CS 1511 Homework 4

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Monday, Jan 28

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Assume the definition in (b) is true. Then for a recursively enumerable language L, there exists a Turing machine M with a read/write tape that is initially empty and a write-nly output tape, such that only elements of L are written to the output tape, and every element of L is eventually written to the output tape.

Now construct a turing machine M'.

$M' =$ "On input w:

1. Run M until it produces a new output on its output tape.
2. Check if w was the item written onto the output tape. If yes, accept.
3. Otherwise, go to step 1. If M is halted, loop indefinitely.

Machine M' is the same machine defined in part (a). If $x \in L$, then x will show up on the output tape of machine M and will accept. If $x \notin L$, M will loop indefinitely on x.

8 (a) Assume there is a function to compute the Kolmogorov complexity of a string. This function can be written into a program which has some arbitrary length x. Since this program will decide whether any string is compressible then it is infinite and

8. b)

The number of bits to represent any string with an equal amount of 0's and 1's will be 2^n. The probability of finding an incompressible string with this property is $1/\sqrt{n}$. With this in mind, the K(x) Kolmogorov complexity will be greater than or equal to the length of the string if it's incompressible. In this case, the formula will look like this.

$n - 1/2 \log_2 n + c >$ string length

As the size of n grows, eventually $1/2 \log_2 n$ will grow larger than the constant c, and the equation will eventually no longer hold. Thus, there is a finite range that n can exist within where the string will be incompressible. Therefore, there are a finite amount of strings that are incompressible and have this property.

8 (d) A set is recursively enumerable if $\exists$ a Turing Machine that outputs each element in the