

## DAA Assignment - 4

Qus 1 soln:-

$$T(n) = 3T(n/2) + n^2$$

$$a=3, b=2, f(n)=n^2$$

$\therefore a \neq b$  are constant &  $f(n)$  is the function

$\therefore$  Master's theorem is applicable

$$c = \log_b a$$

$$= \log_2 3 = 1.58$$

$$n^c = n^{1.58} \text{ which is } < n^2$$

$\therefore$  Case 3 is applicable

$$T(n) = O(n^2)$$

Q2 soln:-

$$T(n) = 4T(n/2) + n^2$$

$$a=4, b=2, f(n)=n^2$$

$$c = \log_b a = \log_2 4 = 2$$

$$n^c = n^2$$

$$n^c = f(n)$$

Case 2

$$T(n) = O(n^2 \log n)$$

Q3 soln:-

$$T(n) = T(n/2) + 2^n$$

$$a=1, b=2, f(n)=2^n$$

$$c = \log_b a = \log_2 1 = 0 \quad n^c = n^0 = 1$$

$$f(n) > n^c$$

$\therefore$  Case 3

$$T(n) = O(2^n)$$

Qus 4 soln:-

$$T(n) = 2^n T(n/2) + n^n$$

$$a=2^n, b=2, f(n)=n^n$$

$\therefore a$  is not constant, its value depends on  $n$

$\therefore$  Master's theorem not applicable.

Ques soln:-

$$T(n) = 16T(n/4) + n$$

$$a = 16, b = 4, f(n) = n$$

$$c = \log_b a = \log_4 16 = 2$$

$$n^c > f(n)$$

Case 1

$$T(n) = O(n^2)$$

Ques soln:-

$$T(n) = 2T(n/2) + n \log n$$

$$a = 2, b = 2, f(n) = n \log n$$

$$c = \log_b a = \log_2 2 = 1$$

$$n^c = n$$

$$f(n) \geq n^c$$

Case 3 is applied

$$T(n) = O(n \log n)$$

Q7 soln:-

$$T(n) = 2T(n/2) + n/\log n$$

$$a = 2, b = 2, f(n) = n/\log n$$

$$c = \log_2 2 = 1$$

$$n^c = n$$

non polynomial diff b/w  $n^c$  &  $f(n)$  $\therefore$  masters theorem not applicable.

Ques soln:-

$$T(n) = 2T(n/4) + n^{0.5}$$

$$a = 2, b = 4, f(n) = n^{0.5}$$

$$c = \log_b a = \log_4 2 = 0.5$$

$$n^c = n^{0.5}$$

$$\therefore f(n) > n^c$$

Case 3 is applied

$$T(n) = O(n^{0.5})$$



Q9 soln:-

$$T(n) = 0.5T(n/2) + \frac{1}{n}$$

$a < 1$   $\therefore$  Master's theorem applicable

Q10 soln:-

$$T(n) = 16T(n/4) + n!$$

$$a = 16, b = 4, f(n) = n!$$

$$c = \log_b a$$

$$= \log_4 16 = 2$$

$$n^c = n^2$$

$$f(n) > n^c$$

Case 3

$$T(n) = O(n!)$$

Q11 soln:-

$$T(n) = 4T(n/2) + \log n$$

$$a = 4, b = 2, f(n) = \log n$$

$$c = \log_2 4 = 2$$

$$n^c = n^2$$

$$n^c > f(n)$$

$\therefore$  Case 1 is applied

$$T(n) = O(n^2)$$

Q12 soln:-

$$T(n) = \sqrt{n} T(n/2) + \log n$$

$a$  is not constant, therefore master's theorem not applicable.

Q13 soln:-

$$T(n) = 3T(n/2) + n$$

$$a = 3, b = 2, f(n) = n$$

$$c = \log_b a = \log_2 3 = 1.58$$

$$n^c = n^{1.58} > f(n)$$

Case 1

$$\therefore T(n) = O(n^{1.58})$$

Q14 soln:-

$$T(n) = 3T(n/3) + \sqrt{n}$$

$$a = 3, b = 3, f(n) = \sqrt{n}$$

$$c = \log_b a = 1$$

$$n^c = n > \sqrt{n}$$

Case 1 is applied  $T(n) = O(n)$ 

Q15 soln:-

$$T(n) = 4T(n/2) + cn$$

$$a = 4, b = 2, f(n) = c \cdot n$$

$$n^c = n^2 > f(n)$$

 $\therefore$  Case 1 is applied

$$T(n) = O(n^2)$$

Q16 soln:-

$$T(n) = 3T(n/4) + n \log n$$

$$a = 3, b = 4, f(n) = n \log n$$

$$c = \log_b a = \log_4 3 = 0.78$$

$$n^c = n^{0.78} < f(n)$$

 $\therefore$  Case 3 is applied  $T(n) = O(\log n)$



Q17 soln:-  $T(n) = 3T(n/3) + n/2$   
 $a=3, b=3 \quad f(n) = n/2$   
 $c = \log_b a = 1$

$n^c = n > f(n)$   
 Case 1!  $T(n) = \Theta(n)$

Q18 soln:-  $T(n) = 6T(n/3) + n^2 \log n$   
 $c = \log_3 6 = 1.63$

$n^c = n^{1.63} < f(n)$

Case 3 is applied  $\therefore T(n) = \Theta(n^2 \log n)$

Q19 soln:-  $T(n) = 4T(n/2) + n \log n$

$c = \log_2 4 = 2$

$n^c = n^2 \quad f(n) = n \log 2$

$n^c \not\geq f(n)$

$\therefore$  case 1 is applied  $T(n) = \Theta(n^2)$

Q20 soln:-  $T(n) = 64T(n/8) + n^2 \log n$

$a=64, b=8 \quad f(n) = n^2 \log n$

$c = \log_b a = 2$

$n^c = n^2 < f(n)$

Case 3 is applied  $T(n) = \Theta(n^2 \log n)$

Q21 Sol<sup>n</sup> —

$$T(n) = 7T(n/3) + n^2$$

$$a=7, b=3, f(n)=n^2$$

$$C = \log_b a = \log_3 7 = 1.77$$

$$n^C = n^{1.77} < f(n)$$

Case 3 is applied  $T(n) = O(n^2)$

Q22 Sol<sup>n</sup> —

$$T(n) = T(n/2) + n(2 - \cos n)$$

$\therefore f(n)$  is not regular function

$\therefore$  Master's theorem can't be applied.