

## GATE 2019 CS PAPER Question 06

### Question

Consider the following Boolean identities. Determine which of them are valid:

(A)  $(x \oplus y) \oplus z = x \oplus (y \oplus z)$

(B)  $(x + y) \oplus z = x \oplus (y + z)$

(C)  $x \oplus y = x + y$  if  $xy = 0$

(D)  $x \oplus y = (xy + x'y')'$

### Answer

#### (A) Valid

XOR is associative.  $(x \oplus y) \oplus z = x \oplus (y \oplus z)$  always holds.

#### (B) Not valid

Counterexample:  $x = 1, y = 1, z = 0$ . LHS =  $(1 + 1) \oplus 0 = 1 \oplus 0 = 1$  RHS =  $1 \oplus (1 + 0) = 1 \oplus 1 = 0$  So they differ.

Truth table:

$x$	$y$	$z$	$(x + y) \oplus z$	$x \oplus (y + z)$
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	1
1	0	0	1	1
1	0	1	0	0
1	1	0	1	0
1	1	1	0	0

#### (C) Valid

If  $xy = 0$ , then  $x$  and  $y$  are not both 1. In that case  $x \oplus y = x + y$ .

**(D) Valid**

$$(xy + x'y')' = (x'y + xy') = x \oplus y$$

Hence (A), (C), (D) are valid; (B) is not.