

1)

The Mean Squared Error is calculated as:

$$MSE = \frac{1}{n} \sum_{i=1}^n (Y_i - \hat{Y}_i)^2$$

$$Y_1 = (1 - 4)^2 = 9$$

$$Y_2 = (9 - 10)^2 = 1$$

$$Y_3 = (0 + 1)^2 = 1$$

$$Y_4 = (-3 + 4)^2 = 1$$

The MSE is computed as:

$$\frac{9 + 1 + 1 + 1}{4} = 3$$

2)

Define the matrix A and the template (mask):

$$A = \begin{bmatrix} -3 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix}, \quad \text{Template (mask)} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

The convolution is performed by sliding the template over matrix A and calculating the sum of products at each position.

Step 1: Place the template over the top-left corner of A :

$$\begin{bmatrix} -3 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} = (-3 * 1) + (0 * 0) + (1 * 1) = -3$$

Step 2: Slide the template one position to the right and repeat:

$$\begin{bmatrix} -3 & 0 & 0 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 0 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} = (0 * 0) + (2 * 1) + (1 * 1) = 3$$

Steps 3 and 4 are similar: slide the template down and right, respectively, and compute the sum of products for each new position.

The final convolution matrix is:

$$\begin{bmatrix} -3 & 2 \\ 1 & 2 \end{bmatrix}$$