

CS 1511 Homework 3

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5. (a)

5. (b)

6. (a)

i. $H(X) = \sum P(x) * \log_2 \frac{1}{P(x)}$

$$H(X) = \frac{1}{3} * \log_2 3 + \frac{2}{3} * \log_2 \frac{3}{2} = 0.918$$

ii. $P(y = 1) = P(y = 1|x = 0) * P(x = 0) + P(y = 1|x = 1) * P(x = 1) = \frac{17}{30}$

$$P(y = 0) = P(y = 0|x = 0) * P(x = 0) + P(y = 0|x = 1) * P(x = 1) = \frac{13}{30}$$

iii. $H(Y) = \sum P(y) * \log_2 \frac{1}{P(y)}$

$$H(Y) = \frac{13}{30} * \log_2 \frac{30}{13} + \frac{17}{30} * \log_2 \frac{30}{17} = 0.9871$$

iv. $H(X|Y) = \sum P(x, y) * \log_2 \frac{1}{P(x|y)}$

Using Bayes Formula:

$$(((9/13) * \log_2(13/9) + (4/13)(\log_2(13/4))) * (13/30)) + 17/30 * ((1/17) * (\log_2(17) + 16/17 * \log_2(17/16))) = 0.561$$

v. $H(Y|X) = \sum P(x) * \sum P(y|x) * \log_2 \frac{1}{P(y|x)}$

$$1/3 * ((9/10) * \log_2(10/9) + 1/10 * \log_2(10)) + 2/3 * (2/10 * \log_2(5) + 8/10 * \log_2(10/8)) = 0.63$$

vi. $I(X; Y) = H(X) - H(X|Y) = 0.918 - 0.561 = 0.357$

vii. $I(Y; X) = H(Y) - H(Y|X) = 0.9871 - 0.63 = 0.3571$

viii. In this setting this means that there is the same amount of uncertainty between either what is sent or received. If you know what is sent you are exactly unsure of what is received as the other way around.

6. (b)