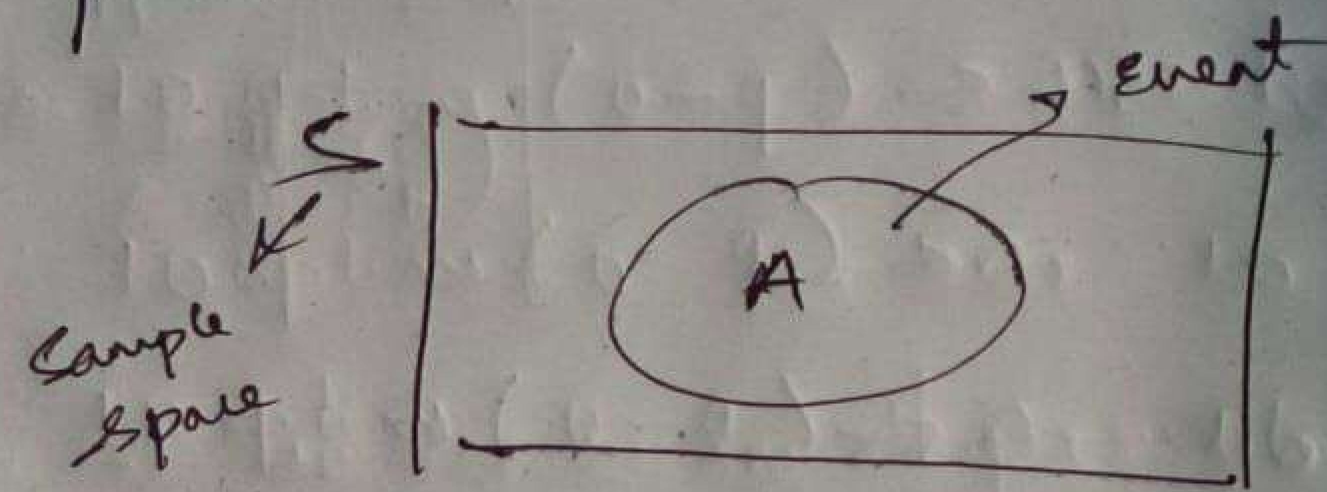
Stat110

Lecture 1: probability & counting

- A **Sample Space** is the set of all possible Outcomes of an Experiment
- An **Event** is a Subset of the Sample space



Naive defn of prob:

$$P(A) = \frac{\#\text{ No of fav outcomes}}{\#\text{ No of possible outcomes}}$$

P(A) → event

No of possible outcomes → Size of Sample Space

flip coin twice

H H
H T
T H
T T

when to use Multiplication Rule :

→ when choice are made in Sequence
→ when Each choice is independent
(no of options at Each step does not depend on prev choices)

Special cases :

1) Dependent Events

ex: A password where no digit repeats

1st digit → 10 options
2nd digit → 9 options (cant repeat the first)
3rd digit → 8 options (")

Total → $10 \times 9 \times 8 = \underline{720}$

2) Exponents (K^r)

ex: Flipping a coin 5 times
↳ Each flip 2 outcomes

Total = $2 \times 2 \times 2 \times 2 \times 2 = 2^5 = \underline{\underline{32}}$

Ex: prob of full house in poker, 5 card hand.
↳ Eg : 3: 7's, 2: 10's

$$\frac{13 \cdot \binom{4}{3} \cdot 12 \binom{4}{2}}{\binom{52}{5}} \approx 0.00144$$

≈ 1 in 694

Binomial coeff: $\binom{n}{k} = \frac{n!}{(n-k)!\,k!}$, 0 if $k > n$

How Ex1) related to definition.

Experiments (appetizer → main course → dessert)

Each experiment has its own no of outcomes

(n_1, n_2, n_3)

Total outcomes are found by multiplying them together, just as the rule.

Ex2) password digits

1st Digit: 10 options (0-9) → $n_1 = 10$

2nd Digit: 10 options (0-9) → $n_2 = 10$

3rd Digit: 10 options (0-9) → $n_3 = 10$

Total password outcomes $= n_1 \times n_2 \times n_3 = 10 \times 10 \times 10$
$= \underline{1000}$

→ The multiplication Rule counts all possible paths through the sequence, like a tree diagram where each level multiplies the total.

Formula:

Total no of possible outcomes for entire sequence $= n_1 \times n_2 \times n_3 \times \cdots \times n_r$

Assumes all outcomes equally ...

finite sample space.

what is naive

Counting:

Multiplication Rule: If we have an Experiment with n_1 possible outcomes and for each outcome of 1st Experiment there are n_2 outcomes for the 2nd Experiment ....., for each ~~exper~~ there are n_r outcomes for rth experiment, then $n_1, n_2 \ldots n_r$ overall possible outcomes

Ex1) Scenario: Building a 3-course meal

1. 1st Exp (Appetizer):
   choose 1 of 2 options (soup or salad)
   $\rightarrow n_1 = 2$ outcomes

2. 2nd Exp (main course):
   for each appetizer choice, there are three main dishes (chicken, fish, pasta).
   $\rightarrow n_2 = 3$ outcomes (independent of the appetizer)

3. 3rd Exp (Dessert):
   for each main course, there are 2 desserts (cake or ice cream)
   $\rightarrow n_3 = 2$ outcomes

Total possible meals $= n_1 \times n_2 \times n_3 = 2 \times 3 \times 2 = \underline{\underline{12}}$

Subsets of size k, of group of n people

$$\frac{n\cdot(n-1)(n-2)\cdots(n-k+1)}{k!} = \frac{n!}{(n-k)!\,k!}$$

Sampling table: choose k objects of n

	order matters	order doesn't
replace	n^k	$\binom{n+k-1}{k}$
don't replace	$n(n-1)\cdots(n-k+1)$	$\binom{n}{k}$

Imagine you're at an ice cream shop with n flavours and you want to buy k scoops. You can choose:

→ The same flavour multiple time

→ Different flavours

→ The order you receive doesn't matter

(Vanilla-choco - same as choco-vanilla)

No of choices → n

No of items to select → k

The formula → $^{n+k-1}C_k = \binom{n+k-1}{k}$

ex: $n \to 3$ (Vanilla, chocolate

$k \to 2$ scoops

1) Vanilla + Vanilla
2) Vanilla + Chocolate
3) Vanilla + Strawberry
4) chocolate + chocolate
5) chocolate + strawberry
6) Strawberry + Strawberry

order doesnt matter

No of ways $= \binom{n+k-1}{k}$

$n \to 3$, $k \to 2$

$$\binom{3+2-1}{2} = \binom{4}{2} = \frac{4!}{2! \times (4-2)!} = \underline{\underline{6}}$$

	order matters
No repetition	$p(n,k) = \frac{n!}{(n-k)!}$
repetition	n^k