Greedy Method

The greedy method is a straightforward design technique to provide a solution of a problem which best suits to immediate needs. The problems have n inputs and require us to obtain a subset that satisfies some constraints, any subset that satisfies that constraints is called a feasible solution. We need to find a feasible solution that either maximizes or minimizes a given objective function. A feasible solution that does this is called an optimal solution.

In general, we consider the following constraints and optimization function:

- Candidate set: From which solution is created
- Selection set: Used to select the best candidate
- Feasible function: Used to check whether constraints are satisfied or not
- Objective function: Used to find minimized or maximised for problem.
- Solution function: It indicates that a complete solution has been reached.

Knapsack Problem

Suppose we given "n" objects and a Knapsack capacity "W". Each object has its weights and has a ~~capacity~~ cost of profit that we need to place in "W".

The objective is to place the objects into the Knapsack, ~~capacity~~ within limits to its capacity, so that for each objects its

$\sum P_i x_i$ to be maximum profit earned; where x_i is, $0 \le x_i \le 1$.

This x_i is dependent on objects will taken full or partial parts of objects.

There are two problem parts of Knapsack either of 0/1 Knapsack (Objects included in solution as full or null consideration) or Fractional Knapsack (Objects included in solution as full or null or partial(s)).

The objective of this problem is to maximize $\sum P_i x_i$, with subject limited to $\sum W_i x_i \le W$.

Fractional Knapsack

Q:1 Consider a Knapsack problem for $n=3$, $W=20$, $(P_1, P_2, P_3) = (25, 14, 15)$ and $(w_1, w_2, w_3) = (18, 15, 10)$ then find the optimal solution.

Sol^n:

- First findout Profit/Weight for each object.
- Place the objects in Knapsack according to higher to lower profit/weight in limits to Knapsack capacity.

$R_1 = \frac{25}{18} = 1.38$, $R_2 = \frac{14}{15} = 0.93$, $R_3 = \frac{15}{10} = 1.5$

order of P/W: 2, 3, 1

S No.	P_i	W_i	X_i	Knapsack $W(20)$	Profit earned for each $\sum P_i X_i$
1.	15	10	1	$20-10=10$	$15 \times 1 = 15$
2.	25	18	10/18	$10-10=0$	$25 \times \frac{10}{18} = 13.8$
3.	14	15	0	0	0

Total profit earned $= \sum P_i X_i$
$= 15 + 13.8 = 28.88$

Q:-2 Find a solution by a Greedy approach of the following items to place in a Knapsack capacity $M = 20$, and its respective weights & profits are as:

Weight $W = \{4, 10, 5, 6, 8, 3\}$
Profit $P = \{20, 15, 30, 18, 16, 21\}$

Sol^n :-

Order highest to lowest

$\frac{\text{Profit}}{\text{Weight}}$ of each items are :

(i) $\frac{20}{4} = 5$ ③

(ii) $\frac{15}{10} = 1.5$ ⑥

(iii) $\frac{30}{5} = 6$ ②

(iv) $\frac{18}{6} = 3$ ④

(v) $\frac{16}{8} = 2$ ⑤

(vi) $\frac{21}{3} = 7$ ①

S.No.	P_i	W_i	X_i	$W(20)$	$\sum P_i X_i$
1.	21	3	1	$20-3=17$	$21 \times 1 = 21$
2.	30	5	1	$17-5=12$	$21 + 30 \times 1 = 51$
3.	20	4	1	$12-4=8$	$51 + 20 \times 1 = 71$
4.	18	6	1	$8-6=2$	$71 + 18 \times 1 = 89$
5.	16	8	2/8	$2-2=0$	$89 + 16 \times \frac{2}{8} = 93$
6.	15	10	0	0	0

Profit $\sum P_i X_i = 93$

Q-3 Consider the Knapsack problem for $n = 5$, $M = 100$, $(P_1, P_2, P_3, P_4, P_5) = (20, 30, 60, 40, 60)$ and $(w_1, w_2, w_3, w_4, w_5) = (10, 20, 30, 40, 50)$, then find the optimal solution.

Ans: $R_1 = \frac{20}{2} = 2$, $R_2 = \frac{30}{20} = 1.5$, $R_3 = \frac{60}{30} = 2$, $R_4 = \frac{40}{40} = 1$, $R_5 = \frac{60}{50} = 1.2$

S.No.	P_i	w_i	Knapsack $M = 100$	Profit Earned $\Sigma P_i x_i$	x_i
1	20	10	$100 - 10 = 90$	20	1
2	60	30	$90 - 30 = 60$	60	1
3	30	20	$60 - 20 = 40$	30	1
4	60	50	$40 - 40 = 0$	$6\cancel{0} \times \frac{\cancel{40}8}{\cancel{50}} = 48$	40/50
5	40	40			

Total Profit Earned $\Sigma P_i x_i = 158$

Q-4 Find the optimal solution to the Knapsack instance $n = 7$, $m = 15$, $(P_1, \dots, P_7) = \{10, 5, 15, 7, 6, 18, 3\}$ and $(w_1, \dots, w_7) = \{2, 3, 5, 7, 1, 4, 1\}$.

Solⁿ: $R1 = \frac{10}{2} = 5$, ② $R2 = \frac{5}{3} = 1.66$, ⑥ $R3 = \frac{15}{5} = 3$ ⑤

$R4 = \frac{7}{7} = 1$, ⑦ $R5 = \frac{6}{1} = 6$, ① $R6 = \frac{18}{4} = 4.5$, ③ $R7 = \frac{3}{1} = 3$ ④

S.No.	P_i	w_i	x_i	Knapsack $M = 15$	Profit Earned $\Sigma P_i x_i$
1.	6	1	1	$15 - 1 = 14$	$6 \times 1 = 6$
2.	10	2	1	$14 - 2 = 12$	10
3.	18	4	1	$12 - 4 = 8$	18
4.	3	1	1	$8 - 1 = 7$	3
5.	15	5	1	$7 - 5 = 2$	15
6.	5	3	2/3	$2 - 2$	$5 \times \frac{2}{3}$
7.	7	7	—	—	

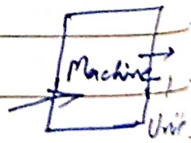
$\Sigma P_i x_i = 6 + 10 + 18 + 3 + 15 + 5 \times \frac{2}{3} = 52 + 3.33 = 55.33$

Job Sequencing with Deadline

Let we have given a set of n jobs, each job in an integer associated with deadline $d_i \geq 0$ and profit $P_i > 0$. For any job i the profit P_i is earned if the job is completed by its deadline. To complete a job, one has to process the job on a machine for one unit of time. Only one job is available for processing jobs. A feasible solution is a subset J of jobs such that each job ~~is~~ must be completed by its deadline. The value of the feasible solution is a ~~subset of~~ sums of ~~subsets~~ ~~completed~~ profits of the jobs in J, or $\sum_{i \in J} P_i$. An optimal solution is a feasible solution with maximum profit value.

Real-life Examples

1. Suppose one shopkeeper having works to complete with its deadline in a day or an limited hour for customer to wait. The feasible solution creates a solutions of all possiblities with its deadline. Where optimal solution find the maximum profit earned.

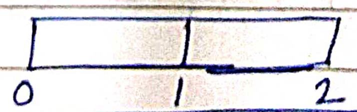


or Typist

Example-1

Let $n = 4$, $(P_1, P_2, P_3, P_4) = (100, 10, 15, 27)$ and ~~d~~ $(d_1, d_2, d_3, d_4) = (2, 1, 2, 1)$. Create a feasible solutions, and find an Optimal solution?

Solⁿ:



Here maximum deadline = 2
{Either one customer can maximum wait for two hour to done ~~is~~ his job

Here n = 4 Jobs.

J_1	100	2
J_2	10	1
J_3	15	2
J_4	27	1

	Feasible Solution	Processing Sequence	Profit Value
1.	(1,2)	2, 1	110
2.	(1,3)	1,3 or 3,1	115
3.	(1,4)	4, 1	127
4.	(2,3)	2, 3	25
5.	(3,4)	4, 3	42
6.	(1)	1	100
7.	(2)	2	10
8.	(3)	3	15
9.	(4)	4	27

(1,2) → 0 | J_2 | J_1 | 2 (slots 0, 1, 2; deadlines 2, 1)

{Here we need to keep in any of the slot, but for maximum profit we keep in maximum ~~of this slot, so that other~~ lower slot period can be best utilized.

(1,3) → | J_1 | J_2 | / | J_2 | J_1 | (0, 1, 2)

(1,4) → | J_4 | J_1 | (0, 1, 2)

(2,3) → | J_2 | J_3 | (0, 1, 2)

(2,4) → (0, 1, 2) ×

(3,4) →

Here all we can observe that the maximum profit gain is combination of (1,4) = 127. Therefore, Optimal solution = 127

Trick:- If the jobs are completed in non increasing of Profit's P_i then one can get the maximum profit.

Algorithm

~~Greedy J~~

```
GreedyJob( d, J, n )
// J is a set of jobs that can be comple-
      ted by their deadline.
{
   J = {1};
   for i = 2 to n do
   {
      if (all jobs in J U {i} can be completed
       by their deadline) then J = J U {i};
   }
}
```

Q-2 Let $n = 5$

Jobs	J_1	J_2	J_3	J_4	J_5
Profits	20	15	10	5	1
Deadlines	2	2	1	3	3

Find ~~Optimal~~ Feasible and an optimal solution.

Solⁿ:-

J	Assigned slots	Job considered	Action	Profit
ϕ	none	1	~~[illegible]~~	

Sol^n:-

J	Assigned slot	Jobs considered	Action		Profits
ϕ	None	1	Assign to	[1, 2]	0
{1}	[1,2]	2	Assign to	[0, 1]	20
{1,2}	[0,1], [1, 2]	3	Can't fit,	reject	35
{1,2}	[0,1], [1,2]	4	Assign to	[2,3]	35
{1,2,3}	[0,1],[1,2], [2,3]	5	Reject		40

J_1	J_2	J_4

0 1 2 3

The optimal solution is $J = \{1, 2, 4\}$ with profit of 40.