

CS 1511 Homework 13

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24.)

25.) Take a language L in PH .

$L = \{ \langle x, k \rangle \mid \text{where } \exists \text{ a } C_{|x|} \text{ Boolean circuit with } |x|^k \text{ gates, and } \forall \text{ circuits } D \text{ with less than } |x|^k \text{ gates, the circuit does not compute the same Boolean function as } C_{|x|}. \}$

This language is clearly in Σ_2^P , so therefore L is a language in PH .

A machine to check if a set of strings x is in L will have a runtime of $O(2^{x^k})$ when simulated on a Turing Machine.

The Turing Machine will have to construct every possible circuit with less than $|x|^k$ gates, which will take $O(2^{x^k})$.

With this in mind, the amount of space necessary (circuit complexity necessary) will be $\Omega(n^k)$.

This is due to our circuit being able to hardwire in all the possibilities from our Turing Machine in polynomial time.

In exponential time, we can use poly-space to model our Turing Machine.

Thus, for every $k > 0$ there is a language in PH whose circuit complexity is $\Omega(n^k)$.