CS 1511 Homework 16

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Monday, March 18

30.)

Let L $\in$ BPL.
$\exists$TM T and $\exists$Integer k such that:
$\forall x \forall R$ T(x,R) halts
if x $\in$ L then prob(T(x,R) accepts) $\geq$ 3/4
if x $\notin$ L then prob(T(x,R) accepts) $\leq$ 1/4
Take an input x that has a length of n. Now we will have C be the number of configurations of TM T with input x. We will combine each configuration with another set of the same configurations to form a matrix. We will create this matrix such that each cell will have a 1/2 probability if the second configuration is reachable from the first in one step, and a probability of 0 otherwise. With this created, each cell W_t with configurations c_1 and c_2 is the probability of reaching configuration c_2 from c_1 in t steps, where W_t is the matrix created by multiplying W by itself t times. By scaling this up, we can compute the accepting probability of T(x, R) and decide if x $\in$ L. With this we can see that each probability is a multiple of $1/2^{poly(n)}$, which we can represent with a poly number of digits. Therefore, L $\in$ P and BPL $\subseteq$ P.

31.)

We already proved that $BP * NP \subseteq NP/Poly$. From here, if we can show that the complement of 3SAT $\in$ NP/poly then have shown that $\Pi_3^p \subseteq \Sigma_3^p$, which would make PH collapse to Σ_3^p.
Since we assuming that the complement of 3SAT $\in$ NP/poly, we can say that there exists a poly-sized circuit family that can decide the complement of 3 SAT with a poly-sized string y that verifies if x $\in$ L. In other words, we have $\exists r \forall a \exists q$ and a y where r is a circuit that decides if ϕ(a,q,c) is true with inputs of phi, y.
This is a Σ_3^p problem, so our proof is complete.