Unit-2

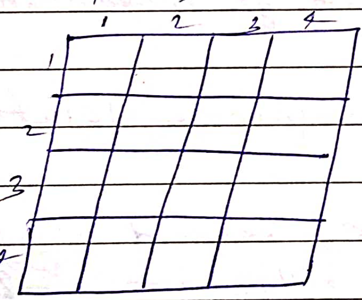
Backtracking

Backtracking is an algorithmic-technique for solving problems recursively by trying to build a solution incrementally, one piece at a time, removing those solutions that fail to satisfy the constraints of the problem at any point of time.

N- Queen's Problem

Place n queens in $n \times n$ board so that two queens doesn't comes in same row, same column and same diagonal. Otherewise this will be treated as attack, then we need to backtrack and need to find another possible solutions.

4- Queen's Problems



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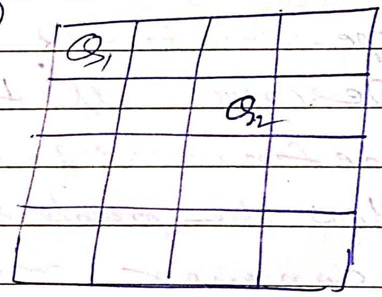
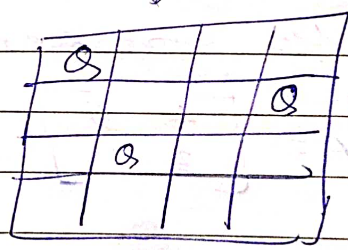
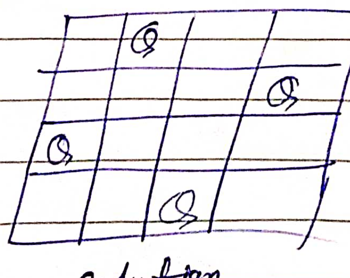




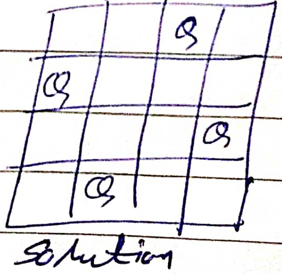
No Solution

2 Second Column - 1st Queen



Solution

3 Column - 1st Quee

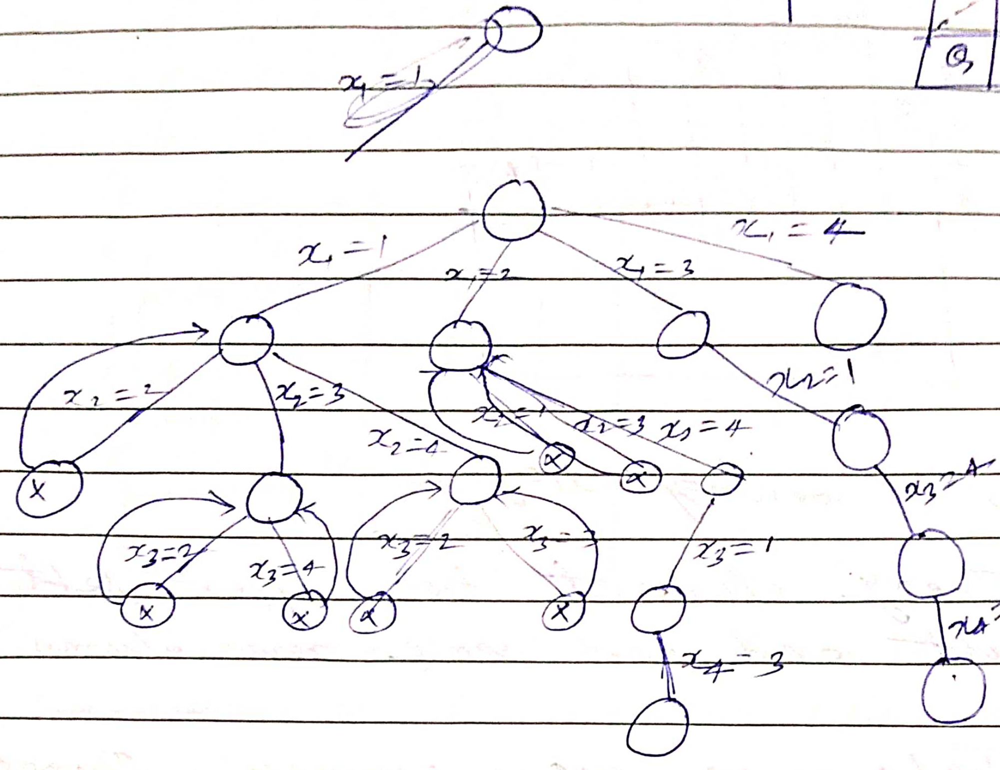


Solution

State-Space Tree

4th column Queen

No Solution (only 3-Queen's)

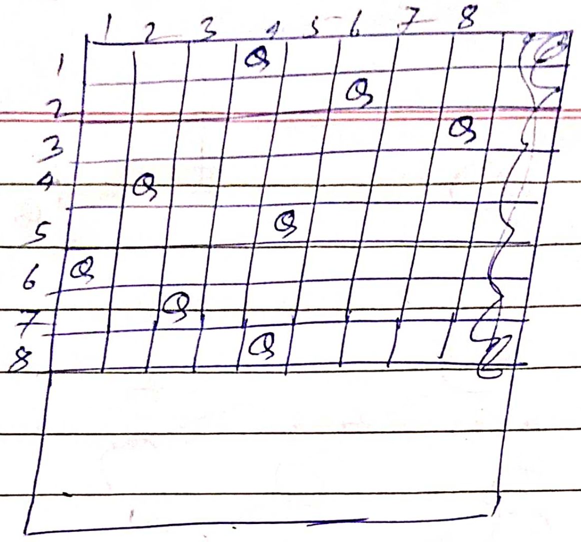


2, 4, 1, 3 is one of the solution & 3, 1, 4, 2 is another solution.

8-Queen's Problem

Place 8-queens on a 8X8 chessboard so that no two "attack". Let us number the rows & columns of the chessboard 1 through 8. Since each queen must be placed on a different row, we can assume queen i is to placed on row i. All solutions to the 8-queens problem represented through as 8-tuples $(x_1, x_2 - - x_8)$, where x_i is the column on which queen i is placed. Therefore solution space consists of 8^8 8-tuples. One of the possible solution is

4, 6, 8, 2, 7, 1, 3, 5



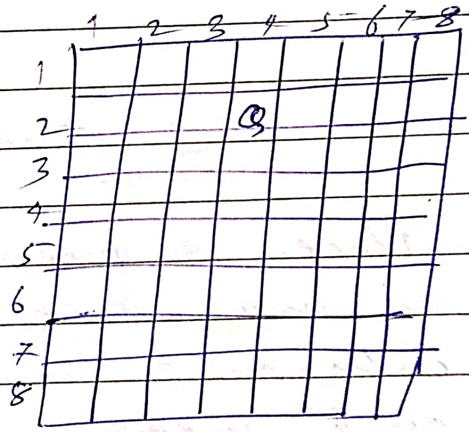
How to identify Diagonal

Every element from upper left to lower right and has same row-column value.

Take on one example, Queen is placed at [4,2] 4th column & 2nd row

[3,1]

[5,3],
[6,4]
[7,5]
[8,6]



[4,2] used as [i,j]

[3,1] used as [k,l]

i,j — Initial position of Q, ith row, jth column

k,l — New position of Q, kth row, lth column

$\boxed{i-j = k-l \quad \text{or} \quad i+j = k+l}$

One possible solution of 8-queens problem is

	1	2	3	4	5	6	7	8
1				q_1				
2						q_2		
3								q_3
4		q_4						
5							q_5	
6	q_6							
7			q_7					
8					q_8			

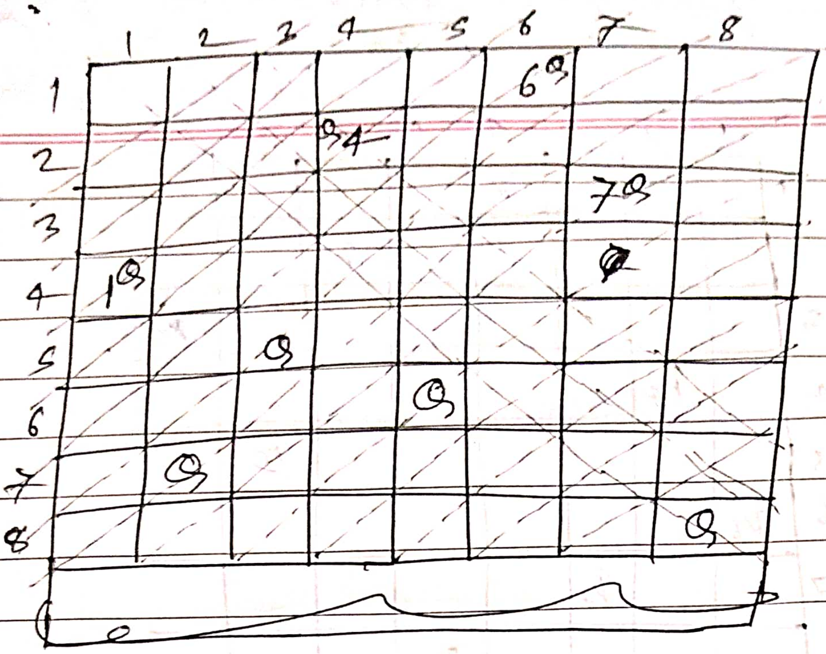
The 8-queens problem is frequently used as an example of AI courses. The objective is to place 8-queens on an empty chess board so that non of them can take the other. For 8-queens we have 92 solutions. The solution space consists of ~~8~~ 18 permutations i.e 40,320.

The General solution ~~of an~~

Some times we are provided a feasible sequence in the problem, i.e the positions of first four queens for the first rows are given and next four positions are found to solve 8-queens problem.

Example Solve 8-queens problem for a feasible sequence (6, 4, 7, 1)

Solⁿ:



Now we have to see if we put the fifth queen at a particular place then we need to avoid conflict of diagonal placing, the following formula is followed up:

Let $P_1 = (i, j)$ and $P_2 = (k, l)$ are two positions. These positions will be on same diagonal if

$$i+j = k+l \quad \text{or} \quad i-j = k-l$$

Suppose a queen is placed to (5, 2) position

$\boxed{\text{Upper Left or Down Right}}$

then the position (5, 2) will be a diagonal conflict because there is already a queen placed on position (4, 1). If we check, we find that these positions are in conflict:

By formula $\quad i-j = k-l$

$$5-2 = 4-1$$

$$3 = 3$$

So, the new queen can't be placed on the position (5, 2).

The fifth queen is now placed in 3rd column since $5-3 \neq 4-1$ or $5+3 \neq 4+1$, so the solution is feasible and queen has been placed in 3rd row.

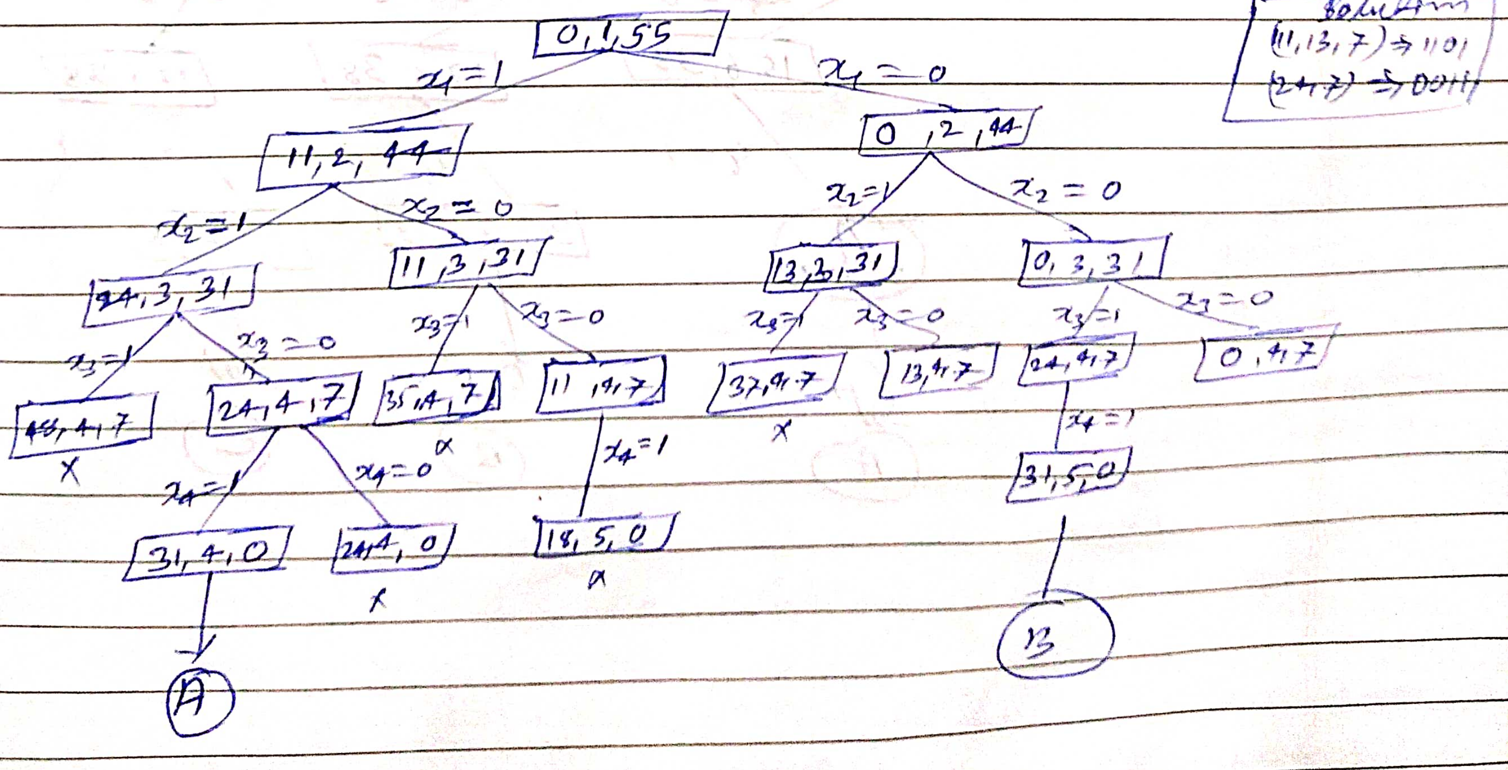
Sum of Subsets

Suppose we have given n distinct positive numbers (usually called weight) and we desire to find all combinations of these numbers whose sums are m. This is called the sum of subsets problem. The solution element x_i is either one or zero depending upon its weight w_i is included in solution or not.

For example, suppose $(w_1, w_2, w_3, w_4) = (11, 13, 24, 7)$ and $m = 7$. The subsets of W which makes the sum 31 are (11, 13, 7) and (24, 7). Its solution vector is represented (1, 2, 4) and (3, 4). Another way of representing solution vector is in binary form. The solution vector can be either 0 or 1. If w_i is chosen then $x_i = 1$ otherwise $x_i = 0$. For example, the solution vector is (1, 1, 0, 1) and (0, 0, 1, 1)

The solution space is created with the use of Backtracking algorithm

$(w_1, w_2, w_3, w_4) = (11, 13, 24, 7)$, $W = 31$, Total $w = 55$



Q- Let W = {5, 7, 10, 12, 15, 18, 20} and m = 35

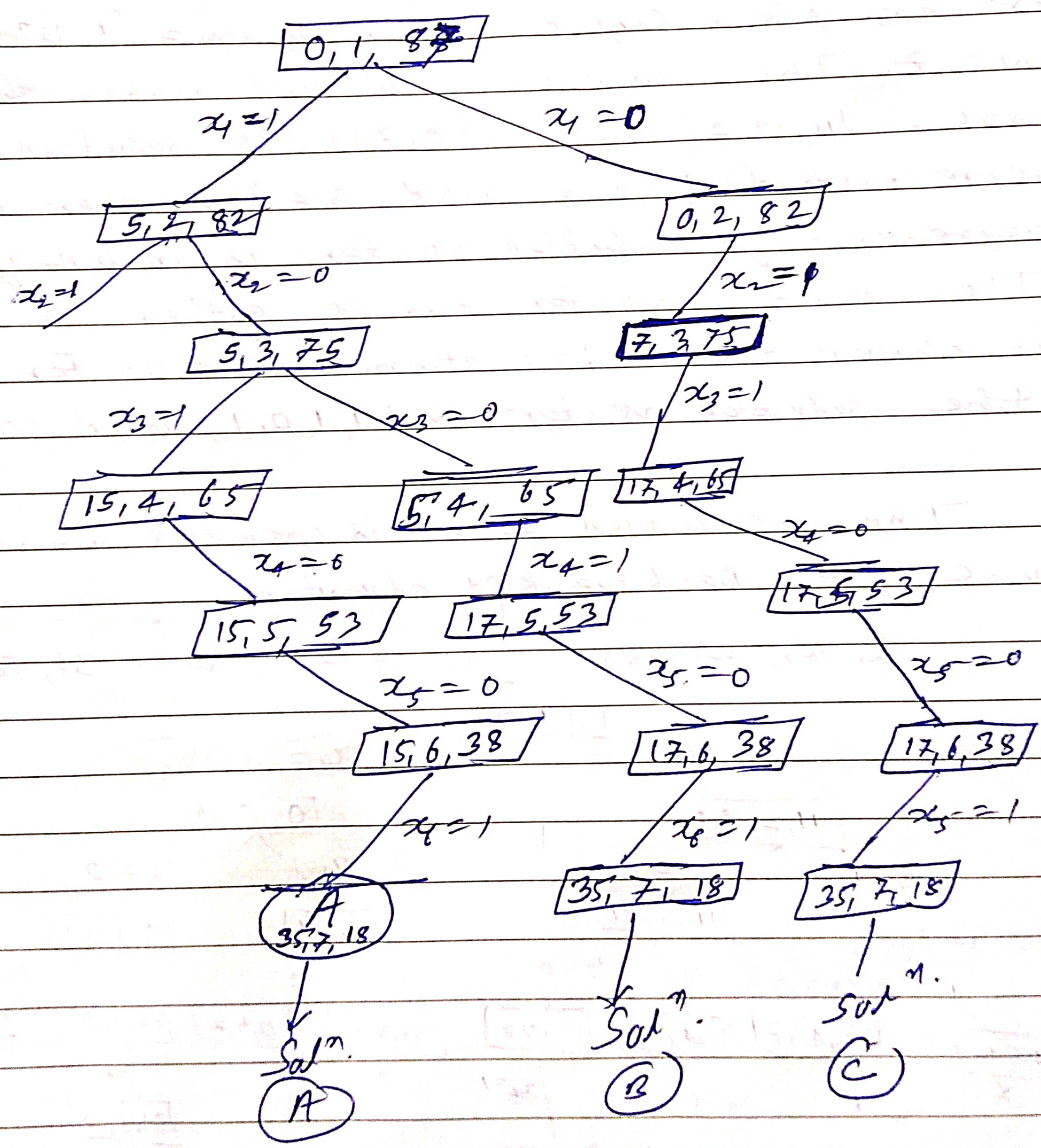
Find all possible subsets of W that sum to m using backtracking. Draw the portion of the search tree.

Solⁿ:

(A) 5, 10, 20 (1010001)

(B) 5, 12, 18 (1001010)

(C) 7, 10, 18 (0110010)



$n = 6, \quad w[1:6] = \{5, 10, 12, 13, 15, 18\}$ → total $w = 73$

$m = 30$

(I) 1 1 0 0 1 0

(II) 1 0 1 1 0 0

(III) 0 0 1 1 0 0

{5, 10, 15}
{5, 12, 13}
{12, 18}