2013 Nirma University International Conference on Engineering (NUiCONE)

Image Denoising using Wavelet Transform and Median Filtering

Ajay Boyat[1]
[1]MITM, Indore, India.
[1]a_boyat@yahoo.co.in

Brijendra Kumar Joshi[2]
[2]MCTE, Mhow, Indore, India
[2]brijendrajoshi@yahoo.com

***Abstract*-- During acquisition of an image, from its source, noise becomes integral part of it, which is very difficult to remove. Various algorithms have been used in past to denoise images. Image denoising still has scope for improvement. In this paper we present a new image denoising algorithm based on combined effect of wavelet transform and median filtering. The algorithm removes most of the noisy part from the image and maintains the quality. The level of wavelet decomposition is restricted to three. The renowned index Peak Signal to Noise Ratio (PSNR) and Root Mean Square Error (RMSE) demonstrate marked improvement of image denoising over other methods.**

***Index Terms*-- Haar wavelet, Median filter, PSNR and RMSE.**

I. Introduction

Image distortion is often a common issue due to various types of noise. Gaussian noise, Salt and Pepper noise, Poisson noise, Speckle noise etc are fundamental noise types in case of images. The noise may come from a noise source present in the vicinity of image capturing location or may be introduced due to imperfection/inaccuracy inherent in the image capturing devices like cameras. For example, lenses may be misaligned, focal length may be weak, scattering and other adverse conditions may be present in the atmosphere, etc. This makes careful study of noise and noise approximation an essential ingredient of image denoising. This leads to selection of proper noise model for image processing systems [1].

Noise gets introduced in images during image acquisition or transmission. This may be from Electronic or photometric sources. Blurring occurs due to imperfect image formation process such as spreading of focal length, non-stationary camera placement etc, resulting in bandwidth reduction of images [2].

To remove noise, traditionally, linear techniques were used. However, these techniques do not perform well in case of impulsive noise. They also obviously do not fit where non-linear operation is required. Linear filters generally introduce noise during image formation and transmission process. Image signals deal with low and high frequency contents. Lines, junctions, edges, corners and other fine details of an image are represented in high-frequency components. Therefore these high frequency components become very important for visual perception. However most of linear filters have only low pass characteristics and hence edges, lines and other fine details are lost due to filtering.

Several non-linear techniques have been proposed for image restoration both in the spatial domain and multi scale (wavelet) domain. Multi scale approaches have proved to be superior [3-4].

Traditionally, Pixel based image denoising schemes have been used for quite long, However, in recent years, wavelet based image denoising algorithms have achieved remarkable results [5].

Yang et al. [6] have proposed use of median filters for noise reduction under structural constraints. Family of non-linear filters includes filters based on order statistics. Among order statistics based filters, median filters are best known [7].

II. Wavelet Transform

Short Time Fourier Transform (STFT) has been used for time frequency analysis with limitations, which are overcome by the wavelet transform. The wavelet transform enables multi resolution and helps analyze the frequency content of an image. Dilation and Translation are the two essential steps of wavelet transform that give a group of template functions. Unlike Fourier transform, wavelet transform function offers more flexibility. It is an analyzing function and can be chosen with more freedom without the need of using sine and cosine forms [8].

2.1.1. Continuous wavelet transforms (CWT)

To find out the detailed coefficients of a continuous time signal, CWT can be used for its efficiency. Limited in time domain mother wavelet function is $\psi(n) \in L^2(R)$ where, $\psi(n)$ has values in a certain range and is zero elsewhere. Gaussian model with zero-mean and known variance is used. Properties of the mother wavelet are normalized as shown in equations (1) and (2).

$$\int_{-\infty}^{\infty} \psi(n)dn = 0 \quad (1)$$

$$\left\|\psi(n)\right\|^2 = \int_{-\infty}^{\infty} \psi(n)\psi^*(n)dn = 1 \qquad (2)$$

The CWT of $f_{(n)}$ with respect to $\psi(n)$ is shown in equation (3).

$$W_{(j,k)} \equiv \int_{-\infty}^{\infty} f_{(n)} \frac{1}{\sqrt{|j|}} \psi^*\left(\frac{n-k}{j}\right) dn \qquad (3)$$

2.1.2. Discrete wavelet transforms (DWT)

Discrete signal is in the time domain converted (transformed) into the time-frequency domain with the helf of DWT. DWT arranges coefficients in a manner that enables both spectrum analysis and spectral behavior of the signal in time. When decomposed, signal breaks it into two parts, low pass and high pass; carrying information about original signal. Combinations of up sampling and down sampling are repeatedly used as per requirements as shown in Fig. 1.

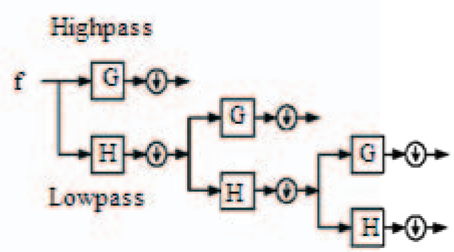

Figure 1. 2-D for discrete wavelet transforms

Low pass component represents smoothness in the image where as high pass component represents drastic change in the image i.e. edges [9-11]. In a DWT from [12] the values $j = 2^l$ and $k = 2^l p$ implies dilation and translation respectively, where l and p are integers. Two dimensional squares $d(l, p)$ referred as DWT of $f_{(n)}$, which is shown in equation (4).

$$f_{(n)} = \sum_{l=-\infty}^{\infty} \sum_{p=-\infty}^{\infty} d_{(l,p)} 2^{-\frac{l}{2}} \psi_{\left(2^{-l} n - p\right)} \qquad (4)$$

2.2. Haar Wavelet Transform

Alfred Haar invented Haar wavelet. The Haar function ψ(m) is shown in Fig. 2 and it represented by equation (5). It is commonly known as mother wavelet. Orthogonality is the property of the Haar wavelets and the requirements of additions and subtractions are met by their forward and inverse transforms. The Haar wavelets are one of the easiest to implement [13].

$$\psi(m) = \begin{cases} 1; \ 0 \geq m \geq 0.5 \\ -1; 0.5 \leq m \leq 1 \\ 0; \ otherwise \end{cases} \qquad (5)$$

Haar wavelet scaling function is $\phi(m)$, which is shown in equation (6).

$$\phi(m) = \begin{cases} 1 \ ; \ 0 \leq m \leq 1 \\ 0 \ ; \ otherwise \end{cases} \qquad (6)$$

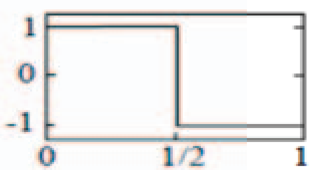

Figure 2. Haar function {ψ (m)}

III. MEDIAN FILTER

Non-linear filter classification starts with a median filter. The medial pixel value among all the neighbors in a window unlike the mean filter (linear filter) are generally used to reduce noise in image. To preserve the smoothness in a resultant image iterative Median filter is most prominent choice. According to the Order statistics the intensity value of an image is critical in deciding the ranking of the neighboring pixels and this value is replaced by the median value of surrounding pixel values [14-18].

IV. PROPOSED METHOD

A Wavelet decomposes the original and noisy image to determine the level of satisfaction for image denoising. We exploit Haar wavelet decomposition at level three is as shown in the Fig. 1 to Fig. 5 using reference Lena image and first author's image.

Edge preservation requires Wavelets operation in much more depth for which point discontinuities wavelet is reasonable choice. A combination of wavelet based iterative noise density and Median filter is the best approach for several noise standards and is used for Gaussian noise. We decomposes an image into different segments up to the level 3 called HH3, HL3, LH3 and LL3 sub bands as shown in Fig. 3 through Fig. 5. These Four sub bands, namely HH, HL, LH and LL are shown the diagonal details, horizontal features, vertical structures and approximation of the image.

We further decompose The LL sub band into higher levels [19]. This type of adjustment does not fulfill all the requirements of image denoising. So wavelet based iterative noise density and median filter model considering low frequency contents of image signal is used to further decompose it into three levels.

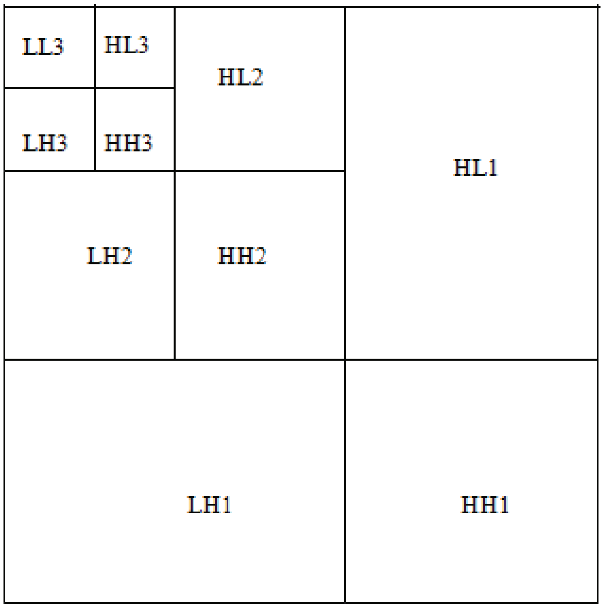

Fig. 3 Wavelet Decomposition of image at three levels

First wavelet transform decomposed the final noisy image $Z(j,k)$ into approximate and detail components. Keeping the detail component as zero. Further process the approximate part using the median filter [20-22]. The following equations represent the proposed algorithm.

$$Z(j,k) = D[f(j,k)] + I(j,k)$$

Where $f(j,k)$ is original image, $I(j,k)$ is Noise signal, $Z(j,k)$ is degraded image and D is degrading function.

The DWT of degraded image $Z(j,k)$ is

$$w(j,k) = W(j,k)Z(j,k)$$

Which implies the approximation and details signals

$$w(j,k) = \{A_{2^j}^d f, D_{2^j}^1 f, D_{2^j}^2 f, D_{2^j}^3 f\}$$

Whereas $A_{2^j}^d f$ is the approximation corresponds to Low frequencies and $D_{2^j}^1 f, D_{2^j}^2 f$ *and* $D_{2^j}^3 f$ are details of the wavelet $w(j,k)$ set to zero.

$D_{2^j}^1 f = 0,\ D_{2^j}^2 f = 0$ and $D_{2^j}^3 f = 0$

Apply median filtering upon the approximation coefficients of wavelet

$$\hat{v} = MED\{A_{2^j,1}^d, A_{2^j,2}^d, A_{2^j,3}^d ......, A_{2^j,N}^d\}$$

Taking Inverse Discrete Wavelet Transform (IDWT)

$$\hat{f}(j,k) = W^T w(j,k)$$

Our objective is to obtain the denoised image $\hat{f}(j,k)$ as close to original image $f(j,k)$ as possible.

Error is $e(j,k) = \hat{f}(j,k) - f(j,k)$

That is why Mean square error is

$$MSE = \frac{1}{PQ}\sum_{p=0}^{P-1}\sum_{q=0}^{Q-1}[f(j,k) - \hat{f}(j,k)]^2$$

The Peak signal to noise ratio is given by

$$PSNR = 20\log_{10}\frac{255^2 PQ}{\sum_{p=0}^{P-1}\sum_{q=0}^{Q-1}[f(j,k) - \hat{f}(j,k)]^2} dB$$

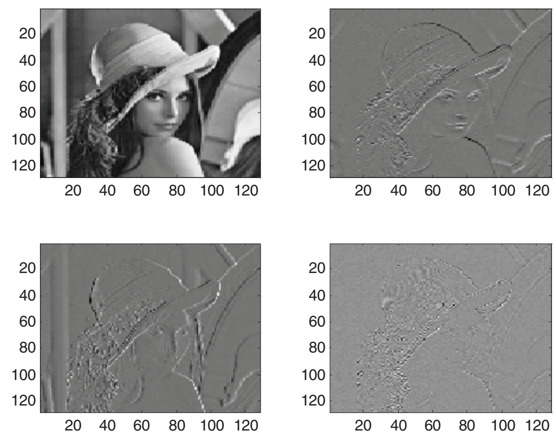

Fig. 4.Wavelet decomposition of reference image

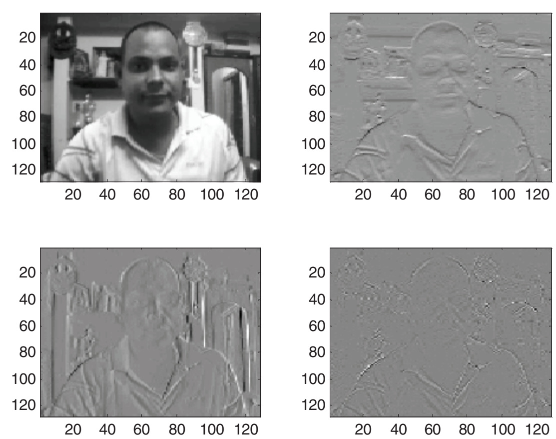

Fig. 5.Wavelet decomposition of first author's image

The filter size is initially set to 3×3 at each processing step and then gradually expand until it meets desired number of potential denoised pixels including local mean, local maximum value, local minimum value or local median value. This phenomenon is not always true under restriction of the expansion of the filter size. Result needs the adaptation in a given filter according to optimum choice.

Although adaptive median filters are good choice in image restoration corrupted by noise but considerable amount of computational time is required when the image is highly corrupted. To get the denoised image the final result of filter is recombined. The proposed flow chart of the algorithm is shown below in Fig. 6.

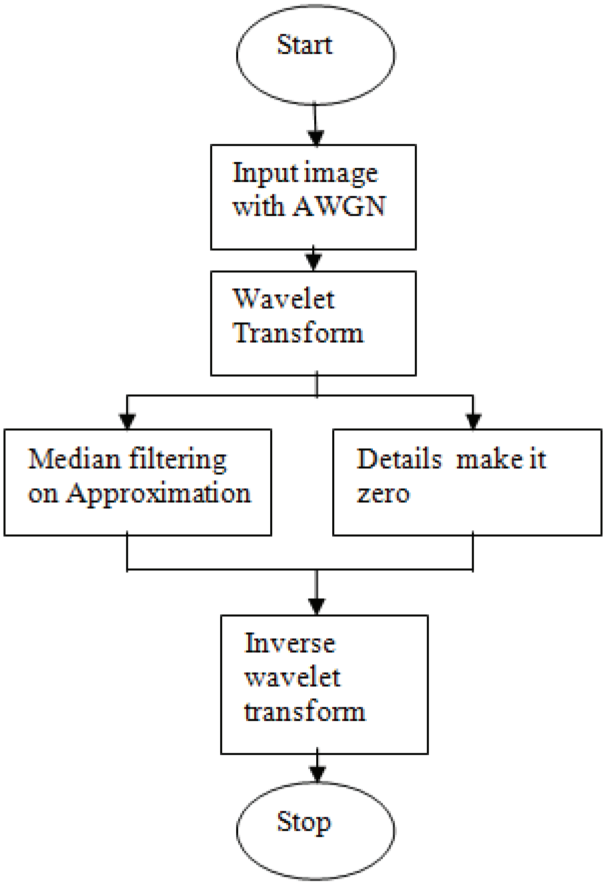

Fig. 6. The Denoising algorithm.

V. SIMULATION AND RESULTS

The proposed method was implemented using MATLAB R2012a on a Dell Laptop with 2GB RAM, Core i-3 processor and windows 7 operating system. The results are demonstrated in Fig. 9 through 12.

In this paper we have mainly focused on wavelet based iterative noise density and median filter for determining the PSNR and RMSE using Gaussian noise model. The locally adaptive zero mean and known variance nearly approximated 0.01 to 0.02 for which 0.005 is the fixed interval for all iteration. Proposed method assumes the nearly optimal reasonable variance for each wavelet coefficient has been one of the important concepts in the image denoising.

The paper not only gives the mathematical modeling of composite approach but also gives certain level of satisfactory results which one shown in Fig. 7 through 14. The comparisons have been performed for Haar wavelet with Median filter and without Median filter, the different data used as parameters for simulation are shown in Table 1.

The algorithm minimizes the structural constraints such as horizontal, vertical and details signals, because the algorithms only deals with approximation signals and denoise using median filter. Though the first author's image is not a standard one but it was only used for the purpose of novelty in the work. It is observed that, as we slightly increase the noise density PSNR decreases slowly.

Table 1 Simulation parameters

S.no.	*Parameter*	*Specification*
1.	Noise type	Gaussian
2.	Wavelet	Haar
3.	Decompose Level	3
4.	Filter	Median
5.	Filter size	3X3
6.	Iteration	21
7.	Noise mean (μ)	Zero value
8.	Noise density(σ)	0.01 to 0.02 for which 0.0005 is the fixed interval for all iteration

Fig. 7 First Reference (Lena) image [23] when Noise density is 100% improved.

Fig. 8 Second Reference (First Author's) image when Noise density is 100% improved.

As we move up to the 40% increase in the density the algorithm gives optimum results shown in Fig. 7 through Fig. 14. From 40% to 100% resultant denoised image would start to become more blurred. Up to the 100% increase in noise density the denoised image is totally corrupt. Composite approach achieved closer image as compared to original one and almost 05 dB improvement in PSNR and 20 dB falls in RMSE.

Fig. 9 Reference Leena image when Noise density is improved from 20% to 100%.

Fig. 10 Second Reference (First Author's) image when Noise density is improved from 20% to 100%.

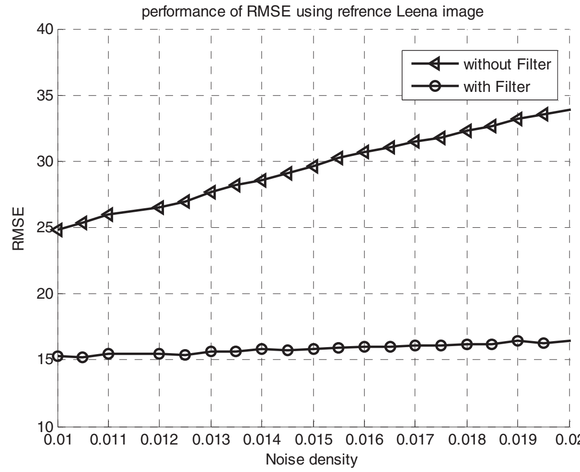

Fig. 11 Graph for Comparing RMSE and Noise density of reference image with and without median filter.

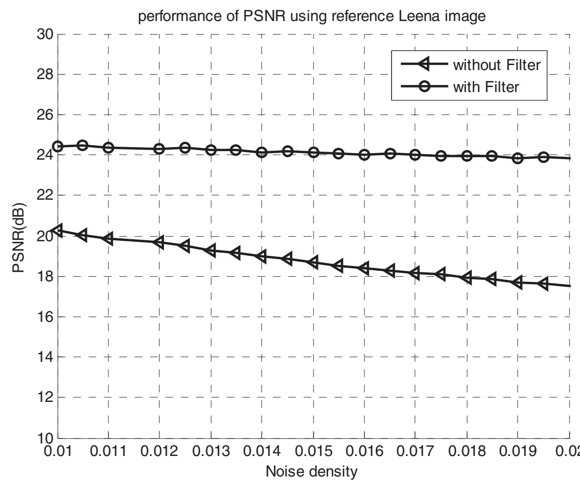

Fig. 12 Graph for Comparing PSNR and Noise density of reference image with and without median filter

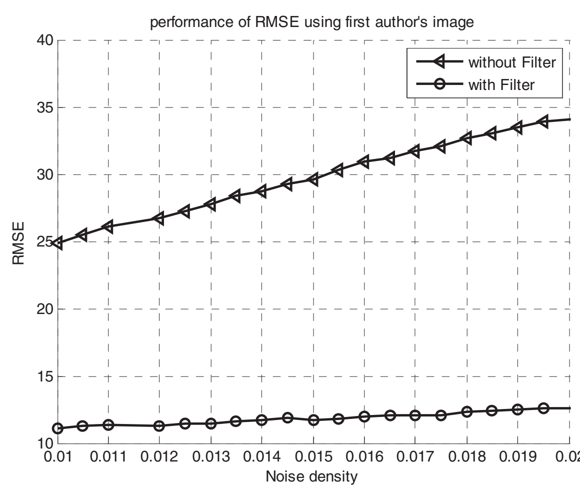

Fig.13. Graph for Comparing RMSE and Noise density of First author's image with and without median filter

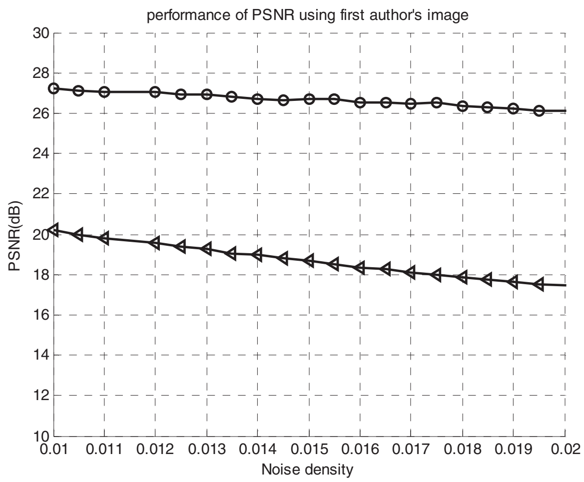

Fig. 14 Graph for Comparing PSNR and Noise density of First author's image with and without median filter

VI. CONCLUSION

It is observed that the wavelet domain iterative noise density and median filter based model produces the nearly constant result for PSNR and RMSE. The detail coefficients of wavelet have been put as zero and only analysis is done of the approximation coefficients without any thresholding. Almost 5 dB PSNR has been improved and 20dB down fall in the RMSE, which make result better.

Hence with low noise density the improvement is significant and remarkable. But as we increase the noise density from 40% to 100% this algorithm gives promising results which is shown in Fig. 7 through Fig. 10. This is because of almost double the noise density taken. This implies that the algorithm worked up to the 100% improvement in variations from the default variance and gives the optimum result.

Since wavelet transform itself does the thresholding in one sense, that is why in this algorithm we have not used any kind of thresholding. In spite of this, results are promising, though another level of thresholding can be thought of.

REFERENCES

[1] R. C. Gonzalez and R. E. Woods, "Digital Image Processing," second ed., Prentice Hall, Englewood, Englewood, Cliffs, NJ, 2002.

[2] J. Patil, and S. Jadhav "A Comparative Study of Image Denoising Techniques," *International Journal of Innovative Research in Science, Engineering and Technology,* vol. 2, issue 3, March 2013.

[3] S Mallet "A Wavelet Tour of Signal Processing," Academic Press, New York, 1998.

[4] V. Zlokolica, A. Pizurica and W. Philips, "Wavelet-Domain Video Denoising Based on Reliability Measures" *Circuits and Systems for Video Technology, IEEE Transactions,* vol. 16 , no. *8*, pp. 993-1007, *August*, *2006.*

[5] S. M. M. Rahman and M. K. Hasan "Wavelet-domain iterative center weighted median for image denoising," *Signal Processing, Elsevier Science,* vol. *83*, no. 3, *pp.*603-610, Feb. *2003*.

[6] R. Yang, L. Yin, M. Gabbouji, J. Astola, and Y. Neuvo "Optimal Weighted Median Filters Under Structural Constraints," *IEEE Transactions on Signal Processing*, vol. 43, no. 3, pp. 591-604 March 1995.

[7] S. N. Sandra and S. N. Ivan, "Application of Genetic Algorithm in Median Filtering," *Proceedings of the International Multiconference on Computer Science and Information Technology,* pp. 127 – 139 © 2007 PIPS.

[8] N. Debande, Y. Souissi, M. A. E. Aabid, S. Guilley and J. L. Danger, "Wavelet transform based pre-processing for side channel analysis," *45th Annual IEEE/ACM International Symposium on Microarchitecture Workshops,* 1-5 Dec. 2012.

[9] A. A. Blessie, J. Nalini and S.C.Ramesh, "Image Compression Using Wavelet Transform Based on the Lifting Scheme and its Implementation" *International Journal of Computer Science Issues,* vol. 8, issue 3, no. 1, May 2011.

[10] S. G. Chang, B. Yu, and M. Vetterli, "Adaptive wavelet thresholding for image denoising and compression," *IEEE Transactions on Image Processing,* vol. 9, no. 9, pp. 1532-1546, Sep., 2010.

[11] A. Fathi and A. R. N. Nilchi, "Efficient Image Denoising Method Based on a New Adaptive Wavelet Packet thresholding Function,"*IEEE Trans. Image Processing*, vol. 21, no. 9, September 2012.

[12] S. Salivahanan, A. Vallavaraj, and C. Gnanapriya "Digital Signal Processing," Tata Mcgraw-Hill, vol. 23, NewDelhi, 2008.

[13] M. A. Pasnur and P. S. Malge, "Image Retrieval Using Modified Haar Wavelet Transform and K Means Clustering" *International Journal of Emerging Technology and Advanced Engineering,* vol. 3, issue 3, March 2013.

[14] T. Chhabra, G. Dua and T. Malhotra "Comparative Analysis of Denoising Methods in CT Images" *International Journal of Emerging Trends in Electrical and Electronics,* vol. 3, issue. 2, May-2013.

[15] David L. Donoho and Iain M. Johnstone, "Ideal Spatial Adaption Via Wavelet Shrikage," Biometrika,, vol. 3, pp.192–206, Mar. 1994.5 Sep., 1994.

[16] Y. M. M. Babu, M.V. Subramanyam, and M.N. Giri Prasad "PCA based image denoising," *Signal & Image Processing An International Journal (SIPIJ)* vol.3, no.2, April 2012.

[17] S. Tong, G. Moncef and N. Yrjö "Analysis of two-dimensional center weighted Median filter," *Multidimensional Systems and Signal Processing,* vol. 6, pp. 159-172, 01 April 1995.

[18] S. H. Teoh and H. Ibrahim "Median Filtering Frameworks for Reduction Impulse Noise from Grayscale Digital Images: A Literature SurveyInternational," *Journal of Future Computer and Communication*, vol. 1, no. 4, December 2012.

[19] T. kaur, M. Sandhu and P. Goel, "Performance Comparison of Transform Domain for Speckle Reduction in Ultrasound Image" *International Journal of Engineering Research and Application,* vol. 2, issue 1, pp.184-188.

[20] P. Subashini, M. Krishnaveni, B. K. Ane, and D. Roller "Image Denoising Based on Wavelet Analysis for Satellite Imagery," Soft Computing Models in Industrial and Environmental Applications, *springer,7th International Conference, SOCO'12,* Ostrava, Czech Republic, pp. 399-407, September 5th-7th, 2012-13.

[21] D. L. Donoho and I. M. Johnstone "Adaptive to unknown smoothness via Wavelet shrinkage", *Journal of the American Statistical Association,*vol. 90, no 432, Dec., 1995

[22] S. G. Mallet "A Theory for Multiresolution Signal Decomposition: The Wavelet Representation," *IEEE Transactions on Pattern Analysis and Machine Intelligance,* vol. 11, no. 7, July 1989.

[23] L. Zhang, W. Dong, D. Zhang and G. Shi "Two Stage denoising by principal component analysis with local pixel grouping," *Elsevier Pattern Recognition,* vol. 43, issue 4, pp. 1531-1549, April 2010.