

# CS 1511 Homework 17

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## 32.)

The language  $\text{QNR} = \{ (a, p) \mid a \text{ is not a quadratic residue modulo } p \text{ where } p \text{ is a prime.} \}$

From Euler's criterion, we know that there are  $(p+1)/2$  quadratic residues and  $(p-1)/2$  quadratic nonresidues.

In the private coin protocol, the verifier takes a random number  $r \bmod p$  and a random bit  $b \in \{0,1\}$  and sends the prover  $r^2 \bmod p$  if  $b = 0$  and sends the prover  $ar^2 \bmod p$  if  $b = 1$ .

If  $a$  is a quadratic residue, then the prover has a  $1/2$  chance of guessing  $b$  correctly. Otherwise, the prover is certain of the value of  $b$ .

We know from Euler's criterion that if  $a$  is not a quadratic residue, then  $|S|$  will be equal to  $(p-1)/2$ . If it is a quadratic residue,  $|S|$  will be equal to  $(p+1)/2$ . With the set  $S$  being the set of quadratic or non-quadratic residues.