Merge Sort

classmate
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Example of Divide and Conquer, and its worst case complexity is $O(n\log n)$. The key idea of sorting the elements using Merge sort is:

- Given a sequence of n elements/keys in $a[1], a[2], \ldots, a[n]$.
- General idea is to split elements in two parts. $a[1], \ldots, a[n/2]$ and $a[\frac{n}{2}+1], \ldots, a[n]$.
- Each part is individually sorted, and
- The resulting sorted sequences are merge to produce a single sorted sequence of n elements.

$\lfloor x \rfloor$ = higest integer not less than x

Therefore, Merge sort describes the above process using recursion and function Merge, which merge two sorted sets.

Write here Algo'

Example An array of 10 elements $a[1:10]$
$= (310, 285, 179, 652, 351, 423, 861, 254, 450, 520)$

Merge sort begins by split elements into two equal parts ~~a[5]~~ $a[1:5]$ & $a[6:10]$. ~~Further~~ ~~a[1:5]~~

310, 285, 179, 652, 351 | 423, 861, 254, 450, 520

Further $a[1:5]$, then splitted into two parts, $a[1:3]$ & $a[4:5]$

310, 285, 179, | 652, 351 | 423, 861, 254, 450, 520

Further $a[1:3]$ is splitted into two as $a[1:2]$ & $a[3]$

310, 285 | 179 | 652, 351 | 423, 861, 254, 450, 520

Further be in $a[1], a[2]$

310 | 285 | 179 | 652, 351 | 425, 861, 254, 450, 520

Now calls merge, elements a[1] and a[2] are merge to yield

285, 310 | 179 | 652, 351 | 423, 861, 254, 450, 520

Then a[3] is merged with a[1:2] and
179, 285, 310 | 652, 351 | 423, 861, 254, 450, 520
is produced. Next, a[4] and a[5] are merged

179, 285, 310 | 351, 652 | 423, 861, 254, 450, 520

and then a[1:3] and a[4:5]:
179, 285, 310, 351, 652 | 423, 861, 254, 450, 520

At this point, the algorithm returns to merge sort for second half list process. Repeat the second half ~~recursive~~ calls

179, 285, 310, 351, 652 | 423 | 861 | 254 | 450, 520

a[6] & a[7] are merged. Then a[8] is merged with a[1:7]

179, 285, 310, 351, 652 | 254, 423, 861 | 450, 520

Next, a~~[6:8] merged to~~ a[9] & a[10] are merged, and then a[6:8] and a[9:10]

179, 285, 310, 351, 652 | 254, 423, 450, 520, 861

At this point there are two sorted subarrays and finally merge produces the fully sorted result

179, 254, 285, 310, 351, 423, 450, 520, 652, 861

Algorithm (Merge Sort)

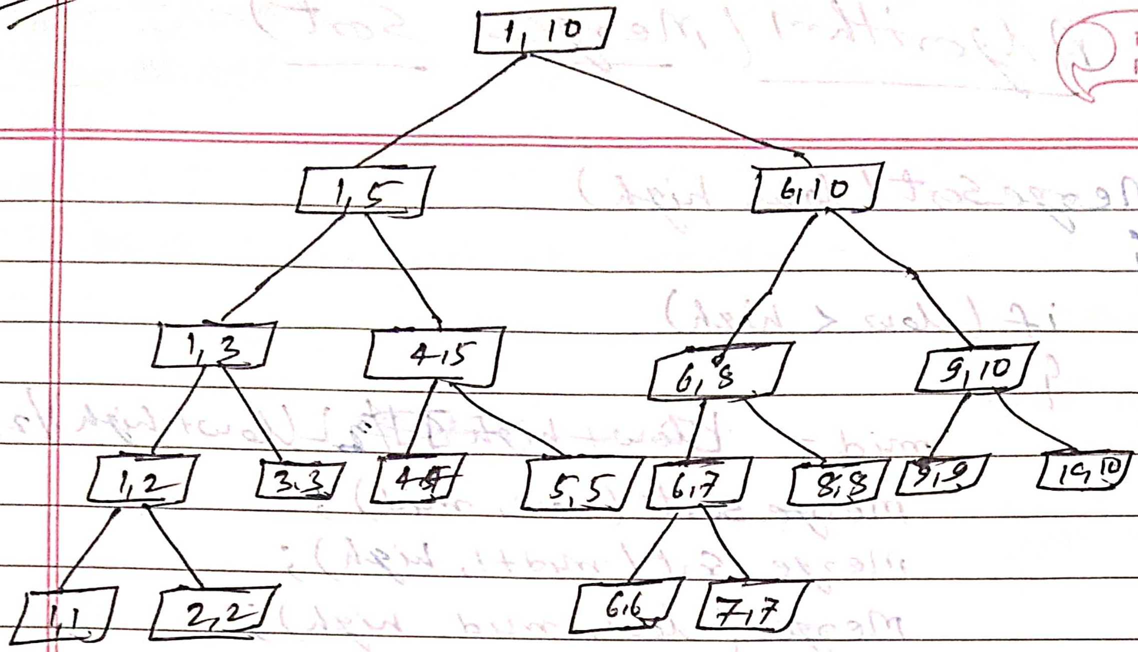
```
MergeSort(low, high)
{
    if(low < high)
    {
        mid = ⌊(low+high)/2⌋
        MergeSort(low, mid);
        MergeSort(mid+1, high);
        Merge(low, mid, high);
    }
}
```

Algorithm Merge

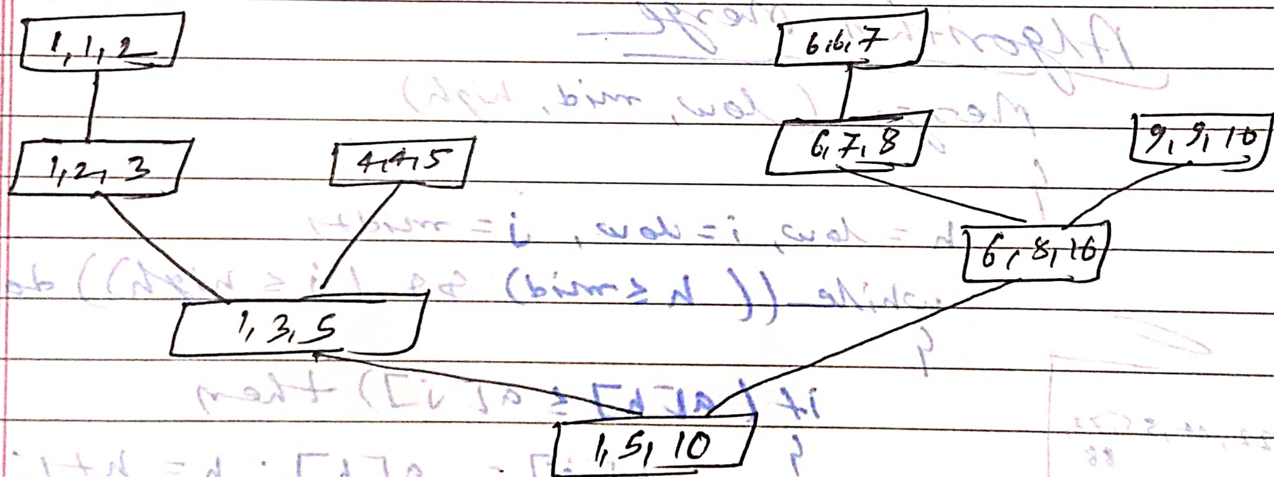
```
Merge(low, mid, high)
{
    h = low, i = low, j = mid+1
    while((h ≤ mid) && (j ≤ high)) do
    {
        if(a[h] ≤ a[j]) then
        {       b[i] = a[h]; h = h+1;
        }
        else
        {  b[i] = a[j]; j = j+1;
        }
        i = i+1;
    }
    if(h > mid) then
        for k = j to high do
        {       b[i] = a[k]; i = i+1;
        }
    else
        for k = h to mid do
        {       b[i] = a[k]; i = i+1;
        }
    for k = low to high do   a[k] = b[k];
}
```

A = 22, 44, 55, 77, 88

B = 33, 66, 70



Tree of Calls of MergeSort(1, 10)



Tree of Calls of Merge

Notations

$f(n) \le m|g(n)|$, then

$f(n) = O(g(n))$

$\frac{1}{2}(cn/c\,n/2)$

$2(n)$

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Analysis

The time for merging operation is propotional to n, then the computing time for merge sort is described by recurrence relation

$$T(n) = \begin{cases} a & ; n=1, a \text{ a constant} \\ 2T(n/2) + cn & ; n>1, c \text{ a constant} \end{cases}$$

When n is a power of 2, $n = 2^K$, we can solve this eqn. by successive substitution

$$T(n) = 2\left(2T\left(\tfrac{n}{4}\right) + cn/2\right) + cn$$

$$= 4T(n/4) + 2cn$$

$$= 4\left(2T(n/8) + \tfrac{cn}{4}\right) + 2cn$$

for n no. of partitions.

$$= 2^K \, T(1) + Kcn$$

$$= an + cn\log_2 n$$

$$= n(a + c\log n)$$

$$= O(n\log n)$$

$2^K = n$

$\boxed{K = \log_2 n}$

It is easy to see that if $2^K < n \le 2^{K+1}$, then $T(n) \le T(2^{K+1})$

$2^K = n$

K

$2^K = n$

$\log_2 2^K = \log_2 n$

$\boxed{K = \log_2 n}$

$\boxed{an + cn\log_2 n}$

Highest order is

$\boxed{n\log_2 n}$

Rough

$$T(n) = 2T(n/2) + cn$$

$$= 2\left(2T(n/4) + \tfrac{cn}{2}\right) + cn$$

$$= 4T(n/4) + 2cn$$

$$= 4\left(2T(n/8) + \tfrac{cn}{4}\right) + 2cn$$

$$= 2^K\, T(1) + Kcn$$

$a.n$