×10^2 MeV of energy, so the energy liberated by 1 kg of U-235 is

$$E = (2.56 \times 10^{24}\text{ nuclei})(2.0 \times 10^8\text{ eV/nucleus})(1.6 \times 10^{-19}\text{ J/1 eV}) = 8.2 \times 10^{13}\text{ J}$$

The amount of U-235 needed to produce 9.3×10^{19} J is, therefore,

$$m = (9.3 \times 10^{19}\text{ J})(1\text{ kg})/(8.2 \times 10^{13}\text{ J}) = \boxed{1.1 \times 10^6\text{ kg}}$$

25. ***REASONING AND SOLUTION*** The binding energy per nucleon for a nucleus with $A = 239$ is about 7.6 MeV per nucleon, according to Figure 32.8 The nucleus fragments into two pieces of mass ratio 0.32 : 0.68. These fragments thus have nucleon numbers

$$A_1 = (0.32)(A) = 76 \qquad \text{and} \qquad A_2 = (0.68)(A) = 163$$

Using Figure 32.8 we can estimate the binding energy per nucleon for A_1 and A_2. We find that the binding energy per nucleon for A_1 is about 8.8 MeV, representing an increase of 8.8 MeV – 7.6 MeV = 1.2 MeV per nucleon. Since there are 76 nucleons present, the energy released for A_1 is

$$76 \times 1.2 \text{ MeV} = 91 \text{ MeV}$$

Similarly, for A_2, we see that the binding energy increases to 8.0 MeV per nucleon, the difference being 8.0 MeV – 7.6 MeV = 0.4 MeV per nucleon. Since there are 163 nucleons present, the energy released for A_2 is

$$163 \times 0.4 \text{ MeV} = 70 \text{ MeV}$$

The energy released per fission is $E = 91 \text{ MeV} + 70 \text{ MeV} = \boxed{160 \text{ MeV}}$.

26. ***REASONING AND SOLUTION***

a. We know that

$$(2.0 \times 10^8 \text{ eV})(1.6 \times 10^{-19} \text{ J/1 eV}) = 3.2 \times 10^{-11} \text{ J}$$

is released for each nucleus. The number N of nuclei needed to produce the equivalent of 20.0 kilotons of TNT is, therefore,

$$N = (20.0 \text{ kilotons})[(5.0 \times 10^{12} \text{ J})/(1 \text{ kiloton})]/(3.2 \times 10^{-11} \text{ J}) = \boxed{3.1 \times 10^{24}}$$

b. We know there are 6.02×10^{23} nuclei in 235 g of U-235. Therefore, 3.1×10^{24} nuclei have a mass of

$$m = (3.1 \times 10^{24})(235 \text{ g/mole})/(6.02 \times 10^{23}/\text{mole}) = \boxed{1.2 \times 10^3 \text{ g}}$$

c. The equivalent mass of the bomb's energy can be obtained from $E_0 = mc^2$ as follows:

$$m = E_0/c^2 = (1.0 \times 10^{14} \text{ J})/(3.0 \times 10^8 \text{ m/s})^2 = 1.1 \times 10^{-3} \text{ kg} = \boxed{1.1 \text{ g}}$$

27. **SSM** **WWW** ***REASONING*** We first determine the total power generated (used and wasted) by the plant. Energy is power times the time, according to Equation 6.10, and given the energy, we can determine how many kilograms of $^{235}_{92}\text{U}$ are fissioned to produce this energy.

SOLUTION Since the power plant produces energy at a rate of 8.0×10^8 W when operating at 25 % efficiency, the total power produced by the power plant is

$$\left(8.0 \times 10^8 \text{ W}\right)4 = 3.2 \times 10^9 \text{ W}$$

The energy equivalent of one atomic mass unit is given in the text (see Section 31.3) as

$$1 \text{ u} = 1.4924 \times 10^{-10} \text{ J} = 931.5 \text{ MeV}$$

Since each fission produces 2.0×10^2 MeV of energy, the total mass of $^{235}_{92}\text{U}$ required to generate 3.2×10^9 W for a year (3.156×10^7 s) is

$$\overbrace{\left(3.2 \times 10^9 \text{ J/s}\right)\left(3.156 \times 10^7 \text{ s}\right)}^{\text{Power times time gives energy in joules}}$$

$$\times \underbrace{\left(\frac{931.5 \text{ MeV}}{1.4924 \times 10^{-10} \text{ J}}\right)}_{\text{Converts joules to MeV}} \underbrace{\left(\frac{1.0 \ ^{235}_{92}\text{U nucleus}}{2.0 \times 10^2 \text{ MeV}}\right)}_{\text{Converts MeV to number of nuclei}} \underbrace{\left(\frac{0.235 \text{ kg}}{6.022 \times 10^{23} \ ^{235}_{92}\text{U nuclei}}\right)}_{\text{Converts number of nuclei to kilograms}} = \boxed{1200 \text{ kg}}$$

28. ***REASONING*** The conservation of nucleon number states that the total number of nucleons (protons plus neutrons) before the reaction occurs must be equal to the total number of nucleons after the reaction. This conservation law will allow us to find the atomic mass number A of the unknown particle Y. The conservation of electric charge states that the net electric charge of the particles before the reaction must be equal to the net charge after the reaction. This conservation law will allow us to find the atomic number Z of the unknown particle X.

SOLUTION

a. The total number of nucleons before the reaction is $1 + A$. The total number of nucleons after the reaction is 3. Setting these two numbers equal to each other yields $\boxed{A = 2}$. The net electric charge before the reaction is $Z + 1$. The net electric charge after the reaction is 1. Setting these two numbers equal to each other yields $\boxed{Z = 0}$. The nucleon ${}^{1}_{Z}\text{X} = {}^{1}_{0}\text{n}$ is a $\boxed{\text{neutron}}$. The nucleon ${}^{A}_{1}\text{Y} = {}^{2}_{1}\text{H}$ is a $\boxed{\text{hydrogen nucleus}}$.

b. The sum of the atomic masses before the reaction is 1.0087 u + 2.0141 u = 3.0228 u. The sum of the atomic mass after the reaction is 3.0161 u. The difference between the sums is 3.0228 u – 3.0161 u = 0.0067 u. This mass difference is equivalent to an energy of

$$(0.0067\ \text{u})\left(\frac{931.5\ \text{MeV}}{1\ \text{u}}\right) = \boxed{6.2\ \text{MeV}}$$

29. ***REASONING AND SOLUTION*** The mass defect Δm for the fusion process is

$$\Delta m = 2.0141 \text{ u} + 2.0141 \text{ u} - 3.0161 \text{ u} - 1.0078 \text{ u} = 0.0043 \text{ u}$$

Since 1 u = 931.5 MeV, the energy released is

$$(0.0043 \text{ u})(931.5 \text{ MeV})/(1 \text{ u}) = \boxed{4.0 \text{ MeV}}$$

30. ***REASONING AND SOLUTION*** The energy released in the reaction is found from the mass defect between 2 deuterium nuclei and a helium nucleus plus neutron;

$$\Delta m = 2(2.0141\text{ u}) - (3.0160\text{ u} + 1.0087\text{ u}) = 0.0035\text{ u}$$

The energy is, therefore,

$$E = (0.0035\text{ u})(931.5\text{ MeV})/(1\text{ u}) = \boxed{3.3\text{ MeV}}$$

31. **SSM** ***REASONING*** To find the energy released per reaction, we follow the usual procedure of determining how much the mass has decreased because of the fusion process. Once the energy released per reaction is determined, we can determine the mass of lithium ${}^{6}_{3}\text{Li}$ needed to produce 3.8×10^{10} J.

SOLUTION The reaction and the masses are shown below:

$$\underbrace{{}^{2}_{1}\text{H}}_{2.014\text{ u}} + \underbrace{{}^{6}_{3}\text{Li}}_{6.015\text{ u}} \rightarrow \underbrace{2\ {}^{4}_{2}\text{He}}_{2(4.003\text{ u})}$$

The mass defect is, therefore, $2.014\text{ u} + 6.015\text{ u} - 2(4.003\text{ u}) = 0.023\text{ u}$. Since 1 u is equivalent to 931.5 MeV, the released energy is 21 MeV, or since the energy equivalent of one atomic mass unit is given in Section 31.3 as $1\text{ u} = 1.4924 \times 10^{-10}\text{ J} = 931.5\text{ MeV}$,

$$(21\text{ MeV})\left(\frac{1.4924 \times 10^{-10}\text{ J}}{931.5\text{ MeV}}\right) = 3.4 \times 10^{-12}\text{ J}$$

In 1.0 kg of lithium ${}^{6}_{3}\text{Li}$, there are

$$(1.0\text{ kg of }{}^{6}_{3}\text{Li})\left(\frac{1.0 \times 10^{3}\text{ g}}{1.0\text{ kg}}\right)\left(\frac{6.022 \times 10^{23}\text{ nuclei/mol}}{6.015\text{ g/mol}}\right) = 1.0 \times 10^{26}\text{ nuclei}$$

Therefore, 1.0 kg of lithium ${}^{6}_{3}\text{Li}$ would produce an energy of

$$(3.4 \times 10^{-12}\text{ J/nuclei})(1.0 \times 10^{26}\text{ nuclei}) = 3.4 \times 10^{14}\text{ J}$$

If the energy needs of one household for a year is estimated to be 3.8×10^{10} J, then the amount of lithium required is

$$\frac{3.8 \times 10^{10}\text{ J}}{3.4 \times 10^{14}\text{ J/kg}} = \boxed{1.1 \times 10^{-4}\text{ kg}}$$

32. ***REASONING AND SOLUTION***

a. The number n of atoms of hydrogen and its isotopes in 1 kg of water (molecular mass = 18 u) is

$$n = 2(6.02 \times 10^{23}/\text{mol})(1.00 \times 10^{3}\ \text{g})/(18.015\ \text{g/mol}) = 6.68 \times 10^{25}$$

If deuterium makes up 0.015% of hydrogen in this number of atoms, the number N of deuterium atoms is

$$N = (1.5 \times 10^{-4})(6.68 \times 10^{25}) = \boxed{1.0 \times 10^{22}}$$

b. Each deuterium nucleus provides 7.2 MeV of energy, so the energy from 1 kg of water is

$$E = (1.0 \times 10^{22})(7.2 \times 10^{6}\ \text{eV})(1.6 \times 10^{-19}\ \text{J})/(1\ \text{eV}) = 1.15 \times 10^{10}\ \text{J}$$

To supply 9.3×10^{19} J of energy, we would need

$$m = (9.3 \times 10^{19}\ \text{J})/(1.15 \times 10^{10}\ \text{J/kg}) = \boxed{8.1 \times 10^{9}\ \text{kg}}$$

33. ***REASONING AND SOLUTION*** We will use the atomic mass of hydrogen ${}^{1}_{1}\text{H}$ (1.007 825 u), deuterium ${}^{2}_{1}\text{H}$ (2.014 102 u), helium ${}^{3}_{2}\text{He}$ (3.016 030 u), helium ${}^{4}_{2}\text{He}$ (4.002 603 u), a positron ${}^{0}_{1}\text{e}$ (0.000 549 u), and an electron (0.000 549 u). The mass defect Δm for two reactions of type (1) is

$$\Delta m = 2\,(1.007\,825\text{ u} + 1.007\,825\text{ u} - 2.014\,102\text{ u} - 0.000\,549\text{ u} - 0.000\,549\text{ u}) = 0.000\,900\text{ u}$$

In this result, one of the two values of 0.000 549 u accounts for the positron. The other is present because the two hydrogen atoms contain a total of two electrons, whereas a single deuterium atom contains only one electron. The mass defect for two reactions of type (2) is

$$\Delta m = 2\,(1.007\,825\text{ u} + 2.014\,102\text{ u} - 3.016\,030\text{ u}) = 0.011\,794\text{ u}$$

The mass defect for one reaction of type (3) is

$$\Delta m = 3.016\,030\text{ u} + 3.016\,030\text{ u} - 4.002\,603\text{ u} - 1.007\,825\text{ u} - 1.007\,825\text{ u} = 0.013\,807\text{ u}$$

The total mass defect for the proton-proton cycle is the sum of three values just calculated:

$$\Delta m_{\text{total}} = 0.000\,900\text{ u} + 0.011\,794\text{ u} + 0.013\,807\text{ u} = 0.026\,501\text{ u}$$

Since 1 u = 931.5 MeV, the energy released is

$$(0.026\,501\text{ u})[(931.5\text{ MeV})/(1\text{ u})] = \boxed{24.7\text{ MeV}}$$

34. ***REASONING*** The energy released in the decay is the difference between the rest energy of the π^- and the sum of the rest energies of the μ^- and $\bar{\nu}_\mu$. Each of these energies can be found in Table 32.3

SOLUTION The rest energies of the particles are π^- (139.6 MeV), μ^- (105.7 MeV) and $\bar{\nu}_\mu$ (≈ 0 MeV). The energy released is 139.6 MeV – 105.7 MeV = $\boxed{33.9 \text{ MeV}}$.

35. **SSM** ***REASONING AND SOLUTION*** The lambda particle contains three different quarks, one of which is the up quark u, and contains no antiquarks. Therefore, the remaining two quarks must be selected from the down quark d, the strange quark s, the charmed quark c, the top quark t, and the bottom quark b. Since the lambda particle has an electric charge of zero and since u has a charge of $+2e/3$, the charges of the remaining two quarks must add up to a total charge of $-2e/3$. This eliminates the quarks c and t as choices, because they each have a charge of $+2e/3$. We are left, then, with d, s, and b as choices for the remaining two quarks in the lambda particle. The three possibilities are as follows:

(1) u,d,s	(2) u,d,b	(3) u,s,b

36. ***REASONING AND SOLUTION***

a. The electron and its antiparticle (the positron) both have the same mass. Therefore, each electron produces one gamma ray of the same energy.

$$E_0 = mc^2 = (9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^8\ \text{m/s})^2 = 8.20 \times 10^{-14}\ \text{J} \qquad (28.5)$$

or

$$E_0 = (8.20 \times 10^{-14}\ \text{J})/(1.60 \times 10^{-19}\ \text{J})/(1\ \text{eV}) = \boxed{0.513\ \text{MeV}}$$

b. The wavelength can be found by combining Equations 29.2 ($E_0 = hf$) and 16.1 ($\lambda = c/f$) so that $\lambda = c/f = hc/E_0$:

$$\lambda = hc/E_0 = (6.63 \times 10^{-34}\ \text{J·s})(3.00 \times 10^8\ \text{m/s})/(8.20 \times 10^{-14}\ \text{J}) = \boxed{2.43 \times 10^{-12}\ \text{m}}$$

c. The magnitude of the momentum of each photon is

$$p = h/\lambda = (6.63 \times 10^{-34}\ \text{J·s})/(2.43 \times 10^{-12}\ \text{m}) = \boxed{2.73 \times 10^{-22}\ \text{kg·m/s}} \qquad (29.8)$$

37. ***REASONING AND SOLUTION*** The proton is constructed from the quarks d, u, and u. Therefore, the antiproton is constructed from $\boxed{\bar{\mathrm{d}},\ \bar{\mathrm{u}},\ \text{and}\ \bar{\mathrm{u}}}$.

38. ***REASONING AND SOLUTION*** Let us consider the difference in rest energies between the two incident protons and the outgoing proton, neutron and π^+. This would be equivalent to the minimum energy of the incident proton. We have

$$E = (1.0087\text{ u})(931.5\text{ MeV})/(1\text{ u}) + 139.6\text{ MeV} - (1.0073\text{ u})(931.5\text{ MeV})/(1\text{ u})$$

$$E = 939.6\text{ MeV} + 139.6\text{ MeV} - 938.3\text{ MeV} = \boxed{140.9\text{ MeV}}$$

39. **SSM** **WWW** ***REASONING*** The momentum of a photon is given in the text as $p = E/c$ (see the discussion leading to Equation 29.6). This expression applies to any massless particle that travels at the speed of light. In particular, if the neutrino has no mass and travels at the speed of light, it applies to the neutrino. Once the momentum of the neutrino is determined, the de Broglie wavelength can be calculated from Equation 29.6 ($p = h/\lambda$).

SOLUTION

a. The momentum of the neutrino is, therefore,

$$p = \frac{E}{c} = \left(\frac{35\ \text{MeV}}{3.00 \times 10^{8}\ \text{m/s}}\right)\left(\frac{1.4924 \times 10^{-10}\ \text{J}}{931.5\ \text{MeV}}\right) = \boxed{1.9 \times 10^{-20}\ \text{kg} \cdot \text{m/s}}$$

where we have used the fact that 1.4924×10^{-10} J = 931.5 MeV (see Section 31.3).

b. According to Equation 29.6, the de Broglie wavelength of the neutrino is

$$\lambda = \frac{h}{p} = \frac{6.63 \times 10^{-34}\ \text{J}\times\text{s}}{1.9 \times 10^{-20}\ \text{kg} \cdot \text{m/s}} = \boxed{3.5 \times 10^{-14}\ \text{m}}$$

40. ***REASONING AND SOLUTION*** In order for the proton to come within a distance r of the second proton, it must overcome the Coulomb repulsive force. Therefore, the kinetic energy of the incoming proton must equal the Coulombic potential energy of the system.

$$\text{KE} = \text{EPE} = ke^2/r$$

$$\text{KE} = (8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6 \times 10^{-19}\ \text{C})^2/(8.0 \times 10^{-15}\ \text{m}) = 2.9 \times 10^{-14}\ \text{J}$$

$$\text{KE} = (2.9 \times 10^{-14}\ \text{J})(1\ \text{eV})/(1.6 \times 10^{-19}\ \text{J}) = 1.8 \times 10^5\ \text{eV} = \boxed{0.18\ \text{MeV}}$$

41. ***REASONING AND SOLUTION*** The absorbed dose (AD) is

$$\text{AD} = (3.1 \times 10^{-5}\ \text{Gy/s})(0.10\ \text{s}) = 3.1 \times 10^{-6}\ \text{Gy}$$

The energy absorbed is given by

$$\text{Energy absorbed} = \text{AD} \times (\text{Mass of absorbing material}) \tag{32.2}$$

$$= (3.1 \times 10^{-6}\ \text{Gy})(1.2\ \text{kg}) = \boxed{3.7 \times 10^{-6}\ \text{J}}$$

42. ***REASONING AND SOLUTION*** The additional matter comes from the kinetic energy of the incoming protons. We can find the amount by considering the difference in rest energies as follows:

$$\Delta E = (1.007\,276\text{ u})(931.5\text{ MeV})/(1\text{ u}) + 139.6\text{ MeV} + 1116\text{ MeV} + 497.7\text{ MeV}$$

$$- 2(1.007\,276\text{ u})(931.5\text{ MeV})/(1\text{ u}) = \boxed{815\text{ MeV}}$$

43. **SSM** **WWW** ***REASONING*** The reaction given in the problem statement is written in the shorthand form: ${}^{17}_{8}\mathrm{O}\,(\gamma,\alpha\,\mathrm{n})\,{}^{12}_{6}\mathrm{C}$. The first and last symbols represent the initial and final nuclei, respectively. The symbols inside the parentheses denote the incident particles or rays (left side of the comma) and the emitted particles or rays (right side of the comma).

SOLUTION Using the reasoning above and noting that an α particle is a helium nucleus, ${}^{4}_{2}\mathrm{He}$, we have

$$\boxed{\gamma + {}^{17}_{8}\mathrm{O} \rightarrow {}^{12}_{6}\mathrm{C} + {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n}}$$

44. ***REASONING AND SOLUTION*** The energy liberated can be found from the mass defect of the reaction, which is

$$\Delta m = 235.044 \text{ u} - 2(1.009 \text{ u}) - 140.914 \text{ u} - 91.926 \text{ u} = 0.186 \text{ u}$$

The equivalent energy is

$$\Delta E = (0.186 \text{ u})(931.5 \text{ MeV})/(1 \text{ u}) = \boxed{173 \text{ MeV}}$$

45. **SSM** ***REASONING*** To find the energy released per reaction, we follow the usual procedure of determining how much the mass has decreased because of the fusion process. Once the energy released per reaction is determined, we can determine the amount of gasoline that must be burned to produce the same amount of energy.

SOLUTION The reaction and the masses are shown below:

$$\underbrace{3\ {}^{2}_{1}\text{H}}_{3(\,2.0141\text{ u})} \rightarrow \underbrace{{}^{4}_{2}\text{He}}_{4.0026\text{ u}} + \underbrace{{}^{1}_{1}\text{H}}_{1.0078\text{ u}} + \underbrace{{}^{1}_{0}\text{n}}_{1.0087\text{ u}}$$

The mass defect is, therefore, $3(2.0141\text{ u}) - 4.0026\text{ u} - 1.0078\text{ u} - 1.0087\text{ u} = 0.0232\text{ u}$. Since 1 u is equivalent to 931.5 MeV, the released energy is 21.6 MeV, or since it is shown in Section 31.3 that $931.5\text{ MeV} = 1.4924 \times 10^{-10}\text{ J}$, the energy released per reaction is

$$(21.6\text{ MeV})\left(\frac{1.4924 \times 10^{-10}\text{ J}}{931.5\text{ MeV}}\right) = 3.46 \times 10^{-12}\text{ J}$$

To find the total energy released by all the deuterium fuel, we need to know the number of deuterium nuclei present. The number of deuterium nuclei is

$$(6.1 \times 10^{-3}\text{ g})\left(\frac{6.022 \times 10^{23}\text{ nuclei/mol}}{2.0141\text{ g/mol}}\right) = 1.8 \times 10^{21}\text{ nuclei}$$

Since each reaction consumes three deuterium nuclei, the total energy released by the deuterium fuel is

$$\frac{1}{3}(3.46 \times 10^{-12}\text{ J/nuclei})(1.8 \times 10^{21}\text{ nuclei}) = 2.1 \times 10^{9}\text{ J}$$

If one gallon of gasoline produces $2.1 \times 10^{9}\text{ J}$ of energy, then the number of gallons of gasoline that would have to be burned to equal the energy released by all the deuterium fuel is

$$(2.1 \times 10^{9}\text{ J})\left(\frac{1.0\text{ gal}}{2.1 \times 10^{9}\text{ J}}\right) = \boxed{1.0\text{ gal}}$$

46. ***REASONING AND SOLUTION*** The absorbed dose is

$$750 \text{ rad} = 7.5 \text{ Gy} = 7.5 \text{ J/kg} = Q/m$$

The rise in temperature of the water is determined from $Q = cm\Delta T$, Equation 12.4, so that the temperature rise is

$$\Delta T = (Q/m)/c = (7.5 \text{ J/kg})/[4186 \text{ J/(kg}\cdot\text{C}^\circ)] = \boxed{1.8 \times 10^{-3} \text{ C}^\circ}$$

47. ***REASONING AND SOLUTION*** We have $Q = cm\Delta T$, Equation 12.4. The power is the energy per unit time, so $P = Q/t$. The mass of the water passing through the core each second is, therefore,

$$\frac{m}{t} = \frac{\frac{Q}{t}}{c\,\Delta T} = \frac{5.6 \times 10^{9}\ \text{J/s}}{[4420\ \text{J/(kg}\cdot\text{C°)}](287\ \text{°C} - 216\ \text{°C})} = \boxed{1.8 \times 10^{4}\ \text{kg/s}}$$

48. ***REASONING AND SOLUTION***

a. The number of nuclei in one gram of U-235 can be obtained as follows:

$$1 \text{ gram of U-235} = [(1/235) \text{ mol}] \times (6.02 \times 10^{23} \text{ g/mol}) = 2.56 \times 10^{21} \text{ nuclei}$$

Each nucleus yields 2.0×10^2 MeV of energy, so we have

$$E = (2.0 \times 10^8 \text{ eV})[(1.60 \times 10^{-19} \text{ J})/1 \text{ eV})](2.56 \times 10^{21}) = \boxed{8.2 \times 10^{10} \text{ J}}$$

b. If 30.0 kWh of energy are used per day, the total energy use per year is

$$E_{\text{total}} = (30.0 \text{ kWh/d})[(3.60 \times 10^6 \text{ J})/(1 \text{ kWh})][(365 \text{ d})/(1 \text{ yr})] = 3.94 \times 10^{10} \text{ J/yr}$$

The amount of U-235 needed in a year, then, is

$$m = (3.94 \times 10^{10} \text{ J})/(8.2 \times 10^{10} \text{ J/g}) = \boxed{0.48 \text{ g}}$$

49. ***REASONING AND SOLUTION*** Using Equation 32.1,

$$\text{Exposure} = \frac{q}{\left(2.58 \times 10^{-4}\right) m}$$

we can get the amount of charge produced in 1 kilogram of dry air when exposed to 1.0 R of X-rays as follows:

$$q = (2.58 \times 10^{-4})(1.0\ \text{R})(1\ \text{kg}) = 2.58 \times 10^{-4}\ \text{C}$$

Alternatively, we know that 1.0 R deposits 8.3×10^{-3} J of energy in 1 kg of dry air. Thus, 8.3×10^{-3} J of energy produces 2.58×10^{-4} C. To find how much energy E is required to produce 1.60×10^{-19} C, we set up a ratio as follows:

$$\frac{E}{8.3 \times 10^{-3}\ \text{J}} = \frac{1.60 \times 10^{-19}\ \text{C}}{2.58 \times 10^{-4}\ \text{C}} \qquad \text{or} \qquad E = 5.1 \times 10^{-18}\ \text{J}$$

Converting to eV, we find

$$E = (5.1 \times 10^{-18}\ \text{J})(1\ \text{eV})/(1.6 \times 10^{-19}\ \text{J}) = \boxed{32\ \text{eV}}$$

CHAPTER 18 | *ELECTRIC FORCES AND ELECTRIC FIELDS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** In Figure 18.8, the grounding wire is removed first, followed by the rod, and the sphere is left with a positive charge. If the rod were removed first, followed by the grounding wire, the sphere would not be left with a charge. Once the rod is removed, the repulsive force caused by the presence of the rubber rod is no longer present. Since the wire is still attached, free electrons will enter the sphere from the ground until the sphere is once again neutral.

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2. ***REASONING AND SOLUTION*** A metallic rod is given a positive charge by the process of induction as illustrated in Figure 18.8.

   a. The metallic object becomes positive because, during the induction process, electrons are forced from the object to the earth. The mass of the object will decrease by an amount equal to the mass of the electrons that left the metallic object.

   b. The metallic object becomes negative because, during the induction process, electrons are pulled onto the object from the earth. The mass of the object will increase by an amount that is equal to the mass of the "excess" electrons that are pulled onto the object.

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3. ***REASONING AND SOLUTION*** When the charged insulating rod is brought near to (but not touching) the sphere, the free electrons in the sphere will move. If the rod is negatively charged, the free electrons will move to the side of the sphere that is opposite to the side where the rod is; if the rod is positively charged, the free electrons will migrate to the side of the sphere where the rod is. In either case, the region of the sphere near the vicinity of the rod will acquire a charge that has the opposite sign as the charge on the rod.

   a. Since oppositely charged objects always attract each other, the rod and sphere will always experience a mutual attraction.

   b. Since the side of the sphere in the vicinity of the rod will always have charge that is opposite in sign to the charge on the rod, the rod and the sphere will always attract each other. They never repel each other.

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4. ***REASONING AND SOLUTION*** On a dry day, just after washing your hair to remove natural oils and drying it thoroughly, you run a plastic comb through it. As the surface of the comb rubs against your hair, the comb becomes electrically charged. If the comb is brought near small bits of paper, the charge on the comb causes a separation of charge on the bits of paper. Since the paper is neutral, it contains equal amounts of positive and negative charge. The charge on the comb causes the regions of the bits of paper that are

closest to the comb to become oppositely charged; therefore, the bits of paper are attracted to the comb. This situation is analogous to that in Figure 18.9.

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5. ***REASONING AND SOLUTION*** A balloon is blown up and rubbed against a person's shirt a number of times. The balloon is then touched to a ceiling. Upon being released, the balloon remains stuck to the ceiling. The balloon is charged by contact. The ceiling is neutral. The charged balloon will induce a slight surface charge on the ceiling that is opposite in sign to the charge on the balloon. Since the charge on the balloon and the ceiling are opposite in sign, they will attract each other. Since both the balloon and the ceiling are insulators, charge cannot flow from one to the other. The charge on the balloon remains fixed on the balloon, while the charge on the ceiling remains fixed on the ceiling. The electrostatic force that the ceiling exerts on the balloon is sufficient to hold the balloon in place.

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6. ***REASONING AND SOLUTION*** A proton and electron are held in place on the x axis. The proton is at $x = \ d$, while the electron is at $x = +d$. They are released simultaneously, and the only force that affects their motions is the electrostatic force of attraction that each applies to the other. According to Newton's third law, the force F_{pe} exerted on the proton by the electron is equal in magnitude and opposite in direction to the force F_{ep} exerted on the electron by the proton. In other words, $F_{\text{pe}} = \ F_{\text{ep}}$. According to Newton's second law, this equation can be written

$$m_{\text{p}} a_{\text{p}} = \ m_{\text{e}} a_{\text{e}} \tag{1}$$

where m_{p} and m_{e} are the respective masses and a_{p} and a_{e} are the respective accelerations of the proton and the electron. Since the mass of the electron is considerably smaller than the mass of the proton, the acceleration of the electron at any instant must be considerably greater than the acceleration of the proton at that instant in order for Equation (1) to hold. Since the electron has a much greater acceleration than the proton, it will attain greater velocities than the proton and, therefore, reach the origin first.

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7. ***REASONING AND SOLUTION*** In order for a particle to execute simple harmonic motion, it must obey a force law of the form of Equation 10.2, $F = -kx$, where x is the displacement of the object from its equilibrium position, and k is the spring constant. The force described by Equation 10.2 is a restoring force in the sense that it always pulls the particle toward its equilibrium position.

The force of repulsion that a charge q_1 feels when it is pushed toward another charge q_2 of the same polarity is given by Coulomb's law, Equation 18.1, $F = kq_1q_2/r^2$. Clearly, Coulomb's law is an inverse square law. It does not have the same mathematical form as Equation 10.2. Therefore, a charged particle that is pushed toward another charged particle of the same polarity that is fixed in position, will not exhibit simple harmonic motion when

it is released. Coulomb's law does *not* describe a restoring force. When q_1 is released, it simply "flies away" from q_2 and never returns.

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8. ***REASONING AND SOLUTION*** Identical point charges are fixed to opposite corners of a square, as shown in the figure at the right.

There exists an electric field at each point in space in the vicinity of this configuration. The electric field at a given point is the resultant of the electric field at that point due to the charge at corner 1 and the charge at corner 3.

The magnitude of the electric field due to a single point charge of strength q is given by Equation 18.3: $E = kq/r^2$, where r is the distance from the charge to the point in question.

Since the point charges at corners 1 and 3 are identical, they each have the same value of q. Furthermore, they are equidistant from the center point C. Therefore, at the center of the square, each charge gives rise to an electric field that is equal in magnitude and opposite in direction. The resultant electric field at the center of the square is, therefore, zero. Note that this result is independent of the polarity of the charges.

The distance from either empty corner, 2 or 4, to either of the charges is the same. Since the charges are equidistant from either empty corner, each charge gives rise to an electric field that is equal in magnitude. Since the direction of the electric field due to a point charge is radial (radially inward for negative charges and radially outward for positive charges), we see that the electric fields due to each of the two point charges will be mutually perpendicular. Their resultant can be found by using the Pythagorean theorem.

The magnitude of the force experienced by a third point charge placed in this system is, from Equation 18.2: $F = q_0 E$, where q_0 is the magnitude of the charge and E is the magnitude of the electric field at the location of the charge. Since the electric field is zero at the center of the square, a third point charge will experience no force there. Thus, a third point charge will experience the greater force at one of the empty corners of the square.

---

9. ***REASONING AND SOLUTION*** The figure shows a thin, nonconducting rod, on which positive charges are spread evenly, so that there is the same amount of charge per unit length at every point. We wish to deduce the direction of the electric field at the point P, which is directly above the midpoint of the rod.

We first imagine dividing the rod up into a large number of small length elements. Since the charge per unit length is the same along the rod, each element will contain an equal amount of charge q. If the elements are chosen small enough, then the amount of charge on each may be treated as a point charge. That is, the charge on each element gives rise to

an electric field at the point P given by $E = k\ q/r^2$, where r is the distance between that particular element and the point P. The electric field at P is found by adding, vectorially, the effects of all the elements. Since the rod is positively charged, the direction of $\mathbf{E}$ is away from the rod and along the line that connects the point P to the particular element of charge. In general, each element q is a different distance r from the point P, and, therefore, gives rise to a different electric field $\mathbf{E}$ at the point P. However, every element on the left half of the rod can be paired with an element on the right half of the rod that is the same distance r from the point P. Thus, each element on the left can be paired with an element on the right that gives rise to an electric field at P that has the same magnitude E.

We notice that each $\mathbf{E}$ has both x and y components; however, for charge elements on the right, the x component points in the negative x direction, while for elements on the left, the x component points in the positive x direction, as shown below.

Thus, the x field components from the charges on the left cancel pair-wise with the x field components from the charges on the right. The y field components from all of the charge elements point in the positive y direction. Therefore, the net electric field at the point P will point only in the positive y direction, that is, perpendicularly away from the rod.

On another identical rod, positive charges are spread evenly over only the left half of the rod, and the same amount of negative charges are spread evenly over the right half. Again, we wish to deduce the direction of the electric field at the point P, which is located above the midpoint of the rod.

We proceed as we did previously. Since the left half of the rod is positively charged, the direction of $\mathbf{E}$ is away from the rod and along the line that connects the point P to the particular element of charge on the left half. Conversely, the right half of the rod is negatively charged. Therefore, the direction of $\mathbf{E}$ is toward the rod and along the line that connects the point P to the particular element of charge on the right half. As with the first rod, each element of charge q gives rise to a different electric field $\mathbf{E}$ at the point P. But once again, each element on the left half of the rod can be paired with an element on the right half of the rod that is the same distance r from the point P and, therefore, gives rise to an electric field at P of the same magnitude E.

Each $\mathbf{E}$ has both x and y components; however, for charge elements on the right, the y component points in the negative y direction, while for charge elements on the left, the y component points in the positive y direction, as shown in the following drawings.

Thus, the y field components that arise from the charges on the left cancel pair-wise with the y field components that arise from the charges on the right. The x field components from all of the charge elements point in the positive x direction. Therefore, the net electric field at the point P will point only in the positive x direction, that is, parallel to the rod, pointing from the positive toward the negative side.

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10. ***REASONING AND SOLUTION*** From Equation 18.2, we see that the electric field **E** that exists at a point in space is the force per unit charge, $\mathbf{F}/q_0$, experienced by a small test charge q_0 placed at that point. When a positive or negative charge is placed at a point where there is an electric field, the charge will experience a force given by $\mathbf{F} = q_0\mathbf{E}$. If the charge q_0 is positive, then **F** and **E** are in the same direction; that is, a positive charge experiences a force in the direction of the electric field. Similarly, if the charge q_0 is negative, then **F** and **E** are in the opposite direction; a negative charge experiences a force that is opposite to the direction of the electric field.

If two different charges experience forces that differ in both magnitude and direction when they are separately placed at the same point in space, we can conclude that the two charges differ in both magnitude and polarity.

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11. ***REASONING AND SOLUTION*** Three point charges are fixed to the corners of a square (side length L and diagonal length d), one to a corner, in such a way that the net electric field at the empty corner is zero.

a. Each charge produces an electric field that is radially toward (positive charges) or radially away from (negative charges) the charge. The net electric field at corner 1 is the vector sum of the electric fields due to each of the charges at corners 2, 3 and 4. Since the electric field at corner 1 is zero, then the charges in corners 2 and 4 must have the same sign and have the opposite sign to the charge in corner 3. Suppose, for example, that the charge in corner 3 is positive. Then the electric field at corner 1 due to the charge in corner 3 must point away from corner 3 along the diagonal. In order for the vector sum of the fields due to all three charges to vanish at corner 1, the electric field due to the charge in corner 2 must cancel the x component of the electric field due to the charge in corner 3. Likewise, the electric field due to the charge in corner 4 must cancel the y component of the electric field due to the

charge in corner 3. Thus, at corner 1, the electric field due to the charge at corner 2 must point to the right, while the electric field due to the charge in corner 4 must point toward the bottom of the page. The charges in corners 2 and 4, therefore, must be negative. Similar arguments apply if the charge in corner 3 is negative; we can then deduce that the charges in corners 2 and 4 must be positive. We conclude, therefore, that the three charges *cannot* all have the same sign.

b. Let all three charges have magnitude Q. Then the magnitude of the electric field at corner 1 due to the charge in corner 3 is given by Equation 18.3:

$$E_{13} = \frac{kQ}{d^2} = \frac{kQ}{2L^2}$$

As discussed in part (a), this electric field vector points along the diagonal. Therefore, its x and y components have the following magnitudes

$$(E_{13})_x = \frac{kQ}{2L^2}\cos 45° = \frac{kQ}{2L^2}\ \frac{\sqrt{2}}{2} \div = \frac{kQ}{(2\sqrt{2})L^2}$$

$$(E_{13})_y = \frac{kQ}{2L^2}\sin 45° = \frac{kQ}{2L^2}\ \frac{\sqrt{2}}{2} \div = \frac{kQ}{(2\sqrt{2})L^2}$$

In order for the net electric field at corner 1 to be zero, the charges in corners 2 and 4 must have the opposite sign to the charge in corner 3 [as discussed in part (a)], and the electric field at corner 1 due to the charge in corner 2 must be equal to $(E_{13})_x$. Similarly, the electric field at corner 1 due to the charge in corner 4 must be equal to $-(E_{13})_y$. If the charges in all three corners have the same magnitude, however, the electric field at corner 1 due to the charge at corner 2 and the electric field at corner 1 due to the charge at corner 4 have magnitudes given by Equation 18.3 as

$$E_{12} = E_{14} = \frac{kQ}{L^2}$$

Clearly, E_{12} $(E_{13})_x$ and E_{14} $(E_{13})_y$; therefore, all three charges *cannot* have the same magnitude.

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12. ***REASONING AND SOLUTION*** The charges at corners 1 and 3 are both $+q$ while the charges at corners 2 and 4 are both $-q$. Let's concentrate first on the electric field produced at the center C of the rectangle by the positive charges. The positive charge at corner 1 produces an electric field at C that points toward corner 3. In contrast, the positive charge at corner 3 produces an electric field at C that points toward corner 1. Thus, the two fields have opposite directions. The magnitudes of the fields are identical because the charges have the same magnitude and are equally far from the center. Therefore, the fields from the

two positive charges cancel. In a similar manner, the fields due to the negative charges on corners 2 and 4 also cancel. Consequently, the net electric field at the center *C* due to all four charges is zero.

---

13. ***REASONING AND SOLUTION*** We can think of dividing the line into three distinct regions. Region I to the left of the positive charge, region II between the charges, and region III to the right of the negative charge. The electric field $\mathbf{E}_+$ from the positive charge always points away from the positive charge, while the electric field $\mathbf{E}_-$ always points toward the negative charge.

In order for the electric field to be zero, the resultant of the electric field due to the positive charge $\mathbf{E}_+$ and the electric field due to the negative charge $\mathbf{E}_-$ must be zero. This can only occur in regions I or III since those are the only two regions where the vectors $\mathbf{E}_+$ and $\mathbf{E}_-$ point in opposite directions.

The magnitude of the electric field at a distance *r* from either charge is given by Equation 18.3: $E = kq/r^2$. When the two charges have equal magnitudes, the point on the line where the magnitude of $\mathbf{E}_+$ is equal to the magnitude of $\mathbf{E}_-$ is midway between the charges, since that point is equidistant from either charge (*q* and *r* are the same for both $\mathbf{E}_+$ and $\mathbf{E}_-$). The field will not be zero, however, because both $\mathbf{E}_+$ and $\mathbf{E}_-$ point to the right. If the magnitude of the negative charge is greater than the magnitude of the positive charge, then, the position where the magnitude of $\mathbf{E}_+$ is equal to the magnitude of $\mathbf{E}_-$ will occur closer to the positive charge. There will be a point in region I where the magnitudes of $\mathbf{E}_+$ and $\mathbf{E}_-$ are the same. Therefore, a point of zero electric field can occur on the line in region I.

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14. ***REASONING AND SOLUTION*** In general, electric field lines are always directed away from positive charges and toward negative charges. At a point in space, the direction of the electric field is tangent to the electric field line that passes through that point. The magnitude of the electric field is strongest in regions where the field lines are closest together. Furthermore, the text states that electric field lines created by a positive point charge are directed radially outward, while those created by a negative point charge are directed radially inward.

a. *In both regions I and II the electric field is the same everywhere.* This statement is false. By inspection, we see that the field lines in II get closer together as we proceed from left to right in the drawing; hence, the magnitude of the electric field is getting stronger.

The direction of the electric field at any point along a field line can be found by constructing a line tangent to the field line. Clearly, the direction of the electric field in II changes as we proceed from left to right in the drawing.

b. *As you move from left to right in each case, the electric field becomes stronger.* This statement is false. While this is true for the field in II by reasons that are discussed in part (a) above, we can see by inspection that the field lines in I remain equally spaced and point in the same direction as we go from left to right. Thus, the field in I remains constant in both magnitude and direction.

c. *The electric field in I is the same everywhere but becomes stronger in II as you move from left to right.* This statement is true for the reasons discussed in parts (a) and (b) above.

d. *The electric fields in both I and II could be created by negative charges located somewhere on the left and positive charges located somewhere on the right.* Since electric field lines are always directed away from positive charges and toward negative charges, this statement is false.

e. *Both I and II arise from a single point charge located somewhere on the left.* Since the electric field lines are radial in the vicinity of a single point charge, this statement is false.

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15. ***REASONING AND SOLUTION*** The effects of air resistance and gravity are ignored throughout this discussion. As shown in Figure 18.25, the electric field lines between the plates of a parallel plate capacitor are parallel and equally spaced, except near the edges. Ignoring edge effects, then, we can treat the electric field between the plates as a constant electric field. Since the force on the charge is given by Equation 18.2 as $\mathbf{F} = q_0\mathbf{E}$, we can deduce that the force on the positive charge, and therefore its acceleration, is constant in magnitude and is directed toward the negative plate. Since the charge enters the region with a horizontal velocity and the acceleration is downward, we can conclude that the horizontal component of the velocity will remain constant while the vertical component of the velocity will increase in the downward direction as the particle moves from left to right. This situation is analogous to that of a projectile fired horizontally near the surface of the earth. We can deduce, therefore, that the charged particle will follow a parabolic trajectory.

a. Two possible trajectories for the particle are shown below. For case 1, the particle hits the bottom plate and never emerges from the capacitor. The point at which the particle hits the bottom plate (when indeed it does) depends on the initial speed v of the particle when it enters the capacitor and the magnitude E of the electric field between the plates. For certain values of v and E, the particle may exit from the right side of the capacitor without striking the bottom plate. This is shown in case 2. Note that after the particle leaves the capacitor, it will move in a straight line at constant speed according to Newton's first law (assuming that no other forces are present).

Case 1

Case 2

b. The direction of the particle's instantaneous displacement vector and velocity vector are in the instantaneous direction of motion of the particle. Furthermore, the direction of the linear momentum vector is always in the same direction as the particle's velocity, and, therefore, in the instantaneous direction of motion. Since the particle travels in a parabolic path as discussed in part (a), we know that the directions of the particle's displacement vector, velocity vector, and linear momentum vector are always tangent to the path shown in part (a). They are not, therefore, parallel to the electric field inside the capacitor. From Newton's second law, we know that the acceleration vector of the particle is in the direction of the net force that acts on the particle. Since the particle is positively charged, we know that the net force on the particle at any point is in the same direction as the direction of the electric field at that point. Therefore, the acceleration vector is parallel to the electric field in the capacitor.

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16. ***REASONING AND SOLUTION*** Refer to Figure 18.27 which shows the electric field lines for two identical positive point charges. Imagine a plane that is perpendicular to the line that passes through the charges and is half into and half out of the paper. The plane is midway between the charges. The electric flux through this plane is given by Equation 18.6: $_{E} = (E\cos f)\ A$, where E is the magnitude of the electric field at a point on the plane, f is the angle that the electric field makes with the normal to the plane at

that point, A is a tiny area that just surrounds the point, and the summation is carried out over the entire plane. Midway between the charges, the electric field is zero (see Figure 18.27). Therefore, the electric field is zero for all points on the plane. The electric flux through the plane is, therefore, also zero.

---

17. ***REASONING AND SOLUTION*** Two charges + q and – q are inside a Gaussian surface. Since the net charge inside the Gaussian surface is zero, Gauss' law states that the electric flux $_E$ through the surface is also zero. However, the fact that $_E$ is zero does not imply that **E** is zero at any point on the Gaussian surface.

The flux through a Gaussian surface of any shape that surrounds the two point charges is given by text Equation 18.6: $_E = (E\cos f)\ A$, where E is the magnitude of the electric field at a point on the surface, f is the angle that the electric field makes with the normal to the surface at that point, A is a tiny area that surrounds the point, and the summation is carried out over the entire Gaussian surface. The individual terms $(E\cos f)\ A$ can be positive or negative, depending on the sign of the factor (cos f).

We can see from Figure 18.26 that the electric field in the vicinity of the two point charges changes from point to point in both magnitude and direction. Furthermore, as Figure 18.26 indicates, the electric field lines are directed away from the positive charge and toward the negative charge. In general, the value of E is non-zero at every point on any arbitrary Gaussian surface that surrounds the charges. The magnitude and direction of **E** at each point is such that when the summation in Equation 18.6 is carried out over the entire Gaussian surface, the electric flux through the entire surface is zero.

---

18. ***REASONING AND SOLUTION*** Gauss' law states that the electric flux $_E$ through a Gaussian surface is equal to Q/e_0, where Q is the net charge enclosed by the surface and e_0 is the permittivity of free space.

The drawing shows three charges, labeled q_1, q_2, and q_3.

a. Since the charges labeled q_1 and q_2 lie within the Gaussian surface, they determine the electric flux through the surface. From Gauss' law, the flux through the surface is

$$_E = (q_1 + q_2)/e_0$$

b. Each of the three charges produces its own electric field. The magnitude of the field at the point P due to any one of the charges is given by Equation 18.3: $E = kq/r^2$, where q is the magnitude of the charge and r is the distance between the charge and the point P. The

net electric field at the point *P* is the resultant of the electric fields that are produced by each charge individually. Thus, all three charges contribute to the electric field at the point *P*.

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19. ***REASONING AND SOLUTION*** Gauss' law states that the electric flux $_E$ through a Gaussian surface is equal to Q/e_0, where *Q* is the net charge enclosed by the surface and e_0 is the permittivity of free space. Gauss' law make no reference to position or location other than to specify that *Q* refers to the net charged *enclosed* by the Gaussian surface.

Suppose that a charge $+q$ is placed inside a spherical Gaussian surface. The charge is positioned so that it is *not* at the center of the sphere.

a. In order for Gauss' law to tell us anything about the charge within the Gaussian surface, the electric flux through the surface must be known. Since Gauss' law makes no reference to the position of the net charge *Q*, Gauss' law cannot tell us where the charge is located. If the flux through the surface is known, Gauss' law can only tell us that a net amount of charge $Q = +q$ is located somewhere within the surface.

b. If we know that the net charge enclosed by the sphere is $Q = +q$, then, by the statement of Gauss' law, we know that the flux through the surface is $_E = +q/e_0$ (regardless of where the enclosed charges are positioned relative to the center of the sphere).

CHAPTER 19 *ELECTRIC POTENTIAL ENERGY AND THE ELECTRIC POTENTIAL*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The work done in moving a charge q_0 from A to B is given by Equation 19.4: $W_{AB} = q_0(V_A \quad V_B)$. For the cases in the drawing, we have

Case 1: $W_{AB} = q_0(150\text{ V} \quad 100\text{ V}) = q_0(50\text{ V})$

Case 2: $W_{AB} = q_0[25\text{ V} \quad (-25\text{ V})] = q_0(50\text{ V})$

Case 3: $W_{AB} = q_0[-10\text{ V} \quad (-60\text{ V})] = q_0(50\text{ V})$

The work done on the charge by the electric force is the same in each case.

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2. ***REASONING AND SOLUTION*** The potential at a point in space that is a distance r from a point charge q is given by Equation 19.6: $V = kq/r$. When more than one point charge is present, the total potential at any location is the algebraic sum of the individual potentials created by each charge at that location.

A positive point charge and a negative point charge have equal magnitudes. One of the charges is fixed to one corner of a square. If the other charge is placed opposite to the first charge along the diagonal of the square, then each charge will be the same distance L from the empty corners. The potential at each of the empty corners will be

$$V = \frac{k(+Q)}{L} + \frac{k(\quad Q)}{L} = 0$$

Therefore, if the potential at each empty corner is to be the same, then the charges must be placed at diagonally opposite corners as shown in the figure.

---

3. ***REASONING AND SOLUTION*** The potential at a point in space that is a distance r from a point charge q is given by Equation 19.6: $V = kq/r$. When more than one point charge is present, the total potential at any location is the algebraic sum of the individual potentials created by each charge at that location.

Three point charges have identical magnitudes, but two of the charges are positive and one is negative. These charges are fixed to the corners of a square, one to a corner, as shown in the figure.

The potential at the empty corner is given by

$$V = \frac{q_1}{L} + \frac{q_2}{L\sqrt{2}} + \frac{q_3}{L}$$

Using q to denote the magnitude of each charge, we have the following possibilities:

q_1 and q_2 are positive: $$V = \frac{kq}{L} + \frac{kq}{L\sqrt{2}} \quad \frac{kq}{L} = \frac{kq}{L\sqrt{2}}$$

q_1 and q_3 are positive: $$V = \frac{kq}{L} \quad \frac{kq}{L\sqrt{2}} + \frac{kq}{L} = \frac{kq}{L}\left(2 \quad \frac{1}{\sqrt{2}}\right)$$

q_2 and q_3 are positive: $$V = \quad \frac{kq}{L} + \frac{kq}{L\sqrt{2}} + \frac{kq}{L} = \frac{kq}{L\sqrt{2}}$$

In each case, the potential at the empty corner is positive.

---

4. ***REASONING AND SOLUTION*** Four point charges of equal magnitude are placed at the corners of a square as shown in the figure at the right.

The electric field at the center of the square is the *vector sum* of the electric field at the center due to each of the charges individually. The potential at the center of the square is equal to the *algebraic sum* of the potentials at the center due to each of the charges individually.

All four charges are equidistant from the center (a distance r in the figure). If two diagonal charges have the same magnitude and sign, then the electric field at the center due to these two charges have equal magnitude and opposite directions. Their resultant is, therefore, zero. Thus, the electric field at the center will be zero if each diagonal pair of charges has the same magnitude and sign.

If a diagonal pair of charges has the same magnitude and sign, they will give rise to a non-zero potential at the center. Thus, the potential due to one diagonal pair of charges must cancel the potential due to the other diagonal pair of charges. This will be the case if the two pairs of diagonal charges have opposite signs.

Thus, both the electric field and the electric potential will be zero at the center of the square if all four charges have the same magnitude, q_1 and q_3 have the same sign, and q_2 and q_4 have the same sign, which is opposite to the signs of q_1 and q_3.

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5. ***REASONING AND SOLUTION***
Positive charge is spread uniformly around a circular ring, as shown in the figure, with q representing the total amount of charge on the ring. Equation 19.6 gives the correct potential at points along the line perpendicular to the plane of the ring at its center. The equation does not, however, give the correct potential at points that do not lie on this line.

We can consider the ring to be composed of point charges of magnitude q spread uniformly around the ring so that $q = \quad q$. According to Equation 19.6, each "point charge" gives rise to a potential given by $V = k \quad q/r$ at a distance r from the "point charge." The potential due to the ring at any point in space is the algebraic sum of the potentials of all the "point charges". For any point along the line perpendicular to the plane of the ring at its center, the distance r is the same for all "point charges" on the ring. Therefore, for any point on that line, $V = \quad V = \quad k \quad q/r = kq/r$; thus, we obtain Equation 19.6. For points that are not on the line, the distance r will, in general, be different for each "point charge" that makes up the total charge q; therefore, the potential V at that point will not be given by Equation 19.6 with a single value for r.

---

6. ***REASONING AND SOLUTION*** The electric field at a single location is zero. This does not necessarily mean that the electric potential at the same place is zero. According to Equation 19.7, $E = - \quad V/ \quad s$. Therefore, the fact that **E** = 0 means that the potential gradient, $V/ \quad s$, is zero, and the potential difference does not change with distance. In other words, in the vicinity of a point where **E** is zero, the electric potential is constant, but not necessarily zero

To illustrate this fact, consider the situation of two identical point charges. The electric field at each location in the vicinity of the charges is the resultant of the electric field at that location due to each of the two charges. The magnitude of the field due to each charge is given by Equation 18.3: $E = kq/r^2$. At the point exactly halfway between the charges, along the line that joins them, r is the same for the field contribution from each charge; therefore, the magnitudes of the field contributions at this point are the same. Since the charges are identical, the directions of the field contributions at this point are in opposite directions. The following figure shows the situation for both positive and negative charges. The resultant of the two field contributions is zero. Thus, the electric field is zero exactly

halfway between the charges on the line that joins them. The electric potential at this point, however, is the algebraic sum of the potential at that point due to each individual charge. Therefore, the potential at the point exactly halfway between the charges is

$$V = \frac{kq}{r} + \frac{kq}{r} = \frac{2kq}{r}$$

which is clearly non-zero.

Positive point charges

Negative point charges

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7. ***REASONING AND SOLUTION*** An electric potential energy exists when two protons are separated by a certain distance. It is equal to the work that must be done by an external agent to assemble the configuration. Suppose that we imagine assembling the system, one particle at a time. If there are no other charges in the region, there are no existing electric fields; therefore, no work is required to put the first proton in place. That proton, however, gives rise to an electric field that fills the region. Its magnitude at a distance r from the proton is given by Equation 18.3, $E = ke/r^2$, where $+e$ is the magnitude of the charge on the proton. Since the region contains an electric field due to the first proton, there also exists an electric potential, and the external agent must do work to place the second proton at a distance d from the first proton. The electric potential energy of the final configuration is equal to the work that must be done to bring the second proton from infinity and place it at a distance d from the first proton. The electric potential at a distance d from the first proton is $V_{\text{proton}} = +ke/d$ (Equation 19.6). According to Equation 19.3, the electric potential energy of the final configuration is therefore

$$\text{EPE} = V_{\text{proton}}(+e) = +\frac{ke^2}{d}$$

a. If both protons are replaced by electrons, similar arguments apply. However, since the electron carries a negative charge ($-e$), the electric potential at a distance d from the first electron is $V_{\text{electron}} = \quad ke/d$. The electric potential energy of the final configuration is now given by

$$\text{EPE} = V_{\text{electron}}(\quad e) = \quad \frac{ke}{d}(\quad e) = +\frac{ke^2}{d}$$

Therefore, if both protons are replaced by electrons, the electric potential energy *remains the same*.

b. When only one of the protons is replaced by an electron, we find that

$$\text{EPE} = V_{\text{proton}}(\quad e) = \left(+\frac{ke}{d}\right)(\quad e) = \quad \frac{ke^2}{d}$$

Thus, when only one of the protons is replaced by an electron, the electric potential energy *decreases* from $+ke^2/d$ to $\quad ke^2/d$.

---

8. ***REASONING AND SOLUTION*** A proton (charge = +*e*) is fixed in place. An electron (charge = –*e*) is released from rest and allowed to collide with the proton. Since the electron is released from rest, its initial kinetic energy is zero. Since the electron is in the vicinity of the proton, it has electric potential energy. The initial total energy of the electron is, therefore, electric potential energy. After the electron is released, the potential energy decreases, while the kinetic energy increases. Assuming that the charges are point charges and that r_{i} is the initial separation between the charges, while r_{f} is the separation just before the collision, the conservation of energy gives

$$E_{\text{initial}} = E_{\text{final}}$$

$$\frac{ke^2}{r_{\text{i}}} = \frac{ke^2}{r_{\text{f}}} + \tfrac{1}{2}m_{\text{electron}}v^2$$

$$ke^2\left(\frac{1}{r_{\text{f}}} \quad \frac{1}{r_{\text{i}}}\right) = \quad (\text{EPE}) = \tfrac{1}{2}m_{\text{electron}}v^2$$

where $\quad (\text{EPE}) = ke^2\left(\frac{1}{r_{\text{f}}} \quad \frac{1}{r_{\text{i}}}\right)$ is the change in the electric potential energy.

Therefore, the speed of the electron when it collides with the proton is

$$v = \sqrt{\frac{2 \quad (\text{EPE})}{m_{\text{electron}}}} \tag{1}$$

When the roles of the electron and proton are interchanged and the same experiment repeated, similar arguments apply.

It can be shown that the speed of the proton when it collides with the electron is

$$v = \sqrt{\frac{2 \quad (\text{EPE})}{m_{\text{proton}}}} \tag{2}$$

Since the mass of the electron is much less than the mass of the proton, we can conclude from Equations 1 and 2 that the electron is traveling faster when the collision occurs.

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9. ***REASONING AND SOLUTION*** The electric potential is constant throughout a given region of space. Since the potential does not change from point to point in this region, the potential gradient, $V/\ s$, is zero throughout this region. From Equation 19.7, we see that the electric field is $E = \ V/\ s$ and must also be zero throughout this region.

---

10. ***REASONING AND SOLUTION*** In a region where the electric field is constant, we see from Equation 19.7, $E = \ V/\ s$, that the potential gradient is constant. In other words, the rate at which the electric potential changes with distance in the region is constant. The potential, therefore, is not zero, but varies from point to point in a linear fashion with distance.

---

11. ***REASONING AND SOLUTION*** A positive charge is placed in an electric field. At each location in the electric field, the field is perpendicular to an equipotential surface. Therefore, in order for the charge to experience a constant electric potential, it must be moved along a path that is perpendicular to the electric field.

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12. ***REASONING AND SOLUTION*** The electric potential at any location in the vicinity of the charges is given by

$$V = \frac{k(+q)}{r_+} + \frac{k(\ \ q)}{r}$$

where r_+ is the distance from the positive charge to the point in question and r_- is the distance from the negative charge to the point in question. All points for which $r_+ = r_-$ correspond to points where the potential is zero, according to the above expression. Such points would lie on a plane that is midway between the two charges. Therefore, we can conclude that line *B* in the figure is the edge-on view of an equipotential surface.

As stated in the text, the electric field created by a group of charges is everywhere perpendicular to the associated equipotential surfaces. Lines *A* and *C* can be ruled out as possible edge-on views of equipotential surfaces, because they are not perpendicular to the electric field lines that would arise from the dipole configuration (see Figure 19.15).

---

13. ***REASONING AND SOLUTION*** A positive test charge is moved along the line between two identical point charges.

a. If the two identical point charges are positive, then the test charge experiences a repulsive force from each of the two fixed charges. Halfway between the charges, the resultant of these two repulsive forces will be zero since the distance from the test charge to either fixed charge is the same. As the test charge is moved from the midpoint, it will feel a net repulsive force which tends to push the charge back to the midpoint. The work done by this repulsive force in pushing the positive charge back to the midpoint is positive. According to the discussion in Section 19.4, the charge therefore goes from a higher potential (away from the midpoint) toward a lower potential (at the midpoint). Thus, with regard to the electric potential, the midpoint is analogous to the bottom of a valley.

b. If the two identical point charges are negative, then the test charge experiences an attractive force from each of the two fixed charges. Halfway between the charges, the resultant of these two attractive forces will be zero since the distance from the test charge to either fixed charge is the same. As the test charge is moved from the midpoint, it will feel a net attractive force which tends to pull the charge away from the midpoint. The work done by this attractive force in pulling the positive charge away from the midpoint is positive. According to the discussion in Section 19.4, the charge therefore goes from a higher potential (at the midpoint) toward a lower potential (away from the midpoint). Thus, with regard to the electric potential, the midpoint is analogous to the top of a hill.

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14. ***REASONING AND SOLUTION*** A negative test charge is moved along the line between two identical point charges.

a. If the two identical point charges are positive, then the test charge experiences an attractive force from each of the two fixed charges. Halfway between the charges, the resultant of these two attractive forces will be zero since the distance from the test charge to either fixed charge is the same. As the test charge is moved from the midpoint, it will feel a net attractive force which tends to pull the charge away from the midpoint. The work done by this attractive force in pulling the test charge away from the midpoint is positive. According to the discussion in Section 19.4, the charge therefore goes from a higher potential (at the midpoint) toward a lower potential (away from the midpoint). Thus, with regard to the electric potential, the midpoint is analogous to the top of a hill.

b. If the two identical point charges are negative, then the test charge experiences a repulsive force from each of the two fixed charges. Halfway between the charges, the resultant of these two repulsive forces will be zero since the distance from the test charge to either fixed charge is the same. As the test charge is moved from the midpoint, it will feel a net repulsive force which tends to push the charge toward the midpoint. The work done by this repulsive force in pushing the test charge back to the midpoint is positive. According to the discussion in Section 19.4, the charge therefore goes from a higher potential (away from the midpoint) toward a lower potential (at the midpoint). Thus, with regard to the electric potential, the midpoint is analogous to the bottom of a valley.

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15. ***REASONING AND SOLUTION*** According to Equation 19.3, the electric potential energy is $\text{EPE} = q_0 V$, where V is the potential relative to a zero value at infinity and q_0 is the charge. For a given value of the potential, the electric potential energy depends on the value of the charge. Thus, the electric potential energy is not the same for every charge placed at the point in question.

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16. ***REASONING AND SOLUTION*** Since both particles are released from rest, their initial kinetic energies are zero. They both have electric potential energy by virtue of their respective positions in the electric field between the plates. Since the particles are oppositely charged, they move in opposite directions toward opposite plates of the capacitor. As they move toward the plates, the particles gain kinetic energy and lose potential energy. Using $(\text{EPE})_0$ and $(\text{EPE})_\text{f}$ to denote the initial and final electric potential energies of the particle, respectively, we find from energy conservation that

$$(\text{EPE})_0 = \tfrac{1}{2} m_{\text{particle}} v_\text{f}^2 + (\text{EPE})_\text{f}$$

The final speed of each particle is given by

$$v_\text{f} = \sqrt{\frac{2\left[(\text{EPE})_0 \quad (\text{EPE})_\text{f}\right]}{m_{\text{particle}}}}$$

Since both particles travel through the same distance between the plates of the capacitor, the change in the electric potential energy is the same for both particles. Since the mass of the electron is smaller than the mass of the proton, the final speed of the electron will be greater than that of the proton. Therefore, the electron travels faster than the proton as the particles move toward the respective plates. The electron, therefore, strikes the capacitor plate first.

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17. ***REASONING AND SOLUTION*** A parallel plate capacitor is charged up by a battery. The battery is then disconnected, but the charge remains on the plates. The plates are then pulled apart.

a. The capacitance of the system is given by Equation 19.10 with $k = 1$, $C = \epsilon_0 A / d$, where A is the area of either plate and d is the distance between the plates. Since d increases when the plates are pulled apart and A remains the same, the capacitance *decreases*.

b. According to Equation 19.8, the potential difference between the plates of the capacitor is given by $V = q / C$. Since q remains the same and C decreases, the potential difference V must *increase*.

c. The magnitude of the electric field between the plates is given by $E = V / d$. Since $V = q / C$, and $C = \epsilon_0 A / d$, the electric field between the plates is given by

$$E = \frac{(q / C)}{d} = \frac{q(d / \epsilon_0 A)}{d} = \frac{q}{\epsilon_0 A}$$

Since both q and A remain the same, the electric field *remains the same*.

d. The energy stored in the capacitor is given by Equation 19.11, $\text{Energy} = (1/2)CV^2$. Since $V = q/C$, we have

$$\text{Energy} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}C\left(\frac{q}{C}\right)^2 = \tfrac{1}{2}\frac{q^2}{C}$$

Since q remains the same and C decreases, the electric potential energy stored by the capacitor *increases*. This increase will be equal to the work that is done by the external agent in pulling the plates apart.

CHAPTER 20 | *ELECTRIC CIRCUITS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION***
   a. When S_1 is set to position A, current can flow from the generator only along the path that requires S_2 to be set to position A. Hence, the light will be on when S_2 is in position A.

   b. When S_1 is set to position B, current can flow from the generator only along the path that requires S_2 to be set to position B. Hence, the light will be on when S_2 is in position B.

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2. ***REASONING AND SOLUTION*** When an incandescent light bulb is turned on, the tungsten filament becomes white hot. Since the voltage is constant, the power delivered to the light bulb is given by Equation 20.6c: $P = V^2 / R$. From Equation 20.5, $R = R_0 [1 + a(T \quad T_0)]$, where a is the temperature coefficient of resistivity and is a positive number. Thus, as the filament temperature increases, the resistance of the wire increases, and as the filament heats up, the power delivered to the bulb decreases.

---

3. ***REASONING AND SOLUTION*** Two materials have different resistivities. Two wires of the same length are made, one from each of the materials. The resistance of each wire is given by Equation 20.3: $R = rL / A$, where r is the resistivity of the wire material, and L and A are, respectively, the length and cross-sectional area of the wire. Even when the wires have the same length, they may have the same resistance, if the cross-sectional areas of the wires are chosen so that the ratio rL / A is the same for each.

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4. ***REASONING AND SOLUTION*** The resistance of a wire is given by Equation 20.3: $R = rL / A$, where r is the resistivity of the wire material, L is the length of the wire, and A is its cross-sectional area. Since the cross-sectional area is proportional to the square of the diameter, a doubling of the diameter causes the cross-sectional area to be increased four-fold. From Equation 20.3, we see that doubling both the diameter and length causes the resistance of the wire to be reduced by a factor of 2.

---

5. ***REASONING AND SOLUTION*** One electrical appliance operates with a voltage of 120 V, while another operates with a voltage of 240 V. The power used by either appliance is given by Equation 20.6c: $P = V^2 / R$. Without knowing the resistance R of each appliance, no conclusion can be reached as to which appliance, if either, uses more power.

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6. ***REASONING AND SOLUTION*** Two light bulbs are designed for use at 120 V and are rated at 75 W and 150 W. The power used by either bulb is given by Equation 20.6c: $P = V^2 / R$. We see from Equation 20.6c that, at constant voltage, the power used by a bulb is inversely proportional to the resistance of the filament. Therefore, the filament resistance is greater for the 75-W bulb.

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7. ***REASONING AND SOLUTION*** Rather than state how many watts of power an appliance uses, appliance instructions often give statements such as "10 A, 120 V" instead. This statement gives the current used by the appliance for a specific voltage. It provides all the information necessary to determine the power usage of the appliance from Equation 20.6a: $P = IV$. Thus, the "current-voltage" rating given in the instructions is equivalent to a statement of the power consumption of the appliance.

---

8. ***REASONING AND SOLUTION*** When the switch is initially closed, a current appears in the circuit, because charges flow through the heater wire, the bimetallic strip, the contact point, and the light bulb. The bulb glows in response. As charges flow through the heater wire, it becomes hot thereby heating the bimetallic strip. As the strip is heated, the brass, having a larger coefficient of thermal expansion, expands more than the steel. Thus, the bimetallic strip bends into an arc away from the contact point, and the electric current drops to zero because the charges no longer have a continuous path along which to flow. The bulb ceases to glow. Since there is no longer a current, the resistance wire, and hence, the bimetallic strip begin to cool down to room temperature. As the bimetallic strip cools, it bends back to its initial position, as shown in the drawing in the text. When it reaches room temperature it will again touch the contact point, and a current will begin to flow again. The wire and the bimetallic strip will become hot and the bimetallic strip will bend away from the contact point and the current will again drop to zero. This cycle will continue as long as the switch remains closed. Therefore, as long as the switch remains closed, the bulb will flash on and off.

---

9. ***REASONING AND SOLUTION*** The power rating of a 1000-W heater specifies the power consumed when the heater is connected to an ac voltage of 120 V. The power consumed by the heater is given by Equation 20.6c: $P_1 = V^2 / R$. When two of these heaters are connected in series, the equivalent resistance of the combination is $R + R = 2R$. The power consumed by two of the heaters connected in series is, therefore, $P_2 = V^2/(2R) = P_1/2 = (1000\text{ W})/2 = 500\text{ W}$.

---

10. ***REASONING AND SOLUTION*** A number of light bulbs are to be connected to a single electrical outlet. The bulbs will provide more brightness when they are connected in such a way that their power output is greatest. Since the voltage at the outlet is constant, the power delivered by the bulbs is given by Equation 20.6c: $P = V^2 / R$, where R is the equivalent resistance of the combination of light bulbs. When the light bulbs are connected in series, their equivalent resistance is given by Equation 20.16: $R_S = R_1 + R_2 + R_3 + \ldots$ When the

light bulbs are connected in parallel, their equivalent resistance is given by Equation 20.17: $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$ Clearly, the equivalent resistance will be less when the bulbs are connected in parallel. From Equation 20.6c, we can conclude that the power output will be greatest, and the light bulbs will provide more brightness, when they are connected in parallel.

---

11. ***REASONING AND SOLUTION*** A car has two headlights. The filament of one burns out. However, the other headlight stays on. We can immediately conclude that the bulbs are *not* connected in series. The figure below shows the series arrangement of two such bulbs.

When the bulbs are connected in series, charges must flow through the filaments of *both* lights in order to have a complete circuit. Since the filament of the second bulb is burned out, charges will not be able to flow around the circuit, and neither headlight will stay on.

On the other hand, if the bulbs are connected in parallel, as shown at the right, the current will split at the junction *J*. Charges will be able to flow through the branch of the circuit that contains the good bulb, and that headlight will stay on. Notice that the order of the bulbs does not matter in either case. The results are the same.

---

12. ***REASONING AND SOLUTION*** When two or more circuit elements are connected in series, they are connected such that the same electric current flows through each element. When two or more circuit elements are connected in parallel, they are connected such that the same voltage is applied across each element.

The circuit in Figure (*a*) can be shown to be a combination of series and parallel arrangements of resistors. The circuit can be redrawn as shown below.

We can see in the redrawn figure that the current through resistors 2 and 3 is the same; therefore, resistors 2 and 3 are in series and can be represented by an equivalent resistance 23 as shown in the following drawing.

The voltage across resistance 23 and resistor 4 is the same, so these two resistances are in parallel; they can be represented by an equivalent resistance 234. The current through resistance 234 is the same as that through resistor 7, so resistance 234 is in series with resistor 7; they can be represented by an equivalent resistance 2347 as shown in the figure below.

The voltage across 2347 is the same as that across resistor 5; therefore, resistance 2347 is in parallel with resistor 5. They can be represented by an equivalent resistance 23475. Similarly, resistance 23475 is in series with resistor 8, giving an equivalent resistance 234758. Resistance 234758 is in parallel with resistor 6, giving an equivalent resistance 2347586.

Finally, the current through resistor 1 and resistance 2347586 is the same, so they are in series as shown at the right.

The circuit in Figure (*b*) can also be shown to be a combination of series and parallel arrangements of resistors. Since both ends of resistors 2 and 3 are connected, the voltage across resistors 2 and 3 is the same. The same statement can be made for resistors 4 and 5, and resistors 6 and 7. Therefore, resistor 2 is in parallel with the resistor 3 to give an equivalent resistance labeled 23. Resistor 4 is in parallel with resistor 5 to give an equivalent resistance 45, and resistor 6 is in parallel with resistor 7 to give an equivalent resistance 67. From the right-hand portion of the drawing below, it is clear that the resistances 23, 45, and 67 are in series with resistor 1.

The drawing at the right shows the circuit in Figure (*c*). No such simplifying arguments can be made for this circuit. No two resistors carry the same current; thus, no two of the resistors are in series. Furthermore, no two resistors have the same voltage applied across them; thus, no two of the resistors are in parallel. Circuit (*c*) contains resistors that are neither in series nor in parallel.

---

13. ***REASONING AND SOLUTION*** One way is to make two combinations that consist of two resistors in parallel. The resistance of each combination is

$$R_p = [(1/R) + (1/R)]^{-1} = R/2$$

These two combinations can then be placed in series, as shown at the right. The equivalent resistance of the series combination is

$$R_s = (R/2) + (R/2) = R$$

A second way is to make two combinations that consist of two resistors in series. The resistance of each combination is

$$R_s = R + R = 2R$$

These two combinations can then be placed in parallel, as shown at the right. The equivalent resistance of the parallel combination is

$$R_p = [1/(2R) + 1/(2R)]^{-1} = R$$

---

14. ***REASONING AND SOLUTION*** An ammeter is used to measure the current in a particular branch of a circuit, while a voltmeter is used to measure the voltage across two points in a circuit. An ideal ammeter must have a low resistance so that its presence in the circuit does not affect the current measurement. Conversely, an ideal voltmeter must have a large resistance so that it does not draw current and change the voltage between the two points in question.

---

15. ***REASONING AND SOLUTION*** A voltmeter is inadvertently mistaken for an ammeter and placed in a circuit. A voltmeter is a high resistance instrument. Since an ammeter is placed in series with the circuit, we would be placing a large resistance in series with the circuit, and the resistance of the circuit would greatly increase. From Ohm's law, $V = IR$, the current in the circuit would drop markedly.

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16. ***REASONING AND SOLUTION***
From Ohm's law, $V = IR$, 1 ohm = 1 volt/ampere = 1 volt/(coulomb/second). From the definition of capacitance, $q = CV$, 1 farad = 1 coulomb/volt. Then

$$1 \text{ ohm} \times \text{farad} = \frac{1 \text{ volt}}{(\text{coulomb/second})} \times \frac{1 \text{ coulomb}}{\text{volt}} = \frac{1 \text{ volt} \times \text{second}}{\text{coulomb}} \times \frac{1 \text{ coulomb}}{\text{volt}} = 1 \text{ second}$$

CHAPTER 21 | *MAGNETIC FORCES AND MAGNETIC FIELDS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** Magnetic field lines, like electric field lines, never intersect. When a moving test charge is placed in a magnetic field so that its velocity vector has a component perpendicular to the field, the particle will experience a force. That force is perpendicular to both the direction of the field and the direction of the velocity. If it were possible for magnetic field lines to intersect, then there would be a different force associated with each of the two intersecting field lines; the particle could be pushed in two directions. Since the force on a particle always has a unique direction, we can conclude that magnetic field lines can never cross.

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2. ***REASONING AND SOLUTION*** If you accidentally use your left hand, instead of your right hand, to determine the direction of the magnetic force on a positive charge moving in a magnetic field, the direction that you determine will be exactly opposite to the correct direction.

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3. ***REASONING AND SOLUTION*** A charged particle, passing through a certain region of space, has a velocity whose magnitude and direction remain constant.

   a. If it is known that the external magnetic field is zero everywhere in the region, we can conclude that the electric field is also zero. Any charged particle placed in an electric field will experience a force given by $\mathbf{F} = q\mathbf{E}$, where q is the charge and $\mathbf{E}$ is the electric field. If the magnitude and direction of the velocity of the particle are constant, then the particle has zero acceleration. From Newton's second law, we know that the net force on the particle is zero. But there is no magnetic field and, hence, no magnetic force. Therefore, the net force is the electric force. Since the electric force is zero, the electric field must be zero.

   b. If it is known that the external electric field is zero everywhere, we *cannot* conclude that the external magnetic field is also zero. In order for a moving charged particle to experience a magnetic force when it is placed in a magnetic field, the velocity of the moving charge must have a component that is perpendicular to the direction of the magnetic field. If the moving charged particle enters the region such that its velocity is parallel or antiparallel to the magnetic field, it will experience no magnetic force, even though a magnetic field is present. In the absence of an external electric field, there is no electric force either. Thus, there is no net force, and the velocity vector will not change in any way.

---

4. ***REASONING AND SOLUTION*** Suppose that the positive charge in Figure 21.10b were launched from the south pole toward the north pole, in a direction opposite to the magnetic field. Regardless of the strength of the magnetic field, the particle will always reach the

north pole. Since the charge is launched directly opposite to the magnetic field, its velocity will be antiparallel to the field and have no component perpendicular to the field. Therefore, there will be no magnetic force on the particle. The particle will move at constant velocity from the south pole of the magnet to the north pole.

---

5. ***REASONING AND SOLUTION*** Since the paths of the particles are perpendicular to the magnetic field, we know that the velocities of the particles are perpendicular to the field. Since the velocity of particle #2 is perpendicular to the magnetic field and it passes through the field undeflected, we can conclude that particle #2 is neutral.

Particles #1 and #3 move in circular paths. The figure at the right shows the direction of the (centripetal) magnetic force that acts on the particles. If the fingers of the right hand are pointed into the page so that the thumb points in the direction of motion of particle #1, the palm of the hand points toward the center of the circular path traversed by the particle. We can conclude, therefore, from RHR-1 that particle #1 is positively charged. If the fingers of the right hand are pointed into the page so that the thumb points in the direction of motion of particle #3, the palm of the hand points away from the center of the circular path traversed by the particle. We conclude, therefore, from RHR-1 that particle #3 is negatively charged.

---

6. ***REASONING AND SOLUTION*** Three particles have identical charges and masses. They enter a constant magnetic field and follow the paths as shown. The magnitude of the magnetic force on each particle is given by Equation 21.1 as $F = q_0 (v \sin q) B$, where q_0 is the magnitude of the charge on the particle, v is the speed of the particle and B is the magnitude of the magnetic field. Since the particle paths are perpendicular to the magnetic field, $q = 90°$ and $\sin q = 1$, so that $F = q_0 vB$. This magnetic force supplies the centripetal force that is necessary for the particles to move on the observed circular paths, so $q_0 vB = mv^2 / r$, where m is the mass of the particle and r is the radius of its circular path. Solving for v, we find $v = q_0 Br / m$. Since all three particles have the same charge and mass, we can conclude that the speed of the particle is directly proportional to the radius of the particle's path. Therefore, particle #1 is traveling the fastest, while particle #2 is traveling the slowest.

---

7. ***REASONING AND SOLUTION*** A proton follows the path shown in Figure 21.12. The magnitude of the magnetic force on the proton is given by Equation 21.1 as $F = q_0 (v \sin q) B$, where q_0 is the magnitude of the charge on the proton, v is the speed of the proton and B is the magnitude of the magnetic field. Since the proton's path is perpendicular to the magnetic field, q = 90° and sin q = 1, so that $F = q_0 vB$. This magnetic force supplies the centripetal force that is necessary for the proton to move on the observed circular path, so $q_0 vB = m_{\text{proton}} v^2 / r$, where m is the mass of the particle and r is the radius of its circular path. The magnitude of the magnetic field is, therefore, $B = m_{\text{proton}} v / (q_0 r)$.

If we want an electron to follow exactly the same path, we must adjust the magnetic field. The magnitude of the charge on the electron is the same as that on the proton; however, the electron is negatively charged and the proton is positively charged. As stated in the text, the direction of the force on a negative charge is *opposite* to that predicted by RHR-1 for a positive charge. Therefore, the direction of the magnetic field must be reversed. In order for the electron to travel with the same speed v in a circular path of the same radius r, the magnitude of the magnetic field must be changed to the value $B = m_{\text{electron}} v / (q_0 r)$.

Thus, the electron will travel in the same path as the proton in Figure 21.12 if the direction of the magnetic field is reversed and the magnitude of the magnetic field is reduced by a factor $(m_{\text{electron}} / m_{\text{proton}})$.

---

8. ***REASONING AND SOLUTION*** The drawing shows a top view of four interconnected chambers. A negative charge is fired into chamber 1. By turning on separate magnetic fields in each chamber, the charge is made to exit from chamber 4.

a. In each chamber the path of the particle is one- quarter of a circle. The drawing at the right also shows the direction of the centripetal force that must act on the particle in each chamber in order for the particle to traverse the path. The charged particle can be made to move in a circular path by launching it into a region in which there exists a magnetic field that is perpendicular to the velocity of the particle.

Using RHR-1, we see that if the palm of the right hand were facing in the direction of **F** in chamber 1 so that the thumb points along the path of the particle, the fingers of the right hand must point out of the page. This is the direction that the magnetic field must have to make a *positive* charge move along the path shown in chamber 1. Since the particle is *negatively* charged, the field must point opposite to that direction or into the page.

Similar reasoning using RHR-1, and remembering that the particle is negatively charged, leads to the following conclusions: in region 2 the field must point out of the page, in region 3 the field must point out of the page, and in region 4 the field must point into the page.

b. If the speed of the particle is v when it enters chamber 1, it will emerge from chamber 4 with the same speed v. The magnetic force is always perpendicular to the velocity of the particle; therefore, it cannot do work on the particle and cannot change the kinetic energy of the particle, according to the work-energy theorem. Since the kinetic energy is unchanged, the speed remains constant.

---

9. ***REASONING AND SOLUTION*** A positive charge moves along a circular path under the influence of a magnetic field. The magnetic field is perpendicular to the plane of the circle, as in Figure 21.12. If the velocity of the particle is reversed at some point along the path, the particle will *not* retrace its path. If the velocity of the particle is suddenly reversed, then from RHR-1 we see that the force on the particle reverses direction. The particle will travel on a different circle that intersects the point where the direction of the velocity changes. The direction of motion of the particle (clockwise or counterclockwise) will be the same as that in the original circle. This is suggested in the figure below.

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10. ***REASONING AND SOLUTION*** A television tube consists of an evacuated tube that contains an electron gun that sends a narrow beam of high-speed electrons toward the screen of the tube (see text Figure 21.37). The inner surface of the screen is covered with a phosphor coating, and when the electrons strike it, they generate a spot of visible light. The electron beam is deflected by the magnetic fields produced by electromagnets placed around the neck of the tube, between the electron gun and the screen. The magnetic fields produced by the electromagnets exert forces on the moving electrons, causing their trajectories to bend and reach different points on the screen, thereby producing the picture. If one end of a bar magnet is placed near a TV screen, the magnetic field of the bar magnet alters the trajectories of the electrons. As a result, the picture becomes distorted.

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11. ***REASONING AND SOLUTION*** When the particle is launched in the x, y plane, its initial velocity will be perpendicular to the magnetic field; therefore, the particle will travel on a circular path in the x, y plane. In order for the charged particle to hit the target, the target must lie on the circular path of the moving particle. This will occur if the particle moves in a counterclockwise circle that passes through the third quadrant of the coordinate system. RHR-1 can be used to determine possible trajectories in the following way. Place the fingers of the right hand into the page (direction of the magnetic field) and orient the thumb

along one of the coordinate axes (direction of the particle's initial velocity). The palm of the right hand will face the direction in which the force on the charged particle is directed. Since the particle travels on a circle, the direction of the force will point toward the center of the particle's trajectory. The figures below show the results for all four possible cases.

Clearly, the charged particle can hit the target only if the initial velocity of the particle points either in the negative x direction or the positive y direction.

---

12. ***REASONING AND SOLUTION*** Refer to Figure 21.17. We can use RHR-1 with the modification that the direction of the velocity of a positive charge is the same as the direction of the conventional current I.

a. If the direction of the current I in the wire is reversed, the thumb of the right hand (direction of I) will be directed into the page, and the palm of the hand (direction of **F**) will face the left side of the page. Therefore, the wire will be pushed to the left.

b. If *both* the current and the magnetic poles are reversed, then the fingers of the right hand must point toward the top of the page (direction of **B**), while the thumb will be directed into the page (direction of I). The palm of the hand will face the right (direction of **F**); therefore, the wire will be pushed to the right.

---

13. ***REASONING AND SOLUTION*** The bulb flashes like a turn signal on a car. When current appears in the circuit after the switch is closed, the bulb glows. Adjacent coils of the helix have current in them in the same direction. We have seen that parallel wires carrying current in the same direction attract one another due to magnetic forces. These forces squeeze the coils together, shortening the helix in the process. The bottom end of the wire is withdrawn from the mercury, thus interrupting the current in the circuit, and the bulb goes

out. Without the current, there are no magnetic forces to squeeze the coils together, and the helix, acting like a spring, expands. The bottom end of the wire dips back into the mercury, and current reappears in the circuit, causing the bulb to glow again. As this process repeats itself, the bulb flashes on and off.

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14. ***REASONING AND SOLUTION*** In Figure 21.28, assume that current I_1 is larger than current I_2. Consider three regions: Region I lies to the left of both wires, region II lies between the wires, and region III lies to the right of both wires. Let $\mathbf{B}_1$ represent the magnetic field due to current I_1, and let $\mathbf{B}_2$ represent the magnetic field due to current I_2.

a. *The currents point in opposite directions.* From RHR-2, the vectors $\mathbf{B}_1$ and $\mathbf{B}_2$ point in the same direction in region II. Therefore, regardless of the magnitudes of $\mathbf{B}_1$ and $\mathbf{B}_2$, the resultant of the fields can never be zero between the wires. The fields $\mathbf{B}_1$ and $\mathbf{B}_2$ point in opposite directions in regions I and III. Therefore, the resultant of $\mathbf{B}_1$ and $\mathbf{B}_2$ could be zero only in these two regions. The resultant of $\mathbf{B}_1$ and $\mathbf{B}_2$ will be zero at the point where their magnitudes are equal. The magnitude of the magnetic field at a distance r from a long straight wire is given by Equation 21.5: $B = m_0 I / (2pr)$, where I is the magnitude of the current in the wire and m_0 is the permeability of free space. If the current I_1 is larger than the current I_2, the magnitude of $\mathbf{B}_1$ will be equal to the magnitude of $\mathbf{B}_2$ somewhere in region III. This is because region III is closer to the smaller current I_2, and the smaller value for r in Equation 21.5 allows the effect of I_2 to offset the effect of the greater and more distant current I_1. Therefore, there is a point to the right of both wires where the total magnetic field is zero.

b. *The currents point in the same direction.* From RHR-2, the vectors $\mathbf{B}_1$ and $\mathbf{B}_2$ point in the same direction in both regions I and III; therefore, the resultant of $\mathbf{B}_1$ and $\mathbf{B}_2$ cannot be zero in these regions. In region II, the vectors $\mathbf{B}_1$ and $\mathbf{B}_2$ point in opposite directions; therefore, there is a place where the total magnetic field is zero between the wires. It is located closer to the wire carrying the smaller current I_2.

---

15. ***REASONING AND SOLUTION*** The drawing shows an end-on view of three parallel wires that are perpendicular to the plane of the paper. In two of the wires, the current is directed into the paper, while in the remaining wire the current is directed

out of the paper. The two outermost wires are held rigidly in place. From Example 8, we know that two parallel currents that point in the same direction attract each other while two parallel currents that point in opposite directions repel each other. Therefore, the middle wire will be attracted to the wire on the left and repelled from the wire on the right. Since each of the fixed wires exerts a force to the left on the middle wire, the net force on the middle wire will be to the left. Thus, the middle wire will move to the left.

---

16. ***REASONING AND SOLUTION*** The figure below shows the arrangements of electromagnets and magnets.

We can determine the polarity of the electromagnets by using RHR-2. Imagine holding the current-carrying wire of the electromagnet in the right hand as the wire begins to coil around the iron core. The thumb points in the direction of the current. For the electromagnet in figure (*a*), the fingers of the right hand wrap around the wire on the left end so that they point, inside the coil, toward the right end. Thus, the right end of the coil must be a north pole. Similar reasoning can be used to identify the north and south poles of the electromagnet in figure (*b*). The results are shown in the figure above. Since the like poles of two different magnets repel each other and the dissimilar poles of two different magnets attract each other, we can conclude that in both arrangements, the electromagnet is repelled from the permanent magnet at the right.

---

17. ***REASONING AND SOLUTION*** The figure below shows the arrangements of electromagnets.

We can determine the polarity of each electromagnet by using RHR-2. Imagine holding the current-carrying wire of the electromagnet in the right hand as the wire begins to coil around the iron core. The thumb points in the direction of the current. For the

electromagnet on the left in figure (*a*), the fingers of the right hand wrap around the wire on the left end so that they point, inside the coil, toward the right end. Thus, the right end of the coil must be a north pole. Similar reasoning can be used to identify the north and south poles of the other remaining electromagnets. The results are shown in the figure above. Since the like poles of two different magnets repel each other and the dissimilar poles of two different magnets attract each other, we can conclude that only the arrangement shown in (*a*) results in attraction. The electromagnets shown in arrangement (*b*) result in repulsion.

---

18. ***REASONING AND SOLUTION*** Refer to Figure 21.5. If the earth's magnetism is assumed to originate from a large circular loop of current within the earth, the plane of the current loop must be perpendicular to the magnetic axis of the earth, as suggested in the figure at the right. Using RHR-2, the current must flow clockwise when viewed looking down at the loop from the north magnetic pole.

---

19. ***REASONING AND SOLUTION*** The total magnetic field at the point P is the resultant of the magnetic field at P due to each individual wire. If the current in all four wires is directed into the page, then from RHR-2, the magnetic field at P due to the current in wire 1 must point toward wire 3, the magnetic field at P due to the current in wire 2 must point toward wire 1, the magnetic field at P due to the current in wire 3 must point toward wire 4, and the magnetic field at P due to the current in wire 4 must point toward wire 2, as shown at the right.

The magnetic field at a distance r from a long straight wire that carries a current I is $B = m_0 I/(2pr)$. Since the current in all four wires has the same magnitude and all four wires are equidistant from the point P, each wire gives rise to a magnetic field at P of the same magnitude. When current is flowing through all four wires, the total magnetic field at P is zero. If the current in any single wire is turned off, the total magnetic field will point

toward one of the corners. For example, if the current in wire 1 is turned off, the resultant of $\mathbf{B}_2$ and $\mathbf{B}_3$ is still zero and the total magnetic field is $\mathbf{B}_4$. If the current in wire 2 is turned off, then the total magnetic field is $\mathbf{B}_3$, and so on.

We could have achieved similar results if the current in all four wires was directed out of the page. If fact, as long as opposite wires along the diagonal have currents in the same direction (for example, wires 1 and 4 with outward currents and wires 2 and 3 with inward currents) the total magnetic field will point toward one of the corners when one of the currents is turned off.

---

20. ***REASONING AND SOLUTION*** You have two bars, one of which is a permanent magnet and the other of which is not a magnet, but is made from a ferromagnetic material like iron. The two bars look exactly alike.

a. A third bar (the test bar) that is a permanent magnet can be used to distinguish which of the look-alike bars is the permanent magnet and which is the ferromagnetic bar. Both ends of each of the look-alike bars, in succession, should be brought next to one end of the test bar. The look-alike permanent magnet will be attracted to the test bar when opposite poles are brought together, while it will be repelled from the test bar when like poles are brought together. Both ends of the ferromagnetic bar, however, will be attracted to the test bar. The magnetic field of the test bar will always induce a pole that is opposite in polarity to the pole of the magnet. Ferromagnetism always results in attraction.

b. The identities of the look-alike bars can be determined from a third bar that is not a magnet, but is made from a ferromagnetic material. Either end of the look-alike permanent magnet will be attracted to both ends of the ferromagnetic test bar, while the look-alike ferromagnetic bar will have no effect on the ferromagnetic test bar.

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21. ***REASONING AND SOLUTION*** A strong electromagnet picks up a delivery truck carrying cans of soda pop. Inside the truck cans of the soft drink are seen to fly upward and stick to the roof just beneath the electromagnet. We can conclude that the cans must be made from a ferromagnetic material, at least in part. Since aluminum is a nonferromagnetic material, we know that the cans are not entirely made of aluminum.

CHAPTER 22 | *ELECTROMAGNETIC INDUCTION*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** If the coil and the magnet in Figure 22.1*a* were each moving with the same velocity relative to the earth, there would be no relative motion between the magnet and the coil. The magnetic flux through the coil due to the bar magnet would be constant and, therefore, the combined motion of the bar magnet and the coil would *not* result in an induced current in the coil. We are ignoring here any effect due to the earth's magnetic field.

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2. ***REASONING AND SOLUTION*** In the discussion concerning Figure 22.5, we saw that a force of 0.086 N from an external agent was required to keep the rod moving at constant speed. Suppose the light bulb in the figure is unscrewed from its socket. Once the light bulb is unscrewed, the conducting rails and the rod are no longer part of a complete circuit (the resistance of the empty socket is infinite). Therefore, even though there will be a charge separation and a motional emf in the rod given by vBL, there will be no current in the rod. Since there is no current, there is no magnetic force to resist the motion of the rod. From Newton's first law, the rod will continue to move with constant velocity **v** without the application of an external force.

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3. ***REASONING AND SOLUTION*** A metal sheet moves to the right at a velocity **v** in a magnetic field **B** that is directed into the sheet. At the instant shown in the figure, the magnetic field only extends over half of the sheet. An induced emf leads to the eddy current shown.

We can apply RHR-1 (modified for currents) to the portion of the eddy current that exists in the portion of the sheet that is in the magnetic field. With the thumb of the right hand pointing toward the top of the page (direction of I), and the fingers of the right hand pointing into the page (direction of **B**), the palm of the right hand faces the left (direction of **F**). Thus, there is a retarding magnetic force **F** that acts on the sheet due to the interaction of the eddy current with the magnetic field. Hence, the eddy current causes the sheet to slow down.

---

4. ***REASONING AND SOLUTION*** A magnetic field **B** is necessary if there is to be a magnetic flux passing through a coil of wire. Yet, just because there is a magnetic field does not mean that a magnetic flux will pass through a coil.

The general expression for magnetic flux is given by Equation 22.2: $= (B \cos f)A$, where B is the magnitude of the magnetic field, A is the cross-sectional area of the coil, and f is the angle between the magnetic field **B** and the normal to the surface of the coil. Equation 22.2 shows that the flux depends only on the component of the magnetic field that is perpendicular to the surface of the coil. As shown in Example 4 and in Figure 22.11, when the coil is oriented so that it is parallel to the field, $f = 90°$, **B** has no component normal to the surface of the coil, and the magnetic flux through the coil is zero. Therefore, the magnetic flux through a coil can be zero even though there is a magnetic field present.

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5. ***REASONING AND SOLUTION*** It is known that the magnetic flux through a 1-m^2 flat surface is 2 Wb. From this data alone, it is possible to determine certain information about the average magnetic field at the surface, but *not the magnitude and direction of the field.*

The general expression for the magnetic flux through the surface is given by Equation 22.2: $= (B \cos f)A$, where B is the magnitude of the magnetic field, A is the cross-sectional area of the surface, and f is the angle between the magnetic field **B** and the normal to the surface. Equation 22.2 shows that the flux depends only on the component of the magnetic field that is perpendicular to the surface. Therefore, from the information given, we can only ascertain that the component of the magnetic field that is perpendicular to the surface has an average magnitude of 2 T.

---

6. ***REASONING AND SOLUTION*** Initially, before the switch is closed, neither the conducting rails nor the rod carries a current. When the switch is closed, a conventional current will flow along the conducting rails from the positive toward the negative terminal of the battery. Since the rod is a conducting rod, current will flow through the rod, from top to bottom. According to RHR-1, there will be a force that points to the right on the conducting rod due to the magnetic field; therefore, the rod will be pushed and accelerate to the right. As the rod moves to the right, the area bound by the "loop" increases, thereby increasing the magnetic flux through the loop. As the magnetic flux increases, an induced emf appears around the "loop." According to Lenz's law, the induced emf that appears, will appear in such a way so as to oppose the increase in the magnetic flux. This will occur if the induced emf opposes the battery emf, with the result that the current in the rod begins to decrease and reaches zero when the induced emf exactly offsets the battery emf. With no current in the rod, there is no longer a magnetic force applied to the rod. With no force, there is zero acceleration. In other words, from this point on, the rod moves with a constant velocity.

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7. ***REASONING AND SOLUTION*** A bolt of lightning contains moving charges, and hence, is a current. This current is surrounded by its magnetic field. Since the charges in a bolt of lightning move erratically, the current is a time-dependent and gives rise to a time-dependent magnetic field. Many household appliances contain coils. If the time-dependent magnetic field passes through these coils, the magnetic flux through the coils will be time-dependent. From Faraday's law, there will be an induced emf and, hence, an induced

current in the coils of the appliance. This will result in an induced current in the circuit that contains the appliance. Therefore, a bolt of lightning can produce a current in the circuit of an electrical appliance, even when the lightning does not directly strike the appliance.

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8. ***REASONING AND SOLUTION*** A solenoid is connected to an ac source. A copper ring is placed inside the solenoid, with the normal to the ring parallel to the axis of the solenoid.

When a current passes through the coils of the solenoid, a magnetic field appears inside the solenoid. Near the center of the solenoid, the magnetic field lines are nearly parallel to the solenoid axis. Since the normal of the copper ring is parallel to the axis, there is a flux through the copper ring when the solenoid carries a current. When the solenoid is connected to an ac source, the current alternates direction, giving rise to a magnetic field inside the solenoid that continuously reverses direction. Therefore, when the solenoid is connected to an ac source, the flux through the copper ring changes with time. From Faraday's law, we know that the changing flux results in an induced emf and, therefore, an induced current around the ring. Since the copper ring has a finite resistance, heat will be generated in the ring at a rate of I^2R, where I is the magnitude of the induced current and R is the resistance of the ring. Therefore, the copper ring will get hot even though nothing touches it.

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9. ***REASONING AND SOLUTION*** The cable under the floor carries a steady current I; therefore, it will be surrounded by a magnetic field **B** that does not change with time. The magnitude of the field is given by Equation 21.5, $B = m_0 I/(2pr)$, where r is the distance from the center of the cable. Since the robot's sensor coil is close to and parallel to the floor, magnetic field lines from the current-carrying cable will pass through the coil's cross-section. As long as the robot moves parallel to the cable, the distance from each point on the coil's cross section and the center of the cable remains constant, and the magnetic flux through the coil does not change. If the robot deviates from its parallel path, the distance from the coil to the cable will change. The magnitude of the magnetic field through the coil will change as the distance r changes. Therefore, the flux through the coil will change, and, from Faraday's law, there will be an induced emf in the coil.

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10. ***REASONING AND SOLUTION*** The generator-like action of the alternator in a car occurs while the engine is running and keeps the battery fully charged. The headlights would discharge an old and failing battery quickly if it were not for the alternator action. When a battery is in bad shape, the engine and the alternator must do more work to keep the battery

charged when the headlights are on. Therefore, when the battery is failing, the engine of a parked car runs more "quietly" when the headlights are off than when they are on.

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11. ***REASONING AND SOLUTION*** Lenz's Law states that the polarity of an induced emf is such that the induced current produces an induced magnetic field that opposes the change in the flux that causes the emf. In Figure 22.3 a coil of wire is being stretched.

a. As the coil of wire is stretched, the cross-sectional area of the coil decreases. Therefore, the magnetic flux that penetrates the coil decreases. In order to oppose this decrease in flux, a magnetic field must be induced so as to enhance the existing magnetic field. The induced magnetic field, therefore, must point into the page in the region that is surrounded by the coil. RHR-2 can be applied to determine the direction of the induced current. If the wire is held in the right hand with the fingers pointing into the page in the interior of the coil (direction of induced **B**), then the thumb of the right hand (direction of induced I) points so that the induced current is clockwise. This is the direction shown in the drawing.

b. If the direction of the magnetic field were reversed, then using the same reasoning as in part (a), we can reason that the induced magnetic field must point out of the page to oppose the change in flux through the coil as the coil is stretched. Using RHR-2 we deduce that the induced current must be counterclockwise around the loop.

---

12. ***REASONING AND SOLUTION*** The figure shows the set-up.

a. When the switch is open, there is no current in the coil and no magnetic field is present. When the switch in the circuit is closed, a current is established in the coil. From RHR-2, the magnetic field associated with the current in the coil leaves the bottom of the coil and enters the top of the coil (see drawing below). In other words, when there is a current in the coil, the bottom of the coil acts like the north pole of a bar magnet, and the top of the coil acts like the south pole of a bar magnet. The magnetic field of the coil causes a magnetic flux through the metal ring.

In the very short time that it takes for the current to rise from zero to its steady value, the magnetic field associated with the current also rises from zero to its steady value. As the magnetic field increases, the magnetic flux through the metal ring changes, and there is an induced voltage and an induced current around the ring.

From Lenz's law, the polarity of the induced voltage will be such that it opposes the changing flux through the metal ring. The induced magnetic field of the metal ring will be directed upward through the ring to subtract from the magnetic field of the coil that points downward through the ring. This is illustrated in the figure at the right. To preserve clarity, the iron core and a portion of the circuit have not been shown. The induced magnetic field of the metal ring is similar to that of a bar magnet, with the north pole above the ring, and the south pole below the ring. Thus, the situation is similar to that of two bar magnets with their south poles next to each other. The two south poles repel,
and the ring "jumps" upward.

b. If the polarity of the battery were reversed, the ring still "jumps" upward when the switch is closed. When the polarity of the battery is reversed, and the switch is closed, the current moves through the coil in the opposite direction.
The magnetic field associated with the current points in a direction that is opposite to the direction of the magnetic field in part (a). In other words, the top of the coil acts like the north pole of a bar magnet, and the bottom of the coil acts like the south pole of a bar magnet.

When the switch is closed, the magnetic flux through the metal ring changes, and there is an induced voltage and an induced current around the ring.

In accord with Lenz's law, the induced magnetic field of the metal ring will be directed downward through the ring to subtract from the magnetic field of the coil that points upward through the ring. The induced magnetic field of the metal ring is similar to that of a bar magnet, with the south pole above the ring, and the north pole below the ring. The two north poles repel, and the ring "jumps" upward.

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13. ***REASONING AND SOLUTION*** The string of an electric guitar is made of a magnetizable material. As shown in Figure 22.18, the presence of a permanent magnet inside the pickup coil causes the string to become magnetized with north and south poles. When the magnetized string is plucked, it oscillates above the pickup coil, thereby changing the magnetic flux that passes through the coil. The guitar string vibrates in a standing wave

pattern that consists of nodes and antinodes. The magnetic flux will change most rapidly where the oscillations of the magnetized string are the largest; therefore, the electromagnetic pickup should be placed at an antinode to produce a maximum emf.

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14. ***REASONING AND SOLUTION*** The current drawn by an electric motor is given by Equation 22.5: $I = (V - \text{emf})/R$, where V is the applied emf, "emf" is the back emf developed because the coil of the motor is rotating, and R is the resistance of the wire in the coil.

An electric motor in a hair drier is running at normal speed; therefore, the back emf is comparable to the applied emf, and the motor is drawing a relatively small current. If the shaft of the motor is prevented from turning, the back emf is suddenly reduced to zero. We see from Equation 22.5 that the current drawn by the motor will increase to the value V/R. Heat will be produced in the coil at a rate of $I^2R = (V/R)^2 R = V^2/R$, and the temperature of the coil will increase.

---

15. ***REASONING AND SOLUTION*** One transformer is a step-up device, while another is step-down. These two units have the same voltage across and the same current in their primary coils.

If we ignore any heat loss within the transformers, then, according to the principle of energy conservation, the average power delivered to the circuit attached to the secondary coil is the same for either transformer.

This conclusion can be verified using Equations 20.15a, and Equation 22.13. According to Equation 20.15a, the average power delivered to the secondary coil, and therefore, the average power that the secondary coil delivers to the circuit is $\overline{P}_s = I_s V_s$. Solving Equation 22.13 for I_s and substituting the resulting expression into Equation 20.15a, we have

$$\overline{P}_s = I_s V_s = \frac{I_p V_p}{V_s} \div V_s = I_p V_p$$

This result holds for both the step-up, as well as, the step-down device. Since both devices have the same voltage across and the same current in their primary coils, the average power delivered to the secondary coil, and therefore, the average power delivered to the circuit attached to the secondary coil, are the same.

CHAPTER 23 | *ALTERNATING CURRENT CIRCUITS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A light bulb and a parallel plate capacitor (including a dielectric material between the plates) are connected in series to the 60-Hz ac voltage present at a wall outlet. Since the capacitor and the light bulb are in series, the rms current at any instant is the same through each element, and is given by Equation 23.6: $I_{\text{rms}} = V_{\text{rms}} / Z$, where Z is the impedance of the circuit. The impedance of the circuit is given by Equation 23.7 with $X_{\text{L}} = 0$ (since there is no inductance in the circuit): $Z = \sqrt{R^2 + (-X_{\text{C}})^2} = \sqrt{R^2 + X_{\text{C}}^{\ 2}}$. According to Equation 19.10, if the dielectric between the plates of the capacitor is removed, the capacitance decreases by a factor of k, where k is the dielectric constant. From Equation 23.2, $X_{\text{C}} = 1/(2pfC)$, we see that decreasing the capacitance increases the capacitive reactance X_{C}. Therefore, the impedance of the circuit, $Z = \sqrt{R^2 + X_{\text{C}}^{\ 2}}$, increases. The rms current is $I_{\text{rms}} = V_{\text{rms}} / Z$ and will, therefore, be *less* than it was before the dielectric was removed. Thus, the brightness will *decrease*.

---

2. ***REASONING AND SOLUTION*** The ends of a long straight wire are connected to the terminals of an ac generator, and the current is measured. The wire is then disconnected, wound into the shape of a multiple-turn coil, and reconnected to the generator. After the wire is wound into a coil, it has a greater inductance. When the generator is turned on, the coil will develop a voltage that opposes a change in the current according to Faraday's law. Since the induced voltage opposes the rise in current, the rms current in the circuit will be less than it was before the wire was wound into a coil.

   This can also be seen by considering Equations 23.6 and 23.7. Before the wire was wound into a coil, its primary property was that of resistance, and the current through the wire was given by $I_{\text{rms}} = V_{\text{rms}} / Z$ with $Z = R$, or $I_{\text{rms}} = V_{\text{rms}} / R$. After the wire is wound into a coil, the wire possesses both resistance and inductance. Now the current in the coil is given by $I_{\text{rms}} = V_{\text{rms}} / Z$, where $Z = \sqrt{R^2 + X_{\text{L}}^{\ 2}}$. Since Z is necessarily greater than R, the rms current is less when the wire is wound into a coil.

---

3. ***REASONING AND SOLUTION*** An air-core inductor is connected in series with a light bulb of resistance R. This circuit is plugged into an electrical outlet. The current in the circuit is given by Equation 23.6, $I_{\text{rms}} = V_{\text{rms}} / Z$, where, from Equation 23.7, $Z = \sqrt{R^2 + X_{\text{L}}^{\ 2}}$. When a piece of iron is inserted in the inductor, the magnetic field in the inductor is enhanced relative to that in air, and the inductance increases. Equation 23.4, $X_{\text{L}} = 2pfL$, shows that when the inductance L increases, the inductive reactance X_{L} also

increases. The impedance of the circuit, therefore, increases, and the current in the circuit, $I_{\text{rms}} = V_{\text{rms}} / Z$, decreases. Thus, the brightness of the bulb decreases.

---

4. ***REASONING AND SOLUTION*** It is possible that the current in the simplified circuit has the same rms value as that in the original circuit. This would be the case if the original circuit were at resonance. Then $X_C = X_L$, and Equation 23.7 for the impedance would simplify to $Z = R$. As a result, the currents in the original and the simplified circuits would each be $I_{\text{rms}} = V_{\text{rms}}/R$.

---

5. ***REASONING AND SOLUTION*** Consider the series RCL circuit shown in Figure 23.9. The impedance of the circuit is given by Equation 23.7: $Z = \sqrt{R^2 + (X_L \quad X_C)^2}$, where the capacitive reactance is given by Equation 23.2 [$X_C = 1/(2pfC)$] and the inductive reactance is given by Equation 23.4 [$X_L = 2pfL$]. One example of this expression for Z is plotted in Figure 23.15 (see the blue curve). The vertical axis in this figure gives the impedance, while the horizontal axis gives the frequency. A horizontal line drawn to intersect the vertical axis above the minimum in the curve will intersect the curve at two places. These places correspond to two different frequencies on the horizontal axis. Thus, the circuit in Figure 23.9 can have the same impedance at two different frequencies.

---

6. ***REASONING AND SOLUTION*** An inductor and a capacitor are connected in parallel across the terminals of a generator.

a. *The frequency of the generator becomes very large.* From Equation 23.2 (or Figure 23.2) we see that, in the high frequency limit, the capacitive reactance approaches zero. Therefore, in the high frequency limit, the capacitor behaves as if it were replaced by a wire with zero resistance, and the generator delivers a very large current to the capacitor. From Equation 23.4 (or Figure 23.6) we see that, in the high frequency limit, the inductive reactance becomes very large. In this limit, the inductor behaves as if it has been cut out of the circuit, leaving a gap in the wire, and the generator delivers no current to the inductor. Therefore, when the frequency of the generator becomes very large, the current through the capacitor becomes large, and no current flows through the inductor. The total current delivered by the generator is large.

b. *The frequency of the generator becomes very small.* From Equation 23.2 we see that, in the low frequency limit, the capacitive reactance approaches infinity. The capacitor acts as if it has been cut from the circuit, leaving a gap in the wire. As a result, the generator delivers no current to the capacitor. From Equation 23.4 we see that, in the low frequency limit, the inductive reactance approaches zero. In this limit, the inductor acts as if it has been replaced by a wire with zero resistance, and the generator delivers a very large current to the inductor. Therefore, when the frequency of the generator becomes very small, the current through the inductor becomes very large and the current through the capacitor is zero. The total current delivered by the generator is again large.

---

7. ***REASONING AND SOLUTION*** The rms current is given by Equation 23.6: $I_{\text{rms}} = V_{\text{rms}} / Z$, where V_{rms} is the rms voltage of the generator and Z is the impedance of the circuit. Since the rms voltage of the generator is the same in both cases, the greater current is delivered to the circuit with the smaller impedance. We see from Equation 23.2 [$X_{\text{C}} = 1/(2pfC)$] and Equation 23.4 [$X_{\text{L}} = 2pfL$] that in the high frequency limit, the capacitive reactance is nearly zero and the inductive reactance is essentially infinite. In other words, in the high frequency limit, the capacitors behave as if they have been replaced by wires of zero resistance and the inductors behave as if they have been cut from the circuit leaving gaps in the wires. The figure below shows the two circuits in this limit.

Circuit I behaves as if it consists only of two identical resistors in parallel; therefore, the impedance of circuit I is $1/Z = 1/R + 1/R$, or $Z = R/2$. Circuit II behaves as if it consists only of two identical resistors in series; therefore, the impedance of circuit II is $Z = 2R$. At a very high frequency, then, circuit I has the smaller impedance, and, therefore, its generator supplies the greater rms current.

8. ***REASONING AND SOLUTION*** The phase angle between the current in and the voltage across a series RCL combination is the angle f between the current phasor I_0 and the voltage phasor V_0 in Figure 23.11. According to Equation 23.8, the tangent of this angle is related to the resistance *R*, the inductive reactance X_L, and the capacitive reactance X_C, according to the relation

$$\tan f = \frac{X_L - X_C}{R}$$

At resonance, $X_L = X_C$, and so $\tan f = 0$. Therefore, $f = \tan^{1}(0) = 0$; in other words, the phase angle between the current phasor and the voltage phasor is zero. Since the phase angle between the current phasor and the voltage phasor is zero, we can conclude that the current is in phase with the voltage.

---

9. ***REASONING AND SOLUTION*** The resonant frequency of a series RCL circuit is given by Equation 23.10:

$$f_0 = \frac{1}{2p\sqrt{LC}}$$

a. Since the resonant frequency of a RCL circuit does not depend on the value of *R*, it is possible for two series RCL circuits to have the same resonant frequencies and yet have different *R* values.

b. The resonant frequency of a series RCL circuit is inversely proportional to $\sqrt{LC}$. It is possible for two series RCL circuits to have the same resonant frequencies and yet have different values of *C* and *L*, provided that the product *LC* is the same in both circuits.

---

10. ***REASONING AND SOLUTION*** The generator connected to a series RCL circuit has a frequency that is greater than the resonant frequency of the circuit. Suppose that it is necessary to match the resonant frequency of the circuit to the frequency of the generator. To accomplish this, a second capacitor will be added to the circuit. In order to match the resonant frequency of the circuit to the frequency of the generator, the resonant frequency of the circuit must be increased. The resonant frequency of a series RCL circuit is given by Equation 23.10: $f_0 = 1/(2p\sqrt{LC})$. Since the resonant frequency is inversely proportional to $\sqrt{LC}$, the capacitor must be added to the circuit so that it *decreases* the capacitance. This can be accomplished by placing the second capacitor *in series* with the first capacitor (see Equation 20.19). With the proper choice of the value of *C* for the second capacitor, the capacitance of the series combination will be smaller than the capacitance of the first capacitor alone, and the resonant frequency of the circuit will increase to match the generator frequency.

---

11. ***REASONING AND SOLUTION*** The drawing at the right shows a full-wave rectifier circuit, in which the direction of the current through the load resistor R is the same for both positive and negative halves of the generator's voltage cycle. The points A and B are connected directly to the top and bottom of the generator, respectively. The diodes are labeled a, b, c, and d. The direction of the current through the circuit can be found by recalling the fact that charge flows through a diode only when the diode is in a forward bias condition. When the diode is in a forward bias condition, the side of the diode symbol that contains the arrowhead has a positive potential relative to the other side.

a. When the top of the generator is positive and the bottom is negative, point A in the figure has the positive potential and point B the negative potential. Since point A has the positive potential and is connected directly to the arrowhead for diode a, diode a is in a forward bias condition. In contrast, we see that for diode b, the side opposite the arrowhead is connected to point A, so diode b is in a reverse bias condition. Similarly, since point B is negative, and is connected directly to the side of diode c opposite the arrowhead, the arrowhead must be positive, and diode c is in a forward bias condition. In contrast, the arrowhead of diode d is negative, so diode d is in a reverse bias condition. Thus, when the top of the generator is positive and the bottom is negative, only diodes a and c are in the forward bias condition and allow charge to flow through the resistance R. Drawing 1 below shows the path taken by the current under these circumstances.

b. When the top of the generator is negative and the bottom is positive, reasoning similar to that given above in part (a) indicates that diodes b and d are now forward biased, while diodes a and c are in reverse bias. Furthermore, the forward bias diodes allow charge to flow from left to right through the resistance R. Drawing 2 below shows the path taken by the current under these circumstances.

Drawing 1

Drawing 2

CHAPTER 24 | *ELECTROMAGNETIC WAVES*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION***

a. The intensity of a wave refers to the power P carried by the wave that passes perpendicularly through a surface in the path of the wave, divided by the area A of the surface. It is given for sound waves by Equation 16.8 and for electromagnetic waves by Equation 24.4. The concept of intensity applies to both sound waves and electromagnetic waves, because both types of wave transfer energy and, therefore, power away from their respective sources.

b. In a transverse wave, the particles of the medium vibrate perpendicular to the direction of propagation of the wave. In the case of electromagnetic waves, there are no particle vibrations, but rather the electric and magnetic fields oscillate perpendicular to the direction of propagation of the wave. When a wave is produced or altered so that the vibrations or oscillations take place in a particular direction perpendicular to the wave velocity, the wave is said to be *polarized*.

In a longitudinal wave, the notion of polarization has no meaning, as discussed in Section 24.6 of the text. Therefore, *transverse waves can be polarized*, while *longitudinal waves cannot be polarized*.

Since sound waves are longitudinal waves, while electromagnetic waves are transverse waves, the concept of polarization applies only to electromagnetic waves.

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2. ***REASONING AND SOLUTION*** Refer to Figure 24.2. Between the times indicated in parts *c* and *d* in the drawing, negative charges have moved to the top of the antenna, leaving a net positive charge of equal magnitude on the bottom of the antenna. Therefore, as the negative charges flow, the conventional current points toward the bottom of the antenna. Using RHR-2, the magnetic field for the electromagnetic wave at P must point out of the page.

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3. ***REASONING AND SOLUTION*** A transmitting antenna is located at the origin of an x, y, z axis system, and broadcasts an electromagnetic wave whose electric field oscillates along the y axis. The wave travels along the $+x$ axis. Three possible loops can be used with an LC-tuned circuit to detect the wave: One loop lies in the x, y plane, another in the x, z plane, and the third in the y, z plane.

The loop that will detect the electromagnetic wave must be oriented so that the normal to the loop is parallel to the magnetic field. Then, as the wave passes by the loop, the changing magnetic field penetrates the loop and results in an induced emf and current, as predicted by Faraday's law. Since the electromagnetic wave travels along the $+x$ direction, and the electric field oscillations of the electromagnetic wave are along the y axis, the magnetic field oscillations will be along the z axis. For optimum reception, therefore, the loop should lie in the x, y plane so that the normal to the loop is in the z direction.

---

4. ***REASONING AND SOLUTION*** When a straight-wire antenna is used as the receiving antenna to detect electromagnetic waves, the wires must be oriented parallel to the electric field, as shown in Figure 24.5. The electric field exerts a force on the electrons in the wire and causes them to oscillate up and down along the length of the antenna. Since the electric field drives the electrons directly, the peak emf that causes the ac current in the receiving antenna depends on the amplitude of the electric field. It does not depend on the frequency of the wave.

When a loop antenna such as that shown in Figure 24.6, is used as the receiving antenna, the induced emf around the loop depends on the rate at which the magnetic flux changes with time, $/ \quad t$. If B represents the instantaneous magnitude of the magnetic field and A represents the cross-sectional area of the loop, and the normal of the loop points in the same direction as **B**, then $= BA$. Then $/ \quad t = \quad (BA)/ \quad t$. Since the area of the loop, A, does not change, it follows that $/ \quad t = (\quad B/ \quad t)A$. Hence, the induced emf in the receiving loop depends on the rate at which the magnetic field of the wave changes with time, and therefore, on the frequency of the electromagnetic wave. Thus, the peak value of the emf induced in a loop antenna depends on the frequency of the wave.

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5. ***REASONING AND SOLUTION*** The electric field of an electromagnetic wave is related to the magnetic field through the relation $E = cB$ (Equation 24.3). Therefore, if the electric field of a wave decreases in magnitude, the magnetic field will also decrease.

This conclusion can also be reached by considering the fact that in an electromagnetic wave propagating through air or vacuum, the electric field and the magnetic field carry equal amounts of energy per unit volume of space. If the electric field of a wave decreases in magnitude, the electric energy per unit volume must decrease. The magnetic energy per unit volume must decrease to the same amount; therefore, the magnitude of the associated magnetic field must also decrease.

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6. ***REASONING AND SOLUTION*** The same Doppler effect arises for electromagnetic waves when either the source or the observer of the waves moves; it is only the relative motion of the source and the observer with respect to one another that is important (see Section 24.5). Therefore, when an astronomer measures the Doppler change in frequency for the light reaching earth from a distant star, the astronomer *cannot* tell whether the star is moving away from the earth or whether the earth is moving away from the star.

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7. ***REASONING AND SOLUTION*** The only real difference between a polarizer and an analyzer is the purpose for which they are used. Both consist of a piece of polarizing material. When the piece of polarizing material is used to produce a desired polarization direction, it is referred to as a *polarizer*. When the piece of polarizing material is used to change the polarization direction and to adjust the intensity so that the polarization direction of the incident light can be determined, it is referred to as an *analyzer*. The same piece of polarizing material can be used as the polarizer in one situation and the analyzer in another.

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8. ***REASONING AND SOLUTION*** Malus' law applies to the setup in Figure 24.20, which shows the analyzer rotated through an angle q and the polarizer held fixed. When the analyzer is held fixed and the polarizer is rotated, Malus' law still applies. Malus' law states that the average intensity $\bar{S}$ depends on the angle q between the polarizer and analyzer. It does not matter whether the polarizer is fixed and the analyzer is rotated, or vice-versa.

---

9. ***REASONING AND SOLUTION*** In Example 7, we saw that, when the angle between the polarizer and analyzer is 63.4°, the intensity of the transmitted light drops to one-tenth of that of the incident unpolarized light. The light intensity that is not transmitted is absorbed by both the polarizer and the analyzer. The polarizer absorbs one-half of the incident intensity. The analyzer absorbs four-tenths, or two-fifths, of the original incident light. This absorbed energy results in an increase in the temperature of the polarizer and analyzer.

---

10. ***REASONING AND SOLUTION*** Light is incident from the left on two pieces of polarizing material, 1 and 2. As part *a* of the following drawing illustrates, the transmission axis of material 1 is along the vertical direction, while that of material 2 makes an angle of q with respect to the vertical. In part (*b*) of the drawing, the two polarizing materials are interchanged.

a. If the incident light is unpolarized and has an average intensity $\bar{I}$, then the average intensity of the light that leaves polarizer #1 and strikes polarizer #2 in Figure (*a*) is $\bar{S}_0 = \bar{I}/2$. According to Malus' law, the average intensity of the light leaving polarizer #2 is $\bar{S} = \bar{S}_0 \cos^2 q = (\bar{I}\cos^2 q)/2$.

Similarly, the average intensity of the light that leaves polarizer #2 and strikes polarizer #1 in Figure (*b*) is $\bar{S}_0 = \bar{I}/2$. The angle between the transmission axis of polarizer #2 and polarizer #1 is still q, so that the average intensity of the light leaving polarizer #1 in (*b*) is $\bar{S} = \bar{S}_0 \cos^2 q = (\bar{I}\cos^2 q)/2$. Therefore, the transmitted light in Figure (*a*) is the same as that in Figure (*b*).

b. If the incident light in Figure (*a*) is polarized along the vertical direction with average intensity $\bar{I}$, then, since the polarization direction of the light is the same as the transmission axis of the polarizer, all of the incident light is transmitted. The average intensity of the light that leaves polarizer #1 and strikes polarizer #2 is $\bar{S}_0 = \bar{I}$. According to Malus' law, the average intensity of the light leaving polarizer #2 is $\bar{S} = \bar{S}_0 \cos^2 q = \bar{I}\cos^2 q$.

In Figure (*b*) we have from Malus' law that the average intensity of the light that leaves polarizer #2 and strikes polarizer #1 is $\bar{S}_0 = \bar{I}\cos^2 q$. The angle between the transmission axis of polarizer #2 and polarizer #1 in Figure (*b*) is q. Therefore, according to Malus' law, the average intensity of the light leaving polarizer #1 in Figure (*b*) is $\bar{S} = \bar{S}_0 \cos^2 q = (\bar{I}\cos^2 q)\cos^2 q = \bar{I}\cos^4 q$. Therefore, the intensity of the transmitted light in Figure (*a*) is greater than the intensity of the transmitted light in Figure (*b*).

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11. ***REASONING AND SOLUTION*** Light from the sun is unpolarized; however, when the sunlight is reflected from horizontal surfaces such as the surface of a swimming pool, lake, or ocean, the reflected light is partially polarized in the horizontal direction. Polaroid sunglasses are constructed with lenses made of Polaroid (a polarizing material) with the transmission axis oriented vertically. Thus, the horizontally polarized light that is reflected from horizontal surfaces is blocked from the eyes.

Suppose you are sitting on the beach near a lake on a sunny day, wearing Polaroid sunglasses. When you are sitting upright, the horizontally polarized light that is reflected from the lake is blocked from your eyes, as discussed above. When you lay down on your side, facing the lake, the transmission axis of the Polaroid sunglasses is now oriented in a nearly horizontal direction. Most of the horizontally polarized light that is reflected from the lake is transmitted through the sunglasses and reaches your eyes. Therefore, the sunglasses don't work as well as they did when you were sitting in an upright position.

CHAPTER 25 *THE REFLECTION OF LIGHT: MIRRORS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A sign painted on a store window is reversed when viewed from inside the store. A person inside the store views the sign in a plane mirror. As discussed in the text, the image of an object formed in a plane mirror is upright, has the same size as the object, is located as far behind the mirror as the object is in front of it, and is reversed left to right. Therefore, since the image is reversed left to right, the image of the sign painted on the store window, when viewed in a plane mirror in the store, will appear as it does when viewed from outside the store.

---

2. ***REASONING AND SOLUTION*** As discussed in the text, the image of an object formed in a plane mirror is reversed left to right. If a clock is held in front of a mirror, its image is reversed left to right, but not up and down. In order to understand the appearance of the image of the second hand, as viewed by a person looking into the mirror, imagine replacing the second hand by a rotating vector that always points away from its rotation axis. At any instant, this vector can be resolved into horizontal and vertical components. Since the image is reversed left to right, the image of the horizontal component will be reversed in direction. In contrast, since the image is *not reversed* up and down, the image of the vertical component will point in the same direction as the object. Therefore, when the horizontal and vertical components of the image are combined, the resultant will rotate *counterclockwise*. Thus, from the point of view of a person looking into the mirror, the image of the second hand of the clock rotates in the counterclockwise direction.

---

3. ***REASONING AND SOLUTION*** When parallel rays of light strike a concave mirror, they are reflected; these reflected rays converge at the focal point of the mirror. When parallel rays of light strike a convex mirror, they are also reflected; these reflected rays diverge from the mirror's surface and appear to originate from the focal point located behind the mirror.

   a. The earth-sun distance is very large; therefore, when rays of light from the sun reach the earth, they are essentially parallel. If it is desired to start a fire with sunlight, it is necessary to focus the parallel light rays from the sun on a very small area, preferably a point, on the piece of paper. Since a concave mirror reflects parallel rays so that they converge in front of the mirror, a concave mirror, rather than a convex mirror, should be used.

   b. For best results, the piece of paper should be placed at the focal point of the mirror since this is the location where the rays converge to a point and the heating would be greatest.

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4. ***REASONING AND SOLUTION*** The photograph in the text shows an experimental device at Sandia National laboratories in New Mexico. The device is a mirror that focuses sunlight to heat sodium to a boil.

a. The earth-sun distance is very large; therefore, when rays of sunlight reach the earth, they are essentially parallel. A concave mirror reflects parallel rays of light so that the reflected rays converge at the focal point of the mirror. It is reasonable to conclude, therefore, that the mirror in the photograph is a concave mirror.

b. Since the rays of light converge at the focal point of the mirror, it is reasonable to conclude that the sodium unit is located at the mirror's focal point. Therefore, the distance between the mirror and the sodium unit is equal to the focal length of the mirror.

---

5. ***REASONING AND SOLUTION*** When parallel rays of light strike a concave mirror, they are reflected. For rays that lie close to the principal axis of the mirror, these rays will converge at a single point, namely the focal point of the mirror. Rays that are far from the principal axis do not converge to a single point after reflection. Each reflected ray obeys the law of reflection; namely, the angle of reflection, as measured with respect to the normal to the surface, is equal to the angle of incidence. Since the reflecting surface is spherical, the direction of the lines that are normal to the surface varies from point to point; however, they are directed radially with respect to the center of the "sphere." Rays that are farther from the principal axis have greater angles of incidence and greater angles of reflection. Therefore, when such rays reflect, they cross the principal axis to the right of the focal point *F* in Figure 25.14. The top ray in this drawing could be directed through the focal point if the angle of reflection, and therefore, the angle of incidence were decreased. This can be accomplished by changing the shape of the mirror so that the line normal to the surface at the point of incidence is rotated upward as shown in the figure below.

Spherical Concave mirror *Reshaped mirror*

Therefore, to bring the top ray closer to the focal point, the mirror must be "opened up" to produce a more gently curving surface.

---

6. ***REASONING AND SOLUTION***

a. For a real image, light rays actually pass through the points on the image. Such an image can be projected onto a screen directly. This can be accomplished by placing the screen at the location of the image.

If the object is placed between the focal point and the center of curvature of a concave mirror, a real image is formed beyond the center of curvature. The real image is enlarged and inverted (see Figure 25.19*a*). If the object is placed beyond the center of curvature, a real image is formed between the center of curvature and the focal point. This real image is reduced in size and inverted with respect to the object (see Figure 25.19*b*). In these cases, the image is real; therefore, it can be projected directly onto a screen (placed at the location of the image) without the help of other mirrors or lenses.

If, however, the object is placed between the focal point and the concave mirror, an enlarged, upright *virtual* image is produced (see Figure 25.20*a*). The image is behind the mirror and cannot be projected directly onto a screen without the help of other optical components.

b. A convex mirror always produces a virtual image that is behind the mirror; therefore the image cannot be projected directly onto a screen without the help of other optical components.

---

7. ***REASONING AND SOLUTION***

a. The back side of a shiny teaspoon acts like a convex mirror. When an object is placed in front of a convex mirror, a virtual image is produced that is reduced in size and upright, as shown in Figure 25.22. Thus, when you look at the back side of a shiny teaspoon held at arms length, you will see yourself upright.

b. The concave side of a shiny teaspoon acts like a concave mirror. If the teaspoon is held at arm's length, then you (the object) are farther from the reflecting surface than the center of curvature of the surface. The situation is similar to that of an object located beyond the center of curvature of a concave mirror. A real image is formed that is reduced in size and *inverted* with respect to the object, as shown in Figure 25.19*b*. Therefore, when you look at the concave side of a shiny teaspoon, you will see yourself upside down.

---

8. ***REASONING AND SOLUTION*** If you stand between two parallel plane mirrors, you see an infinite number of images of yourself. This occurs because an image in one mirror is reflected in the other mirror to produce another image, which is then re-reflected, and so-forth. The multiple images are equally spaced.

The image produced by a convex mirror is reduced in size relative to the object. Like a plane mirror, the image is virtual and lies behind the mirror; however, the virtual image in a convex mirror is closer to the mirror than it would be if the mirror were planar. If you stand facing a convex mirror, with a plane mirror behind you, you will see an infinite number of images of yourself, as you do in the case of two parallel plane mirrors. The reason is that the first image in the convex mirror is reflected in the plane mirror, to produce an identical image behind it. The image behind the plane mirror is re-reflected in the convex mirror to produce another smaller image closer to the focal point of the convex mirror. This second image in the convex mirror is smaller than the first one, because the image behind the plane

mirror is farther from the convex mirror than you are. Further reflections and re-reflections occur, leading to a series of images in the convex mirror that decrease in size as they occur closer and closer to the focal point. The images "pile up" so to speak, becoming closer and closer together as they approach the focal point. The size of the image becomes zero at the focal point. Thus, the series of images appears to vanish at the focal point of the convex mirror.

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9. ***REASONING AND SOLUTION*** The microphone arrangement shown in the figure is used to pick up weak sounds. It consists of a "hollowed-out" shell behind the mike. The shell acts like a mirror for sound waves. Therefore, when parallel rays of sound hit the inside surface of the shell, they will be reflected from it. Since the shell is concave, the reflected rays will converge at the focal point of the shell. Presumably, weak sounds will originate far from the microphone; therefore, when the they reach the microphone arrangement, these rays will be essentially parallel to the principal axis of the shell. Since the reflected rays will converge at the focal point, the microphone should be located at the focal point to detect them optimally.

---

10. ***REASONING AND SOLUTION*** When you see the image of yourself formed by a mirror, it is because (1) light rays actually coming from a real image enter your eyes or (2) light rays appearing to come from a virtual image enter your eyes. If the light rays from the image do not enter your eyes, you do not see yourself.

a. If you stand in front of a convex mirror on its principal axis, your virtual image is behind the mirror. The image will be reduced in size and upright regardless of your location. You will be able to see yourself at any location on the principal axis when you are in front of a convex mirror, because rays that appear to come from this image can enter your eyes.

b. If you stand in front of a concave mirror on its principal axis, and you are beyond the center of curvature of the mirror, you will see a real image of yourself that is inverted and reduced in size relative to your size and orientation (Figure 25.19*b*). If you stand between the mirror and its focal point, you will see a virtual image of yourself that is upright and enlarged in size (Figure 25.20). In the former case, rays emanating from the real image can reach your eyes. In the latter case, rays that appear to emanate from the virtual image can reach your eyes.

When you stand on the principal axis of a concave mirror so that you are between the center of curvature and the focal point of the mirror, the image is formed beyond the center of curvature (Figure 25.19*a*). You cannot see this image, because it is behind you.

---

11. ***REASONING AND SOLUTION*** Plane mirrors and convex mirrors form virtual images. With a plane mirror, the image may be infinitely far behind the mirror, depending on where the object is located in front of the mirror.

For an object in front of a single convex mirror, the greatest distance behind the mirror at which the image can be found is equal to the magnitude of the focal length of the mirror. This can be confirmed using the mirror equation (Equation 25.3): $1/d_{\mathrm{o}} + 1/d_{\mathrm{i}} = 1/f$, where f is taken to be a negative number because the mirror is a convex mirror. The largest

image distance occurs when the object is at infinity. Then $1/d_o = 1/\quad = 0$, and the mirror equation gives $d_i = f$. Since f is a negative number, d_i is a negative number, indicating that the image is behind the mirror. Therefore, the image will never be located beyond the focal point, behind the mirror.

---

12. ***REASONING AND SOLUTION*** As shown in Figure 25.22, an object placed in front of a convex mirror produces a virtual image behind the mirror that is reduced in size and is upright in orientation for all object distances. Therefore, it is *not* possible to use a convex mirror to produce an image that is larger than the object.

---

13. ***REASONING AND SOLUTION*** Suppose you stand in front of a spherical mirror (concave or convex). As shown in Figures 25.19 and 25.20, a concave mirror can form a real, inverted image (Figures 25.19*a* and 25.19*b*) or a virtual, upright image (Figure 25.20). As shown in Figure 25.22, a convex mirror can only form a virtual, upright image.

a. Therefore, it is not possible for your image to be real and upright.

b. Similarly, it is not possible for your image to be virtual and inverted.

CHAPTER 26 *THE REFRACTION OF LIGHT: LENSES AND OPTICAL INSTRUMENTS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** Since the index of refraction of water is greater than that of air, the ray in Figure 26.2*a* is bent toward the normal at the angle q_{w1} when it enters the water. According to Snell's law (Equation 26.2), the sine of q_{w1} is given by

$$\sin q_{\text{w1}} = \frac{n_{\text{air}} \sin q_1}{n_{\text{water}}} = \frac{\sin q_1}{n_{\text{water}}} \tag{1}$$

where we have taken $n_{\text{air}} = 1.000$. When a layer of oil is added on top of the water, the angle of refraction at the air/oil interface is q_{oil} and, according to Snell's law, we have

$$\sin q_{\text{oil}} = \frac{n_{\text{air}} \sin q_1}{n_{\text{oil}}} = \frac{\sin q_1}{n_{\text{oil}}} \tag{2}$$

But q_{oil} is also the angle of incidence at the oil/water interface. At this interface the angle of refraction is q_{w2} and is given by Snell's law as follows:

$$\sin q_{\text{w2}} = \frac{n_{\text{oil}} \sin q_{\text{oil}}}{n_{\text{water}}} = \frac{n_{\text{oil}}}{n_{\text{water}}} \frac{\sin q_1}{n_{\text{oil}}} = \frac{\sin q_1}{n_{\text{water}}} \tag{3}$$

where we have substituted Equation (2) for $\sin q_{\text{oil}}$. According to Equation (1), this result is equal to $\sin q_{\text{w1}}$. Therefore, we can conclude that the angle of refraction as the ray enters the water does *not* change due to the presence of the oil.

---

2. ***REASONING AND SOLUTION*** When light travels from a material with refractive index n_1 into a material with refractive index n_2, the angle of refraction q_2 is related to the angle of incidence q_1 by Equation 26.2: $n_1 \sin q_1 = n_2 \sin q_2$ or $q_2 = \sin^{1}(n_1/n_2)\sin q_1$. When $n_1 < n_2$, the angle of refraction will be less than the angle of incidence. The larger the value of n_2, the smaller the angle of refraction for the same angle of incidence. The angle of refraction is smallest for slab B; therefore, slab B has the greater index of refraction.

---

3. ***REASONING AND SOLUTION*** When light travels from a material with refractive index n_1 into a material with refractive index n_2, the angle of refraction q_2 is related to the angle of incidence q_1 by Equation 26.2: $n_1 \sin q_1 = n_2 \sin q_2$. Therefore, $n_1 = n_2 \sin q_2 / (\sin q_1)$. Both blocks are made from the same material; therefore, n_2 is the same for each system. Furthermore, the angle of incidence is the same for each system; therefore, $\sin q_1$ is the same in both liquids. Thus, the liquid with the greater index of refraction will be the one for which $\sin q_2$, and therefore q_2, is the largest. This occurs in liquid A. Therefore, the index of refraction of liquid A is greater than that of liquid B.

---

4. ***REASONING AND SOLUTION*** When an observer peers over the edge of a deep empty bowl, he does not see the entire bottom surface, so a small object lying on the bottom is hidden from view. However, when the bowl is filled with water, the object can be seen.

When the object is viewed from the edge of the bowl, light rays from the object pass upward through the water. Since $n_{\text{air}} < n_{\text{water}}$, the light rays from the object refract away from the normal when they enter air. The refracted rays travel to the observer, as shown in the figure at the right. When the rays entering the air are extended back into the water, they show that the observer sees a virtual image of the object at an apparent depth that is less than the actual depth, as indicated in the drawing. Therefore, the apparent position of the object in the water is in the line of sight of the observer, even though the object could not be seen before the water was added.

---

5. ***REASONING AND SOLUTION*** Two identical containers, one filled with water (n = 1.33) and the other filled with ethyl alcohol (n = 1.36) are viewed from directly above. According to Equation 26.3, when viewed from directly above in a medium of refractive index n_2, the apparent depth $d\ell$ in a medium of refractive index n_1 is related to the actual depth d by the relation $d = d(n_2 / n_1)$. Assuming that the observer is in air, n_2 = 1.00. Since n_1 refers to the refractive index of the liquid in the containers, we see that the apparent depth in each liquid is inversely proportional to the refractive index of the liquid. The index of refraction of water is smaller than that of ethyl alcohol; therefore, the container filled with water appears to have the greater depth of fluid.

---

6. ***REASONING AND SOLUTION*** When you look through an aquarium window at a fish, the fish appears to be closer than it actually is. When light from the fish leaves the water and enters the air, it is bent away from the normal as shown below. Therefore, the apparent location of the image is closer to the observer than the actual location of the fish.

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7. ***REASONING AND SOLUTION*** At night, when it is dark outside and you are standing in a brightly lit room, it is easy to see your reflection in a window. During the day it is not so easy. If we assume that the room is brightly lit by the same amount in both cases, then the light reflected from the window is the same during the day as it is at night. However, during the day, light is coming through the window from the outside. In addition to the reflection, the observer also sees the light that is refracted through the window from the outside. The light from the outside is so intense that it obscures the reflection in the glass.

---

8. ***REASONING AND SOLUTION***

a. The man is using a bow and arrow to shoot a fish. The light from the fish is refracted away from the normal when it enters the air; therefore, the apparent depth of the image of the fish is less than the actual depth of the fish. When the arrow enters the water, it will continue along the same straight line path from the bow. Therefore, in order to strike the fish, the man must aim below the image of the fish. The situation is similar to that shown in Figure 26.5*a*; we can imagine replacing the boat by a dock and the chest by a fish.

b. Now the man is using a laser gun to shoot the fish. When the laser beam enters the water it will be refracted. From the principle of reversibility, we know that if the laser beam travels along one of the rays of light emerging from the water that originates on the fish, it will follow exactly the same path in the water as that of the ray that originates on the fish. Therefore, in order to hit the fish, the man must aim directly at the image of the fish.

---

9. ***REASONING AND SOLUTION*** Two rays of light converge to a point on a screen, as shown below.

A plane-parallel plate of glass is placed in the path of this converging light, and the glass plate is parallel to the screen, as shown below. As discussed in the text, when a ray of light passes through a pane of glass that has parallel surfaces, and is surrounded by air, the

emergent ray is parallel to the incident ray, but is laterally displaced from it. The extent of the displacement depends on the angle of incidence, on the thickness, and on the refractive index of the glass.

As shown in the scale drawing above, the point of convergence does not remain on the screen. It will move away from the glass as shown.

---

10. ***REASONING AND SOLUTION*** Light from the sun is unpolarized; however, when the sunlight is reflected from horizontal surfaces, such as the surface of an ocean, the reflected light is partially polarized in the horizontal direction. Polaroid sunglasses are constructed with lenses made of Polaroid (a polarizing material) with the transmission axis oriented vertically. Thus, the horizontally polarized light that is reflected from horizontal surfaces is blocked from the eyes.

Suppose you are sitting on the beach near a lake on a sunny day, wearing Polaroid sunglasses. When a person is sitting upright, the horizontally polarized light that is reflected from the water is blocked from her eyes, as discussed above, and she notices little discomfort due to the glare from the water. When she lies on her side, the transmission axis of the Polaroid sunglasses is now oriented in a nearly horizontal direction. Most of the horizontally polarized light that is reflected from the water is transmitted through the sunglasses and reaches her eyes. Therefore, when the person lies on her side, she will notice that the glare increases.

---

11. ***REASONING AND SOLUTION*** Light from the sun is unpolarized; however, when the sunlight is reflected from horizontal surfaces such as the surface of a swimming pool, lake, or ocean, the reflected light is partially polarized in the horizontal direction. Polaroid sunglasses are constructed with lenses made of Polaroid (a polarizing material) with the transmission axis oriented vertically. Thus, the horizontally polarized light that is reflected from horizontal surfaces is blocked from the eyes.

If you are sitting by the shore of a lake on a sunny and windless day, you will notice that the effectiveness of your Polaroid sunglasses in reducing the glare of the sunlight reflected from the lake varies depending on the time of the day. As the angle of incidence of the sun's rays increases from 0°, the degree of polarization of the rays in the horizontal direction increases. Since Polaroid sunglasses are designed so that the transmission axes are aligned in the vertical direction when they are worn normally, they become more effective as the sun gets lower in the sky. When the angle of incidence is equal to Brewster's angle, the reflected light is completely polarized parallel to the surface, and the sunglasses are most effective. For angles of incidence greater than Brewster's angle, the glasses again become less effective.

---

12. ***REASONING AND SOLUTION*** According to the *principle of reversibility* (see Section 25.5), if the direction of a light ray is reversed, the light retraces its original path. While the principle of reversibility was discussed in Section 25.5 in connection with the reflection of light rays, it is equally valid when the light rays are refracted. Imagine constructing a mixture of colored rays by passing a beam of sunlight through a prism in the usual fashion. By orienting a second prism so that the rays of colored light are incident on the second prism with angles of incidence that are equal to their respective angles of refraction as they emerge from the first prism, we have a perfectly symmetric situation. The rays through the second prism will follow the reverse paths of the rays through the first prism, and the light emerging from the second prism will be sunlight.

---

13. ***REASONING AND SOLUTION*** For glass (refractive index n_g), the critical angle for the glass/air interface can be determined from Equation 26.4:

$$\sin q_c = \frac{1.0}{n_g} \qquad (1)$$

In Figure 26.7 the angle of incidence at the upper glass/air interface is q_2. Total internal reflection will occur there only if $q_2 \quad q_c$. But q_2 is also the angle of refraction at the lower air/glass interface and can be obtained using Snell's law as given in Equation 26.2:

$$(1.0)\sin q_1 = n_g \sin q_2 \quad \text{or} \quad \sin q_2 = \frac{1.0}{n_g}\sin q_1$$

Using Equation (1) for $1.0/n_g$, we obtain

$$\sin q_2 = \sin q_c \ \sin q_1 \qquad (2)$$

For all incident angles q_1 that are less than $90°$, Equation (2) indicates that $\sin q_2 \quad \sin q_c$, since $\sin q_1 < 1$. Therefore, $q_2 < q_c$ and total internal reflection can not occur at the upper glass/air interface.

---

14. ***REASONING AND SOLUTION***

a. When a rainbow is formed, light from the sun enters a spherical water droplet and is refracted by an amount that depends on the refractive index of water for that wavelength. Light that is reflected from the back of the droplet is again refracted at it reenters the air, as suggested in Figure 26.21. Although all colors are refracted for any given droplet, the observer sees only one color, because only one color travels at the proper angle to reach the observer. The observer sees the full spectrum in the rainbow because each color originates from water droplets that lie at different elevation angles.

As shown in Figure 26.21, the sun must be located *behind* the observer, if the observer is to see the rainbow. Therefore, if you want to make a rainbow by spraying water from a

garden hose into the air, you must stand with the sun behind you, and adjust the hose so that it sprays a fine mist of water in front of you. The distance between the observer and the droplets is not crucial. The important factor is the angle formed by the intersection of the line that extends from the sun to the droplet with the line that extends from the droplet to the observer. *Remark:* When the distance is only a few meters, as it would be in the case of a "garden-hose rainbow", each eye would receive rays from different parts of the mist. Therefore, the observer could see two rainbows that cross over each other.

b. Each color of light that leaves a given droplet travels in a specific direction that is governed by Snell's law. You can't ever walk under a rainbow, because each color that originates from a single droplet travels in a unique direction. To walk under a rainbow, all the colors would have to be refracted vertically downward, which is not the case. Therefore, you can't walk under a rainbow, because the rays are traveling in the wrong directions to reach the observer's eyes.

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15. ***REASONING AND SOLUTION*** A person is floating on an air mattress in the middle of a swimming pool. His friend is sitting on the side of the pool. The person on the air mattress claims that there is a light shining up from the bottom of the pool directly beneath him. His friend insists, however, that she cannot see any light from where she sits on the side.

Rays from a light source on the bottom of the pool will radiate outward from the source in all directions. However, only rays for which the angle of incidence is less than the critical angle will emerge from the water. Rays with an angle of incidence equal to, or greater than, the critical angle will undergo total internal reflection back into the water, as shown in the following figure.

Because of the geometry, the rays that leave the water lie within a cone whose apex lies at the light source. Thus, rays of light that leave the water emerge from within an illuminated circle just above the source. If the mattress is just over the source, it could cover the area through which the light would emerge. A person sitting on the side of the pool would not see any light emerging. Therefore, the statements made by both individuals are correct.

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16. ***REASONING AND SOLUTION*** Total internal reflection occurs only when light travels from a higher-index medium (refractive index = n_1) toward a lower-index medium (refractive index = n_2). Total internal reflection does not occur when light propagates from

a lower-index to a higher-index medium. The smallest angle of incidence for which total internal reflection will occur at the higher-index/lower-index interface is called the critical angle and is given by Equation 26.4: $\sin q_c = n_2 / n_1$ where $n_1 > n_2$.

A beam of blue light is propagating in glass. When the light reaches the boundary between the glass and the surrounding air, the beam is totally reflected back into the glass. However, red light with the same angle of incidence is not totally reflected and some of the light is refracted into the air. According to Table 26.2, the index of refraction of glass is greater for blue light than it is for red light. From Snell's law, therefore, we can conclude that the critical angle is greater for red light than it is for blue light. Therefore, if the angle of incidence is equal to or greater than the critical angle for blue light, but less than the critical angle for red light, blue light will be totally reflected back into the glass, while some of the red light will be refracted into the air.

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17. ***REASONING AND SOLUTION*** A beacon light in a lighthouse is to produce a parallel beam of light. The beacon consists of a bulb and a converging lens. As shown in Figure 26.22*b*, paraxial rays that are parallel to the principal axis converge to the focal point after passing through the lens. From the principle of reversibility, we can deduce that if a point source of light were placed at the focal point, the emitted light would travel in parallel rays after passing through the lens. Therefore, in the construction of the beacon light, the bulb should be placed at the focal point of the lens.

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18. ***REASONING AND SOLUTION*** The figure at the right shows a converging lens (in air). The normal to the surface of the lens is shown at five locations on each side of the lens.

A ray of light bends toward the normal when it travels from a medium with a lower refractive index into a medium with a higher refractive index. Likewise, a ray of light bends away from the normal when it travels from a medium with a higher refractive index into a medium with a lower refractive index. When rays of light traveling in air enter a converging lens, they are bent toward the normal. When these rays leave the right side of the lens, they are bent away from the normal; however, since the normals diverge on the right side of the lens, the rays again converge.

If this lens is surrounded by a medium which has a higher index of refraction than the lens, then when rays of light enter the lens, the rays are bent away from the normal, and, therefore, they diverge. When the rays leave the right side of the lens, they are bent toward the normal; however, since the normals diverge on the right side of the lens, the rays diverge further. Therefore, a converging lens (in air) will behave as a diverging lens when it is surrounded by a medium that has a higher index of refraction than the lens.

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19. ***REASONING AND SOLUTION*** A spherical mirror and a lens are immersed in water. The effect of the mirror on rays of light is governed by the law of reflection; namely $q_r = q_i$. The effect of the lens on rays of light is governed by Snell's law; namely, $n_1 \sin q_1 = n_2 \sin q_2$. The law of reflection, as it applies to the mirror, does not depend on the index of refraction of the material in which it is immersed. Snell's law, however, as

it applies to the lens, depends on both the index of refraction of the lens and the index of refraction of the material in which it is immersed. Therefore, compared to the way they work in air, the lens will be more affected by the water.

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20. ***REASONING AND SOLUTION*** A converging lens is used to project a real image onto a screen, as in Figure 26.27*b*. A piece of black tape is then placed on the upper half of the lens.

The ray diagram above shows the rays from two points on the object, one point at the top of the object and one point on the lower half of the object. As shown in the diagram, rays from both points converge to form the image on the right side of the lens. Therefore, the entire image will be formed. However, since fewer rays reach the image when the tape is present, the intensity of the image will be less than it would be without the tape.

---

21. ***REASONING AND SOLUTION*** When light travels from a material with refractive index n_1 into a material with refractive index n_2, the angle of refraction q_2 is related to the angle of incidence q_1 by Snell's law (Equation 26.2): $n_1 \sin\ q_1 = n_2 \sin\ q_2$.

A converging lens is made from glass whose index of refraction is *n*. The lens is surrounded by a fluid whose index of refraction is also *n*. This situation is known as *index matching* and is discussed in Conceptual Example 7. Since the refractive index of the surrounding fluid is the same as that of the lens, $n_1 = n_2$, and Snell's law reduces to $\sin\ q_1 = \sin\ q_2$. The angle of refraction is equal to the angle of incidence at both surfaces of the lens; the path of light rays is unaffected as the rays travel through the lens. Therefore, this lens cannot form an image, either real or virtual, of an object.

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22. ***REASONING AND SOLUTION*** The expert claims that the height of the window can be calculated from only two pieces of information : (1) the measured height on the film, and (2) the focal length of the camera. The expert is not correct. According to the thin-lens equation (Equation 26.6), $1/d_o + 1/d_i = 1/f$, where d_o is the object distance, d_i is the image distance, and *f* is the focal length of the lens. The magnification equation (Equation 26.7), relates the image and object heights to the image and object distances: $h_o/h_i = d_o/d_i$. These two equations contain five unknowns. To determine any one of

the unknowns, three of the other unknowns must be known. In this case, all that we know is the height of the image, h_i, and the focal length of the camera, f. Therefore, we do not have enough information given to determine the distance from the ground to the window (the height of the object in this case), h_o. We still need to know either the distance from the photographer to the house (the object distance, d_o), or the distance from the center of the lens to the film (the image distance, d_i). We can conclude, therefore, that the expert is incorrect.

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23. ***REASONING AND SOLUTION*** Suppose two people who wear glasses are camping. One is nearsighted, and the other is farsighted. It is desired to start a fire with the sun's rays. A converging lens can be used to focus the nearly parallel rays of the sun on a sheet of paper. If the paper is placed at the focal point of the lens, the sun's rays are concentrated to give a large intensity, so that the paper heats up rapidly and ignites. As shown in Figures 26.36 and 26.37, nearsightedness can be corrected with diverging lenses, and farsightedness can be corrected using converging lenses. Therefore, the glasses of the farsighted person would be useful in starting a fire, while the glasses of the nearsighted person would not be useful.

---

24. ***REASONING AND SOLUTION*** A 21-year-old with normal vision (near point = 25 cm) is standing in front of a plane mirror. The near point is the point nearest the eye at which an object can be placed and still produce a sharp image on the retina. Therefore, if the 21-year old wants to see himself in focus, he can stand no closer to the mirror than 25 cm from his image. As discussed in Chapter 25, the image in a plane mirror is located as far behind the mirror as the observer is in front of the mirror. If the 21-year-old is 25 cm from his image, he must be 25 cm/2 = 12.5 cm in front of the mirror's surface. Therefore, he can stand no closer than 12.5 cm in front of the mirror and still see himself in focus.

---

25. ***REASONING AND SOLUTION*** The distance between the lens of the eye and the retina is constant; therefore, the eye has a fixed image distance. The only way for images to be produced on the retina for objects located at different distances is for the focal length of the lens to be adjusted. This is accomplished through with the ciliary muscles. If we read for a long time, our eyes become "tired," because the ciliary muscle must be tensed so that the focal length is shortened enough to bring the print into focus. When the eye looks at a distant object, the ciliary muscle is fully relaxed. Therefore, when your eyes are "tired" from reading, it helps to stop and relax the ciliary muscle by looking at a distant object.

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26. ***REASONING AND SOLUTION*** As discussed in the text, for light from an object in air to reach the retina of the eye, it must travel through five different media, each with a different index of refraction. About 70 % of the refraction occurs at the air/cornea interface where the refractive index of air is taken to be unity and the refractive index of the cornea is 1.38.

To a swimmer under water, objects look blurred and out of focus. However, when the swimmer wears goggles that keep the water away from the eyes, the objects appear sharp and in focus. Without the goggles, light from objects must undergo the first refraction at a

water/cornea interface. Since the index of refraction of water is 1.33 while that of the cornea is 1.38, the amount of refraction is smaller than it is when the person is in air, and the presence of the water prevents the image from being formed on the retina. Consequently, objects look blurred and out of focus. When the swimmer wears goggles, incoming light passes through the volume of air contained in the goggles before it reaches the eyes of the swimmer. The first refraction of the light in the eye occurs at an air/cornea interface. The refraction occurs to the proper extent, so that the image is formed on the retina. Therefore, when the swimmer wears the goggles, objects appear to be sharp and in focus.

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27. ***REASONING AND SOLUTION*** The refractive power of the lens of the eye is 15 diopters when surrounded by the aqueous and vitreous humors. If this lens is removed from the eye and surrounded by air, its refractive power increases to about 150 diopters. From Snell's law, we know that the effect of the lens on incoming light depends not only on the refractive index of the lens, but also on the refractive index of the materials on either side of the lens. The refractive index of the lens is 1.40, while that of the aqueous humor is 1.33, and that of the vitreous humor is 1.34. Light that leaves the lens has been refracted twice, once when it enters the lens and again when it leaves the lens. Since the refractive indices of these three media are not very different, the amount of refraction at each interface is small. When the lens is surrounded by air, the light is again doubly refracted. In this case, however, the refractive indices at each interface differ substantially, so the amount of refraction at each interface is much larger. Therefore, when the lens is in air, its focal length is much smaller than it is when the lens is in place in the eye. According to Equation 26.8, the refractive power of a lens is equal to $1/f$, where the refractive power is expressed in diopters when the focal length is in meters. The smaller the focal length of the lens, the larger its refractive power. Consequently, the refractive power of the lens is much greater when the lens is surrounded by air.

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28. ***REASONING AND SOLUTION*** A full glass of wine acts, approximately, as a converging lens and focuses the light to a spot on the table. An empty glass consists only of thin glass layers on opposite sides, which do not refract the light enough to act as a lens and produce a focused image.

---

29. ***REASONING AND SOLUTION*** The angle q subtended by the image as measured from the principal axis of the lens of the eye is equal to the angle subtended by the object. This angle is called the angular size of both the image and the object and is given by $q \quad h_o / d_o$, where q is expressed in radians.

Jupiter is the largest planet in our solar system. Yet to the naked eye, it looks smaller than Venus. This occurs because the distance from Earth to Jupiter is about 15 times greater than the distance from Earth to Venus, while the diameter of Jupiter is only about 12 times larger than that of Venus. Consequently, the angular size of Jupiter is about 12/15 or 0.80 times as large as that of Venus. Therefore, Jupiter looks smaller than Venus.

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30. ***REASONING AND SOLUTON***
a. The figure below is a ray diagram that shows that the eyes of a person wearing glasses appear to be smaller when the glasses use diverging lenses.

b. The figure below is a ray diagram that shows that the eyes of a person wearing glasses appear to be larger when the glasses use converging lenses.

Notice that in both cases, the eye lies between the focal length of the lens and the lens, and that both images are virtual images.

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31. ***REASONING AND SOLUTION*** As discussed in the text, regardless of the position of a real object, a diverging lens always forms a virtual image that is upright and smaller relative to the object. The figures below show this for two cases: one in which the object is within the focal point, and the other in which the object is beyond the focal point. In each case, the image is smaller than the object. Therefore, a diverging lens cannot be used as a magnifying glass.

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32. ***REASONING AND SOLUTION*** A person whose near point is 75 cm from the eyes, must hold a printed page at least 75 cm from his eyes in order to see the print without blurring, while a person whose near point is 25 cm can hold the page as close as 25 cm and still find the print in focus. If the size of the print is small, it will be more difficult to see the print at 75 cm than at 25 cm, even though the print is in focus. Therefore, the person whose near point is located 75 cm from the eyes will benefit more by using a magnifying glass.

---

33. ***REASONING AND SOLUTION*** The angular magnification of a telescope is given by Equation 26.12: $M \quad f_o / f_e$, where f_o is the focal length of the objective, and f_e is the focal length of the eyepiece. In order to produce a final image that is magnified, f_o must be greater than f_e. Therefore if two lenses, whose focal lengths are 3.0 and 45 cm are to be used to build a telescope, the lens with the 45 cm focal length should be used for the objective, and the lens with the 3.0 cm focal length should be used for the eyepiece.

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34. ***REASONING AND SOLUTION*** A telescope consists of an objective and an eyepiece. The objective focuses nearly parallel rays of light that enter the telescope from a distant object to form an image just beyond its focal point. The image is real, inverted, and reduced in size relative to the object. The eyepiece acts like a magnifying glass. It is positioned so that the image formed by the objective lies just within the focal point of the eyepiece. The final image formed by the eyepiece is virtual, upright and enlarged.

Two refracting telescopes have identical eyepieces, although one is twice as long as the other. Since the eyepiece is positioned so that the image formed by the objective lies just within the focal point of the eyepiece, the longer telescope has an objective with a longer focal length. The angular magnification of a telescope is given by Equation 26.12: $M \quad f_o / f_e$, where f_o is the focal length of the objective, and f_e is the focal length of the eyepiece. Both telescopes have the same value for f_e. The longer telescope has the larger value of f_o; therefore, the longer telescope has the greater angular magnification.

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35. ***REASONING AND SOLUTION*** In a telescope the objective forms a first image just beyond the focal point of the objective and just within the focal point of the eyepiece. Thus, as Figure 26.42 shows, the distance between the two converging lenses is $L \quad f_o + f_e$. For the two lenses specified, this would mean that L 4.5 cm + 0.60 cm = 5.1 cm. But L is given as L = 14 cm, which means that there is a relatively large separation between the focal points of the objective and the eyepiece. This arrangement is like that for a microscope shown in Figure 26.33. Thus, the instrument described in the question is a microscope.

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36. ***REASONING AND SOLUTION***

a. A projector produces a real image at the location of the screen.

b. A camera produces a real image at the location of the film.

c. A magnifying glass produces a virtual image behind the lens.

d. Eyeglasses produce virtual images that the eye then sees in focus.

e. A compound microscope produces a virtual image.

f. A telescope produces a virtual image.

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37. ***REASONING AND SOLUTION*** Chromatic aberration occurs when the index of refraction of the material from which a lens is made varies with wavelength. Lenses obey Snell's law. If the index of refraction of a lens varies with wavelength, then different colors of light that pass through the lens refract by different amounts. Therefore, different colors come to a focus at different points. Mirrors obey the law of reflection. The angle of reflection depends only on the angle of incidence, regardless of the wavelength of the incident light; therefore, chromatic aberration occurs in lenses, but not in mirrors.

CHAPTER 27 | *INTERFERENCE AND THE WAVE NATURE OF LIGHT*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A radio station broadcasts simultaneously from two transmitting antennas at two different locations. The radio that receives the broadcast could have better reception depending on the location of the receiving antenna. According to the principle of linear superposition, when the electromagnetic waves from the transmitting antenna arrive at the same point, the resultant wave is the sum of the individual waves. The amplitude of the resultant wave depends on the relative phase between the two waves. The relative phase between the waves depends on the path length difference between the two waves. If the two waves arrive at the receiving antenna so that the path length difference between them is equal to an integer number of wavelengths, the two waves will be in phase, they will reinforce each other, and constructive interference will occur. The resulting amplitude of the radio wave will be larger than it would be from either transmitting antenna alone; therefore, the radio reception will be better. If the two waves arrive at the receiving antenna so that the path length difference between them is equal to an odd multiple of half of a wavelength, the two waves will be exactly out of phase, they will mutually cancel, and destructive interference will occur. The receiving antenna will receive little or no signal in this situation. As a result, the radio reception will be very bad. Thus, having two transmitting antennas does not necessarily lead to better reception.

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2. ***REASONING AND SOLUTION*** Constructive interference occurs between waves from two in-phase sources when the difference in travel distances from the sources to the point where the waves combine is the same or equal to an integer number of wavelengths. For a square the distance from any corner to the center is the same. Hence, constructive interference occurs at the center.

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3. ***REASONING AND SOLUTION***

   a. The light waves coming from *both* slits in a Young's double slit experiment have their phases shifted by an amount equivalent to half a wavelength. Since the light from both slits is changed by the same amount, the relative phase difference between the light from the two slits is zero when the light leaves the slits. When the light reaches the screen, the relative phase difference between light waves from the two slits will be the same as if the phase of the waves had not been shifted at the slits. Therefore, the pattern will be exactly the same as that described in the text.

   b. Light coming from *only one* of the slits in a Young's double slit experiment has its phase shifted by an amount equivalent to half of a wavelength. Now, there is a relative phase difference of one half of a wavelength between the light leaving the slits. When the light reaches the screen, there will be a relative phase difference due to the fact that light from each slit, in general, traveled along different paths. In addition, there will be the initial

phase difference that is equivalent to half of a wavelength. Therefore, the pattern will be similar to that described in the text; however, the points of constructive interference will be points of destructive interference, and the points of destructive interference will be points of constructive interference. In other words, the positions of the light and dark fringes are interchanged.

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4. ***REASONING AND SOLUTION*** According to Equation 27.1, the bright fringes of a double slit experiment occur at values of q for which $\sin q = m l / d$ where m = 0, 1, 2, 3, ... According to Equation 27.2, the dark fringes occur at values of q for which $\sin q = [m + (1/2)] l / d$ where m = 0, 1, 2, 3, . . . The slits S_1 and S_2 in Figure 27.4 are replaced with identical loudspeakers and use the same ac electrical signal to drive them. The two sound waves produced will then be identical, and we have the audio equivalent of Young's double slit experiment. Sound waves that reach the center of the screen have traveled along paths of equal length; therefore, the sound waves from each loudspeaker arrive there in phase and constructive interference occurs. Consequently, at the center point on the screen, the sound will have maximum loudness. As you proceed to walk away from the center (in either direction), the sound will decrease in volume. When the angle q is such that $\sin q = (1/2) l / d$, the sound will decrease to zero amplitude. If you continue to walk beyond this point, the loudness of the sound will increase. It will reach a maximum (a point of constructive interference) when the angle q is such that $\sin q = l / d$. The sound will again decrease in loudness until the next point of destructive interference occurs. This point occurs when $\sin q = (3/2) l / d$. In general, there will a pattern of alternating loud and soft sounds heard as you walk away from the center of the screen. The intensity at the points of constructive interference varies in a manner that is similar to that shown in Figure 27.6. The central maximum has the greatest intensity. The sound at each region of constructive interference decreases in intensity as one proceeds away from the center (in either direction).

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5. ***REASONING AND SOLUTION*** The angle q for the interference maximum in Young's double-slit experiment is given by Equation 27.1: $\sin q = m l / d$ where m = 0, 1, 2, 3, . . . When the wavelength l of the light is greater than the distance d between the slits, the ratio l / d is greater than one; however, $\sin q$ cannot be greater than one. Therefore, it is *not* possible to see interference fringes when the wavelength of the light is greater than the distance between the slits.

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6. ***REASONING AND SOLUTION*** A camera lens is covered with a nonreflective coating that eliminates the reflection of perpendicularly incident green light. If the light were incident on the nonreflective coating at 45°, rather than perpendicularly, it would not be eliminated by the coating and an observer would see it.

Using reasoning similar (but not identical to) to that in Example 3, if the refractive index of the film is less than the refractive index of the lens, the minimum nonzero thickness of the nonreflecting coating is given by $t = l_{\text{coating}} / 4$. This result follows from the fact that

the extra distance traveled by the wave that travels through the coating is $2t$ when the wave strikes the film at normal incidence.

If the wave strikes the thin film at 45°, then the extra distance traveled by the wave that travels through the coating is no longer $2t$. Instead, it is $2d$, where d is the distance shown in the figure at the right. The value of d depends on the value for q_2, the angle of refraction. This angle, in turn, depends on the refractive index n_1 for air and n_2 for the lens coating, according to Snell's law (Equation 26.2). To eliminate the light that is incident at 45°, the coating would have to have a minimum nonzero thickness such that $d = l_{\text{coating}}/4$. Since the coating is designed such that $t = l_{\text{coating}}/4$, it does not eliminate the light incident at 45°.

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7. ***REASONING AND SOLUTION*** When sunlight reflects from a thin film of soapy water, the film appears multicolored, in part because destructive interference removes different wavelengths from the light reflected at different places, depending on the thickness of the film. As the film becomes thinner and thinner, the path length difference between the light reflected from the top of the film and the light reflected from the bottom of the film becomes closer to zero. The light reflected from the top of the film undergoes a phase change equivalent to half of a wavelength. Therefore, as the thickness of the film approaches zero, the light reflected from the top of the film is almost exactly out of phase with the light reflected from the bottom of the film. Therefore, as the film becomes thinner and thinner, it looks darker and darker in reflected light. It appears black just before it breaks, because at that point destructive interference occurs between the light reflected from the top and the light reflected from the bottom of the film.

---

8. ***REASONING AND SOLUTION*** Two pieces of the same glass are covered with thin films of different materials. The thickness t of each film is the same. In reflected sunlight, however, the films have different colors.

This could occur because, in general, different materials have different refractive indices. Using reasoning similar to (but not identical to) that in Example 3, the wavelength l_{air} that is removed from the reflected light corresponds to a value of l_{film} in the film that satisfies either of the following conditions:

$$2t = m\,l_{\text{film}}, \quad m = 0,\ 1,\ 2,\ 3,\ \ldots \qquad n_{\text{film}} > n_{\text{glass}}, \tag{1}$$

and

$$2t = \left(m + \tfrac{1}{2}\right) l_{\text{film}}, \quad m = 0,\ 1,\ 2,\ 3,\ \ldots \qquad n_{\text{film}} < n_{\text{glass}}. \tag{2}$$

However, the value of the wavelength l_{film} of the light in the film depends on the index of refraction of the film. According to Equation 27.3, $l_{\text{film}} = l_{\text{vacuum}} / n$, where n is the index of refraction of the film. If the index of refraction of the two materials are different, then conditions (1) and (2) will be satisfied for two different values of l_{film}. These values of l_{film} correspond to different values of l_{air}, and, therefore, different wavelengths will be removed from the reflected light.

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9. ***REASONING AND SOLUTION*** When a spherical surface is put in contact with an optically flat plate, circular interference patterns, called Newton's rings, can be observed in reflected light, as shown in Figure 27.14*b*. There is a dark spot at the center of the pattern of Newton's rings. The dark spot occurs for the following reason. At the center of the plate, the spherical surface is in contact with the plate. Destructive interference occurs there, because the thickness of the "air wedge" is essentially zero at this point. Therefore, the only difference in phase between the light reflected from the bottom of the spherical surface and the light reflected from the top of the optical flat is the half-wavelength phase change due to reflection from the optical flat. This situation is similar to that discussed in part (b) of Example 5 in the text.

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10. ***REASONING AND SOLUTION*** A thin film of material is floating on water (n = 1.33). When the material has a refractive index of n = 1.20, the film looks bright in reflected light as its thickness approaches zero. But when the material has a refractive index of n = 1.45, the film looks black in reflected light as its thickness approaches zero.

When the material has a refractive index of n = 1.20, both the light reflected from the top of the film and the light reflected from the bottom occur under conditions where light is traveling through a material with a smaller refractive index toward a material with a larger refractive index. Therefore, both the light reflected from the top and the bottom undergoe a phase change upon reflection. Both phase changes are equivalent to half of a wavelength. Therefore, the reflections introduce no net phase difference between the light reflected from the top and bottom. As the thickness of the film approaches zero, the path difference between the light reflected from the top and the bottom of the film approaches zero. Since the path difference is close to zero, and there is no relative phase difference due to reflection, the light reflected from the top will be in phase with the light reflected from the bottom of the film; constructive interference will occur, and the film looks bright in reflected light.

When the material has a refractive index of n = 1.45, only the light reflected from the top of the film is traveling through a material with lower refractive index toward a material with a larger refractive index. Therefore, only the light reflected from the top undergoes a phase change upon reflection. This phase change is equivalent to half of a wavelength. Therefore, there is a net phase difference equivalent to half of a wavelength between the light reflected from the top and bottom of the film. As the thickness of the film approaches zero, the path difference between the light reflected from the top and the light reflected from the bottom of the film approaches zero. Since the path difference is close to zero, and there is a relative phase difference due to reflection that is equal to half of a wavelength, the light reflected from the top will be exactly out of phase with the light reflected from the bottom of the film, and destructive interference will occur. Therefore, the film looks black in reflected light.

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11. ***REASONING AND SOLUTION*** A transparent coating is deposited on a glass plate and has a refractive index that is larger than that of the glass. For a certain wavelength within the coating, the thickness of the coating is a quarter wavelength. The coating enhances the reflection of the light corresponding to this wavelength.

As discussed in Example 3, only the light reflected from the top of the coating is traveling in through a material (air) with lower refractive index toward a material coating with a larger refractive index. Therefore, only the light reflected from the top of the coating undergoes a phase change upon reflection. This change in phase is equivalent to half of a wavelength. Therefore, there is a net phase difference equivalent to half of a wavelength between the light reflected from the top and light reflected from the bottom of the coating. Since the thickness of the coating is a quarter wavelength, the light reflected from the bottom of the coating traverses a distance equal to two times a quarter wavelength, or half a wavelength, as it travels through the coating. The overall relative phase difference between the light reflected from the top and the light reflected from the bottom of the coating is equivalent to half of a wavelength due to the reflection from the top of the coating and another half of a wavelength due to the path difference between the light reflected from the top and the light reflected from the bottom. Thus, the overall phase difference is equivalent to a whole wavelength, and the light reflected from the top is in phase with the light reflected from the bottom. Constructive interference occurs, and the coating enhances light corresponding to that wavelength.

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12. ***REASONING AND SOLUTION*** Rayleigh's criterion states that two point objects are just resolved when the first dark fringe in the diffraction pattern of one falls directly on the central bright fringe in the diffraction pattern of the other. Equation 27.6 approximates the minimum angular displacement between the two objects so that they are just resolved by an optical instrument: $q_{\min} \quad 1.22\, l / D$, where l is the wavelength of light used and D is the diameter of the aperture of the optical instrument. According to Equation 27.6, optical instruments with the highest resolution should have the largest possible diameter D and utilize light of the shortest possible wavelength.

The *f*-number setting on a camera gives the ratio of the focal length of the camera lens to the diameter of the aperture through which the light enters the camera. Therefore, smaller *f*-numbers mean larger aperture diameters. According to Equation 27.6, if we wish to resolve two closely spaced objects in a picture, we should adjust the aperture to have the largest possible diameter. As a result, we should use a small *f*-number setting.

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13. ***REASONING AND SOLUTION*** As discussed in the text, the extent to which a wave bends or diffracts around the edges of an obstacle or opening is determined by the ratio l/W, where l is the wavelength of the wave and W is the width of the obstacle or opening. As the ratio l/W becomes larger, diffraction effects become more pronounced.

The wavelength of light is extremely small and is normally expressed in nanometers ($1\text{ nm} = 10^{-9}\text{ m}$). Therefore, the ratio l/W is very small for light, and when light passes through a typical doorway, diffraction effects are not noticeable. Many sound waves

encountered in the everyday world have wavelengths that are comparable to the dimensions of a typical doorway. Therefore, the ratio l/W is nearly unity for these sounds, and when sound passes through a doorway, diffraction effects are noticeable; you can hear around corners, although you cannot see around corners.

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14. ***REASONING AND SOLUTION*** The Rayleigh criterion is $q_{\min}$ 1.22 l/D, where l is the wavelength of the light, D is the diameter of the opening into the eye, and $q_{\min}$ is the minimum angle that two point objects must subtend at the opening in order to be resolved by the eye. In this case D is the opening through which light from the painting enters the eye. Before squinting, the angle subtended at the eye is greater than $q_{\min}$, since the dots are distinguishable. Because of the squinting, light reaches the retina through an opening that is smaller than the normal size of the eye's pupil. A smaller value for D, with a given wavelength, means that the value for $q_{\min}$ is larger. Thus, after squinting, the angle subtended at the eye is smaller than the larger value for $q_{\min}$, and the dots of color can no longer be individually resolved, which gives the painting a more normal look.

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15. ***REASONING AND SOLUTION*** Four light bulbs are arranged at the corners of a rectangle that is three times longer than it is wide. You look at this arrangement perpendicular to the plane of the rectangle. From very far away, your eyes cannot resolve the individual bulbs and you see a single rectangular "smear" of light. From close in, you see the individual bulbs. The figures below illustrate what you would see at two positions between these two extremes.

In the first case, you are still far away, but you have moved close enough so that you can begin to resolve the bulbs at opposite ends of the diagonal. Essentially, you see a rectangular "smear" of light with a gap in the middle.

In the second case, you have moved closer still and now can resolve the two bulbs on one of the short sides of the rectangle from the two on the other short side. Essentially, you see two "smears" of light, each smear consisting of the two bulbs at the ends of a short side of the rectangle.

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16. ***REASONING AND SOLUTION*** The resolving power, or the ability to resolve two closely spaced images, through any circular aperture (such as the pupil of your eye) is limited by diffraction effects. The Rayleigh criterion for resolution focuses on these effects and is given by $q_{\min} \quad 1.22\, l / D$ (Equation 27.6), where l is the wavelength of light, D is the diameter of the aperture, and $q_{\min}$ is the minimum angle that can be resolved between two point objects by a circular aperture. A smaller value for $q_{\min}$ or l/D means a greater resolving power.

a. Suppose the pupil of your eye were elliptical instead of circular in shape, with the long axis of the ellipse oriented in the vertical direction. Since the resolving power depends on the diameter of the aperture and since the ellipse is longer than it is wide, the resolving power of your eye would not be the same in the horizontal and vertical directions.

b. If the pupil of your eye were elliptical instead of circular in shape, with the long axis of the ellipse oriented in the vertical direction, the "diameter" or "width" would be larger in the vertical direction and smaller in the horizontal direction. Consider two "planes" of light rays. Plane 1 is oriented so that neighboring rays are horizontal, and plane 2 is oriented so that neighboring rays are stacked vertically. Since the "diameter" of your eye is largest in the vertical direction, the ratio l/D is smaller for light in plane 2 than it is for light in plane 1. Thus, diffraction effects will be less noticeable for light rays stacked vertically. Therefore, the resolving power will be greater in the vertical direction.

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17. ***REASONING AND SOLUTION*** Rayleigh's criterion states that two point objects are just resolved when the first dark fringe in the diffraction pattern of one falls directly on the central bright fringe in the diffraction pattern of the other. Equation 27.6 approximates the minimum angular displacement $q_{\min}$ between the two objects so that they are just resolved by an optical instrument: $q_{\min} \quad 1.22\, l / D$, where l is the wavelength of light used and D is the diameter of the aperture of the optical instrument. According to Equation 27.6, optical instruments with the highest resolution should have the largest possible diameter D and utilize light of the shortest possible wavelength.

Suppose you were designing an eye and could select the size of the pupil and the wavelength of the electromagnetic waves to which the eye is sensitive. As far as the limitation created by diffraction is concerned, the following list ranks the design choices in order of decreasing resolving power (greatest first): (a) large pupil (large D) and ultraviolet wavelengths (small l); (c) small pupil (small D) and ultraviolet wavelengths (small l); (b) small pupil (small D) and infrared wavelength (larger l).

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18. ***REASONING AND SOLUTION*** In our discussion of single-slit diffraction, we ignored the height of the slit, in effect assuming that the height was much larger than the width. Suppose that the height and width were the same size, so that diffraction occurred in both dimensions. The resulting pattern would change in the following way. The height of the horizontal fringes would be reduced. Furthermore, there would be two sets of fringes. One set would be along the horizontal axis, and the other set would be along the vertical axis. The two sets would share a common central maximum. The intensity of the central

maximum will be greatest. The intensity will decrease on either side of the central maximum, in both the horizontal and the vertical directions. The figure below shows the central maximum and the less intense higher order on either side of the central maximum in both dimensions.

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19. ***REASONING AND SOLUTION*** According to Equation 27.7, the angles q that locate principal maxima of a diffraction grating satisfy the condition $\sin q = ml/d$, where $m = 0$, 1, 2, 3, . . .

If the entire interference apparatus of a diffraction grating (light source, grating, and screen) were immersed in water, the wavelength of the light would *decrease* from the value l_{air} to the value l_{water}. This is because these values are related by Equation 27.3, $l_{\text{water}} = l_{\text{vacuum}}/n_{\text{water}} = l_{\text{air}}/n_{\text{water}}$, where we have used the fact that l_{vacuum} and l_{air} are nearly the same, since n_{air} is nearly 1. Thus, angles q that locate principal maxima of a diffraction grating immersed in water will be smaller than when the apparatus is immersed in air. Consequently, the principle maxima will be closer together when the apparatus is immersed in water relative to their location when the entire apparatus is immersed in air.

CHAPTER 28 | *SPECIAL RELATIVITY*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The speed of light postulate states that the speed of light in *a vacuum*, measured in any inertial reference frame, always has the same value of c, no matter how fast the source of the light and the observer are moving relative to each other.

   The speed of light in water is c/n, where $n = 1.33$ is the refractive index of water. Thus, the speed of light in water is less than c. This does not violate the speed of light postulate, because the postulate refers to the speed of light in a vacuum, not in a physical medium.

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2. ***REASONING AND SOLUTION*** A baseball player at home plate hits a pop fly straight up (the beginning event) that is caught by the catcher at home plate (the ending event).

   a. A spectator in the stands is at rest relative to these events; therefore, this spectator would record the proper time interval between them t_0.

   b. A spectator sitting on the couch and watching the game on TV is at rest relative to these events; therefore, this spectator would also record the proper time interval between them t_0.

   c. The third baseman running in to cover the play is moving relative to the events; therefore, the third baseman *will not* record the proper time interval between them. He will record a dilated time interval t given by the time-dilation equation (Equation 28.1): $t = t_0 \Big/ \sqrt{1 \quad v^2/c^2}$, where v is the relative speed between the observer who measures t_0 and the observer who measures t.

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3. ***REASONING AND SOLUTION*** The earth spins on its axis once each day. To a person viewing the earth from an inertial reference frame in space, a clock at the equator would run more slowly than a clock at the north (magnetic) pole. The observer in the inertial reference frame is not in the rest frame of either clock, so he will measure a dilated time for both clocks. According to the time dilation equation (Equation 28.1): $t = t_0 \Big/ \sqrt{1 \quad v^2/c^2}$, where v is the relative speed between the observer who measures t_0 and the observer who measures t. Both clocks move around the earth's rotation axis with the same angular speed w as that of the earth. The linear speed of each clock is given by $v = rw$, where r is the distance from the clock to the rotation axis. The clock at the equator has the greater value of r, and, therefore, the greater linear speed v. According to Equation 28.1, the observer in the inertial reference frame will see the clock with the larger linear speed v register the longer time interval t. Therefore, the clock at the equator will appear to run slower when viewed by the inertial observer.

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4. ***REASONING AND SOLUTION***

a. You are standing at a railroad crossing watching a train go by. Both you and a passenger in the train are looking at a clock on the train. The passenger on the train is at rest relative to the clock; therefore, the passenger on the train measures the proper time interval.

b. A passenger in the train car is at rest relative to the car; therefore, the passenger on the train measures the proper length of the train car.

c. Since you are standing at the railroad crossing, you are at rest relative to the track; therefore, you measure the proper distance between the railroad ties under the track.

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5. ***REASONING AND SOLUTION*** There are tables that list data for the various particles of matter that physicists have discovered. Often, such tables list the masses of the particles in units of energy, such as MeV, rather than kilograms. This is possible because of the equivalence between mass and energy. The total energy E of a moving object is related to its mass m and its speed v by Equation 28.4: $E = mc^2 \Big/ \sqrt{1 \quad v^2/c^2}$. The mass of a particle listed in the table is actually the total energy of the particle when it is at rest (v = 0 m/s); the rest energy of an object of mass m is given by Equation 28.5: $E_0 = mc^2$.

---

6. ***REASONING AND SOLUTION*** When an object is accelerated from rest to a speed v, the object acquires kinetic energy in addition to its rest energy. The total energy E of the object is the sum of its rest energy E_0 and its kinetic energy KE. According to Equation 28.6, the kinetic energy of an object of mass m moving at speed v is $\text{KE} = E \quad E_0 = mc^2 \left(1 \Big/ \sqrt{1 \quad v^2/c^2}\right) \quad 1$. The work required to accelerate an object from rest to speed v is, from the work-energy theorem (Chapter 6), equal to the change in the kinetic energy of the object. Since the object begins at rest, its change in kinetic energy is equal to its final kinetic energy and is given by Equation 28.6.

According to Equation 28.6, it is easier to accelerate an electron to a speed that is close to the speed of light, compared to accelerating a proton to the same speed. This is because the mass of the electron is much less than that of the proton. Hence, the final kinetic energy of the proton, will be greater than that of the electron at the same speed v. Therefore, the proton will require more work to get it to the same speed as the electron.

---

7. ***REASONING AND SOLUTION*** Light travels in water at a speed of $2.26 \quad 10^8$ m/s. It is possible for a particle that has mass to travel through water at a faster speed than this. But objects that have mass *cannot reach the speed of light in a vacuum*, that is, they cannot attain a speed of $c = 3.00 \quad 10^8 \text{ m/s}$. It is the speed of light in a vacuum that is the ultimate speed, not the speed of light in a material like water.

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8. ***REASONING AND SOLUTION*** Two positive, electric charges separated by a finite distance have more mass than when they are infinitely far apart. The electric potential energy of the two-particle system is zero when the particles are infinitely far apart. Work is required to bring two positive charges in from infinity, so that there is a finite separation between them; therefore, the electric potential energy and consequently the total energy of the two-particle system increases as they are brought in from infinity. Consistent with the theory of special relativity, any increase in the total energy of the system, including a change in the electric potential energy, is equivalent to an increase in the mass of the system. Therefore, the two-particle system has more mass when the particles have a finite distance between them, than when they are infinitely far apart.

---

9. ***REASONING AND SOLUTION*** One system consists of two stationary electrons separated by a distance r. Another consists of a positron and an electron, both stationary and separated by the same distance r. A positron has the same mass as an electron, but it has a positive electric charge $+e$. Since the charges of unlike sign attract each other, and the charges of like sign repel each other, more work is required to bring the charges from infinity to assemble the system that consists of the two stationary electrons. Since more work is required to assemble the system of two stationary electrons, that system has the greater electric potential energy. Consistent with mass-energy equivalence predicted by the theory of special relativity, the system with the larger total energy will have the greater mass. Therefore, the system that consists of two stationary electrons has the greater mass.

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10. ***REASONING AND SOLUTION*** A parallel plate capacitor is initially uncharged. The capacitor is charged up by removing electrons from one plate and placing them on the other plate. Work and, therefore, energy is required to charge the capacitor. It is stored in the electric field between the plates of the charged capacitor in the form of electric potential energy. Since the charged capacitor has more electric potential energy than the uncharged capacitor, its also has more total energy than the uncharged capacitor. Therefore, from the equivalence of mass and energy, we can conclude that the charged capacitor has more mass.

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11. ***REASONING AND SOLUTION*** The speed limit on many interstate highways is 65 miles per hour. If the speed of light were 65 miles per hour, you would never be able to drive at the speed limit. When the speed, v, of an object is equal to the speed of light, c, the term $1\Big/\sqrt{1 \quad v^2/c^2}$ becomes infinitely large. Therefore, according to Equation 28.6, the kinetic energy of the object would be infinitely large. According to the work-energy theorem, an infinite amount of work would be required to increase the speed of the object to $v = c$. Since an infinite amount of work is not available, it would *not* be possible for the car to reach the speed limit.

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12. ***REASONING AND SOLUTION*** A person is approaching you in a truck that is traveling very close to the speed of light. This person throws a baseball toward you. Relative to the truck, the ball is thrown with a speed nearly equal to the speed of light, so the person on the truck sees the baseball move away from the truck at a very high speed. Yet you see the baseball move away from the truck very slowly.

The relative velocities in this problem are:

v_{BY} = velocity of the **B**aseball relative to **Y**ou
v_{BT} = velocity of the **B**aseball relative to the **T**ruck
v_{TY} = velocity of the **T**ruck relative to **Y**ou

These velocities are related by the velocity-addition formula (Equation 28.7),

$$v_{BY} = \frac{v_{BT} + v_{TY}}{1 + \frac{v_{BT} v_{TY}}{c^2}}$$

If both v_{BT} and v_{TY} are close to the speed of light, $v_{BY} \quad (c+c)/[1+c^2/c^2] = 2c/2 = c$, and we can conclude that v_{BY} is very close to c. Since both v_{BY} and v_{TY} are measured relative to the *same* inertial reference frame, namely your reference frame, you see the baseball leaving the truck at a relative speed that is the difference between the two speeds, that is, $v_{BY} - v_{TY}$. Since v_{BY} and v_{TY} are unequal, but close to the speed of light, the difference $v_{BY} - v_{TY}$ will be very small. Therefore, you see the baseball move away from the truck very slowly.

---

13. ***REASONING AND SOLUTION*** The quantities listed are (a) the time interval between two events, (b) the length of an object, (c) the speed of light in a vacuum, and (d) the relative speed between the observers.

a. The time interval between two events depends on the reference frame in which the time measurements are made. An observer in the rest frame of the two events will measure the proper time interval. In any other inertial reference frame in motion relative to the rest frame of the events, the time interval will be longer or dilated. Therefore, two observers in relative motion will not, in general, agree on the measured value of the time interval between two events.

b. The length of an object depends on the reference frame in which the length measurement is made. An observer in the rest frame of the object will measure the proper length of the object. In any other inertial reference frame in motion relative to the rest frame of the object, the length will be shorter or contracted in the direction of motion. Therefore, two observers in relative motion will not, in general, agree on the measured value of the length of the object.

c. According to the speed of light postulate, the speed of light in a vacuum will be the same in all inertial reference frames that are in motion relative to each other. Therefore, two observers will always measure the speed of light to be the same, regardless of the relative velocity between them.

d. Since there is no preferred inertial reference frame, two observers in inertial reference frames that are in motion relative to each other will always measure the same relative speed between them, regardless of the relative velocity.

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14. ***REASONING AND SOLUTION*** If the speed of light were infinitely large, instead of $3 \quad 10^8$ m/s, the effects of time dilation and length contraction would *not* be observable. According to the time dilation equation (Equation 28.1), $t = \quad t_0 \Big/ \sqrt{1 \quad v^2/c^2}$, where v is the relative speed between the observer who measures t_0 and the observer who measures t. According to the length contraction equation (Equation 28.2), $L = L_0\sqrt{1 \quad v^2/c^2}$, where v is the relative speed between the observer who measures L_0 and the observer who measures L. If c were infinite, then the factor v^2/c^2 would be equal to zero, $\sqrt{1 \quad v^2/c^2}$ would be equal to unity, and it would follow that $t = \quad t_0$ and $L = L_0$. In other words, the time interval for an event would be the same in all inertial reference frames, regardless of the relative speed between them. Similarly, the length of an object would be the same in all inertial reference frames, regardless of the relative speed between them. Therefore, if c were infinitely large, time dilation and length contraction would not be observable.

CHAPTER 29 | *PARTICLES AND WAVES*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A monochromatic light source emits photons of a single frequency. According to Equation 29.2, the energy, E, of a single photon is related to its frequency, f, by the relation $E = hf$, where h is Planck's constant.

   The photons emitted by a source of light do *not* all have the same energy. Since the photons do not all have the same energy, then, from Equation 29.2, we can conclude that the photons do not all have the same frequency. Therefore, the source is not monochromatic.

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2. ***REASONING AND SOLUTION*** According to the data given in Example 1, Chapter 24, the frequency of visible light ranges from $4.0 \quad 10^{14}$ Hz (red light) to $7.9 \quad 10^{14}$ Hz (violet light). According to Equation 29.2, the energy, E, of a photon is related to its frequency, f, by the relation $E = hf$, where h is Planck's constant. According to Equation 29.2, the energy of a photon is directly proportional to its frequency.

   a. The red-colored light bulb emits photons with the lowest frequency compared to light bulbs of other colors (orange, yellow, green, or blue); therefore, the red-colored light bulb emits photons with the lowest energy.

   b. The color blue appears next to violet in the continuous visible spectrum; therefore, the frequency of blue light is slightly smaller than that of violet, but greater than the frequency of other colors of the visible spectrum. Thus, the blue-colored light bulb emits photons with the highest frequency compared to the other light bulbs; therefore, the blue-colored light bulb emits photons with the greatest energy.

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3. ***REASONING AND SOLUTION*** A photon emitted by a higher-wattage red light bulb *does not* have more energy than a photon emitted by a lower-wattage red bulb. The wattage of a bulb describes the power output of a bulb. Since average power is defined as energy per unit time, the power output of a light bulb tells us the *rate* at which the light bulb produces energy. According to Equation 29.2, the energy, E, of a photon is related to its frequency, f, by the relation $E = hf$, where h is Planck's constant. Thus, the energy of a photon depends *only* on the frequency of the associated light wave. The frequency of red light is the same, regardless of the rate of energy production; therefore, all photons of red light have the same energy, regardless of the nature of their source.

   Remark: Since the higher-wattage bulb provides more energy per unit time than the lower-wattage bulb, we can conclude that the higher-wattage bulb produces *more photons* per unit time. All of the "red photons," however, have the same energy.

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4. ***REASONING AND SOLUTION*** When a sufficient number of visible light photons strike a piece of photographic film, the film becomes exposed. An X-ray photon is more energetic than a visible light photon. Yet, most photographic films are not exposed by the X-ray machines used at airport security checkpoints. Since a single X-ray photon is more energetic than a single photon of visible light, we can conclude that the number of X-ray photons per unit time emitted by airport security machines is much smaller than the number of visible light photons per unit time produced by normal lighting fixtures.

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5. ***REASONING AND SOLUTION*** When radiation strikes a metallic surface, a photon of the radiation can give up its energy to an electron in the metal. If the photon has enough energy to perform the minimum amount of work required to remove the electron from the metal, the electron can be emitted from the metal. The minimum amount of work required to remove an electron from a metal is called its *work function*. If the photon does not have enough energy to remove an electron, the energy of the photon is absorbed by the electron; the increase in energy is manifested as thermal motion.

   Radiation of a given wavelength causes electrons to be emitted from the surface of one metal (metal 1) but not from the surface of another metal (metal 2). We can conclude that the individual photons of the radiation have sufficient energy to do the work required to remove the electrons from the surface of metal 1, but not enough energy to remove the electrons from the surface of metal 2. Therefore, the work function of metal 1 must be less than the work function of metal 2.

---

6. ***REASONING AND SOLUTION*** In a photoelectric effect experiment, the intensity of the light is increased while the frequency is kept constant. The frequency is greater than the minimum frequency f_0, so that photoelectrons are emitted from the negatively charged metal plate (see Figure 29.4).

   Light intensity is the energy per second per unit area that crosses a transverse surface. According to Einstein's photon hypothesis (see Equation 29.2), a beam of light is composed of individual photons, each having an energy $E = hf$, where h is Planck's constant and f is the frequency of the light. From this viewpoint, the intensity of the light is equal to the energy of each photon multiplied by the number of photons per second per unit area crossing a transverse surface. An increase in the intensity of the light beam, therefore, corresponds to an increase in the number of photons per second that strikes the negatively charged metal plate. This will result in the emission of an increased number of photoelectrons per second at the negative plate.

   a. As the number of photoelectrons emitted per second from the negative plate increases, the number of photoelectrons collected per second at the positive plate increases. Since the current in the phototube is proportional to the number of photoelectrons per second collected at the positive plate, increasing the light intensity will cause the current in the phototube to increase.

   b. As discussed above, an increase in intensity will cause the number of photoelectrons emitted per second from the metal surface to increase.

c. According to Equation 29.3, the maximum kinetic energy that a photoelectron could have is given by $\mathrm{KE}_{\max} = hf \quad W_0$, where W_0 is the work function of the metal. Since the frequency is kept constant, and W_0 is a property of the metal that is independent of the intensity, an increase in intensity will have no effect on $\mathrm{KE}_{\max}$. The maximum kinetic energy that an electron could have remains the same.

d. The kinetic energy of an object of mass m moving non-relativistically with speed v can be expressed in terms of its momentum p as follows:

$$\mathrm{KE} = \tfrac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

Therefore, treating the electron classically, the maximum momentum that a photoelectron could have is $p_{\max} = \sqrt{2m\,(\mathrm{KE}_{\max})}$. Since the maximum kinetic energy that an electron could have remains the same as the intensity is increased at constant frequency, the maximum momentum that an electron could have remains the same as well.

e. According to Equation 29.8, the minimum de Broglie wavelength of an object of maximum momentum $p_{\max}$ is given by

$$l_{\min} = \frac{h}{p_{\max}}$$

Since the maximum momentum that a photoelectron could have remains the same as the intensity is increased at constant frequency, we can conclude that the minimum de Broglie wavelength that an electron could have also remains the same.

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7. ***REASONING AND SOLUTION*** As the result of a Compton scattering experiment, an electron is accelerated straight ahead in the same direction as that of the incident X-ray photon. Momentum conservation requires that the total initial momentum be equal to the total final momentum. The total initial momentum consists only of the forward momentum of the incoming photon, which we assume to be traveling in the $+x$ direction. The electron is initially at rest. Therefore, the direction of the final total momentum of the recoiling electron and the scattered photon must also point in the $+x$ direction. There can be no component of the final total momentum along the $+y$ or $-y$ direction. We know that the recoiling electron has momentum only in the $+x$ direction. Thus, in order for the y component of the total final momentum to be absent, the scattered photon must move either in the $+x$ or $-x$ direction. To distinguish between these two choices, it is necessary to refer to energy conservation, as well as momentum conservation.

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8. ***REASONING AND SOLUTION*** A photon can undergo Compton scattering from a molecule such as nitrogen, just as it does from an electron. However, the change in photon wavelength is much less than when an electron is scattered. To see why, let us examine Equation 29.7 which gives the difference between the wavelength l' of the scattered photon and the wavelength l of the incident photon in terms of the scattering angle q:

$l' \quad l = [h/(mc)](1 \quad \cos q)$, where h is Planck's constant, m is the mass of the target particle, and c is the speed of light in a vacuum. The mass of a nitrogen molecule is much greater than the mass of an electron. Therefore, the factor $h/(mc)$ will be much smaller if the target particle is a nitrogen molecule. Consequently, the change in the photon wavelength, $l' - l$, is much less than it is when the target particle is an electron.

---

9. ***REASONING AND SOLUTION*** When the speed of a particle with mass doubles, its momentum doubles, and its kinetic energy becomes four times greater. A photon, however, does not behave the same as a particle with mass. The momentum of a photon is $p = E/c$ (see the derivation of Equation 29.6); therefore, the energy of the photon can be written as $E = pc$. Clearly, when the momentum of a photon doubles, its energy also doubles. It does *not* become four times greater.

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10. ***REASONING AND SOLUTION*** When bright light is incident on the radiometer, photons strike both the black and the shiny surfaces. The photons are absorbed by the black surfaces and are reflected by the shiny surfaces. Linear momentum is transferred to the panels because of the photon collisions. Since the orientation of the light is arbitrary, photons will strike the panels at arbitrary angles of incidence. Only the component of the photon's momentum that is perpendicular to the face of the panel contributes to its motion. As discussed in Conceptual Example 3, the momentum transfer to the panels is a maximum for the shiny surfaces and is twice as large in magnitude than it is for the black surfaces. Following the reasoning of Conceptual Example 3, we can use the impulse-momentum theorem and Newton's third law to deduce that the force on the shiny side is greater than the force on the black side. Therefore, in bright light, the arrangement would spin in the direction from the shiny side toward the black side. The observed spinning is in the opposite direction. Thus, photon collisions with the panels *cannot* be the cause of the spinning.

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11. ***REASONING AND SOLUTION*** The linear momentum of an object of mass m traveling with velocity $\mathbf{v}$ is $\mathbf{p} = m\mathbf{v}$. The de Broglie wavelength of an object moving with momentum $\mathbf{p}$ is given by Equation 29.8: $l = h/p$, where h is Planck's constant.

A stone is dropped from the top of a building. As the stone falls, it is uniformly accelerated, and its velocity increases uniformly in the downward direction. Consequently, the linear momentum of the stone also increases uniformly in the downward direction. According to Equation 29.8, the de Broglie wavelength of an object or particle is inversely proportional to the magnitude of its linear momentum. Therefore, as the stone falls, the de Broglie wavelength of the stone must *decrease*.

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12. ***REASONING AND SOLUTION*** An electron and a neutron have different masses. According to Equation 29.8, $l = h / p$, the de Broglie wavelength of a particle is inversely proportional to the magnitude of its linear momentum. Therefore, if the electron and the neutron have different speeds such that the magnitudes of their respective momenta are the same, they will have the same de Broglie wavelength.

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13. ***REASONING AND SOLUTION*** Suppose that in Figure 29.1, the electrons are replaced with protons that have the same speed. The speed of the protons is the same as that of the electrons; however, the mass of a proton is greater than the mass of an electron. Therefore, the magnitude of the linear momentum of a proton is greater than the linear momentum of an electron traveling at the same speed. According to Equation 29.8, $l = h / p$, the de Broglie wavelength of the "proton beam" will be smaller than that of the "electron beam." Equation 27.1 gives the condition that must be satisfied by the angle q that locates the interference maxima in a Young's double-slit experiment: $\sin q = m l / d$, where m takes on integer values. According to Equation 27.1, the sine of the angle q is directly proportional to the wavelength of the incident beam. Since the proton beam has a smaller wavelength than the electron beam, the values of sin q that correspond to maxima will be smaller. Consequently, the difference in the corresponding values of q will be smaller, and the angular separation between the bright fringes will *decrease* when the electrons are replaced by protons.

CHAPTER 30 | *THE NATURE OF THE ATOM*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A tube is filled with atomic hydrogen at room temperature. Electromagnetic radiation with a continuous spectrum of wavelengths, including those in the Lyman, Balmer, and Paschen series, enters one end of the tube and leaves the other end. The exiting radiation is found to contain absorption lines.

   At room temperature, most of the atoms of atomic hydrogen contain electrons that are in the ground state ($n = 1$) energy level. Since the radiation contains a continuous spectrum of wavelengths, it contains photons with a wide range of energies ($E = hf = hc/l$). In particular, it will contain photons with energies that are equal to the energy difference between the atomic states in the Lyman series. When the radiation is incident on the atoms in the tube, these photons are absorbed by the electrons. When a photon, whose energy is equal to the energy difference for a transition in the Lyman series, is absorbed by a ground state electron, that electron will make a transition to a higher energy level. Every photon of this energy will be absorbed by a ground state electron and cause a transition. The wavelength of radiation that corresponds to that particular photon energy will, therefore, be removed from the radiation. When the radiation is analyzed, the wavelengths that correspond to transitions in the Lyman series will be absent. Since most of the atoms in the tube are in the ground state ($n = 1$), the electron populations in the $n = 2$ and $n = 3$ states are extremely small. Therefore, any absorption lines resulting from Balmer or Paschen transitions will be extremely weak. When the radiation is analyzed, the only predominant absorption lines in the exiting radiation will correspond to wavelengths in the Lyman series.

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2. ***REASONING AND SOLUTION*** Refer to the situation described in Question 1. Suppose the electrons in the atoms are mostly in excited states. Most of the electrons are in states with $n \geq 2$; therefore, Balmer and Paschen series transitions will occur, and the absorption lines in the exiting radiation will correspond to wavelengths in the Balmer and Paschen series. Since there are relatively few electrons in the ground state, only a relatively few number of photons that correspond to wavelengths in the Lyman series will be absorbed. Most of the "Lyman photons" will remain in the radiation; therefore, the exiting radiation will *not* contain absorption lines that correspond to wavelengths in the Lyman series. Although the absorption lines that correspond to transitions in the Lyman series are not present, there will be more absorption lines in the exiting radiation compared to the situation when the electrons are in the ground state, because absorption lines corresponding to both the Balmer and Paschen series will be present.

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3. ***REASONING AND SOLUTION*** According to Equation 30.13, the energy E_n of the nth atomic state in a Bohr atom is given by $E_n = -(13.6\text{ eV})Z^2/n^2$, where $n = 1, 2, 3, \ldots$, and Z is equal to the number of protons in the nucleus. The energy that must be absorbed by the electron to cause an upward transition from the initial state n_i to the final

state n_f is $E_{fi} = E_f - E_i$. Using Equation 30.13, we find that the required energy is $E_{fi} = -(13.6\text{ eV})Z^2\left(1/n_f^2 - 1/n_i^2\right)$, with n_i, n_f = 1, 2, 3, . . . and $n_f > n_i$. The energy required to ionize the atom from when the outermost electron is in the state n_i can be found by letting the value of n_f approach infinity. The ionization energy is then

$$E = -(13.6\text{ eV})Z^2\left(1/\quad -1/n_i^2\right) = -(13.6\text{ eV})Z^2\left(0 - 1/n_i^2\right) = (13.6\text{ eV})Z^2/n_i^2$$

From this expression, we see that the ionization energy is inversely proportional to the square of the principal quantum number n_i of the initial state of the electron; thus, less energy is required to remove the outermost electron in an atom when the atom is in an excited state. Therefore, when the atom is in an excited state, it is more easily ionized than when it is in the ground state.

---

4. ***REASONING AND SOLUTION*** In the Bohr model for the hydrogen atom, the closer the electron is to the nucleus, the smaller is the total energy of the atom. This is not true in the quantum mechanical picture of the hydrogen atom.

In the Bohr model, the *n*th orbit is a circle of definite radius r_n; every time that the position of the electron in this orbit is measured, the electron is found exactly a distance r_n from the nucleus. In contrast, according to the quantum mechanical picture, when an atom is in the *n*th state, the position of the electron is uncertain. Even though the atomic state is well defined by the principal quantum number *n*, the location of the electron is not definite. When the atom is in the *n*th state, the electron can sometimes be found close to the nucleus, while, at other times, it can be found far from the nucleus, or at some intermediate position. In the absence of any external magnetic fields, the energy of the state is determined by the value of *n*, and for a given *n*, the position of the electron is uncertain. Therefore, it is not correct to say that in the quantum mechanical picture of the hydrogen atom, the closer the electron is to the nucleus, the smaller is the total energy of the atom.

---

5. ***REASONING AND SOLUTION***

a. Consider two different hydrogen atoms. The electron in each atom is in an excited state. The Bohr model uses the same quantum number *n* to specify both the energy and the orbital angular momentum. According to the Bohr model, if an atom is in its *n*th state, the corresponding energy is $E_n = (13.6\text{ eV})Z^2/n^2$, where *n* = 1, 2, 3, . . ., and *Z* is equal to the number of protons in the nucleus (Equation 30.13). The integer *n* that specifies the energy also specifies the orbital angular momentum according to $L_n = nh/(2p)$, where *h* is Planck's constant (Equation 30.8). Therefore, according to the Bohr model, it is *not* possible for the electrons to have different energies, but the same orbital angular momentum.

b. The quantum mechanical model uses two <u>different</u> quantum numbers to specify the energy and the orbital angular momentum of an atomic electron. In the absence of magnetic fields, the energy is determined by the principal quantum number *n*, while the orbital

angular momentum is determined by the orbital quantum number ℓ. According to quantum mechanics, if an atom is in its nth state with orbital quantum number ℓ, the energy is identical to that of the Bohr model, and the magnitude of the orbital angular momentum is $h/(2p)\sqrt{\ell(\ell+1)}$, where $\ell = 0, 1, 2, \ldots, (n-1)$. Since $\ell = 0, 1, 2, \ldots, (n-1)$, different values of ℓ are compatible with the same value of n. For example, when $n = 3$, the possible values of ℓ are 0, 1, 2, and when $n = 4$, the possible values of ℓ are 0, 1, 2, 3. Thus, the electron in one of the atoms could have $n = 3$, $\ell = 2$, while the electron in the other atom could have $n = 4$, $\ell = 2$. Therefore, according to quantum mechanics, it is possible for the electrons to have different energies but have the same orbital angular momentum.

---

6. ***REASONING AND SOLUTION*** In the quantum mechanical picture of the hydrogen atom, the orbital angular momentum of the electron *may* be zero in any of the possible energy states. This is true because, if an atom is in its nth state with orbital quantum number ℓ, the magnitude of the orbital angular momentum is given by $h/(2p)\sqrt{\ell(\ell+1)}$, where $\ell = 0, 1, 2, \ldots, (n-1)$; therefore, for any value of n, the value of ℓ *may* be zero. However, when the hydrogenic electron is in the ground state, $n = 1$, and the only possible value for the orbital quantum number is $\ell = 0$. Therefore, when the electron is in the ground state, its orbital angular momentum *must* be zero.

---

7. ***REASONING AND SOLUTION***

a. According to the Pauli exclusion principle, no two electrons in an atom can have the same set of values for the four quantum numbers n, ℓ, m_ℓ, and m_s. For a given value of the orbital quantum number ℓ, the magnetic quantum number m_ℓ, can have $2\ell + 1$ possible values. Each of these values can be combined with two possible values for the spin quantum number m_s. Therefore, the maximum number of electrons that the ℓth subshell can hold is $2(2\ell + 1)$. All of the electrons in a 5g subshell have $n = 5$, and $\ell = 4$. Thus, the maximum number of electrons that can be contained in a 5g subshell is $2(2\ell + 1) = 2[2(4) + 1] = 18$. Therefore, a 5g subshell can contain at most 18 electrons and cannot contain 22 electrons.

b. Since 17 is less than 18, a 5g subshell could contain 17 electrons.

---

8. ***REASONING AND SOLUTION*** An electronic configuration for manganese ($Z = 25$) is written as $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^4\,4s^2\,4p^1$. According to Figure 30.17, the ground state electronic configuration for manganese is written as $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^5\,4s^2$. Clearly, the electronic configuration given in the question corresponds to *an excited state* in which the last electron in the 3d subshell has been excited to the 4p subshell.

---

9. ***REASONING AND SOLUTION*** Characteristic X-rays are produced when an electron with enough energy strikes the target atom and knocks one of the K-shell ($n = 1$) electrons

entirely out of the atom. An electron in one of the outer shells ($n \quad 2$) then drops into the K-shell and emits an X-ray photon in the process.

The ground state of hydrogen contains one electron in the $n = 1$ state, whereas, the ground state of helium contains two electrons in the $n = 1$ state. In both atoms, the ground state configuration contains only K-shell electrons. Even if one of the K-shell electrons were knocked out by an impinging electron, there are no electrons in higher levels to fall into the K-shell vacancy. Therefore, we would not expect hydrogen and helium atoms in their ground state to emit characteristic X-rays.

---

10. ***REASONING AND SOLUTION*** The characteristic lines in the X-ray spectrum depend on the target material. They occur at the wavelength value that corresponds to the wavelength of the X-ray photon that was emitted when an electron from the outer shell of the atom in the target material drops down to the K-shell. Since all identical atoms of the same element contain the same energy levels, the K and K lines will occur at the same wavelength values for all X-ray targets made of those atoms, regardless of the voltage applied across the X-ray tube.

The cutoff wavelength l_0, however, depends on the voltage V across the X-ray tube. According to Equation 30.17, $l_0 = (hc)/(eV)$; the cutoff wavelength is inversely proportional to the voltage across the tube. Therefore, when a smaller voltage is used to operate the tube, the cutoff wavelength increases.

We can conclude, therefore, that when the tube is operated at a smaller voltage, the characteristic lines should occur at the same wavelengths as they did at a higher voltage, and the cutoff wavelength should increase (i.e., shift to the right on the wavelength axis). The drawing in the text, shows the X-ray spectra produced by an X-ray tube when the tube is operated at two different voltages. The features of the drawing are consistent with the conclusions reached above.

---

11. ***REASONING AND SOLUTION*** Bremsstrahlung X-rays are produced when the electrons decelerate upon hitting the target. In the production of X-rays, it is possible to create Bremsstrahlung X-rays without producing the characteristic X-rays. The wavelengths of the characteristic lines depend on the target material; the cutoff wavelength, however, is independent of the target material and depends only on the energy of the impinging electrons. In order to produce Bremsstrahlung X-rays without producing the characteristic X-rays, the potential difference V applied to the tube must be chosen so that the cutoff wavelength is greater than the wavelengths of the characteristic X-rays for the target material. Therefore, using Equation 30.17, the value of V must be chosen so that $V = (hc)/(e l_0)$, where $l_0 > l_{K_a}$.

---

12. ***REASONING AND SOLUTION*** The short side of X-ray spectra ends abruptly at a cutoff wavelength l_0. This cutoff wavelength depends only on the energy of the impinging electrons. Regardless of the composition of the target, the incident electron cannot give up more than all of its kinetic energy when it is decelerated by the target. The kinetic energy that the electron has acquired upon reaching the target is given by eV (see Section 19.2),

where V is the potential difference that is applied to the X-ray tube. Therefore, the cutoff wavelength depends on the value of V, and not on the composition of the target material used in the X-ray tube.

---

13. ***REASONING AND SOLUTION*** A laser beam focused to a small spot can cut through a piece of metal. Light rays leaving the laser are nearly parallel, so the light can be focused to a very small spot. The intensity is the electromagnetic energy per unit time per unit area (see Equations 16.8 and 24.4). Therefore, when the beam is focused to a spot with a very small area, the intensity delivered by the beam is very large. For example, a pulsed ruby laser with a peak power of 10^8 W can be focused to a peak intensity of 10^{20} W/cm^2. This intensity is large enough to ionize air or burn a hole in a piece of metal.

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14. ***REASONING AND SOLUTION*** The energy of a photon of light of frequency f is given by Equation 29.2: $E = hf$. Since $c = lf$ for light, the energy of a photon can be written in terms of the wavelength l as $E = hc / l$. According to Table 26.2, the wavelength of green light is smaller than the wavelength of red light; therefore, the laser that produces green light emits photons that are more energetic than those emitted by the helium/neon laser.

CHAPTER 31 | *NUCLEAR PHYSICS AND RADIOACTIVITY*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** Isotopes are nuclei that contain the same number of protons, but a different number of neutrons. A material is known to be an isotope of lead, although the particular isotope is not known.

   a. The atomic number Z of an atom is equal to the number of protons in its nucleus. It is different for different elements, and all isotopes of the same element have the same atomic number. Therefore, if we know that a material is an isotope of lead, we can specify its atomic number ($Z = 82$).

   b. The various isotopes of the same element differ in the number of neutrons in the nucleus. Since we do not know which particular isotope of lead makes up the material, we cannot specify the neutron number.

   c. The atomic mass number is equal to the total number of protons and neutrons in the nucleus. Since we do not know which isotope of lead makes up the material, we do not know how many neutrons are in the nucleus. Therefore, we cannot specify the atomic mass number.

---

2. ***REASONING AND SOLUTION*** Two nuclei have different nucleon numbers A_1 and A_2. The two nuclei may or may not be isotopes of the same element. Two isotopes of the same element have the same number of protons in the nucleus, but differ in the number of neutrons. The nucleon number is equal to the total number of protons and neutrons in the nucleus. If the nucleon numbers differ only because the neutron numbers differ, then the two nuclei are isotopes of the same element. However, if the nucleon numbers differ because the nuclei contain a different number of protons, then the two nuclei are *not* isotopes of the same element (regardless of their respective neutron numbers). Therefore, two nuclei with different nucleon numbers are *not* necessarily isotopes of the same element.

---

3. ***REASONING AND SOLUTION*** Two nuclei that contain different numbers of protons and different numbers of neutrons can have the same radius. According to Equation 31.2, the approximate radius of the nucleus depends on the atomic mass number A and is given by $r \quad (1.2 \quad 10^{-15} \text{ m}) A^{1/3}$. The atomic mass number A is equal to the total number of protons and neutrons. Therefore, if two nuclei have different numbers of protons and different numbers of neutrons, but the sum of the protons and neutrons is the same for each nucleus, then the two nuclei have the same radius.

---

4. ***REASONING AND SOLUTION*** Using Figure 31.6, the following nuclei have been ranked in ascending order (smallest first) according to the binding energy per nucleon: ${}^{232}_{90}\text{Th}$, ${}^{184}_{74}\text{W}$, ${}^{31}_{15}\text{P}$, ${}^{59}_{27}\text{Co}$.

---

5. ***REASONING AND SOLUTION*** The general form for decay is

$$\underbrace{{}^{A}_{Z}\text{P}}_{\substack{\text{Parent}\\\text{nucleus}}} \qquad \underbrace{{}^{A\ \ 4}_{Z\ \ 2}\text{D}}_{\substack{\text{Daughter}\\\text{nucleus}}} \quad + \quad \underbrace{{}^{4}_{2}\text{He}}_{\substack{a\text{ particle}\\(\text{helium nucleus})}}$$

while the general form for the two types of beta decay are

[$^{-}$ decay]

$$\underbrace{{}^{A}_{Z}\text{P}}_{\substack{\text{Parent}\\\text{nucleus}}} \qquad \underbrace{{}^{A}_{Z+1}\text{D}}_{\substack{\text{Daughter}\\\text{nucleus}}} \quad + \quad \underbrace{{}^{0}_{-1}\text{e}}_{\substack{b^{-}\text{ particle}\\(\text{electron})}}$$

and

[$^{+}$ decay]

$$\underbrace{{}^{A}_{Z}\text{P}}_{\substack{\text{Parent}\\\text{nucleus}}} \qquad \underbrace{{}^{A\ \ 0}_{Z\ \ 1}\text{D}}_{\substack{\text{Daughter}\\\text{nucleus}}} \quad + \quad \underbrace{{}^{0}_{1}\text{e}}_{\substack{b^{+}\text{ particle}\\(\text{positron})}}$$

According to Equation 31.2, the approximate radius of the nucleus depends on the atomic mass number A and is given by $r \quad (1.2 \quad 10^{-15}\ \text{m})\,A^{1/3}$.

a. For decay, the mass number of the daughter nucleus is four nucleons smaller than that of the parent nucleus. Therefore, according to Equation 31.2, the daughter nucleus should have a smaller radius than the parent nucleus.

b. For both types of decay, the mass number of the daughter nucleus is identical to that of the parent nucleus. Therefore, according to Equation 31.2, the radius of the daughter nucleus should be the same as that of the parent nucleus.

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6. ***REASONING AND SOLUTION*** The general forms for decay, decay, and decay are, respectively:

$$\underbrace{{}^{A}_{Z}\text{P}}_{\substack{\text{Parent}\\\text{nucleus}}} \qquad \underbrace{{}^{A\ \ 4}_{Z\ \ 2}\text{D}}_{\substack{\text{Daughter}\\\text{nucleus}}} \quad + \quad \underbrace{{}^{4}_{2}\text{He}}_{a\text{ particle}}$$

$$\underbrace{{}^{A}_{Z}\text{P}}_{\substack{\text{Parent}\\\text{nucleus}}} \qquad \underbrace{{}^{A}_{Z+1}\text{D}}_{\substack{\text{Daughter}\\\text{nucleus}}} \quad + \quad \underbrace{{}^{0}_{-1}\text{e}}_{b\text{ particle}}$$

$$\underbrace{{}^{A}_{Z}\mathrm{P}^{*}}_{\substack{\text{Excited}\\ \text{energy state}}} \qquad \underbrace{{}^{A}_{Z}\mathrm{P}}_{\substack{\text{Lower}\\ \text{energy state}}} + \underbrace{g}_{g\text{ ray}}$$

As discussed in Section 31.1, the identity of an element depends on the number of protons in the nucleus. Clearly, in both decay and decay, the number of protons of the parent nucleus changes as the parent nucleus emits a charged particle. In contrast, in the process of decay, the parent nucleus emits only energy and the number of protons in the nucleus remains the same. Therefore, and decay result in transmutations, while decay does not.

---

7. ***REASONING AND SOLUTION*** In order for a nuclear decay to occur, it must bring the parent nucleus toward a more stable state by allowing the release of energy. The total mass of the decay products must be less than the mass of the parent nucleus. The difference in mass between the products and the parent nucleus represents an equivalent amount of energy that is given by Equation 28.5: $E_0 = mc^2$. This is the energy that is released during the decay.

As discussed in Example 4, uranium ${}^{238}_{92}\mathrm{U}$ decays into thorium ${}^{234}_{90}\mathrm{Th}$ by means of decay. The ${}^{238}_{92}\mathrm{U}$ nucleus never decays by just emitting a single proton, instead of an alpha particle. The hypothetical decay scheme for the unobserved decay, is given below along with the pertinent atomic masses:

$$\underbrace{{}^{238}_{92}\mathrm{U}}_{\substack{\text{Uranium}\\ 238.050\ 78\text{ u}}} \qquad \underbrace{{}^{237}_{91}\mathrm{Pa}}_{\substack{\text{Protactinium}\\ 237.051\ 14\text{ u}}} + \underbrace{{}^{1}_{1}\mathrm{H}}_{\substack{\text{Proton}\\ 1.007\ 83\text{ u}}}$$

The total mass of the decay products is $(237.051\ 14\text{ u}) + (1.007\ 83\text{ u}) = 238.058\ 97\text{ u}$. This is greater, not smaller, than the mass of the parent nucleus ${}^{238}_{92}\mathrm{U}$; therefore, the daughter nucleus is less stable than the parent nucleus. Consequently, the hypothetical decay never occurs.

---

8. ***REASONING AND SOLUTION*** According to Equation 31.5, if there are N_0 radioactive nuclei present at time $t = 0$, then the number N of radioactive nuclei present at time t is $N = N_0 e^{\ \ lt}$, where the decay constant l is given by Equation 31.6: $l = (\ln\ 2)/T_{1/2}$, where $T_{1/2}$ is the half-life of the nuclei. According to Equation 31.6, unstable nuclei with short half-lives have large decay constants. When l is large, the term $e^{\ \ lt}$ approaches zero even for very short times. Therefore, unstable nuclei with short half-lives typically have only a small or zero abundance after a short period of time.

---

9. ***REASONING AND SOLUTION*** The half-life of a radioactive isotope is the time required for one-half of the nuclei present to disintegrate. The disintegrations do not occur continuously at regular time intervals. Individual disintegrations occur randomly; we cannot predict *a priori* which particular nucleus will decay. Any given nucleus may decay at any time. The time may be much shorter than the half-life or it may be much longer than the half-life. Therefore, even though the half-life of indium $^{115}_{49}\text{In}$ is $4.41 \quad 10^{14}\ \text{yr}$, it is possible for any single nucleus in the sample to decay after only one second has passed.

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10. ***REASONING AND SOLUTION*** The activity of a radioactive sample is the number of disintegrations per second that occur. In other words, it is the magnitude of $N/\ t$. According to Equation 31.4, $N/\ t = \ lN$, where l is the decay constant that is characteristic of the isotope in question, and N is the number of radioactive nuclei that are present.

a. While each isotope has its own characteristic decay constant, the activity is proportional to the product lN, according to Equation 31.4. Therefore, two samples with different decay constants l, and different numbers of radioactive nuclei N, can have identical activities if the product lN is the same for each. Thus, it is possible for two different samples that contain different radioactive elements to have the same activity.

b. Since each isotope has its own characteristic decay constant l, two different samples of the same radioactive isotope have the same decay constant. If the two samples contain different numbers of radioactive nuclei, the product lN will be different for each sample. Therefore, they will have different activities.

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11. ***REASONING AND SOLUTION*** Radiocarbon dating utilizes the $^{14}_{6}\text{C}$ isotope that undergoes b^- decay with a half-life of 5730 yr. If the present activity of $^{14}_{6}\text{C}$ can be measured in a carbon-containing object, the age of the object can be determined. Example 11 shows how to determine the age of an object, once the present activity is known.

Consider the following objects, each about 1000 yr old: a wooden box, a gold statue, and some plant seeds. Since 1000 yr is of the same order of magnitude as the half-life of $^{14}_{6}\text{C}$, the change in activity should be significant, and the present activity should be easily measured. However, the sample must contain $^{14}_{6}\text{C}$ atoms. Since both the wooden box and the seeds contain $^{14}_{6}\text{C}$, radiocarbon dating can be used to determine their ages. If the gold statue is made of pure gold, it will not contain any $^{14}_{6}\text{C}$ atoms. Therefore, radiocarbon dating could not be applied to the gold statue.

---

12. ***REASONING AND SOLUTION*** Two isotopes have half-lives that are short relative to the age of the earth. Today, one of these isotopes (isotope A) is found in nature and the other (isotope B) is not. It is reasonable to conclude that the supply of isotope A has been replenished by some natural process that off-sets the radioactive decay, while the supply of isotope B has not been replenished. The replenishing of isotope A could occur, for example, as the result of the interaction of cosmic rays with the earth's upper atmosphere or the radioactive decay of some other isotope.

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13. ***REASONING AND SOLUTION*** In radiocarbon dating, the age of the sample is determined from the ratio of the present activity to the original activity. Suppose that there were a greater number of ${}^{14}_{6}\mathrm{C}$ atoms in a plant living 5000 yr ago than is currently believed. According to Equation 31.4: $N/\ \ t = \ \ lN$, and the original activity would be greater than is currently believed. A longer time would be required for the activity of a larger number of ${}^{14}_{6}\mathrm{C}$ nuclei to decrease to its present value. Therefore, the age of the seeds obtained from radiocarbon dating would be too small.

---

14. ***REASONING AND SOLUTION*** Tritium is an isotope of hydrogen and undergoes b^{-} decay with a half-life of 12.33 yr. Like carbon ${}^{14}_{6}\mathrm{C}$, tritium is produced in the atmosphere because of cosmic rays and can be used in a radioactive dating technique. The age cannot be significantly smaller or significantly larger than the half-life of the radioisotope. During a 700 yr period, 700/12.33 or roughly 57 half-lives would occur, and the activity of a given sample would decrease to an immeasurably small fraction of its initial value (on the order of 10^{-17}). Therefore, tritium dating could *not* be used to determine a reliable date for a 700 yr old sample.

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15. ***REASONING AND SOLUTION*** Because of radioactive decay, one element can be transmuted into another. Thus, a container of uranium ${}^{238}_{92}\mathrm{U}$ ultimately becomes a container of lead ${}^{206}_{82}\mathrm{Pb}$. According to Figure 31.16, the first step is the transmutation from uranium ${}^{238}_{92}\mathrm{U}$ to thorium ${}^{234}_{90}\mathrm{Th}$. This step has the greatest half-life, with a value of $4.5\ \ 10^{9}\ \mathrm{y}$. Therefore, we can conclude that the complete transmutation from uranium ${}^{238}_{92}\mathrm{U}$ to lead ${}^{206}_{82}\mathrm{Pb}$ would take at least billions of years.

---

16. ***REASONING AND SOLUTION*** According to Equation 21.2, the radius r of curvature for a particle with charge q and mass m, that moves perpendicular to a magnetic field of magnitude B, is given by

$$r = \frac{mv}{qB}$$

Thus, the ratio of the radii of curvature for the motion of the a particle and the b^- particle is

$$\frac{r_b}{r_a} = \frac{m_b v / q_b B}{m_a v / q_a B} = \frac{m_b / q_b}{m_a / q_a} = \frac{m_b}{m_a}\frac{q_a}{q_b}$$

The mass of the a particle is 1.0026 u, while the mass of a b^- particle (an electron) is

$$(9.109\ \ 389\ \ 731\ \ \ 10^{-31}\ \text{kg})\left(\frac{1.0073\ \text{u}}{1.6726\ \ \ 10^{-27}\ \text{kg}}\right) = 5.486\ \ \ 10^{-4}\ \text{u}$$

Since the magnitudes of the charge on the a and b^- particles are $2e$ and e, respectively, the ratio of the radii of curvature is

$$\frac{r_b}{r_a} = \left(\frac{5.486\ \ \ 10^{-4}\ \text{u}}{4.0026\ \text{u}}\right)\left(\frac{2e}{e}\right) = 2.7\ \ \ 10^{-4}$$

Therefore, the radius of curvature of the path of the b^- particle is smaller than the radius of curvature of the path of the a particle. Therefore, the trajectory of the b^- particle has a greater curvature than that of the a particle.

CHAPTER 32 *IONIZING RADIATION, NUCLEAR ENERGY, AND ELEMENTARY PARTICLES*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The RBE, or relative biological effectiveness, of a particular type of radiation compares the dose of that radiation needed to produce a certain biological effect to the dose of 200 keV X-rays needed to produce the same biological effect. The RBE depends on the nature of the ionizing radiation and the type of tissue that is being irradiated. Therefore, if two different types of radiation have the same RBE, this *does not* mean that each type delivers the same amount of energy to the tissue that it irradiates. Rather, it implies that the two different types of radiation are equally effective in producing the same biological effect on the same kind of tissue.

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2. ***REASONING AND SOLUTION*** When a dentist X-rays your teeth, a lead apron is placed over your chest and lower body. The purpose of this apron is to protect the tissues on your chest and lower body from the ionizing effect of the X-rays. The thickness of the lead is chosen so that the apron blocks the penetration of the X-rays.

---

3. ***REASONING AND SOLUTION***
a. The units "rads" and "grays" are used to express the *absorbed dose* of ionizing radiation. According to Equation 32.2, the absorbed dose is equal to the energy absorbed from the radiation per unit mass of the absorbing material. The gray (Gy) is the SI unit for absorbed dose and is equal to 1 J/kg. The relationship between rads and grays is: 1 rad = 0.01 gray.

b. The units "rads" and "roentgens" are *not* related. The rad is used to express the *absorbed dose*, and 1 rad = 0.01 J/kg. The roentgen is used to express the *exposure*, and 1 roentgen = $2.58 \quad 10^{-4}$ C/kg. The exposure expresses the ionizing abilities of X-rays or g rays in air, while the absorbed dose specifies the amount of energy absorbed per kilogram of absorbing material.

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4. ***REASONING AND SOLUTION*** In any nuclear reaction, the following quantities are conserved: (1) mass / energy; (2) electric charge, (3) linear momentum, (4) angular momentum, and (5) nucleon number.

a. ${}^{60}_{28}\text{Ni}\ (a, p)\ {}^{62}_{29}\text{Cu}$ corresponds to the reaction

$$ {}^{4}_{2}\text{He} + {}^{60}_{28}\text{Ni} \qquad {}^{62}_{29}\text{Cu} + {}^{1}_{1}\text{H} $$

The nucleon number for the reactants is 4 + 60 = 64, while the nucleon number of the products is 62 + 1 = 63. Therefore, this reaction does *not* occur, because it violates the conservation of nucleon number.

b. ${}^{27}_{13}\text{Al}\ (n,\ n)\ {}^{28}_{13}\text{Al}$ corresponds to the reaction

$$ {}^{1}_{0}\text{n} \quad + \quad {}^{27}_{13}\text{Al} \qquad\qquad {}^{28}_{13}\text{Al} \quad + \quad {}^{1}_{0}\text{n} $$

The nucleon number for the reactants is 1 + 27 = 28, while the nucleon number of the products is 28 + 1 = 29. Therefore, this reaction does *not* occur, because it violates the conservation of nucleon number.

c. ${}^{39}_{19}\text{K}\ (p,\ a)\ {}^{36}_{17}\text{Cl}$ corresponds to the reaction

$$ {}^{1}_{1}\text{H} \quad + \quad {}^{39}_{19}\text{K} \qquad\qquad {}^{36}_{17}\text{Cl} \quad + \quad {}^{4}_{2}\text{He} $$

The nucleon number for the reactants is 1 + 39 = 40, while the nucleon number of the products is 36 + 4 = 40. Nucleon number is conserved. The proton number on the left is $1 + 19 = 20$, while the proton number on the right is $17 + 2 = 19$. Since each proton carries a unit of charge +*e*, the total electric charge on the left is +20 *e*, while the total electric charge on the right is +19 *e*. Therefore, this reaction does *not* occur because it violates the conservation of electric charge.

---

5. ***REASONING AND SOLUTION*** Thermal neutrons and thermal protons have nearly the same speed. But thermal neutrons and thermal electrons have very different speeds. As discussed in Section 32.3, the adjective "thermal" applies to particles that have a kinetic energy of about 0.04 eV or less. According to Equation 6.2, the kinetic energy of a particle of mass *m* traveling with speed *v* is given by $\text{KE} = \frac{1}{2}mv^2$. Solving for *v* we have $v = \sqrt{2(\text{KE})/m}$. Therefore, for particles that have the same average kinetic energy, the speed of the particle is inversely proportional to the square root of its mass. Since the mass of the neutron is very nearly equal to the mass of a proton, thermal neutrons and thermal protons have nearly the same speed. The mass of the electron is much smaller than the mass of the neutron; therefore, thermal electrons will have a higher average speed than thermal neutrons.

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6. ***REASONING AND SOLUTION*** Consider the hypothetical fission of a nucleus of nucleon number $A = 25$ into two fragments of about equal mass ($A \quad 12.5$). The curve in Figure 32.8 shows that the binding energy of a nucleus with $A = 25$ is about 8.0 MeV per nucleon. Further inspection of Figure 32.8 shows that the binding energy of the fragments *has decreased* to about 6.5 MeV per nucleon. The difference between the binding energy per nucleon of the parent and the binding energy per nucleon of the daughter nucleus is about $6.5\ \text{MeV} - 8.0\ \text{MeV} = -1.5\ \text{MeV}$ of energy per nucleon, where the minus sign

indicates that the energy is absorbed. Therefore, a release of energy *would not* accompany the hypothetical fission reaction.

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7. ***REASONING AND SOLUTION*** In the fission of $^{235}_{92}\text{U}$, there are, on the average, 2.5 neutrons released per fission. The fact that the fission of $^{235}_{92}\text{U}$ releases on the average, 2.5 neutrons, makes it possible for a self-sustaining series of fissions to occur. Each neutron released by the fission process can initiate another fission reaction, resulting in the release of still more neutrons. These neutrons initiate more fission reactions, and the process results in a self-sustaining chain reaction.

   Suppose a *different* element is being fissioned and, on the average, only 1.0 neutron is released per fission. If a small fraction of the thermal neutrons absorbed by the nuclei does *not* produce a fission, a self-sustaining chain reaction *cannot* be produced using this element. Since each fission produces only one neutron, the product of a fission reaction can, at best, initiate only one more fission reaction. However, since a small fraction of the thermal neutrons absorbed by the nuclei does *not* produce a fission, at some point, the absorbed neutron will not produce a fission and, since there are no other neutrons to initiate a fission, the process will eventually stop.

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8. ***REASONING AND SOLUTION*** Much more energy is released when a uranium $^{235}_{92}\text{U}$ nucleus fissions than when a carbon $^{12}_{6}\text{C}$ atom in coal is combined with oxygen during the coal-burning process. It is true that the mass of the uranium nucleus is about 20 times greater than that of a carbon nucleus (235/12 = 20). But even 20 carbon atoms, when burned, release much less energy than a single $^{235}_{92}\text{U}$ nucleus releases when it fissions. In fact, they release about 2 million times less energy. Thus, the mass of coal needed is about 2 million times greater than the mass of uranium.

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9. ***REASONING AND SOLUTION*** Figure 32.8 shows that the binding energy of a nucleus with $A = 60$ is about 9.0 MeV per nucleon. If two such nuclei fuse, the daughter nucleus would have $A = 120$. According to Figure 32.8, the binding energy of the daughter nucleus *decreases* to about 8.3 MeV per nucleon. Consequently, the difference between the binding energy per nucleon of the initial nuclei and the binding energy per nucleon of the fused nucleus is about $8.3\text{ MeV} - 9.0\text{ MeV} = -0.7\text{ MeV}$ of energy per nucleon, where the minus sign indicates that the energy is absorbed. Therefore, the fusion of two nuclei, each with $A = 60$, would *not* release energy.

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10. ***REASONING AND SOLUTION*** Nuclear fission occurs when a massive nucleus captures a slow moving neutron and is split into two less-massive fragments that have a greater binding energy per nucleon than the massive parent nucleus. Since the binding energy per nucleon of the daughter nuclei is greater than that of the parent nucleus, energy is released. It appears primarily as kinetic energy of the fission products.

Nuclear fusion occurs when two low-mass nuclei with relatively small binding energies per nucleon combine or "fuse" into a single, more massive nucleus that has a greater binding energy per nucleon. Since the binding energy per nucleon of the more massive nucleus is greater than that of the parent nuclei, a substantial amount of energy is released. The energy released per nucleon in a fusion reaction is greater than that released in a fission reaction. Therefore, for a given mass of fuel, a fusion reaction will yield more energy than a fission reaction.

CHAPTER 19 *ELECTRIC POTENTIAL ENERGY AND THE ELECTRIC POTENTIAL*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The work done in moving a charge q_0 from A to B is given by Equation 19.4: $W_{AB} = q_0(V_A \quad V_B)$. For the cases in the drawing, we have

Case 1: $W_{AB} = q_0(150\text{ V} \quad 100\text{ V}) = q_0(50\text{ V})$

Case 2: $W_{AB} = q_0[25\text{ V} \quad (-25\text{ V})] = q_0(50\text{ V})$

Case 3: $W_{AB} = q_0[-10\text{ V} \quad (-60\text{ V})] = q_0(50\text{ V})$

The work done on the charge by the electric force is the same in each case.

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2. ***REASONING AND SOLUTION*** The potential at a point in space that is a distance r from a point charge q is given by Equation 19.6: $V = kq/r$. When more than one point charge is present, the total potential at any location is the algebraic sum of the individual potentials created by each charge at that location.

A positive point charge and a negative point charge have equal magnitudes. One of the charges is fixed to one corner of a square. If the other charge is placed opposite to the first charge along the diagonal of the square, then each charge will be the same distance L from the empty corners. The potential at each of the empty corners will be

$$V = \frac{k(+Q)}{L} + \frac{k(\quad Q)}{L} = 0$$

Therefore, if the potential at each empty corner is to be the same, then the charges must be placed at diagonally opposite corners as shown in the figure.

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3. ***REASONING AND SOLUTION*** The potential at a point in space that is a distance r from a point charge q is given by Equation 19.6: $V = kq/r$. When more than one point charge is present, the total potential at any location is the algebraic sum of the individual potentials created by each charge at that location.

Three point charges have identical magnitudes, but two of the charges are positive and one is negative. These charges are fixed to the corners of a square, one to a corner, as shown in the figure.

The potential at the empty corner is given by

$$V = \frac{q_1}{L} + \frac{q_2}{L\sqrt{2}} + \frac{q_3}{L}$$

Using q to denote the magnitude of each charge, we have the following possibilities:

q_1 and q_2 are positive: $$V = \frac{kq}{L} + \frac{kq}{L\sqrt{2}} \quad \frac{kq}{L} = \frac{kq}{L\sqrt{2}}$$

q_1 and q_3 are positive: $$V = \frac{kq}{L} \quad \frac{kq}{L\sqrt{2}} + \frac{kq}{L} = \frac{kq}{L}\left(2 \quad \frac{1}{\sqrt{2}}\right)$$

q_2 and q_3 are positive: $$V = \quad \frac{kq}{L} + \frac{kq}{L\sqrt{2}} + \frac{kq}{L} = \frac{kq}{L\sqrt{2}}$$

In each case, the potential at the empty corner is positive.

---

4. ***REASONING AND SOLUTION*** Four point charges of equal magnitude are placed at the corners of a square as shown in the figure at the right.

The electric field at the center of the square is the *vector sum* of the electric field at the center due to each of the charges individually. The potential at the center of the square is equal to the *algebraic sum* of the potentials at the center due to each of the charges individually.

All four charges are equidistant from the center (a distance r in the figure). If two diagonal charges have the same magnitude and sign, then the electric field at the center due to these two charges have equal magnitude and opposite directions. Their resultant is, therefore, zero. Thus, the electric field at the center will be zero if each diagonal pair of charges has the same magnitude and sign.

If a diagonal pair of charges has the same magnitude and sign, they will give rise to a non-zero potential at the center. Thus, the potential due to one diagonal pair of charges must cancel the potential due to the other diagonal pair of charges. This will be the case if the two pairs of diagonal charges have opposite signs.

Thus, both the electric field and the electric potential will be zero at the center of the square if all four charges have the same magnitude, q_1 and q_3 have the same sign, and q_2 and q_4 have the same sign, which is opposite to the signs of q_1 and q_3.

---

5. ***REASONING AND SOLUTION***
Positive charge is spread uniformly around a circular ring, as shown in the figure, with q representing the total amount of charge on the ring. Equation 19.6 gives the correct potential at points along the line perpendicular to the plane of the ring at its center. The equation does not, however, give the correct potential at points that do not lie on this line.

We can consider the ring to be composed of point charges of magnitude q spread uniformly around the ring so that $q = \quad q$. According to Equation 19.6, each "point charge" gives rise to a potential given by $V = k \quad q/r$ at a distance r from the "point charge." The potential due to the ring at any point in space is the algebraic sum of the potentials of all the "point charges". For any point along the line perpendicular to the plane of the ring at its center, the distance r is the same for all "point charges" on the ring. Therefore, for any point on that line, $V = \quad V = \quad k \quad q/r = kq/r$; thus, we obtain Equation 19.6. For points that are not on the line, the distance r will, in general, be different for each "point charge" that makes up the total charge q; therefore, the potential V at that point will not be given by Equation 19.6 with a single value for r.

---

6. ***REASONING AND SOLUTION*** The electric field at a single location is zero. This does not necessarily mean that the electric potential at the same place is zero. According to Equation 19.7, $E = - \quad V/ \quad s$. Therefore, the fact that **E** = 0 means that the potential gradient, $V/ \quad s$, is zero, and the potential difference does not change with distance. In other words, in the vicinity of a point where **E** is zero, the electric potential is constant, but not necessarily zero

To illustrate this fact, consider the situation of two identical point charges. The electric field at each location in the vicinity of the charges is the resultant of the electric field at that location due to each of the two charges. The magnitude of the field due to each charge is given by Equation 18.3: $E = kq/r^2$. At the point exactly halfway between the charges, along the line that joins them, r is the same for the field contribution from each charge; therefore, the magnitudes of the field contributions at this point are the same. Since the charges are identical, the directions of the field contributions at this point are in opposite directions. The following figure shows the situation for both positive and negative charges. The resultant of the two field contributions is zero. Thus, the electric field is zero exactly

halfway between the charges on the line that joins them. The electric potential at this point, however, is the algebraic sum of the potential at that point due to each individual charge. Therefore, the potential at the point exactly halfway between the charges is

$$V = \frac{kq}{r} + \frac{kq}{r} = \frac{2kq}{r}$$

which is clearly non-zero.

Positive point charges

Negative point charges

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7. ***REASONING AND SOLUTION*** An electric potential energy exists when two protons are separated by a certain distance. It is equal to the work that must be done by an external agent to assemble the configuration. Suppose that we imagine assembling the system, one particle at a time. If there are no other charges in the region, there are no existing electric fields; therefore, no work is required to put the first proton in place. That proton, however, gives rise to an electric field that fills the region. Its magnitude at a distance r from the proton is given by Equation 18.3, $E = ke/r^2$, where $+e$ is the magnitude of the charge on the proton. Since the region contains an electric field due to the first proton, there also exists an electric potential, and the external agent must do work to place the second proton at a distance d from the first proton. The electric potential energy of the final configuration is equal to the work that must be done to bring the second proton from infinity and place it at a distance d from the first proton. The electric potential at a distance d from the first proton is $V_{\text{proton}} = +ke/d$ (Equation 19.6). According to Equation 19.3, the electric potential energy of the final configuration is therefore

$$\text{EPE} = V_{\text{proton}}(+e) = +\frac{ke^2}{d}$$

a. If both protons are replaced by electrons, similar arguments apply. However, since the electron carries a negative charge ($-e$), the electric potential at a distance d from the first electron is $V_{\text{electron}} = \quad ke/d$. The electric potential energy of the final configuration is now given by

$$\text{EPE} = V_{\text{electron}}(\quad e) = \quad \frac{ke}{d}(\quad e) = +\frac{ke^2}{d}$$

Therefore, if both protons are replaced by electrons, the electric potential energy *remains the same*.

b. When only one of the protons is replaced by an electron, we find that

$$\text{EPE} = V_{\text{proton}}(\quad e) = \left(+\frac{ke}{d}\right)(\quad e) = \quad \frac{ke^2}{d}$$

Thus, when only one of the protons is replaced by an electron, the electric potential energy *decreases* from $+ke^2/d$ to $\quad ke^2/d$.

---

8. ***REASONING AND SOLUTION*** A proton (charge = +*e*) is fixed in place. An electron (charge = –*e*) is released from rest and allowed to collide with the proton. Since the electron is released from rest, its initial kinetic energy is zero. Since the electron is in the vicinity of the proton, it has electric potential energy. The initial total energy of the electron is, therefore, electric potential energy. After the electron is released, the potential energy decreases, while the kinetic energy increases. Assuming that the charges are point charges and that r_{i} is the initial separation between the charges, while r_{f} is the separation just before the collision, the conservation of energy gives

$$E_{\text{initial}} = E_{\text{final}}$$

$$\frac{ke^2}{r_{\text{i}}} = \frac{ke^2}{r_{\text{f}}} + \tfrac{1}{2}m_{\text{electron}}v^2$$

$$ke^2\left(\frac{1}{r_{\text{f}}} \quad \frac{1}{r_{\text{i}}}\right) = \quad (\text{EPE}) = \tfrac{1}{2}m_{\text{electron}}v^2$$

where $\quad (\text{EPE}) = ke^2\left(\frac{1}{r_{\text{f}}} \quad \frac{1}{r_{\text{i}}}\right)$ is the change in the electric potential energy.

Therefore, the speed of the electron when it collides with the proton is

$$v = \sqrt{\frac{2 \quad (\text{EPE})}{m_{\text{electron}}}} \tag{1}$$

When the roles of the electron and proton are interchanged and the same experiment repeated, similar arguments apply.

It can be shown that the speed of the proton when it collides with the electron is

$$v = \sqrt{\frac{2 \quad (\text{EPE})}{m_{\text{proton}}}} \tag{2}$$

Since the mass of the electron is much less than the mass of the proton, we can conclude from Equations 1 and 2 that the electron is traveling faster when the collision occurs.

---

9. ***REASONING AND SOLUTION*** The electric potential is constant throughout a given region of space. Since the potential does not change from point to point in this region, the potential gradient, $V/\ s$, is zero throughout this region. From Equation 19.7, we see that the electric field is $E = \ V/\ s$ and must also be zero throughout this region.

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10. ***REASONING AND SOLUTION*** In a region where the electric field is constant, we see from Equation 19.7, $E = \ V/\ s$, that the potential gradient is constant. In other words, the rate at which the electric potential changes with distance in the region is constant. The potential, therefore, is not zero, but varies from point to point in a linear fashion with distance.

---

11. ***REASONING AND SOLUTION*** A positive charge is placed in an electric field. At each location in the electric field, the field is perpendicular to an equipotential surface. Therefore, in order for the charge to experience a constant electric potential, it must be moved along a path that is perpendicular to the electric field.

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12. ***REASONING AND SOLUTION*** The electric potential at any location in the vicinity of the charges is given by

$$V = \frac{k(+q)}{r_+} + \frac{k(\ \ q)}{r}$$

where r_+ is the distance from the positive charge to the point in question and r_- is the distance from the negative charge to the point in question. All points for which $r_+ = r_-$ correspond to points where the potential is zero, according to the above expression. Such points would lie on a plane that is midway between the two charges. Therefore, we can conclude that line *B* in the figure is the edge-on view of an equipotential surface.

As stated in the text, the electric field created by a group of charges is everywhere perpendicular to the associated equipotential surfaces. Lines *A* and *C* can be ruled out as possible edge-on views of equipotential surfaces, because they are not perpendicular to the electric field lines that would arise from the dipole configuration (see Figure 19.15).

---

13. ***REASONING AND SOLUTION*** A positive test charge is moved along the line between two identical point charges.

a. If the two identical point charges are positive, then the test charge experiences a repulsive force from each of the two fixed charges. Halfway between the charges, the resultant of these two repulsive forces will be zero since the distance from the test charge to either fixed charge is the same. As the test charge is moved from the midpoint, it will feel a net repulsive force which tends to push the charge back to the midpoint. The work done by this repulsive force in pushing the positive charge back to the midpoint is positive. According to the discussion in Section 19.4, the charge therefore goes from a higher potential (away from the midpoint) toward a lower potential (at the midpoint). Thus, with regard to the electric potential, the midpoint is analogous to the bottom of a valley.

b. If the two identical point charges are negative, then the test charge experiences an attractive force from each of the two fixed charges. Halfway between the charges, the resultant of these two attractive forces will be zero since the distance from the test charge to either fixed charge is the same. As the test charge is moved from the midpoint, it will feel a net attractive force which tends to pull the charge away from the midpoint. The work done by this attractive force in pulling the positive charge away from the midpoint is positive. According to the discussion in Section 19.4, the charge therefore goes from a higher potential (at the midpoint) toward a lower potential (away from the midpoint). Thus, with regard to the electric potential, the midpoint is analogous to the top of a hill.

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14. ***REASONING AND SOLUTION*** A negative test charge is moved along the line between two identical point charges.

a. If the two identical point charges are positive, then the test charge experiences an attractive force from each of the two fixed charges. Halfway between the charges, the resultant of these two attractive forces will be zero since the distance from the test charge to either fixed charge is the same. As the test charge is moved from the midpoint, it will feel a net attractive force which tends to pull the charge away from the midpoint. The work done by this attractive force in pulling the test charge away from the midpoint is positive. According to the discussion in Section 19.4, the charge therefore goes from a higher potential (at the midpoint) toward a lower potential (away from the midpoint). Thus, with regard to the electric potential, the midpoint is analogous to the top of a hill.

b. If the two identical point charges are negative, then the test charge experiences a repulsive force from each of the two fixed charges. Halfway between the charges, the resultant of these two repulsive forces will be zero since the distance from the test charge to either fixed charge is the same. As the test charge is moved from the midpoint, it will feel a net repulsive force which tends to push the charge toward the midpoint. The work done by this repulsive force in pushing the test charge back to the midpoint is positive. According to the discussion in Section 19.4, the charge therefore goes from a higher potential (away from the midpoint) toward a lower potential (at the midpoint). Thus, with regard to the electric potential, the midpoint is analogous to the bottom of a valley.

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15. ***REASONING AND SOLUTION*** According to Equation 19.3, the electric potential energy is $\text{EPE} = q_0 V$, where V is the potential relative to a zero value at infinity and q_0 is the charge. For a given value of the potential, the electric potential energy depends on the value of the charge. Thus, the electric potential energy is not the same for every charge placed at the point in question.

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16. ***REASONING AND SOLUTION*** Since both particles are released from rest, their initial kinetic energies are zero. They both have electric potential energy by virtue of their respective positions in the electric field between the plates. Since the particles are oppositely charged, they move in opposite directions toward opposite plates of the capacitor. As they move toward the plates, the particles gain kinetic energy and lose potential energy. Using $(\text{EPE})_0$ and $(\text{EPE})_\text{f}$ to denote the initial and final electric potential energies of the particle, respectively, we find from energy conservation that

$$(\text{EPE})_0 = \tfrac{1}{2} m_{\text{particle}} v_\text{f}^2 + (\text{EPE})_\text{f}$$

The final speed of each particle is given by

$$v_\text{f} = \sqrt{\frac{2\left[(\text{EPE})_0 \quad (\text{EPE})_\text{f}\right]}{m_{\text{particle}}}}$$

Since both particles travel through the same distance between the plates of the capacitor, the change in the electric potential energy is the same for both particles. Since the mass of the electron is smaller than the mass of the proton, the final speed of the electron will be greater than that of the proton. Therefore, the electron travels faster than the proton as the particles move toward the respective plates. The electron, therefore, strikes the capacitor plate first.

---

17. ***REASONING AND SOLUTION*** A parallel plate capacitor is charged up by a battery. The battery is then disconnected, but the charge remains on the plates. The plates are then pulled apart.

a. The capacitance of the system is given by Equation 19.10 with $k = 1$, $C = \epsilon_0 A / d$, where A is the area of either plate and d is the distance between the plates. Since d increases when the plates are pulled apart and A remains the same, the capacitance *decreases*.

b. According to Equation 19.8, the potential difference between the plates of the capacitor is given by $V = q / C$. Since q remains the same and C decreases, the potential difference V must *increase*.

c. The magnitude of the electric field between the plates is given by $E = V / d$. Since $V = q / C$, and $C = \epsilon_0 A / d$, the electric field between the plates is given by

$$E = \frac{(q / C)}{d} = \frac{q(d / \epsilon_0 A)}{d} = \frac{q}{\epsilon_0 A}$$

Since both q and A remain the same, the electric field *remains the same*.

d. The energy stored in the capacitor is given by Equation 19.11, $\text{Energy} = (1/2)CV^2$. Since $V = q/C$, we have

$$\text{Energy} = \tfrac{1}{2}CV^2 = \tfrac{1}{2}C\left(\frac{q}{C}\right)^2 = \tfrac{1}{2}\frac{q^2}{C}$$

Since q remains the same and C decreases, the electric potential energy stored by the capacitor *increases*. This increase will be equal to the work that is done by the external agent in pulling the plates apart.

CHAPTER 20 | *ELECTRIC CIRCUITS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION***
   a. When S_1 is set to position A, current can flow from the generator only along the path that requires S_2 to be set to position A. Hence, the light will be on when S_2 is in position A.

   b. When S_1 is set to position B, current can flow from the generator only along the path that requires S_2 to be set to position B. Hence, the light will be on when S_2 is in position B.

---

2. ***REASONING AND SOLUTION*** When an incandescent light bulb is turned on, the tungsten filament becomes white hot. Since the voltage is constant, the power delivered to the light bulb is given by Equation 20.6c: $P = V^2 / R$. From Equation 20.5, $R = R_0 [1 + a(T \quad T_0)]$, where a is the temperature coefficient of resistivity and is a positive number. Thus, as the filament temperature increases, the resistance of the wire increases, and as the filament heats up, the power delivered to the bulb decreases.

---

3. ***REASONING AND SOLUTION*** Two materials have different resistivities. Two wires of the same length are made, one from each of the materials. The resistance of each wire is given by Equation 20.3: $R = rL / A$, where r is the resistivity of the wire material, and L and A are, respectively, the length and cross-sectional area of the wire. Even when the wires have the same length, they may have the same resistance, if the cross-sectional areas of the wires are chosen so that the ratio rL / A is the same for each.

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4. ***REASONING AND SOLUTION*** The resistance of a wire is given by Equation 20.3: $R = rL / A$, where r is the resistivity of the wire material, L is the length of the wire, and A is its cross-sectional area. Since the cross-sectional area is proportional to the square of the diameter, a doubling of the diameter causes the cross-sectional area to be increased four-fold. From Equation 20.3, we see that doubling both the diameter and length causes the resistance of the wire to be reduced by a factor of 2.

---

5. ***REASONING AND SOLUTION*** One electrical appliance operates with a voltage of 120 V, while another operates with a voltage of 240 V. The power used by either appliance is given by Equation 20.6c: $P = V^2 / R$. Without knowing the resistance R of each appliance, no conclusion can be reached as to which appliance, if either, uses more power.

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6. ***REASONING AND SOLUTION*** Two light bulbs are designed for use at 120 V and are rated at 75 W and 150 W. The power used by either bulb is given by Equation 20.6c: $P = V^2 / R$. We see from Equation 20.6c that, at constant voltage, the power used by a bulb is inversely proportional to the resistance of the filament. Therefore, the filament resistance is greater for the 75-W bulb.

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7. ***REASONING AND SOLUTION*** Rather than state how many watts of power an appliance uses, appliance instructions often give statements such as "10 A, 120 V" instead. This statement gives the current used by the appliance for a specific voltage. It provides all the information necessary to determine the power usage of the appliance from Equation 20.6a: $P = IV$. Thus, the "current-voltage" rating given in the instructions is equivalent to a statement of the power consumption of the appliance.

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8. ***REASONING AND SOLUTION*** When the switch is initially closed, a current appears in the circuit, because charges flow through the heater wire, the bimetallic strip, the contact point, and the light bulb. The bulb glows in response. As charges flow through the heater wire, it becomes hot thereby heating the bimetallic strip. As the strip is heated, the brass, having a larger coefficient of thermal expansion, expands more than the steel. Thus, the bimetallic strip bends into an arc away from the contact point, and the electric current drops to zero because the charges no longer have a continuous path along which to flow. The bulb ceases to glow. Since there is no longer a current, the resistance wire, and hence, the bimetallic strip begin to cool down to room temperature. As the bimetallic strip cools, it bends back to its initial position, as shown in the drawing in the text. When it reaches room temperature it will again touch the contact point, and a current will begin to flow again. The wire and the bimetallic strip will become hot and the bimetallic strip will bend away from the contact point and the current will again drop to zero. This cycle will continue as long as the switch remains closed. Therefore, as long as the switch remains closed, the bulb will flash on and off.

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9. ***REASONING AND SOLUTION*** The power rating of a 1000-W heater specifies the power consumed when the heater is connected to an ac voltage of 120 V. The power consumed by the heater is given by Equation 20.6c: $P_1 = V^2 / R$. When two of these heaters are connected in series, the equivalent resistance of the combination is $R + R = 2R$. The power consumed by two of the heaters connected in series is, therefore, $P_2 = V^2/(2R) = P_1/2 = (1000\text{ W})/2 = 500\text{ W}$.

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10. ***REASONING AND SOLUTION*** A number of light bulbs are to be connected to a single electrical outlet. The bulbs will provide more brightness when they are connected in such a way that their power output is greatest. Since the voltage at the outlet is constant, the power delivered by the bulbs is given by Equation 20.6c: $P = V^2 / R$, where R is the equivalent resistance of the combination of light bulbs. When the light bulbs are connected in series, their equivalent resistance is given by Equation 20.16: $R_S = R_1 + R_2 + R_3 + \ldots$ When the

light bulbs are connected in parallel, their equivalent resistance is given by Equation 20.17: $\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots$ Clearly, the equivalent resistance will be less when the bulbs are connected in parallel. From Equation 20.6c, we can conclude that the power output will be greatest, and the light bulbs will provide more brightness, when they are connected in parallel.

---

11. ***REASONING AND SOLUTION*** A car has two headlights. The filament of one burns out. However, the other headlight stays on. We can immediately conclude that the bulbs are *not* connected in series. The figure below shows the series arrangement of two such bulbs.

When the bulbs are connected in series, charges must flow through the filaments of *both* lights in order to have a complete circuit. Since the filament of the second bulb is burned out, charges will not be able to flow around the circuit, and neither headlight will stay on.

On the other hand, if the bulbs are connected in parallel, as shown at the right, the current will split at the junction *J*. Charges will be able to flow through the branch of the circuit that contains the good bulb, and that headlight will stay on. Notice that the order of the bulbs does not matter in either case. The results are the same.

---

12. ***REASONING AND SOLUTION*** When two or more circuit elements are connected in series, they are connected such that the same electric current flows through each element. When two or more circuit elements are connected in parallel, they are connected such that the same voltage is applied across each element.

The circuit in Figure (*a*) can be shown to be a combination of series and parallel arrangements of resistors. The circuit can be redrawn as shown below.

We can see in the redrawn figure that the current through resistors 2 and 3 is the same; therefore, resistors 2 and 3 are in series and can be represented by an equivalent resistance 23 as shown in the following drawing.

The voltage across resistance 23 and resistor 4 is the same, so these two resistances are in parallel; they can be represented by an equivalent resistance 234. The current through resistance 234 is the same as that through resistor 7, so resistance 234 is in series with resistor 7; they can be represented by an equivalent resistance 2347 as shown in the figure below.

The voltage across 2347 is the same as that across resistor 5; therefore, resistance 2347 is in parallel with resistor 5. They can be represented by an equivalent resistance 23475. Similarly, resistance 23475 is in series with resistor 8, giving an equivalent resistance 234758. Resistance 234758 is in parallel with resistor 6, giving an equivalent resistance 2347586.

Finally, the current through resistor 1 and resistance 2347586 is the same, so they are in series as shown at the right.

The circuit in Figure (*b*) can also be shown to be a combination of series and parallel arrangements of resistors. Since both ends of resistors 2 and 3 are connected, the voltage across resistors 2 and 3 is the same. The same statement can be made for resistors 4 and 5, and resistors 6 and 7. Therefore, resistor 2 is in parallel with the resistor 3 to give an equivalent resistance labeled 23. Resistor 4 is in parallel with resistor 5 to give an equivalent resistance 45, and resistor 6 is in parallel with resistor 7 to give an equivalent resistance 67. From the right-hand portion of the drawing below, it is clear that the resistances 23, 45, and 67 are in series with resistor 1.

The drawing at the right shows the circuit in Figure (*c*). No such simplifying arguments can be made for this circuit. No two resistors carry the same current; thus, no two of the resistors are in series. Furthermore, no two resistors have the same voltage applied across them; thus, no two of the resistors are in parallel. Circuit (*c*) contains resistors that are neither in series nor in parallel.

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13. ***REASONING AND SOLUTION*** One way is to make two combinations that consist of two resistors in parallel. The resistance of each combination is

$$R_p = [(1/R) + (1/R)]^{-1} = R/2$$

These two combinations can then be placed in series, as shown at the right. The equivalent resistance of the series combination is

$$R_s = (R/2) + (R/2) = R$$

A second way is to make two combinations that consist of two resistors in series. The resistance of each combination is

$$R_s = R + R = 2R$$

These two combinations can then be placed in parallel, as shown at the right. The equivalent resistance of the parallel combination is

$$R_p = [1/(2R) + 1/(2R)]^{-1} = R$$

---

14. ***REASONING AND SOLUTION*** An ammeter is used to measure the current in a particular branch of a circuit, while a voltmeter is used to measure the voltage across two points in a circuit. An ideal ammeter must have a low resistance so that its presence in the circuit does not affect the current measurement. Conversely, an ideal voltmeter must have a large resistance so that it does not draw current and change the voltage between the two points in question.

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15. ***REASONING AND SOLUTION*** A voltmeter is inadvertently mistaken for an ammeter and placed in a circuit. A voltmeter is a high resistance instrument. Since an ammeter is placed in series with the circuit, we would be placing a large resistance in series with the circuit, and the resistance of the circuit would greatly increase. From Ohm's law, $V = IR$, the current in the circuit would drop markedly.

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16. ***REASONING AND SOLUTION***
From Ohm's law, $V = IR$, 1 ohm = 1 volt/ampere = 1 volt/(coulomb/second). From the definition of capacitance, $q = CV$, 1 farad = 1 coulomb/volt. Then

$$1 \text{ ohm} \times \text{farad} = \frac{1 \text{ volt}}{(\text{coulomb/second})} \times \frac{1 \text{ coulomb}}{\text{volt}} = \frac{1 \text{ volt} \times \text{second}}{\text{coulomb}} \times \frac{1 \text{ coulomb}}{\text{volt}} = 1 \text{ second}$$

CHAPTER 21 | *MAGNETIC FORCES AND MAGNETIC FIELDS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** Magnetic field lines, like electric field lines, never intersect. When a moving test charge is placed in a magnetic field so that its velocity vector has a component perpendicular to the field, the particle will experience a force. That force is perpendicular to both the direction of the field and the direction of the velocity. If it were possible for magnetic field lines to intersect, then there would be a different force associated with each of the two intersecting field lines; the particle could be pushed in two directions. Since the force on a particle always has a unique direction, we can conclude that magnetic field lines can never cross.

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2. ***REASONING AND SOLUTION*** If you accidentally use your left hand, instead of your right hand, to determine the direction of the magnetic force on a positive charge moving in a magnetic field, the direction that you determine will be exactly opposite to the correct direction.

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3. ***REASONING AND SOLUTION*** A charged particle, passing through a certain region of space, has a velocity whose magnitude and direction remain constant.

   a. If it is known that the external magnetic field is zero everywhere in the region, we can conclude that the electric field is also zero. Any charged particle placed in an electric field will experience a force given by $\mathbf{F} = q\mathbf{E}$, where q is the charge and $\mathbf{E}$ is the electric field. If the magnitude and direction of the velocity of the particle are constant, then the particle has zero acceleration. From Newton's second law, we know that the net force on the particle is zero. But there is no magnetic field and, hence, no magnetic force. Therefore, the net force is the electric force. Since the electric force is zero, the electric field must be zero.

   b. If it is known that the external electric field is zero everywhere, we *cannot* conclude that the external magnetic field is also zero. In order for a moving charged particle to experience a magnetic force when it is placed in a magnetic field, the velocity of the moving charge must have a component that is perpendicular to the direction of the magnetic field. If the moving charged particle enters the region such that its velocity is parallel or antiparallel to the magnetic field, it will experience no magnetic force, even though a magnetic field is present. In the absence of an external electric field, there is no electric force either. Thus, there is no net force, and the velocity vector will not change in any way.

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4. ***REASONING AND SOLUTION*** Suppose that the positive charge in Figure 21.10b were launched from the south pole toward the north pole, in a direction opposite to the magnetic field. Regardless of the strength of the magnetic field, the particle will always reach the

north pole. Since the charge is launched directly opposite to the magnetic field, its velocity will be antiparallel to the field and have no component perpendicular to the field. Therefore, there will be no magnetic force on the particle. The particle will move at constant velocity from the south pole of the magnet to the north pole.

---

5. ***REASONING AND SOLUTION*** Since the paths of the particles are perpendicular to the magnetic field, we know that the velocities of the particles are perpendicular to the field. Since the velocity of particle #2 is perpendicular to the magnetic field and it passes through the field undeflected, we can conclude that particle #2 is neutral.

Particles #1 and #3 move in circular paths. The figure at the right shows the direction of the (centripetal) magnetic force that acts on the particles. If the fingers of the right hand are pointed into the page so that the thumb points in the direction of motion of particle #1, the palm of the hand points toward the center of the circular path traversed by the particle. We can conclude, therefore, from RHR-1 that particle #1 is positively charged. If the fingers of the right hand are pointed into the page so that the thumb points in the direction of motion of particle #3, the palm of the hand points away from the center of the circular path traversed by the particle. We conclude, therefore, from RHR-1 that particle #3 is negatively charged.

---

6. ***REASONING AND SOLUTION*** Three particles have identical charges and masses. They enter a constant magnetic field and follow the paths as shown. The magnitude of the magnetic force on each particle is given by Equation 21.1 as $F = q_0 (v \sin q) B$, where q_0 is the magnitude of the charge on the particle, v is the speed of the particle and B is the magnitude of the magnetic field. Since the particle paths are perpendicular to the magnetic field, $q = 90°$ and $\sin q = 1$, so that $F = q_0 vB$. This magnetic force supplies the centripetal force that is necessary for the particles to move on the observed circular paths, so $q_0 vB = mv^2 / r$, where m is the mass of the particle and r is the radius of its circular path. Solving for v, we find $v = q_0 Br / m$. Since all three particles have the same charge and mass, we can conclude that the speed of the particle is directly proportional to the radius of the particle's path. Therefore, particle #1 is traveling the fastest, while particle #2 is traveling the slowest.

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7. ***REASONING AND SOLUTION*** A proton follows the path shown in Figure 21.12. The magnitude of the magnetic force on the proton is given by Equation 21.1 as $F = q_0 (v \sin q) B$, where q_0 is the magnitude of the charge on the proton, v is the speed of the proton and B is the magnitude of the magnetic field. Since the proton's path is perpendicular to the magnetic field, q = 90° and sin q = 1, so that $F = q_0 vB$. This magnetic force supplies the centripetal force that is necessary for the proton to move on the observed circular path, so $q_0 vB = m_{\text{proton}} v^2 / r$, where m is the mass of the particle and r is the radius of its circular path. The magnitude of the magnetic field is, therefore, $B = m_{\text{proton}} v / (q_0 r)$.

If we want an electron to follow exactly the same path, we must adjust the magnetic field. The magnitude of the charge on the electron is the same as that on the proton; however, the electron is negatively charged and the proton is positively charged. As stated in the text, the direction of the force on a negative charge is *opposite* to that predicted by RHR-1 for a positive charge. Therefore, the direction of the magnetic field must be reversed. In order for the electron to travel with the same speed v in a circular path of the same radius r, the magnitude of the magnetic field must be changed to the value $B = m_{\text{electron}} v / (q_0 r)$.

Thus, the electron will travel in the same path as the proton in Figure 21.12 if the direction of the magnetic field is reversed and the magnitude of the magnetic field is reduced by a factor $(m_{\text{electron}} / m_{\text{proton}})$.

---

8. ***REASONING AND SOLUTION*** The drawing shows a top view of four interconnected chambers. A negative charge is fired into chamber 1. By turning on separate magnetic fields in each chamber, the charge is made to exit from chamber 4.

a. In each chamber the path of the particle is one- quarter of a circle. The drawing at the right also shows the direction of the centripetal force that must act on the particle in each chamber in order for the particle to traverse the path. The charged particle can be made to move in a circular path by launching it into a region in which there exists a magnetic field that is perpendicular to the velocity of the particle.

Using RHR-1, we see that if the palm of the right hand were facing in the direction of **F** in chamber 1 so that the thumb points along the path of the particle, the fingers of the right hand must point out of the page. This is the direction that the magnetic field must have to make a *positive* charge move along the path shown in chamber 1. Since the particle is *negatively* charged, the field must point opposite to that direction or into the page.

Similar reasoning using RHR-1, and remembering that the particle is negatively charged, leads to the following conclusions: in region 2 the field must point out of the page, in region 3 the field must point out of the page, and in region 4 the field must point into the page.

b. If the speed of the particle is v when it enters chamber 1, it will emerge from chamber 4 with the same speed v. The magnetic force is always perpendicular to the velocity of the particle; therefore, it cannot do work on the particle and cannot change the kinetic energy of the particle, according to the work-energy theorem. Since the kinetic energy is unchanged, the speed remains constant.

---

9. ***REASONING AND SOLUTION*** A positive charge moves along a circular path under the influence of a magnetic field. The magnetic field is perpendicular to the plane of the circle, as in Figure 21.12. If the velocity of the particle is reversed at some point along the path, the particle will *not* retrace its path. If the velocity of the particle is suddenly reversed, then from RHR-1 we see that the force on the particle reverses direction. The particle will travel on a different circle that intersects the point where the direction of the velocity changes. The direction of motion of the particle (clockwise or counterclockwise) will be the same as that in the original circle. This is suggested in the figure below.

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10. ***REASONING AND SOLUTION*** A television tube consists of an evacuated tube that contains an electron gun that sends a narrow beam of high-speed electrons toward the screen of the tube (see text Figure 21.37). The inner surface of the screen is covered with a phosphor coating, and when the electrons strike it, they generate a spot of visible light. The electron beam is deflected by the magnetic fields produced by electromagnets placed around the neck of the tube, between the electron gun and the screen. The magnetic fields produced by the electromagnets exert forces on the moving electrons, causing their trajectories to bend and reach different points on the screen, thereby producing the picture. If one end of a bar magnet is placed near a TV screen, the magnetic field of the bar magnet alters the trajectories of the electrons. As a result, the picture becomes distorted.

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11. ***REASONING AND SOLUTION*** When the particle is launched in the x, y plane, its initial velocity will be perpendicular to the magnetic field; therefore, the particle will travel on a circular path in the x, y plane. In order for the charged particle to hit the target, the target must lie on the circular path of the moving particle. This will occur if the particle moves in a counterclockwise circle that passes through the third quadrant of the coordinate system. RHR-1 can be used to determine possible trajectories in the following way. Place the fingers of the right hand into the page (direction of the magnetic field) and orient the thumb

along one of the coordinate axes (direction of the particle's initial velocity). The palm of the right hand will face the direction in which the force on the charged particle is directed. Since the particle travels on a circle, the direction of the force will point toward the center of the particle's trajectory. The figures below show the results for all four possible cases.

Clearly, the charged particle can hit the target only if the initial velocity of the particle points either in the negative x direction or the positive y direction.

---

12. ***REASONING AND SOLUTION*** Refer to Figure 21.17. We can use RHR-1 with the modification that the direction of the velocity of a positive charge is the same as the direction of the conventional current I.

a. If the direction of the current I in the wire is reversed, the thumb of the right hand (direction of I) will be directed into the page, and the palm of the hand (direction of **F**) will face the left side of the page. Therefore, the wire will be pushed to the left.

b. If *both* the current and the magnetic poles are reversed, then the fingers of the right hand must point toward the top of the page (direction of **B**), while the thumb will be directed into the page (direction of I). The palm of the hand will face the right (direction of **F**); therefore, the wire will be pushed to the right.

---

13. ***REASONING AND SOLUTION*** The bulb flashes like a turn signal on a car. When current appears in the circuit after the switch is closed, the bulb glows. Adjacent coils of the helix have current in them in the same direction. We have seen that parallel wires carrying current in the same direction attract one another due to magnetic forces. These forces squeeze the coils together, shortening the helix in the process. The bottom end of the wire is withdrawn from the mercury, thus interrupting the current in the circuit, and the bulb goes

out. Without the current, there are no magnetic forces to squeeze the coils together, and the helix, acting like a spring, expands. The bottom end of the wire dips back into the mercury, and current reappears in the circuit, causing the bulb to glow again. As this process repeats itself, the bulb flashes on and off.

---

14. ***REASONING AND SOLUTION*** In Figure 21.28, assume that current I_1 is larger than current I_2. Consider three regions: Region I lies to the left of both wires, region II lies between the wires, and region III lies to the right of both wires. Let $\mathbf{B}_1$ represent the magnetic field due to current I_1, and let $\mathbf{B}_2$ represent the magnetic field due to current I_2.

a. *The currents point in opposite directions.* From RHR-2, the vectors $\mathbf{B}_1$ and $\mathbf{B}_2$ point in the same direction in region II. Therefore, regardless of the magnitudes of $\mathbf{B}_1$ and $\mathbf{B}_2$, the resultant of the fields can never be zero between the wires. The fields $\mathbf{B}_1$ and $\mathbf{B}_2$ point in opposite directions in regions I and III. Therefore, the resultant of $\mathbf{B}_1$ and $\mathbf{B}_2$ could be zero only in these two regions. The resultant of $\mathbf{B}_1$ and $\mathbf{B}_2$ will be zero at the point where their magnitudes are equal. The magnitude of the magnetic field at a distance r from a long straight wire is given by Equation 21.5: $B = m_0 I / (2pr)$, where I is the magnitude of the current in the wire and m_0 is the permeability of free space. If the current I_1 is larger than the current I_2, the magnitude of $\mathbf{B}_1$ will be equal to the magnitude of $\mathbf{B}_2$ somewhere in region III. This is because region III is closer to the smaller current I_2, and the smaller value for r in Equation 21.5 allows the effect of I_2 to offset the effect of the greater and more distant current I_1. Therefore, there is a point to the right of both wires where the total magnetic field is zero.

b. *The currents point in the same direction.* From RHR-2, the vectors $\mathbf{B}_1$ and $\mathbf{B}_2$ point in the same direction in both regions I and III; therefore, the resultant of $\mathbf{B}_1$ and $\mathbf{B}_2$ cannot be zero in these regions. In region II, the vectors $\mathbf{B}_1$ and $\mathbf{B}_2$ point in opposite directions; therefore, there is a place where the total magnetic field is zero between the wires. It is located closer to the wire carrying the smaller current I_2.

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15. ***REASONING AND SOLUTION*** The drawing shows an end-on view of three parallel wires that are perpendicular to the plane of the paper. In two of the wires, the current is directed into the paper, while in the remaining wire the current is directed

out of the paper. The two outermost wires are held rigidly in place. From Example 8, we know that two parallel currents that point in the same direction attract each other while two parallel currents that point in opposite directions repel each other. Therefore, the middle wire will be attracted to the wire on the left and repelled from the wire on the right. Since each of the fixed wires exerts a force to the left on the middle wire, the net force on the middle wire will be to the left. Thus, the middle wire will move to the left.

---

16. ***REASONING AND SOLUTION*** The figure below shows the arrangements of electromagnets and magnets.

We can determine the polarity of the electromagnets by using RHR-2. Imagine holding the current-carrying wire of the electromagnet in the right hand as the wire begins to coil around the iron core. The thumb points in the direction of the current. For the electromagnet in figure (*a*), the fingers of the right hand wrap around the wire on the left end so that they point, inside the coil, toward the right end. Thus, the right end of the coil must be a north pole. Similar reasoning can be used to identify the north and south poles of the electromagnet in figure (*b*). The results are shown in the figure above. Since the like poles of two different magnets repel each other and the dissimilar poles of two different magnets attract each other, we can conclude that in both arrangements, the electromagnet is repelled from the permanent magnet at the right.

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17. ***REASONING AND SOLUTION*** The figure below shows the arrangements of electromagnets.

We can determine the polarity of each electromagnet by using RHR-2. Imagine holding the current-carrying wire of the electromagnet in the right hand as the wire begins to coil around the iron core. The thumb points in the direction of the current. For the

electromagnet on the left in figure (*a*), the fingers of the right hand wrap around the wire on the left end so that they point, inside the coil, toward the right end. Thus, the right end of the coil must be a north pole. Similar reasoning can be used to identify the north and south poles of the other remaining electromagnets. The results are shown in the figure above. Since the like poles of two different magnets repel each other and the dissimilar poles of two different magnets attract each other, we can conclude that only the arrangement shown in (*a*) results in attraction. The electromagnets shown in arrangement (*b*) result in repulsion.

---

18. ***REASONING AND SOLUTION*** Refer to Figure 21.5. If the earth's magnetism is assumed to originate from a large circular loop of current within the earth, the plane of the current loop must be perpendicular to the magnetic axis of the earth, as suggested in the figure at the right. Using RHR-2, the current must flow clockwise when viewed looking down at the loop from the north magnetic pole.

---

19. ***REASONING AND SOLUTION*** The total magnetic field at the point P is the resultant of the magnetic field at P due to each individual wire. If the current in all four wires is directed into the page, then from RHR-2, the magnetic field at P due to the current in wire 1 must point toward wire 3, the magnetic field at P due to the current in wire 2 must point toward wire 1, the magnetic field at P due to the current in wire 3 must point toward wire 4, and the magnetic field at P due to the current in wire 4 must point toward wire 2, as shown at the right.

The magnetic field at a distance r from a long straight wire that carries a current I is $B = m_0 I/(2pr)$. Since the current in all four wires has the same magnitude and all four wires are equidistant from the point P, each wire gives rise to a magnetic field at P of the same magnitude. When current is flowing through all four wires, the total magnetic field at P is zero. If the current in any single wire is turned off, the total magnetic field will point

toward one of the corners. For example, if the current in wire 1 is turned off, the resultant of $\mathbf{B}_2$ and $\mathbf{B}_3$ is still zero and the total magnetic field is $\mathbf{B}_4$. If the current in wire 2 is turned off, then the total magnetic field is $\mathbf{B}_3$, and so on.

We could have achieved similar results if the current in all four wires was directed out of the page. If fact, as long as opposite wires along the diagonal have currents in the same direction (for example, wires 1 and 4 with outward currents and wires 2 and 3 with inward currents) the total magnetic field will point toward one of the corners when one of the currents is turned off.

---

20. ***REASONING AND SOLUTION*** You have two bars, one of which is a permanent magnet and the other of which is not a magnet, but is made from a ferromagnetic material like iron. The two bars look exactly alike.

a. A third bar (the test bar) that is a permanent magnet can be used to distinguish which of the look-alike bars is the permanent magnet and which is the ferromagnetic bar. Both ends of each of the look-alike bars, in succession, should be brought next to one end of the test bar. The look-alike permanent magnet will be attracted to the test bar when opposite poles are brought together, while it will be repelled from the test bar when like poles are brought together. Both ends of the ferromagnetic bar, however, will be attracted to the test bar. The magnetic field of the test bar will always induce a pole that is opposite in polarity to the pole of the magnet. Ferromagnetism always results in attraction.

b. The identities of the look-alike bars can be determined from a third bar that is not a magnet, but is made from a ferromagnetic material. Either end of the look-alike permanent magnet will be attracted to both ends of the ferromagnetic test bar, while the look-alike ferromagnetic bar will have no effect on the ferromagnetic test bar.

---

21. ***REASONING AND SOLUTION*** A strong electromagnet picks up a delivery truck carrying cans of soda pop. Inside the truck cans of the soft drink are seen to fly upward and stick to the roof just beneath the electromagnet. We can conclude that the cans must be made from a ferromagnetic material, at least in part. Since aluminum is a nonferromagnetic material, we know that the cans are not entirely made of aluminum.

CHAPTER 22 | *ELECTROMAGNETIC INDUCTION*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** If the coil and the magnet in Figure 22.1*a* were each moving with the same velocity relative to the earth, there would be no relative motion between the magnet and the coil. The magnetic flux through the coil due to the bar magnet would be constant and, therefore, the combined motion of the bar magnet and the coil would *not* result in an induced current in the coil. We are ignoring here any effect due to the earth's magnetic field.

---

2. ***REASONING AND SOLUTION*** In the discussion concerning Figure 22.5, we saw that a force of 0.086 N from an external agent was required to keep the rod moving at constant speed. Suppose the light bulb in the figure is unscrewed from its socket. Once the light bulb is unscrewed, the conducting rails and the rod are no longer part of a complete circuit (the resistance of the empty socket is infinite). Therefore, even though there will be a charge separation and a motional emf in the rod given by vBL, there will be no current in the rod. Since there is no current, there is no magnetic force to resist the motion of the rod. From Newton's first law, the rod will continue to move with constant velocity **v** without the application of an external force.

---

3. ***REASONING AND SOLUTION*** A metal sheet moves to the right at a velocity **v** in a magnetic field **B** that is directed into the sheet. At the instant shown in the figure, the magnetic field only extends over half of the sheet. An induced emf leads to the eddy current shown.

We can apply RHR-1 (modified for currents) to the portion of the eddy current that exists in the portion of the sheet that is in the magnetic field. With the thumb of the right hand pointing toward the top of the page (direction of I), and the fingers of the right hand pointing into the page (direction of **B**), the palm of the right hand faces the left (direction of **F**). Thus, there is a retarding magnetic force **F** that acts on the sheet due to the interaction of the eddy current with the magnetic field. Hence, the eddy current causes the sheet to slow down.

---

4. ***REASONING AND SOLUTION*** A magnetic field **B** is necessary if there is to be a magnetic flux passing through a coil of wire. Yet, just because there is a magnetic field does not mean that a magnetic flux will pass through a coil.

The general expression for magnetic flux is given by Equation 22.2: $= (B \cos f)A$, where B is the magnitude of the magnetic field, A is the cross-sectional area of the coil, and f is the angle between the magnetic field **B** and the normal to the surface of the coil. Equation 22.2 shows that the flux depends only on the component of the magnetic field that is perpendicular to the surface of the coil. As shown in Example 4 and in Figure 22.11, when the coil is oriented so that it is parallel to the field, $f = 90°$, **B** has no component normal to the surface of the coil, and the magnetic flux through the coil is zero. Therefore, the magnetic flux through a coil can be zero even though there is a magnetic field present.

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5. ***REASONING AND SOLUTION*** It is known that the magnetic flux through a 1-m^2 flat surface is 2 Wb. From this data alone, it is possible to determine certain information about the average magnetic field at the surface, but *not the magnitude and direction of the field.*

The general expression for the magnetic flux through the surface is given by Equation 22.2: $= (B \cos f)A$, where B is the magnitude of the magnetic field, A is the cross-sectional area of the surface, and f is the angle between the magnetic field **B** and the normal to the surface. Equation 22.2 shows that the flux depends only on the component of the magnetic field that is perpendicular to the surface. Therefore, from the information given, we can only ascertain that the component of the magnetic field that is perpendicular to the surface has an average magnitude of 2 T.

---

6. ***REASONING AND SOLUTION*** Initially, before the switch is closed, neither the conducting rails nor the rod carries a current. When the switch is closed, a conventional current will flow along the conducting rails from the positive toward the negative terminal of the battery. Since the rod is a conducting rod, current will flow through the rod, from top to bottom. According to RHR-1, there will be a force that points to the right on the conducting rod due to the magnetic field; therefore, the rod will be pushed and accelerate to the right. As the rod moves to the right, the area bound by the "loop" increases, thereby increasing the magnetic flux through the loop. As the magnetic flux increases, an induced emf appears around the "loop." According to Lenz's law, the induced emf that appears, will appear in such a way so as to oppose the increase in the magnetic flux. This will occur if the induced emf opposes the battery emf, with the result that the current in the rod begins to decrease and reaches zero when the induced emf exactly offsets the battery emf. With no current in the rod, there is no longer a magnetic force applied to the rod. With no force, there is zero acceleration. In other words, from this point on, the rod moves with a constant velocity.

---

7. ***REASONING AND SOLUTION*** A bolt of lightning contains moving charges, and hence, is a current. This current is surrounded by its magnetic field. Since the charges in a bolt of lightning move erratically, the current is a time-dependent and gives rise to a time-dependent magnetic field. Many household appliances contain coils. If the time-dependent magnetic field passes through these coils, the magnetic flux through the coils will be time-dependent. From Faraday's law, there will be an induced emf and, hence, an induced

current in the coils of the appliance. This will result in an induced current in the circuit that contains the appliance. Therefore, a bolt of lightning can produce a current in the circuit of an electrical appliance, even when the lightning does not directly strike the appliance.

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8. ***REASONING AND SOLUTION*** A solenoid is connected to an ac source. A copper ring is placed inside the solenoid, with the normal to the ring parallel to the axis of the solenoid.

When a current passes through the coils of the solenoid, a magnetic field appears inside the solenoid. Near the center of the solenoid, the magnetic field lines are nearly parallel to the solenoid axis. Since the normal of the copper ring is parallel to the axis, there is a flux through the copper ring when the solenoid carries a current. When the solenoid is connected to an ac source, the current alternates direction, giving rise to a magnetic field inside the solenoid that continuously reverses direction. Therefore, when the solenoid is connected to an ac source, the flux through the copper ring changes with time. From Faraday's law, we know that the changing flux results in an induced emf and, therefore, an induced current around the ring. Since the copper ring has a finite resistance, heat will be generated in the ring at a rate of I^2R, where I is the magnitude of the induced current and R is the resistance of the ring. Therefore, the copper ring will get hot even though nothing touches it.

---

9. ***REASONING AND SOLUTION*** The cable under the floor carries a steady current I; therefore, it will be surrounded by a magnetic field **B** that does not change with time. The magnitude of the field is given by Equation 21.5, $B = m_0 I/(2pr)$, where r is the distance from the center of the cable. Since the robot's sensor coil is close to and parallel to the floor, magnetic field lines from the current-carrying cable will pass through the coil's cross-section. As long as the robot moves parallel to the cable, the distance from each point on the coil's cross section and the center of the cable remains constant, and the magnetic flux through the coil does not change. If the robot deviates from its parallel path, the distance from the coil to the cable will change. The magnitude of the magnetic field through the coil will change as the distance r changes. Therefore, the flux through the coil will change, and, from Faraday's law, there will be an induced emf in the coil.

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10. ***REASONING AND SOLUTION*** The generator-like action of the alternator in a car occurs while the engine is running and keeps the battery fully charged. The headlights would discharge an old and failing battery quickly if it were not for the alternator action. When a battery is in bad shape, the engine and the alternator must do more work to keep the battery

charged when the headlights are on. Therefore, when the battery is failing, the engine of a parked car runs more "quietly" when the headlights are off than when they are on.

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11. ***REASONING AND SOLUTION*** Lenz's Law states that the polarity of an induced emf is such that the induced current produces an induced magnetic field that opposes the change in the flux that causes the emf. In Figure 22.3 a coil of wire is being stretched.

a. As the coil of wire is stretched, the cross-sectional area of the coil decreases. Therefore, the magnetic flux that penetrates the coil decreases. In order to oppose this decrease in flux, a magnetic field must be induced so as to enhance the existing magnetic field. The induced magnetic field, therefore, must point into the page in the region that is surrounded by the coil. RHR-2 can be applied to determine the direction of the induced current. If the wire is held in the right hand with the fingers pointing into the page in the interior of the coil (direction of induced **B**), then the thumb of the right hand (direction of induced I) points so that the induced current is clockwise. This is the direction shown in the drawing.

b. If the direction of the magnetic field were reversed, then using the same reasoning as in part (a), we can reason that the induced magnetic field must point out of the page to oppose the change in flux through the coil as the coil is stretched. Using RHR-2 we deduce that the induced current must be counterclockwise around the loop.

---

12. ***REASONING AND SOLUTION*** The figure shows the set-up.

a. When the switch is open, there is no current in the coil and no magnetic field is present. When the switch in the circuit is closed, a current is established in the coil. From RHR-2, the magnetic field associated with the current in the coil leaves the bottom of the coil and enters the top of the coil (see drawing below). In other words, when there is a current in the coil, the bottom of the coil acts like the north pole of a bar magnet, and the top of the coil acts like the south pole of a bar magnet. The magnetic field of the coil causes a magnetic flux through the metal ring.

In the very short time that it takes for the current to rise from zero to its steady value, the magnetic field associated with the current also rises from zero to its steady value. As the magnetic field increases, the magnetic flux through the metal ring changes, and there is an induced voltage and an induced current around the ring.

From Lenz's law, the polarity of the induced voltage will be such that it opposes the changing flux through the metal ring. The induced magnetic field of the metal ring will be directed upward through the ring to subtract from the magnetic field of the coil that points downward through the ring. This is illustrated in the figure at the right. To preserve clarity, the iron core and a portion of the circuit have not been shown. The induced magnetic field of the metal ring is similar to that of a bar magnet, with the north pole above the ring, and the south pole below the ring. Thus, the situation is similar to that of two bar magnets with their south poles next to each other. The two south poles repel,
and the ring "jumps" upward.

b. If the polarity of the battery were reversed, the ring still "jumps" upward when the switch is closed. When the polarity of the battery is reversed, and the switch is closed, the current moves through the coil in the opposite direction.
The magnetic field associated with the current points in a direction that is opposite to the direction of the magnetic field in part (a). In other words, the top of the coil acts like the north pole of a bar magnet, and the bottom of the coil acts like the south pole of a bar magnet.

When the switch is closed, the magnetic flux through the metal ring changes, and there is an induced voltage and an induced current around the ring.

In accord with Lenz's law, the induced magnetic field of the metal ring will be directed downward through the ring to subtract from the magnetic field of the coil that points upward through the ring. The induced magnetic field of the metal ring is similar to that of a bar magnet, with the south pole above the ring, and the north pole below the ring. The two north poles repel, and the ring "jumps" upward.

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13. ***REASONING AND SOLUTION*** The string of an electric guitar is made of a magnetizable material. As shown in Figure 22.18, the presence of a permanent magnet inside the pickup coil causes the string to become magnetized with north and south poles. When the magnetized string is plucked, it oscillates above the pickup coil, thereby changing the magnetic flux that passes through the coil. The guitar string vibrates in a standing wave

pattern that consists of nodes and antinodes. The magnetic flux will change most rapidly where the oscillations of the magnetized string are the largest; therefore, the electromagnetic pickup should be placed at an antinode to produce a maximum emf.

---

14. ***REASONING AND SOLUTION*** The current drawn by an electric motor is given by Equation 22.5: $I = (V - \text{emf})/R$, where V is the applied emf, "emf" is the back emf developed because the coil of the motor is rotating, and R is the resistance of the wire in the coil.

An electric motor in a hair drier is running at normal speed; therefore, the back emf is comparable to the applied emf, and the motor is drawing a relatively small current. If the shaft of the motor is prevented from turning, the back emf is suddenly reduced to zero. We see from Equation 22.5 that the current drawn by the motor will increase to the value V/R. Heat will be produced in the coil at a rate of $I^2R = (V/R)^2 R = V^2/R$, and the temperature of the coil will increase.

---

15. ***REASONING AND SOLUTION*** One transformer is a step-up device, while another is step-down. These two units have the same voltage across and the same current in their primary coils.

If we ignore any heat loss within the transformers, then, according to the principle of energy conservation, the average power delivered to the circuit attached to the secondary coil is the same for either transformer.

This conclusion can be verified using Equations 20.15a, and Equation 22.13. According to Equation 20.15a, the average power delivered to the secondary coil, and therefore, the average power that the secondary coil delivers to the circuit is $\overline{P}_s = I_s V_s$. Solving Equation 22.13 for I_s and substituting the resulting expression into Equation 20.15a, we have

$$\overline{P}_s = I_s V_s = \frac{I_p V_p}{V_s} \div V_s = I_p V_p$$

This result holds for both the step-up, as well as, the step-down device. Since both devices have the same voltage across and the same current in their primary coils, the average power delivered to the secondary coil, and therefore, the average power delivered to the circuit attached to the secondary coil, are the same.

CHAPTER 23 | *ALTERNATING CURRENT CIRCUITS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A light bulb and a parallel plate capacitor (including a dielectric material between the plates) are connected in series to the 60-Hz ac voltage present at a wall outlet. Since the capacitor and the light bulb are in series, the rms current at any instant is the same through each element, and is given by Equation 23.6: $I_{\text{rms}} = V_{\text{rms}} / Z$, where Z is the impedance of the circuit. The impedance of the circuit is given by Equation 23.7 with $X_{\text{L}} = 0$ (since there is no inductance in the circuit): $Z = \sqrt{R^2 + (-X_{\text{C}})^2} = \sqrt{R^2 + X_{\text{C}}^{\ 2}}$. According to Equation 19.10, if the dielectric between the plates of the capacitor is removed, the capacitance decreases by a factor of k, where k is the dielectric constant. From Equation 23.2, $X_{\text{C}} = 1/(2pfC)$, we see that decreasing the capacitance increases the capacitive reactance X_{C}. Therefore, the impedance of the circuit, $Z = \sqrt{R^2 + X_{\text{C}}^{\ 2}}$, increases. The rms current is $I_{\text{rms}} = V_{\text{rms}} / Z$ and will, therefore, be *less* than it was before the dielectric was removed. Thus, the brightness will *decrease*.

---

2. ***REASONING AND SOLUTION*** The ends of a long straight wire are connected to the terminals of an ac generator, and the current is measured. The wire is then disconnected, wound into the shape of a multiple-turn coil, and reconnected to the generator. After the wire is wound into a coil, it has a greater inductance. When the generator is turned on, the coil will develop a voltage that opposes a change in the current according to Faraday's law. Since the induced voltage opposes the rise in current, the rms current in the circuit will be less than it was before the wire was wound into a coil.

   This can also be seen by considering Equations 23.6 and 23.7. Before the wire was wound into a coil, its primary property was that of resistance, and the current through the wire was given by $I_{\text{rms}} = V_{\text{rms}} / Z$ with $Z = R$, or $I_{\text{rms}} = V_{\text{rms}} / R$. After the wire is wound into a coil, the wire possesses both resistance and inductance. Now the current in the coil is given by $I_{\text{rms}} = V_{\text{rms}} / Z$, where $Z = \sqrt{R^2 + X_{\text{L}}^{\ 2}}$. Since Z is necessarily greater than R, the rms current is less when the wire is wound into a coil.

---

3. ***REASONING AND SOLUTION*** An air-core inductor is connected in series with a light bulb of resistance R. This circuit is plugged into an electrical outlet. The current in the circuit is given by Equation 23.6, $I_{\text{rms}} = V_{\text{rms}} / Z$, where, from Equation 23.7, $Z = \sqrt{R^2 + X_{\text{L}}^{\ 2}}$. When a piece of iron is inserted in the inductor, the magnetic field in the inductor is enhanced relative to that in air, and the inductance increases. Equation 23.4, $X_{\text{L}} = 2pfL$, shows that when the inductance L increases, the inductive reactance X_{L} also

increases. The impedance of the circuit, therefore, increases, and the current in the circuit, $I_{\text{rms}} = V_{\text{rms}} / Z$, decreases. Thus, the brightness of the bulb decreases.

---

4. ***REASONING AND SOLUTION*** It is possible that the current in the simplified circuit has the same rms value as that in the original circuit. This would be the case if the original circuit were at resonance. Then $X_C = X_L$, and Equation 23.7 for the impedance would simplify to $Z = R$. As a result, the currents in the original and the simplified circuits would each be $I_{\text{rms}} = V_{\text{rms}}/R$.

---

5. ***REASONING AND SOLUTION*** Consider the series RCL circuit shown in Figure 23.9. The impedance of the circuit is given by Equation 23.7: $Z = \sqrt{R^2 + (X_L \quad X_C)^2}$, where the capacitive reactance is given by Equation 23.2 [$X_C = 1/(2pfC)$] and the inductive reactance is given by Equation 23.4 [$X_L = 2pfL$]. One example of this expression for Z is plotted in Figure 23.15 (see the blue curve). The vertical axis in this figure gives the impedance, while the horizontal axis gives the frequency. A horizontal line drawn to intersect the vertical axis above the minimum in the curve will intersect the curve at two places. These places correspond to two different frequencies on the horizontal axis. Thus, the circuit in Figure 23.9 can have the same impedance at two different frequencies.

---

6. ***REASONING AND SOLUTION*** An inductor and a capacitor are connected in parallel across the terminals of a generator.

a. *The frequency of the generator becomes very large.* From Equation 23.2 (or Figure 23.2) we see that, in the high frequency limit, the capacitive reactance approaches zero. Therefore, in the high frequency limit, the capacitor behaves as if it were replaced by a wire with zero resistance, and the generator delivers a very large current to the capacitor. From Equation 23.4 (or Figure 23.6) we see that, in the high frequency limit, the inductive reactance becomes very large. In this limit, the inductor behaves as if it has been cut out of the circuit, leaving a gap in the wire, and the generator delivers no current to the inductor. Therefore, when the frequency of the generator becomes very large, the current through the capacitor becomes large, and no current flows through the inductor. The total current delivered by the generator is large.

b. *The frequency of the generator becomes very small.* From Equation 23.2 we see that, in the low frequency limit, the capacitive reactance approaches infinity. The capacitor acts as if it has been cut from the circuit, leaving a gap in the wire. As a result, the generator delivers no current to the capacitor. From Equation 23.4 we see that, in the low frequency limit, the inductive reactance approaches zero. In this limit, the inductor acts as if it has been replaced by a wire with zero resistance, and the generator delivers a very large current to the inductor. Therefore, when the frequency of the generator becomes very small, the current through the inductor becomes very large and the current through the capacitor is zero. The total current delivered by the generator is again large.

---

7. ***REASONING AND SOLUTION*** The rms current is given by Equation 23.6: $I_{\text{rms}} = V_{\text{rms}} / Z$, where V_{rms} is the rms voltage of the generator and Z is the impedance of the circuit. Since the rms voltage of the generator is the same in both cases, the greater current is delivered to the circuit with the smaller impedance. We see from Equation 23.2 [$X_{\text{C}} = 1/(2pfC)$] and Equation 23.4 [$X_{\text{L}} = 2pfL$] that in the high frequency limit, the capacitive reactance is nearly zero and the inductive reactance is essentially infinite. In other words, in the high frequency limit, the capacitors behave as if they have been replaced by wires of zero resistance and the inductors behave as if they have been cut from the circuit leaving gaps in the wires. The figure below shows the two circuits in this limit.

Circuit I behaves as if it consists only of two identical resistors in parallel; therefore, the impedance of circuit I is $1/Z = 1/R + 1/R$, or $Z = R/2$. Circuit II behaves as if it consists only of two identical resistors in series; therefore, the impedance of circuit II is $Z = 2R$. At a very high frequency, then, circuit I has the smaller impedance, and, therefore, its generator supplies the greater rms current.

8. ***REASONING AND SOLUTION*** The phase angle between the current in and the voltage across a series RCL combination is the angle f between the current phasor I_0 and the voltage phasor V_0 in Figure 23.11. According to Equation 23.8, the tangent of this angle is related to the resistance *R*, the inductive reactance X_L, and the capacitive reactance X_C, according to the relation

$$\tan f = \frac{X_L - X_C}{R}$$

At resonance, $X_L = X_C$, and so $\tan f = 0$. Therefore, $f = \tan^{1}(0) = 0$; in other words, the phase angle between the current phasor and the voltage phasor is zero. Since the phase angle between the current phasor and the voltage phasor is zero, we can conclude that the current is in phase with the voltage.

---

9. ***REASONING AND SOLUTION*** The resonant frequency of a series RCL circuit is given by Equation 23.10:

$$f_0 = \frac{1}{2p\sqrt{LC}}$$

a. Since the resonant frequency of a RCL circuit does not depend on the value of *R*, it is possible for two series RCL circuits to have the same resonant frequencies and yet have different *R* values.

b. The resonant frequency of a series RCL circuit is inversely proportional to $\sqrt{LC}$. It is possible for two series RCL circuits to have the same resonant frequencies and yet have different values of *C* and *L*, provided that the product *LC* is the same in both circuits.

---

10. ***REASONING AND SOLUTION*** The generator connected to a series RCL circuit has a frequency that is greater than the resonant frequency of the circuit. Suppose that it is necessary to match the resonant frequency of the circuit to the frequency of the generator. To accomplish this, a second capacitor will be added to the circuit. In order to match the resonant frequency of the circuit to the frequency of the generator, the resonant frequency of the circuit must be increased. The resonant frequency of a series RCL circuit is given by Equation 23.10: $f_0 = 1/(2p\sqrt{LC})$. Since the resonant frequency is inversely proportional to $\sqrt{LC}$, the capacitor must be added to the circuit so that it *decreases* the capacitance. This can be accomplished by placing the second capacitor *in series* with the first capacitor (see Equation 20.19). With the proper choice of the value of *C* for the second capacitor, the capacitance of the series combination will be smaller than the capacitance of the first capacitor alone, and the resonant frequency of the circuit will increase to match the generator frequency.

---

11. ***REASONING AND SOLUTION*** The drawing at the right shows a full-wave rectifier circuit, in which the direction of the current through the load resistor R is the same for both positive and negative halves of the generator's voltage cycle. The points A and B are connected directly to the top and bottom of the generator, respectively. The diodes are labeled a, b, c, and d. The direction of the current through the circuit can be found by recalling the fact that charge flows through a diode only when the diode is in a forward bias condition. When the diode is in a forward bias condition, the side of the diode symbol that contains the arrowhead has a positive potential relative to the other side.

a. When the top of the generator is positive and the bottom is negative, point A in the figure has the positive potential and point B the negative potential. Since point A has the positive potential and is connected directly to the arrowhead for diode a, diode a is in a forward bias condition. In contrast, we see that for diode b, the side opposite the arrowhead is connected to point A, so diode b is in a reverse bias condition. Similarly, since point B is negative, and is connected directly to the side of diode c opposite the arrowhead, the arrowhead must be positive, and diode c is in a forward bias condition. In contrast, the arrowhead of diode d is negative, so diode d is in a reverse bias condition. Thus, when the top of the generator is positive and the bottom is negative, only diodes a and c are in the forward bias condition and allow charge to flow through the resistance R. Drawing 1 below shows the path taken by the current under these circumstances.

b. When the top of the generator is negative and the bottom is positive, reasoning similar to that given above in part (a) indicates that diodes b and d are now forward biased, while diodes a and c are in reverse bias. Furthermore, the forward bias diodes allow charge to flow from left to right through the resistance R. Drawing 2 below shows the path taken by the current under these circumstances.

Drawing 1

Drawing 2

CHAPTER 24 | *ELECTROMAGNETIC WAVES*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION***

a. The intensity of a wave refers to the power P carried by the wave that passes perpendicularly through a surface in the path of the wave, divided by the area A of the surface. It is given for sound waves by Equation 16.8 and for electromagnetic waves by Equation 24.4. The concept of intensity applies to both sound waves and electromagnetic waves, because both types of wave transfer energy and, therefore, power away from their respective sources.

b. In a transverse wave, the particles of the medium vibrate perpendicular to the direction of propagation of the wave. In the case of electromagnetic waves, there are no particle vibrations, but rather the electric and magnetic fields oscillate perpendicular to the direction of propagation of the wave. When a wave is produced or altered so that the vibrations or oscillations take place in a particular direction perpendicular to the wave velocity, the wave is said to be *polarized*.

In a longitudinal wave, the notion of polarization has no meaning, as discussed in Section 24.6 of the text. Therefore, *transverse waves can be polarized*, while *longitudinal waves cannot be polarized*.

Since sound waves are longitudinal waves, while electromagnetic waves are transverse waves, the concept of polarization applies only to electromagnetic waves.

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2. ***REASONING AND SOLUTION*** Refer to Figure 24.2. Between the times indicated in parts *c* and *d* in the drawing, negative charges have moved to the top of the antenna, leaving a net positive charge of equal magnitude on the bottom of the antenna. Therefore, as the negative charges flow, the conventional current points toward the bottom of the antenna. Using RHR-2, the magnetic field for the electromagnetic wave at P must point out of the page.

---

3. ***REASONING AND SOLUTION*** A transmitting antenna is located at the origin of an x, y, z axis system, and broadcasts an electromagnetic wave whose electric field oscillates along the y axis. The wave travels along the $+x$ axis. Three possible loops can be used with an LC-tuned circuit to detect the wave: One loop lies in the x, y plane, another in the x, z plane, and the third in the y, z plane.

The loop that will detect the electromagnetic wave must be oriented so that the normal to the loop is parallel to the magnetic field. Then, as the wave passes by the loop, the changing magnetic field penetrates the loop and results in an induced emf and current, as predicted by Faraday's law. Since the electromagnetic wave travels along the $+x$ direction, and the electric field oscillations of the electromagnetic wave are along the y axis, the magnetic field oscillations will be along the z axis. For optimum reception, therefore, the loop should lie in the x, y plane so that the normal to the loop is in the z direction.

---

4. ***REASONING AND SOLUTION*** When a straight-wire antenna is used as the receiving antenna to detect electromagnetic waves, the wires must be oriented parallel to the electric field, as shown in Figure 24.5. The electric field exerts a force on the electrons in the wire and causes them to oscillate up and down along the length of the antenna. Since the electric field drives the electrons directly, the peak emf that causes the ac current in the receiving antenna depends on the amplitude of the electric field. It does not depend on the frequency of the wave.

When a loop antenna such as that shown in Figure 24.6, is used as the receiving antenna, the induced emf around the loop depends on the rate at which the magnetic flux changes with time, $/ \quad t$. If B represents the instantaneous magnitude of the magnetic field and A represents the cross-sectional area of the loop, and the normal of the loop points in the same direction as **B**, then $= BA$. Then $/ \quad t = \quad (BA)/ \quad t$. Since the area of the loop, A, does not change, it follows that $/ \quad t = (\quad B/ \quad t)A$. Hence, the induced emf in the receiving loop depends on the rate at which the magnetic field of the wave changes with time, and therefore, on the frequency of the electromagnetic wave. Thus, the peak value of the emf induced in a loop antenna depends on the frequency of the wave.

---

5. ***REASONING AND SOLUTION*** The electric field of an electromagnetic wave is related to the magnetic field through the relation $E = cB$ (Equation 24.3). Therefore, if the electric field of a wave decreases in magnitude, the magnetic field will also decrease.

This conclusion can also be reached by considering the fact that in an electromagnetic wave propagating through air or vacuum, the electric field and the magnetic field carry equal amounts of energy per unit volume of space. If the electric field of a wave decreases in magnitude, the electric energy per unit volume must decrease. The magnetic energy per unit volume must decrease to the same amount; therefore, the magnitude of the associated magnetic field must also decrease.

---

6. ***REASONING AND SOLUTION*** The same Doppler effect arises for electromagnetic waves when either the source or the observer of the waves moves; it is only the relative motion of the source and the observer with respect to one another that is important (see Section 24.5). Therefore, when an astronomer measures the Doppler change in frequency for the light reaching earth from a distant star, the astronomer *cannot* tell whether the star is moving away from the earth or whether the earth is moving away from the star.

---

7. ***REASONING AND SOLUTION*** The only real difference between a polarizer and an analyzer is the purpose for which they are used. Both consist of a piece of polarizing material. When the piece of polarizing material is used to produce a desired polarization direction, it is referred to as a *polarizer*. When the piece of polarizing material is used to change the polarization direction and to adjust the intensity so that the polarization direction of the incident light can be determined, it is referred to as an *analyzer*. The same piece of polarizing material can be used as the polarizer in one situation and the analyzer in another.

---

8. ***REASONING AND SOLUTION*** Malus' law applies to the setup in Figure 24.20, which shows the analyzer rotated through an angle q and the polarizer held fixed. When the analyzer is held fixed and the polarizer is rotated, Malus' law still applies. Malus' law states that the average intensity $\bar{S}$ depends on the angle q between the polarizer and analyzer. It does not matter whether the polarizer is fixed and the analyzer is rotated, or vice-versa.

---

9. ***REASONING AND SOLUTION*** In Example 7, we saw that, when the angle between the polarizer and analyzer is 63.4°, the intensity of the transmitted light drops to one-tenth of that of the incident unpolarized light. The light intensity that is not transmitted is absorbed by both the polarizer and the analyzer. The polarizer absorbs one-half of the incident intensity. The analyzer absorbs four-tenths, or two-fifths, of the original incident light. This absorbed energy results in an increase in the temperature of the polarizer and analyzer.

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10. ***REASONING AND SOLUTION*** Light is incident from the left on two pieces of polarizing material, 1 and 2. As part *a* of the following drawing illustrates, the transmission axis of material 1 is along the vertical direction, while that of material 2 makes an angle of q with respect to the vertical. In part (*b*) of the drawing, the two polarizing materials are interchanged.

a. If the incident light is unpolarized and has an average intensity $\bar{I}$, then the average intensity of the light that leaves polarizer #1 and strikes polarizer #2 in Figure (*a*) is $\bar{S}_0 = \bar{I}/2$. According to Malus' law, the average intensity of the light leaving polarizer #2 is $\bar{S} = \bar{S}_0 \cos^2 q = (\bar{I}\cos^2 q)/2$.

Similarly, the average intensity of the light that leaves polarizer #2 and strikes polarizer #1 in Figure (*b*) is $\bar{S}_0 = \bar{I}/2$. The angle between the transmission axis of polarizer #2 and polarizer #1 is still q, so that the average intensity of the light leaving polarizer #1 in (*b*) is $\bar{S} = \bar{S}_0 \cos^2 q = (\bar{I}\cos^2 q)/2$. Therefore, the transmitted light in Figure (*a*) is the same as that in Figure (*b*).

b. If the incident light in Figure (*a*) is polarized along the vertical direction with average intensity $\bar{I}$, then, since the polarization direction of the light is the same as the transmission axis of the polarizer, all of the incident light is transmitted. The average intensity of the light that leaves polarizer #1 and strikes polarizer #2 is $\bar{S}_0 = \bar{I}$. According to Malus' law, the average intensity of the light leaving polarizer #2 is $\bar{S} = \bar{S}_0 \cos^2 q = \bar{I}\cos^2 q$.

In Figure (*b*) we have from Malus' law that the average intensity of the light that leaves polarizer #2 and strikes polarizer #1 is $\bar{S}_0 = \bar{I}\cos^2 q$. The angle between the transmission axis of polarizer #2 and polarizer #1 in Figure (*b*) is q. Therefore, according to Malus' law, the average intensity of the light leaving polarizer #1 in Figure (*b*) is $\bar{S} = \bar{S}_0 \cos^2 q = (\bar{I}\cos^2 q)\cos^2 q = \bar{I}\cos^4 q$. Therefore, the intensity of the transmitted light in Figure (*a*) is greater than the intensity of the transmitted light in Figure (*b*).

---

11. ***REASONING AND SOLUTION*** Light from the sun is unpolarized; however, when the sunlight is reflected from horizontal surfaces such as the surface of a swimming pool, lake, or ocean, the reflected light is partially polarized in the horizontal direction. Polaroid sunglasses are constructed with lenses made of Polaroid (a polarizing material) with the transmission axis oriented vertically. Thus, the horizontally polarized light that is reflected from horizontal surfaces is blocked from the eyes.

Suppose you are sitting on the beach near a lake on a sunny day, wearing Polaroid sunglasses. When you are sitting upright, the horizontally polarized light that is reflected from the lake is blocked from your eyes, as discussed above. When you lay down on your side, facing the lake, the transmission axis of the Polaroid sunglasses is now oriented in a nearly horizontal direction. Most of the horizontally polarized light that is reflected from the lake is transmitted through the sunglasses and reaches your eyes. Therefore, the sunglasses don't work as well as they did when you were sitting in an upright position.

CHAPTER 25 *THE REFLECTION OF LIGHT: MIRRORS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A sign painted on a store window is reversed when viewed from inside the store. A person inside the store views the sign in a plane mirror. As discussed in the text, the image of an object formed in a plane mirror is upright, has the same size as the object, is located as far behind the mirror as the object is in front of it, and is reversed left to right. Therefore, since the image is reversed left to right, the image of the sign painted on the store window, when viewed in a plane mirror in the store, will appear as it does when viewed from outside the store.

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2. ***REASONING AND SOLUTION*** As discussed in the text, the image of an object formed in a plane mirror is reversed left to right. If a clock is held in front of a mirror, its image is reversed left to right, but not up and down. In order to understand the appearance of the image of the second hand, as viewed by a person looking into the mirror, imagine replacing the second hand by a rotating vector that always points away from its rotation axis. At any instant, this vector can be resolved into horizontal and vertical components. Since the image is reversed left to right, the image of the horizontal component will be reversed in direction. In contrast, since the image is *not reversed* up and down, the image of the vertical component will point in the same direction as the object. Therefore, when the horizontal and vertical components of the image are combined, the resultant will rotate *counterclockwise*. Thus, from the point of view of a person looking into the mirror, the image of the second hand of the clock rotates in the counterclockwise direction.

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3. ***REASONING AND SOLUTION*** When parallel rays of light strike a concave mirror, they are reflected; these reflected rays converge at the focal point of the mirror. When parallel rays of light strike a convex mirror, they are also reflected; these reflected rays diverge from the mirror's surface and appear to originate from the focal point located behind the mirror.

   a. The earth-sun distance is very large; therefore, when rays of light from the sun reach the earth, they are essentially parallel. If it is desired to start a fire with sunlight, it is necessary to focus the parallel light rays from the sun on a very small area, preferably a point, on the piece of paper. Since a concave mirror reflects parallel rays so that they converge in front of the mirror, a concave mirror, rather than a convex mirror, should be used.

   b. For best results, the piece of paper should be placed at the focal point of the mirror since this is the location where the rays converge to a point and the heating would be greatest.

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4. ***REASONING AND SOLUTION*** The photograph in the text shows an experimental device at Sandia National laboratories in New Mexico. The device is a mirror that focuses sunlight to heat sodium to a boil.

a. The earth-sun distance is very large; therefore, when rays of sunlight reach the earth, they are essentially parallel. A concave mirror reflects parallel rays of light so that the reflected rays converge at the focal point of the mirror. It is reasonable to conclude, therefore, that the mirror in the photograph is a concave mirror.

b. Since the rays of light converge at the focal point of the mirror, it is reasonable to conclude that the sodium unit is located at the mirror's focal point. Therefore, the distance between the mirror and the sodium unit is equal to the focal length of the mirror.

---

5. ***REASONING AND SOLUTION*** When parallel rays of light strike a concave mirror, they are reflected. For rays that lie close to the principal axis of the mirror, these rays will converge at a single point, namely the focal point of the mirror. Rays that are far from the principal axis do not converge to a single point after reflection. Each reflected ray obeys the law of reflection; namely, the angle of reflection, as measured with respect to the normal to the surface, is equal to the angle of incidence. Since the reflecting surface is spherical, the direction of the lines that are normal to the surface varies from point to point; however, they are directed radially with respect to the center of the "sphere." Rays that are farther from the principal axis have greater angles of incidence and greater angles of reflection. Therefore, when such rays reflect, they cross the principal axis to the right of the focal point *F* in Figure 25.14. The top ray in this drawing could be directed through the focal point if the angle of reflection, and therefore, the angle of incidence were decreased. This can be accomplished by changing the shape of the mirror so that the line normal to the surface at the point of incidence is rotated upward as shown in the figure below.

Spherical Concave mirror *Reshaped mirror*

Therefore, to bring the top ray closer to the focal point, the mirror must be "opened up" to produce a more gently curving surface.

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6. ***REASONING AND SOLUTION***

a. For a real image, light rays actually pass through the points on the image. Such an image can be projected onto a screen directly. This can be accomplished by placing the screen at the location of the image.

If the object is placed between the focal point and the center of curvature of a concave mirror, a real image is formed beyond the center of curvature. The real image is enlarged and inverted (see Figure 25.19*a*). If the object is placed beyond the center of curvature, a real image is formed between the center of curvature and the focal point. This real image is reduced in size and inverted with respect to the object (see Figure 25.19*b*). In these cases, the image is real; therefore, it can be projected directly onto a screen (placed at the location of the image) without the help of other mirrors or lenses.

If, however, the object is placed between the focal point and the concave mirror, an enlarged, upright *virtual* image is produced (see Figure 25.20*a*). The image is behind the mirror and cannot be projected directly onto a screen without the help of other optical components.

b. A convex mirror always produces a virtual image that is behind the mirror; therefore the image cannot be projected directly onto a screen without the help of other optical components.

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7. ***REASONING AND SOLUTION***

a. The back side of a shiny teaspoon acts like a convex mirror. When an object is placed in front of a convex mirror, a virtual image is produced that is reduced in size and upright, as shown in Figure 25.22. Thus, when you look at the back side of a shiny teaspoon held at arms length, you will see yourself upright.

b. The concave side of a shiny teaspoon acts like a concave mirror. If the teaspoon is held at arm's length, then you (the object) are farther from the reflecting surface than the center of curvature of the surface. The situation is similar to that of an object located beyond the center of curvature of a concave mirror. A real image is formed that is reduced in size and *inverted* with respect to the object, as shown in Figure 25.19*b*. Therefore, when you look at the concave side of a shiny teaspoon, you will see yourself upside down.

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8. ***REASONING AND SOLUTION*** If you stand between two parallel plane mirrors, you see an infinite number of images of yourself. This occurs because an image in one mirror is reflected in the other mirror to produce another image, which is then re-reflected, and so-forth. The multiple images are equally spaced.

The image produced by a convex mirror is reduced in size relative to the object. Like a plane mirror, the image is virtual and lies behind the mirror; however, the virtual image in a convex mirror is closer to the mirror than it would be if the mirror were planar. If you stand facing a convex mirror, with a plane mirror behind you, you will see an infinite number of images of yourself, as you do in the case of two parallel plane mirrors. The reason is that the first image in the convex mirror is reflected in the plane mirror, to produce an identical image behind it. The image behind the plane mirror is re-reflected in the convex mirror to produce another smaller image closer to the focal point of the convex mirror. This second image in the convex mirror is smaller than the first one, because the image behind the plane

mirror is farther from the convex mirror than you are. Further reflections and re-reflections occur, leading to a series of images in the convex mirror that decrease in size as they occur closer and closer to the focal point. The images "pile up" so to speak, becoming closer and closer together as they approach the focal point. The size of the image becomes zero at the focal point. Thus, the series of images appears to vanish at the focal point of the convex mirror.

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9. ***REASONING AND SOLUTION*** The microphone arrangement shown in the figure is used to pick up weak sounds. It consists of a "hollowed-out" shell behind the mike. The shell acts like a mirror for sound waves. Therefore, when parallel rays of sound hit the inside surface of the shell, they will be reflected from it. Since the shell is concave, the reflected rays will converge at the focal point of the shell. Presumably, weak sounds will originate far from the microphone; therefore, when the they reach the microphone arrangement, these rays will be essentially parallel to the principal axis of the shell. Since the reflected rays will converge at the focal point, the microphone should be located at the focal point to detect them optimally.

---

10. ***REASONING AND SOLUTION*** When you see the image of yourself formed by a mirror, it is because (1) light rays actually coming from a real image enter your eyes or (2) light rays appearing to come from a virtual image enter your eyes. If the light rays from the image do not enter your eyes, you do not see yourself.

a. If you stand in front of a convex mirror on its principal axis, your virtual image is behind the mirror. The image will be reduced in size and upright regardless of your location. You will be able to see yourself at any location on the principal axis when you are in front of a convex mirror, because rays that appear to come from this image can enter your eyes.

b. If you stand in front of a concave mirror on its principal axis, and you are beyond the center of curvature of the mirror, you will see a real image of yourself that is inverted and reduced in size relative to your size and orientation (Figure 25.19*b*). If you stand between the mirror and its focal point, you will see a virtual image of yourself that is upright and enlarged in size (Figure 25.20). In the former case, rays emanating from the real image can reach your eyes. In the latter case, rays that appear to emanate from the virtual image can reach your eyes.

When you stand on the principal axis of a concave mirror so that you are between the center of curvature and the focal point of the mirror, the image is formed beyond the center of curvature (Figure 25.19*a*). You cannot see this image, because it is behind you.

---

11. ***REASONING AND SOLUTION*** Plane mirrors and convex mirrors form virtual images. With a plane mirror, the image may be infinitely far behind the mirror, depending on where the object is located in front of the mirror.

For an object in front of a single convex mirror, the greatest distance behind the mirror at which the image can be found is equal to the magnitude of the focal length of the mirror. This can be confirmed using the mirror equation (Equation 25.3): $1/d_{\mathrm{o}} + 1/d_{\mathrm{i}} = 1/f$, where f is taken to be a negative number because the mirror is a convex mirror. The largest

image distance occurs when the object is at infinity. Then $1/d_o = 1/\quad = 0$, and the mirror equation gives $d_i = f$. Since f is a negative number, d_i is a negative number, indicating that the image is behind the mirror. Therefore, the image will never be located beyond the focal point, behind the mirror.

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12. ***REASONING AND SOLUTION*** As shown in Figure 25.22, an object placed in front of a convex mirror produces a virtual image behind the mirror that is reduced in size and is upright in orientation for all object distances. Therefore, it is *not* possible to use a convex mirror to produce an image that is larger than the object.

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13. ***REASONING AND SOLUTION*** Suppose you stand in front of a spherical mirror (concave or convex). As shown in Figures 25.19 and 25.20, a concave mirror can form a real, inverted image (Figures 25.19*a* and 25.19*b*) or a virtual, upright image (Figure 25.20). As shown in Figure 25.22, a convex mirror can only form a virtual, upright image.

a. Therefore, it is not possible for your image to be real and upright.

b. Similarly, it is not possible for your image to be virtual and inverted.

CHAPTER 26 *THE REFRACTION OF LIGHT: LENSES AND OPTICAL INSTRUMENTS*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** Since the index of refraction of water is greater than that of air, the ray in Figure 26.2*a* is bent toward the normal at the angle q_{w1} when it enters the water. According to Snell's law (Equation 26.2), the sine of q_{w1} is given by

$$\sin q_{\text{w1}} = \frac{n_{\text{air}} \sin q_1}{n_{\text{water}}} = \frac{\sin q_1}{n_{\text{water}}} \tag{1}$$

where we have taken $n_{\text{air}} = 1.000$. When a layer of oil is added on top of the water, the angle of refraction at the air/oil interface is q_{oil} and, according to Snell's law, we have

$$\sin q_{\text{oil}} = \frac{n_{\text{air}} \sin q_1}{n_{\text{oil}}} = \frac{\sin q_1}{n_{\text{oil}}} \tag{2}$$

But q_{oil} is also the angle of incidence at the oil/water interface. At this interface the angle of refraction is q_{w2} and is given by Snell's law as follows:

$$\sin q_{\text{w2}} = \frac{n_{\text{oil}} \sin q_{\text{oil}}}{n_{\text{water}}} = \frac{n_{\text{oil}}}{n_{\text{water}}} \frac{\sin q_1}{n_{\text{oil}}} = \frac{\sin q_1}{n_{\text{water}}} \tag{3}$$

where we have substituted Equation (2) for $\sin q_{\text{oil}}$. According to Equation (1), this result is equal to $\sin q_{\text{w1}}$. Therefore, we can conclude that the angle of refraction as the ray enters the water does *not* change due to the presence of the oil.

---

2. ***REASONING AND SOLUTION*** When light travels from a material with refractive index n_1 into a material with refractive index n_2, the angle of refraction q_2 is related to the angle of incidence q_1 by Equation 26.2: $n_1 \sin q_1 = n_2 \sin q_2$ or $q_2 = \sin^{1}(n_1/n_2)\sin q_1$. When $n_1 < n_2$, the angle of refraction will be less than the angle of incidence. The larger the value of n_2, the smaller the angle of refraction for the same angle of incidence. The angle of refraction is smallest for slab B; therefore, slab B has the greater index of refraction.

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3. ***REASONING AND SOLUTION*** When light travels from a material with refractive index n_1 into a material with refractive index n_2, the angle of refraction q_2 is related to the angle of incidence q_1 by Equation 26.2: $n_1 \sin q_1 = n_2 \sin q_2$. Therefore, $n_1 = n_2 \sin q_2 / (\sin q_1)$. Both blocks are made from the same material; therefore, n_2 is the same for each system. Furthermore, the angle of incidence is the same for each system; therefore, $\sin q_1$ is the same in both liquids. Thus, the liquid with the greater index of refraction will be the one for which $\sin q_2$, and therefore q_2, is the largest. This occurs in liquid A. Therefore, the index of refraction of liquid A is greater than that of liquid B.

---

4. ***REASONING AND SOLUTION*** When an observer peers over the edge of a deep empty bowl, he does not see the entire bottom surface, so a small object lying on the bottom is hidden from view. However, when the bowl is filled with water, the object can be seen.

When the object is viewed from the edge of the bowl, light rays from the object pass upward through the water. Since $n_{\text{air}} < n_{\text{water}}$, the light rays from the object refract away from the normal when they enter air. The refracted rays travel to the observer, as shown in the figure at the right. When the rays entering the air are extended back into the water, they show that the observer sees a virtual image of the object at an apparent depth that is less than the actual depth, as indicated in the drawing. Therefore, the apparent position of the object in the water is in the line of sight of the observer, even though the object could not be seen before the water was added.

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5. ***REASONING AND SOLUTION*** Two identical containers, one filled with water (n = 1.33) and the other filled with ethyl alcohol (n = 1.36) are viewed from directly above. According to Equation 26.3, when viewed from directly above in a medium of refractive index n_2, the apparent depth $d\ell$ in a medium of refractive index n_1 is related to the actual depth d by the relation $d = d(n_2 / n_1)$. Assuming that the observer is in air, n_2 = 1.00. Since n_1 refers to the refractive index of the liquid in the containers, we see that the apparent depth in each liquid is inversely proportional to the refractive index of the liquid. The index of refraction of water is smaller than that of ethyl alcohol; therefore, the container filled with water appears to have the greater depth of fluid.

---

6. ***REASONING AND SOLUTION*** When you look through an aquarium window at a fish, the fish appears to be closer than it actually is. When light from the fish leaves the water and enters the air, it is bent away from the normal as shown below. Therefore, the apparent location of the image is closer to the observer than the actual location of the fish.

---

7. ***REASONING AND SOLUTION*** At night, when it is dark outside and you are standing in a brightly lit room, it is easy to see your reflection in a window. During the day it is not so easy. If we assume that the room is brightly lit by the same amount in both cases, then the light reflected from the window is the same during the day as it is at night. However, during the day, light is coming through the window from the outside. In addition to the reflection, the observer also sees the light that is refracted through the window from the outside. The light from the outside is so intense that it obscures the reflection in the glass.

---

8. ***REASONING AND SOLUTION***

a. The man is using a bow and arrow to shoot a fish. The light from the fish is refracted away from the normal when it enters the air; therefore, the apparent depth of the image of the fish is less than the actual depth of the fish. When the arrow enters the water, it will continue along the same straight line path from the bow. Therefore, in order to strike the fish, the man must aim below the image of the fish. The situation is similar to that shown in Figure 26.5*a*; we can imagine replacing the boat by a dock and the chest by a fish.

b. Now the man is using a laser gun to shoot the fish. When the laser beam enters the water it will be refracted. From the principle of reversibility, we know that if the laser beam travels along one of the rays of light emerging from the water that originates on the fish, it will follow exactly the same path in the water as that of the ray that originates on the fish. Therefore, in order to hit the fish, the man must aim directly at the image of the fish.

---

9. ***REASONING AND SOLUTION*** Two rays of light converge to a point on a screen, as shown below.

A plane-parallel plate of glass is placed in the path of this converging light, and the glass plate is parallel to the screen, as shown below. As discussed in the text, when a ray of light passes through a pane of glass that has parallel surfaces, and is surrounded by air, the

emergent ray is parallel to the incident ray, but is laterally displaced from it. The extent of the displacement depends on the angle of incidence, on the thickness, and on the refractive index of the glass.

As shown in the scale drawing above, the point of convergence does not remain on the screen. It will move away from the glass as shown.

---

10. ***REASONING AND SOLUTION*** Light from the sun is unpolarized; however, when the sunlight is reflected from horizontal surfaces, such as the surface of an ocean, the reflected light is partially polarized in the horizontal direction. Polaroid sunglasses are constructed with lenses made of Polaroid (a polarizing material) with the transmission axis oriented vertically. Thus, the horizontally polarized light that is reflected from horizontal surfaces is blocked from the eyes.

Suppose you are sitting on the beach near a lake on a sunny day, wearing Polaroid sunglasses. When a person is sitting upright, the horizontally polarized light that is reflected from the water is blocked from her eyes, as discussed above, and she notices little discomfort due to the glare from the water. When she lies on her side, the transmission axis of the Polaroid sunglasses is now oriented in a nearly horizontal direction. Most of the horizontally polarized light that is reflected from the water is transmitted through the sunglasses and reaches her eyes. Therefore, when the person lies on her side, she will notice that the glare increases.

---

11. ***REASONING AND SOLUTION*** Light from the sun is unpolarized; however, when the sunlight is reflected from horizontal surfaces such as the surface of a swimming pool, lake, or ocean, the reflected light is partially polarized in the horizontal direction. Polaroid sunglasses are constructed with lenses made of Polaroid (a polarizing material) with the transmission axis oriented vertically. Thus, the horizontally polarized light that is reflected from horizontal surfaces is blocked from the eyes.

If you are sitting by the shore of a lake on a sunny and windless day, you will notice that the effectiveness of your Polaroid sunglasses in reducing the glare of the sunlight reflected from the lake varies depending on the time of the day. As the angle of incidence of the sun's rays increases from 0°, the degree of polarization of the rays in the horizontal direction increases. Since Polaroid sunglasses are designed so that the transmission axes are aligned in the vertical direction when they are worn normally, they become more effective as the sun gets lower in the sky. When the angle of incidence is equal to Brewster's angle, the reflected light is completely polarized parallel to the surface, and the sunglasses are most effective. For angles of incidence greater than Brewster's angle, the glasses again become less effective.

---

12. ***REASONING AND SOLUTION*** According to the *principle of reversibility* (see Section 25.5), if the direction of a light ray is reversed, the light retraces its original path. While the principle of reversibility was discussed in Section 25.5 in connection with the reflection of light rays, it is equally valid when the light rays are refracted. Imagine constructing a mixture of colored rays by passing a beam of sunlight through a prism in the usual fashion. By orienting a second prism so that the rays of colored light are incident on the second prism with angles of incidence that are equal to their respective angles of refraction as they emerge from the first prism, we have a perfectly symmetric situation. The rays through the second prism will follow the reverse paths of the rays through the first prism, and the light emerging from the second prism will be sunlight.

---

13. ***REASONING AND SOLUTION*** For glass (refractive index n_g), the critical angle for the glass/air interface can be determined from Equation 26.4:

$$\sin q_c = \frac{1.0}{n_g} \qquad (1)$$

In Figure 26.7 the angle of incidence at the upper glass/air interface is q_2. Total internal reflection will occur there only if $q_2 \quad q_c$. But q_2 is also the angle of refraction at the lower air/glass interface and can be obtained using Snell's law as given in Equation 26.2:

$$(1.0)\sin q_1 = n_g \sin q_2 \quad \text{or} \quad \sin q_2 = \frac{1.0}{n_g}\sin q_1$$

Using Equation (1) for $1.0/n_g$, we obtain

$$\sin q_2 = \sin q_c \ \sin q_1 \qquad (2)$$

For all incident angles q_1 that are less than $90°$, Equation (2) indicates that $\sin q_2 \quad \sin q_c$, since $\sin q_1 < 1$. Therefore, $q_2 < q_c$ and total internal reflection can not occur at the upper glass/air interface.

---

14. ***REASONING AND SOLUTION***

a. When a rainbow is formed, light from the sun enters a spherical water droplet and is refracted by an amount that depends on the refractive index of water for that wavelength. Light that is reflected from the back of the droplet is again refracted at it reenters the air, as suggested in Figure 26.21. Although all colors are refracted for any given droplet, the observer sees only one color, because only one color travels at the proper angle to reach the observer. The observer sees the full spectrum in the rainbow because each color originates from water droplets that lie at different elevation angles.

As shown in Figure 26.21, the sun must be located *behind* the observer, if the observer is to see the rainbow. Therefore, if you want to make a rainbow by spraying water from a

garden hose into the air, you must stand with the sun behind you, and adjust the hose so that it sprays a fine mist of water in front of you. The distance between the observer and the droplets is not crucial. The important factor is the angle formed by the intersection of the line that extends from the sun to the droplet with the line that extends from the droplet to the observer. *Remark:* When the distance is only a few meters, as it would be in the case of a "garden-hose rainbow", each eye would receive rays from different parts of the mist. Therefore, the observer could see two rainbows that cross over each other.

b. Each color of light that leaves a given droplet travels in a specific direction that is governed by Snell's law. You can't ever walk under a rainbow, because each color that originates from a single droplet travels in a unique direction. To walk under a rainbow, all the colors would have to be refracted vertically downward, which is not the case. Therefore, you can't walk under a rainbow, because the rays are traveling in the wrong directions to reach the observer's eyes.

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15. ***REASONING AND SOLUTION*** A person is floating on an air mattress in the middle of a swimming pool. His friend is sitting on the side of the pool. The person on the air mattress claims that there is a light shining up from the bottom of the pool directly beneath him. His friend insists, however, that she cannot see any light from where she sits on the side.

Rays from a light source on the bottom of the pool will radiate outward from the source in all directions. However, only rays for which the angle of incidence is less than the critical angle will emerge from the water. Rays with an angle of incidence equal to, or greater than, the critical angle will undergo total internal reflection back into the water, as shown in the following figure.

Because of the geometry, the rays that leave the water lie within a cone whose apex lies at the light source. Thus, rays of light that leave the water emerge from within an illuminated circle just above the source. If the mattress is just over the source, it could cover the area through which the light would emerge. A person sitting on the side of the pool would not see any light emerging. Therefore, the statements made by both individuals are correct.

---

16. ***REASONING AND SOLUTION*** Total internal reflection occurs only when light travels from a higher-index medium (refractive index = n_1) toward a lower-index medium (refractive index = n_2). Total internal reflection does not occur when light propagates from

a lower-index to a higher-index medium. The smallest angle of incidence for which total internal reflection will occur at the higher-index/lower-index interface is called the critical angle and is given by Equation 26.4: $\sin q_c = n_2 / n_1$ where $n_1 > n_2$.

A beam of blue light is propagating in glass. When the light reaches the boundary between the glass and the surrounding air, the beam is totally reflected back into the glass. However, red light with the same angle of incidence is not totally reflected and some of the light is refracted into the air. According to Table 26.2, the index of refraction of glass is greater for blue light than it is for red light. From Snell's law, therefore, we can conclude that the critical angle is greater for red light than it is for blue light. Therefore, if the angle of incidence is equal to or greater than the critical angle for blue light, but less than the critical angle for red light, blue light will be totally reflected back into the glass, while some of the red light will be refracted into the air.

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17. ***REASONING AND SOLUTION*** A beacon light in a lighthouse is to produce a parallel beam of light. The beacon consists of a bulb and a converging lens. As shown in Figure 26.22*b*, paraxial rays that are parallel to the principal axis converge to the focal point after passing through the lens. From the principle of reversibility, we can deduce that if a point source of light were placed at the focal point, the emitted light would travel in parallel rays after passing through the lens. Therefore, in the construction of the beacon light, the bulb should be placed at the focal point of the lens.

---

18. ***REASONING AND SOLUTION*** The figure at the right shows a converging lens (in air). The normal to the surface of the lens is shown at five locations on each side of the lens.

A ray of light bends toward the normal when it travels from a medium with a lower refractive index into a medium with a higher refractive index. Likewise, a ray of light bends away from the normal when it travels from a medium with a higher refractive index into a medium with a lower refractive index. When rays of light traveling in air enter a converging lens, they are bent toward the normal. When these rays leave the right side of the lens, they are bent away from the normal; however, since the normals diverge on the right side of the lens, the rays again converge.

If this lens is surrounded by a medium which has a higher index of refraction than the lens, then when rays of light enter the lens, the rays are bent away from the normal, and, therefore, they diverge. When the rays leave the right side of the lens, they are bent toward the normal; however, since the normals diverge on the right side of the lens, the rays diverge further. Therefore, a converging lens (in air) will behave as a diverging lens when it is surrounded by a medium that has a higher index of refraction than the lens.

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19. ***REASONING AND SOLUTION*** A spherical mirror and a lens are immersed in water. The effect of the mirror on rays of light is governed by the law of reflection; namely $q_r = q_i$. The effect of the lens on rays of light is governed by Snell's law; namely, $n_1 \sin q_1 = n_2 \sin q_2$. The law of reflection, as it applies to the mirror, does not depend on the index of refraction of the material in which it is immersed. Snell's law, however, as

it applies to the lens, depends on both the index of refraction of the lens and the index of refraction of the material in which it is immersed. Therefore, compared to the way they work in air, the lens will be more affected by the water.

---

20. ***REASONING AND SOLUTION*** A converging lens is used to project a real image onto a screen, as in Figure 26.27*b*. A piece of black tape is then placed on the upper half of the lens.

The ray diagram above shows the rays from two points on the object, one point at the top of the object and one point on the lower half of the object. As shown in the diagram, rays from both points converge to form the image on the right side of the lens. Therefore, the entire image will be formed. However, since fewer rays reach the image when the tape is present, the intensity of the image will be less than it would be without the tape.

---

21. ***REASONING AND SOLUTION*** When light travels from a material with refractive index n_1 into a material with refractive index n_2, the angle of refraction q_2 is related to the angle of incidence q_1 by Snell's law (Equation 26.2): $n_1 \sin\ q_1 = n_2 \sin\ q_2$.

A converging lens is made from glass whose index of refraction is *n*. The lens is surrounded by a fluid whose index of refraction is also *n*. This situation is known as *index matching* and is discussed in Conceptual Example 7. Since the refractive index of the surrounding fluid is the same as that of the lens, $n_1 = n_2$, and Snell's law reduces to $\sin\ q_1 = \sin\ q_2$. The angle of refraction is equal to the angle of incidence at both surfaces of the lens; the path of light rays is unaffected as the rays travel through the lens. Therefore, this lens cannot form an image, either real or virtual, of an object.

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22. ***REASONING AND SOLUTION*** The expert claims that the height of the window can be calculated from only two pieces of information : (1) the measured height on the film, and (2) the focal length of the camera. The expert is not correct. According to the thin-lens equation (Equation 26.6), $1/d_o + 1/d_i = 1/f$, where d_o is the object distance, d_i is the image distance, and *f* is the focal length of the lens. The magnification equation (Equation 26.7), relates the image and object heights to the image and object distances: $h_o/h_i = d_o/d_i$. These two equations contain five unknowns. To determine any one of

the unknowns, three of the other unknowns must be known. In this case, all that we know is the height of the image, h_i, and the focal length of the camera, f. Therefore, we do not have enough information given to determine the distance from the ground to the window (the height of the object in this case), h_o. We still need to know either the distance from the photographer to the house (the object distance, d_o), or the distance from the center of the lens to the film (the image distance, d_i). We can conclude, therefore, that the expert is incorrect.

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23. ***REASONING AND SOLUTION*** Suppose two people who wear glasses are camping. One is nearsighted, and the other is farsighted. It is desired to start a fire with the sun's rays. A converging lens can be used to focus the nearly parallel rays of the sun on a sheet of paper. If the paper is placed at the focal point of the lens, the sun's rays are concentrated to give a large intensity, so that the paper heats up rapidly and ignites. As shown in Figures 26.36 and 26.37, nearsightedness can be corrected with diverging lenses, and farsightedness can be corrected using converging lenses. Therefore, the glasses of the farsighted person would be useful in starting a fire, while the glasses of the nearsighted person would not be useful.

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24. ***REASONING AND SOLUTION*** A 21-year-old with normal vision (near point = 25 cm) is standing in front of a plane mirror. The near point is the point nearest the eye at which an object can be placed and still produce a sharp image on the retina. Therefore, if the 21-year old wants to see himself in focus, he can stand no closer to the mirror than 25 cm from his image. As discussed in Chapter 25, the image in a plane mirror is located as far behind the mirror as the observer is in front of the mirror. If the 21-year-old is 25 cm from his image, he must be 25 cm/2 = 12.5 cm in front of the mirror's surface. Therefore, he can stand no closer than 12.5 cm in front of the mirror and still see himself in focus.

---

25. ***REASONING AND SOLUTION*** The distance between the lens of the eye and the retina is constant; therefore, the eye has a fixed image distance. The only way for images to be produced on the retina for objects located at different distances is for the focal length of the lens to be adjusted. This is accomplished through with the ciliary muscles. If we read for a long time, our eyes become "tired," because the ciliary muscle must be tensed so that the focal length is shortened enough to bring the print into focus. When the eye looks at a distant object, the ciliary muscle is fully relaxed. Therefore, when your eyes are "tired" from reading, it helps to stop and relax the ciliary muscle by looking at a distant object.

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26. ***REASONING AND SOLUTION*** As discussed in the text, for light from an object in air to reach the retina of the eye, it must travel through five different media, each with a different index of refraction. About 70 % of the refraction occurs at the air/cornea interface where the refractive index of air is taken to be unity and the refractive index of the cornea is 1.38.

To a swimmer under water, objects look blurred and out of focus. However, when the swimmer wears goggles that keep the water away from the eyes, the objects appear sharp and in focus. Without the goggles, light from objects must undergo the first refraction at a

water/cornea interface. Since the index of refraction of water is 1.33 while that of the cornea is 1.38, the amount of refraction is smaller than it is when the person is in air, and the presence of the water prevents the image from being formed on the retina. Consequently, objects look blurred and out of focus. When the swimmer wears goggles, incoming light passes through the volume of air contained in the goggles before it reaches the eyes of the swimmer. The first refraction of the light in the eye occurs at an air/cornea interface. The refraction occurs to the proper extent, so that the image is formed on the retina. Therefore, when the swimmer wears the goggles, objects appear to be sharp and in focus.

---

27. ***REASONING AND SOLUTION*** The refractive power of the lens of the eye is 15 diopters when surrounded by the aqueous and vitreous humors. If this lens is removed from the eye and surrounded by air, its refractive power increases to about 150 diopters. From Snell's law, we know that the effect of the lens on incoming light depends not only on the refractive index of the lens, but also on the refractive index of the materials on either side of the lens. The refractive index of the lens is 1.40, while that of the aqueous humor is 1.33, and that of the vitreous humor is 1.34. Light that leaves the lens has been refracted twice, once when it enters the lens and again when it leaves the lens. Since the refractive indices of these three media are not very different, the amount of refraction at each interface is small. When the lens is surrounded by air, the light is again doubly refracted. In this case, however, the refractive indices at each interface differ substantially, so the amount of refraction at each interface is much larger. Therefore, when the lens is in air, its focal length is much smaller than it is when the lens is in place in the eye. According to Equation 26.8, the refractive power of a lens is equal to $1/f$, where the refractive power is expressed in diopters when the focal length is in meters. The smaller the focal length of the lens, the larger its refractive power. Consequently, the refractive power of the lens is much greater when the lens is surrounded by air.

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28. ***REASONING AND SOLUTION*** A full glass of wine acts, approximately, as a converging lens and focuses the light to a spot on the table. An empty glass consists only of thin glass layers on opposite sides, which do not refract the light enough to act as a lens and produce a focused image.

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29. ***REASONING AND SOLUTION*** The angle q subtended by the image as measured from the principal axis of the lens of the eye is equal to the angle subtended by the object. This angle is called the angular size of both the image and the object and is given by $q \quad h_o / d_o$, where q is expressed in radians.

Jupiter is the largest planet in our solar system. Yet to the naked eye, it looks smaller than Venus. This occurs because the distance from Earth to Jupiter is about 15 times greater than the distance from Earth to Venus, while the diameter of Jupiter is only about 12 times larger than that of Venus. Consequently, the angular size of Jupiter is about 12/15 or 0.80 times as large as that of Venus. Therefore, Jupiter looks smaller than Venus.

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30. ***REASONING AND SOLUTON***
a. The figure below is a ray diagram that shows that the eyes of a person wearing glasses appear to be smaller when the glasses use diverging lenses.

b. The figure below is a ray diagram that shows that the eyes of a person wearing glasses appear to be larger when the glasses use converging lenses.

Notice that in both cases, the eye lies between the focal length of the lens and the lens, and that both images are virtual images.

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31. ***REASONING AND SOLUTION*** As discussed in the text, regardless of the position of a real object, a diverging lens always forms a virtual image that is upright and smaller relative to the object. The figures below show this for two cases: one in which the object is within the focal point, and the other in which the object is beyond the focal point. In each case, the image is smaller than the object. Therefore, a diverging lens cannot be used as a magnifying glass.

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32. ***REASONING AND SOLUTION*** A person whose near point is 75 cm from the eyes, must hold a printed page at least 75 cm from his eyes in order to see the print without blurring, while a person whose near point is 25 cm can hold the page as close as 25 cm and still find the print in focus. If the size of the print is small, it will be more difficult to see the print at 75 cm than at 25 cm, even though the print is in focus. Therefore, the person whose near point is located 75 cm from the eyes will benefit more by using a magnifying glass.

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33. ***REASONING AND SOLUTION*** The angular magnification of a telescope is given by Equation 26.12: $M \quad f_o / f_e$, where f_o is the focal length of the objective, and f_e is the focal length of the eyepiece. In order to produce a final image that is magnified, f_o must be greater than f_e. Therefore if two lenses, whose focal lengths are 3.0 and 45 cm are to be used to build a telescope, the lens with the 45 cm focal length should be used for the objective, and the lens with the 3.0 cm focal length should be used for the eyepiece.

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34. ***REASONING AND SOLUTION*** A telescope consists of an objective and an eyepiece. The objective focuses nearly parallel rays of light that enter the telescope from a distant object to form an image just beyond its focal point. The image is real, inverted, and reduced in size relative to the object. The eyepiece acts like a magnifying glass. It is positioned so that the image formed by the objective lies just within the focal point of the eyepiece. The final image formed by the eyepiece is virtual, upright and enlarged.

Two refracting telescopes have identical eyepieces, although one is twice as long as the other. Since the eyepiece is positioned so that the image formed by the objective lies just within the focal point of the eyepiece, the longer telescope has an objective with a longer focal length. The angular magnification of a telescope is given by Equation 26.12: $M \quad f_o / f_e$, where f_o is the focal length of the objective, and f_e is the focal length of the eyepiece. Both telescopes have the same value for f_e. The longer telescope has the larger value of f_o; therefore, the longer telescope has the greater angular magnification.

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35. ***REASONING AND SOLUTION*** In a telescope the objective forms a first image just beyond the focal point of the objective and just within the focal point of the eyepiece. Thus, as Figure 26.42 shows, the distance between the two converging lenses is $L \quad f_o + f_e$. For the two lenses specified, this would mean that L 4.5 cm + 0.60 cm = 5.1 cm. But L is given as L = 14 cm, which means that there is a relatively large separation between the focal points of the objective and the eyepiece. This arrangement is like that for a microscope shown in Figure 26.33. Thus, the instrument described in the question is a microscope.

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36. ***REASONING AND SOLUTION***

a. A projector produces a real image at the location of the screen.

b. A camera produces a real image at the location of the film.

c. A magnifying glass produces a virtual image behind the lens.

d. Eyeglasses produce virtual images that the eye then sees in focus.

e. A compound microscope produces a virtual image.

f. A telescope produces a virtual image.

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37. ***REASONING AND SOLUTION*** Chromatic aberration occurs when the index of refraction of the material from which a lens is made varies with wavelength. Lenses obey Snell's law. If the index of refraction of a lens varies with wavelength, then different colors of light that pass through the lens refract by different amounts. Therefore, different colors come to a focus at different points. Mirrors obey the law of reflection. The angle of reflection depends only on the angle of incidence, regardless of the wavelength of the incident light; therefore, chromatic aberration occurs in lenses, but not in mirrors.

CHAPTER 27 | *INTERFERENCE AND THE WAVE NATURE OF LIGHT*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A radio station broadcasts simultaneously from two transmitting antennas at two different locations. The radio that receives the broadcast could have better reception depending on the location of the receiving antenna. According to the principle of linear superposition, when the electromagnetic waves from the transmitting antenna arrive at the same point, the resultant wave is the sum of the individual waves. The amplitude of the resultant wave depends on the relative phase between the two waves. The relative phase between the waves depends on the path length difference between the two waves. If the two waves arrive at the receiving antenna so that the path length difference between them is equal to an integer number of wavelengths, the two waves will be in phase, they will reinforce each other, and constructive interference will occur. The resulting amplitude of the radio wave will be larger than it would be from either transmitting antenna alone; therefore, the radio reception will be better. If the two waves arrive at the receiving antenna so that the path length difference between them is equal to an odd multiple of half of a wavelength, the two waves will be exactly out of phase, they will mutually cancel, and destructive interference will occur. The receiving antenna will receive little or no signal in this situation. As a result, the radio reception will be very bad. Thus, having two transmitting antennas does not necessarily lead to better reception.

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2. ***REASONING AND SOLUTION*** Constructive interference occurs between waves from two in-phase sources when the difference in travel distances from the sources to the point where the waves combine is the same or equal to an integer number of wavelengths. For a square the distance from any corner to the center is the same. Hence, constructive interference occurs at the center.

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3. ***REASONING AND SOLUTION***

   a. The light waves coming from *both* slits in a Young's double slit experiment have their phases shifted by an amount equivalent to half a wavelength. Since the light from both slits is changed by the same amount, the relative phase difference between the light from the two slits is zero when the light leaves the slits. When the light reaches the screen, the relative phase difference between light waves from the two slits will be the same as if the phase of the waves had not been shifted at the slits. Therefore, the pattern will be exactly the same as that described in the text.

   b. Light coming from *only one* of the slits in a Young's double slit experiment has its phase shifted by an amount equivalent to half of a wavelength. Now, there is a relative phase difference of one half of a wavelength between the light leaving the slits. When the light reaches the screen, there will be a relative phase difference due to the fact that light from each slit, in general, traveled along different paths. In addition, there will be the initial

phase difference that is equivalent to half of a wavelength. Therefore, the pattern will be similar to that described in the text; however, the points of constructive interference will be points of destructive interference, and the points of destructive interference will be points of constructive interference. In other words, the positions of the light and dark fringes are interchanged.

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4. ***REASONING AND SOLUTION*** According to Equation 27.1, the bright fringes of a double slit experiment occur at values of q for which $\sin q = m l / d$ where m = 0, 1, 2, 3, ... According to Equation 27.2, the dark fringes occur at values of q for which $\sin q = [m + (1/2)] l / d$ where m = 0, 1, 2, 3, . . . The slits S_1 and S_2 in Figure 27.4 are replaced with identical loudspeakers and use the same ac electrical signal to drive them. The two sound waves produced will then be identical, and we have the audio equivalent of Young's double slit experiment. Sound waves that reach the center of the screen have traveled along paths of equal length; therefore, the sound waves from each loudspeaker arrive there in phase and constructive interference occurs. Consequently, at the center point on the screen, the sound will have maximum loudness. As you proceed to walk away from the center (in either direction), the sound will decrease in volume. When the angle q is such that $\sin q = (1/2) l / d$, the sound will decrease to zero amplitude. If you continue to walk beyond this point, the loudness of the sound will increase. It will reach a maximum (a point of constructive interference) when the angle q is such that $\sin q = l / d$. The sound will again decrease in loudness until the next point of destructive interference occurs. This point occurs when $\sin q = (3/2) l / d$. In general, there will a pattern of alternating loud and soft sounds heard as you walk away from the center of the screen. The intensity at the points of constructive interference varies in a manner that is similar to that shown in Figure 27.6. The central maximum has the greatest intensity. The sound at each region of constructive interference decreases in intensity as one proceeds away from the center (in either direction).

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5. ***REASONING AND SOLUTION*** The angle q for the interference maximum in Young's double-slit experiment is given by Equation 27.1: $\sin q = m l / d$ where m = 0, 1, 2, 3, . . . When the wavelength l of the light is greater than the distance d between the slits, the ratio l / d is greater than one; however, $\sin q$ cannot be greater than one. Therefore, it is *not* possible to see interference fringes when the wavelength of the light is greater than the distance between the slits.

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6. ***REASONING AND SOLUTION*** A camera lens is covered with a nonreflective coating that eliminates the reflection of perpendicularly incident green light. If the light were incident on the nonreflective coating at 45°, rather than perpendicularly, it would not be eliminated by the coating and an observer would see it.

Using reasoning similar (but not identical to) to that in Example 3, if the refractive index of the film is less than the refractive index of the lens, the minimum nonzero thickness of the nonreflecting coating is given by $t = l_{\text{coating}} / 4$. This result follows from the fact that

the extra distance traveled by the wave that travels through the coating is $2t$ when the wave strikes the film at normal incidence.

If the wave strikes the thin film at 45°, then the extra distance traveled by the wave that travels through the coating is no longer $2t$. Instead, it is $2d$, where d is the distance shown in the figure at the right. The value of d depends on the value for q_2, the angle of refraction. This angle, in turn, depends on the refractive index n_1 for air and n_2 for the lens coating, according to Snell's law (Equation 26.2). To eliminate the light that is incident at 45°, the coating would have to have a minimum nonzero thickness such that $d = l_{\text{coating}}/4$. Since the coating is designed such that $t = l_{\text{coating}}/4$, it does not eliminate the light incident at 45°.

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7. ***REASONING AND SOLUTION*** When sunlight reflects from a thin film of soapy water, the film appears multicolored, in part because destructive interference removes different wavelengths from the light reflected at different places, depending on the thickness of the film. As the film becomes thinner and thinner, the path length difference between the light reflected from the top of the film and the light reflected from the bottom of the film becomes closer to zero. The light reflected from the top of the film undergoes a phase change equivalent to half of a wavelength. Therefore, as the thickness of the film approaches zero, the light reflected from the top of the film is almost exactly out of phase with the light reflected from the bottom of the film. Therefore, as the film becomes thinner and thinner, it looks darker and darker in reflected light. It appears black just before it breaks, because at that point destructive interference occurs between the light reflected from the top and the light reflected from the bottom of the film.

---

8. ***REASONING AND SOLUTION*** Two pieces of the same glass are covered with thin films of different materials. The thickness t of each film is the same. In reflected sunlight, however, the films have different colors.

This could occur because, in general, different materials have different refractive indices. Using reasoning similar to (but not identical to) that in Example 3, the wavelength l_{air} that is removed from the reflected light corresponds to a value of l_{film} in the film that satisfies either of the following conditions:

$$2t = m\,l_{\text{film}}, \quad m = 0,\ 1,\ 2,\ 3,\ \ldots \qquad n_{\text{film}} > n_{\text{glass}}, \tag{1}$$

and

$$2t = \left(m + \tfrac{1}{2}\right) l_{\text{film}}, \quad m = 0,\ 1,\ 2,\ 3,\ \ldots \qquad n_{\text{film}} < n_{\text{glass}}. \tag{2}$$

However, the value of the wavelength l_{film} of the light in the film depends on the index of refraction of the film. According to Equation 27.3, $l_{\text{film}} = l_{\text{vacuum}} / n$, where n is the index of refraction of the film. If the index of refraction of the two materials are different, then conditions (1) and (2) will be satisfied for two different values of l_{film}. These values of l_{film} correspond to different values of l_{air}, and, therefore, different wavelengths will be removed from the reflected light.

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9. ***REASONING AND SOLUTION*** When a spherical surface is put in contact with an optically flat plate, circular interference patterns, called Newton's rings, can be observed in reflected light, as shown in Figure 27.14*b*. There is a dark spot at the center of the pattern of Newton's rings. The dark spot occurs for the following reason. At the center of the plate, the spherical surface is in contact with the plate. Destructive interference occurs there, because the thickness of the "air wedge" is essentially zero at this point. Therefore, the only difference in phase between the light reflected from the bottom of the spherical surface and the light reflected from the top of the optical flat is the half-wavelength phase change due to reflection from the optical flat. This situation is similar to that discussed in part (b) of Example 5 in the text.

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10. ***REASONING AND SOLUTION*** A thin film of material is floating on water (n = 1.33). When the material has a refractive index of n = 1.20, the film looks bright in reflected light as its thickness approaches zero. But when the material has a refractive index of n = 1.45, the film looks black in reflected light as its thickness approaches zero.

When the material has a refractive index of n = 1.20, both the light reflected from the top of the film and the light reflected from the bottom occur under conditions where light is traveling through a material with a smaller refractive index toward a material with a larger refractive index. Therefore, both the light reflected from the top and the bottom undergoe a phase change upon reflection. Both phase changes are equivalent to half of a wavelength. Therefore, the reflections introduce no net phase difference between the light reflected from the top and bottom. As the thickness of the film approaches zero, the path difference between the light reflected from the top and the bottom of the film approaches zero. Since the path difference is close to zero, and there is no relative phase difference due to reflection, the light reflected from the top will be in phase with the light reflected from the bottom of the film; constructive interference will occur, and the film looks bright in reflected light.

When the material has a refractive index of n = 1.45, only the light reflected from the top of the film is traveling through a material with lower refractive index toward a material with a larger refractive index. Therefore, only the light reflected from the top undergoes a phase change upon reflection. This phase change is equivalent to half of a wavelength. Therefore, there is a net phase difference equivalent to half of a wavelength between the light reflected from the top and bottom of the film. As the thickness of the film approaches zero, the path difference between the light reflected from the top and the light reflected from the bottom of the film approaches zero. Since the path difference is close to zero, and there is a relative phase difference due to reflection that is equal to half of a wavelength, the light reflected from the top will be exactly out of phase with the light reflected from the bottom of the film, and destructive interference will occur. Therefore, the film looks black in reflected light.

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11. ***REASONING AND SOLUTION*** A transparent coating is deposited on a glass plate and has a refractive index that is larger than that of the glass. For a certain wavelength within the coating, the thickness of the coating is a quarter wavelength. The coating enhances the reflection of the light corresponding to this wavelength.

As discussed in Example 3, only the light reflected from the top of the coating is traveling in through a material (air) with lower refractive index toward a material coating with a larger refractive index. Therefore, only the light reflected from the top of the coating undergoes a phase change upon reflection. This change in phase is equivalent to half of a wavelength. Therefore, there is a net phase difference equivalent to half of a wavelength between the light reflected from the top and light reflected from the bottom of the coating. Since the thickness of the coating is a quarter wavelength, the light reflected from the bottom of the coating traverses a distance equal to two times a quarter wavelength, or half a wavelength, as it travels through the coating. The overall relative phase difference between the light reflected from the top and the light reflected from the bottom of the coating is equivalent to half of a wavelength due to the reflection from the top of the coating and another half of a wavelength due to the path difference between the light reflected from the top and the light reflected from the bottom. Thus, the overall phase difference is equivalent to a whole wavelength, and the light reflected from the top is in phase with the light reflected from the bottom. Constructive interference occurs, and the coating enhances light corresponding to that wavelength.

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12. ***REASONING AND SOLUTION*** Rayleigh's criterion states that two point objects are just resolved when the first dark fringe in the diffraction pattern of one falls directly on the central bright fringe in the diffraction pattern of the other. Equation 27.6 approximates the minimum angular displacement between the two objects so that they are just resolved by an optical instrument: $q_{\min} \quad 1.22\, l / D$, where l is the wavelength of light used and D is the diameter of the aperture of the optical instrument. According to Equation 27.6, optical instruments with the highest resolution should have the largest possible diameter D and utilize light of the shortest possible wavelength.

The *f*-number setting on a camera gives the ratio of the focal length of the camera lens to the diameter of the aperture through which the light enters the camera. Therefore, smaller *f*-numbers mean larger aperture diameters. According to Equation 27.6, if we wish to resolve two closely spaced objects in a picture, we should adjust the aperture to have the largest possible diameter. As a result, we should use a small *f*-number setting.

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13. ***REASONING AND SOLUTION*** As discussed in the text, the extent to which a wave bends or diffracts around the edges of an obstacle or opening is determined by the ratio l/W, where l is the wavelength of the wave and W is the width of the obstacle or opening. As the ratio l/W becomes larger, diffraction effects become more pronounced.

The wavelength of light is extremely small and is normally expressed in nanometers ($1\text{ nm} = 10^{-9}\text{ m}$). Therefore, the ratio l/W is very small for light, and when light passes through a typical doorway, diffraction effects are not noticeable. Many sound waves

encountered in the everyday world have wavelengths that are comparable to the dimensions of a typical doorway. Therefore, the ratio l/W is nearly unity for these sounds, and when sound passes through a doorway, diffraction effects are noticeable; you can hear around corners, although you cannot see around corners.

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14. ***REASONING AND SOLUTION*** The Rayleigh criterion is $q_{\min}$ 1.22 l/D, where l is the wavelength of the light, D is the diameter of the opening into the eye, and $q_{\min}$ is the minimum angle that two point objects must subtend at the opening in order to be resolved by the eye. In this case D is the opening through which light from the painting enters the eye. Before squinting, the angle subtended at the eye is greater than $q_{\min}$, since the dots are distinguishable. Because of the squinting, light reaches the retina through an opening that is smaller than the normal size of the eye's pupil. A smaller value for D, with a given wavelength, means that the value for $q_{\min}$ is larger. Thus, after squinting, the angle subtended at the eye is smaller than the larger value for $q_{\min}$, and the dots of color can no longer be individually resolved, which gives the painting a more normal look.

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15. ***REASONING AND SOLUTION*** Four light bulbs are arranged at the corners of a rectangle that is three times longer than it is wide. You look at this arrangement perpendicular to the plane of the rectangle. From very far away, your eyes cannot resolve the individual bulbs and you see a single rectangular "smear" of light. From close in, you see the individual bulbs. The figures below illustrate what you would see at two positions between these two extremes.

In the first case, you are still far away, but you have moved close enough so that you can begin to resolve the bulbs at opposite ends of the diagonal. Essentially, you see a rectangular "smear" of light with a gap in the middle.

In the second case, you have moved closer still and now can resolve the two bulbs on one of the short sides of the rectangle from the two on the other short side. Essentially, you see two "smears" of light, each smear consisting of the two bulbs at the ends of a short side of the rectangle.

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16. ***REASONING AND SOLUTION*** The resolving power, or the ability to resolve two closely spaced images, through any circular aperture (such as the pupil of your eye) is limited by diffraction effects. The Rayleigh criterion for resolution focuses on these effects and is given by $q_{\min} \quad 1.22\, l / D$ (Equation 27.6), where l is the wavelength of light, D is the diameter of the aperture, and $q_{\min}$ is the minimum angle that can be resolved between two point objects by a circular aperture. A smaller value for $q_{\min}$ or l/D means a greater resolving power.

a. Suppose the pupil of your eye were elliptical instead of circular in shape, with the long axis of the ellipse oriented in the vertical direction. Since the resolving power depends on the diameter of the aperture and since the ellipse is longer than it is wide, the resolving power of your eye would not be the same in the horizontal and vertical directions.

b. If the pupil of your eye were elliptical instead of circular in shape, with the long axis of the ellipse oriented in the vertical direction, the "diameter" or "width" would be larger in the vertical direction and smaller in the horizontal direction. Consider two "planes" of light rays. Plane 1 is oriented so that neighboring rays are horizontal, and plane 2 is oriented so that neighboring rays are stacked vertically. Since the "diameter" of your eye is largest in the vertical direction, the ratio l/D is smaller for light in plane 2 than it is for light in plane 1. Thus, diffraction effects will be less noticeable for light rays stacked vertically. Therefore, the resolving power will be greater in the vertical direction.

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17. ***REASONING AND SOLUTION*** Rayleigh's criterion states that two point objects are just resolved when the first dark fringe in the diffraction pattern of one falls directly on the central bright fringe in the diffraction pattern of the other. Equation 27.6 approximates the minimum angular displacement $q_{\min}$ between the two objects so that they are just resolved by an optical instrument: $q_{\min} \quad 1.22\, l / D$, where l is the wavelength of light used and D is the diameter of the aperture of the optical instrument. According to Equation 27.6, optical instruments with the highest resolution should have the largest possible diameter D and utilize light of the shortest possible wavelength.

Suppose you were designing an eye and could select the size of the pupil and the wavelength of the electromagnetic waves to which the eye is sensitive. As far as the limitation created by diffraction is concerned, the following list ranks the design choices in order of decreasing resolving power (greatest first): (a) large pupil (large D) and ultraviolet wavelengths (small l); (c) small pupil (small D) and ultraviolet wavelengths (small l); (b) small pupil (small D) and infrared wavelength (larger l).

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18. ***REASONING AND SOLUTION*** In our discussion of single-slit diffraction, we ignored the height of the slit, in effect assuming that the height was much larger than the width. Suppose that the height and width were the same size, so that diffraction occurred in both dimensions. The resulting pattern would change in the following way. The height of the horizontal fringes would be reduced. Furthermore, there would be two sets of fringes. One set would be along the horizontal axis, and the other set would be along the vertical axis. The two sets would share a common central maximum. The intensity of the central

maximum will be greatest. The intensity will decrease on either side of the central maximum, in both the horizontal and the vertical directions. The figure below shows the central maximum and the less intense higher order on either side of the central maximum in both dimensions.

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19. ***REASONING AND SOLUTION*** According to Equation 27.7, the angles q that locate principal maxima of a diffraction grating satisfy the condition $\sin q = ml/d$, where $m = 0$, 1, 2, 3, . . .

If the entire interference apparatus of a diffraction grating (light source, grating, and screen) were immersed in water, the wavelength of the light would *decrease* from the value l_{air} to the value l_{water}. This is because these values are related by Equation 27.3, $l_{\text{water}} = l_{\text{vacuum}}/n_{\text{water}} = l_{\text{air}}/n_{\text{water}}$, where we have used the fact that l_{vacuum} and l_{air} are nearly the same, since n_{air} is nearly 1. Thus, angles q that locate principal maxima of a diffraction grating immersed in water will be smaller than when the apparatus is immersed in air. Consequently, the principle maxima will be closer together when the apparatus is immersed in water relative to their location when the entire apparatus is immersed in air.

CHAPTER 28 | *SPECIAL RELATIVITY*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The speed of light postulate states that the speed of light in *a vacuum*, measured in any inertial reference frame, always has the same value of c, no matter how fast the source of the light and the observer are moving relative to each other.

   The speed of light in water is c/n, where $n = 1.33$ is the refractive index of water. Thus, the speed of light in water is less than c. This does not violate the speed of light postulate, because the postulate refers to the speed of light in a vacuum, not in a physical medium.

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2. ***REASONING AND SOLUTION*** A baseball player at home plate hits a pop fly straight up (the beginning event) that is caught by the catcher at home plate (the ending event).

   a. A spectator in the stands is at rest relative to these events; therefore, this spectator would record the proper time interval between them t_0.

   b. A spectator sitting on the couch and watching the game on TV is at rest relative to these events; therefore, this spectator would also record the proper time interval between them t_0.

   c. The third baseman running in to cover the play is moving relative to the events; therefore, the third baseman *will not* record the proper time interval between them. He will record a dilated time interval t given by the time-dilation equation (Equation 28.1): $t = t_0 \Big/ \sqrt{1 \quad v^2/c^2}$, where v is the relative speed between the observer who measures t_0 and the observer who measures t.

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3. ***REASONING AND SOLUTION*** The earth spins on its axis once each day. To a person viewing the earth from an inertial reference frame in space, a clock at the equator would run more slowly than a clock at the north (magnetic) pole. The observer in the inertial reference frame is not in the rest frame of either clock, so he will measure a dilated time for both clocks. According to the time dilation equation (Equation 28.1): $t = t_0 \Big/ \sqrt{1 \quad v^2/c^2}$, where v is the relative speed between the observer who measures t_0 and the observer who measures t. Both clocks move around the earth's rotation axis with the same angular speed w as that of the earth. The linear speed of each clock is given by $v = rw$, where r is the distance from the clock to the rotation axis. The clock at the equator has the greater value of r, and, therefore, the greater linear speed v. According to Equation 28.1, the observer in the inertial reference frame will see the clock with the larger linear speed v register the longer time interval t. Therefore, the clock at the equator will appear to run slower when viewed by the inertial observer.

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4. ***REASONING AND SOLUTION***

a. You are standing at a railroad crossing watching a train go by. Both you and a passenger in the train are looking at a clock on the train. The passenger on the train is at rest relative to the clock; therefore, the passenger on the train measures the proper time interval.

b. A passenger in the train car is at rest relative to the car; therefore, the passenger on the train measures the proper length of the train car.

c. Since you are standing at the railroad crossing, you are at rest relative to the track; therefore, you measure the proper distance between the railroad ties under the track.

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5. ***REASONING AND SOLUTION*** There are tables that list data for the various particles of matter that physicists have discovered. Often, such tables list the masses of the particles in units of energy, such as MeV, rather than kilograms. This is possible because of the equivalence between mass and energy. The total energy E of a moving object is related to its mass m and its speed v by Equation 28.4: $E = mc^2 \Big/ \sqrt{1 \quad v^2/c^2}$. The mass of a particle listed in the table is actually the total energy of the particle when it is at rest (v = 0 m/s); the rest energy of an object of mass m is given by Equation 28.5: $E_0 = mc^2$.

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6. ***REASONING AND SOLUTION*** When an object is accelerated from rest to a speed v, the object acquires kinetic energy in addition to its rest energy. The total energy E of the object is the sum of its rest energy E_0 and its kinetic energy KE. According to Equation 28.6, the kinetic energy of an object of mass m moving at speed v is $\text{KE} = E \quad E_0 = mc^2 \left(1 \Big/ \sqrt{1 \quad v^2/c^2}\right) \quad 1$. The work required to accelerate an object from rest to speed v is, from the work-energy theorem (Chapter 6), equal to the change in the kinetic energy of the object. Since the object begins at rest, its change in kinetic energy is equal to its final kinetic energy and is given by Equation 28.6.

According to Equation 28.6, it is easier to accelerate an electron to a speed that is close to the speed of light, compared to accelerating a proton to the same speed. This is because the mass of the electron is much less than that of the proton. Hence, the final kinetic energy of the proton, will be greater than that of the electron at the same speed v. Therefore, the proton will require more work to get it to the same speed as the electron.

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7. ***REASONING AND SOLUTION*** Light travels in water at a speed of $2.26 \quad 10^8$ m/s. It is possible for a particle that has mass to travel through water at a faster speed than this. But objects that have mass *cannot reach the speed of light in a vacuum*, that is, they cannot attain a speed of $c = 3.00 \quad 10^8 \text{ m/s}$. It is the speed of light in a vacuum that is the ultimate speed, not the speed of light in a material like water.

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8. ***REASONING AND SOLUTION*** Two positive, electric charges separated by a finite distance have more mass than when they are infinitely far apart. The electric potential energy of the two-particle system is zero when the particles are infinitely far apart. Work is required to bring two positive charges in from infinity, so that there is a finite separation between them; therefore, the electric potential energy and consequently the total energy of the two-particle system increases as they are brought in from infinity. Consistent with the theory of special relativity, any increase in the total energy of the system, including a change in the electric potential energy, is equivalent to an increase in the mass of the system. Therefore, the two-particle system has more mass when the particles have a finite distance between them, than when they are infinitely far apart.

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9. ***REASONING AND SOLUTION*** One system consists of two stationary electrons separated by a distance r. Another consists of a positron and an electron, both stationary and separated by the same distance r. A positron has the same mass as an electron, but it has a positive electric charge $+e$. Since the charges of unlike sign attract each other, and the charges of like sign repel each other, more work is required to bring the charges from infinity to assemble the system that consists of the two stationary electrons. Since more work is required to assemble the system of two stationary electrons, that system has the greater electric potential energy. Consistent with mass-energy equivalence predicted by the theory of special relativity, the system with the larger total energy will have the greater mass. Therefore, the system that consists of two stationary electrons has the greater mass.

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10. ***REASONING AND SOLUTION*** A parallel plate capacitor is initially uncharged. The capacitor is charged up by removing electrons from one plate and placing them on the other plate. Work and, therefore, energy is required to charge the capacitor. It is stored in the electric field between the plates of the charged capacitor in the form of electric potential energy. Since the charged capacitor has more electric potential energy than the uncharged capacitor, its also has more total energy than the uncharged capacitor. Therefore, from the equivalence of mass and energy, we can conclude that the charged capacitor has more mass.

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11. ***REASONING AND SOLUTION*** The speed limit on many interstate highways is 65 miles per hour. If the speed of light were 65 miles per hour, you would never be able to drive at the speed limit. When the speed, v, of an object is equal to the speed of light, c, the term $1\Big/\sqrt{1 \quad v^2/c^2}$ becomes infinitely large. Therefore, according to Equation 28.6, the kinetic energy of the object would be infinitely large. According to the work-energy theorem, an infinite amount of work would be required to increase the speed of the object to $v = c$. Since an infinite amount of work is not available, it would *not* be possible for the car to reach the speed limit.

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12. ***REASONING AND SOLUTION*** A person is approaching you in a truck that is traveling very close to the speed of light. This person throws a baseball toward you. Relative to the truck, the ball is thrown with a speed nearly equal to the speed of light, so the person on the truck sees the baseball move away from the truck at a very high speed. Yet you see the baseball move away from the truck very slowly.

The relative velocities in this problem are:

v_{BY} = velocity of the **B**aseball relative to **Y**ou
v_{BT} = velocity of the **B**aseball relative to the **T**ruck
v_{TY} = velocity of the **T**ruck relative to **Y**ou

These velocities are related by the velocity-addition formula (Equation 28.7),

$$v_{BY} = \frac{v_{BT} + v_{TY}}{1 + \frac{v_{BT} v_{TY}}{c^2}}$$

If both v_{BT} and v_{TY} are close to the speed of light, $v_{BY} \quad (c+c)/[1+c^2/c^2] = 2c/2 = c$, and we can conclude that v_{BY} is very close to c. Since both v_{BY} and v_{TY} are measured relative to the *same* inertial reference frame, namely your reference frame, you see the baseball leaving the truck at a relative speed that is the difference between the two speeds, that is, $v_{BY} - v_{TY}$. Since v_{BY} and v_{TY} are unequal, but close to the speed of light, the difference $v_{BY} - v_{TY}$ will be very small. Therefore, you see the baseball move away from the truck very slowly.

---

13. ***REASONING AND SOLUTION*** The quantities listed are (a) the time interval between two events, (b) the length of an object, (c) the speed of light in a vacuum, and (d) the relative speed between the observers.

a. The time interval between two events depends on the reference frame in which the time measurements are made. An observer in the rest frame of the two events will measure the proper time interval. In any other inertial reference frame in motion relative to the rest frame of the events, the time interval will be longer or dilated. Therefore, two observers in relative motion will not, in general, agree on the measured value of the time interval between two events.

b. The length of an object depends on the reference frame in which the length measurement is made. An observer in the rest frame of the object will measure the proper length of the object. In any other inertial reference frame in motion relative to the rest frame of the object, the length will be shorter or contracted in the direction of motion. Therefore, two observers in relative motion will not, in general, agree on the measured value of the length of the object.

c. According to the speed of light postulate, the speed of light in a vacuum will be the same in all inertial reference frames that are in motion relative to each other. Therefore, two observers will always measure the speed of light to be the same, regardless of the relative velocity between them.

d. Since there is no preferred inertial reference frame, two observers in inertial reference frames that are in motion relative to each other will always measure the same relative speed between them, regardless of the relative velocity.

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14. ***REASONING AND SOLUTION*** If the speed of light were infinitely large, instead of $3 \quad 10^8$ m/s, the effects of time dilation and length contraction would *not* be observable. According to the time dilation equation (Equation 28.1), $t = \quad t_0 \Big/ \sqrt{1 \quad v^2/c^2}$, where v is the relative speed between the observer who measures t_0 and the observer who measures t. According to the length contraction equation (Equation 28.2), $L = L_0\sqrt{1 \quad v^2/c^2}$, where v is the relative speed between the observer who measures L_0 and the observer who measures L. If c were infinite, then the factor v^2/c^2 would be equal to zero, $\sqrt{1 \quad v^2/c^2}$ would be equal to unity, and it would follow that $t = \quad t_0$ and $L = L_0$. In other words, the time interval for an event would be the same in all inertial reference frames, regardless of the relative speed between them. Similarly, the length of an object would be the same in all inertial reference frames, regardless of the relative speed between them. Therefore, if c were infinitely large, time dilation and length contraction would not be observable.

CHAPTER 29 | *PARTICLES AND WAVES*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A monochromatic light source emits photons of a single frequency. According to Equation 29.2, the energy, E, of a single photon is related to its frequency, f, by the relation $E = hf$, where h is Planck's constant.

   The photons emitted by a source of light do *not* all have the same energy. Since the photons do not all have the same energy, then, from Equation 29.2, we can conclude that the photons do not all have the same frequency. Therefore, the source is not monochromatic.

---

2. ***REASONING AND SOLUTION*** According to the data given in Example 1, Chapter 24, the frequency of visible light ranges from $4.0 \quad 10^{14}$ Hz (red light) to $7.9 \quad 10^{14}$ Hz (violet light). According to Equation 29.2, the energy, E, of a photon is related to its frequency, f, by the relation $E = hf$, where h is Planck's constant. According to Equation 29.2, the energy of a photon is directly proportional to its frequency.

   a. The red-colored light bulb emits photons with the lowest frequency compared to light bulbs of other colors (orange, yellow, green, or blue); therefore, the red-colored light bulb emits photons with the lowest energy.

   b. The color blue appears next to violet in the continuous visible spectrum; therefore, the frequency of blue light is slightly smaller than that of violet, but greater than the frequency of other colors of the visible spectrum. Thus, the blue-colored light bulb emits photons with the highest frequency compared to the other light bulbs; therefore, the blue-colored light bulb emits photons with the greatest energy.

---

3. ***REASONING AND SOLUTION*** A photon emitted by a higher-wattage red light bulb *does not* have more energy than a photon emitted by a lower-wattage red bulb. The wattage of a bulb describes the power output of a bulb. Since average power is defined as energy per unit time, the power output of a light bulb tells us the *rate* at which the light bulb produces energy. According to Equation 29.2, the energy, E, of a photon is related to its frequency, f, by the relation $E = hf$, where h is Planck's constant. Thus, the energy of a photon depends *only* on the frequency of the associated light wave. The frequency of red light is the same, regardless of the rate of energy production; therefore, all photons of red light have the same energy, regardless of the nature of their source.

   Remark: Since the higher-wattage bulb provides more energy per unit time than the lower-wattage bulb, we can conclude that the higher-wattage bulb produces *more photons* per unit time. All of the "red photons," however, have the same energy.

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4. ***REASONING AND SOLUTION*** When a sufficient number of visible light photons strike a piece of photographic film, the film becomes exposed. An X-ray photon is more energetic than a visible light photon. Yet, most photographic films are not exposed by the X-ray machines used at airport security checkpoints. Since a single X-ray photon is more energetic than a single photon of visible light, we can conclude that the number of X-ray photons per unit time emitted by airport security machines is much smaller than the number of visible light photons per unit time produced by normal lighting fixtures.

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5. ***REASONING AND SOLUTION*** When radiation strikes a metallic surface, a photon of the radiation can give up its energy to an electron in the metal. If the photon has enough energy to perform the minimum amount of work required to remove the electron from the metal, the electron can be emitted from the metal. The minimum amount of work required to remove an electron from a metal is called its *work function*. If the photon does not have enough energy to remove an electron, the energy of the photon is absorbed by the electron; the increase in energy is manifested as thermal motion.

   Radiation of a given wavelength causes electrons to be emitted from the surface of one metal (metal 1) but not from the surface of another metal (metal 2). We can conclude that the individual photons of the radiation have sufficient energy to do the work required to remove the electrons from the surface of metal 1, but not enough energy to remove the electrons from the surface of metal 2. Therefore, the work function of metal 1 must be less than the work function of metal 2.

---

6. ***REASONING AND SOLUTION*** In a photoelectric effect experiment, the intensity of the light is increased while the frequency is kept constant. The frequency is greater than the minimum frequency f_0, so that photoelectrons are emitted from the negatively charged metal plate (see Figure 29.4).

   Light intensity is the energy per second per unit area that crosses a transverse surface. According to Einstein's photon hypothesis (see Equation 29.2), a beam of light is composed of individual photons, each having an energy $E = hf$, where h is Planck's constant and f is the frequency of the light. From this viewpoint, the intensity of the light is equal to the energy of each photon multiplied by the number of photons per second per unit area crossing a transverse surface. An increase in the intensity of the light beam, therefore, corresponds to an increase in the number of photons per second that strikes the negatively charged metal plate. This will result in the emission of an increased number of photoelectrons per second at the negative plate.

   a. As the number of photoelectrons emitted per second from the negative plate increases, the number of photoelectrons collected per second at the positive plate increases. Since the current in the phototube is proportional to the number of photoelectrons per second collected at the positive plate, increasing the light intensity will cause the current in the phototube to increase.

   b. As discussed above, an increase in intensity will cause the number of photoelectrons emitted per second from the metal surface to increase.

c. According to Equation 29.3, the maximum kinetic energy that a photoelectron could have is given by $\mathrm{KE}_{\max} = hf \quad W_0$, where W_0 is the work function of the metal. Since the frequency is kept constant, and W_0 is a property of the metal that is independent of the intensity, an increase in intensity will have no effect on $\mathrm{KE}_{\max}$. The maximum kinetic energy that an electron could have remains the same.

d. The kinetic energy of an object of mass m moving non-relativistically with speed v can be expressed in terms of its momentum p as follows:

$$\mathrm{KE} = \tfrac{1}{2}mv^2 = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

Therefore, treating the electron classically, the maximum momentum that a photoelectron could have is $p_{\max} = \sqrt{2m\,(\mathrm{KE}_{\max})}$. Since the maximum kinetic energy that an electron could have remains the same as the intensity is increased at constant frequency, the maximum momentum that an electron could have remains the same as well.

e. According to Equation 29.8, the minimum de Broglie wavelength of an object of maximum momentum $p_{\max}$ is given by

$$l_{\min} = \frac{h}{p_{\max}}$$

Since the maximum momentum that a photoelectron could have remains the same as the intensity is increased at constant frequency, we can conclude that the minimum de Broglie wavelength that an electron could have also remains the same.

---

7. ***REASONING AND SOLUTION*** As the result of a Compton scattering experiment, an electron is accelerated straight ahead in the same direction as that of the incident X-ray photon. Momentum conservation requires that the total initial momentum be equal to the total final momentum. The total initial momentum consists only of the forward momentum of the incoming photon, which we assume to be traveling in the $+x$ direction. The electron is initially at rest. Therefore, the direction of the final total momentum of the recoiling electron and the scattered photon must also point in the $+x$ direction. There can be no component of the final total momentum along the $+y$ or $-y$ direction. We know that the recoiling electron has momentum only in the $+x$ direction. Thus, in order for the y component of the total final momentum to be absent, the scattered photon must move either in the $+x$ or $-x$ direction. To distinguish between these two choices, it is necessary to refer to energy conservation, as well as momentum conservation.

---

8. ***REASONING AND SOLUTION*** A photon can undergo Compton scattering from a molecule such as nitrogen, just as it does from an electron. However, the change in photon wavelength is much less than when an electron is scattered. To see why, let us examine Equation 29.7 which gives the difference between the wavelength l' of the scattered photon and the wavelength l of the incident photon in terms of the scattering angle q:

$l' \quad l = [h/(mc)](1 \quad \cos q)$, where h is Planck's constant, m is the mass of the target particle, and c is the speed of light in a vacuum. The mass of a nitrogen molecule is much greater than the mass of an electron. Therefore, the factor $h/(mc)$ will be much smaller if the target particle is a nitrogen molecule. Consequently, the change in the photon wavelength, $l' - l$, is much less than it is when the target particle is an electron.

---

9. ***REASONING AND SOLUTION*** When the speed of a particle with mass doubles, its momentum doubles, and its kinetic energy becomes four times greater. A photon, however, does not behave the same as a particle with mass. The momentum of a photon is $p = E/c$ (see the derivation of Equation 29.6); therefore, the energy of the photon can be written as $E = pc$. Clearly, when the momentum of a photon doubles, its energy also doubles. It does *not* become four times greater.

---

10. ***REASONING AND SOLUTION*** When bright light is incident on the radiometer, photons strike both the black and the shiny surfaces. The photons are absorbed by the black surfaces and are reflected by the shiny surfaces. Linear momentum is transferred to the panels because of the photon collisions. Since the orientation of the light is arbitrary, photons will strike the panels at arbitrary angles of incidence. Only the component of the photon's momentum that is perpendicular to the face of the panel contributes to its motion. As discussed in Conceptual Example 3, the momentum transfer to the panels is a maximum for the shiny surfaces and is twice as large in magnitude than it is for the black surfaces. Following the reasoning of Conceptual Example 3, we can use the impulse-momentum theorem and Newton's third law to deduce that the force on the shiny side is greater than the force on the black side. Therefore, in bright light, the arrangement would spin in the direction from the shiny side toward the black side. The observed spinning is in the opposite direction. Thus, photon collisions with the panels *cannot* be the cause of the spinning.

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11. ***REASONING AND SOLUTION*** The linear momentum of an object of mass m traveling with velocity $\mathbf{v}$ is $\mathbf{p} = m\mathbf{v}$. The de Broglie wavelength of an object moving with momentum $\mathbf{p}$ is given by Equation 29.8: $l = h/p$, where h is Planck's constant.

A stone is dropped from the top of a building. As the stone falls, it is uniformly accelerated, and its velocity increases uniformly in the downward direction. Consequently, the linear momentum of the stone also increases uniformly in the downward direction. According to Equation 29.8, the de Broglie wavelength of an object or particle is inversely proportional to the magnitude of its linear momentum. Therefore, as the stone falls, the de Broglie wavelength of the stone must *decrease*.

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12. ***REASONING AND SOLUTION*** An electron and a neutron have different masses. According to Equation 29.8, $l = h / p$, the de Broglie wavelength of a particle is inversely proportional to the magnitude of its linear momentum. Therefore, if the electron and the neutron have different speeds such that the magnitudes of their respective momenta are the same, they will have the same de Broglie wavelength.

---

13. ***REASONING AND SOLUTION*** Suppose that in Figure 29.1, the electrons are replaced with protons that have the same speed. The speed of the protons is the same as that of the electrons; however, the mass of a proton is greater than the mass of an electron. Therefore, the magnitude of the linear momentum of a proton is greater than the linear momentum of an electron traveling at the same speed. According to Equation 29.8, $l = h / p$, the de Broglie wavelength of the "proton beam" will be smaller than that of the "electron beam." Equation 27.1 gives the condition that must be satisfied by the angle q that locates the interference maxima in a Young's double-slit experiment: $\sin q = m l / d$, where m takes on integer values. According to Equation 27.1, the sine of the angle q is directly proportional to the wavelength of the incident beam. Since the proton beam has a smaller wavelength than the electron beam, the values of sin q that correspond to maxima will be smaller. Consequently, the difference in the corresponding values of q will be smaller, and the angular separation between the bright fringes will *decrease* when the electrons are replaced by protons.

CHAPTER 30 | *THE NATURE OF THE ATOM*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** A tube is filled with atomic hydrogen at room temperature. Electromagnetic radiation with a continuous spectrum of wavelengths, including those in the Lyman, Balmer, and Paschen series, enters one end of the tube and leaves the other end. The exiting radiation is found to contain absorption lines.

   At room temperature, most of the atoms of atomic hydrogen contain electrons that are in the ground state ($n = 1$) energy level. Since the radiation contains a continuous spectrum of wavelengths, it contains photons with a wide range of energies ($E = hf = hc/l$). In particular, it will contain photons with energies that are equal to the energy difference between the atomic states in the Lyman series. When the radiation is incident on the atoms in the tube, these photons are absorbed by the electrons. When a photon, whose energy is equal to the energy difference for a transition in the Lyman series, is absorbed by a ground state electron, that electron will make a transition to a higher energy level. Every photon of this energy will be absorbed by a ground state electron and cause a transition. The wavelength of radiation that corresponds to that particular photon energy will, therefore, be removed from the radiation. When the radiation is analyzed, the wavelengths that correspond to transitions in the Lyman series will be absent. Since most of the atoms in the tube are in the ground state ($n = 1$), the electron populations in the $n = 2$ and $n = 3$ states are extremely small. Therefore, any absorption lines resulting from Balmer or Paschen transitions will be extremely weak. When the radiation is analyzed, the only predominant absorption lines in the exiting radiation will correspond to wavelengths in the Lyman series.

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2. ***REASONING AND SOLUTION*** Refer to the situation described in Question 1. Suppose the electrons in the atoms are mostly in excited states. Most of the electrons are in states with $n \geq 2$; therefore, Balmer and Paschen series transitions will occur, and the absorption lines in the exiting radiation will correspond to wavelengths in the Balmer and Paschen series. Since there are relatively few electrons in the ground state, only a relatively few number of photons that correspond to wavelengths in the Lyman series will be absorbed. Most of the "Lyman photons" will remain in the radiation; therefore, the exiting radiation will *not* contain absorption lines that correspond to wavelengths in the Lyman series. Although the absorption lines that correspond to transitions in the Lyman series are not present, there will be more absorption lines in the exiting radiation compared to the situation when the electrons are in the ground state, because absorption lines corresponding to both the Balmer and Paschen series will be present.

---

3. ***REASONING AND SOLUTION*** According to Equation 30.13, the energy E_n of the nth atomic state in a Bohr atom is given by $E_n = -(13.6\text{ eV})Z^2/n^2$, where $n = 1, 2, 3, \ldots$, and Z is equal to the number of protons in the nucleus. The energy that must be absorbed by the electron to cause an upward transition from the initial state n_i to the final

state n_f is $E_{fi} = E_f - E_i$. Using Equation 30.13, we find that the required energy is $E_{fi} = -(13.6\text{ eV})Z^2\left(1/n_f^2 - 1/n_i^2\right)$, with n_i, n_f = 1, 2, 3, . . . and $n_f > n_i$. The energy required to ionize the atom from when the outermost electron is in the state n_i can be found by letting the value of n_f approach infinity. The ionization energy is then

$$E = -(13.6\text{ eV})Z^2\left(1/\quad -1/n_i^2\right) = -(13.6\text{ eV})Z^2\left(0 - 1/n_i^2\right) = (13.6\text{ eV})Z^2/n_i^2$$

From this expression, we see that the ionization energy is inversely proportional to the square of the principal quantum number n_i of the initial state of the electron; thus, less energy is required to remove the outermost electron in an atom when the atom is in an excited state. Therefore, when the atom is in an excited state, it is more easily ionized than when it is in the ground state.

---

4. ***REASONING AND SOLUTION*** In the Bohr model for the hydrogen atom, the closer the electron is to the nucleus, the smaller is the total energy of the atom. This is not true in the quantum mechanical picture of the hydrogen atom.

In the Bohr model, the *n*th orbit is a circle of definite radius r_n; every time that the position of the electron in this orbit is measured, the electron is found exactly a distance r_n from the nucleus. In contrast, according to the quantum mechanical picture, when an atom is in the *n*th state, the position of the electron is uncertain. Even though the atomic state is well defined by the principal quantum number *n*, the location of the electron is not definite. When the atom is in the *n*th state, the electron can sometimes be found close to the nucleus, while, at other times, it can be found far from the nucleus, or at some intermediate position. In the absence of any external magnetic fields, the energy of the state is determined by the value of *n*, and for a given *n*, the position of the electron is uncertain. Therefore, it is not correct to say that in the quantum mechanical picture of the hydrogen atom, the closer the electron is to the nucleus, the smaller is the total energy of the atom.

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5. ***REASONING AND SOLUTION***

a. Consider two different hydrogen atoms. The electron in each atom is in an excited state. The Bohr model uses the same quantum number *n* to specify both the energy and the orbital angular momentum. According to the Bohr model, if an atom is in its *n*th state, the corresponding energy is $E_n = (13.6\text{ eV})Z^2/n^2$, where *n* = 1, 2, 3, . . ., and *Z* is equal to the number of protons in the nucleus (Equation 30.13). The integer *n* that specifies the energy also specifies the orbital angular momentum according to $L_n = nh/(2p)$, where *h* is Planck's constant (Equation 30.8). Therefore, according to the Bohr model, it is *not* possible for the electrons to have different energies, but the same orbital angular momentum.

b. The quantum mechanical model uses two <u>different</u> quantum numbers to specify the energy and the orbital angular momentum of an atomic electron. In the absence of magnetic fields, the energy is determined by the principal quantum number *n*, while the orbital

angular momentum is determined by the orbital quantum number ℓ. According to quantum mechanics, if an atom is in its nth state with orbital quantum number ℓ, the energy is identical to that of the Bohr model, and the magnitude of the orbital angular momentum is $h/(2p)\sqrt{\ell(\ell+1)}$, where $\ell = 0, 1, 2, \ldots, (n-1)$. Since $\ell = 0, 1, 2, \ldots, (n-1)$, different values of ℓ are compatible with the same value of n. For example, when $n = 3$, the possible values of ℓ are 0, 1, 2, and when $n = 4$, the possible values of ℓ are 0, 1, 2, 3. Thus, the electron in one of the atoms could have $n = 3$, $\ell = 2$, while the electron in the other atom could have $n = 4$, $\ell = 2$. Therefore, according to quantum mechanics, it is possible for the electrons to have different energies but have the same orbital angular momentum.

---

6. ***REASONING AND SOLUTION*** In the quantum mechanical picture of the hydrogen atom, the orbital angular momentum of the electron *may* be zero in any of the possible energy states. This is true because, if an atom is in its nth state with orbital quantum number ℓ, the magnitude of the orbital angular momentum is given by $h/(2p)\sqrt{\ell(\ell+1)}$, where $\ell = 0, 1, 2, \ldots, (n-1)$; therefore, for any value of n, the value of ℓ *may* be zero. However, when the hydrogenic electron is in the ground state, $n = 1$, and the only possible value for the orbital quantum number is $\ell = 0$. Therefore, when the electron is in the ground state, its orbital angular momentum *must* be zero.

---

7. ***REASONING AND SOLUTION***

a. According to the Pauli exclusion principle, no two electrons in an atom can have the same set of values for the four quantum numbers n, ℓ, m_ℓ, and m_s. For a given value of the orbital quantum number ℓ, the magnetic quantum number m_ℓ, can have $2\ell + 1$ possible values. Each of these values can be combined with two possible values for the spin quantum number m_s. Therefore, the maximum number of electrons that the ℓth subshell can hold is $2(2\ell + 1)$. All of the electrons in a 5g subshell have $n = 5$, and $\ell = 4$. Thus, the maximum number of electrons that can be contained in a 5g subshell is $2(2\ell + 1) = 2[2(4) + 1] = 18$. Therefore, a 5g subshell can contain at most 18 electrons and cannot contain 22 electrons.

b. Since 17 is less than 18, a 5g subshell could contain 17 electrons.

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8. ***REASONING AND SOLUTION*** An electronic configuration for manganese ($Z = 25$) is written as $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^4\,4s^2\,4p^1$. According to Figure 30.17, the ground state electronic configuration for manganese is written as $1s^2\,2s^2\,2p^6\,3s^2\,3p^6\,3d^5\,4s^2$. Clearly, the electronic configuration given in the question corresponds to *an excited state* in which the last electron in the 3d subshell has been excited to the 4p subshell.

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9. ***REASONING AND SOLUTION*** Characteristic X-rays are produced when an electron with enough energy strikes the target atom and knocks one of the K-shell ($n = 1$) electrons

entirely out of the atom. An electron in one of the outer shells ($n \quad 2$) then drops into the K-shell and emits an X-ray photon in the process.

The ground state of hydrogen contains one electron in the $n = 1$ state, whereas, the ground state of helium contains two electrons in the $n = 1$ state. In both atoms, the ground state configuration contains only K-shell electrons. Even if one of the K-shell electrons were knocked out by an impinging electron, there are no electrons in higher levels to fall into the K-shell vacancy. Therefore, we would not expect hydrogen and helium atoms in their ground state to emit characteristic X-rays.

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10. ***REASONING AND SOLUTION*** The characteristic lines in the X-ray spectrum depend on the target material. They occur at the wavelength value that corresponds to the wavelength of the X-ray photon that was emitted when an electron from the outer shell of the atom in the target material drops down to the K-shell. Since all identical atoms of the same element contain the same energy levels, the K and K lines will occur at the same wavelength values for all X-ray targets made of those atoms, regardless of the voltage applied across the X-ray tube.

The cutoff wavelength l_0, however, depends on the voltage V across the X-ray tube. According to Equation 30.17, $l_0 = (hc)/(eV)$; the cutoff wavelength is inversely proportional to the voltage across the tube. Therefore, when a smaller voltage is used to operate the tube, the cutoff wavelength increases.

We can conclude, therefore, that when the tube is operated at a smaller voltage, the characteristic lines should occur at the same wavelengths as they did at a higher voltage, and the cutoff wavelength should increase (i.e., shift to the right on the wavelength axis). The drawing in the text, shows the X-ray spectra produced by an X-ray tube when the tube is operated at two different voltages. The features of the drawing are consistent with the conclusions reached above.

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11. ***REASONING AND SOLUTION*** Bremsstrahlung X-rays are produced when the electrons decelerate upon hitting the target. In the production of X-rays, it is possible to create Bremsstrahlung X-rays without producing the characteristic X-rays. The wavelengths of the characteristic lines depend on the target material; the cutoff wavelength, however, is independent of the target material and depends only on the energy of the impinging electrons. In order to produce Bremsstrahlung X-rays without producing the characteristic X-rays, the potential difference V applied to the tube must be chosen so that the cutoff wavelength is greater than the wavelengths of the characteristic X-rays for the target material. Therefore, using Equation 30.17, the value of V must be chosen so that $V = (hc)/(e l_0)$, where $l_0 > l_{K_a}$.

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12. ***REASONING AND SOLUTION*** The short side of X-ray spectra ends abruptly at a cutoff wavelength l_0. This cutoff wavelength depends only on the energy of the impinging electrons. Regardless of the composition of the target, the incident electron cannot give up more than all of its kinetic energy when it is decelerated by the target. The kinetic energy that the electron has acquired upon reaching the target is given by eV (see Section 19.2),

where V is the potential difference that is applied to the X-ray tube. Therefore, the cutoff wavelength depends on the value of V, and not on the composition of the target material used in the X-ray tube.

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13. ***REASONING AND SOLUTION*** A laser beam focused to a small spot can cut through a piece of metal. Light rays leaving the laser are nearly parallel, so the light can be focused to a very small spot. The intensity is the electromagnetic energy per unit time per unit area (see Equations 16.8 and 24.4). Therefore, when the beam is focused to a spot with a very small area, the intensity delivered by the beam is very large. For example, a pulsed ruby laser with a peak power of 10^8 W can be focused to a peak intensity of 10^{20} W/cm^2. This intensity is large enough to ionize air or burn a hole in a piece of metal.

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14. ***REASONING AND SOLUTION*** The energy of a photon of light of frequency f is given by Equation 29.2: $E = hf$. Since $c = lf$ for light, the energy of a photon can be written in terms of the wavelength l as $E = hc / l$. According to Table 26.2, the wavelength of green light is smaller than the wavelength of red light; therefore, the laser that produces green light emits photons that are more energetic than those emitted by the helium/neon laser.

CHAPTER 31 | *NUCLEAR PHYSICS AND RADIOACTIVITY*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** Isotopes are nuclei that contain the same number of protons, but a different number of neutrons. A material is known to be an isotope of lead, although the particular isotope is not known.

   a. The atomic number Z of an atom is equal to the number of protons in its nucleus. It is different for different elements, and all isotopes of the same element have the same atomic number. Therefore, if we know that a material is an isotope of lead, we can specify its atomic number ($Z = 82$).

   b. The various isotopes of the same element differ in the number of neutrons in the nucleus. Since we do not know which particular isotope of lead makes up the material, we cannot specify the neutron number.

   c. The atomic mass number is equal to the total number of protons and neutrons in the nucleus. Since we do not know which isotope of lead makes up the material, we do not know how many neutrons are in the nucleus. Therefore, we cannot specify the atomic mass number.

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2. ***REASONING AND SOLUTION*** Two nuclei have different nucleon numbers A_1 and A_2. The two nuclei may or may not be isotopes of the same element. Two isotopes of the same element have the same number of protons in the nucleus, but differ in the number of neutrons. The nucleon number is equal to the total number of protons and neutrons in the nucleus. If the nucleon numbers differ only because the neutron numbers differ, then the two nuclei are isotopes of the same element. However, if the nucleon numbers differ because the nuclei contain a different number of protons, then the two nuclei are *not* isotopes of the same element (regardless of their respective neutron numbers). Therefore, two nuclei with different nucleon numbers are *not* necessarily isotopes of the same element.

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3. ***REASONING AND SOLUTION*** Two nuclei that contain different numbers of protons and different numbers of neutrons can have the same radius. According to Equation 31.2, the approximate radius of the nucleus depends on the atomic mass number A and is given by $r \quad (1.2 \quad 10^{-15} \text{ m}) A^{1/3}$. The atomic mass number A is equal to the total number of protons and neutrons. Therefore, if two nuclei have different numbers of protons and different numbers of neutrons, but the sum of the protons and neutrons is the same for each nucleus, then the two nuclei have the same radius.

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4. ***REASONING AND SOLUTION*** Using Figure 31.6, the following nuclei have been ranked in ascending order (smallest first) according to the binding energy per nucleon: ${}^{232}_{90}\text{Th}$, ${}^{184}_{74}\text{W}$, ${}^{31}_{15}\text{P}$, ${}^{59}_{27}\text{Co}$.

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5. ***REASONING AND SOLUTION*** The general form for decay is

$$\underbrace{{}^{A}_{Z}\text{P}}_{\substack{\text{Parent}\\\text{nucleus}}} \qquad \underbrace{{}^{A\ \ 4}_{Z\ \ 2}\text{D}}_{\substack{\text{Daughter}\\\text{nucleus}}} \quad + \quad \underbrace{{}^{4}_{2}\text{He}}_{\substack{a\text{ particle}\\(\text{helium nucleus})}}$$

while the general form for the two types of beta decay are

[$^{-}$ decay]

$$\underbrace{{}^{A}_{Z}\text{P}}_{\substack{\text{Parent}\\\text{nucleus}}} \qquad \underbrace{{}^{A}_{Z+1}\text{D}}_{\substack{\text{Daughter}\\\text{nucleus}}} \quad + \quad \underbrace{{}^{0}_{-1}\text{e}}_{\substack{b^{-}\text{ particle}\\(\text{electron})}}$$

and

[$^{+}$ decay]

$$\underbrace{{}^{A}_{Z}\text{P}}_{\substack{\text{Parent}\\\text{nucleus}}} \qquad \underbrace{{}^{A\ \ 0}_{Z\ \ 1}\text{D}}_{\substack{\text{Daughter}\\\text{nucleus}}} \quad + \quad \underbrace{{}^{0}_{1}\text{e}}_{\substack{b^{+}\text{ particle}\\(\text{positron})}}$$

According to Equation 31.2, the approximate radius of the nucleus depends on the atomic mass number A and is given by $r \quad (1.2 \quad 10^{-15}\ \text{m})\,A^{1/3}$.

a. For decay, the mass number of the daughter nucleus is four nucleons smaller than that of the parent nucleus. Therefore, according to Equation 31.2, the daughter nucleus should have a smaller radius than the parent nucleus.

b. For both types of decay, the mass number of the daughter nucleus is identical to that of the parent nucleus. Therefore, according to Equation 31.2, the radius of the daughter nucleus should be the same as that of the parent nucleus.

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6. ***REASONING AND SOLUTION*** The general forms for decay, decay, and decay are, respectively:

$$\underbrace{{}^{A}_{Z}\text{P}}_{\substack{\text{Parent}\\\text{nucleus}}} \qquad \underbrace{{}^{A\ \ 4}_{Z\ \ 2}\text{D}}_{\substack{\text{Daughter}\\\text{nucleus}}} \quad + \quad \underbrace{{}^{4}_{2}\text{He}}_{a\text{ particle}}$$

$$\underbrace{{}^{A}_{Z}\text{P}}_{\substack{\text{Parent}\\\text{nucleus}}} \qquad \underbrace{{}^{A}_{Z+1}\text{D}}_{\substack{\text{Daughter}\\\text{nucleus}}} \quad + \quad \underbrace{{}^{0}_{-1}\text{e}}_{b\text{ particle}}$$

$$\underbrace{{}^{A}_{Z}\mathrm{P}^{*}}_{\substack{\text{Excited}\\ \text{energy state}}} \qquad \underbrace{{}^{A}_{Z}\mathrm{P}}_{\substack{\text{Lower}\\ \text{energy state}}} + \underbrace{g}_{g\text{ ray}}$$

As discussed in Section 31.1, the identity of an element depends on the number of protons in the nucleus. Clearly, in both decay and decay, the number of protons of the parent nucleus changes as the parent nucleus emits a charged particle. In contrast, in the process of decay, the parent nucleus emits only energy and the number of protons in the nucleus remains the same. Therefore, and decay result in transmutations, while decay does not.

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7. ***REASONING AND SOLUTION*** In order for a nuclear decay to occur, it must bring the parent nucleus toward a more stable state by allowing the release of energy. The total mass of the decay products must be less than the mass of the parent nucleus. The difference in mass between the products and the parent nucleus represents an equivalent amount of energy that is given by Equation 28.5: $E_0 = mc^2$. This is the energy that is released during the decay.

As discussed in Example 4, uranium ${}^{238}_{92}\mathrm{U}$ decays into thorium ${}^{234}_{90}\mathrm{Th}$ by means of decay. The ${}^{238}_{92}\mathrm{U}$ nucleus never decays by just emitting a single proton, instead of an alpha particle. The hypothetical decay scheme for the unobserved decay, is given below along with the pertinent atomic masses:

$$\underbrace{{}^{238}_{92}\mathrm{U}}_{\substack{\text{Uranium}\\ 238.050\ 78\text{ u}}} \qquad \underbrace{{}^{237}_{91}\mathrm{Pa}}_{\substack{\text{Protactinium}\\ 237.051\ 14\text{ u}}} + \underbrace{{}^{1}_{1}\mathrm{H}}_{\substack{\text{Proton}\\ 1.007\ 83\text{ u}}}$$

The total mass of the decay products is $(237.051\ 14\text{ u}) + (1.007\ 83\text{ u}) = 238.058\ 97\text{ u}$. This is greater, not smaller, than the mass of the parent nucleus ${}^{238}_{92}\mathrm{U}$; therefore, the daughter nucleus is less stable than the parent nucleus. Consequently, the hypothetical decay never occurs.

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8. ***REASONING AND SOLUTION*** According to Equation 31.5, if there are N_0 radioactive nuclei present at time $t = 0$, then the number N of radioactive nuclei present at time t is $N = N_0 e^{\ \ lt}$, where the decay constant l is given by Equation 31.6: $l = (\ln\ 2)/T_{1/2}$, where $T_{1/2}$ is the half-life of the nuclei. According to Equation 31.6, unstable nuclei with short half-lives have large decay constants. When l is large, the term $e^{\ \ lt}$ approaches zero even for very short times. Therefore, unstable nuclei with short half-lives typically have only a small or zero abundance after a short period of time.

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9. ***REASONING AND SOLUTION*** The half-life of a radioactive isotope is the time required for one-half of the nuclei present to disintegrate. The disintegrations do not occur continuously at regular time intervals. Individual disintegrations occur randomly; we cannot predict *a priori* which particular nucleus will decay. Any given nucleus may decay at any time. The time may be much shorter than the half-life or it may be much longer than the half-life. Therefore, even though the half-life of indium $^{115}_{49}\text{In}$ is $4.41 \quad 10^{14}\ \text{yr}$, it is possible for any single nucleus in the sample to decay after only one second has passed.

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10. ***REASONING AND SOLUTION*** The activity of a radioactive sample is the number of disintegrations per second that occur. In other words, it is the magnitude of $N/\ t$. According to Equation 31.4, $N/\ t = \ lN$, where l is the decay constant that is characteristic of the isotope in question, and N is the number of radioactive nuclei that are present.

a. While each isotope has its own characteristic decay constant, the activity is proportional to the product lN, according to Equation 31.4. Therefore, two samples with different decay constants l, and different numbers of radioactive nuclei N, can have identical activities if the product lN is the same for each. Thus, it is possible for two different samples that contain different radioactive elements to have the same activity.

b. Since each isotope has its own characteristic decay constant l, two different samples of the same radioactive isotope have the same decay constant. If the two samples contain different numbers of radioactive nuclei, the product lN will be different for each sample. Therefore, they will have different activities.

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11. ***REASONING AND SOLUTION*** Radiocarbon dating utilizes the $^{14}_{6}\text{C}$ isotope that undergoes b^- decay with a half-life of 5730 yr. If the present activity of $^{14}_{6}\text{C}$ can be measured in a carbon-containing object, the age of the object can be determined. Example 11 shows how to determine the age of an object, once the present activity is known.

Consider the following objects, each about 1000 yr old: a wooden box, a gold statue, and some plant seeds. Since 1000 yr is of the same order of magnitude as the half-life of $^{14}_{6}\text{C}$, the change in activity should be significant, and the present activity should be easily measured. However, the sample must contain $^{14}_{6}\text{C}$ atoms. Since both the wooden box and the seeds contain $^{14}_{6}\text{C}$, radiocarbon dating can be used to determine their ages. If the gold statue is made of pure gold, it will not contain any $^{14}_{6}\text{C}$ atoms. Therefore, radiocarbon dating could not be applied to the gold statue.

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12. ***REASONING AND SOLUTION*** Two isotopes have half-lives that are short relative to the age of the earth. Today, one of these isotopes (isotope A) is found in nature and the other (isotope B) is not. It is reasonable to conclude that the supply of isotope A has been replenished by some natural process that off-sets the radioactive decay, while the supply of isotope B has not been replenished. The replenishing of isotope A could occur, for example, as the result of the interaction of cosmic rays with the earth's upper atmosphere or the radioactive decay of some other isotope.

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13. ***REASONING AND SOLUTION*** In radiocarbon dating, the age of the sample is determined from the ratio of the present activity to the original activity. Suppose that there were a greater number of ${}^{14}_{6}\mathrm{C}$ atoms in a plant living 5000 yr ago than is currently believed. According to Equation 31.4: $N/\ \ t = \ \ lN$, and the original activity would be greater than is currently believed. A longer time would be required for the activity of a larger number of ${}^{14}_{6}\mathrm{C}$ nuclei to decrease to its present value. Therefore, the age of the seeds obtained from radiocarbon dating would be too small.

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14. ***REASONING AND SOLUTION*** Tritium is an isotope of hydrogen and undergoes b^{-} decay with a half-life of 12.33 yr. Like carbon ${}^{14}_{6}\mathrm{C}$, tritium is produced in the atmosphere because of cosmic rays and can be used in a radioactive dating technique. The age cannot be significantly smaller or significantly larger than the half-life of the radioisotope. During a 700 yr period, 700/12.33 or roughly 57 half-lives would occur, and the activity of a given sample would decrease to an immeasurably small fraction of its initial value (on the order of 10^{-17}). Therefore, tritium dating could *not* be used to determine a reliable date for a 700 yr old sample.

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15. ***REASONING AND SOLUTION*** Because of radioactive decay, one element can be transmuted into another. Thus, a container of uranium ${}^{238}_{92}\mathrm{U}$ ultimately becomes a container of lead ${}^{206}_{82}\mathrm{Pb}$. According to Figure 31.16, the first step is the transmutation from uranium ${}^{238}_{92}\mathrm{U}$ to thorium ${}^{234}_{90}\mathrm{Th}$. This step has the greatest half-life, with a value of $4.5\ \ 10^{9}\ \mathrm{y}$. Therefore, we can conclude that the complete transmutation from uranium ${}^{238}_{92}\mathrm{U}$ to lead ${}^{206}_{82}\mathrm{Pb}$ would take at least billions of years.

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16. ***REASONING AND SOLUTION*** According to Equation 21.2, the radius r of curvature for a particle with charge q and mass m, that moves perpendicular to a magnetic field of magnitude B, is given by

$$r = \frac{mv}{qB}$$

Thus, the ratio of the radii of curvature for the motion of the a particle and the b^- particle is

$$\frac{r_b}{r_a} = \frac{m_b v / q_b B}{m_a v / q_a B} = \frac{m_b / q_b}{m_a / q_a} = \frac{m_b}{m_a}\frac{q_a}{q_b}$$

The mass of the a particle is 1.0026 u, while the mass of a b^- particle (an electron) is

$$(9.109\ \ 389\ \ 731\ \ \ 10^{-31}\ \text{kg})\left(\frac{1.0073\ \text{u}}{1.6726\ \ \ 10^{-27}\ \text{kg}}\right) = 5.486\ \ \ 10^{-4}\ \text{u}$$

Since the magnitudes of the charge on the a and b^- particles are $2e$ and e, respectively, the ratio of the radii of curvature is

$$\frac{r_b}{r_a} = \left(\frac{5.486\ \ \ 10^{-4}\ \text{u}}{4.0026\ \text{u}}\right)\left(\frac{2e}{e}\right) = 2.7\ \ \ 10^{-4}$$

Therefore, the radius of curvature of the path of the b^- particle is smaller than the radius of curvature of the path of the a particle. Therefore, the trajectory of the b^- particle has a greater curvature than that of the a particle.

CHAPTER 32 *IONIZING RADIATION, NUCLEAR ENERGY, AND ELEMENTARY PARTICLES*

CONCEPTUAL QUESTIONS

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1. ***REASONING AND SOLUTION*** The RBE, or relative biological effectiveness, of a particular type of radiation compares the dose of that radiation needed to produce a certain biological effect to the dose of 200 keV X-rays needed to produce the same biological effect. The RBE depends on the nature of the ionizing radiation and the type of tissue that is being irradiated. Therefore, if two different types of radiation have the same RBE, this *does not* mean that each type delivers the same amount of energy to the tissue that it irradiates. Rather, it implies that the two different types of radiation are equally effective in producing the same biological effect on the same kind of tissue.

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2. ***REASONING AND SOLUTION*** When a dentist X-rays your teeth, a lead apron is placed over your chest and lower body. The purpose of this apron is to protect the tissues on your chest and lower body from the ionizing effect of the X-rays. The thickness of the lead is chosen so that the apron blocks the penetration of the X-rays.

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3. ***REASONING AND SOLUTION***
a. The units "rads" and "grays" are used to express the *absorbed dose* of ionizing radiation. According to Equation 32.2, the absorbed dose is equal to the energy absorbed from the radiation per unit mass of the absorbing material. The gray (Gy) is the SI unit for absorbed dose and is equal to 1 J/kg. The relationship between rads and grays is: 1 rad = 0.01 gray.

b. The units "rads" and "roentgens" are *not* related. The rad is used to express the *absorbed dose*, and 1 rad = 0.01 J/kg. The roentgen is used to express the *exposure*, and 1 roentgen = $2.58 \quad 10^{-4}$ C/kg. The exposure expresses the ionizing abilities of X-rays or g rays in air, while the absorbed dose specifies the amount of energy absorbed per kilogram of absorbing material.

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4. ***REASONING AND SOLUTION*** In any nuclear reaction, the following quantities are conserved: (1) mass / energy; (2) electric charge, (3) linear momentum, (4) angular momentum, and (5) nucleon number.

a. ${}^{60}_{28}\text{Ni}\ (a, p)\ {}^{62}_{29}\text{Cu}$ corresponds to the reaction

$$ {}^{4}_{2}\text{He} + {}^{60}_{28}\text{Ni} \qquad {}^{62}_{29}\text{Cu} + {}^{1}_{1}\text{H} $$

The nucleon number for the reactants is 4 + 60 = 64, while the nucleon number of the products is 62 + 1 = 63. Therefore, this reaction does *not* occur, because it violates the conservation of nucleon number.

b. ${}^{27}_{13}\text{Al}\ (n,\ n)\ {}^{28}_{13}\text{Al}$ corresponds to the reaction

$$ {}^{1}_{0}\text{n} \quad + \quad {}^{27}_{13}\text{Al} \qquad\qquad {}^{28}_{13}\text{Al} \quad + \quad {}^{1}_{0}\text{n} $$

The nucleon number for the reactants is 1 + 27 = 28, while the nucleon number of the products is 28 + 1 = 29. Therefore, this reaction does *not* occur, because it violates the conservation of nucleon number.

c. ${}^{39}_{19}\text{K}\ (p,\ a)\ {}^{36}_{17}\text{Cl}$ corresponds to the reaction

$$ {}^{1}_{1}\text{H} \quad + \quad {}^{39}_{19}\text{K} \qquad\qquad {}^{36}_{17}\text{Cl} \quad + \quad {}^{4}_{2}\text{He} $$

The nucleon number for the reactants is 1 + 39 = 40, while the nucleon number of the products is 36 + 4 = 40. Nucleon number is conserved. The proton number on the left is $1 + 19 = 20$, while the proton number on the right is $17 + 2 = 19$. Since each proton carries a unit of charge +*e*, the total electric charge on the left is +20 *e*, while the total electric charge on the right is +19 *e*. Therefore, this reaction does *not* occur because it violates the conservation of electric charge.

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5. ***REASONING AND SOLUTION*** Thermal neutrons and thermal protons have nearly the same speed. But thermal neutrons and thermal electrons have very different speeds. As discussed in Section 32.3, the adjective "thermal" applies to particles that have a kinetic energy of about 0.04 eV or less. According to Equation 6.2, the kinetic energy of a particle of mass *m* traveling with speed *v* is given by $\text{KE} = \frac{1}{2}mv^2$. Solving for *v* we have $v = \sqrt{2(\text{KE})/m}$. Therefore, for particles that have the same average kinetic energy, the speed of the particle is inversely proportional to the square root of its mass. Since the mass of the neutron is very nearly equal to the mass of a proton, thermal neutrons and thermal protons have nearly the same speed. The mass of the electron is much smaller than the mass of the neutron; therefore, thermal electrons will have a higher average speed than thermal neutrons.

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6. ***REASONING AND SOLUTION*** Consider the hypothetical fission of a nucleus of nucleon number $A = 25$ into two fragments of about equal mass ($A \quad 12.5$). The curve in Figure 32.8 shows that the binding energy of a nucleus with $A = 25$ is about 8.0 MeV per nucleon. Further inspection of Figure 32.8 shows that the binding energy of the fragments *has decreased* to about 6.5 MeV per nucleon. The difference between the binding energy per nucleon of the parent and the binding energy per nucleon of the daughter nucleus is about $6.5\ \text{MeV} - 8.0\ \text{MeV} = -1.5\ \text{MeV}$ of energy per nucleon, where the minus sign

indicates that the energy is absorbed. Therefore, a release of energy *would not* accompany the hypothetical fission reaction.

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7. ***REASONING AND SOLUTION*** In the fission of $^{235}_{92}\text{U}$, there are, on the average, 2.5 neutrons released per fission. The fact that the fission of $^{235}_{92}\text{U}$ releases on the average, 2.5 neutrons, makes it possible for a self-sustaining series of fissions to occur. Each neutron released by the fission process can initiate another fission reaction, resulting in the release of still more neutrons. These neutrons initiate more fission reactions, and the process results in a self-sustaining chain reaction.

   Suppose a *different* element is being fissioned and, on the average, only 1.0 neutron is released per fission. If a small fraction of the thermal neutrons absorbed by the nuclei does *not* produce a fission, a self-sustaining chain reaction *cannot* be produced using this element. Since each fission produces only one neutron, the product of a fission reaction can, at best, initiate only one more fission reaction. However, since a small fraction of the thermal neutrons absorbed by the nuclei does *not* produce a fission, at some point, the absorbed neutron will not produce a fission and, since there are no other neutrons to initiate a fission, the process will eventually stop.

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8. ***REASONING AND SOLUTION*** Much more energy is released when a uranium $^{235}_{92}\text{U}$ nucleus fissions than when a carbon $^{12}_{6}\text{C}$ atom in coal is combined with oxygen during the coal-burning process. It is true that the mass of the uranium nucleus is about 20 times greater than that of a carbon nucleus (235/12 = 20). But even 20 carbon atoms, when burned, release much less energy than a single $^{235}_{92}\text{U}$ nucleus releases when it fissions. In fact, they release about 2 million times less energy. Thus, the mass of coal needed is about 2 million times greater than the mass of uranium.

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9. ***REASONING AND SOLUTION*** Figure 32.8 shows that the binding energy of a nucleus with $A = 60$ is about 9.0 MeV per nucleon. If two such nuclei fuse, the daughter nucleus would have $A = 120$. According to Figure 32.8, the binding energy of the daughter nucleus *decreases* to about 8.3 MeV per nucleon. Consequently, the difference between the binding energy per nucleon of the initial nuclei and the binding energy per nucleon of the fused nucleus is about $8.3\text{ MeV} - 9.0\text{ MeV} = -0.7\text{ MeV}$ of energy per nucleon, where the minus sign indicates that the energy is absorbed. Therefore, the fusion of two nuclei, each with $A = 60$, would *not* release energy.

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10. ***REASONING AND SOLUTION*** Nuclear fission occurs when a massive nucleus captures a slow moving neutron and is split into two less-massive fragments that have a greater binding energy per nucleon than the massive parent nucleus. Since the binding energy per nucleon of the daughter nuclei is greater than that of the parent nucleus, energy is released. It appears primarily as kinetic energy of the fission products.

Nuclear fusion occurs when two low-mass nuclei with relatively small binding energies per nucleon combine or "fuse" into a single, more massive nucleus that has a greater binding energy per nucleon. Since the binding energy per nucleon of the more massive nucleus is greater than that of the parent nuclei, a substantial amount of energy is released. The energy released per nucleon in a fusion reaction is greater than that released in a fission reaction. Therefore, for a given mass of fuel, a fusion reaction will yield more energy than a fission reaction.