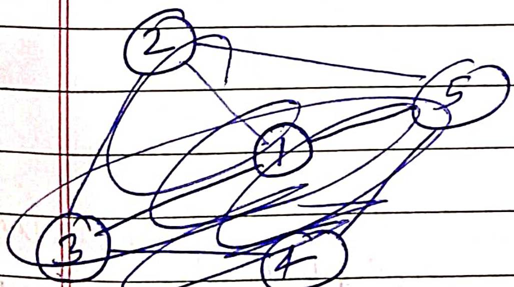
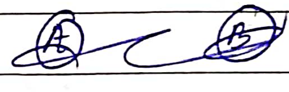
Graph Coloring Problem

- On a graph $G(V,E)$ whose nodes V are to be colored with m colors, where m is positive integer.
- Coloring is to be done in such a way no two adjacent nodes are colored with the same color
- It is special case of graph labeling, this is called a vertex coloring. Similarly an edge coloring (assign color to each edge so that no two adjacent edge share the same color), and a face coloring (assign a color of each face or region so that no two faces share a boundary have the same color.
- It uses a Backtracking technique to solve this problem.
- The solution is given by n-tuple $(x_1, x_2, \ldots, x_n)$ where x_i is the color of the node i, degree of tree m and height $n+1$, where n is the number of nodes. Each node at level i has m children corresponding to the m possible assignment to x_i, $1 \le i \le n$ at least level $n+1$.



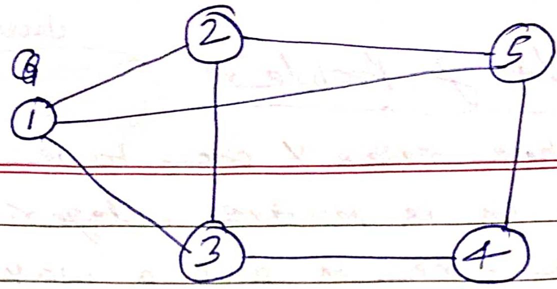


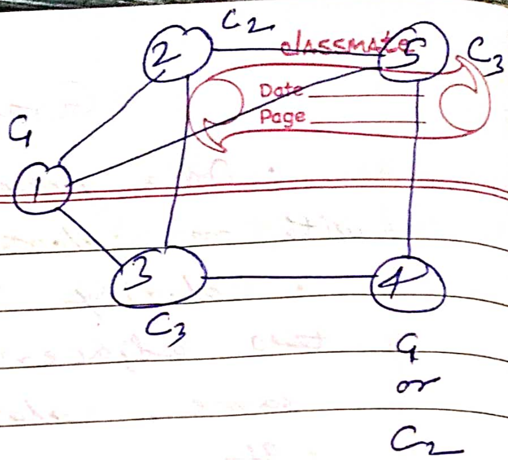
The graph can be color or not on given color

- m Coloring decision Problem
- m Coloring Optimization Problem
(How many of minimum m color the particular problem can be solved)

- This finds all ~~the~~ the feasible solutions

Q: Finding all the possible solutions of a problem in such a way that no two adjacent nodes having same color.

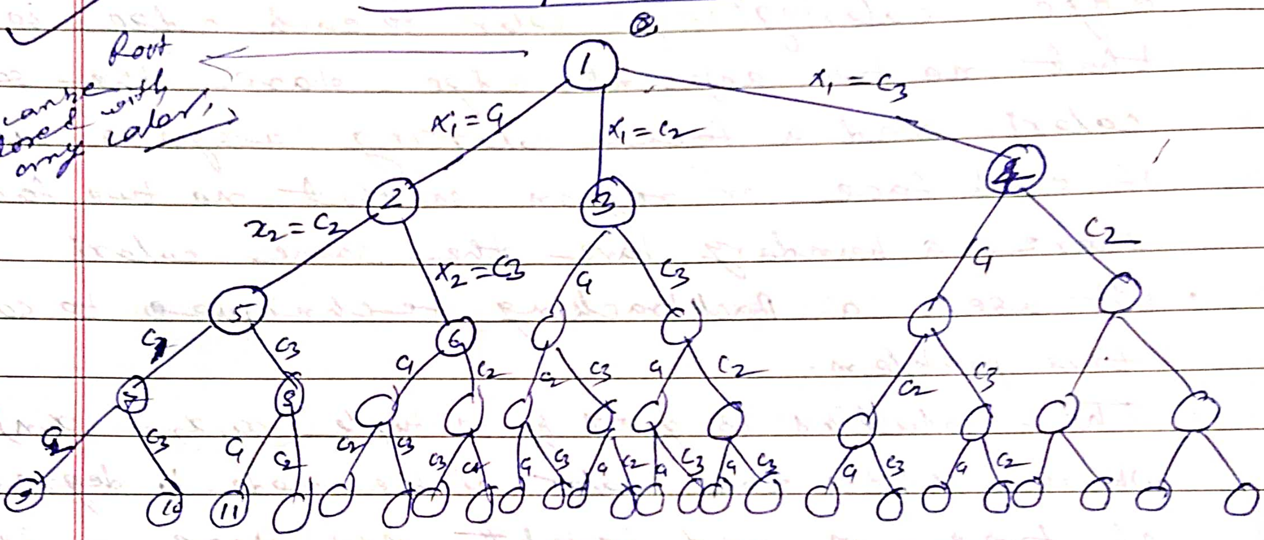


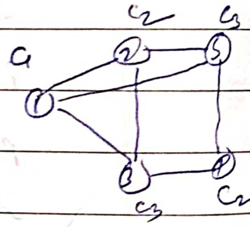


State-Space Tree

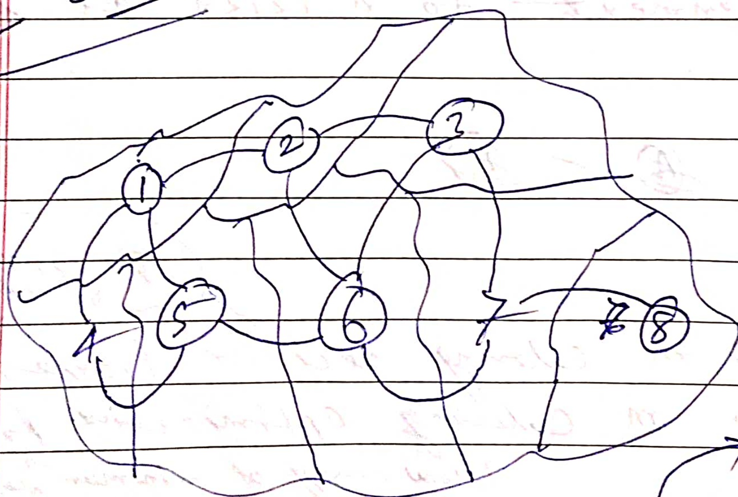
Root

Root can be colored with any color,





One Example



$1 + 3 + 3^2 + 3^3 + 3^4 = 121$

$\dfrac{3^{4+1} - 1}{3 - 1} = \dfrac{243-1}{2} = \dfrac{242}{2} = 121$

1

11

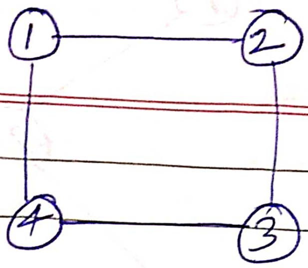
Analysis

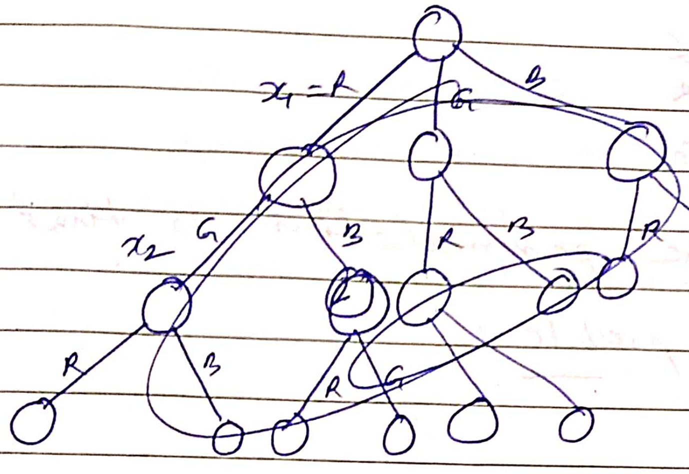
$1 + 3 + 3 \times 3 + 3 \times 3 \times 3 + 3 \times 3 \times 3 \times 3$

$1 + 3 + 3^2 + 3^3 + 3^4 = \dfrac{3^{4+1} - 1}{3 - 1} = \dfrac{3^{4+1} - 1}{2} = 121$

n = no. of vertices. $\boxed{3^{n+1} = C^{n+1}}$

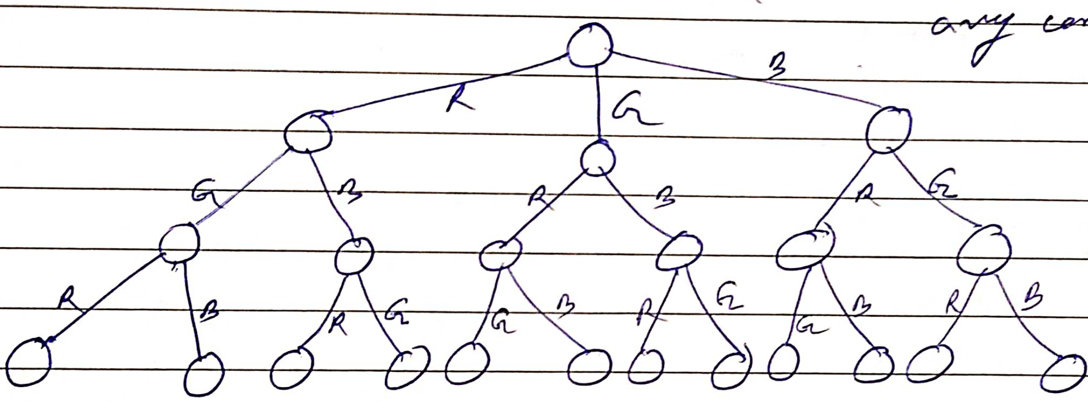
→ C is no. of colors



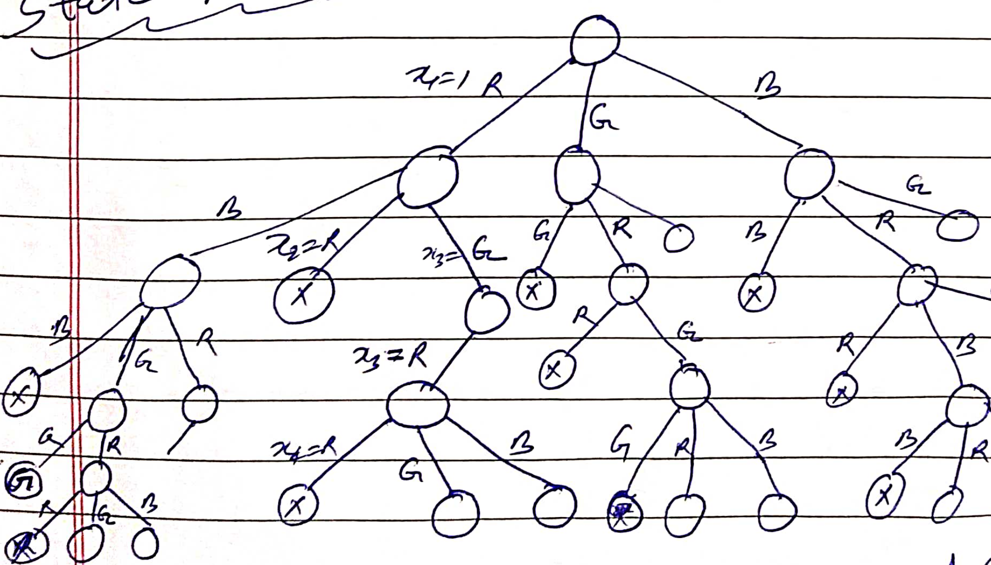


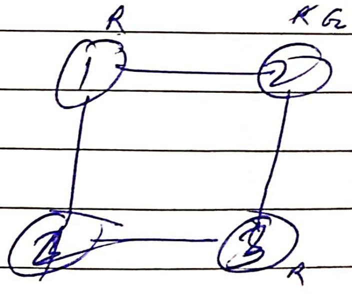
All possible solution.

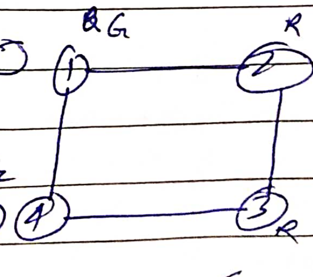
(Without imposing any constraint)



State-Space Diagram







Two possible solution

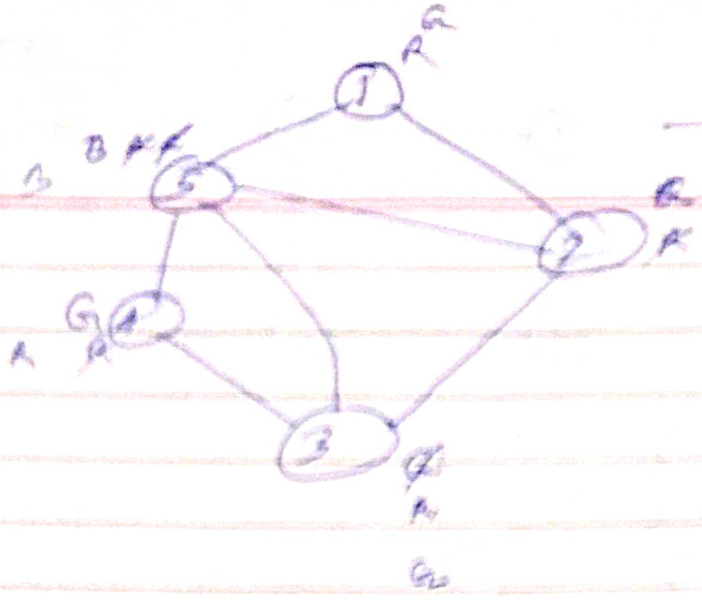
(I) R G R G ✓

(II) R G R B

(III) G R G R

(IV) G R G B

Date 5 - Nov



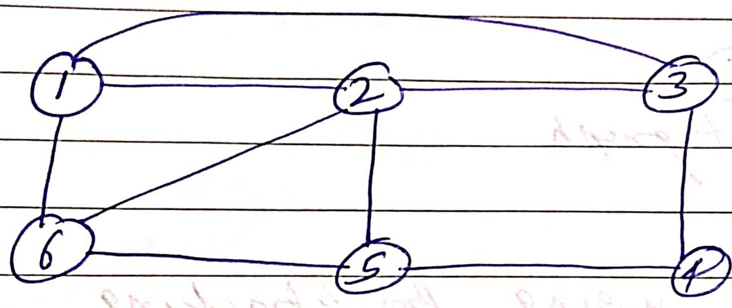
1	2	3	4	5
R	G	R	G	B
G	R	G	R	B

Apply all the permutation so that it satisfy the problem.

Hamiltonian Cycle Problem

Hamiltonian cycle is a cycle through a graph that visits each node or vertex exactly once except the starting vertex which is visited twice.

- We need to find all possible solution.
- This is problem is part of NP Hard problem i.e is in exponential time and it is hard to find Hamiltonian cycle present in the graph.



1, 2, 3, 4, 5, 6

1, 2, 6, 5, 4, 3, 1

~~1, 2, 6~~

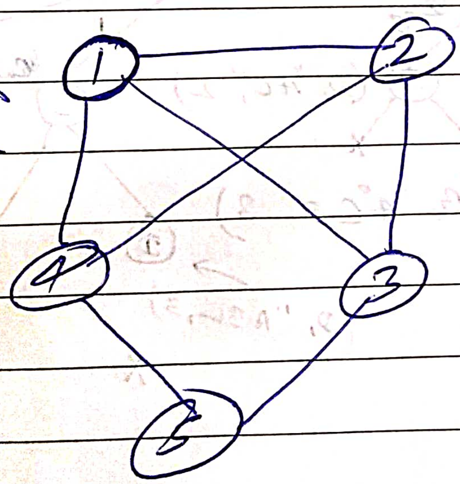
1, 6, 2, 5, 4, 3, 1

2, 3, 4, 5, 6, 1, 2

Same order except the starting point, we should to take care about duplicates

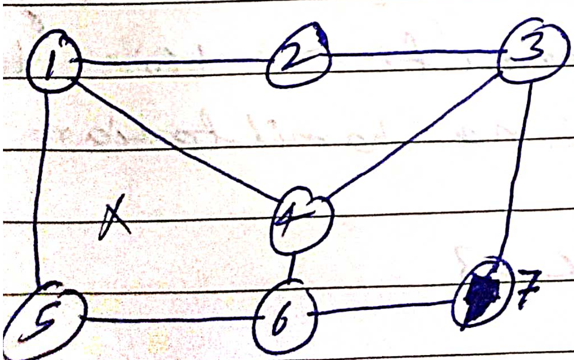
(i) 1, 3, 4, 5, 2, 6, 1

(ii) 1, 3, 4, 5, 6, 2, 1



1, 2, 3, 5, 4, 1

1, 4, 5, 3, 2, 1

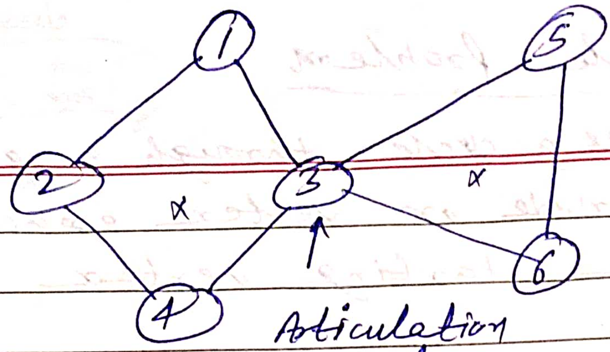


1, 4, 6, 7

1, 4, 5, 1

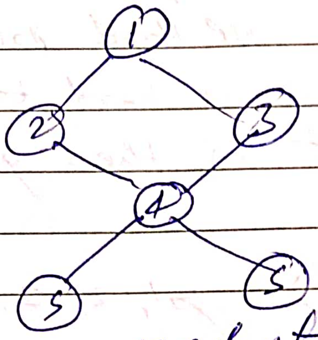
In case of visit node 4 we can't exactly visit each node exactly once except the last node.

polynomial function of the size of



Aticulation point

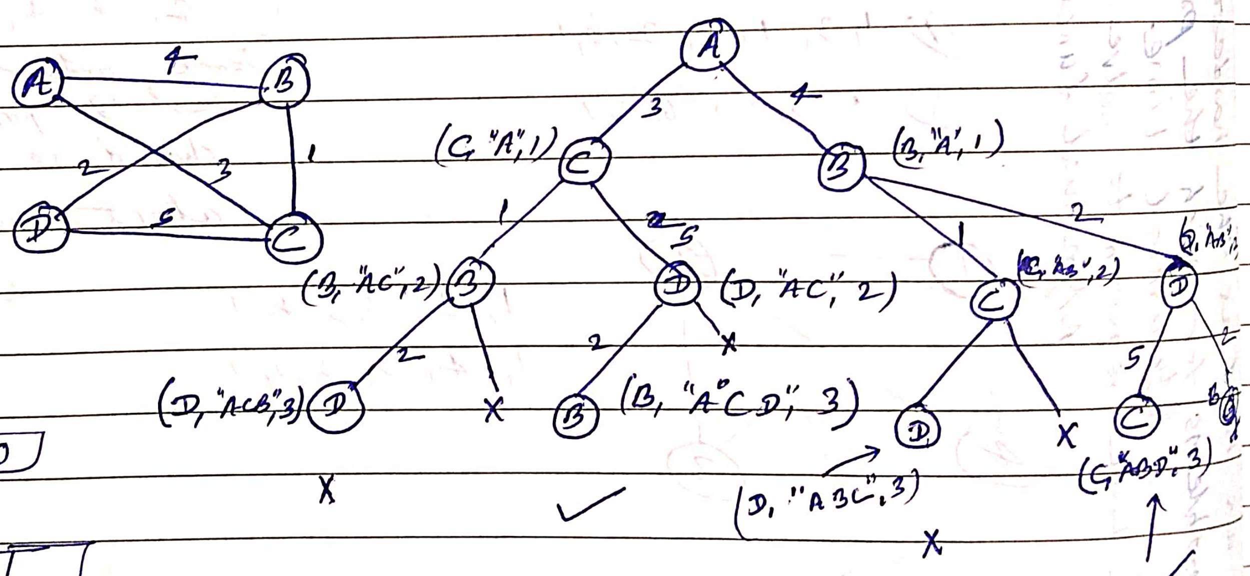
Node 3 is a junction of graph. In any of direction we can't move back. This graph can't be Hamiltonian.



pendent graph

How it works using Backtracking

Q:

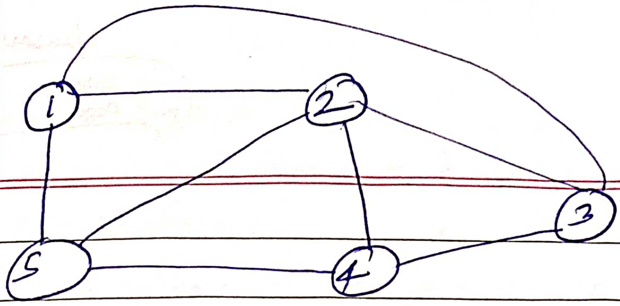


x_i | 0 | 0 | 0 | 0 | 0 |

| A | C | B | D | |

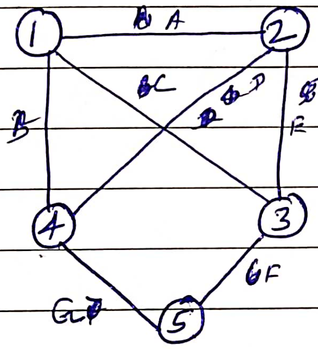
If from last node visit to 1st or starting node there is direct connection this ~~there~~ will hamiltonian otherwise no hamiltonian.

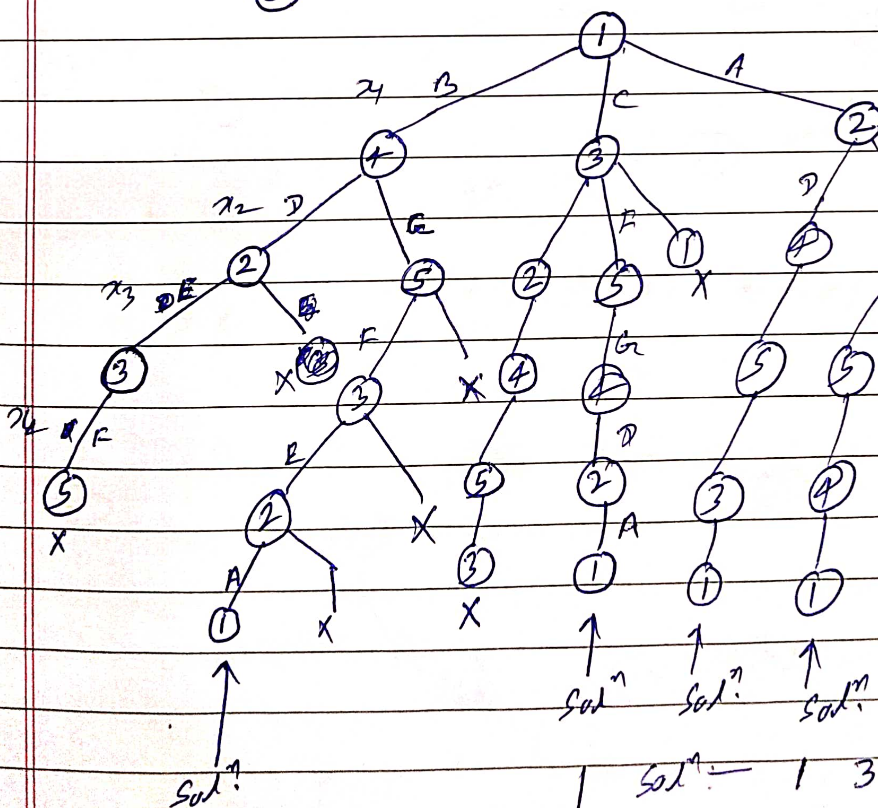
Solⁿ: {A C D B A}, {A, B, D, C, A}



	1	2	3	4	5
1	0	1	1	0	1
2	1	0	1	1	1
3	1	1	0	1	0
4	0	1	1	0	1
5	1	1	0	1	0

	1	2	3	4	5	6
x_i	1	0	0	0	0	0
	1	~~x~~ 2	0	0	0	0
	1	2	~~x~~ 3	0	0	0





All sol^n:-

Sol^n :- 1 3 5 4 2 1

1 2 4 5 3 1

1 2 3 5 4 1

~~1 4 5 3 2 1~~

~~B G F E A~~