CS 1511 Homework 2

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3. (a)

$E_{TM} = \{\langle M \rangle \mid M \text{is a TM and } L(M) = \emptyset\}$
Assume $\exists$ TM R that decides E_{TM}.

Construct Turing Machine S that decides $HALT_{TM}$.
$S =$ "On input $\langle M, w \rangle$ where M is a TM and w is a string:

1. Construct TM M' as follows:

   $M' =$ "On input x:

   (a) Run M on input w
   (b) If M accepts w, <u>reject</u>. Otherwise <u>accept</u>.
2. Run R on input $\langle M' \rangle$
3. If R accepts $\langle M' \rangle$ <u>reject</u> otherwise <u>accept</u>.

Assume that $\langle M, w \rangle \in HALT_{TM}$. Since $\langle M, w \rangle \in HALT_{TM}$, M halts on input w, so $L(M') = \Sigma^*$. Since $L(M') \neq \emptyset$, $\langle M \rangle \notin E_{TM}$. Since R is a decider for E_{TM}, running input $\langle M' \rangle$ will cause R to reject $\langle M' \rangle$, so S will accept $\langle M, w \rangle$.
Assume that $\langle M, w \rangle \notin HALT_{TM}$. Since $\langle M, w \rangle \notin HALT_{TM}$, M does not halt on input w, so $L(M') = \emptyset$. Since $L(M') = \emptyset$, $\langle M \rangle \in E_{TM}$. Since R is a decider for E_{TM}, running input $\langle M' \rangle$ will cause R to accept $\langle M' \rangle$, so S will reject $\langle M, w \rangle$.

3. (b)

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1. Construct TM M' as follows:

   $M' =$ "On input x:

   (a) Run M on input w
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3. If R accepts $\langle M' \rangle$ <u>reject</u> otherwise <u>accept</u>.

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