CS 1511 Homework 17

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32.)

Let L $\in$ BPL.
$\exists$ TM T and $\exists$ integer k such that:
$\forall x \forall R$ T(x,R) halts using log space
if x $\in$ L then prob(T(x,R) accepts) $\geq 3/4$
if x $\notin$ L then prob(T(x,R) accepts) $\leq 1/4$

LOGSPACE $\subset$ P. This is because with $\log n$ space, there are $2^{\log n}$ possible configurations for R. This simplifies to polynomial time.

Now, there exists some u, where u is a random assignment of
Take an input x that has a length of n. Now we will have C be the number of configurations of TM T with input x. We will combine each configuration with another set of the same configurations to form a matrix. We will create this matrix such that each cell will have a 1/2 probability if the second configuration is reachable from the first in one step, and a probability of 0 otherwise. With this created, each cell W_t with configurations c_1 and c_2 is the probability of reaching configuration c_2 from c_1 in t steps, where W_t is the matrix created by multiplying W by itself t times. By scaling this up, we can compute the accepting probability of T(x, R) and decide if x $\in$ L. With this we can see that each probability is a multiple of $1/2^{poly(n)}$, which we can represent with a poly number of digits. Therefore, L $\in$ P and BPL $\subseteq$ P.