

CS 1511 Homework 5

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11 a.

With an input $I = 101$ for a Turing machine M , here is one valid computation history H . In this computation history H , $\text{space} = *$, $q_0 = q_y$, $q_1 = q_p$.

$\#q_y101*\#1q_p01*\#11q_p1*\#111q_p*\#111q_h*$

This configuration thus ends in the halting state.

11 b.

To begin, here are some defined macros for this problem.

$\text{space} = *$, $q_0 = q_y$, $q_1 = q_p$

$\text{BASE} = 7$

$\text{BASE} = 7$ is true in this problem because the sum of the number of states, alphabet size, and $\#$ are equal to 7.

$\text{PLACE}(j) = (H \text{ div}(\text{BASE})^{i+1} \bmod(\text{BASE})^i)$

In this case, H is a number that exists which one can interpret as a computation history of M on I .

$\text{SAME}(i, j) = (\text{PLACE}(i) = \text{PLACE}(j))$

$\text{STATE}(i) = (\text{PLACE}(i) = q_y \vee \text{PLACE}(i) = q_p \vee \text{PLACE}(i) = q_h)$

$\text{TABLE}(i, j) = (\text{STATE}(i+1), \text{PLACE}(i+2) = q_p0 \wedge \text{PLACE}(j+1), \text{STATE}(j+2) = 0q_p \wedge \text{SAME}(i, j)) \vee (\text{STATE}(i+1), \text{PLACE}(i+2) = q_p1 \wedge \text{PLACE}(j+1), \text{STATE}(j+2) = 1q_p \wedge \text{SAME}(i, j)) \vee (\text{STATE}(i+1), \text{PLACE}(i+2) = q_p* \wedge \text{STATE}(j+1), \text{PLACE}(j+2) = q_p* \wedge \text{SAME}(i, j)) \vee (\text{STATE}(i+1), \text{PLACE}(i+2) = q_y1 \wedge \text{PLACE}(j+1), \text{STATE}(j+2) = 0q_y \wedge \text{SAME}(i, j)) \vee (\text{STATE}(i+1), \text{PLACE}(i+2) = q_y0 \wedge \text{PLACE}(j+1), \text{STATE}(j+2) = 1q_y \wedge \text{SAME}(i, j)) \vee (\text{STATE}(i+1), \text{PLACE}(i+2) = q_y* \wedge \text{PLACE}(j+1), \text{STATE}(j+2) = 0q_h* \wedge \text{SAME}(i, j))$

Godel Sentence $S =$

$\text{PLACE}(i) = \# \wedge \text{PLACE}(j) = \# \wedge \text{PLACE}(k) = \# \wedge \text{PLACE}(l) = \# \wedge$

$\forall x \ i < x < j \implies \text{PLACE}(x) \neq \# \wedge$

$\forall x \ j < x < k \implies \text{PLACE}(x) \neq \# \wedge$

$\exists a \ i < i+a < j \implies \text{PLACE}(i+a) = q_p \vee \text{PLACE}(i+a) = q_y \vee \text{PLACE}(i+a) = q_h \wedge$

$\forall x \ 1 \leq x < a-1 \implies \text{PLACE}(i+x) = \text{PLACE}(j+x) \wedge$

$$\forall x \ a + 1 < x < j - 1 \implies \text{PLACE}(i+x) = \text{PLACE}(j+x) \wedge \\ \text{TABLE}(i+a, j+a)$$