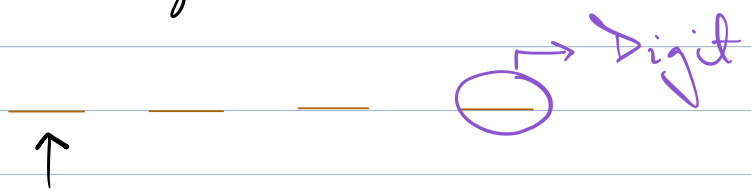
Decimal Number System

No. of unique symbols req to define ⇒ Base any no. system

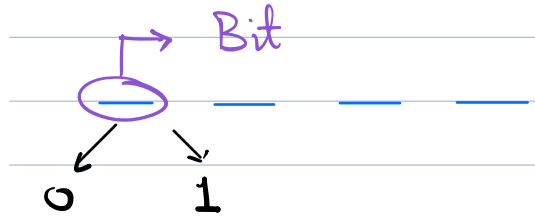


{ 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 }

$3\,2\,4 \Rightarrow 3\times10^2 + 2\times10^1 + 4\times10^0$

$2048 \Rightarrow 2\times10^3 + 0\times10^2 + 4\times10^1 + 8\times10^0$

Binary Number System



Base = 2

{ 0, 1 }

$1\,1\,0 \Rightarrow 1\times(2)^2 + 1\times(2)^1 + 0\times(2)^0$

$= 4 + 2 = 6$

Binary to Decimal

Ex 1 $(1\,1\,0\,1)_2$

$$
\begin{aligned}
1 \times 2^0 &= 2^0 \\
0 \times 2^1 &= 0 \\
1 \times 2^2 &= 2^2 \\
1 \times 2^3 &= 2^3 \\
\hline
8 + 4 + 1 &= 13
\end{aligned}
$$

Ex 2 $(1\,0\,1\,0\,1)_2$

$$
\begin{aligned}
1 \times 2^0 &= 1 \\
0 \times 2^1 &= 0 \\
1 \times 2^2 &= 4 \\
0 \times 2^3 &= 0 \\
1 \times 2^4 &= 16 \\
\hline
&21
\end{aligned}
$$

Ex 3 $(1\,0\,1\,1\,0\,1\,0)_2$

$$1\times2^6 + 0\times2^5 + 1\times2^4 + 1\times2^3 + 0\times2^2 + 1\times2^1 + 0\times2^0$$

$$64 + 16 + 8 + 2 = 90$$

Decimal to Binary

$N = 20$

2	20	0
2	10	0
2	5	1
2	2	0
2	1	1
	0	

$(10100)_2$

$N = 45$

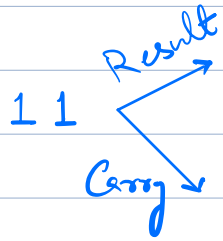
2	45	1
2	22	0
2	11	1
2	5	1
2	2	0
2	1	1
	0	

$(101101)_2$

Addition in Decimal

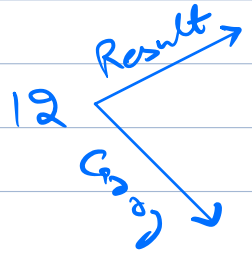
Calculate (368 + 253)

10^2	10^1	10^0
	1	1
3	6	8
2	5	3
6	12	11
6	2	1



11 % 10 = 1

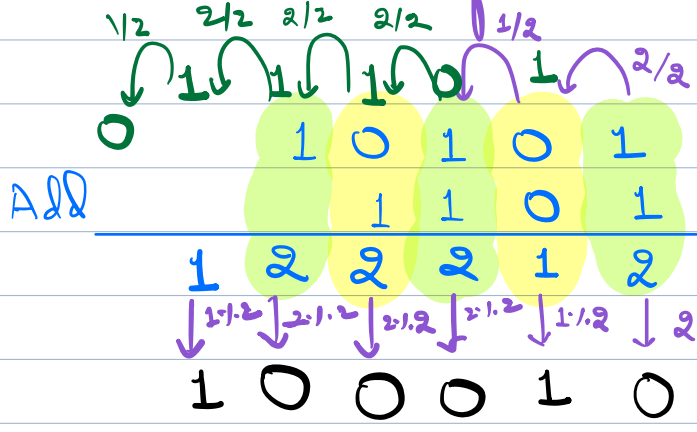
11/10 = 1



12 % 10 = 2

12/10 = 1

Addition of Binary Numbers



⇒ Ans

Result = Sum % Base

Carry = Sum / Base

Ex

```
  0 0 1 1 0
  1 0 1 1 0
  0 0 1 1 1
-------------
  1 1 1 0 1
```

Bit Wise Operators

Exactly 1 bit is true

A	B	AND (&)	OR (|)	XOR (^)
0	0	0	0	0
0	1	0	1	1
1	0	0	1	1
1	1	1	1	0

(A & B & C & D & E) → 1 (If all bits are 1), 0 (Otherwise)

(A | B | C | D | E | F) → 1 (If any of the bit is 1), 0 (If all bits are 0)

(A ^ B ^ C ^ D ^ E ^ F) → 1, 0

NOT (! / ~)

$\Rightarrow \sim A$ → 0 (if A == 1)
→ 1 (if A == 0)

Binary Operators on Multi Bit Numbers

1) AND (&)

		2	1	0
5 →		1	0	1
6 →	&	1	1	0
		1	0	0

$1 \times 2^2 + 0 \times 2^1 + 0 \times 2^0$

5 & 6 = 4

```
int a = 5
int b = 6
int c = a & b;
print (c);
```

↳ Bitwise operators can be directly used on integers w/o converting integers to binary.

2) OR (|)

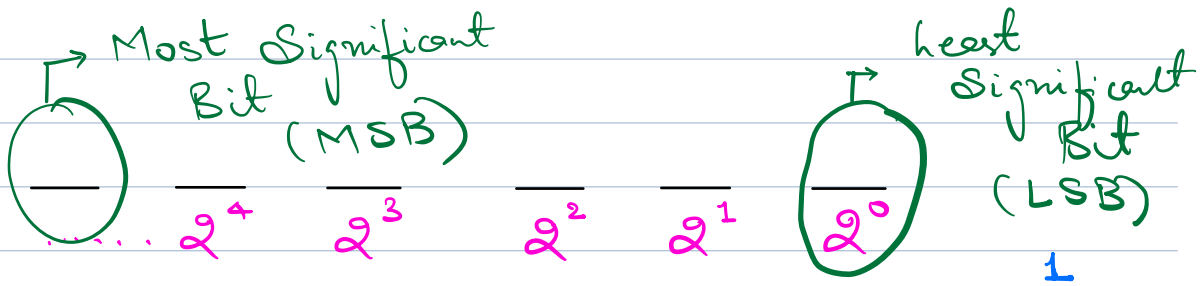
		5	4	3	2	1	0
20	→	0	1	0	1	0	0
45	→	1	0	1	1	0	1
	OR	1	1	1	1	0	1

$32 + 16 + 8 + 4 + 1 = 61$

3) XOR (^)

		5	4	3	2	1	0
20	→	0	1	0	1	0	0
45	→	1	0	1	1	0	1
	XOR	1	1	1	0	0	1

$32 + 16 + 8 + 1 = 57$



$Bit \times 2^4 + Bit \times 2^3 \ldots Bit \times 2^1 + Bit \times 2^0$

$2 \times (\text{Something}) + Bit$

bit = 0 / bit = 1

$2x(x) + 0$ ⇓ Even

$2x(x) + 1$ ⇓ Odd

Eg

$1\ 1\ 0\ 1 = 13$ (Odd)

$1\ 1\ 1\ 0 = 14$ (Even)

if (LSB == 0) ⇒ Even

if (LSB == 1) ⇒ Odd.

(13) ⇒ 1 1 0 1

M ⇒ & 0 0 0 1

Result 0 0 0 1

(10) ⇒ 1 0 1 0

M ⇒ & 0 0 0 1

Result 0 0 0 0

M = 1

Bit (x & 1) → 1 (if x == 1)

Bit (x & 1) → 0 (if x == 0)

To check if a given no. A is odd/Even

(A & 1) → 0 (Even)

(A & 1) → 1 (Odd)

Basic AND Properties

1) (A & 1) → If A is Odd → 1; If A is Even → 0

2) (A & 0) = 0

```
  1 0 1 0 1
& 0 0 0 0 0
-----------
  0 0 0 0 0
```

3) (A & A) = A

```
  1 0 1 0 1
& 1 0 1 0 1
-----------
  1 0 1 0 1
```

Basic OR properties

1) A | 0 = A

```
  1 0 1 0 1
  0 0 0 0 0
-----------
  1 0 1 0 1
```

(1 | 0) = 1

(0 | 0) = 0

2) $A | A = A$

```
 1 0 1 0 1
 1 0 1 0 1
-----------
 1 0 1 0 1
```

$(1|1) = 1$
$(0|0) = 0$

Basic XOR properties.

1) $A \hat{} 0 = A$

```
A → 1 0 1 0 1
^   0 0 0 0 0
-------------
    1 0 1 0 1
```

$(1 \hat{} 0) = 1$
$(0 \hat{} 0) = 0$

2) $A \hat{} A = 0$

$(1 \hat{} 1) = 0$
$(0 \hat{} 0) = 0$

```
A → 1 0 1 0 1
^ A → 1 0 1 0 1
---------------
    0 0 0 0 0
```

Commutative

$A \& B = B \& A$
$A | B = B | A$
$A \hat{} B = B \hat{} A$

Associative

$(A \& B) \& C = A \& (B \& C)$
$= (A \& B) \& C$
$= (A \& C) \& B$

$(A | B) | C = A | (B | C)$

⋮

Q Output of expression

$c \wedge a \wedge d \wedge a \wedge b \wedge d \wedge c \wedge b$

Sol^n Above expression is same as

$(a \wedge a) \wedge (b \wedge b) \wedge (c \wedge c) \wedge (d \wedge d)$

$= 0$

Q

$1 \wedge 3 \wedge 5 \wedge 3 \wedge 2 \wedge 1 \wedge 5$

$= (1 \wedge 1) \wedge (2) \wedge (3 \wedge 3) \wedge (5 \wedge 5)$

$0 \wedge 2 \wedge 0 \wedge 0 = 2$

Given an integer array of +ve elements where all elements occurs twice except one element which occurs one time.
find the value of that unique element.

$A = \{ \underset{0}{2}, \underset{1}{3}, \underset{2}{5}, \underset{3}{6}, \underset{4}{3}, \underset{5}{6}, \underset{6}{2} \}$

Ans = 5

Sol^n Take XOR of all numbers

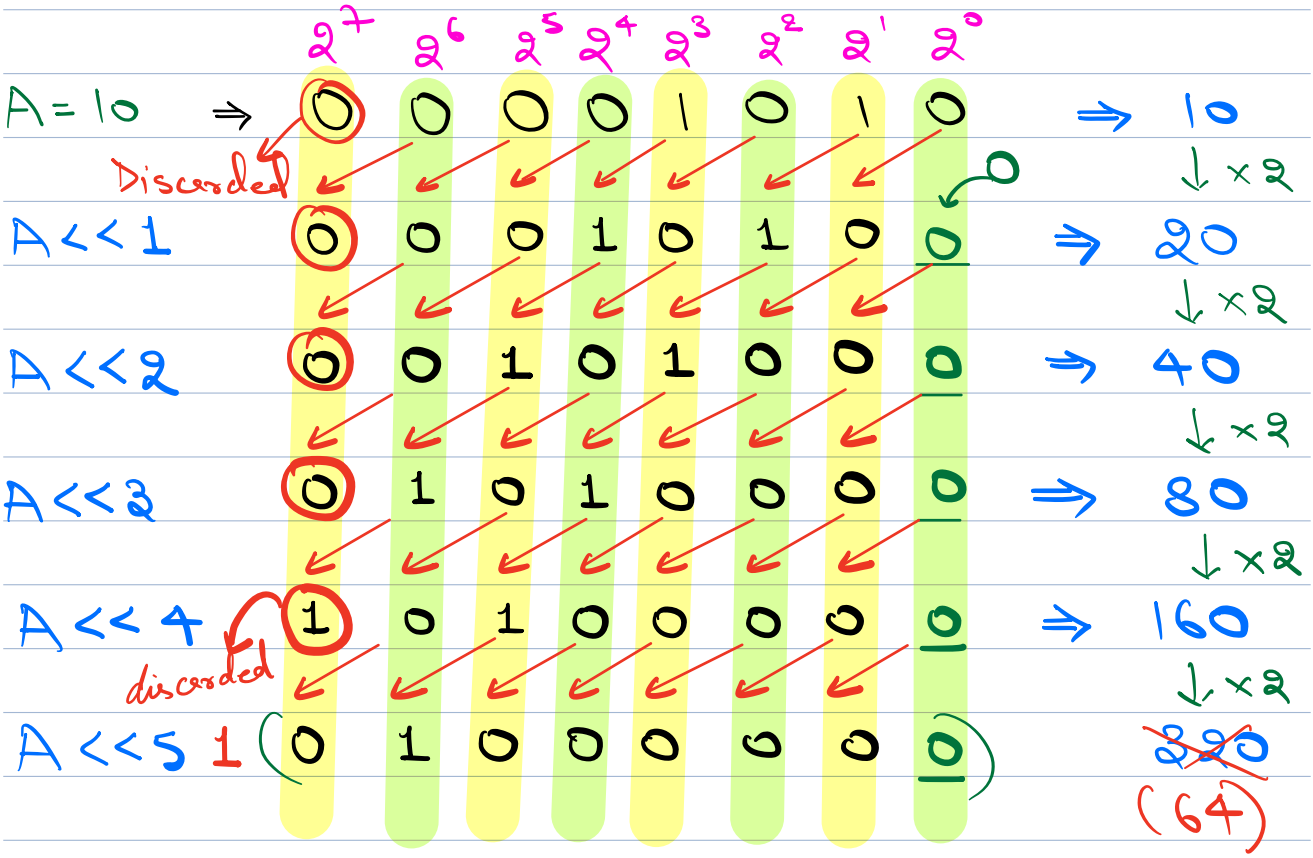
Code

```
int result = 0;
for (i=0; i<N; i++) {
    result = result ^ A[i];
}
return result;
```

T.C. = O(N)
S.C. = O(1)

Left Shift Operator (<<)

Assume 8 bit



$$(A << n) = A \times 2^n$$

↳ as long as no overflow.

$320 - 64 = 256$??

When discarded bit = 1 ⇒ Overflow.