Dynamic Programming
0/1 Knapsack Problem

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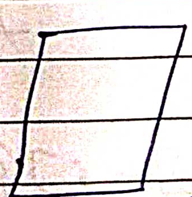
Suppose we are given "n" objects and a Knapsack capacity M, where each objects has its weights W and its ~~weight~~ profits P. The objective is to obtain of filling of a Knapsack that maximizes the total profit earch, such that $\sum\limits_{0 \le i \le n-1} P_i \times x_i$ to be maximum, and capacity W is in the limit of $\sum W_i \times x_i \le M$. The capacity of Knapsack, we require the total weights of all chosen objects must be maximum to M. Therefore, the object is to maximize $\sum P_i \times x_i$ maximum and subject is in limit to $\sum W_i \times x_i \le M$

Example

Suppose we have a Knapsack M=8 and n number of objects we need to fill, where each objects profits & weights are as follows:

$$P = \{1, 2, 5, 6\}$$

$$W = \{2, 3, 4, 5\}$$



M = 8

$$\max \sum P_i \times x_i \ (\text{Max})$$

$$\sum W_i \times x_i \le M$$

For solving a problem, using dynamic approach, we need to create a V table which contains weights (Knapsack weights)

and list of items, with their corresponding Weight W and Profit P. The objective is to fill the V table corresponding to its current capacity of Knapsack and objects selection. If the object(s) are fit in the Knapsack M, then we write its profit P. Here objective is to get (4, 8) → Knapsack M consideration
→ No. of items consideration

P	W	i \ M	0	1	2	3	4	5	6	7	8
		0	0	0	0	0	0	0	0	0	0
1	2	1	0	0	1	1	1	1	1	1	1
2	3	2	0	0	1	2	2	3	3	3	3
5	4	3	0	0	1	2	5	5	6	7	7
6	5	4	0	0	1	2	5	6	6	7	8

The same can also be obtain as

$$V[i, w] = \max\{V[i-1, w], V[i-1, w-W[i]] + P[i]\}$$

Examples

$$V[4, 1] = \max\{V[3, 1], V[3, \underbrace{(1-5)}_{\text{Undefined}}] + 6\}$$

$$= \max\{0, 0\}$$

$$= 0$$

Till $w - w[i]$

$2-5,\ 3-5,\ 4-5$ → This are undefined.

~~so it~~

for ~~&~~ $v[4,2]$, $v[4,3]$ & $v[4,4] = v[3,2] = v[3,3] = v[4,4]$

⇒ 1, 2, 5 respectively

$v[4,5] = \max\{v[3,5], v[3, 5-5] + 6\}$
$\max\{5, 0+6\}$
$= 6$

$v[4,6] = \max\{v[3,6], v[3, 6-5] + 6\}$
$= \max\{6, 0+6\}$
$= 6$

$v[4,7] = \max\{v[3,7], v[3, 7-5] + 6\}$
$= \max\{7, 1+6\} = 7$

$v[4,8] = \max\{v[3,8], v[3, 8-5] + 6\}$
$= \max\{7, 2+6\} = 8$