Research Intern (Dr. Satnesh Singh)

[15 May 2023- 15 July 2023]

Topics Covered:

Conclusions:

We Started The internship by analyzing and studying research Papers on **2-DOF PID Controller for Temperature Control System** and simulated the results on MATLAB. Our main aim was to get familiar with the concepts so as to implement them on MATLAB and get familiar with the MATLAB environment.Consider first-order plus time delay system

P(s)=e^-0.2s/s+1

```
Plant=tf(1,[1,1],'InputDelay',.2)
Kc=6;
Ti=.4;
Td=.1;
b=.6;
c=.63;
C=pid(Kc,1/Ti,Td)
X=pid(Kc*b,0,c*Td)
num1=(C+X)*Plant/(1+C*Plant)
Kc1=4;
Ti1=.4;
Td1=.1;
b1=.6;
c1=.63;
C1=pid(Kc1,1/Ti1,Td1)
X1=pid(Kc1*b1,0,c1*Td1)
num2=(C1+X1)*Plant/(1+C1*Plant)
Kc2=2;
Ti2=.6;
Td2=.1;
b2=.6;
c2=.63;
C2=pid(Kc2,1/Ti2,Td2)
X2=pid(Kc2*b2,0,c2*Td2)
num3=(C2+X2)*Plant/(1+C2*Plant)
Kc3=2;
Ti3=.5;
Td3=.08;
b3=.7;
c3=.63;
C3=pid(Kc3,1/Ti3,Td3)
X3=pid(Kc3*b3,0,c3*Td3)
num4=(C3+X3)*Plant/(1+C3*Plant)
stepplot(num1)
hold on
stepplot(num2)
hold on
stepplot(num3)
```

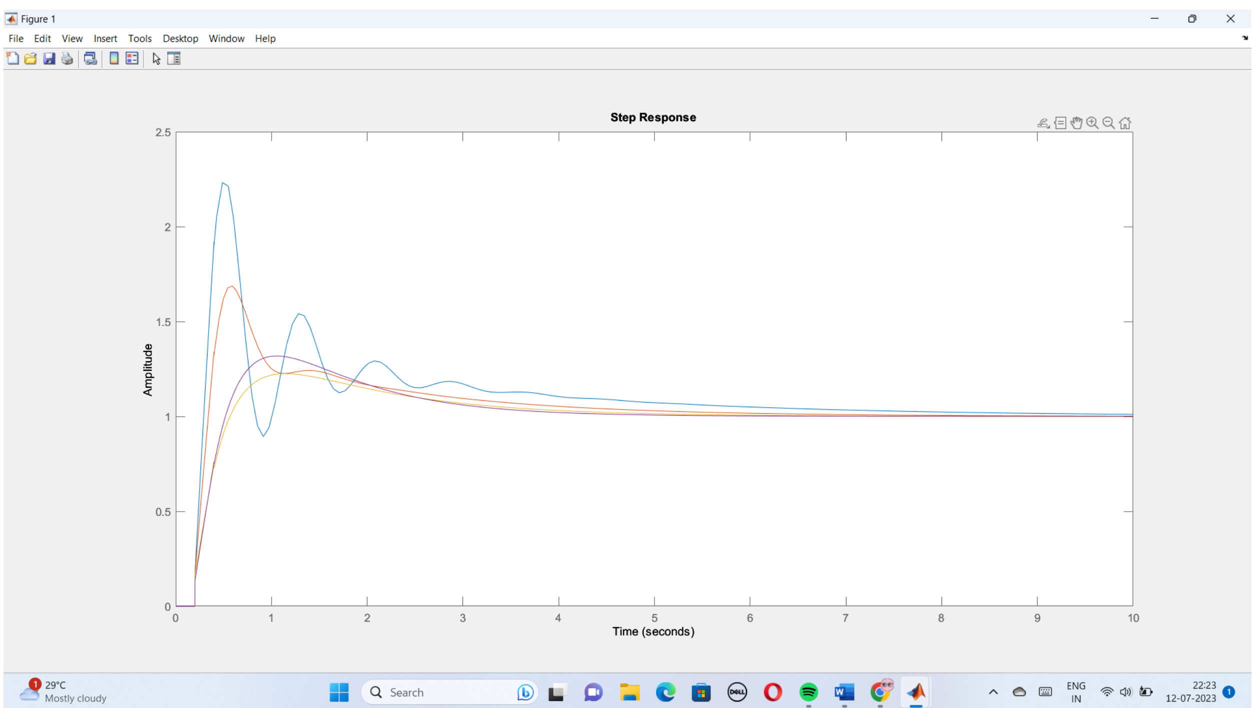
```
hold on
stepplot(num4)
hold on
```

RESULTS:

```
C3 =

            1
  Kp + Ki * --- + Kd * s
            s

  with Kp = 2, Ki = 2, Kd = 0.08

Continuous-time PID controller in parallel form.


X3 =


  Kp + Kd * s


  with Kp = 1.4, Kd = 0.0504

Continuous-time PID controller in parallel form.


num4 =

  A =
        x1    x2    x3    x4    x5
   x1   -1     0     0     0     2
   x2    1     0     0     0     0
   x3    0     0    -1     0     2
   x4    0     0     1     0     0
   x5    0     0  0.96     1  1.08

  B =
       u1
   x1   0
   x2   0
   x3   0
   x4   0
   x5  -1
```

```
C =
        x1      x2      x3      x4      x5
  y1   1.635       1       0       0  0.1304

D =
      u1
  y1   0

E =
      x1  x2  x3  x4  x5
  x1   1   0   0   0   0
  x2   0   1   0   0   0
  x3   0   0   1   0   0
  x4   0   0   0   1   0
  x5   0   0   0   0   0

(values computed with all internal delays set to zero)

Internal delays (seconds): 0.2  0.2
```

Continuous-time state-space model.



Next we tried to learn more about various Control Techniques used for Control System Design.

At First we studied **LQR based Optimal Tuning of PID Controllers**: In this technique we form the Linear Quadratic Regulator (Cost Function) and then use the **Riccati's Equation** to find the values of PID Controller that are optimal to cost function.
Consider the state space system
dx/dt=Ax+Bu
u(t)=-kx(t)
Matrix k of the optimal control vector.
On minimisation of performance index we get reduced-matrix **Riccati Equation**.
This helps us to calculate optimal matrix k.
Then we studied **Robust Sliding Mode Control Design** technique.
In this we got the idea of sliding function and sliding surface.
Then we started our research on how to control attacks on Cyber Physical systems. In this our aim was to study and estimate the attacks on such systems along with forming a controller design for controlling such attacks.
In this we have four categories of attacks namely: **False Data Injection**, **Denial of Service, Replay attacks and Stealthy Attacks**. We studied the control behind these attacks and simulated them in MATLAB. We came across a new concept of **Intermediate Variable** based Estimation and implemented it for estimating Attacks on a system.

```
A=[1.0224,1.2310;
1.6062, -0.0224];
B=[1;
2.5];
C=[1, 0;
1, 2];
D=[1;
0];
sys=ss(A,B,C,D);
K=place(A,B,pole(sys))
mu=.1;
A_=[A, zeros(2,1);
zeros(1,2), eye(1,1)]
B_=[B;
zeros(1)]
C_=[C, D]
M=[zeros(1,2);
eye(size(A))]
setlmis([])
P1= lmivar(2,[3 3])
P3= lmivar(2,[1,1])
H=lmivar(2,[3,2])
gma=lmivar(2,[1,1])
B1=eye(size(B.'*B))-mu*B.'*B
B2= mu*B_.'*(eye(size(A_))-A_-mu*B_*B_.')
M2=[eye(size(B_.'*M))-mu*B_.'*M]
BC=C_+mu*C_*B_*B_.'
G=[zeros(size(M*P1)), M*P1, -M*C_*H]
lmiterm([1,1,1,P3],B2.',B2)
```

```
lmiterm([1,1,1,P1],-1,1)
lmiterm([1,1,2,P3],B2.',B1)
lmiterm([1,1,3,P3],B2.',M2)
lmiterm([1,1,4,P1],(A_+mu*B_*B_.').',1)
%lmiterm([1,1,4,-1], (C_*A_+mu*C_*B_*B_.').', H.')
lmiterm([1,1,5,0],BC.')
lmiterm([1,2,5,0],B_.'*C_.')
lmiterm([1,3,5,1],zeros(size(BC.')),1)
lmiterm([1,4,5,1],zeros(size(BC.')),1)
lmiterm([1,4,4,P1],-1,1)
lmiterm([1,5,5,0],-1)
lmiterm([1,2,2,P3], B1.',B1)
lmiterm([1,2,2,P3],-1,1)
lmiterm([1,2,3,P3],B1.',M2)
lmiterm([1,3,3,P3],M2.',M2)
lmiterm([1,3,3,gma],-1,1)
lmiterm([1,3,4,0],G.')
lmiterm([-1,1,1,P1],1)
lmiterm([-1,2,2,P3],1)
lmiterm([-1,1,2,1],zeros(size(P1)))
Results obtained
```

```
K =

   1.0e-15 *

    0.1531    0.1632


A_ =

    1.0224    1.2310         0
    1.6062   -0.0224         0
         0         0    1.0000


B_ =

    1.0000
    2.5000
         0


C_ =

     1     0     1
     1     2     0
```

M =

```
     0     0
     1     0
     0     1
```

P1 =

```
     1
```

P3 =

```
     2
```

H =

```
     3
```

gma =

```
     4
```

B1 =

```
    0.2750
```

B2 =

```
   -0.4763   -0.0488         0
```

M2 =

```
    0.7500         0
```

BC =

1.1000	0.2500	1.0000
1.6000	3.5000	0

G =

0	0	0	0	0	0	0
0	0	1	0	-3	0	-3
0	0	0	1	-3	-6	0

Then we begin to design controllers for **dynamical hyperchaotic systems** constraining ourselves to use the minimum no. of controllers possible for cost reduction. In an hybrid **Hyperchaotic Lu System** we designed a technique to control disturbances using a single control input and a single sliding surface and compared the results using a single linear control input.

$\dot{x}_{d1} = a(x_{d2} - x_{d1})$ $\dot{e}_{q2}$ e_{q2} $\dot{x}_{r1} = a(x_{r2} - x_{r1})$

$\dot{x}_{d2} = c x_{d2} - x_{d1}x_{d3} + x_{d4}$ D ε_{q2} $\dot{x}_{r2} = c x_{r2} - x_{r1}x_{r3} + x_{r4} + u + D$

$\dot{x}_{d3} = x_{d1}x_{d2} - b x_{d3}$ $\dot{x}_{r3} = x_{r1}x_{r2} - b x_{r3}$

$\dot{x}_{d4} = x_{d3} - d x_{d4}$ $\dot{x}_{r4} = x_{r3} - d x_{r4}$

$$\boxed{\begin{aligned} \xi_i &= x_{ri} + x_{di} \quad ; i = 1,2 \\ \xi_i &= x_{ri} - x_{di} \quad ; i = 3,4 \end{aligned}} \qquad \boxed{\begin{aligned} \dot{\xi}_i &= \dot{x}_{ri} + \dot{x}_{di} \quad ; i = 1,2 \\ \dot{\xi}_i &= \dot{x}_{ri} - \dot{x}_{di} \quad ; i = 3,4 \end{aligned}}$$

$$\Rightarrow \dot{\xi}_1 = a(\xi_2 - \xi_1)$$
$$\dot{\xi}_2 = c\xi_2 + \xi_1\xi_3 - x_{r3}\xi_1 - x_{r1}\xi_3 - \xi_4 + 2x_{r4} + u + D$$
$$\dot{\xi}_3 = x_{r2}\xi_1 + x_{r1}\xi_2 - \xi_1\xi_2 - b\xi_3$$
$$\dot{\xi}_4 = \xi_3 - d\xi_4$$

<u>Lyapunov's</u> <u>Candidate</u>: $0.5\left(\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 + \frac{\tilde{D}^2}{2\gamma}\right)$

$$\dot{V}(t) = \xi_1\dot{\xi}_1 + \xi_2\dot{\xi}_2 + \xi_3\dot{\xi}_3 + \xi_4\dot{\xi}_4 + \frac{\tilde{D}\dot{\hat{D}}}{2\gamma}$$

For finding structure of u:

$$\xi_1 a(\xi_2 - \xi_1) + \xi_2(c\xi_2 + \xi_1\xi_3 - x_{r3}\xi_1 - x_{r1}\xi_3 - \xi_4 + 2x_{r4} + u + \underline{D})$$
$$+ \xi_3(x_{r2}\xi_1 + x_{r1}\xi_2 - \xi_1\xi_2 - b\xi_3) + \xi_4(\xi_3 - d\xi_4) + \frac{\hat{D}\dot{\hat{D}}}{2\gamma}$$

$$\Rightarrow u\xi_2 + c\xi_2^2 + \cancel{\xi_1\xi_3\xi_2} - x_{r3}\xi_1\xi_2 - \cancel{x_{r1}\xi_3\xi_2} - \xi_4\xi_2 + 2x_{r4}\xi_2$$
$$+ D\xi_2 + \cancel{x_{r1}\xi_2\xi_3} + \cancel{x_{r2}\xi} \cancel{\xi_1\xi_2\xi_3} + a\xi_1\xi_2 = -k\xi^2$$

$$\Rightarrow \boxed{u = -(k+c)\xi_2 - a\xi_1 + x_{r3}\xi_1 + \xi_4 - 2x_{r4} - D}$$

$$e_{q1}\dot{e}_{q1} + e_{q2}\dot{e}_{q2} + e_{q3}\dot{e}_{q3} + e_{q4}\dot{e}_{q4} + \frac{\hat{D}\dot{\hat{D}}}{2\gamma} \qquad \boxed{\dot{\hat{D}} = -2\gamma e_{q2}}$$

on comparison $\frac{\dot{\hat{D}}}{2\gamma} + e_{q2} = 0$

Hyper Chaotic Rabinovich System:

$$\dot{x} = -ax + hy + yz$$
$$\dot{y} = hx \cancel{+} - by - xz + \omega + u$$
$$\dot{z} = -dz + xy$$
$$\dot{\omega} = -ky$$

① Linear Feedback Control

$$\boxed{u = -k_1 y}$$

$$\Rightarrow J = \begin{bmatrix} -a & h & 0 & 0 \\ h & -b-k_1 & 0 & 1 \\ 0 & 0 & -d & 0 \\ 0 & -k & 0 & 0 \end{bmatrix}$$

$$\lambda I - J = \begin{bmatrix} \lambda + a & h & 0 & 0 \\ h & \lambda + b + k_1 & 0 & 1 \\ 0 & 0 & \lambda + d & 0 \\ 0 & k & 0 & 0 \end{bmatrix}$$

$$= \lambda^4 + (a+b+d+k_1)\lambda^3 + (k + ab + ak_1 + bd + dk_1 - h^2)\lambda^2 + (ak + dk + adb + adk_1 - dh^2)\lambda + adk = 0$$

For Hyper Chaotic Lü:

$$\cancel{\dot{x} = a(y-x)}$$
$$\cancel{\dot{y} = cy - xz + \omega + u + f}$$
$$\cancel{\dot{z} = xy - bz}$$

$$\dot{\xi}_1 = a(\xi_2 - \xi_1)$$
$$\dot{\xi}_2 = c\xi_2 + \xi_1\xi_3 - x_{r3}\xi_1 - x_{r1}\xi_3 - \xi_4 + 2x_{r4} + u + D$$
$$\dot{\xi}_3 = x_{r2}\xi_1 + x_{r1}\xi_2 - \xi_1\xi_2 - b\xi_3$$
$$\dot{\xi}_4 = \xi_3 - d\xi_4$$

$D\xi_2 - \gamma D\dot{D}$

$\dot{D} = \gamma \xi_2$

$\frac{1}{2\gamma} d \cdot \hat{d}$

$$\boxed{u = k_1 y - 2x_4} \qquad \dot{\hat{D}} = \cancel{x}\, r\, \xi_2$$

$$\dot{\xi}_1 = -a\xi_1 + a\xi_2$$

$$\dot{\xi}_2 = c\xi_2 + \xi_1\xi_3 - x_{r3}\xi_1 - x_{r1}\xi_3 - \xi_4 - k\xi_2$$

$$\dot{\xi}_3 = x_{r2}\xi_1 + x_{r1}\xi_2 - \xi_1\xi_2 - b\xi_3$$

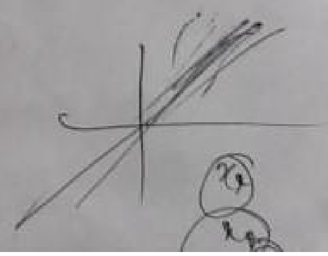
$$\dot{\xi}_4 = \xi_3 - d\xi_4$$

Lyapunov's Candidate is $0.5(\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2)$

$$-a\xi_1^2 + a\xi_1\xi_2 - (k_1 - c)\xi_2^2 + \cancel{\xi_1\xi_2\xi_3} - x_{r3}\xi_1\xi_2$$
$$- \cancel{x_{r1}\xi_2\xi_3} - \xi_2\xi_4 + \xi_1\xi_3 x_{r2} + \cancel{\xi_2\xi_3 x_{r1}}$$
$$- \cancel{\xi_1\xi_2\xi_3} - b\xi_3^2 + \xi_3\xi_4 - d\xi_4$$

$$-a\xi_1^2 - (k_1 - c)\xi_2^2 - b\xi_3^2 - d\xi_4^2 + \underline{a\xi_1\xi_2 - x_{r3}\xi_1\xi}$$
$$- \underline{\xi_2\xi_4} + \xi_1\xi_3 x_{r2} + \xi_3\xi_4 + (r+1)\xi_2\hat{D}$$

$$\begin{bmatrix} -a & \cancel{a/2}\ -x_{r3} & \frac{x_{r2}}{2} & 0 & 0 \\ \cancel{a/2} & -(k_1 - c) & 0 & -0.5 & 1 \\ \frac{x_{r2}}{2} & 0 & -b & 0.5 & 0 \\ 0 & -0.5 & 0.5 & -d & 0 \\ 0 & r & 0 & 0 & 0 \end{bmatrix}$$



Now for Linear feedback control, $\boxed{u = -k\xi_2 - 2x_{r4}}$

Lyapunov's candidate: $0.5\left(\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 + \frac{1}{\gamma}\hat{D}^2\right)$

$$\dot{V}(t) = \xi_1\dot{\xi}_1 + \xi_2\dot{\xi}_2 + \xi_3\dot{\xi}_3 + \xi_4\dot{\xi}_4 + \frac{1}{\gamma}\hat{D}\cdot\dot{\hat{D}} = 0$$

$\Rightarrow \boxed{\dot{\hat{D}} = \gamma\xi_2}$

$$\Rightarrow -a\xi_1^2 - (k-c)\xi_2^2 - b\xi_3^2 - d\xi_4^2 + \boxed{(a - x_{r3})\xi_1\xi_2} - x_{r2}\xi_1\xi_3 - \xi_2\xi_4 + \xi_3\xi_4 + 2\xi_2\hat{D}$$

$$\Rightarrow \begin{bmatrix}\xi_1 & \xi_2 & \xi_3 & \xi_4 & \hat{D}\end{bmatrix} \begin{bmatrix} -a & \underline{\quad} & \frac{x_{r2}}{2} & 0 & 0 \\ \underline{\quad} & -(k-c) & 0 & -0.5 & 1 \\ \frac{x_{r2}}{2} & 0 & -b & 0.5 & 0 \\ 0 & -0.5 & 0.5 & -d & 0 \\ 0 & 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix}\xi_1 \\ \xi_2 \\ \xi_3 \\ \xi_4 \\ \hat{D}\end{bmatrix}$$

$$\dot{D} = (-pD - e_{q_2})\gamma$$

$\Rightarrow$ SMC:

$$\begin{bmatrix} -a & 0 & 0.5|x_{r_2}| & 0 & 0 \\ 0 & -K & 0 & 0 & \frac{1-r}{2} \\ 0.5|x_{r_2}| & 0 & -b & 0.5 & 0 \\ 0 & 0 & 0.5 & -d & 0 \\ 0 & \frac{1-r}{2} & 0 & 0 & -\gamma p \end{bmatrix}$$

<u>Linear:</u> $(-pD - e_{q_2}\gamma)$

$u =$

Lyp $\dot{V}(t) = 0.5\left(\xi_1^2 + \xi_2^2 + \xi_3^2 + \xi_4^2 + \frac{1}{2\gamma}\hat{D}^2\right)$

$\dot{V}(t) = \xi_1\dot{\xi}_1 + \xi_2\dot{\xi}_2 + \dots + \frac{1}{2\gamma}\hat{D}\dot{\hat{D}}$

$= \xi_1(\cancel{d}\xi_2 - \xi_1) + \xi_2(-K\xi_2 + \xi_1\xi_3 - a\xi_1 - x r_1\xi_3 + D)$
$+ \xi_3(x r_2\xi_1 + x r_1\xi_2 - \xi_1\xi_2 - b\xi_3)$
$+ \xi_4(\xi_3 - d\xi_4) + \frac{1}{2\gamma}\hat{D}.\dot{\hat{D}} = 0$

$\Rightarrow \hat{D}$

$\Rightarrow \cancel{+a\xi_1\xi_2} - a\xi_1^2 - K\xi_2^2 + \cancel{\xi_1\xi_2\xi_3} - \cancel{a\xi_1\xi_2} - \cancel{x r_1\xi_2\xi_3}$
$+ \hat{D}\xi_2 + x r_2\xi_1\xi_3 + \cancel{x r_1\xi_2\xi_3} - \cancel{\xi_1\xi_2\xi_3} - b\xi_3^2$
$+ \xi_3\xi_4 - d\xi_4^2 + \frac{1}{2\gamma}\hat{D}\dot{\hat{D}}$

$-a\xi_1^2 - K\xi_2^2 - b\xi_3^2 - d\xi_4^2 + 2\hat{D}\xi_2 + x r_2\xi_1\xi_3$
$+ \xi_3\xi_4 \quad - \gamma p D^2 - \gamma\xi_2 D$

$$\Rightarrow [\xi_1\ \xi_2\ \xi_3\ \xi_4\ \hat{D}]\begin{bmatrix} -a & 0 & 0.5|x r_2| & 0 & 0 \\ 0 & -K & 0 & 0 & \frac{1-\gamma}{2} \\ 0.5|x r_2| & 0 & -b & 0.5 & 0 \\ 0 & 0 & 0.5 & -d & 0 \\ 0 & \frac{1-\gamma}{2} & 0 & 0 & -\gamma p \end{bmatrix}\begin{bmatrix}\xi_1\\ \xi_2\\ \xi_3\\ \xi_4\\ \hat{D}\end{bmatrix}$$

ξ_{ξ_2}

$D\xi_{\xi_2}$

$\Rightarrow -E^T\Omega E$

Bound

$$\boxed{\Rightarrow \Omega = \begin{bmatrix} +a & 0 & -0.5|x r_2| & 0 & 0 \\ 0 & +K & 0 & 0 & -1 \\ -0.5|x r_2| & 0 & +b & -0.5 & 0 \\ 0 & 0 & -0.5 & +d & 0 \\ 0 & -1 & 0 & 0 & 0 \end{bmatrix}}$$

$\xi_2 + \frac{1}{2\gamma}\cancel{D}\dot{D} = -p\cancel{D}$

$\boxed{\dot{D} = (-pD - \xi_2)2\gamma}$