Toward a Genome Scale Dynamic Model of Cell-Free Protein Synthesis in *Escherichia coli*

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Abstract

Cell-free protein expression systems have become widely used in systems and synthetic biology. In this study, we developed an ensemble of dynamic E. coli cell-free protein synthesis (CFPS) models. Model parameters were estimated from measurements of glucose, organic acids, energy species, amino acids, and the protein product, chloramphenicol acetyltransferase (CAT). The ensemble described all of the training data, especially the central carbon metabolism. The model predicted CAT production with a carbon yield equal to 16%, and an energy efficiency equal to 9%, of that of a physiologically realistic case calculated using sequence-specific flux balance analysis. This suggests that CAT production could be further optimized. The dynamic modeling approach predicted that substrate consumption and oxidative phosphorylation were most important to both CAT production and the system as a whole, while CAT production alone depended heavily on the CAT synthesis reaction. Conversely, CAT production was robust to allosteric control, as was most of the network, with the exception of the organic acids in central carbon metabolism. This study is the first to model dynamic protein production in E. coli, and should provide a foundation for genome-scale, dynamic modeling of cell-free E. coli protein synthesis.

Keywords: Biochemical engineering, systems biology, cell-free protein synthesis

Introduction

Cell-free systems offer many advantages for the study, manipulation and modeling of metabolism compared to *in vivo* processes. Central amongst these, is direct access to metabolites and the biosynthetic machinery without the interference of a cell wall, or complications associated with cell growth. This allows us to interrogate the chemical environment while the biosynthetic machinery is operating, potentially at a fine time resolution. Cell-free protein synthesis (CFPS) systems are arguably the most prominent examples of cell-free systems used today [1]. However, CFPS is not new; CFPS in crude *E. coli* extracts has been used since the 1960s to explore fundamentally important biological mechanisms [2, 3]. Today, cell-free systems are used in a variety of applications ranging from therapeutic protein production [4] to synthetic biology [5, 6]. However, if CFPS is to become a mainstream technology for applications such as point of care manufacturing, we must first understand the performance limits of these systems. One tool to address this question is mathematical modeling.

Mathematical modeling has long contributed to our understanding of metabolism. Dec-15 ades before the genomics revolution, mechanistically structured metabolic models arose 16 from the desire to predict microbial phenotypes resulting from changes in intracellular 17 or extracellular states [7]. The single cell E. coli models of Shuler and coworkers pio-18 neered the construction of large-scale, dynamic metabolic models that incorporated multi-19 ple, regulated catabolic and anabolic pathways constrained by experimentally determined 20 kinetic parameters [8]. Shuler and coworkers generated many single cell kinetic mod-21 els, including single cell models of eukaryotes [9, 10], minimal cell architectures [11], as 22 well as DNA sequence based whole-cell models of E. coli [12]. In the post genomics 23 world, large-scale stoichiometric reconstructions of microbial metabolism popularized by techniques such as flux balance analysis (FBA) have become a standard approach [13]. Since the first genome-scale stoichiometric model of E. coli, developed by Edwards and

Palsson [14], well over 100 organisms, including industrially important prokaryotes are now available [15-17]. Stoichiometric models rely on a pseudo-steady-state assumption to reduce unidentifiable genome-scale kinetic models to an underdetermined linear algebraic system, which can be solved efficiently even for large systems. Traditionally, 30 stoichiometric models have also neglected explicit descriptions of metabolic regulation 31 and control mechanisms, instead opting to describe the choice of pathways by prescribing an objective function on metabolism. Interestingly, similar to early cybernetic mod-33 els, the most common metabolic objective function has been the optimization of biomass 34 formation [18], although other metabolic objectives have also been estimated [19]. Re-35 cent advances in constraint-based modeling have overcome the early shortcomings of the platform, including capturing metabolic regulation and control [20]. Thus, modern 37 constraint-based approaches have proven extremely useful in the discovery of metabolic 38 engineering strategies and represent the state of the art in metabolic modeling [21, 22]. However, genome-scale kinetic models of industrial important organisms such as *E. coli* have yet to be constructed. 41

In this study, we developed an ensemble of kinetic cell-free protein synthesis (CFPS)
models using dynamic metabolite measurements in an *E. coli* cell free extract. Model parameters were estimated from measurements of glucose, organic acids, energy species,
amino acids, and the protein product, chloramphenicol acetyltransferase (CAT). Characteristic values for model parameters and initial conditions, estimated from literature, were
used to constrain the parameter estimation problem. The ensemble of parameter sets
described the training data with a median cost that was greater than two orders of magnitude smaller than random sets constructed using the literature parameter constraints. We
then used the ensemble of kinetic models to analyze the CFPS reaction. First, sensitivity analysis of the dynamic model suggested that CAT production was most sensitive to
CAT synthesis parameters, as well as reactions in oxidative phosphorylation and pyruvate

consumption. Sensitivity analysis also showed that the system as a whole was most sensitive to these same parts of the network and glucose consumption. CAT production and 54 other metabolites, specifically organic acid intermediates such as pyruvate, were sensi-55 tive to the presence of allosteric control mechanisms. Next, to gauge the performance 56 of the cell-free reaction, we compared the observed CAT carbon yield with the maximum 57 theoretical CAT carbon yield calculated using sequence-specific flux balance analysis. 58 The CAT yield estimated from the kinetic model was equal to 16% of the theoretical yield 59 when physiologically realistic constraints were used. Taken together, we have integrated 60 traditional kinetics with a logical rule-based description of allosteric control to simulate a 61 comprehensive CFPS dataset. This study provides a foundation for genome-scale, dy-62 namic modeling of cell-free *E. coli* protein synthesis. 63

84 Results

The ensemble of kinetic CFPS models captured the time evolution of CAT biosynthesis (Fig. 1 - 3). The cell-free E. coli metabolic network was constructed by removing growth 66 associated reactions from the iAF1260 reconstruction of K-12 MG1655 E. coli [16], and 67 by adding reactions describing chloramphenical acetyltransferase (CAT) biosynthesis, a 68 model protein for which there exists a comprehensive training dataset [23]. In addition, reactions that were knocked out from the cell extract preparation were removed from the network (\triangle speA, \triangle tnaA, \triangle sdaA, \triangle sdaB, \triangle gshA, \triangle tonA, \triangle endA). The CFPS model equations were formulated using the hybrid cell-free modeling framework of Wayman et al. [24]. An ensemble of model parameters (N > 10,000) was estimated from measurements of glucose, CAT, organic acids (pyruvate, lactate, acetate, succinate, malate), energy species (A(x)P, G(x)P, C(x)P, U(x)P), and 18 of the 20 proteinogenic amino acids using a constrained Markov Chain Monte Carlo (MCMC) approach. The MCMC algorithm 76 minimized the error between the training data and model simulations starting from an initial parameter set assembled from literature and inspection. Parameter sets were selected 78 for the ensemble based upon their error, and the Pearson correlation coefficient between the candidate and existing sets in the ensemble. The parameter set with the lowest error 80 value was defined as the best-fit set. Central carbon metabolism (Fig. 1, top), energy 81 species (Fig. 2), and amino acids (Fig. 3) were captured by the ensemble and the best-82 fit set. The constrained MCMC approach estimated parameter sets with a median error 83 greater than two-order of magnitude less than random parameter sets generated within 84 the same parameter bounds (Fig. 4); thus, we have confidence in the predictive capability 85 of the estimated parameters. The model captured the biphasic CAT production: during the first hour glucose powers production, and CAT is produced at ~10 μ M/h; subsequently, pyruvate and lactate reserves are consumed to power metabolism, and CAT is produced less efficiently at ~5 μ M/h. Allosteric control was important to biphasic CAT production;

without control, the CAT production rate increased and then ceased after 1.5 hr (Fig. 1, bottom). In addition, acetate no longer accumulated after 1.5 hours, in the absence of allosteric control. Interestingly, the simulated malate abundance tracked the experimental measurements during the glucose consumption phase, but increased sharply during the 93 pyruvate consumption phase, without allosteric control. Taken together, we produced an 94 ensemble of kinetic models that was consistent with time series measurements of the 95 production of a model protein. However, while the ensemble described the experimen-96 tal data, it was unclear which kinetic parameters most influenced CAT production, and 97 whether the performance of the CFPS reaction was optimal. 98

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To better understand which parameters and parameter combinations influenced the 99 performance of the kinetic model, we performed sensitivity analysis (Fig. 5). We perturbed each V^{max} parameter, either individually or in pairwise combinations and measured the change in either CAT production or the overall system state. The eigen decomposition of the sensitivity shows that CAT synthesis and oxidative phosphorylation are the most important to overall CAT production, followed by the pyruvate-consuming alanine synthesis reaction. Among the top 20 reactions, we saw a common theme of the cofactors ATP, NADH, NADPH, and coenzyme A, as well as the metabolites pyruvate, glutamate, and α -ketoglutarate. ATP appears 7 times in these 20 reactions, while pyruvate, glutamate, and coenzyme A appear 6 times each. NADH, NADPH, and α -ketoglutarate each appear 5 times. This result makes sense, as the high energy cost of protein synthesis means that energy cofactors played a crucial role. Also, pyruvate served as the primary substrate after glucose ran out, and pyruvate, glutamate, and α -ketoglutarate were all important precursors for the synthesis of amino acids required by CAT production. We performed the same eigen decomposition on the sensitivity of the overall system state to network reactions (Fig. S1). Cytochrome oxidase, part of oxidative phosphorylation, was seen to have the greatest effect on the system state. Next most influential was the forward reaction of

lactate dehydrogenase, followed by NADH:ubiquinone oxidoreductase, another oxidative phosphorylation reaction. The overall system state was also sensitive to cofactors and substrates, specifically NADH and pyruvate. Among the top 20 reactions, NADH appears in 8 reactions, pyruvate in 6, coenzyme A in 5, and ATP in 4. Glutamate, α -ketoglutarate, G3P, and ubiquinone/ubiquinol appear 3 times each. Taken together, sensitivity analysis identified that substrates and energy cofactors, specifically those around oxidative phosphorylation, most influenced model performance.

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To understand whether the CFPS performance was optimal, we calculated the carbon yield and energy efficiency of CAT production (Fig. 8). The best-fit parameter set for the kinetic model predicted a CAT carbon yield of 7.9%, while the experimental dataset had a CAT carbon yield of 8.2%. This was calculated as the increase in CAT concentration times the CAT carbon number, divided by the sum of the consumption terms for glucose and all amino acids except arginine and glutamate, as no data were available for these, weighted by their respective carbon numbers. To explain where the remainder of carbon was going, we performed a carbon balance for the best-fit set (Fig. 8A). Of the other 92% of carbon, 35% accumulated as organic acids (lactate, acetate, succinate and malate) and 9% accumulated as amino acids (alanine and glutamine). The remaining 48% went to the net accumulation of all other metabolites, particularly carbon dioxide. The bestfit set and the experimental dataset both produced CAT with an energy efficiency of 7% (Fig. 8B). This was calculated as the increase in CAT concentration times the CAT number of equivalent ATP molecules, divided by glucose consumption times times the number of equivalent ATP molecules for glucose, equal to 15 in the optimal case. An additional 62% of energy went to the accumulation of glycolysis metabolites, and 31% to organic acids (lactate, acetate, succinate and malate). This shows that there is much room for improvement of the efficiency of CFPS. Gene knockouts in the electron transport chain further reduced the performance of the CFPS reaction (Fig. 6). A key finding of both

the CAT and overall system state sensitivity analysis was the importance of oxidative phosphorylation. To investigate this further, we knocked out key oxidative phosphorylation 143 reactions in the ensemble of kinetic models to examine the effect on CAT production and carbon yield. A single cyd knockout reduced the CAT carbon yield from 7.9% to 2.6% 145 (Table 1). On the other hand, a *nuo* knockout showed a less dramatic decrease in yield, 146 reducing the CAT carbon yield to 6.9%. Knocking out app increased CAT yield to 8.1%, 147 but this increase was not statistically significantly different from that of the control. Lastly, 148 knocking out all three reactions reduced the CAT yield to 0.7%, similar to knocking out 149 the cyd alone. Thus, the model suggested the key step in oxidative phosphorylation 150 was catalyzed by the gene product of cyd. However, while disruption of cyd significantly reduced the CAT carbon yield, it did not completely eliminate the production of CAT. This 152 suggested there was a mixture of energy sources supporting CAT production, with the 153 most significant being oxidative phosphorylation. 154

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Sequence-specific flux balance analysis (ssFBA) predicted optimal CAT yields with no adjustable parameters (Fig. 6). Before exploring CFPS optimality, we first validated the ssFBA approach by comparing simulated and measured concentrations of CAT for the first hour of glucose consumption. We chose this time window (during the first phase of CAT production) because it was approximately linear both in glucose consumption and in the accumulation of most organic acids. As the ssFBA calculation had no adjustable parameters, bounds on transcription and translation rates and biochemical fluxes were either estimated from data or from mechanistic models parameterized from literature. Uncertainty in experimental factors such as RNA polymerase, ribosome concentrations, elongation rates, or the upper bounds for oxygen and glucose consumption rates was addressed by sampling plausible ranges for these parameters. The ensemble of ssFBA simulations predicted CAT formation as a function of time during the first hour of production when constrained by the experimental metabolite data (Fig. 6C). Thus, the molecular description of

transcription and translation were consistent with experimental measurements. Next, to gauge the performance of the CFPS reaction, we next calculated the CAT carbon yield for 169 three classes of constraints: (i) theoretical maximum glucose, amino acid and oxygen up-170 per bounds, and realistic transcriptional/translational constraints; (ii) theoretical maximum 171 glucose, amino acid and oxygen upper bounds, realistic transcriptional/translational con-172 straints and knockouts of amino acid synthesis reactions of amino acids supplemented 173 in the E. coli extract preparation. (iii) metabolite fluxes constrained by the CAT data, and 174 realistic transcriptional/translational constraints and knockouts of amino acid synthesis re-175 actions of amino acids supplemented in the E. coli extract preparation (Fig. 6D). The phys-176 iological theoretical maximum CAT carbon yield (case i) was $49.3\% \pm 3.5\%$ (Fig. 6D, left); 177 this represents optimal network performance if glucose, oxygen and amino acids were 178 produced or consumed at their upper bounds, with bounded transcription and translation 179 rates (96% without glucose contribution in the carbon yield calculation). For case ii, the 180 optimal CAT carbon yield was 48.9% \pm 3.5% (Fig. 6D, middle). Lastly, when metabolite 181 constraints based on experimental measurements were applied (case iii), the predicted 182 carbon yield was $6.4\% \pm 2.9\%$ (Fig. 6D, right). Unsurprisingly, this range of carbon yield 183 encompasses both the best-fit set (7.9%) and the experimental dataset (8.2%). For cases i and ii the energy efficiencies were 72.1% \pm 9.5% and 71.2% \pm 9.6%, respectively, while for case iii it was only $5.1\% \pm 2.4\%$. This dramatic decrease in efficiency when fluxes are constrained to data makes sense, as the network is forced toward a multitude of pathways 187 that may not contribute to CAT production. This range of energy efficiency encompasses 188 both the best-fit set and the experimental dataset (7%). However, the model and the ex-189 perimental dataset fall short of the optimum (cases i and ii). This suggests there is much 190 room for improvement, primarily by reducing byproduct formation. 191

To investigate the differences in carbon yield and energy efficiency, we compared the flux distributions predicted by ssFBA simulations for the different constraint cases (Fig. 7).

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In cases i and ii, glycolysis was used for energy generation, and most of the carbon flux accumulated as acetate. The system produced NADH through lactate dehydrogenase as 195 well as through pyridine nucleotide transhydrogenase (pntAB) to power oxidative phos-196 phorylation. Case iii heavily utilized the first step in the pentose phosphate pathway to 197 generate NADPH; the carbon flux then continued primarily through the Entner-Doudoroff 198 pathway toward pyruvate. Case iii also predicted the accumulation of pyruvate, lacatate, 199 acetate, and carbon dioxide, which contributed to the much lower carbon yield and energy 200 efficiency. In all cases, the energy source was primarily oxidative phosphorylation convert-201 ing cofactors generated throughout the network into ATP. Taken together, this suggested 202 CAT production could be increased by reducing the accumulation of acetate, lactate, and 203 carbon dioxide. 204

5 Discussion

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In this study we present an ensemble of E. coli cell-free protein synthesis (CFPS) mod-206 els that accurately predict a comprehensive CFPS dataset of glucose, CAT, central carbon 207 metabolites, energy species, and amino acid measurements. We used the hybrid cell-free 208 modeling approach of Wayman and coworkers, which integrates traditional kinetic mod-209 eling with a logic-based description of allosteric regulation. CFPS is seen to be biphasic 210 relying on glucose during the first hour and pyruvate and lactate afterward. Allosteric con-211 trol was essential to the maintenance of the network and production of CAT, as without it, 212 central carbon metabolism is exhuasted within 1.5 hours leading to low CAT production. Having captured the experimental data, we investigated if CAT yield and CFPS performance could be further improved. We showed that the model produces CAT with a carbon 215 yield equal to 16% of that of a physiological case in which transcription and translation are constrained, and an energy efficiency equal to 9% of that of the physiological case. The 217 accumulation of waste byproducts, especially acetate and carbon dioxide, is responsible 218 for this sub-optimal performance. Sensitivity analysis showed that certain substrates and 219 energy species are instrumental to CAT production and overall metabolism. The system 220 heavily relied on oxidative phosphorylation for the system's energetic needs as well as for 221 CAT synthesis. A single knockout in oxidative phosphorylation reduced the CAT carbon 222 yield ~3-fold, as well as disrupting the system state, showing its crucial role in CFPS. 223 In comparing flux distributions between low and high yield cases, carbon flux could be 224 potentially diverted toward CAT by reducing acetate overflow and minimizing flux through 225 the Entner-Doudoroff pathway. Taken together, these findings represent the first dynamic 226 model of E. coli cell-free protein synthesis, and an important step toward a functional 227 genome scale description. 228

We present an ensemble of models that quantitatively describes the system behavior of cell-free metabolism and production of CAT. Experimental observations of the metabo-

lites and cometabolites validate the structure of the model and the estimation of kinetic parameters. This is important in applying metabolic engineering principles to rationally 232 design cell-free production processes and predict the redirection of carbon fluxes to prod-233 uct forming pathways. In analyzing the model parameters' effect on CAT production, CAT 234 synthesis is the most important, followed by oxidative phosphorylation and the glutamate 235 and pyruvate consuming reactions, as well as cofactor reactions which are necessary to 236 drive CAT synthesis. For example, the conversion of ATP to GTP shows significance since 237 it is necessary for CAT synthesis. While Jewett and coworkers have shown that ATP may 238 be at saturation in CFPS [1], GTP is also required for CAT synthesis and may be a limiting 239 reactant. Thus, supplementation with additional GTP may improve the efficiency of CAT 240 production. A similar theme is seen in the sensitivity of overall model state, where the most 241 important reactions are glucose and pyruvate consuming reactions and cofactor reactions 242 which are vital to drive CFPS. This can be seen in the biphasic operation of CFPS, with 243 the first phase operating on glucose and the second phase operating on pyruvate. During 244 the first phase, there is an accumulation of byproducts from central carbon with the major-245 ity of flux going toward acetate and some toward pyruvate, lactate, and succinate; with the 246 exception of acetate, these are all consumed in the second phase. This shows that CAT production can be sustained by pyruvate and glutamate in the absence of glucose, which provides alternative strategies to optimize CFPS performance. This is in accordance with literature, which showed pyruvate provided a relatively slow but continuous supply of ATP [25]. Taken together, this shows CFPS can be designed towards a specified application 251 either requiring a slow stable energy source or faster production. This outstanding con-252 trol on model performace was expected as these metabolites are responsible for driving 253 CFPS and represent the first step in the model network. Nevertheless, there are fur-254 ther reactions with considerable impact on model performance. In examining oxidative 255 phosphorylation activity, knockouts in the electron transport pathways disrupt metabolism 256

across the network and drop CAT carbon yield from 7.9% to 2.6%; Jewett and coworkers also saw a similar decrease in CAT yield with pyruvate as the substrate, ranging from 1.5-fold to 4-fold, when knocking out oxidative phosphorylation reactions [1]. Oxidative phosphorylation is vital, since it provides most of the energetic needs of CFPS. While it is unknown how active oxidative phosphorylation is compared to that of *in vivo* systems, both of our modeling approaches suggest its importance to improving CFPS performance and protein yield.

A logical next step for this work would be sequence-specific dynamic modeling, as the kinetic modeling approach in this study used a single reaction to approximate CAT synthesis. Including specific transcription and translation steps for CAT would allow more accurate modeling of the complexity and the resource cost of protein synthesis. Sensitivity analysis could then be performed on these new parameters to determine the robustness of CAT synthesis to the processes of transcription and translation. In addition, further experimentation could also be used to gain a deeper understanding of model performance under a variety of conditions. Specifically, CAT production performed in the absence of amino acids could inform the system's ability to manufacture them, while experimentation in the absence of glucose or oxygen could shed light on the importance of those substrates. Finally, the approach should be extended to other protein products. CAT is only a test protein used for model identification; the modeling framework, and to some extent the parameter values, should be protein agnostic. An important extension of this study would be to apply its insights to other protein applications, where possible.

Materials and Methods

Formulation and solution of the model equations. We used ordinary differential equations (ODEs) to model the time evolution of metabolite (x_i) and scaled enzyme abundance (ϵ_i) in hypothetical cell-free metabolic networks:

$$\frac{dx_i}{dt} = \sum_{j=1}^{\mathcal{R}} \sigma_{ij} r_j(\mathbf{x}, \epsilon, \mathbf{k}) \qquad i = 1, 2, \dots, \mathcal{M}$$
 (1)

$$\frac{d\epsilon_i}{dt} = -\lambda_i \epsilon_i \qquad i = 1, 2, \dots, \mathcal{E}$$
 (2)

where \mathcal{R} denotes the number of reactions, \mathcal{M} denotes the number of metabolites and 282 \mathcal{E} denotes the number of enzymes in the model. The quantity $r_i(\mathbf{x}, \epsilon, \mathbf{k})$ denotes the 283 rate of reaction j. Typically, reaction j is a non-linear function of metabolite and enzyme 284 abundance, as well as unknown kinetic parameters \mathbf{k} ($\mathcal{K} \times 1$). The quantity σ_{ij} denotes 285 the stoichiometric coefficient for species i in reaction j. If $\sigma_{ij} > 0$, metabolite i is produced 286 by reaction j. Conversely, if $\sigma_{ij} < 0$, metabolite i is consumed by reaction j, while $\sigma_{ij} = 0$ 287 indicates metabolite i is not connected with reaction j. Lastly, λ_i denotes the scaled 288 enzyme activity decay constant. The system material balances were subject to the initial 289 conditions $\mathbf{x}(t_o) = \mathbf{x}_o$ and $\epsilon(t_o) = 1$ (initially we have 100% cell-free enzyme abundance). 290 The reaction rate was written as the product of a kinetic term (\bar{r}_j) and a control term 291 (v_i) , $r_i(\mathbf{x}, \mathbf{k}) = \bar{r}_i v_i$. We used multiple saturation kinetics to model the reaction term \bar{r}_i :

$$\bar{r}_j = V_j^{max} \epsilon_i \prod_{s \in m_j^-} \frac{x_s}{K_{js} + x_s} \tag{3}$$

where V_j^{max} denotes the maximum rate for reaction j, ϵ_i denotes the scaled enzyme activity which catalyzes reaction j, K_{js} denotes the saturation constant for species s in reaction j and m_j^- denotes the set of *reactants* for reaction j. On the other hand, the control term $0 \le v_j \le 1$ depended upon the combination of factors which influenced

rate process j. For each rate, we used a rule-based approach to select from competing control factors. If rate j was influenced by $1,\ldots,m$ factors, we modeled this relationship as $v_j = \mathcal{I}_j\left(f_{1j}\left(\cdot\right),\ldots,f_{mj}\left(\cdot\right)\right)$ where $0 \leq f_{ij}\left(\cdot\right) \leq 1$ denotes a transfer function quantifying the influence of factor i on rate j. The function $\mathcal{I}_j\left(\cdot\right)$ is an integration rule which maps the output of regulatory transfer functions into a control variable. We used hill-like transfer functions and $\mathcal{I}_j \in \{min, max\}$ in this study [24].

We included 17 allosteric regulation terms, taken from literature, in the CFPS model. PEP was modeled as an inhibitor for phosphofructokinase [26, 27], PEP carboxykinase [26], PEP synthetase [26, 28], isocitrate dehydrogenase [26, 29], and isocitrate lyase/malate synthase [26, 29, 30], and as an activator for fructose-biphosphatase [26, 31–33]. AKG was modeled as an inhibitor for citrate synthase [26, 34, 35] and isocitrate lyase/malate synthase [26, 30]. 3PG was modeled as an inhibitor for isocitrate lyase/malate synthase [26, 30]. FDP was modeled as an activator for pyruvate kinase [26, 36] and PEP carboxylase [26, 37]. Pyruvate was modeled as an inhibitor for pyruvate dehydrogenase [26, 38, 39] and as an activator for lactate dehydrogenase [40]. Acetyl CoA was modeled as an inhibitor for malate dehydrogenase [26].

Estimation of kinetic model parameters. We estimated an ensemble of diverse parameter sets using a constrained Markov Chain Monte Carlo (MCMC) random walk strategy. Starting from a single best fit parameter set estimated by inspection and literature, we calculated the cost function, equal to the sum-squared-error between experimental data and model predictions:

$$cost = \sum_{i=1}^{\mathcal{D}} \left[\frac{w_i}{\mathcal{Y}_i^2} \sum_{j=1}^{\mathcal{T}_i} \left(y_{ij} - x_i|_{t(j)} \right)^2 \right] \tag{4}$$

where \mathcal{D} denotes the number of datasets (\mathcal{D} = 37), w_i denotes the weight of the i^{th} dataset, \mathcal{T}_i denotes the number of timepoints in the i^{th} dataset, t(j) denotes the j^{th} time-

point, y_{ij} denotes the measurement value of the i^{th} dataset at the j^{th} timepoint, and $x_i|_{t(j)}$ denotes the simulated value of the metabolite corresponding to the i^{th} dataset, interpolated to the j^{th} timepoint. Lastly, the cost calculation was scaled by the maximum experimental value in the i^{th} dataset, $\mathcal{Y}_i = \max_j{(y_{ij})}$. We then perturbed each model parameter between an upper and lower bound that varied by parameter type:

$$k_i^{new} = \min\left(\max\left(k_i \cdot \exp(a \cdot r_i), l_i\right), u_i\right) \qquad i = 1, 2, \dots, \mathcal{P}$$
(5)

where \mathcal{P} denotes the number of parameters ($\mathcal{P} = 815$), which includes 163 maximum re-325 action rates (V^{max}) , 163 enzyme activity decay constants, 455 saturation constants (K_{is}) , 326 and 34 control parameters, k_i^{new} denotes the new value of the i^{th} parameter, k_i denotes 327 the current value of the i^{th} parameter, a denotes a distribution variance, r_i denotes a ran-328 dom sample from the normal distribution, l_i denotes the lower bound for that parameter 329 type, and u_i denotes the upper bound for that parameter type. Maximum reaction rates 330 were bounded between 0 and 500,000 mM/h [41]. Assuming a total enzyme concen-331 tration of 5.0 μ M, this corresponds to catalytic rate bounds of 0 and 27,780 s⁻¹. These 332 bounds resulted in a median catalytic rate of 0.16 s⁻¹ across the ensemble. Enzyme 333 activity decay constants were bounded between 0 and 1 h⁻¹, corresponding to half lives 334 of 42 minutes and infinity; median = 25 h. Saturation constants were bounded between 335 0.001 and 10 mM; median = 0.16 mM. Control parameters (gains and orders) were left unbounded; gain median = 0.076, order median = 0.69. For each newly generated pa-337 rameter set, we re-solved the balance equations and calculated the cost function. All sets 338 with a lower cost (and some with higher cost) were accepted into the ensemble. After 339 generating greater that 10,000 sets, we selected N = 100 sets with minimal set to set 340 correlation to avoid over-sampling any region of parameter space.

Sensitivity analysis of the kinetic CFPS model. We determined the reactions most important to protein production by computing the local sensitivity of CAT concentration (denoted as CAT) to each individual maximum reaction rate, and each pair of maximum reaction rates in the network. The sensitivity index was formulated as:

$$S_{ij}^{\text{CAT}} = \|\text{CAT}(p_i, p_j, t) - \text{CAT}(\alpha \cdot p_i, \alpha \cdot p_j, t)\|_2 \qquad i, j = 1, 2, \dots \mathcal{P}$$
(6)

where $S_{ij}^{\mathtt{CAT}}$ denotes the sensitivity of CAT production to the i^{th} and j^{th} parameters, $\mathtt{CAT}(p_i, p_j, t)$ 346 denotes CAT concentration as a function of time and the i^{th} and j^{th} parameters, α denotes 347 the perturbation factor, and \mathcal{P} denotes the number of maximum reaction rates ($\mathcal{P} = 163$). 348 In calculating the pairwise sensitivities, each parameter was perturbed by 1%; first-order 349 sensitivities (i = j) were subject to two 1% perturbations. Parameters and parameter 350 combinations were stratified into five degrees of importance, from least to most sensitive. 351 Likewise, we determined which reactions were most important to global system per-352 formance by computing the sensitivity of all species for which data exists (denoted as X) 353 to each maximum reaction rate in the network. In this case, each sensitivity index was 354 formulated as: 355

$$S_{ij}^{\mathbf{X}} = \left\| \mathbf{X}(p_i, p_j, t) - \mathbf{X}(\alpha \cdot p_i, \alpha \cdot p_j, t) \right\|_2 \qquad i, j = 1, 2, \dots \mathcal{P}$$
(7)

where $S_{ij}^{\rm X}$ denotes the sensitivity of the system state to the i^{th} and j^{th} parameters, and $X(p_i,p_j,t)$ denotes the system state, an array consisting of the concentration of every species for which data exists as a function of time and the i^{th} and j^{th} parameters. The parameter sensitivities were stratified into five degrees of importance, from least to most sensitive, as above.

Sequence-specific calculation of carbon yield. We estimated the theoretical maximum CAT carbon yield using sequence-specific flux balance analysis (ssFBA) [42]. The sequence-specific flux balance analysis problem was formulated as a linear program:

$$\max_{\boldsymbol{w}} (w_{TL} = \boldsymbol{\theta}^T \boldsymbol{w})$$
Subject to: $\mathbf{S} \mathbf{w} = \mathbf{0}$

$$\alpha_i < w_i < \beta_i \qquad i = 1, 2, \dots, \mathcal{R}$$
(8)

where S denotes the stoichiometric matrix, \mathbf{w} denotes the unknown flux vector, $\boldsymbol{\theta}$ denotes the objective selection vector and α_i and β_i denote the lower and upper bounds on flux w_i , respectively. The stoichiometry of the kinetic model was used for the ssFBA calculations, with the execpetion of the transcription and translation rates. The transcription (TX) and translation (TL) stoichiometry was modeled using the template reactions taken from Allen and Palsson [42]:

$$\begin{aligned} G_{\mathcal{P}} + R_1 & \longrightarrow & G_{\mathcal{P}}^* \\ G_{\mathcal{P}}^* + \sum_{k \in \{A,C,G,U\}} \eta_k \cdot \{k\} \, TP & \xrightarrow{TX} & mRNA + G_{\mathcal{P}} + R_1 + \sum_{k \in \{A,C,G,U\}} 2\eta_k \cdot Pi \\ mRNA & \longrightarrow & \sum_{k \in \{A,C,G,U\}} \eta_k \cdot \{k\} \, MP \\ mRNA + R_2 & \longrightarrow & R_2^* \\ \alpha_j \cdot AA_j + \alpha_j \cdot tRNA + \alpha_j \cdot ATP & \longrightarrow & \alpha_j \cdot AA_j - tRNA_j + \\ & \qquad \qquad \alpha_j \cdot AMP + 2\alpha_j \cdot Pi \qquad j = 1, 2, \dots, 20 \\ R_2^* + \sum_{j \in \{AA\}} \alpha_j \cdot \left(AA_j - tRNA_j + 2 \cdot GTP\right) & \xrightarrow{TL} & \mathcal{P} + R_2 + mRNA + \\ & \qquad \qquad + \sum_{j \in \{AA\}} \alpha_j \cdot \left(tRNA + 2 \cdot GDP + 2 \cdot Pi\right) \end{aligned}$$

where $G_{\mathcal{P}}$ denotes the gene encoding protein product \mathcal{P} , R_1 denotes the concentration of RNA polymerase, $G_{\mathcal{P}}^*$ denotes the gene bounded by the RNA polymerase, η_i and α_j denote the stoichiometric coefficients for nucleotide and amino acid, respectively, P_1 denotes inorganic phosphate, R_2 denotes the ribosome concentration, R_2^* denotes bounded ribosome, and AA_j denotes j^{th} amino acid.

The transcription rate (w_{TX}) was fixed in the ssFBA calculation at:

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$$w_{TX} = V_{TX}^{max} \left(\frac{G}{K_{TX} + G} \right) \tag{9}$$

where G denotes the gene concentration, and K_{TX} denotes a transcription saturation coefficient. The maximum rate of transcription V_{TX}^{max} was formulated as:

$$V_{TX}^{max} \equiv \left[R_1 \left(\frac{v_{TX}}{l_G} \right) \left(\frac{K_{T7}}{1 + K_{T7}} \right) \right] \tag{10}$$

The term R_1 denotes the RNA polymerase abundance, v_{TX} denotes the RNA polymerase elongation rate (nt/hr), l_G denotes the gene length in nucleotides, and the last term describes T7 promoter activity, where K_{T7} denotes a T7 RNA polymerase binding constant [43]. On the other hand, the translation rate (w_{TL}) was bounded by:

$$0 \le w_{TL} \le V_{TL}^{max} \left(\frac{\text{mRNA}_{SS}}{K_{TL} + \text{mRNA}_{SS}} \right)$$
 (11)

where ${
m mRNA_{SS}}$ denotes the steady state mRNA abundance, and K_{TL} denotes the translation saturation constant. The maximum translation rate V_{TL}^{max} was formulated as:

$$V_{TL}^{max} \equiv \left[K_P R_2 \left(\frac{v_{TL}}{l_P} \right) \right] \tag{12}$$

The term K_P denotes the polysome amplification constant, v_{TL} denotes the ribosome

elongation rate (amino acids per hour), l_P denotes the number of amino acids in the protein of interest, and ${
m mRNA_{SS}}$ denotes the steady-state mRNA concentration:

$$mRNA_{SS} \simeq \frac{w_{TX}}{\lambda}$$
 (13)

where λ denotes the rate constant controlling the mRNA degradation rate.

The objective of the sequence-specific flux balance calculation was to maximize the rate of CAT translation, w_{TL} . The total glucose uptake rate was bounded by [0,40 mM/h] according to experimental data; while the amino acid uptake rates were bounded by [0,30 mM/h], but did not reach the maximum flux. The CAT gene and protein sequences were taken from literature. The sequence-specific flux balance linear program was solved using the GNU Linear Programming Kit (GLPK) v4.52 [44].

Quantification of uncertainty. An ensemble of 100 sets of flux distributions was calculated for three different cases: constrained by transcription/translation rates, constrained by transcription/translation rates without amino acid synthesis reactions, and constrained by transcription/translation rates and experimental measurements without amino acid synthesis reactions. For the first case, all rates were left unbounded, except the specific glucose uptake rate, transcription and translation rate. An ensemble of flux distributions was then calculated by randomly sampling the maximum specific glucose uptake rate from within a range of 30 to 40 mM/h, determined from experimental data and randomly sampling RNAP polymerase levels, ribosome levels, and elongation rates in a physiological range determined from literature.. For the second case, an ensemble was generated by randomly sampling the same parameters as the first case, however certain amino acid synthesis reactions were removed from the network. This included all the amino acids that were present in the preparation of the *E. coli* extract (alanine, arginine, aspartate, cysteine, glutamate, glutamine and serine were excluded from the media), thus reactions

producing the excluded amino acids were left in the network. RNA polymerase levels were sampled between 60 and 80 nM, ribosome levels between 7 and 16 μM, the RNA polymerase elongation rate between 20 and 30 nt/sec, and the ribosome elongation rate between 1.5 and 3 aa/sec [45, 46]. For the third case, the ensemble was generated as in the second case, in addition to the lower and upper bounds on the fluxes for the data-informed metabolites were sampled within the range given by the experimental noise. This included the data for glucose, organic acids, energy species, and amino acids; CAT was not constrained by experimental data, but by the transcription/translation rates as stated above.

Calculation of the carbon yield. The CAT carbon yield (Y_C^{CAT}) was calculated as the ratio of carbon produced as CAT divided by the carbon consumed as reactants (glucose and amino acids):

$$Y_C^{CAT} = \frac{\Delta \text{CAT} \cdot C_{CAT}}{\sum_{i=1}^{\mathcal{R}} \max(\Delta m_i, 0) \cdot C_{m_i}}$$
(14)

where Δ CAT denotes the abundance of CAT produced, C_{CAT} denotes carbon number of CAT, \mathcal{R} denotes the number of reactants, Δm_i denotes the amount of the i^{th} reactant consumed (never allowed to be negative), and C_{m_i} denotes the carbon number of the i^{th} reactant. Arginine and glutamate were excluded from the yield calculations, as no experimental measurements were available for these amino acids. Yield of the best-fit parameter set and the experimental data were calculated by setting Δ CAT equal to the final minus the initial CAT concentration, and setting Δm_i equal to the initial minus the final reactant concentration. The individual CAT production and substrate consumption terms for the best-fit set, kinetic models with knockouts, and experimental data are shown in Table 1. Total net consumption of amino acids and amino acid consumption via CAT synthesis were calculated for the best-fit set (Table ??). Total net consumption was cal-

culated as amino acid concentration at 0 hours minus concentration at 3 hours; it was
negative if synthesis outweighed consumption. Consumption toward CAT was calculated
as CAT concentration at 3 hours minus concentration at 0 hours, times the stoichiometric coefficient for that amino acid in the CAT synthesis reaction. The difference between
these was defined as other consumption, equal to consumption from reactions other than
CAT synthesis minus amino acid production.

Calculation of energy efficiency. Energy efficiency was calculated as the ratio of CAT production to glucose consumption, both in terms of equivalent ATP molecules:

where ATP_{TX} , CTP_{TX} , GTP_{TX} , UTP_{TX} denote the stoichiometric coefficients of each energy species for CAT transcription, ATP_{TL} , GTP_{TL} denote the stoichiometric coefficients of ATP and GTP for CAT translation, Δ GLC denotes the glucose consumption, equal to the initial minus the final glucose concentration, and ATP_{GLC} denotes the equivalent ATP number for glucose. $ATP_{TX} = 176$, $CTP_{TX} = 144$, $GTP_{TX} = 151$, $UTP_{TX} = 189$, $ATP_{TL} = 199$, $GTP_{TL} = 438$, $ATP_{GLC} = 15$.

445 Competing interests

The authors declare that they have no competing interests.

447 Author's contributions

J.V directed the modeling study. K.C and J.S conducted the cell free protein synthesis experiments. J.V, J.W, and N.H developed the cell free protein synthesis mathematical model, and parameter ensemble. J.V and M.V performed the sequence-specific flux balance analysis calculations. The manuscript was prepared and edited for publication by J.S, N.H, M.V, J.W and J.V.

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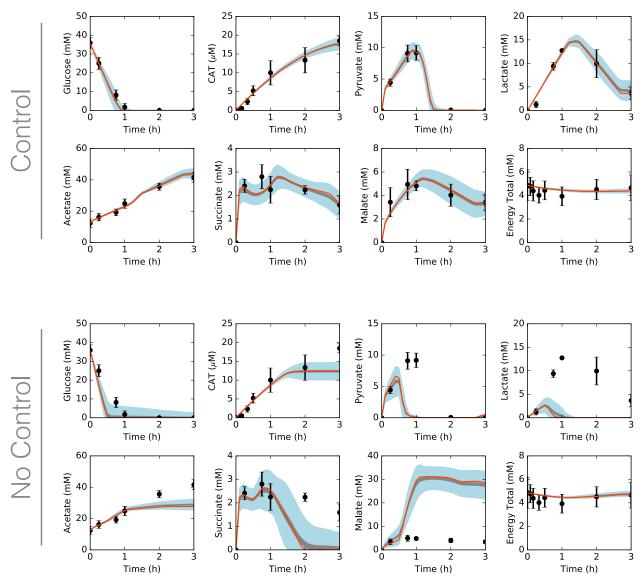


Fig. 1: Central carbon metabolism in the presence (top) and absence (bottom) of allosteric control, including glucose (substrate), CAT (product), and intermediates, as well as total concentration of energy species. Best-fit parameter set (orange line) versus experimental data (points). 95% confidence interval (blue shaded region) and 95% confidence interval of the mean (gray shaded region) over the ensemble of 100 sets.



Fig. 2: Energy species and energy totals by base in the presence of allosteric control. Best-fit parameter set (orange line) versus experimental data (points). 95% confidence interval (blue shaded region) and 95% confidence interval of the mean (gray shaded region) over the ensemble of 100 sets.

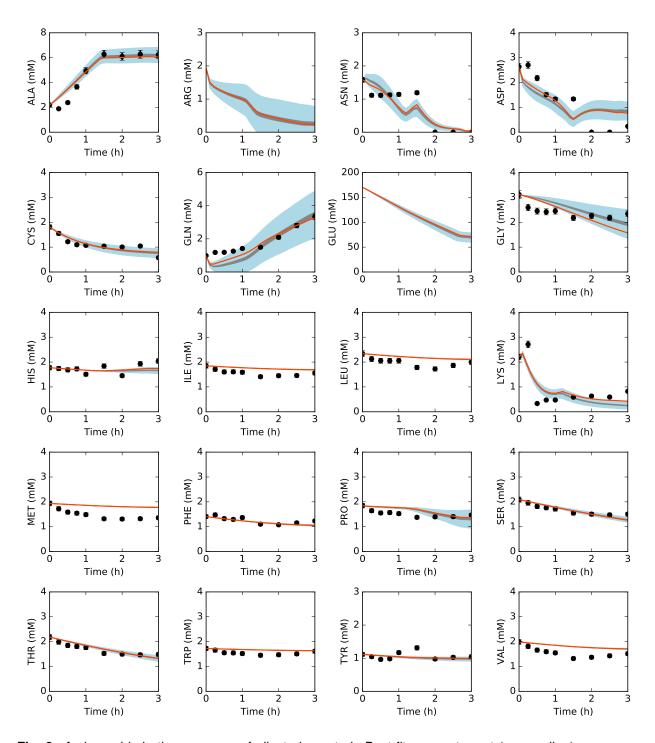


Fig. 3: Amino acids in the presence of allosteric control. Best-fit parameter set (orange line) versus experimental data (points). 95% confidence interval (blue shaded region) and 95% confidence interval of the mean (gray shaded region) over the ensemble of 100 sets.

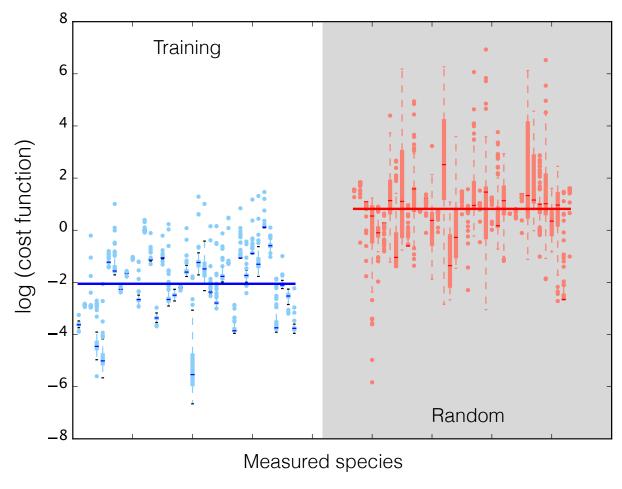


Fig. 4: Log of cost function across 37 datasets for data-trained ensemble (blue) and randomly generated ensemble (red, gray background). Median (bars), interquartile range (boxes), range excluding outliers (dashed lines), and outliers (circles) for each dataset. Median across all datasets (large bar overlaid).

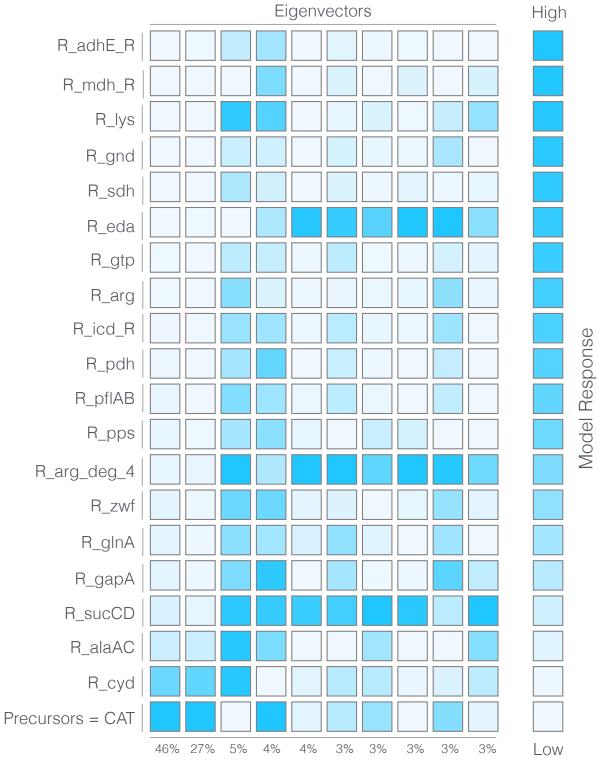


Fig. 5: Sensitivity of CAT production to model parameters, decomposed into eigenmodes. Sensitivity of top 10 eigenvectors (columns, left to right) to top 20 most influential parameters (rows, bottom to top). Relative magnitudes of eigenvalues expressed as percentages.

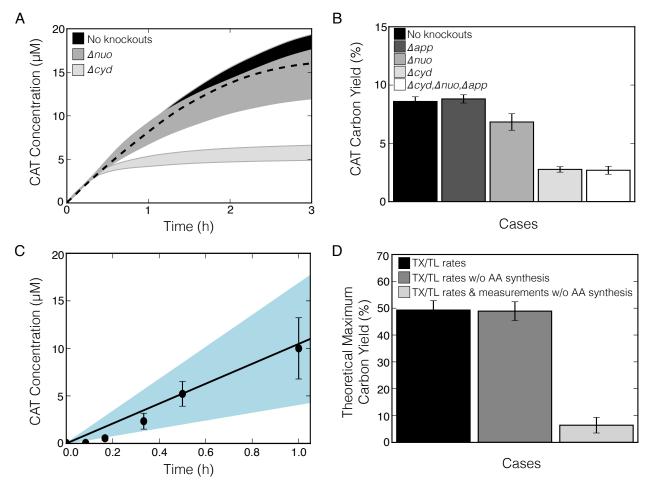


Fig. 6: The effects of oxidative phosphorylation and amino acid synthesis pathways on CAT production and carbon yield. A. 95% confidence interval of the ensemble of kinetic models for CAT concentration versus time, for the best-fit set with no knockouts (black shaded region and dashed line), *nuo* knockout (medium gray), and *cyd* knockout (light gray). B. CAT carbon yield of the ensemble of kinetic models for no knockouts (black), *app* knockout (dark gray), *nuo* knockout (medium gray), *cyd* knockout (light gray), and all three knockouts (white). Error bars represent standard deviation of the ensemble. C. 95% confidence interval of the ensemble of ssFBA simulations (light blue region) of CAT concentration over time, against experimental data (black). D. Theoretical maximum carbon yield of CAT production, calcualted by ssFBA for three different cases: constrained by transcription/translation (TX/TL) rates (black), same as previous but without amino acid synthesis reactions (medium gray), and same as previous but constrained by experimental measurements where available (light gray). Error bars represent standard deviation of the ensemble.

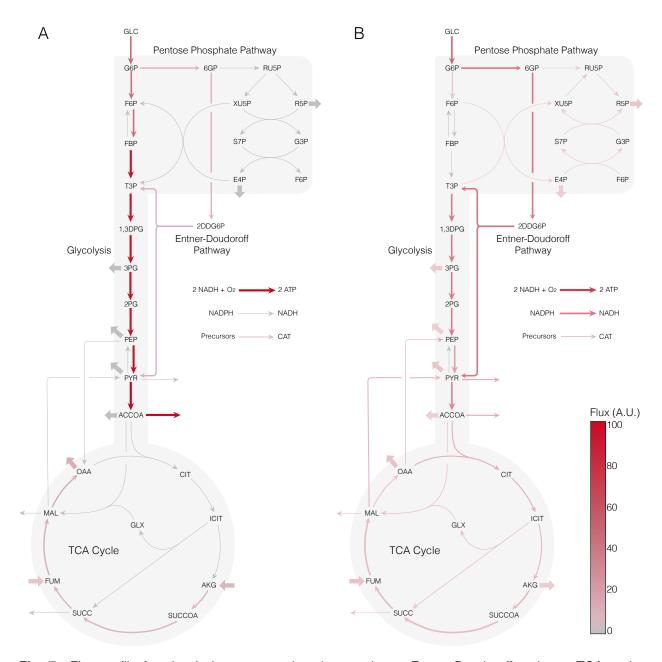


Fig. 7: Flux profile for glycolysis, pentose phosphate pathway, Entner-Doudoroff pathway, TCA cycle, NADPH/NADH transfer, oxidative phosphorylation, and protein synthesis. Thicker and redder arrows signify larger flux values (see colorbar). A. ssFBA constrained by transcription and translation rates, with or without amino acid synthesis reactions. B. ssFBA constrained by transcription and translation rates and experimental measurements where available.

Table 1: CAT carbon yield breakdown for best-fit set, knockouts, and experimental data. Carbon produced as CAT, carbon consumed as glucose and each amino acid, sum of consumed species, and yield. Accumulation of alanine and glutamine (negative consumption terms) was not considered in yield calculation.

Carbon Produced (C-mM)	Best-fit	∆арр	Δ nuo	Δ cyd	Δ app Δ nuo Δ cyd	Data
CAT	20.9	21.4	18.1	6.5	5.1	21.6
Carbon Consumed (C-mM)						
GLC	215.4	215.4	215.4	215.4	159.8	215.4
ALA	-11.6	-11.4	1.7	-3.8	-3.2	-12.1
ASN	6.2	6.2	6.2	6.3	6.3	6.3
ASP	7.5	7.5	3.9	0.0	0.0	9.6
CYS	3.0	3.1	3.0	2.9	2.9	3.7
GLN	-11.4	-11.3	-4.0	1.8	2.7	-11.7
GLY	3.1	3.1	2.6	1.1	0.9	1.5
HIS	0.2	0.2	1.1	0.4	0.3	0.0
ILE	1.0	1.0	0.8	0.3	0.2	1.7
LEU	1.4	1.4	1.2	0.4	0.3	2.0
LYS	10.7	10.7	13.1	13.2	13.2	8.3
MET	0.8	0.8	0.7	0.2	0.2	2.9
PHE	3.2	3.3	2.8	1.0	0.8	1.6
PRO	2.4	2.4	0.7	0.2	0.2	1.9
SER	2.5	2.5	2.4	2.1	2.1	1.8
THR	3.4	3.4	3.3	2.9	2.8	2.8
TRP	1.0	1.0	0.8	0.3	0.2	1.2
TYR	1.1	1.1	1.1	0.4	0.4	0.6
VAL	1.4	1.5	1.2	0.4	0.4	2.4
Sum	264.3	264.6	262.0	249.3	193.7	263.7
Yield	7.9%	8.1%	6.9%	2.6%	2.7%	8.2%

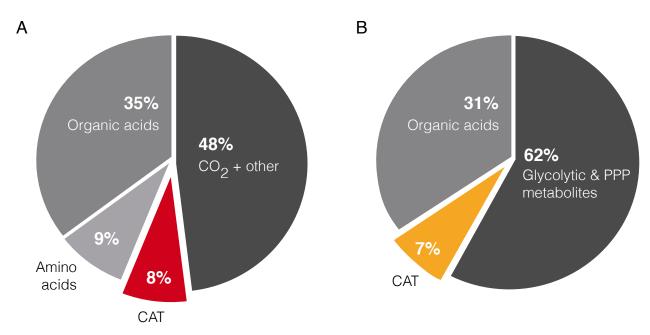


Fig. 8: Carbon and energy balances for the best-fit set. A. Carbon moles produced as CAT, amino acids (alanine and glutamine), organic acids (lactate, acetate, succinate, and malate), and other byproducts including carbon dioxide, as percentages of total carbon consumption (glucose and all other amino acids). B. Energy cost of CAT production, accumulation of organic acids (lactate, acetate, succinate, and malate), and glycolytic and pentose phosphate metabolites, as percentages of total energy utilization from glucose. Energy costs calculated in terms of equivalent ATP molecules.

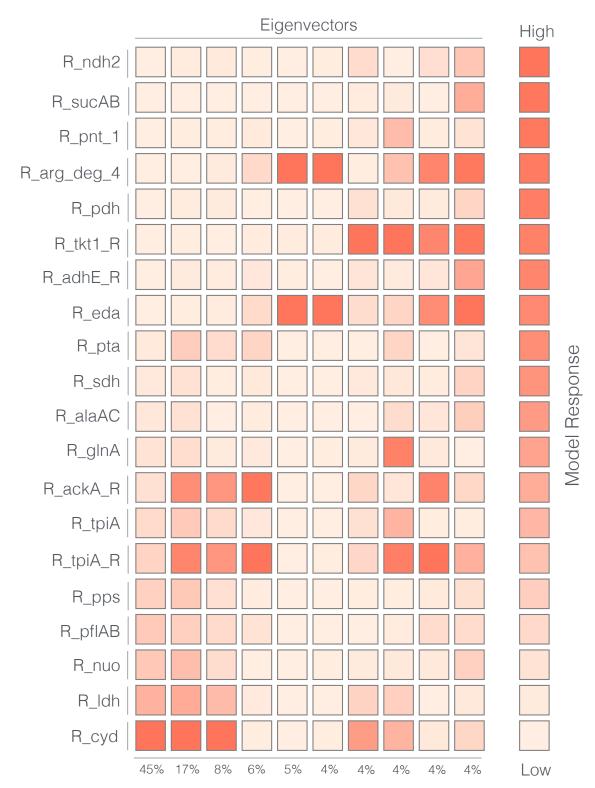


Fig. S1: Sensitivity of system state to model parameters, decomposed into eigenmodes. Sensitivity of top 10 eigenvectors (columns, left to right) to top 20 most influential parameters (rows, bottom to top). Relative magnitudes of eigenvalues expressed as percentages.