# *Über Problem: Jerky Jerry's Jabberwocky Jumper*

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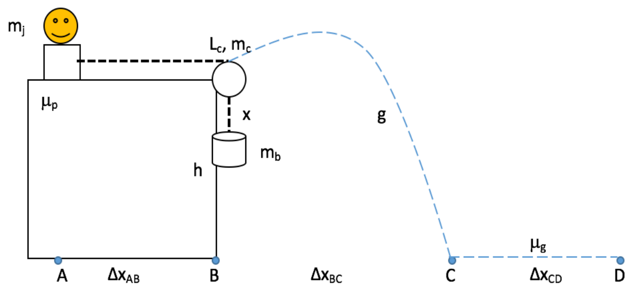
**Project Description:**
"Jerky" Jerry decided to make a jabberwocky jumper using a pulley system (see diagram). His method was to attach one end of a chain to a barrel of rocks, and the other end to the jumper. He placed the barrel and chain over a massless frictionless pulley, and then walked along a platform away from the pulley to point A (the full length of the chain). When he sat in the jumper he accelerated along the platform to point B and then launched off it while releasing the chain from the jumper and avoiding the pulley. He flew through the air as a projectile to point C, transitioning 75% of his speed into the horizontal direction, and eventually slid to a stop at point D. Note: Ignore any heights of the jumper, pulley, and barrel. Ignore any frictional and normal forces of the chain.

**Diagram**

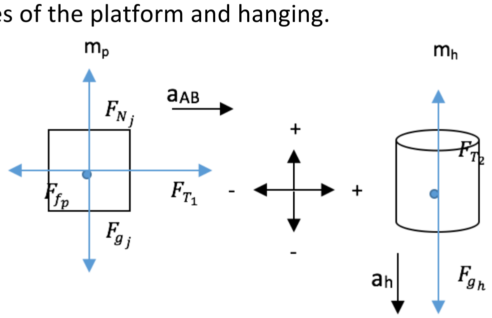



**Givens:**

| | |
|---|---|
| $m_j$ = 79 kg | h = 24 m |
| $m_b$ = 179 kg | $\mu_p$ = 0.29 |
| $m_c$ = 44 kg | $\Delta x_{BD}$ = 82 m |
| $\Delta x_{AB} = L_c$ = 14 m | g = -9.8 m/s$^2$ |

**Strategy:**
Calculate $a_{AB}$ as a function of position using Newton's second law. Then determine $\Delta x_{BC}$ to find $\Delta x_{CD}$ using kinematic equations. Finally calculate $\mu_g$ based on the velocity of C approaching from right to left and using Newton's second law again. Assumptions are $F_{T_1} = F_{T_2}$ and $a_{AB} = -a_h$.

**Solution:**
*Stage AB*
Draw free body diagrams to express the external forces acting on the masses of the platform and hanging.




In the free body diagram $m_p$ and $m_h$ are the total amount of masses on the platform and hanging respectively.

$$m_p = \frac{m_c}{L_c}(L_c - x) + m_j$$
$$m_h = \frac{m_c}{L_c}(x) + m_b$$

Take the net external force acting on the free body diagrams and use Newton's second law to calculate $a_{AB}$.

$$\sum F_{p_y}: F_{N_j} - F_{g_j} = m_p a_y$$
$$F_{N_j} - m_j g = 0$$
$$F_{N_j} = m_j g$$

$$\sum F_{p_x}: F_{T_1} - F_{f_p} = m_p a_{AB}$$
$$F_{T_1} - \mu_p F_{N_j} = m_p a_{AB}$$
$$F_{T_1} - \mu_p m_j g = m_p a_{AB}$$
$$F_{T_1} = m_p a_{AB} + \mu_p m_j g$$

$$\sum F_h: F_{T_2} - F_{g_h} = m_h a_h$$
$$F_{T_2} - m_h g = -m_h a_{AB}$$
$$F_{T_2} = m_h g - m_h a_{AB}$$

Set $F_{T_1} = F_{T_2}$, solve for $a_{AB}$

$$m_p a_{AB} + \mu_p m_j g = m_h g - m_h a_{AB}$$
$$a_{AB}(m_p + m_h) = m_h g - \mu_p m_j g$$
$$a_{AB} = \frac{m_h g - \mu_p m_j g}{m_p + m_h}$$

$a_{AB}$ is a function of position. By substituting the equations of $m_p$ and $m_h$, the equation $a_{AB}[x]$ is formed.

$$a_{AB}[x] = \frac{\left(\frac{m_c}{L_c}(x) + m_b\right) g - \mu_p m_j g}{\left(\frac{m_c}{L_c}(L_c - x) + m_p\right) + \left(\frac{m_c}{L_c}(x) + m_b\right)}$$

Substitute values.

$$a_{AB}[x] = \frac{\left(\frac{44}{14}x + 179\right)(9.8) - (0.29)(79)(9.8)}{\left(\frac{44}{14}(14 - x) + 79\right) + \left(\frac{44}{14}x + 179\right)}$$
$$a_{AB}[x] = 0.102x + 5.065$$

Integrate $a_{AB}[x]$ to $v_{Bx}[x]$ and substitute values to find $v_{Bx}$
Starting off with a well-known identity in physics a[x] dx= v dv.

$$\int_{x_0}^{x} a[x]\,dx = \int_{v_0}^{v} v\,dv$$
$$\int_{x_0}^{x} 0.102x + 5.065\,dx = \int_{v_0}^{v} v\,dv$$
$$0.051x^2 + 5.065x|_0^x = \frac{1}{2}v^2\Big|_0^v$$
$$0.051x^2 + 5.065x = \frac{1}{2}v^2$$
$$v_{B_x}[x] = \sqrt{0.102x^2 + 10.13x}$$

Set x=$\Delta x_{AB}$ to launch Jerry at point with a velocity of $v_{Bx}$.

$$v_{B_x}[14] = \sqrt{0.102(14)^2 + 10.13(14)}$$

<u>$v_{Bx}$ = 12.72 m/s</u>

*Stage BC*

To calculate $\Delta x_{BC}$ use the third kinematic equation to calculate time in air.

$$h = \frac{1}{2} g t_{BC}{}^2 + v_{B_y} t$$
$$h = \frac{1}{2} g t_{BC}{}^2$$
$$t_{BC} = \sqrt{\frac{2h}{g}}$$
$$t_{BC} = \sqrt{\frac{2(-24)}{(-9.8)}}$$
$$\underline{t_{BC} = 2.21 \text{ s}}$$

$$\Delta x_{BC} = v_{C_x} t_{BC}$$
$$\Delta x_{BC} = (12.72)(2.21)$$
$$\underline{\Delta x_{BC} = 28.11 \text{ m}}$$

Calculate $v_{C-}$ to use in Stage CD.

$$v_{C_x} = v_{B_x}$$
$$\underline{v_{Cx} = 12.72 \text{ m/s}}$$

$$v_{C_y} = gt + v_{B_y}$$
$$v_{C_y} = gt$$
$$v_{C_y} = (-9.8)(2.21)$$
$$\underline{v_{Cy} = -21.65 \text{ m/s}}$$

$$v_{C-} = \sqrt{v_{C_y}{}^2 + v_{C_x}{}^2}$$
$$v_{C-} = \sqrt{(-21.65)^2 + (12.72)^2}$$
$$\underline{v_{C-} = 25.08 \text{ m/s}}$$

*Stage CD*

Given $\Delta x_{BD}$ and solved for $\Delta x_{BC}$, $\Delta x_{CD}$ can be calculated.

$$\Delta x_{BD} = \Delta x_{BC} + \Delta x_{CD}$$
$$\Delta x_{CD} = \Delta x_{BD} - \Delta x_{BC}$$
$$\Delta x_{CD} = 82 - 28.11$$
$$\underline{\Delta x_{CD} = 53.88 \text{ m}}$$

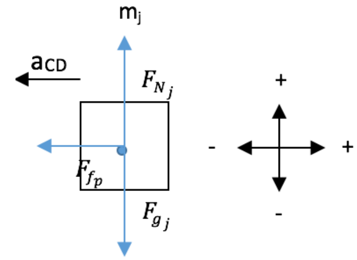
From Stage BC to CD, $v_{C-}$ transitions from a vector quantity both in the x-dir and y-dir to only 75% in the x-dir.

$$v_{C+} = 0.75 v_{C-}$$
$$v_{C+} = 0.75(25.08)$$
$$\underline{v_{C+} = 18.81 \text{ m/s}}$$

Calculate $a_{CD}$ using the fourth kinematic equation to use in Newton's second law to calculate $\mu_g$.

$$v_D{}^2 = v_{C+}{}^2 + 2 a_{CD} \Delta x_{CD}$$
$$0 = v_{C+}{}^2 + 2 a_{CD} \Delta x_{CD}$$
$$a_{CD} = \frac{-v_{C+}{}^2}{2 \Delta x_{CD}}$$
$$a_{CD} = \frac{-(18.81)^2}{2(60.92)}$$
$$\underline{a_{CD} = -3.28 \text{ m/s}^2}$$

Draw a free body diagram to express the external forces acting on Jerry during Stage CD




Take the net external force acting on the free body diagrams and use Newton's second law to calculate $\mu_g$.

$$\sum F_{jy}: F_{N_j} - F_{g_j} = m_j a_y$$
$$F_{N_j} - m_j g = 0$$
$$F_{N_j} = m_j g$$

$$\sum F_{jx}: F_{f_g} = m_j a_{CD}$$
$$\mu_g F_{N_j} = m_j a_{CD}$$
$$\mu_g m_j g = m_j a_{CD}$$
$$\mu_g = \frac{a_{CD}}{g}$$
$$\mu_g = \frac{-3.28}{-9.8}$$

$$\boxed{\mu_g = 0.3351}$$