CSE 6363 - Machine Learning

Homework/Project 3- Spring 2019

Due Date: Apr. 30 2019, 11:59 pm

Hierarchical Clustering

1. Consider an unlabeled version of our height/weight/age data set used in the previous assignments (and shown below).

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D = \{ \quad ((170, 57, 32), \quad W), \\ \quad ((192, 95, 28), \quad M), \\ \quad ((150, 45, 30), \quad W), \\ \quad ((170, 65, 29), \quad M), \\ \quad ((175, 78, 35), \quad M), \\ \quad ((185, 90, 32), \quad M), \\ \quad ((170, 65, 28), \quad W), \\ \quad ((155, 48, 31), \quad W), \\ \quad ((160, 55, 30), \quad W), \\ \quad ((182, 80, 30), \quad M), \\ \quad ((175, 69, 28), \quad W), \\ \quad ((180, 80, 27), \quad M), \\ \quad ((160, 50, 31), \quad W), \\ \quad ((175, 72, 30), \quad M), \quad \}
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a) Apply hierarchical clustering with single (minimum) linkage to this data and show the resulting cluster hierarchy. Indicate the order of the merge operations and the distance (linkage) value between the merged sets at each merge. You can do this on the cluster hierarchy tree if you want.

To perform hierarchical clustering with single linkage we first have every point form its own cluster and compute the distance function between all the clusters. For simplicity we will index each data item and use it as a cluster number and when merging use the smallest index in the cluster as the index of the resulting cluster. Also, when merging only the distances from the new cluster have to be recomputed as the minimum of the distance to any of the merged clusters.

i. Initial clusters and distance function:

		• In	tial Clusters:										
							1:		0, 57, 32)				
							2:		2, 95, 28)}				
							3 : 4 :		0, 45, 30)}				
							5:		$\{0, 65, 29\}$ $\{5, 78, 35\}$				
							6 :		$\{5, 90, 32\}$				
							7 :		0, 65, 28)}				
							8 :	{(155	$5, 48, 31)$ }				
							9 :		0, 55, 30)				
							10		2, 80, 30)}				
							11 12		5, 69, 28)} 0, 80, 27)}				
							13		0, 50, 31)}				
							14		5, 72, 30)				
		• Cl	uster Distance	Matrix:									
	44.0908	23.4094	8.5440	21.7945	36.2491	8.9443	17.5214	10.392	26.019	2 13.6015	25.573	4 12.2474	15.9374
44.0908		65.3299	37.2156	25.0400	9.4868	37.2022	59.8916	51.264	10 18.138	4 31.0644	19.235	4 55.2992	28.6705
23.4094	65.3299		28.3019	41.7013	57.0438	28.3549		14.142					
8.5440	37.2156	28.3019	15 1050	15.1658	29.3087	1.0000	22.7596				18.138		
21.7945 36.2491	25.0400 9.4868	41.7013 57.0438	15.1658 29.3087	15.9060	15.9060	15.5885 29.4279							
8.9443	37.2022	28.3549	1.0000	15.5885	29.4279	20.4210	22.8692				18.055		
17.5214	59.8916	5.9161	22.7596	36.2767	51.6236	22.8692	!	8.660			40.804		
10.3923	51.2640	14.1421	14.1774	27.9106	43.0581	14.2829			33.301				22.6716
26.0192	18.1384	47.4236	19.2354	8.8318	10.6301	19.3132				13.1909			
13.6015 25.5734	31.0644 19.2354	34.7131 46.1952	6.4807 18.1384	11.4018 9.6437	23.6008 12.2474	6.4031 18.0555	29.1548 40.8044				12.124	4 24.3926 36.2767	
12.2474	55.2992	11.2250	18.1384	32.0156	47.1805	18.2757		5.099					26.6458
15.9374	28.6705	36.7967	8.6603	7.8102	20.6882	8.8318	31.2570	22.671	10.630	1 3.6056	9.8995	5 26.6458	3
	i	i Mero	e minim	ım dist	ance clu	isters.	4 7 (dis	st = 1.0)				
	-		tial Clusters:				., / (61)						
		- 0	D:	6				4: {(4, 7)}				
			uster Distance										
44.09	44.09	08 23.40 65.32						10.3923 51.2640			25.5734 19.2354		15.9374 28.6705
23.40			28.301				33.0310				13.2334	33.2332	36.7967
8.54							5.9161	14.1421		34.7131	46.1952	11.2250	
21.79		22 28.30		15.16	58 29.30			14.1421 14.1774	19.2354		46.1952 18.0555	11.2250 18.1384	8.6603
0001	25.04		19		58 29.30 15.90	87 2	22.7596		19.2354				8.6603 7.8102
36.24		00 41.70	19 13 15.165	8	15.90	87 2 60 3	22.7596 36.2767	14.1774	19.2354	6.4031 11.4018	18.0555	$18.1384 \\ 32.0156$	
	9.486	00 41.70 58 57.04	19 13 15.165 38 29.308	8 7 15.90	15.90 60	87 2 60 3	22.7596 36.2767	14.1774 27.9106 43.0581	19.2354 8.8318 10.6301	6.4031 11.4018 23.6008	18.0555 9.6437 12.2474	18.1384 32.0156 47.1805	7.8102 20.6882
17.52	9.486 214 59.89	00 41.70 58 57.04 16 5.916	19 13 15.165 38 29.308 51 22.759	8 7 15.90 6 36.27	15.90 60 67 51.62	87 £	22.7596 36.2767 51.6236	14.1774 27.9106	19.2354 8.8318 10.6301 41.8808	6.4031 11.4018 23.6008 29.1548	18.0555 9.6437 12.2474 40.8044	18.1384 32.0156 47.1805 5.3852	7.8102 20.6882 31.2570
	9.486 214 59.89 223 51.26	00 41.70 58 57.04 16 5.916 40 14.14	19 13 15.165 38 29.308 31 22.759 21 14.177	8 7 15.90 6 36.27 4 27.91	15.90 60 67 51.62 06 43.05	87 2 60 3 36 81	22.7596 36.2767 51.6236 8.6603	14.1774 27.9106 43.0581	19.2354 8.8318 10.6301 41.8808 33.3017	6.4031 11.4018 23.6008 29.1548	18.0555 9.6437 12.2474	18.1384 32.0156 47.1805 5.3852 5.0990	7.8102 20.6882
17.52 10.39 26.01 13.60	9.486 214 59.89 223 51.26 92 18.13 215 31.06	00 41.70 57.04 16 5.916 40 14.14 84 47.42 44 34.71	19 13 15.165 38 29.308 51 22.759 21 14.177 36 19.235 31 6.403	8 7 15.90 6 36.27 4 27.91 4 8.831 1 11.40	15.90 60 67 51.62 06 43.05 8 10.63 18 23.60	87 36 81 01 4 08 3	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155	19.2354 8.8318 10.6301 41.8808 33.3017	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909	18.0555 9.6437 12.2474 40.8044 32.1559	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056
17.52 10.39 26.01 13.60 25.57	91 9.486 214 59.89 223 51.26 .92 18.13 215 31.06 234 19.23	00 41.70 68 57.04 16 5.916 40 14.14 84 47.42 44 34.71 54 46.19	19 13 15.168 38 29.308 61 22.759 21 14.177 36 19.238 31 6.403 52 18.058	8 7 15.90 6 36.27 4 27.91 4 8.831 1 11.40 5 9.643	15.90 60 67 51.62 66 43.05 8 10.63 18 23.60 17 12.24	87	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995
17.52 10.39 26.01 13.60 25.57 12.24	91 9.486 214 59.89 223 51.26 .92 18.13 31.5 31.06 334 19.23 174 55.29	00 41.70 58 57.04 16 5.916 40 14.14 84 47.42 44 34.71 54 46.19 92 11.22	19 13 15.165 38 29.308 61 22.755 21 14.177 36 19.235 31 6.403 52 18.055 50 18.138	8 7 15.90 6 36.27 4 27.91 4 8.831 11.40 5 9.643 4 32.01	15.90 60 67 51.62 06 43.05 8 10.63 18 23.60 17 12.24 47.18	87 2 660 3 36 81 01 4 08 2 74 4	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056
17.52 10.39 26.01 13.60 25.57	9.486 9.486 9.486 9.23 51.26 9.92 18.13 9.15 31.06 9.34 19.23 174 55.29 1874 28.67	00 41.70 58 57.04 16 5.916 40 14.14 84 47.42 44 34.71 54 46.19 92 11.22 05 36.79	19 13 15.168 38 29.308 51 22.758 21 14.177 36 19.238 31 6.403 52 18.058 50 18.138 67 8.660	8 7 15.90 6 36.27 4 27.91 4 8.831 11.40 5 9.643 4 32.01 3 7.810	15.90 60 67 51.62 06 43.05 8 10.63 18 23.60 17 12.24 47.18 22 20.68	87 2 60 3 36 81 01 4 08 2 74 4 05 82 3	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995
17.52 10.39 26.01 13.60 25.57 12.24	9.486 9.486 9.486 9.23 51.26 9.92 18.13 9.15 31.06 9.34 19.23 174 55.29 1874 28.67	00 41.70 68 57.04 16 5.916 40 14.14 84 47.42 44 34.71 54 46.19 92 11.22 05 36.79 i. Merg	19 13 15.168 38 29.308 51 22.758 21 14.177 36 19.238 31 6.403 52 18.058 550 18.138 667 8.660 e minim	8 7 15.90 6 36.27 4 27.91 4 8.831 11.40 5 9.643 4 32.01 3 7.810	15.90 60 67 51.62 06 43.05 8 10.63 18 23.60 17 12.24 47.18 22 20.68	87 2 60 3 36 81 01 4 08 2 74 4 05 82 3	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995
17.52 10.39 26.01 13.60 25.57 12.24	9.486 9.486 9.486 9.23 51.26 9.92 18.13 9.15 31.06 9.34 19.23 174 55.29 1874 28.67	00 41.70 68 57.04 16 5.916 40 14.14 84 47.42 44 34.71 54 46.19 92 11.22 05 36.79 i. Merg	19 13 15.168 38 29.308 51 22.758 21 14.177 36 19.238 31 6.403 52 18.058 50 18.138 67 8.660	8 7 15.90 6 36.27 4 27.91 4 8.831 11.40 5 9.643 4 32.01 3 7.810	15.90 60 67 51.62 06 43.05 8 10.63 18 23.60 17 12.24 47.18 22 20.68	87 2 60 3 36 81 01 4 08 2 74 4 05 82 3	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301 3.6056)	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995
17.52 10.39 26.01 13.60 25.57 12.24	9.486 9.486 9.486 9.23 51.26 9.92 18.13 9.15 31.06 9.34 19.23 174 55.29 1874 28.67	00 41.70 68 57.04 16 5.916 40 14.14 84 47.42 44 34.71 54 46.19 92 11.22 05 36.79 i. Merg	19 13 15.168 38 29.308 51 22.758 21 14.177 36 19.238 31 6.403 52 18.058 550 18.138 667 8.660 e minim	8 7 15.90 6 36.27 4 27.91 4 8.831 1 11.40 5 9.643 4 32.01 3 7.810 LIM dist	15.90 60 67 51.62 06 43.05 8 10.63 18 23.60 17 12.24 47.18 22 20.68	87 2 60 3 36 81 01 4 08 2 74 4 05 82 3	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716 dist = 3	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301 3.6056)	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995
17.52 10.39 26.01 13.60 25.57 12.24 15.93	9.486 9.486 9.23 51.26 9.92 18.13 31.06 334 19.23 174 55.29 174 28.67	00 41.70 68 57.04 16 5.916 40 14.14 84 47.42 44 34.71 55 36.79 6 In 6 Cl 4.0908 23	19 13 15.165 38 29.308 51 22.759 21 14.177 36 19.235 31 6.403 52 18.055 50 18.138 67 8.660 e minim tial Clusters: uster Distance 3.4094 8.	8 7 15.90 6 36.27 4 27.91 4 8.831 1 11.40 5 9.643 4 32.01 3 7.810 LIM dist	15.90 60 67 51.62 06 43.05 8 10.63 18 23.60 17 12.24 56 47.18 12 20.68 ance clu	87 2 60 3 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570 10, 12 (14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716 dist = 3 10: {1	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301 3.6056) 0,12}	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926 3.6056	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767 9.8995	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767 26.6458	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995 26.6458
17.52 10.39 26.01 13.60 25.57 12.24 15.93	9.486 214 59.89 223 51.26 9.92 18.13 31.06 334 19.23 174 55.29 174 28.67 115 4.0908	00 41.70 68 57.04 16 5.916 40 14.14 84 47.42 44 34.71 54 46.19 92 11.22 05 36.79 i. Merg • In • Cl 4.0908 23	19 13 15.165 38 29.308 51 22.755 21 14.177 36 19.235 31 6.403 52 18.055 50 18.138 67 8.660 e minim tial Clusters: uster Distance 3.4094 8. 5.3299 37	8 7 15.90 6 36.27 4 27.91 4 8.831 1 11.40 5 5 9.643 4 32.01 3 7.810 tim dist	15.90 60 67 51.62 66 43.05 88 10.63 18 23.60 17 12.24 56 47.18 12 20.68 ance clu 1.7945 36 5.0400 9	87 2 60 3 36 81 01 4 08 2 74 4 05 82 3 Sters:	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570 10, 12 (14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716 dist = 3 10: {1	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301 3.6056) 0,12}	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926 3.6056 4 13.6018 4 31.0644	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767 9.8995	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767 26.6458	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995 26.6458
17.52 10.39 26.01 13.60 25.57 12.24 15.93	9.486 214 59.89 223 51.26 92 18.13 31.06 334 19.23 174 55.29 174 28.67 115 4.0908 3.4094 63	00 41.70 68 57.04 16 5.916 40 14.14 84 47.42 44 34.71 54 46.19 92 11.22 05 36.79 i. Merg	19 13 15.165 38 29.308 51 22.755 21 14.177 36 19.235 31 6.403 52 18.055 50 18.138 67 8.660 e minim tial Clusters: uster Distance 3.4094 8. 5.3299 37 28	8 7 15.90 6 36.27 4 27.91 4 8.831 1 11.40 5 9.643 4 32.01 3 7.810 LIM dist. Matrix: **Matrix:** **Matrix:* **Matrix:** **Matrix:** **Matrix:** **Matrix:** **Matrix:*	15.90 60 67 51.62 66 43.05 88 10.63 18 23.60 17 12.24 56 47.18 12 20.68 ance clu 1.7945 36 5.0400 9 1.7013 55	87 2 60 3 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5 5	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570 10, 12 (17.5214 59.8916 5.9161	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716 dist = 3 10: {1 4 10.392 6 51.264 14.142	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301 3.6056) 0,12} 23 25.573 10 18.138 21 46.195	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926 3.6056 4 13.6018 4 31.0644 2 34.7131	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767 9.8995 5 12.:	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767 26.6458	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995 26.6458
17.52 10.39 26.01 13.60 25.57 12.24 15.93	9.486 214 59.89 223 51.26 92 18.13 31.06 334 19.23 474 55.29 474 28.67 11 4.0908 3.4094 63 3.5440 3	00 41.70 68 57.04 16 5.916 40 14.14 84 47.42 44 34.71 54 46.19 92 11.22 05 36.79 i. Merg	19 13	8 7 15.90 6 36.27 4 27.91 4 8.831 1 11.40 5 9.643 4 32.01 3 7.810 LIM dist. Matrix: 5440 2: 2022 2: 3019 4: 18	15.90 60 67 51.62 66 43.05 8 10.63 18 23.60 47.18 20.68 ance clu 1.7945 6.0400 9 1.7013 57 6.1658 28	87 2 60 3 36 81 01 4 08 2 74 4 05 82 3 Sters:	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570 10, 12 (17.5214 59.8916 5.9161 22.7596	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716 dist = 3 10: {1 4 10.392 6 51.264 14.142 6 14.177	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301 8.6056) 0,12} 23 25.573 10 18.138 21 46.195 24 18.055	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926 3.6056 4 13.6018 4 31.0644 2 34.7131 5 6.4031	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767 9.8995 5 12.5 1 1.5 1 1.5 1 1.5	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767 26.6458 2474 15.9 2992 28.6 2250 36.7 1384 8.6	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995 26.6458
17.52 10.39 26.01 13.60 25.57 12.24 15.93	9.486 9.486 9.486 9.486 9.23 9.123 9.126 9.23 9.126 9.24 9.23 9.24 9.23 9.24 9.28 9.27 11 4.0908 9.486	00 41.70 68 57.04 16 5.916 40 14.14 47.42 44 34.71 54 46.19 92 11.22 05 36.79 i. Merg • In • Cl 4.0908 2: 5.3299 7.2022 28 5.0400 42	19 13	8 7 15.90 6 36.27 4 27.91 4 8.831 1 11.40 5 9.643 4 32.01 3 7.810 1 1 55440 2: 2.2022 2: 3.3019 4: 1.658	15.90 60 67 51.62 66 43.05 8 10.63 18 23.60 47.18 20.68 ance clu 1.7945 6.0400 9 1.7013 57 6.1658 28	87 2 60 3 8 8 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570 10, 12 (17.5214 59.8916 5.9161	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716 dist = 3 10: {1 1	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301 8.6056) 0, 12} 23 25.573- 40 18.138- 21 46.195: 74 18.055: 76 8.8318	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926 3.6056 4 13.6018 4 31.0644 2 34.7131 5 6.4031 11.4018	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767 9.8995 6 12.1 4 55.1 18.1 18.1 18.1	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767 26.6458	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995 26.6458
17.52 10.39 26.01 13.60 25.57 12.24 15.93	9.486 214 59.89 223 51.26 .92 18.13 .915 31.06 .34 19.23 .74 55.29 .74 28.67 .11 .44.0908 .3.4094 63 .3.4094 63 .3.5440 33 .1.7945 22 .6.2491 9	00 41.70 68 57.04 16 5.916 40 14.14 47.42 44 34.71 54 46.19 92 11.22 05 36.79 i. Merg 4.0908 2: 5.3299 7.2022 2: 5.0400 4: 0.4868 5	19 13	8 7 15.90 6 36.27 4 27.91 4 8.81 6 3 7.810 1 3 7.810 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1	15.90 60 67 51.62 8 10.63 18 23.60 17 12.24 56 47.18 12 20.68 ance clu 1.7945 6.0400 9 1.7013 57 6.1658 22 6.9060	87 2 60 3 8 8 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570 10, 12 (17.5214 59.8916 5.9161 22.7596 36.2767	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716 dist = 3 10: {1 1	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301 3.6056) 0, 12} 23 25.573 40 18.138 21 46.195 44 18.055 46 8.8318 31 10.630	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926 3.6056 4 13.6015 4 31.0644 2 34.7131 5 6.4031 6 11.4018 1 23.6008	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767 9.8995 5 12.5 1 55.5 1 1.5 1 8.5 3 32.4	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767 26.6458 2474 15.9 2992 28.6 2250 36.7 1384 8.6 0156 7.8 1805 20.6	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995 26.6458
17.52 10.39 26.01 13.60 25.57 12.24 15.93	191 9.486 191 59.89 1923 51.26 192 18.13 105 31.06 134 19.23 174 55.29 174 28.67 117 4.0908 3.4094 63 3.4094 63 3.5440 3 1.7945 21 6.2491 9	00 41.70 68 57.04 16 5.916 40 14.14 84 47.42 44 34.71 54 46.19 92 11.22 05 36.79 i. Merg 4.0908 2: 5.3299 7.2022 28 5.0400 4: 4.4868 5	19 13 15.165 38 29.308 31 22.759 21 14.177 36 19.235 31 6.403 55 18.138 67 8.660 e minim tital Clusters: uster Distance 3.4094 8. 5.3299 37 28 3.3019 1.7013 15 7.0438 29	8	15.90 60 67 51.62 66 43.05 8 10.63 18 23.60 17 12.24 56 47.18 12 20.68 ance clu 1.7945 6.0400 9 1.7013 5.1658 23 6.1658 25 6.9060 6.2767 51	87 2 60 3 6 8 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570 10, 12 (17.5214 59.8916 5.9161 22.7596 36.2763 51.6236	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716 dist = 3 10: {1 1 10.392 6 51.264 14.142 6 14.177 7 27.916 6 43.058 8.660	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301 3.6056) 0, 12} 23 25.573 40 18.138 21 46.195 24 18.055 26 8.8318 31 10.630 3 40.804	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926 3.6056 4 13.6018 4 31.0644 2 34.7131 5 6.4031 8 11.4018 1 23.6008 4 29.1548	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767 9.8995 5 12.5 11.5 18.5 18.5 3 32.6 3 47.5 3 5.3	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767 26.6458 2474 15.9 2992 28.6 22250 36.7 1384 8.6 0156 7.8 1805 20.6	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995 26.6458
17.52 10.39 26.01 13.60 25.57 12.24 15.93	191 9.486 191 59.89 123 51.26 192 18.13 105 31.06 134 19.23 174 55.29 174 28.67 115 4.0908 3.4094 63 3.5440 3 1.7945 25 6.2491 9 7.5214 55 0.3923 5	00 41.70 68 57.04 16 5.916 40 14.14 84 47.42 44 34.71 54 46.19 92 11.22 05 36.79 i. Merg 4.0908 2: 63 5.3299 7.2022 2: 5.0400 4: 6.4868 55 9.8916 5 1.2640 14	19 13	8	15.90 60 67 51.62 66 43.05 8 10.63 18 23.60 17 12.24 56 47.18 12 20.68 ance clu 1.7945 36 5.0400 9 1.7013 57 5.1658 29 5.9060 6.2767 51 7.9106 43	87 2 60 3 8 8 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570 10, 12 (17.5214 59.8916 5.9161 22.7596 36.2767	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716 dist = 3 10: {1 1 10.392 6 51.264 14.142 6 14.177 7 27.910 6 43.058 8.660	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301 3.6056) .0, 12} 23 25.573 10 18.1388 11 46.1955 14 18.055 16 8.8318 10.630 3 40.804 32.1555	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926 3.6056 4 13.6018 4 31.0644 2 34.7131 5 6.4031 8 11.4018 1 23.6008 4 29.1548	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767 9.8995 5 12.5 1 1.5 1 18.5 3 2.6 3 47.5 3 5.3 5 5.0	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767 26.6458 2474 15.9 2992 28.6 2250 36.7 1384 8.6 0156 7.8 1805 20.6	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995 26.6458 3374 4705 967 603 102 8882
17.52 10.39 26.01 13.60 25.57 12.24 15.93 4 2.88 2.3 3.1	191 9.486 191 59.89 123 51.26 192 18.13 105 31.06 134 19.23 174 55.29 174 28.67 115 4.0908 3.4094 63 3.5440 3 1.7945 23 6.2491 9 7.5214 58 0.3923 55 5.5734 18	00 41.70 68 57.04 16 5.916 40 14.14 84 47.42 44 34.71 55 36.79 i. Merg • In • Cl 4.0908 2: 5.3299 7.2022 2: 5.0400 4: 0.4868 5: 9.8916 5 1.2640 14 8.1384 46	19 13	8	15.90 60 67 51.62 66 43.05 8 10.63 18 23.60 17 12.24 56 47.18 12 20.68 ance clu 1.7945 36 5.0400 9 1.7013 57 5.1658 28 5.9060 63.2767 51 7.9106 43 8318 10	87 2 60 3 8 8 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9 1 9	22.7596 36.2767 51.6236 8.6603 41.8808 29.1548 40.8044 5.3852 31.2570 10, 12 (17.5214 59.8916 59.9161 22.7596 36.2767 51.6236	14.1774 27.9106 43.0581 8.6603 33.3017 20.6155 32.1559 5.0990 22.6716 dist = 3 10: {1 4 10.3926 5 51.264 14.1426 6 14.177 7 27.910 6 43.058 8.660	19.2354 8.8318 10.6301 41.8808 33.3017 13.1909 3.6056 37.2156 10.6301 3.6056) 0, 12} 23 25.573 10 18.138 21 46.195 24 18.055 26 8.8318 31 10.630 3 40.804 32.155	6.4031 11.4018 23.6008 29.1548 20.6155 13.1909 12.1244 24.3926 3.6056 4 13.6018 4 31.0644 2 34.7131 5 6.4031 1 1.4018 1 23.6008 4 29.1548 9 20.6158 12.1244	18.0555 9.6437 12.2474 40.8044 32.1559 3.6056 12.1244 36.2767 9.8995 4 55.3 11.3 18.3 2.6 47.3 3 5.0 4 36.3	18.1384 32.0156 47.1805 5.3852 5.0990 37.2156 24.3926 36.2767 26.6458 2474 15.9 2992 28.6 2250 36.7 1384 8.6 0156 7.8 1805 20.6	7.8102 20.6882 31.2570 22.6716 10.6301 3.6056 9.8995 26.6458 3374 4705 9967 603 102 1882 1570 1716 1995

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26.6458

26.6458

 12.2474
 55.2992
 11.2250
 18.1384
 32.0156
 47.1805
 5.3852
 5.0990
 36.2767
 24.3926

 15.9374
 28.6705
 36.7967
 8.6603
 7.8102
 20.6882
 31.2570
 22.6716
 9.8995
 3.6056

iv. Merge minimum distance clusters: 11, 14 (dist = 3.6056)

Initial Clusters:

11: {11,14}

• Cluster Distance Matrix:

	44.0908	23.4094	8.5440	21.7945	36.2491	17.5214	10.3923	25.5734	13.6015	12.2474
44.0908		65.3299	37.2022	25.0400	9.4868	59.8916	51.2640	18.1384	28.6705	55.2992
23.4094	65.3299		28.3019	41.7013	57.0438	5.9161	14.1421	46.1952	34.7131	11.2250
8.5440	37.2022	28.3019		15.1658	29.3087	22.7596	14.1774	18.0555	6.4031	18.1384
21.7945	25.0400	41.7013	15.1658		15.9060	36.2767	27.9106	8.8318	7.8102	32.0156
36.2491	9.4868	57.0438	29.3087	15.9060		51.6236	43.0581	10.6301	20.6882	47.1805
17.5214	59.8916	5.9161	22.7596	36.2767	51.6236		8.6603	40.8044	29.1548	5.3852
10.3923	51.2640	14.1421	14.1774	27.9106	43.0581	8.6603		32.1559	20.6155	5.0990
25.5734	18.1384	46.1952	18.0555	8.8318	10.6301	40.8044	32.1559		9.8995	36.2767
13.6015	28.6705	34.7131	6.4031	7.8102	20.6882	29.1548	20.6155	9.8995		24.3926
12.2474	55.2992	11.2250	18.1384	32.0156	47.1805	5.3852	5.0990	36.2767	24.3926	

v. Merge minimum distance clusters: 9, 13 (dist = 5.099)

Initial Clusters:

 $9: \{9, 13\}$

Cluster Distance Matrix:

	44.0908	23.4094	8.5440	21.7945	36.2491	17.5214	10.3923	25.5734	13.6015
44.0908		65.3299	37.2022	25.0400	9.4868	59.8916	51.2640	18.1384	28.6705
23.4094	65.3299		28.3019	41.7013	57.0438	5.9161	11.2250	46.1952	34.7131
8.5440	37.2022	28.3019		15.1658	29.3087	22.7596	14.1774	18.0555	6.4031
21.7945	25.0400	41.7013	15.1658		15.9060	36.2767	27.9106	8.8318	7.8102
36.2491	9.4868	57.0438	29.3087	15.9060		51.6236	43.0581	10.6301	20.6882
17.5214	59.8916	5.9161	22.7596	36.2767	51.6236		5.3852	40.8044	29.1548
10.3923	51.2640	11.2250	14.1774	27.9106	43.0581	5.3852		32.1559	20.6155
25.5734	18.1384	46.1952	18.0555	8.8318	10.6301	40.8044	32.1559		9.8995
13.6015	28.6705	34.7131	6.4031	7.8102	20.6882	29.1548	20.6155	9.8995	

vi. Merge minimum distance clusters: 8, 9 (dist = 5.3852)

Initial Clusters:

8: {8,9}

•	Cluster Dista	nce Matrix:						
	44.0908	23.4094	8.5440	21.7945	36.2491	10.3923	25.5734	13.6015
44.0908		65.3299	37.2022	25.0400	9.4868	51.2640	18.1384	28.6705
23.4094	65.3299		28.3019	41.7013	57.0438	5.9161	46.1952	34.7131
8.5440	37.2022	28.3019		15.1658	29.3087	14.1774	18.0555	6.4031
21.7945	25.0400	41.7013	15.1658		15.9060	27.9106	8.8318	7.8102
36.2491	9.4868	57.0438	29.3087	15.9060		43.0581	10.6301	20.6882
10.3923	51.2640	5.9161	14.1774	27.9106	43.0581		32.1559	20.6155
25.5734	18.1384	46.1952	18.0555	8.8318	10.6301	32.1559		9.8995
13.6015	28.6705	34.7131	6.4031	7.8102	20.6882	20.6155	9.8995	

vii. Merge minimum distance clusters: 3, 8 (dist = 5.9161)

Initial Clusters:

 $3: \{3, 8\}$

 Clu 	ster Distance I	Matrix:					
	44.0908	10.3923	8.5440	21.7945	36.2491	25.5734	13.6015
44.0908		51.2640	37.2022	25.0400	9.4868	18.1384	28.6705
10.3923	51.2640		14.1774	27.9106	43.0581	32.1559	20.6155
8.5440	37.2022	14.1774		15.1658	29.3087	18.0555	6.4031
21.7945	25.0400	27.9106	15.1658		15.9060	8.8318	7.8102
36.2491	9.4868	43.0581	29.3087	15.9060		10.6301	20.6882
25.5734	18.1384	32.1559	18.0555	8.8318	10.6301		9.8995
13.6015	28.6705	20.6155	6.4031	7.8102	20.6882	9.8995	

viii. Merge minimum distance clusters: 4, 11 (dist = 6.4031)

Initial Clusters:

 $4: \{4,11\}$

Cluster I	Distance Matrix	X:				
	44.0908	10.3923	8.5440	21.7945	36.2491	25.5734
44.0908		51.2640	28.6705	25.0400	9.4868	18.1384
10.3923	51.2640		14.1774	27.9106	43.0581	32.1559
8.5440	28.6705	14.1774		7.8102	20.6882	9.8995
21.7945	25.0400	27.9106	7.8102		15.9060	8.8318
36.2491	9.4868	43.0581	20.6882	15.9060		10.6301
25.5734	18.1384	32.1559	9.8995	8.8318	10.6301	

ix. Merge minimum distance clusters: 4, 5 (dist = 7.8102)

Initial Clusters:

 $4: \{4,5\}$

Cluster Distan	ce Matrix:				
	44.0908	10.3923	8.5440	36.2491	25.5734
44.0908		51.2640	25.0400	9.4868	18.1384
10.3923	51.2640		14.1774	43.0581	32.1559
8.5440	25.0400	14.1774		15.9060	8.8318
36.2491	9.4868	43.0581	15.9060		10.6301
25.5734	18.1384	32.1559	8.8318	10.6301	

x. Merge minimum distance clusters: 1, 4 (dist = 8.5440)

Initial Clusters:

 $1: \{1,4\}$

• Cluster Distance Matrix: 25.040010.3923 15.9060 8.8318 9.4868 25.040018.138451.2640 $10.3923 51.2640$ 43.058132.1559 15.9060 9.486843.0581 10.6301 8.8318 18.1384 32.1559 10.6301

xi. Merge minimum distance clusters: 1, 10 (dist = 8.8318)

Initial Clusters:

 $1: \{1, 10\}$

Cluster Distance Matrix:

	18.1384	10.3923	10.6301
18.1384		51.2640	9.4868
10.3923	51.2640		43.0581

 $10.6301 \quad 9.4868 \quad 43.0581$

xii. Merge minimum distance clusters: 2, 6 (dist = 9.4868)

Initial Clusters:

 $2: \{2, 6\}$

• Cluster Distance Matrix:

 $\begin{array}{ccc} & 10.6301 & 10.3923 \\ 10.6301 & & 43.0581 \\ 10.3923 & 43.0581 & \end{array}$

xiii. Merge minimum distance clusters: 1, 3 (dist = 10.3923)

Initial Clusters:

 $1:\quad \{1,3\}$

Cluster Distance Matrix: 10.6301
 10.6301

xiv. Merge minimum distance clusters: 1, 2 (dist = 10.6301)

• Initial Clusters:

 $1: \{1,2\}$

b) Repeat the clustering using complete (maximum) linkage. Again, make sure you indicate the linkage value for every cluster merge.

To perform hierarchical clustering with complete linkage we first have every point form its own cluster and compute the distance function between all the clusters. For simplicity we will index each data item and use it as a cluster number and when merging use the smallest index in the cluster as the index of the resulting cluster. Also, when merging only the distances from the new cluster have to be recomputed as the maximum of the distance to any of the merged clusters.

i. Initial clusters and distance function:

• Initial Clusters:

 $\{(170, 57, 32)\}$ 2: $\{(192, 95, 28)\}$ 3: $\{(150, 45, 30)\}$ {(170, 65, 29)} $\{(175, 78, 35)\}$ 5: $\{(185, 90, 32)\}$ {(170, 65, 28)} 7: 8: $\{(155, 48, 31)\}$ {(160, 55, 30)} $10: \quad \{(182, 80, 30)\}$ $11: \{(175, 69, 28)\}$ $12: \{(180, 80, 27)\}$ $13: \{(160, 50, 31)\}$ $14: \{(175, 72, 30)\}$

• Cluster Distance Matrix:

	44.0908	23.4094	8.5440	21.7945	36.2491	8.9443	17.5214	10.3923	26.0192	13.6015	25.5734	12.2474	15.9374
44.0908		65.3299	37.2156	25.0400	9.4868	37.2022	59.8916	51.2640	18.1384	31.0644	19.2354	55.2992	28.6705
23.4094	65.3299		28.3019	41.7013	57.0438	28.3549	5.9161	14.1421	47.4236	34.7131	46.1952	11.2250	36.7967
8.5440	37.2156	28.3019		15.1658	29.3087	1.0000	22.7596	14.1774	19.2354	6.4807	18.1384	18.1384	8.6603
21.7945	25.0400	41.7013	15.1658		15.9060	15.5885	36.2767	27.9106	8.8318	11.4018	9.6437	32.0156	7.8102
36.2491	9.4868	57.0438	29.3087	15.9060		29.4279	51.6236	43.0581	10.6301	23.6008	12.2474	47.1805	20.6882
8.9443	37.2022	28.3549	1.0000	15.5885	29.4279		22.8692	14.2829	19.3132	6.4031	18.0555	18.2757	8.8318
17.5214	59.8916	5.9161	22.7596	36.2767	51.6236	22.8692		8.6603	41.8808	29.1548	40.8044	5.3852	31.2570
10.3923	51.2640	14.1421	14.1774	27.9106	43.0581	14.2829	8.6603		33.3017	20.6155	32.1559	5.0990	22.6716
26.0192	18.1384	47.4236	19.2354	8.8318	10.6301	19.3132	41.8808	33.3017		13.1909	3.6056	37.2156	10.6301
13.6015	31.0644	34.7131	6.4807	11.4018	23.6008	6.4031	29.1548	20.6155	13.1909		12.1244	24.3926	3.6056
25.5734	19.2354	46.1952	18.1384	9.6437	12.2474	18.0555	40.8044	32.1559	3.6056	12.1244		36.2767	9.8995
12.2474	55.2992	11.2250	18.1384	32.0156	47.1805	18.2757	5.3852	5.0990	37.2156	24.3926	36.2767		26.6458
15.9374	28.6705	36.7967	8.6603	7.8102	20.6882	8.8318	31.2570	22.6716	10.6301	3.6056	9.8995	26.6458	

ii. Merge minimum distance clusters: 4, 7 (dist = 1.0)

Initial Clusters:

 $4: \{(4,7)\}$

• Cluster Distance Matrix:

	44.0908	23.4094	8.9443	21.7945	36.2491	17.5214	10.3923	26.0192	13.6015	25.5734	12.2474	15.9374
44.0908		65.3299	37.2156	25.0400	9.4868	59.8916	51.2640	18.1384	31.0644	19.2354	55.2992	28.6705
23.4094	65.3299		28.3549	41.7013	57.0438	5.9161	14.1421	47.4236	34.7131	46.1952	11.2250	36.7967
8.9443	37.2156	28.3549		15.5885	29.4279	22.8692	14.2829	19.3132	6.4807	18.1384	18.2757	8.8318
21.7945	25.0400	41.7013	15.5885		15.9060	36.2767	27.9106	8.8318	11.4018	9.6437	32.0156	7.8102
36.2491	9.4868	57.0438	29.4279	15.9060		51.6236	43.0581	10.6301	23.6008	12.2474	47.1805	20.6882
17.5214	59.8916	5.9161	22.8692	36.2767	51.6236		8.6603	41.8808	29.1548	40.8044	5.3852	31.2570
10.3923	51.2640	14.1421	14.2829	27.9106	43.0581	8.6603		33.3017	20.6155	32.1559	5.0990	22.6716
26.0192	18.1384	47.4236	19.3132	8.8318	10.6301	41.8808	33.3017		13.1909	3.6056	37.2156	10.6301
13.6015	31.0644	34.7131	6.4807	11.4018	23.6008	29.1548	20.6155	13.1909		12.1244	24.3926	3.6056
25.5734	19.2354	46.1952	18.1384	9.6437	12.2474	40.8044	32.1559	3.6056	12.1244		36.2767	9.8995
12.2474	55.2992	11.2250	18.2757	32.0156	47.1805	5.3852	5.0990	37.2156	24.3926	36.2767		26.6458
15.9374	28.6705	36.7967	8.8318	7.8102	20.6882	31.2570	22.6716	10.6301	3.6056	9.8995	26.6458	

iii. Merge minimum distance clusters: 10, 12 (dist = 3.6056)

Initial Clusters:

 $10: \{(10,12)\}$

• Cluster Distance Matrix:

	44.0908	23.4094	8.9443	21.7945	36.2491	17.5214	10.3923	26.0192	13.6015	12.2474	15.9374
44.090	08	65.3299	37.2156	25.0400	9.4868	59.8916	51.2640	19.2354	31.0644	55.2992	28.6705
23.409	94 65.3299		28.3549	41.7013	57.0438	5.9161	14.1421	47.4236	34.7131	11.2250	36.7967
8.944	3 37.2156	28.3549		15.5885	29.4279	22.8692	14.2829	19.3132	6.4807	18.2757	8.8318
21.794	45 25.0400	41.7013	15.5885		15.9060	36.2767	27.9106	9.6437	11.4018	32.0156	7.8102
36.249	91 9.4868	57.0438	29.4279	15.9060		51.6236	43.0581	12.2474	23.6008	47.1805	20.6882
17.52	14 59.8916	5.9161	22.8692	36.2767	51.6236		8.6603	41.8808	29.1548	5.3852	31.2570
10.39	23 51.2640	14.1421	14.2829	27.9106	43.0581	8.6603		33.3017	20.6155	5.0990	22.6716
26.019	92 19.2354	47.4236	19.3132	9.6437	12.2474	41.8808	33.3017		13.1909	37.2156	10.6301
13.60	15 31.0644	34.7131	6.4807	11.4018	23.6008	29.1548	20.6155	13.1909		24.3926	3.6056
12.24'	74 55.2992	11.2250	18.2757	32.0156	47.1805	5.3852	5.0990	37.2156	24.3926		26.6458
15.93'	74 28.6705	36.7967	8.8318	7.8102	20.6882	31.2570	22.6716	10.6301	3.6056	26.6458	

iv. Merge minimum distance clusters: 11, 14 (dist = 3.6056)

Initial Clusters:

 $11: \{(11,14)\}$

Cluster Distance Matrix:

	44.0908	23.4094	8.9443	21.7945	36.2491	17.5214	10.3923	26.0192	15.9374	12.2474
44.0908		65.3299	37.2156	25.0400	9.4868	59.8916	51.2640	19.2354	31.0644	55.2992
23.4094	65.3299		28.3549	41.7013	57.0438	5.9161	14.1421	47.4236	36.7967	11.2250
8.9443	37.2156	28.3549		15.5885	29.4279	22.8692	14.2829	19.3132	8.8318	18.2757
21.7945	25.0400	41.7013	15.5885		15.9060	36.2767	27.9106	9.6437	11.4018	32.0156
36.2491	9.4868	57.0438	29.4279	15.9060		51.6236	43.0581	12.2474	23.6008	47.1805
17.5214	59.8916	5.9161	22.8692	36.2767	51.6236		8.6603	41.8808	31.2570	5.3852
10.3923	51.2640	14.1421	14.2829	27.9106	43.0581	8.6603		33.3017	22.6716	5.0990
26.0192	19.2354	47.4236	19.3132	9.6437	12.2474	41.8808	33.3017		13.1909	37.2156
15.9374	31.0644	36.7967	8.8318	11.4018	23.6008	31.2570	22.6716	13.1909		26.6458
12.2474	55.2992	11.2250	18.2757	32.0156	47.1805	5.3852	5.0990	37.2156	26.6458	

v. Merge minimum distance clusters: 9, 13 (dist = 5.0990)

Initial Clusters:

 $9: \{(9,13)\}$

• Cluster Distance Matrix:

	44.0908	23.4094	8.9443	21.7945	36.2491	17.5214	12.2474	26.0192	15.9374
44.0908		65.3299	37.2156	25.0400	9.4868	59.8916	55.2992	19.2354	31.0644
23.4094	65.3299		28.3549	41.7013	57.0438	5.9161	14.1421	47.4236	36.7967
8.9443	37.2156	28.3549		15.5885	29.4279	22.8692	18.2757	19.3132	8.8318
21.7945	25.0400	41.7013	15.5885		15.9060	36.2767	32.0156	9.6437	11.4018
36.2491	9.4868	57.0438	29.4279	15.9060		51.6236	47.1805	12.2474	23.6008
17.5214	59.8916	5.9161	22.8692	36.2767	51.6236		8.6603	41.8808	31.2570
12.2474	55.2992	14.1421	18.2757	32.0156	47.1805	8.6603		37.2156	26.6458
26.0192	19.2354	47.4236	19.3132	9.6437	12.2474	41.8808	37.2156		13.1909
15.9374	31.0644	36.7967	8.8318	11.4018	23.6008	31.2570	26.6458	13.1909	

vi. Merge minimum distance clusters: 3, 8 (dist = 5.9161)

Initial Clusters:

 $3: \{(3,8)\}$

•	Cluster Dista	nce Matrix:						
	44.0908	23.4094	8.9443	21.7945	36.2491	12.2474	26.0192	15.9374
44.0908		65.3299	37.2156	25.0400	9.4868	55.2992	19.2354	31.0644
23.4094	65.3299		28.3549	41.7013	57.0438	14.1421	47.4236	36.7967
8.9443	37.2156	28.3549		15.5885	29.4279	18.2757	19.3132	8.8318
21.7945	25.0400	41.7013	15.5885		15.9060	32.0156	9.6437	11.4018
36.2491	9.4868	57.0438	29.4279	15.9060		47.1805	12.2474	23.6008
12.2474	55.2992	14.1421	18.2757	32.0156	47.1805		37.2156	26.6458
26.0192	19.2354	47.4236	19.3132	9.6437	12.2474	37.2156		13.1909
15.9374	31.0644	36.7967	8.8318	11.4018	23.6008	26.6458	13.1909	

vii. Merge minimum distance clusters: 4, 11 (dist = 8.8318)

Initial Clusters:

 $4: \{(4,11)\}$

Cluster Distance Matrix:								
	44.0908	23.4094	15.9374	21.7945	36.2491	12.2474	26.0192	
44.0908		65.3299	37.2156	25.0400	9.4868	55.2992	19.2354	
23.4094	65.3299		36.7967	41.7013	57.0438	14.1421	47.4236	
15.9374	37.2156	36.7967		15.5885	29.4279	26.6458	19.3132	
21.7945	25.0400	41.7013	15.5885		15.9060	32.0156	9.6437	
36.2491	9.4868	57.0438	29.4279	15.9060		47.1805	12.2474	
12.2474	55.2992	14.1421	26.6458	32.0156	47.1805		37.2156	
26.0192	19.2354	47.4236	19.3132	9.6437	12.2474	37.2156		

viii. Merge minimum distance clusters: 2, 6 (dist = 9.4868)

Initial Clusters:

 $2: \{(2,6)\}$

 Cluster I 	Distance Matrix	x:				
	44.0908	23.4094	15.9374	21.7945	12.2474	26.0192
44.0908		65.3299	37.2156	25.0400	55.2992	19.2354
23.4094	65.3299		36.7967	41.7013	14.1421	47.4236
15.9374	37.2156	36.7967		15.5885	26.6458	19.3132
21.7945	25.0400	41.7013	15.5885		32.0156	9.6437
12.2474	55.2992	14.1421	26.6458	32.0156		37.2156
26.0192	19.2354	47.4236	19.3132	9.6437	37.2156	

ix. Merge minimum distance clusters: 5, 10 (dist = 9.6437)

Initial Clusters:

 $5: \{(5,10)\}$

•	Cluster Distar	nce Matrix:				
		44.0908	23.4094	15.9374	26.0192	12.2474
	44.0908		65.3299	37.2156	25.0400	55.2992
	23.4094	65.3299		36.7967	47.4236	14.1421
	15.9374	37.2156	36.7967		19.3132	26.6458
	26.0192	25.0400	47.4236	19.3132		37.2156
	12.2474	55.2992	14.1421	26.6458	37.2156	

x. Merge minimum distance clusters: 1, 9 (dist = 12.2474)

Initial Clusters:

 $1: \{(1,9)\}$

Cluster Distance Matrix:

ci Distance ivi	iutia.			
	55.2992	23.4094	26.6458	37.2156
55.2992		65.3299	37.2156	25.0400
23.4094	65.3299		36.7967	47.4236
26.6458	37.2156	36.7967		19.3132
37.2156	25.0400	47.4236	19.3132	

xi. Merge minimum distance clusters: 4, 5 (dist = 19.3132)

Initial Clusters:

 $4: \{(4,5)\}$

Cluster Distance Matrix:

 55.2992
 23.4094
 37.2156

 55.2992
 65.3299
 37.2156

 23.4094
 65.3299
 47.4236

 37.2156
 47.4236

xii. Merge minimum distance clusters: 1, 3 (dist = 23.4094)

Initial Clusters:

 $1: \{(1,3)\}$

Cluster Distance Matrix:

 $\begin{array}{ccc} & 65.3299 & 47.4236 \\ 65.3299 & 37.2156 \end{array}$

 $47.4236 \quad \ \ 37.2156$

xiii. Merge minimum distance clusters: 2, 4 (dist = 37.2156)

• Initial Clusters:

 $2: \{(2,4)\}$

• Cluster Distance Matrix:

65.3299

65.3299

xiv. Merge minimum distance clusters: 1, 2 (dist = 65.3299)

Initial Clusters:

 $1: \{(1,3)\}$

Self-Training

1. Consider the following linearly separable training data set:

```
D_s = \{ (170, 57, 32), W \},
                        ((190, 95, 28), M),
                        ((150, 45, 35), W),
                        ((168, 65, 29), M),
                        ((175, 78, 26), M),
                        ((185, 90, 32), M),
                        ((171, 65, 28), W),
                        ((155, 48, 31), W),
                        ((165, 60, 27), W)
                                               }
D_u = \{ (182, 80, 30), 
                                               (178, 80, 27),
                             (175, 69, 28),
          (160, 50, 31),
                             (170, 72, 30),
                                               (152, 45, 29),
          (177, 79, 28),
                             (171, 62, 27),
                                               (185, 90, 30),
          (181, 83, 28),
                             (168, 59, 24),
                                               (158, 45, 28),
          (178, 82, 28),
                             (165, 55, 30),
                                               (162, 58, 28),
          (180, 80, 29),
                             (173, 75, 28),
                                               (172, 65, 27),
                                               (182, 84, 27),
          (160, 51, 29),
                             (178, 77, 28),
          (175, 67, 28),
                             (163, 50, 27),
                                               (177, 80, 30),
          (170,65,28)
```

- a) Implement a self-training system using a logistic regression classifier for this problem.
- b) Learn a classifier using the semi-supervised learning algorithm and compare it agains a classifier learned only from the labeled data D_s using the following test set:

```
D_{t} = \{ ((169, 58, 30), W), \\ ((185, 90, 29), M), \\ ((148, 40, 31), W), \\ ((177, 80, 29), M), \\ ((170, 62, 27), W), \\ ((172, 72, 30), M), \\ ((175, 68, 27), W), \\ ((178, 80, 29), M) \}
```