State and Parameter Estimation for Application in Model Predictive Control of Buildings

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Contents

1 Abstract

With the need to make buildings more energy efficient through intelligent control algorithms arises another need for efficient and simple joint state-parameter estimation methods so that we come up with simple yet accurate building thermal dynamics model. We explore the State Augmentation method with various nonlinear Bayesian Filters/Smoothers for this task. The building data is generated from a known fourth-order thermal RC network model.To accelerate the development and implementation of a building MPC we have developed and tested a holistic framework for state estimation in the form of a Python package titled *bayesian_filters_smoothers* which includes implementation of Kalman Filter-Smoother (KFS), Extended Kalman Filter-Smoother (EKFS), Unscented Kalman Filter-Smoother (UKFS), and Gaussian Filter-Smoother (GFS). Moreover, we have developed and tested three parameter estimation techniques of Bayesian Augmentation, Least Squares (LS) Estimation, Batch Estimation, and Maximum Likelihood Estimation (MLE) under the supervision and guidance of Mr. Ninad Kiran Gaikwad. Ninad is a 3rd year PhD student in the EECS department at WSU working as a research assistant at the SCALE Lab. The three optimisation techniques are based on solving a constrained nonlinear programming problem.

2 Introduction

Buildings are the biggest consumers of electrical energy in the USA - according to the US Energy Information Administration (EIA), buildings account for 40 % of the total electricity consumed during the year 2020; out of which commercial and residential buildings account for 18 % and 22 % electricity consumption respectively. As a result, buildings become a potential asset for energy usage optimization from the point of view of electric grids. However, energy usage optimization is not the only problem that buildings lead to, but the coupled problems of providing Quality of Service (QoS) to the occupants and supporting the grid with ancillary services also need to be dealt with for complete utilization of buildings as an asset to the electric grid.

In order to effectively control the building operation so as to maximize occupant comfort, minimize energy consumption, and be of some utility to the grid through providing ancillary services, we need advanced control strategies. Model Predictive Control (MPC) is the most studied and implemented advanced control strategy for building control. It is an approximate version (computationally tractable) of optimal control with the flexibility of incorporating numerous varied constraints and cost function formulations, which presents itself as a discretized mathematical program. Implementation of a successful MPC is not stand alone; however, it depends on the accurate state estimation of the hidden system states (inference of the hidden states of the system model from observed system data) for initializing the system equations within the MPC formulation, and on the accuracy of the mathematical system model incorporated within the MPC as dynamic constraints for which we need robust parameter estimation methods to infer system parameters from the observed system data.

In this work, we have considered grey-box-based modeling of the building thermal model where the model structure is an RC-Network-based ODE (Ordinary Differential Equation) formulation. Furthermore, for parameter estimation, we have developed methods based on the Bayesian Filters/Smoothers and Constrained Nonlinear Programming.

The rest of the report is organized as follows: Section-3 describes the grey-box building thermal models, 4 illustrates the formulation of the different parameter estimation methods, **??** illustrates the results and provides the discussions for the same, and 6 provides the conclusion to this report.

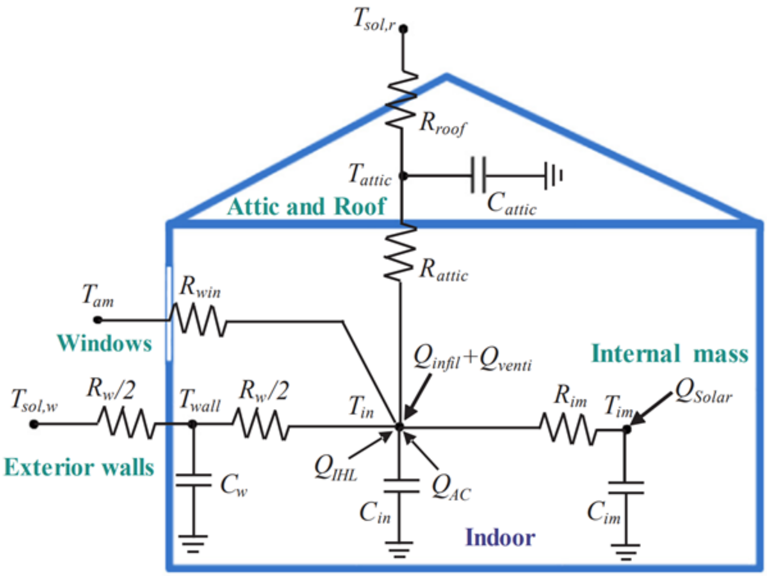

Figure 1: Four state model

3 Residential Building RC Network Models

3.1 4-State Single-Family House Model Development

[1] have come up with a four-state thermal model for a single-family house, which is given as follows;

$$\dot{T}_{ave}(t) = \frac{1}{Cin} * \frac{T_{wall}(t) - T_{ave}(t)}{R_w/2} - \frac{T_{am}(t) - T_{ave}(t)}{R_w in} \tag{1}$$

$$\dot{T}_{ave}(t) = \frac{T_{wall}(t) - T_{ave}(t)}{C_{in}R_w/2} + \frac{T_{attic}(t) - T_{ave}(t)}{C_{in}R_{attic}} + \frac{T_{im}(t) - T_{ave}(t)}{C_{in}R_{im}} + \frac{T_{am}(t) - T_{ave}(t)}{C_{in}R_{win}} + \tag{2}$$

$$C_1Q_{IHL} + C_2Q_{AC} + Q_{venti} + Q_{infil} \tag{3}$$

$$\dot{T}_{attic}(t) = \frac{T_{sol,r}(t) - T_{attic}(t)}{C_{attic}R_{roof}} - \frac{T_{attic}(t) - T_{ave}(t)}{C_{attic}R_{attic}} \tag{4}$$

$$\dot{T}_{im}(t) = \frac{T_{im}(t) - T_{ave}(t)}{C_{im}R_{im}} + C_3Q_{solar} \tag{5}$$

Where,
T_{wall} : Wall Surface Temperature
T_{ave} : House Average Air Temperature
T_{attic} : Attic Air Temperature
T_{im} : Internal Mass Temperature
C_w : Thermal Capacitance of Wall
C_{in} : Thermal Capacitance of Indoor Air

C_{attic} : Thermal Capacitance of Air in Attic
C_{im} : Thermal Capacitance of Internal Mass
R_w : Thermal Resistance of Walls
R_{attic} : Thermal Resistance of Attic
R_{roof} : Thermal Resistance of Roof
R_{im} : Thermal Resistance of Internal Mass
R_{win} : Thermal Resistance of Windows
Q_{IHL} : Internal Heat Gain from People and Equipment
Q_{AC} : Heating and Cooling gain from AC
Q_{venti} : Heat gain from Ventilation
Q_{infil} : Heat Gain from Infiltration
Q_{solar} : Heat Gain from Solar Radiation
C_1, C_2, C_3 : Heat Gain Fractions

3.2 2-State Single Family House Development

The above four-state model can be simplified further into a two state model:

$$\dot{T}_{ave}(t) = \frac{1}{Cin} * \frac{T_{wall}(t) - T_{ave}(t)}{R_w/2} - \frac{T_{am}(t) - T_{ave}(t)}{R_w in} + C_1 * Q_{ih} + C_2 * Q_{ac} + Q_{venti} + Q_i nfil$$
$$\dot{T}_{wall}(t) = \frac{1}{Cw} * \frac{T_{ave}(t) - T_{wall}(t)}{R_w/2} - \frac{T_{sol_w}(t) - T_{wall}(t)}{R_w/2} + C_3 * Q_{solar}$$

Where,
T_{wall} : Wall Surface Temperature
T_{ave} : House Average Air Temperature
T_{am} : Ambient Temperature
T_{attic} : Attic Air Temperature
C_w : Thermal Capacitance of Wall
R_w : Thermal Resistance of Walls
R_{attic} : Thermal Resistance of Attic
R_{win} : Thermal Resistance of Windows
Q_{AC} : Heating and Cooling gain from AC
Q_{venti} : Heat gain from Ventilation
Q_{infil} : Heat Gain from Infiltration
Q_{solar} : Heat Gain from Solar Radiation
C_1, C_2, C_3 : Heat Gain Fractions

3.3 1-State Single Family House Development

The above four-state model can be simplified further into a single state modelas well:

$$\dot{T}_{ave}(t) = \frac{1}{Cin} * \frac{T_{am}(t) - T_{ave}(t)}{R_{win}} + C_1 * Q_{ih} + C_2 * Q_{ac} + C_3 * Q_{solar} + Q_{venti} + Q_i nfil$$

Where,
T_{ave} : House Average Air Temperature
T_{am} : Ambient Temperature
R_{win} : Thermal Resistance of Windows
Q_{AC} : Heating and Cooling gain from AC
Q_{venti} : Heat gain from Ventilation
Q_{infil} : Heat Gain from Infiltration
Q_{solar} : Heat Gain from Solar Radiation
C_1, C_2, C_3 : Heat Gain Fractions

3.4 ODE Discretization Methods

Let us consider a linear ODE system as follows;

$$\dot{x} = Ax + Bu \ , \tag{6}$$
$$y = Cx \ , \tag{7}$$

where the state is x, the measurement/output is y, and the input u.

Now we can convert the above continuous time linear system into a discrete time linear system by applying Euler Discretization with a discrete time-step $\triangle t$ as follows;

$$x_k = x_{k-1} + \triangle t \left(A x_{k-1} + B u_{k-1} \right) \ , \tag{8}$$
$$y_k = C x_k \ . \tag{9}$$

Rearranging the above we get;

$$x_k = \left(I + \triangle t A \right) x_{k-1} + \left(\triangle t B \right) u_{k-1} \ , \tag{10}$$
$$y_k = C x_k \ . \tag{11}$$

We get the discrete time matrices as $A_d = I + \triangle t A$, $B_d = \triangle t B$, and $C_d = C$ and we can rewrite the discrete time linear system as;

$$x_k = A_d x_{k-1} + B_d u_{k-1} \ , \tag{12}$$
$$y_k = C_d x_k \ . \tag{13}$$

We can discretize any linear or nonlinear system with our choice of discretization method. Euler method is most commonly used and should be considered as the first option, while other advanced discretization schemes such as Runge-Kutta can be used if the results from Euler method are not satisfactory.

4 Bayesian Parameter Estimation Methods

In this section, we will describe the common methods for parameter estimation using the Bayesian Estimation Framework [2]

4.1 Bayesian Filter Estimation via State Augmentation

Let us consider a nonlinear process-measurement model with unknown parameter θ as follows;

$$\begin{aligned} x_k &= f_k(x_{k-1}, u_{k-1}, \theta) + q_{k-1} \ , \\ y_k &= h_k(x_k, \theta) + r_k \ . \end{aligned} \tag{14}$$

Now, the complete model with time-invariant parameter θ can be given as;

$$\begin{aligned} \theta_k &= \theta_{k-1} \ , \\ x_k &= f_k(x_{k-1}, u_{k-1}, \theta_{k-1}) + q_{k-1} \ , \\ y_k &= h_k(x_k, \theta_{k-1}) + r_k \ . \end{aligned} \tag{15}$$

Now, let us define the augmented state as $\tilde{x}_k \triangleq [x_k, \theta_k]^T$. We can now rewrite the model in terms of the augmented state as follows;

$$\begin{aligned} \tilde{x}_k &= \tilde{f}_k(\tilde{x}_{k-1}, u_{k-1}) + \tilde{q}_{k-1} \ , \\ y_k &= \tilde{h}_k(\tilde{x}_k) + r_k \ . \end{aligned} \tag{16}$$

Where we have $\tilde{q}_k \triangleq [q_k, \underline{0}]^T$, $\underline{0} \in \mathbb{R}^n$.

Now, the above model as suggested in [3] can be solved using any filtering algorithm described in the previous sections, and we will get recursive estimates of the state and parameters jointly. However, in the current form of the model there are the following disadvantages:

1. As no noise is present in the parameter equation i.e. the parameter is time-invariant, the dynamic model of the parameter is singular, which problems with non-linear filtering algorithms causing it to diverge.
2. We can use the above formulation of the model when the entire system is linear and the parameters also appear linearly in the system.

To avoid the above disadvantages for a model which is non-linear in both state and parameters, we introduce artificial noise in the parameter dynamic equation as follows;

$$\theta_k = \theta_{k-1} + q_{k-1_\theta} \ . \tag{17}$$

Now, the state augmented process/measurement model (16) with $\tilde{q}_k \triangleq [q_{k_x}, q_{k_\theta}]^T$, $q_{k_\theta} \in \mathbb{R}^n$, can be used with any of the appropriate filtering algorithms described in the previous section to provide for a more stable and robust implementation to compute recursively both the state and parameters of the model. However, even this modified problem with a time-variant parameter with a small noise process can be inapplicable but it is advised to start from this method due to its implementation simplicity before moving on to more complicated methods.

4.2 Nonlinear Optimization based Least Squares Estimation

Now the parameter estimation problem can be formulated as an optimization problem as follows;

$$\min_{\theta} \sum_{k=1}^{N} [y(k) - \tilde{y}(k)]^2 , \tag{18}$$

subject to the following constraints:

$$\tilde{x}(k+1) = \tilde{x}(k) + t_s [A(\theta)\tilde{x}(k) + B(\theta)u(k)] , \tag{19}$$
$$\tilde{y}(k) = C\tilde{x}(k) + Du(k), \tag{20}$$
$$\tilde{x}(k) \in \mathbb{X}, \tag{21}$$
$$\theta \in \Theta, \tag{22}$$

For the parameter estimation problem as given in [4] the cost function given in (18) minimizes the sum of squared error between the actual measured output y and output produced by the estimated system $\tilde{y}$. the equality constraint (19) is the discretized (Euler Forward) system state equation, while the equality constraint (20) is discretized system output equation. (21) and (22) constrains the states $\tilde{x}$ which are part of the decision variables of the optimization problem and the parameter vector θ respectively, to a physically meaningful set of values.

One other important point to note is that the above method doesn't consider the noise due to process error or noise due to measurement error. This method believes that the model used is perfect as well as the measurements it is being trained on are error-free, which is not the case and this becomes a very naive simplification of the problem at hand. In the next section, we will discuss techniques developed to handle the process and measurement noises; hence, the setup of the problem will be based in a probabilistic setting leading us to a more complete formulation of the problem.

4.3 Batch Estimation

The batch estimation method is formulated as a constrained nonlinear optimization problem that reflects a Maximum A Posteriori (MAP) Estimation technique and is given as follows;

$$\min_{X_1^e, w_1, \ldots, w_{N-1}} (X_1^e)^T P_{1|0}^{-1} X_1^e + \sum_{k=1}^{N} \left[v_k^T R^{-1} v_k \right] + \sum_{k=1}^{N-1} \left[w_k^T Q^{-1} w_k \right] , \tag{23}$$

subject to the following constraints:

$$v_k = y_k - h(x_k, \theta_k), \tag{24}$$
$$x_k = f(x_{k-1}, \theta_{k-1}, u_{k-1}) + w_{k-1}^x, \tag{25}$$
$$\theta_k = \theta_{k-1} + w_{k-1}^\theta. \tag{26}$$

Where, $X_1^e \coloneqq X_1 - X_{1|0}$, $X \coloneqq [x, \theta]^T$, and $w \coloneqq [w^x, w^\theta]^T$. Also, $w \sim N(0, Q)$ and $v \sim N(0, R)$.

4.4 Maximum Likelihood Estimation (MLE)

The MLE is developed as a nonlinear optimization problem that tries to minimize the one-step prediction errors produced by the appropriate Bayesian Filter. We illustrate the formulation of such an MLE method utilizing the Kalman Filter as follows;

$$\min_{\theta,e,S,\tilde{x},P} \sum_{k=0}^{N} \left[e_k^T S^{-1} e_k\right] + \log |S_k|, \tag{27}$$

subject to the following constraints:

$$S_k = C_k P_k C_k^T + R_k, \tag{28}$$

$$e_k = y_k - C_k \tilde{x_k}, \tag{29}$$

$$\tilde{x}_{k+1} = A_k(\theta)(\tilde{x_k} + P_k C_k^T S_k^{-1} e_k) + B(\theta) u_k, \tag{30}$$

$$P_{k+1} = A_k(\theta)(P_k - P_k C_k^T S_k^{-1} C_k P_k) A_k(\theta)^T + Q_k, \tag{31}$$

(32)

5 Results and Discussion

The results below shows the graphs that we got for generalised cases of the three algorithms to be more specific, the case of a single pendulum that we have implemented from [5] We have designed the entire structure of the code for all the three type of algorithms here at ***Code Base OneDrive Link*** but due to the time constraint we couldn't run it to generate results for specified model cases which we plan on doing in the future.
Figure 2,3 and 4 represents the graph that we got from the code for State Augmentation. Here in the topmost graph graph Angle (theta) and angular velocity (omega) are the two state variables of the pendulum system displayed on this graph. Plotting of the true states (theta true and omega true) and filtered estimates (theta filter and omega filter) is done. Estimating the true states from noisy measurements is the filter's aim. The fact that the filtered states closely resemble the genuine states indicates that the filter is working effectively. The gravitational parameter estimate is shown in the middle graph (parameter g). The filtered g (g filter) is a dotted line that shows little fluctuation from the actual value and looks to be a close estimate of the constant true value of g (g true). The figure at bottom represents the estimated pendulum length; the true value is constant (L true). The filtered estimate (L filter), which shows how well the UKF performed in estimating this parameter, stays constant and is relatively close to the genuine value.

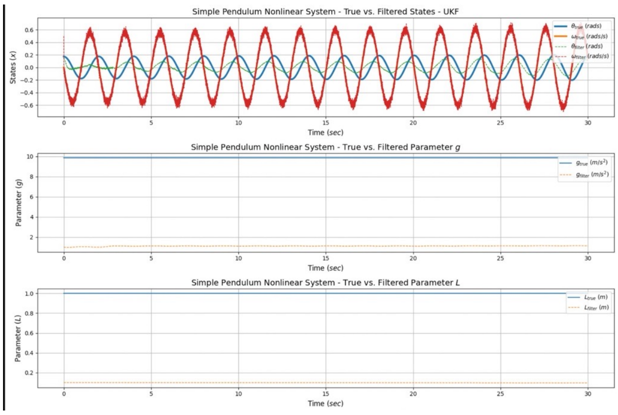

Figure 2: State Augmentation for UKF

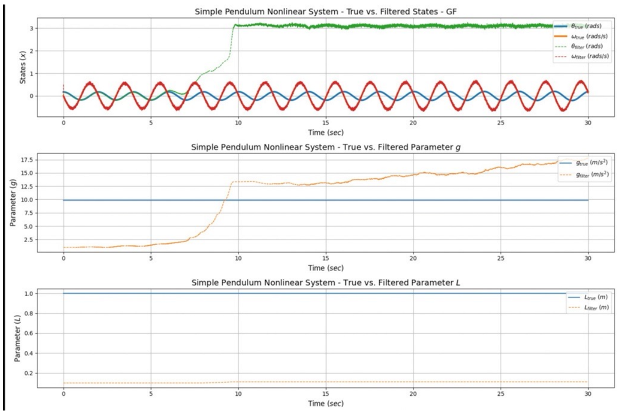

Figure 3: State Augmentation for GF

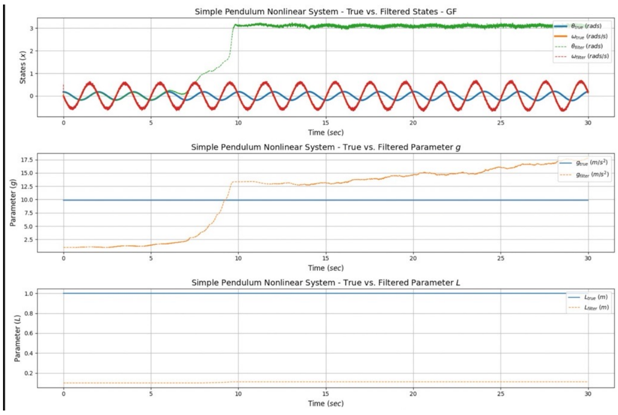

Figure 4: State Augmentation for EKF

Figure 5 has three graphs showing the real states of a basic pendulum system over a period of two seconds:The True States Graph in the first graph displays the true states of the pendulum as smooth sinusoidal functions, denoted by (theta, in radians) and (omega, in radians per second). The angular velocity is represented by omega, a sine wave, and the angle of the pendulum by theta, which seems to be a cosine wave. This is to be expected as the position and velocity in a basic harmonic oscillator, such as a pendulum, are sinusoidal and 90 degrees out of phase. The same states are shown on the noisy states graph, but there is more noise in the same graph. The second graph depicts parameter estimation through Non-linear programming. The third graph shows a single graph that provides solution for non-linear programming (NLP) that compares the true and estimated states (theta and omega) of the pendulum system. Similarly we got figures for Batch estimations depicted in Figure 6.

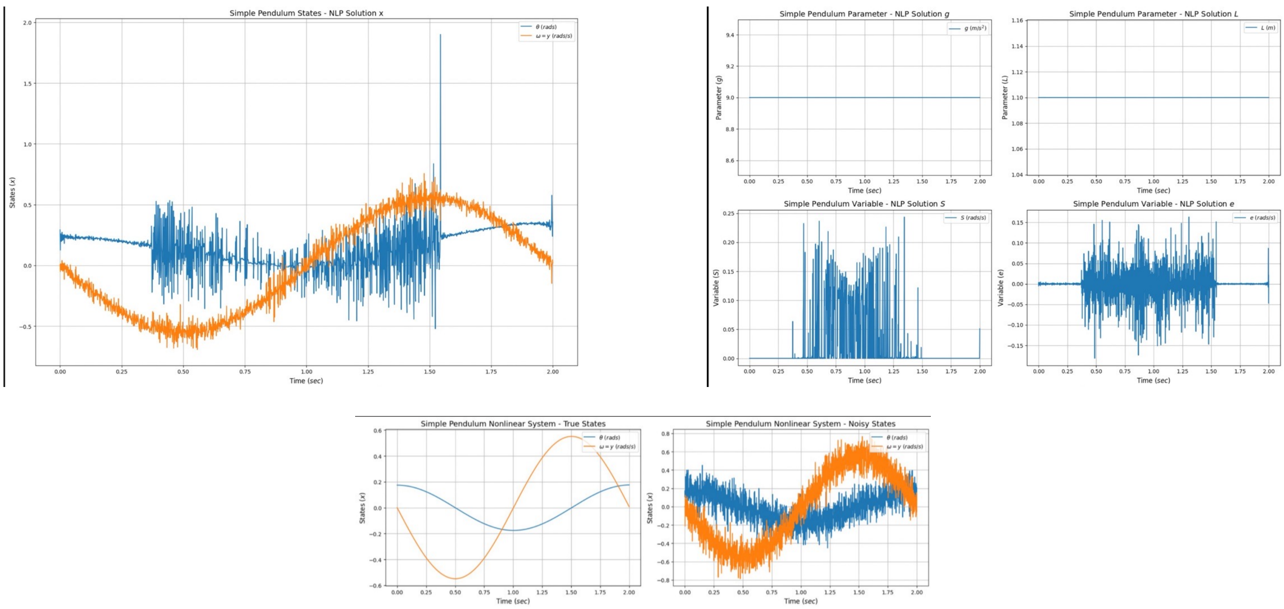

Figure 5: Maximum Likelihood Estimation

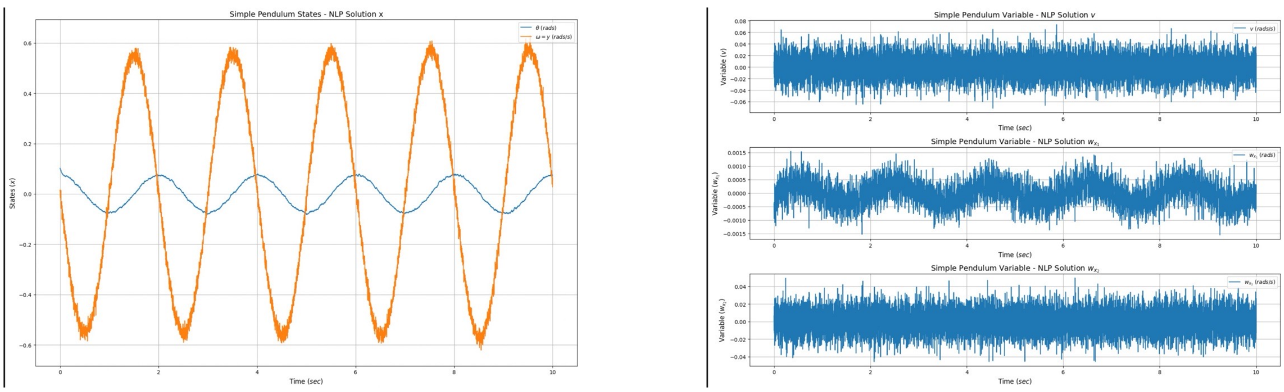

Figure 6: Batch Estimation

6 Conclusion

We have developed and tested state and parameter estimation algorithms. These preliminary results indicate the utility of these techniques in the broader scheme of Model Predictive Control of buildings. Moreover, the developed algorithms form the building blocks for developing more sophisticated algorithms as the need arises in the future. In the near future, we will be testing these algorithms on data generated from EnergyPlus, Modelica simulations, and actual building data. Moreover, we will be developing Particle Filter/Smoother and parameter estimation algorithms based on Energy Function and Expectation Maximization approach so as to have all the major tools to tackle state-parameter estimation problems in relation to MPC.We have written the entire code for the three different house models for all the three type of optimization techniques i.e. Maximum Likelihood, Least Squares Estimation and Batch Estimation. We plan on executing it in the future by tuning in the several parameters and obtaining results for a wide range of conditions.

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