CS 1511 Homework 12

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21. b)

EXACT INDSET = $\{\langle G, k\rangle$ | the largest independent set in G has size exactly k$\}$.

INDSET = $\{\langle G, k\rangle$ | there exists an independent set in G with size k$\}$.

LE INDSET = $\{\langle G, k\rangle$ | all sets in G have size $\leq$ k $\}$.

EXACT INDSET = INDSET $\cap$ LE INDSET

INDSET $\in$ NP. This is shown in a previous theorem. The polynomial verifier would just check the given set and check if it is independent, if it is in G, and if it is of length k.

LE INDSET is in CoNP. This is because it is a "for all" (for all set in G ...). More formally, the Turing Machine verifier would be the same as that above.

EXACT INDSET is the union of these. This is by definition. Every element in EXACT INDSET will be in INDSET. If the largest independent set in G has size exactly k, then there definitely exists a set of size k. Every element is also in LE INDSET. If the largest set has size exactly k, then it all sets will have size less than or equal to k.

So, EXACT INDSET is in DP. L_1 = INDSET, $L_1 \in$ NP, L_2 = LE INDSET . $L_2 \in$ CoNP, L = $L_1 \cap L_2$.

21 c)

We need to show $\forall L \in DP \leq EXACTINDSET$