

Rachel LeCover¹ and Jeffrey D. Varner^{1,*}

ABSTRACT

We used the MIMIC II Waveform Database as the source of the pressure and heart rate data used to test this model.⁷ MIMIC contains de-identified data from patients who visited the Beth Israel Deaconess Medical Center ICU. This database contains a wealth of information: diagnoses, demographics, clinical data, such as heart rate and intravenous medications as well as laboratory results. A subset of the MIMIC II Waveform Database has been linked to patients found in the MIMIC II Database, allowing us to combine the waveforms with demographic information. From this subset, we selected patients that had numerics records with data recorded once per second with more than five data points available in the first 10 minutes of recorded data, giving us 273 patients meeting this criteria. To our knowledge, this is the first study to attempt to model heart rate as a function of blood pressure for such a large number of patients. Additionally, many previous studies were performed with healthy patients, in contrast, this study focuses on intensive care unit patients.

Results

One Dimensional Optimization of Parameters

We minimized averaged the mean squared error per patient using the Nelder Mead algorithm as implemented in Julia package NLOpt. We forced all of the parameters to be non-negative as to be biologically correct. The original and estimated values of the parameters are shown in Table 1.

Within the estimated parameter set, the value of N , the baseline firing rate increased, and both k_1 and τ_1 , representing a change in the response rate of the faster firing baroreceptors in the model. Baroreceptors are generally divided into three groups, depending on their transmission speeds (on the order of half a second, five seconds, and five hundred seconds), but this model only includes two types.² The increase in τ_1 and the decrease in τ_2 in the estimated parameter set may be the result of oversimplification.

The baseline heart rate (h_0) decreased slightly, to 88 bpm, from the initial 100 bpm. Both τ_{ach} and m_{ach} , which determine how acetylcholine plays into the model, decreased dramatically in the estimated parameter set, with the reduction in m_{ach} reducing the effects of a change in acetylcholine on heart rate, and the decrease in τ_{ach} increasing the rate of acetylcholine change. The estimated parameter set reduced the averaged mean squared error to 352 from 1167. As seen in Figure 1, the estimated parameters slightly overestimate heart rate for this patient, but they perform far better than the original parameters, which severely underestimate the heart rate. This behavior is typical of the model on patients in this study.

k_2 and β were held constant as to reduce the search space, as done by Olufsen and Ottesen. α , the parameter used to smooth the pressure data was held constant at 1.5 to smooth the data with a minimal phase shift. The maximum firing rate, M , was held constant, as it only appears in this system of equations by normalizing n . The sympathetic nervous system delay, τ_d , was held constant, owing to the propagation of discontinuities.⁸

Clustering

We clustered the patients based on their age, average heart rate, and SAPS (Simplified Acute Physiology) score, a measure that estimates a patient's risk of death within an intensive care unit.⁹ We used the patient's average SAPS since some patients had multiple admissions resulting in more than one blood pressure-heart rate track. Through the use of the Clustering Julia package, we created up to 26 clusters based on these variables, and scored each cluster with the sum of its silhouettes, where a higher score means that each member of the cluster is more similar to the other members of the clusters.¹⁰ We found that grouping the patients into two clusters gave the highest score and therefore the best clustering. The sum of silhouettes as a function of number of clusters is shown in 5, and the patients by cluster in 2. Cluster 1 patients ($n = 165$), shown in white, tend to have a lower average heart rate (73 BPM compared to 88 BPM), and be older than patients in cluster 2 ($n = 108$) (69 years vs 59 years, on average).

Multidimensional Optimization

We used the Julia language POETs package, which combines simulated annealing with Pareto optimality to generate families of best parameters.¹¹ Using the two clusters formed by k-means, we minimized the averaged mean squared error to create these parameter families. We utilized a monotonically decreasing cooling schedule with five iterations at each temperature. The trade off curve generated using $\alpha = .9$ is shown in 8, and with $\alpha = .5$ in 9. We used the parameter families from the slower cooling for further analysis. To decrease the time necessary to perform the simulated annealing, we utilized Julia's transparent parallelization capabilities. With the @parallel (+) operator, we were able to calculate patient's mean squared errors in parallel. The speed up from parallelization is shown in 6. We then selected the ten best sets of parameters from each cluster to examine the performance of the model with the new parameters. A sample patient from cluster 1 is shown in 3, and from cluster 2 in 4. The new parameter families reduced the averaged mean squared error even further than the parameters estimated using Nelder Mead-the ten best families for cluster 1 reduced the averaged mean squared error to 135 or less, and for cluster two, the averaged mean squared error was reduced to 178, or less.

Both of the families had fairly similar values for m_{nor} and m_{ach} , however, cluster 1 had a larger value of N , the resting firing rate than cluster 2. Cluster 1 contains the older patients, on average, suggesting that the resting firing rate may be age dependent. The differences in average heart rate between the clusters apparent in the clustering are apparent in the difference in h_0 between the two clusters, with cluster 2 having a faster base line heart rate, at $1.749 \pm .05$ beats per second, corresponding to 104.95 ± 3 beats per minute, compared to 74.4 ± 4 beats per minute for cluster 1.

Sensitivity Using Finite Differences

The derivatives of all parameters were estimated using central differences.

$$\frac{dh}{d\theta_j} = \frac{h(\theta_0 + \frac{e_j}{2}) - h(\theta_0 - \frac{e_j}{2})}{\delta} \quad (1)$$

where $\delta = 10^{-8} * \theta_j$ and e_j is a vector of length δ in the j^{th} direction. To collapse the time dimension, we calculated overall state sensitivity coefficients.¹²

$$S_{0j}(t) = \frac{1}{n_s} p_j \left(\sum_{k=1}^{n_t} \sum_{i=1}^{n_s} \left[\frac{1}{x_i} \frac{dx_i(t_k)}{dp_j} \right]^2 \right)^{1/2} \quad (2)$$

where $n_s = 1$, as h is the only state variable and n_t is the number of time points available for that patient. We calculated sensitivities of parameters that were optimized and not that of those held constant.

Single Objective Sensitivity

From the original values provided by Olufsen and Ottesen, we found that h_0 was the most sensitive parameter, followed by N , m_{ach} , and m_{nor} in that order, of the parameters estimated. Of the four most sensitive equations, three of them (h_0 , m_{ach} and m_{nor}) appear directly in the equation for heart rate, and N indirectly appears, as both c_{ach} and c_{nor} are functions of N . The four estimated parameters that were the most sensitive in our study were among the five most sensitive parameters found by Olufsen and Ottesen, as seen in 3. However, k_2 showed a very high degree of sensitivity in both the original and estimated parameter set, even more so than any of the estimated parameters. It also had the highest variance in sensitivity, due to the fact that some patients exhibited a sensitivity to k_2 that was an order of magnitude larger than the mean.

Between the estimated and the original parameters, the sensitivity of h_0 barely changed, but N became much less sensitive in the estimated parameter set and m_{ach} increased in sensitivity. This apparent change in sensitivity originates from the method of calculation, where the sum is divided by the value of the parameter. As N increased in the estimated parameter set, dividing by this new value decreased the overall state sensitivity coefficient, while the opposite is true for m_{ach} .

Multiobjective Sensitivity

For the multiobjective case, we used the same finite differences, but averaged them not only over the patients, but over the families of parameters-the ten best for each cluster. As with the single objective case, h_0 is the most sensitive parameter, followed by N , as seen in 7 for both clusters, but N is about two times as sensitive in cluster 2 as in cluster one, whereas h_0 has nearly the same sensitivity for both clusters. If one were to rank the parameters by sensitivity, the ordering of the four most sensitive parameters between the two clusters is identical, however, τ_{nor} in cluster 2 is approximately an order of magnitude less sensitive than in cluster 1 and τ_{ach} is nearly three orders of magnitude less sensitive in cluster 2 compared to cluster 1.

If we were to compare to the parameters estimated by Nelder Mead, we find similar sensitivities in h_0 and N as well as in m_{ach} and m_{nor} . The sensitivities in all of the τ s included in the model is much smaller in the multiobjective case than in the single objective case. Part of this difference in sensitivity may arise from the differences in value found for these parameters by the different methods of optimization.

Discussion

We estimated three sets of parameters that can be used to predict heart rates of patients in intensive care units. The accuracy of the model was much improved when patients were grouped by their characteristics.

The model used has several shortcomings. This model oversimplifies the process of heart rate regulation. It does not include cardiac sympathetic afferent reflex, which is sensitive to both mechanical and chemical inputs.¹³ It can only be used for short time scales, as it does not include the slower acting renin-angiotensin-aldosterone system, which determines blood volume, and therefore blood pressure.¹ Nevertheless, it does capture the dynamics of the system at short time scales.

Methods

All calculations were carried out Ubuntu 16.04 using Julia version .0.4.5, with 7.7 GB of available RAM on a Intel Core i7-6700 CPU @ 3.40GHz. Differential equations were solved using the ODE package, with solvers ode23 and ode78, with an absolute and relative tolerance of 10^{-8} .

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Author contributions statement

Must include all authors, identified by initials, for example: A.A. conceived the experiment(s), A.A. and B.A. conducted the experiment(s), C.A. and D.A. analysed the results. All authors reviewed the manuscript.

Additional information

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Table 1. Original and estimated parameter values. The bolded parameters were held constant.

Parameter	α	N	M	k_1	k_2	τ_1	τ_2	τ_{ach}	τ_{nor}	β	h_0	m_{nor}	m_{ach}	τ_d
Original Value	1.5	75	120	1.5	0.5	0.5	250	0.5	0.5	6	1.67	0.96	0.7	7
Parameter Estimations from MIMIC II Data	1.5	107.33	120	41.139	0.5	14.062	202.925	2.94E-05	8.156	6	1.474	0.165	5.20E-02	7

Table 2. Average parameter values for the 10 best sets per cluster. Data is reported as mean \pm stdev. Bolded columns were held constant.

Cluster	α	N	M	k_1	k_2	τ_1	τ_2	τ_{ach}	τ_{nor}	β	h_0	m_{nor}	m_{ach}	τ_d
1	1.5	50.36 \pm 9	120	.9566 \pm .3	0.5	.1854 \pm .13	127.31 \pm 46	.5968 \pm .4	.3176 \pm .06	6	1.245 \pm .006	.1816 \pm .02	.0434 \pm .004	7
2	1.5	37.98 \pm 9	120	1.379 \pm .7	0.5	.3621 \pm .11	82.88 \pm 33	.1410 \pm .06	.1688 \pm .04	6	1.749 \pm .05	.1609 \pm .02	.0962 \pm .06	7

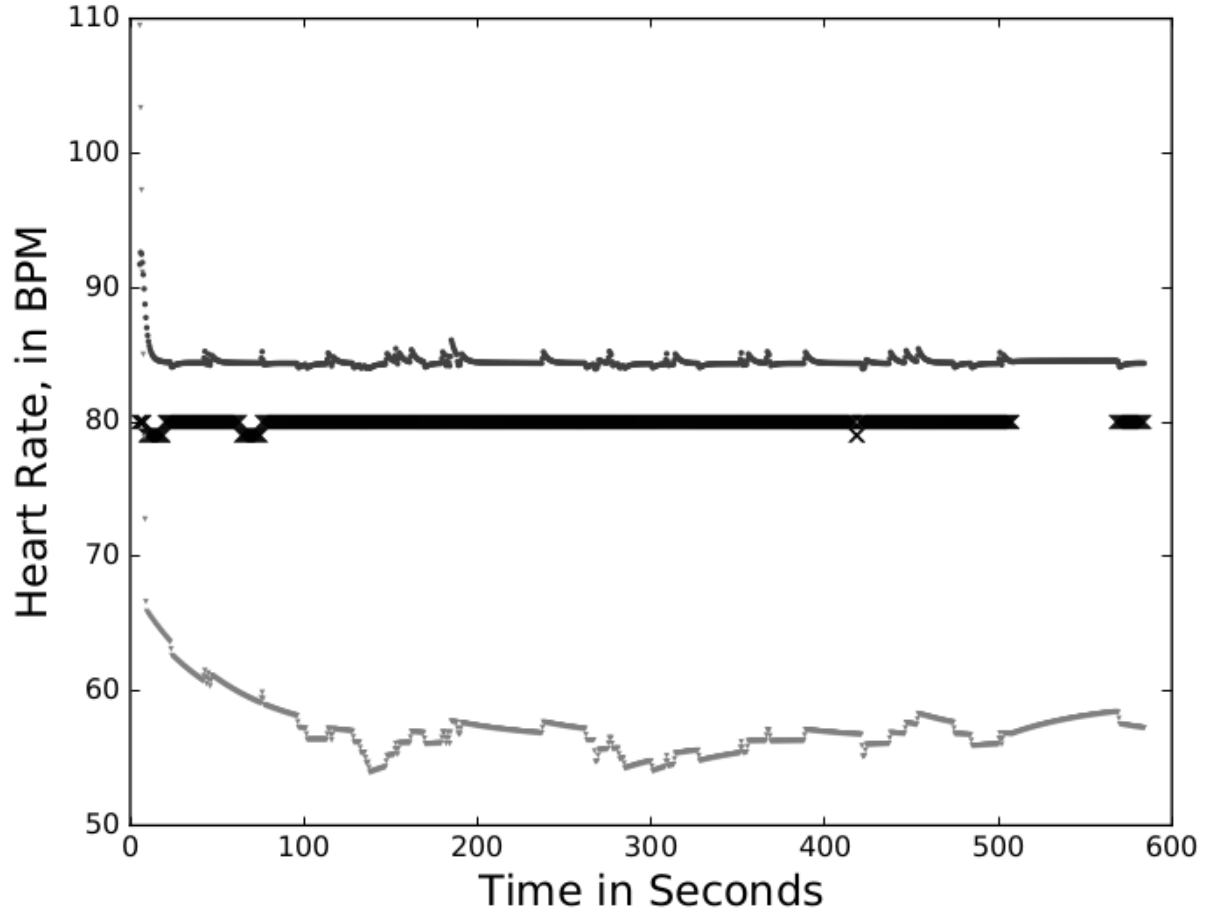


Figure 1. The black xs are the measured heart rate, the dark gray is the model prediction using the original parameters and the light gray is the model prediction using the original parameters.

Table 3. Average Overall State Sensitivity Coefficients For Single Objective Case

Parameter set	S_{α}	S_N	S_M	S_{k_1}	S_{k_2}	S_{τ_1}	S_{τ_2}	$S_{\tau_{ach}}$	$S_{\tau_{por}}$	S_{β}	S_{h_0}	$S_{m_{ach}}$	$S_{m_{nor}}$	S_{τ_d}
original parameters	3.99 ± 2	15.26 ± 4	$.0069 \pm .002$	$.0792 \pm .04$	142.61 ± 30	$.0739 \pm .04$	$.0156 \pm .004$	$.0540 \pm .013$	$0.00055 \pm 4E-4$	$.113 \pm .13$	19.9 ± 6	$.574 \pm .7$	14.6 ± 4	$.085 \pm .09$
estimated parameters	70 ± 200	2.24 ± 5	$.322 \pm .6$	$.89 \pm 3$	108 ± 400	1.20 ± 4	$.61 \pm 3$	0.52 ± 2	4.34 ± 1.9	$.234 \pm .19$	20.1 ± 6	6.79 ± 2	$.83 \pm 1.9$	$2.08E-7 \pm 2E-7$

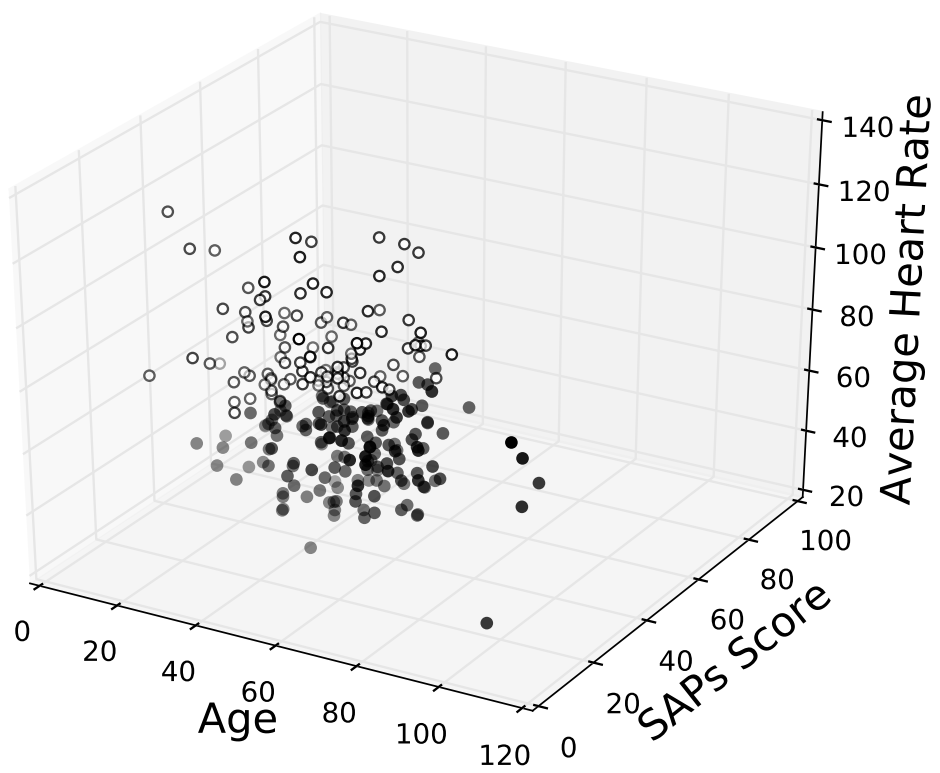


Figure 2. The patients, clustered into two groups.

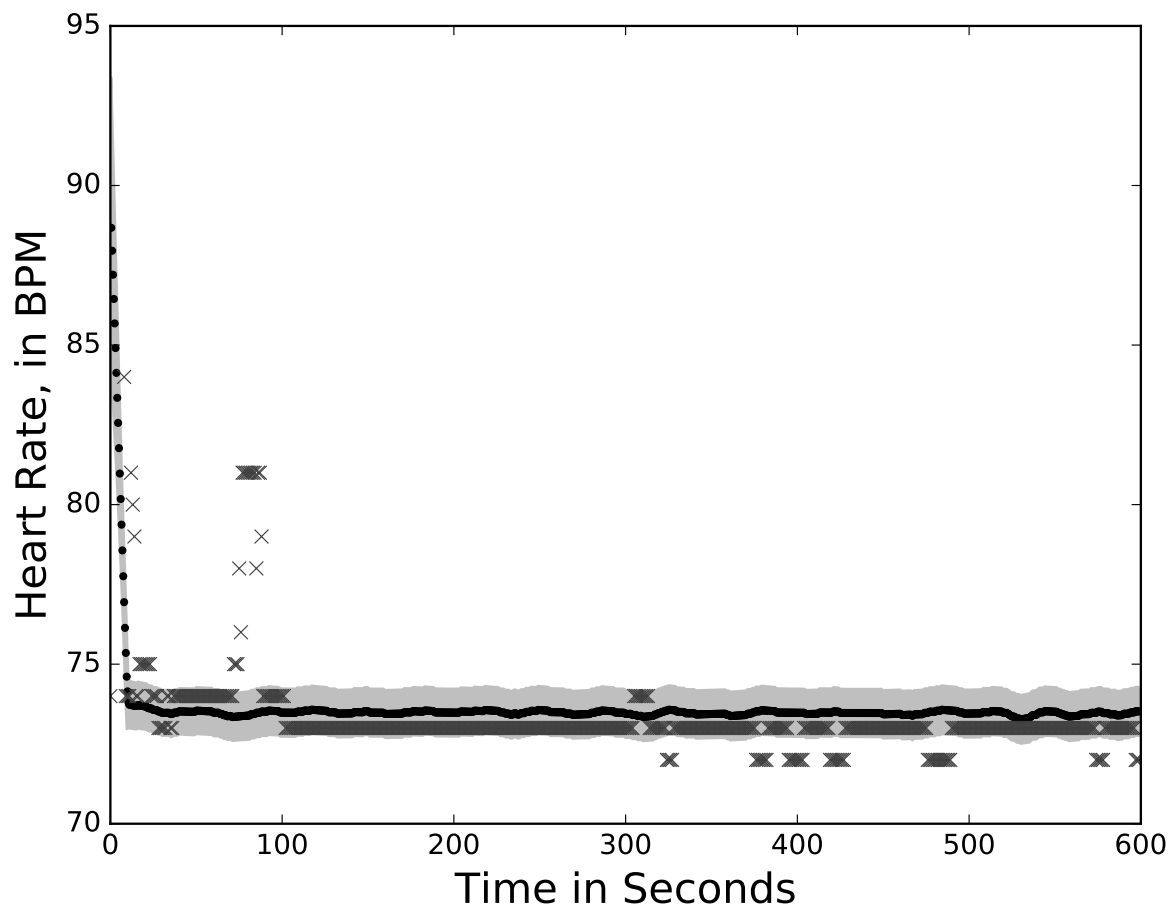


Figure 3. Performance of the model on a patient in cluster 1. The x represent the true heart rate, the black dots are the mean model prediction from the family of best parameter sets, and the grey gives a 95% confidence interval.

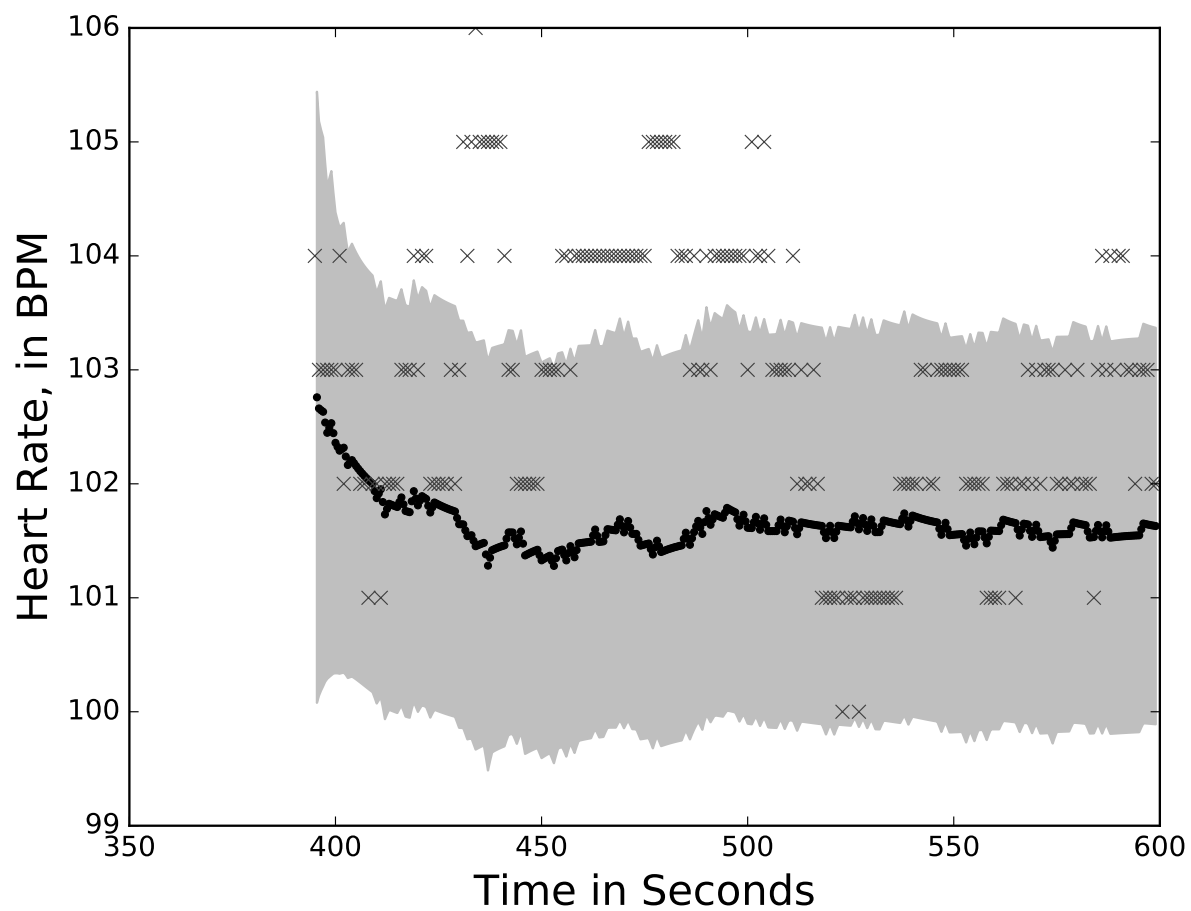


Figure 4. Performance of the model on a patient in cluster 2. The x represent the true heart rate, the black dots are the mean model prediction from the family of best parameter sets, and the grey gives a 95% confidence interval.

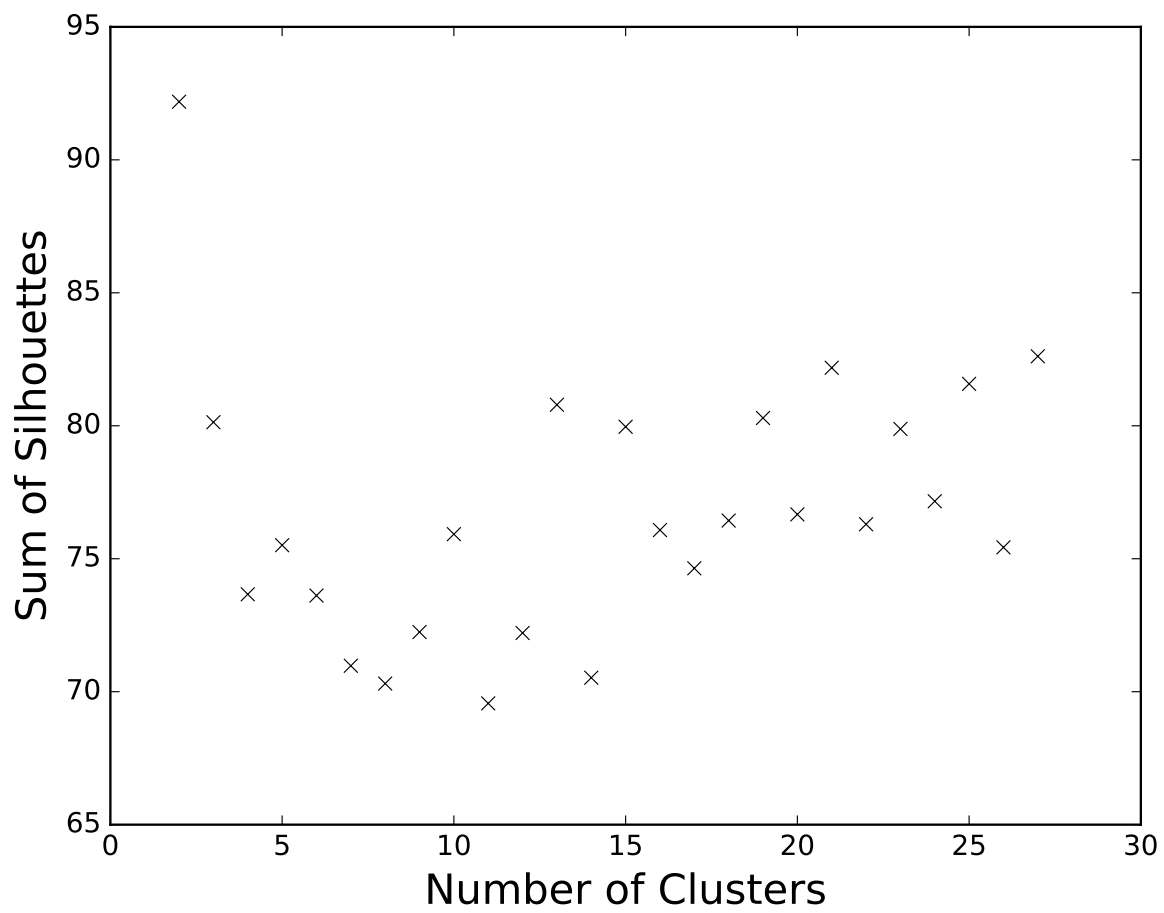


Figure 5. Sum of silhouettes as function of number of clusters

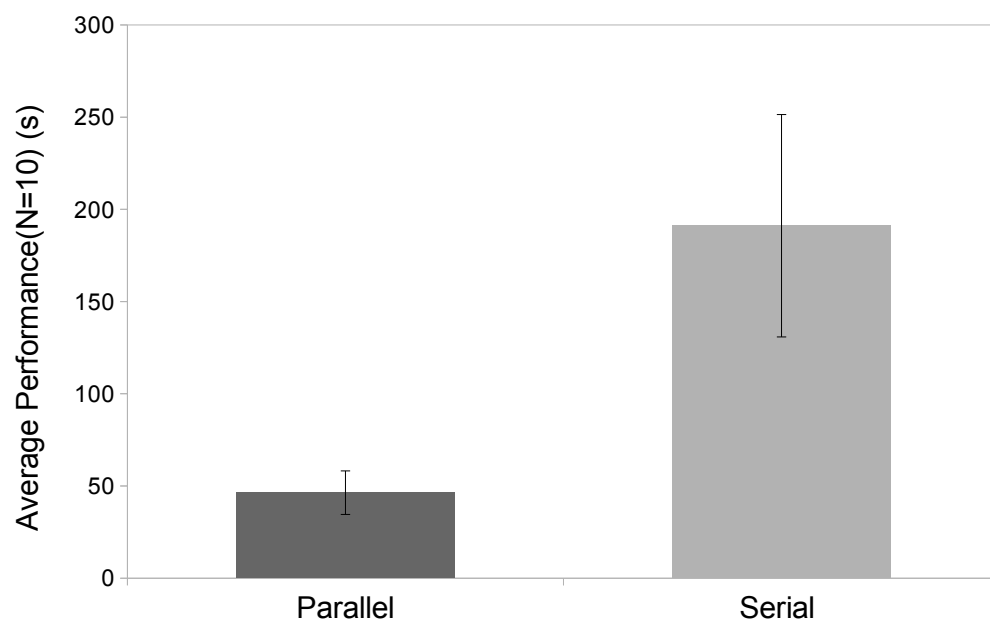


Figure 6. Switching from serial to parallel computation resulted in a significant speed up. In parallel operation, six cores were used.

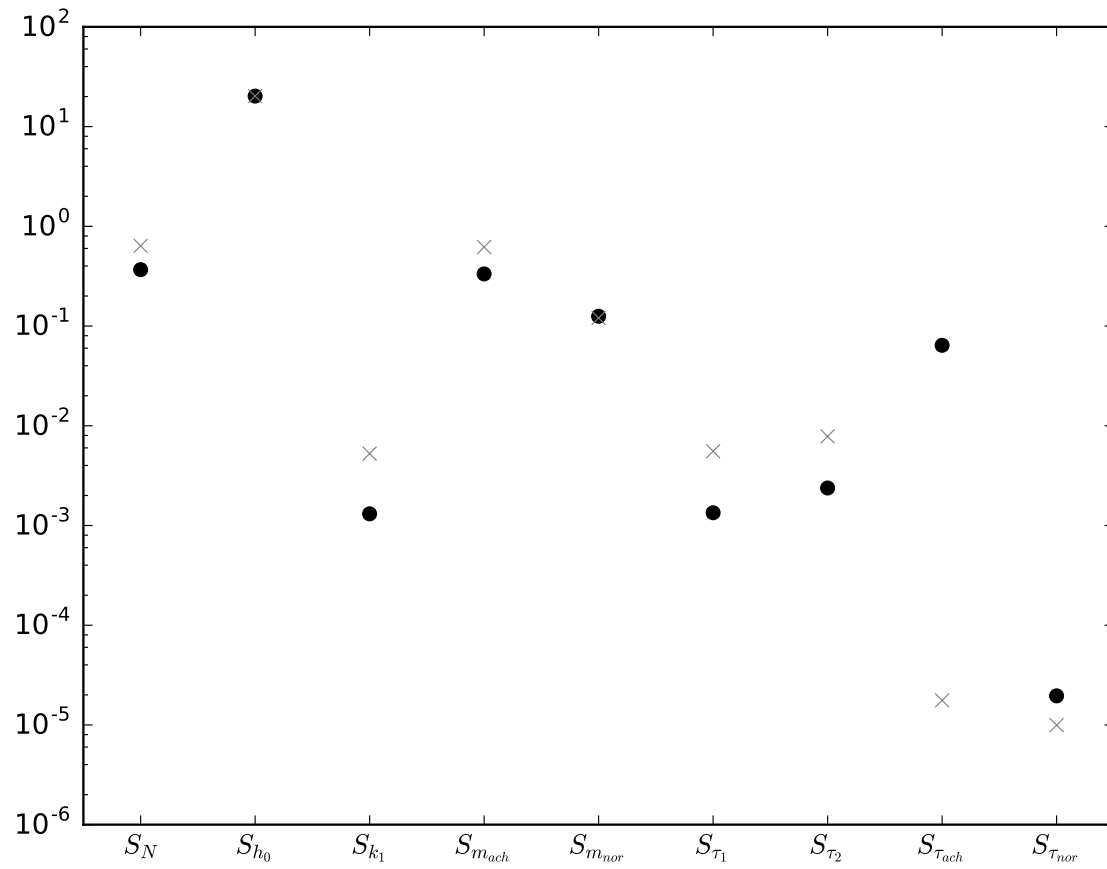


Figure 7. The black circles are from cluster 1, the grey x's represent cluster 2. Error bars are omitted for the sake of clarity.

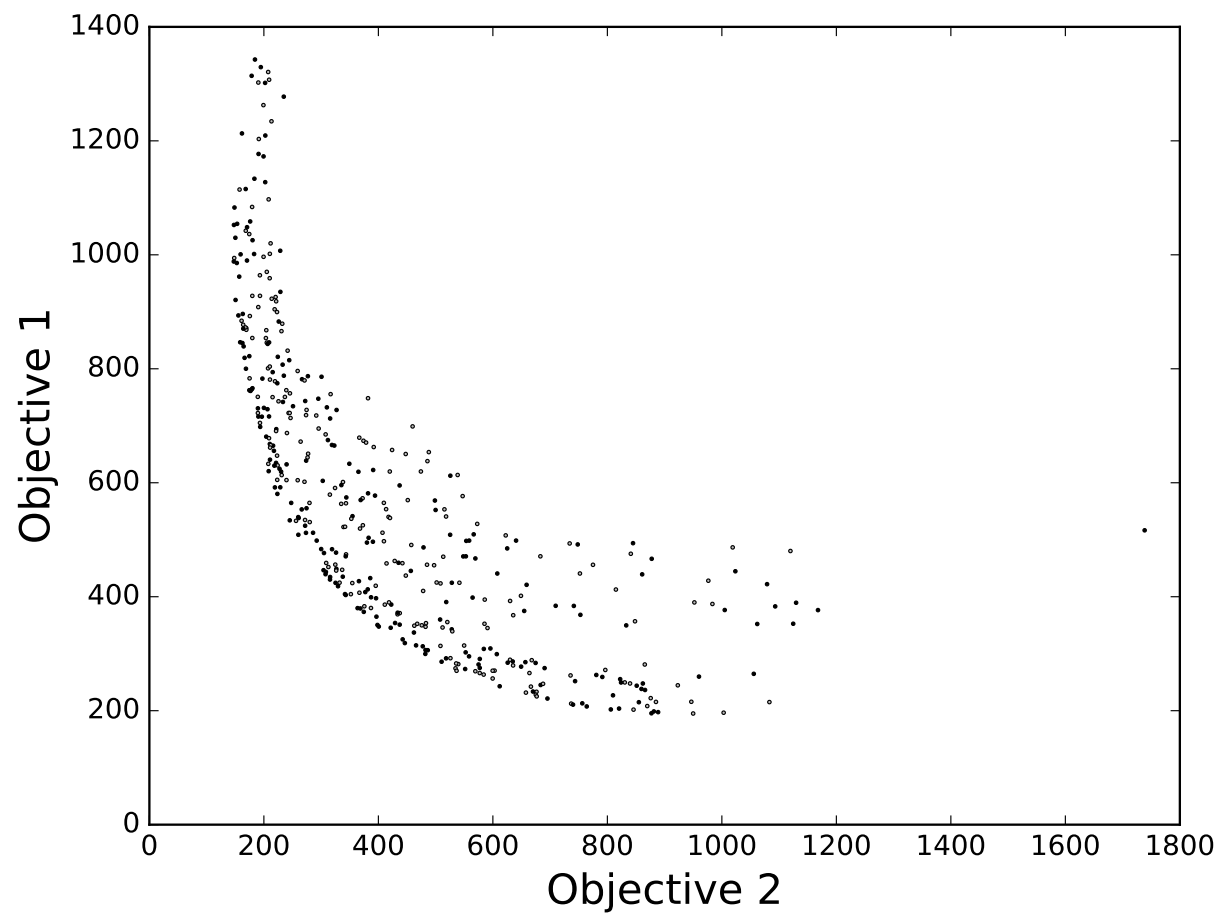


Figure 8. The black dots represent the rank 0, or best parameter sets, and the grey dots represent rank 1-4 parameter sets.

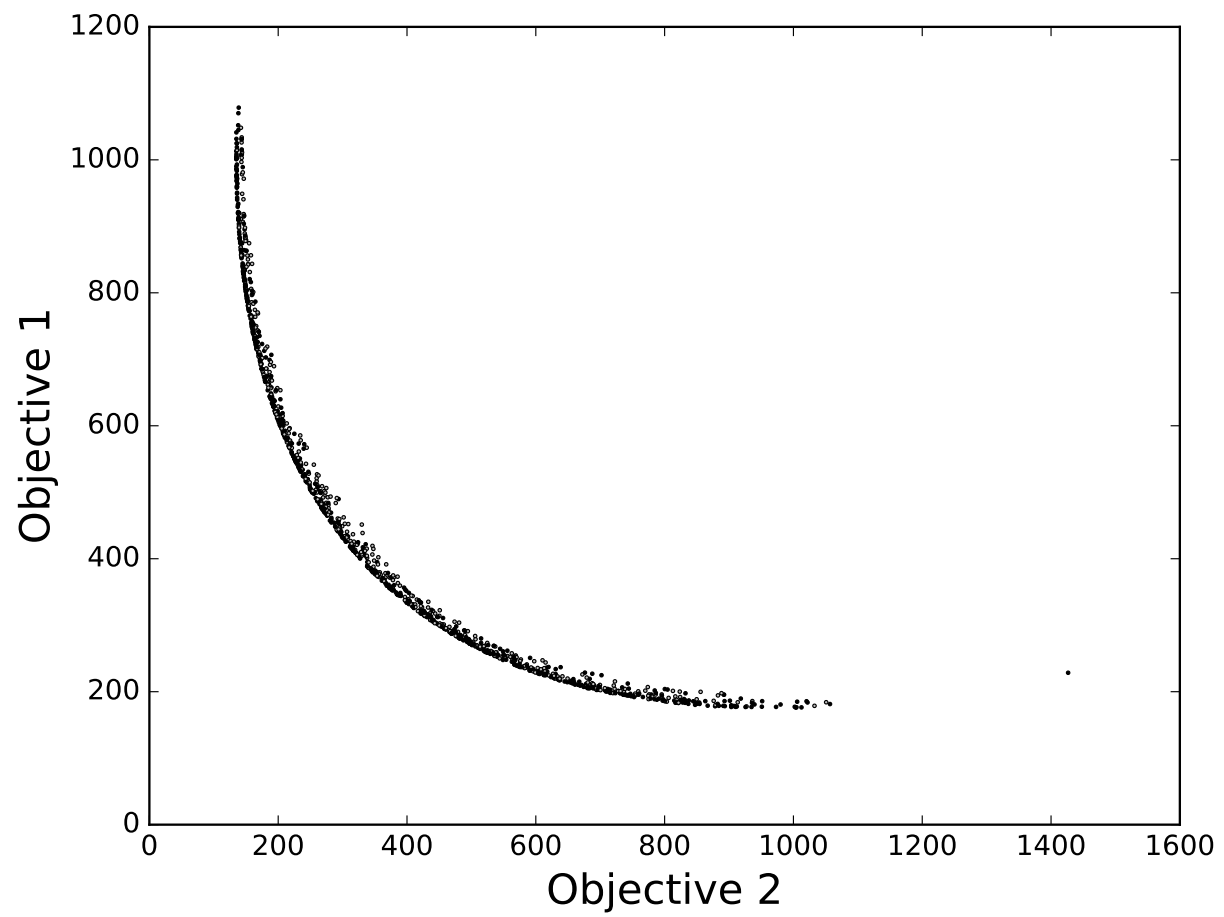


Figure 9. The black dots represent the rank 0, or best parameter sets, and the grey dots represent rank 1-4 parameter sets.