Control Systems Assignment-1

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EE19BTECH11029

September 10, 2020

Question :

For the rotational mechanical system shown in Figure P2.17, find the transfer function G(s) = $\theta_2(s) \backslash T(s)$

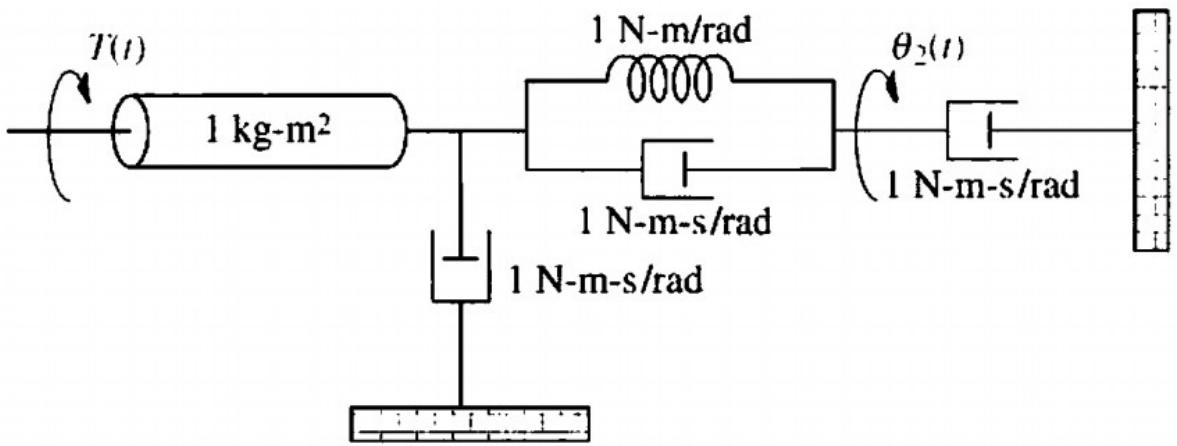

FIGURE P2.17

Rotational Mechanical Systems

$*$*It goes around a constant fixed axis*.

$*$*These systems mainly consists of three basic elements*.

1. Moment of interia
2. Dashpot
3. Torsional Spring

$*$*If a torque is applied to a rotational mechanical system*,

then it is opposed by opposing torques due to moment

of interia, elasticity and friction of the system.

Rotational Mechanical Systems

$*$*Then the applied torque and the opposing torques*

are in opposite directions.

$*$*Then the algebraic sum of torques acting*

on the system is zero.

$\star$*Moment of Ineria* :

Torque, T(t) = J($\frac{d\omega}{dt}$) = $J(\frac{d^2\theta}{dt^2})$

$$Impedance, T(s)/\theta(s) = Js^2 \tag{1}$$

$\star$*Torsional Spring* :

Torque, T(t) = Kθ(t)

$$Impedance, T(s)/\theta(s) = K \tag{2}$$

Rotational Mechanical Systems

$\star$*Dashpot* :

1) One free end (i.e if one end is fixed to a reference):

$$\mathrm{T(t)} = \mathrm{D}\omega(t) = D(\frac{d\theta}{dt})$$

2) For two free ends,

$$\mathrm{T(t)} = \mathrm{D}(\ \frac{d(\theta_1 - \theta_2)}{dt})$$

$$Impedance, T(s)/\theta(s) = Ds \tag{3}$$

Solution

Given rotational mechanical system shown in Figure P2.17,

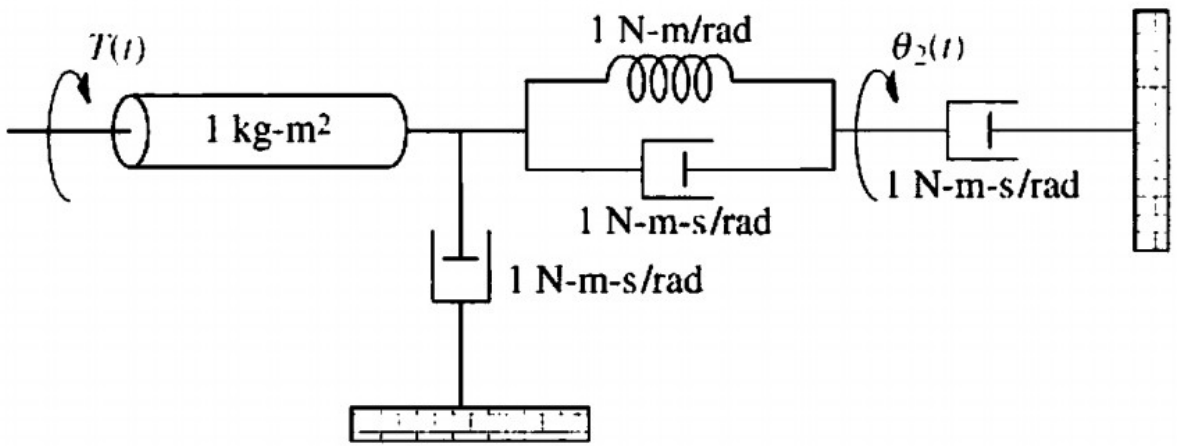

FIGURE P2.17

Solution

Writing the equations of motion,

$$(s^2 + 2s + 1)\theta_1(s) - (s + 1)\theta_2(s) = T(s)$$
$$-(s + 1)\theta_1(s) + (2s + 1)\theta_2(s) = 0 \qquad (4)$$

By solving the equation for $\theta_2(s)$

$$\theta_2(s) = \frac{\begin{vmatrix} (s^2 + 2s + 1) & T(s) \\ -(s + 1) & 0 \end{vmatrix}}{\begin{vmatrix} (s^2 + 2s + 1) & -(s + 1) \\ -(s + 1) & (2s + 1) \end{vmatrix}}$$

Solution

$$\Rightarrow \theta_2(s) = \frac{T(s)}{2s(s+1)} \tag{5}$$

Hence , substituting the T(s)

$\Rightarrow$ *We get*,

$$\Longrightarrow \frac{\theta_2(s)}{T(s)} = \frac{1}{2s(s+1)} \tag{6}$$

Hence, the transfer function is

$$\frac{1}{2s(s+1)} \tag{7}$$