CS 1511 Homework 19

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35. a) The polynomial to replace z would be $\sum_{x=0}^{1}\prod_{y=0}^{1}\sum_{z=0}^{1} 1-(1-(xy(1-z)))*(1-((1-x)(1-y)z))$ After multiplication, this becomes $\sum_{x=0}^{1}\prod_{y=0}^{1}\sum_{z=0}^{1} 1-(1-(xy-xyz))*(1-(z(1-y-x+xy)$

$1-(1-(xy-xyz))*(1-(z-zy-zx+zxy))$

$1-((1-(z-zy-zx+zxy))-(xy+xyz)+(xyz-xy^2z-x^2zy+zx^2y^2-xyz^2+xy^2z^2+x^2yz^2-x^2y^2z^2)$

$1-(1-z+zy+zx-zxy-xy-xyz+xyz-xy^2z-x^2zy+zx^2y^2-xyz^2+xy^2z^2+x^2yz^2-x^2y^2z^2$

Which all finally becomes
$\sum_{x=0}^{1}\prod_{y=0}^{1}\sum_{z=0}^{1} 1-(1-z-yz+xz-xyz-xy-xy^2z-x^2zy+zx^2y^2-xyz^2+xy^2z^2+x^2yz^2-x^2y^2z^2)$

35. b) i. The integer S would start with the polynomial $s(x)=\prod_{y=0}^{1}\sum_{z=0}^{1} 1-(1-z-yz+xz-xyz-xy-xy^2z-x^2zy+zx^2y^2-xyz^2+xy^2z^2+x^2yz^2-x^2y^2z^2)$ added together having 0 and 1 for x to get S. Here would be those steps. Since F is satisfiable, Merlin does not need to lie.

$S(x)=\prod_{y=0}^{1}(1-(1-(0)-y(0)+x(0)-xy(0)-xy-xy^2(0)-x^2(0)y+(0)x^2y^2-xy(0)^2+xy^2(0)^2+x^2y(0)^2-x^2y^2(0)^2))+(1-(1-(1)-(1)(1)+x(1)-x(1)(1)-x(1)-x(1)^2(1)-x^2(1)(1)+(1)x^2(1)^2-x(1)(1)^2+x(1)^2(1)^2+x^2(1)(1)^2-x^2(1)^2(1)^2))$

$S(x)=[(1-(1-(0)-(0)(0)+x(0)-x(0)(0)-x(0)-x(0)^2(0)-x^2(0)(0)+(0)x^2(0)^2-x(0)(0)^2+x(0)^2(0)^2+x^2(0)(0)^2-x^2(0)^2(0)^2))+(1-(1-(1)-(1)(1)+x(1)-x(1)(1)-x(1)-x(1)^2(1)-x^2(1)(1)+(1)x^2(1)^2-x(1)(1)^2+x(1)^2(1)^2+x^2(1)(1)^2-x^2(1)^2(1)^2))]*[(1-(1-(0)-(1)(0)+x(0)-x(1)(0)-x(1)-x(1)^2(0)-x^2(0)(1)+(0)x^2(1)^2-x(1)(0)^2+x(1)^2(0)^2+x^2(1)(0)^2-x^2(1)^2(0)^2))+(1-(1-(1)-(1)(1)+x(1)-x(1)(1)-x(1)-x(1)^2(1)-x^2(1)(1)+(1)x^2(1)^2-x(1)(1)^2+x(1)^2(1)^2+x^2(1)(1)^2-x^2(1)^2(1)^2))]$
This value would equal S.

$S(0)+S(1)=[(1-(1-(0)-(0)(0)+(0)(0)-(0)(0)(0)-(0)(0)-(0)(0)^2(0)-(0)^2(0)(0)+(0)(0)^2(0)^2-(0)(0)(0)^2+(0)(0)^2(0)^2+(0)^2(0)(0)^2-(0)^2(0)^2(0)^2))+(1-(1-(1)-(1)(1)+(0)(1)-(0)(1)(1)-(0)(1)-(0)(1)^2(1)-(0)^2(1)(1)+(1)(0)^2(1)^2-(0)(1)(1)^2+(0)(1)^2(1)^2+(0)^2(1)(1)^2-(0)^2(1)^2(1)^2))]*[(1-(1-(0)-(1)(0)+(0)(0)-(0)(1)(0)-(0)(1)-(0)(1)^2(0)-$

$(0)^2(0)(1) + (0)(0)^2(1)^2 - (0)(1)(0)^2 + (0)(1)^2(0)^2 + (0)^2(1)(0)^2 - (0)^2(1)^2(0)^2)) + (1 - (1 - (1) - (1)(1) + (0)(1) - (0)(1)(1) - (0)(1) - (0)(1)^2(1) - (0)^2(1)(1) + (1)(0)^2(1)^2 - (0)(1)(1)^2 + (0)(1)^2(1)^2 + (0)^2(1)(1)^2 - (0)^2(1)^2(1)^2))] + [(1 - (1 - (0) - (0)(0) + 1(0) - 1(0)(0) - 1(0) - 1(0)^2(0) - 1^2(0)(0) + (0)1^2(0)^2 - 1(0)(0)^2 + 1(0)^2(0)^2 + 1^2(0)(0)^2 - 1^2(0)^2(0)^2)) + (1 - (1 - (1) - (1)(1) + 1(1) - 1(1)(1) - 1(1) - 1(1)^2(1) - 1^2(1)(1) + (1)1^2(1)^2 - 1(1)(1)^2 + 1(1)^2(1)^2 + 1^2(1)(1)^2 - 1^2(1)^2(1)^2))] * [(1 - (1 - (0) - (1)(0) + 1(0) - 1(1)(0) - 1(1) - 1(1)^2(0) - 1^2(0)(1) + (0)1^2(1)^2 - 1(1)(0)^2 + 1(1)^2(0)^2 + 1^2(1)(0)^2 - 1^2(1)^2(0)^2)) + (1 - (1 - (1) - (1)(1) + 1(1) - 1(1)(1) - 1(1) - 1(1)^2(1) - 1^2(1)(1) + (1)1^2(1)^2 - 1(1)(1)^2 + 1(1)^2(1)^2 + 1^2(1)(1)^2 - 1^2(1)^2(1)^2))]$

ii. The polynomial is g(y) = $\sum_{z=0}^{1} 1 - (1 - z - yz + (1/3)z - (1/3)yz - (1/3)y - (1/3)y^2z - (1/3)^2zy + z(1/3)^2y^2 - (1/3)yz^2 + (1/3)y^2z^2 + (1/3)^2yz^2 - (1/3)^2y^2z^2)$

iii. Arthur is checking if this second polynomial works such that g(0) * g(1) = s(1/3) to see if there is any issue with s(1/3). If there is, then there is proof that s(1/3) is inccorect.

35. c) i. The polynomial to replace z would now be
$\sum_{x=0}^{1} \prod_{y=0}^{1} \sum_{z=0}^{1} 1 - (1 - z - yz + xz - xyz - xy - xyz - xzy + zxy - xyz + xyz + xyz - xyz)$

ii. The integer S would be generated the same way as above, with the polynomial $s(x) = \prod_{y=0}^{1} \sum_{z=0}^{1} 1 - (1 - z - yz + xz - xyz - xy - xyz - xzy + zxy - xyz + xyz + xyz - xyz)$

$s(x) = \prod_{y=0}^{1}(1 - (1 - (0) - y(0) + x(0) - xy(0) - xy - xy(0) - x(0)y + (0)xy - xy(0) + xy(0) + xy(0) - xy(0))) + (1 - (1 - (1) - y(1) + x(1) - xy(1) - xy - xy(1) - x(1)y + (1)xy - xy(1) + xy(1) + xy(1) - xy(1)))$

$s(x) = [(1 - (1 - (0) - (0)(0) + x(0) - x(0)(0) - x(0) - x(0)(0) - x(0)(0) + (0)x(0) - x(0)(0) + x(0)(0) + x(0)(0) - x(0)(0))) + (1 - (1 - (1) - (0)(1) + x(1) - x(0)(1) - x(0) - x(0)(1) - x(1)(0) + (1)x(0) - x(0)(1) + x(0)(1) + x(0)(1) - x(0)(1)))] * [(1 - (1 - (0) - (1)(0) + x(0) - x(1)(0) - x(1) - x(1)(0) - x(0)(1) + (0)x(1) - x(1)(0) + x(1)(0) + x(1)(0) - x(1)(0))) + (1 - (1 - (1) - (1)(1) + x(1) - x(1)(1) - x(1) - x(1)(1) - x(1)(1) + (1)x(1) - x(1)(1) + x(1)(1) + x(1)(1) - x(1)(1)))]$

$s(0) = [(1 - (1 - (0) - (0)(0) + (0)(0) - (0)(0)(0) - (0)(0) - (0)(0)(0) - (0)(0)(0) + (0)(0)(0) - (0)(0)(0) + (0)(0)(0) + (0)(0)(0) - (0)(0)(0))) + (1 - (1 - (1) - (0)(1) + (0)(1) - (0)(0)(1) - (0)(0) - (0)(0)(1) - (0)(1)(0) + (1)(0)(0) - (0)(0)(1) + (0)(0)(1) + (0)(0)(1) - (0)(0)(1)))] * [(1 - (1 - (0) - (1)(0) + (0)(0) - (0)(1)(0) - (0)(1) - (0)(1)(0) - (0)(0)(1) + (0)(0)(1) - (0)(1)(0) + (0)(1)(0) + (0)(1)(0) - (0)(1)(0))) + (1 - (1 - (1) - (1)(1) + (0)(1) - (0)(1)(1) - (0)(1) - (0)(1)(1) - (0)(1)(1) + (1)(0)(1) - (0)(1)(1) + (0)(1)(1) + (0)(1)(1) - (0)(1)(1)))]$

$s(0) = 1 * (0 + 2)$

$s(1) = [(1 - (1 - (0) - (0)(0) + (1)(0) - (1)(0)(0) - (1)(0) - (1)(0)(0) - (1)(0)(0) + (0)(1)(0) - (1)(0)(0) + (1)(0)(0) + (1)(0)(0) - (1)(0)(0))) + (1 - (1 - (1) - (0)(1) + (1)(1) - (1)(0)(1) - (1)(0) - (1)(0)(1) - (1)(1)(0) + (1)(1)(0) - (1)(0)(1) + (1)(0)(1) + (1)(0)(1) - (1)(0)(1)))] * [(1 - (1 - (0) - (1)(0) + (1)(0) - (1)(1)(0) - (1)(1) - (1)(1)(0) - (1)(0)(1) + (0)(1)(1) - (1)(1)(0) + (1)(1)(0) + (1)(1)(0) - (1)(1)(0))) + (1 - (1 - (1) - (1)(1) + (1)(1) - (1)(1)(1) - (1)(1) - (1)(1)(1) - (1)(1)(1) + (1)(1)(1) - (1)(1)(1) + (1)(1)(1) + (1)(1)(1) - (1)(1)(1)))] = 0$

$s(0) + s(1) = 2$

1. With x = 0, y = 0, z = 1

Here are the steps to find S:

iii. The polynomial is g(y) = $\sum_{z=0}^{1} 1-(1-z-yz+(1/3)z-(1/3)yz-(1/3)y-(1/3)yz-(1/3)zy+z(1/3)y-(1/3)yz+(1/3)yz+(1/3)yz-(1/3)yz)$

iv. Arthur is checking if this second polynomial works such that g(0) * g(1) = s(1/3) to see if there is any issue with s(1/3). If there is, then there is proof that s(1/3) is inccorect.