Concepts of Physics

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BHARATI BHAWAN (P & D)

CONCEPTS OF PHYSICS

[VOLUME 1]

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Dedicated to
Indian Philosophy & Way of Life
of which
my parents were
an integral part

FOREWORD

A few years ago I had an occasion to go through the book *Calculus* by L V Terasov. It unravels intricacies of the subject through a dialogue between Teacher and Student. I thoroughly enjoyed reading it. For me this seemed to be one of the few books which teach a difficult subject through inquisition, and using programmed concept for learning. After that book, Dr Harish Chandra Verma's book on physics, *CONCEPTS OF PHYSICS* is another such attempt, even though it is not directly in the dialogue form. I have thoroughly appreciated it. It is clear that Dr Verma has spent considerable time in formulating the structure of the book, besides its contents. I think he has been successful in this attempt. Dr Verma's book has been divided into two parts because of the size of the total manuscript. There have been several books on this subject, each one having its own flavour. However, the present book is a totally different attempt to teach physics, and I am sure it will be extremely useful to the undergraduate students. The exposition of each concept is extremely lucid. In carefully formatted chapters, besides problems and short questions, a number of objective questions have also been included. This book can certainly be extremely useful not only as a textbook, but also for preparation of various competitive examinations.

Those who have followed Dr Verma's scientific work always enjoyed the outstanding contributions he has made in various research areas. He was an outstanding student of Physics Department of IIT Kanpur during his academic career. An extremely methodical, sincere person as a student, he has devoted himself to the task of educating young minds and inculcating scientific temper amongst them. The present venture in the form of these two volumes is another attempt in that direction. I am sure that young minds who would like to *learn physics in an appropriate manner* will find these volumes extremely useful.

I must heartily congratulate Dr Harish Chandra Verma for the magnificent job he has done.

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PREFACE

Why a new book?

Excellent books exist on physics at an introductory college level so why a new one? Why so many books exist at the same level, in the first place, and why each of them is highly appreciated? It is because each of these books has the privilege of having an author or authors who have *experienced* physics and have their own method of communicating with the students. During my years as a physics teacher, I have developed a somewhat different methodology of presenting physics to the students. *Concepts of Physics* is a translation of this methodology into a textbook.

Prerequisites

The book presents a calculus-based physics course which makes free use of algebra, trigonometry and co-ordinate geometry. The level of the latter three topics is quite simple and high school mathematics is sufficient. Calculus is generally done at the introductory college level and I have assumed that the student is enrolled in a concurrent first calculus course. The relevant portions of calculus have been discussed in Chapter 2 so that the student may start using it from the beginning.

Almost no knowledge of physics is a prerequisite. I have attempted to start each topic from the zero level. A receptive mind is all that is needed to use this book.

Basic philosophy of the book

The motto underlying the book is *physics is enjoyable*.

Being a description of the nature around us, physics is our best friend from the day of our existence. I have extensively used this aspect of physics to introduce the physical principles starting with common day occurrences and examples. The subject then appears to be friendly and enjoyable. I have taken care that numerical values of different quantities used in problems correspond to real situations to further strengthen this approach.

Teaching and training

The basic aim of physics teaching has been to let the student know and understand the principles and equations of physics and their applications in real life.

However, to be able to use these principles and equations correctly in a given physical situation, one needs further training. A large number of *questions and solved and unsolved problems* are given for this purpose. Each question or problem has a specific purpose. It may be there to bring out a subtle point which might have passed unnoticed while doing the text portion. It may be a further elaboration of a concept developed in the text. It may be there to make the student react when several concepts introduced in different chapters combine and show up as a physical situation and so on. Such tools have been used to develop a culture: *analyse the situation, make a strategy to invoke correct principles and work it out.*

Conventions

I have tried to use symbols, names, etc., which are popular nowadays. SI units have been consistently used throughout the book. SI prefixes such as *micro, milli, mega,* etc., are used whenever they make the presentation more readable. Thus, 20 µF is preferred over 20×10^{-6} F. Co-ordinate sign convention is used in geometrical optics. Special emphasis has been given to dimensions of physical quantities. Numerical values of physical quantities have been mentioned with the units even in equations to maintain dimensional consistency.

I have tried my best to keep errors out of this book. I shall be grateful to the readers who point out any errors and/or make other constructive suggestions.

H C Verma

ACKNOWLEDGEMENTS

The work on this book started in 1984. Since then, a large number of teachers, students and physics lovers have made valuable suggestions which I have incorporated in this work. It is not possible for me to acknowledge all of them individually. I take this opportunity to express my gratitude to them. However, to Dr S B Mathur, who took great pains in going through the entire manuscript and made valuable comments, I am specially indebted. I am also beholden to my colleagues Dr A Yadav, Dr Deb Mukherjee, Mr M M R Akhtar, Dr Arjun Prasad, Dr S K Sinha and others who gave me valuable advice and were good enough to find time for fruitful discussions. To Dr T K Dutta of B E College, Sibpur I am grateful for having taken time to go through portions of the book and making valuable comments.

I thank my student Mr Shailendra Kumar who helped me in checking the answers. I am grateful to Dr B C Rai, Mr Sunil Khijwania & Mr Tejaswi Khijwania for helping me in the preparation of rough sketches for the book.

Finally, I thank the members of my family for their support and encouragement.

H C Verma

TO THE STUDENTS

Here is a brief discussion on the organisation of the book which will help you in using the book most effectively. The book contains 47 chapters divided in two volumes. Though I strongly believe in the underlying unity of physics, a broad division may be made in the book as follows:

Chapters 1–14: Mechanics
15–17: Waves including wave optics
18–22: Optics
23–28: Heat and thermodynamics
29–40: Electric and magnetic phenomena
41–47: Modern physics

Each chapter contains a description of the physical principles related to that chapter. It is well supported by mathematical derivations of equations, descriptions of laboratory experiments, historical background, etc. There are "in-text" solved examples. These examples explain the equation just derived or the concept just discussed. These will help you in fixing the ideas firmly in your mind. Your teachers may use these in-text examples in the classroom to encourage students to participate in discussions.

After the theory section, there is a section on *Worked Out Examples*. These numerical examples correspond to various thinking levels and often use several concepts introduced in that chapter or even in previous chapters. You should read the statement of a problem and try to solve it yourself. In case of difficulty, look at the solution given in the book. Even if you solve the problem successfully, you should look into the solution to compare it with your method of solution. You might have thought of a better method, but knowing more than one method is always beneficial.

Then comes the part which tests your understanding as well as develops it further. *Questions for Short Answer* generally touch very minute points of your understanding. It is not necessary that you answer these questions in a single sitting. They have great potential to initiate very fruitful dicussions. So, freely discuss these questions with your friends and see if they agree with your answer. Answers to these questions are not given for the simple reason that the answers could have cut down the span of such discussions and that would have sharply reduced the utility of these questions.

There are two sections on multiple-choice questions, namely OBJECTIVE I and OBJECTIVE II. There are four options following each of these questions. Only one option is correct for OBJECTIVE I questions. Any number of options, zero to four, may be correct for OBJECTIVE II questions. Answers to all these questions are provided.

Finally, a set of numerical problems are given for your practice. Answers to these problems are also provided. The problems are generally arranged according to the sequence of the concepts developed in the chapter but they are not grouped under section-headings. I don't want to bias your ideas beforehand by telling you that this problem belongs to that section and hence use that particular equation. You should yourself look into the problem and decide which equations or which methods should be used to solve it. Many of the problems use several concepts developed in different sections of the chapter. Many of them even use the concepts from the previous chapters. Hence, you have to plan out the strategy after understanding the problem.

Remember, no problem is difficult. Once you understand the theory, each problem will become easy. So, don't jump to exercise problems before you have gone through the theory, the worked-out problems and the objectives. Once you feel confident in theory, do the exercise problems. The exercise problems are so arranged that they gradually require more thinking.

I hope you will enjoy *Concepts of Physics*.

H C Verma

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CHAPTER 1

INTRODUCTION TO PHYSICS

1.1 WHAT IS PHYSICS ?

The nature around us is colourful and diverse. It contains phenomena of large varieties. The winds, the sands, the waters, the planets, the rainbow, heating of objects on rubbing, the function of a human body, the energy coming from the sun and the nucleus ...... there are a large number of objects and events taking place around us.

Physics is the study of nature and its laws. We expect that all these different events in nature take place according to some basic laws and *revealing these laws of nature from the observed events* is physics. For example, the orbiting of the moon around the earth, falling of an apple from a tree and tides in a sea on a full moon night can all be explained if we know the Newton's law of gravitation and Newton's laws of motion. Physics is concerned with the basic rules which are applicable to all domains of life. Understanding of physics, therefore, leads to applications in many fields including bio and medical sciences.

The great physicist Dr R. P. Feynman has given a wonderful description of what is "understanding the nature". Suppose we do not know the rules of chess but are allowed to watch the moves of the players. If we watch the game for a long time, we may make out some of the rules. With the knowledge of these rules we may try to understand why a player played a particular move. However, this may be a very difficult task. Even if we know all the rules of chess, it is not so simple to understand all the complications of a game in a given situation and predict the correct move. Knowing the basic rules is, however, the minimum requirement if any progress is to be made.

One may guess at a wrong rule by partially watching the game. The experienced player may make use of a rule for the first time and the observer of the game may get surprised. Because of the new move some of the rules guessed at may prove to be wrong and the observer will frame new rules.

Physics goes the same way. The nature around us is like a big chess game played by Nature. The events in the nature are like the moves of the great game. We are allowed to watch the events of nature and guess at the basic rules according to which the events take place. We may come across new events which do not follow the rules guessed earlier and we may have to declare the old rules inapplicable or wrong and discover new rules.

Since physics is the study of nature, it is real. No one has been given the authority to frame the rules of physics. We only *discover* the rules that are operating in nature. Aryabhat, Newton, Einstein or Feynman are great physicists because from the observations available at that time, they could guess and frame the laws of physics which explained these observations in a convincing way. But there can be a new phenomenon any day and if the rules discovered by the great scientists are not able to explain this phenomenon, no one will hesitate to change these rules.

1.2 PHYSICS AND MATHEMATICS

The description of nature becomes easy if we have the freedom to use mathematics. To say that the gravitational force between two masses is proportional to the product of the masses and is inversely proportional to the square of the distance apart, is more difficult than to write

$$F \propto \frac{m_1 m_2}{r^2} \cdot \qquad \ldots \quad (1.1)$$

Further, the techniques of mathematics such as algebra, trigonometry and calculus can be used to make predictions from the basic equations. Thus, if we know the basic rule (1.1) about the force between two particles, we can use the technique of integral calculus to find what will be the force exerted by a uniform rod on a particle placed on its perpendicular bisector.

Thus, mathematics is the language of physics. Without knowledge of mathematics it would be much more difficult to discover, understand and explain the laws of nature. The importance of mathematics in today's world cannot be disputed. However, mathematics itself is not physics. We use a language to express our ideas. But the idea that we want to express has the main attention. If we are poor at grammar and vocabulary, it would be difficult for us to communicate our feelings but while doing so our basic interest is in the feeling that we want to express. It is nice to board a deluxe coach to go from Delhi to Agra, but the sweet memories of the deluxe coach and the video film shown on way are next to the prime goal of reaching Agra. "To understand nature" is physics, and mathematics is the deluxe coach to take us there comfortably. This relationship of physics and mathematics must be clearly understood and kept in mind while doing a physics course.

1.3 UNITS

Physics describes the laws of nature. This description is quantitative and involves measurement and comparison of physical quantities. To measure a physical quantity we need some standard unit of that quantity. An elephant is heavier than a goat but exactly how many times ? This question can be easily answered if we have chosen a standard mass calling it a *unit mass*. If the elephant is 200 times the unit mass and the goat is 20 times we know that the elephant is 10 times heavier than the goat. If I have the knowledge of the unit length and some one says that Gandhi Maidan is 5 times the unit length from here, I will have the idea whether I should walk down to Gandhi Maidan or I should ride a rickshaw or I should go by a bus. Thus, the physical quantities are quantitatively expressed in terms of a unit of that quantity. The measurement of the quantity is mentioned in two parts, the first part gives how many times of the standard unit and the second part gives the name of the unit. Thus, suppose I have to study for 2 hours. The numeric part 2 says that it is 2 *times* of the unit of time and the second part *hour* says that the unit chosen here is an hour.

Who Decides the Units ?

How is a standard unit chosen for a physical quantity ? The first thing is that it should have international acceptance. Otherwise, everyone will choose his or her own unit for the quantity and it will be difficult to communicate freely among the persons distributed over the world. A body named *Conférence Générale des Poids et Mesures* or CGPM also known as *General Conference on Weight and Measures* in English has been given the authority to decide the units by international agreement. It holds its meetings and any changes in standard units are communicated through the publications of the Conference.

Fundamental and Derived Quantities

There are a large number of physical quantities which are measured and every quantity needs a definition of unit. However, not all the quantities are independent of each other. As a simple example, if a unit of length is defined, a unit of area is automatically obtained. If we make a square with its length equal to its breadth equal to the unit length, its area can be called the unit area. All areas can then be compared to this standard unit of area. Similarly, if a unit of length and a unit of time interval are defined, a unit of speed is automatically obtained. If a particle covers a unit length in unit time interval, we say that it has a unit speed. We can define a set of *fundamental quantities* as follows :

(a) the fundamental quantities should be independent of each other, and

(b) all other quantities may be expressed in terms of the fundamental quantities.

It turns out that the number of fundamental quantities is only seven. All the rest may be derived from these quantities by multiplication and division. Many different choices can be made for the fundamental quantities. For example, one can take speed and time as fundamental quantities. Length is then a derived quantity. If something travels at unit speed, the distance it covers in unit time interval will be called a unit distance. One may also take length and time interval as the fundamental quantities and then speed will be a derived quantity. Several systems are in use over the world and in each system the fundamental quantities are selected in a particular way. The units defined for the fundamental quantities are called *fundamental units* and those obtained for the derived quantities are called the *derived units*.

Fundamental quantities are also called base quantities.

SI Units

In 1971 CGPM held its meeting and decided a system of units which is known as the *International System of Units*. It is abbreviated as SI from the French name *Le Systéme International d'Unités*. This system is widely used throughout the world.

Table (1.1) gives the fundamental quantities and their units in SI.

Table 1.1 : *Fundamental or Base Quantities*

Quantity	*Name of the Unit*	*Symbol*
Length	**metre**	m
Mass	**kilogram**	kg
Time	**second**	s
Electric Current	**ampere**	A
Thermodynamic Temperature	**kelvin**	K
Amount of Substance	**mole**	mol
Luminous Intensity	**candela**	cd

Besides the seven fundamental units two supplementary units are defined. They are for plane angle and solid angle. The unit for plane angle is *radian* with the symbol *rad* and the unit for the solid angle is *steradian* with the symbol *sr*.

SI Prefixes

The magnitudes of physical quantities vary over a wide range. We talk of separation between two protons inside a nucleus which is about 10^{-15} m and the distance of a quasar from the earth which is about 10^{26} m. The mass of an electron is $9{\cdot}1 \times 10^{-31}$ kg and that of our galaxy is about $2{\cdot}2 \times 10^{41}$ kg. The CGPM recommended standard prefixes for certain powers of 10. Table (1.2) shows these prefixes.

Table 1.2 : *SI prefixes*

Power of 10	*Prefix*	*Symbol*
18	**exa**	E
15	**peta**	P
12	**tera**	T
9	**giga**	G
6	**mega**	M
3	**kilo**	k
2	**hecto**	h
1	**deka**	da
– 1	**deci**	d
– 2	**centi**	c
– 3	**milli**	m
– 6	**micro**	μ
– 9	**nano**	n
– 12	**pico**	p
– 15	**femto**	f
– 18	**atto**	a

1.4 DEFINITIONS OF BASE UNITS

Any standard unit should have the following two properties :

(a) *Invariability* : The standard unit must be invariable. Thus, defining distance between the tip of the middle finger and the elbow as a unit of length is not invariable.

(b) *Availability* : The standard unit should be easily made available for comparing with other quantities.

CGPM decided in its 2018 meeting that all the SI base quantities will be defined in terms of certain universal constants and these constants will be assigned fixed numerical values by definition. In this case both the criteria of invariability and availability are automatically satisfied. The new definitions became operative since 20th May 2019. We give below the definitions of the these quantities. The fixed values given to the universal constants will appear in the definitions only. The definitions carry certain physical quantities and concepts that are beyond the scope of this book but you need not worry about it.

Second

1 second is the time that makes the unperturbed ground state hyperfine transition frequency $\Delta\nu_{Cs}$ to be 9192631770 when expressed in the unit Hz which is equal to s^{-1}.

Metre

1 metre is the length that makes the speed of light in vacuum to be 299792458 when expressed in the unit $m{\cdot}s^{-1}$, where the second is defined in terms of the caesium frequency $\Delta\nu_{Cs}$.

Kilogram

1 kilogram is the mass that makes the Planck's constant h to be $6.62607015 \times 10^{-34}$ when expressed in the unit J·s which is equal to $kg{\cdot}m^2{\cdot}s^{-1}$, where the metre and the second are defined in terms of c and $\Delta\nu_{Cs}$.

Ampere

1 ampere is the current which makes the elementary charge e to be $1.602176634 \times 10^{-19}$ when expressed in the unit C which is equal to A·s, where the second is defined in terms of $\Delta\nu_{Cs}$.

Kelvin

1 kelvin is the temperature that makes the Boltzmann constant to be 1.380649×10^{-23} when expressed in the unit $J{\cdot}K^{-1}$ which is equal to $kg{\cdot}m^2{\cdot}s^{-2}{\cdot}K^{-1}$, where kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$.

Mole

1 mole of a substance is defined to contain exactly $6.02214076 \times 10^{23}$ elementary entities. This number is the fixed numerical value of the Avogadro constant N_A when expressed in the unit mol^{-1} and is called Avogadro number.

Candela

The candela is the SI unit of luminous intensity. 1 candela is the luminous intensity that makes the luminous efficacy of monochromatic radiation of frequency 540×10^{12} Hz, K_{cd} to be 683 when expressed in the unit $lm\,W^{-1}$ which is equal to $cd \cdot sr \cdot kg^{-1} m^{2} s^{3}$, where kilogram, metre and second are defined in terms of h, c and $\Delta\nu_{Cs}$.

1.5 DIMENSION

All the physical quantities of interest can be derived from the base quantities. When a quantity is expressed in terms of the base quantities, it is written as a product of different powers of the base quantities. The exponent of a base quantity that enters into the expression, is called the *dimension of the quantity in that base*. To make it clear, consider the physical quantity force. As we shall learn later, force is equal to mass times acceleration. Acceleration is change in velocity divided by time interval. Velocity is length divided by time interval. Thus,

$$\begin{aligned} force &= mass \times acceleration \\ &= mass \times \frac{velocity}{time} \\ &= mass \times \frac{length/time}{time} \\ &= mass \times length \times (time)^{-2}. \qquad \text{... (1.2)} \end{aligned}$$

Thus, the dimensions of force are 1 in mass, 1 in length and –2 in time. The dimensions in all other base quantities are zero. Note that in this type of calculation, the magnitudes are not considered. It is equality of the type of quantity that enters. Thus, change in velocity, initial velocity, average velocity, final velocity all are equivalent in this discussion, each one is length/time.

For convenience the base quantities are represented by one letter symbols. Generally, mass is denoted by M, length by L, time by T and electric current by I. The thermodynamic temperature, the amount of substance and the luminous intensity are denoted by the symbols of their units K, mol and cd respectively. The physical quantity that is expressed in terms of the base quantities is enclosed in square brackets to remind that the equation is among the dimensions and not among the magnitudes. Thus equation (1.2) may be written as [force] = MLT^{-2}.

Such an expression for a physical quantity in terms of the base quantities is called the *dimensional formula*. Thus, the dimensional formula of force is MLT^{-2}. The two versions given below are equivalent and are used interchangeably.

(a) The dimensional formula of force is MLT^{-2}.

(b) The dimensions of force are 1 in mass, 1 in length and –2 in time.

Example 1.1

Calculate the dimensional formula of energy from the equation $E = \frac{1}{2} mv^2$.

Solution : Dimensionally, $E = mass \times (velocity)^2$, since $\frac{1}{2}$ is a number and has no dimension.

$$\therefore \quad [E] = M \times \left(\frac{L}{T}\right)^2 = ML^2 T^{-2}$$

1.6 USES OF DIMENSION

A. Homogeneity of Dimensions in an Equation

An equation contains several terms which are separated from each other by the symbols of equality, plus or minus. The dimensions of all the terms in an equation must be identical. This is another way of saying that one can add or subtract similar physical quantities. Thus, a velocity cannot be added to a force or an electric current cannot be subtracted from the thermodynamic temperature. This simple principle is called the *principle of homogeneity of dimensions* in an equation and is an extremely useful method to check whether an equation may be correct or not. If the dimensions of all the terms are not same, the equation must be wrong. Let us check the equation

$$x = ut + \frac{1}{2} at^2$$

for the dimensional homogeneity. Here x is the distance travelled by a particle in time t which starts at a speed u and has an acceleration a along the direction of motion.

$$[x] = L$$

$$[ut] = velocity \times time = \frac{length}{time} \times time = L$$

$$\left[\frac{1}{2} at^2\right] = [at^2] = acceleration \times (time)^2$$

$$= \frac{velocity}{time} \times (time)^2 = \frac{length/time}{time} \times (time)^2 = L$$

Thus, the equation is correct as far as the dimensions are concerned.

Limitation of the Method

Note that the dimension of $\frac{1}{2}at^2$ is same as that of at^2. Pure numbers are dimensionless. Dimension does not depend on the magnitude. Due to this reason the equation $x = ut + at^2$ is also dimensionally correct. Thus, a dimensionally correct equation need not be actually correct but a dimensionally wrong equation must be wrong.

Example 1.2

Test dimensionally if the formula $t = 2\pi\sqrt{\frac{m}{F/x}}$ may be correct, where t is time period, m is mass, F is force and x is distance.

Solution : The dimension of force is MLT^{-2}. Thus, the dimension of the right-hand side is

$$\sqrt{\frac{M}{MLT^{-2}/L}} = \sqrt{\frac{1}{T^{-2}}} = T.$$

The left-hand side is time period and hence the dimension is T. The dimensions of both sides are equal and hence the formula may be correct.

B. Conversion of Units

When we choose to work with a different set of units for the base quantities, the units of all the derived quantities must be changed. Dimensions can be useful in finding the conversion factor for the unit of a derived physical quantity from one system to other. Consider an example. When SI units are used, the unit of pressure is 1 pascal. Suppose we choose 1 cm as the unit of length, 1 g as the unit of mass and 1 s as the unit of time (this system is still in wide use and is called CGS system). The unit of pressure will be different in this system. Let us call it for the time-being 1 CGS pressure. Now, how many CGS pressure is equal to 1 pascal ?

Let us first write the dimensional formula of pressure.

We have $$P = \frac{F}{A}$$

Thus, $$[P] = \frac{[F]}{[A]} = \frac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}$$

so, $$1 \text{ pascal} = (1 \text{ kg})\,(1 \text{ m})^{-1}\,(1 \text{ s})^{-2}$$

and $$1 \text{ CGS pressure} = (1 \text{ g})\,(1 \text{ cm})^{-1}\,(1 \text{ s})^{-2}$$

Thus, $$\frac{1 \text{ pascal}}{1 \text{ CGS pressure}} = \left(\frac{1 \text{ kg}}{1 \text{ g}}\right)\left(\frac{1 \text{ m}}{1 \text{ cm}}\right)^{-1}\left(\frac{1 \text{ s}}{1 \text{ s}}\right)^{-2} = \left(10^3\right)\left(10^2\right)^{-1} = 10$$

or, $$1 \text{ pascal} = 10 \text{ CGS pressure}.$$

Thus, knowing the conversion factors for the base quantities, one can work out the conversion factor for any derived quantity if the dimensional formula of the derived quantity is known.

C. Deducing Relation among the Physical Quantities

Sometimes dimensions can be used to deduce a relation between the physical quantities. If one knows the quantities on which a particular physical quantity depends and if one guesses that this dependence is of product type, method of dimension may be helpful in the derivation of the relation. Taking an example, suppose we have to derive the expression for the time period of a simple pendulum. The simple pendulum has a bob, attached to a string, which oscillates under the action of the force of gravity. Thus, the time period may depend on the length of the string, the mass of the bob and the acceleration due to gravity. We assume that the dependence of time period on these quantities is of product type, that is,

$$t = kl^a m^b g^c \qquad \ldots \; (1.3)$$

where k is a dimensionless constant and a, b and c are exponents which we want to evaluate. Taking the dimensions of both sides,

$$T = L^a M^b (LT^{-2})^c = L^{a+c} M^b T^{-2c}.$$

Since the dimensions on both sides must be identical, we have

$$a + c = 0$$
$$b = 0$$
and $$-2c = 1$$

giving $a = \frac{1}{2}$, $b = 0$ and $c = -\frac{1}{2}$.

Putting these values in equation (1.3)

$$t = k\sqrt{\frac{l}{g}}. \qquad \ldots \; (1.4)$$

Thus, by dimensional analysis we can deduce that the time period of a simple pendulum is independent of its mass, is proportional to the square root of the length of the pendulum and is inversely proportional to the square root of the acceleration due to gravity at the place of observation.

Limitations of the Dimensional Method

Although dimensional analysis is very useful in deducing certain relations, it cannot lead us too far. First of all we have to know the quantities on which a particular physical quantity depends. Even then the method works only if the dependence is of the product type. For example, the distance travelled by a uniformly accelerated particle depends on the initial velocity u, the acceleration a and the time t. But the method of dimensions cannot lead us to the correct expression for x because the expression is not of product type. It is equal to the sum of two terms as

$$x = ut + \frac{1}{2}at^2.$$

Secondly, the numerical constants having no dimensions cannot be deduced by the method of dimensions. In the example of time period of a simple pendulum, an unknown constant k remains in equation (1.4). One has to know from somewhere else that this constant is 2π.

Thirdly, the method works only if there are as many equations available as there are unknowns. In mechanical quantities, only three base quantities length, mass and time enter. So, dimensions of these three may be equated in the guessed relation giving at most three equations in the exponents. If a particular quantity (in mechanics) depends on more than three quantities we shall have more unknowns and less equations. The exponents cannot be determined uniquely in such a case. Similar constraints are present for electrical or other nonmechanical quantities.

1.7 ORDER OF MAGNITUDE

In physics, we come across quantities which vary over a wide range. We talk of the size of a mountain and the size of the tip of a pin. We talk of the mass of our galaxy and the mass of a hydrogen atom. We talk of the age of the universe and the time taken by an electron to complete a circle around the proton in a hydrogen atom. It becomes quite difficult to get a feel of largeness or smallness of such quantities. To express such widely varying numbers, one uses the *powers of ten* method.

In this method, each number is expressed as $a \times 10^b$ where $1 \le a < 10$ and b is a positive or negative integer. Thus the diameter of the sun is expressed as $1{\cdot}39 \times 10^9$ m and the diameter of a hydrogen atom as $1{\cdot}06 \times 10^{-10}$ m. To get an approximate idea of the number, one may round the number a to 1 if it is less than or equal to 5 and to 10 if it is greater than 5. The number can then be expressed approximately as 10^b. We then get the *order of magnitude* of that number. Thus, the diameter of the sun is *of the order of* 10^9 m and that of a hydrogen atom is *of the order of* 10^{-10} m. More precisely, the exponent of 10 in such a representation is called the order of magnitude of that quantity. Thus, the diameter of the sun is 19 *orders of magnitude larger* than the diameter of a hydrogen atom. This is because the order of magnitude of 10^9 is 9 and of 10^{-10} is -10. The difference is $9 - (-10) = 19$.

To quickly get an approximate value of a quantity in a given physical situation, one can make an *order of magnitude calculation.* In this all numbers are approximated to 10^b form and the calculation is made.

Let us estimate the number of persons that may sit in a circular field of radius 800 m. The area of the field is

$$A = \pi r^2 = 3{\cdot}14 \times (800 \text{ m})^2 \approx 10^6 \text{ m}^2.$$

The average area one person occupies in sitting $\approx 50 \text{ cm} \times 50 \text{ cm} = 0{\cdot}25 \text{ m}^2 = 2{\cdot}5 \times 10^{-1} \text{ m}^2 \approx 10^{-1} \text{ m}^2$. The number of persons who can sit in the field is

$$N \approx \frac{10^6 \text{ m}^2}{10^{-1} \text{ m}^2} = 10^7.$$

Thus of the order of 10^7 persons may sit in the field.

1.8 THE STRUCTURE OF WORLD

Man has always been interested to find how the world is structured. Long long ago scientists suggested that the world is made up of certain indivisible small particles. The number of particles in the world is large but the varieties of particles are not many. Old Indian philosopher Kanadi derives his name from this proposition (In Sanskrit or Hindi *Kana* means a small particle). After extensive experimental work people arrived at the conclusion that the world is made up of just three types of ultimate particles, the proton, the neutron and the electron. All objects which we have around us, are aggregation of atoms and molecules. The molecules are composed of atoms and the atoms have at their heart a nucleus containing protons and neutrons. Electrons move around this nucleus in special arrangements. It is the number of protons, neutrons and electrons in an atom that decides all the properties and behaviour of a material. Large number of atoms combine to form an object of moderate or large size. However, the laws that we generally deduce for these macroscopic objects are not always applicable to atoms, molecules, nuclei or the elementary particles. These laws known as *classical physics* deal with large size objects only. When we say a particle in classical physics we mean an object which is small as compared to other moderate or large size objects and for which the classical physics is valid. It may still contain millions and millions of atoms in it. Thus, a particle of dust dealt in classical physics may contain about 10^{18} atoms.

Twentieth century experiments have revealed another aspect of the construction of world. There are perhaps no ultimate indivisible particles. Hundreds of elementary particles have been discovered and there are free transformations from one such particle to the other. Nature is seen to be a well-connected entity.

Worked Out Examples

1. *Find the dimensional formulae of the following quantities :*
(a) the universal constant of gravitation G,
(b) the surface tension S,
(c) the thermal conductivity k and
(d) the coefficient of viscosity η.
Some equations involving these quantities are

$$F = \frac{Gm_1 m_2}{r^2}, \qquad S = \frac{\rho g r h}{2},$$

$$Q = k\frac{A(\theta_2 - \theta_1)t}{d} \quad \text{and} \quad F = -\eta A\frac{v_2 - v_1}{x_2 - x_1}$$

where the symbols have their usual meanings.

Solution : (a) $F = G\dfrac{m_1 m_2}{r^2}$

or, $G = \dfrac{Fr^2}{m_1 m_2}$

or, $[G] = \dfrac{[F]L^2}{M^2} = \dfrac{MLT^{-2}\,.\,L^2}{M^2} = M^{-1}L^3T^{-2}.$

(b) $S = \dfrac{\rho g r h}{2}$

or, $[S] = [\rho]\,[g]L^2 = \dfrac{M}{L^3}\cdot\dfrac{L}{T^2}\cdot L^2 = MT^{-2}.$

(c) $Q = k\dfrac{A(\theta_2 - \theta_1)t}{d}$

or, $k = \dfrac{Qd}{A(\theta_2 - \theta_1)\,t}.$

Here, Q is the heat energy having dimension ML^2T^{-2}, $\theta_2 - \theta_1$ is temperature, A is area, d is thickness and t is time. Thus,

$$[k] = \frac{ML^2T^{-2}L}{L^2KT} = MLT^{-3}K^{-1}.$$

(d) $F = -\eta A\dfrac{v_2 - v_1}{x_2 - x_1}$

or, $MLT^{-2} = [\eta]L^2\dfrac{L/T}{L} = [\eta]\dfrac{L^2}{T}$

or, $[\eta] = ML^{-1}T^{-1}.$

2. *Find the dimensional formulae of*
(a) the charge Q,
(b) the potential V,
(c) the capacitance C, and
(d) the resistance R.
Some of the equations containing these quantities are

$$Q = It,\ U = VIt,\ Q = CV \text{ and } V = RI;$$

where I denotes the electric current, t is time and U is energy.

Solution : (a) $Q = It.$ Hence, $[Q] = IT.$

(b) $U = VIt$

or, $ML^2T^{-2} = [V]IT$ or, $[V] = ML^2I^{-1}T^{-3}.$

(c) $Q = CV$

or, $IT = [C]ML^2I^{-1}T^{-3}$ or, $[C] = M^{-1}L^{-2}I^2T^4.$

(d) $V = RI$

or, $R = \dfrac{V}{I}$ or, $[R] = \dfrac{ML^2I^{-1}T^{-3}}{I} = ML^2I^{-2}T^{-3}.$

3. *The SI and CGS units of energy are joule and erg respectively. How many ergs are equal to one joule ?*

Solution : Dimensionally, Energy = *mass* × (*velocity*)^2

$$= mass \times \left(\frac{length}{time}\right)^2 = ML^2T^{-2}.$$

Thus, 1 joule = (1 kg) (1 m)^2 (1 s)$^{-2}$

and 1 erg = (1 g) (1 cm)^2 (1 s)$^{-2}$

$$\frac{1\text{ joule}}{1\text{ erg}} = \left(\frac{1\text{ kg}}{1\text{ g}}\right)\left(\frac{1\text{ m}}{1\text{ cm}}\right)^2\left(\frac{1\text{ s}}{1\text{ s}}\right)^{-2}$$

$$= \left(\frac{1000\text{ g}}{1\text{ g}}\right)\left(\frac{100\text{ cm}}{1\text{ cm}}\right)^2 = 1000 \times 10000 = 10^7.$$

So, 1 joule = 10^7 erg.

4. *Young's modulus of steel is 19×10^{10} N/m^2. Express it in dyne/cm^2. Here dyne is the CGS unit of force.*

Solution : The unit of Young's modulus is N/m^2.

This suggests that it has dimensions of $\dfrac{Force}{(distance)^2}$.

Thus, $[Y] = \dfrac{[F]}{L^2} = \dfrac{MLT^{-2}}{L^2} = ML^{-1}T^{-2}.$

N/m^2 is in SI units.

So, 1 N/m^2 = (1 kg)(1 m)$^{-1}$ (1 s)$^{-2}$

and 1 dyne/cm^2 = (1 g)(1 cm)$^{-1}$(1 s)$^{-2}$

so, $$\frac{1\text{ N/m}^2}{1\text{ dyne/cm}^2} = \left(\frac{1\text{ kg}}{1\text{ g}}\right)\left(\frac{1\text{ m}}{1\text{ cm}}\right)^{-1}\left(\frac{1\text{ s}}{1\text{ s}}\right)^{-2}$$

$$= 1000 \times \frac{1}{100} \times 1 = 10$$

or, 1 N/m^2 = 10 dyne/cm^2

or, 19×10^{10} N/m^2 = 19×10^{11} dyne/cm^2.

5. *If velocity, time and force were chosen as basic quantities, find the dimensions of mass.*

Solution : Dimensionally, *Force* = *mass* × *acceleration*

$$= mass \times \frac{velocity}{time}$$

or, $$mass = \frac{force \times time}{velocity}$$

or, $$[mass] = FTV^{-1}.$$

6. *Test dimensionally if the equation $v^2 = u^2 + 2ax$ may be correct.*

Solution : There are three terms in this equation v^2, u^2 and $2ax$. The equation may be correct if the dimensions of these three terms are equal.

$$[v^2] = \left(\frac{\mathrm{L}}{\mathrm{T}}\right)^2 = \mathrm{L}^2\,\mathrm{T}^{-2};$$

$$[u^2] = \left(\frac{\mathrm{L}}{\mathrm{T}}\right)^2 = \mathrm{L}^2\,\mathrm{T}^{-2};$$

and $$[2ax] = [a]\,[x] = \left(\frac{\mathrm{L}}{\mathrm{T}^2}\right)\mathrm{L} = \mathrm{L}^2\,\mathrm{T}^{-2}.$$

Thus, the equation may be correct.

7. *The distance covered by a particle in time t is given by $x = a + bt + ct^2 + dt^3$; find the dimensions of a, b, c and d.*

Solution : The equation contains five terms. All of them should have the same dimensions. Since $[x]$ = *length*, each of the remaining four must have the dimension of length.

Thus, $[a] = \mathit{length} = \mathrm{L}$

$[bt] = \mathrm{L}$, or, $[b] = \mathrm{LT}^{-1}$

$[ct^2] = \mathrm{L}$, or, $[c] = \mathrm{LT}^{-2}$

and $[dt^3] = \mathrm{L}$, or, $[d] = \mathrm{LT}^{-3}$.

8. *If the centripetal force is of the form $m^a\, v^b\, r^c$, find the values of a, b and c.*

Solution : Dimensionally,

$$\mathit{Force} = (\mathit{Mass})^a \times (\mathit{velocity})^b \times (\mathit{length})^c$$

or, $\mathrm{MLT}^{-2} = \mathrm{M}^a(\mathrm{L}^b\,\mathrm{T}^{-b})\,\mathrm{L}^c = \mathrm{M}^a\,\mathrm{L}^{b+c}\,\mathrm{T}^{-b}$

Equating the exponents of similar quantities,

$a = 1,\ b + c = 1,\ -b = -2$

or, $a = 1,\ b = 2,\ c = -1$ or, $F = \dfrac{mv^2}{r}$.

9. *When a solid sphere moves through a liquid, the liquid opposes the motion with a force F. The magnitude of F depends on the coefficient of viscosity η of the liquid, the radius r of the sphere and the speed v of the sphere. Assuming that F is proportional to different powers of these quantities, guess a formula for F using the method of dimensions.*

Solution : Suppose the formula is $F = k\,\eta^a\, r^b\, v^c$.

Then, $$\mathrm{MLT}^{-2} = [\mathrm{ML}^{-1}\mathrm{T}^{-1}]^a\,\mathrm{L}^b\left(\frac{\mathrm{L}}{\mathrm{T}}\right)^c$$
$$= \mathrm{M}^a\,\mathrm{L}^{-a+b+c}\,\mathrm{T}^{-a-c}.$$

Equating the exponents of M, L and T from both sides,

$$a = 1$$
$$-a + b + c = 1$$
$$-a - c = -2$$

Solving these, $a = 1$, $b = 1$, and $c = 1$.

Thus, the formula for F is $F = k\eta rv$.

10. *The heat produced in a wire carrying an electric current depends on the current, the resistance and the time. Assuming that the dependence is of the product of powers type, guess an equation between these quantities using dimensional analysis. The dimensional formula of resistance is $\mathrm{ML}^2\,\mathrm{I}^{-2}\mathrm{T}^{-3}$ and heat is a form of energy.*

Solution : Let the heat produced be H, the current through the wire be I, the resistance be R and the time be t. Since heat is a form of energy, its dimensional formula is $\mathrm{ML}^2\,\mathrm{T}^{-2}$.

Let us assume that the required equation is

$$H = kI^a\,R^b\,t^c,$$

where k is a dimensionless constant.

Writing dimensions of both sides,

$$\mathrm{ML}^2\mathrm{T}^{-2} = \mathrm{I}^a(\mathrm{ML}^2\mathrm{I}^{-2}\mathrm{T}^{-3})^b\,\mathrm{T}^c$$
$$= \mathrm{M}^b\,\mathrm{L}^{2b}\,\mathrm{T}^{-3b+c}\,\mathrm{I}^{a-2b}$$

Equating the exponents,

$$b = 1$$
$$2b = 2$$
$$-3b + c = -2$$
$$a - 2b = 0$$

Solving these, we get, $a = 2$, $b = 1$ and $c = 1$.

Thus, the required equation is $H = kI^2Rt$.

□

QUESTIONS FOR SHORT ANSWER

1. The metre is defined as the distance travelled by light in $\dfrac{1}{299{,}792{,}458}$ second. Why didn't people choose some easier number such as $\dfrac{1}{300{,}000{,}000}$ second? Why not 1 second?

2. What are the dimensions of :
(a) volume of a cube of edge a,
(b) volume of a sphere of radius a,
(c) the ratio of the volume of a cube of edge a to the volume of a sphere of radius a?

3. Suppose you are told that the linear size of everything in the universe has been doubled overnight. Can you test this statement by measuring sizes with a metre stick ? Can you test it by using the fact that the speed of light is a universal constant and has not changed ? What will happen if all the clocks in the universe also start running at half the speed ?

4. If all the terms in an equation have same units, is it necessary that they have same dimensions ? If all the terms in an equation have same dimensions, is it necessary that they have same units ?

5. If two quantities have same dimensions, do they represent same physical content ?

6. It is desirable that the standards of units be easily available, invariable, indestructible and easily reproducible. If we use foot of a person as a standard unit of length, which of the above features are present and which are not ?

7. Suggest a way to measure :
(a) the thickness of a sheet of paper,
(b) the distance between the sun and the moon.

OBJECTIVE I

1. Which of the following sets cannot enter into the list of fundamental quantities in any system of units ?
(a) length, mass and velocity,
(b) length, time and velocity,
(c) mass, time and velocity,
(d) length, time and mass.

2. A physical quantity is measured and the result is expressed as nu where u is the unit used and n is the numerical value. If the result is expressed in various units then
(a) $n \propto$ size of u (b) $n \propto u^2$
(c) $n \propto \sqrt{u}$ (d) $n \propto \frac{1}{u}$.

3. Suppose a quantity x can be dimensionally represented in terms of M, L and T, that is, $[x] = \mathrm{M}^a \mathrm{L}^b \mathrm{T}^c$. The quantity mass
(a) can always be dimensionally represented in terms of L, T and x,
(b) can never be dimensoinally represented in terms of L, T and x,
(c) may be represented in terms of L, T and x if $a = 0$,
(d) may be represented in terms of L, T and x if $a \neq 0$.

4. A dimensionless quantity
(a) never has a unit, (b) always has a unit,
(c) may have a unit, (d) does not exist.

5. A unitless quantity
(a) never has a nonzero dimension,
(b) always has a nonzero dimension,
(c) may have a nonzero dimension,
(d) does not exist.

6. $\int \frac{dx}{\sqrt{2ax - x^2}} = a^n \sin^{-1}\left[\frac{x}{a} - 1\right]$.
The value of n is
(a) 0 (b) −1
(c) 1 (d) none of these.
You may use dimensional analysis to solve the problem.

OBJECTIVE II

1. The dimensions $\mathrm{ML}^{-1}\mathrm{T}^{-2}$ may correspond to
(a) work done by a force
(b) linear momentum
(c) pressure
(d) energy per unit volume.

2. Choose the correct statement(s) :
(a) A dimensionally correct equation may be correct.
(b) A dimensionally correct equation may be incorrect.
(c) A dimensionally incorrect equation may be correct.
(d) A dimensionally incorrect equation may be incorrect.

3. Choose the correct statement(s) :
(a) All quantities may be represented dimensionally in terms of the base quantities.
(b) A base quantity cannot be represented dimensionally in terms of the rest of the base quantities.
(c) The dimension of a base quantity in other base quantities is always zero.
(d) The dimension of a derived quantity is never zero in any base quantity.

EXERCISES

1. Find the dimensions of
(a) linear momentum,
(b) frequency and
(c) pressure.

2. Find the dimensions of
(a) angular speed ω, (b) angular acceleration α,
(c) torque Γ and (d) moment of interia I.
Some of the equations involving these quantities are

$\omega = \dfrac{\theta_2 - \theta_1}{t_2 - t_1}$, $\alpha = \dfrac{\omega_2 - \omega_1}{t_2 - t_1}$, $\Gamma = F.r$ and $I = mr^2$.

The symbols have standard meanings.

3. Find the dimensions of
(a) electric field E, (b) magnetic field B and
(c) magnetic permeability μ_0.
The relevant equations are
$F = qE$, $F = qvB$, and $B = \dfrac{\mu_0 I}{2\pi a}$;
where F is force, q is charge, v is speed, I is current, and a is distance.

4. Find the dimensions of
(a) electric dipole moment p and
(b) magnetic dipole moment M.
The defining equations are $p = q.d$ and $M = IA$;
where d is distance, A is area, q is charge and I is current.

5. Find the dimensions of Planck's constant h from the equation $E = h\nu$ where E is the energy and ν is the frequency.

6. Find the dimensions of
(a) the specific heat capacity c,
(b) the coefficient of linear expansion α and
(c) the gas constant R.
Some of the equations involving these quantities are
$Q = mc(T_2 - T_1)$, $l_t = l_0[1 + \alpha(T_2 - T_1)]$ and $PV = nRT$.

7. Taking force, length and time to be the fundamental quantities find the dimensions of
(a) density, (b) pressure,
(c) momentum and (d) energy.

8. Suppose the acceleration due to gravity at a place is 10 m/s^2. Find its value in cm/(minute)^2.

9. The average speed of a snail is 0·020 miles/hour and that of a leopard is 70 miles/hour. Convert these speeds in SI units.

10. The height of mercury column in a barometer in a Calcutta laboratory was recorded to be 75 cm. Calculate this pressure in SI and CGS units using the following data : Specific gravity of mercury = 13·6, Density of water = 10^3 kg/m^3, g = 9·8 m/s^2 at Calcutta. Pressure = $h\rho g$ in usual symbols.

11. Express the power of a 100 watt bulb in CGS unit.

12. The normal duration of I.Sc. Physics practical period in Indian colleges is 100 minutes. Express this period in microcenturies. 1 microcentury = $10^{-6} \times 100$ years. How many microcenturies did you sleep yesterday ?

13. The surface tension of water is 72 dyne/cm. Convert it in SI unit.

14. The kinetic energy K of a rotating body depends on its moment of inertia I and its angular speed ω. Assuming the relation to be $K = kI^a\omega^b$ where k is a dimensionless constant, find a and b. Moment of inertia of a sphere about its diameter is $\frac{2}{5}Mr^2$.

15. Theory of relativity reveals that mass can be converted into energy. The energy E so obtained is proportional to certain powers of mass m and the speed c of light. Guess a relation among the quantities using the method of dimensions.

16. Let I = current through a conductor, R = its resistance and V = potential difference across its ends. According to Ohm's law, product of two of these quantities equals the third. Obtain Ohm's law from dimensional analysis. Dimensional formulae for R and V are $ML^2I^{-2}T^{-3}$ and $ML^2T^{-3}I^{-1}$ respectively.

17. The frequency of vibration of a string depends on the length L between the nodes, the tension F in the string and its mass per unit length m. Guess the expression for its frequency from dimensional analysis.

18. Test if the following equations are dimensionally correct :
(a) $h = \dfrac{2S\cos\theta}{\rho rg}$, (b) $v = \sqrt{\dfrac{P}{\rho}}$,
(c) $V = \dfrac{\pi P r^4 t}{8\eta l}$, (d) $\nu = \dfrac{1}{2\pi}\sqrt{\dfrac{mgl}{I}}$;
where h = height, S = surface tension, ρ = density, P = pressure, V = volume, η = coefficient of viscosity, ν = frequency and I = moment of inertia.

19. Let x and a stand for distance. Is $\displaystyle\int \frac{dx}{\sqrt{a^2 - x^2}} = \frac{1}{a}\sin^{-1}\frac{a}{x}$ dimensionally correct ?

□

ANSWERS

OBJECTIVE I

1. (b) 2. (d) 3. (d) 4. (c) 5. (a) 6. (a)

OBJECTIVE II

1. (c), (d) 2. (a), (b), (d) 3. (a), (b), (c)

EXERCISES

1. (a) MLT^{-1} (b) T^{-1} (c) $ML^{-1}T^{-2}$
2. (a) T^{-1} (b) T^{-2} (c) ML^2T^{-2} (d) ML^2
3. (a) $MLT^{-3}I^{-1}$ (b) $MT^{-2}I^{-1}$ (c) $MLT^{-2}I^{-2}$
4. (a) LTI (b) L^2I
5. ML^2T^{-1}
6. (a) $L^2T^{-2}K^{-1}$ (b) K^{-1} (c) $ML^2T^{-2}K^{-1}(\text{mol})^{-1}$

7. (a) $FL^{-4}T^2$ (b) FL^{-2} (c) FT (d) FL
8. 36×10^5 cm/(minute)^2
9. 0·0089 m/s, 31 m/s
10. 10×10^4 N/m^2, 10×10^5 dyne/cm^2
11. 10^9 erg/s
12. 1·9 microcenturies
13. 0·072 N/m
14. $a = 1$, $b = 2$
15. $E = kmc^2$
16. $V = IR$
17. $\frac{k}{L}\sqrt{\frac{F}{m}}$
18. all are dimensionally correct
19. no

□