BMAT201L - Complex Variables and Linear Algebra
Linear Algebra Toolkit

Submitted to Prof. Vijay Kumar

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Introduction

The Linear Algebra Toolkit is a computational aid for exploring and solving core problems in linear algebra. The toolkit integrates several foundational algorithms and methods that are central to both theoretical understanding and practical applications in mathematics, engineering, and computer science. The toolkit is developed with clarity, modularity, and educational accessibility in mind, making it a valuable resource for students.

Key Features

1. **Accurate Results** – All computations are executed with mathematical rigor, ensuring reliable and precise outcomes for every problem.
2. **Step-by-Step Solutions** – Each method is broken down into clear, logical steps to facilitate a deeper understanding of the underlying mathematical processes.
3. **Time-Saving Computation** – Automates laborious calculations, allowing users to concentrate on conceptual understanding rather than manual arithmetic.
4. **User-Friendly Interface** – Designed with accessibility in mind, the intuitive interface and clear prompts enable users from diverse backgrounds to engage with the toolkit effectively.
5. **Stepwise Navigation** – Users can proceed through each computational step at their own pace, reinforcing their grasp of linear algebra techniques.
6. **Quick Access to Final Answers** – For efficiency, users may opt to skip directly to final results, with the option to revisit the detailed steps for review and learning.

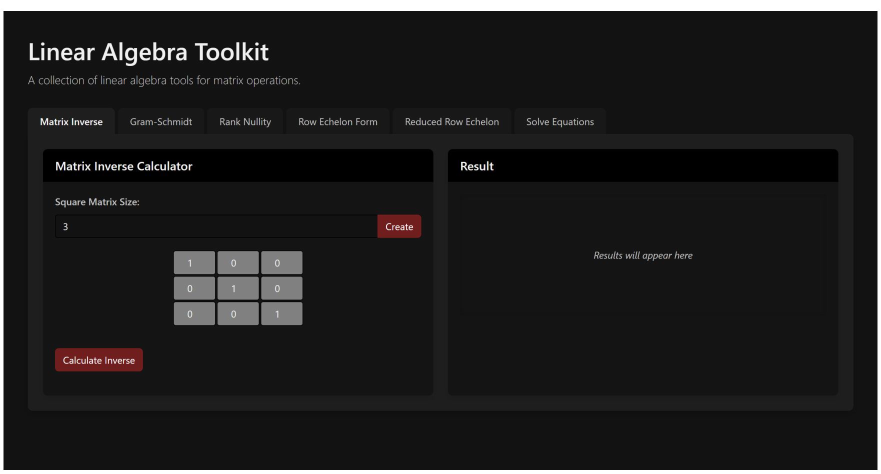


Figure 1: Home page

Main Sections and Topics Implemented

This project incorporates six essential topics from Linear Algebra, each selected for their conceptual importance and time consuming process. The toolkit automates these processes while preserving value through detailed step-by-step solutions.

Matrix Inversion

- Computes the inverse of a square matrix using the Gauss-Jordan elimination technique.
- Displays each row operation used in the augmentation and reduction process for clarity and learning support.

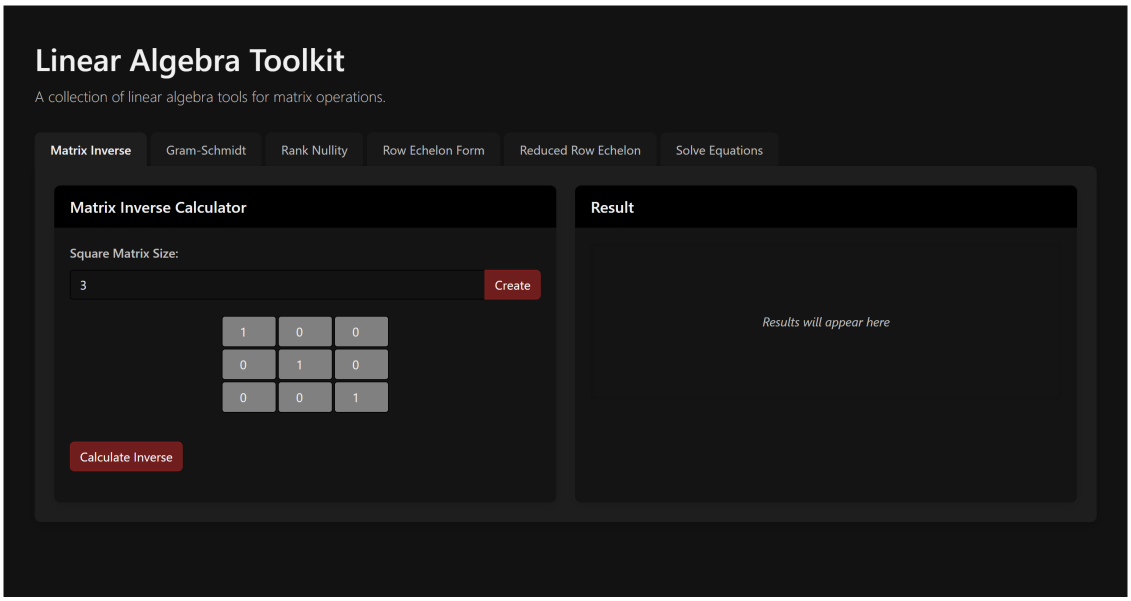
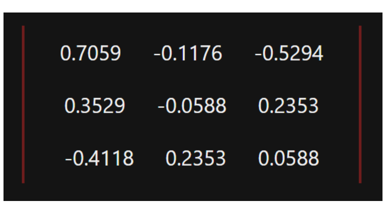


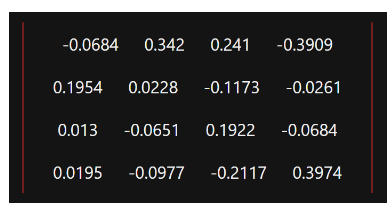
Figure 2: Matrix Inverse

Examples

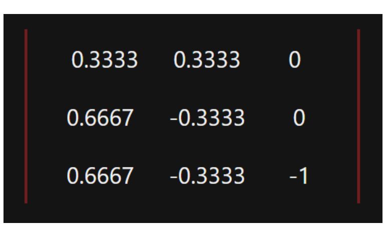
$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 6 \\ -1 & 2 & 0 \end{bmatrix}$$



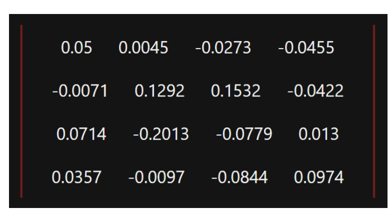
$$\begin{bmatrix} 1 & 5 & 4 & 2 \\ 4 & 1 & 0 & 4 \\ 2 & 0 & 6 & 3 \\ 2 & 0 & 3 & 5 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 1 & -1 \\ 2 & -1 & 0 \\ 0 & 1 & -1 \end{bmatrix}$$



$$\begin{bmatrix} 9 & 8 & 5 & 7 \\ 5 & 0 & -5 & 3 \\ -6 & 8 & 5 & 0 \\ -8 & 4 & 2 & 8 \end{bmatrix}$$



Orthonormalization

- Converts a linearly independent set of vectors into an orthogonal or orthonormal basis using the Gram-Schmidt method.
- Each projection, subtraction, and normalization step is detailed to illustrate the underlying vector space transformations.

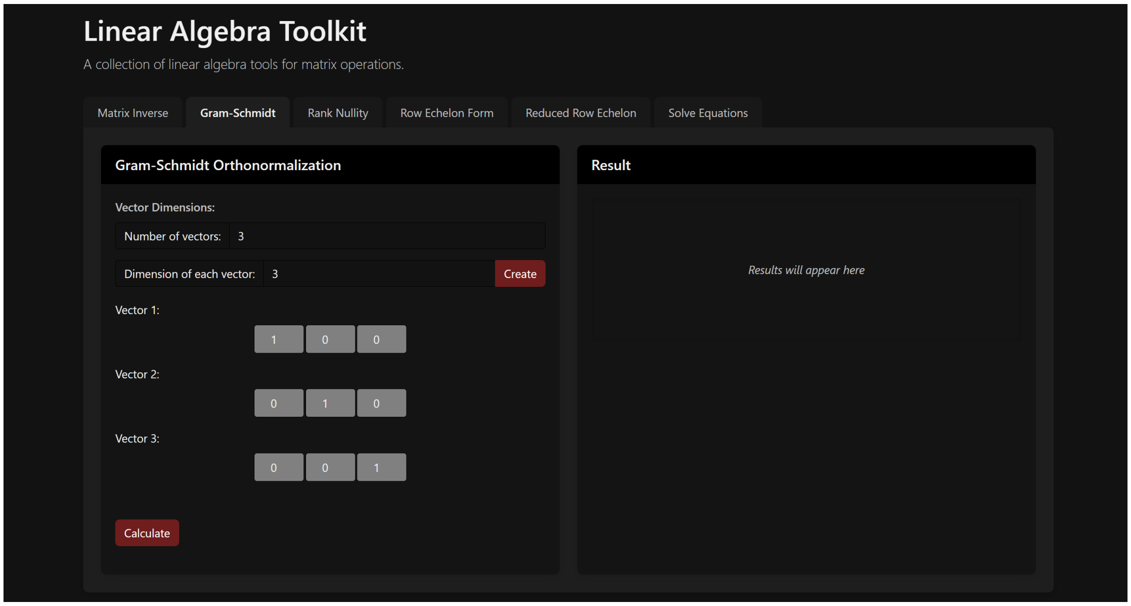
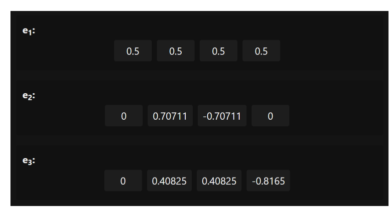


Figure 3: Gram-Schmidt

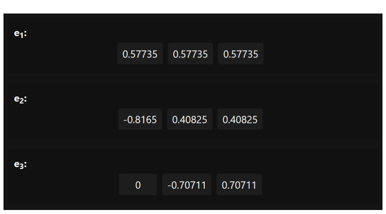
Examples

$$\mathbf{u}_1 = \begin{pmatrix} 1 & 1 & 0 & 1 \end{pmatrix}$$
$$\mathbf{u}_2 = \begin{pmatrix} 0 & 1 & 1 & 1 \end{pmatrix}$$
$$\mathbf{u}_3 = \begin{pmatrix} -1 & 0 & 0 & 1 \end{pmatrix}$$

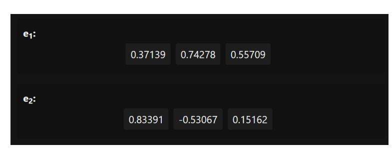
$$\mathbf{u}_1 = \begin{pmatrix} 1 & 1 & 1 & 1 \end{pmatrix}$$
$$\mathbf{u}_2 = \begin{pmatrix} 1 & 2 & 0 & 1 \end{pmatrix}$$
$$\mathbf{u}_3 = \begin{pmatrix} 2 & 2 & 4 & 0 \end{pmatrix}$$



$$\mathbf{u}_1 = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$
$$\mathbf{u}_2 = \begin{pmatrix} 0 & 1 & 1 \end{pmatrix}$$
$$\mathbf{u}_3 = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}$$



$$\mathbf{u}_1 = \begin{pmatrix} 2 & 4 & 3 \end{pmatrix}$$
$$\mathbf{u}_2 = \begin{pmatrix} 1 & 1 & 1 \end{pmatrix}$$



Rank and Nullity (Matrix Spaces)

- Identifies the basis for the column space, row space, and null space of a given matrix.
- Includes computation of matrix rank and nullity, followed by verification of the Rank-Nullity Theorem.
- The methodology highlights dependencies and dimensionality in linear mappings.

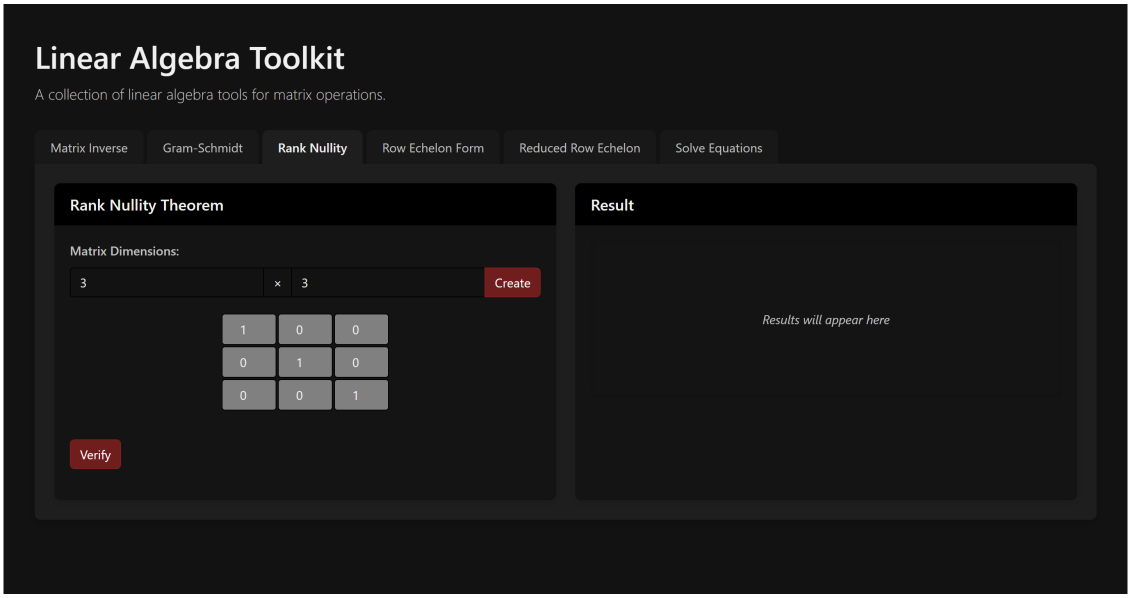
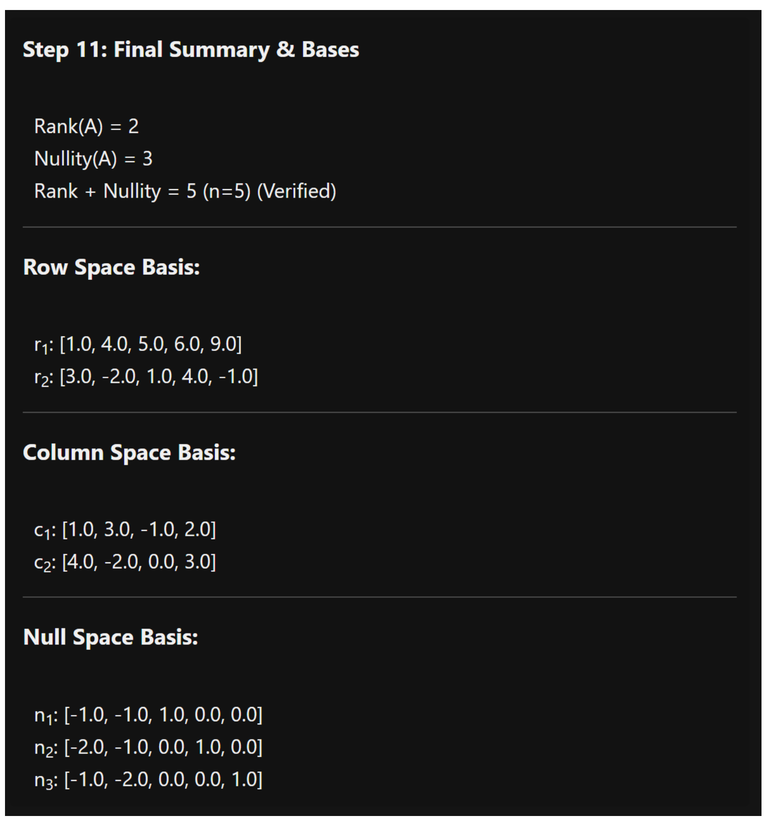


Figure 3: Rank Nullity

Example

$$\begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$$



Row Echelon Form (REF)

- Reduces matrices to row echelon form using a structured sequence of elementary row operations.
- This transformation facilitates solving systems of linear equations and understanding pivot structure.

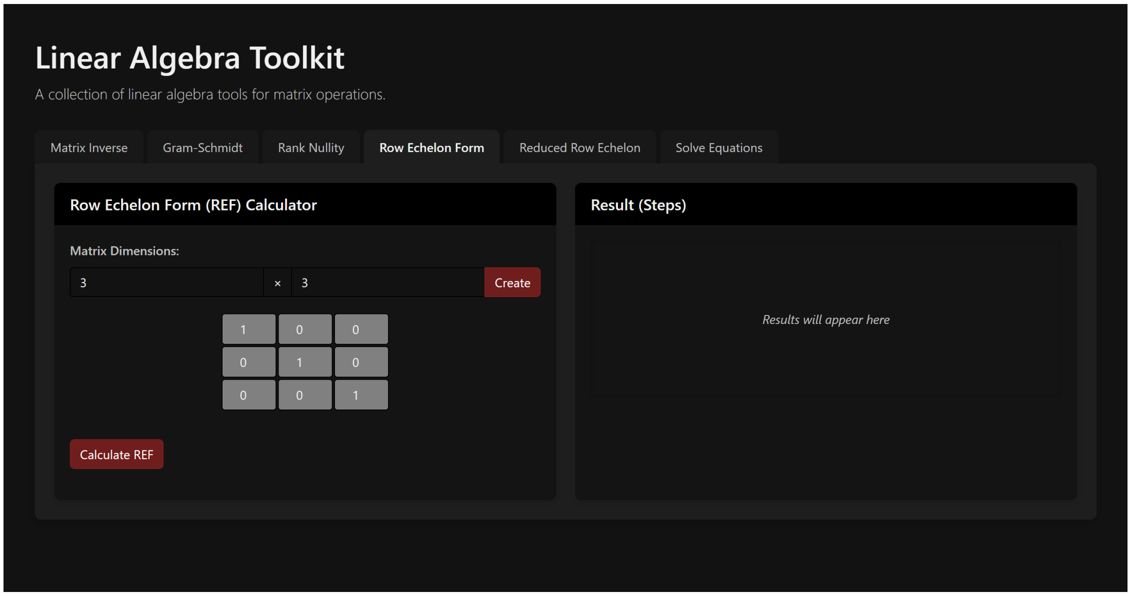
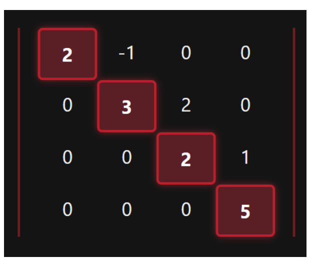


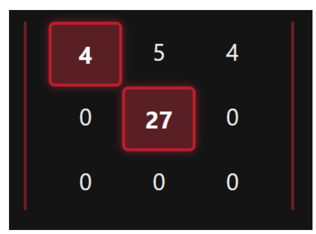
Figure 4: REF

Examples

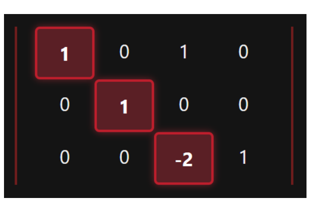
$$\begin{bmatrix} 2 & -1 & 0 & 0 \\ 0 & 3 & 2 & 0 \\ 0 & 0 & 2 & 1 \\ 2 & 0 & 1 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 4 & 5 & 4 \\ 5 & 13 & 5 \\ 4 & 5 & 4 \end{bmatrix}$$



$$\begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 \\ 2 & 0 & 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$$

Reduced Row Echelon Form (RREF)

- Further simplifies matrices to reduced row echelon form.
- Used to determine solution types (unique, infinite, or none) for linear systems.
- Emphasizes algorithmic transparency with intermediate row states.

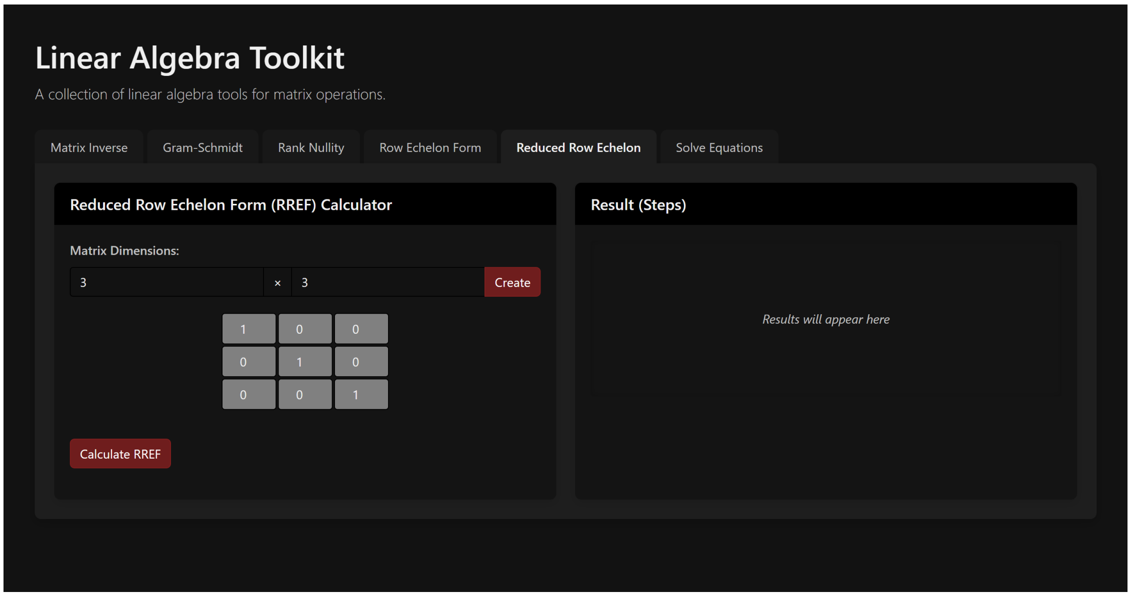
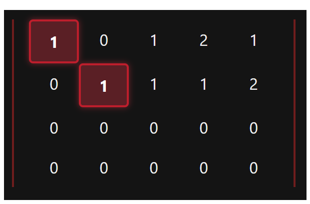


Figure 3: RREF

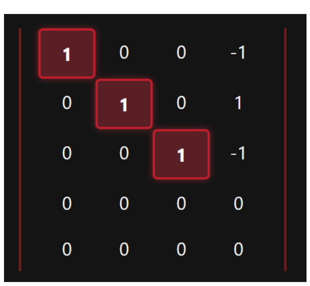
Examples

$$\begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 17 & 8 \end{bmatrix}$$

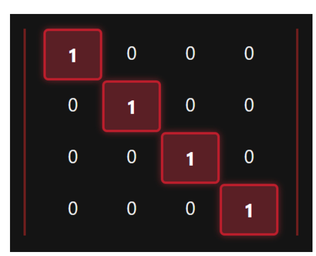


$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 0 & 0 & 1 \end{bmatrix}$$



$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$



Solving Linear Systems

- Solves both homogeneous and non-homogeneous systems using augmented matrices and row-reduction techniques.
- Offers detailed step-by-step reduction and solution classification based on matrix rank analysis.
- Includes an option for direct solution output for quick verification.

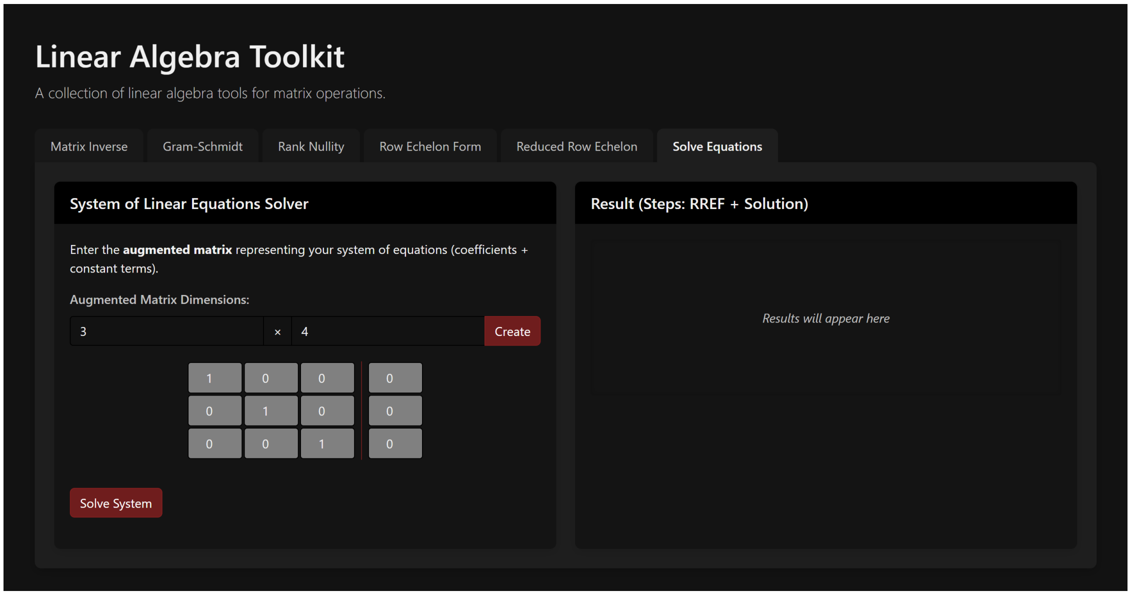
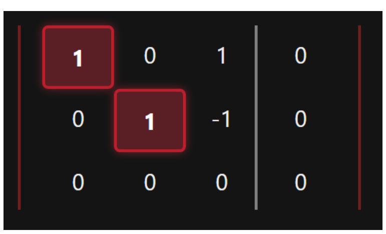


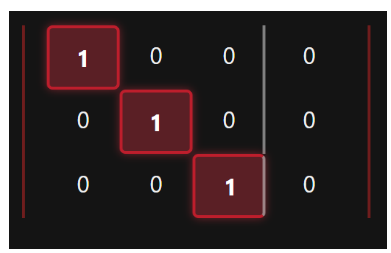
Figure 3: Equation Solver

Examples

$$
\begin{aligned}
x + 2y - z &= 0 \\
3x - y + 4z &= 0 \\
2x + y + z &= 0
\end{aligned}
$$



$$
\begin{aligned}
x + y + z &= 0 \\
2x - y + 3z &= 0 \\
-x + 4y - 2z &= 0
\end{aligned}
$$



$$
\begin{aligned}
2x + 3y &= 5 \\
x - y &= 1
\end{aligned}
$$

$$
\begin{aligned}
x + 2y + z &= 3 \\
2x + 1y + 4z &= 1 \\
-x + 2y &= 1
\end{aligned}
$$

How to Use

1. Select the desired linear algebra operation from the main menu.
2. Input the matrix or system of equations using the designated input fields.
3. Choose between a detailed, step-by-step solution walkthrough or immediate access to the final result.
4. Progress through the solution steps at your own pace, with the ability to revisit previous steps for deeper review.

Conclusion

This project is designed to significantly enhance the accessibility, accuracy, and efficiency of learning and applying linear algebra. By automating complex and time-consuming computations, it allows users to focus on understanding key concepts rather than getting bogged down in tedious arithmetic.