Multidisciplinary System Design Optimization (MSDO)

Multiobjective Optimization (I)

Lecture 14

by

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Where in Framework ?

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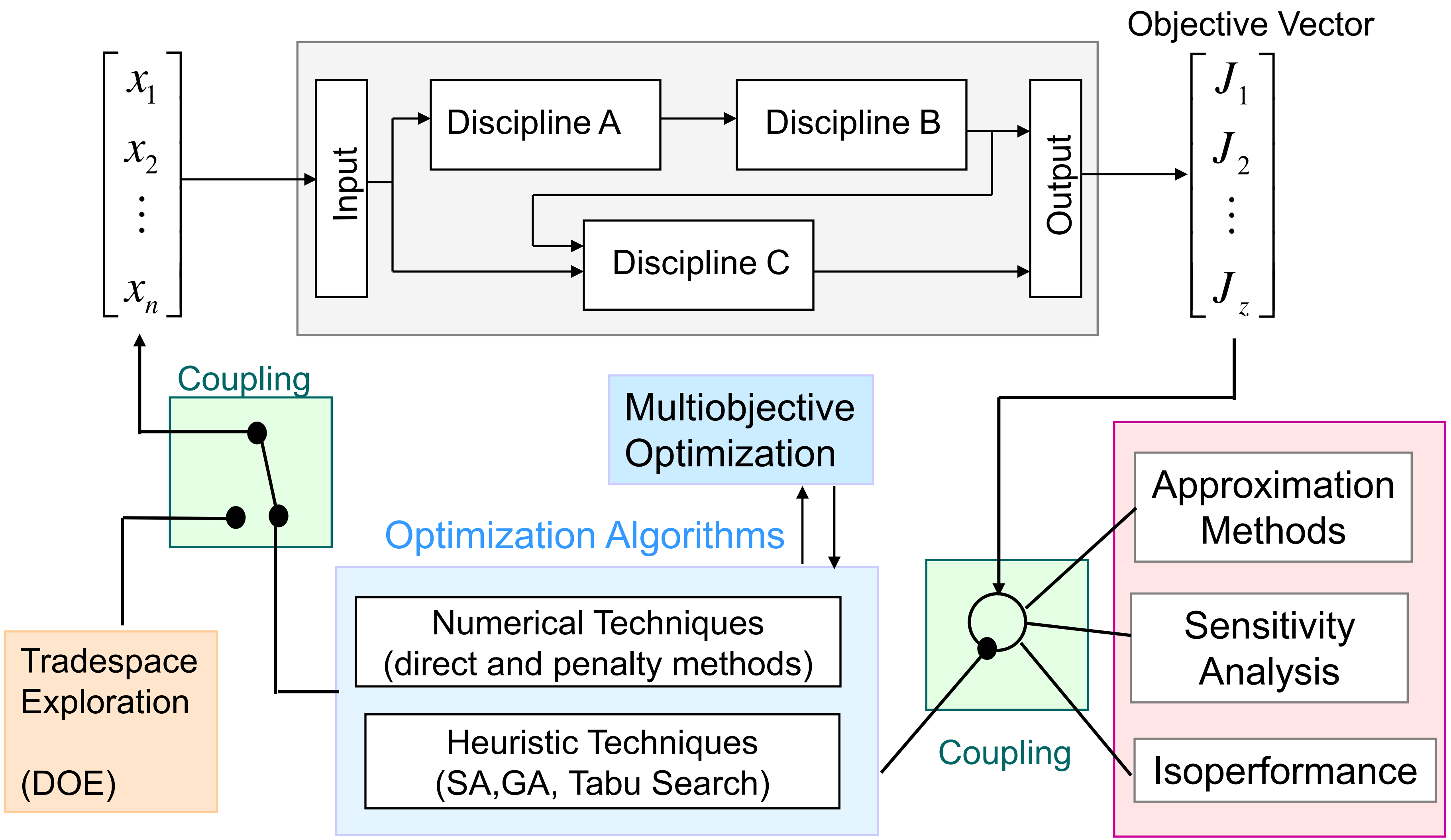

Lecture Content

- Why multiobjective optimization?
- Example – twin peaks optimization
- History of multiobjective optimization
- Weighted Sum Approach
- Pareto-Optimality
- Dominance and Pareto Filtering

Multiobjective Optimization Problem Formal Definition

Design problem may be formulated
as a problem of Nonlinear Programming (NLP). When
Multiple objectives (criteria) are present we have a MONLP

$$\min \mathbf{J} \quad \mathbf{x},\mathbf{p}$$

$$\text{s.t.} \quad \mathbf{g}(\mathbf{x},\mathbf{p}) \leq 0$$

$$\mathbf{h}(\mathbf{x},\mathbf{p}) = 0$$

$$x_{i,LB} \leq x_i \leq x_{i,UB} \quad (i = 1,\ldots,n)$$

where

$$\mathbf{J} = \left[J_1 \quad \mathbf{x} \quad \cdots \quad J_z \quad \mathbf{x} \right]^T$$

$$\mathbf{x} = \quad x_1 \quad \cdots \quad x_i \quad \cdots \quad x_n \quad {}^T$$

$$\mathbf{g} = \left[g_1(\mathbf{x}) \cdots g_{m_1}(\mathbf{x}) \right]^T$$

$$\mathbf{h} = \left[h_1(\mathbf{x}) \cdots h_{m_2}(\mathbf{x}) \right]^T$$

Multiple Objectives

The objective can be a vector **J** of *z* system responses or characteristics we are trying to maximize or minimize

$$\mathbf{J} = \begin{bmatrix} J_1 \\ J_2 \\ J_3 \\ J_i \\ \vdots \\ J_z \end{bmatrix} = \begin{bmatrix} \text{cost} \quad [\$] \\ -\text{ range} \quad [\text{km}] \\ \text{weight } [\text{kg}] \\ -\text{ data rate } [\text{bps}] \\ \vdots \\ -\text{ ROI} \quad [\%] \end{bmatrix}$$

Often the objective is a scalar function, but for real systems often we attempt multi-objective optimization:

$$\mathbf{x} \mapsto \mathbf{J}(\mathbf{x})$$

Objectives are usually conflicting.

Why multiobjective optimization ?

While multidisciplinary design can be associated with the traditional disciplines such as aerodynamics, propulsion, structures, and controls there are also the lifecycle areas of manufacturability, supportability, and cost which require consideration.

After all, it is the balanced design with equal or weighted treatment of performance, cost, manufacturability and supportability which has to be the ultimate goal of multidisciplinary system design optimization.

⇨ Design attempts to satisfy multiple, possibly conflicting objectives at once.

Design Decisions

Aspect Ratio
Dihedral Angle
Vertical Tail Area
Engine Thrust
Skin Thickness

# of Engines
Fuselage Splices
Suspension Points
Location of Mission Computer
Access Door Locations

Example: F/A-18 Aircraft

Objectives

Speed
Range
Payload Capability
Radar Cross Section
Stall Speed
Stowed Volume

Acquisition cost
Cost/Flight hour
MTBF
Engine swap time
Assembly hours

Avionics growth Potential

Multiobjective Examples

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Design Optimization

Aircraft Design

max {range}
max {passenger volume}
max {payload mass}
min {specific fuel consumption}
max {cruise speed}
min {lifecycle cost}

$$\mathbf{J} = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_z \end{bmatrix}$$

Operations Research

Production Planning

max {total net revenue}
max {min net revenue in any time period}
min {backorders}
min {overtime}
min {finished goods inventory}

Multiobjective vs. Multidisciplinary

- Multiobjective Optimization
  - Optimizing conflicting objectives
  - e.g., Cost, Mass, Deformation
  - Issues: Form Objective Function that represents designer preference! Methods used to date are largely primitive.

- Multidisciplinary Design Optimization
  - Optimization involves several disciplines
  - e.g. Structures, Control, Aero, Manufacturing
  - Issues: Human and computational infrastructure, cultural, administrative, communication, software, computing time, methods

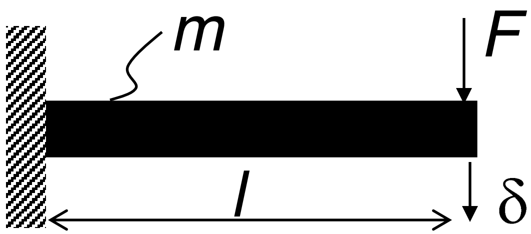
- All optimization is (or should be) multiobjective
  - Minimizing mass alone, as is often done, is problematic

Multidisciplinary vs. Multiobjective

single objective

single discipline

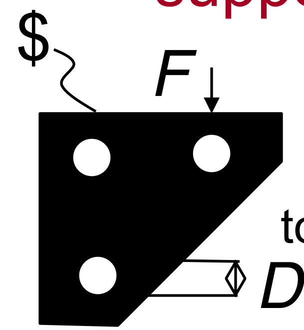
cantilever beam



Minimize displacement
s.t. mass and loading constraint

multiple disciplines

support bracket

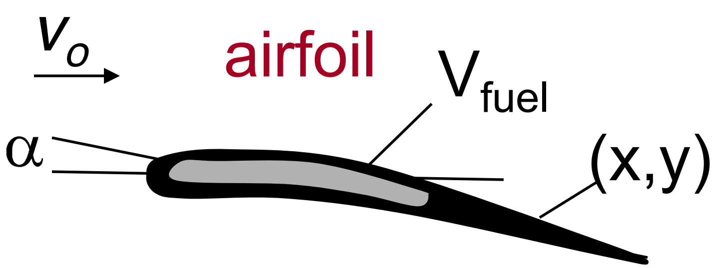


Minimize stamping costs (mfg) subject to loading and geometry constraint

multiple obj.

single discipline

airfoil



Maximize C_L/C_D and maximize wing fuel volume for specified α, v_o

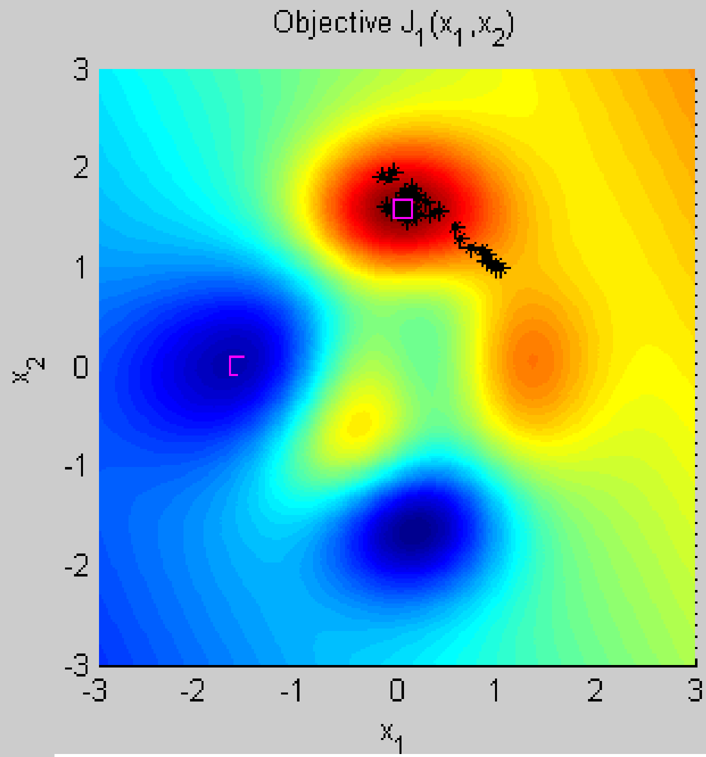
multiple disciplines

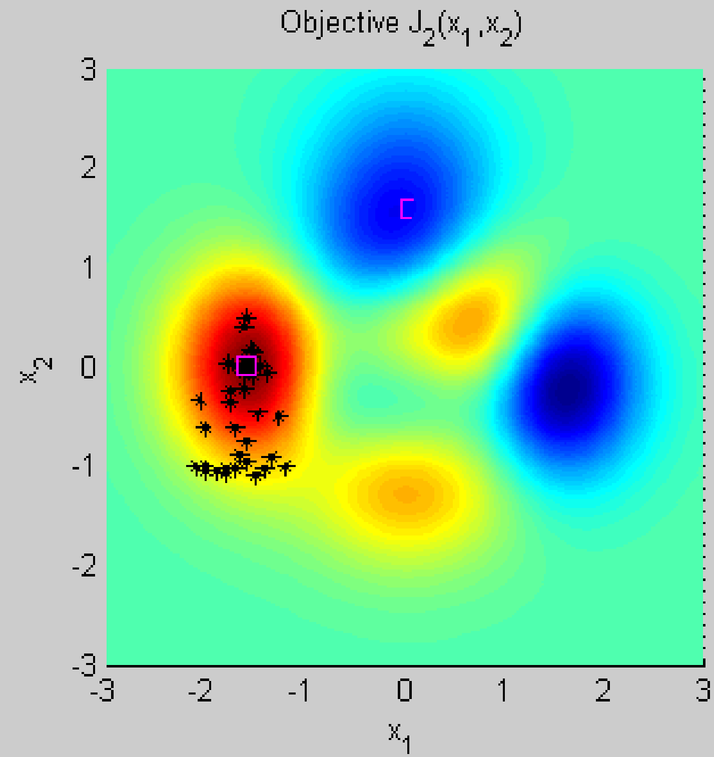
commercial aircraft

Minimize SFC and maximize cruise speed s.t. fixed range and payload

Example: Double Peaks Optimization

Objective: max **J**= [J_1 J_2]^T (demo)





$$J_1 = 3(1-x_1)^2 e^{-x_1^2-(x_2+1)^2}$$
$$-10\left(\frac{x_1}{5} - x_1^3 - x_2^5\right) e^{-x_1^2-x_2^2}$$
$$-3e^{-(x_1+2)^2-x_2^2} + 0.5(2x_1 + x_2)$$

$$J_2 = 3(1+x_2)^2 e^{x_2^2-(x_1+1)^2}$$
$$-10\left(-\frac{x_2}{5} + x_2^3 - x_1^5\right) e^{x_2^2-x_1^2} - 3e^{-(2-x_2)^2-x_1^2}$$

Double peaks optimization

Optimum for J_1 alone:

$$\mathbf{x}^{1*} = \begin{Bmatrix} 0.0532 \\ 1.5973 \end{Bmatrix}$$

$$\boxed{J_1^* = 8.9280}$$

$$J_2(\mathbf{x}^{1*}) = -4.8202$$

Optimum for J_2 alone:

$$\mathbf{x}^{2*} = \begin{Bmatrix} -1.5808 \\ 0.0095 \end{Bmatrix}$$

$$J_1(\mathbf{x}^{2*}) = -6.4858$$

$$\boxed{J_2^* = 8.1118}$$

Each point **x1*** and **x2*** optimizes objectives J_1 and J_2 individually. Unfortunately, at these points the other objective exhibits a low objective function value. There is <u>no single point</u> that simultaneously optimizes both objectives J_1 and J_2!

Tradeoff between J_1 and J_2

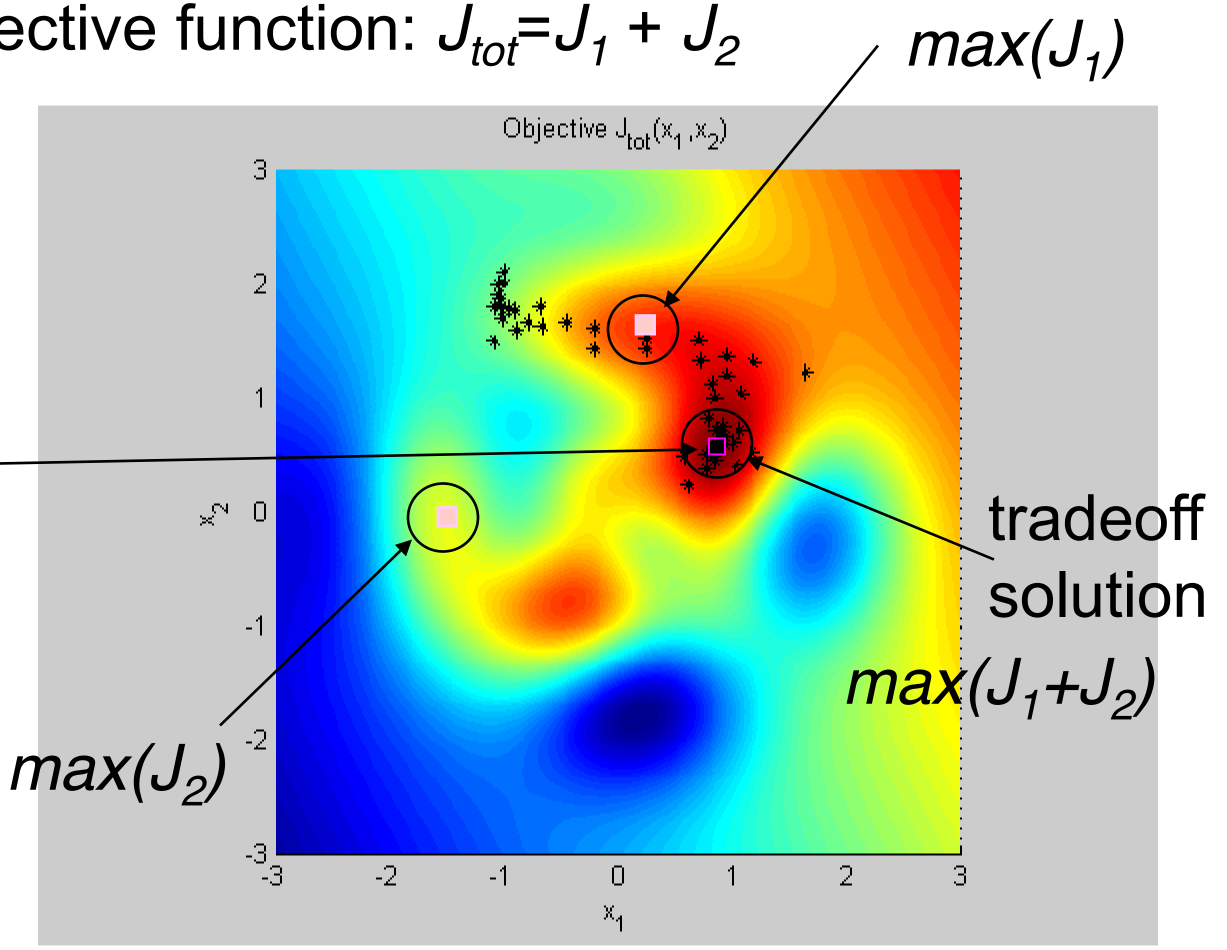
- Want to do well with respect to <u>both</u> J_1 and J_2
- Define new objective function: $J_{tot}=J_1 + J_2$
- Optimize J_{tot}

Result:

$$\mathbf{x}^{tot*} = \begin{Bmatrix} 0.8731 \\ 0.5664 \end{Bmatrix}$$

$$J_{tot}^* = 6.1439$$

$$\mathbf{J}(\mathbf{x}^{tot*}) = \begin{Bmatrix} 3.0173 \\ 3.1267 \end{Bmatrix} = \begin{Bmatrix} J1 \\ J2 \end{Bmatrix}$$



History (1) – Multicriteria Decision Making

- Life is about making decisions. Most people attempt to make the "best" decision within a specified set of possible decisions.
- In 1881, King's College (London) and later Oxford Economics Professor **F.Y. Edgeworth** is the first to define an optimum for multicriteria economic decision making. He does so for the multiutility problem within the context of two consumers, P and π:
  - *"It is required to find a point (x,y,) such that in whatever direction we take an infinitely small step, P and π do not increase together but that, while one increases, the other decreases."*
  - **Reference:** Edgeworth, F.Y., *Mathematical Psychics,* P. Keagan, London, England, 1881.

History (2) – Vilfredo Pareto

- **Born in Paris in 1848** to a French Mother and Genovese Father
- Graduates from the **University of Turin** in 1870 with a degree in **Civil Engineering**
  - Thesis Title: "The Fundamental Principles of **Equilibrium** in Solid Bodies"
- While working in Florence as a Civil Engineer from 1870-1893, Pareto takes up the study of philosophy and politics and is one of the **first to analyze economic problems with mathematical tools**.
- In 1893, Pareto becomes the Chair of Political Economy at the **University of Lausanne in Switzerland**, where he creates his two most famous theories:
  - Circulation of the Elites
  - **The Pareto Optimum**
    - ***"The optimum allocation of the resources of a society is not attained so long as it is possible to make at least one individual better off in his own estimation while keeping others as well off as before in their own estimation."***
    - **Reference:** Pareto, V., *Manuale di Economia Politica*, Societa Editrice Libraria, Milano, Italy, 1906. Translated into English by A.S. Schwier as *Manual of Political Economy*, Macmillan, New York, 1971.

History (3) – Extension to Engineering

- After the translation of Pareto's ***Manual of Political Economy*** into English, **Prof. Wolfram Stadler** of San Francisco State University begins to apply the notion of Pareto Optimality to the fields of engineering and science in the **middle 1970's**.
- The applications of multi-objective optimization in engineering design grew over the following decades.
- **References:**

  - Stadler, W., "A Survey of Multicriteria Optimization, or the Vector Maximum Problem," Journal of Optimization Theory and Applications, Vol. 29, pp. 1-52, 1979.
  - Stadler, W. "Applications of Multicriteria Optimization in Engineering and the Sciences (A Survey)," *Multiple Criteria* Decision Making – Past Decade and Future Trends, ed. M. Zeleny, JAI Press, Greenwich, Connecticut, 1984.
  - Ralph E. Steuer, "Multicriteria Optimization - Theory, Computation and Application", 1985

Notation and Classification

Traditionally - single objective constrained optimization:

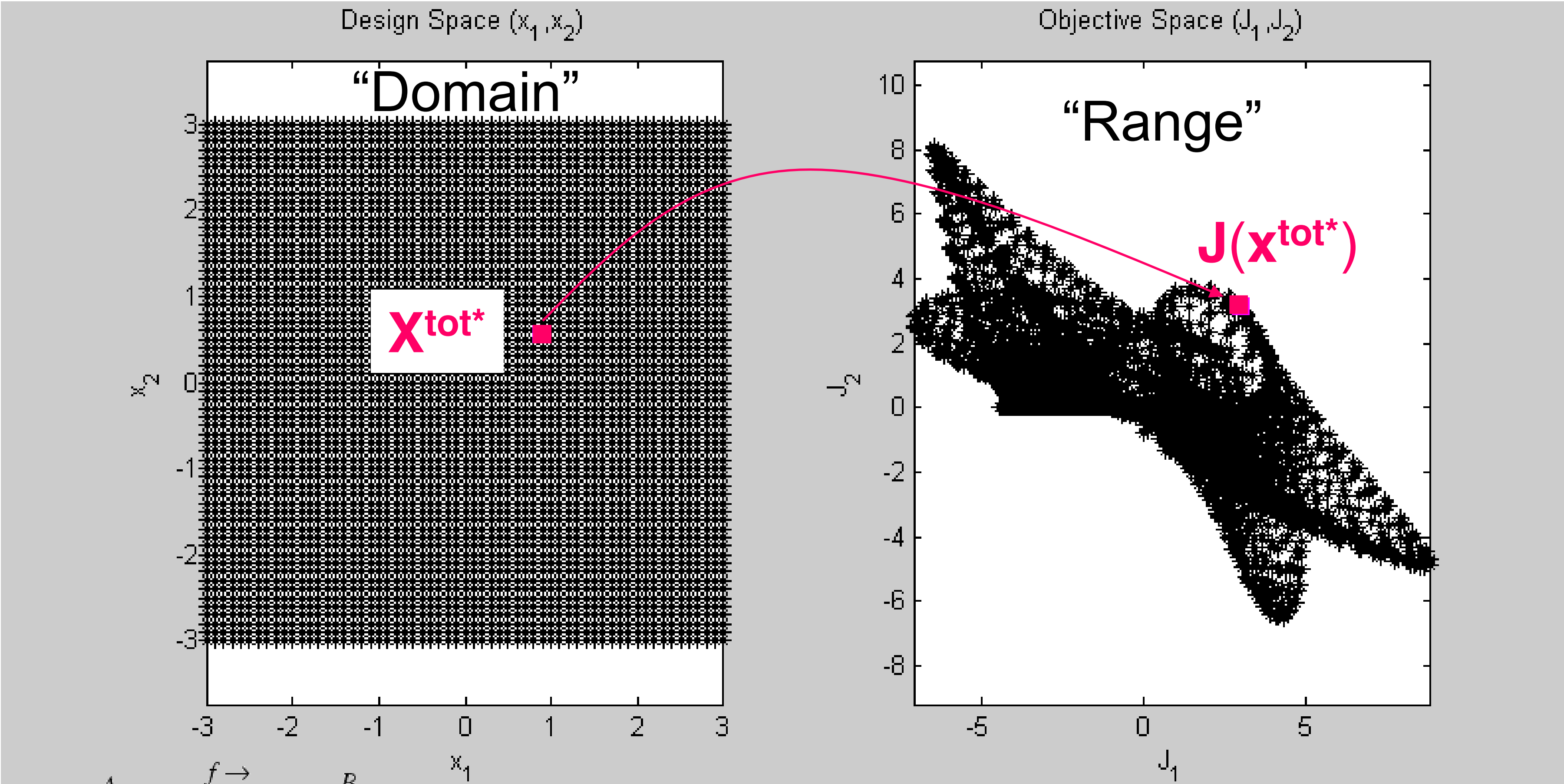
$$\max \mathbf{J} = f(\mathbf{x}) \qquad f(\mathbf{x}) \mapsto J \quad \text{objective function}$$

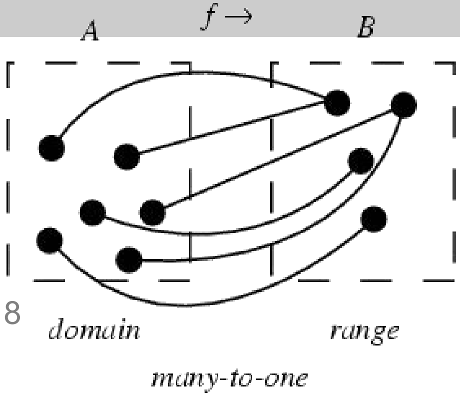
$$s.t. \quad \mathbf{x} \in S \qquad S \mapsto \quad \text{feasible region}$$

If $f(\mathbf{x})$ linear & constraints linear & single objective = LP
If $f(\mathbf{x})$ linear & constraints linear & multiple obj. = MOLP
If $f(\mathbf{x})$ and/or constraints nonlinear & single obj.= NLP
If $f(\mathbf{x})$ and/or constraints nonlinear & multiple obj.= MONLP

Ref: Ralph E. Steuer, "Multicriteria Optimization - Theory, Computation and Application", 1985

Design Space vs Objective Space





A function *f* which may (but does not necessarily) associate a given member of the range of *f* with more than one member of the domain of *f*.

Formal Solution of a MOO Problem

Trivial Case:

There is a point $x^* \in S$ that simultaneously optimizes all objectives J_i, where $1 \leq i \leq z$

Such a point almost never exists - i.e. there is no point that will simultaneously optimizes all objectives at once

Two fundamental approaches:

Scalarization Approaches

$$\max \; U \;\; J_1, J_2, \ldots, J_z$$

$$s.t. \;\; J_i = f_i \;\; \mathbf{x} \qquad 1 \leq \mathrm{i} \leq \mathrm{z}$$

$$\mathbf{x} \in S$$

Preferences included upfront

Pareto Approaches

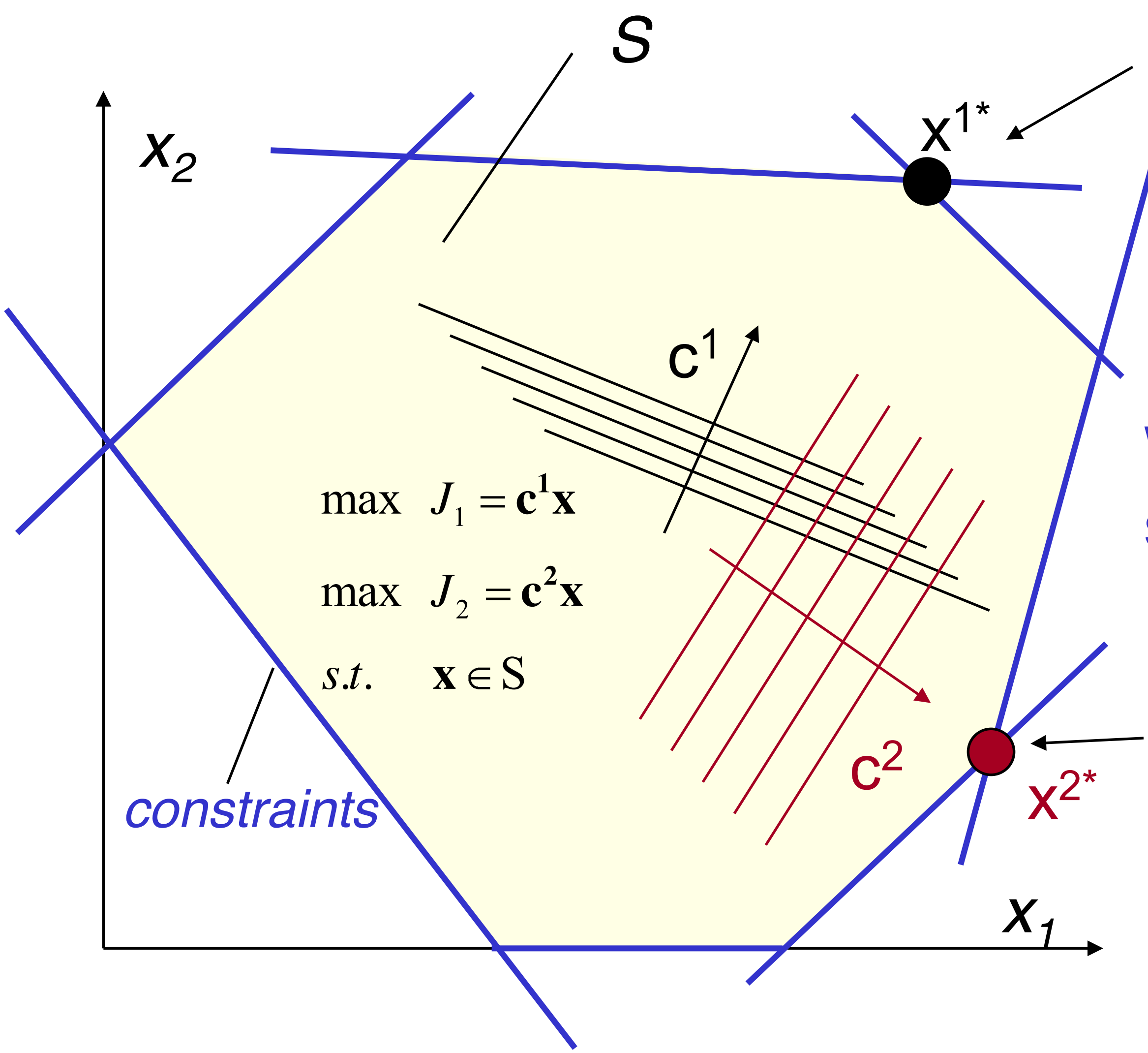
$J_i^1 \geq J_i^2 \quad \forall \; i$

and $J_i^1 > J_i^2$ for

at least one i

Preferences included a posteriori

SOLP versus MOLP



Optimal solution if only J_1 considered

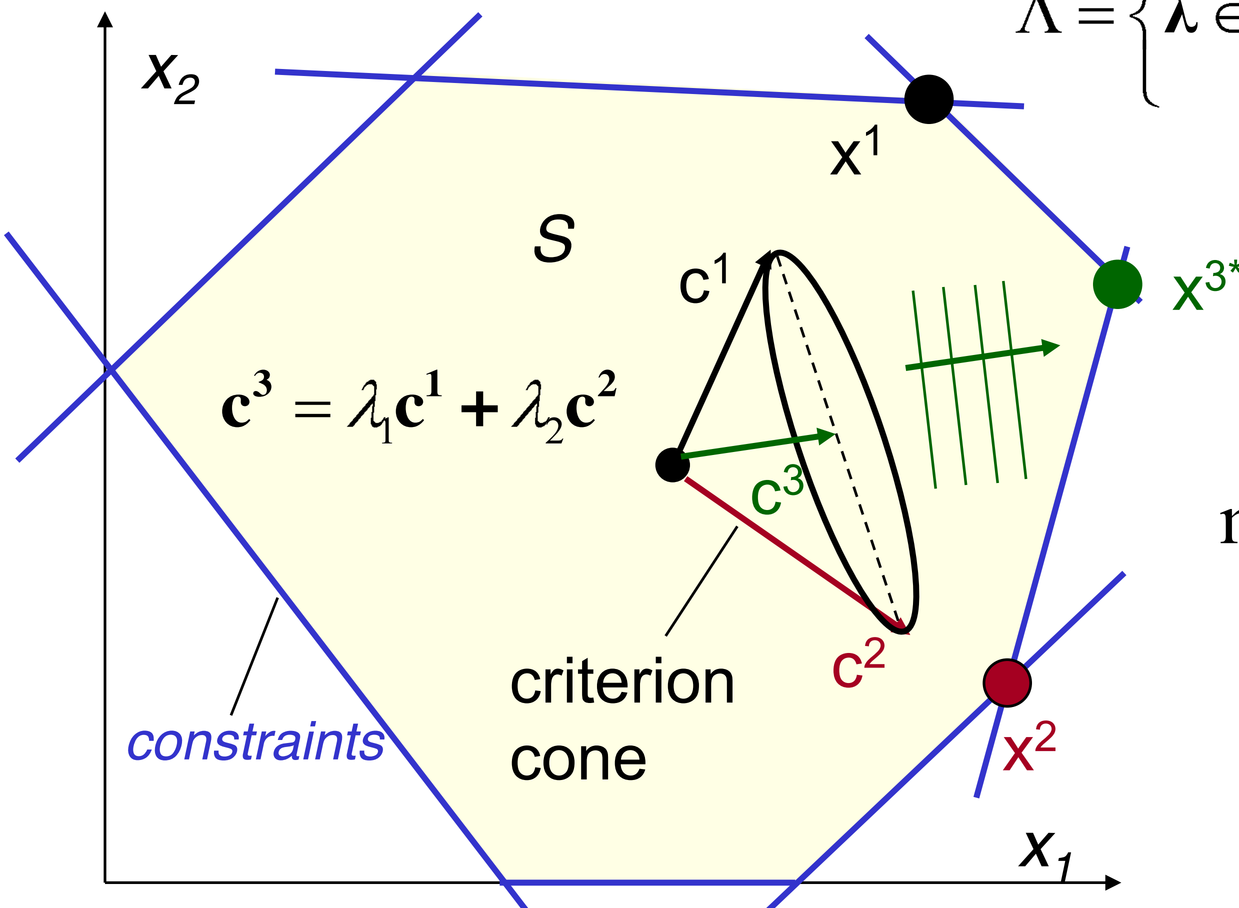
What is the optimal solution of a MOLP?

Optimal solution if only J_2 considered

Weighted-Sum Approach

Each objective *i* is multiplied by a strictly positive scalar λ_i

$$\Lambda = \left\{ \boldsymbol{\lambda} \in \mathbb{R}^z \,\middle|\, \lambda_i > 0, \sum_{i=1}^{k} \lambda_i = 1 \right\}$$



Solve the composite or WSLP:

$$\max \;\; \boldsymbol{\lambda}^T \mathbf{C}\mathbf{x} \,\middle|\, \mathbf{x} \in S$$

Strictly convex combination of objectives

Group Exercise: Weights (5 min)

We are trying to build the "optimal" automobile

There are six consumer groups:

- -G1: "25 year old single" (Cannes, France)
- -G2: "family w/3 kids" (St. Louis, MO)
- -G3: "electrician/entrepreneur" (Boston, MA)
- -G4: "traveling salesman" (Montana, MT)
- -G5: "old lady" (Rome, Italy)
- -G6: "taxi driver" (Hong Kong, China)

Objective Vector:

J1: Turning Radius [m]
J2: Acceleration [0-60mph]
J3: Cargo Space [m^3]
J4: Fuel Efficiency [mpg]
J5: Styling [Rating 0-10]
J6: Range [km]
J7: Crash Rating [poor-excellent]
J8: Passenger Space [m^3]
J9: Mean Time to Failure [km]

Assignment: Determine λ_i , i =1…9

$$\sum_{i=1}^{9} \lambda_i = 1000$$

What are the scale factors sf_i?

- Scaling is critical in multiobjective optimization
- Scale each objective by sf_i: $\bar{J}_i = J_i / sf_i$
- Common practice is to scale by $sf_i = J_i^*$
- Alternatively, scale to initial guess $\bar{\mathbf{J}}(\mathbf{x_o})=[1..1]^T$
- Multiobjective optimization then takes place in a non-dimensional, unit-less space
- Recover original objective function values by reverse scaling

Saab 9-5

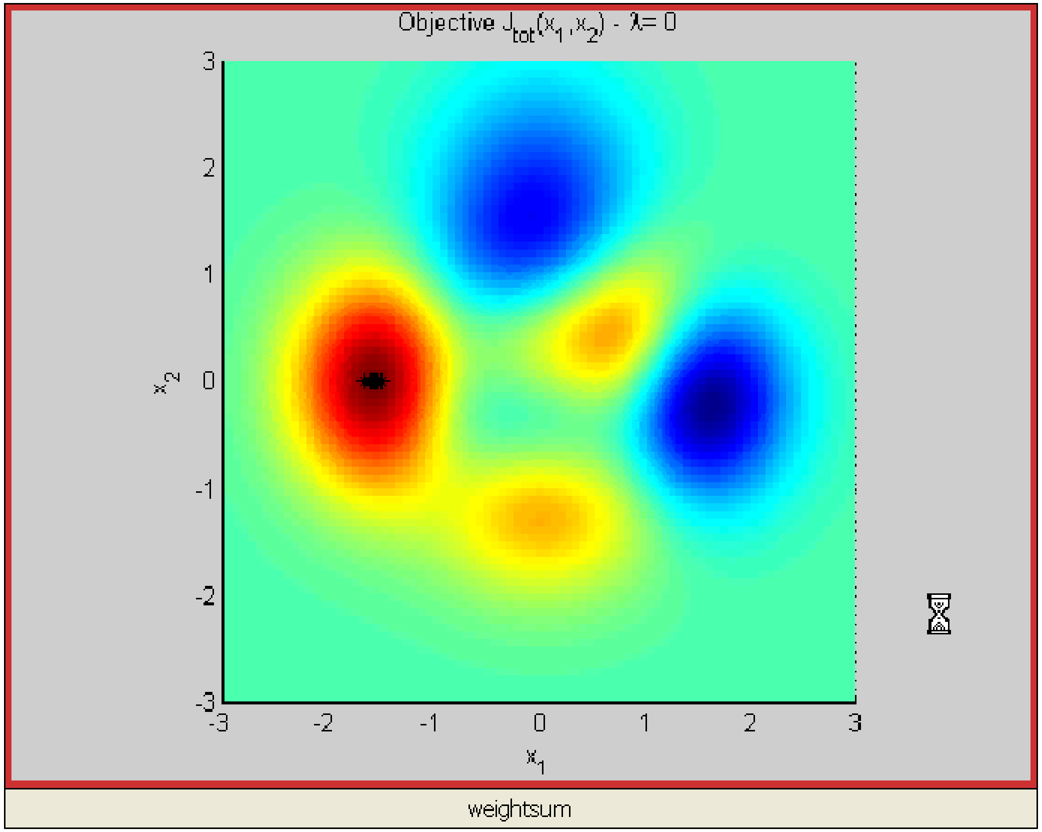
Example: J_1=range [sm] sf_1=573.5 [sm]

J_2=fuel efficiency [mpg] sf_2=36 [mpg]

Suzuki "Swift"

Weighted Sum: Double Peaks

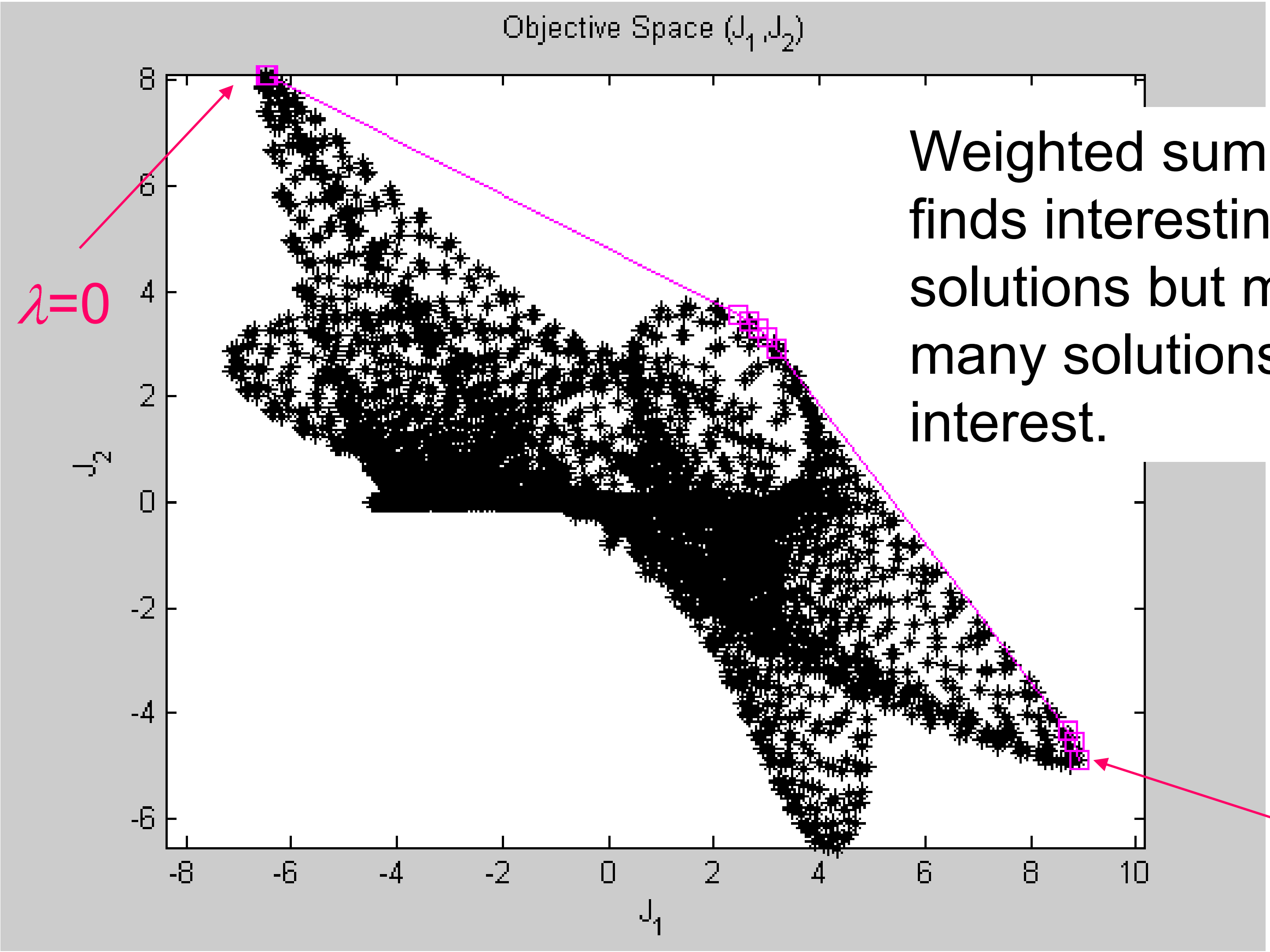
$$J_{tot} = \lambda J_1 + (1-\lambda) J_2 \quad \text{where} \quad \lambda \in [0,1]$$



$\Delta\lambda$=0.05

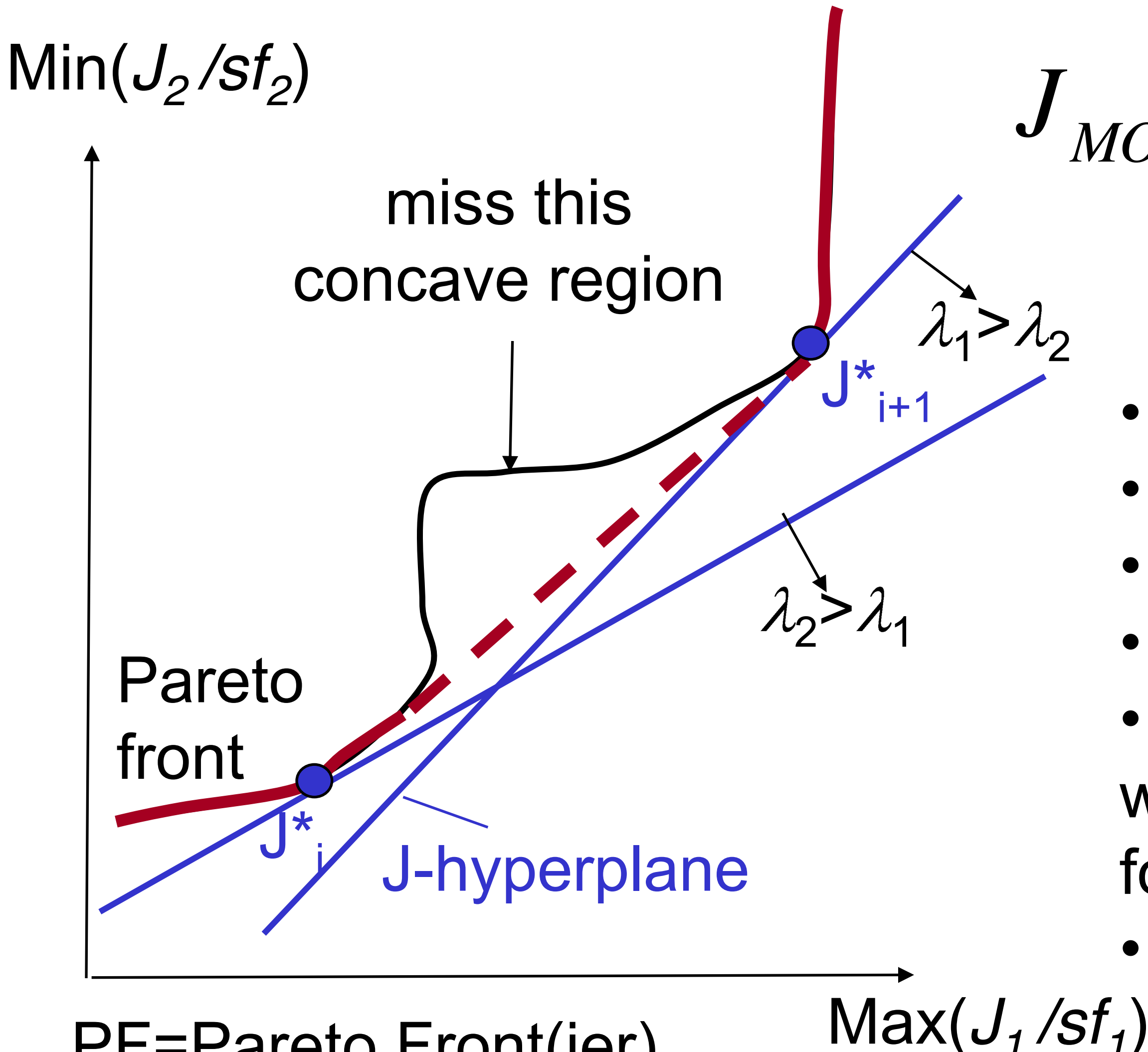
Demo: At each setting of λ we solve a new single objective optimization problem – the underlying function changes at each increment of λ

Weighted Sum Approach (II)



Weighted sum finds interesting, solutions but misses many solutions of interest.

Weighted Sum (WS) Approach



$$J_{MO} = \sum_{i=1}^{z} \frac{\lambda_i}{sf_i} J_i$$

weight

scale factor

- convert back to SOP
- LP in J-space
- easy to implement
- scaling important !
- weighting determines which point along PF is found
- misses concave PF

PF=Pareto Front(ier)

Properties of optimal solution

$\mathbf{x}^*$ optimal if $\mathbf{J}(\mathbf{x}^*) \geq \mathbf{J}(\mathbf{x})$ (maximization)

for $\mathbf{x}^* \in S$ and for $\mathbf{x} \neq \mathbf{x}^*$

This is why multiobjective optimization is also sometimes referred to as vector optimization

$\mathbf{x}^*$ must be an efficient solution

$\mathbf{x} \in S$ is efficient if and only if (iff) its objective vector (criteria) J($\mathbf{x}$) is non-dominated

A point $\mathbf{x} \in S$ is efficient if it is not possible to move feasibly from it to increase an objective without decreasing at least one of the others

Dominance (assuming maximization)

Let $\mathbf{J}^1, \mathbf{J}^2 \in \mathbb{R}^z$ be two objective (criterion) vectors.

$$J^i = \begin{bmatrix} J_1 \\ J_2 \\ \vdots \\ J_z \end{bmatrix}$$

Then $\mathbf{J}^1$ dominates $\mathbf{J}^2$ (weakly) iff

$$\mathbf{J}^1 \geq \mathbf{J}^2 \quad \text{and} \quad \mathbf{J}^1 \neq \mathbf{J}^2$$

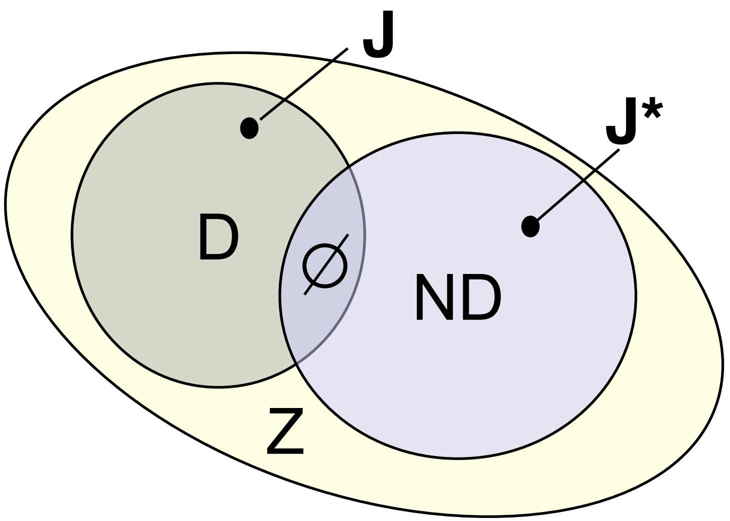
More precisely: $J_i^1 \geq J_i^2 \quad \forall\, i \quad$ and $\; J_i^1 > J_i^2 \;$ for at least one i

Also $\mathbf{J}^1$ strongly dominates $\mathbf{J}^2$ iff

$$\mathbf{J}^1 > \mathbf{J}^2$$

More precisely: $J_i^1 > J_i^2 \quad \forall\, i$

Set Theory



Set Theory:

$\mathbf{x} \in S$

$\mathbf{J}, \mathbf{J}^* \in Z$ A solution must be feasible

$D \cap ND = \varnothing$ A solution is either dominated or non-dominated but cannot be both at the same time

$D \subset Z, ND \subset Z$ All dominated and non-dominated solutions must be feasible

$D \cup ND = Z$ All feasible solutions are either non-dominated or dominated

$\mathbf{J}^* \quad \mathbf{x}^* \quad \in ND$ Pareto-optimal solutions are non-dominated

Dominance versus Efficiency

- Whereas the idea of dominance refers to vectors in criterion space *J*, the idea of efficiency refers to points in decision space *x*.

- Can use this criterion as a Pareto Filter if the design space has been explored (e.g. DoE).

Dominance - Exercise

max{range} [km]
min{cost} [$/km]
max{passengers} [-]
max{speed} [km/h]

Multiobjective Aircraft Design

#1	#2	#3	#4	#5	#6	#7	#8
7587	6695	3788	8108	5652	6777	5812	7432
321	211	308	278	223	355	401	208
112	345	450	88	212	90	185	208
950	820	750	999	812	901	788	790

⇨ Which designs are non-dominated ? (5 min)

Dominance - Exercise

Algorithm for extracting non-dominated solutions:

Pairwise comparison

#1		#2	Score #1	Score #2
7587	>	6695	1	0
321	>	211	0	1
112	<	345	0	1
950	>	820	1	0
			2 vs	2

Neither #1 nor #2 dominate each other

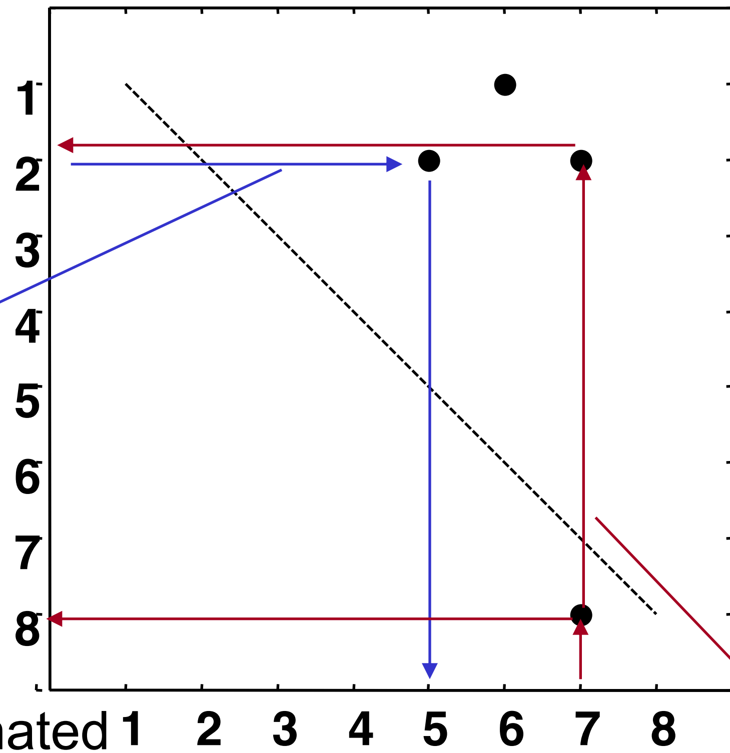
#1		#6	Score #1	Score #6
7587	>	6777	1	0
321	<	355	1	0
112	>	90	1	0
950	>	901	1	0
			4 vs	0

Solution #1 dominates solution #6

In order to be dominated a solution must have a”score” of 0 in pairwise comparison

Domination Matrix

Shows which solution dominates which other solution (horizontal rows) and (vertical columns)



Row Σ

1
2
0
0
0
0
0
1

j-th row indicates how many solutions j-th solution dominates

Solution 2 dominates Solution 5

Solution 7 is dominated by Solutions 2 and 8

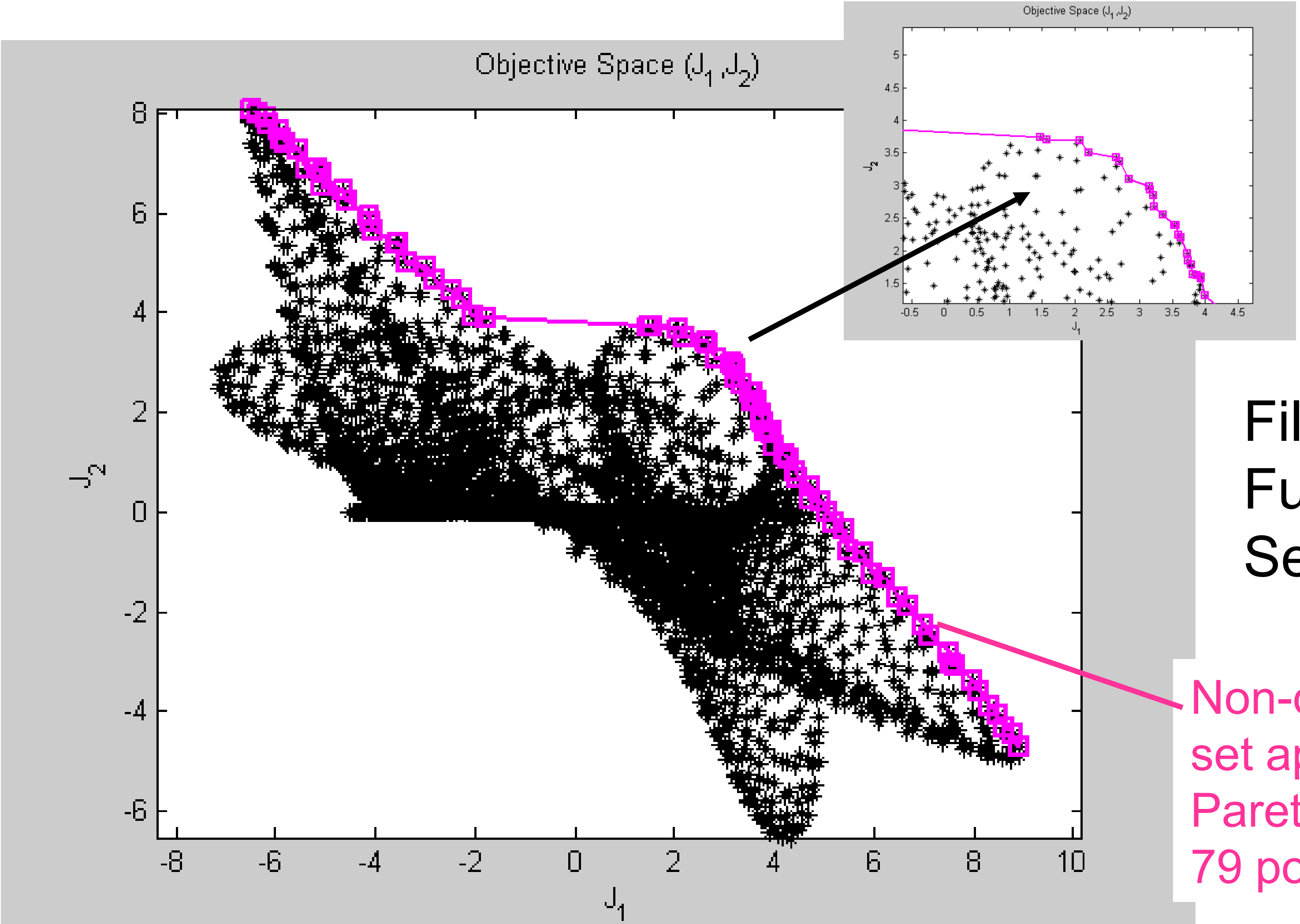
k-th column indicates by how many solutions the k-th solution is dominated

Column Σ

0	0	0	0	1	1	2	0

Non-dominated solutions have a zero in the column Σ !

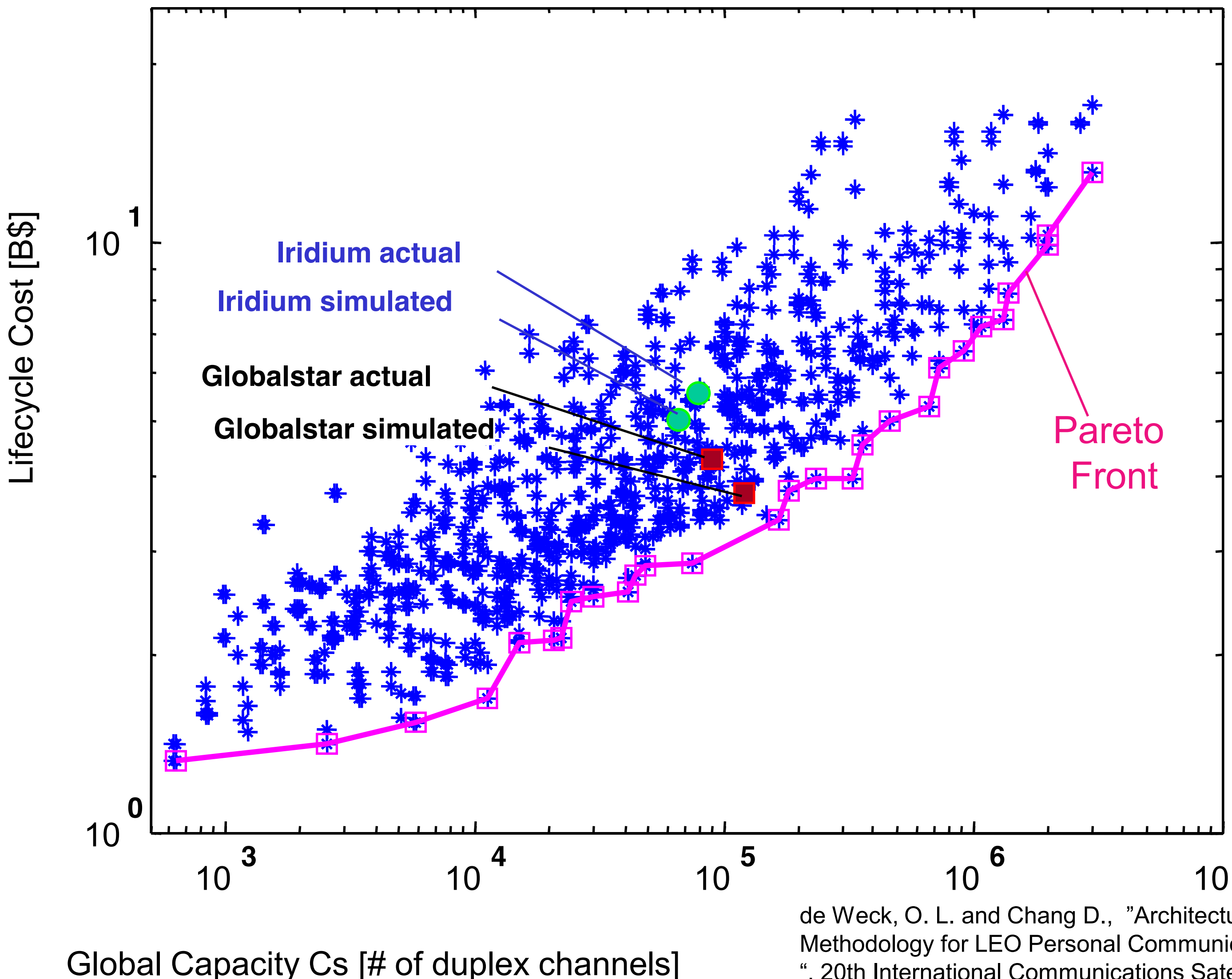
Double Peaks: Non-dominated Set



Filtered the Full Factorial Set: 3721

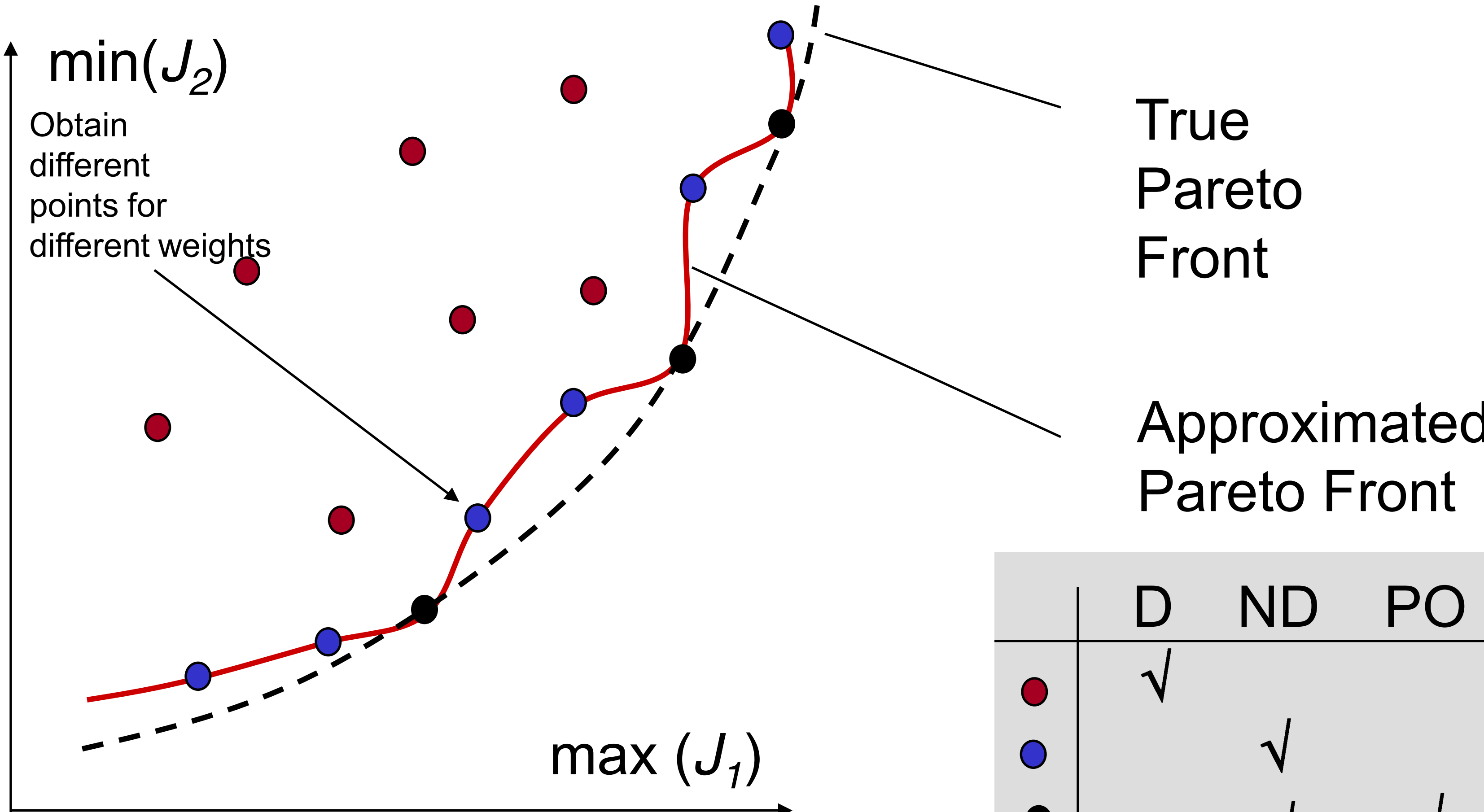
Non-dominated set approximates Pareto frontier: 79 points (2.1%)

Simulation Results - Satellites



de Weck, O. L. and Chang D., "Architecture Trade Methodology for LEO Personal Communication Systems ", 20th International Communications Satellite Systems Conference, Paper No. AIAA-2002-1866, Montréal, Québec, Canada, May 12-15, 2002.

Pareto-Optimal vs ND



	D	ND	PO
● (red)	√		
● (blue)		√	
● (black)		√	√

All pareto optimal points are non-dominated
Not all non-dominated points are pareto-optimal

It's easier to show dominatedness than non-dominatedness !!!

Lecture Summary

- A multiobjective problem has more than one optimal solution
- All points on Pareto Front are non-dominated
- Methods:
  - Weighted Sum Approach (Caution: Scaling !)
  - Pareto-Filter Approach
  - Methods for direct Pareto Frontier calculation next time:
    - AWS (Adaptive Weighted Sum)
    - NBI (Normal Boundary Intersection)

The key difference between multiobjective optimization methods can be found in how and when designer preferences are brought into the process.

…. More in next lecture

Remember ….

Pareto Optimal means …..

"Take from Peter to pay Paul"

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