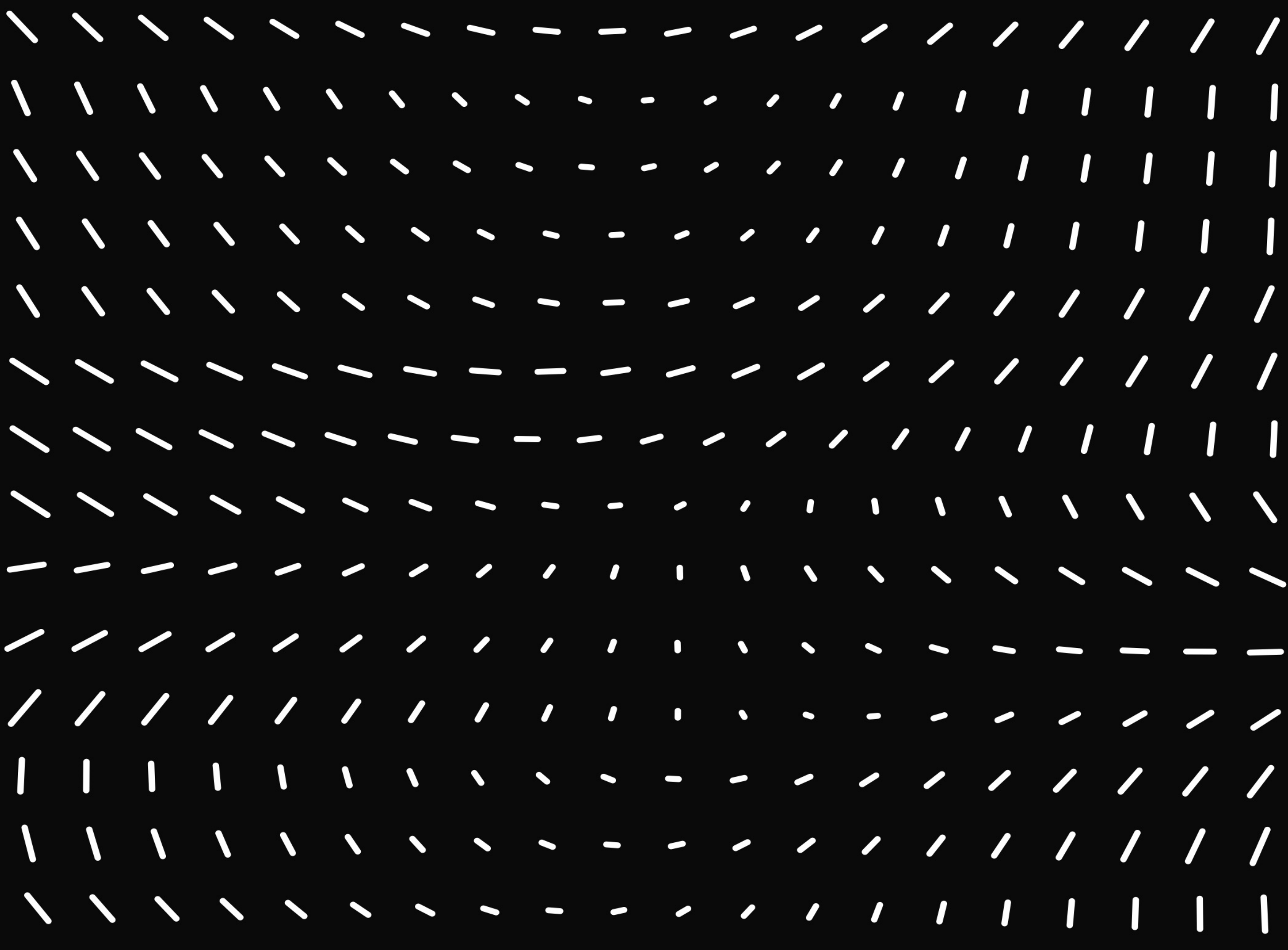
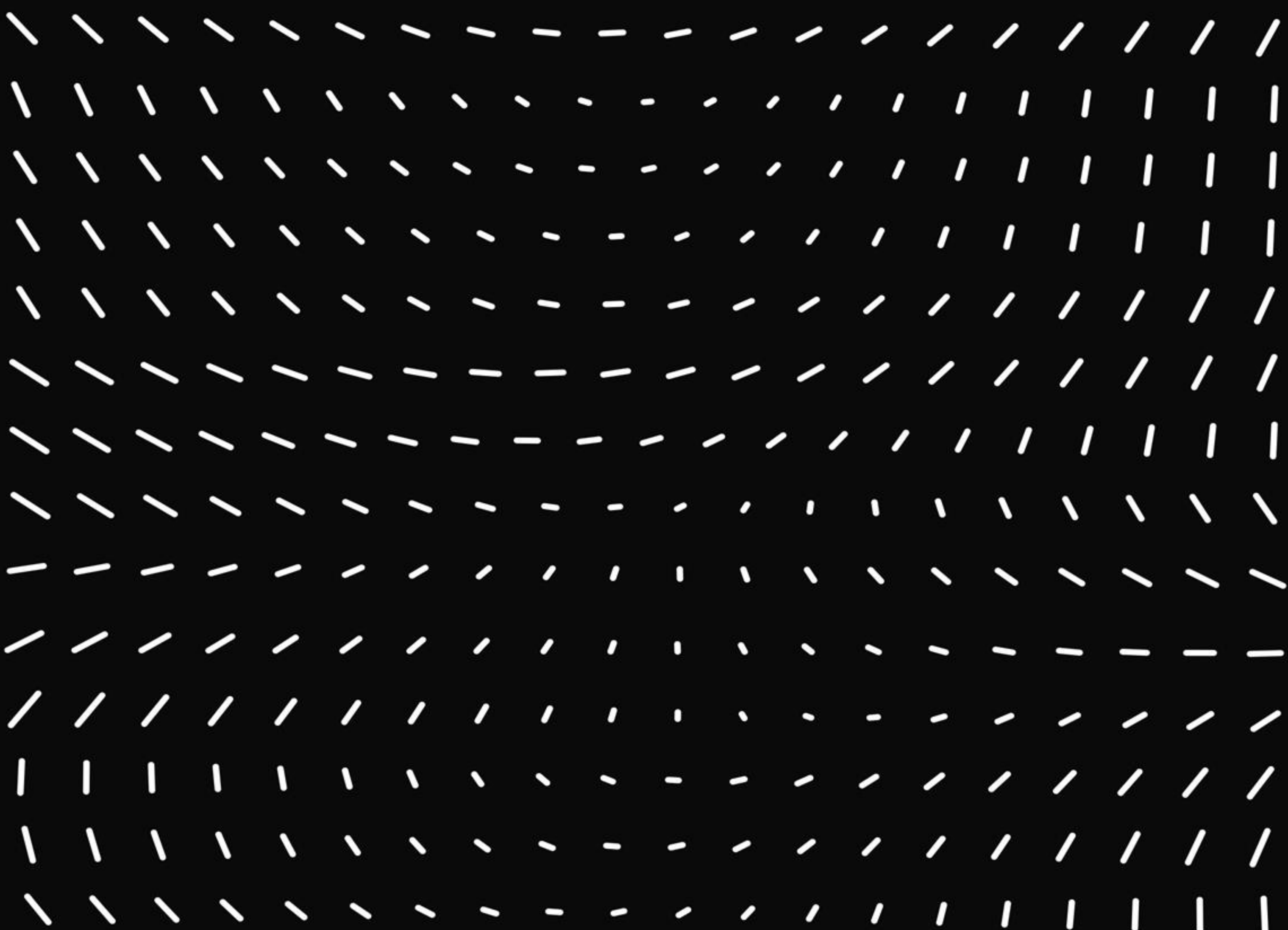


# Vector

# Analysis

advanced 



## Pseudovector

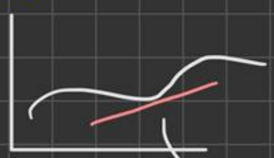
- Product of 2 vectors
- Some anomolous behaviour
- Examples?
- what about product of 2 pseudovectors

## Pseudoscalar

- scalar triple product result

# Differential Calculus

## 'Ordinary' Derivative



Derivative → variation of magnitude of quantity

## Gradient

- what if direction of movt. matters?
- How?

$$dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial y} dy + \frac{\partial Q}{\partial z} dz$$

- what does this tell us?

if $dx, dy, dz \rightarrow$ small position variations
$dT \rightarrow$ Temp. change acc. w/ above

why will only 3 derivatives suffice?

Prev. eq$^n$ ≈ $$dQ = \left(\frac{\partial Q}{\partial x}\hat{x} + \frac{\partial Q}{\partial y}\hat{y} + \frac{\partial Q}{\partial z}\hat{z}\right) \cdot (dx \cdot \hat{x} + dy \cdot \hat{y} + dz \cdot \hat{z})$$
$$= (\nabla T) \cdot (d\ell)$$

$\nabla T$ → Gradient of T (vector w/ 3 components)

## Geometric representat$^n$ of Gradient:

Let's rewrite dot product: $dT = \nabla T \cdot dl = |\nabla T||dl| \cos\theta$

$\theta$ → Angle between

Max. change of T occurs along movt. of direction.

$|\nabla T| \rightarrow$ Slope along maximal direction
↓
- $\nabla T$ points here
- Dir. of max. increase of T

Direction → Along maximal direction
Magnit. → Slope along direction

Surfaces that don't follow these have undifferentiable functions assc. w/ them

## Vanishing of a gradient

if $\nabla T = 0 \implies dT = 0$ for a SMALL displacement about the point. We call this a stationary.

## $\nabla$ Operator

Gradient → Formal appearance of vector $\nabla \times T$ (scalar)

$$dQ = \frac{\partial Q}{\partial x} dx + \frac{\partial Q}{\partial y} dy + \frac{\partial Q}{\partial z} dz$$

$$\nabla = \frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}$$

- Not a vector!
  w/o any quantity, eg Q or T, it has no meaning
- $\nabla T \neq \nabla$ times t
  ↪ Instruction to differentiate upon

| Normal vector mul. | Grad operator mult (Not. mul → operation) |
|---|---|
| → Scalar × Vector<br>→ Vector × Vector (Dot product)<br>→ Vector × Vector (Cross | → On scalar func$^n$ → eg. $\nabla T$<br>→ On vector func w/ dot prod. → Divergence<br>→ On vector func. w/ cross pr. → Curl. |

## Divergence

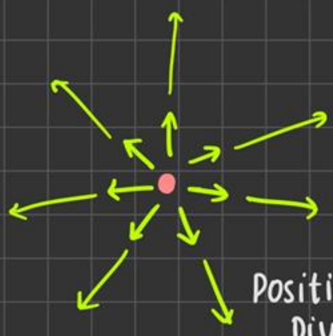
We can't have divergence of a scalar quantity but we can of a gradient
In gradient we provide direction of change, however, the quantity itself is scalar.

Constructing divergence: ($\nabla$ dot product vector)

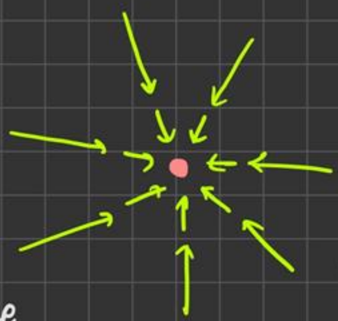
$$\nabla \cdot v = \left(\frac{\partial}{\partial x}\hat{x} + \frac{\partial}{\partial y}\hat{y} + \frac{\partial}{\partial z}\hat{z}\right) \cdot (\vec{V}_x + \vec{V}_y + \vec{V}_z) = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z}$$

## Geometric Interpretation

Divergence - measure of how much $\vec{v}$ spreads from pt. taken.
(Divergence is of a vector FUNCTION not vector)
Outward flux from a surface.



Positive Divergence



Negative Divergence

Positive divergence.

0 divergence.

## Curl

$$\nabla \times v = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ v_x & v_y & v_z \end{vmatrix} = \hat{x}\left(\frac{\partial}{\partial y}\cdot v_z - \frac{\partial v_y}{\partial z}\right) + \hat{y}\left(\frac{\partial v_x}{\partial z} - \frac{\partial v_z}{\partial x}\right) + \hat{z}\left(\frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y}\right)$$

Similar to cross product.

## Geometric Interpretation

Measure of how much vector v 'curls around' a point.
Above figures have no curl.

(Refer to divergence & curls, grads' product rule)
LINK:

But importantly, we note the laplacian.

Laplacian: $\nabla^2 v$ or $\nabla \cdot (\nabla v)$

## Second Derivative Class

(In Griffith's)

- D of G
- C of G
- G of D
- C of C
- D of C

(No other possibility)

Given in book