# CS 1511 Homework 18

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Sunday, March 25

**31. a)**

**31. b)**

**31. c)**

**31. d)**

**34** Prove that there exists a perfectly complete $AM[O(1)]$ protocol for proving a lower bound on set size.

Hint: First note that in the current set lower bound protocol we can have the prover choose the hash function. Consider the easier case of constructing a protocol to distinguish between the case $|S| \geq$ K and $|S| \leq$ 1K where c¿2 can even be a function of K. c If c is large enough, we can allow the prover to use several hash functions h1 , . . . , hi , and it can be proven that if $i$ is large enough, well have $\cup_i hi(S) = 0, 1k$.The gap can be increased by considering instead of S the set Sl, that is the $l$ times cartesian product of S.