CS 1511 Homework 23

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45. Showing that final state of qubit b is the initial state of qubit x.

When Alice measures, qubit b collapses to one of four possible states. As shown in the previous problem, there are four ways to take the final state of qubit b and find the initial state x. According to our last homework, the values of the two classical bits can decide what operation to proceed with.

For example, in one case (x = 0 and y = 0) that qubit x was not modified at all, so the initial and final state of x = b.

If (x = 0 and y = 1) we could take qubit b, apply a Hadamard operation and multiply by $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ to get the initial state of x.

If (x = 1 and y = 0), we could take qubit b and apply a CNOT operation to it to get qubit x.

If (x = 1 and y =1), we could take qubit b and apply a CNOT operation to it before multiplying by $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ to get qubit x at its initial state.

This is possible because all the operations are reversible in quantum.

After negation of x,

a = 1

b = $| 0\rangle/\sqrt{2} + | 1\rangle/\sqrt{2}$

x = $\alpha | 1\rangle + \beta | 0\rangle$

After Hadamard gate,

a = $| 0\rangle/\sqrt{2} + | 1\rangle/\sqrt{2}$

b = $| 0\rangle/\sqrt{2} + | 1\rangle/\sqrt{2}$

x = $\alpha | 1\rangle + \beta | 0\rangle$

After measuring a and x

a = what's measured

b = $| 0\rangle/\sqrt{2} + | 1\rangle/\sqrt{2}$

x = what's measured

Bob can run b through a Hadamard gate and then negate x if a = 1 to get the original state of x.

46. BQP is the complexity class that contains languages that are solvable by a quantum computer in polynomial time with an error probability of 1/3.

Add another qubit to the register. When the qubit is zero, all amplitudes correspond to the real part of the amplitudes in the original algorithm. When it is one, the amplitudes correspond to the imaginary part of the amplitudes of the original algorithm.

This means that the states can be either the real part or imaginary part of the matricies. The state of them depends of the state of the qubit. This is because of the linearity of the matrix operations. The real or imaginary part can be chosen based on the qubit.