CHEME 131 Module 1: Zero Coupon Treasury Securities

Jeffrey D. Varner*

*R. F. Smith School of Chemical and Biomolecular Engineering
Cornell University, Ithaca NY 14853

Introduction

United States Marketable Treasury Securities, are issued by the U.S. Department of the Treasury to fund its operations and meet financial obligations. These debt securities are structured loan agreements between a borrower, i.e., the U.S. government, and a lender (you) that allows the government to fund its operations and obligations (Fig. 1). The debt holder and the U.S. Treasury have a marketable repayment agreement, which can be held by the lender (you) until the completion of the contract or resold on a secondary market. Although there are various types of U.S. government debt securities, they all share a few common characteristics. First, U.S. Treasury debt securities have a predetermined term length; thus, the contract duration between the borrower and lender is fixed. Second, U.S. Treasury debt securities have a par value, representing the instrument's face value, a price (which may differ from the par value), and an interest rate paid to the lender. Next, some U.S. Treasury debt securities have interest payments commonly called coupons. These payments give the lender fixed cashflows on a predetermined schedule throughout the debt instrument's term. Finally, income from interest on U.S. Treasury debt securities is free of state and local income taxes but subject to federal income taxes. You can purchase U.S. Treasury debt directly from the United States Treasury via TreasuryDirect or through a bank or broker to lend money to the U.S. government.

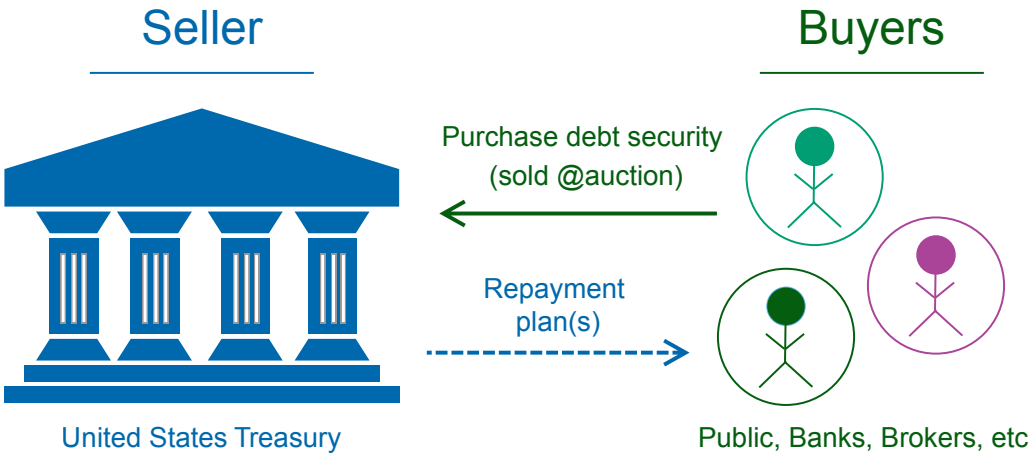

Fig. 1: Schematic of U.S. Government Debt Securities.

Treasury Bills, Notes and Bonds

United States Treasury Bills, or T-bills are Treasury debt instruments with short-term maturity periods T = 4, 8, 13, 26, and 52 weeks and zero coupon payments Thus, Treasury bills (T-bills) are short-term debt securities issued by the U.S. Treasury. United States Treasury Notes or T-notes, are debt instruments that provide a stable interest payment every six months until maturity, called a coupon payment. These notes are offered in terms of T = 2, 3, 5, 7, and 10 years and can be bought for more or less than their face (par) value. Upon maturity, the lender receives the entire par value. Treasury notes and bonds are coupon debt instruments, which means that the lender

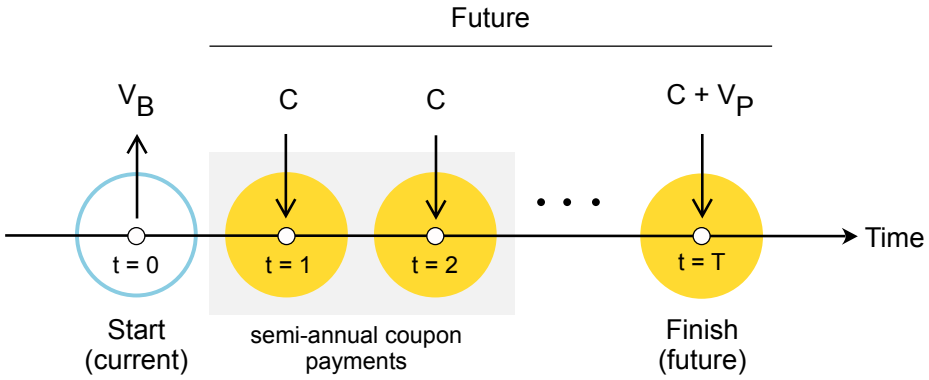

Fig. 2: Asbtract asset schematic of coupon-bearing Treasury note (bond). The lender (you) gives the United States Treasury the price V_B of the note (bond) at auction. In return, the Treasury pays semi-annual coupon paymemts to holder (you) over the life of the note (bond). At maturity, the lender (you) receives the final coupon payment and the par value of the note (bond). Schematic written from the note (bond) holders perspective.

receives periodic interest payments based on a coupon rate during the lifespan of the security. The coupon rate is fixed at the time of issuance and is calculated as a percentage of the par value. At maturity, the lender receives the par value of the bond (or note) and a final coupon payment. The pricing of these securities is based on the present value of the future cash flows. Thus, we need to develop a mathematical framework to compute the present value of these cash flows which we can use for treasury securities and other financial instruments.

Abstract Asset and the Time Value of Money

The traditional view of an asset is a resource with economic value that an individual, corporation, or country owns or controls with the expectation that it will provide a future benefit. However, in a more general sense, we can define an abstract asset as a sequence of current and future cash flows demarcated in some currency, for example, Euros, Dollars, Yuan, or cryptocurrencies such as Bitcoin. The cash flows can be positive or negative, and the asset can be either tangible or intangible. The challenge with this perspective is that current and future cash flows are not directly comparable, i.e., we cannot add or subtract cash flows that occur at different times.

Concept 1: Time value of money

The value of money is not conserved over time. One dollar today is not worth the same as one dollar tomorrow. The change in the value of money over time is called the time value of money.

The time value of money is an empirical observation that has been seen over hundreds of years. But why is this the case? The short answer: money today has a greater Utility than the same amount tomorrow; because we have an extra day to invest that money.

Pricing Zero-Coupon Treasury Securities

The price of a zero-coupon Treasury bill V_B with an effective interest rate of $\bar{r}$ and a maturity of T-years at auction is the discounted face (par) value V_P such that the net present value (NPV) of the bill is zero:

$$\text{NPV}(T, r) = -V_B + \mathcal{D}_{T,0}^{-1}(\bar{r}) \cdot V_P = 0 \tag{1}$$

or equivalently:

$$V_B = \mathcal{D}_{T,0}^{-1}(\bar{r}) \cdot V_P \tag{2}$$

The quantity T denotes the duration of the bill (in years), $\bar{r}$ is the effective annualized interest rate, and $\mathcal{D}_{T,0}^{-1}(\bar{r})$ is the inverse multistep discount factor for period $0 \to T$. The discount factor $\mathcal{D}_{T,0}^{-1}(\bar{r})$ can be either computed on a discrete or continuous basis. In the case of treasury securities, the discount factor is computed on a discrete basis, assuming m compounding periods per year.

Multiperiod Discrete Discount Factor

Fill me in.