

Gardner's Minichess Variant is solved

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Abstract

A 5×5 board is the smallest board on which one can set up all kind of chess pieces as a start position. We consider Gardner's minichess variant in which all pieces are set as in a standard chessboard (from Rook to King). This game has roughly 9×10^{18} legal positions and is comparable in this respect with checkers. We weakly solve this game, that is we prove its game-theoretic value and give a strategy to draw against best play for White and Black sides. Our approach requires surprisingly small computing power. We give a human readable proof. The way the result is obtained is generic and could be generalized to bigger chess settings or to other games.

1 Introduction

Solving popular games like Othello, Checkers or Chess tantamount to the grail search in the field of computer games. The resolution of checkers [SBB⁺07] put a mark in the field in the sense that the space search of this game is enormous (5×10^{20}) and the difficulty to make correct move decisions fairly high.

The game of chess have always been recognized as the ultimate challenge in artificial intelligence. Since the early days of computer science chess and computers have interacted together [Pro12]. Nowadays computers have superhuman strength and the game is partially solved: endgame databases up to few pieces have been computed. The most famous ones being the Nalimov tables (6 pieces). Recently Lomonosov endgame tablebases [Ltd13] have been computed and give perfect play for 7 pieces (the size of the tablebase is 140 Terabytes). Nevertheless, the resolution of chess remains too difficult to be imagined: the number of legal positions is something around 10^{45} [All94] and decision complexity is very high (the amount of chess literature is a proof by itself).

Some studies have been done to resolve particular cases of chess on smaller board. Notably, 3×3 , 3×4 and 4×4 (limited to 9 pieces on the board) chess variants have been solved by K. Kryukov [Kry04, Kry09, Kry11]. In these variants there is no starting position as in traditional chess. Positions are treated as puzzles. Each variant is strongly solved in the sense that the game-theoretic value of all legal position is determined together with the perfect play associated. The number of legal positions is roughly 3×10^{15} for the 4×4 variant [Kry11].

In this paper we study the variant called Gardner's Chess. It is played on a 5×5 board, the initial position is the initial position of chess but for the three pieces on the King side that are removed. The rules are the ones of classical chess without the two squares move for Pawns, en passant moves and castling. This variant has roughly 9×10^{18} legal positions. This variant has been played extensively notably in Italy by correspondence [Pri07]. The results of finished games were the following:

- White victory 40%
- draw 32%
- Black victory 28 %

2 Results

The game-theoretic value of Gardner’s Chess is draw. We prove this by giving two oracles, one for White and one for Black. Both oracles can force draw versus best play. The intersection of the two oracles gives flawless games. Thus Gardner’s chess is weakly solved.

The proof is surprisingly small and can be totally checked by a human. Oracles are given in appendix 3 for the White side and appendix 4 for the Black side. From this point of view our result strongly differs from the resolution of checkers despite the fact that space search and difficulty of decision are of the same order of magnitude in both games. Indeed, the proof of [SBB⁺07] is not checkable by human eyes: it has required an enormous computing power (hundreds of computers in parallel over a decade).

Most of our work was achieved with consumer-grade laptop computers. We have adapted the open source Stockfish chess engine [RCK⁺10] to play Gardner’s Chess mainly by restricting the movements to the part of the board and changing the promotion ranks. Sources, executables for several environments and various files, including the oracles in PGN format as well as the list of the perfect openings for Gardner’s Chess, can be found at the author’s Minichess Resolution page: “<http://membres-lig.imag.fr/prost/MiniChessResolution/>”.

The main line of oracles were computed in a semi-automated way: we were mainly following the most equalizing line. It turns out that most of the deviations from the main line can be quickly decided. It is mainly due to the fact that in Gardner’s chess pawns are immediately exchanged or blocked. Moreover, pieces cannot develop naturally since almost all free squares are controlled by pawns. Also the fact that promotion happens quickly leads to some very rapid checkmates that allow to prune the game tree.

Using these Oracles it is impossible to lose. Oracle for White (resp. for Black) does not examine alternative choices for White (resp. Black) decision nodes but indicates how to answer every possible Black (resp. White) “reasonable” move. Unreasonable moves, i.e. moves that obviously lead to a position where it is clear that Black (resp. White) cannot win can be dealt with our engine. We provide the maximal number of moves required to mate for our engine (not necessarily the distance to mate). Nevertheless, in these positions, from a human point of view, it is easy not to lose.

As a by-product of our study on Gardner’s Chess the analysis of perfect openings shows the positions in which the evaluation of Stockfish is tricked. Indeed for some positions while showing largely “won” evaluation (up to +6) the position is completely equal. What is interesting is that these evaluation bugs can be found on a 8x8 board as well. Thus the analysis of these positions may help to improve the evaluation of Stockfish for standard chess games.

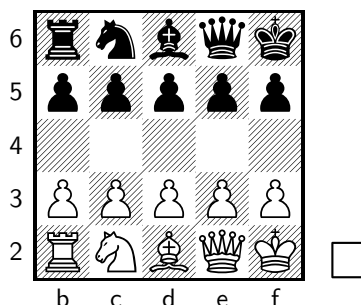
A complete description of the openings in gardner mini chess as well as a sample of tricky draws and difficult checkmates can be found at “<http://membres-lig.imag.fr/prost/MiniChessResolution/>”

3 Gardner : Oracle for White Draw

We give an oracle for the White side of the Gardner variation. The objective is to force a draw versus the best play. Therefore, we give it as a tree of variations that needs no explanations on White nodes: it is maybe possible to find shorter draw (or even win) but our aim is to have an oracle the most readable from a human point of view: the definitive judgment on the leafs of this tree are clear.

Since there are no choices to be explored for White nodes we adopt the following convention to name sub-variations: first we note the depth in the oracle, then we enumerate deviations from the main line by enumerating Black (relevant) moves from left to right, pawns come first, after we enumerate moves of the pieces following the lexical order going from left to right and top to bottom. Thus the variation [3|1.3.7] is the one obtained by following the oracle until depth 3 and selecting as sub-variation move 1 as the first move for Black, then move 3 as second move for Black and 7 as the third move. We write $+-$ (resp. $-+$) when it is obvious that Black (resp. White) cannot win. We write $\#x\#_\bullet$ (resp. $\#x\#_\circ$) when there exists a forced checkmate of the Black King (resp. White King) in x moves (though it is possible that shorter checkmates exist). Very often positions that look lost (because one side has a piece advantage for instance) can be fully

decided by our engine as forced checkmates. Justifying lines are written like this: $\triangle 1 \text{ b4 } 2 \text{ cxb4 d4}$. Finally, the coordinate of the lower left square is **b2**. Hence the starting position is:



In this position the Black move identified by 1 is $\dots \text{b4}$ and move number 6 is $\dots \text{b4}$, move number 7 is $\dots \text{c4}$.

We give the White oracle as a variation tree. After each move of the oracle we start by giving all lines in which a forced checkmate can be found using our engine.

1 b4

($1 \dots \text{d4 } 2 \text{ bxc5 } \#47\#$, \triangle after $2 \dots \text{bxc5 } 3 \text{ f4}$ both the pressure on the b file and on diagonal b2 f6 are too strong to be sustained by Black. $1 \dots \text{e4 } 2 \text{ bxc5 } \#28\#$, the point is that on $2 \dots \text{bxc5 } 3 \text{ d4}$ the threat of bxb5 combined with the lack of space for Black is too hard to be met. Other moves just lose a piece at least. $1 \dots \text{f4 } 2 \text{ bxc5 } \#24\#$, $\triangle 2 \dots \text{Bxc5 } 3 \text{ d4}$ the threat of bxb5 . $1 \dots \text{cxb4 } 2 \text{ cxb4 } \#24\#$. White is a piece up for nothing. $1 \dots \text{cxd4 } 2 \text{ bxc5 } \#17\#$. White is a piece up for nothing.):

- [1|1] $1 \dots \text{c4 } 2 \text{ d4 } (2 \dots \text{bxb4 } 3 \text{ dxe5+ } \#29\#, 2 \dots \text{bxc5 } 3 \text{ bxc5 } \#8\#, 2 \dots \text{cxd4 } \#29\#, 2 \dots \text{f4 } 3 \text{ e4 } \#38\#)$.

- [1|1.1] $2 \dots \text{exd4 } 3 \text{ exd4 } (3 \dots \text{bxc5 } \#6\#, 3 \dots \text{cxe5 } \#14\#, 3 \dots \text{cxe3+ } \#6\#, 3 \dots \text{cxe4 } \#10\#, 3 \dots \text{cxe5 } \#12\#, 3 \dots \text{bxe5 } \#31\#)$:

- * [1|1.1.1] $3 \dots \text{f4 } 4 \text{ cxe6+ } \text{cxe6}$ = White just has to move his King on e2-f2 and Black cannot break through. No matter what is the relative position of the two Kings, if black Knight takes on d4 or b4 White takes back with the Knight and the position is still blocked for Black and if Black plays $\dots \text{bxb4}$ the position is $\#17\#$ when kings on file e and $\#24\#$ when kings are in file f. Finally, if Black plays $\dots \text{cxe5}$ white just takes it with dxe5 and if Black plays $\dots \text{bxe5}$ White just continue to move his king.

- * [1|1.1.2] $3 \dots \text{cxe2+ } 4 \text{ cxe2}$ = for the same reason as line [1|1.1.1].

- [1|1.2] $2 \dots \text{e4 } 3 \text{ f4}$ = Black is in zugzwang and must give a piece. Due to the blocked nature of the position he can do it without losing but he cannot break through e.g. $3 \dots \text{bxe5 } 4 \text{ fxe5+ } \text{cxe5 } 5 \text{ dxe5+ } \text{cxe5 } 6 \text{ cxd4}$ and White can simply moves back and forth with the Knight.

- [1|1.3] $2 \dots \text{cxb4 } 3 \text{ dxe5+ } (3 \dots \text{bxe5 } 4 \text{ bxb4 } \#20\#, 3 \dots \text{cxe5 } 4 \text{ cxd4 } \#13\#) + -$:

- * [1|1.3.1] $3 \dots \text{cxe5 } 4 \text{ cxb4 } (4 \dots \text{bxb4 } 5 \text{ bxb4 } \#25\#, 4 \dots \text{cxe6 } 5 \text{ e4 } \#17\#, 4 \dots \text{cxd4 } 5 \text{ exd4 } \#3\#, 4 \dots \text{cxe4 } 5 \text{ fxe4 } \#9\#, 4 \dots \text{cxf4 } 5 \text{ exf4 } \#4\#, 4 \dots \text{cxd3 } 5 \text{ exf4 } \#4\#, 4 \dots \text{cxb3 } 5 \text{ bxb3+ } \#5\#, 4 \dots \text{d4 } 5 \text{ cxd4 } \#10\#, 4 \dots \text{f4 } 5 \text{ exf4 } \#10\#, 4 \dots \text{bxc6 } 5 \text{ cxc6 } \#10\#)$:

- [1|1.3.1.1] $4 \dots \text{bxc5 } 5 \text{ cxc2}$ = White blocks the position on the dark squares with cxd4 and bxb4 (and moves his Rook between b2-b4 if Black does not move. $\triangle 5 \dots \text{f4 } 6 \text{ cxd4 b4}$ (other moves leads to a loss for Black) $7 \text{ exf4 bxd4+ } 8 \text{ cxd4 cxe2+}$ (other moves lead to direct checkmate).

- [1|1.3.1.2] $4 \dots \text{cxe6 } 5 \text{ cxc2} + -$ similar to line [1|1.3.1.1].

1...cxb4 2 cxb4 All Knight and Bishop moves lose a piece and end up in a position where clearly Black cannot win (2...Qd4 #23, 2...Qxb4 #18, 2...Qxb4 #15, 2...e4 3 Qc3+ #15, 2...f4 3 bxc5 #29, the idea is that the b6 pawn is lost and Black is lacking space \triangle 3...Qxc5 4 d4 Q6d5.Rxb5)
2...d4 3 e4 (3...fxe4 4 fxe4 #8, since the threat of Qf3+ is too strong, 3...Qxb4 #15, 3...Qxb4 #15, 3...Qb3 #12, 3...Qc4 #8, 3...Qc6 #8.)
3...f4 4 Qxf4 (All Black's alternatives lead to forced checkmate since they lose a piece for nothing 4...Qc5 #8, 4...Qb3 #11, 4...Qc4 #8, 4...Qd5 #5, 4...Qf5 #8, 4...Qxb4 #25, 4...Qxb4 #25.)
4...exf4 5 Qd2 (5...Qe5 6 Qxf4+ #2, 5...Qxb4 6 Qxb4 +- d5 cannot be protected and the threat of Qxf4+ forbids 6...Qxb4)
5...Qe5 6 Qe2 = White just moves his King on e2-f2 and Black cannot untangle by ...Qe5 because of Qxf4 and must otherwise give a piece and cannot win.

4 Gardner : Oracles for Black Draw

We give, for each of the White seven legal first move, an oracle from the Black point of view that forces the draw. So here we give no explanations for Black decision nodes and we explore all reasonable moves (as explained earlier) at White decision nodes. These oracles are sometimes much simpler than the White oracle for draw since, rather curiously, it is sometimes more difficult for White to achieve draw. It means that often even slight deviations from the main line directly lead to positions that can be decided as forced checkmates.

4.1 White moves b4

1 b4 cxb4 (2 Qb3 d4 #17, 2 Qxb4 Qxb4 #21, 2 c4 bxc4 #15, the b4 c4 pawn duo is too strong, 2 Qd4 bxc3 #20, 2 e4 bxc3 #25)

- [1|1] 2 d4 bxc3 (3 dxe5+ #17, 3 e4 #9, 3 f4 #10, 3 Qb3 #17, 3 Qb4 #12, 3 Qxb5 #12, 3 Qa4 #9, 3 Qd3 #10, 3 Qc4 #10, 3 Qxb5 #7)

3 Qxc3 b4 (4 e4 dxe4 #19, 4 f4 exf4 #30, 4 Qxb4 Qxb4 #26, 4 Qxb4 Qxb4 #24, 4 Qd2 b3 #23, 4 Qd2 bxc3 #18, 4 Qd3 e4 #26, 4 Qc4 dxc4 #10, 4 Qb5 Qxb5 #9)

- [1|1.1] 4 dxe5+ Qxe5 (5 e4 #13, 5 f4 #15, 5 Qb3 #15, 5 Qxb4 #22, 5 Qd4 #10, 5 Qd2 #8, 5 Qd3 #15, 5 Qc4 #10, 5 Qb5 #8, 5 Qxb4 #17, 5 Qd2 #11, 5 Qd4 #29, 5 Qxe5+ Qxe5 #39)

5 Qxb4 Qxb4

(6 e4 #9, 6 f4 #9, 6 Qb2 #8, 6 Qd2 #10, 6 Qd4 #9, 6 Qxe5+ #51?, 6 Qxb4 #27, 6 Qd4 #12, 6 Qd2 #11, 6 Qd3 #20, 6 Qc4 #6, 6 Qb5 #7)

6 Qxb4 Qxb4 (7 e4 #7, 7 Qd4 #14, 7 Qb5 #9, 7 Qc4 #6, 7 Qd3 #6, 7 Qd2 #17)

- * [1|1.1.1] 7 f4 Qc3 (8 e4 #8, 8 Qd4 #13, 8 Qb5 #10, 8 Qc4 #5, 8 Qd3 #5, 8 Qd2 #4, 8 Qe2 #8, 8 Qe2 #7) 8 Qxb4 d4 (9 exd4 #18, 9 Qd5 #17, 9 Qd3 #16, 9 Qf3 #16, 9 Qc6 #14, 9 Qf3 #10, 9 e4 #7, 9 Qd2 #5, 9 Qb2 Qxe3 checkmate, 9 Qc2 Qxe3 checkmate, 9 Qc4 Qxe3 checkmate, 9 Qb5 Qxe3 checkmate)
= since on both Qc2 and Qd3 black exchanges Queen on e3 and the remaining position is draw \triangle 9 Qc2 dxe3+ 10 Qxe3 Qd4 11 Qe2 Qxe3+ 12 Qxe3 Qxe3+ 13 Qxe3.

- * [1|1.1.2] 7 Qxb4 d4 = the only move to avoid ...dxe3+ and the liquidation of all pawns is 8 e3 Qb3 9 Qd5+ Qe6 10 exf5+ Qxd5 11 f4 Qe3+ 12 Qxe3 dxe3+ 13 Qxe3.

- [1|1.2] 4 Qb3 f4 (5 e4 #19, 5 Qb2 #33, 5 Qxb4 #27, 5 Qxb4 #22, 5 Qxb4 #27, 5 Qb5 #10, 5 Qc4 #8, 5 Qd2 #20)

- * [1|1.2.1] 5 dxe5+ ♗xe5 (6 e4 ♗9♖, 6 exf4 ♗17♖, 6 ♖b2 ♗11♖, 6 ♖xb4 ♗32♖, 6 ♗xb4 ♗39♖, 6 ♗b2 ♗22♖, 6 ♗d2 ♗21♖, 6 ♗xe5+ ♗60♖, 6 ♗d4 ♗9♖, 6 ♗d2 ♗14♖, 6 ♗d3 ♗12♖, 6 ♗c4 ♗7♖, 6 ♗b5 ♗10♖, 6 ♗xb4 ♗20♖)

6 ♗d4 ♗xd4 (7 ♗xd4 ♗25♖, 7 ♗d2 ♗9♖, 7 ♗d3 ♗9♖, 7 ♗xb4 ♗9♖, 7 exf4 ♗8♖, 7 ♖xb4 ♗7♖, 7 ♖b2 ♗7♖, 7 ♖d3 ♗7♖, 7 e4 ♗7♖, 7 ♗b5 ♗7♖, 7 ♖c3 ♗6♖, 7 ♗c4 ♗6♖)

7 exd4 ♗d6 (8 ♗d3 ♗26♖, 8 ♖b2 ♗23♖, 8 ♗d2 ♗22♖, 8 ♗xb4 ♗18♖, 8 ♖xb4 ♗13♖, 8 ♗e3 ♗11♖, 8 ♖c3 ♗10♖, 8 ♗e3 ♗7♖, 8 ♗e4 ♗6♖, 8 ♗e5 ♗7♖, 8 ♖d3 ♗10♖, 8 ♖e3 ♗8♖, 8 ♗b5 ♗6♖)

8 ♗xe6+ ♗xe6 = White is blocked by Black pawns and cannot progress. The Black King may just move on f6 f5 squares.
- * [1|1.2.2] 5 exf4 exf4 = △ 6 ♗d3 ♗f5 7 ♗e2 ♗e6 with repetition. White cannot play ♗d2 due to ... bxc3. Once queen have been exchanged the position is blocked and Black can just move his King ad lib.
- * [1|1.2.3] 5 ♗b2 fxe3+ 6 ♗xe3 exd4 7 ♗xd4+ exd4 8 ♗xd4+ ♗e5 = Black can play ... ♗c3 on any queen moves and blocks the position.
- * [1|1.2.4] 5 ♗d2 exd4 6 exd4 ♗xe2+ 7 ♗xe2 = White cannot untangle and Black may just move his King around.
- * [1|1.2.5] 5 ♗d3 exd4 = similar as line [1|2.8.4.1].
- [1|1.3] 4 ♖xb4 ♖xb4 = △ 5 dxe5+ ♗xe5 6 ♗xb4 ♗xb4 7 ♗xb4 d4.
- [1|2] 2 f4 bxc3 (3 d4 ♗10♖, 3 e4 ♗9♖, 3 fxe5+ ♗xe5 ♗15♖, 3 ♖b3 ♗15♖, 3 ♖b4 ♗15♖, 3 ♖xb5 ♗14♖, 3 ♗b4 ♗10♖, 3 ♗d4 ♗14♖, 3 ♗f3 ♗8♖, 3 ♗f3 ♗8♖)

3 ♗xc3 b4 (4 d4 ♗33♖, 4 e4 ♗12♖, 4 ♗xb4 ♗26♖, 4 ♗d4 ♗22♖, 4 ♗d2 exf4 ♗23♖, 4 ♗f3 ♗22♖, 4 ♗d2 ♗22♖, 4 ♗d4 ♗19♖)
- [1|2.1] 4 fxe5+ ♗xe5 (5 e4 ♗12♖, 5 ♖b3 ♗19♖, 5 ♖xb4 ♗27♖, 5 ♗xb4 ♗26♖, 5 ♗d4 ♗10♖, 5 ♗xb4 ♗14♖, 5 ♗d4 ♗23♖, 5 ♗d2 ♗11♖, 5 ♗d2 ♗14♖, 5 ♗f3 ♗12♖, 5 ♗f3 ♗11♖)
 - * [1|2.1.1] 5 ♗xe5+ ♗xe5 (6 e4 ♗10♖, 6 ♖xb4 ♗12♖, 6 ♗xb4 ♗15♖, 6 ♗d4 ♗14♖, 6 ♗d2 ♗8♖, 6 ♗f3 ♗8♖, 6 ♗f3 ♗8♖)
 - [1|2.1.1.1] 6 ♖b3 d4 (7 ♗f3 ♗20♖, 7 exd4 ♗17♖, 7 ♗d2 ♗14♖, 7 e4 ♗13♖, 7 ♗f3 ♗12♖, 7 ♖b2 ♗12♖, 7 ♗xb4 ♗11♖, 7 ♖xb4 ♗11♖, 7 ♖c3 ♗8♖)

7 ♗xd4 ♗xd4 (8 ♗b2 ♗9♖, 8 ♖xb4 ♗8♖, 8 ♗d2 ♗7♖, 8 ♖b2 ♗7♖, 8 ♗c2 ♗5♖, 8 ♖c3 ♗4♖, 8 e4 ♗2♖)

8 exd4 ♗xd4+ (9 ♗f3 ♗12♖)

9 ♗e3 ♗xe3+ 10 ♗xe3 ♗e5 -+ Black can easily achieve draw since the White Rook has to keep an eye on the b pawn and the Black King is in front of the White d pawn.

- [1|2.1.1.2] 6 d4 ♗e4 7 ♖b3 f4 = since the only moves that not lose for White are either 8 exf4 ♗xe2+ 9 ♗xe2 ♗f5 and Black and White King move ad lib., or 8 ♗d2 f3 9 ♗xb4 ♖xb4 10 ♖xb4 ♗xb4 11 ♗xb4 ♗c2+ and perpetual check or 12 ♗c5 ♗b3 13 ♗d6 ♗xe3+ 14 ♗xe3 stalemate.
 - * [1|2.1.2] 5 d4 ♗d6 (6 e4 ♗15♖, 6 ♗xb4 ♗26♖, 6 ♗xb4 ♗30♖, 6 ♗d2 ♗45♖, 6 ♖xb4 ♗14♖, 6 ♗d2 ♗21♖, 6 ♗d3 ♗24♖, 6 ♗c4 ♗18♖, 6 ♗b5 ♗8♖, 6 ♗f3 ♗32♖, 6 ♗f3 ♗16♖)

6 ♖b3 ♗e4 -+ since White cannot do anything to untangle and Black may just move his Rook on b5 b6.
- [1|2.2] 4 ♖b3 d4 (5 e4 ♗10♖, 5 ♖b2 ♗10♖, 5 ♖xb4 ♗20♖, 5 ♗xb4 ♗13♖, 5 ♗xd4 ♗14♖, 5 ♗b2 ♗14♖, 5 ♗xb4 ♗15♖, 5 ♗xd4 ♗14♖, 5 ♗d2 ♗11♖, 5 ♗d2 ♗11♖, 5 ♗f3 ♗10♖, 5 ♗f3 ♗9♖)
 - * [1|2.2.1] 5 exd4 exd4 6 ♗xe6+, other White moves lose straightforwardly since the Bishop is lost, 6... ♗xe6 7 ♗xd4, otherwise Black just moves his Rook on b5 b6 and White cannot break through 7... ♗xd4 8 ♗xd4+ ♗d5 = the f4 pawn is going to fall and White cannot win this position.

- * [1|2.2.2] 5 fxe5+ ♟xe5 6 ♚xb4 other moves lose the Rook and lead to quick White defeat 6... ♚xb4 7 ♟xb4 ♜xb4 8 ♜xb4 dxe3+ 9 ♚xe3 ♚d6 10 ♜c2 ♚c6 = the best for White is to repeat moves with 11 ♜b4 ♚d6.
- [1|2.3] 4 ♚xb4 ♚xb4 (5 d4 ♠20♠, 5 e4 ♠12♠, 5 ♜d4 ♠12♠, 5 ♟xb4 ♠20♠, 5 ♟b2 ♠9♠, 5 ♟d4 ♠14♠, 5 ♟d2 ♠11♠, 5 ♟xe5+ ♠18♠, 5 ♚d2 ♠16♠, 5 ♚f3 ♠12♠, 5 ♟f3 ♠12♠)
 - * [1|2.3.1] 5 fxe5+ ♟xe5 6 ♟xb4 ♜xb4 7 ♜xb4 d4 = last pawns will soon be exchanged and White cannot force any advantage △ 8 e4 ♚b3 9 ♜d5+ ♟e6 10 ♚f3 fxe4.
 - * [1|2.3.2] 5 ♜xb4 ♜xb4 6 fxe5+ ♟xe5 7 ♟xb4 d4 = for the same reasons as line [1|4.3.1].
- [1|2.4] 4 ♟xe5+ ♟xe5 (5 d4 ♠24♠, 5 e4 ♠10♠, 5 ♚b3 ♠21♠, 5 ♚xb4 ♠13♠, 5 ♜xb4 ♠14♠, 5 ♜d4 ♠14♠, 5 ♚d2 ♠9♠, 5 ♚f3 ♠9♠, 5 ♟f3 ♠9♠)

5 fxe5+ ♚xe5 6 d4 ♚e4 = the position is blocked and Black can just move his King to e6 f6 White can't remove his Rook from the b file and if he tries to break through the ending will be a clear draw.
- [1|2.5] 4 ♜xb4 ♚xb4 (5 d4 ♠20♠, 5 e4 ♠12♠, 5 ♚b3 ♠10♠, 5 ♚c2 ♠24♠, 5 ♚d2 ♠21♠, 5 ♟xb4 ♠31♠, 5 ♟d2 ♠9♠, 5 ♟d4 ♠12♠, 5 ♟xe5+ ♠18♠, 5 ♚c2 ♠20♠, 5 ♚d2 ♠17♠, 5 ♚f3 ♠22♠, 5 ♟f3 ♠21♠)
 - * [1|2.5.1] 5 fxe5+ ♟xe5 6 ♚xb4 ♜xb4 7 ♟xb4 d4 = this endgame is completely draw since a couple of pawns will be exchanged and the remaining ones are mutually blocked.
 - * [1|2.5.2] 5 ♚xb4 ♟xb4 6 fxe5+ ♜xe5 7 ♟xb4 ♚b6 = .
- [1|2.6] 4 ♚f3 d4 (5 exd4 ♠20♠, 5 e4 ♠10♠, 5 ♚b3 ♠10♠, 5 ♚xb4 ♠20♠, 5 ♜xb4 ♠10♠, 5 ♜xd4 ♠10♠, 5 ♟xb4 ♠19♠, 5 ♟d2 ♠22♠, 5 ♟xd4 ♠15♠, 5 ♚e2 ♠10♠, 5 ♚e4 ♠8♠, 5 ♚d5 ♠9♠, 5 ♚xc6 ♠11♠, 5 ♟e2 ♠9♠)

5 fxe5+ ♜xe5 6 ♟xd4 b3 –+ △ 7 ♟e2 bxc2 ♚+ 8 ♚xc2 ♚b3 White can hold the balance due to the pin on the Knight and of the threat e4 which forces Black to move back his Queen to e6.
- [1|3] 2 ♜xb4 ♜xb4 (3 c4 bxc4 ♠12♠, 3 e4 ♜c6 ♠33♠, 3 f4 ♜c6 ♠33♠, 3 ♚b3 d4 ♠18♠, 3 ♚xb4 ♟xb4 ♠25♠, 3 ♚c2 ♜xc2 ♠11♠)
 - [1|3.1] 3 cxb4 d4 (4 ♚b3 ♚xb3 ♠12♠, 4 ♚c2 ♚b3 ♠16♠, 4 ♟c3 dxc3 ♠11♠)
 - * [1|3.1.1] 4 exd4 exd4 (5 ♚b3 ♠9♠, 5 ♚c2 ♠13♠, 5 ♟c3 ♠11♠, 5 ♟e3 ♠29♠, 5 ♟f4 ♠15♠, 5 ♚e3 ♠8♠, 5 ♚e4 ♠10♠, 5 ♚e5 ♠7♠)
 - [1|3.1.1.1] 5 f4 ♚xe2+ 6 ♟xe2 ♚c6 = the position is totally blocked on dark squares and White can only play his King or his Rook on b2 b3.
 - [1|3.1.1.2] 5 ♚xe6+ ♟xe6 = for the same reasons as line [1|3.1.1].
 - * [1|3.1.2] 4 e4 f4 (5 ♚b3 ♠6♠, 5 ♚c2 ♠16♠, 5 ♟c3 ♠8♠, 5 ♟e3 ♠10♠, 5 ♚e3 ♠9♠)

the only move leads to a type of drawn position already seen in line [1|1.2] of the White oracle 5 ♟xf4 exf4 6 ♚c4 ♚e5 = White can just move his Rook on b2-b3 or his King other over moves are losing (he cannot give up the control of the c file).
 - * [1|3.1.3] 4 f4 exf4 (5 e4 ♠29♠, 5 ♚b3 ♠12♠, 5 ♚c2 ♠17♠, 5 ♟c3 ♠2♠, 5 ♚f3 ♠19♠, 5 ♟f3 ♠12♠)
 - [1|3.1.3.1] 5 exd4 ♚xe2+ 6 ♟xe2 ♟e6 = Black King will seat on d5 and White cannot get through.
 - [1|3.1.3.2] 5 exf4 ♚xe2+ 6 ♟xe2 ♟e6 = same as line above, the Black King seats on d5 and Black may just move his Rook between b6 c6.
 - * [1|3.2] 3 d4 e4 (4 c4 ♠17♠, 4 f4 ♠10♠, 4 ♚xb4 ♠9♠, 4 ♚b3 ♠9♠, 4 ♚xb5 ♠8♠, 4 ♚c2 ♠8♠, 4 ♚c4 ♠8♠)

4 cxb4 exf3 (5 ♚d3 ♠33♠, 5 ♚c2 ♠9♠, 5 ♚xb5 ♠10♠, 5 ♚b3 ♠9♠, 5 ♟c3 ♠8♠, 5 e4 ♠7♠, 5 ♚c4 ♠7♠)

- [1|3.2.1] 5 ♖xf3 ♜c6 (6 ♜b3 ♠31♠, 6 e4 ♠14♠, 6 ♙c3 ♠12♠, 6 ♖f4 ♠9♠, 6 ♖xd5 ♠8♠, 6 ♜c2 ♠7♠)
 - [1|3.2.1.1] 6 ♖e2 ♜c4 (7 ♜b3 ♠29♠, 7 ♙f3 ♠27♠, 7 ♖xc4 ♠17♠)
 - * [1|3.2.1.1.1] 7 ♖f3 ♖e4 (8 ♙e2 ♠32♠, 8 ♖e2 ♠25♠, 8 ♜b3 ♠21♠)

8 ♖xe4 fxe4 = White cannot get through since his Bishop is limited by his pawns. If the White Rook moves to the third row then ... ♜c2 limits the White choice to ♜c3 after the Rook exchange the position is an easy draw.

 - * [1|3.2.1.1.2] 7 ♖d3 ♖e4 = if White takes on e4 we have the same position as variation [1|3.2.1.1.1] otherwise Black just moves his king on e6-f6.
 - [1|3.2.1.2] 6 ♙e2 ♜c4 = similarly to lines [1|3.2.1.1.1] and [1|3.2.1.1.2] Black will play ... ♖e4 and block the position.
 - [1|3.2.2] 5 ♙xf3 ♖e4 6 ♙f2 ♙e6 (7 ♖c4 ♠7♠, 7 ♖xb5 ♠8♠, 7 ♖d3 ♠8♠, 7 ♜c2 ♠12♠, 7 ♖f3 ♜c6 = see line [1|3.2.1.1.2])

7 ♜b3 ♜c6 (8 ♖xb5 ♠34♠) = if White does not take the b5 pawn the position is similar to line [1|3.2.1.1.1].

2 cxb4 d4 (3 ♙c3 dxc3 ♠8♠, 3 ♜b3 ♖xb3 ♠11♠)

- [2|1] 3 exd4 exd4 (4 ♜b3 ♠10♠, 4 ♙e3 ♠29♠, 4 ♙c3 ♠15♠, 4 ♙e3 ♠19♠, 4 ♙f4 ♠11♠)
 - [2|1.1] 4 f4 ♖xe2+ 5 ♙xe2 f4 = since only Kings can move without losing a piece and leading to a lost position (♜b3 is possible but changes nothing to the evaluation of the position).
 - [2|1.2] 4 ♖xe6+ ♙xe6 5 f4 = similar as line [2|1.1].
 - [2|1.3] 4 ♙xd4 ♙xd4 (5 ♖e3 ♠22♠, 5 ♖xe6 ♠16♠, 5 ♙e3 ♠10♠)

5 ♙c3
- [2|2] 3 f4 exf4 (4 e4 ♠24♠, 4 ♜b3 ♠14♠, 4 ♙c3 ♠11♠, 4 ♖f3 ♠23♠, 4 ♙f3 ♠15♠)
 - [2|2.1] 4 exd4 ♖xe2+ 5 ♙xe2 ♙e6 = the Black King will move to d5-e6.
 - [2|2.2] 4 exf4 ♖xe2+ 5 ♙xe2 = see variation [2|1.2].
 - [2|2.3] 4 ♙xd4 ♙xd4 5 exd4 (5 ♙c3 fxe3+ 6 ♖xe3 ♖xe3+ 7 ♙xe3 ♙e5 followed by exchanges to a completely drawn endgame does not change the assesment of the position) 5... ♖xe2+ 6 ♙xe2 ♙e6 = the Black King will seat on d5.
- [2|4] 3 ♙xd4 ♙xd4
 - [2|4.1] 4 exd4 exd4 (5 ♜b3 ♠9♠, 5 ♜c2 ♠13♠, 5 ♙c3 ♠11♠, 5 ♙e3 ♠19♠, 5 ♙f4 ♠15♠, 5 ♖e3 ♠8♠, 5 ♖e4 ♠10♠, 5 ♖e5 ♠7♠)
 - * [2|4.1.1] 5 f4 ♖xe2+ 6 ♙xe2 = see variation [1|3.1.1.1]
 - * [2|4.1.2] 5 ♖xe6+ ♙xe6 = see variation [1|3.1.1.2]

3 e4 f4 (4 ♜b3 ♖xb3 ♠7♠, 4 ♙xd4 ♙xd4 ♠16♠, 4 ♙e3 fxe3 ♠18♠, 4 ♙c3 dxc3 ♠9♠, 4 ♙e3 ♠18♠, 4 ♖e3 ♠12♠)

4 ♙xf4 exf4 (5 e5 ♠16♠, 5 ♜b3 ♠7♠, 5 ♙xd4 ♠9♠, 5 ♙e3 ♠7♠, 5 ♖e3 ♠7♠)

5 ♖d2 ♙e5 = since the only non losing moves for White are limited to the King and Queen moves over the d2, e2 and f2 squares.

4.2 White moves c4

1 c4 bxc4 The pin on the b file leads to forced mate ♠27♠.

4.3 White moves d4

1 d4 e4 (2 ♖b4 ♖21♖, 2 ♗xb5 ♖12♖, 2 ♗c4 ♖9♖, 2 ♗d3 ♖10♖, 2 fxe4 ♖13♖)

- [1|1] 2 b4 c4 (3 fxe4 ♖9♖, 3 ♗b3 ♖8♖, 3 ♗xc4 ♖16♖, 3 ♗d3 ♖9♖) 3 f4 ♗xb4 (4 ♗b3 ♖8♖, 4 ♗xb4 ♗xb4 ♖21♖, 4 ♗xc4 ♖15♖, 4 ♗d3 ♖10♖, 4 ♗f3 ♖11♖)
 - [1|1.1]4 cxb4 ♗d6 = despite his extra piece White cannot win since he is blocked by his own pawns on dark squares.
 - [1|1.2]4 ♗xb4 ♗xb4 = △ 5 ♗xb4 ♗d6 and White may only move his Rook, on 5 bxc4 ♗d6 is similar to [1|1.1].
- [1|2] 2 c4 bxc4 (3 b4 ♖15♖, 3 bxc4 ♖19♖, 3 ♗b4 ♖14♖, 3 ♗b4 ♖10♖, 3 fxe4 ♖10♖, 3 f4 ♖17♖, 3 ♗d3 ♖8♖, 3 ♗xc4 ♖12♖)
 - [1|2.1] 3 dxc5 ♗xc5 (4 b4 ♖20♖, 4 ♗b4 ♖12♖, 4 ♗d4 ♖15♖, 4 ♗b4 ♖14♖, 4 ♗d3 ♖8♖, 4 ♗xc4 ♖11♖, 4 f4 ♖12♖)
 - 4 ♗c3 ♗e5 (5 ♗xe5+ ♖19♖, 5 bxc4 ♖16♖, 5 fxe4 ♖13♖, 5 b4 ♖9♖, 5 ♗d4 ♖12♖, 5 ♗b4 ♖12♖, 5 f4 ♖7♖, 5 ♗d4 ♖9♖, 5 ♗xc4 ♖9♖, 5 ♗c3 ♖7♖)
 - 5 ♗d2 f4 (6 ♗e2 ♖12♖, 6 fxe4 ♖13♖, 6 b4 ♖17♖, 6 ♗d4 ♖10♖, 6 ♗d4 ♖8♖, 6 bxc4 ♖8♖, 6 ♗b4 ♖8♖, 6 ♗d4 ♖7♖, 6 ♗b4 ♖7♖, 6 ♗e2 ♖6♖, 6 ♗xd5 ♖6♖, 6 ♗d3 ♖5♖)
 - 6 ♗xe5 ♗xe5 (7 ♗d4 ♖16♖, 7 ♗c3 ♖12♖, 7 fxe4 ♖20♖, 7 ♗d4 ♖11♖, 7 ♗e2 ♖11♖, 7 ♗b4 ♖10♖, 7 ♗xd5 ♖8♖, 7 ♗b4 ♖6♖, 7 bxc4 ♖5♖, 7 ♗e2 ♖5♖, 7 ♗d3 ♖4♖)
 - 7 b4 fxe3+ (8 ♗xe3 ♖14♖, 8 ♗e2 ♖1♖)
 - 8 ♗xe3 ♗d3+ (9 ♗xd3 ♖6♖)
 - 9 ♗e2 ♗f4+ 10 ♗f2 ♗f4+ = draw by repetition.
 - [1|2.2] 3 ♗c3 ♗xb3
 - 4 ♗xb3 cxb3 (5 fxe4 ♖9♖, 5 f4 ♖15♖, 5 ♗b4 ♖11♖, 5 ♗b4 ♖8♖, 5 ♗b2 ♖10♖, 5 ♗d2 ♖6♖, 5 ♗b5 ♖6♖, 5 ♗c4 ♖6♖, 5 ♗d3 ♖5♖, 5 ♗d2 ♖15♖)
 - 5 dxc5+ ♗e5 (6 fxe4 ♖8♖, 6 f4 ♖6♖, 6 ♗b4 ♖7♖, 6 ♗b2 ♖8♖, 6 ♗b4 ♖9♖, 6 ♗d2 ♖7♖, 6 ♗d4 ♖9♖, 6 ♗d2 ♖16♖, 6 ♗d3 ♖4♖, 6 ♗c4 ♖6♖, 6 ♗b5 ♖6♖, 6 ♗xe5+ ♖13♖)
 - 6 ♗d4 ♗xd4 (7 fxe4 ♖7♖, 7 f4 ♖6♖, 7 ♗b2 ♖11♖, 7 ♗b4 ♖5♖, 7 ♗d2 ♖6♖, 7 ♗d2 ♖11♖, 7 ♗d3 ♖3♖, 7 ♗c4 ♖8♖, 7 ♗b5 ♖10♖, 7 ♗b2 ♖11♖, 7 ♗c2 ♖2♖)
 - [1|2.2.1] 7 ♗xd4+ ♗xd4 (8 ♗d2 ♖11♖, 8 fxe4 ♖6♖, 8 c6♗ ♖5♖ (other promotions as well), 8 f4 ♖3♖, 8 ♗c2 bxc2♗+ checkmate)
 - 8 ♗b2 f4 (9 ♗xd4+ ♖17♖, 9 c6♗ ♖11♖ (other promotions as well), 9 fxe4 ♖7♖, 9 ♗d2 ♖6♖, 9 ♗c2 bxc2♗+ checkmate)
 - 9 exd4 exf3 (10 ♗d2 ♖3♖, 10 c6♗ ♗e3 checkmate, 10 c6♗ ♗e3 checkmate, 10 ♗c2 ♗e3 checkmate, 10 ♗e2 fxe2♗+ checkmate, 10 ♗xb3 ♗e2 checkmate, 10 ♗c3 ♗e2 checkmate)
 - 10 c6♗ ♗xc6, promotion to Rook is handled similarly, (11 ♗xf3 ♖7♖, 11 ♗c3 ♖4♖, 11 ♗d2 ♖4♖, 11 ♗c2 ♖3♖, 11 ♗e2 ♖3♖)
 - 11 ♗xb3 ♗e6 = Black will play ... ♗e2+ and after Queen exchange the pawn endgame is draw.
 - [1|2.2.2] 7 exd4 e3+ 8 ♗xe3 ♗xe3+ 9 ♗xe3 = the Black King just moves to e6-f6 and White King cannot break through. If the White Biwhop goes to e5 either Black can play f4 and get room for his King or it means that White played f4 hence after ... ♗b4 the Knight cannot be taken without stalemating the Black King.
- [1|3] 2 dxc5 ♗xc5 (3 c4 b4 ♖23♖, 3 fxe4 ♖14♖, 3 f4 ♗d6 ♖14♖, 3 ♗b4 f4 ♖16♖, 3 ♗xb5 ♖11♖, 3 ♗c4 ♖9♖, 3 ♗d3 ♖9♖, 3 b4 ♖16♖)
 - 3 ♗d4 ♗xd4 (4 b4 ♖12♖, 4 c4 ♖9♖, 4 fxe4 ♖10♖, 4 f4 ♖9♖, 4 ♗c2 ♖9♖, 4 ♗xb5 ♖8♖, 4 ♗c4 ♖8♖, 4 ♗d3 ♖7♖)

- * [1|3.1] 4 cxd4 exf3 (5 b4 #12#o, 5 dxc5 #14#o, 5 e4 #5#o, 5 ♖c2 #10#o, 5 ♜c3 #10#o, 5 ♜b4 #8#o, 5 ♞c4 #12#o, 5 ♞xb5 #16#o)
 - [1|3.1.1] 5 ♞d3 ♜d6 = on any reasonable move (6 ♞e4 #6#o, 6 ♞c2 #32#o, 6 b4 #34#o, 6 ♜c3 #24#o, 6 ♜b4 #15#o, 6 ♞c3 #24#o, 6 e4 #12#o, 6 ♞xb5 #8#o, 6 ♞e2 #8#o, 6 ♞c4 #7#o, 6 ♞xf5+ #6#o) Black plays ... ♞e4 and locks the position as in variation [1|3.1.3.1].
 - [1|3.1.2] 5 ♞xf3 ♜d6 = on any reasonable move (6 ♞xf5+ #7#o, 6 ♞f4 #8#o, 6 ♞xd5 #9#o, 6 ♞e4 #4#o, 6 e4 #37#o, 6 ♜b4 #22#o) Black plays ... ♞e4 and locks the position as in variation [1|3.2.3.1].
 - [1|3.1.3] 5 ♞xf3 ♞e4+ 6 ♞f2 ♜d6 (7 ♖c2 #10#o, 7 ♜b4 #12#o, 7 ♞d3 #7#o, 7 ♞c4 #6#o, 7 ♞xb5 #8#o)
 - [1|3.1.3.1] 7 b4 ♞e6 = Black just moves his King on e6-f6 and the position is blocked on the dark squares △ 8 ♞f2 ♞f6 9 ♞xe3 fxe3
 - [1|3.1.3.2] 7 ♜c3 ♞e6 = see line [1|3.2.3.1].
 - [1|3.1.3.3] 7 ♞f3 ♞e6 = see line [1|3.2.3.1].
- * [1|3.2] 4 exd4 ♜d6 (5 ♖c2 f4 #23#o, 5 b4 f4 #18#o, 5 ♜e3 f4 #22#o, 5 ♞e3 f4 #22#o, 5 c4 bxc4 #23#o, 5 ♞e3 f4+ #23#o, 5 ♜f4 #11#o, 5 ♞xb5 #10#o, 5 ♞c4 #8#o, 5 ♞d3 #4#o, 5 ♜f4 #11#o)
 - [1|3.2.1] 5 fxe4 ♞xe4 = if White exchanges Queen on e4 then with ... fxe4 Black closes the position and with ... ♖c6 White cannot do anything. If White does not exchange Queens then Black may just play his King (on 6 b4 f4 is #28#o).
 - [1|3.2.2] 5 f4 e3+ = since Black follows with ... ♞e4 and blocks the position.
- [1|4] 2 f4 c4 (3 ♞d3 #12#o, 3 ♞xc4 bxc4 #15#o, 3 ♞f3 #8#o)
 - * [1|4.1] 3 b4 ♜xb4 = due to the blocked position White cannot achieve anything, this type of position has already been treated in line [1|1] of this oracle for instance.
 - * [1|4.2] 3 bxc4 dxc4 (4 d5 ♞xd5 #22#o, 4 ♖xb5 ♖xb5 #10#o, 4 ♖b3 #9#o, 4 ♞xc4 #8#o, 4 ♞d3 #9#o, 4 ♞f3 #11#o, 4 ♜xb4 ♜c5 #12#o this surprising move lead to direct checkmate since White is completely blocked and will eventually, due to his lack of space, have to gite his Queen within a few moves.)
 - 4 ♖b4 ♜xb4 (5 d5 #8#o, 5 ♜xb4 #23#o, 5 ♞xc4 #5#o, 5 ♞d3 #7#o, 5 ♞f3 #6#o) 5 cxb4 ♞d5 = because the position is totally blocked and Black just moves his King to e6 f6. The only way to untangle for White is to sacrifice the Queen on c4 which lead to quick checkmate.
 - * [1|4.3] 3 ♜b4 ♜xb4 (4 ♖c2 #8#o, 4 ♞xc4 #9#o, 4 ♞d3 #7#o, 4 bxc4 #15#o, 4 ♞f3 #7#o) = The draw is tricky to understand at first sight but becomes clear with the following variation 4 cxb4 ♜xf4 (5 ♜c3 #16#o, 5 ♖c2 #15#o, 5 ♞d3 #6#o, 5 ♞xc4 #9#o, 5 ♞f3 #7#o). From here the idea is to build a blockade on dark squares.
 - After 5 exf4 ♖c6 (in order to be able to take with the Rook in the case of bxc4) = The blockade has been achieved and Black just moves his Queen on d6 and his King on e6 f6.
 - 5 bxc4 bxc4 6 exf4 ♖b5 = another blockade is built on dark squares and White cannot break through.

4.4 White moves e4

- 1 e4 f4 (2 d4 #25#o, 2 ♜b4 #17#o, 2 ♜d4 #25#o, 2 ♜xf4 exf4 #22#o, 2 ♞e3 fxe3+ #25#o)
 - [1|1] 2 b4 cxb4 (3 c4 #15#o, 3 d4 #27#o, 3 exd5 #27#o, 3 ♖b3 #17#o, 3 ♖xb4 #34#o, 3 ♜xb4 #19#o, 3 ♜d4 #14#o, 3 ♜e3 #17#o, 3 ♜e3 #14#o, 3 ♜xf4 #16#o, 3 ♞e3 #17#o)
 - 3 cxb4 d4 = This position is draw for the same reason as position [1|1.2] of the White oracle (see section 3). White is in zugzwang and must give a piece, the only non losing way to do it is by 4 ♜xf4 exf4 5 ♞d2.

- [1|2] 2 c4 bxc4 (3 b4 #22#, 3 dxc4 #19#, 3 d4 #15#, 3 ♖b4 #12#, 3 ♖d4 #15#, 3 ♖e3 #16#, 3 ♜c3 #20#, 3 ♜b4 #12#, 3 ♜e3 #13#, 3 ♜xf4 #12#, 3 ♞e3 #14#.)
 - * [1|2.1] 3 bxc4 ♞xb2 = is a tricky draw in which White appear to be losing but can hold. The mainline is the following 4 cxd5 ♞xd5 5 exd5 ♖d4 6 ♖xd4 cxd4 at this point Black will regain the Queen and the bishop by force (otherwise White get mated) and end up in a ending like this one 7 ♜b4 ♞xe2+ 8 ♞xe2 ♜xb4 and the position is a curious draw (clearly White cannot win which is enough for our oracle).
 - * [1|2.2] 3 cxd5 ♞xd5 (4 b4 #11#, 4 d4 #10#, 4 ♖b4 #9#, 4 ♖d4 #8#, 4 ♖e3 #11#, 4 ♜b4 #7#, 4 ♜c3 #11#, 4 ♜e3 #7#, 4 ♜xf4 #9#, 4 ♞e3 #11#, 4 ♞e4 #11#, 4 ♞xe5+ #3#.)
 - [1|2.2.1] 4 bxc4 ♞xb2 = see line [1|2.1].
 - [1|2.2.2] 4 dxc4 ♞e6 –+ since White is restricted by Black pawns that completely control the dark squares and cannot move his Knight, hence his Rook. Black may just move his Queen between e6 f5. △ (5 b4 cxb4 #22#).
- [1|3] 2 ♖e3 fxe3+ (3 ♞xe3 #25#, 3 ♞xe3 #22#.)

3 ♜xe3 d4 –+ White cannot win △ 4 ♜c2 dxc3 5 ♜xc3 b4 6 ♜d2 ♖d4 7 ♞e3 ♞c6 White is in zugzwang and must give another piece.
- [1|4] 2 ♜e3 d4 (3 ♖b4 fxe3+ #9#, 3 ♖xd4 fxe3+ #17#, 3 ♜d2 #25#, 3 ♜xd4 cxd4 #13#, 3 ♜xf4 dxc3 #16#, 3 ♞d2 #28#. 3 b4 dxc3 #11#. Black either promotes c pawn or is a Rook up (and 4 ♜b2 ♞xb2 5 ♜xc5 ♖d4 is not helping))
 - * [1|4.1] 3 c4 fxe3+ (4 ♞xe3 #13#.)

4 ♖xe3 dxe3+ = because on each recapture by White Black closes the position with ... b4 and white cannot break through since f4 leads to a quick defeat.
 - * [1|4.2] 3 cxd4 cxd4 = (4 b4 #17#, 4 ♖b4 #9#, 4 ♖xd4 #14#, 4 ♜d2 #14#, 4 ♜xd4 ♖xd4 #12#, 4 ♞d2 #15#.) △ 4 ♜xf4 exf4 5 b4 (otherwise White is a piece down and will lose) 5... ♜e5 and the position is completely blocked on the dark squares.
- 2 exd5 ♞xd5 (3 d4 exd4 #21#, 3 ♖b4 cxb4 #11#, 3 ♖d4 exd4 #16#, 3 ♖e3 fxe3+ #52#, 3 ♜e3 fxe3+ #44#, 3 ♜xf4 exf4 #15#, 3 ♞e3 fxe3 #10#, 3 ♞xe5+ ♖xe5 #2#.)
 - [2|1] 3 b4 cxb4 (4 c4 #16#, 4 d4 #15#, 4 ♞b3 #12#, 4 ♞xb4 #14#, 4 ♖xb4 #13#, 4 ♖d4 #12#, 4 ♖e3 #14#, 4 ♜e3 #10#, 4 ♜xf4 #10#, 4 ♞e3 #9#, 4 ♞e4 #14#, 4 ♞xe5+ #5#.)

4 cxb4 ♖d4 (5 ♞b3 #7#, 5 ♖xd4 #8#, 5 ♖e3 #14#, 5 ♜c3 #9#, 5 ♜e3 #9#, 5 ♞e3 #2#, 5 ♞xe5+ #5#, 5 ♜xf4 #8#.)

5 ♞e4 ♞xe4 = (6 ♞b3 ♞e2 checkmate, 6 ♖xd4 #5#, 6 ♖e3 ♞xf3 checkmate, 6 ♜c3 ♞xf3 checkmate, 6 ♜e3 ♞xf3 checkmate, 6 ♜xf4 ♞xf3 checkmate)

on either d-pawn or f-pawn capture of the Queen Black plays ... ♖xc2 and then his King on e6-f6 squares. The position is completely blocked.
 - [2|2] 3 c4 bxc4 by transposition we have reached line [1|2.2].
- 3 ♞e4 ♞xe4 (4 b4 #7#, 4 c4 #6#, 4 d4 #7#, 4 ♖b4 #10#, 4 ♖d4 #6#, 4 ♖e3 #7#, 4 ♜e3 ♞xe3 checkmate, 4 ♜xf4 #8#.)
 - [3|1] 4 dxe4 b4 (5 ♖xb4 #27#, 5 ♖d4 #9#, 5 ♜xf4 #7#, 5 ♜e3 #6#.)
 - * [3|1.1] 5 c4 ♞e6 = the position is completely blocked and Black can just move his King on e6-f6.
 - * [3|1.2] 5 cxb4 cxb4 = White pieces are blocked, his only active plan is to bring the King on c4 but Black can play its bishop on c5-f2 △ 6 ♞e2 ♜c5 7 ♞d3 ♞e6 8 ♞c4 ♞d6.

- * [3|1.3] 5 ♖e3 fxe3+ (6 ♜xe3 ♔10♖, 6 ♜e2 ♔14♖)
 - 6 ♜xe3 bxc3 (7 ♜b4 ♔5♖, 7 ♜c2 ♔11♖, 7 ♜d3 ♔5♖, 7 ♜e2 ♔5♖, 7 ♜f2 ♔5♖, 7 ♜f4 ♔5♖)
 - 7 ♜xc3 c4 (8 ♜b4 ♔23♖, 8 ♜e2 ♔15♖, 8 ♜d2 ♔28♖, 8 ♜f2 ♔13♖, 8 ♜f4 ♔25♖, 8 ♜b4 ♔9♖, 8 ♜d4 ♔7♖, 8 ♜xe5+ ♔8♖, 8 ♜d2 ♔9♖, 8 ♜c2 ♔10♖, 8 ♜d2 ♔10♖, 8 ♜e2 ♔8♖, 8 ♜f2 ♔8♖)
 - 8 bxc4 ♜xb2 –+ △ 9 ♜xb2 ♜e6 white cannot win since after ... ♜c5 White clearly cannot progress.
- * [3|1.4] 5 ♜e2 bxc3 (6 ♜d3 ♔5♖, 6 ♜xf4 ♔5♖, 6 ♜b4 ♔5♖, 6 ♜e3 ♔5♖, 6 ♜b4 ♔5♖, 6 ♜e3 ♔5♖, 6 ♜d4 ♔3♖, 6 ♜f2 cxd2 ♜+ checkmate)
 - 6 ♜xc3 ♜d4+ (7 ♜d2 ♔39♖, 7 ♜f2 ♔16♖, 7 ♜xd4 ♔21♖)
 - [3|1.4.1] 7 ♜d3 ♜xf3 (8 ♜c4 ♔40♖, 8 ♜e2 ♔21♖, 8 ♜d2 ♔14♖, 8 ♜b4 ♔11♖, 8 ♜b4 ♔13♖, 8 ♜d4 ♔13♖, 8 ♜xe5 ♔10♖, 8 ♜d4 ♔10♖)
 - [3|1.4.1.1] 8 ♜b4 ♜c6 (9 ♜e2 ♔41♖, 9 ♜e2 ♔24♖, 9 ♜d2 ♔14♖, 9 ♜c4 ♔12♖, 9 ♜d4 ♔12♖, 9 ♜d4 ♔11♖, 9 ♜xe5 ♔10♖, 9 ♜b3 ♔9♖)
 - * [3|1.4.1.1.1] 9 ♜b5 ♜b6 =
 - * [3|1.4.1.1.2] 9 bxc5 ♜xc5 =
 - * [3|1.4.1.1].3 9 ♜e3 fxe3 =
 - [3|1.4.1.2] 8 ♜e3 fxe3 (9 ♜e2 ♔41♖, 9 ♜e2 ♔24♖, 9 ♜b4 ♔17♖, 9 ♜c2 ♔15♖, 9 ♜c4 ♔14♖, 9 ♜b4 ♔13♖, 9 ♜xe5 ♔8♖, 9 ♜d2 ♔7♖, 9 ♜d4 ♔7♖, 9 ♜d2 ♔7♖, 9 ♜f2 ♔5♖, 9 ♜c2 ♔2♖)
 - 9 ♜xe3 c4 = since Rook exchanges is unavoidable (otherwise White lose) and the Bishop's ending is draw.
 - [3|1.4.2] 7 ♜xd4 exd4 (8 ♜xd4 ♔18♖, 8 ♜d2 ♔18♖, 8 ♜e5 ♔13♖, 8 ♜b4 ♔13♖, 8 ♜e3 ♔10♖, 8 ♜f2 ♔8♖,) = △ 8.b4 c4 9.Nxd4 Be5 and the resulting Rook ending is draw.

4 fxe4 ♜b4 (5 ♜d4 ♔14♖, 5 ♜xb4 ♔21♖, 5 ♜d4 ♔13♖, 5 ♜e3 ♔7♖, 5 ♜xf4 ♔7♖)

- [4|1] 5 ♜e2 ♜e6 6 ♜f2 (or 6 ♜f3) 6... ♜f6 = since other moves than King loses (see previous lines) or rejoin the mainline (c4).
- [4|2] 5 ♜f3 ♜e6 same as line [4|1].

5 c4 ♜d4 = the position is totally locked on dark squares and White cannot progress.

4.5 White moves f4

1 f4 exf4 (2 ♜b4 ♔20♖, 2 ♜d4 ♔24♖, 2 ♜f3 ♔24♖, 2 ♜f3 ♜e5+ ♔17♖, 2 c4 ♜e5 ♔26♖, 2 e4 fxe4 ♔18♖)

- [1|1] 2 ♜b4 ♜f3 (3 bxc5 ♔14♖, 3 c4 ♔11♖, 3 ♜d4 ♔11♖, 3 ♜e4 ♔9♖, 3 ♜b3 ♔9♖, 3 ♜d4 ♔11♖, 3 ♜xf3 ♔24♖)
 - 3 ♜xf3 ♜e5 (4 c4 ♔14♖, 4 ♜d4 ♔14♖, 4 ♜e4 ♔11♖, 4 ♜b3 ♔13♖, 4 ♜d4 ♔19♖, 4 ♜e2 ♔23♖, 4 ♜f4 ♔9♖, 4 ♜xf5+ ♔2♖, 4 ♜e4 ♔7♖, 4 ♜xd5 ♔6♖, 4 ♜e2 ♔20♖)
 - 4 bxc5 ♜xd3+ 5 ♜e2 ♜xc5 (6 ♜b4 ♔37♖, 6 ♜c4 ♔14♖, 6 ♜e4 ♔11♖, 6 ♜b3 ♔12♖, 6 ♜xb5 ♔14♖, 6 ♜f2 ♔21♖, 6 ♜f4 ♔12♖, 6 ♜e4 ♔8♖, 6 ♜xd5 ♔10♖, 6 ♜xf5+ ♔9♖)
 - * [1|1.1] 6 ♜b4 ♜e4 (7 ♜c4 ♔12♖, 7 ♜b3 ♔21♖, 7 ♜b2 ♔37♖, 7 ♜d3 ♔29♖, 7 ♜f2 ♔13♖, 7 ♜f4 ♔15♖)
 - 7 ♜d4 ♜xd2 8 ♜xd2 ♜e4 = △ 9 ♜xe4 fxe4 10 ♜xb5 ♜xb5 11 ♜xb5 and the ending Knight vs. Bishop is draw or △ 9 ♜xb5 ♜xf3 10 ♜xb6 ♜f2+ 11 ♜d3 ♜e6 12 ♜c6+ and perpetual check.

- * [1|1.2] 6 ♖d4 ♗e4 Because of the threat to the White king, White's move is forced 7 ♗xe4 fxe4 → White is blocked and must take the b pawn with his Rook if he looks for any progress after the rook exchanges Black easily draw. If 8 ♖xb4 ♖c2 9 ♗b3 ♗c5 White is in zugzwang and lose.
- [1|3] 2 d4 fxe3+ (3 ♖xe3 cxd4 #26#, 3 ♗xe3 #41#, 3 ♖f3 ♗e4 checkmate)
3 ♗xe3 ♗e4 (4 ♗f3 #48#, 4 ♗d2 #30#, 4 b4 #35#, 4 c4 #16#, 4 ♖b4 #15#, 4 ♗d2 #24#,
4 ♗f4 #18#, 4 ♗d3 #9#, 4 ♗c4 #8#, 4 ♗xb5 #8#)
4 dxc5 ♗xc5 (5 c4 #4#, 5 ♖b4 #14#, 5 ♖d4 #14# 5 ♗d2 #5#, 5 ♗d3 #5#, 5 ♗c4 #5#, 5
♗xb5 #5#, 5 ♗f3 #32#, 5 ♗c4 #15#)
- * [1|3.1] 5 b4 ♗d6 the only non losing move is 6 ♗xb6 ♗f4+ = perpetual check on d2 f4.
- * [1|3.2] 5 ♗xc5 ♗f4+ = perpetual check on d2 f4.
- 2 exf4 ♗xe2+ 3 ♖xe2 d4 (4 ♖b4 cxb4 #24#, 4 ♖xd4 cxd4 #31#, 4 ♖e3 dxe3 #31#, 4 ♗e3
dxe3 #8#)
- [4|1] 4 b4 ♖e6 (5 ♖e3 #32#, 5 ♗e3 #40#, 5 ♖xd4 #28#)
 - * [4|1.1] 5 bxc5 ♗xc5 (6 ♖f3 #23#, 6 ♗b4 #23#, 6 ♖e3 #22#, 6 ♗xb5 #12#, 6 ♖f2 #7#)
 - [4|1.1.1] 6 c4 bxc4 (7 ♗b3 #6#, 7 ♗b4 #10#, 7 ♗b5 #8#, 7 ♖b4 #9#, 7 ♖xd4 #6#,
7 ♖e3 #10#, 7 dxc4 #8#, 7 ♗c3 #12#, 7 ♗b4 #10#, 7 ♗e3 #7#, 7 ♖f2 #6#, 7 ♖f3
#7#)
7 ♗xb6 ♗xb6 (8 ♖b4 #11#, 8 ♖xd4 #13#, 8 ♖e3 #14#, 8 ♗b4 #15#, 8 ♗c3
#14#, 8 ♗e3 #14#, 8 ♖f2 #10#, 8 ♖f3 #10#)
8 dxc4 ♗c5 → White is blocked and cannot untangle if Black just moves his King to
d6-e6.
 - [4|1.1.2] 6 cxd4 ♖xd4 (7 ♖f2 #13#, 7 ♖e3 #13#) 7 ♖xd4+ ♗xd4 = the Black King
blocks the position on d5.
 - [4|1.1.3] 6 ♗b3 dxc3 (7 ♗xb5 #11#, 7 d4 #15#, 7 ♗b4 #11#, 7 ♗b2 #5#, 7 ♖b4 #4#,
7 ♖d4+ #4#, 7 ♖e3 #2#, 7 ♖f3 #4#)
 - [4|1.1.3.1] 7 ♗xc3 b4 Black moves his King to d5 and blocks the position.
 - [4|1.1.3.2] 7 ♗xc3 ♖d4+ (8 ♖e3 #11#)
 - * 8 ♖xd4+ ♗xd4 = Black King comes to d5 and blocks the position.
 - * 8 ♖f2 ♖d5 =
 - [4|1.1.3.3] 7 ♗e3 ♗xe3 = after ... b4 and ... ♖d5 Black locks down the position
and White cannot progress.
 - [4|1.1.4] 6 ♖b4 dxc3 (7 ♗c2 #8#, 7 ♖xc6 #7#, 7 d4 #6#, 7 ♖f3 #5#, 7 ♗b3 #4#, 7
♖d5 #5#, 7 ♖c2 #5#)
7 ♗xc3 ♖xb4 (8 ♗xb4 #17#, 8 ♖e2 #17#, 8 ♗d2 #16#, 8 ♗e5 #16#, 8 ♗b3 #15#,
8 ♗d2 #14#, 8 ♖f3 #14#, 8 ♗f6 #13#, 8 ♗d4 #11#, 8 ♗c3 #9#)
 - [4|1.1.4.1] 8 ♗xb4 ♗xb4 = this Rook ending is clearly draw.
 - [4|1.1.4.2] 8 d4 ♗d6 (9 d5+ #18#, 9 ♗b3 #16#, 9 ♗xb4 #15#, 9 ♗c2 #10#, 9
♗d2 #14#, 9 ♗d2 #18#, 9 ♖d2 #18#, 9 ♖e3 #16#, 9 ♖f3 #19#, 9 ♖f2 #12#)
9 ♗xb4 ♗xb4 = this is the same ending as variation [1|1.1.4.1].
 - [4|1.1.5] 6 ♖xd4+ ♖xd4 (7 ♖e3 #23#, 7 ♖f2 #25#)
7 cxd4 ♗xd4 (8 ♗c2 #20#, 8 ♗xb5 #10#, 8 ♗c3 #11#, 8 ♗b4 #11#, 8 ♗e3 #21#, 8
♖f3 #10#) = since Black King may move to seat on d5 and block the position. If Bishops
are exchanged the resulting Rook ending is clearly draw.
 - [4|1.1.6] 6 ♗e3 dxc3 (7 d4 #9#, 7 ♗b4 #10#, 7 ♗xb5 #12#, 7 ♖b4 #5#, 7 ♖d4+
#9#, 7 ♖f2 #7#, 7 ♖f3 #7#, 7 ♗d2 #5#, 7 ♗f2 #7#, 7 ♗d4 #9#, 7 ♗xc5 #7#)
7 ♗b3 ♗xe3 (8 d4 #11#, 8 ♗b2 #5#, 8 ♗b4 #9#, 8 ♗xb5 #7#, 8 ♖b4 #10#, 8
♖d4+ #5#, 8 ♖xe3 #8#, 8 ♖f3 #9#, 8 ♗xc3 #13#)

- 8 ♖xe3 b4 (9 ♜xb4 #7♖, 9 ♜xc3 #9♖, 9 ♜b2 #5♖, 9 ♜xb4 #11♖, 9 ♜d4 #6♖, 9 ♜e2 #15♖, 9 ♜f2 #12♖, 9 ♜f3 #18♖)
- 9 d4 ♜d5 (10 ♜xb4 #5♖, 10 ♜xc3 #8♖, 10 ♜b2 #4♖, 10 ♜xb4+ #14♖, 10 ♜e2 #9♖, 10 ♜f2 #9♖, 10 ♜f3 #8♖)
- 10 ♜d3 ♜b5 (11 ♜xb4 #8♖, 11 ♜xc3 #17♖, 11 ♜b2 #6♖, 11 ♜e2 #6♖, 11 ♜xb4+ #15♖, 11 ♜e3 #8♖)
- 11 ♜e3+ ♜d6 (12 ♜b2 #8♖, 12 ♜xb4 #10♖, 12 ♜xc3 #16♖, 12 ♜c2 #10♖, 12 ♜e2 #9♖, 12 ♜c4 #16♖, 12 ♜c4+ #19♖, 12 ♜d5 #7♖, 12 ♜xf5+ #9♖, 12 d5 #14♖)
- 12 ♜c2 ♜d5 = by repetition.
- * [4|1.2] 5 c4 bxc4 (6 bxc6 #10♖, 6 b5 #19♖, 6 ♜b3 #7♖, 6 ♜xd4 #♖, 6 ♜e3 #12♖, 6 ♜c3 #8♖, 6 ♜e3 #11♖, 6 ♜f2 #12♖, 6 ♜f3 #11♖)
- 6 dxc4 cxb4 (7 c5 #20♖, 7 ♜xb4 #12♖, 7 ♜xb4 #24♖, 7 ♜xd4+ #22♖, 7 ♜c3 #12♖, 7 ♜xb4 #20♖, 7 ♜e3 #16♖, 7 ♜f3 #21♖, 7 ♜f2 #22♖)
- [4|1.2.1] 7 ♜e3 dxe3 (8 ♜b3 #8♖, 8 ♜xb4 #7♖, 8 ♜c2 #10♖, 8 ♜c3 #7♖, 8 ♜xb4 #10♖, 8 ♜d3 #11♖, 8 ♜f3 #11♖, 8 ♖xe3 #♖, 8 c5 #11♖)

8 ♜xe3 ♜e5 (9 ♜b3 #19♖, 9 ♜xb4 #13♖, 9 ♜c2 #12♖, 9 ♜d2 #23♖, 9 ♜d2 #13♖, 9 ♜f2 #20♖, 9 ♜d4 #15♖, 9 ♜c5 #12♖, 9 ♜xb6 #19♖, 9 ♜d2 #12♖, 9 ♜f2 #12♖)

[4|1.2.1.1] 9 c5 ♜xc5 = △ 10 ♜xc5 ♜c4 11 ♜xb4 ♜xb4 12 ♜xb4.

[4|1.2.1.1] 9 fxe5 ♜xe5 = △ 10.c5 Rc6 11.Rxb4 Kd5
 - [4|1.2.2] 7 ♜d3 ♜c5 (8 ♜xb4 #24♖, 8 ♜xb4 #24♖, 8 ♜xd4+ #24♖, 8 ♜e3 #21♖, 8 ♜xb4 #24♖, 8 ♜c3 #11♖, 8 ♜e3 #21♖, 8 ♜e2 #18♖)

8 ♜b3 ♜d6 9 ♜e2 ♜e6 = White is blocked and cannot do anything concrete in this position.
 - [4|1.2.3] 7 ♜b3 ♜c5 = this move simply transposes to variation [4|1.2.2]
- * [4|1.3] 5 cxd4 cxd4 = the position is totally blocked Black just moves his King on e6 d5.
- * [4|1.4] 5 ♜b3 c4 (6 dxc4 #24♖, 6 cxd4 #13♖, 6 ♜xd4 #10♖)
- 6 ♜b2 dxc3 (7 dxc4 #10♖, 7 ♜e3 #6♖, 7 ♜e3 #6♖)
- 7 ♜xc3 cxd3+ (8 ♜f3 #15♖, 8 ♜e3 #19♖, 8 ♜d2 #13♖)
- 8 ♜xd3 ♜xf4 (9 ♜b3 #33♖, 9 ♜d4 #29♖, 9 ♜e2 #26♖, 9 ♜e5 #9♖, 9 ♜f6 #18♖, 9 ♜d2 #22♖)
- 9 ♜d4+ ♜xd4 (10 ♜b3 #9♖, 10 ♜c2 #10♖, 10 ♜d2 #11♖, 10 ♜e2+ #9♖, 10 ♜f2 #17♖, 10 ♜d2 #14♖, 10 ♜xd4 #11♖)
- 10 ♜xd4 ♜c6 = Black will place his King on d5 and White cannot progress.
- * [4|1.5] 5 ♜f2 dxc3 (6 bxc5 #4♖, 6 d4 #5♖, 6 ♜b3 #4♖, 6 ♜d4 #10♖, 6 ♜e3 #10♖, 6 ♜e3 #7♖, 6 ♜e2 #6♖, 6 ♜e3 #6♖, 6 ♜f3 #5♖)
- 6 ♜xc3 cxb4 (7 d4 #13♖, 7 ♜b3 #16♖, 7 ♜xb4 #23♖, 7 ♜d4+ #24♖, 7 ♜e3 #11♖, 7 ♜f6 #18♖, 7 ♜e5 #13♖, 7 ♜e2 #12♖, 7 ♜e3 #13♖, 7 ♜f3 #13♖)
- [4|1.5.1] 7 ♜xb4 ♜xb4 (8 d4 #11♖, 8 ♜b3 #10♖, 8 ♜xb4 #18♖, 8 ♜d4 #13♖, 8 ♜e3 #13♖, 8 ♜e2 #12♖, 8 ♜e3 #13♖, 8 ♜f3 #12♖)

8 ♜xb4 ♜xb4 (9 d4 #11♖, 9 ♜b3 #12♖, 9 ♜c2 #11♖, 9 ♜d2 #7♖, 9 ♜e2+ #12♖, 9 ♜e2 #11♖, 9 ♜e3 #10♖, 9 ♜f3 #11♖) 9 ♜xb4 ♜d5 = this Rook endgame is a draw.
 - [4|1.5.2] 7 ♜d2 b3 (8 ♜e3 #33♖, 8 ♜b4 #14♖, 8 ♜e3 #23♖, 8 d4 #14♖, 8 ♜c3 #14♖, 8 ♜e3 #21♖, 8 ♜b4 #11♖, 8 ♜d4 #12♖, 8 ♜f3 #12♖, 8 ♜e2 #12♖)

8 ♜xb3 ♜c5+ (9 ♜e3 #23♖, 9 ♜d4+ #12♖)

 - * [4|1.5.2.1] 9 ♜e3 ♜xe3+ = the Black King will block the position on d5.
 - * [4|1.5.2.2] 9 d4 ♜xd4 = △ 10 ♜xd4+ ♜xd4 11 ♜d3 b4 12 ♜e3 b3 13 ♜xd4 the White Bishop must be exchanged vs the b pawn and the Rook ending is draw.
 - * [4|1.5.2.3] 9 ♜e2 ♜d4+ = △ 10 ♜xd4+ ♜xd4 and Black locks the position by ... ♜d5.

- * [4|1.5.2.4] **9 ♖f3 ♗d5** = the reason has to be seen in previous lines. Black exchange the Bishop vs the Knight and the remaining position is blocked.
- [4|1.5.3] **7 ♙d4 ♜xd4** (**8 ♚b3 ♖8♖, 8 ♚xb4 ♖10♖, 8 ♜xb4 ♖10♖, 8 ♜e3 ♖9♖, 8 ♗e3 ♖8♖**)
8 ♜xd4+ ♗d5 (**9 ♚b3 ♖9♖, 9 ♚xb4 ♖10♖, 9 ♜b3 ♖31♖, 9 ♜xb5 ♖10♖, 9 ♜c6 ♖11♖, 9 ♜e2 ♖22♖, 9 ♜e6 ♖12♖, 9 ♜f3 ♖23♖, 9 ♚c2 ♖11♖, 9 ♚d2 ♖12♖, 9 ♚e2 ♖12♖, 9 ♗e2 ♖11♖, 9 ♗e3 ♖11♖, 9 ♗f3 ♖11♖**)
- [4|1.5.3.1] **9 ♜c2 ♙c5+** = for the same reasons as lines [4|1.5.2.1] / [4|1.5.2.2] / [4|1.5.2.3] / [4|1.5.2.4]
- [4|1.5.3.2] **9 ♜xf5 ♙c5+** = once again the Black Bishop is exchanged vs the Knight and the remaining Rook ending is draw.
- * [4|1.5.4] **7 ♜xb4 ♙xb4** all moves but one lead to White checkmate (**8 d4 ♖10♖, 8 ♚b3 ♖17♖, 8 ♚xb4 ♖11♖, 8 ♚c2 ♖11♖, 8 ♚d2 ♖10♖, 8 ♚e2+ ♖13♖, 8 ♙d2 ♖11♖, 8 ♙d4 ♖7♖, 8 ♙e5 ♖11♖, 8 ♙f6 ♖12♖, 8 ♗e2 ♖9♖, 8 ♗e3 ♖10♖, 8 ♗f3 ♖9♖**) **8 ♙xb4 ♜xb4** = (**9 d4 ♖8♖, 9 ♚b3 ♖11♖, 9 ♚c2 ♖7♖, 9 ♚d2 ♖12♖, 9 ♚e2+ ♖12♖, 9 ♗e3 ♖12♖, 9 ♗e2 ♖12♖, 9 ♗f3 ♖11♖**)
9 ♚xb4 leads to the same drawn Rook endgame as [4|1.5.1].
- [4|1.6] **5 ♗f3 cxb4** (**6 ♗f2 ♖13♖, 6 c4 ♖27♖, 6 ♚xb4 ♖17♖, 6 ♚b3 ♖14♖, 6 ♙e3 ♖13♖, 6 ♗e2 ♖14♖, 6 ♜e3 ♖11♖**)
 - * [4|1.6.1] **6 cxd4 b3** (**7 ♗f2 ♖15♖, 7 ♗e2 ♖11♖, 7 ♗e3 ♖14♖, 7 ♙c3 ♖19♖, 7 ♙b4 ♖11♖, 7 ♙e3 ♖26♖, 7 ♜e3 ♖15♖, 7 ♜b4 ♖14♖**)
[4|1.6.1.1] **7 ♚xb3 b4** (**8 ♙xb4 ♖35♖, 8 ♚c3 ♖17♖, 8 ♚xb4 ♖20♖, 8 ♜xb4 ♖15♖, 8 ♜e3 ♖11♖**) = White cannot progress without giving a piece or moving d5+ after which the Black King blocks the position. If White moves around Black simply plays his Rook on b5-b6. \triangle **8 d5+ ♗xd5 9 ♜e3+ ♗e6** (**10 ♗e2 ♖11♖, 10 ♜d5 ♖11♖, 10 ♚c3 ♖14♖, 10 ♙c3 ♖33♖, 10 ♚b2 ♖33♖, 10 ♚xb4 ♖20♖, 10 ♙xb4 ♖17♖, 10 ♜xf5 ♖24♖, 10 ♜c4 ♖14♖, 10 ♗f2 ♖29♖, 10 d4 ♖11♖**) and the only non losing line is to repeat with **10 ♜c2**.
[4|1.6.1.2] **7 d5+** = see line [4|1.6.1.1].
 - * [4|1.6.2] **6 cxb4 ♗d5** = position is totally blocked.
 - * [4|1.6.3] **6 ♜xd4+ ♜xd4** = \triangle **7 cxd4** Black puts his King on d5 and the White position is totally blocked.
 - * [4|1.6.4] **6 ♜xb4 ♜xb4** = \triangle **7 cxb4 ♗d5** is similar to line [4|1.6.2].

- [4|2] **4 cxd4 cxd4** = see line [4|3].

- [4|3] **4 ♗f2 b4** = the position is totally blocked and Black can just move his King on e6-f6 \triangle **5 cxb4 cxb4 6 ♗f3 ♗e6** etc.

- [4|4] **4 ♗f3 b4** = for the same reasons as in line [4|3].

4 c4 b4 = see line [1|5.2]

4.6 White moves ♜b4

1 ♜b4 cxb4 ♖21♖. White is a piece down.

4.7 White moves ♜d4

1 ♜d4 exd4 ♖25♖. White is a piece down.

5 Conclusion

The game-theoretical value of Gardner's chess has been proved to be a draw. The proof was done in a semi-automated way in which humans were guiding the engine. The authors were 'pushing' lines for which it was thought that the exact distance to checkmate could be computed and backtracked once leaves were showing perfect distance to checkmate. This meta-algorithm leads to a very asymmetric way of selecting moves. For instance, when a position is thought to be decidable as a White win, very few time is spent on White decision nodes (since we 'know' the game to be won more or less no matter what). The idea is that enormous time and energy can be saved when the game theoretic value of a position, rather than the most precise move or the shortest path to checkmate, is looked for. Indeed, when a game is thought to be winning, e.g. for White, one has only to provide one forced line (even if it is not the 'best' one) and thus can avoid exhaustive search at White decision nodes. It can be seen as a form of meta-negascout [Fis81]. Nevertheless it is very different in the sense that the process is very asymmetric and guided by the fact that the overall evaluation of the position is known.

This procedure can be fully automated and tuned to some given degree of precision (basically what is the threshold after which a position is considered as decided). For future works we plan to implement it and test it for larger chess variants in order to compute their game theoretic values. Other games could also be considered.

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