CS 1511 Homework 5

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11 a.

With an input I = 101 for a Turing machine M, here is one valid computation history H. In this computation history H, space = *, q_0 = q_y, q_1 = q_p .

#q_y101*#1q_p01*#11q_p1*#111q_p*#111q_h*

This configuration thus ends in the halting state.

11 b.

To begin, here are some defined macros for this problem.

space = *, q_0 = q_y, q_1 = q_p

BASE = 7

BASE = 7 is true in this problem because the sum of the number of states, alphabet size, and # are equal to 7.

PLACE(j) = (H div(BASE)$^{i+1}$ mod(BASE)^{i})

In this case, H is a number that exists which one can interpret as a computation history of M on I.

SAME(i, j) = (PLACE(i) = PLACE(j))

STATE(i) = (PLACE(i) = q_y $\vee$ PLACE(i) = q_p $\vee$ PLACE(i) = q_h)

TABLE(i,j) = (STATE(i+1), PLACE(i+2) = q_p0 $\wedge$ PLACE(j+1), STATE(j+2) = 0q_p $\wedge$ SAME(i, j)) $\vee$ (STATE(i+1), PLACE(i+2) = q_p1 $\wedge$ PLACE(j+1), STATE(j+2) = 1q_p $\wedge$ SAME(i, j)) $\vee$ (STATE(i+1), PLACE(i+2) = q_p* $\wedge$ STATE(j+1), PLACE(j+2) = q_p* $\wedge$ SAME(i, j)) $\vee$ (STATE(i+1), PLACE(i+2) = q_y1 $\wedge$ PLACE(j+1), STATE(j+2) = 0q_y $\wedge$ SAME(i, j)) $\vee$ (STATE(i+1), PLACE(i+2) = q_y0 $\wedge$ PLACE(j+1), STATE(j+2) = 1q_y $\wedge$ SAME(i, j)) $\vee$ (STATE(i+1), PLACE(i+2) = q_y* $\wedge$ PLACE(j+1), STATE(j+2) = 0q_h* $\wedge$ SAME(i, j))

Proof:

PLACE(i) = # $\wedge$ PLACE(j) = # $\wedge$ PLACE(k) = # $\wedge$ PLACE(l) = # $\wedge$

$\forall$x $i < x < j \implies$ PLACE(x) $\neq$ # $\wedge$

$\forall$x $j < x < k \implies$ PLACE(x) $\neq$ # $\wedge$

$\exists$a $i < i + a < j \implies$ PLACE(i+a) = q_p $\vee$ PLACE(i+a) = q_y $\vee$ PLACE(i+a) = q_h $\wedge$

$\forall$x $1 \leq x < a - 1 \implies$ PLACE(i+x) = PLACE(j+x) $\wedge$

∀x $a + 1 < x < j - 1 \implies$ PLACE(i+x) = PLACE(j+x) $\wedge$ TABLE(i+a, j+a)