The Gas Station Problem

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Problem Statement

Your car signals that your gas tank is almost empty. You wonder where you should buy gas. There is a gas station right next to you, but the price is too high for your frugal tastes. You remember there is a gas station across town, suitable for your wallet. Is it better to stay and fill up your gas tank without traveling the extra distance, or should you make the trek across town to fill up your gas tank?

Assumptions

- Your car uses regular gas
- You have one gallon left in your tank
- The more expensive gas costs $2.25 per gallon, while the cheaper, further away gas costs $2.10.
- You're going for a casual drive with no reason to be anywhere at a specific time.

Process

Approaches

First we used specific examples of cars. We chose the most popular cars in America and two extremes for mileage to see if that affected our results. We chose to use Worcester as our generic town, since that's where we are. We used the longest distance through the town, with some extra distance for Realism™. From there we calculated the gas usage based on the car's mileage and how much it would cost for each gas station.
Once we had values for the specific examples, we generalized the problem for any distance, mileage, and gas tank size.
We believe that these methods are successful because the specific examples helped us create the generalization.

Solution

Specific Examples

We took three cars, the first being the most popular and having an average mileage and gas tank size, and the other two being the two extremes of both. The formula for the more expensive station was obtained by multiplying the amount of gas you would need to buy (tank-1) by the price. The formula for the less expensive one was obtained in a similar way, however the amount of gas it would take to travel to the further station was included in the amount to buy.

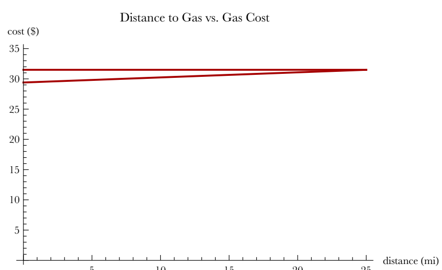
Example 1: Popular Car (2017 Toyota Camry)

- Based on the assumptions, the prices and formulae were made.

In[1]:=

```
price1 = 2.25;
price2 = 2.1;
tank = 15;
mileage = 25;
cost1[distance_] := (tank - 1) * 2.25;
cost2[distance_] := (tank - (1 - distance / mileage)) * 2.1;
p1 = Plot[cost1[x], {x, 0, mileage}, AxesLabel → {"distance (mi)", "cost ($)"},
   PlotLabel → "Distance to Gas vs. Gas Cost", Axes → True];
p2 = Plot[cost2[x], {x, 0, mileage}];
Show[p1, p2, PlotRange → {{0, mileage}, {0, 35}}]
```

Out[9]=

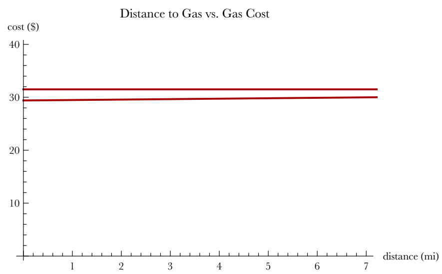


- The point where the two lines intersect is the distance where it is no longer better to go for the cheaper gas.

Example 2: Inefficient, Large Tank (Bugatti Veyron)

In[10]:=
```
tank = 25;
mileage = 7;
Show[p1, p2, PlotRange → {{0, mileage}, {0, 40}}]
```

Out[12]=

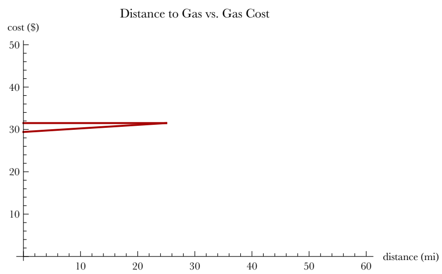


- For this model, the point of intersection is beyond the maximum range of the car on one gallon of gas. Therefore, as long as you can go on one tank of gas, you should drive for the cheaper gas.

Example 3: Very Efficient, Small Tank (2017 Toyota Prius)

In[13]:=
```
tank = 11;
mileage = 60;
Show[p1, p2, PlotRange → {{0, mileage}, {0, 50}}]
```

Out[15]=



- For this model, however, the point of intersection occurs well before the car runs out of gas.

Results

- Based on these models, one can see that it is typically better to drive for the less expensive gas, even if it takes more time.
- For cars with average and low gas mileage, it is always better to drive as far as you can.
- For cars with high mileage, however, there is a point where it is no longer worth it to drive.

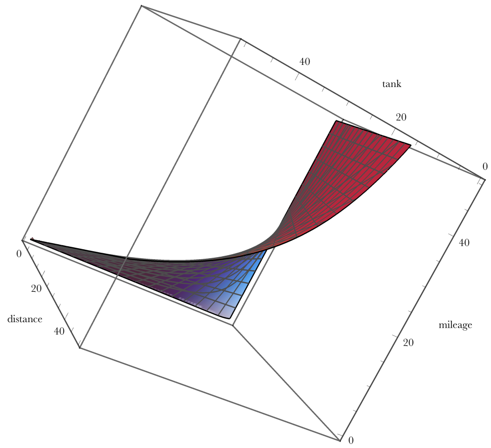
Generalization

Here we cleared the variables and graphed it using only variables to provide a view of any combinations of distance to gas station, car mileage, and gas tank size.

In[16]:=
```
Clear[tank, mileage]
cost3[distance_, mileage_, tank_] = (tank - (1 - distance / mileage)) * 2.1;
cost4[distance_, mileage_, tank_] = (tank - 1) * 2.25;
```

```
In[19]:= cp1 = ContourPlot3D[cost3[x, y, z] == cost4[x, y, z], {x, 0, 50}, {y, 0, 50},
  {z, 0, 50}, AxesLabel → {"distance", "mileage", "tank"}, Axes → True]
```

Out[19]=



Each point in this plot represents the combination of values giving the point where the two gas stations' costs were equal. This does not allow the difference between the two prices to change.

Self-Assessment!

What We Learned

- How to simplify a problem
  - Choosing specific examples before making it abstract
  - Controlling as many factors as possible
- How to follow the mathematical modeling process
- How to use Mathematica more efficiently

Assistance

- We asked the internet how to graph in 4 dimensions to add the price changing but stack overflow couldn't help us there.
- We used the internet to get values for both the assumptions and the specific examples.
  - This helped us to simplify the problem and understand our basic solution.