CS 1511 Homework 16

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30.)

Let L $\in$ BPL.
$\exists$ TM T and $\exists$ integer k such that:
$\forall x \forall R$ T(x,R) halts using log space
if x $\in$ L then prob(T(x,R) accepts) $\geq 3/4$
if x $\notin$ L then prob(T(x,R) accepts) $\leq 1/4$

LOGSPACE $\subset$ P. This is because with $\log n$ space, there are $2^{\log n}$ possible configurations for R. This simplifies to polynomial time.

Now, there exists some u, where u is a random assignment of
Take an input x that has a length of n. Now we will have C be the number of configurations of TM T with input x. We will combine each configuration with another set of the same configurations to form a matrix. We will create this matrix such that each cell will have a 1/2 probability if the second configuration is reachable from the first in one step, and a probability of 0 otherwise. With this created, each cell W_t with configurations c_1 and c_2 is the probability of reaching configuration c_2 from c_1 in t steps, where W_t is the matrix created by multiplying W by itself t times. By scaling this up, we can compute the accepting probability of T(x, R) and decide if x $\in$ L. With this we can see that each probability is a multiple of $1/2^{poly(n)}$, which we can represent with a poly number of digits. Therefore, L $\in$ P and BPL $\subseteq$ P.

31.)

If $L \in$ BP $\cdot$ NP, this means that we can probabliscally reduce $\overline{3SAT}$ to 3SAT. So, for most reductions, this reduction will work. We wish to change this to be a $\exists\forall\exists$ problem.
We already proved that BP $\cdot$ NP $\subseteq NP/Poly$. Therefore we know since co3SAT is in this language there is a family of circuits ($\exists C_n$) that decides 3SAT nondeterministically.

From here, if we can show by repeatedly performing pairings of random strings, we can increase the probability to show that

If the length of the random string R is m, we can say there are 2^m different possible m's. The probability that T is incorrect is less than $\frac{1}{4^n}$ where n is the length of x.

Now in the BPP $\subset \Sigma_2^p$ proof we married (pair) each R with another R to decrease the expected number of R's that cause the machine to error. When we perform k marriages, the expected number of Rs that cause the machine to error will be less than 1. This occurs when $k = \frac{m}{2n}$. Here we do something similar. We perform k marriages of reductions so that the number of "incorrect" reductions becomes less than 1. Then there must exist some random pairing of reductions such that all reductions work.

Again, since we assuming that the complement of 3SAT $\in$ NP/poly, we can say that there exists a poly-sized circuit family that can decide the complement of 3 SAT with a poly-sized string y that verifies if x $\in$ L.
In other words, we have $\exists C_n \forall u \exists r$ and C_n is a circuit family that reduces if $\overline{3SAT}$ to 3SAT with either input C_n(r, u) or C_n(r, $r \oplus u$).
Since $\overline{3SAT}$ can be reduced to 3SAT by a Σ_3^p algorithm, this means $\overline{3SAT}$ would be complete for Π_3^p. This means that $\Pi_3^p \subseteq \Sigma_3^p$, which would make PH collapse to Σ_3^p.