CS 1511 Homework 2 1511

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1 Question 1

a. $E_{TM} = \{\langle M \rangle |\ M\ is\ a\ TM\ and\ L(M) = \emptyset\}$
Assume there exists a function S that maps E_{TM} from $HALT_{TM}$
Construct function S that is a mapping reducible function from $HALT_{TM}$ as follows:

S = "On input $\langle M, w \rangle$ where M is a TM and w is a string:
1. Construct TM P as follows:
   P = "On input x:
   1. Run M on input w
   2. If M halts on input w, accept
   Otherwise, reject
2. Output $\langle P \rangle$

Assume that $\langle M, w \rangle$ exists within $HALT_{TM}$, so M halts on input w. Since M halts on input w, L(P) = Σ*. Thus, $\langle P \rangle$ does not exist within E_{TM}. This will be the output of S$\langle M, w \rangle$.
Assume that $\langle M, w \rangle$ doesn't exist within $HALT_{TM}$, so M doesn't halt on input w. Since M doesn't halt on input w, L(P) = $\emptyset$. Thus, $\langle P \rangle$ exists within E_{TM}. This will be the output of S$\langle M, w \rangle$.

Thus, E_{TM} is reducible from $HALT_{TM}$ and undecideable.

b. $ALL_{TM} = \{\langle M \rangle |\ M\ is\ a\ TM\ and\ L(M) = \emptyset\}$
Assume there exists a function S that maps ALL_{TM} from $HALT_{TM}$

Construct function S that is a mapping reducible function from $HALT_{TM}$ as follows:

S = "On input $\langle M, w\rangle$ where M is a TM and w is a string:

1. Construct TM P as follows:

   P = "On input x:

   1. Run M on input w
   2. If M halts on input w, reject

   Otherwise, accept
2. Output $\langle P\rangle$

Assume that $\langle M, w\rangle$ exists within $HALT_{TM}$, so M halts on input w. Since M halts on input w, L(P) = $\emptyset$. Thus, $\langle P\rangle$ does not exist within ALL_{TM}. This will be the output of S$\langle M, w\rangle$. Assume that $\langle M, w\rangle$ doesn't exist within $HALT_{TM}$, so M doesn't halt on input w. Since M doesn't halt on input w, L(P) = Σ*. Thus, $\langle P\rangle$ exists within ALL_{TM}. This will be the output of S$\langle M, w\rangle$.

Thus, ALL_{TM} is reducible from $HALT_{TM}$ and undecideable.

c. $BIN_{TM} = \{\langle M\rangle |\ M\ is\ a\ TM\ and\ L(M) = \emptyset\}$

Assume there exists a function S that maps BIN_{TM} from $HALT_{TM}$
Construct function S that is a mapping reducible function from $HALT_{TM}$ as follows:

S = "On input $\langle M, w\rangle$ where M is a TM and w is a string:

1. Construct TM P as follows:

   P = "On input x:

   1. Run M on input w
   2. If M halts on input w, reject

   Otherwise, accept
2. Output $\langle P\rangle$

Assume that $\langle M, w\rangle$ exists within $HALT_{TM}$, so M halts on input w. Since M halts on input w, L(P) = $\emptyset$. Thus, $\langle P\rangle$ does not exist within ALL_{TM}. This will be the output of S$\langle M, w\rangle$. Assume that $\langle M, w\rangle$ doesn't exist within $HALT_{TM}$, so M doesn't halt on input w. Since M doesn't halt on input w, L(P) = Σ^*. Thus, $\langle P \rangle$ exists within ALL_{TM}. This will be the output of S$\langle M, w \rangle$.

Thus, ALL_{TM} is reducible from $HALT_{TM}$ and undecideable.

2 Question 2

Answer: