Topics Covered by Exam:

1. Linear Equations (Chapter 1, all.)

   To review: Make sure you can solve a system of equations using RREF. Also studying later sections will review this information.

2. Linear Transformations (Chapter 2, all)

   To review: Make sure you can show a function is a linear transformation and that you can find the inverse of a matrix.

   Look also at problems from the book, particularly section 2.4 and the odd T/F up to 49 in the end of the chapter.

3. Subspaces of $\mathbb{R}^n$ (Chapter 3, all)

   To review: Learn the definitions (know the precise statements, know examples, know how to use the definitions).

   Do exercises from 3.4 1-15 odd. Do T/F at the end of the chapter up to 43 (give explanations).

4. Linear Spaces (Chapter 4, all)

   To review: Learn definitions and practice showing sets are linearly independent and span in linear spaces.

   Do extra book problems from 4.3, particularly 14 and a bunch from 5-29 odd (answers are in the back of the book). Also, do Chapter 4 review: 1-11, 13, 14, 16-23, 28-31, 36, 42, 45, 49.

5. Determinants (Chapter 6)

   Know how to take the determinant of 2x2 and 3x3 matrices. Know all the properties of the determinant: Facts: 6.2.1, 6.2.4, 6.2.5, 6.2.6, 6.2.7, 6.2.8, and definition 6.2.11.

   To review: Try problems 6.1 11-22 and 6.2 problems 17-27 and see T/F: # 1, 2, 4, 6, 7, 8, 9, 10, 12, 15, 16, 17, 19, 24.

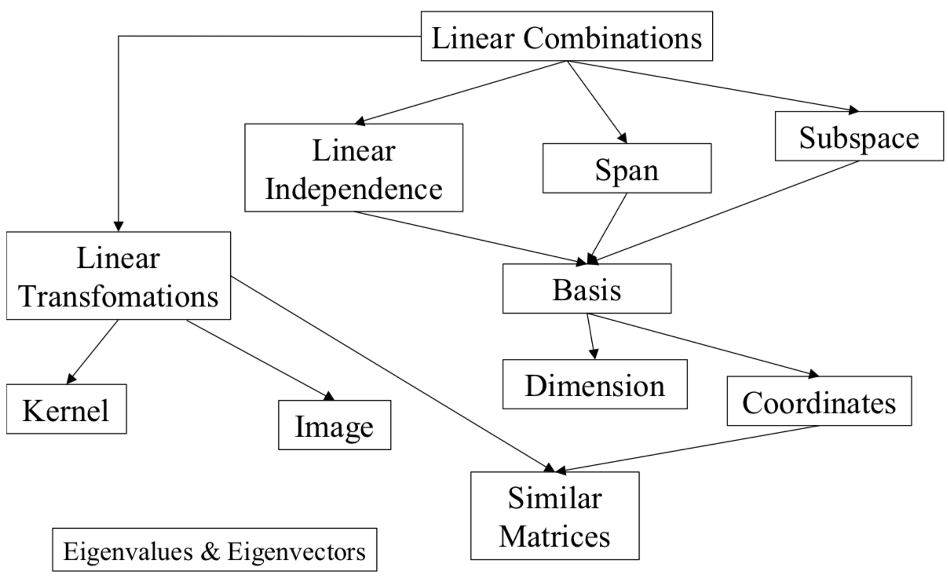
6. Eigenvalues and Eigenvectors (Chapter 7, Sections 1-3)

   To review: Review Eigenstuff practice quiz and quiz and redo with different matrices like those found in 7.3 # 1-11 **be sure to do examples that are and are not diagonalizable and be sure to find the change of basis matrix in each case that it is.**

   Also look at the following T/F Questions for chapter 7, #1-7, 15, 17, 18, 19, 21, 22, 27, 28, 33, 36, 38, 39, 44, 46.

Know and understand this diagram:

Basic Concepts for Linear Algebra



Example 1 *A practice example: Recall that P_2 is the set of polynomials of degree less than or equal to 2. Consider a linear transformation $T : P_2 \to P_2$ defined by $T(f(t)) = f(t-3)$.*

1. *Show that this is a linear transformation.*
2. $T(a_0 + a_1 t + a_2 t^2) =$
3. $T(1) =$
4. $T(t) =$
5. $T(t^2) =$
6. *If $\mathfrak{A}$ is the standard basis for P_2. What is $A = [T]_{\mathfrak{A}}$?* $\begin{bmatrix} 1 & -3 & 9 \\ 0 & 1 & -6 \\ 0 & 0 & 1 \end{bmatrix}$
7. *What is the kernel of A? What is its nullity T?*
8. *What is the rank of T? What is its Image?*
9. *How are rank and nullity related?*
10. *Is $A = [T]_{\mathfrak{A}}$ invertible? Is it an isomorphism?*
11. *What is the characteristic polynomial of A?*
12. *What are the eigenvalues of A? What are their algebraic multiplicities?*
13. *What are the eigenvectors of A?*
14. *For each eigenvalue, find the eigenspaces and the and their dimensions of each eigenvalue.*
15. *Is A diagonalizable?*

Example 2 *Fill in the blanks:*

1. *A subset W of a linear space V is called a subspace of V if*

   (a) _______________ $\in W$

   (b) *If* $\vec{u}, \vec{v} \in W$ *and* $k \in \mathbb{R}$ *then* ____

2. *Let* $\mathfrak{B} = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ *be a set of elements in a linear space* V.

   (a) *A* **linear combination** *of* $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ *is a vector* $\vec{v} =$ _________________________ *where the* $c_i \in \mathbb{R}$.

   (b) $\text{Span}\mathfrak{B} = \{$ _________________________ $|$ _________________________ $\}$, *that is the set of all linear combinations of vectors in* $\mathfrak{B}$.

   (c) *The set* $\mathfrak{B} = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ *is said to be* **linearly independent** *if* _______________

   (d) *The set* $\mathfrak{B} = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_k\}$ *is said to be* **a basis** *for a subspace W of V if*

      i. *The vectors of* $\mathfrak{B}$ _______________ W.

      ii. *The vectors of* $\mathfrak{B}$ *are* _____________________________.

      iii. *If* $\mathfrak{B}$ *is a basis for W then* $\dim(W) =$ _______________

   (e) *If* $\dim(W) = k$ *then any linearly independent set of k vectors is a basis.*

   (f) *If* $\dim(W) = k$ *then any spanning set of k vectors is a basis.*

   (g) *If* $\vec{v} \in W$ *and* $\mathfrak{B}$ *is a basis, how to you find* $[\vec{v}]_{\mathfrak{B}}$?

Example 3 *Let $T : V \to W$ be a linear transformation.*

1. *What is the kernel of T? What is the nullity of T?*

2. *What is the image of T? What is the rank of T?*

3. *If $\mathfrak{B} = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ is a basis for V then*

   $[T]_{\mathfrak{B}} =$

   From here on assume $T : \mathbb{R}^n \to \mathbb{R}^m$ via $T(\vec{x}) = A\vec{x}$

4. *What is the size of A?*

5. *How do you know if T is one to one?*

6. *How do you know if T is onto?*

7. *How do you know if T is invertible?*

From here on assume $T : \mathbb{R}^n \to \mathbb{R}^m$ (i.e. $m = n$).

8. *How do you find the characteristic polynomial?*

9. *Is the characteristic polynomial the same for $[T]_{\mathfrak{A}}$ where $\mathfrak{A}$ is another basis for $\mathbb{R}^n$?*

10. *What is an eigenvalue λ of T? How do you find it? What is the algebraic multiplicity of λ?*

11. *What is an eigenvector of T associated to eigenvalue λ? How do you find it?*

12. *What is the eigenspace* E_λ *of* T *associated to eigenvalue* λ*? How do you find it?*

13. *What do we mean when we say that* $\mathfrak{B} = \{\vec{w}_1, \vec{w}_2, \ldots, \vec{w}_n\}$ *is an eigenbasis for* $\mathbb{R}^n$ *with respect to* T*?*

14. *What does* $[T]_{\mathfrak{B}}$ *look like?*

15. *How is it related to* $A = [T]_S$ *where* S *is the standard basis for* $\mathbb{R}^n$*?*

16. *If* $\{\lambda_1, \lambda_2, \ldots, \lambda_k\}$ *are eigenvalues for* T*, what are the eigenvalues for* T^{25}*?*