Non-overshooting PD and PID controllers design for First And Second Order Closed Loop Systems

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***Abstract*—This study focuses on devising PD and PID controllers tailored for unique plant systems. The parameters for the PID controller are optimized to ensure a stable closed-loop system with a frequency response that consistently decreases over time. Through this approach, we identify specific regions within the parameter space of the controller. Criteria like gain crossover frequency and phase isodamping characteristics guide the selection of the most suitable solution from the identified options. We evaluate the performance of these PD and PID controllers in controlling roll rates using aileron deflection inputs and provide code implementations to support our findings.**

***Index Terms*—non-overshooting step response, PID controllers, PID controllers, Gain crossover frequency, isodamping property, styling, insert**

I. Introduction

PID controllers stand out for their straightforward design and ease of tuning, making them a favorite in industrial control settings [1]. The literature is brimming with diverse methodologies for PID controller design. One popular approach involves tweaking the PID coefficients to fine-tune a performance metric for the closed-loop system using optimization techniques. Another angle focuses on tuning PID controllers according to specific frequency criteria like gain margin, phase margin, and gain crossover frequency [5]. A noteworthy study, [6], introduced a design strategy centered on phase margin and gain crossover frequency, with the crossover frequency being determined through an integral performance index.

The field has also seen advancements in adaptive control and auto-tuning techniques for PID controllers [7]. Control engineers have not overlooked transient response control, aiming for either a non-overshooting or minimal overshoot in step responses for real-world applications. Various methods have been proposed to tackle this challenge. For example, [8] laid out conditions for state-space models to yield a non-overshooting step response, while [9] used a min-max optimization method to strategically position zeros for minimal overshoot. In another approach, [10] designed a compensator tailored for specific minimum phase systems to ensure a non-overshooting closed-loop step response.

Other innovative solutions include a two-parameter controller aimed at reducing overshoot in closed-loop step responses [11], as well as state feedback designs focused on non-overshooting step responses [12]. Additionally, research in [16] explored the relationship between transfer function pole and zero locations and step response overshoot, informing the development of a PID controller to mitigate overshoot.

The magnitude optimum method has also gained traction in PID controller design [25–28]. Here, the goal is to adjust controller parameters so that the magnitude of the closed-loop system frequency response approaches 1. While this method fosters a uniform frequency response across a wide range, it doesn't necessarily guarantee a non-overshooting step response.

In this study, our objective is to craft PD and PID controllers tailored to specific system types to achieve non-overshooting step responses. Based on empirical observations, systems with a monotonically decreasing frequency response tend to exhibit minimal overshoot in step responses [29]. With this in mind, we select PD and PID controller coefficients to shape a closed-loop frequency response with this desirable property. This approach yields certain parameter inequalities for PID controllers, which we solve numerically to pinpoint specific regions in the parameter space. From these regions, a range of controllers can be developed to achieve a non-overshooting step response. Controllers meeting set criteria for gain crossover frequency and phase isodamping are given priority. The isodamping property ensures that the phase of the loop gain frequency response remains consistent around the gain crossover frequency, bolstering the closed-loop system's resilience to gain fluctuations.

To validate our approach, we conduct simulations focusing on roll rate and aileron deflection, demonstrating the efficacy of our PD and PID controller designs. The subsequent sections of this paper are organized as follows: Section 2 outlines our proposed PD and PID controllers designed for non-overshooting step responses. In Section 3, we delve into the performance assessment of these controllers, presenting both simulation findings and theoretical insights related to roll rate and aileron input. Finally, Section 4 wraps up with our conclusions.

II. PROBLEM FORMULATION

In this section, we unveil our innovative PD and PID controller designs tailored to ensure stability in closed-loop systems while achieving non-overshooting step responses. Our design approach also incorporates criteria like gain crossover frequency and phase isodamping. We've selected specific plant models to test and implement these controllers effectively.

Let's delve into the control structure illustrated in Figure 1, which showcases a unit negative feedback configuration. The controller, denoted as $C(s)$ and potentially either a PD or PID controller, needs to be meticulously designed to suit a minimum phase plant, $G(s)$. The ultimate aim here is to prevent any overshoot in the step response of the closed-loop system.

To realize this objective, we fine-tune the controller parameters in such a way that the magnitude of the closed-loop system's frequency response exhibits a monotonically decreasing trend. This ensures that the system remains stable while also minimizing overshoot, aligning perfectly with our design goals. Or

$$\mid (Hj\omega 2) \mid < \mid H(j\omega 1) \mid \text{ , for : } \omega 2 > \omega 1 \text{ ,} \qquad (1)$$

where $H(j\omega)$ is the frequency response of the closedloop system. The following remarks could be expressed for using relation (1).

Remark 1: Condition (1) means that when the frequency ω increases, the numerator of $|H(j\omega)|$ increases smaller than its denominator.

Remark 2: Condition (1) holds true specifically for minimum phase plants, emphasizing the importance of ensuring the closed-loop system itself is minimum phase. Consequently, both the plant and the controller need to be designed in a manner that guarantees the closed-loop system maintains this minimum phase characteristic.

Remark 3: When condition (1) is met, the resulting step response typically exhibits either minimal overshoot or no overshoot at all. For instance, a second-order system featuring complex conjugate poles and a damping ratio ranging between 0.7 and 1 fulfills condition (1), yet it may still display a minor overshoot in its step response, generally below 5%. In practical control engineering scenarios, such a slight overshoot is often deemed acceptable. It's worth noting that for transfer functions characterized by complex conjugate poles, satisfying condition (1) can lead to a step response with only a marginal overshoot.

III. MATHEMATICAL FORMULATIONS

A. *Non-overshooting PID controller design for first-order systems*

The plant transfer function is considered as

$$G(s) = \frac{K}{1+Ts} \qquad (2)$$

The PID controller with the following transfer function is considered:

$$C(s) = K_p\left(1 + \frac{1}{T_i s} + T_d s\right) \qquad (3)$$

where Kp, Ti and Td are the proportional gain, integrator and derivative time constants, respectively.
The closed-loop transfer function is calculated as

$$H(s) = \frac{KK_p(T_iT_ds^2 + T_is + 1)}{T_i(T + KK_pT_d)s^2 + T_i(1 + KK_p)s + KK_p} \qquad (4)$$

According to (4), the closed-loop system is stable if the following relations are satisfied

$$\begin{aligned} KK_p &> 0, \\ Ti(1 + KK_p) &> 0, \\ Ti(T + KK_pT_d) &> 0. \end{aligned} \qquad (5)$$

Since $K > 0$, the following relations could be derived from (5)

$$K_p > 0, \qquad T_i > 0, \qquad T + KK_pT_d > 0. \qquad (6)$$

This implies that for ensuring stability in the closed-loop system, both K_p and T_i need to be positive, while T_d can potentially be negative. Conversely, to prevent undershoot in the step response of the closed-loop system, the coefficients in the numerator of equation (4) should all be positive. This means that

$$K_pT_iT_d > 0, \quad K_pT_i > 0, \quad K_p > 0. \qquad (7)$$

Thus, to attain a step response without undershoot, Td must be positive, too. Finally, we have

$$K_p > 0, \quad T_i > 0, \quad T_d > 0. \qquad (8)$$

The magnitude square of the closed-loop system frequency response is calculated as

$$|H(j\omega)|^2 = K^2K_p\frac{(T_dT_i\omega^4 + T_i(T_i - 2T_d)\omega^2 + 1)}{\alpha(\omega)} \qquad (9)$$

where

$$\begin{aligned} \alpha(\omega) = T_i^2(T + KK_pT_d)^2\omega^4 + T_i(T_i(1 + KK_p)^2 \\ -2KK_p(T + KK_pT_d))\omega^2 + K^2K_p^2 \end{aligned} \qquad (10)$$

To realize monotonically decreasing condition (1), the following inequality should be fulfilled

$$\begin{aligned}T_i^3\{Ti(T-T_d)(T+T_d+2KK_pT_d)\\ -2TT_d(T+KK_pT_d)\}\omega_1^2\omega_2^2(\omega_2^2-\omega_1^2)\\ +T_i^2T(T+2KK_pT_d)(\omega_2^4-\omega_1^4)\\ +Ti\{Ti(1+2KK_p)-2KK_pT\}(\omega_2^2-\omega_1^2)>0. \quad (11)\end{aligned}$$

According to (1),$(\omega_2^2-\omega_1^2)$ and $(\omega_2^4-\omega_1^4)$ are positive. On the other hand, considering positive values for Ti, Kp and Td, the term $Ti^2T(T+2KK_pTd)$ is always positive. Thus the sufficient condition for realization of (11) is that the first and third terms in (11) are greater than or equal to zero. Thus we have

$$Ti(T-T_d)(T+T_d+2KK_pT_d)\geq 2TT_d(T+KK_pT_d) \quad (12)$$

$$Ti(1+2KK_p)\geq 2KK_pT. \quad (13)$$

The realization of (12) requires the satisfaction of the following two relations:

$$T_d\leq T. \quad (14)$$

$$Ti\geq\frac{2T_d(T+KK_pT_d)}{(T-T_d)(T+T_d+2KK_pT_d)} \quad (15)$$

To establish (13), the following condition should be met

$$Ti\geq\frac{2KK_pT}{1+2KK_p} \quad (16)$$

Finally, attaining a stable closed-loop system with nonovershooting and non-undershooting step response requires the following conditions:

$$0<T_d\leq T,\quad K_p>0,\quad T_i>0 \quad (17a)$$

$$Ti\geq\frac{2KK_pT}{1+2KK_p} \quad (17b)$$

$$Ti\geq\frac{2T_d(T+KK_pT_d)}{(T-T_d)(T+T_d+2KK_pT_d)} \quad (17c)$$

Finally, the PID controller design for plant (2) is summarized in Algorithm 1.

Algorithm 1.

Step 1. Choose an arbitrary positive value for Kp and an arbitrary Td, where $0<Td\leq T$.

Step 2. Now, find an appropriate value for Ti ensuring (17b) and (17c).

B. Non-overshooting PD controller design for integrating systems

In this section, the following type 1 second-order plant is considered:

$$G(s)=\frac{K}{s(1+Ts)} \quad (18)$$

where K and T are arbitrary positive parameters. To track constant reference values, a PD controller with the following transfer function could be utilized for plant (18):

$$C(s)=K_p(1+T_ds). \quad (19)$$

Considering plant (18) and controller (19), the following closed-loop system transfer function is obtained:

$$H(s)=\frac{KK_p(T_ds+1)}{Ts^2+(1+KK_pT_d)s+KK_p}. \quad (20)$$

According to (20), the necessary and sufficient conditions to reach a stable closed-loop system without undershoot in its step response occurs when:

$$K_p>0,\quad T_d>0. \quad (21)$$

The magnitude square of the closed-loop system frequency response is given by:

$$|H(j\omega)|^2=\frac{K^2K_p^2(T_d^2\omega^2+1)}{\psi(\omega)}, \quad (22)$$

where

$$\psi(\omega)=T^2\omega^4+K^2(K_p^2T_d^2+2KK_p(T_d-T)+1)\omega^2+K^2K_p^2. \quad (23)$$

According to (1), the function (22) is monotonically decreasing if the following inequalities are satisfied:

$$\begin{aligned}T^2(\omega_2^4-\omega_1^4)+T_d^2T^2\omega_1^2\omega_2^2(\omega_2^2-\omega_1^2)\\ +(2KK_p(T_d-T)+1)(\omega_2^2-\omega_1^2)>0. \quad (24)\end{aligned}$$

Considering (21), the sufficient condition for establishment of (24) is:

$$T_d\geq\frac{2KK_pT-1}{2KK_p}. \quad (25)$$

Finally, if the following conditions are fulfilled, a closed-loop system step response with zero overshoot and undershoot could be obtained:

$$\begin{aligned}K_p>0,\quad T_d>0,\\ T_d\geq\frac{2KK_pT-1}{2KK_p}. \quad (26)\end{aligned}$$

The PD controller design for plant (18) is illustrated in the following algorithm.

Algorithm 2.

Step 1. Select an arbitrary positive value for K_p.

Step 2. Now, choose an appropriate value for T_d satisfying (26).

C. *Non-overshooting PID controller design for second-order systems*

Consider the following stable second-order plant:

$$G(s) = \frac{K}{s^2 + As + B} \tag{27}$$

where K, A and B are arbitrary positive constants. The closed-loop system transfer function for plant (18) and controller (3) becomes

$$H(s) = \frac{KK_p(T_iT_ds^2 + T_is + 1)}{T_is^3 + T_i(A + KK_pT_d)s^2 + T_i(B + KK_p)s + KK_p}. \tag{28}$$

It could be easily verified that the closed-loop system is a stable system without undershoot in its transient response, if

$$K_p > 0, \quad T_i > 0, \quad T_d > 0, \tag{29}$$

$$Ti \geq \frac{KK_p}{(A + KK_pT_d)(B + KK_p)} \tag{30}$$

Substituting s = jω in (28) gives

$$|H(j\omega)|^2 = \frac{K^2K_p^2(T_i^2T_d^2\omega^4 + T_i(T_i - 2T_d)\omega^2 + 1)}{\beta(\omega)}, \tag{31}$$

where,

$$\begin{aligned}\beta(\omega) = {} & T_2\omega^6 + T_2\left((A + K_KT_d)^2\right.\\ & - 2(B + K_KK_p)\omega^4 + T_i\left(T_i(B + K_KK_p)^2\right.\\ & - 2K_KK_p(A + K_KK_pT_d)\omega^2 + K^2K_p^2.\end{aligned} \tag{32}$$

The monotonically decreasing condition (1) gives

$$\begin{aligned}& T_i^2(\omega_2^6 - \omega_1^6) + T_i^4T_d^2\omega_1^4\omega_2^4(\omega_2^4 - \omega_1^4) + \\ & + T_i^3(T_i - 2T_d)\omega_1^2\omega_2(\omega_2^4 - \omega_1^4) + \\ & + T_i^2(2AKK_pT_d + A^2 - 2B - 2KK_p)(\omega_2^4 - \omega_1^4) + \\ & + T_i^3(-2T_d(A^2 + AKK_pT_d - 2B - 2KK_p) + \\ & + Ti(A^2 + 2AKK_pT_d - 2B - 2KK_p - B^2T_d^2 \\ & - 2BKK_pT_d^2))\omega_2^2\omega_2^2(\omega_2^2 - \omega_1^2) + \\ & + T_i(T_i(B^2 + 2BKK_p) - 2AKK_p)(\omega_2^2 - \omega_1^2) > 0\end{aligned} \tag{33}$$

The sufficient conditions to realize (33) are =

$$T_i \geq 2T_d$$

$$T_i \geq \frac{2AKK_p}{(B^2 + 2BKK_p)}$$

$$T_d \geq \frac{2KK_p + 2B - A^2}{2AKK_p}$$

$$A^2 + 2AKK_pT_d - 2B - 2KK_p - B^2T_d^2 - 2BKK_pT_d \geq 0$$

$$T_i \geq \frac{2T_d(A^2 + AKK_pT_d - 2B - 2KK_p)}{A^2}$$

$$+2KK_pT_d(A - BT_d) - 2B - 2KK_p - B^2T_d^2. \tag{34}$$

Incorporating conditions (30) and (34) yields the following non-overshooting step response conditions for plant (27)

$$K_p > 0, \quad T_i > 0, \quad T_d > 0, \tag{35a}$$

$$T_i \geq 2T_d \tag{35b}$$

$$Ti \geq \frac{KK_p}{(A + KK_pT_d)(B + KK_p)} \tag{35c}$$

$$T_i \geq \frac{2AKK_p}{(B^2 + 2BKK_p)} \tag{35d}$$

$$T_d \geq \frac{2KK_p + 2B - A^2}{2AKK_p} \tag{35e}$$

$$A^2 + 2KK_pT_d(A - BT_d) - 2B - 2KK_p - B^2T_d^2 > 0 \tag{35f}$$

$$Ti \geq \frac{2T_d(A^2 + AKK_pT_d - 2B - 2KK_p)}{A^2 + 2KK_pT_d(A - BT_d) - 2B - 2KK_p - B^2T_d^2} \tag{35g}$$

Inequality (34f) could be rewritten as

$$(B^2 + 2BKK_p)T_d^2 - 2AKK_pT_d - A^2 + 2B + 2KK_p < 0 \tag{36}$$

If $A^2 \leq 2B$ is considered, then we have

$$\begin{aligned}& A^2K^2K_p^2 + (B^2 + 2BKK_p)(A^2 - 2B - 2KK_p) \\ & = (A^2 - 4B)K^2K_p^2 + 2B(A^2 - 3B)KK_p \\ & \quad + B^2(A^2 - 2B) \leq 0\end{aligned} \tag{37}$$

On the other hand, $B^2 + 2BKK_p > 0$. Thus, according to (37), the left side of (36) should be positive. Therefore, the following limitation for the parameters of the plant (27) should be realized

$$A > 2B. \tag{38}$$

Considering the constraint (38), relation (36) will be satisfied if the following relations are fulfilled:

$$K_p \leq \frac{A^2 - 2B}{2K} \tag{39}$$

$$T_d < \frac{AKK_p}{B^2 + 2BKK_p} + \frac{\sqrt{A^2K^2K_p^2 + (B^2 + 2BKK_p)(A^2 - 2B - 2KK_p)}}{B^2 + 2BKK_p} \tag{40}$$

Moreover, according to (39), the left side of (35e) is negative. This means that inequality (35e) will be automatically fulfilled. Thus relation (35) could be rewritten as

$$K_p > 0, \quad T_i > 0, \quad T_d > 0, \tag{41a}$$

$$K_p \leq \frac{A^2 - 2B}{2K} \tag{41b}$$

$$T_d < \frac{AKK_p}{B^2 + 2BKK_p}$$

$$+ \frac{\sqrt{A^2K^2K_p^2 + (B^2 + 2BKK_p)(A^2 - 2B - 2KK_p)}}{B^2 + 2BKK_p} \quad (41c)$$

$$Ti \geq \frac{KK_p}{(A + KK_pT_d)(B + KK_p)} \quad (41d)$$

$$T_i \geq \frac{2AKK_p}{(B^2 + 2BKK_p)} \quad (41e)$$

$$T_i \geq 2T_d \quad (41f)$$

$$Ti \geq \frac{2T_d(A^2 + AKK_pT_d - 2B - 2KK_p)}{A^2 + 2KK_pT_d(A - BT_d) - 2B - 2KK_p - B^2T_d^2} \quad (41g)$$

The following algorithm describes the details of the PID controller design for plant (27).

Algorithm 3.

Step 1. Choose a positive value for K_p ensuring (41b).

Step 2. Select a positive value for T_d satisfying (41b), (41c).

Step 3. Now, find an appropriate positive value for T_i satisfying (41d) – (41g).

IV. CONTROL LAW FORMULATION

While multiple controllers might meet the criteria outlined in equations (14), (23), or (31), additional criteria are necessary to identify the most suitable controllers from this pool. To refine the PID design procedure, we introduce two new conditions. The first condition is the gain crossover criterion, ensuring that the loop gain frequency response maintains a unit magnitude at a predetermined frequency. ω_c.Or

$$|(G(j\omega_c)C(j\omega_c))| = 1. \quad (41)$$

Condition (41) is essential for achieving the desired speed in the transient response of the closed-loop system. For a PD controller, which has just two design parameters, satisfying the additional condition (41) suffices. However, for a PID controller with three design parameters, it's also crucial to meet the following isodamping property:

$$\frac{d}{d\omega}\angle[G(j\omega)C(j\omega)]\bigg|_{\omega=\omega_c} = 0. \quad (42)$$

This indicates that phase variations around the crossover frequency are minimal, resulting in a closed-loop system that is resilient to fluctuations in gain. By applying conditions (41) and (42) to plant (2), we derive the following relationships among the PID controller parameters:

$$K_p = \frac{Ti\omega_c\sqrt{1 + T^2\omega_c^2}}{K\sqrt{(1 - TiT_d\omega_c^2)^2 + T_i^2\omega_c^2}} \quad (43)$$

$$T_i = \frac{-a_1 \pm \sqrt{a_1^2 + 4a_2T}}{2a_2} \quad (44)$$

where

$$a_1 = 1 + T^2\omega_c^2 + 2TT_d\omega_c^2,$$
$$a_2 = \omega_c^2(T - T_d)(TT_d\omega_c^2 - 1). \quad (45)$$

By integrating equations (17), (43), and (44), we can determine the PID controller parameters tailored for plant (2). This process essentially transforms the three-dimensional region defined by condition (17) into a one-dimensional region with respect to the parameter T_d. While multiple sets of PID controller parameters can satisfy these relationships, designers have the flexibility to choose one that best suits their needs. Notably, the solutions obtained can differ based on adjustments to the crossover frequency parameter.

Remark 4: The gain crossover frequency ω_c should be appropriately selected such that inequalities (17) are fulfilled. This could be realized by trial and error. Moreover, this parameter determines the transient response speed of the closed-loop system. Increasing ω_c decreases the settling time of the closed-loop system step response. Moreover, increasing ω_c increases the control signal amplitude. Thus this parameter should be selected such that a satisfactory transient response with permissible control signal will be obtained. For plant (18), applying (41) causes the following constraint on the PD controller parameters in (19)

$$K_p = \frac{\omega_c\sqrt{1 + T^2\omega_c^2}}{K\sqrt{1 + T_d^2\omega_c^2}} \quad (46)$$

This converts the admissible two-dimensional parameter region obtained from (26) to a one-dimensional region in terms of parameter Td.

Remark 5:For the PD controller design for plant (18), any arbitrary gain crossover frequency ω_c could be selected. Only the value of Td should be selected such that inequality (d2) will be realized. However, this parameter determines the settling time of the closedloop system step response and the maximum amplitude of the control signal.

Finally, for the second-order plant (27), the gain crossover frequency and isodamping property lead to the following relations between PID controller parameters:

$$K_p = \frac{Ti\omega_c\sqrt{(B - \omega_c^2)^2 + A^2\omega_c^2}}{K\sqrt{(1 - TiT_d\omega_c^2)^2 + T_i^2\omega_c^2}} \quad (47)$$

$$T_i = \frac{-b_1 \pm \sqrt{b_2^2 - 4b_0b_2}}{2b_2} \quad (48)$$

Where

$$b_0 = \frac{A(B + \omega_c^2)}{(B - \omega_c^2)^2 + A^2\omega_c^2},$$
$$b_1 = -(1 + 2b_0T_d\omega_c^2),$$
$$b_2 = \omega_c^2\left(b_0(1 + T_d\omega_c^2) - T_d\right). \quad (49)$$

Conditions (40) and constraints (47) and (48) yield a one-dimensional parameter region in terms of parameter T_d which

could be considered as the solution region.
Remark 6: The gain crossover frequency ω_c should be chosen such that inequalities in (40) are satisfied. Substituting (47) and (48) in (40) will not lead to straightforward inequalities in terms of Kp, Ti , Td and ω_c. Thus finding an appropriate value for ω_c for satisfaction of conditions (40) could be performed by software. In the next section, the ability of the so-designed non-overshooting PD and PID controllers for the control of the mentioned plants is verified through some experimental and simulation tests.

V. RESULTS AND DISCUSSION

In this section, the performance of the proposed PID and PD controllers is investigated. Three examples are given to show the effectiveness of non-overshooting PID controllers.

Example 1: Consider a modular DC servomotor system.A permanent magnet DC motor coupled with a tachometer to measure its angular velocity and a position potentiometer to measure its position is considered.The open loop transfer function of the speed servo system is

$$G_s = \frac{\omega(s)}{V(s)} = \frac{3.0}{0.1s+1}$$

where $\omega(t)$ is the motor angular velocity and V(t) is the voltage applied.

Controller Parameters:

	K_p	T_i	T_d	$\omega(c)$
Example 1	0.364	0.6265	0.02	5.0
Example 2	0.348	2.658	0.05	3.0

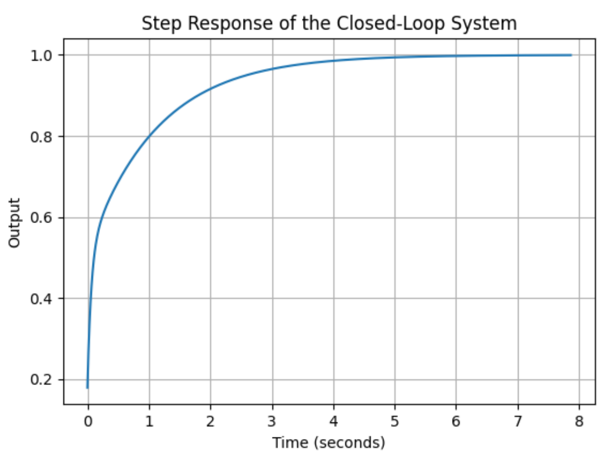

Fig. 1. Simulation for Example 1

Example 2: Roll Control of an Aircraft
Consider the roll control of an aircraft where the aileron input directly controls the roll rate of the aircraft. Let's assume a simplified transfer function for the roll dynamics of the aircraft given by:

$$G_s = \frac{\phi(s)}{\delta_a(s))} = \frac{30}{s(0.2s+1)}$$

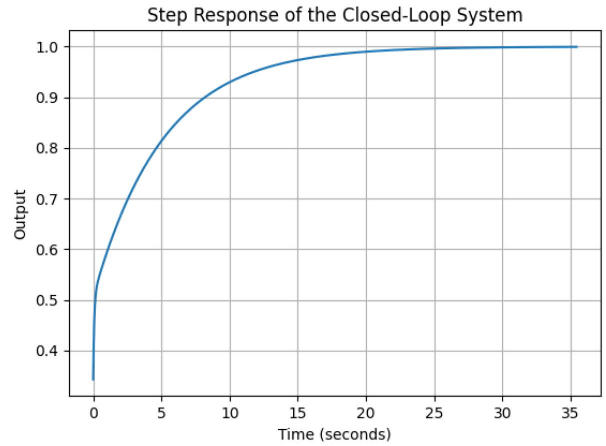

Fig. 2. Simulation for Example 2

Where $\phi(s)$ is the Laplace transform of the roll angle (in degrees). $\delta_a(s)$ is the Laplace transform of the aileron deflection angle (in degrees).

Controller Parameters:

	K_p	T_d	$\omega(c)$
Example 1	0.04622	1.0	5.0
Example 2	0.00066	1.0	0.02

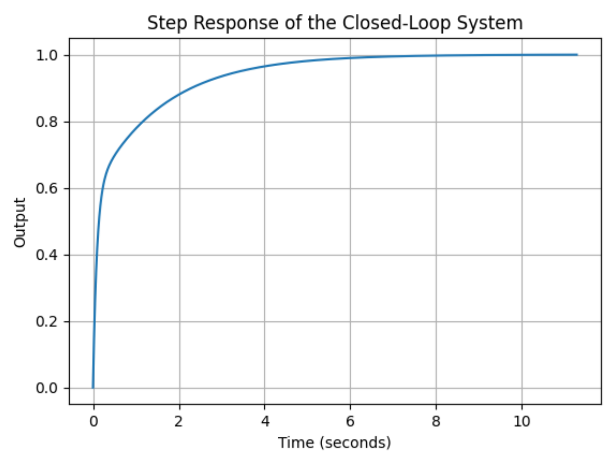

Fig. 3. Simulation for Example 1

Example 3:
In this example, the ability of the non-overshooting PID controller for controlling a second-order plant is verified. Consider a second-order plant with the following transfer function:

$$G_s = \frac{K}{s^2 + As + B}$$

We take the values of K=**1.0** A=**2.0** B=**1** $T_d = \mathbf{0.5}$

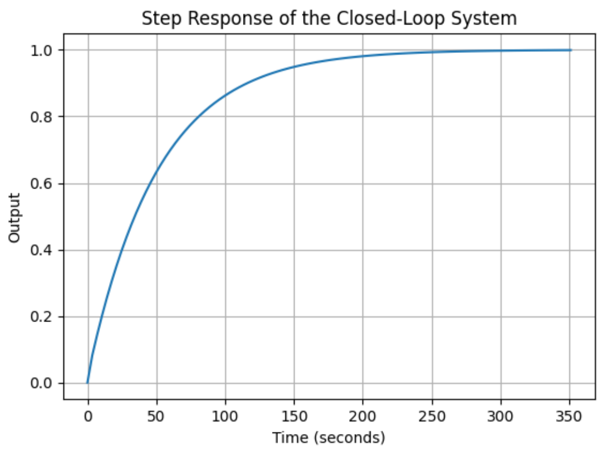

Fig. 4. Simulation for Example 2

We get the transfer function to be:

$$G_s = \frac{K}{s^2 + 2s + 1}$$

The input range for ω_c is given to be from **0.1** to **0.5**
We get the valid ω_c values to be **[0.1, 0.2, 0.30000000000000004, 0.4]**

Controller Parameters:

	K_p	T_i	T_d	$\omega(c)$
Example 1	1.005	66.438	0.5	**0.1**
Example 2	1.8031	2.379	0.5	**1.0**

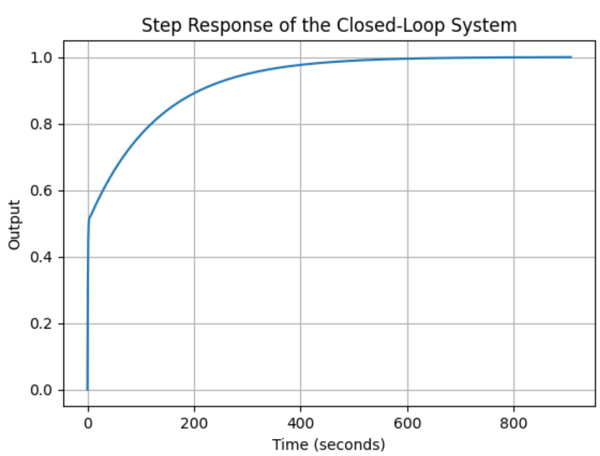

Fig. 5. Simulation for Example 1

CONCLUSION

This study presents an innovative analytical tuning approach specifically designed for PID controllers, aimed at reducing overshoot in step responses of closed-loop systems. The core of this method lies in the precise adjustment of PID parameters to achieve a closed-loop system frequency response that decreases monotonically, effectively eliminating overshoot. Furthermore, the methodology tactically modifies the phase of the loop gain to ensure stability around the selected gain crossover frequency, bolstering the closed-loop system's resilience to fluctuations in gain. Through simulations, we confirm the efficacy of the newly proposed PID controller that minimizes overshoot.

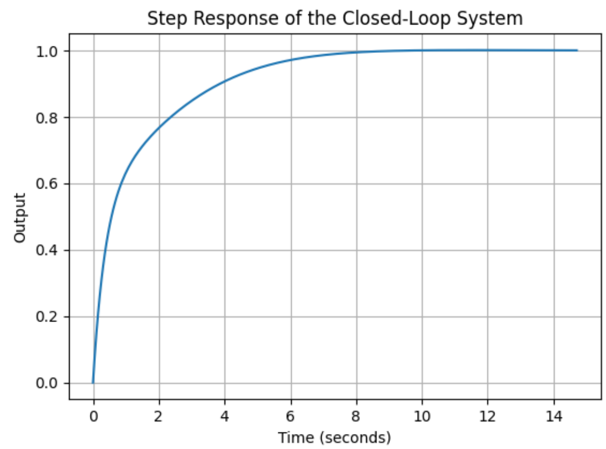

Fig. 6. Simulation for Example 2

Work Done

All controllers were implemented in Python, enabling automated determination of parameter values that promote non-overshooting conditions. The scripts not only detect cases where parameter tuning falls short of achieving non-overshooting characteristics but also generate and display corresponding step responses and closed-loop transfer functions, juxtaposed with plant functions. This extensive output facilitates in-depth analysis of system dynamics and enhances comprehension of how parameter modifications influence closed-loop performance.

For the full scripts, guidelines, and supplementary materials, please refer to our GitHub repository. Visit the link below for access to the codes and further details:

- https://github.com/varun-sappa/ae322a-term-project

Future Work

Enhanced Parameter Analysis: Enhance the Python script to provide a comprehensive range of parameter values compatible with non-overshooting conditions, allowing for more nuanced controller adjustments.

Adaptive Control Strategies: Explore adaptive control strategies that dynamically adjust PID parameters based on system response characteristics and environmental changes, enhancing controller robustness and adaptability across varying operating conditions.

Experimental Validation: Conduct experimental validation of the proposed non-overshooting PID controller in real-world scenarios to assess its practical applicability and performance compared to traditional PID tuning methods. This validation would provide valuable insights into real-world challenges and validate the effectiveness of the proposed methodology

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