

CONCEPTS OF PHYSICS

[VOLUME 2]

H C VERMA, PhD

Retired Professor
Department of Physics
IIT, Kanpur



Bharati Bhawan
PUBLISHERS & DISTRIBUTORS

for more buy it

*Dedicated to
Indian Philosophy & Way of Life
of which
my parents were
an integral part*

for more buy it

FOREWORD

A few years ago I had an occasion to go through the book *Calculus* by L V Terasov. It unravels intricacies of the subject through a dialogue between Teacher and Student. I thoroughly enjoyed reading it. For me this seemed to be one of the few books which teach a difficult subject through inquisition, and using programmed concept for learning. After that book, Dr Harish Chandra Verma's book on physics, *CONCEPTS OF PHYSICS* is another such attempt, even though it is not directly in the dialogue form. I have thoroughly appreciated it. It is clear that Dr Verma has spent considerable time in formulating the structure of the book, besides its contents. I think he has been successful in this attempt. Dr Verma's book has been divided into two parts because of the size of the total manuscript. There have been several books on this subject, each one having its own flavour. However, the present book is a totally different attempt to teach physics, and I am sure it will be extremely useful to the undergraduate students. The exposition of each concept is extremely lucid. In carefully formatted chapters, besides problems and short questions, a number of objective questions have also been included. This book can certainly be extremely useful not only as a textbook, but also for preparation of various competitive examinations.

Those who have followed Dr Verma's scientific work always enjoyed the outstanding contributions he has made in various research areas. He was an outstanding student of Physics Department of IIT Kanpur during his academic career. An extremely methodical, sincere person as a student, he has devoted himself to the task of educating young minds and inculcating scientific temper amongst them. The present venture in the form of these two volumes is another attempt in that direction. I am sure that young minds who would like to *learn physics in an appropriate manner* will find these volumes extremely useful.

I must heartily congratulate Dr Harish Chandra Verma for the magnificent job he has done.

Y R Waghmare
Professor of Physics
IIT Kanpur.

for more buy it

P R E F A C E

Why a new book ?

Excellent books exist on physics at an introductory college level so why a new one ? Why so many books exist at the same level, in the first place, and why each of them is highly appreciated ? It is because each of these books has the privilege of having an author or authors who have *experienced* physics and have their own method of communicating with the students. During my years as a physics teacher, I have developed a somewhat different methodology of presenting physics to the students. *Concepts of Physics* is a translation of this methodology into a textbook.

Prerequisites

The book presents a calculus-based physics course which makes free use of algebra, trigonometry and co-ordinate geometry. The level of the latter three topics is quite simple and high school mathematics is sufficient. Calculus is generally done at the introductory college level and I have assumed that the student is enrolled in a concurrent first calculus course. The relevant portions of calculus have been discussed in Chapter 2 so that the student may start using it from the beginning.

Almost no knowledge of physics is a prerequisite. I have attempted to start each topic from the zero level. A receptive mind is all that is needed to use this book.

Basic philosophy of the book

The motto underlying the book is *physics is enjoyable*.

Being a description of the nature around us, physics is our best friend from the day of our existence. I have extensively used this aspect of physics to introduce the physical principles starting with common day occurrences and examples. The subject then appears to be friendly and enjoyable. I have taken care that numerical values of different quantities used in problems correspond to real situations to further strengthen this approach.

Teaching and training

The basic aim of physics teaching has been to let the student know and understand the principles and equations of physics and their applications in real life.

However, to be able to use these principles and equations correctly in a given physical situation, one needs further training. A large number of *questions and solved and unsolved problems* are given for this purpose. Each question or problem has a specific purpose. It may be there to bring out a subtle point which might have passed unnoticed while doing the text portion. It may be a further elaboration of a concept developed in the text. It may be there to make the student react when several concepts introduced in different chapters combine and show up as a physical situation and so on. Such tools have been used to develop a culture: *analyse the situation, make a strategy to invoke correct principles and work it out*.

Conventions

I have tried to use symbols, names, etc., which are popular nowadays. SI units have been consistently used throughout the book. SI prefixes such as *micro*, *milli*, *mega*, etc., are used whenever they make the presentation more readable. Thus, $20 \mu\text{F}$ is preferred over $20 \times 10^{-6} \text{ F}$. Co-ordinate sign convention is used in geometrical optics. Special emphasis has been given to dimensions of physical quantities. Numerical values of physical quantities have been mentioned with the units even in equations to maintain dimensional consistency.

I have tried my best to keep errors out of this book. I shall be grateful to the readers who point out any errors and/or make other constructive suggestions.

H C Verma

ACKNOWLEDGEMENTS

The work on this book started in 1984. Since then, a large number of teachers, students and physics lovers have made valuable suggestions which I have incorporated in this work. It is not possible for me to acknowledge all of them individually. I take this opportunity to express my gratitude to them. However, to Dr S B Mathur, who took great pains in going through the entire manuscript and made valuable comments, I am specially indebted. I am also beholden to my colleagues Dr A Yadav, Dr Deb Mukherjee, Mr M M R Akhtar, Dr Arjun Prasad, Dr S K Sinha and others who gave me valuable advice and were good enough to find time for fruitful discussions. To Dr T K Dutta of B E College, Sibpur I am grateful for having taken time to go through portions of the book and making valuable comments.

I thank my student Mr Shailendra Kumar who helped me in checking the answers. I am grateful to Dr B C Rai, Mr Sunil Khijwania & Mr Tejaswi Khijwania for helping me in the preparation of rough sketches for the book.

Finally, I thank the members of my family for their support and encouragement.

H C Verma

TO THE STUDENTS

Here is a brief discussion on the organisation of the book which will help you in using the book most effectively. The book contains 47 chapters divided in two volumes. Though I strongly believe in the underlying unity of physics, a broad division may be made in the book as follows:

Chapters 1–14: Mechanics

- 15–17: Waves including wave optics
- 18–22: Optics
- 23–28: Heat and thermodynamics
- 29–40: Electric and magnetic phenomena
- 41–47: Modern physics

Each chapter contains a description of the physical principles related to that chapter. It is well supported by mathematical derivations of equations, descriptions of laboratory experiments, historical background, etc. There are "in-text" solved examples. These examples explain the equation just derived or the concept just discussed. These will help you in fixing the ideas firmly in your mind. Your teachers may use these in-text examples in the classroom to encourage students to participate in discussions.

After the theory section, there is a section on *Worked Out Examples*. These numerical examples correspond to various thinking levels and often use several concepts introduced in that chapter or even in previous chapters. You should read the statement of a problem and try to solve it yourself. In case of difficulty, look at the solution given in the book. Even if you solve the problem successfully, you should look into the solution to compare it with your method of solution. You might have thought of a better method, but knowing more than one method is always beneficial.

Then comes the part which tests your understanding as well as develops it further. *Questions for Short Answer* generally touch very minute points of your understanding. It is not necessary that you answer these questions in a single sitting. They have great potential to initiate very fruitful discussions. So, freely discuss these questions with your friends and see if they agree with your answer. Answers to these questions are not given for the simple reason that the answers could have cut down the span of such discussions and that would have sharply reduced the utility of these questions.

There are two sections on multiple-choice questions, namely OBJECTIVE I and OBJECTIVE II. There are four options following each of these questions. Only one option is correct for OBJECTIVE I questions. Any number of options, zero to four, may be correct for OBJECTIVE II questions. Answers to all these questions are provided.

Finally, a set of numerical problems are given for your practice. Answers to these problems are also provided. The problems are generally arranged according to the sequence of the concepts developed in the chapter but they are not grouped under section-headings. I don't want to bias your ideas beforehand by telling you that this problem belongs to that section and hence use that particular equation. You should yourself look into the problem and decide which equations or which methods should be used to solve it. Many of the problems use several concepts developed in different sections of the chapter. Many of them even use the concepts from the previous chapters. Hence, you have to plan out the strategy after understanding the problem.

Remember, no problem is difficult. Once you understand the theory, each problem will become easy. So, don't jump to exercise problems before you have gone through the theory, the worked-out problems and the objectives. Once you feel confident in theory, do the exercise problems. The exercise problems are so arranged that they gradually require more thinking.

I hope you will enjoy *Concepts of Physics*.

H C Verma

for more buy it

Table of Contents

Chapters 1–22

Chapter 23

Heat and Temperature

- 23.1 Hot and Cold Bodies
- 23.2 Zeroth Law of Thermodynamics
- 23.3 Defining Scale of Temperature : Mercury and Resistance Thermometers
- 23.4 Constant Volume Gas Thermometer
- 23.5 Ideal Gas Temperature Scale
- 23.6 Celsius Temperature Scale
- 23.7 Ideal Gas Equation
- 23.8 Callender's Compensated Constant Pressure Thermometer
- 23.9 Adiabatic and Diathermic Walls
- 23.10 Thermal Expansion
- Worked Out Examples
- Questions for Short Answer
- Objective I
- Objective II
- Exercises

Chapter 24

Kinetic Theory of Gases

- 24.1 Introduction
- 24.2 Assumptions of Kinetic Theory of Gases
- 24.3 Calculation of the Pressure of an Ideal Gas
- 24.4 rms Speed
- 24.5 Kinetic Interpretation of Temperature
- 24.6 Deductions from Kinetic Theory
- 24.7 Ideal Gas Equation
- 24.8 Maxwell's Speed Distribution Law
- 24.9 Thermodynamic State
- 24.10 Brownian Motion
- 24.11 Vapour
- 24.12 Evaporation
- 24.13 Saturated and Unsaturated Vapour : Vapour Pressure
- 24.14 Boiling
- 24.15 Dew Point
- 24.16 Humidity
- 24.17 Determination of Relative Humidity
- 24.18 Phase Diagrams : Triple Point
- 24.19 Dew and Fog
- Worked Out Examples
- Questions for Short Answer
- Objective I
- Objective II
- Exercises

Volume-1

Chapter 25

Calorimetry

- | | | |
|------|--|----|
| 25.1 | Heat as a Form of Energy | 39 |
| 1 | Units of Heat | 39 |
| 1 | Principle of Calorimetry | 39 |
| 1 | Specific Heat Capacity and Molar Heat Capacity | 39 |
| 1 | Determination of Specific Heat Capacity in Laboratory | 40 |
| 3 | Specific Latent Heat of Fusion and Vaporization | 41 |
| 5 | Measurement of Specific Latent Heat of Fusion of Ice | 41 |
| 5 | Measurement of Specific Latent Heat of Vaporization of Water | 42 |
| 5 | Mechanical Equivalent of Heat | 43 |
| 6 | Worked Out Examples | 44 |
| 6 | Questions for Short Answer | 46 |
| 6 | Objective I | 46 |
| 7 | Objective II | 46 |
| 11 | Exercises | 47 |

Chapter 26

Laws of Thermodynamics

- | | | |
|------|---------------------------------------|----|
| 26.1 | The First Law of Thermodynamics | 49 |
| 26.2 | Work Done by a Gas | 50 |
| 26.3 | Heat Engines | 51 |
| 26.4 | The Second Law of Thermodynamics | 53 |
| 26.5 | Reversible and Irreversible Processes | 54 |
| 26.6 | Entropy | 55 |
| 26.7 | Carnot Engine | 55 |
| 16 | Worked Out Examples | 57 |
| 17 | Questions for Short Answer | 60 |
| 18 | Objective I | 61 |
| 19 | Objective II | 61 |
| 20 | Exercises | 62 |

Chapter 27

Specific Heat Capacities of Gases

- | | | |
|------|---|----|
| 27.1 | Two Kinds of Specific Heat Capacities of Gases | 65 |
| 27.2 | Relation Between C_p and C_v for an Ideal Gas | 66 |
| 27.3 | Determination of C_p of a Gas | 67 |
| 27.4 | Determination of C_v of a Gas | 68 |
| 27.5 | Isothermal and Adiabatic Processes | 68 |
| 27.6 | Relations between p , V , T in a Reversible Adiabatic Process | 69 |
| 27.7 | Work Done in an Adiabatic Process | 70 |
| 27.8 | Equipartition of Energy | 70 |
| 26 | Worked Out Examples | 72 |
| 32 | Questions for Short Answer | 76 |
| 32 | Objective I | 76 |
| 33 | Objective II | 77 |
| 34 | Exercises | 77 |

Chapter 28

Heat Transfer

28.1 Thermal Conduction	81
28.2 Series and Parallel Connection of Rods	82
28.3 Measurement of Thermal Conductivity of a Solid	83
28.4 Convection	84
28.5 Radiation	84
28.6 Prevost Theory of Exchange	85
28.7 Blackbody Radiation	85
28.8 Kirchhoff's Law	86
28.9 Nature of Thermal Radiation	87
28.10 Stefan–Boltzmann Law	87
28.11 Newton's Law of Cooling	88
28.12 Detection and Measurement of Radiation	89
Worked Out Examples	96
Questions for Short Answer	97
Objective I	97
Objective II	98
Exercises	98

Chapter 29

Electric Field and Potential

29.1 What Is Electric Charge ?	104
29.2 Coulomb's Law	104
29.3 Electric Field	105
29.4 Lines of Electric Force	106
29.5 Electric Potential Energy	107
29.6 Electric Potential	107
29.7 Electric Potential due to a Point Charge	108
29.8 Relation between Electric Field and Potential	109
29.9 Electric Dipole	109
29.10 Torque on an Electric Dipole Placed in an Electric Field	110
29.11 Potential Energy of a Dipole Placed in a Uniform Electric Field	112
29.12 Conductors, Insulators and Semiconductors	112
29.13 The Electric Field inside a Conductor	113
Worked Out Examples	114
Questions for Short Answer	119
Objective I	119
Objective II	120
Exercises	121

Chapter 30

Gauss's Law

30.1 Flux of an Electric Field through a Surface	127
30.2 Solid Angle	127
30.3 Gauss's Law and Its Derivation from Coulomb's Law	128
30.4 Applications of Gauss's Law	129
30.5 Spherical Charge Distributions	131
30.6 Earthing a Conductor	134
Worked Out Examples	135
Questions for Short Answer	136
Objective I	139
Objective II	139
Exercises	140

Chapter 31

Capacitors

31.1 Capacitor and Capacitance	144
31.2 Calculation of Capacitance	144
31.3 Combination of Capacitors	145
31.4 Force between the Plates of a Capacitor	147
31.5 Energy Stored in a Capacitor and Energy Density in Electric Field	150
31.6 Dielectrics	150
31.7 Parallel-plate Capacitor with a Dielectric	151
31.8 An Alternative Form of Gauss's Law	152
31.9 Electric Field due to a Point Charge q Placed in an Infinite Dielectric	153
31.10 Energy in the Electric field in a Dielectric	154
31.11 Corona Discharge	154
31.12 High-voltage Generator	155
Worked Out Examples	156
Questions for Short Answer	164
Objective I	165
Objective II	165
Exercises	166

104

Electric Current In Conductors	172
32.1 Electric Current and Current Density	172
32.2 Drift Speed	173
32.3 Ohm's Law	174
32.4 Temperature Dependence of Resistivity	175
32.5 Battery and emf	176
32.6 Energy Transfer in an Electric Circuit	177
32.7 Kirchhoff's Laws	178
32.8 Combination of Resistors in Series and Parallel	179
32.9 Grouping of Batteries	180
32.10 Wheatstone Bridge	181
32.11 Ammeter and Voltmeter	182
32.12 Stretched-wire Potentiometer	183
32.13 Charging and Discharging of Capacitors	185
32.14 Atmospheric Electricity	186
Worked Out Examples	187
Questions for Short Answer	196
Objective I	196
Objective II	197
Exercises	198

127

Thermal and Chemical Effects of Electric Current	206
33.1 Joule's Laws of Heating	206
33.2 Verification of Joule's Laws	207
33.3 Seebeck Effect	207
33.4 Peltier Effect	209
33.5 Thomson Effect	210
33.6 Explanation of Seebeck, Peltier and Thomson Effects	210
33.7 Electrolysis	211
33.8 Faraday's Laws of Electrolysis	211
33.9 Voltmeter or Coulombmeter	213

33.10 Primary and Secondary Cells	214
33.11 Primary Cells	214
33.12 Secondary Cell : Lead Accumulator	215
Worked Out Examples	215
Questions for Short Answer	215
Objective I	217
Objective II	217
Exercises	218

Chapter 34

Magnetic Field

34.1 Introduction	221
34.2 Definition of Magnetic Field \vec{B}	221
34.3 Relation between Electric and Magnetic Fields	221
34.4 Motion of a Charged Particle in a Uniform Magnetic Field	222
34.5 Magnetic Force on a Current-carrying Wire	222
34.6 Torque on a Current Loop	223
Worked Out Examples	224
Questions for Short Answer	225
Objective I	228
Objective II	229
Exercises	230

Chapter 35

Magnetic Field due to a Current

35.1 Biot-Savart Law	237
35.2 Magnetic Field due to Current in a Straight Wire	238
35.3 Force between Parallel Currents	239
35.4 Field due to a Circular Current	239
35.5 Ampere's Law	241
35.6 Magnetic Field at a Point due to a Long, Straight Current	241
35.7 Solenoid	242
35.8 Toroid	243
Worked Out Examples	243
Questions for Short Answer	244
Objective I	245
Objective II	246
Exercises	247

Chapter 36

Permanent Magnets

36.1 Magnetic Poles and Bar Magnets	255
36.2 Torque on a Bar Magnet Placed in a Magnetic Field	255
36.3 Magnetic Field due to a Bar Magnet	255
36.4 Magnetic Scalar Potential	258
36.5 Terrestrial Magnetism	258
36.6 Determination of Dip at a Place	258
36.7 Neutral Point	258
36.8 Tangent Galvanometer	258
36.9 Moving-coil Galvanometer	258
36.10 Shunt	258
36.11 Tangent Law of Perpendicular Fields	260
36.12 Deflection Magnetometer	261
36.13 Oscillation Magnetometer	263

36.14 Determination of M and B_H	270
36.15 Gauss's Law for Magnetism	270
Worked Out Examples	270
Questions for Short Answer	275
Objective I	276
Objective II	277
Exercises	277

Chapter 37

Magnetic Properties of Matter

37.1 Magnetization of Materials : Intensity of Magnetization	279
37.2 Paramagnetism, Ferromagnetism and Diamagnetism	279
37.3 Magnetic Intensity H	280
37.4 Magnetic Susceptibility	281
37.5 Permeability	281
37.6 Curie's Law	281
37.7 Properties of Dia-, Para- and Ferromagnetic Substances	282
37.8 Hysteresis	283
37.9 Soft Iron and Steel	283
Worked Out Examples	284
Questions for Short Answer	285
Objective I	285
Objective II	286
Exercises	286

Chapter 38

Electromagnetic Induction

38.1 Faraday's Law of Electromagnetic Induction	288
38.2 Lenz's Law	288
38.3 The Origin of Induced emf	289
38.4 Eddy Current	291
38.5 Self-induction	291
38.6 Growth and Decay of Current in an LR Circuit	292
38.7 Energy Stored in an Inductor	294
38.8 Mutual Induction	295
38.9 Induction Coil	295
Worked Out Examples	296
Questions for Short Answer	303
Objective I	304
Objective II	305
Exercises	306

Chapter 39

Alternating Current

39.1 Alternating Current	316
39.2 AC Generator or AC Dynamo	316
39.3 Instantaneous and rms Current	317
39.4 Simple AC Circuits	318
39.5 Vector Method to Find the Current in an AC Circuit	320
39.6 More AC Circuits	320
39.7 Power in AC Circuits	322
39.8 Choke Coil	323
39.9 Hot-wire Instruments	323

38.10	DC Dynamo	369
39.11	DC Motor	370
39.12	Transformer	370
	Worked Out Examples	370
	Questions for Short Answer	370
	Objective I	373
	Objective II	374
	Exercises	375
<hr/> Chapter 40 <hr/>		
Electromagnetic Waves		
40.1	Introduction	378
40.2	Maxwell's Displacement Current	382
40.3	Continuity of Electric Current	383
40.4	Maxwell's Equations and Plane Electromagnetic Waves	383
40.5	Energy Density and Intensity	384
40.6	Momentum	384
40.7	Electromagnetic Spectrum and Radiation in Atmosphere	384
	Worked Out Examples	384
	Questions for Short Answer	384
	Objective I	384
	Objective II	384
	Exercises	384
<hr/> Chapter 41 <hr/>		
Electric Current through Gases		
41.1	Discharge through Gases at Low Pressure	388
41.2	Cathode Rays	388
41.3	Canal Rays or Positive Rays	388
41.4	Discovery and Properties of Electron	390
41.5	Thermionic Emission	390
41.6	Diode Valve	391
41.7	Triode Valve	391
41.8	Triode as an Amplifier	391
	Worked Out Examples	392
	Questions for Short Answer	392
	Objective I	393
	Objective II	393
	Exercises	394
<hr/> Chapter 42 <hr/>		
Photoelectric Effect and Wave-Particle Duality		
42.1	Photon Theory of Light	395
42.2	Photoelectric Effect	395
42.3	Matter Waves	397
	Worked Out Examples	397
	Questions for Short Answer	397
	Objective I	398
	Objective II	398
	Exercises	398
<hr/> Chapter 43 <hr/>		
Bohr's Model and Physics of the Atom		
43.1	Early Atomic Models	409
	Worked Out Examples	409
	Questions for Short Answer	409
	Objective I	410
	Objective II	410
	Exercises	410
<hr/> Chapter 44 <hr/>		
X-rays		
332	332	388
332	332	388
333	333	388
	Worked Out Examples	388
	Questions for Short Answer	388
	Objective I	388
	Objective II	388
	Exercises	388
<hr/> Chapter 45 <hr/>		
Semiconductors and Semiconductor Devices		
341	341	397
343	343	397
343	343	397
344	344	397
345	345	400
346	346	400
347	347	401
349	349	402
349	349	403
351	351	405
351	351	406
352	352	409
352	352	414
355	355	416
355	355	417
356	356	418
	Worked Out Examples	418
	Questions for Short Answer	418
	Objective I	419
	Objective II	419
	Exercises	419
<hr/> Chapter 46 <hr/>		
The Nucleus		
359	359	422
359	359	422
363	363	424
363	363	424
364	364	425
364	364	425
365	365	427
	Worked Out Examples	427
	Questions for Short Answer	427
	Objective I	429
	Objective II	429
	Exercises	429
<hr/> Chapter 47 <hr/>		
Modern Physics		
368	368	431
368	368	431
	Worked Out Examples	431
	Questions for Short Answer	431
	Objective I	432
	Objective II	432
	Exercises	432

46.10 Nuclear Fusion	435
46.11 Fusion in Laboratory	436
Worked Out Examples	437
Questions for Short Answer	440
Objective I	440
Objective II	441
Exercises	442

Chapter 47

The Special Theory of Relativity

47.1 The Principle of Relativity	446
47.2 Are Maxwell's Laws Independent of Frame ?	446
47.3 Kinematical Consequences	447
47.4 Dynamics at Large Velocity	451
47.5 Energy and Momentum	452
47.6 The Ultimate Speed	453
47.7 Twin Paradox	453
Worked Out Examples	455
Questions for Short Answer	456
Objective I	457
Objective II	457
Exercises	458

APPENDIX A

APPENDIX B

INDEX

CHAPTER 23

HEAT AND TEMPERATURE

23.1 HOT AND COLD BODIES

When we rub our hands for some time, they become warm. When a block slides on a rough surface, it becomes warm. Press against a rapidly spinning wheel. The wheel slows down and becomes warm. While going on a bicycle, touch the road with your shoe. The bicycle slows down and the shoe becomes warm. When two vehicles collide with each other during an accident, they become very hot. When an aeroplane crashes, it becomes so hot that it catches fire.

In each of these examples, mechanical energy is lost and the bodies in question become hot. Where does the mechanical energy vanish? It goes into the internal energy of the bodies. We conclude that the cold bodies absorb energy to become hot. In other words, a hot body has more internal energy than an otherwise identical cold body.

When a hot body is kept in contact with a cold body, the cold body warms up and the hot body cools down. The internal energy of the hot body decreases and the internal energy of the cold body increases. Thus, energy is transferred from the hot body to the cold body when they are placed in contact. Notice that no mechanical work is done during this transfer of energy (neglect any change in volume of the body). This is because there are no displacements involved. This is different from the case when we lift a ball vertically and the energy of the ball–earth system increases or when a compressed spring relaxes and a block attached to its end speeds up. In the case of lifting the ball, we do some work on the ball and the energy is increased by that amount. In the case of spring–block example, the spring does some work and the kinetic energy of the block increases.

The transfer of energy from a hot body to a cold body is a nonmechanical process. The energy that is transferred from one body to the other, without any mechanical work involved, is called *heat*.

23.2 ZEROTH LAW OF THERMODYNAMICS

Two bodies are said to be in *thermal equilibrium* if no transfer of heat takes place when they are placed in contact. We can now state the Zeroth law of thermodynamics as follows:

If two bodies A and B are in thermal equilibrium and A and C are also in thermal equilibrium then B and C are also in thermal equilibrium.

It is a matter of observation and experience that is described in the Zeroth law. It should not be taken as obvious. For example, if two persons A and B know each other and A and C know each other, it is not necessary that B and C know each other.

The Zeroth law allows us to introduce the concept of temperature to measure the hotness or coldness of a body. All bodies in thermal equilibrium are assigned equal temperature. A hotter body is assigned higher temperature than a colder body. Thus, the temperatures of two bodies decide the direction of heat-flow when the two bodies are put in contact. Heat flows from the body at higher temperature to the body at lower temperature.

23.3 DEFINING SCALE OF TEMPERATURE: MERCURY AND RESISTANCE THERMOMETERS

We are now in a position to say whether two given bodies are at the same temperature or not. If they are not at the same temperature, we also know which is at higher temperature and which is at lower temperature. Our next task is to define a scale of temperature so that we can give numerical value to the temperature of a body. To do this, we can choose a substance and look for a measurable property of the substance which monotonically changes with temperature. The temperature can then be defined as a chosen function of this property. As an example, take a mass of mercury in a glass bulb terminating in a long capillary. The length of the mercury column in the capillary changes with temperature. Each length

corresponds to a particular temperature of the mercury. How can we assign a numerical value corresponding to an observed length of the mercury column?



Figure 23.1

The earlier method was to choose two fixed points of temperature which can be easily reproduced in laboratory. The temperature of melting ice at 1 atm (called ice point) and the temperature of boiling water at 1 atm (called steam point) are often chosen as the fixed points. We arbitrarily assign a temperature t_1 to the ice point and t_2 to the steam point. Suppose the length of the mercury column is l_1 when the bulb is kept in melting ice and it is l_2 when the bulb is kept in boiling water. Thus, a length l_1 of mercury column means that the temperature of the bulb is t_1 and a length l_2 of the column means that the temperature is t_2 . The temperature corresponding to any length l may be defined by assuming a linear relation between l and t ,

$$t = al + b. \quad \dots (23.1)$$

A change of one degree in temperature will mean a change of $\frac{l_2 - l_1}{t_2 - t_1}$ in the length of mercury column. Thus, we can graduate the length of the capillary directly in degrees.

The centigrade system assumes ice point at 0°C and the steam point at 100°C . If the length of the mercury column between its values for 0°C and 100°C is divided equally in 100 parts, each part will correspond to a change of 1°C .

Let l_0 and l_{100} denote the lengths of the mercury column at 0°C and 100°C respectively. From equation (23.1),

$$0 = al_0 + b$$

$$\text{and } 100 = al_{100} + b$$

$$\text{giving } b = -al_0$$

$$\text{and } a = \frac{100}{l_{100} - l_0}.$$

Putting in equation (23.1), the temperature corresponding to a length l is given by

$$t = \frac{l - l_0}{l_{100} - l_0} \times 100 \text{ degree} \quad \dots (23.2)$$

To measure the temperature of a body, the bulb containing mercury is kept in contact with the body and sufficient time is allowed so that the mercury comes to thermal equilibrium with the body. The temperature of the body is then the same as that of the mercury. The length of the column then gives the temperature according to equation (23.2).

Another popular system known as Fahrenheit system assumes 32°F for the ice point and 212°F for the steam point. A change of 1°F means $\frac{1}{180}$ of the interval between the steam point and the ice point. The average temperature of a normal human body is around 98°F . The conversion formula from centigrade to Fahrenheit scale is

$$F = 32 + \frac{9}{5} C.$$

The expansion of mercury is just one thermometric property that can be used to define a temperature scale and prepare thermometers. There may be many others. Electric resistance of a metal wire increases monotonically with temperature and may be used to define a temperature scale. If R_0 and R_{100} denote the resistances of a metal wire at ice point and steam point respectively, we can define temperature t corresponding to the resistance R_t as

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 \text{ degree}. \quad \dots (23.3)$$

The temperatures of the ice point and the steam point are chosen to be 0°C and 100°C as in centigrade scale. A platinum wire is often used to construct a thermometer based on this scale. Such a thermometer is called platinum resistance thermometer and the temperature scale is called the *platinum scale*.

Platinum Resistance Thermometer

The platinum resistance thermometer works on the principle of *Wheatstone bridge* used to measure a resistance. In a Wheatstone bridge, four resistances P , Q , R and X are joined in a loop as shown in figure (23.2a). A galvanometer and a battery are also joined as shown. If

$$\frac{P}{Q} = \frac{R}{X},$$

there is no deflection in the galvanometer and the bridge is called balanced. If the condition is not fulfilled, there is a deflection.

Figure (23.2b) represents the arrangement for a platinum resistance thermometer. A thin platinum wire is coiled on a mica base and placed in a glass tube. Two connecting wires YY going through the

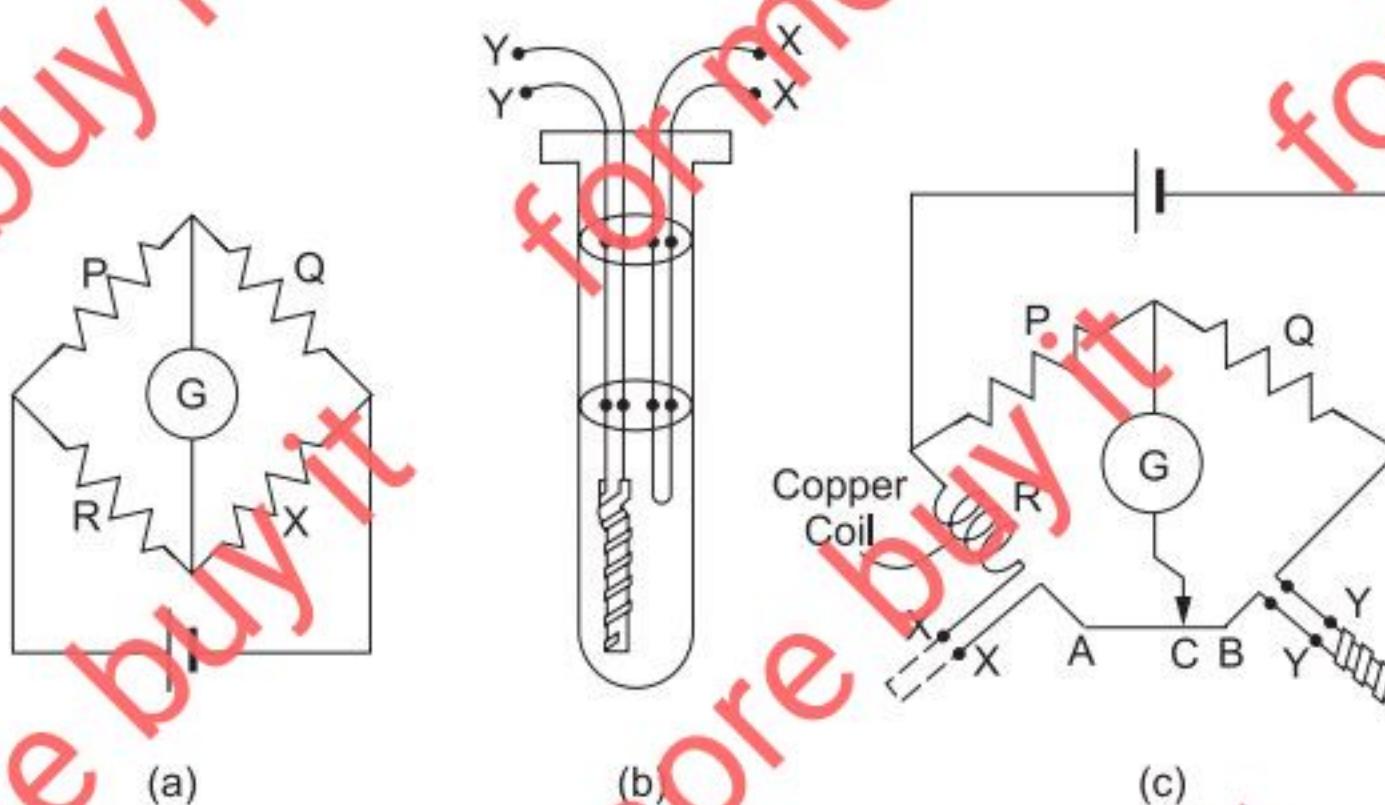


Figure 23.2

ebonite lid of the tube are connected to the platinum coil. A similar copper wire XX , called compensating wire, also goes into the tube as shown in the figure. A Wheatstone bridge is arranged as shown in figure (23.2c). Two equal resistances P and Q are connected in two arms of the bridge. A copper coil having resistance roughly equal to that of the platinum coil is connected in the third arm through the compensating wire XX . The platinum coil is inserted in the fourth arm of the bridge by connecting the points Y, Y in that arm. The end C of the wire connected to the galvanometer can slide on a uniform wire AB of length l . The end A of this wire is connected to the wire XX and the end B is connected to the wire YY . Thus, the copper coil, the compensating wire and the wire AC are in the third arm and the platinum coil, the connecting wire YY and the wire CB are in the fourth arm of the bridge. The test tube containing the platinum coil is immersed in the bath of which we want to measure the temperature. The end C is slid on AB till the deflection in the galvanometer becomes zero. Let $AC = x$.

Suppose the resistance of the copper coil connected in the third arm is R , that of the compensating wire is R_c and that of the wire AB is r . The resistance of the connecting wire YY is the same as R_c and that of the platinum coil is R_t . The resistance of the wire $AC = \frac{r}{l}x$ and that of the wire $CB = \frac{r}{l}(l-x)$. The net resistance in the third arm is $R + R_c + \frac{r}{l}x$ and that in the fourth arm is $R_t + R_c + \frac{r}{l}(l-x)$. For no deflection in the galvanometer,

$$\frac{P}{Q} = \frac{R + R_c + \frac{r}{l}x}{R_t + R_c + \frac{r}{l}(l-x)}.$$

As $P = Q$, we get

$$R_t + \frac{r}{l}(l-x) = R + \frac{r}{l}x$$

$$\text{or, } R_t = (R - r) + \frac{2r}{l}x.$$

If x_0 , x_{100} and x_t are the values of x at the ice point, steam point and the temperature t respectively,

$$R_{100} = (R - r) + \frac{2r}{l}x_{100}$$

$$R_0 = (R - r) + \frac{2r}{l}x_0$$

$$\text{and } R_t = (R - r) + \frac{2r}{l}x_t.$$

This gives

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100 = \frac{x_t - x_0}{x_{100} - x_0} \times 100.$$

Absolute Scale and Ideal Gas Scale

Equation (23.3) tells us that the change in resistance ΔR when the temperature increases by one degree ($\Delta t = 1^\circ$) is

$$\Delta R = \frac{R_{100} - R_0}{100}$$

and is the same for all temperatures. This means that the resistance increases uniformly as the temperature increases. Note that this conclusion follows from the particular definition (23.3) of temperature and is not the property of platinum or the other metal used to form the resistance. Similarly, it follows that the expansion of mercury is uniform on the mercury scale defined by (23.2). If we measure the change in resistance of a platinum wire against the temperature measured on mercury scale, we shall find that the resistance varies slowly at lower temperatures and slightly more rapidly at higher temperatures. Similarly, if the expansion of mercury is measured against the temperature defined on platinum scale (23.3), we shall find that mercury does not expand at uniform rate as the temperature varies.

Thus, the two scales of temperature do not agree with each other. They are forced to agree at ice point and steam point but for other physical states the readings of the two thermometers will be different. In general, the scale depends on the properties of the thermometric substance used to define the scale.

It is possible to define an absolute scale of temperature which does not depend on the thermometric substance and its properties. We now define the ideal gas temperature scale which happens to be identical to the absolute temperature scale.

23.4 CONSTANT VOLUME GAS THERMOMETER

A gas enclosed in a container has a definite volume and a definite pressure in any given state. If the volume is kept constant, the pressure of a gas

increases monotonically with increasing temperature. This property of a gas may be used to construct a thermometer. Figure (23.3) shows a schematic diagram of a constant volume gas thermometer.

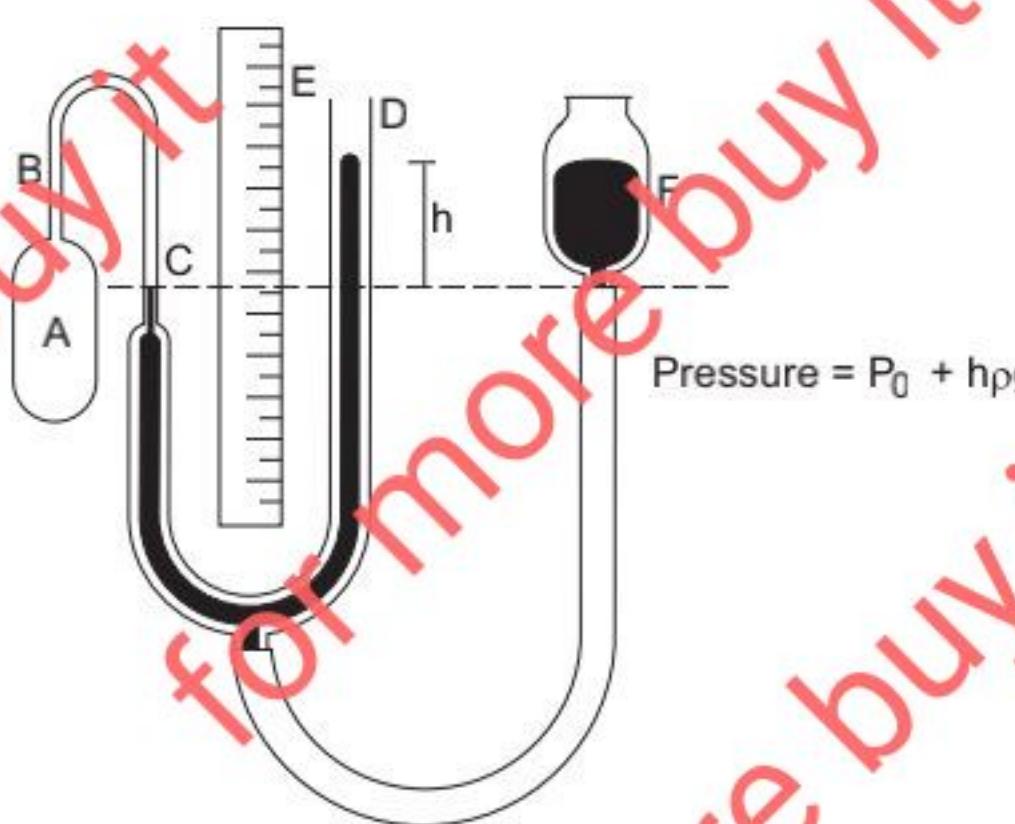


Figure 23.3

A mass of gas is enclosed in a bulb A connected to a capillary BC . The capillary is connected to the manometer CD which contains mercury. The other end of the manometer is open to atmosphere. A vertical metre scale E is fixed in such a way that the height of the mercury column in the tube D can easily be measured. The mercury in the manometer is connected to the mercury reservoir F through a rubber tube.

The capillary BC has a fixed mark at C . By raising or lowering the reservoir F , the mercury level in the left part of the manometer is maintained at C . This ensures that the volume of the gas enclosed in the bulb A (and the capillary BC) remains constant. The pressure of the gas is equal to the atmospheric pressure plus the pressure due to the difference of the mercury columns in the manometer. Thus, $p = p_0 + h\rho g$, where p_0 = atmospheric pressure, h = difference of mercury levels in the manometer tube, ρ = density of mercury and g = acceleration due to gravity.

If the temperature of the bulb is increased and its volume is kept constant by adjusting the height of the reservoir F , the pressure of the gas $p = p_0 + h\rho g$ increases. Thus, a temperature scale may be defined by choosing some suitable function of this pressure. Let us assume that the temperature is proportional to the pressure, i.e.,

$$T = cp, \quad \dots (23.4)$$

where c is some constant.

In addition, the temperature of triple point of water is assigned a value 273.16 K. (Triple point is a state in which ice, water and water vapour can stay together in equilibrium.) The unit is called a kelvin and is denoted by the symbol K. To get the value of the constant c in equation (23.4), we can put the bulb

A in a triple point cell and measure the pressure p_{tr} of the gas. From equation (23.4),

$$273.16 \text{ K} = cp_{tr}$$

or,

$$c = \frac{273.16 \text{ K}}{p_{tr}}$$

The temperature of the gas when the pressure is p is obtained by putting this value of c in equation (23.4). It is

$$T = \frac{p}{p_{tr}} \times 273.16 \text{ K} \quad \dots (23.5)$$

To use the thermometer, we must first determine the pressure of the gas p_{tr} at the triple point. This is a fixed value for the thermometer and is used in any measurement. To measure the temperature of a bath of extended volume, the bulb A is dipped in the bath. Sufficient time is allowed so that the gas in the bulb comes to thermal equilibrium with the bath. The reservoir F is adjusted to bring the volume of the gas to its original value and the pressure p of the gas is measured with the manometer. The temperature T on the gas scale is then obtained from equation (23.5).

One can also define a centigrade scale with gas thermometers. Suppose the pressure of the gas is p_0 when the bulb A is placed in melting ice (ice point) and it is p_{100} when the bulb is placed in a steam bath (steam point). We assign 0°C to the temperature of the ice point and 100°C to the steam point. The temperature t corresponding to a pressure p of the gas is defined by

$$t = \frac{p - p_0}{p_{100} - p_0} \times 100^\circ\text{C}. \quad \dots (23.6)$$

The constant volume gas thermometer allows several errors in the temperature measurement. The main sources of error are the following.

- (a) The space in the capillary tube BC generally remains out of the heat bath in which the bulb A is placed. The gas in BC is, therefore, not at the same temperature as the gas in A .
- (b) The volume of the glass bulb changes slightly with temperature allowing the volume of the gas to change.

Example 23.1

The pressure of air in the bulb of a constant volume gas thermometer is 73 cm of mercury at 0°C, 100.3 cm of mercury at 100°C and 77.8 cm of mercury at room temperature. Find the room temperature in centigrades.

$$\begin{aligned} \text{Solution : We have } t &= \frac{p - p_0}{p_{100} - p_0} \times 100^\circ\text{C} \\ &= \frac{77.8 - 73}{100.3 - 73} \times 100^\circ\text{C} = 17^\circ\text{C}. \end{aligned}$$

23.5 IDEAL GAS TEMPERATURE SCALE

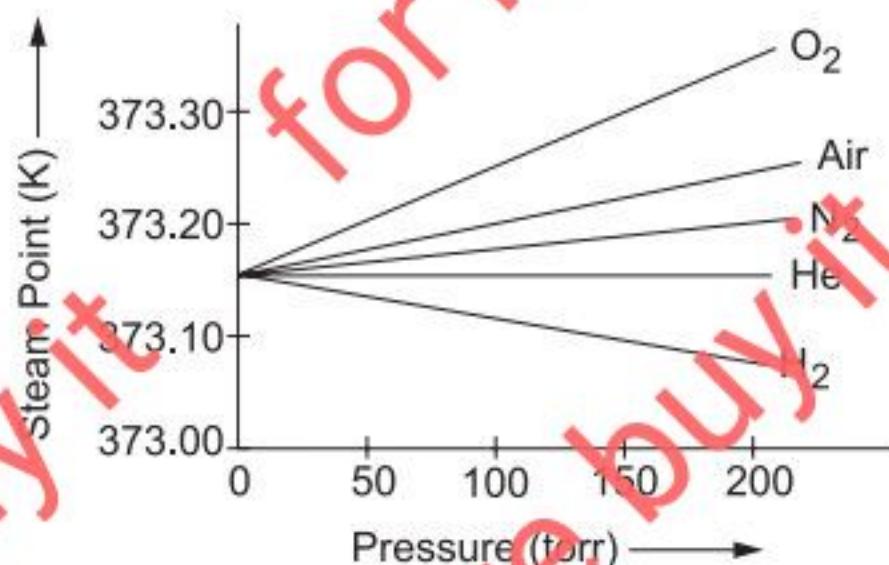


Figure 23.4

The temperature scale defined by equation (23.5) depends slightly on the gas used and its amount present in the thermometer. Figure (23.4) shows the temperature of a steam bath (steam point) as measured by placing different gases in different amounts. On the horizontal axis we have plotted the pressure p_{tr} of the gas in the thermometer at the triple point of water. This is almost proportional to the mass of the gas present. From the figure, we see that although different gas thermometers differ in their measurement of steam point but the difference decreases as the amount of the gas and hence p_{tr} decreases. In the limit $p_{tr} \rightarrow 0$, all the different gas thermometers give the same value 373.15 K for steam point. So we define a temperature scale by the equation

$$T = \lim_{p_{tr} \rightarrow 0} \frac{p}{p_{tr}} \times 273.16 \text{ K}. \quad (23.7)$$

When we use small amount of gas in a gas thermometer, the scale (23.5) is almost identical to (23.7).

The temperature scale defined by (23.7) is called *ideal gas temperature scale* and is independent of the gas chosen. However, it may depend on the properties of gases in general. As mentioned above, it is possible to define an *absolute temperature scale* which does not depend on any property of any substance. Such a temperature scale has been defined and is in use. The unit of this temperature is called Kelvin and is abbreviated as K. The ideal gas temperature scale (23.7) happens to be identical with the absolute scale and hence we have used K to denote the unit.

23.6 CELSIUS TEMPERATURE SCALE

The temperature of the ice point on the ideal gas scale is 273.15 K and of the steam point is $T = 373.15 \text{ K}$. The interval between the two is 100 K. The centigrade scale discussed earlier has 0°C for the ice point and 100°C for the steam point. However, dividing the temperature interval into 100 parts needs a thermometric substance like mercury in glass or resistance of platinum wire etc. Thus, the scales are

different for different thermometers. The use of the term "centigrade scale" is now replaced by "Celsius scale". Celsius scale is defined to have the ice point at 0°C and the steam point at 100°C. The size of a degree in Celsius scale is defined to be the same as the size of a degree in the ideal gas scale. The Celsius scale is shifted from the ideal gas scale by -273.15. If θ denotes the Celsius temperature and T denotes the kelvin temperature,

$$\theta = T - 273.15 \text{ K}. \quad (23.8)$$

The mercury centigrade scale described earlier is quite close to the Celsius scale.

23.7 IDEAL GAS EQUATION

We have seen that the pressure of all gases changes with temperature in a similar fashion for low pressures. Many of the properties of gases are common at low pressures (and high temperatures, that is, far above their condensation point). The pressure, volume and the temperature in kelvin of such a gas obey the equation

$$pV = nRT, \quad (23.9)$$

where n is the amount of the gas in number of moles and R is a universal constant having value $8.314 \text{ J K}^{-1} \text{ mol}^{-1}$. The constant R is called the gas constant. Equation (23.9) is known as *ideal gas equation*. A gas obeying this equation is called an *ideal gas*.

23.8 CALLENDAR'S COMPENSATED CONSTANT PRESSURE THERMOMETER

In constant volume gas thermometer, the gas in the capillary tube connecting the bulb and the manometer remains outside the heat bath. The temperature of this part is different from the main bulk of the gas and this introduces some error. Callender's compensated constant pressure thermometer avoids this problem by a special design.

Figure (23.5) shows a schematic diagram of the Callender's thermometer. An ideal gas is filled in a bulb A connected to a manometer M and another bulb B through a capillary cd. The bulb B is filled with mercury and is graduated in volume. Mercury may be

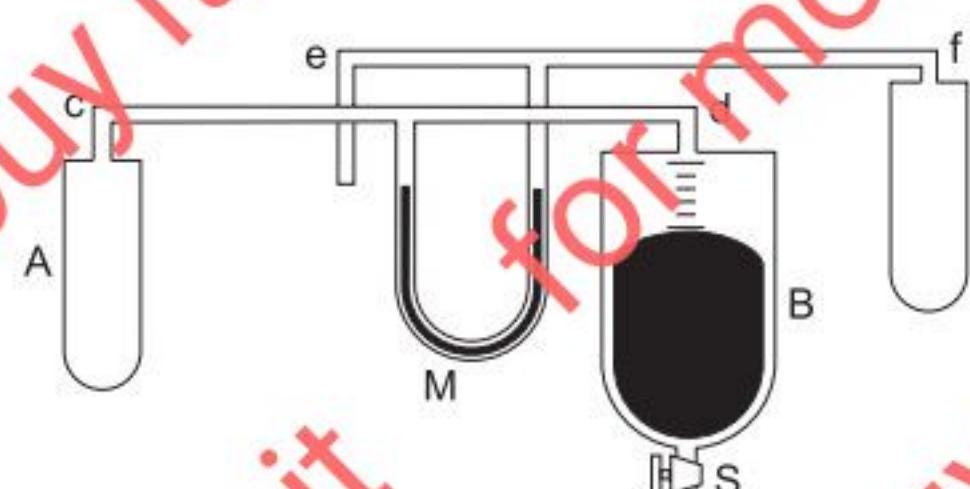


Figure 23.5

taken out from the bulb by opening a stopcock fitted in the lower part of *B*. The volume of the mercury taken out may be measured with the help of the graduations on *B*. The other arm of the manometer is connected to a capillary tube *ef* and a bulb *C*. The capillary *ef* and the capillary *cd* have equal volumes. The bulb *A* and the bulb *C* also have equal volumes.

The same ideal gas (as filled in *A*) is filled in the bulb *C*. The amount of gas in the bulb *C* (and the capillary *ef*) is kept the same as the amount in the bulb *A* (and the capillary *cd*). When all the parts of the thermometer are at the same temperature and the bulb *B* is completely filled with mercury, the levels of the liquid in the two arms of the manometer are equal.

To measure the temperature of a heat bath (at temperature larger than the ice point), the bulb *A* is placed in the heat bath and the bulbs *B* and *C* are placed in melting-ice baths. The pressure in the bulb *A* becomes more than the pressure in the bulb *C* and the levels in the manometer tubes become different. Mercury is taken out from *B* till the levels in the manometer tubes become equal. When mercury is taken out, the total volume of the gas in bulb *A*, capillary *cd* and bulb *B* increases. This decreases the pressure on this side. When sufficient amount of mercury is taken out, the pressure becomes equal to the pressure in the bulb *C* and hence, the levels in the manometer tubes again become equal.

Suppose,

the volume of *A* = volume of *C* = V ,

volume of capillary *cd* = volume of capillary *ef* = v_0 ,

volume of the mercury taken out = v' ,

temperature of the heat bath = T ,

temperature of the ice bath = T_0 ,

and the temperature of *cd* and *ef* = T' .

Using $pV = nRT$, the number of moles in the bulb *A*, the capillary *cd* and the bulb *B*

$$= \frac{pV}{RT} + \frac{pv_0}{RT'} + \frac{pv'}{RT_0}$$

and, the number of moles in the bulb *C* and the capillary *ef*

$$= \frac{pV}{RT_0} + \frac{pv_0}{RT'}$$

As equal amounts of the gas are filled on the two sides,

$$\frac{pV}{RT} + \frac{pv_0}{RT'} + \frac{pv'}{RT_0} = \frac{pV}{RT_0} + \frac{pv_0}{RT'}$$

or,

$$\frac{V}{T} = \frac{V}{T_0} - \frac{v'}{T_0}$$

or,

$$T = \frac{V}{V - v'} T_0 \quad \dots (23.10)$$

All the quantities on the right side are known and hence, the temperature of the bath is obtained.

23.9 ADIABATIC AND DIATHERMIC WALLS

We have seen that heat flows from a high-temperature body to a low-temperature body when they are put in contact. There are certain materials which resist the flow of heat through them. When two bodies at different temperatures are separated by such a material, the heat-flow is very slow. We assume an idealised wall or separator which does not allow any heat flow through it. The bodies on the two sides of such a wall may remain at different temperatures. Such a wall is called an *adiabatic wall*.

Opposite is the concept of a *diathermic wall* which allows heat transfer through it rapidly. If two bodies at different temperatures are separated by such a wall, their temperatures will become equal in a short time.

23.10 THERMAL EXPANSION

If the temperature of a body increases, in general, its size also increases. We used this expansion property to define a temperature scale. Now, we have an absolute scale of temperature independent of any property of any substance. We can study the thermal expansion of a body as a function of temperature. Consider a rod at temperature T and suppose its length at this temperature is L . As the temperature is changed to $T + \Delta T$, the length is changed to $L + \Delta L$. We define *average coefficient of linear expansion* in the temperature range ΔT as

$$\bar{\alpha} = \frac{1}{L} \frac{\Delta L}{\Delta T}$$

The *coefficient of linear expansion* at temperature T is limit of average coefficient as $\Delta T \rightarrow 0$, i.e.,

$$\alpha = \lim_{\Delta T \rightarrow 0} \frac{1}{L} \frac{\Delta L}{\Delta T} = \frac{1}{L} \frac{dL}{dT}$$

Suppose the length of a rod is L_0 at 0°C and L_θ at temperature θ measured in Celsius. If α is small and constant over the given temperature interval,

$$\alpha = \frac{L_\theta - L_0}{L_0 \theta}$$

$$\text{or, } L_\theta = L_0 (1 + \alpha \theta) \quad \dots (23.11)$$

This equation is widely used in solving problems.

The coefficient of volume expansion γ is defined in a similar way. If V is the volume of a body at temperature T , the *coefficient of volume expansion* at temperature T is

$$\gamma = \frac{1}{V_0} \frac{dV}{dT}$$

It is also known as coefficient of cubical expansion.

If V_0 and V_θ denote the volumes at 0°C and θ (measured in Celsius) respectively and γ is small and constant over the given temperature range, we have

$$V_\theta = V_0 (1 + \gamma \theta) \quad \dots (23.12)$$

It is easy to show that $\gamma = 30$.

The change in volume of water as temperature increases is slightly complicated. Figure (23.6) shows the volume of 1 g of water as the temperature increases from 0°C to 10°C . The volume of water

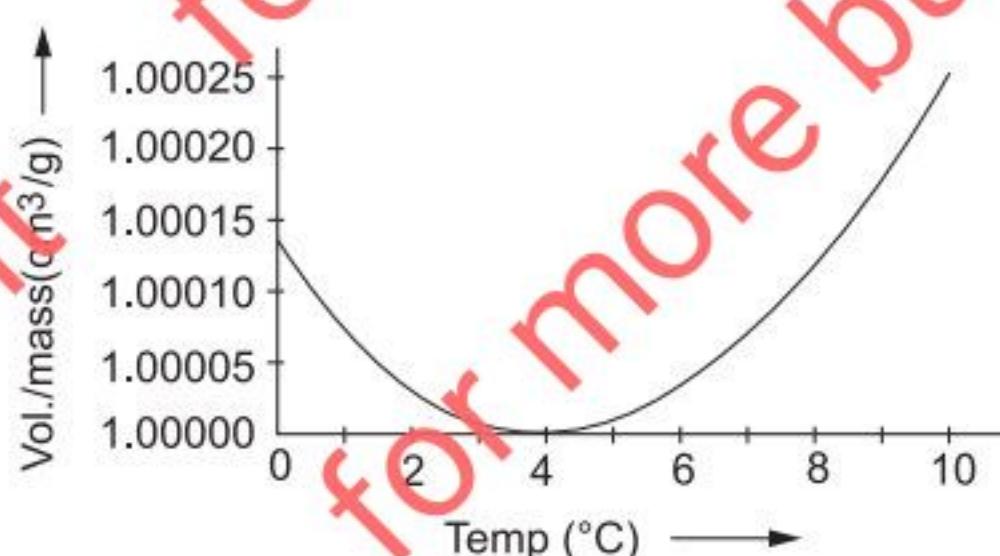


Figure 23.6

1. The pressure of the gas in a constant volume gas thermometer at steam point ($373\cdot15\text{ K}$) is $1\cdot50 \times 10^4\text{ Pa}$. What will be the pressure at the triple point of water?

Solution : The temperature in kelvin is defined as

$$T = \frac{p}{p_{tr}} \times 273\cdot16\text{ K}$$

Thus,

$$373\cdot15 = \frac{1\cdot50 \times 10^4\text{ Pa}}{p_{tr}} \times 273\cdot16$$

$$p_{tr} = 1\cdot50 \times 10^4\text{ Pa} \times \frac{273\cdot16}{373\cdot15} \\ = 1\cdot10 \times 10^4\text{ Pa.}$$

2. The pressure of air in the bulb of a constant volume gas thermometer at 0°C and 100°C are $73\cdot00\text{ cm}$ and 100 cm of mercury respectively. Calculate the pressure at the room temperature 20°C .

Solution : The room temperature on the scale measured by the thermometer is

$$t = \frac{p_t - p_0}{P_{100} - P_0} \times 100^\circ\text{C.}$$

Thus,

$$20^\circ\text{C} = \frac{p_t - 73\cdot00 \text{ cm of Hg}}{100 \text{ cm of Hg} - 73\cdot00 \text{ cm of Hg}} \times 100^\circ\text{C}$$

$$\text{or } p_t = 78\cdot4 \text{ cm of mercury.}$$

decreases from about $1\cdot00013\text{ cm}^3$ to $1\cdot00000\text{ cm}^3$ as the temperature increases from 0°C to 4°C . This means γ is negative in this temperature range. The volume again increases as the temperature is increased further from 4°C . The density (mass/volume) of water is thus maximum at 4°C .

The anomalous expansion of water has a favourable effect for animals living in water. Since the density of water is maximum at 4°C , water at the bottom of lakes remain at 4°C in winter even if that at the surface freezes. This allows marine animals to remain alive and move near the bottom.

The study of expansion of a liquid presents another difficulty due to the expansion of the container. Consider a liquid kept in a flask with a graduated stem. As the temperature is increased, the volume of the flask expands faster than the liquid in the beginning and the level of liquid in the stem goes down as if the liquid has contracted. As the temperature of the liquid increases, the volume of the liquid increases and rises in the stem. The apparent increase in the volume is equal to the real increase in the volume of the liquid minus the increase in the volume of the container.

Worked Out Examples

3. The pressure of the gas in a constant volume gas thermometer is 80 cm of mercury in melting ice at 1 atm . When the bulb is placed in a liquid, the pressure becomes 160 cm of mercury. Find the temperature of the liquid.

Solution : For an ideal gas at constant volume,

$$\frac{T_1}{T_2} = \frac{p_1}{p_2}$$

$$\text{or, } T_2 = \frac{p_2}{p_1} T_1.$$

The temperature of melting ice at 1 atm is $273\cdot15\text{ K}$. Thus, the temperature of the liquid is

$$T_2 = \frac{160}{80} \times 273\cdot15\text{ K} = 546\cdot30\text{ K.}$$

4. In a constant volume gas thermometer, the pressure of the working gas is measured by the difference in the levels of mercury in the two arms of a U-tube connected to the gas at one end. When the bulb is placed at the room temperature $27\cdot0^\circ\text{C}$, the mercury column in the arm open to atmosphere stands $5\cdot00\text{ cm}$ above the level of mercury in the other arm. When the bulb is placed in a hot liquid, the difference of mercury levels becomes $45\cdot0\text{ cm}$. Calculate the temperature of the liquid. (Atmospheric pressure = $75\cdot0\text{ cm}$ of mercury.)

Solution : The pressure of the gas = atmospheric pressure + the pressure due to the difference in mercury levels.

At 27°C , the pressure is $75 \text{ cm} + 5 \text{ cm} = 80 \text{ cm}$ of mercury. At the liquid temperature, the pressure is $75 \text{ cm} + 45 \text{ cm} = 120 \text{ cm}$ of mercury. Using $T_2 = \frac{P_2}{P_1} T_1$, the temperature of the liquid is

$$T = \frac{120}{80} \times (27.0 + 273.15) \text{ K} = 350.22 \text{ K.}$$

$$= 177.07^\circ\text{C} \approx 177^\circ\text{C}.$$

5. The resistances of a platinum resistance thermometer at the ice point, the steam point and the boiling point of sulphur are 2.50 , 3.50 and 6.50Ω respectively. Find the boiling point of sulphur on the platinum scale. The ice point and the steam point measure 0° and 100° respectively.

Solution : The temperature on the platinum scale is defined as

$$t = \frac{R_t - R_0}{R_{100} - R_0} \times 100^\circ.$$

The boiling point of sulphur on this scale is

$$t = \frac{6.50 - 2.50}{3.50 - 2.50} \times 100^\circ = 400^\circ.$$

6. A platinum resistance thermometer reads 0° and 100° at the ice point and the boiling point of water respectively. The resistance of a platinum wire varies with Celsius temperature θ as $R_t = R_0 (1 + \alpha\theta + \beta\theta^2)$, where $\alpha = 3.8 \times 10^{-3} \text{ }^\circ\text{C}^{-1}$ and $\beta = -5.6 \times 10^{-7} \text{ }^\circ\text{C}^{-2}$. What will be the reading of this thermometer if it is placed in a liquid bath maintained at 50°C ?

Solution : The resistances of the wire in the thermometer at 100°C and 50°C are

$$R_{100} = R_0 [1 + \alpha \times 100^\circ\text{C} + \beta \times (100^\circ\text{C})^2]$$

$$\text{and, } R_{50} = R_0 [1 + \alpha \times 50^\circ\text{C} + \beta \times (50^\circ\text{C})^2].$$

The temperature t measured on the platinum thermometer is given by

$$t = \frac{R_{50} - R_0}{R_{100} - R_0} \times 100^\circ$$

$$= \frac{\alpha \times 50^\circ\text{C} + \beta \times (50^\circ\text{C})^2}{\alpha \times 100^\circ\text{C} + \beta \times (100^\circ\text{C})^2} \times 100^\circ$$

$$= 50.4^\circ.$$

7. A platinum resistance thermometer is constructed which reads 0° at ice point and 100° at steam point. Let t_p denote the temperature on this scale and let t denote the temperature on a mercury thermometer scale. The resistance of the platinum coil varies with t as $R_t = R_0 (1 + \alpha t + \beta t^2)$. Derive an expression for the resistance as a function of t_p .

Solution : Let R_{t_p} denote the resistance of the coil at the platinum scale temperature t_p . Then

$$t_p = \frac{R_{t_p} - R_0}{R_{100} - R_0} \times 100$$

$$\text{or, } R_{t_p} = \frac{t_p}{100} (R_{100} - R_0) + R_0$$

$$= \frac{t_p}{100} [R_0 \{1 + \alpha \times 100 + \beta \times (100)^2\} - R_0] + R_0$$

$$= \frac{t_p}{100} [\alpha \times 100 + \beta \times (100)^2] R_0 + R_0$$

$$= R_0 \left[1 + \left\{ \alpha \times 100 + \beta \times (100)^2 \right\} \frac{t_p}{100} \right]$$

$$= R_0 \left[1 + \alpha t_p + \beta \times (100) t_p \right].$$

Only numerical values of α and β are to be used.

8. An iron rod of length 50 cm is joined at an end to an aluminium rod of length 100 cm . All measurements refer to 20°C . Find the length of the composite system at 100°C and its average coefficient of linear expansion. The coefficient of linear expansion of iron and aluminium are $12 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$ and $24 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$ respectively.

Solution : The length of the iron rod at 100°C is

$$l_1 = (50 \text{ cm}) [1 + (12 \times 10^{-6} \text{ }^\circ\text{C}^{-1})(100^\circ\text{C} - 20^\circ\text{C})]$$

$$= 50.048 \text{ cm.}$$

The length of the aluminium rod at 100°C is

$$l_2 = (100 \text{ cm}) [1 + (24 \times 10^{-6} \text{ }^\circ\text{C}^{-1})(100^\circ\text{C} - 20^\circ\text{C})]$$

$$= 100.192 \text{ cm.}$$

The length of the composite system at 100°C is

$$50.048 \text{ cm} + 100.192 \text{ cm} = 150.24 \text{ cm.}$$

The length of the composite system at 20°C is 150 cm . So, the average coefficient of linear expansion of the composite rod is

$$\alpha = \frac{0.24 \text{ cm}}{150 \text{ cm} \times (100^\circ\text{C} - 20^\circ\text{C})}$$

$$= 20 \times 10^{-6} \text{ }^\circ\text{C}^{-1}.$$

9. An iron ring measuring 15.00 cm in diameter is to be shrunk on a pulley which is 15.05 cm in diameter. All measurements refer to the room temperature 20°C . To what minimum temperature should the ring be heated to make the job possible? Calculate the strain developed in the ring when it comes to the room temperature. Coefficient of linear expansion of iron = $12 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$.

Solution : The ring should be heated to increase its diameter from 15.00 cm to 15.05 cm .

Using $l_2 = l_1 (1 + \alpha \Delta \theta)$,

$$= \frac{0.05 \text{ cm}}{15.00 \text{ cm} \times 12 \times 10^{-6} \text{ }^{\circ}\text{C}^{-1}}$$

 278°C The temperature $= 20^{\circ}\text{C} + 278^{\circ}\text{C} = 298^{\circ}\text{C}$.The strain developed $= \frac{l_2 - l_1}{l_1} = 3.33 \times 10^{-1}$.

10. A pendulum clock consists of an iron rod connected to a small, heavy bob. If it is designed to keep correct time at 20°C , how fast or slow will it go in 24 hours at 40°C ?

Coefficient of linear expansion of iron $= 1.2 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}$.**Solution :** The time period at temperature θ is

$$\begin{aligned} T &= 2\pi\sqrt{l_0/g} \\ &= 2\pi\sqrt{l_0(1+\alpha\theta)/g} \\ &= 2\pi\sqrt{l_0/g}(1+\alpha\theta)^{1/2} \\ &\approx T_0(1+\frac{1}{2}\alpha\theta). \end{aligned}$$

Thus,

$$T_{20} = T_0[1 + \frac{1}{2}\alpha(20^{\circ}\text{C})]$$

and,

$$T_{40} = T_0[1 + \frac{1}{2}\alpha(40^{\circ}\text{C})]$$

or,

$$\begin{aligned} \frac{T_{40}}{T_{20}} &= [1 + (20^{\circ}\text{C})\alpha][1 + (10^{\circ}\text{C})\alpha]^{-1} \\ &\approx [1 + (20^{\circ}\text{C})\alpha][1 - (10^{\circ}\text{C})\alpha] \\ &\approx 1 + (10^{\circ}\text{C})\alpha \end{aligned}$$

or,

$$\frac{T_{40} - T_{20}}{T_{20}} = (10^{\circ}\text{C})\alpha = 1.2 \times 10^{-4}. \quad \dots \text{ (i)}$$

This is fractional loss of time. As the temperature increases, the time period also increases. Thus, the clock goes slow. The time lost in 24 hours is, by (i),

$$\Delta t = (24 \text{ hours}) (1.2 \times 10^{-4}) = 10.4 \text{ s.}$$

11. A pendulum clock having copper rod keeps correct time at 20°C . It gains 15 seconds per day if cooled to 0°C . Calculate the coefficient of linear expansion of copper.

Solution : The time period at temperature θ is

$$\begin{aligned} T &= 2\pi\sqrt{l_0/g} \\ &\approx T_0(1 + \frac{1}{2}\alpha\theta) \end{aligned}$$

Thus,

$$T_{20} = T_0[1 + \alpha(10^{\circ}\text{C})]$$

or,

$$\frac{(T_{20} - T_0)}{T_0} = \alpha(10^{\circ}\text{C}). \quad \dots \text{ (i)}$$

T_{20} is the correct time period. The period at 0°C is smaller so that the clock runs fast. Equation (i) gives approximately the fractional gain in time. The time gained in 24 hours is

$$\Delta T = (24 \text{ hours})[(10^{\circ}\text{C})\alpha]$$

or,

$$15 \text{ s} = (24 \text{ hours})[(10^{\circ}\text{C})\alpha]$$

or,

$$\begin{aligned} \alpha &= \frac{15 \text{ s}}{(24 \text{ hours}) (10^{\circ}\text{C})} \\ &= 1.7 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}. \end{aligned}$$

12. A piece of metal weighs 46 g in air and 30 g in a liquid of density $1.24 \times 10^3 \text{ kg m}^{-3}$ kept at 27°C . When the temperature of the liquid is raised to 42°C , the metal piece weighs 30.5 g. The density of the liquid at 42°C is $1.20 \times 10^3 \text{ kg m}^{-3}$. Calculate the coefficient of linear expansion of the metal.

Solution : Let the volume of the metal piece be V_0 at 27°C and V_θ at 42°C . The density of the liquid at 27°C is $\rho_0 = 1.24 \times 10^3 \text{ kg m}^{-3}$ and the density of the liquid at 42°C is $\rho_\theta = 1.20 \times 10^3 \text{ kg m}^{-3}$.

The weight of the liquid displaced = apparent loss in the weight of the metal piece when dipped in the liquid. Thus,

$$V_0 \rho_0 = 46 \text{ g} - 30 \text{ g} = 16 \text{ g}$$

and

$$V_\theta \rho_\theta = 46 \text{ g} - 30.5 \text{ g} = 15.5 \text{ g}.$$

Thus,

$$\frac{V_\theta}{V_0} = \frac{\rho_0}{\rho_\theta} \times \frac{15.5}{16}$$

$$\text{or}, \quad 1 + 3\alpha\Delta\theta = \frac{1.24 \times 10^3 \times 15.5}{1.20 \times 10^3 \times 16}$$

$$\text{or}, 1 + 3\alpha(42^{\circ}\text{C} - 27^{\circ}\text{C}) = 1.00104$$

$$\text{or}, \quad \alpha = 2.3 \times 10^{-5} \text{ }^{\circ}\text{C}^{-1}.$$

13. A sphere of diameter 7.0 cm and mass 266.5 g floats in a bath of liquid. As the temperature is raised, the sphere begins to sink at a temperature of 35°C . If the density of the liquid is 1.327 g cm^{-3} at 0°C , find the coefficient of cubical expansion of the liquid. Neglect the expansion of the sphere.

Solution : It is given that the expansion of the sphere is negligible as compared to the expansion of the liquid. At 0°C , the density of the liquid is $\rho_0 = 1.327 \text{ g cm}^{-3}$. At 35°C , the density of the liquid equals the density of the sphere. Thus,

$$\begin{aligned} \rho_{35} &= \frac{266.5 \text{ g}}{\frac{4}{3}\pi(3.5 \text{ cm})^3} \\ &= 1.484 \text{ g cm}^{-3} \end{aligned}$$

We have

$$\frac{\rho_\theta}{\rho_0} = \frac{V_0}{V_\theta} = \frac{1}{(1 + \gamma\theta)}$$

or,

$$\rho_\theta = \frac{\rho_0}{1 + \gamma\theta}.$$

Thus,

$$\gamma = \frac{\rho_0 - \rho_{35}}{\rho_{35}(35^{\circ}\text{C})}$$

$$= \frac{(1.527 - 1.484) \text{ g cm}^{-3}}{(1.484 \text{ g cm}^{-3})(35^\circ\text{C})} \\ = 8.28 \times 10^{-4} \text{ }^\circ\text{C}^{-1}.$$

14. An iron rod and a copper rod lie side by side. As the temperature is changed, the difference in the lengths of the rods remains constant at a value of 10 cm. Find the lengths at 0°C . Coefficients of linear expansion of iron and copper are $1.1 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ and $1.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ respectively.

Solution : Suppose the length of the iron rod at 0°C is l_{i0} and the length of the copper rod at 0°C is l_{c0} . The lengths at temperature θ are

$$l_{i\theta} = l_{i0} (1 + \alpha_i \theta)$$

and

$$l_{c\theta} = l_{c0} (1 + \alpha_c \theta).$$

Subtracting,

$$l_{i\theta} - l_{c\theta} = (l_{i0} - l_{c0}) + (\alpha_i - \alpha_c) \theta.$$

Now,

$$l_{i\theta} - l_{c\theta} = l_{i0} - l_{c0} (= 10 \text{ cm}).$$

Thus, from (iii), $l_{i0} \alpha_i = l_{c0} \alpha_c$

or,

$$\frac{l_{i0}}{l_{c0}} = \frac{\alpha_c}{\alpha_i}$$

or,

$$\frac{l_{i0}}{l_{i0} - l_{c0}} = \frac{\alpha_c}{\alpha_c - \alpha_i} \\ = \frac{1.7 \times 10^{-5} \text{ }^\circ\text{C}^{-1}}{0.6 \times 10^{-5} \text{ }^\circ\text{C}^{-1}} = \frac{17}{6}.$$

This shows that $l_{i0} - l_{c0}$ is positive. Its value is 10 cm as given in the question.

Hence,

$$l_{i0} = \frac{17}{6} \times (l_{i0} - l_{c0}) \\ = \frac{17}{6} \times 10 \text{ cm} = 28.3 \text{ cm}$$

and

$$l_{c0} = l_{i0} - 10 \text{ cm} = 18.3 \text{ cm}.$$

15. A uniform steel wire of cross-sectional area 0.20 mm^2 is held fixed by clamping its two ends. Find the extra force exerted by each clamp on the wire if the wire is cooled from 100°C to 0°C . Young's modulus of steel = $2.0 \times 10^{11} \text{ N m}^{-2}$. Coefficient of linear expansion of steel = $1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$.

Solution : Let us assume that the tension is zero at 100°C so that l_0 is the natural length of the wire at 100°C . As the wire cools down, its natural length decreases to l_θ . As the wire is fixed at the clamps, its length remains the same as the length at 100°C . Thus, the extension of the wire over its natural length at 0°C is

$$l_\theta - l_0 = l_0 (1 + \alpha \theta) - l_0 = l_0 \alpha \theta$$

The strain developed is $\frac{l_\theta - l_0}{l_0} \approx \frac{l_\theta - l_0}{l_0} = \alpha \theta$.

The stress developed = $Y \times \text{strain} = Y \alpha \theta$.

The tension in the wire at 0°C is

$$T = \text{stress} \times \text{area} \\ = Y \alpha \theta \times 0.20 \text{ mm}^2 \\ = (2.0 \times 10^{11} \text{ N m}^{-2}) \times (1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}) \\ \times 100^\circ\text{C} \times (0.20 \times 10^{-6} \text{ m}^{-2}) \\ = 48 \text{ N}.$$

This is equal to the extra force exerted by each clamp.

16. A glass vessel of volume 100 cm^{-3} is filled with mercury and is heated from 25°C to 75°C . What volume of mercury will overflow? Coefficient of linear expansion of glass = $1.8 \times 10^{-6} \text{ }^\circ\text{C}^{-1}$ and coefficient of volume expansion of mercury is $1.8 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$.

Solution : The volume of mercury at 25°C is

$$V_0 = 100 \text{ cm}^{-3}.$$

The coefficient of volume expansion of mercury

$$\gamma_L = 1.8 \times 10^{-4} \text{ }^\circ\text{C}^{-1}.$$

The coefficient of volume expansion of glass

$$\gamma_s = 3 \times 1.8 \times 10^{-6} \text{ }^\circ\text{C}^{-1} \\ = 5.4 \times 10^{-6} \text{ }^\circ\text{C}^{-1}.$$

Thus, the volume of mercury at 75°C is

$$V_{L\theta} = V_0 (1 + \gamma_L \Delta \theta)$$

and the volume of the vessel at 75°C is

$$V_{s\theta} = V_0 (1 + \gamma_s \Delta \theta).$$

The volume of mercury overflowed

$$= V_{L\theta} - V_{s\theta} = V_0 (\gamma_L - \gamma_s) \Delta \theta \\ = (100 \text{ cm}^{-3}) (1.8 \times 10^{-4} - 5.4 \times 10^{-6}) / \text{ }^\circ\text{C} \times (50^\circ\text{C}) \\ = 0.87 \text{ cm}^3.$$

Note that $\gamma_a = (\gamma_L - \gamma_s)$ acts as the effective coefficient of expansion of the liquid with respect to the solid. The expansion of mercury 'as seen from the glass' can be written as

$$V_\theta - V_0 = V_0 \gamma_a \theta$$

$$\text{or, } V_\theta = V_0 (1 + \gamma_a \theta).$$

The constant γ_a is called the 'apparent coefficient of expansion' of the liquid with respect to the solid.

17. A barometer reads 75.0 cm on a steel scale. The room temperature is 30°C . The scale is correctly graduated for 0°C . The coefficient of linear expansion of steel is $\alpha = 1.2 \times 10^{-5} \text{ }^\circ\text{C}^{-1}$ and the coefficient of volume expansion of mercury is $\gamma = 1.8 \times 10^{-4} \text{ }^\circ\text{C}^{-1}$. Find the correct atmospheric pressure.

Solution : The 75 cm length of steel at 0°C will become l_θ at 30°C where,

$$l_\theta = (75 \text{ cm}) [1 + \alpha (30^\circ\text{C})]. \quad \dots \text{ (i)}$$