CS 1511 Homework 4

Mathew Varughese, Justin Kramer, Zach Smith

Monday, Jan 28

7

Assume the definition in (b) is true. Then for a recursively enumerable language L, there exists a Turing machine M with a read/write tape that is initially empty and a write-nly output tape, such that only elements of L are written to the output tape, and every element of L is eventually written to the output tape.

Now construct a turing machine M'.

$M' =$ "On input w:

1. Run M until it produces a new output on its output tape.
2. Check if w was the item written onto the output tape. If yes, accept.
3. Otherwise, go to step 1. If M is halted, loop indefinitely.

Machine M' is the same machine defined in part (a). If $x \in L$, then x will show up on the output tape of machine M and will accept. If $x \notin L$, M will loop indefinitely on x.

8 (a)

8 (b)

8 (c) Assume that the set of incompressible strings contains a infinite subset that is recursively enumerable. This means there is a mapping from the Natural Numbers this infinite subset. For example, 1 maps to w_1, 2 maps to w_2, 3 maps to w_3, etc. Then construct a Turing Machine M that outputs each of these strings. The subset is infinite, so $\exists x$ such that $|x| > |<M>| + a + c$. x where a is the natural number that x corresponds to and c is a constant. Then create M' such that M' outputs x on input a. $|<M'>|$ is less than $|x|$ because M' was constructed so that it is of similar length to M. This is a contradiction because x is incompressible so no program shorter than it should be able to output it.