

# CS 1511 Homework 21

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## 42 a.

We begin with the EPR experiment from the book. We give Alice  $x$  and Bob  $y$ . Both of them see that their variables equal 1. In this case, they will both decide to rotate their qubits according to the book's protocol.

The qubits of Alice and Bob begin as  $|ij\rangle = 1/\sqrt{2} |00\rangle + 1/\sqrt{2} |11\rangle$

To start, Alice rotates her qubit by  $\pi/8$ .

The matrix representing a rotation of  $\pi/8$  to our qubit is:

$$\begin{bmatrix} \cos(\pi/8) & \cos(5\pi/8) \\ \sin(\pi/8) & \sin(5\pi/8) \end{bmatrix}$$

This matrix comes from taking our original  $\pi/8$  matrix which is :

$$\begin{bmatrix} \cos(\pi/8) & 0 \\ \sin(\pi/8) & 0 \end{bmatrix}$$

and multiplying by:

$$\begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

to get:

$$\begin{bmatrix} \cos(5\pi/8) \\ \sin(5\pi/8) \end{bmatrix}$$

When combined, we get the overall rotation we have above.

After this rotation is applied, our qubit  $|ij\rangle = 1/\sqrt{2} * \cos(\pi/8) |00\rangle + 1/\sqrt{2} * \sin(\pi/8) |10\rangle + 1/\sqrt{2} * \cos(5\pi/8) |01\rangle + 1/\sqrt{2} * \sin(5\pi/8) |11\rangle$

Now Bob decides to rotate by  $-\pi/8$  since he sees  $y = 1$ , and the protocol says that causes this rotation.

By the same process as above, we get the rotation of  $-\pi/8$  to be:

$$\begin{bmatrix} \cos(-\pi/8) & \cos(-5\pi/8) \\ \sin(-\pi/8) & \sin(-5\pi/8) \end{bmatrix}$$

Now we apply this rotation to our already rotated qubit, making our qubit  $|ij\rangle = (1/\sqrt{2} * \cos(\pi/8) |0\rangle + 1/\sqrt{2} * \sin(\pi/8) |1\rangle)(1/\sqrt{2} * \cos(\pi/8) |0\rangle - 1/\sqrt{2} * \sin(\pi/8) |1\rangle) + (1/\sqrt{2} * -\sin(\pi/8) |0\rangle + 1/\sqrt{2} * \cos(\pi/8) |1\rangle)(1/\sqrt{2} * \sin(\pi/8) |0\rangle + 1/\sqrt{2} * \sin(\pi/8) |1\rangle)$

Now we do the multiplications to simplify. Since  $1/\sqrt{2}$  is in all parts, we can simplify it out of our math.

After simplifying, we get the following state of our qubit.

$$|ij\rangle = (\cos^2(\pi/8) - \sin^2(\pi/8)) |00\rangle - 2\sin(\pi/8)\cos(\pi/8) |01\rangle + 2\sin(\pi/8)\cos(\pi/8) |10\rangle + (\cos^2(\pi/8) - \sin^2(\pi/8)) |11\rangle$$

Now we just have the measuring of Alice and Bob. To start, let's find out what we need to win the game.

In order to win the game with  $x = 1$  and  $y = 1$ , we will need  $a$  and  $b$  to be different. Thus, Alice and Bob must give different answers.

We can calculate the chance of each possible path down the possibilities tree. We start with the probability that we get  $|00\rangle$ , which is

equal to the probability of  $|11\rangle$ . This probability is the square of the amplitude, which is  $(\cos^2(\pi/8) - \sin^2(\pi/8))^2$ . This is equal to .5.

Also, the probability of getting  $|01\rangle$  or  $|10\rangle$  are the same, and the squares of their amplitudes are  $(2\sin(\pi/8)\cos(\pi/8))^2$ , which is also .5.

Since everything has equal probabilities, the probability of having different values for  $a$  and  $b$  are .5.

This means that the chance of Alice and Bob winning in this case is  $1/2$ .

**42 b.**