CS 1511 Homework 22

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Fri, April 5

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If a = 0^n, Simon's algorithm still works. This is because if the function is one-to-one, and $a = 0^n$, after we compute $\mid xz\rangle - >\mid x(y \oplus f(x))\rangle$ we can measure $\mid (x \oplus a)$ and see that it's equivalent to x. This will let us know that $a = 0^n$. We will therefore have correctly computed a. Or, if we continue Simon's algorithm, we will eventually be finding k linear equations for $y \odot a = 0$ with a uniform string for y that makes this true. In this case, every single one of these y's will work. Solving the linear equations will give us that all values of a are 0, which is true.

44 a. To get the Bell state $1/\sqrt{2} \mid 0\rangle + 1/\sqrt{2} \mid 1\rangle$, Alice would do nothing to her qubit. This will be when x = 0 and y = 0.

To get the Bell state $1/\sqrt{2} \mid 0\rangle - 1/\sqrt{2} \mid 1\rangle$, Alice can perform a multiplication by $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ to her qubit. This will be when x = 0 and y = 1.

To get the Bell state $-1/\sqrt{2} \mid 0\rangle + 1/\sqrt{2} \mid 1\rangle$, Alice can perform a CNOT gate operation to her qubit. This will be when x = 1 and y = 0.

To get the Bell state $-1/\sqrt{2} \mid 0\rangle - 1/\sqrt{2} \mid 1\rangle$, Alice can perform a multiplication by $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ to her qubit before a CNOT gate operation. This will be when x = 1 and y = 1.

44 b.

If x = 0 and y = 0, we get the original state $1/\sqrt{2} \mid 00\rangle + 1/\sqrt{2} \mid 11\rangle$
If x = 0 and y = 1, we get the state $1/\sqrt{2} \mid 00\rangle - 1/\sqrt{2} \mid 11\rangle$
If x = 1 and y = 0, we get the state $1/\sqrt{2} \mid 10\rangle + 1/\sqrt{2} \mid 01\rangle$
If x = 1 and y = 1, we get the state $1/\sqrt{2} \mid 01\rangle - 1/\sqrt{2} \mid 10\rangle$

44 c.

When Bob measures his qubit b, the state of qubit a will collapse to state 0 or 1 with equal probability 1/2 of each occurring. If we then take a CNOT gate to qubits a and b and then a Hadamard gate operation on qubit a, we will see the x value as the "x coordinate" of the

resulting vector and the y value as the "y coordinate" of the resulting vector.