CS 1511 Homework 2

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Wednesday, Jan 16

3. (a)

$E_{TM} = \{\langle M \rangle \mid M \text{is a TM and } L(M) = \emptyset\}$
Assume $\exists$ TM R that decides E_{TM}.

Construct Turing Machine S that decides $HALT_{TM}$.
$S =$ "On input $\langle M, w \rangle$ where M is a TM and w is a string:

1. Construct TM M' as follows:

   $M' =$ "On input x:

   (a) Run M on input w
   (b) If M accepts w, <u>reject</u>. Otherwise <u>accept</u>.
2. Run R on input $\langle M' \rangle$
3. If R accepts $\langle M' \rangle$ <u>reject</u> otherwise <u>accept</u>.

Assume that $\langle M, w \rangle \in HALT_{TM}$. Since $\langle M, w \rangle \in HALT_{TM}$, M halts on input w, so $L(M') = \Sigma^*$. Since $L(M') \neq \emptyset$, $\langle M \rangle \notin E_{TM}$. Since R is a decider for E_{TM}, running input $\langle M' \rangle$ will cause R to reject $\langle M' \rangle$, so S will accept $\langle M, w \rangle$.
Assume that $\langle M, w \rangle \notin HALT_{TM}$. Since $\langle M, w \rangle \notin HALT_{TM}$, M does not halt on input w, so $L(M') = \emptyset$. Since $L(M') = \emptyset$, $\langle M \rangle \in E_{TM}$. Since R is a decider for E_{TM}, running input $\langle M' \rangle$ will cause R to accept $\langle M' \rangle$, so S will reject $\langle M, w \rangle$.

3. (b)

$E_{TM} = \{\langle M \rangle \mid M \text{is a TM and } L(M) = \Sigma^*\}$
Assume $\exists$ TM R that decides E_{TM}.

Construct Turing Machine S that decides $HALT_{TM}$.
$S =$ "On input $\langle M, w \rangle$ where M is a TM and w is a string:

1. Construct TM M' as follows:

   $M' =$ "On input x:

   (a) Run M on input w
   (b) If M accepts w, <u>reject</u>. Otherwise <u>accept</u>.

2. Run R on input $\langle M' \rangle$
3. If R $\langle M' \rangle$ reject otherwise accept.

Assume that $\langle M, w \rangle \in HALT_{TM}$. Since $\langle M, w \rangle \in HALT_{TM}$, M halts on input w, so $L(M') = \Sigma^*$. Since $L(M') \neq \emptyset$, $\langle M \rangle \notin E_{TM}$. Since R is a decider for E_{TM}, running input $\langle M' \rangle$ will cause R to reject $\langle M' \rangle$, so S will accept $\langle M, w \rangle$.

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3. (e)

The first three subproblems are based upon the idea of $HALT_{TM}$ being undecidable. The fourth subproblem demonstrates that A_{TM} is undecidable. With this in mind, A_{TM} says that with a TM M and an input w, there is no way to decide M. The first three subproblems are basically versions of this problem, but with differing ways of stating w. For example, part (a.) includes checking every input to find out that nothing is accepted by M. The w in this case would be each string that is checked. The same goes for part (c.), where the problem involves checking if all strings in the language have the property of including the string 11110. Just like the fourth subproblem, checking for this property of strings in the language is undecidable. Thus, these subproblems are rooted in the fourth subproblem.

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The holographic principle was developed after thorough debates about black holes between Stephen Hawking and Leonard Susskind. Susskind developed the principle, which states that a black hole takes in a three-dimensional and spreads it around its entire event horizon from an outside view. This is similar to a hologram where an object in one place is spread across a film. This principle also applies to the three-dimensional universe, where everything is laid out across a one-dimensional film at the edge of the universe. Furthermore, Hawking argued that black holes destroy information when objects pass into them. Susskind, who was correct, argued that black holes take in and outwardly radiate the information they take in, preserving it in its entirety.