

# Don't Roll Your Own FO: Challenges in Proving Post-Quantum CCA Security of Real-World KEMs

Varun Maram  
Cybersecurity Group  
SandboxAQ



: <https://varun-maram.github.io/>

: varun-maram-pqc

Based on joint works with Paul Grubbs, Kenny Paterson, and Keita Xagawa.



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- CRYSTALS-Kyber
- Streamlined NTRU Prime

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Fujisaki-Okamoto transform.

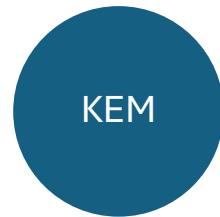
In the Quantum Random Oracle Model (QROM).

Based on joint works with Paul Grubbs, Kenny Paterson, and Keita Xagawa.



# Fujisaki-Okamoto Transformation

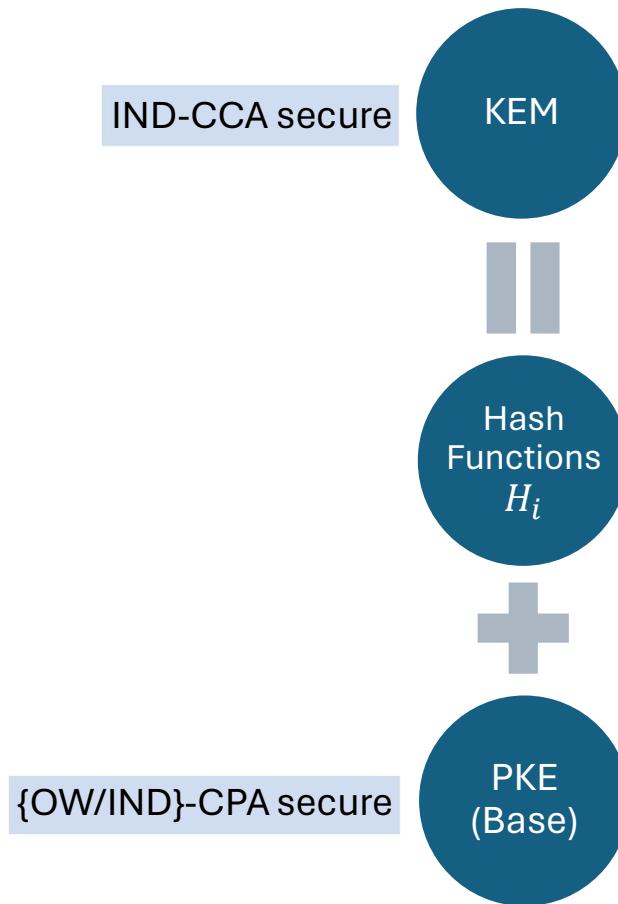
IND-CCA secure



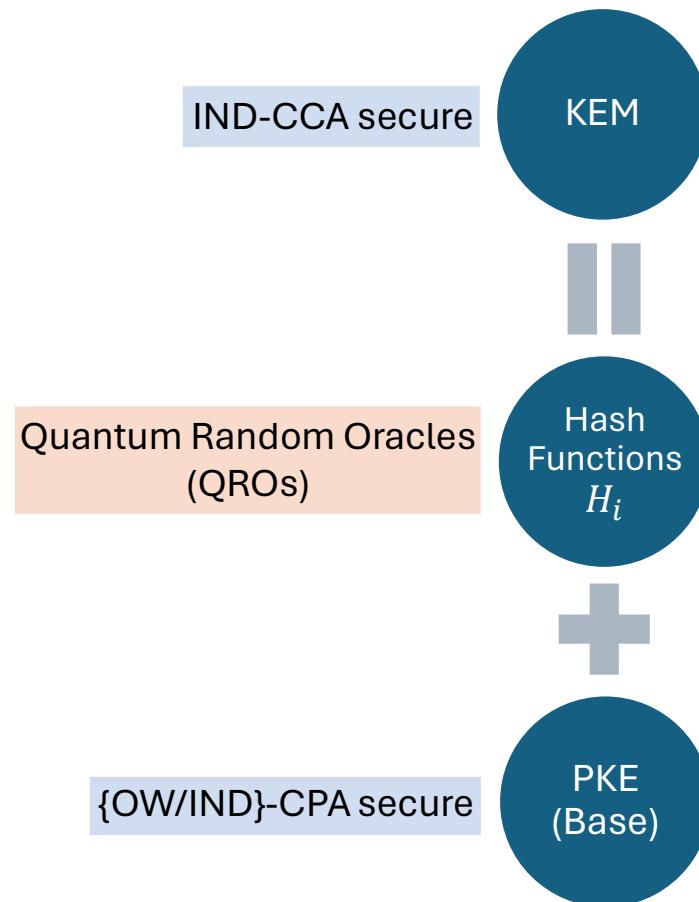
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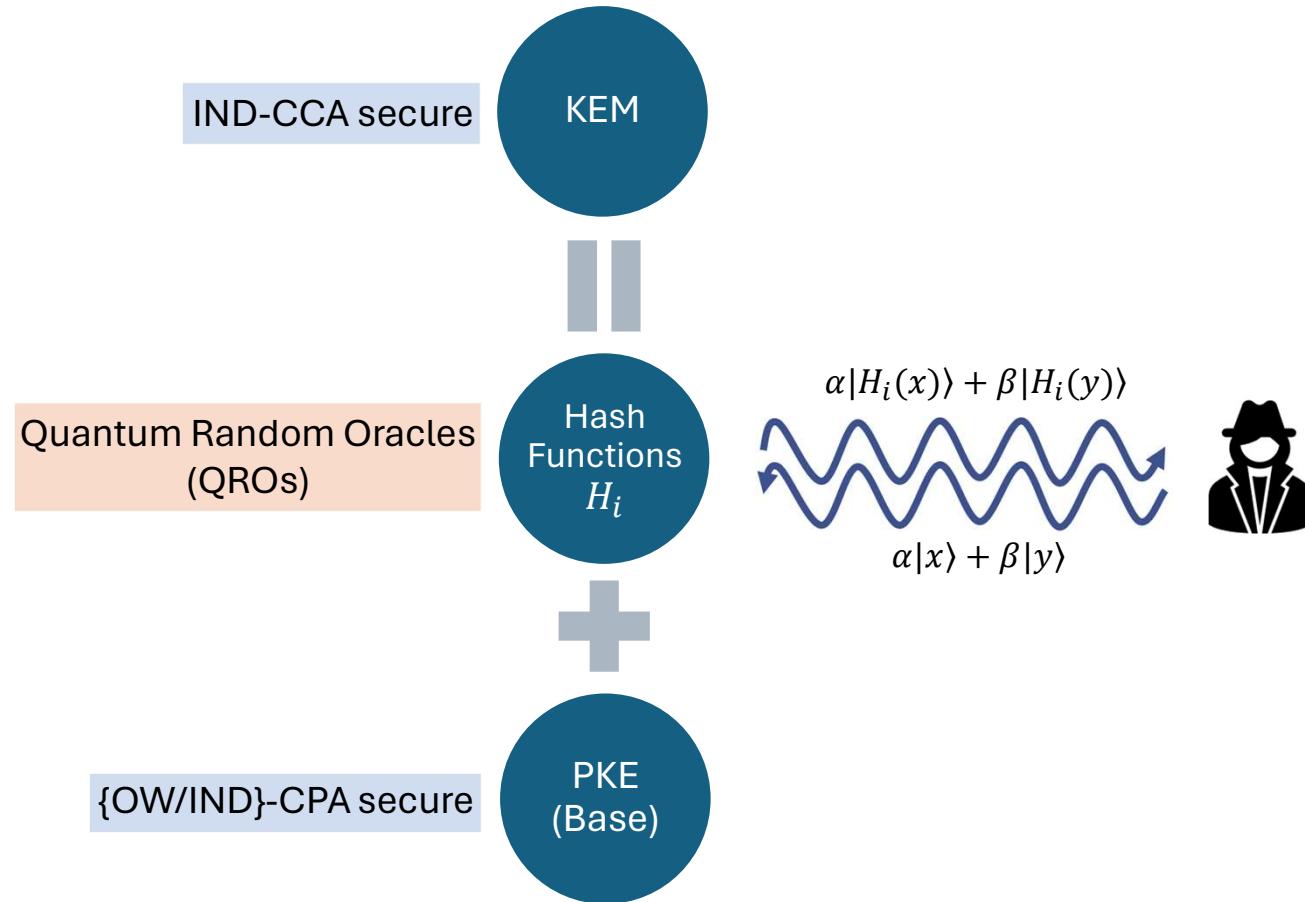
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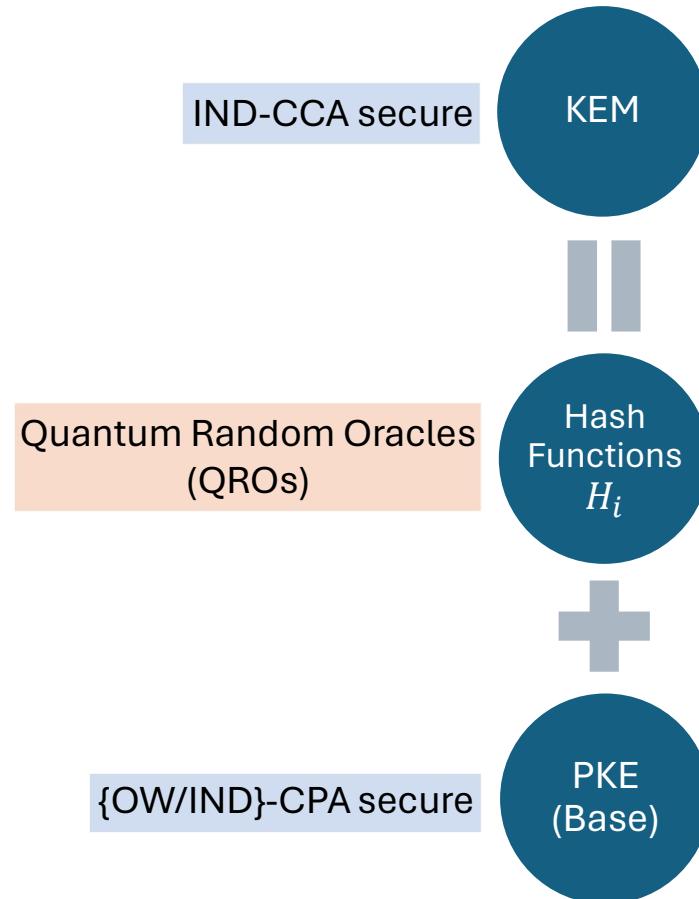
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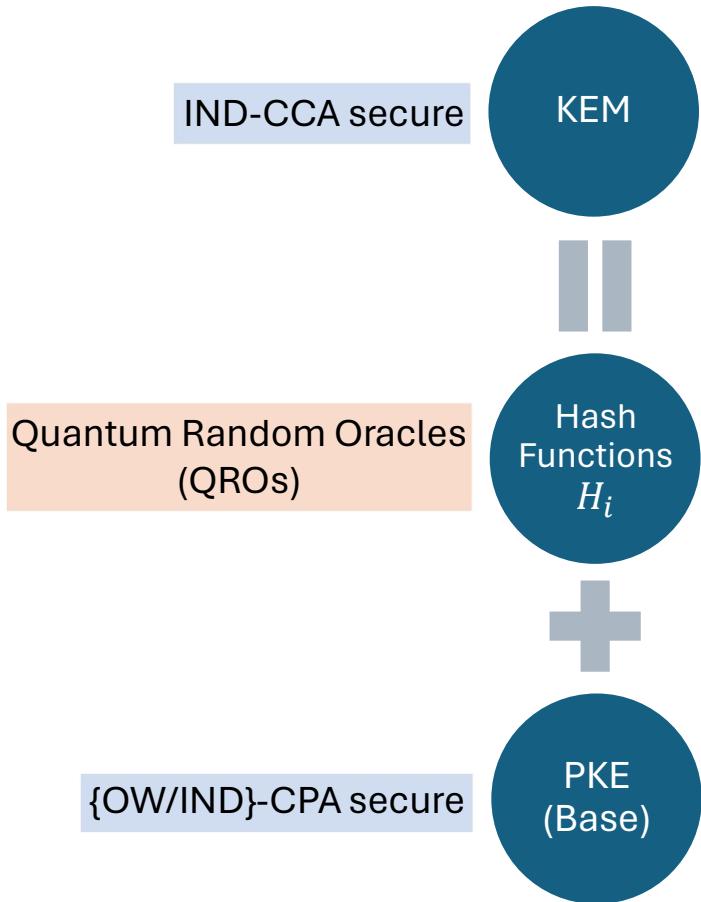
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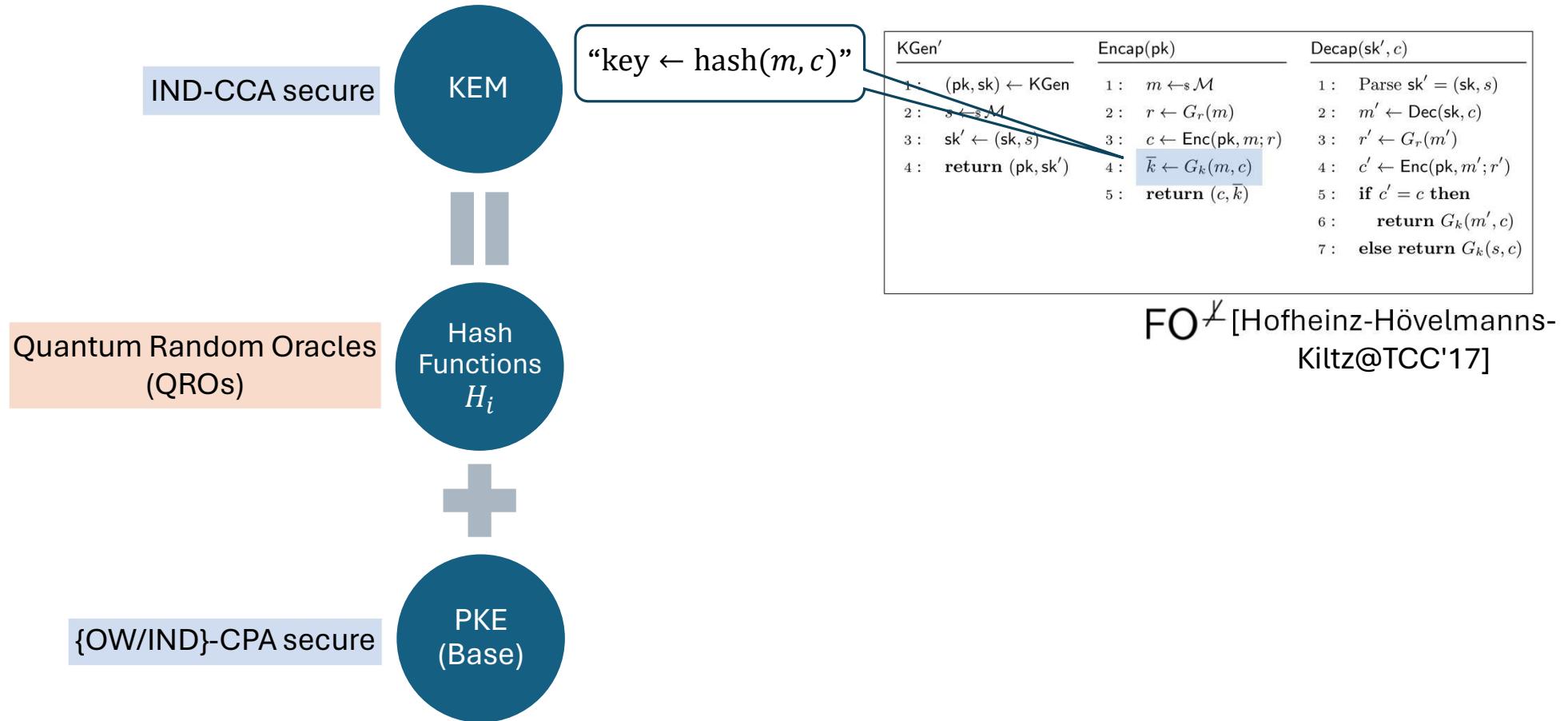
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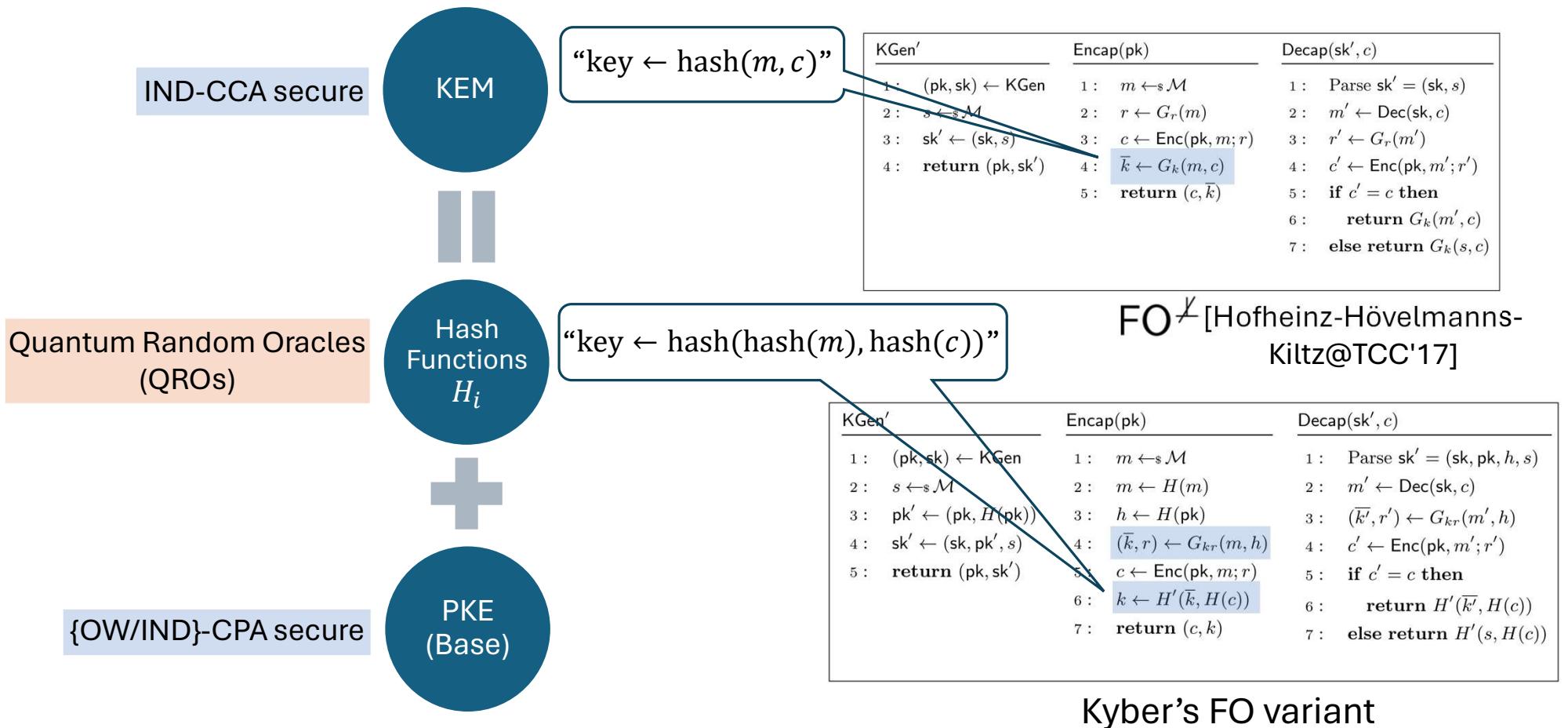
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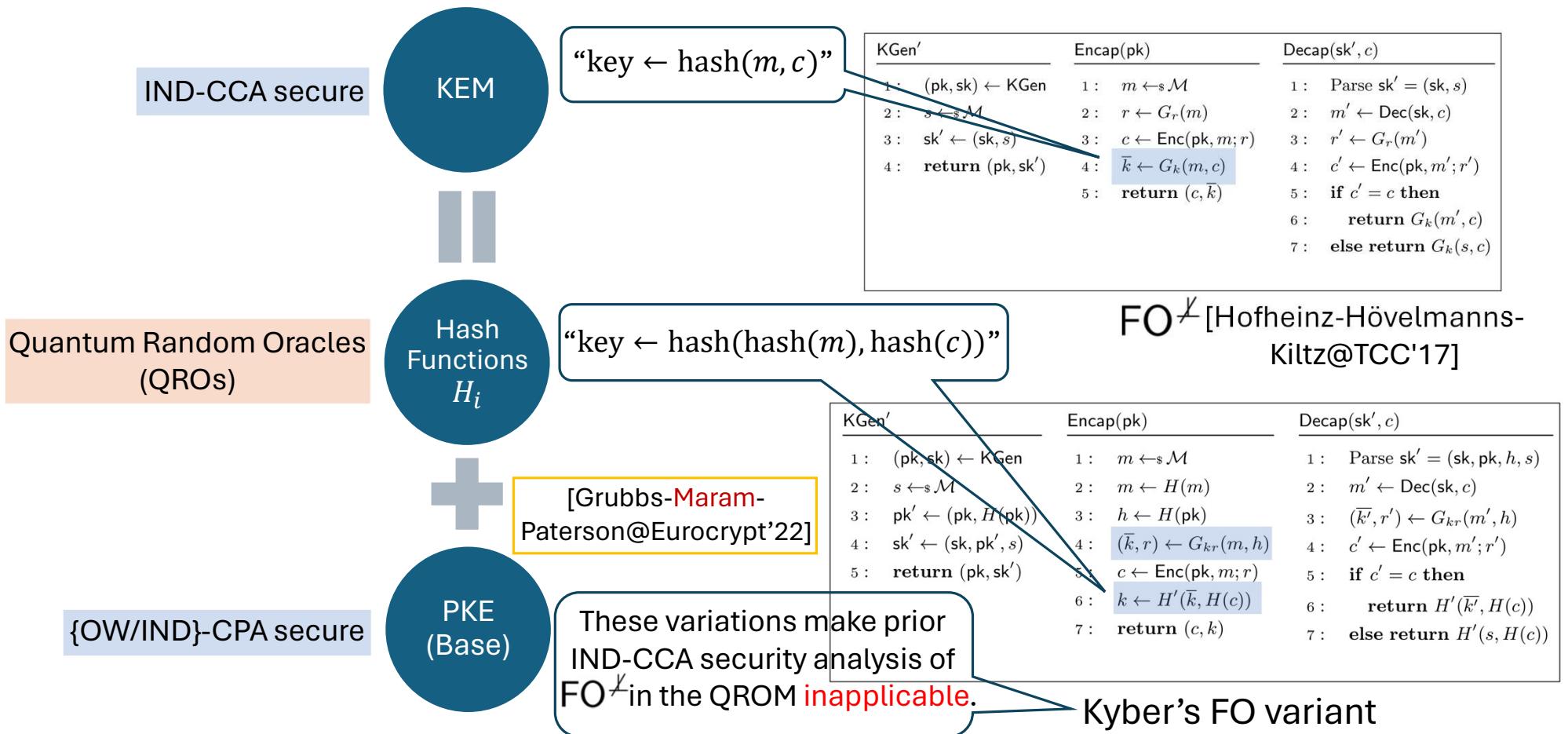
# FO Variants



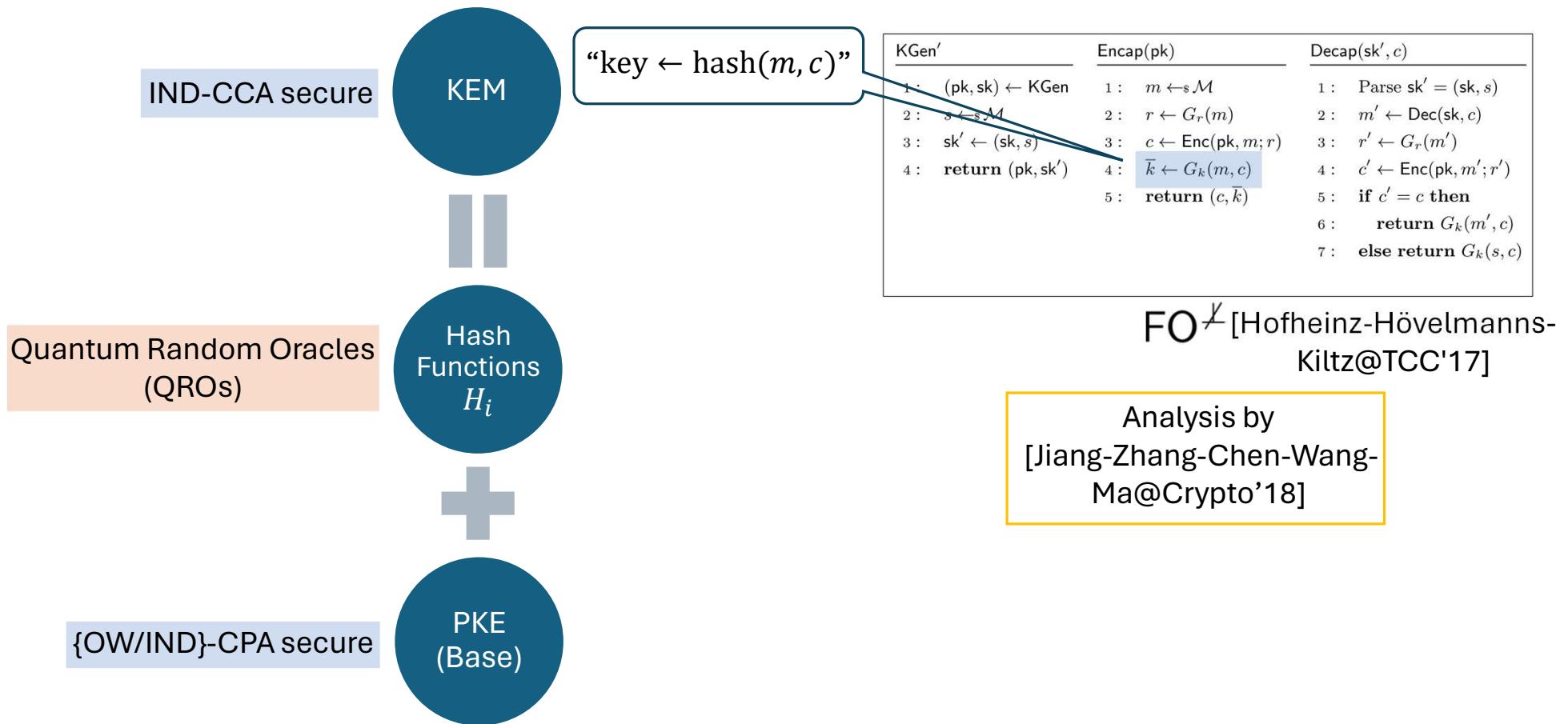
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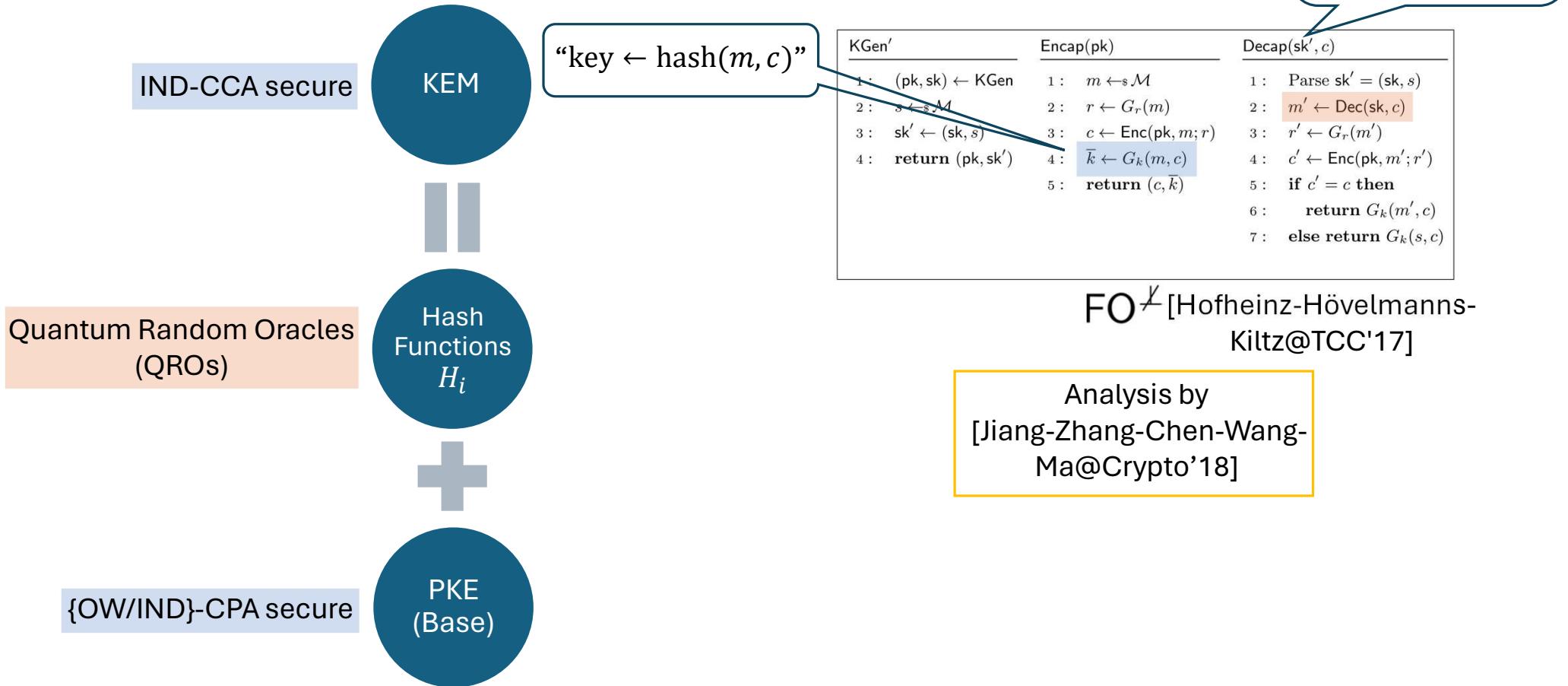
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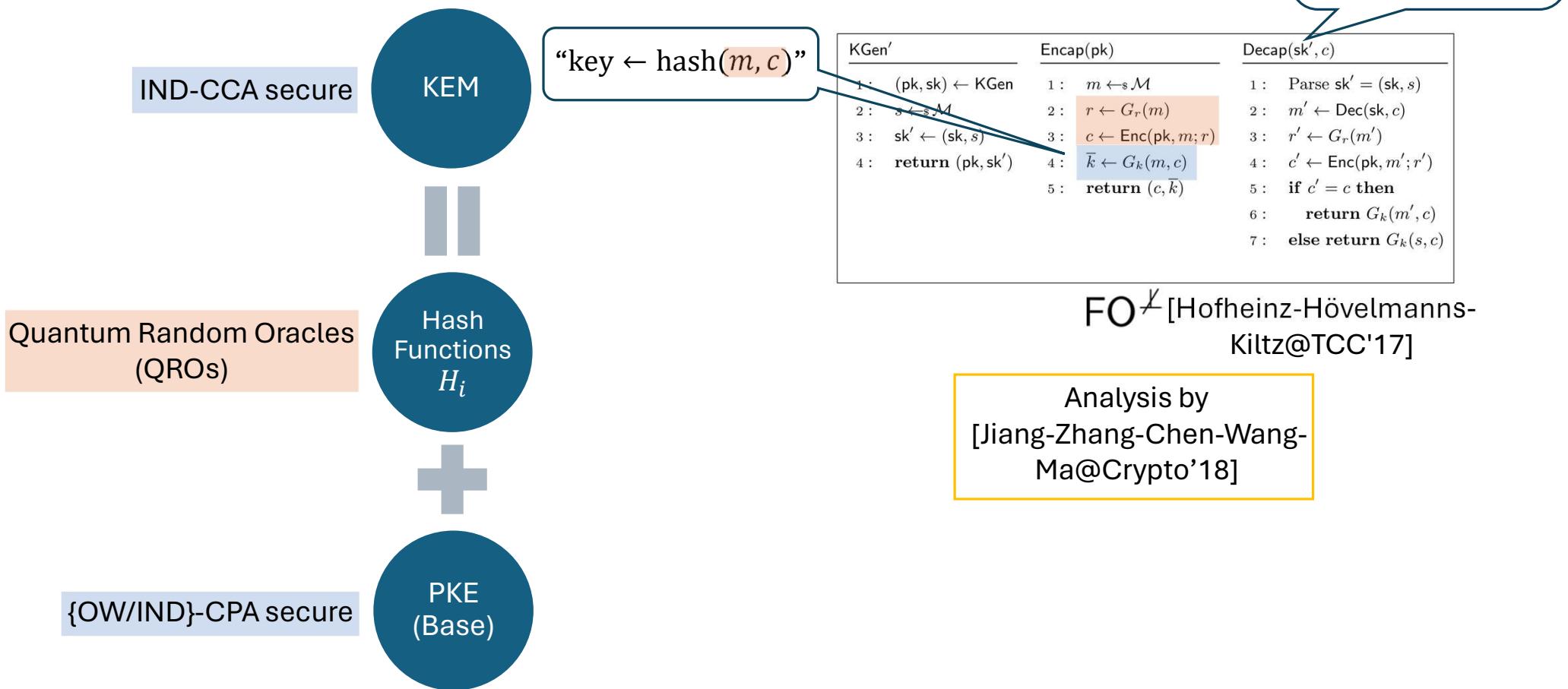
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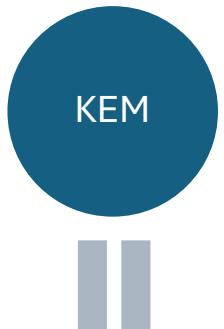


# FO Variants



# FO Variants

IND-CCA secure



Hash Functions  
 $H_i$



{OW/IND}-CPA secure



"key  $\leftarrow \text{hash}(m, c)$ "

"key  $\leftarrow \text{hash}'(c)$ "

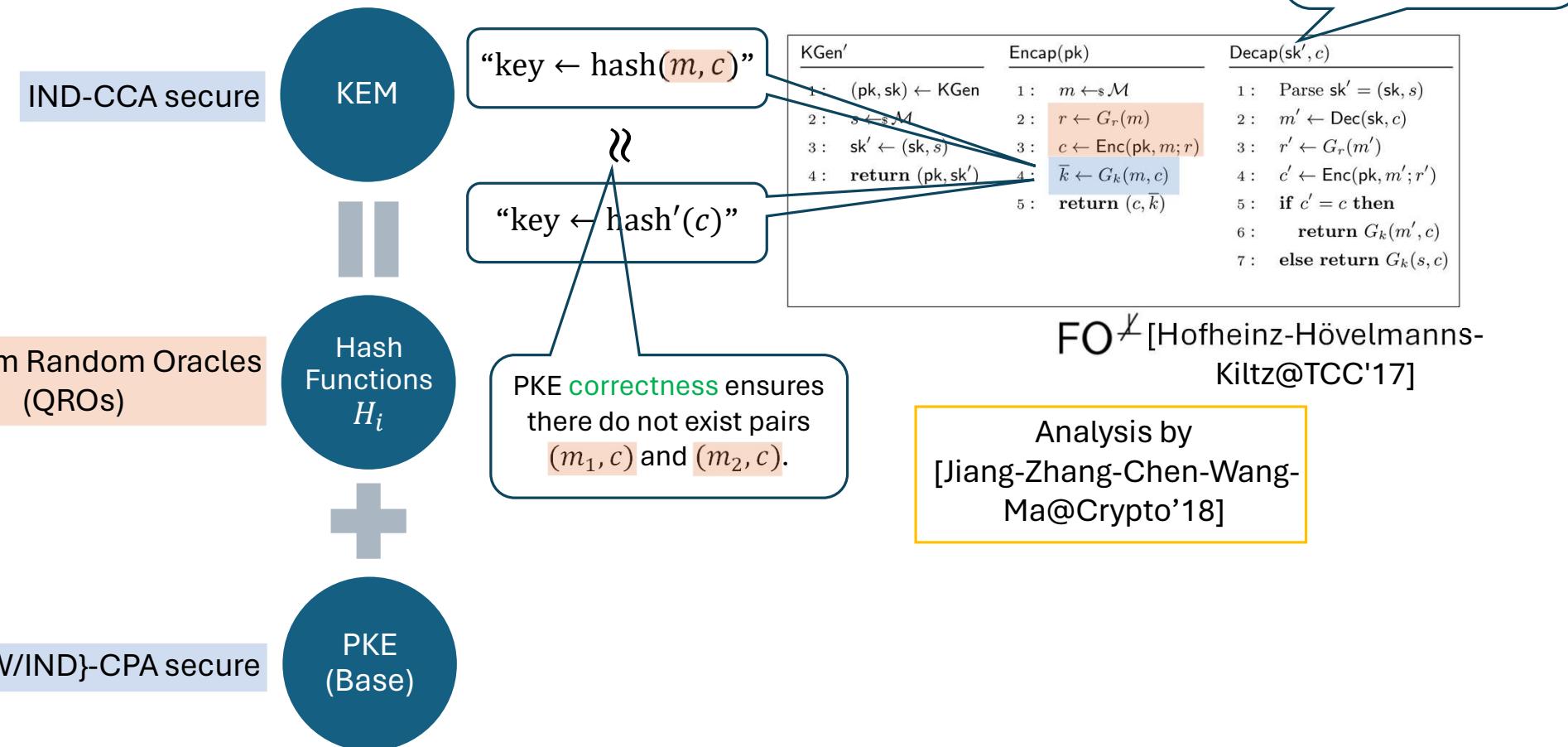
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FO $^{\neq}$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

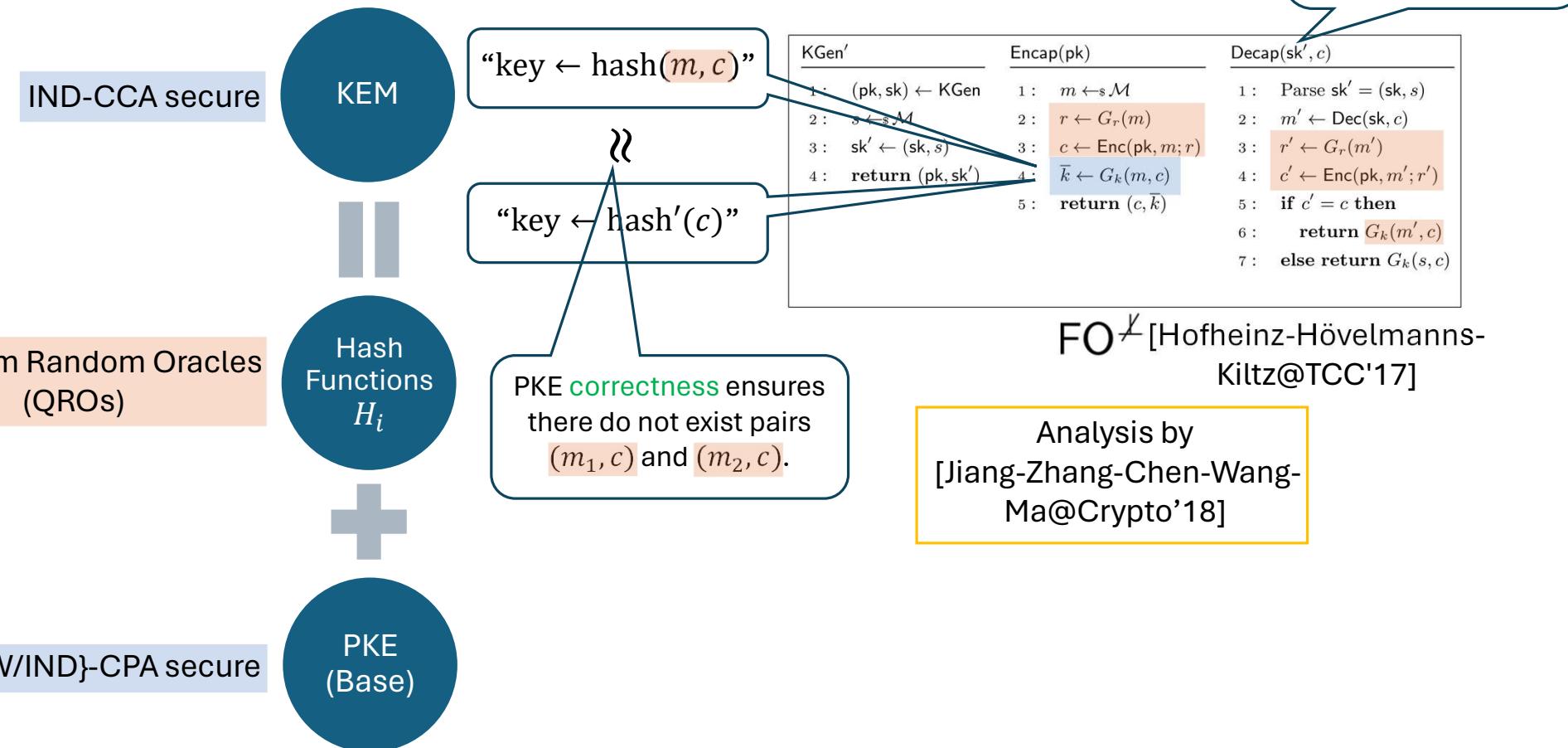
Analysis by  
[Jiang-Zhang-Chen-Wang-Ma@Crypto'18]

CPA-reduction  
needs  $\text{sk}'$  to answer  
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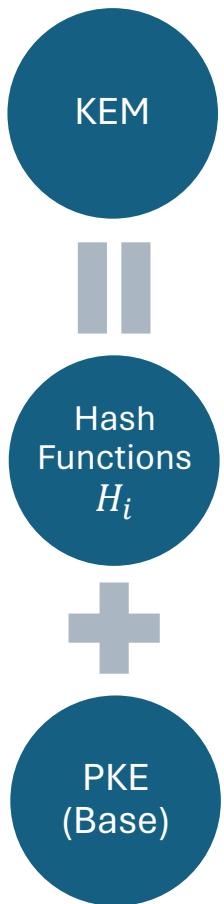


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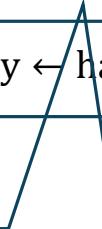
IND-CCA secure



“key  $\leftarrow \text{hash}(m, c)$ ”

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PKE correctness ensures there do not exist pairs  $(m_1, c)$  and  $(m_2, c)$ .



$\text{KGen}'$

- 1 :  $(\text{pk}, \text{sk}) \leftarrow \text{KGen}$
- 2 :  $s \leftarrow \mathbb{S} \setminus M$
- 3 :  $\text{sk}' \leftarrow (\text{sk}, s)$
- 4 : **return**  $(\text{pk}, \text{sk}')$

$\text{Encap}(\text{pk})$

- 1 :  $m \leftarrow \mathbb{s} \setminus M$
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- 5 : **return**  $(c, \bar{k})$

$\text{Decap}(\text{sk}', c)$

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- 2 :  $m' \leftarrow \text{Dec}(\text{sk}, c)$
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$\text{FO}^{\neq}[\text{Hofheinz-Hövelmanns-Kiltz@TCC'17}]$

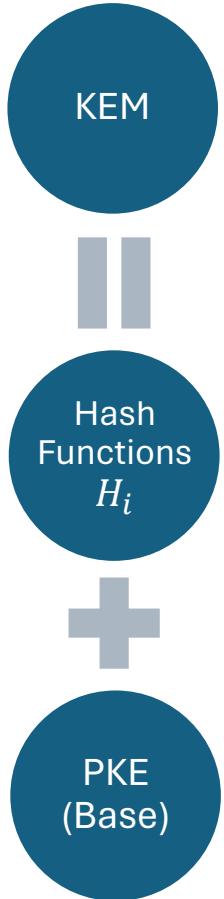
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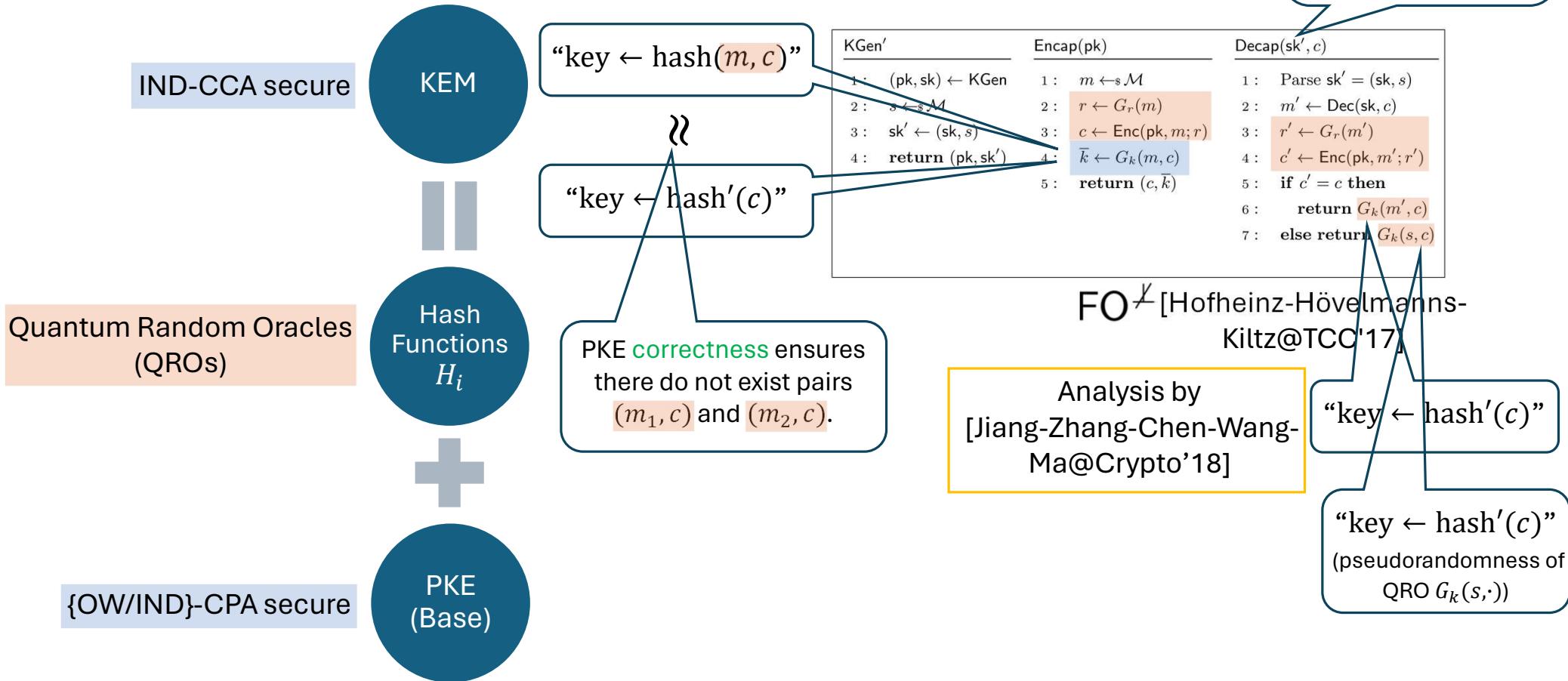
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CPA-reduction needs  $\text{sk}'$  to answer CCA-adversary's **Decap** queries.

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(pseudorandomness of QRO  $G_k(s, \cdot)$ )

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CPA-reduction can just answer with  $\text{key} := \text{hash}'(c)$ !

IND-CCA secure



Quantum Random Oracles (QROs)

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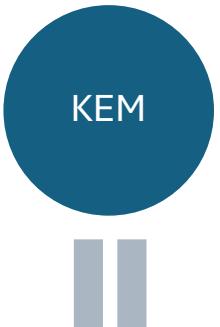
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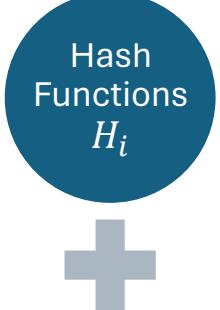
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Quantum Random Oracles  
(QROs)



“key  $\leftarrow \text{hash}(\text{hash}(m), \text{hash}(c))$ ”

{OW/IND}-CPA secure



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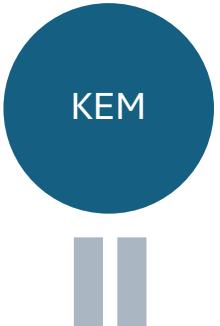
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Kyber's FO variant

# FO Variants

CPA-reduction can just answer with  $\text{key} := \text{hash}'(c)$ !

IND-CCA secure



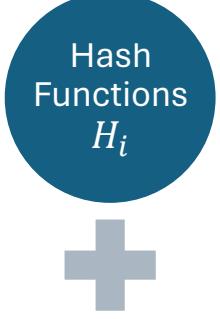
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Quantum Random Oracles (QROs)



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{OW/IND}-CPA secure

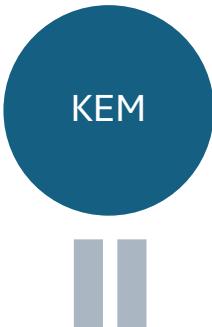


Kyber's FO variant

# FO Variants

CPA-reduction can just answer with key  $\coloneqq \text{hash}'(c)$ !

IND-CCA secure



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Quantum Random Oracles  
(QROs)



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{OW/IND}-CPA secure



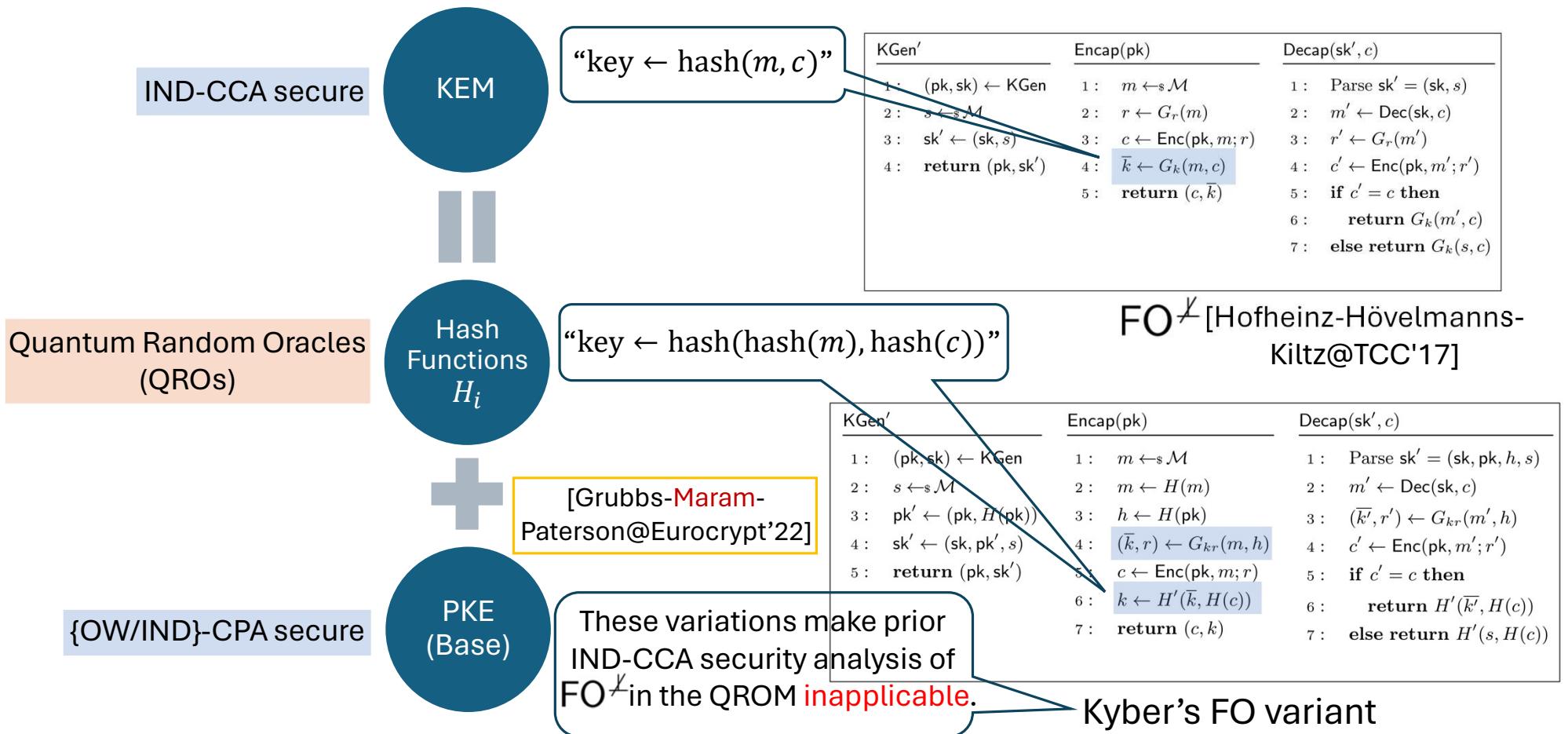
Given “ $\text{hash}(m)$ ” and “ $\text{hash}(c)$ ”, how to extract  $(m, c)$  for checking this programming condition?

FO $^{\neq}$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

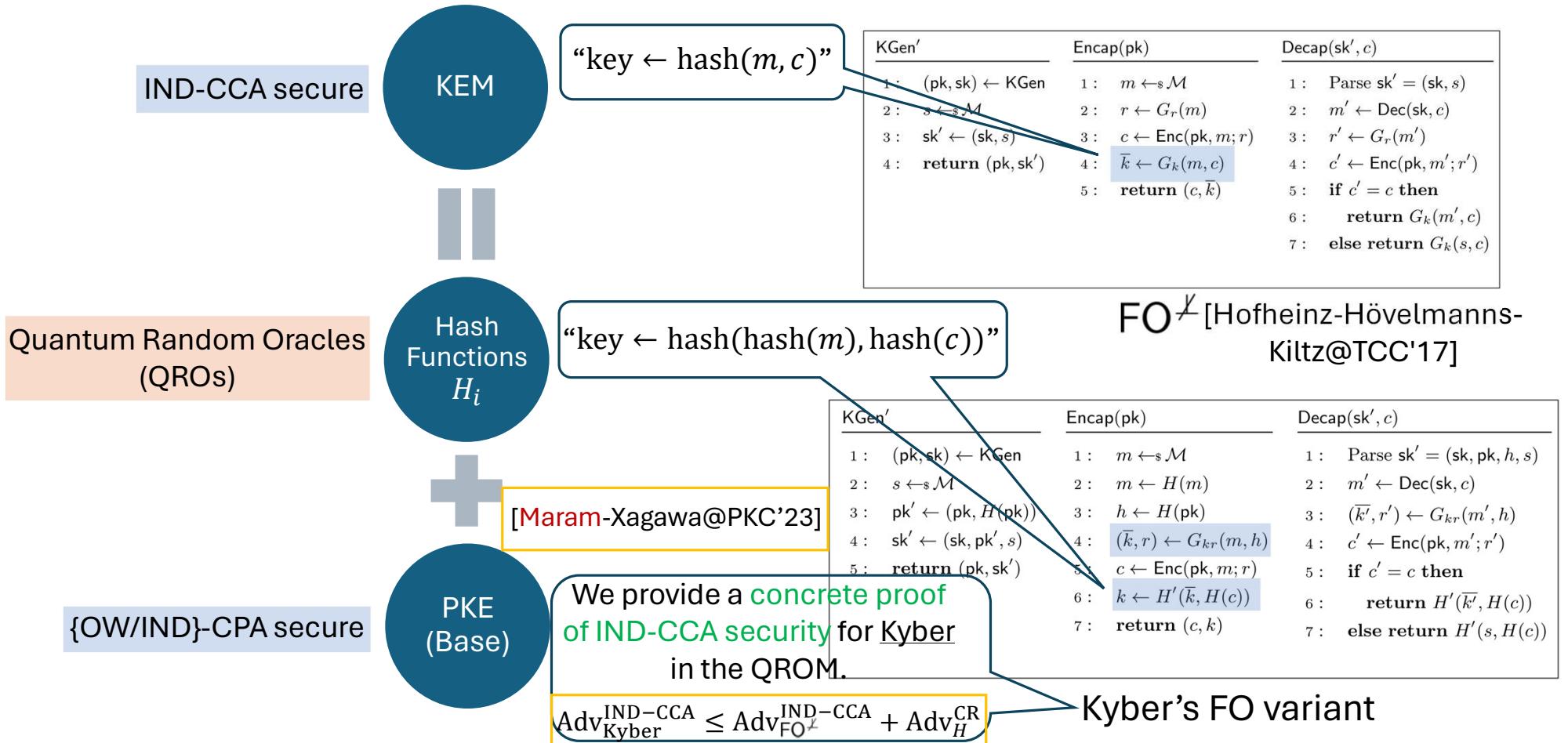
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Kyber's FO variant

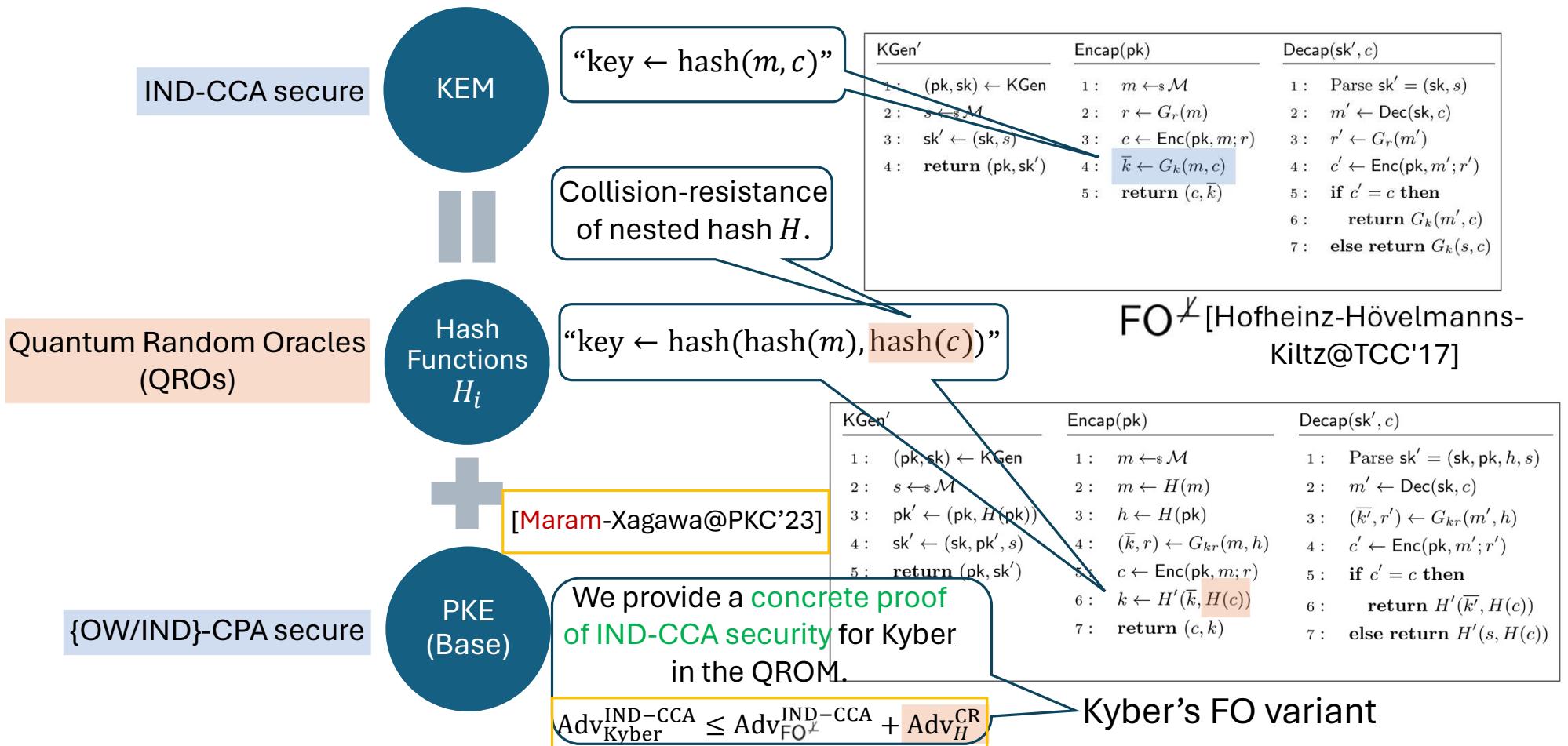
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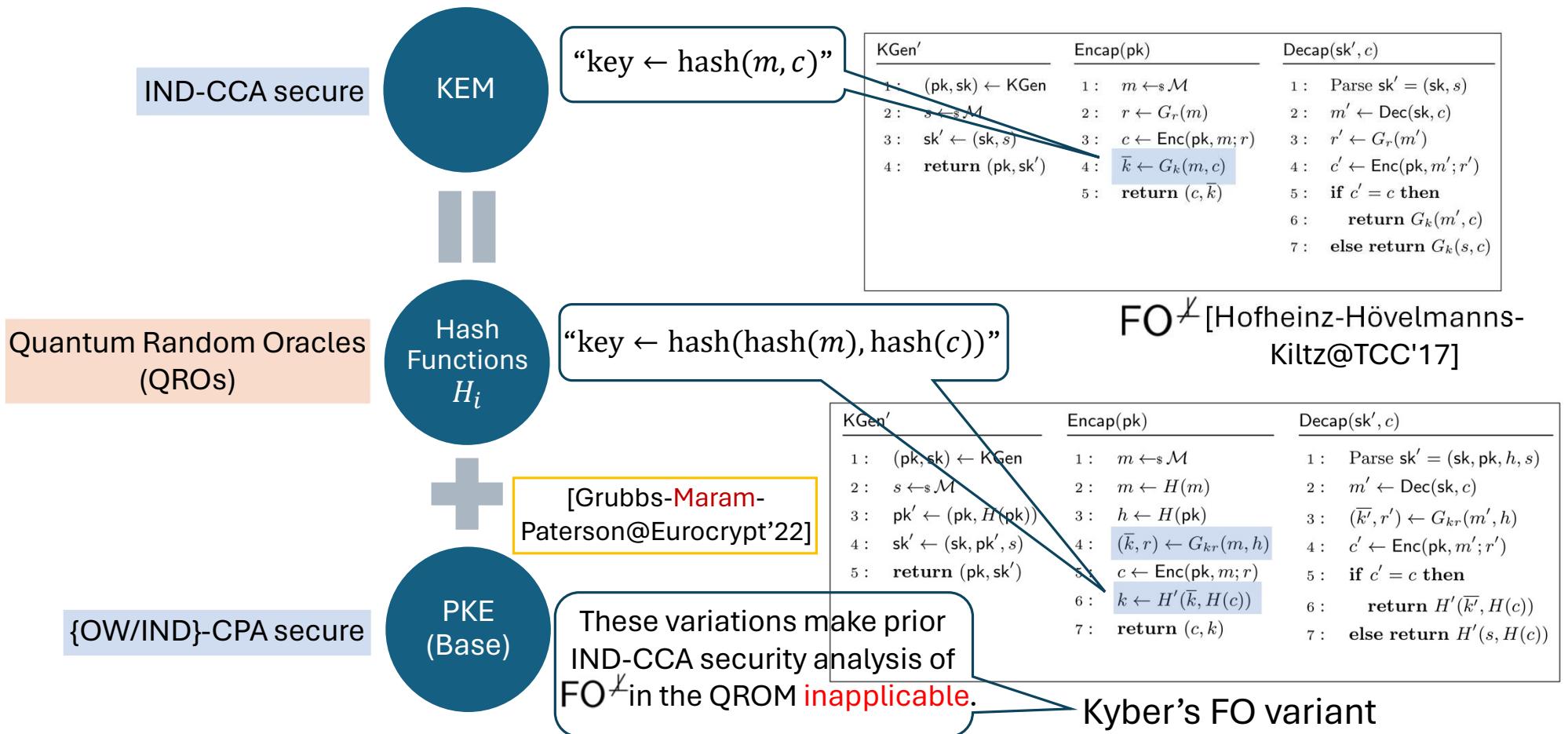
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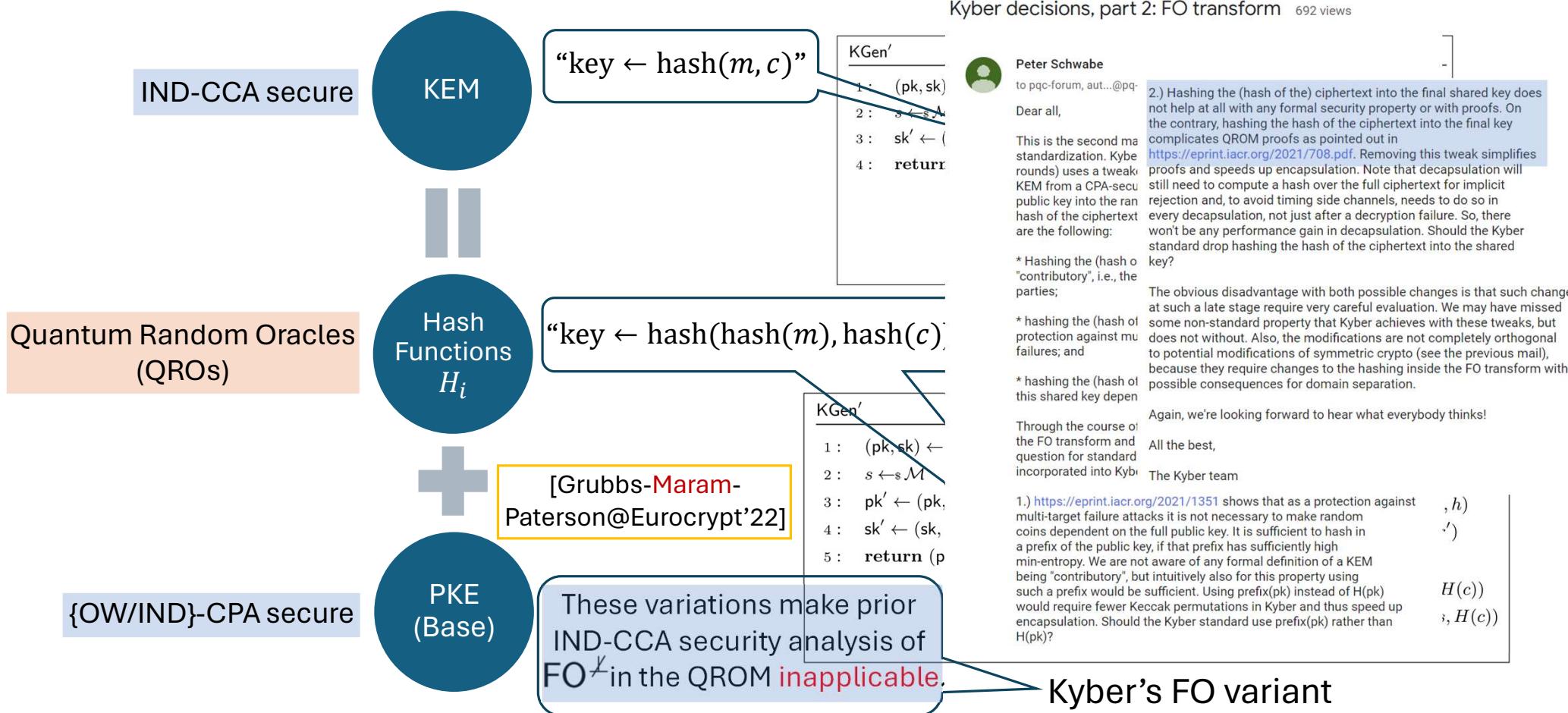
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# FO Variants

Kyber decisions, part 2: FO transform 692 views

IND-CCA secure



“key  $\leftarrow \text{hash}(m, c)$ ”

```

KGen'
1 : (pk, sk)
2 : s  $\leftarrow$  A
3 : sk'  $\leftarrow$  (
4 : return

```



Peter Schwabe

to pqc-forum, aut...@pqc-forum, b...@cloudflare.com, pqc-...@list.nist.gov, Andreas Hülsing

This is the second major standardization. Kyber (r) uses a tweak KEM from a CPA-secure public key into the random hash of the ciphertext are the following:

\* Hashing the (hash of) "contributory", i.e., the parties;

\* hashing the (hash of protection against multi-target failure); and

\* hashing the (hash of this shared key) dependent

Through the course of the FO transform and question for standard incorporated into Kyber

2.) Hashing the (hash of the) ciphertext into the final shared key does not help at all with any formal security property or with proofs. On the contrary, hashing the hash of the ciphertext into the final key complicates QROM proofs as pointed out in <https://eprint.iacr.org/2021/708.pdf>. Removing this tweak simplifies proofs and speeds up encapsulation. Note that decapsulation will still need to compute a hash over the full ciphertext for implicit rejection and, to avoid timing side channels, needs to do so in every decapsulation, not just after a decryption failure. So, there won't be any performance gain in decapsulation. Should the Kyber standard drop hashing the hash of the ciphertext into the shared key?

The obvious disadvantage with both possible changes is that such changes at such a late stage require very careful evaluation. We may have missed some non-standard property that Kyber achieves with these tweaks, but does not without. Also, the modifications are not completely orthogonal to potential modifications of symmetric crypto (see the previous mail), because they require changes to the hashing inside the FO transform with possible consequences for domain separation.

Again, we're looking forward to hear what everybody thinks!

All the best,

The Kyber team

1.) <https://eprint.iacr.org/2021/1351> shows that as a protection against multi-target failure attacks it is not necessary to make random coins dependent on the full public key. It is sufficient to hash in a prefix of the public key, if that prefix has sufficiently high min-entropy. We are not aware of any formal definition of a KEM being "contributory", but intuitively also for this property using such a prefix would be sufficient. Using prefix(pk) instead of H(pk) would require fewer Keccak permutations in Kyber and thus speed up encapsulation. Should the Kyber standard use prefix(pk) rather than H(pk)?

dustin...@nist.gov

to pqc-forum, b...@cloudflare.com, pqc-...@list.nist.gov, Andreas Hülsing

All,

NIST is planning to incorporate the CRYSTAL-Kyber team's suggestion of tweaking the FO transform into the draft standard, as described in [Peter Schwabe's earlier post](#). Any comments or feedback are welcome.

Dustin

Apr 28, 2023, 2:50:13 PM

Quant

{OW/IND}-CPA secure

PKE  
(Base)

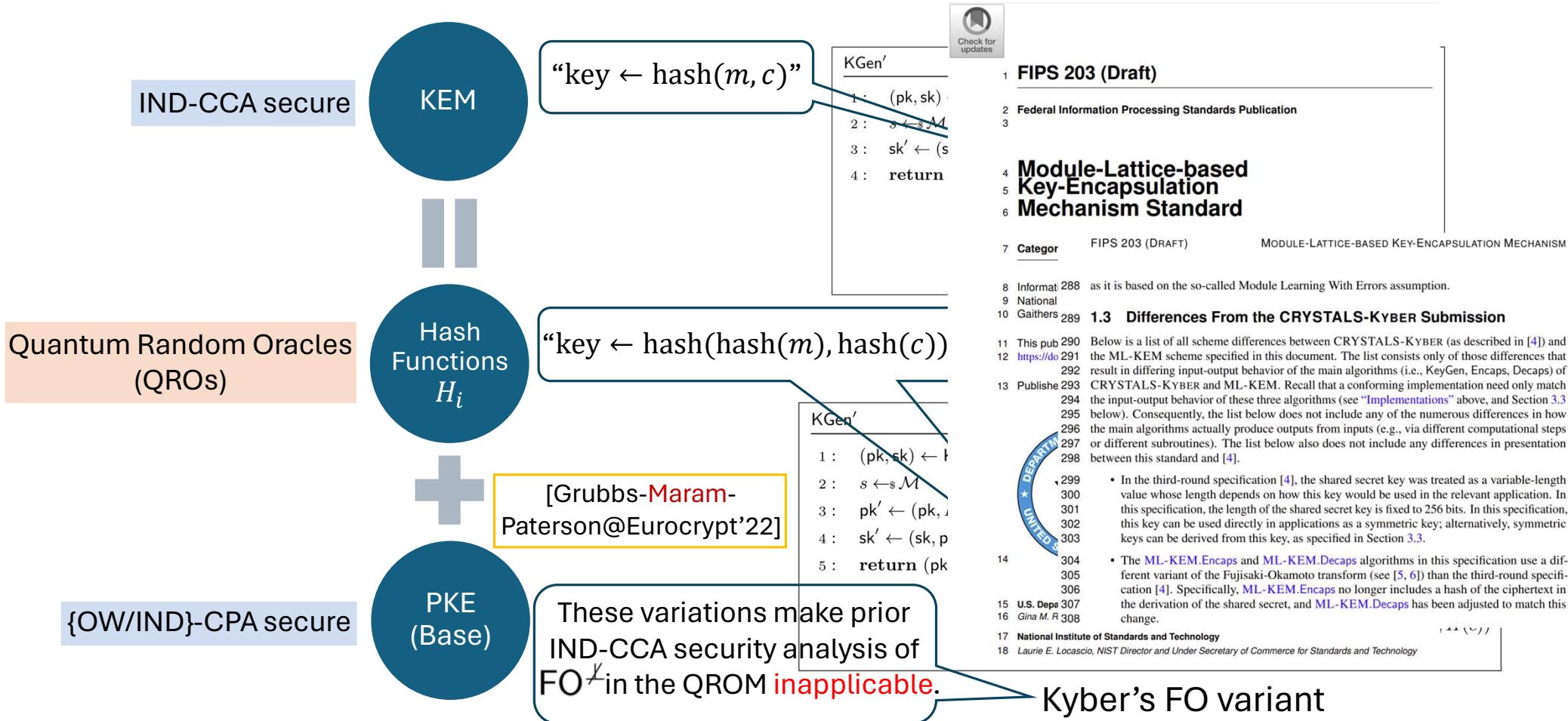
[Grubbs-Maram-  
Paterson@Eurocrypt'22]

These variations make prior  
IND-CCA security analysis of  
FO<sup>†</sup> in the QROM **inapplicable**.

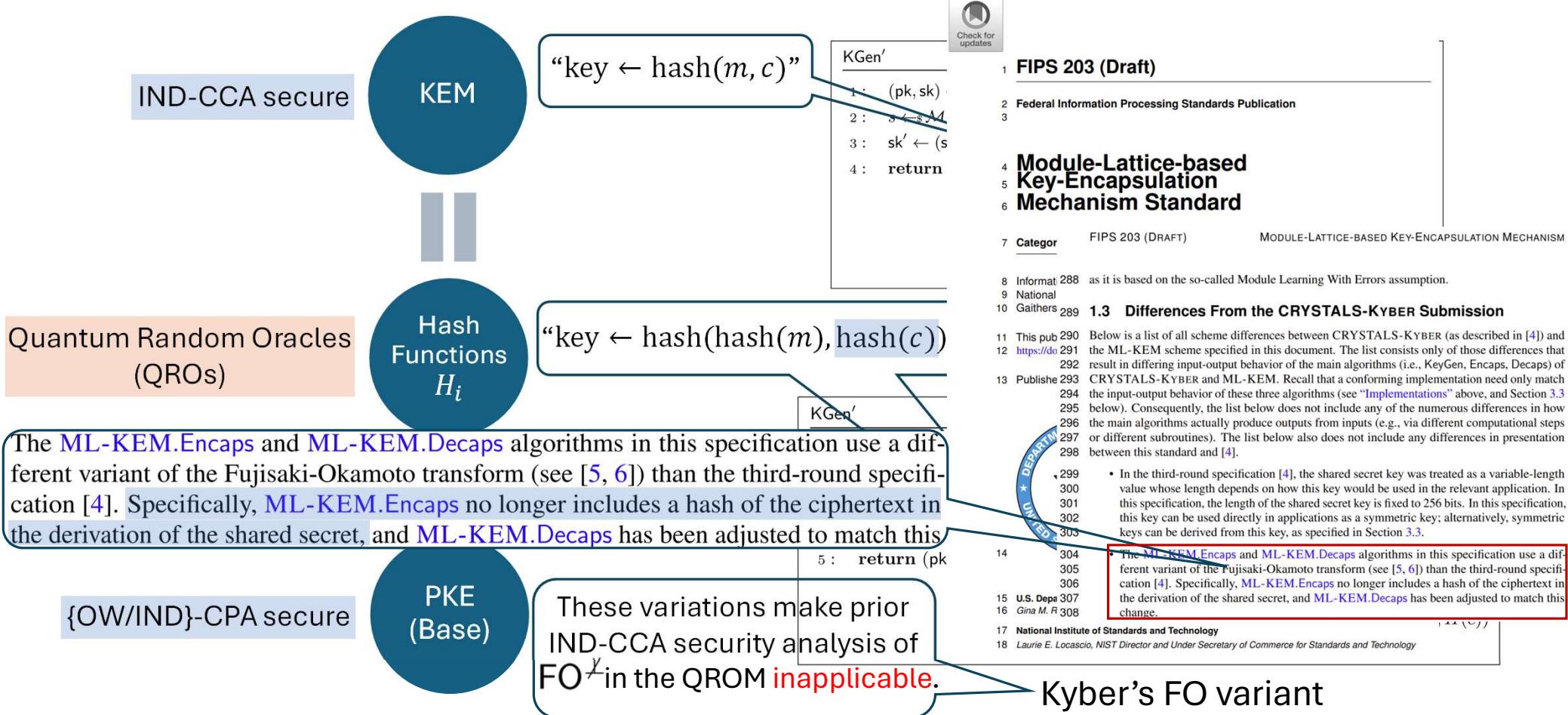
Kyber's FO variant

$H(c)$   
 $\vdash, H(c)$

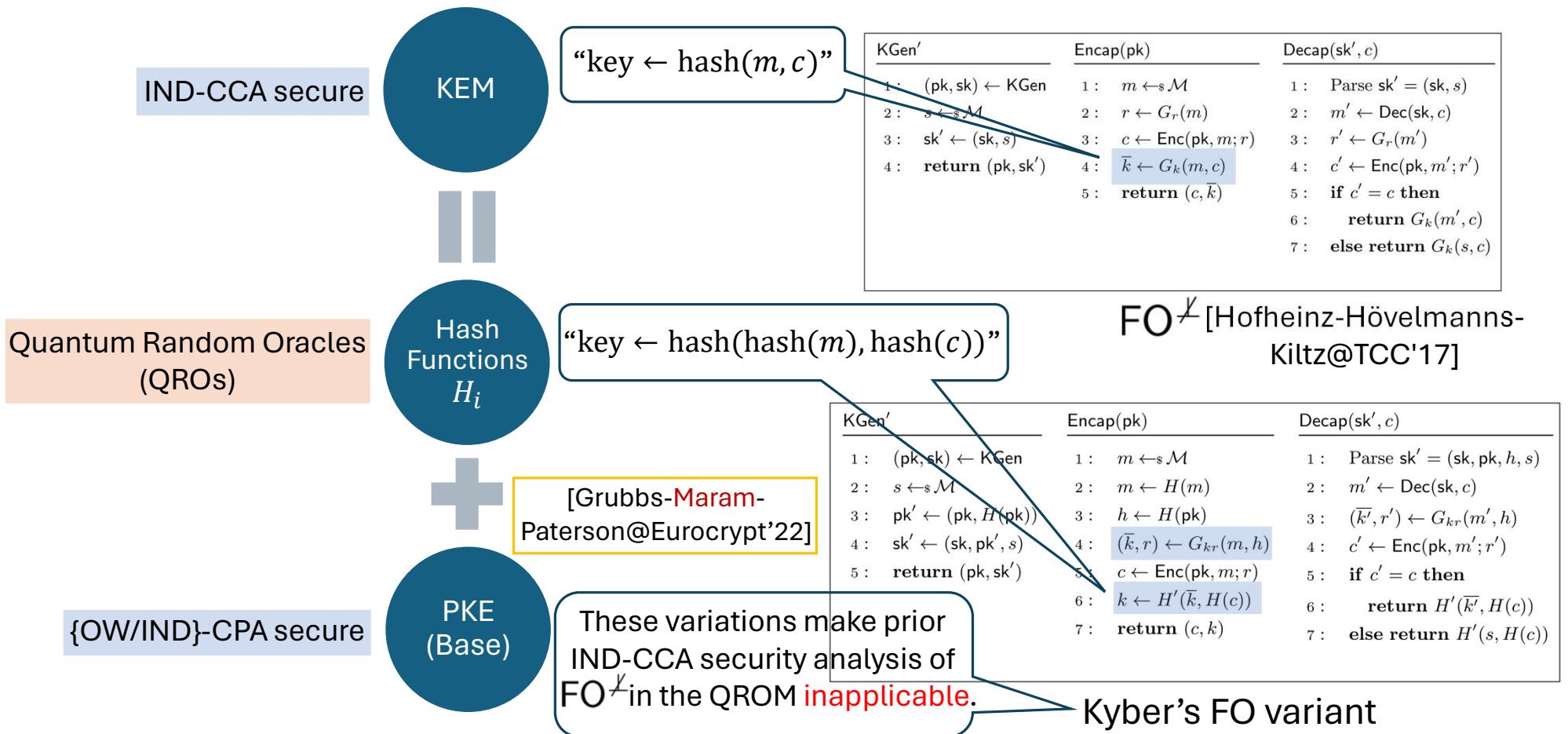
# FO Variants



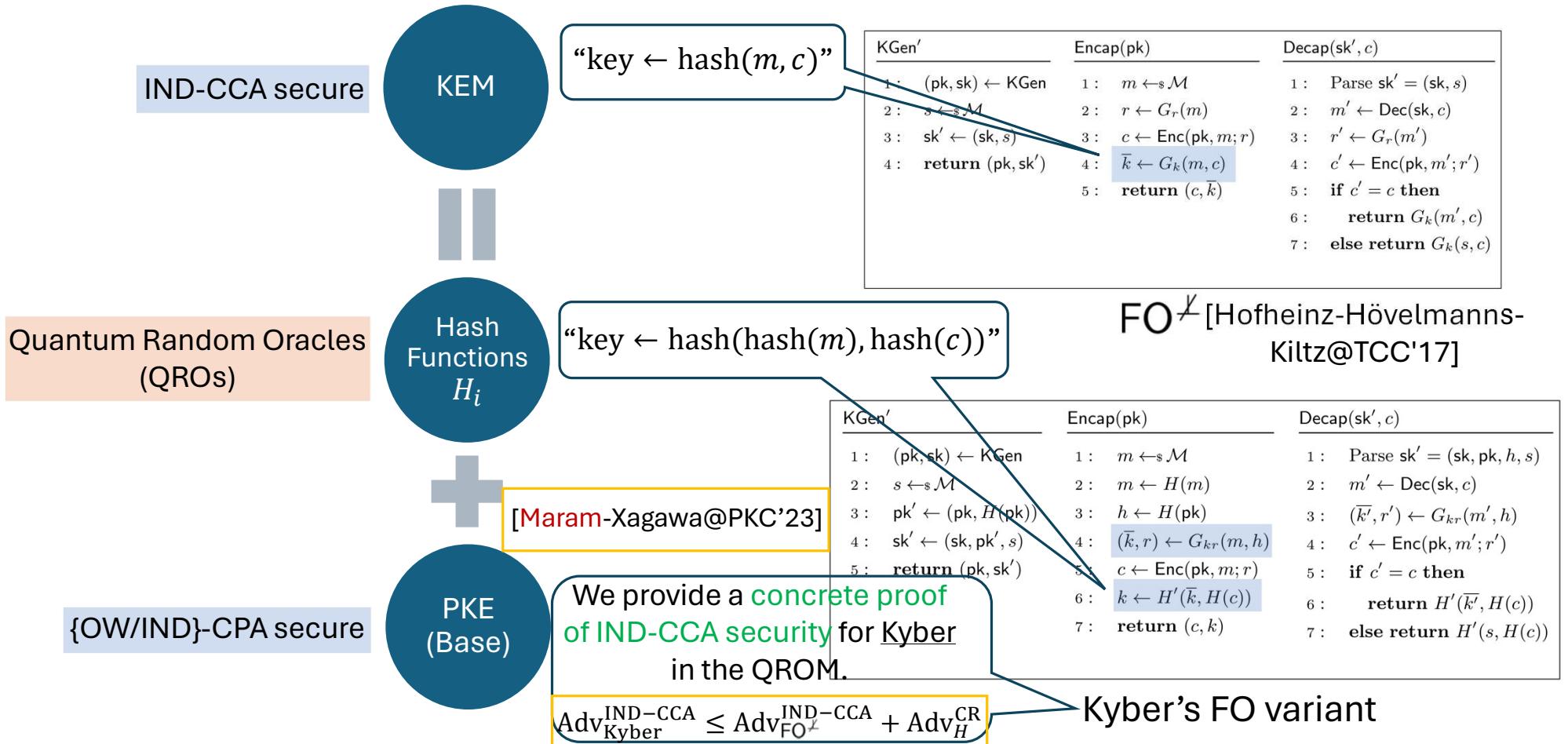
# FO Variants



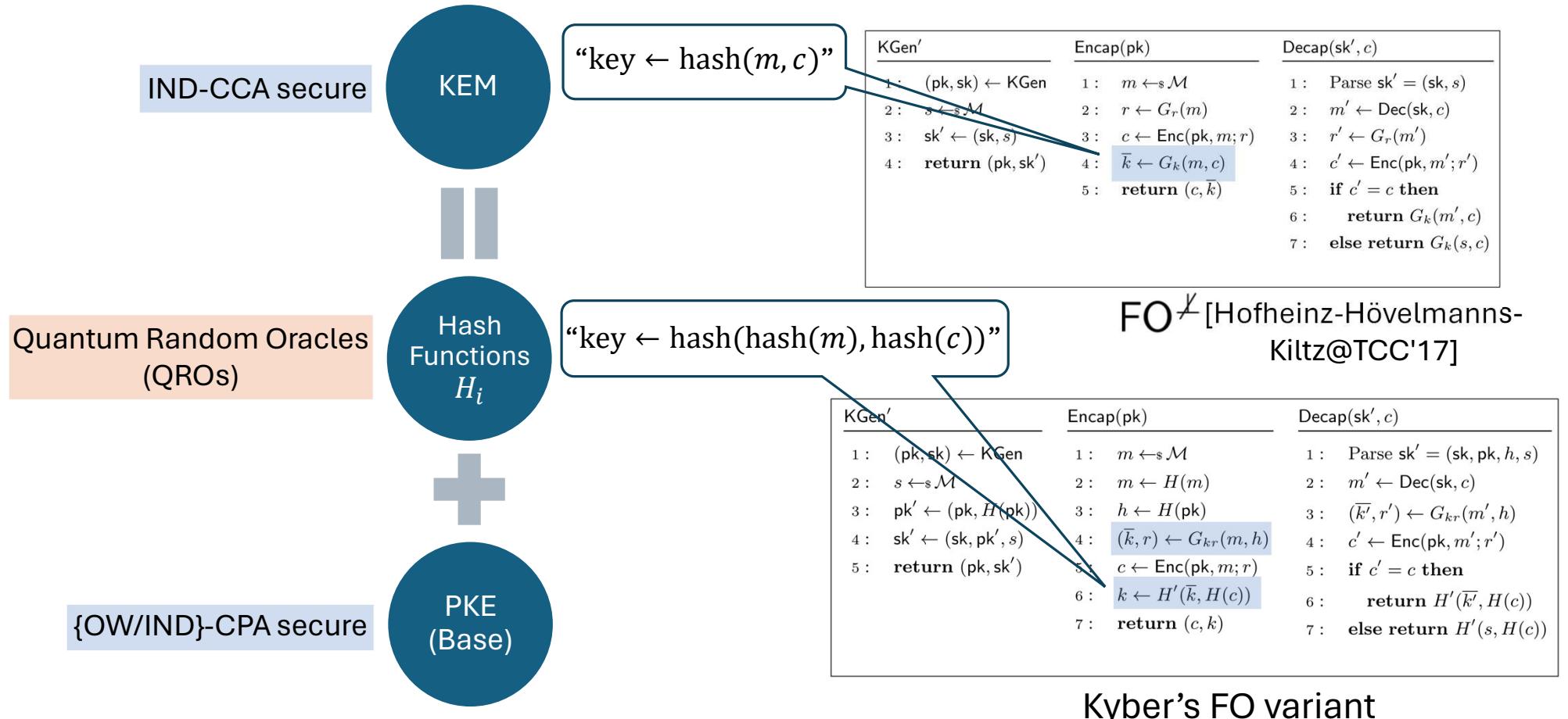
# FO Variants



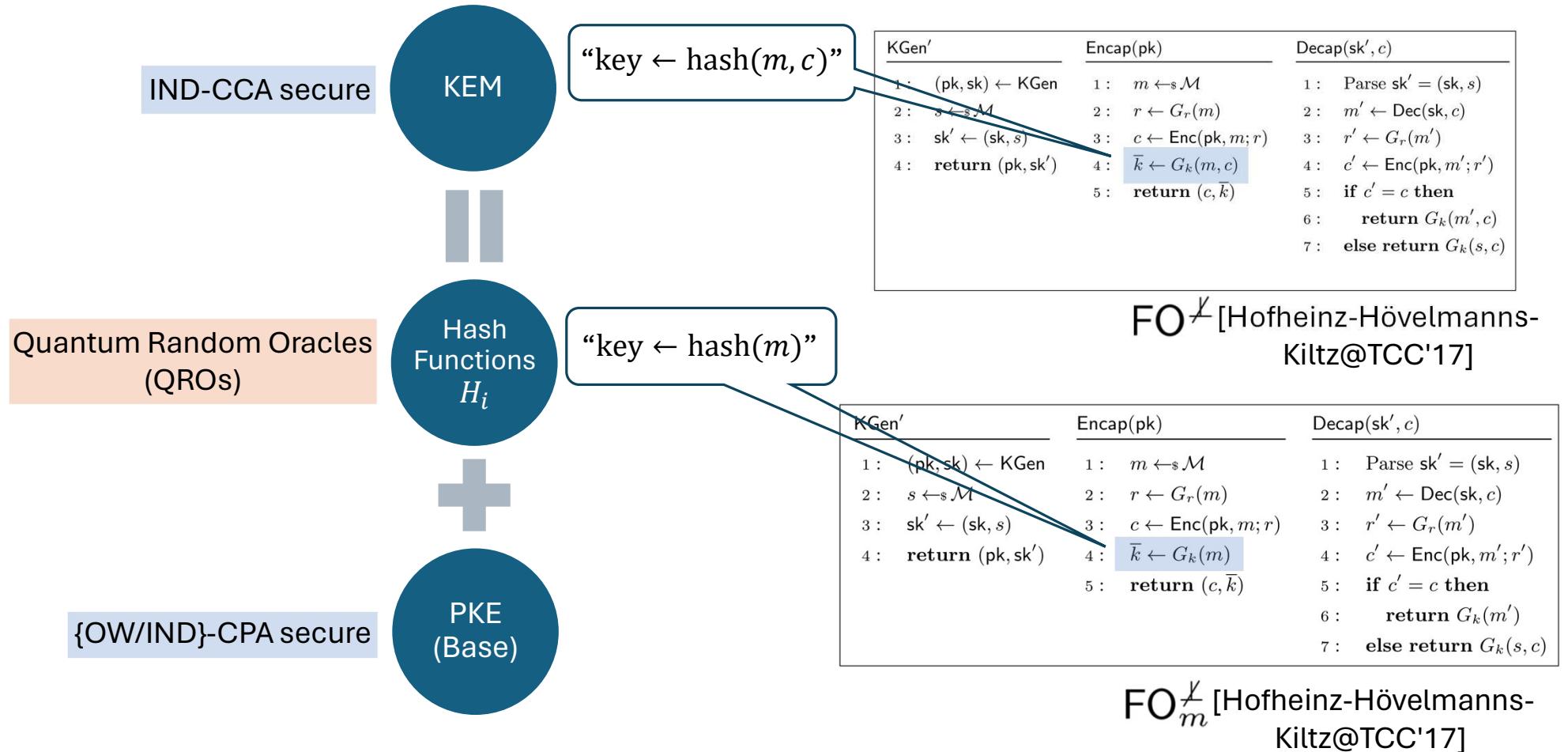
# FO Variants



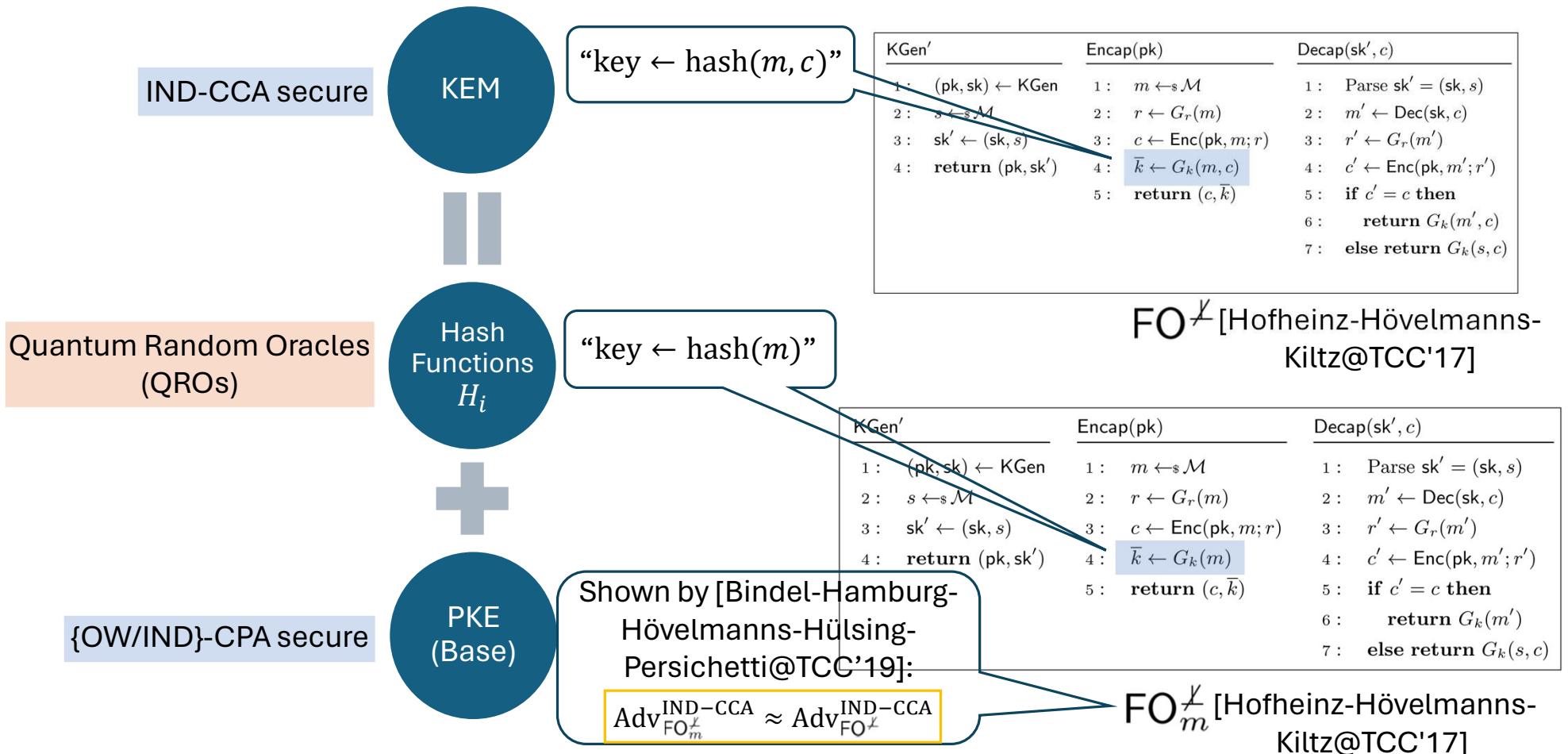
# Analysis of Kyber



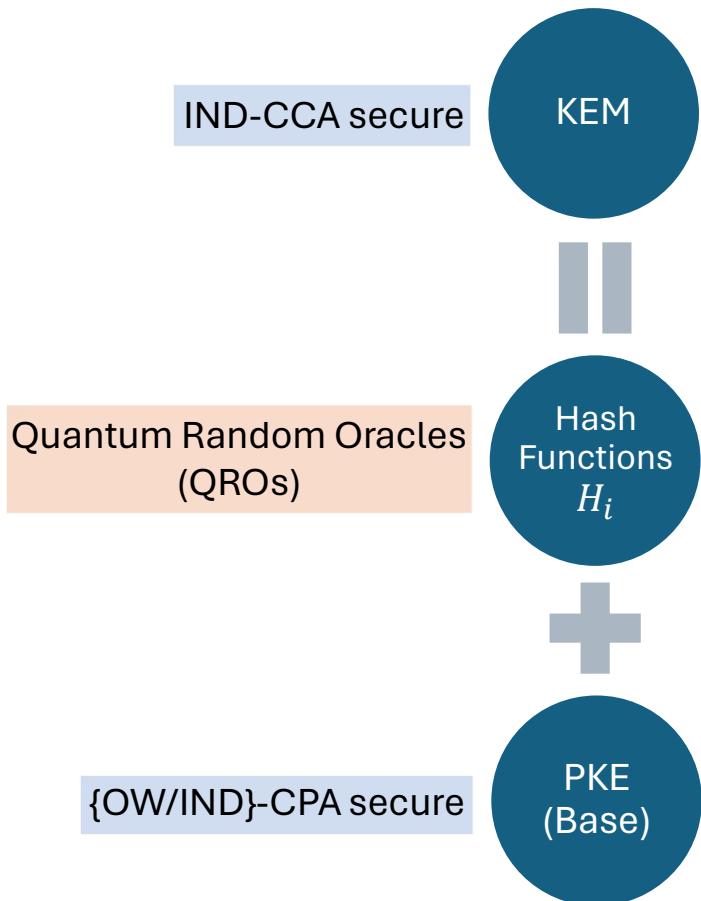
# Analysis of Kyber



# Analysis of Kyber



# Analysis of Kyber



KGen'	Encap(pk)	Decap( $\text{sk}', c$ )
1 : $(\text{pk}, \text{sk}) \leftarrow \text{KGen}$	1 : $m \leftarrow_{\$} \mathcal{M}$	1 : Parse $\text{sk}' = (\text{sk}, s)$
2 : $s \leftarrow_{\$} \mathcal{M}$	2 : $r \leftarrow G_r(m)$	2 : $m' \leftarrow \text{Dec}(\text{sk}, c)$
3 : $\text{sk}' \leftarrow (\text{sk}, s)$	3 : $c \leftarrow \text{Enc}(\text{pk}, m; r)$	3 : $r' \leftarrow G_r(m')$
4 : <b>return</b> $(\text{pk}, \text{sk}')$	4 : $\bar{k} \leftarrow G_k(m)$	4 : $c' \leftarrow \text{Enc}(\text{pk}, m'; r')$
	5 : <b>return</b> $(c, \bar{k})$	5 : <b>if</b> $c' = c$ <b>then</b>
		6 : <b>return</b> $G_k(m')$
		7 : <b>else return</b> $G_k(s, c)$

$\text{FO}_m^{\not\perp}$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

# Analysis of Kyber

KGen'	Encap(pk)	Decap(sk, c)
1 : $(\text{pk}, \text{sk}) \leftarrow \text{KGen}$	1 : $m \leftarrow_{\$} \mathcal{M}$	1 : $m' \leftarrow \text{Dec}(\text{sk}, c)$
2 : <b>return</b> ( $\text{pk}, \text{sk}$ )	2 : $r \leftarrow G_r(m)$	2 : $r' \leftarrow G_r(m')$
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	6 : <b>return</b> $\bar{k}'$	
	7 : <b>else return</b> $\perp$	

$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

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$\text{FO}_m^{\not\perp}$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

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$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

“Explicit rejection”

“Implicit rejection”

KGen'	Encap(pk)	Decap(sk', c)
1 : $(\text{pk}, \text{sk}) \leftarrow \text{KGen}$	1 : $m \leftarrow_{\$} \mathcal{M}$	1 : Parse $\text{sk}' = (\text{sk}, s)$
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$\text{FO}_m^{\perp\!\!\perp}$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

# Analysis of Kyber

KGen'	Encap(pk)	Decap(sk, c)
1 : $(\text{pk}, \text{sk}) \leftarrow \text{KGen}$	1 : $m \leftarrow_{\$} \mathcal{M}$	1 : $m' \leftarrow \text{Dec}(\text{sk}, c)$
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$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

# Analysis of Kyber

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1 : $(\text{pk}, \text{sk}) \leftarrow \text{KGen}$	1 : $m \leftarrow_{\$} \mathcal{M}$	1 : $m' \leftarrow \text{Dec}(\text{sk}, c)$
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$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]



# Analysis of Kyber

KGen'	Encap(pk)	Decap(sk, c)
1 : $(\text{pk}, \text{sk}) \leftarrow \text{KGen}$	1 : $m \leftarrow_s \mathcal{M}$	1 : $m' \leftarrow \text{Dec}(\text{sk}, c)$
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$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]



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		7 : <b>return</b> $H'(\bar{k}', H(c))$
		8 : <b>else return</b> $H'(s, H(c))$

Kyber (simplified)

Ignoring initial hashes “ $H(m)$ ” and “ $H(pk)$ ” in *Encap*.

# Analysis of Kyber

KGen'	Encap(pk)	Decap(sk, c)
1 : $(\text{pk}, \text{sk}) \leftarrow \text{KGen}$	1 : $m \leftarrow_{\$} \mathcal{M}$	1 : $m' \leftarrow \text{Dec}(\text{sk}, c)$
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$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

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Kyber (simplified)



# Analysis of Kyber

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$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

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Kyber (simplified)



# Analysis of Kyber

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	6 : <b>return</b> $\bar{k}'$	6 : <b>else return</b> $\perp$
	7 :	

$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

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Kyber (simplified)



$(c, \bar{k})$



$(c, k)$



$H, H'$   
 $k \leftarrow H'(\bar{k}, H(c))$

# Analysis of Kyber

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5 : <b>return</b> $(c, \bar{k})$	5 : <b>if</b> $c' = c$ <b>then</b> 6 : <b>return</b> $\bar{k}'$ 7 : <b>else return</b> $\perp$	5 : <b>else return</b> $\perp$

$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

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Kyber (simplified)



# Analysis of Kyber

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5 : <b>return</b> $(c, \bar{k})$	5 : <b>if</b> $c' = c$ <b>then</b>	5 : $\bar{k}'$
	6 : <b>return</b> $\bar{k}'$	6 : <b>else return</b> $\perp$
	7 : <b>else return</b> $\perp$	

$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

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Kyber (simplified)

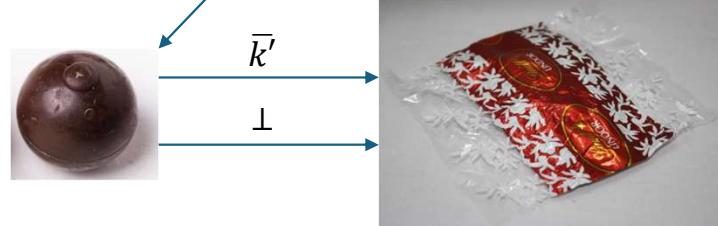


IND-CCA security of  $\text{FO}_m^\perp$  KEMs in the QROM  $\Rightarrow$  IND-CCA security of Kyber in the QROM

# Analysis of Kyber

KGen'	Encap(pk)	Decap(sk, c)
1 : $(\text{pk}, \text{sk}) \leftarrow \text{KGen}$	1 : $m \leftarrow_{\$} \mathcal{M}$	1 : $m' \leftarrow \text{Dec}(\text{sk}, c)$
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$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]



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4 : <b>return</b> $(\text{pk}, \text{sk}')$	4 : $\bar{k} \leftarrow G_k(m)$	4 : $c' \leftarrow \text{Enc}(\text{pk}, m'; r')$
	5 : $k \leftarrow H'(\bar{k}, H(c))$	5 : $\bar{k}' \leftarrow G_k(m')$
	6 : <b>return</b> $(c, k)$	6 : <b>if</b> $c' = c$ <b>then</b> 7 : <b>return</b> $H'(\bar{k}', H(c))$ 8 : <b>else return</b> $H'(s, H(c))$

Kyber (simplified)



IND-CCA security of  $\text{FO}_m^\perp$  KEMs in the QROM  $\Rightarrow$  IND-CCA security of Kyber in the QROM

# Analysis of Kyber

Tighter Proofs of CCA Security  
in the Quantum Random Oracle Model  
Measure-Rewind-Measure: Tighter  
Quantum Random Oracle Model Proofs  
for One-Way to Hiding and CCA Security

Nina Veronika Kuchta<sup>1</sup>, Amin Sakzad<sup>1(✉)</sup>, Damien Stehlé<sup>2,3</sup>, Ron Steinfeld<sup>1(✉)</sup>  
and Shi-Feng Sun<sup>1,4</sup>

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<sup>2</sup> {amin.sakzad, ron.steinfield}@monash.edu

<sup>2</sup> Univ. Lyon, EnsL, UCBL, CNRS, Inria, LIP, 69342 Lyon Cedex 07, France

<sup>3</sup> Institut Universitaire de France, Paris, France

<sup>4</sup> Data61, CSIRO, Canberra, Australia [Eurocrypt'20]

Non-tight proofs,  
compared to  $\text{FO}_m^\perp$ .

IND-CCA security of  $\text{FO}_m^\perp$  KEMs  
in the QROM



IND-CCA security of Kyber  
in the QROM

# Analysis of Kyber

Tighter Proofs of CCA Security  
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for One-Way to Hiding and CCA Security  
Nina

Veronika Kuchta<sup>1</sup>, Amin Sakzad<sup>1(✉)</sup>, Damien Stehlé<sup>2,3</sup>, Ron Steinfeld<sup>1(✉)</sup>  
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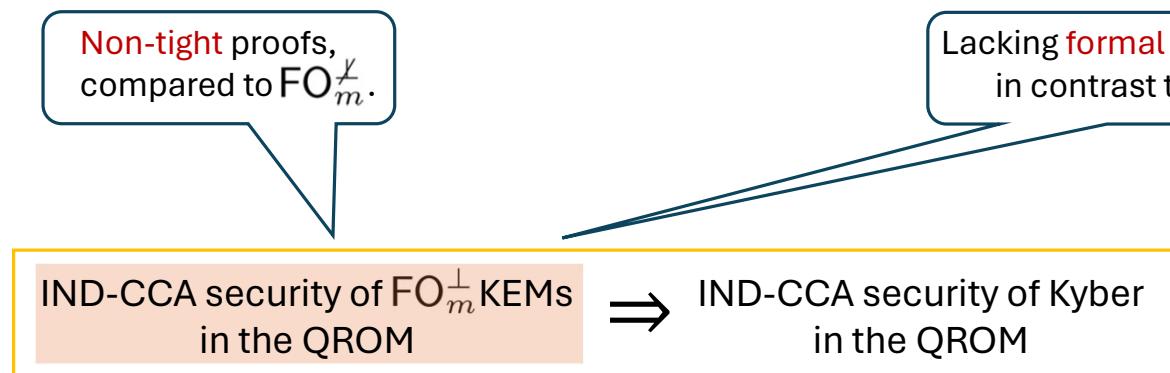
<sup>4</sup> Data61, CSIRO, Canberra, Australia [Eurocrypt'20]

Post-Quantum Verification  
of Fujisaki-Okamoto

Dominique Unruh<sup>(✉)</sup>

University of Tartu, Tartu, Estonia  
unruh@ut.ee

**Abstract.** We present a computer-verified formalization of the post-quantum security proof of the Fujisaki-Okamoto transform (as analyzed by Hövelmanns, Kiltz, Schäge, and Unruh, PKC 2020). The formalization is done in quantum relational Hoare logic and checked in the `qrhl-tool` (Unruh, POPL 2019). [Asiacrypt'20]



# Analysis of Kyber

KGen'	Encap(pk)	Decap(sk, c)
1 : $(\text{pk}, \text{sk}) \leftarrow \text{KGen}$	1 : $m \leftarrow_{\$} \mathcal{M}$	1 : $m' \leftarrow \text{Dec}(\text{sk}, c)$
2 : <b>return</b> $(\text{pk}, \text{sk})$	2 : $r \leftarrow G_r(m)$	2 : $r' \leftarrow G_r(m')$
	3 : $c \leftarrow \text{Enc}(\text{pk}, m; r)$	3 : $c' \leftarrow \text{Enc}(\text{pk}, m'; r')$
	4 : $\bar{k} \leftarrow G_k(m)$	4 : <b>if</b> $c' = c$ <b>then</b>
	5 : <b>return</b> $(c, \bar{k})$	5 : <b>return</b> $G_k(m')$
		6 : <b>else return</b> $\perp$

$\text{FO}_m^\perp$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

“Explicit rejection”



KGen'	Encap(pk)	Decap( $\text{sk}', c$ )
1 : $(\text{pk}, \text{sk}) \leftarrow \text{KGen}$	1 : $m \leftarrow_{\$} \mathcal{M}$	1 : Parse $\text{sk}' = (\text{sk}, s)$
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4 : <b>return</b> $(\text{pk}, \text{sk}')$	4 : $\bar{k} \leftarrow G_k(m)$	4 : $c' \leftarrow \text{Enc}(\text{pk}, m'; r')$
	5 : $k \leftarrow H'(\bar{k}, H(c))$	5 : $\bar{k}' \leftarrow G_k(m')$
	6 : <b>return</b> $(c, k)$	6 : <b>if</b> $c' = c$ <b>then</b>
		7 : <b>return</b> $H'(\bar{k}', H(c))$
		8 : <b>else return</b> $H'(s, H(c))$

Kyber (simplified)



IND-CCA security of  $\text{FO}_m^\perp$  KEMs  
in the QROM



⇒ IND-CCA security of Kyber  
in the QROM

# Analysis of Kyber

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$\text{FO}_m^{\perp}$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

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Kyber (simplified)



IND-CCA security of  $\text{FO}_m^{\perp}$  KEMs  
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?

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$\text{FO}_m^{\not\perp}$  [Hofheinz-Hövelmanns-Kiltz@TCC'17]

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Kyber (simplified)



IND-CCA security of  $\text{FO}_m^{\not\perp}$  KEMs  
in the QROM  $\Rightarrow$

?

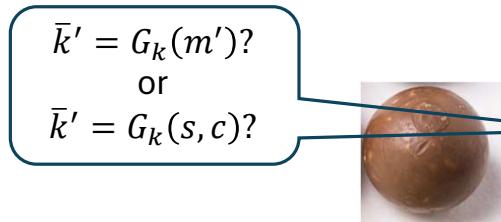
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Kyber (simplified)



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Kyber (simplified)



IND-CCA security of  $\text{FO}_m^{\not\perp}$  KEMs  
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$\text{FO}_m^{\not\perp}$   
modified



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Kyber (simplified)  
modified



IND-CCA security of  $\text{FO}_m^{\not\perp}$  KEMs  
in the QROM



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# Analysis of Kyber

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$\text{FO}_m^{\not\perp}$   
modified



$\bar{k}'$



$H'(\bar{k}', H(c))$



IND-CCA security of  $\text{FO}_m^{\not\perp}$  KEMs  
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Kyber (simplified)  
modified

# Analysis of Kyber

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$\text{FO}_m^{\cancel{Y}}$   
modified

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Kyber (simplified)  
modified

$$\bar{k}' = G_k(m')$$



$$\bar{k}'$$



$$H'(\bar{k}', H(c))$$



IND-CCA security of  $\text{FO}_m^{\cancel{Y}}$  KEMs  
in the QROM



?

# Analysis of Kyber

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$\text{FO}_m^{\cancel{Y}}$   
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Kyber (simplified)  
modified

$$\bar{k}' = \bar{H}(H(c))$$



$$\bar{k}'$$



$$H'(\bar{k}', H(c))$$



IND-CCA security of  $\text{FO}_m^{\cancel{Y}}$  KEMs  
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$\text{FO}_m^{\not\perp}$   
modified



$\bar{k}'$



$H'(\bar{k}', H(c))$



IND-CCA security of  $\text{FO}_m^{\not\perp}$  KEMs  
in the QROM



?

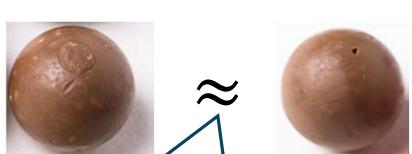
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Kyber (simplified)  
modified

# Analysis of Kyber

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Kyber (simplified)  
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IND-CCA security of  $\text{FO}_m^{\cancel{Y}}$  KEMs  
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$\text{FO}_m^{\neq}$



IND-CCA  
adversary

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$\text{FO}_m^{\neq}$



IND-CCA  
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$\text{FO}_m^{\neq}$ modified		6 : <b>return</b> $G_k(m')$
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# Analysis of Kyber

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$\text{FO}_m^{\neq}$



IND-CCA  
adversary



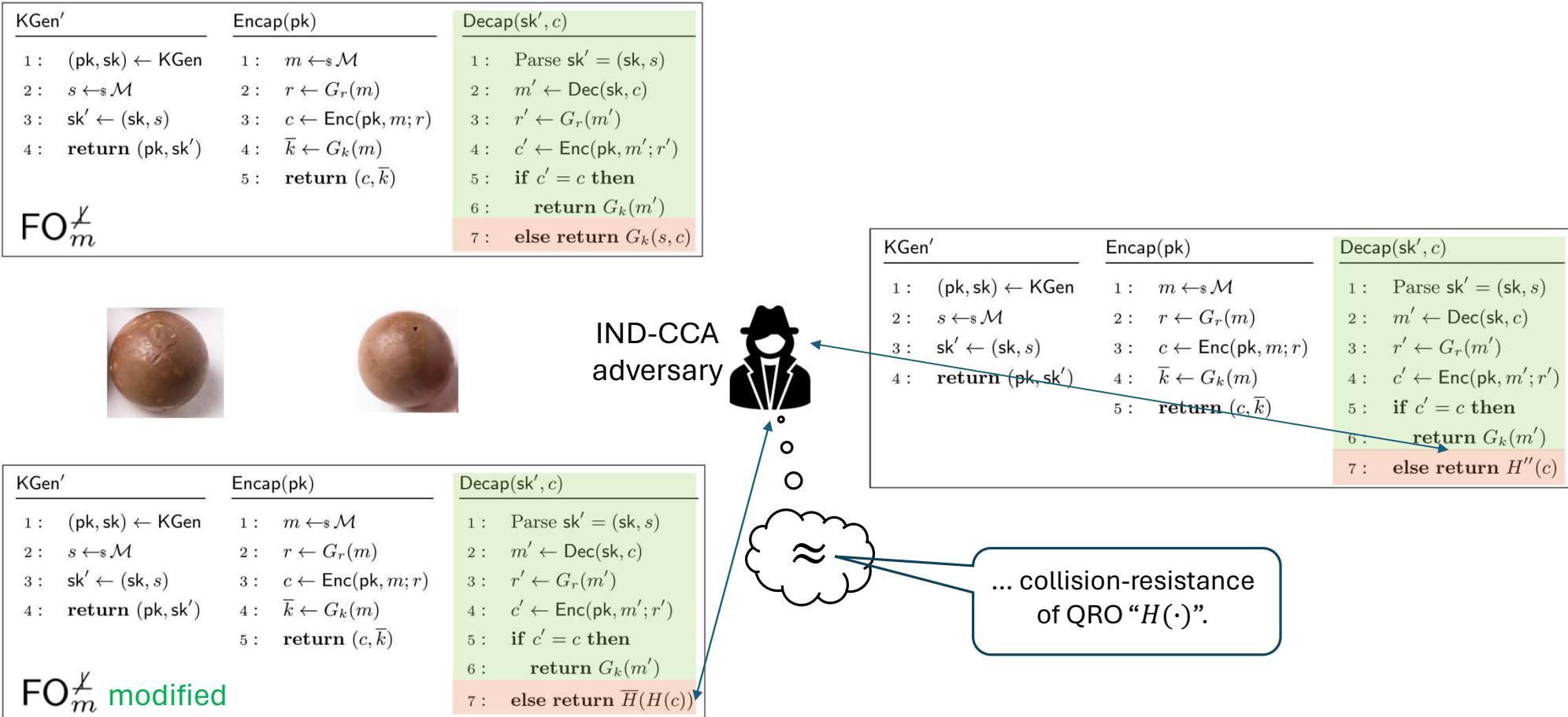
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$\text{FO}_m^{\neq}$  modified

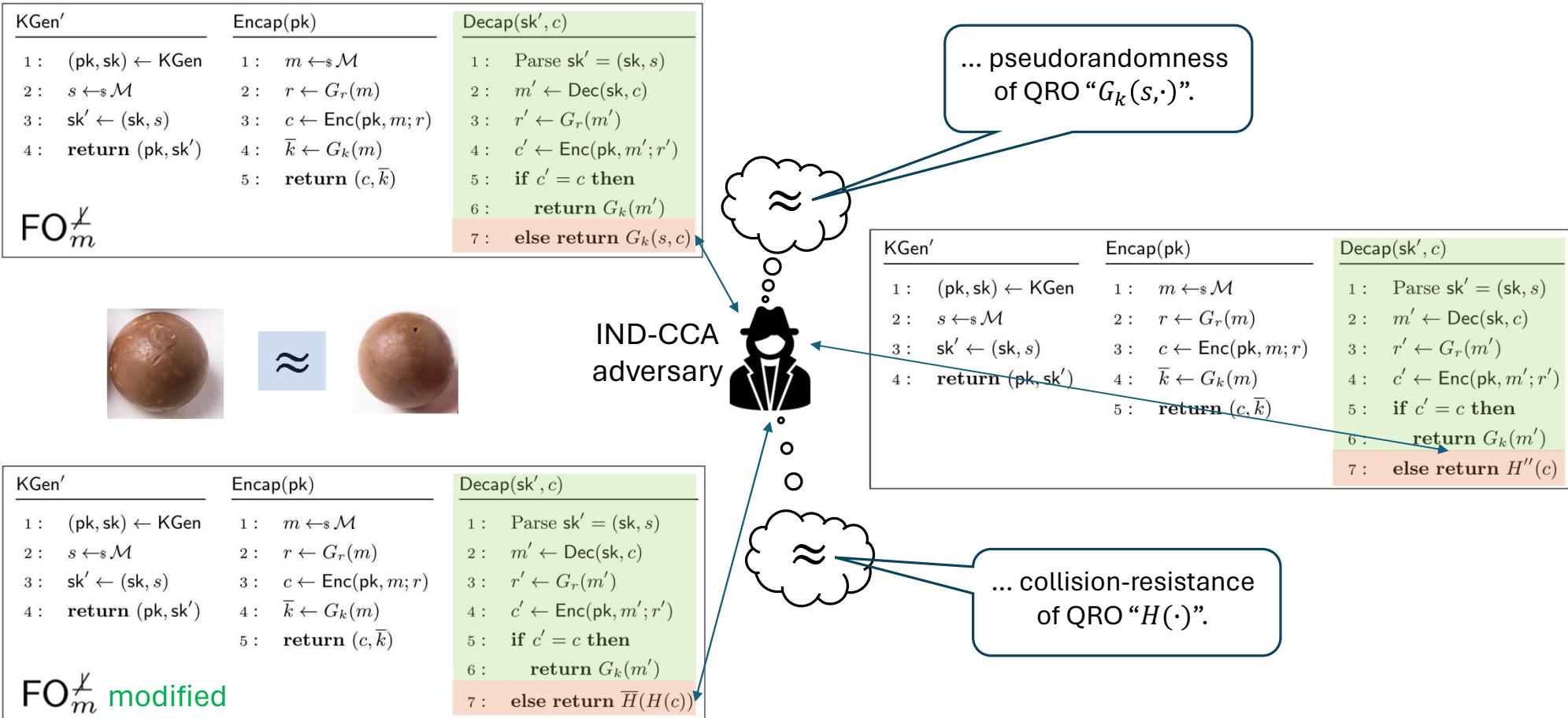
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# Analysis of Kyber



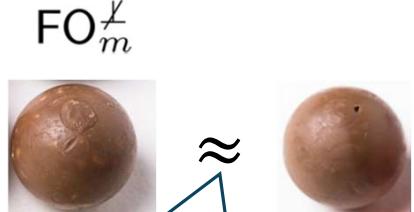
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$\text{FO}_m^{\cancel{Y}}$   
modified

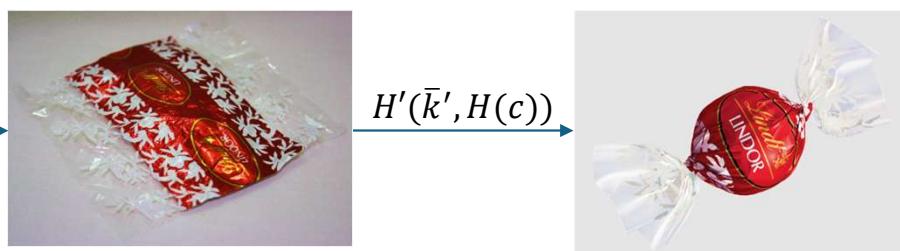


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IND-CCA security of  $\text{FO}_m^{\cancel{Y}}$  KEMs  
in the QROM

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Kyber (simplified)  
modified



$\bar{k}'$

$H'(\bar{k}', H(c))$

?

# Analysis of Kyber

KGen'	Encap(pk)	Decap( $\text{sk}', c$ )
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$\text{FO}_m^{\cancel{Y}}$   
modified

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		8 : <b>else return</b> $H'(\overline{H}(H(c)), H(c))$

Kyber (simplified)  
modified

Kyber (simplified)



$\bar{k}'$

$H'(\bar{k}', H(c))$



We also show that IND-CCA  
security of both KEMs are  
computationally equivalent.

IND-CCA security of  $\text{FO}_m^{\cancel{Y}}$  KEMs  
in the QROM

$\Rightarrow$

?

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(simplified)		
Kyber		
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# Analysis of Kyber

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IND-CCA  
adversary

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(simplified) Kyber modified		
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# Analysis of Kyber

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(simplified)  
Kyber



IND-CCA  
adversary

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(simplified)  
Kyber modified

# Analysis of Kyber

KGen'

---

```

1 : (pk, sk) ← KGen
2 : s ←$ M
3 : sk' ← (sk, s)
4 : return (pk, sk')

```

(simplified)  
Kyber



Encap(pk)

---

```

1 : m ←$ M
2 : r ← Gr(m)
3 : c ← Enc(pk, m; r)
4 : k ← Gk(m)
5 : k ← H'(k̄, H(c))
6 : return (c, k)

```

Decap(sk', c)

---

```

1 : Parse sk' = (sk, s)
2 : m' ← Dec(sk, c)
3 : r' ← Gr(m')
4 : c' ← Enc(pk, m'; r')
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6 : if c' = c then
7 :   return H'(k̄', H(c))
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(simplified)  
Kyber modified



Encap(pk)

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```

... pseudorandomness  
of QRO "H'(s, ·)".

KGen'

---

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# Analysis of Kyber

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(simplified)  
Kyber



IND-CCA  
adversary



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(simplified)  
Kyber modified

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(simplified)  
Kyber



IND-CCA  
adversary



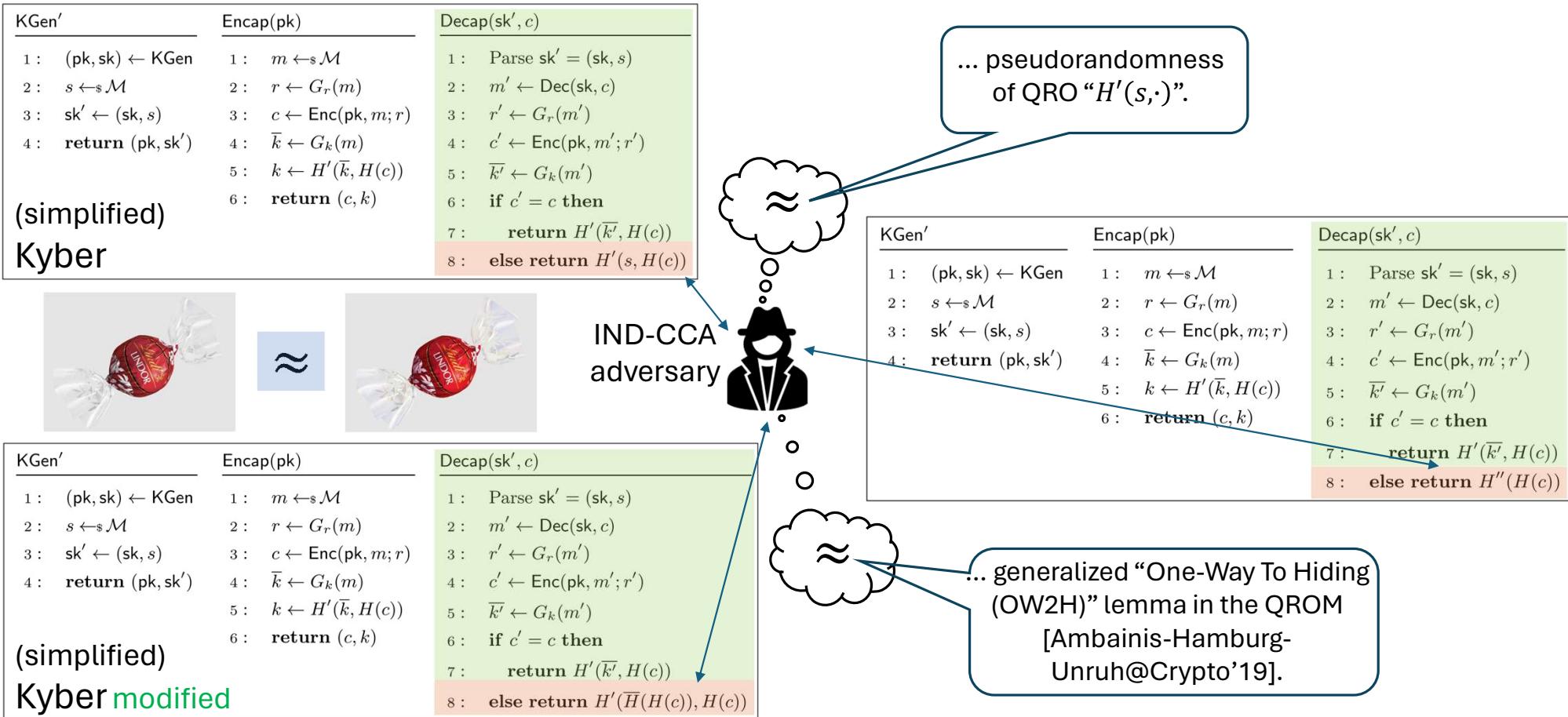
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(simplified)  
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... generalized “One-Way To Hiding  
(OW2H)” lemma in the QROM  
[Ambainis-Hamburg-  
Unruh@Crypto’19].

# Analysis of Kyber



# Analysis of Kyber

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$\text{FO}_m^{\cancel{Y}}$   
modified

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Kyber (simplified)  
modified

Kyber (simplified)



$\bar{k}'$

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IND-CCA security of  $\text{FO}_m^{\cancel{Y}}$  KEMs in the QROM

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$\text{FO}_m^{\cancel{\chi}}$   
modified

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Kyber (simplified)  
modified

Kyber (simplified)



$\bar{k}'$



$H'(\bar{k}', H(c))$



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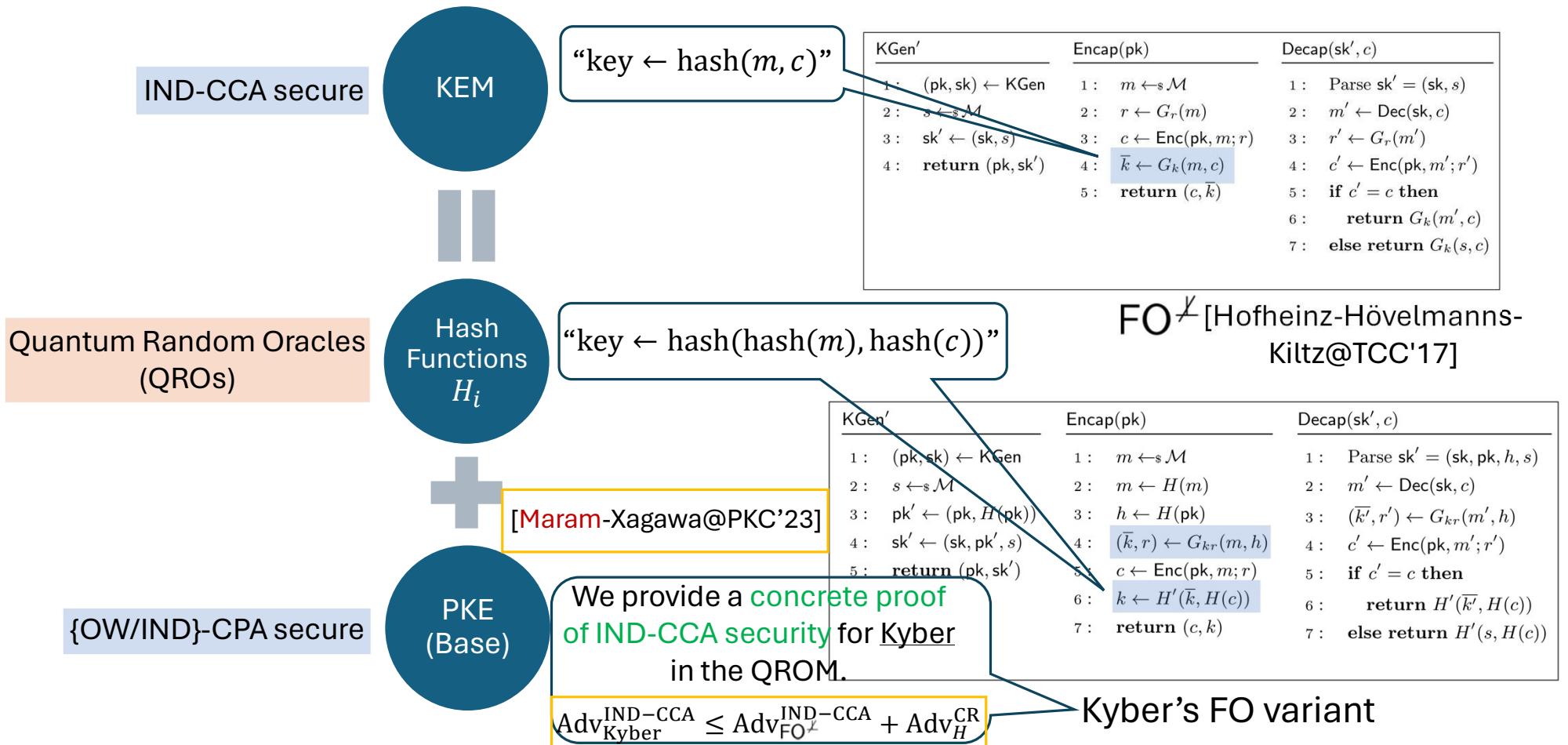
IND-CCA security of  $\text{FO}_m^{\cancel{\chi}}$  KEMs  
in the QROM



IND-CCA security of Kyber  
in the QROM

We also show that IND-CCA  
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# Analysis of Kyber



# Analysis of Kyber

$$\text{Adv}_{\text{Kyber}}^{\text{IND-C}} \leq \text{Adv}_{\text{FO}^{\not\perp}}^{\text{IND-CCA}} + \text{Adv}_H^{\text{CR}}$$

# Analysis of Kyber

$$\text{Adv}_{\text{Kyber}}^{\text{IND-CCA}} \leq \text{Adv}_{\text{FO}_m^{\not\perp}}^{\text{IND-CCA}} + \text{Adv}_H^{\text{CR}}$$

# Analysis of Kyber

$$\text{Adv}_{\text{Kyber}}^{\text{IND-CCA}} \leq \text{Adv}_{\text{FO}_m^{\not\perp}}^{\text{IND-CCA}} + \text{Adv}_H^{\text{CR}}$$

Shown by [Bindel-Hamburg-Hövelmanns-Hülsing-Persichetti@TCC'19]:

$$\text{Adv}_{\text{FO}_m^{\not\perp}}^{\text{IND-C}} \approx \text{Adv}_{\text{FO}_m^{\not\perp}}^{\text{IND-CCA}}$$

# Analysis of Kyber

$$\text{Adv}_{\text{Kyber}}^{\text{IND-CCA}} \leq \text{Adv}_{\text{FO}_m^{\neq}}^{\text{IND-CCA}} + \text{Adv}_H^{\text{CR}}$$



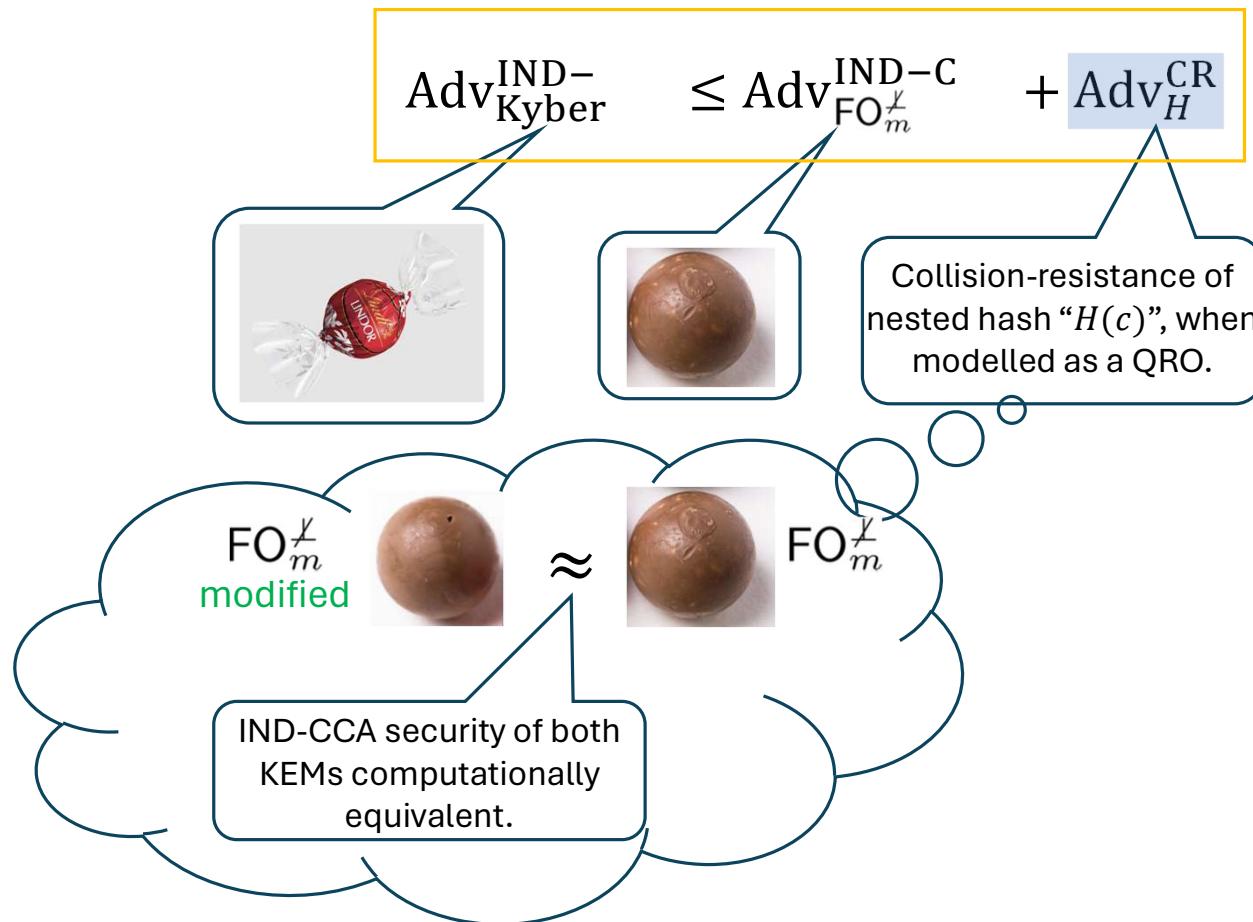
# Analysis of Kyber

$$\text{Adv}_{\text{Kyber}}^{\text{IND-CCA}} \leq \text{Adv}_{\text{FO}_m^{\neq}}^{\text{IND-C}} + \text{Adv}_H^{\text{CR}}$$



Collision-resistance of  
nested hash “ $H(c)$ ”, when  
modelled as a QRO.

# Analysis of Kyber



# Don't Roll Your Own FO: Challenges in Proving Post-Quantum CCA Security of Real-World KEMs

- CRYSTALS-Kyber
- Streamlined NTRU Prime

Varun Maram  
Cybersecurity Group  
SandboxAQ

Fujisaki-Okamoto transform.

In the Quantum Random Oracle Model (QROM).

Based on joint works with Paul Grubbs, Kenny Paterson, and Keita Xagawa.



# Don't Roll Your Own FO: Challenges in Proving Post-Quantum CCA Security of Real-World KEMs

- CRYSTALS-Kyber
- Streamlined NTRU Prime

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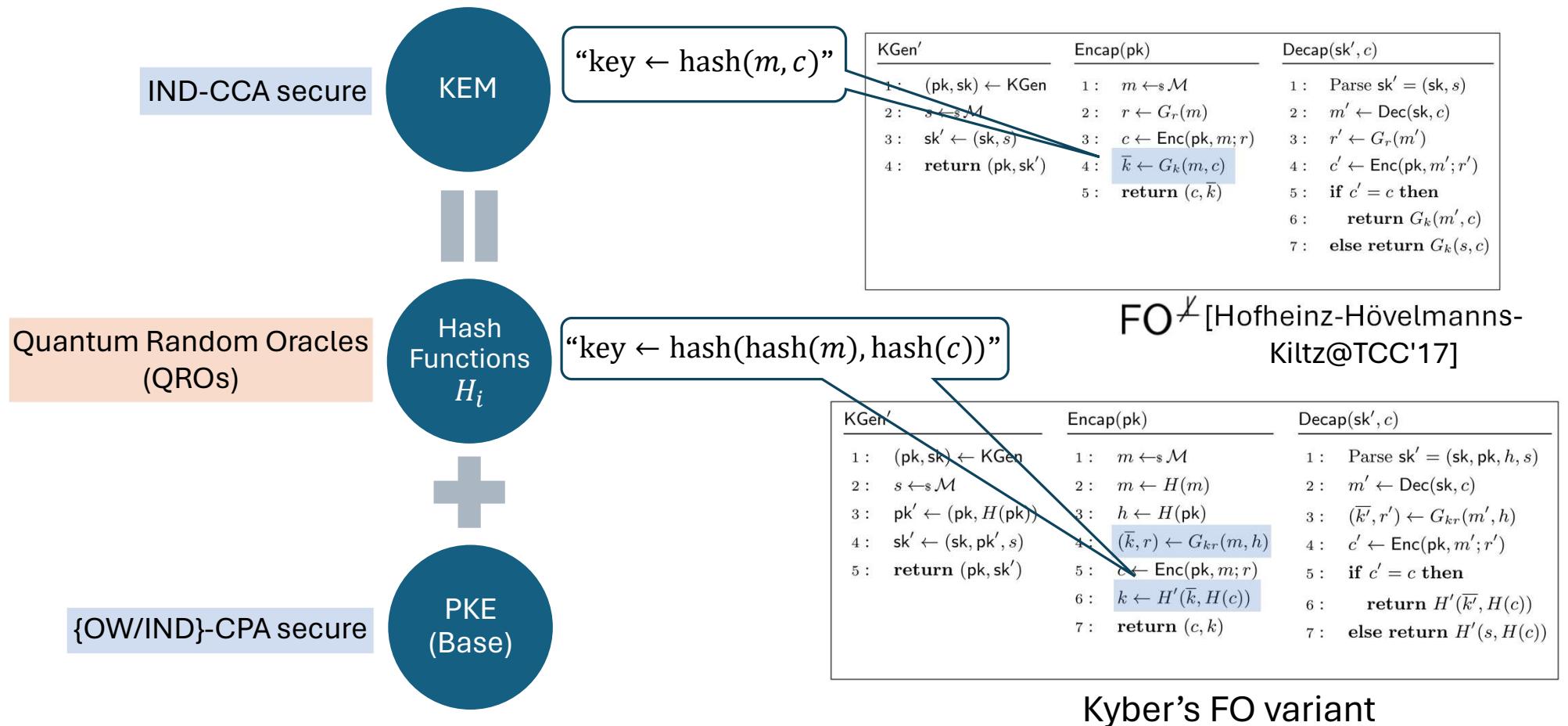
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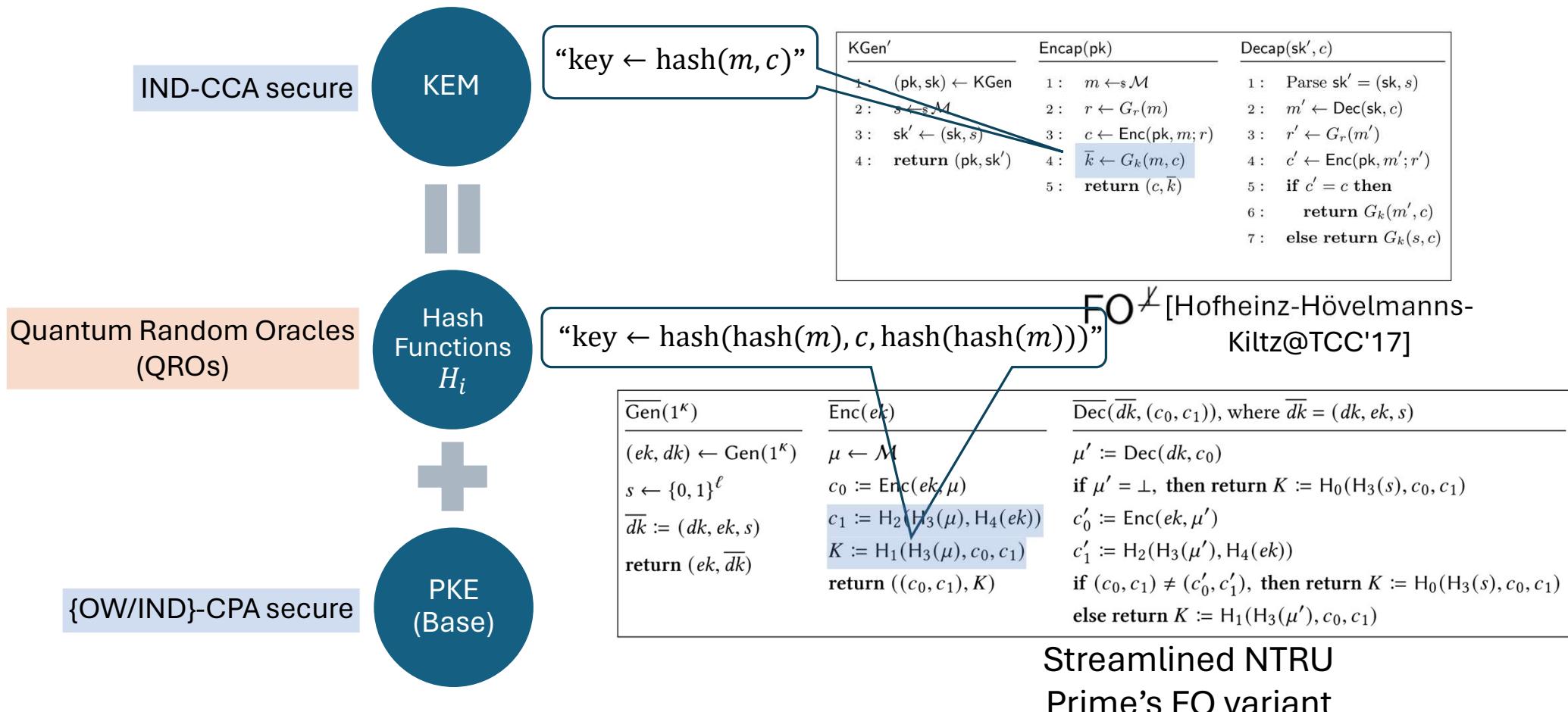
Based on joint works with Paul Grubbs, Kenny Paterson, and Keita Xagawa.



# FO Variants



# FO Variants



# Streamlined NTRU Prime (SNTRUp)

## NTRU Prime

Intro	Warnings	Papers	Security	Software
Speed	NISTPQC	FAQ		

NTRU Prime is a third-round candidate in NIST's [Post-Quantum Cryptography Standardization Project](#). The third-round submission package specifies twelve KEMs:

- Streamlined NTRU Prime: sntrup653, sntrup761, sntrup857, sntrup953, sntrup1013, sntrup1277.
- NTRU LPRime: ntrulpr653, ntrulpr761, ntrulpr857, ntrulpr953, ntrulpr1013, ntrulpr1277.

# Streamlined NTRU Prime (SNTRUp)

## OpenSSH 9.0/9.0p1 (2022-04-08)

### NTRU Prime

Intro	Warnings	Papers	Security	Softw
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OpenSSH 9.0 was released on 2022-04-08. It is available from the mirrors listed at <https://www.openssh.com/>.

OpenSSH is a 100% complete SSH protocol 2.0 implementation and includes sftp client and server support.

This creates one area of potential incompatibility: [scp\(1\)](#) when using Once again, we would like the SFTP protocol no longer requires this finicky and brittle quoting, continued support of the `\`` and attempts to use it may cause transfers to fail. We consider the code or patches, reported removal of the need for double-quoting shell characters in file names project. More information to be a benefit and do not intend to introduce bug-compatibility for legacy scp/rpc in [scp\(1\)](#) when using the SFTP protocol.

Changes since OpenSSH 8.9  
===== Another area of potential incompatibility relates to the use of remote paths relative to other user's home directories, for example - "scp host:~user/file /tmp". The SFTP protocol has no native way to expand a `\~user` path. However, [sftp-server\(8\)](#) in OpenSSH 8.7 and later support a protocol extension "`expand-path@openssh.com`" to support this.

Potentially-incompatible  
-----

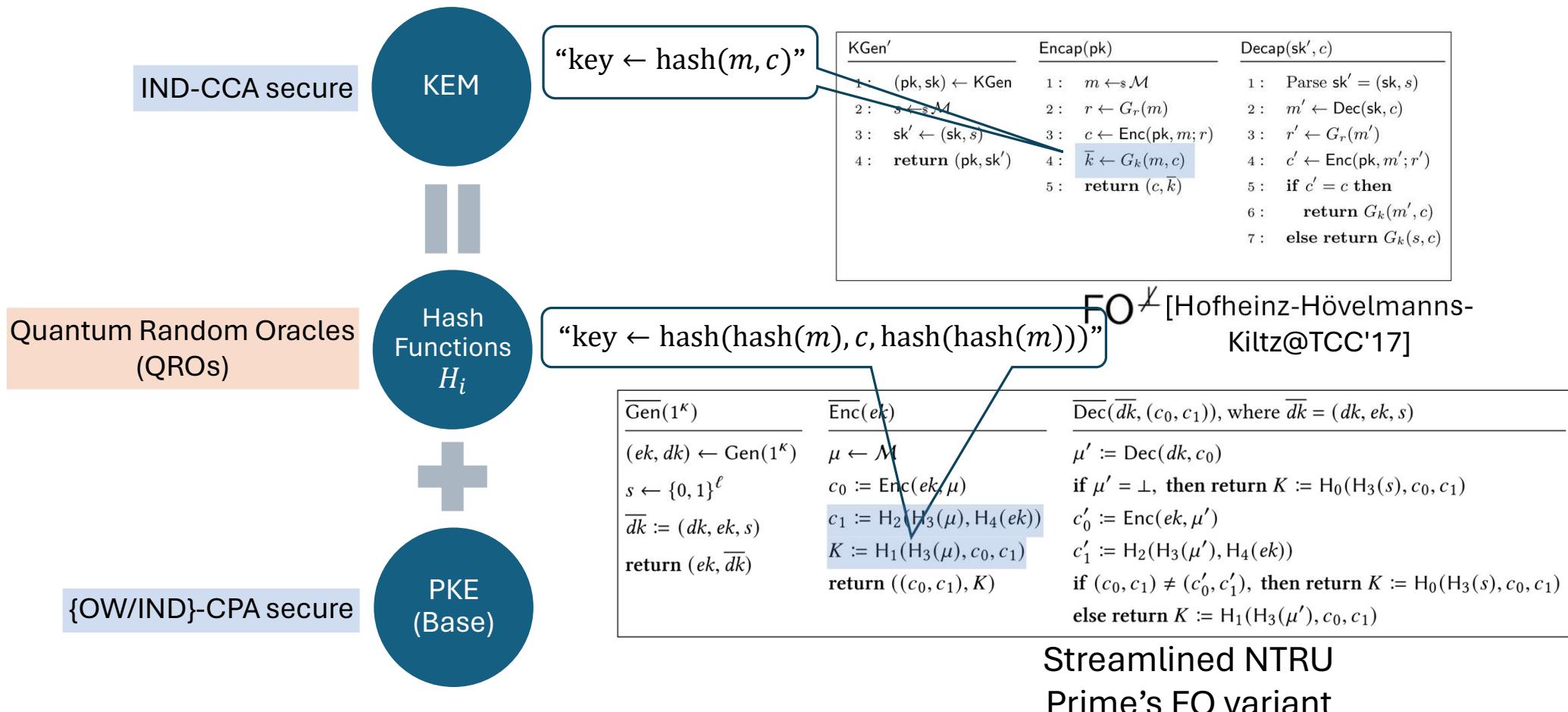
In case of incompatibility, the [scp\(1\)](#) client may be instructed to use the legacy scp/rpc using the `-O` flag.

This release switches [scp\(1\)](#) to using the SFTP protocol.

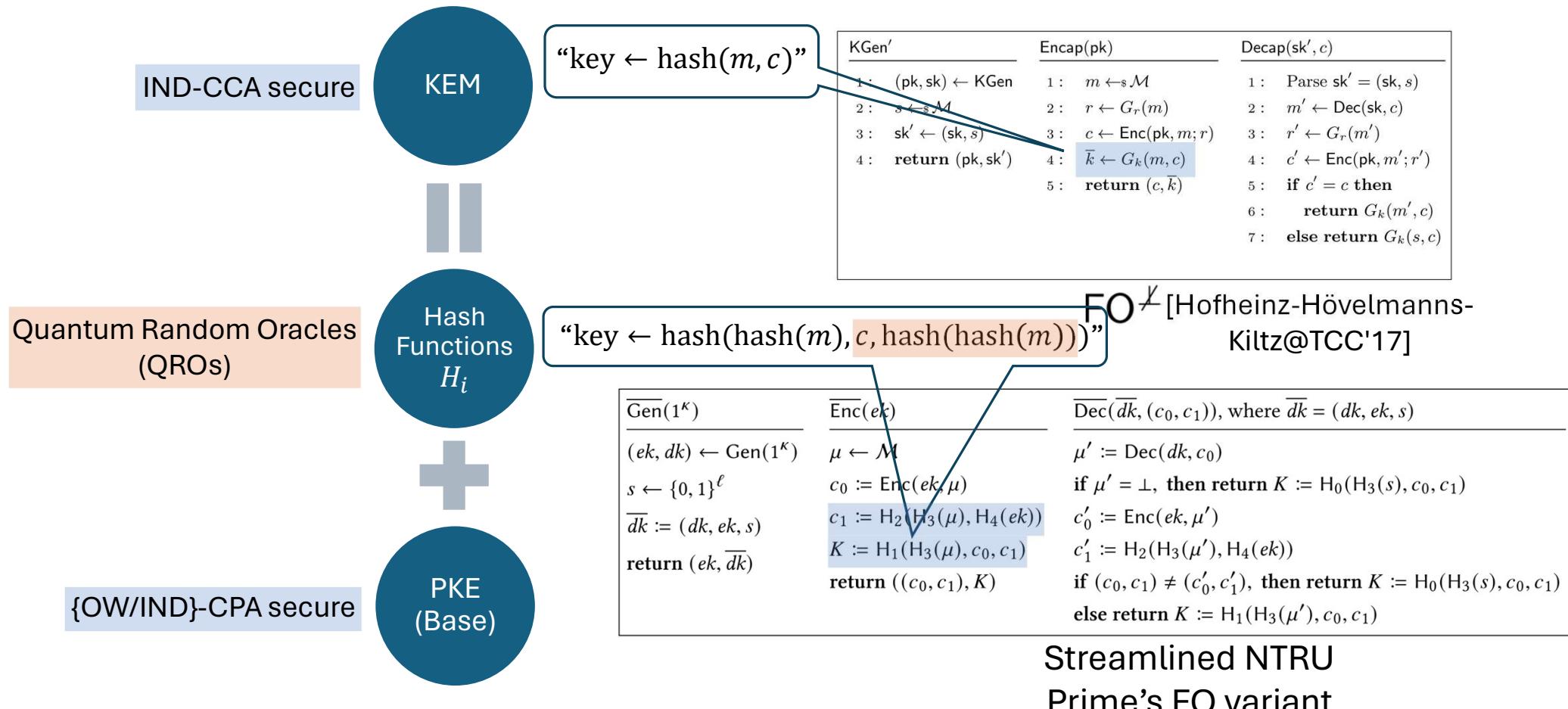
New features  
-----

Legacy scp/rpc performs well with "scp host:\* .") through the requiring double quoting ( included on [scp\(1\)](#) command as shell commands on the command line. \* [ssh\(1\)](#), [sshd\(8\)](#): use the hybrid Streamlined NTRU Prime + x25519 key exchange method by default ("sntrup761x25519-sha512@openssh.com"). The NTRU algorithm is believed to resist attacks enabled by future quantum computers and is paired with the X25519 ECDH key exchange (the previous default) as a backstop against any weaknesses in NTRU Prime that may be discovered in the future. The combination ensures that the hybrid exchange offers at least as good security as the status quo.

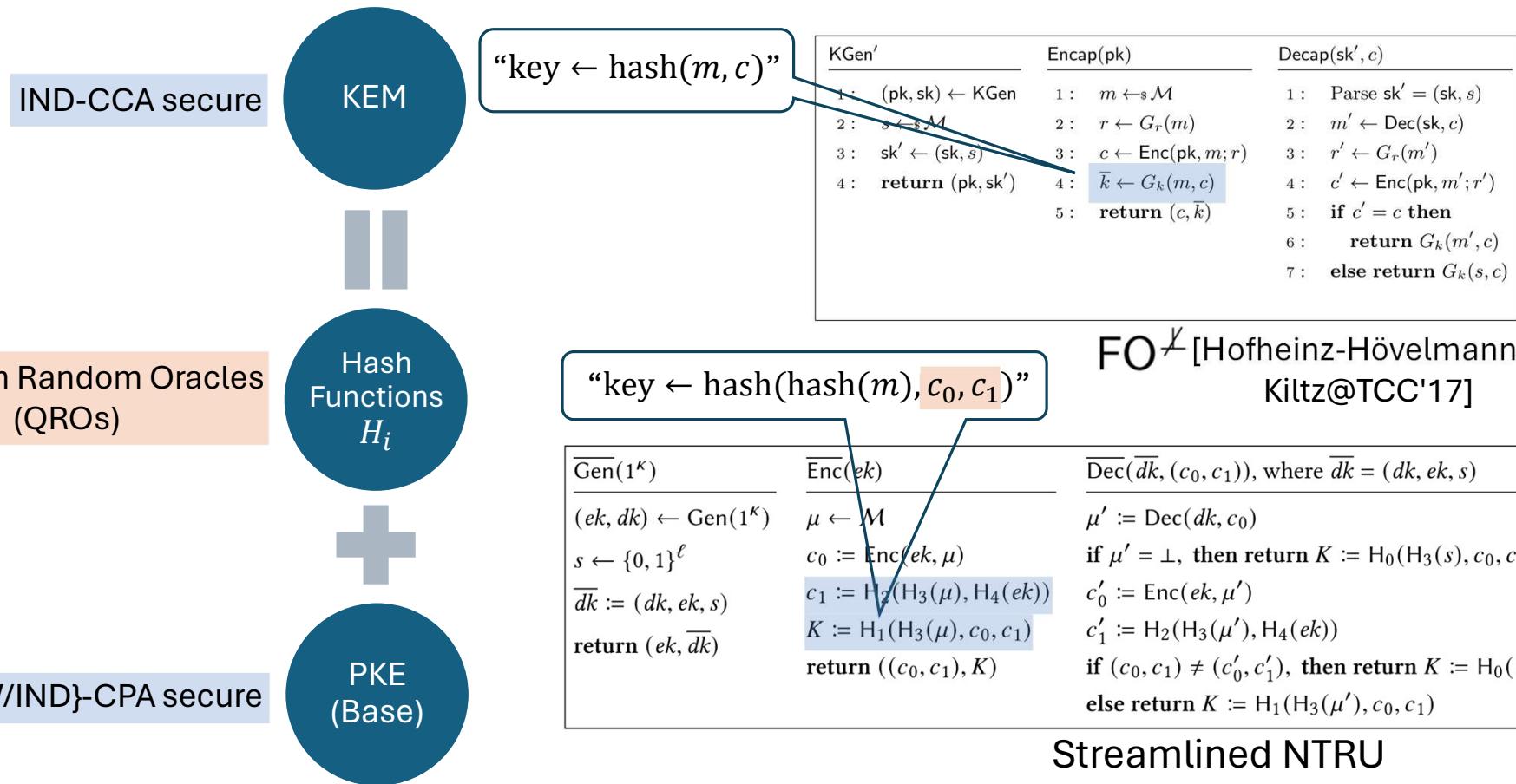
# FO Variants



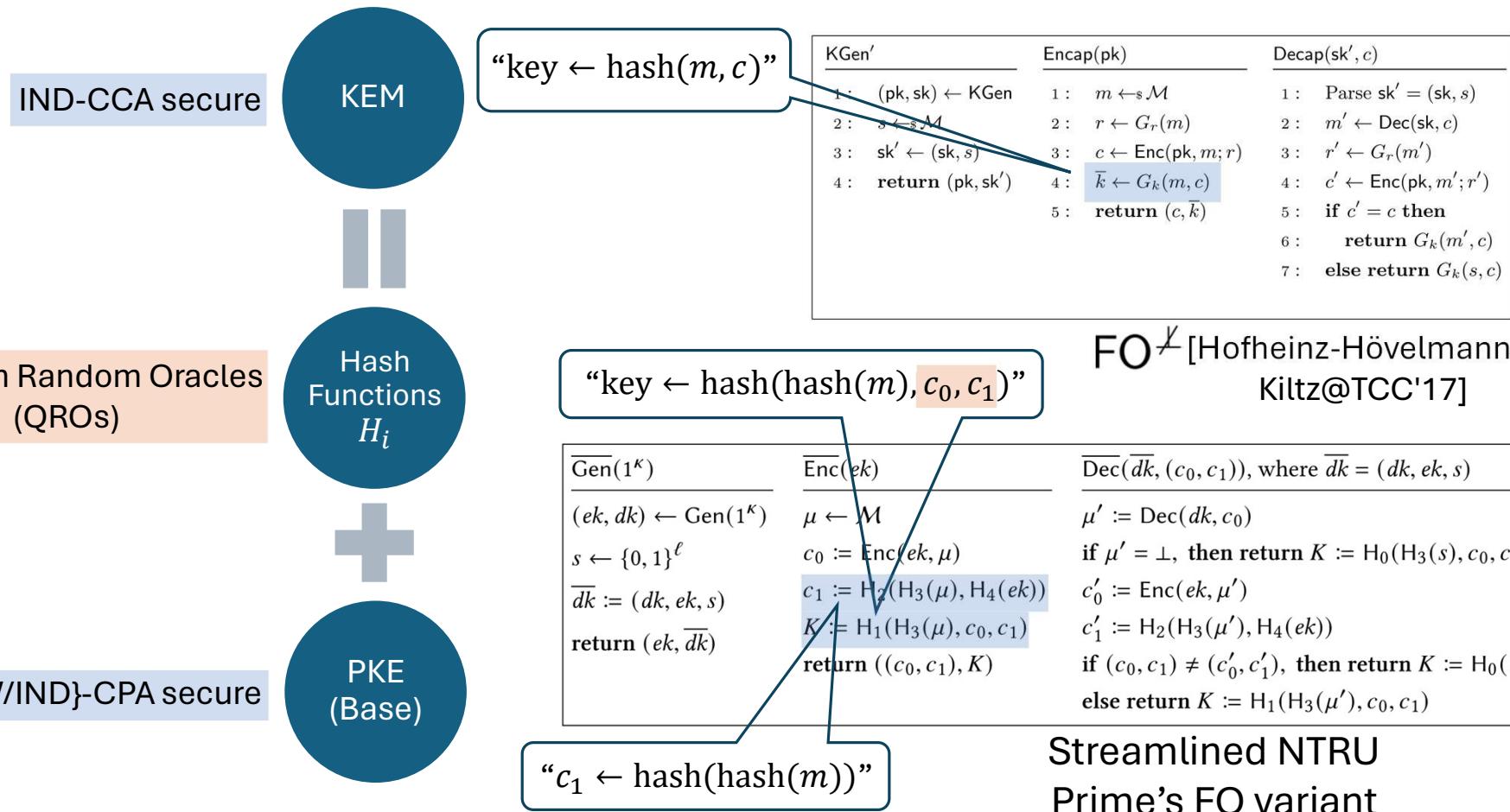
# Analysis of SNTRUp



# Analysis of SNTRUp



# Analysis of SNTRUp



# Analysis of SNTRUp

“key  $\leftarrow \text{hash}(m, c_0, c_1)$ ”

IND-CCA secure

KEM



Hash Functions  
 $H_i$



PKE  
(Base)

Quantum Random Oracles  
(QROs)

{OW/IND}-CPA secure

“ $c_1 \leftarrow \text{hash}(m)$ ”

$\overline{\text{Gen}}(1^\kappa)$

---

1 :  $(ek, dk) \leftarrow \text{Gen}(1^\kappa)$   
 2 :  $s \leftarrow \mathcal{M}$   
 3 :  $\overline{dk} := (dk, ek, s)$   
 4 : return  $(ek, \overline{dk})$

$\overline{\text{Enc}}(ek)$

---

1 :  $\mu \leftarrow \mathcal{D}_M$   
 2 :  $c_0 := \text{Enc}(ek, \mu)$   
 3 :  $c_1 := F(\mu[, ek])$   
 4 :  $K := H(\mu, c_0, c_1)$   
 5 : return  $((c_0, c_1), K)$

$\overline{\text{Dec}}(\overline{dk}, (c_0, c_1))$ , where  $\overline{dk} = (dk, ek, s)$

---

1 :  $\mu' \leftarrow \text{Dec}(dk, c_0)$   
 2 : if  $\mu' = \perp$  or  $c_0 \neq \text{Enc}(ek, \mu')$  or  $c_1 \neq F(\mu'[, ek])$   
 3 : then return  $K := H(s, c_0, c_1)$   
 4 : else return  $K := H(\mu', c_0, c_1)$

“key  $\leftarrow \text{hash}(\text{hash}(m), c_0, c_1)$ ”

$H \cup \perp$  [Xagawa@Eurocrypt’22]

$\overline{\text{Gen}}(1^\kappa)$

---

$(ek, dk) \leftarrow \text{Gen}(1^\kappa)$   
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 $\overline{dk} := (dk, ek, s)$   
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$\overline{\text{Enc}}(ek)$

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$\mu \leftarrow \mathcal{M}$   
 $c_0 := \text{Enc}(ek, \mu)$   
 $c_1 := H_2(H_3(\mu), H_4(ek))$   
 $K := H_1(H_3(\mu), c_0, c_1)$   
 return  $((c_0, c_1), K)$

$\overline{\text{Dec}}(\overline{dk}, (c_0, c_1))$ , where  $\overline{dk} = (dk, ek, s)$

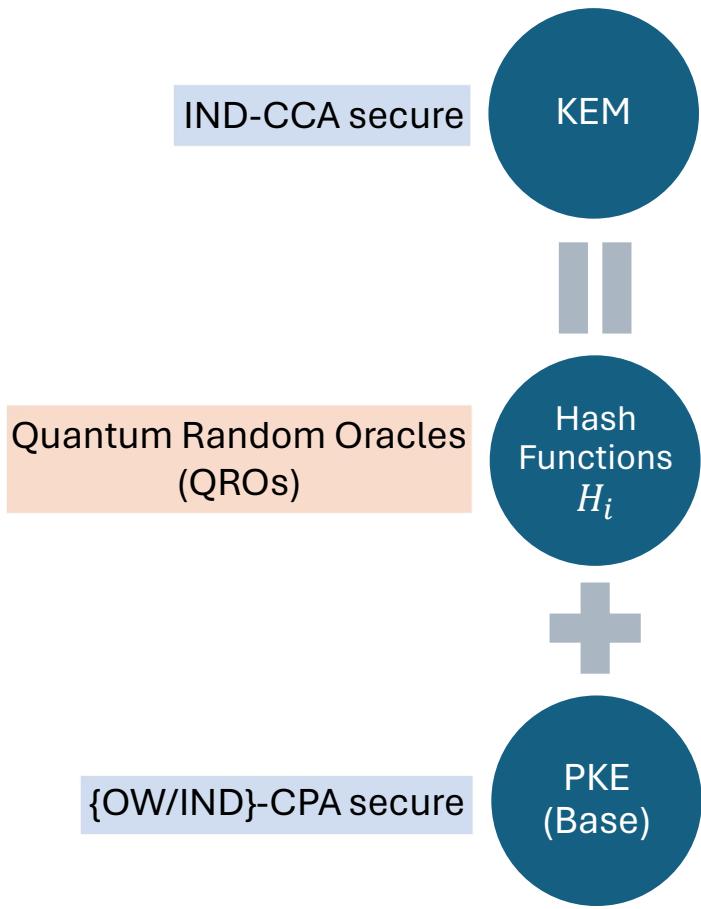
---

$\mu' := \text{Dec}(dk, c_0)$   
 if  $\mu' = \perp$ , then return  $K := H_0(H_3(s), c_0, c_1)$   
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 if  $(c_0, c_1) \neq (c'_0, c'_1)$ , then return  $K := H_0(H_3(s), c_0, c_1)$   
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“ $c_1 \leftarrow \text{hash}(\text{hash}(m))$ ”

Streamlined NTRU  
Prime’s FO variant

# Analysis of SNTRUp



$\overline{\text{Gen}}(1^\kappa)$	$\overline{\text{Enc}}(ek)$	$\overline{\text{Dec}}(\overline{dk}, (c_0, c_1))$ , where $\overline{dk} = (dk, ek, s)$
1 : $(ek, dk) \leftarrow \text{Gen}(1^\kappa)$	1 : $\mu \leftarrow \mathcal{D}_{\mathcal{M}}$	1 : $\mu' \leftarrow \text{Dec}(dk, c_0)$
2 : $s \leftarrow \mathcal{M}$	2 : $c_0 := \text{Enc}(ek, \mu)$	2 : if $\mu' = \perp$ or $c_0 \neq \text{Enc}(ek, \mu')$ or $c_1 \neq \mathbb{F}(\mu', ek)$
3 : $\overline{dk} := (dk, ek, s)$	3 : $c_1 := \mathbb{F}(\mu, ek)$	3 : then return $K := \mathbb{H}(s, c_0, c_1)$
4 : return $(ek, \overline{dk})$	4 : $K := \mathbb{H}(\mu, c_0, c_1)$	4 : else return $K := \mathbb{H}(\mu', c_0, c_1)$
	5 : return $((c_0, c_1), K)$	

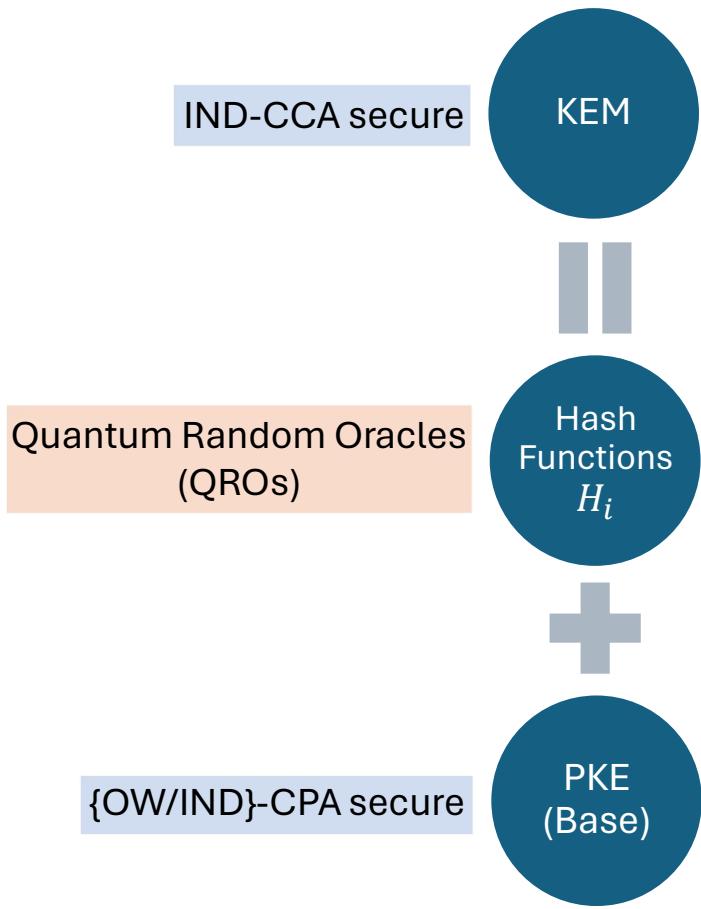
$\mathsf{HU}^I$  [Xagawa@Eurocrypt'22]

$\overline{\text{Gen}}(1^\kappa)$	$\overline{\text{Enc}}(ek)$	$\overline{\text{Dec}}(\overline{dk}, (c_0, c_1))$ , where $\overline{dk} = (dk, ek, s)$
$(ek, dk) \leftarrow \text{Gen}(1^\kappa)$	$\mu \leftarrow \mathcal{M}$	$\mu' := \text{Dec}(dk, c_0)$
$s \leftarrow \{0, 1\}^\ell$	$c_0 := \text{Enc}(ek, \mu)$	if $\mu' = \perp$ , then return $K := \mathbb{H}_0(\mathbb{H}_3(s), c_0, c_1)$
$\overline{dk} := (dk, ek, s)$	$c_1 := \mathbb{H}_2(\mathbb{H}_3(\mu), \mathbb{H}_4(ek))$	$c'_0 := \text{Enc}(ek, \mu')$
return $(ek, \overline{dk})$	$K := \mathbb{H}_1(\mathbb{H}_3(\mu), c_0, c_1)$	$c'_1 := \mathbb{H}_2(\mathbb{H}_3(\mu'), \mathbb{H}_4(ek))$
	return $((c_0, c_1), K)$	if $(c_0, c_1) \neq (c'_0, c'_1)$ , then return $K := \mathbb{H}_0(\mathbb{H}_3(s), c_0, c_1)$
		else return $K := \mathbb{H}_1(\mathbb{H}_3(\mu'), c_0, c_1)$

Streamlined NTRU  
Prime's FO variant

" $c_1 \leftarrow \text{hash}(\text{hash}(m))$ "

# Analysis of SNTRUp



$\overline{\text{Gen}}(1^\kappa)$	$\overline{\text{Enc}}(ek)$	$\overline{\text{Dec}}(\overline{dk}, (c_0, c_1))$ , where $\overline{dk} = (dk, ek, s)$
1 : $(ek, dk) \leftarrow \text{Gen}(1^\kappa)$	1 : $\mu \leftarrow \mathcal{D}_M$	1 : $\mu' \leftarrow \text{Dec}(dk, c_0)$
2 : $s \leftarrow \mathcal{M}$	2 : $c_0 := \text{Enc}(ek, \mu)$	2 : if $\mu' = \perp$ or $c_0 \neq \text{Enc}(ek, \mu')$ or $c_1 \neq \mathbb{F}(\mu', ek)$
3 : $\overline{dk} := (dk, ek, s)$	3 : $c_1 := \mathbb{F}(\mu, ek)$	3 : then return $K := H(s, c_0, c_1)$
4 : return $(ek, \overline{dk})$	4 : $K := H(\mu, c_0, c_1)$	4 : else return $K := H(\mu', c_0, c_1)$
	5 : return $((c_0, c_1), K)$	

$c_1 \leftarrow \text{hash}(m)$  and  
 $c_1 \leftarrow \text{hash}(\text{hash}(m))$   
 are quantum **indifferentiable**. [Zhandry@Crypto'19]

$\overline{\text{Gen}}(1^\kappa)$	$\overline{\text{Enc}}(ek)$	$\overline{\text{Dec}}(\overline{dk}, (c_0, c_1))$ , where $\overline{dk} = (dk, ek, s)$
$(ek, dk) \leftarrow \text{Gen}(1^\kappa)$	$\mu \leftarrow \mathcal{M}$	$\mu' := \text{Dec}(dk, c_0)$
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return $(ek, \overline{dk})$	$K := H_1(H_3(\mu), c_0, c_1)$	$c'_1 := H_2(H_3(\mu'), H_4(ek))$
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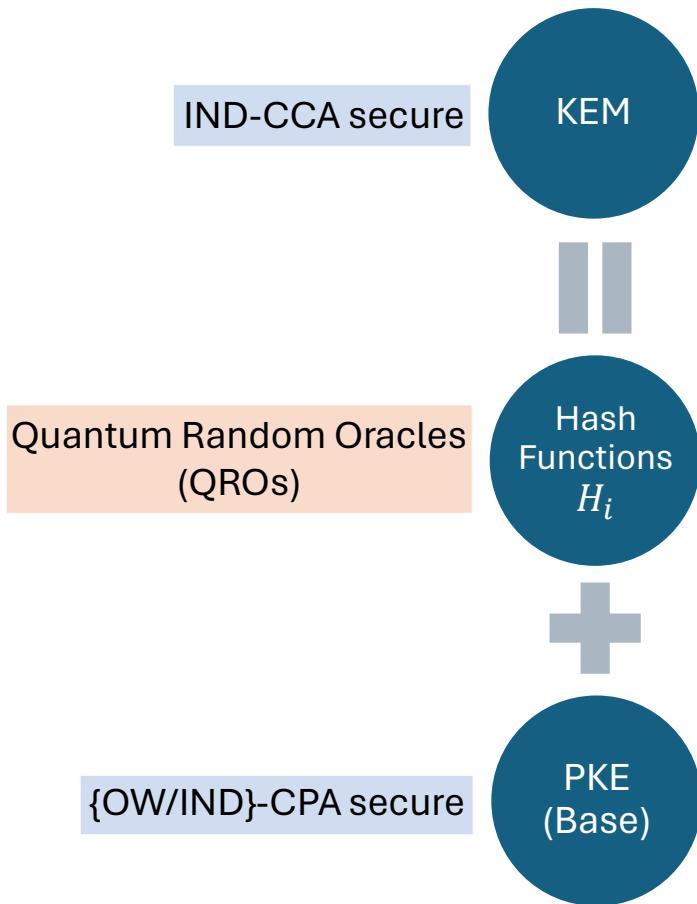
Streamlined NTRU Prime's FO variant

$c_1 \leftarrow \text{hash}(\text{hash}(m))$

$c_1 \leftarrow \text{hash}(m)$

$H \cup \perp$  [Xagawa@Eurocrypt'22]

# Analysis of SNTRUp



$\overline{\text{Gen}}(1^\kappa)$

---

- 1 :  $(ek, dk) \leftarrow \text{Gen}(1^\kappa)$
- 2 :  $s \leftarrow \mathcal{M}$
- 3 :  $\overline{dk} := (dk, ek, s)$
- 4 : **return**  $(ek, \overline{dk})$

$\overline{\text{Enc}}(ek)$

---

- 1 :  $\mu \leftarrow \mathcal{D}_{\mathcal{M}}$
- 2 :  $c_0 := \text{Enc}(ek, \mu)$
- 3 :  $c_1 := F(\mu[, ek])$
- 4 : **return**  $(c_0, c_1)$
- 5 : **return**  $((c_0, c_1), K)$

$\overline{\text{Dec}}(\overline{dk}, (c_0, c_1))$ , where  $\overline{dk} = (dk, ek, s)$

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- 1 :  $\mu' \leftarrow \text{Dec}(dk, c_0)$
- 2 : if  $\mu' = \perp$  or  $c_0 \neq \text{Enc}(ek, \mu')$  or  $c_1 \neq F(\mu'[, ek])$
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“key  $\leftarrow \text{hash}(m, c_0, c_1)$ ” and  
“key  $\leftarrow \text{hash}(\text{hash}(m), c_0, c_1)$ ”  
are quantum **indifferentiable**.

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$\overline{\text{Enc}}(ek)$

---

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$\overline{\text{Dec}}(\overline{dk}, (c_0, c_1))$ , where  $\overline{dk} = (dk, ek, s)$

---

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- $c'_1 := H_2(H_3(\mu'), H_4(ek))$
- if  $(c_0, c_1) \neq (c'_0, c'_1)$ , then return  $K := H_0(H_3(s), c_0, c_1)$
- else return  $K := H_1(H_3(\mu'), c_0, c_1)$

“key  $\leftarrow \text{hash}(\text{hash}(m), c_0, c_1)$ ”

Streamlined NTRU  
Prime’s FO variant



D. J. Bernstein  
to pqc-...@list.nist.gov

Oct 8, 2021, 4:55:56 PM

To say a bit more about the indifferentiability perspective: The point of the results in, e.g.,

<https://cs.nyu.edu/~dodis/ps/merkle.pdf> Lemma 3.2  
<https://hal.inria.fr/file/index/docid/765883/filename/main.pdf>

is that a ROM proof of any protocol using  $H(x,y)$  for fixed-length  $x,y$  implies a ROM proof of the same protocol using  $H_2(H_1(x),y)$ ; and

<https://eprint.iacr.org/2018/276> Theorem 4

says something analogous for QROM. The results are tight when  $H_1$  has reasonably long output. (If  $H_1$  is modeled as collision-free then  $H_2(H_1(x),y)$  is a perfect simulation of  $H(x,y)$ , although formalizing this is harder than doing ROM proofs.)



Quantum Random Oracles  
(QROs)



PKE  
(Base)

{OW/IND}-CPA secure

“key  $\leftarrow \text{hash}(\text{hash}(m), c_0, c_1)$ ”

# SNTRUp

“key  $\leftarrow \text{hash}(m, c_0, c_1)$ ”

$\overline{\text{Enc}}(ek)$	$\overline{\text{Dec}}(\overline{dk}, (c_0, c_1))$ , where $\overline{dk} = (dk, ek, s)$
$\kappa)$	
1 : $\mu \leftarrow \mathcal{D}_M$	1 : $\mu' \leftarrow \text{Dec}(dk, c_0)$
2 : $c_0 := \text{Enc}(ek, \mu)$	2 : if $\mu' = \perp$ or $c_0 \neq \text{Enc}(ek, \mu')$ or $c_1 \neq F(\mu', ek)$
3 : $c_1 := F(\mu, ek)$	3 : then return $K := H(s, c_0, c_1)$
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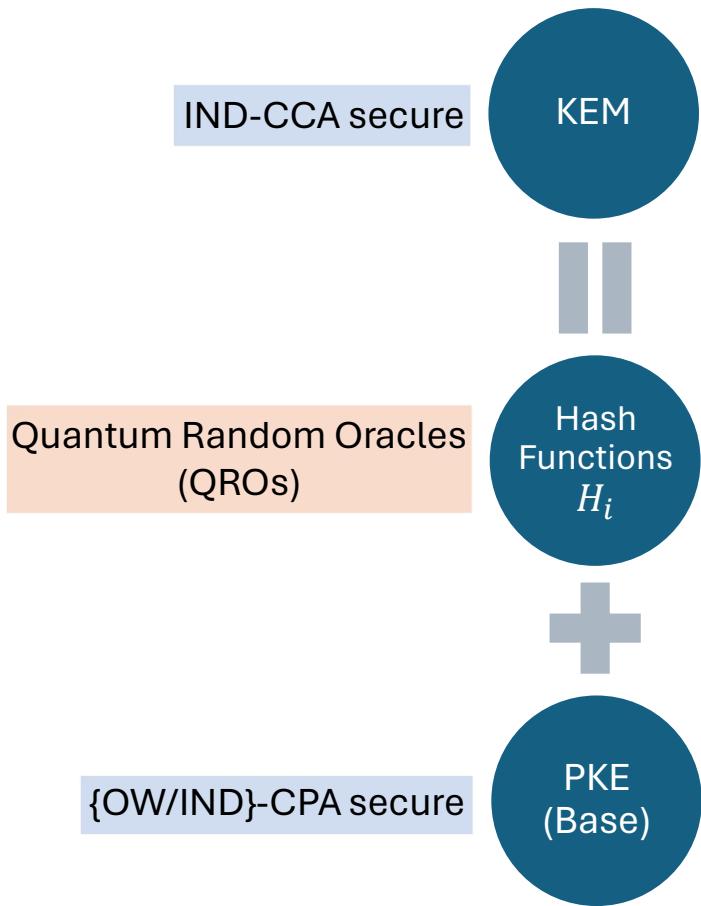
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$s \leftarrow \{0, 1\}^\ell$	$c_0 := \text{Enc}(ek, \mu)$	if $\mu' = \perp$ , then return $K := H_0(H_3(s), c_0, c_1)$
$\overline{dk} := (dk, ek, s)$	$c_1 := H_2(H_3(\mu), H_4(ek))$	$c'_0 := \text{Enc}(ek, \mu')$
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Streamlined NTRU  
Prime's FO variant

# Analysis of SNTRUp



$\overline{\text{Gen}}(1^\kappa)$

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“key  $\leftarrow \text{hash}(m, c_0, c_1)$ ”

$\overline{\text{Dec}}(\overline{dk}, (c_0, c_1))$ , where  $\overline{dk} = (dk, ek, s)$

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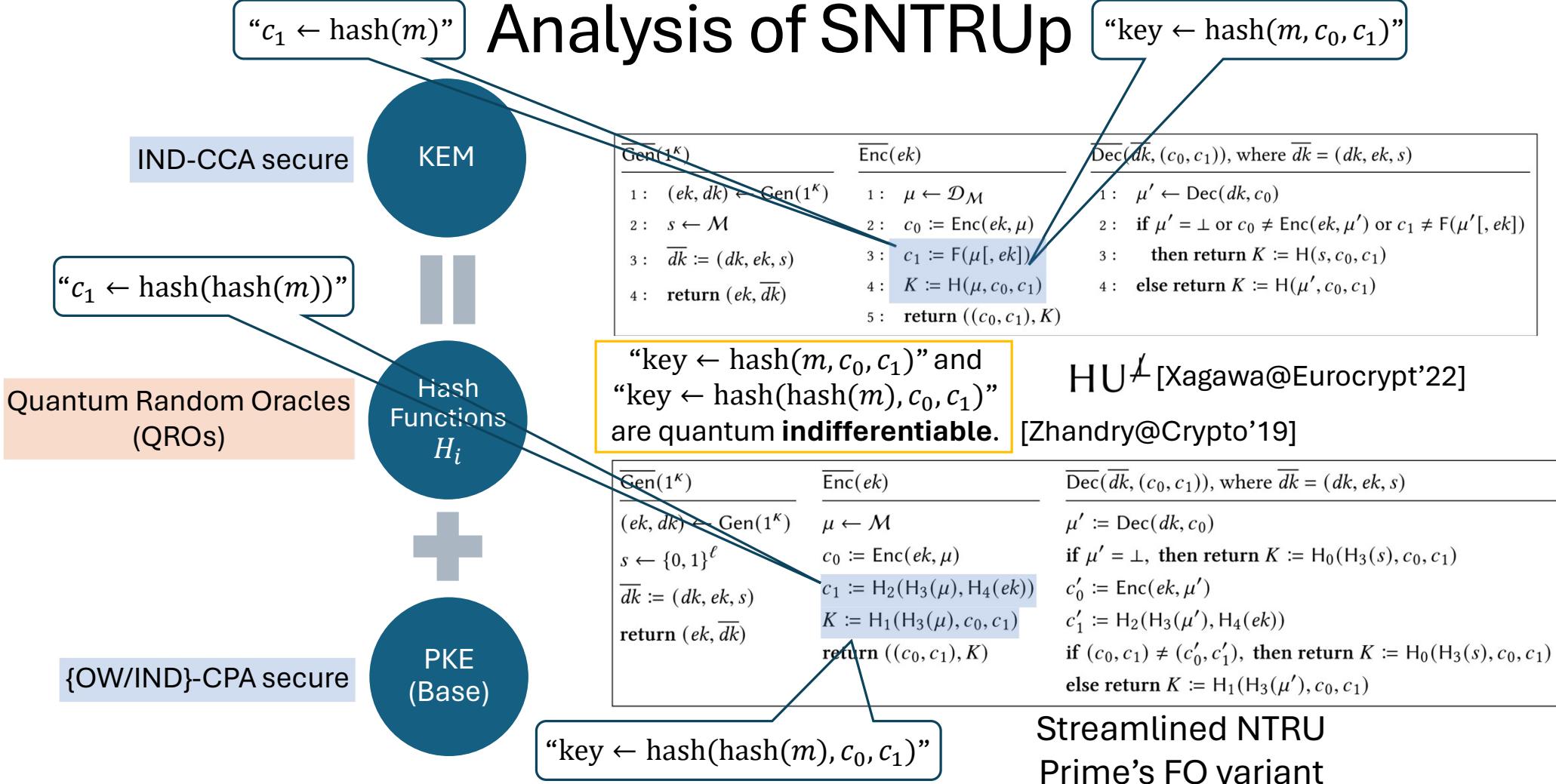
---

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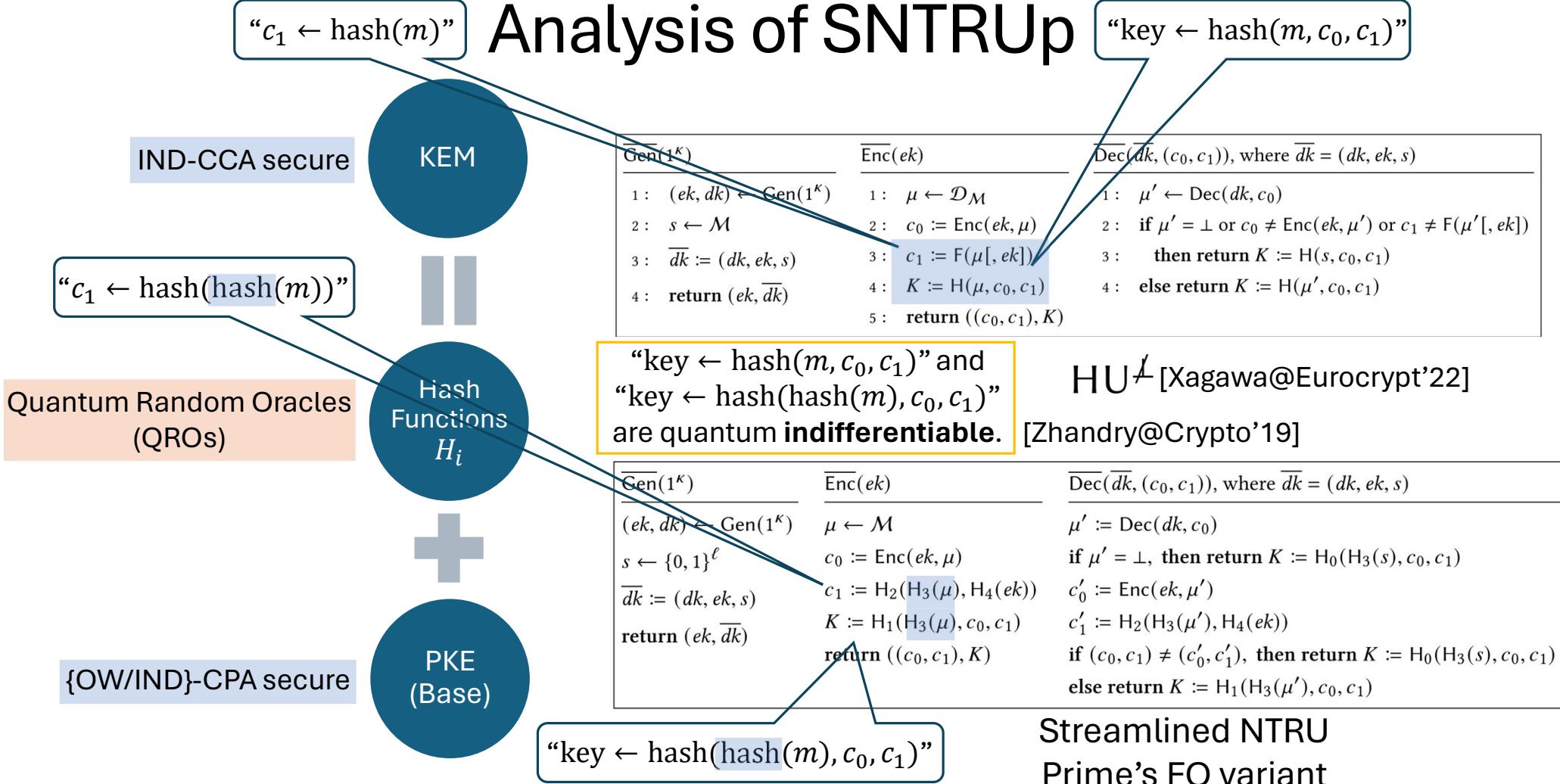
“key  $\leftarrow \text{hash}(\text{hash}(m), c_0, c_1)$ ”

Streamlined NTRU  
Prime’s FO variant

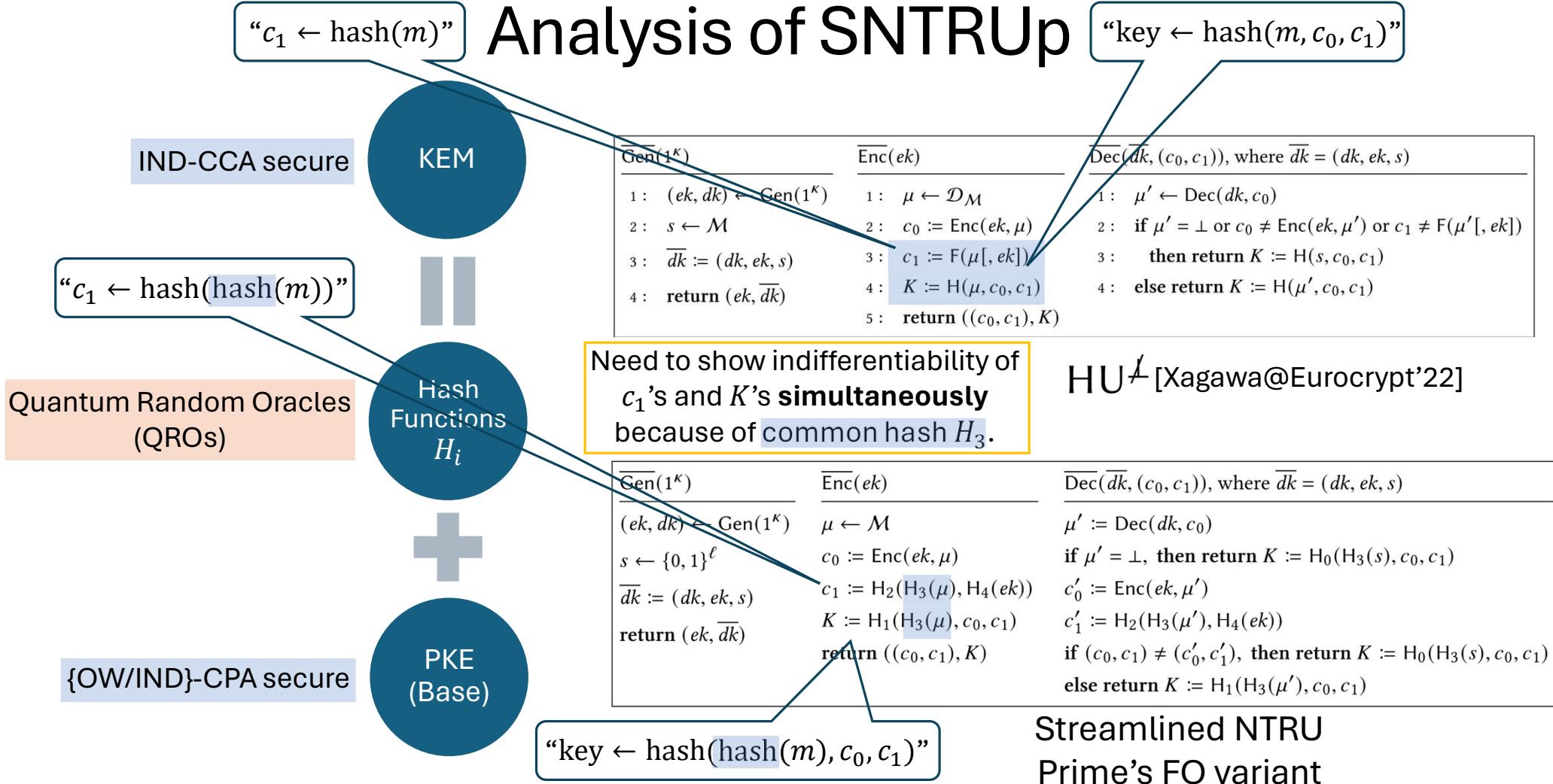
# Analysis of SNTRUp



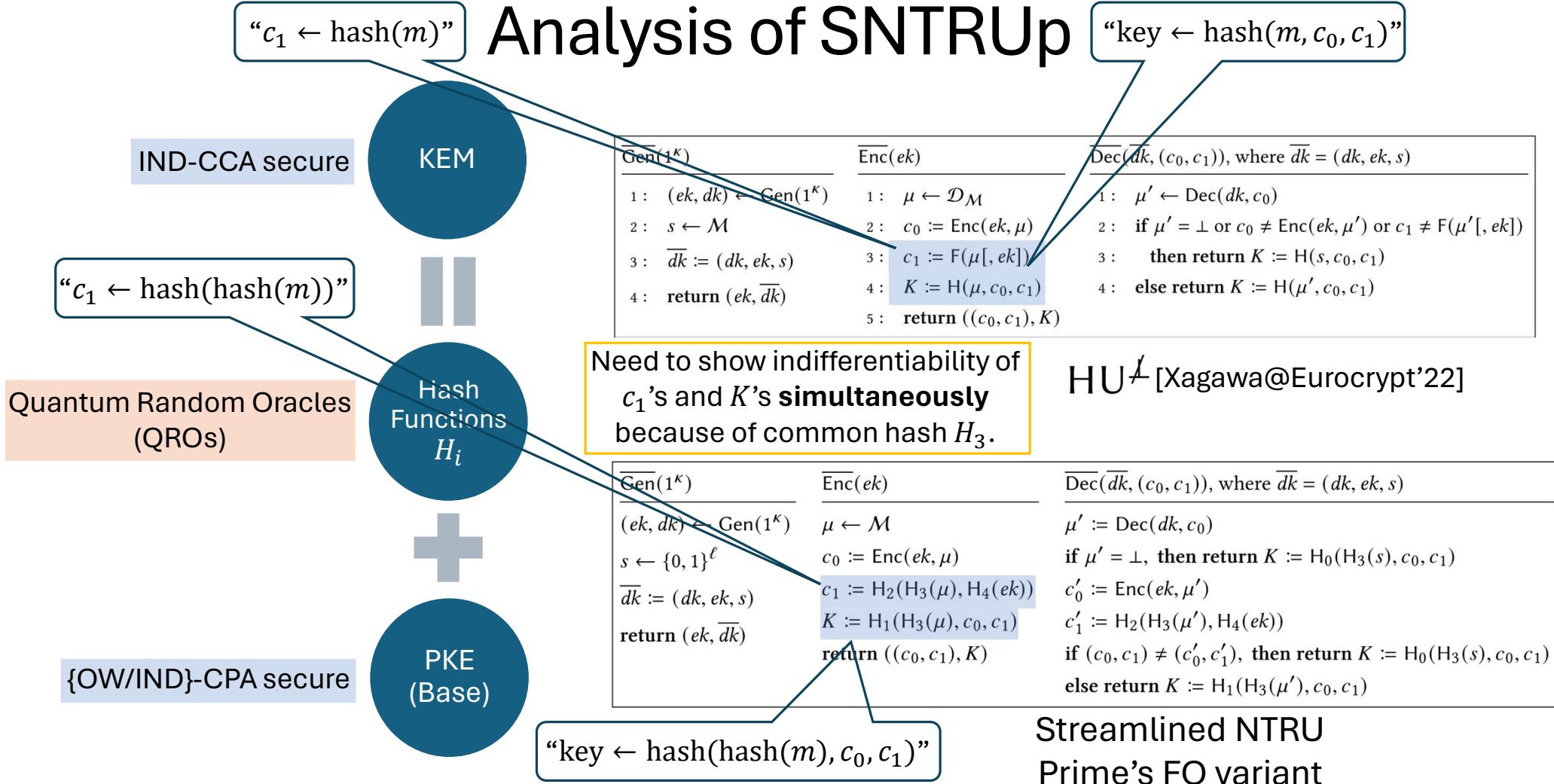
# Analysis of SNTRUp



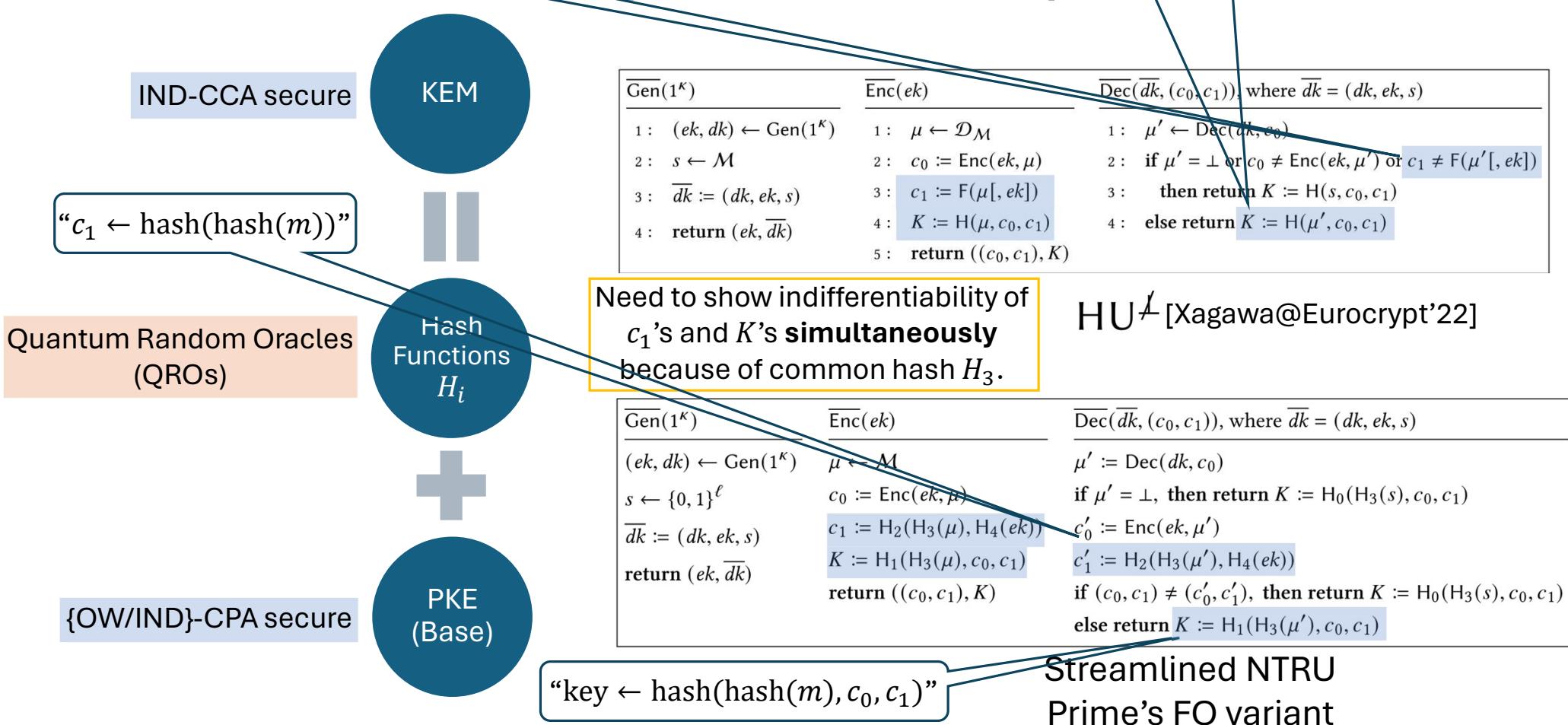
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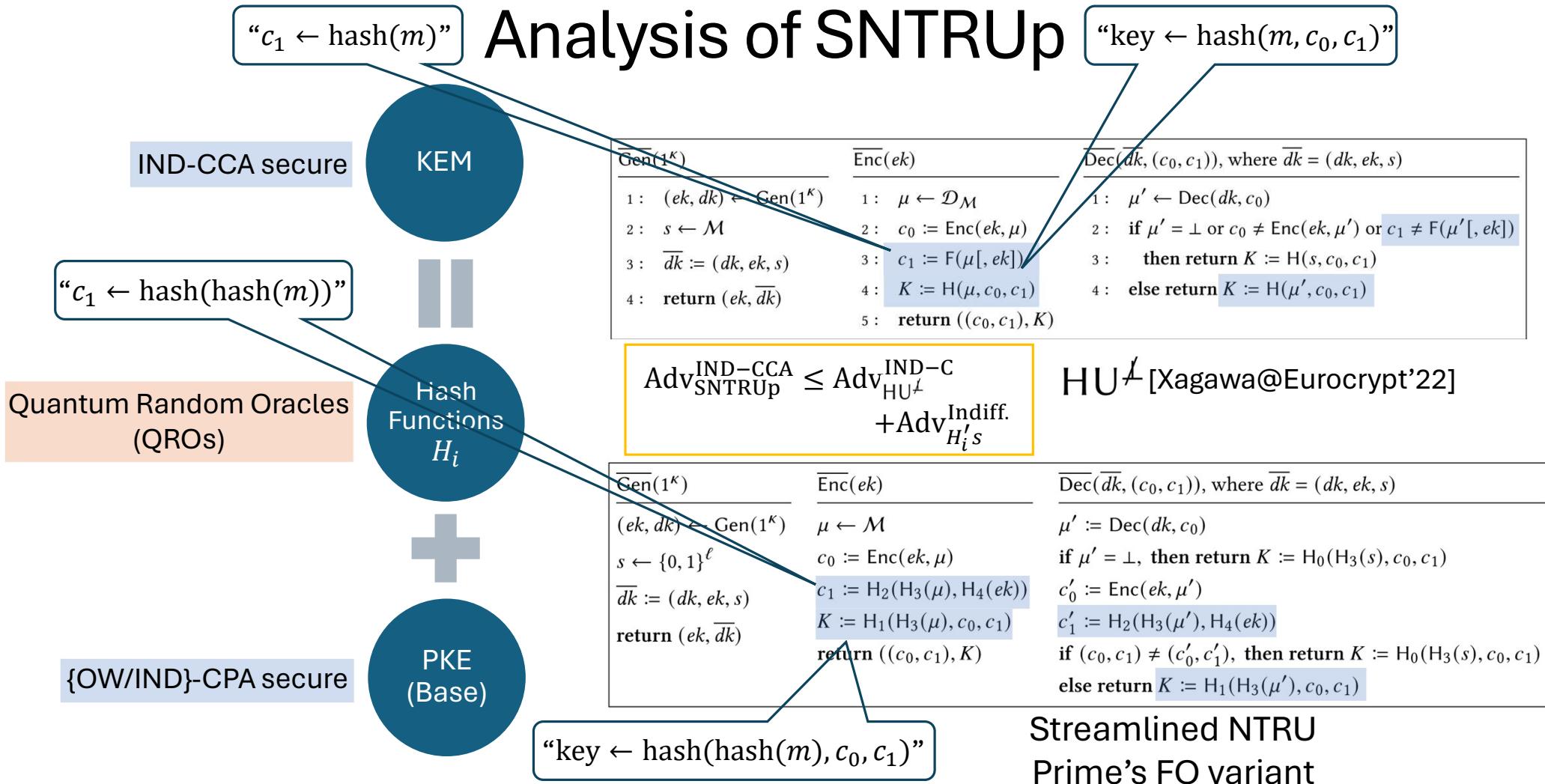
# Analysis of SNTRUp



# Analysis of SNTRUp



# Analysis of SNTRUp



# Don't Roll Your Own FO: Challenges in Proving Post-Quantum CCA Security of Real-World KEMs

- CRYSTALS-Kyber
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Varun Maram  
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Fujisaki-Okamoto transform.

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# Don't Roll Your Own Need FO(?) Challenges in Proving Post-Quantum CCA Security of Real-World KEMs

- CRYSTALS-Kyber
- Streamlined NTRU Prime

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Fujisaki-Okamoto transform.

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# Post-quantum SSH

## OpenSSH 9.0/9.0p1 (2022-04-08)

### NTRU Prime

Intro	Warnings	Papers	Security	Softw
Speed	NISTPQC	FAQ		

NTRU Prime is a third-round candidate in NIST's [Post-Quantum Cryptography Standardization Project](#). The third-round submission specifies twelve KEMs:

- Streamlined NTRU Prime: sntrup653, sntrup761, sntrup953, sntrup1013, sntrup1277.
- NTRU LPRime: ntrulpr653, ntrulpr761, ntrulpr857, ntrulpr1013, ntrulpr1277.

OpenSSH 9.0 was released on 2022-04-08. It is available from the mirrors listed at <https://www.openssh.com/>.

OpenSSH is a 100% complete SSH protocol 2.0 implementation and includes sftp client and server support.

This creates one area of potential incompatibility: [scp\(1\)](#) when using SFTP protocol no longer requires this finicky and brittle quoting, continued support of the `\`` and attempts to use it may cause transfers to fail. We consider the removal of the need for double-quoting shell characters in file names to be a benefit and do not intend to introduce bug-compatibility for legacy scp/rpc in [scp\(1\)](#) when using the SFTP protocol.

Changes since OpenSSH 8.9  
=====

Another area of potential incompatibility relates to the use of remote paths relative to other user's home directories, for example - "scp host:~user/file /tmp". The SFTP protocol has no native way to expand a ~user path. However, [sftp-server\(8\)](#) in OpenSSH 8.7 and later support a protocol extension "expand-path@openssh.com" to support this.

Potentially-incompatible  
-----

In case of incompatibility, the [scp\(1\)](#) client may be instructed to use the legacy scp/rpc using the -O flag.

This release switches [scp\(1\)](#) to using the SFTP protocol.

Legacy scp/rpc performs well with "scp host:\* .") through the requiring double quoting ( included on [scp\(1\)](#) command as shell commands on the |

\* [ssh\(1\)](#), [sshd\(8\)](#): use the hybrid Streamlined NTRU Prime + x25519 key exchange method by default ("sntrup761x25519-sha512@openssh.com"). The NTRU algorithm is believed to resist attacks enabled by future quantum computers and is paired with the X25519 ECDH key exchange (the previous default) as a backstop against any weaknesses in NTRU Prime that may be discovered in the future. The combination ensures that the hybrid exchange offers at least as good security as the status quo.

# Post-quantum SSH

## 7.4 Final result: Multi-ciphersuite SSH

Combining Lemmas 5 and 6 from the previous subsection with the composition theorem (Theorem 3) immediately yields that the SSH protocol is multi-ciphersuite secure, even with key re-use across ciphersuites.

**Corollary 1** (SSH is multi-ciphersuite secure). *Let  $\vec{\text{SSH}}$  be the multi-ciphersuite SSH protocol with each of the  $n_{\text{SP}}$  ciphersuites  $\text{SSH}_c$  being a signed-Diffie–Hellman ciphersuite as in Section 4. The algorithms  $\mathcal{B}_1, \dots, \mathcal{B}_5$  inferred from the proof are such that, for all algorithms  $\mathcal{A}$ :*

$$\text{Adv}_{\vec{\text{SSH}}, c}^{\text{mcs-acce-so-auth}}(\mathcal{A}) \leq n_{\text{SP}} \left( \frac{(n_P n_S)^2}{2^\mu} + \text{Adv}_{H_c}^{\text{cr}}(\mathcal{B}_1^{\mathcal{A}}) + n_P \text{Adv}_{\text{SIG}_c}^{\text{suf-cma}}(\mathcal{B}_2^{\mathcal{A}}) \right)$$

and

$$\begin{aligned} \text{Adv}_{\vec{\text{SSH}}, c}^{\text{mcs-acce-so-aenc}}(\mathcal{A}) &\leq \text{Adv}_{\vec{\text{SSH}}, c}^{\text{mcs-acce-so-auth}}(\mathcal{A}) \\ &+ n_P n_S \left( \text{Adv}_{g_c, q_q}^{\text{ddh}}(\mathcal{B}_3^{\mathcal{A}}) + \text{Adv}_{\text{PRF}_c}^{\text{prf}}(\mathcal{B}_4^{\mathcal{A}}) + \text{Adv}_{\text{StE}_c}^{\text{bsae}}(\mathcal{B}_5^{\mathcal{A}}) \right) \end{aligned}$$

and  $\mathcal{B}_1^{\mathcal{A}}, \dots, \mathcal{B}_5^{\mathcal{A}}$  have approximately the same running time as  $\mathcal{A}$ . Moreover, analogous versions of the theorem apply for mutual authentication.

“Pre-quantum” analysis by [Bergsma-Dowling-Kohlar-Schwenk-Stebila@CCS’14]

# Post-quantum SSH

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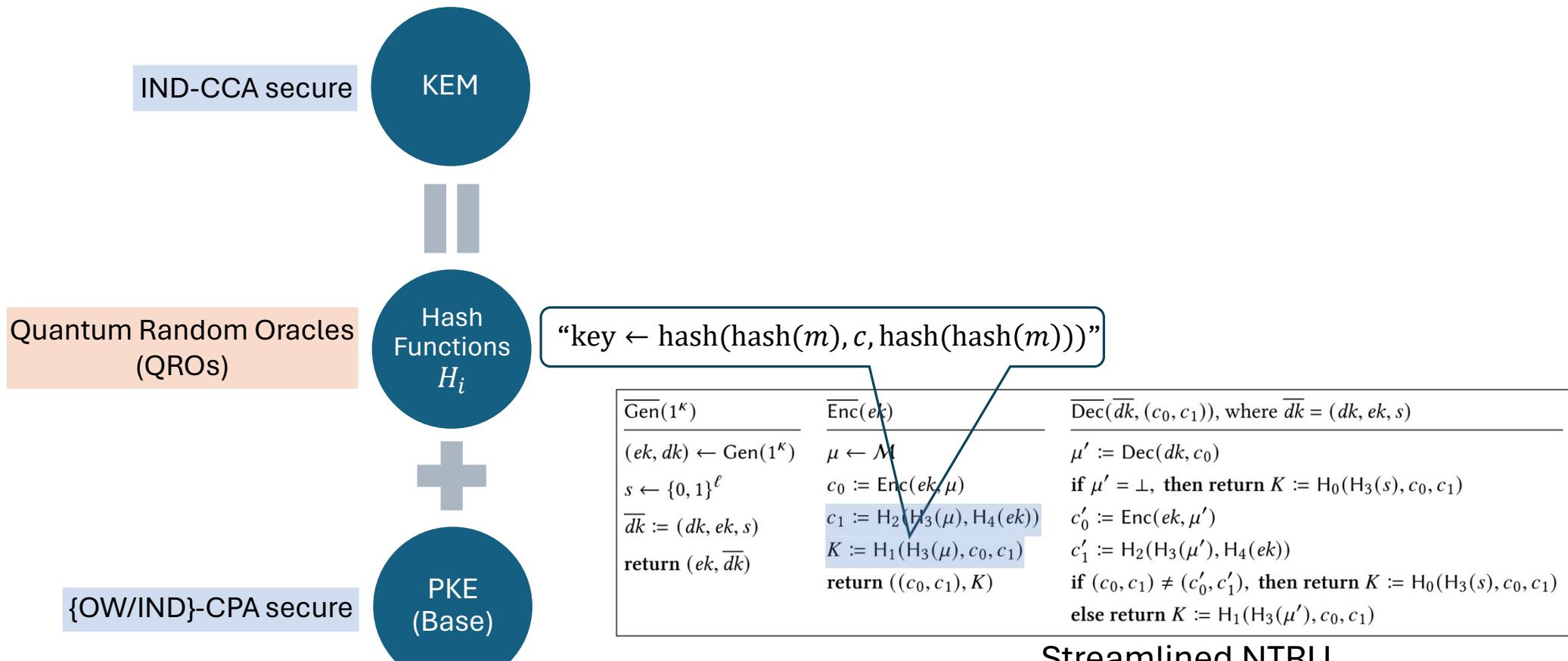
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CPA security of (hybrid)  
KEM should suffice.

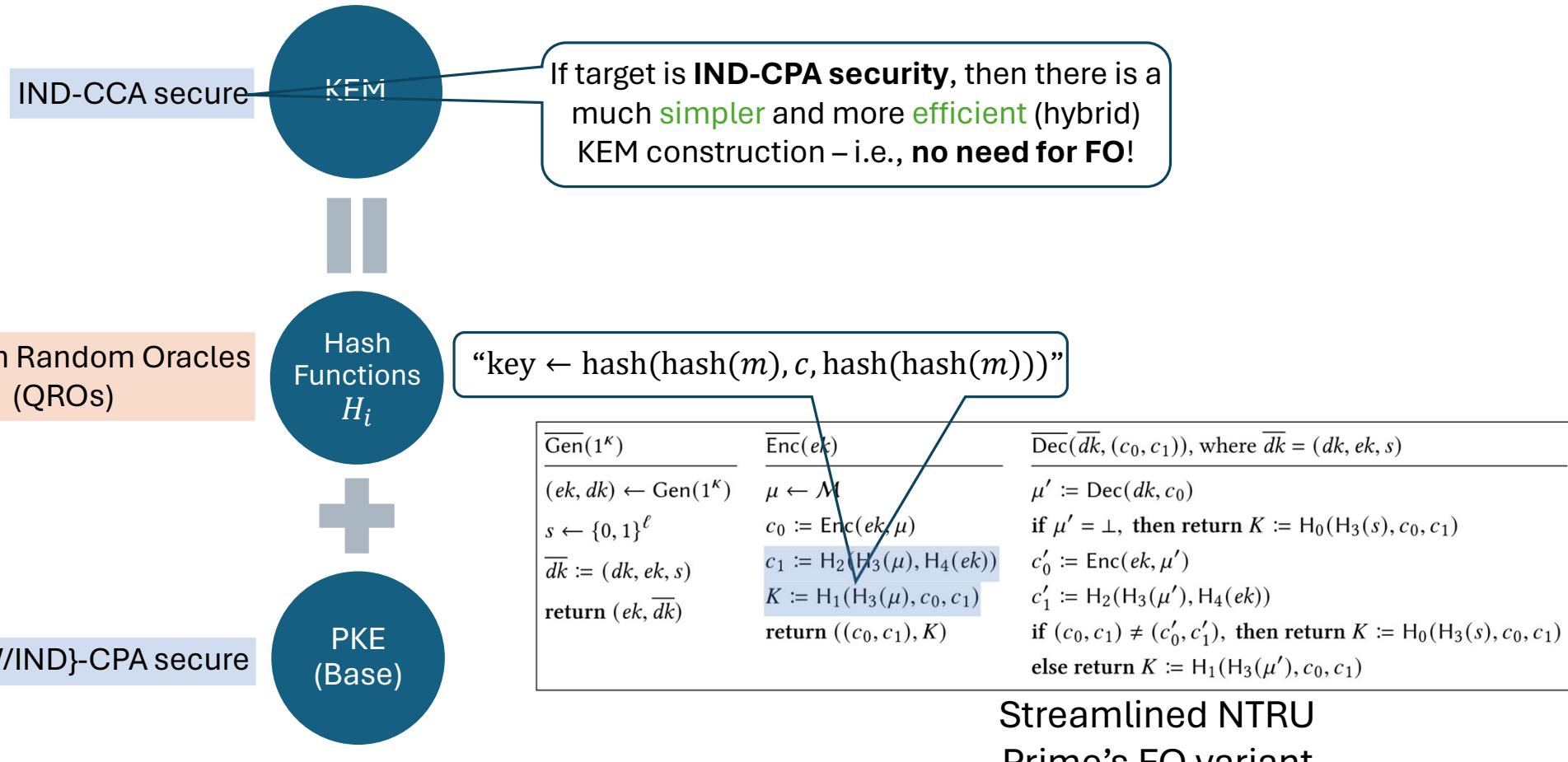
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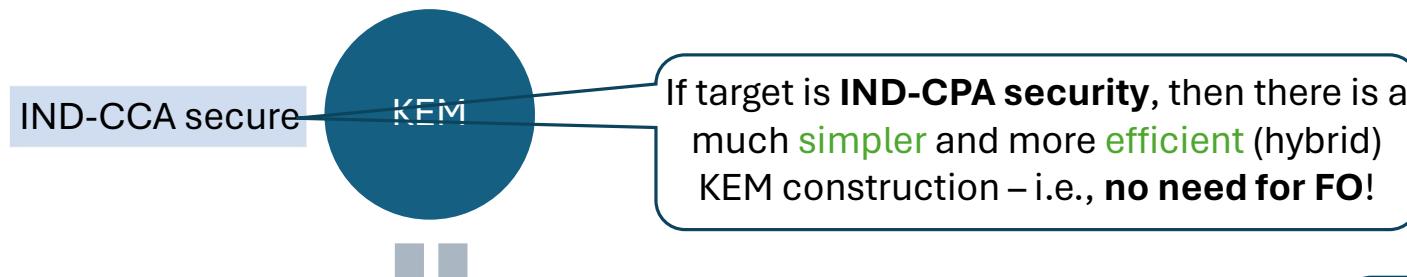
# FO Variants



# FO Variants



# FO Variants



## 7. Security Considerations

The shared secrets computed in the hybrid key exchange should be computed in a way that achieves the "hybrid" property: the resulting secret is secure as long as at least one of the component key exchange algorithms is unbroken. See [GTACON] and [BINDEL] for an investigation of these issues. Under the assumption that shared secrets are fixed length once the combination is fixed, the construction from [Section 3.3](#) corresponds to the dual-PRF combiner of [BINDEL] which is shown to preserve security under the assumption that the hash function is a dual-PRF.

As noted in [Section 2](#), KEMs used in the manner described in this document MUST explicitly be designed to be secure in the event that the public key is reused, such as achieving IND-CCA2 security or having a transform like the Fujisaki-Okamoto transform applied. Kyber has such security properties. However, some other post-quantum KEMs are designed to be IND-CPA-secure (i.e., without countermeasures such as the FO transform) are completely insecure under public key reuse; for example, some lattice-based IND-CPA-secure KEMs are vulnerable to attacks that recover the private key after just a few thousand samples [FLUHRER].

Datatracker

draft-ietf-tls-hybrid-design-09

Internet-Draft

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(m))),"

Doesn't defend against **key-reuse attacks** in bad SSH implementations, however...

$\text{Dec}(\overline{dk}, (c_0, c_1))$ , where  $\overline{dk} = (dk, ek, s)$

$\mu' := \text{Dec}(dk, c_0)$

if  $\mu' = \perp$ , then return  $K := H_0(H_3(s), c_0, c_1)$

$c'_0 := \text{Enc}(ek, \mu')$

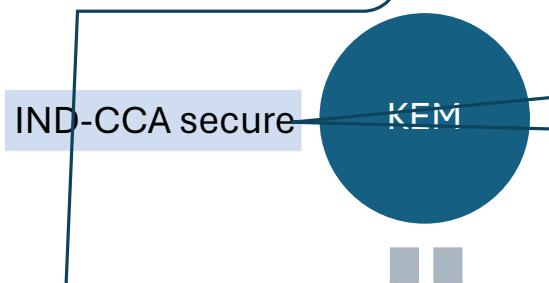
$c'_1 := H_2(H_3(\mu'), H_4(ek))$

if  $(c_0, c_1) \neq (c'_0, c'_1)$ , then return  $K := H_0(H_3(s), c_0, c_1)$

else return  $K := H_1(H_3(\mu'), c_0, c_1)$

Streamlined NTRU  
Prime's FO variant

Is there a notion **weaker than IND-CCA2** which captures key-reuse attacks while allowing for more **efficient** schemes?



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# FO Variants

If target is **IND-CPA security**, then there is a much **simpler** and more **efficient** (hybrid) KEM construction – i.e., **no need for FO!**

Doesn't defend against **key-reuse attacks** in bad SSH implementations, however...

---

$\text{Dec}(\overline{dk}, (c_0, c_1))$ , where  $\overline{dk} = (dk, ek, s)$

$$\begin{aligned} \mu' &:= \text{Dec}(dk, c_0) \\ \text{if } \mu' &= \perp, \text{ then return } K := H_0(H_3(s), c_0, c_1) \\ c'_0 &:= \text{Enc}(ek, \mu') \\ c'_1 &:= H_2(H_3(\mu'), H_4(ek)) \\ \text{if } (c_0, c_1) &\neq (c'_0, c'_1), \text{ then return } K := H_0(H_3(s), c_0, c_1) \\ \text{else return } K &:= H_1(H_3(\mu'), c_0, c_1) \end{aligned}$$


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Streamlined NTRU Prime's FO variant

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