

# Decision Tree classifier

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## **Decision Tree based on Iterative Dichotomiser 3 (ID3) algorithm**

1. Decision Tree classifier is a type of supervised learning algorithm.
2. Used in Machine learning to model or predict outcomes based on input data.
3. It's a tree like structure
  - Internal node: Test on attribute
  - Branch: Attribute value
  - Leaf node: Final decision/outcome or prediction (Label)
4. Can solve both classification and regression problems.

- Decision Tree: A tree structure where each internal node represents a feature or attribute, each branch represents value of the attribute, and each leaf node represents the outcome (class label).
- Entropy: A measure of the uncertainty or impurity in a set of data. Entropy helps determine how mixed the data is in terms of different classes.
- Information Gain: A measure of the effectiveness of an attribute in classifying data. It is calculated as the reduction in entropy after splitting the data on that attribute.

Dataset where you want to classify whether an individual plays a sport based on attributes like "Weather," "Temperature," "Humidity," and "Wind"

ID3 would use these attributes to construct a decision tree by selecting the attribute that provides the highest information gain at each step.

Day	Outlook	Temp	Humidity	Wind	PlayTennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

### Attribute: Outlook

Values (Outlook) = Sunny, Overcast, Rain

$$S = [9+, 5-]$$

$$Entropy(S) = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94$$

$$S_{Sunny} \leftarrow [2+, 3-]$$

$$Entropy(S_{Sunny}) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.971$$

$$S_{Overcast} \leftarrow [4+, 0-]$$

$$Entropy(S_{Overcast}) = -\frac{4}{4} \log_2 \frac{4}{4} - \frac{0}{4} \log_2 \frac{0}{4} = 0$$

$$S_{Rain} \leftarrow [3+, 2-]$$

$$Entropy(S_{Rain}) = -\frac{3}{5} \log_2 \frac{3}{5} - \frac{2}{5} \log_2 \frac{2}{5} = 0.971$$

$$Gain(S, Outlook) = Entropy(S) - \sum_{v \in \{Sunny, Overcast, Rain\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Outlook)$$

$$= Entropy(S) - \frac{5}{14} Entropy(S_{Sunny}) - \frac{4}{14} Entropy(S_{Overcast}) - \frac{5}{14} Entropy(S_{Rain})$$

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

## Attribute: Temp

Values (Temp) = Hot, Mild, Cool

$$S = [9+, 5-]$$

$$Entropy(S) = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94$$

$$S_{Hot} \leftarrow [2+, 2-]$$

$$Entropy(S_{Hot}) = -\frac{2}{4} \log_2 \frac{2}{4} - \frac{2}{4} \log_2 \frac{2}{4} = 1.0$$

$$S_{Mild} \leftarrow [4+, 2-]$$

$$Entropy(S_{Mild}) = -\frac{4}{6} \log_2 \frac{4}{6} - \frac{2}{6} \log_2 \frac{2}{6} = 0.9183$$

$$S_{Cool} \leftarrow [3+, 1-]$$

$$Entropy(S_{Cool}) = -\frac{3}{4} \log_2 \frac{3}{4} - \frac{1}{4} \log_2 \frac{1}{4} = 0.8113$$

$$Gain(S, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Temp)$$

$$= Entropy(S) - \frac{4}{14} Entropy(S_{Hot}) - \frac{6}{14} Entropy(S_{Mild})$$

$$- \frac{4}{14} Entropy(S_{Cool})$$

Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

## Attribute: Humidity

Values (Humidity) = High, Normal

$$S = [9+, 5-]$$

$$Entropy(S) = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94$$

$$S_{High} \leftarrow [3+, 4-]$$

$$Entropy(S_{High}) = -\frac{3}{7} \log_2 \frac{3}{7} - \frac{4}{7} \log_2 \frac{4}{7} = 0.9852$$

$$S_{Normal} \leftarrow [6+, 1-]$$

$$Entropy(S_{Normal}) = -\frac{6}{7} \log_2 \frac{6}{7} - \frac{1}{7} \log_2 \frac{1}{7} = 0.5916$$

$$Gain(S, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Humidity)$$

$$= Entropy(S) - \frac{7}{14} Entropy(S_{High}) - \frac{7}{14} Entropy(S_{Normal})$$



Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

## Attribute: Wind

Values (Wind) = Strong, Weak

$$S = [9+, 5-]$$

$$Entropy(S) = -\frac{9}{14} \log_2 \frac{9}{14} - \frac{5}{14} \log_2 \frac{5}{14} = 0.94$$

$$S_{Strong} \leftarrow [3+, 3-]$$

$$Entropy(S_{Strong}) = 1.0$$

$$S_{Weak} \leftarrow [6+, 2-]$$

$$Entropy(S_{Weak}) = -\frac{6}{8} \log_2 \frac{6}{8} - \frac{2}{8} \log_2 \frac{2}{8} = 0.8113$$

$$Gain(S, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S, Wind) = Entropy(S) - \frac{6}{14} Entropy(S_{Strong}) - \frac{8}{14} Entropy(S_{Weak})$$



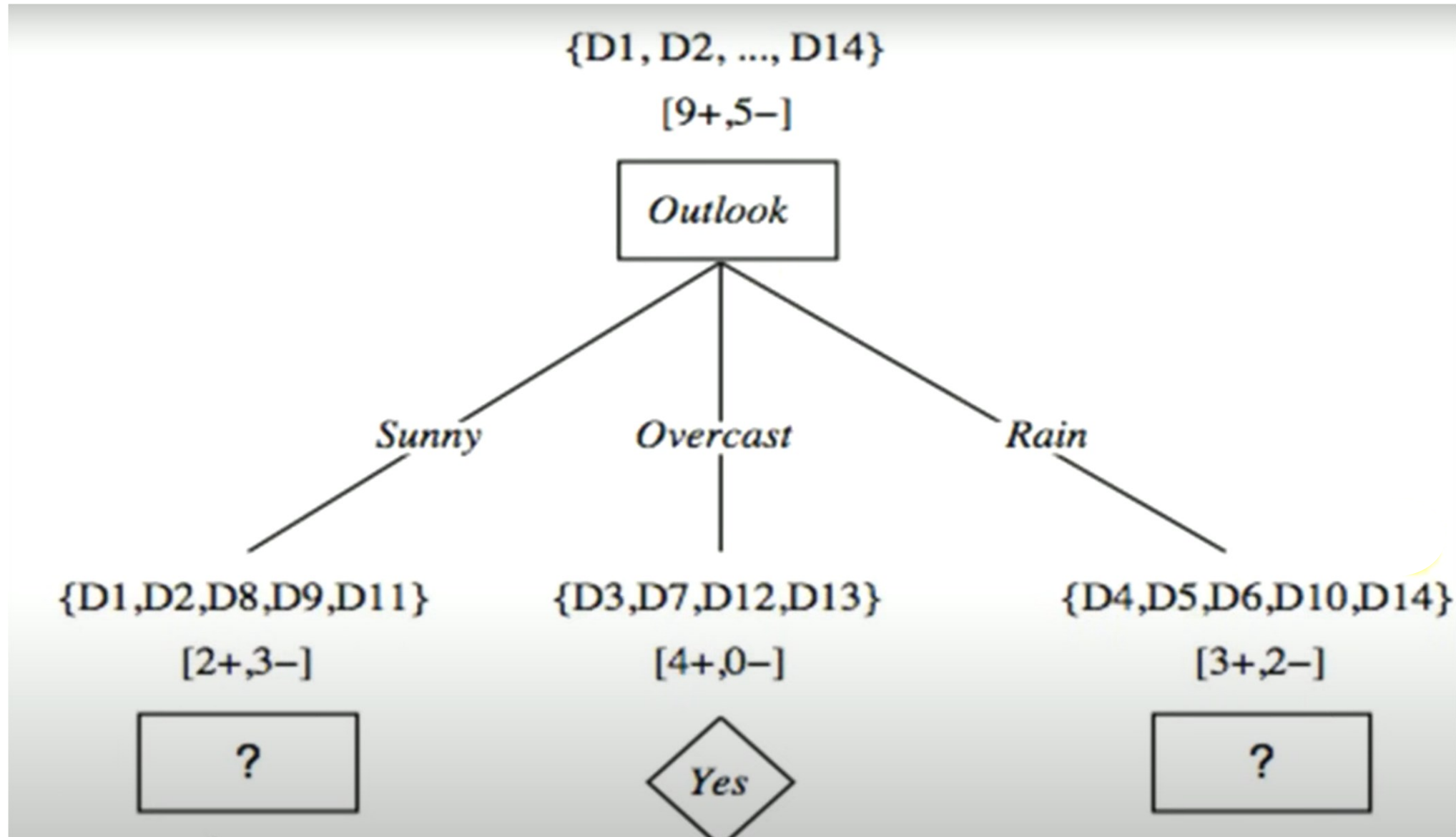
Day	Outlook	Temp	Humidity	Wind	Play Tennis
D1	Sunny	Hot	High	Weak	No
D2	Sunny	Hot	High	Strong	No
D3	Overcast	Hot	High	Weak	Yes
D4	Rain	Mild	High	Weak	Yes
D5	Rain	Cool	Normal	Weak	Yes
D6	Rain	Cool	Normal	Strong	No
D7	Overcast	Cool	Normal	Strong	Yes
D8	Sunny	Mild	High	Weak	No
D9	Sunny	Cool	Normal	Weak	Yes
D10	Rain	Mild	Normal	Weak	Yes
D11	Sunny	Mild	Normal	Strong	Yes
D12	Overcast	Mild	High	Strong	Yes
D13	Overcast	Hot	Normal	Weak	Yes
D14	Rain	Mild	High	Strong	No

$$Gain(S, Outlook) = 0.2464$$

$$Gain(S, Temp) = 0.0289$$

$$Gain(S, Humidity) = 0.1516$$

$$Gain(S, Wind) = 0.0478$$



Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

## Attribute: Temp

Values (Temp) = Hot, Mild, Cool

$$S_{Sunny} = [2+, 3-]$$

$$Entropy(S_{Sunny}) = -\frac{2}{5}\log_2\frac{2}{5} - \frac{3}{5}\log_2\frac{3}{5} = 0.97$$

$$S_{Hot} \leftarrow [0+, 2-]$$

$$Entropy(S_{Hot}) = 0.0$$

$$S_{Mild} \leftarrow [1+, 1-]$$

$$Entropy(S_{Mild}) = 1.0$$

$$S_{Cool} \leftarrow [1+, 0-]$$

$$Entropy(S_{Cool}) = 0.0$$

$$Gain(S_{Sunny}, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Temp)$$

$$= Entropy(S) - \frac{2}{5} Entropy(S_{Hot}) - \frac{2}{5} Entropy(S_{Mild})$$

$$- \frac{1}{5} Entropy(S_{Cool})$$

$$Gain(S_{sunny}, Temp) = 0.97 - \frac{2}{5} 0.0 - \frac{2}{5} 1 - \frac{1}{5} 0.0 = 0.570$$

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

### Attribute: Humidity

Values (Humidity) = High, Normal

$$S_{Sunny} = [2+, 3-]$$

$$Entropy(S) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$

$$S_{high} \leftarrow [0+, 3-]$$

$$Entropy(S_{High}) = 0.0$$

$$S_{Normal} \leftarrow [2+, 0-]$$

$$Entropy(S_{Normal}) = 0.0$$

$$Gain(S_{Sunny}, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Humidity) = Entropy(S) - \frac{3}{5} Entropy(S_{High}) - \frac{2}{5} Entropy(S_{Normal})$$

Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

## Attribute: Wind

Values (Wind) = Strong, Weak

$$S_{Sunny} = [2+, 3-]$$

$$Entropy(S) = -\frac{2}{5} \log_2 \frac{2}{5} - \frac{3}{5} \log_2 \frac{3}{5} = 0.97$$

$$S_{Strong} \leftarrow [1+, 1-]$$

$$Entropy(S_{Strong}) = 1.0$$

$$S_{Weak} \leftarrow [1+, 2-]$$

$$Entropy(S_{Weak}) = -\frac{1}{3} \log_2 \frac{1}{3} - \frac{2}{3} \log_2 \frac{2}{3} = 0.918$$

$$Gain(S_{Sunny}, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Sunny}, Wind) = Entropy(S) - \frac{2}{5} Entropy(S_{Strong}) - \frac{3}{5} Entropy(S_{Weak})$$

$$Gain(S_{Sunny}, Wind) = 0.97 - \frac{2}{5} 1.0 - \frac{3}{5} 0.918 = 0.0192$$

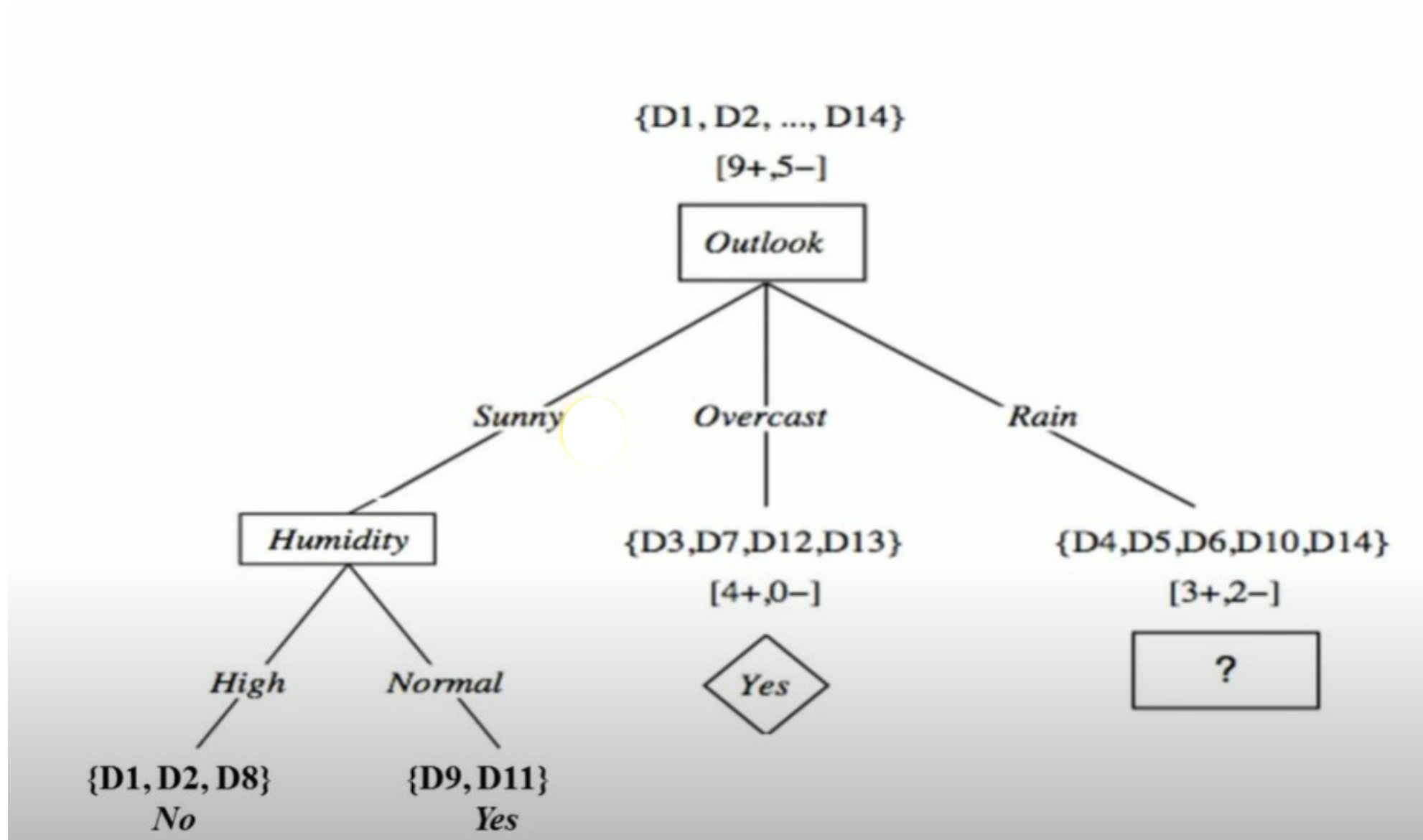
Day	Temp	Humidity	Wind	Play Tennis
D1	Hot	High	Weak	No
D2	Hot	High	Strong	No
D8	Mild	High	Weak	No
D9	Cool	Normal	Weak	Yes
D11	Mild	Normal	Strong	Yes

$$Gain(S_{sunny}, Temp) = 0.570$$

$$Gain(S_{sunny}, Humidity) = 0.97$$

$$Gain(S_{sunny}, Wind) = 0.0192$$





Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
D10	Mild	Normal	Weak	Yes
D14	Mild	High	Strong	No

## Attribute: Temp

Values (Temp) = Hot, Mild, Cool

$$S_{Rain} = [3+, 2-]$$

$$Entropy(S_{Sunny}) = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5} = 0.97$$

$$S_{Hot} \leftarrow [0+, 0-]$$

$$Entropy(S_{Hot}) = 0.0$$

$$S_{Mild} \leftarrow [2+, 1-]$$

$$Entropy(S_{Mild}) = -\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3} = 0.9183$$

$$S_{Cool} \leftarrow [1+, 1-]$$

$$Entropy(S_{Cool}) = 1.0$$

$$Gain(S_{Rain}, Temp) = Entropy(S) - \sum_{v \in \{Hot, Mild, Cool\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Temp)$$

$$= Entropy(S) - \frac{0}{5} Entropy(S_{Hot}) - \frac{3}{5} Entropy(S_{Mild})$$

$$- \frac{2}{5} Entropy(S_{Cool})$$

$$Gain(S_{Rain}, Temp) = 0.97 - \frac{0}{5} 0.0 - \frac{3}{5} 0.918 - \frac{2}{5} 1.0 = 0.0192$$

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
D10	Mild	Normal	Weak	Yes
D14	Mild	High	Strong	No

### Attribute: Humidity

Values (Humidity) = High, Normal

$$S_{Rain} = [3+, 2-]$$

$$Entropy(S_{Sunny}) = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5} = 0.97$$

$$S_{High} \leftarrow [1+, 1-]$$

$$Entropy(S_{High}) = 1.0$$

$$S_{Normal} \leftarrow [2+, 1-]$$

$$Entropy(S_{Normal}) = -\frac{2}{3}\log_2\frac{2}{3} - \frac{1}{3}\log_2\frac{1}{3} = 0.9183$$

$$Gain(S_{Rain}, Humidity) = Entropy(S) - \sum_{v \in \{High, Normal\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Humidity) = Entropy(S) - \frac{2}{5} Entropy(S_{High}) - \frac{3}{5} Entropy(S_{Normal})$$

$$Gain(S_{Rain}, Humidity) = 0.97 - \frac{2}{5} 1.0 - \frac{3}{5} 0.918 = 0.0192$$

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
D10	Mild	Normal	Weak	Yes
D14	Mild	High	Strong	No

## Attribute: Wind

Values (wind) = Strong, Weak

$$S_{Rain} = [3+, 2-]$$

$$Entropy(S_{Sunny}) = -\frac{3}{5}\log_2\frac{3}{5} - \frac{2}{5}\log_2\frac{2}{5} = 0.97$$

$$S_{Strong} \leftarrow [0+, 2-]$$

$$Entropy(S_{Strong}) = 0.0$$

$$S_{Weak} \leftarrow [3+, 0-]$$

$$Entropy(S_{Weak}) = 0.0$$

$$Gain(S_{Rain}, Wind) = Entropy(S) - \sum_{v \in \{Strong, Weak\}} \frac{|S_v|}{|S|} Entropy(S_v)$$

$$Gain(S_{Rain}, Wind) = Entropy(S) - \frac{2}{5} Entropy(S_{Strong}) - \frac{3}{5} Entropy(S_{Weak})$$

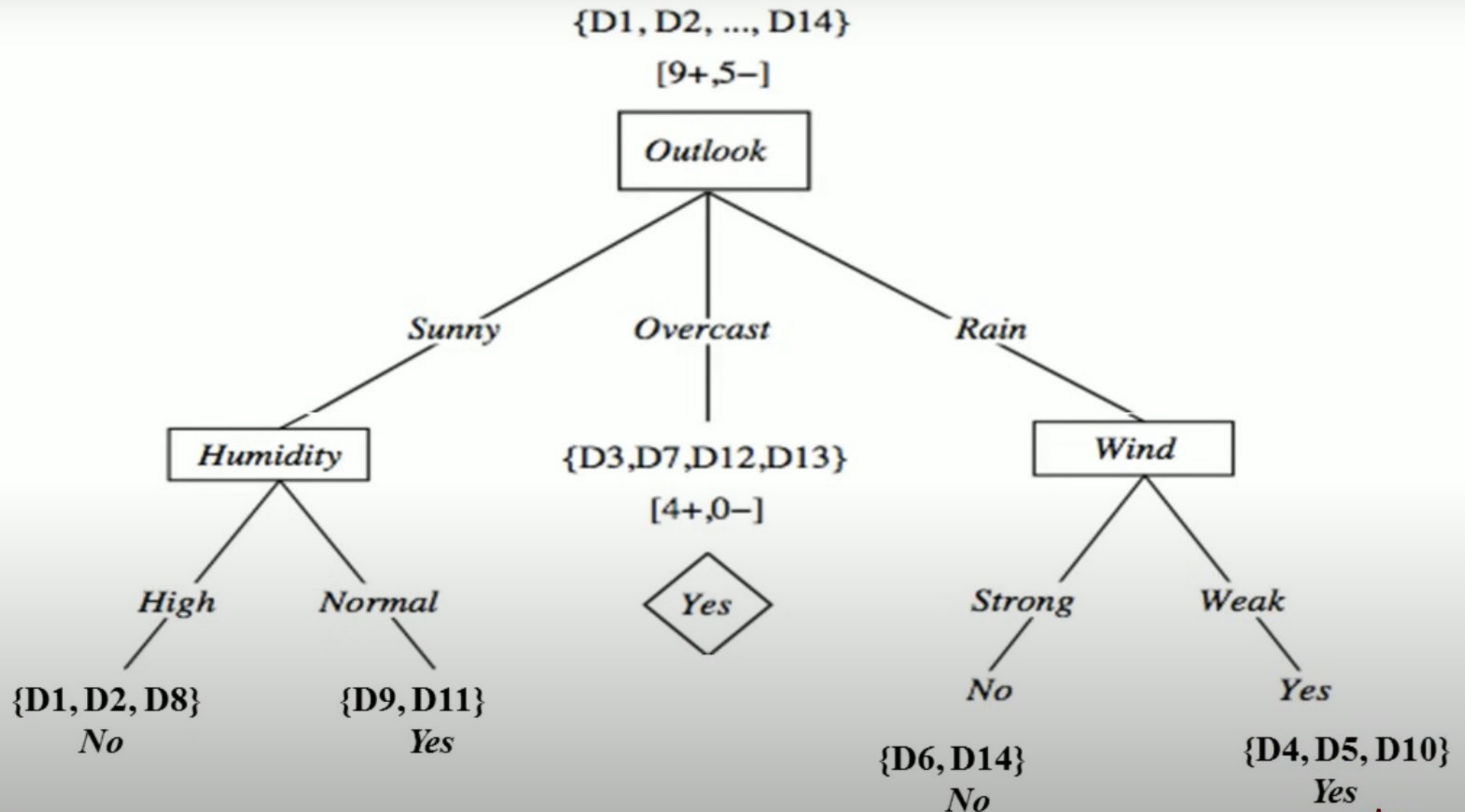
$$Gain(S_{Rain}, Wind) = 0.97 - \frac{2}{5} 0.0 - \frac{3}{5} 0.0 = 0.97$$

Day	Temp	Humidity	Wind	Play Tennis
D4	Mild	High	Weak	Yes
D5	Cool	Normal	Weak	Yes
D6	Cool	Normal	Strong	No
D10	Mild	Normal	Weak	Yes
D14	Mild	High	Strong	No

$$Gain(S_{Rain}, Temp) = 0.0192$$

$$Gain(S_{Rain}, Humidity) = 0.0192$$

$$Gain(S_{Rain}, Wind) = 0.97$$





# Pruning

Pruning is used to reduce size of a tree or to get the optimal size of the tree (why tree should have optimum size?)

2. It removes nodes that do not provide additional information.
3. Pruning means shortening the branches of a tree. In this process some branch nodes are converted into leaf node and removing the leaf node under the original branch.

# Regularization

- Decision tree do not assume a particular form of a relationship between the independent and dependent variables unlike linear models (?)
- DT is a nonparameterized algorithm unlike linear models where we supply the input parameters.
- Decision tree tries to fit the train data perfectly which leads to overfitting.
- To avoid overfitting, we need to restrict the DT's freedom during the tree creation. This is called regularization.

# Regularization

- The regularization hyperparameters depends on the algorithm used.
- Max\_depth – is the maximum length of a path from root to leaf (in terms of number of decision points)
- Min\_sample\_split- A limit to stop further splitting of nodes when the number of observations in the node is lower than this value.
- Max\_leaf \_nodes – maximum number of leaf nodes in a tree.
- Max\_feature\_size – max number of features that are evaluated for splitting each node.

### **Advantages of ID3:**

- Simple and Intuitive: ID3 is easy to understand and implement.
- Efficient for small datasets: The algorithm works efficiently with smaller datasets and categorical data.

### **Limitations of ID3:**

- Overfitting: ID3 tends to overfit the training data, especially if the tree becomes too complex.
- Bias towards attributes with many values: ID3 can be biased towards attributes with more distinct/unique values, even if they are not the most informative.
- Requires categorical data.

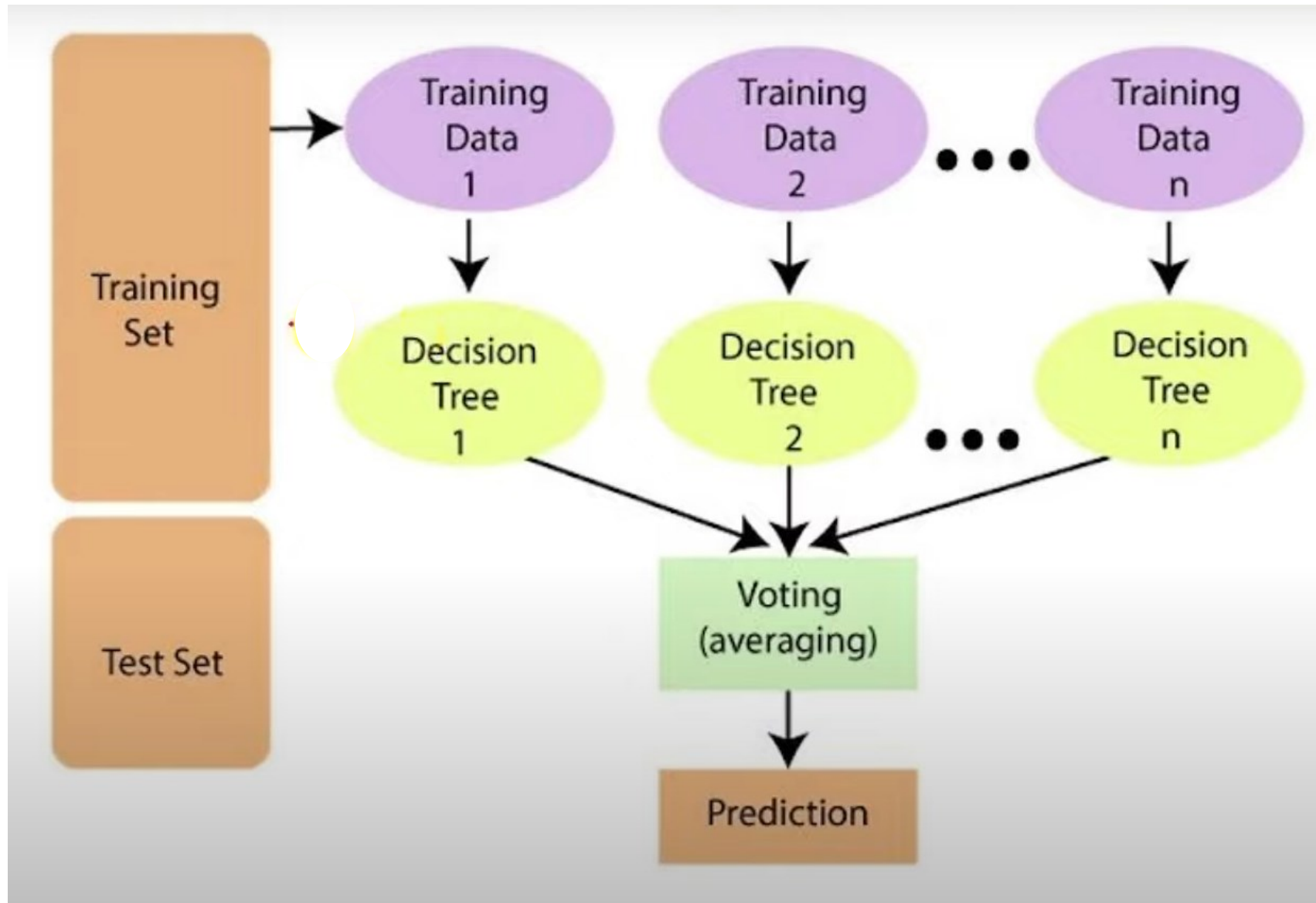
# Random forest classifier

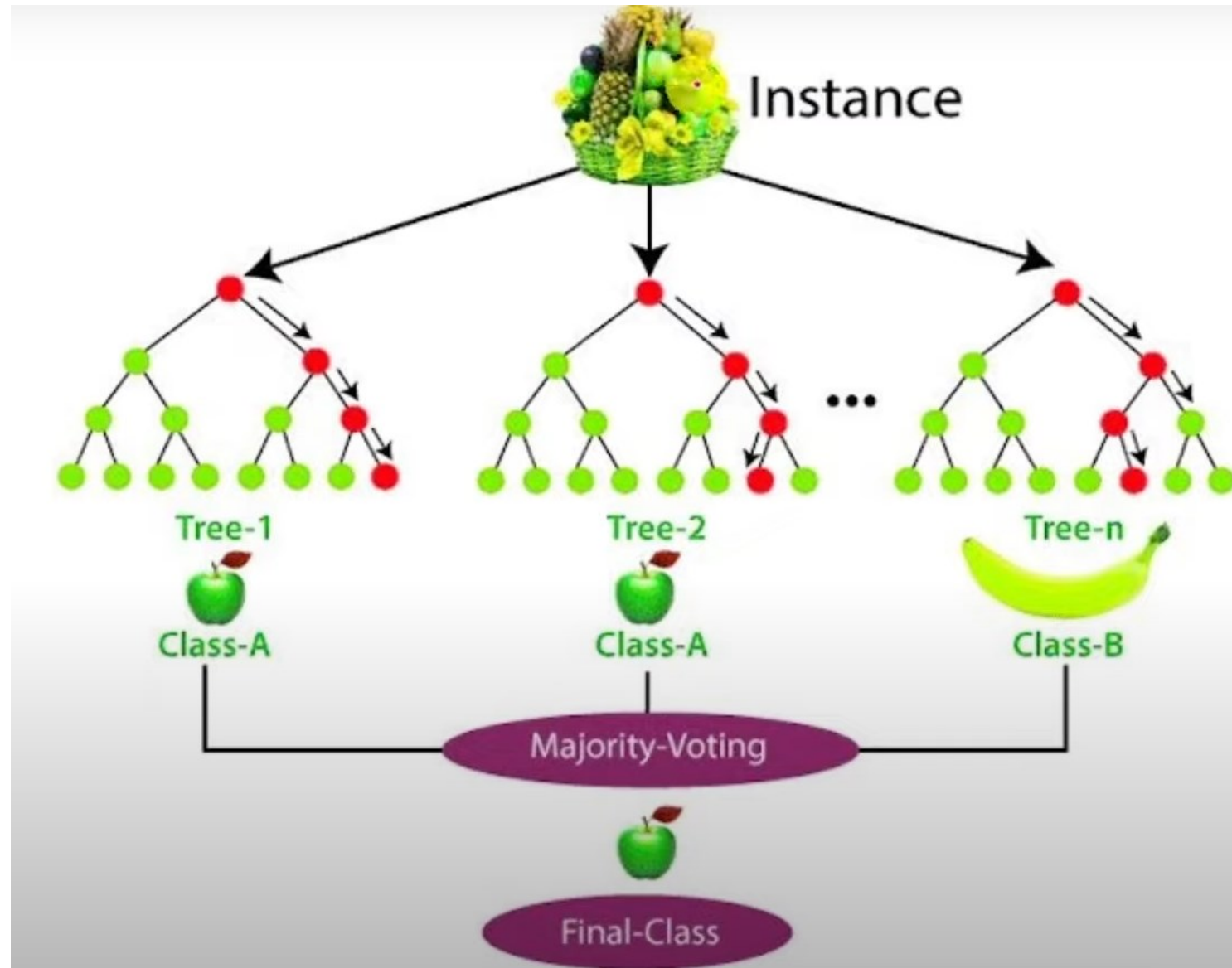
- Random forest is a commonly-used machine learning algorithm.
- A random forest is an ensemble learning method where multiple decision trees are constructed and then they are merged to get a more accurate prediction.
- Random forest became popular because of its ease of use and flexibility in handling both classification and regression problems.

## 1. Build random forests

- a) If the number of examples in the training set is  $N$ , take a sample of  $n$  examples at random - but with replacement, from the original data. This sample will be the training set for generating the tree.
- b) If there are  $M$  input variables,  $m$  variables are selected at random out of the  $M$  and the best split on these  $m$  is used to split the node. The value of  $m$  is held constant during the generation of the various trees in the forest.
- c) Each tree is grown to the largest extent possible.





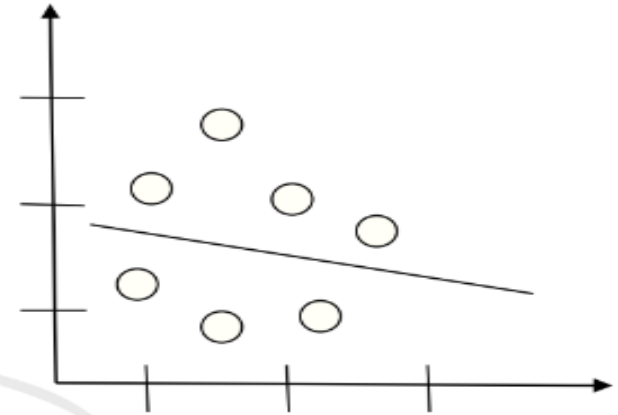


1. It takes less training time as compared to other algorithms.
2. It predicts output with high accuracy, even for the large dataset it runs efficiently.
3. It can also maintain accuracy when a large proportion of data is missing.

1. A weakness of random forest algorithms is that when used for regression they cannot predict beyond the range in the training data, and that they may over-fit data sets that are particularly noisy.
2. The sizes of the models created by random forests may be very large. It may take hundreds of megabytes of memory and may be slow to evaluate.
3. Random forest models are black boxes that are very hard to interpret.

# Underfitting

- When the ML model built is not able to completely learn the trend from the available data.
- Example- accuracy is 70% where expectation is to get 90%
- It usually occurs:
  - When the model is too simple and not able to learn the trend properly
  - While trying to avoid overfitting by stopping the training data feed to the model at an early stage;
  - Due to insufficient features considered while feeding training data



# Overfitting

- Occurs when the ML model built tries to learn more than required and results in a complex model.
- Overfitting usually occurs in a model while trying to learn all data points in a training data and end up memorizing them.
- Ex. training error is 95% but test error is 80%.
- Overfitting can be avoided by following methods:
  - Ensemble techniques
  - Removing unwanted features
  - Increase data size for training
  - Regularization
  - Stop feeding train data earlier

