CS 1511 Homework 20

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37.

38. The Toffoli gate takes 3 bits. If the first bit are set 1, it performs a NOT on the third bit. Otherwise, they stay the same.

It is Universal.

NOT = 1, 1, A -¿ performs NOT A.

AND = I_1, I_2, 0 -¿ performs an AND on I_1, I_2

We can make a NAND gate because we take an AND gate and a NOT gate after that.

OR = OR can be built from a NAND gate (NOT AND). Take the two inputs and put them into two NAND gates. Take the outputs of those NAND gates and input them into another NAND gate. This will output the value of an OR gate.

39 a. $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} = \begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} =$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} a \\ -b \end{bmatrix}$

39 b. a^2

39 c. $(-b)^2$

40 a. (10.6 in the text)

When you measure the register, you are not changing the state. You will output the value with the probability defined in v.

40 b. The same logic applies. Measuring the first qubit will reveal the output with the probability defined in v. This does not change the second qubit.

40 c. Same logic applies. Opening the second qubit does not change the logic of the first one.

41 a. $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

41 b. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

41 c. $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix}$

41 d. $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} * \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a/\sqrt{2} + b/\sqrt{2} \\ a/\sqrt{2} - b/\sqrt{2} \\ c/\sqrt{2} + d/\sqrt{2} \\ c/\sqrt{2} - d/\sqrt{2} \end{bmatrix}$

41 e. $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} = \begin{bmatrix} a/\sqrt{2} + b/\sqrt{2} \\ a/\sqrt{2} - b/\sqrt{2} \\ c \\ d \end{bmatrix}$

41 f. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & 0 & 1/\sqrt{2} & -1/\sqrt{2} \end{bmatrix} * \begin{bmatrix} a/\sqrt{2} + b/\sqrt{2} \\ a/\sqrt{2} - b/\sqrt{2} \\ c \\ d \end{bmatrix} = \begin{bmatrix} a/\sqrt{2} + b/\sqrt{2} \\ a/\sqrt{2} - b/\sqrt{2} \\ c/\sqrt{2} + d/\sqrt{2} \\ c/\sqrt{2} - d/\sqrt{2} \end{bmatrix}$