CS 1511 Homework 11

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19. a

EXACT INDSET = $\{< G, k >|$ the largest independent set in G has size exactly k$\}$.

To show EXACT INDSET $\in \pi_2^p$, we need to demonstrate two things.

$\forall x \forall y \forall z$ T(x, y, z) runs in time $\mid x \mid^k$

$\forall x \exists y$ T(x, y, z) accepts iff x $\in$ EXACT INDSET

Our language will now become $\forall x$, with x being an independent set of G, $\exists w$ where w is the largest independent set of size exactly k.

To solve this in poly-time, we will need 3 read-only tapes and 1 work tape, which is based on a simple scaling from our 2 read-only 1 work tape model with π_1^p.

Basically, it reads on the second tape the set of vertices. It goes through the tape and checks each vertex in the set and marks it on the graph (on the first tape). When it does this, it makes sure adjacent vertices are not touching to ensure it is an independent set.

Then, it checks the third tape, which is another set. It just needs to validate that this set is also a indepdent set and that it is larger or equal to in size to the set on the 2nd tape.

19. b

SUCCINCT SET-COVER = $\{< S, n, k >= :$ There is a subset of the collection of 3-DNF formulas S on n variables called S' where S' evaluates to 1 for every assignment to its k variables$\}$.

To show SUCCINCT SET-COVER $\in \Sigma_2^p$, we need to demonstrate two things.

$\forall x \forall y \forall z$ T(x, y, z) runs in time $\mid x \mid^k$

$\exists x \forall y$ T(x, y, z) accepts iff x $\in$ SUCCINCT SET-COVER

Our language will now become $\exists S'$, with S' being a subset of S, where $\forall w$ combinations of k variable assignments, the formula evaluates to 1.

To solve this in poly-time, we will need 3 read-only tapes and 1 work tape, which is based on a simple scaling from our 2 read-only 1 work tape model with Σ_1^p.

19. c

VC-DIMENSION = $\{< C, k >= :$ C represents a collection S s.t. VC(S) $\geq$ k$\}$.

To show VC-DIMENSION $\in \Sigma_3^p$, we need to demonstrate two things.

$\forall x \forall y \forall z \forall w$ T(x, y, z, w) runs in time $\mid x \mid^k$

$\exists x \forall y \exists z$ T(x, y, z, w) accepts iff x $\in$ VC-DIMENSION

Our language will now become $\exists C$, a Boolean circuit that $\forall X'$, where X' a subset of the size of the largest set, $\exists i$ such that subset Si of S has an intersection with X' that equals k and C(i, x) = 1.

To solve this in poly-time, we will need 4 read-only tapes and 1 work tape, which is based on a simple scaling from our 3 read-only 1 work tape model with Σ_2^p.

20. a

If 3SAT is reducible to its complement in Karp reductions then there exists a polynomial function F such that $x \in 3SAT \iff F(x) \in \overline{3SAT}$.

By this logic, since 3SAT is NP complete, every language L in NP can be reduced to 3SAT. And since this F exists, every language in NP could be reduced to $\overline{3SAT}$ in poly time.

$\overline{3SAT} \in$ CoNP. So every language in NP would be in CoNP, because then there would exists a reduction to $\overline{3SAT}$. This means every L in NP would be in CoNP.

Also, if there is a reduction from 3SAT to its complement, then there exists a reduction from the complement of 3SAT to 3SAT. This is because if $A \leq B$, then $\overline{A} \leq \overline{B}$. This is because if there is a function to reduce a language to another in polynomial time, another function can be constructed for the complement of that language which negates the membership. By this logic and the previous ideas, it can be shown every language in CoNP would be in NP. Therefore, CoNP = NP.

20. b

With the same logic as the previous problem, the hierarchy would eventually collapse so that any Σ_n^p could equal any π_n^p for the n=1 that the hierarchy collapses to. This would be a proof by induction that repeats the previous problem for all complexity classes. In this complete collapse to the bottom of the hierarchy, the union of all class PH would now be able to equal NP.