CHEME 133 Module 4: Analysis of American-Style Composite Options Contracts at Expiration

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Introduction

Composite options contracts are financial instruments that are composed of two or more individual options contracts. The payoff and profite of a composite contract at expiration is the sum of the values of the individual contracts. The advantage of constructing composite contracts is that they can be used to construct complex payoffs and profits startegies from simple contract components. In this module, we will analyze composite contracts at expiration, where the composite contract is composed of two or more American-style options.

General Formulation

Call and put contracts can be combined to develop composite contract structures with interesting payoff diagrams. Let $\mathcal{C}$ be a composite contract with d legs (individual contracts) where each leg is written with respect to the same underlying asset XYZ and same expiration date. Then, the payoff of the composite contract $\hat{V}(S(T),K_1,\dots,K_d)$ at time T (expiration) is given by:

$$\hat{V}(S(T),K_1,\dots,K_d)=\sum_{i\in\mathcal{C}}\theta_i\cdot n_i\cdot V_i(S(T),K_i) \tag{1}$$

where K_i denotes the strike price of contract i, θ_i denotes the contract orientation i: $\theta_i=-1$ if contract i is short (sold), otherwise $\theta_i=1$, and the quantity n_i denotes the copy number of contract i. The profit of the composite contract $\hat{P}$ at time T (expiration) is given by:

$$\hat{P}(S(T),K_1,\dots,K_d)=\sum_{i\in\mathcal{C}}\theta_i\cdot n_i\cdot P_i(S(T),K_i) \tag{2}$$

where $P_i(S(T),K_i)$ denotes the profit of contract i. Finally, the profit for contract of type $\star$ is given by:

$$P_{\star}(K,S(T))=V_{\star}(K,S(T))-\mathcal{P}_{\star}(K,S(0)) \tag{3}$$

where $\mathcal{P}_{\star}(K,S(0))$ denotes the premium of contract $\star$, and $V_{\star}(K,S(T))$ denotes the payoff of contract $\star$ at expiration.

Defined-Risk Directional Composite Contracts

Directional composite contracts make a directional assumption about the price movement of the underlying asset, and can be opened for a credit or a debit. A common directional composite contract is a *spread*.

Put Vertical Spread

A `put` vertical spread is constructed by combining 2 $\times$ `put` contracts, a short `put` contract generates income while a `long` put contract controls downside risk. Let contract j have a strike price of K_j and premium $\mathcal{P}_j$. The share price at expiration is given by S. Finally, let contract 1 be the short `put` leg $\theta_1 = -1$ and contract 2 be the long `put` leg $\theta_2 = 1$. Then, the profit for a single `put` vertical spread at expiration is given by:

$$\hat{P} = -P_1 + P_2 \tag{4}$$

which, after substitution of the profit functions for a put contract, gives:

$$\hat{P} = (K_2 - S)^+ - (K_1 - S)^+ + (\mathcal{P}_1 - \mathcal{P}_2) \tag{5}$$

where $V_p = (K - S)^+ = \max(K - S, 0)$ is the payoff function for a `put` contract. The first term is the net payout of the two legs of the spread, while the second term is the net cost of the two contracts. The maximum possible profit, loss, and breakeven conditions are given by:

- The maximum possible profit of $(\mathcal{P}_1 - \mathcal{P}_2)$ will occur when $S \geq K_1$.
- The maximum possible loss of $K_2 - K_1 + (\mathcal{P}_1 - \mathcal{P}_2)$ will occur when $S \leq K_2$.
- The vertical put spread will breakeven when $S = K_1 + (\mathcal{P}_2 - \mathcal{P}_1)$.

Bearish call credit spread

A bear `call` credit spread, is an options strategy used when a trader expects a decline in the price of the underlying asset. Assume the initial share price of the underlying asset is S_o (when we are opening the trade). For this trade, we sell a `call` contract at $K_1 < S_o$ for $\mathcal{P}_1$, and buy a `call` contract at $K_2 > S_o$ for $\mathcal{P}_2$. The profit function for the bear `call` credit spread is given by:

$$\hat{P} = (S - K_2)^+ - (S - K_1)^+ + (\mathcal{P}_1 - \mathcal{P}_2) \tag{6}$$

where $V_c = (S - K)^+ = \max(S - K, 0)$ is the payoff function for a `call` contract. The first two terms are the net payout of the two legs of the spread, while the last term is the net cost of the two contracts. The maximum possible profit, loss, and breakeven conditions are given by:

- The maximum possible profit of $(\mathcal{P}_1 - \mathcal{P}_2)$ will occur when $S \leq K_1$.
- The maximum possible loss of $(\mathcal{P}_1 - \mathcal{P}_2) - (K_2 - K_1)$ will occur when $S \geq K_2$.
- The bear `call` spread will breakeven when $S = K_1 + (\mathcal{P}_1 - \mathcal{P}_2)$.

Neutral Composite Contracts

Neutral composite contracts make no directional assumption about the price movement of the underlying asset, and can be opened for a credit or a debit. Two common directional composite contracts are the *straddle* and the *strangle*.

Straddles

A straddle is a neutral strategy constructed by simultaneously buying (or selling) a put and a call option on the same underlying asset XYZ, with the same expiration, and the same strike price. Depending upon the choice of the strike prices and whether an investor buys or sells both legs, a straddle can be initiated as a credit or debit and can potentially have undefined profit or loss. Let K_j denote the strike price of contract j (USD/share), where the price of contract j is $\mathcal{P}_j$ (USD/share). Further, let index $j = 1$ denote the `put` contract, $j = 2$ denote the `call` contract; for a straddle $K_1 = K_2 \equiv K$ (both legs have the same strike). The profit for a single straddle contract $\hat{P}$ at expiration is given by:

$$\hat{P} = \theta \cdot (P_1 + P_2) \tag{7}$$

where $\theta_1 = \theta_2 \equiv \theta$ denotes a direction parameter: $\theta = -1$ if each leg is sold (short), $\theta = 1$ otherwise. After substitution of the profit functions for a `put` and a `call` contract, the overall profit $\hat{P}$ for a `straddle` is given by:

$$\hat{P} = \theta \cdot \Big[(K - S)^{+} + (S - K)^{+} - (\mathcal{P}_1 + \mathcal{P}_2)\Big] \tag{8}$$

where $V_p = (K - S)^{+} = \max(K - S, 0)$ is the payoff function for the `put` contract, and $V_c = (S - K)^{+} = \max(S - K, 0)$ is the payoff function for the `call` contract. The profit (or loss) of a straddle has three regimes given by:

$$\hat{P} = \begin{cases} \theta \cdot \Big[(S(T) - K) - (\mathcal{P}_1 + \mathcal{P}_2)\Big] & S(T) > K \\ -\theta \cdot \Big[\mathcal{P}_1 + \mathcal{P}_2\Big] & S(T) = K \\ \theta \cdot \Big[(K - S(T)) - (\mathcal{P}_1 + \mathcal{P}_2)\Big] & S(T) < K \end{cases} \tag{9}$$

where $S(T)$ denotes the share price of the underlying asset at expiration. Finally, a straddle has two possible breakeven points denoted as S^{+} and S^{-}:

$$\mathcal{B}(T) = \begin{cases} S^{+} = K + (\mathcal{P}_1 + \mathcal{P}_2) & S(T) > K \\ S^{-} = K - (\mathcal{P}_1 + \mathcal{P}_2) & S(T) < K \end{cases} \tag{10}$$

where S^{+} denotes the upper breakeven point, and S^{-} denotes the lower breakeven point.

Strangles

A strangle is a neutral strategy constructed by simultaneously buying (or selling) a `put` and a `call` option on the same underlying asset XYZ, with the same expiration, but with different strike prices. Depending upon the choice of the strike prices and whether an investor buys or sells both legs, a strangle can be initiated for a credit or debit and can potentially have undefined profit or loss. Let K_j denote the strike price of contract j (USD/share), where the price of contract j is $\mathcal{P}_j$ (USD/share). Further, let index $j = 1$ denote the `put` contract, $j = 2$ denote the `call` contract; for a strangle $K_1 < K_2$. The profit for a single strangle contract $\hat{P}$ at expiration is given by:

$$\hat{P} = \theta \cdot (P_1 + P_2) \tag{11}$$

where $\theta_1 = \theta_2 \equiv \theta$ denotes a direction parameter: $\theta = -1$ if each leg is sold (short), $\theta = 1$ otherwise. After substitution of the profit functions for a `put` and a `call` contract, the overall profit $\hat{P}$ for a `strangle` is given by:

$$\hat{P} = \theta \cdot \Big[(K_1 - S)^{+} + (S - K_2)^{+} - (\mathcal{P}_1 + \mathcal{P}_2)\Big] \tag{12}$$

where $V_p = (K_1 - S)^{+} = \max(K_1 - S, 0)$ is the payoff for the `put` contract, and $V_c = (S - K_2)^{+} = \max(S - K_2, 0)$ is the payoff for the `call` contract. The profit (or loss) of a strangle has three regimes given by:

$$\hat{P} = \begin{cases} \theta \cdot \Big[(S(T) - K_2) - (\mathcal{P}_1 + \mathcal{P}_2)\Big] & S(T) > K_2 \\ -\theta \cdot \Big[\mathcal{P}_1 + \mathcal{P}_2\Big] & K_1 \leq S(T) \leq K_2 \\ \theta \cdot \Big[(K_1 - S(T)) - (\mathcal{P}_1 + \mathcal{P}_2)\Big] & S(T) < K_1 \end{cases} \tag{13}$$

where $S(T)$ denotes the share price of the underlying asset at expiration. Finally, a strangle has two possible breakeven points denoted as S^{+} and S^{-}:

$$\mathcal{B}(T) = \begin{cases} S^{+} = K_1 - (\mathcal{P}_1 + \mathcal{P}_2) & S(T) > K_2 \\ S^{-} = K_2 + (\mathcal{P}_1 + \mathcal{P}_2) & S(T) < K_1 \end{cases} \tag{14}$$

where S^{+} denotes the upper breakeven point, and S^{-} denotes the lower breakeven point.

Iron Flys and Condors

Iron flys and condors are examples of defined-risk neutral strategies, i.e., they make no directional assumption about the price movement of the underlying asset, and can be opened for a credit or a debit. However, unlike straddles and strangles, iron flys and condors have a defined maximum loss and maximum possible profit.

Summary

In this module, we have analyzed the profit and loss of American-style composite contracts at expiration. We considered two types of composite contracts: directional and neutral. Directional composite contracts make a directional assumption about the price movement of the underlying asset, and can be opened for a credit or a debit. Directional composite contracts include put vertical spreads and bearish call credit spreads, and are examples of a defined-risk directional strategy. On the other hand, neutral composite contracts make no directional assumption about the price movement of the underlying asset, and can be opened for a credit or a debit. These include straddles, and strangles, which are examples of undefined profit or loss strategies, and iron flys and condors, which are examples of defined-risk neutral strategies.

References