HIDDEN PLAIN SIGHT 4

The uncertain universe

ANDREW THOMAS

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The Uncertain Universe

Andrew Thomas studied physics in the James Clerk Maxwell Building in Edinburgh University, and received his doctorate from Swansea University in 1992.

His *Hidden In Plain Sight* series of books are science bestsellers.

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HIDDEN IN PLAIN SIGHT 4

The Uncertain Universe

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PREFACE

In physics and mathematics, the turbulent 20th century taught us that the universe was a far more uncertain place than was previously realised. Hopefully, the turbulent 20th century was a century in which important lessons were learnt.

Moving into the 21st century, 2014 was a very exciting and controversial year for physics. The result of an experiment was released which potentially represented the most momentous scientific breakthrough of this century. However, if the result was true then it also appeared to open the door to a strange physics of parallel universes which more resembled the physics of *Star Trek* than the physics of Newton.

This book considers the exciting — and perhaps ultimately disappointing — story of 2014, showing that not all the 20th century lessons about uncertainty have been learnt.

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2015

I now have a Twitter account on which I will post updates: twitter.com/andrewthomas101

I have now sold over 100,000 books. I could not do what I do without your support. Thank you so very much.

Oh, and I do know that there are no penguins at the South Pole …

"Uncertainty is the only certainty there is,
and knowing how to live with insecurity
is the only security."

— John Allen Paulos

1

LAS VEGAS

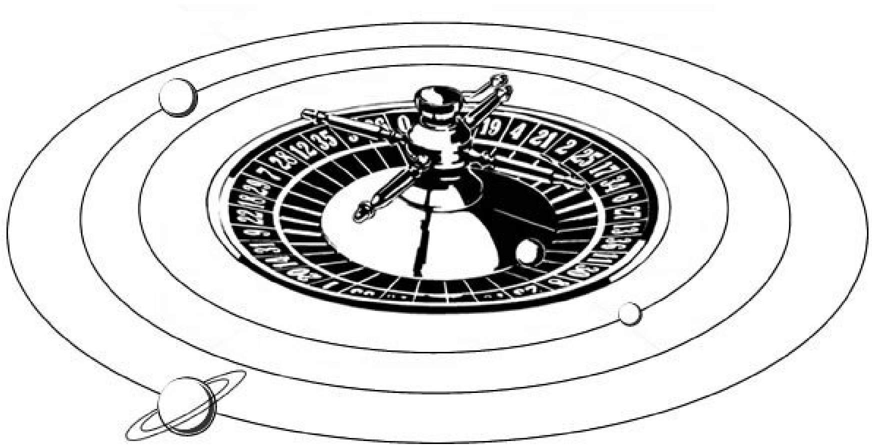
It is the summer of 1975 in Portland, Oregon.

Straight-A physics student Norman Packard is wondering how he might make some easy money over the summer months. With this in mind, he has started reading a book called *Beat The Dealer* written by Edward Thorp. The book details how it can be possible to win at blackjack by counting the cards which have already been played. If enough cards can be remembered, it is possible to gain a small percentage advantage over the house. Packard has an excellent head for numbers, however, when he tried the card-counting method in a casino he found the results to be disappointing — leading eventually to losses.

Feeling despondent over his failure, Packard turned his attention to other games and had a brainwave: he could use his knowledge of physics to win at roulette. Specifically, he knew that the roulette wheel and ball had to move according to Newton's three laws of motion and his law of universal gravitation. If it was possible to measure the speed at which the ball was thrown and the speed at which the wheel was spun then it should be possible to predict the outcome.

Indeed, in his book *The Newtonian Casino*, Thomas Bass describes how a roulette wheel resembles the motion of planets under gravity: "The game of roulette, with a ball revolving around a spinning disk, resembles a model universe governed by the laws of Newtonian mechanics. Planetary ball circles solar disk until gravity sucks it out of orbit and pulls it down to stasis."

The following diagram shows the planets orbiting a spinning roulette wheel:



The first step was to determine the feasibility of the project. Was there enough predictability in the spin of a roulette wheel?

In order to determine this, Packard decided to measure the speed of the ball when it was thrown around the wheel. If there was too much unpredictable variation in the speed of the ball, the plan was not going to work. Packard went to real gaming tables in Las Vegas and clicked a hidden microphone in his hand each time the ball completed one revolution of the wheel. On returning to his hotel room, Packard played back the tape and measured the time between the clicks. In this way, Packard could determine not only the velocity of the ball, but could also calculate the amount the velocity decreased between each revolution. Crucially, Packard discovered that the deceleration of the ball was consistent: "As soon as you realise that there is this layer of predictability in the dynamics of the wheel and the ball then that opens the door to having the possibility to beat the game."

At this point, Packard knew the potential was there. But he also knew that a lot of work was required if he was to beat the casino. Packard called on the help of his best friend from childhood, Doyne Farmer, who was now a physics undergraduate at University of California, Santa Cruz. He managed to sell his idea to Farmer by emphasizing the intellectual aspects of the project. According to Packard: "I was interested in the intellectual interest of conquering randomness, or understanding exactly what were the limits of randomness and predictability."

However, there was also another — less worthy — motivation: "It seemed like a great idea to rip off casinos, who get so much pleasure out of ripping off everyone else."

While the prediction of the particular winning number would be beyond their capabilities, Packard and Farmer aimed to predict the area of the wheel in which the ball would land. They divided the wheel into eight areas called *octants*, each octant containing about five numbers. Ironically, a great advantage was provided by the random arrangement of the numbers on the wheel. If the numbers in a particular octant on the wheel are considered, those numbers appear to be in a completely random arrangement when laid-out on the betting table. This meant it was going to be harder to detect their method.

First, Farmer went to the main supplier of roulette wheels in Nevada and obtained a casino-grade roulette wheel. It cost $1,500 which represented the life savings of the two undergraduates. They arranged for the wheel to be shipped to Portland. However, at this point the FBI became suspicious and interviewed Farmer about his motives.

Farmer tried to convince the FBI that he collected roulette wheels, and he had no interest in using it for gambling. Instead, Farmer protested that this type of table — inlaid with twelve kinds of African wood — was merely a collector's item. Surprisingly, the FBI bought his story. However, unwilling to risk transporting the wheel again, Packard and Farmer based their experiment in Santa Cruz.

In order to accurately track the movement of the ball, a high-speed camera was rigged over the wheel. The forces acting on the moving ball could then be determined. As Farmer explained: "Roulette is a physical system, and if you can measure the initial position and velocity of the ball, and you know the forces acting on it, you should be able to predict what is going to happen." Analysis of the movie footage allowed them to accurately measure the ball's velocity and deceleration. The effect of friction and drag on the ball had to be determined. They could also measure *scatter* and *bounce*. Scatter occurred when the ball hit the diamonds on the sloping sides of the wheel, and bounce occurred when the ball hopped from cup-to-cup before finally coming to rest. The final program included a set of equations which resembled the equations used by NASA for landing spacecraft on the Moon.

But how could complex calculations be performed in the middle of a busy casino? Obviously, they had to be discreet. A small computer was required. [1] For $250 they obtained via mail order a development kit incorporating an 8-bit 6502 microprocessor, which was the same processor used by Apple computers at the time. The computer had access to 5K of RAM, while the program was squeezed into 4K of ROM. The computer was small enough to be hidden under one armpit, with the battery pack hidden under the other armpit. The computer was attached to a microswitch embedded in the sole of the shoe which allowed for data input. This switch was activated by the movement of the big toe.

The result of the calculation was transmitted to the player via three vibrating buzzers attached to the chest. In binary, the three vibrators could encode a digit from 1 to 8, (001, 010, 011, etc.), thus indicating on which octant the bet should be placed.

The first stage was to calculate some parameters which were specific to the table, such as the tilt of the table, and the rate at which the central rotor decelerated. Once this information was obtained, the only other variables depended on the particular croupier working that day. This included the speed at which the croupier threw the ball, and the speed at which the croupier spun the rotor. This information could be obtained by standing next to the table and capturing the data. When a point on the wheel passed, the microswitch in the shoe was clicked. By comparing two successive clicks, the speed of the rotor could be calculated. Similarly, two clicks were made when the ball passed, providing the speed at which the croupier threw the ball.

Once the wheel was spun, there was a window of just fifteen seconds to enter the data and make the calculation: two clicks for the wheel, and two clicks for the ball. After that fifteen seconds, the bet had to be made.

Taking the equipment to Las Vegas, the team favoured the older, faded casinos on Fremont Street such as the Golden Nugget, the Mint, and the Horseshoe Club. These were the first casinos to emerge in Las Vegas in the 1930s, before a New York gangster by the name of Bugsy Siegel built the Flamingo club. That was the start of the modern mega-casinos, now strung-out down the main Strip.

While clearly the computerized system was not completely accurate every time, over a period of time the system provided a big statistical advantage. For every dollar bet, $1.44 was won.

However, this success was bound to draw the attention of the casino security. The increased tension caused the player to sweat excessively, and the increased conductivity resulted in the player receiving occasional electric shocks from the chest mechanism — which resulted in visible spasms. Bear in mind, though, that in the mid 1970s the personal computer was unheard of, and no one would have suspected that a player might be wearing such high-technology. Also, the team agreed never to win more than a few hundred dollars in any given session. For these reasons, the team managed to avoid detection.

One of the most successful evenings came at the Circus Circus casino. The casino's attractions included a flying trapeze over the main floor, and slot machines on a merry-go-round. On that night, a particularly predictable croupier spun the ball in a regular fashion, and the computer worked like a dream: "The predictions were right on target. Playing quarters, we recouped our losses and stacked several hundred dollars in chips in front of us. There was a gut rush of excitement in seeing everything fall into place. After all the time spent testing and troubleshooting it, the computer was finally up and running perfectly. I no longer had the slightest doubt. From that session, I knew the game of roulette had been beaten."

Overall, the team estimate they made about $10,000. It was not a large sum — certainly not as large as they could have made. This is because the motivation of the team was the intellectual challenge of the scientific project, to see if it was possible for physics to beat the system. And in that sense, they were completely successful.

And afterwards in the bar, in celebratory mood, the team raised a toast … to Sir Isaac Newton.

Doyne Farmer is now a professor and a specialist in probability theory at the Santa Fe Institute. Norman Packard now lives in Italy, researching and developing forms of artificial life.

In 1985 the law was changed in Nevada to make it a crime, punishable by up to ten years in prison, to use a computing device in a casino capable of predicting the outcome of a game.

The clockwork universe

The method of Packard and Farmer was successful because Newton's three laws of motion and his law of universal gravitation are completely *deterministic*: there is no mention of chance. Given the state of a system — the position and velocity of all its parts — its movement from that point on is, according to Newton, completely predictable and determined.

As the project team described in the book *The Newtonian Casino*: "Our basic idea was to determine the initial position and velocity for the ball and rotor. We then hoped to predict the final position of the ball in much the same way that a planet's later position around the Sun is predicted from initial conditions."

On this basis, the 18th century French mathematician and philosopher the Marquis de Laplace realised that, if complete knowledge was obtained of the position and velocities of all the objects in the universe, then the future could be forever predicted. The universe — and the future — would be completely determined: "We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which, at any given moment, knew all of the forces that animate Nature and the mutual positions of the beings that compose it, if this intellect were vast enough to submit the data to analysis, could condense into a single formula the movement of the greatest bodies of the universe and that of the lightest atom. For such an intellect nothing could be uncertain and the future, just like the past, would be present before its eyes."

This view became known as the *clockwork universe*. The image of the clockwork universe became the dominant model in physics for 200 years after Newton, and also influenced other areas of society. As Michael Mosley said in the BBC series *The Story of Science*: "The way that Newton had shown that a few universal laws could explain so much of the physical world inspired other intellectuals to look for universal laws that could explain human behaviour, politics, even history. Newton became a hero to revolutionaries who dreamt of utopian societies founded on reason."

The founding fathers of America decided that the laws and political institutions of their new country would be based on science and rational thought. There would be no place for monarchs and traditions. As Edward Dolnick says in his book *The Clockwork Universe*: "As they spelled out the design of America's political institutions, the founders clung to the model of a smooth-running, self-regulating universe. In the eyes of the men who made America, the checks and balances that ensured political stability were directly analogous to the natural pushes and pulls that kept the Solar System in balance." As Woodrow Wilson later wrote: "The Constitution of the United States has been made under the dominion of the Newtonian theory."

Thomas Jefferson installed a portrait of Isaac Newton in a place of honour in his home at Monticello, while Benjamin Franklin's favourite portrait (by the Scottish artist David Martin) showed him reading in front of a bust of Newton:

So, if the clockwork universe model suggests that the future is completely determined by the past, where does probability creep into the equation? If all events are determined with certainty, why are there casinos, and betting shops, and roulette wheels? The answer is that, in this situation, probability arises purely due to **ignorance**. Yes, if we were a super-being with complete knowledge of the position and velocity of every atom in the roulette wheel and the spinning ball, then the clockwork universe suggests we could predict the outcome with certainty. But we inevitably do not have complete knowledge. We only have limited knowledge. And it is as a result of this inevitable ignorance that we cannot predict the outcome of all events with certainty.

The clockwork universe was perhaps a reassuring concept. A universe without surprises, a universe in which progress was assured, a universe in which knowledge could increase without bound.

As we shall see, it was also proved to be completely wrong.

"What hath God wrought?"

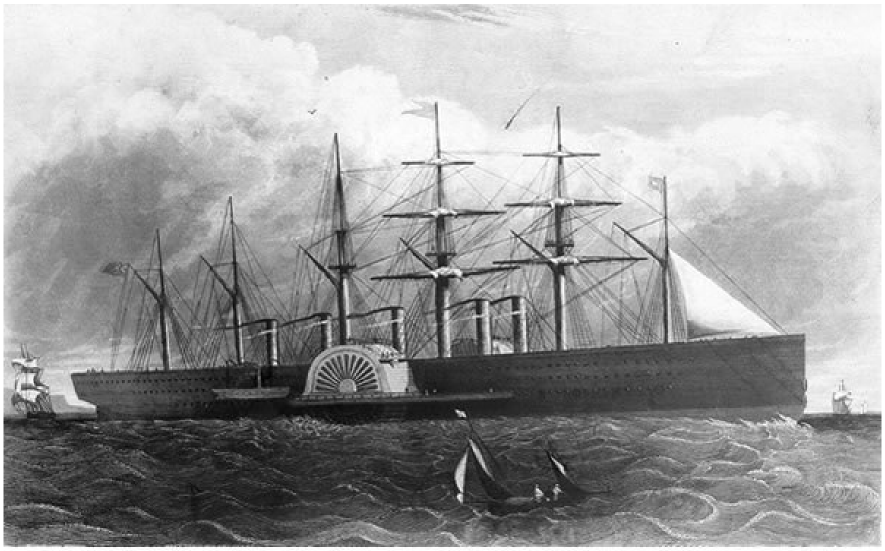
The mid 19th century was a time of great confidence and certainty. Newtonian mechanics had brought the industrial revolution, and Britain and the United States had ridden the wave of the industrial revolution to become the two most powerful nations on Earth.

The industrial revolution was powered by steam. The steam engine powered manufacturing industry, and the steam train spanned continents. However, the steamship still took ten days to connect Britain and America, and this represented the fastest possible communication across the Atlantic. As a result, a group of the greatest scientific minds determined to reduce communication times by physically linking the two continents by a telegraph cable.

In 1844, the American inventor Samuel Morse transmitted the first message on the electronic telegraph. The message travelled down wires 38 miles connecting Washington and Baltimore. Considering this achievement took place during a period of such confidence and certainty, Morse's first message was ominously prophetic. The message simply read: "What hath God wrought?"

The scientific advisor of the transatlantic cable project was the Belfast-born William Thomson, who was known as one of the finest physicists of the age. Thomson had entered the University of Glasgow at the age of ten, and within two years he was publishing scientific papers. He became a professor of natural philosophy by the age of 22, and went on to make important contributions in the field of thermodynamics, playing an important role in establishing physics as a modern scientific discipline.

Thomson designed the transatlantic telegraph cable, improving the core of the electric cable and its insulation. The cable was planned to be laid from the iron steamship the Great Eastern. At 692 feet, this was the largest ship ever built, itself a symbol of the confidence and certainty of the era.



In 1866, the cable successfully linked Britain and America for the first time. *The Times* called the feat "The most wonderful achievement of this victorious century". For his role in this triumph, Queen Victoria knighted William Thomson who later became the first British scientist to enter the House of Lords where he became Lord Kelvin.

It was during this period that Lord Kelvin issued a famous quote which reflected the confidence and certainty of this period: "There is nothing new to be discovered in physics now. All that remains is more and more precise measurement."

This view of Lord Kelvin was certainly not isolated.[2] In 1894, the American physicist Albert Michelson said: "The most important and fundamental laws and facts of physical science have all been discovered. Our future discoveries must be looked for in the sixth place of decimals."

However, Lord Kelvin's pronouncement could not have been more inaccurate. In the following twenty years — the first two decades of the 20^{th} century — science was rocked to the core as the old certainties were swept away.

2

THE UNCERTAIN QUANTUM

Lord Kelvin must have been aware at the time of his pronouncement that there were three major mysteries involving radiation which still required an explanation. In this chapter, we will examine those three mysteries of radiation.

Firstly, if you take a metal bar (for example) and heat it to over 500 degrees Celsius, it will emit electromagnetic radiation: it will begin to glow. It will first become red hot, then it will become white hot. So there is a relationship between the temperature of the bar and the frequency of the emitted radiation (this is how potters used to determine the temperature of their kilns). At lower temperatures, radiation of a lower frequency is emitted (red visible light), whereas at higher temperatures, higher-frequency radiation of a brilliant blue-white colour is emitted.

However, when physicists tried to describe the relationship between temperature and frequency, they found a problem. The classical model agreed well at low frequencies, but it predicted ever-increasing emitted energy at high frequencies (ultraviolet and higher). This was because the model predicted that the power of the radiation was proportional to the square of the frequency of that radiation. Hence, as progressively higher frequencies were considered, the power of the radiation was predicted to rise to an infinitely large amount. This was known as the *ultraviolet catastrophe*.

At the turn of the 20^{th} century, the German physicist Max Planck cast a new light on the mystery. However, things might have turned out differently. When he was 16 years old, Planck had been told by the physicist Philipp von Jolly that he should not study physics because "in this field, almost everything is already discovered, and all that remains is to fill a few unimportant holes." This revealed that the certainty of Lord Kelvin was also shared by other physicists. Planck replied that he did not want to discover anything new, he merely wanted to obtain a greater understanding of what was already known.

In order to solve the ultraviolet catastrophe, Planck took the radical step of suggesting that the emitted radiation was composed of tiny, indivisible packets of energy, with each packet being called a *quantum*. The energy of each packet was proportional to the frequency of the radiation:

$$e = hf$$

Where *e* is the energy of the emitted quantum, *f* is the frequency of the radiation, and *h* is Planck's constant. Planck's constant is equal to 0.0000000000000000000000000006626, a very small number which has a huge importance. It appears in many important equations in physics.

This result indicated that the amount of energy emitted at a certain frequency was proportional to the number of packets of energy being emitted with that frequency.

Using this method, Planck was able to explain why the energy of the emitted radiation did not increase to infinity at higher frequencies: it would take too much energy to make a single quantum at those high frequencies. Hence, at high frequencies, there where simply fewer packets of energy being emitted.

On the 14th December 1900, Planck told his son that he may have made a discovery as important as those of Newton. Later that day, he presented his result to the Berlin Physical Society.

Quantum theory had been born.

However, Planck, like most physicists, believed the physical world was a continuum, with matter being composed of a smooth, continuous substance. This latest result of Planck, however, seemed to suggest that energy could be formed of discrete, discontinuous chunks. Planck was a conservative scientist who was not motivated to further consider the physical significance of his discovery. That would require a different experiment, and a scientist much more ready to consider revolutionary ideas. We have to move on to consider the second major mystery involving radiation: the *photoelectric effect*.

In 1899, the German physicist Philipp Lenard shone a bright light onto metal and discovered that the metal ejected electrons. According to the classical viewpoint, it would be expected that the ejected electrons would have more energy if the light was brighter. However, this was not what was discovered. Instead, it was found that the energy of the ejected electrons was entirely independent of the intensity of the light. In fact, even very low intensity light was capable of ejecting electrons. Instead, the energy of the electrons was proportional to the frequency (i.e., the colour) of the incident light. For example, blue light (high frequency) ejects more energetic electrons that red light (low frequency).

This was a tremendous puzzle which could not be explained by the classical model. How could low intensity light be capable of ejecting electrons? Well, in his *annus mirabilis* ("miracle year") of 1905, Albert Einstein proposed a solution.

Einstein proposed that light was composed of packets of energy, with each packet having an energy equal to Planck's constant multiplied by the frequency of the light:

$$e = hf$$

Hence, this is precisely the same formula discovered five years earlier by Max Planck. This formula now made sense of the photoelectric effect. Low intensity light would be composed of few packets of energy, but the energy of each individual packet was only dependent on the frequency of the light. Hence, even a single packet of light could potentially eject an electron from metal, the energy of the light transferring to the kinetic energy of the electron. This explained how even low intensity light could eject electrons.

High intensity light was composed of many more of these packets of energy, but the energy of each individual packet remained only proportional to the colour of the light. Hence, increasing the brightness of the light did not increase the energy of the ejected electrons: you had to alter the colour of the light to do that.

So, while Max Planck could not understand the physical significance of his earlier result, Einstein's explanation of the photoelectric effect made the meaning clear: light energy really **was** quantized into discrete packets. Each packet of light energy was called a *photon*.

At first, there was considerable resistance to Einstein's theory of light particles. Planck, in particular, could not accept Einstein's theory. In 1913, Planck (and other supporters) wrote a letter of recommendation for Einstein's membership to the Prussian Academy of Sciences. The letter was full of praise, however, Planck included the following passage: "In sum, one can hardly say that there is not one among the great problems, in which modern physics is so rich, to which Einstein has not made a remarkable contribution. That he may have missed the target in his speculations, as, for example, in his hypothesis of the light quanta, cannot really be held too much against him, for it is not possible to introduce really new ideas even in the exact sciences without taking a risk."

However, the existence of the photon was soon experimentally confirmed, and Einstein received his only Nobel prize in 1921 for his discovery of the light quantum.

The third major mystery involving radiation considered the nature of the light emitted from different materials. When a substance is heated, it emits a light with a characteristic colour. For example, when salt (sodium chloride) is heated in a flame, it glows with a yellow colour. This explains the common yellow colour of sodium street lighting. Every chemical element has a different associated colour. If the light from a pure element is passed through a prism, the prism splits the light into its various constituent colours. This spread of colours is called a *spectrum*.

For white light, a rainbow effect is produced as white light contains all colours. However, each particular chemical element has a different spectrum, and it was found that the spectrum contains very sharp peaks in intensity at different light wavelengths. This means the spectrum contains very distinct bright coloured lines. It is as if each element has an identifying barcode. Hydrogen, for example, has four distinct lines in its spectrum. By analysing the light from stars in this manner, it is possible to identify the chemical composition of those stars.

These lines in the spectra of elements was surely revealing something important about the structure of atoms, but the mechanism by which these lines were produced was a mystery.

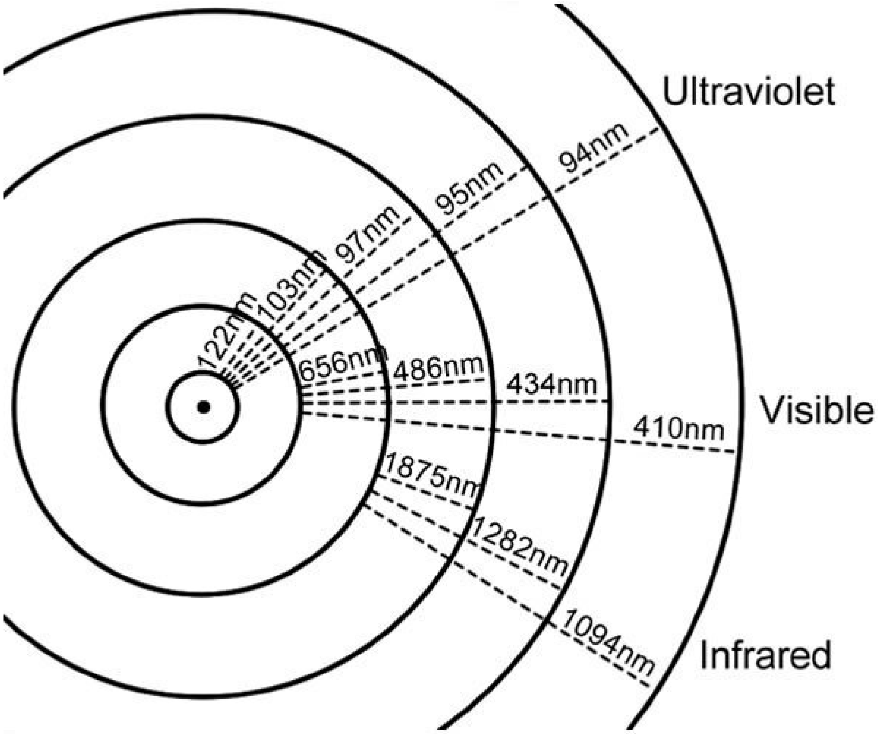
The stage was set for the entrance of a young Danish physicist who has been called the father of quantum physics: Niels Bohr. Bohr was undoubtedly the most important figure in the development of quantum theory. Bohr worked throughout his life to achieve the greatest understanding of quantum behaviour.

Bohr arrived at Manchester University in 1912 to work with Ernest Rutherford, who had just revealed crucial details of the structure of the atom. Rutherford had revealed that the atom consisted of a positively-charged nucleus — only one ten-thousandth the size of the whole atom — surrounded by a cloud of orbiting electrons. For obvious reasons, this was called the *planetary model* of the atom.

However, there were problems with the planetary model. According to classical physics, the orbiting electrons should radiate energy before eventually spiralling into the nucleus (as an analogy, the Moon is losing energy and is therefore slowing down as it orbits the Earth). Bohr considered the previous discovery of Einstein — that light energy was quantized — to suggest that the energy of electrons in their orbits was quantized. This would mean that the electrons could only occupy special orbits with clearly defined energy levels. Hence, there could be no continuous drift towards the nucleus.

Bohr then realised that this could also provide an explanation for the spectral lines. Electrons which were orbiting further away from the nucleus would have higher energy than electrons orbiting nearer the nucleus. When an electron jumped (a *quantum jump*) from a higher orbit to a lower orbit, it would release a photon. Because of the law of conservation of energy, the energy of the photon would be equal to the energy lost by the electron. Hence, electron jumps between certain clearly defined orbits would result in a photon being emitted with a particular colour. This would explain the sharp lines in the spectrum.

The following diagram shows how electrons can jump between different energy levels in the hydrogen atom. The larger the jump, the greater the energy released, and so higher frequency light is produced. The diagram shows the frequency of the light produced for each quantum jump. This explains the four distinct lines in the visible spectrum of hydrogen.



In his final interview, Bohr expressed the difficulties he had faced in trying to determine the structure of the atom merely by considering its colour spectrum: "Just as if you have the wing of a butterfly, then certainly it is very regular with the colours and so on, but nobody thought that one could get the basis of biology from the colouring of the wing of a butterfly."

So all three mysteries associated with radiation could be solved by the realisation that energy can be quantised into packets. But what really made the new quantum theory so devastating was that it introduced uncertainty to science.

The uncertainty principle

In Germany, the young physicist Werner Heisenberg was working to develop a more detailed picture of quantum behaviour. He decided to ignore the classical model of the atom — the planetary model — thus ignoring anything that could not be directly observed. Instead, Heisenberg only considered the features that could be directly observed and measured: the properties of the light emitted by the electrons as they jumped between two energy levels.

But there were many different orbits available for electrons to jump between. For example, an electron might jump from energy level E_3 to energy level E_1, or from energy level E_5 to energy level E_2. What was the best way to denote and analyse all these possible combinations?

When Heisenberg had a attack of hay fever, he retreated to Heligoland, a tiny pollen-free island off the German coast. It was in this distraction-free environment that Heisenberg made his great breakthrough. He plotted all the observed values in the form of a rectangular grid, known as a *matrix*.

In the following matrix, the energy emitted when an electron jumps from energy level E_2 to energy level E_1 is denoted by E_{21}, and so on. This single matrix, therefore, contains information about all the possible quantum jumps:

$$\begin{bmatrix} E_{11} & E_{12} & E_{13} & E_{14} & \cdots \\ E_{21} & E_{22} & E_{23} & E_{24} & \cdots \\ E_{31} & E_{32} & E_{33} & E_{34} & \cdots \end{bmatrix}$$

Heisenberg realised that a matrix operation could be used to extract the quantum properties — such as the position or momentum — of a particle. If, for example, the position of a particle was to be calculated, a particular matrix for quantum position would be used. If momentum was to be extracted, then a different matrix would be used. [3]

However, Heisenberg realised that if matrices ("matrices" is the plural of "matrix") have to be used to determine quantum mechanical behaviour, then that would introduce a remarkable side-effect. This is because in order to extract a particular property of a particle (position or momentum), matrix multiplication would have to be used. And matrix multiplication behaves very differently from conventional multiplication.

In order to see why this is the case, let us consider a simple example. Here is an example of a matrix composed of two rows and two columns:

$$\begin{bmatrix} 2 & 4 \\ 7 & 3 \end{bmatrix}$$

We might ask how matrix multiplication (the multiplication of two matrices) might be achieved. For example, how could we multiply the following two matrices:

$$\begin{bmatrix} 2 & 4 \\ 7 & 3 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 1 & 5 \end{bmatrix}$$

We can multiply these two matrices by the following sequence of steps:

First, consider the first row of the first matrix and the first column of the second matrix (shown in the two dashed ellipses in the following diagram). Take the first number in the first row of the first matrix (which is 2) and multiply it by the first number in the first column of the second matrix (which is 5). This gives us our first intermediate result of 10 (we must remember this). Then take the second number in the first row of the first matrix (which is 4) and multiply it by the second number in the first column of the second matrix (which is 1). This gives us our second intermediate result of 4.

We add our two intermediate results (10 and 4) together to get the answer 14, and that number goes in the first position in our result matrix:

$$\begin{bmatrix} 2 & 4 \\ 7 & 3 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 14 & \\ & \end{bmatrix}$$

To get our next entry in our answer matrix, once again we have to consider the first row of our first matrix, but now we consider the second column of the second matrix. We perform a similar series of steps to the first case, this time we need to calculate (2×6) + (4×5) to give an answer of 32:

$$\begin{bmatrix} 2 & 4 \\ 7 & 3 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 14 & 32 \\ & \end{bmatrix}$$

We continue in a similar fashion until we have filled all the positions in our result matrix. So here is the answer of our matrix multiplication:

$$\begin{bmatrix} 2 & 4 \\ 7 & 3 \end{bmatrix} \times \begin{bmatrix} 5 & 6 \\ 1 & 5 \end{bmatrix} = \begin{bmatrix} 14 & 32 \\ 38 & 57 \end{bmatrix}$$

You might want to check the steps yourself to make sure it is the correct answer. (7×5) + (3×1) = 38 and (7×6) + (3×5) = 57.

So matrix multiplication is fairly straightforward. But Heisenberg realised that if matrices have to be used to determine quantum mechanical behaviour, then that would introduce a remarkable side-effect. And that is because matrix multiplication is *noncommutative*.

Conventional multiplication is commutative in that it does not matter in which order the numbers are multiplied, the answer will be the same. For example, 7×5 will give the same result as 5×7. However, matrix multiplication is noncommutative in that the ordering does matter. If the ordering of the matrices is reversed, the answer will be different.

In order to see this, here is the previous matrix multiplication with the two matrices in reverse order:

$$\begin{bmatrix} 5 & 6 \\ 1 & 5 \end{bmatrix} \times \begin{bmatrix} 2 & 4 \\ 7 & 3 \end{bmatrix} = \begin{bmatrix} 52 & 38 \\ 37 & 19 \end{bmatrix}$$

You will see that the final result matrix is different. You can run through the steps of the calculation yourself if you wish. Clearly, matrix multiplication is noncommutative: the ordering does matter.

Heisenberg realised that the noncommutative properties of matrix mechanics had startling consequences for quantum behaviour. It suggested that if the position of a particle was measured, and then the momentum of the particle was measured, a different result would be obtained than if the momentum had been measured first. The ordering of the measurements mattered. It was as if the measurement of the position of the particle irretrievably "poisoned" the following momentum measurement. The only way to obtain an accurate measure of the particle's momentum would be to measure its momentum first. But that seemed to imply that it was not possible to accurately know **both** a particle's position **and** momentum!

Heisenberg tried to make sense of this startling result by realising that, in order to measure some aspect of a system, we inevitably alter the system being measured in some small way. Our measurement can, therefore, never precisely reflect the true state of the system being measured. For example, if we test the air pressure of a tyre using a gauge, some air has to inevitably leak out of the tyre — thus reducing the pressure. Or, in the case of a particle, we can precisely measure its position by projecting it as a point onto a screen. But that inevitably reduces the particle's velocity to zero — so we cannot obtain a momentum measurement.

This startling result became known as the *Heisenberg uncertainty principle*. For the first time, it appeared that there were fundamental limits on human knowledge about the universe. At a stroke, the idea of the clockwork universe proposed by the Marquis de Laplace was erased: it could never be possible to obtain complete information about the position and velocity of all the particles in the universe, so it could never be possible to predict with certainty the future of the universe.

As Heisenberg said: "'When we know the present precisely, we can predict the future' is as assumption — not the correct conclusion. Even in principle we cannot know the present in all detail."

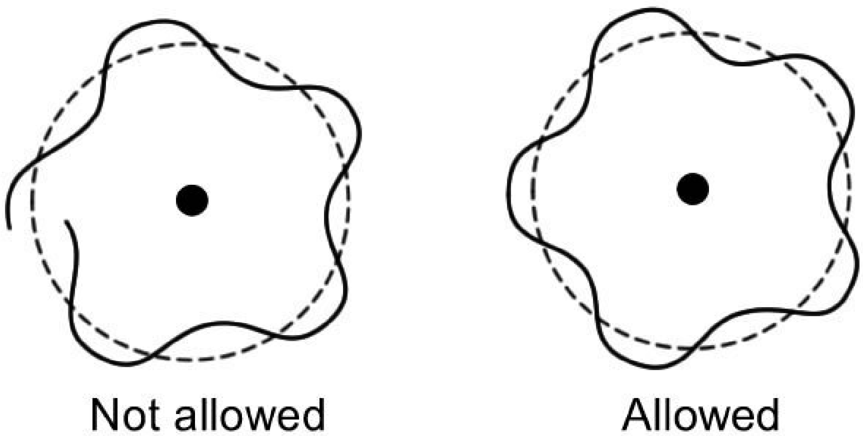
There was a limit on human knowledge. The old certainties were starting to unravel. From now on, the only certainty was uncertainty.

Schrödinger's waves

The development of Heisenberg's *matrix mechanics* was the first time that quantum theory represented a complete description of the quantum behaviour of a particle: the quantum equivalent of Newton's laws of mechanics. This complete theory became known as *quantum mechanics*.

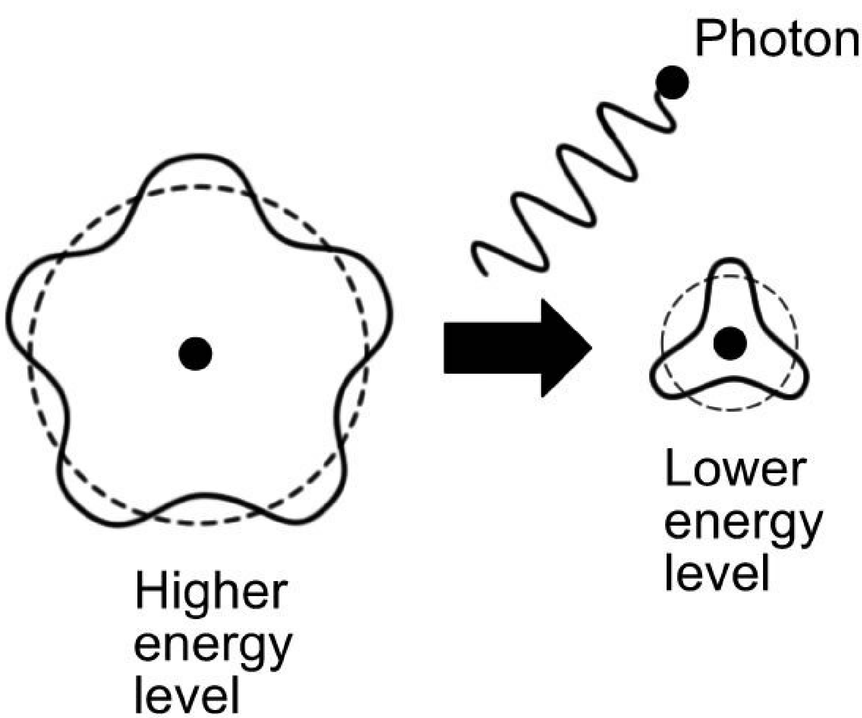
However, matrix mechanics was difficult to use — most physicists had never used matrices before — and difficult to visualise. Adoption of the new technique was slow. So when Erwin Schrödinger presented a new method which substituted the use of waves instead of matrices the method swiftly proved popular (most physicists were familiar with waves, and the model was much easier to visualise). Schrödinger's method became known as *wave mechanics*. [4]

Remember that Bohr had shown that electrons could only occupy special orbits with clearly defined energy levels. Hence, there could be no continuous drift towards the nucleus. Schrödinger used a mathematical model of a wave as a tool for determining these allowed orbits of electrons. An electron orbit was modelled as a wave around the nucleus. An orbit was only allowed if a whole number of wavelengths fitted around the orbit:



This explained why only certain orbits with quantised energy levels were allowed.

When an electron emitted a photon, it could move to a lower, allowed orbit:



This mathematical wave proposed by Schrödinger was called a *wavefunction*. But what did the wavefunction actually represent? Should it only be considered a mathematical tool which could be used for calculating the properties of particles, or did it actually represent some underlying physical wave which had not yet been detected?

The situation was made more complicated by the fact that the values of the wavefunction were *complex numbers*, meaning they had a "real" part and an "imaginary" part. Imaginary numbers are based on the square root of -1. As no real number squared can result in -1, this explained why these numbers were considered "imaginary". Only if the value was squared did the imaginary part disappear (if you square the square root of -1 then you are left with -1, which is a real number).

So the square of the value of the wavefunction represented something "real", something that could be measured in a laboratory. But what did the value represent? It was the German physicist Max Born who provided the incredible answer: the value of the square of the wavefunction represented the **probability** that the electron is found at a particular location. If the value of the wavefunction was larger at a particular location, there was more chance of finding the electron there when an attempt was made to measure its position. Crucially, before the measurement was made, it was simply impossible to obtain a more accurate fix on the electron's position. Before the measurement was taken, it was not possible to use more accurate equipment, or more precise mathematics, to precisely pin down the position of the electron. Before measurement, all that could be known was possibilities.

So this is what the great edifice of physics was finally reduced to: at the most fundamental level, it was only possible to talk about the probability of a particular event occurring. It was not possible to achieve greater certainty. The physicist was reduced to talking about likelihoods and chance, like a common gambling bookmaker.

Max Born's revelation of a fundamentally probabilistic universe meant the end of determinism. In a Newtonian deterministic universe, one event causes another in an entirely predictable manner. Quantum mechanics, however, revealed that the outcome of an event could only be considered in terms of probabilities.

In Chapter One we considered how probability — in a deterministic universe — was merely a product of ignorance. However, quantum probability was revealed to be a fundamental probability which was not a result of ignorance. As Manjit Kumar says in his book *Quantum*: "Quantum probability was not the classical probability of ignorance that could in theory be eliminated. It was an inherent feature of atomic reality. For example, the fact that it was impossible to predict when an individual atom would decay in a radioactive sample, amid the certainty that one would do so, was not due to a lack of knowledge but was the result of the probabilistic nature of the quantum rules that dictate radioactive decay."

Probability which is a result of ignorance can be eliminated. Quantum probability — which lies at the heart of all reality — was here to stay.

3

COPENHAGEN

Ernest Solvay was a Belgian industrialist and philanthropist who had made his fortune from developing a process for manufacturing sodium carbonate. The patents from the process provided Solvay with great wealth, and he used this money to fund a series of physics conferences in Brussels which attracted the greatest minds of the day.

Since the end of the First World War, Germany had been isolated and scientific conferences had excluded German scientists. However, in 1927 the king of Belgium announced that German scientists could be invited to the Solvay conference: "Seven years after the war the feelings which they aroused should be gradually damped down, a better understanding between peoples was absolutely necessary for the future, and science could help to bring this about."

The resulting 1927 conference entered the annals of physics legend by attracting 29 of the greatest physicists of all time, including 17 Nobel Prize winners. Among those attending the conference were Niels Bohr, Werner Heisenberg, Erwin Schrödinger, Max Born, Max Planck, and Albert Einstein.

The fortuitous timing of the 1927 Solvay event meant it occurred at precisely the time when the philosophical argument over the interpretation of quantum mechanics was at its height. In particular, the debates between the two geniuses of Bohr and Einstein are considered some of the most creative and ingenious in scientific history. According to the physicist John Wheeler: "In all the history of human thought, there is no greater dialogue than that which took place over the years between Niels Bohr and Albert Einstein about the meaning of the quantum." The philosopher C.P. Snow went deeper: "No more profound intellectual debate has ever been conducted."

On one side of the debate, Niels Bohr believed quantum mechanics provided a complete description of Nature at the microscopic level, in which case Nature was fundamentally governed by probabilistic processes. On the other side of the debate, Einstein believed that the current quantum theory was merely a temporary contrivance which would be replaced when a deterministic theory was uncovered.

In order to understand Bohr's viewpoint, let us return to the problem of quantum jumps of electrons orbiting the nucleus. When Ernest Rutherford first saw Bohr's revolutionary ideas about the quantum structure of the atom, he immediately raised a question: when does an electron know when to jump, and how does it decide where to jump? Reasonably enough, Rutherford wanted to know what underlying process controlled the quantum jumping: "Bohr's answer was remarkable. Bohr suggested that the whole process was fundamentally random, and could only be considered by statistical methods: every change in the state of an atom should be regarded as an individual process, incapable of more detailed description. We are here so far removed from a causal description that an atom may in general even be said to possess a free choice between various possible transitions."

If Bohr was correct, the implications were staggering: there was no "causal" description of the process, nothing should be considered as causing the quantum jumps. It was only ever possible to talk about the probability of an event occurring. This would imply that Nature, at its root, was random, and no deeper explanation could ever be produced.

Einstein, however, was far from convinced by this explanation. According to Einstein: "I find the idea quite intolerable that an electron exposed to radiation should choose **of its own free will**, not only its moment to jump off, but also its direction. In that case, I would rather be a cobbler, or even an employee in a gaming-house, than a physicist."

Einstein remained convinced that there had to be some underlying deterministic mechanism — hidden from our eyes — which was responsible for determining the timing and direction of the quantum jumps. In this respect, Einstein considered the current theory of quantum mechanics to be incomplete, merely a stop-gap until a more complete theory came along.

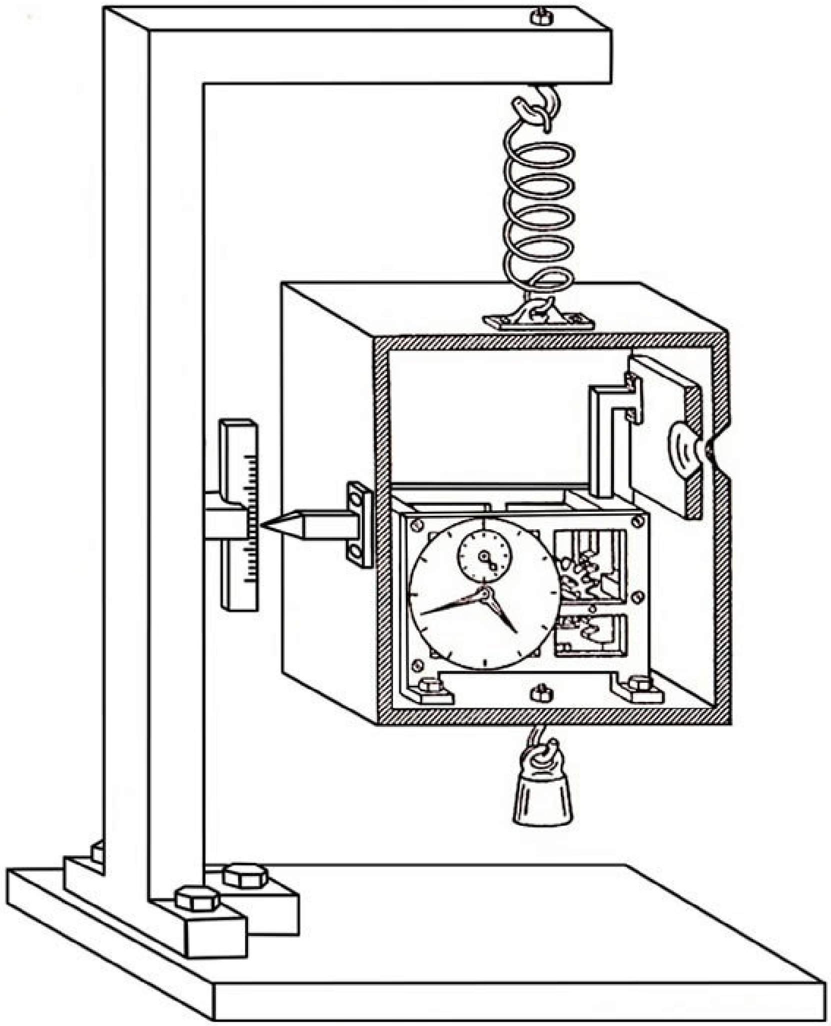
When Einstein arrived at the 1927 Solvay Conference he was initially reluctant to contribute his opinion, stating that he did not feel sufficiently knowledgeable about the latest developments in quantum mechanics. However, Einstein's diffidence disappeared when Heisenberg and Born issued a provocative joint statement: "We consider quantum mechanics to be a closed theory, whose fundamental physical and mathematical assumptions are no longer susceptible of any modification." Essentially, Heisenberg and Born were suggesting that quantum mechanics should be considered to be the final theory, and that the fundamental probabilistic nature of reality just had to be accepted. It was almost as if they were goading Einstein into making a response. In one of his most famous quotes, Einstein replied: "God does not play dice with the universe".

From now on, Einstein would no longer be silent about quantum mechanics.

Einstein's box

During the day, the conference was conducted at the Solvay International Institute for Physics in Brussels. However, all of the participants at the conference were staying at the Hotel Metropole, and it was in the dining room of the hotel that most of the heated discussion occurred. Each morning, Einstein would arrive for breakfast armed with a new thought experiment which he believed revealed a flaw in the uncertainty principle. Einstein and Bohr would continue their discussions as they walked to the institute, and during breaks in the conference. Usually by dinner back in the Metropole, Bohr would reveal a flaw in Einstein's argument, thus saving the uncertainty principle and the prevailing interpretation of quantum mechanics.

However, one of Einstein's thought experiments was so ingenious that Bohr was stunned when he heard of it. Einstein asked Bohr to consider a box with a hole in its side (see the following superbly detailed diagram, actually drawn by Bohr himself). The hole has a shutter which can be opened or closed by a clock mechanism inside the box. The clock inside the box is synchronised with a clock outside the box in the laboratory. The top surface of the box is attached to a spring, and a scale at the side of the box means it is possible to determine the weight of the box.



At the start of the experiment, the box is weighed. At a certain point in time, the clock in the box opens the shutter and a photon of light escapes. After this point, if the box is weighed again then it is possible to determine the mass of the photon. Then, from Einstein's own equation, $E=mc^2$, it is possible to determine the energy of the emitted photon. The time at which the photon was emitted is also known because it is the time at which the shutter opened. It therefore appears possible to know the energy of the photon and the time at which it escaped. But according to the uncertainty principle this should not be possible as time and energy are a pair of properties with a relation similar to position and momentum: the more accurately you know the time, the less accurately you should be able to know the energy. According to the uncertainty principle, you should never be able to know both time and energy with perfect accuracy.

This came as a tremendous shock to Bohr. Einstein's box seemed to undermine the uncertainty principle, and hence the whole edifice of quantum mechanics. As far as Bohr was concerned, it would be "the end of physics" if Einstein was correct.

As Einstein and Bohr walked back to the Hotel Metropole that evening, there was a spring in Einstein's step and a broad grin on his face as he felt he had at last defeated the uncertainty principle. Bohr, however, looked flustered as he struggled to keep up with the confident Einstein.

That evening it was noted that Bohr looked like "a dog who has received a thrashing". Bohr spent a sleepless night going over the experiment time and time again in his head. It was not until the early hours that Bohr believed he had found the flaw in Einstein's reasoning, and he managed to grab just a few hours sleep before confronting Einstein over breakfast.

Bohr realised that Einstein — in his rush to overthrow quantum mechanics — had forgotten the implications of Einstein's own theory of general relativity. General relativity predicted that a clock in a stronger gravitational field would run slower than a clock in a weaker gravitational field. Bohr realised that when the photon was emitted from the box, the spring attached to the top of the box would lift the box slightly higher in the Earth's gravitational field. This would result in the clock inside the box no longer being perfectly synchronised with the clock in the laboratory. Hence, it was no longer possible to obtain the time measurement with perfect accuracy. The uncertainty principle was saved.

When Bohr told Einstein of this solution to the problem over breakfast, it was Einstein's turn to look stunned. Once again, he had failed in his attempt to overthrow quantum mechanics.

From now on, Einstein gave up attacking the uncertainty principle as the weak spot of quantum mechanics. Instead, he turned his attention to the quantum model of reality itself, because it was here that quantum mechanics predicted some very strange things indeed.

Is the Moon there when no one is looking at it?

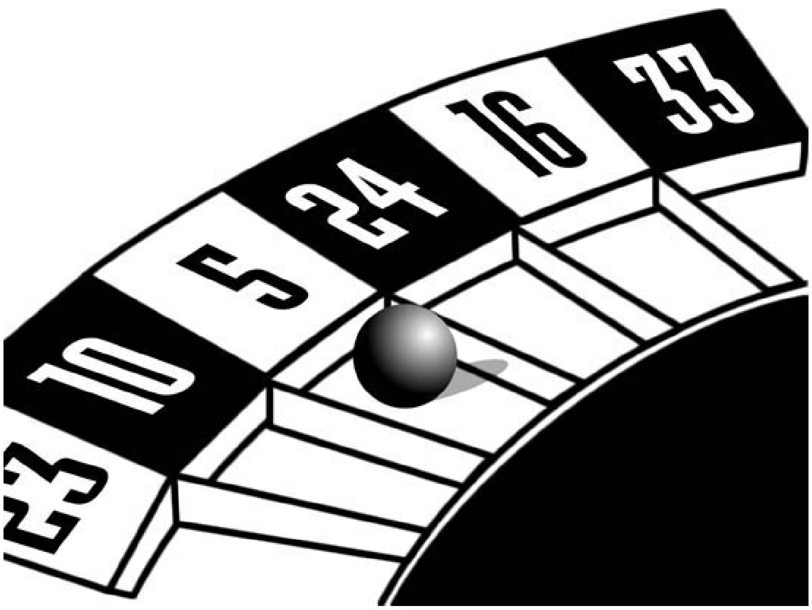
According to the interpretation of Niels Bohr, the uncertainty principle had revealed that it was no longer possible to consider the result of a scientific observation without also considering the effect of that observation on the object being observed. In other words, it was no longer possible to draw a clear boundary between the observer and the object being observed: the effect of the observer became all-important.

With this in mind, Bohr argued that, before an observation or measurement was made, it was scientifically meaningless to talk about the properties of a particle. Bohr said: "It is wrong to think that the task of physics is to find out how Nature is. Physics concerns what we can say about Nature." In other words, it is only our observations which are real. It is meaningless to talk about objective reality in the absence of observation.

Before observation, Bohr insisted, all that existed was the wavefunction (which we considered in the previous chapter). The wavefunction was a mathematical tool for determining probabilities. The wavefunction represented a *superposition* of all possible states which would be possible after measurement. For example, the wavefunction can represent all the possible positions in which an electron might be discovered after a measurement is taken. After the measurement is taken, the wavefunction "collapses" so that only a single value is measured for the location of the electron. [5]

This interpretation of quantum mechanics became known as the *Copenhagen interpretation* (because Niels Bohr's institute of physics was in Copenhagen).

If you read my first book, you will know that this is, yet again, another example of the universe acting like a roulette wheel. Before observation, the state of the system must be considered as being in a superposition state: it acts as though it possesses a combination of all possible property values. The state of the system could then be represented by a roulette ball spinning around the wheel, possessing all possible values. But when an observation is taken, it is as if the roulette ball stops spinning and has to occupy a single value. It is at this stage that probability enters into quantum mechanics: you cannot predict where the ball will stop, you cannot predict what property value you will measure.



So we see that quantum theory cannot correctly predict the outcome of any single experiment! It can only provide the possibility of a particular outcome. This might seem a huge drawback of quantum mechanics. However, knowing the possibility of particular outcomes is an important tool if multiple measurements are made. In this sense, quantum mechanics is a statistical theory: it provides you with the probabilities of a certain result if multiple observations are taken.

It is frequently said that the Copenhagen interpretation implies that the act of observation somehow "creates reality", the motivation for this belief being Niels Bohr's assertion that it is scientifically meaningless to talk about the properties of a particle before observation. For Einstein, this belief was anathema. Einstein believed there had to be an objective reality in the absence of observation. In another famous quote Einstein said "I like to think that the Moon is there even if I am not looking at it."

Unfortunately, this frequently-repeated quote of Einstein has been partly responsible for the production of a number of pseudo-scientific books (and even movies) suggesting that quantum reality is somehow conjured into existence by human observation, or human consciousness. This is surely a complete fallacy, and is based on a lack of understanding of the principles of quantum mechanics. As the physicist Carver Mead has said: "That is probably the biggest misconception that has come out of the Copenhagen view. The idea that the (human) observation of some event makes it somehow more 'real' became entrenched in the philosophy of quantum mechanics. Even the slightest reflection will show how silly it is. An observer is an assembly of atoms. What is different about the observer's atoms from those of any other object? What if the data are taken by computer? Do the events not happen until the scientist gets home from vacation and looks at the printout? It is ludicrous!"

In order to correctly understand the effect of observation at the quantum level, it must be realised that observation can never be responsible for making an object "real", for somehow conjuring it into existence. However, it is the case that the act of observation can have the effect of modifying some properties of an observed object. These properties of a particle — such as spin and location — which are subject to the quantum superposition principle are called *dynamic properties*. But a particle also possesses properties which are fixed and are not subject to the principles of quantum mechanics, such as a particle's electric charge. These properties are called *static properties*. The existence of static, unchanging properties whose values are known prior to observation indicates that the particle does, indeed, exist prior to observation. So quantum mechanics states that the Moon does indeed exist even if no one is looking at it.

So now in order to understand the intended meaning of the earlier Einstein quote, let us consider the quote in its entirety: "I think that a particle must have a separate reality independent of the measurements. That is, an electron has spin, location and so forth even when it is not being measured. I like to think the Moon is there even if I am not looking at it."

So we see from this full quote that Einstein was actually referring to certain **properties** of a particle, in particular its spin and location. He was not referring to the entire **existence** of the particle. So Einstein was merely suggesting that the Moon might be **located** somewhere else if he was not looking at it — he was not suggesting the Moon did not exist at all if he was not looking at it! Unfortunately, the latter part of the quote has been taken out of context and used to launch a pseudo-science industry.

Indeed, we should perhaps not find it too surprising that the act of observation can be responsible for modifying some properties of a particle. Our discussion of the uncertainty principle in the previous chapter considered the measurement of air pressure in a tyre using a gauge, a process which inevitably resulted in some air leaking out of the tyre — thus reducing the pressure. Hence, observation inevitably modifies the object being observed.

In that case, it would appear to be rather meaningless to talk about values of certain properties of a particle before observation. After all, the only way to measure the property value is to perform the observation — and the observation must inevitably alter the value of the property.

So to talk of a certain objective reality before observation clearly appears rather meaningless. Certainly, as we move down to the smallest quantum level, it should probably not be considered surprising that the effect of the observation on the object under observation would become very significant indeed.

So if it is clear that the act of observation inevitably affects the object under observation, this means that the observed object should never be considered as an isolated object. Instead, the observer and the observed object should be considered to be a single, combined system. An object cannot produce a property value without an observer, and an observer cannot produce a property value without an object to observe. It is only by considering a combined system that a property value can be produced.

So, if it requires a combined system to produce a property value, it makes no sense to talk of the property value of a single isolated object. Einstein insisted that there was an "objective reality" in the absence of observation. But if it requires the presence of an observer to **create** the property value, then it really makes no sense to talk of the objective reality of a single, isolated object. It is the combination of observer and observed object which generates reality.

However, Einstein continued to insist that there must be a fixed objective reality in the absence of observation. And, in 1935 Einstein showed he had one more trick up his sleeve, a trick which would shake quantum mechanics — and Niels Bohr — to the core.

The EPR paradox

The Wall Street Crash of 1929 plunged many Americans into penury, and destroyed many businesses. However, Louis Bamberger and his sister Caroline had the good sense (and good fortune) to sell their department store chain just a few weeks before the crash. Flush with money, the two philanthropists donated five million dollars to establish a new research centre. The Institute for Advanced Study was established in Princeton, New Jersey.

The aim of the institute was to provide a relaxed environment in which the best scientists would be free to pursue whichever research direction they chose. There would be no additional pressure on the scientists such as the need to teach pupils, or to publish a certain number of research papers. In fact, there was no experimental facility at all at the institute. Some of the finest minds of the day were left to their own devices, with nothing more than books and a pad of paper.

The belief of the institute was that this unpressurised environment would lead to original thinking and major breakthroughs. However, it appears that the lack of stimulus and pressure had something of a negative impact, and the institute gained a reputation as the place where great minds go to atrophy. As Richard Feynman said: "When I was at Princeton in the 1940s I could see what happened to those great minds at the Institute for Advanced Study who had been specially selected for their tremendous brains and were now given this opportunity to sit in this lovely house by the woods there, with no classes to teach, with no obligations whatsoever. These poor bastards could now sit and think clearly all by themselves, OK? So they don't get any ideas for a while: they have every opportunity to do something, and they're not getting any ideas. I believe that in a situation like this a kind of guilt or depression worms inside of you, and you begin to worry about not getting any ideas. And nothing happens. Still no ideas come. Nothing happens because there's not enough real activity and challenge: you're not in contact with the experimental guys. You don't have to think how to answer questions from the students. Nothing!"

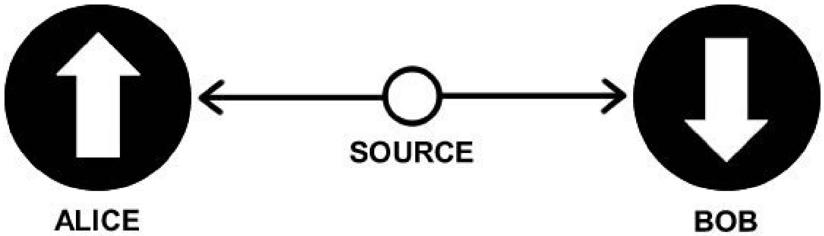
In 1932, Einstein was recruited to join the institute, attracted by its freedoms, and was to spend the rest of his life there. However, the lack of stimulus at the institute certainly seemed to have a negative impact on Einstein. In fact, Einstein's scientific biographer Abraham Pais asserted that after 1925 "Einstein might as well have gone fishing". When Einstein was shown his office at the institute he was asked what equipment he wanted, to which he replied: "A desk, a table, a chair, paper and pencils. Oh yes, and a large wastebasket so I can throw away all my mistakes."

However, while at the institute in 1935, Einstein teamed-up with Nathan Rosen (a 25-year-old who arrived from MIT to become Einstein's assistant) and Boris Podolsky (a 49-year-old physicist who had just moved to the institute) to produce a landmark paper which came to be known as the EPR paper (after the initials of Einstein, Podolsky, and Rosen). The EPR paradox was another attempt by Einstein to show that quantum mechanics could not be a complete description of the workings of Nature.

The basis of the EPR paradox rested on the principle of entanglement. It is possible, under certain circumstances, for the properties of a particle to become correlated with the properties of another particle (i.e., the property values of one particle are dependent on the property values of another particle). For example, an electron has a property called spin, which, if we measure it, we might find the spin to be either upward ("spin up") or downward ("spin down"). When we perform a measurement of electron spin in a particular direction (e.g., the vertical direction), we will only find one of these two states because spin is quantized into one of these two discrete states: it is like a roulette wheel with only two slots into which the ball can drop.

For example, it is possible to produce a pair of electrons using a method which ensures that each electron must have opposite spin to the other electron, i.e., if one electron is "spin down" then the other electron must be "spin up" (this is due to the law of conservation of angular momentum: total angular momentum of the system before the electrons are emitted must equal the total angular momentum of the system after the electrons are emitted). This pair of electrons are then said to be entangled: the property values of one electron is dependent on the property values of the other electron.

So if two people each receive one of the entangled electrons and perform a measurement of spin, they will find that the other person's electron has opposite spin. The following diagram shows two experimenters — Alice and Bob — each receiving one electron from a pair of entangled electrons, and, after measuring the spin, they discover it to have opposite spin to their partner's electron:



But, in the EPR paradox, Einstein realised that quantum mechanics predicted a problem with this picture. According to quantum mechanics, before the spin of an electron is measured that spin value is supposed to be in a multi-valued superposition state: a combination of all possible values, both "spin up" **and** "spin down". This is like the ball spinning around the roulette wheel, with **both** spin values being possible before the measurement is taken. Only after a measurement is taken does the electron possess a definite spin value (either "spin up" **or** "spin down"). So, for example, when Alice measures the spin of her electron and finds it to be "spin up", her electron is no longer in a multi-valued superposition state. It is as if the ball has stopped spinning around the roulette wheel and has dropped into a particular slot: the "spin up" slot.

But as soon as Alice's electron takes a definite value, this means that Bob's electron must also have a definite value — the opposite value: "spin down". So Alice measuring her electron and finding a definite spin value also has the effect of taking Bob's electron out of its superposition state and forcing it to take a definite spin value. It is as if Alice's measurement has an instantaneous effect on Bob's electron.

Einstein realised that quantum mechanics insisted that this effect of one particle onto the other particle must be instantaneous — even if the particles are separated by quite a distance. And here Einstein believed he had found a flaw in quantum mechanics, because Einstein's own theory of special relativity prohibited any such influence over distance acting faster than the speed of light. According to relativity, it was simply not possible for this instantaneous effect predicted by quantum mechanics.

This principle — that an object should only be influenced by its immediate surroundings, and cannot be influenced by some instantaneous effect from a distant object — is called *locality*.

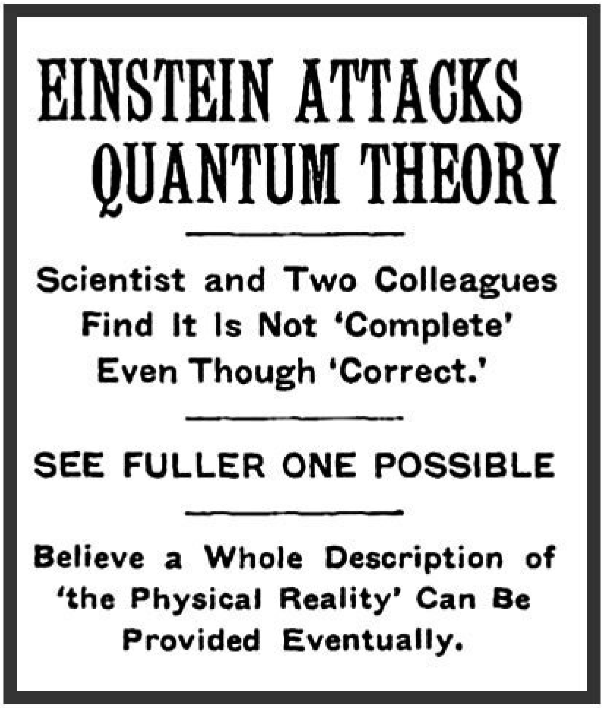
As we have seen, Einstein had created several thought experiments before with the aim of revealing inconsistencies in the Copenhagen interpretation of quantum mechanics. However, each time Niels Bohr had been able to find a flaw in Einstein's argument, thus saving the day for quantum mechanics. This time, however, the situation was different. With the EPR paradox, Einstein had revealed a gaping hole in the Copenhagen interpretation.

According to one of Bohr's colleagues: "The EPR onslaught came down upon us as a bolt from the blue. Its effect on Bohr was remarkable." Bohr abandoned all other work to try to find a solution to the paradox. Bohr worked day and night, becoming increasingly frustrated as he realised the subtlety of Einstein's argument. After six weeks, he presented his solution, but it was less than convincing. Bohr stated that the pair of entangled particles should not be considered separately, but should be considered as forming a single entangled system — even though the two particles might be separated by a considerable distance. Essentially, Bohr was stating that there **was** some form of instantaneous communication between the particles. However, the restriction that nothing can travel faster than light was not broken because it was impossible to send information via this method: the outcome of the particle spin measurements was always random. As Brian Greene said: "Special relativity survived by the skin of its teeth."

Einstein was not satisfied with Bohr's reply, and had no time for this suggestion of instantaneous influences over great distances. Famously, Einstein said: "Physics should represent reality in time and space, free from spooky action at a distance."

Instead, Einstein believed that the answer to the paradox lay in the fact that the particles contained certain property values all along, but these values were hidden from our eyes (so-called *hidden variables*). So when Alice measures her electron and finds it is "spin up", there is no problem with Bob's measurement as Bob's particle knew it was "spin down" all along. In this way, no instantaneous action at a distance is required. The principle of locality would be preserved, but it would mean that the particles contained more information than quantum mechanics said they possessed.

If Einstein was correct, it would mean that quantum mechanics was not a "complete" description of reality. This announcement of Einstein made headlines around the world, such as this one from the *New York Times* on May 4th 1935:



EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'

SEE FULLER ONE POSSIBLE

Believe a Whole Description of 'the Physical Reality' Can Be Provided Eventually.

Unfortunately, there seemed to be no way to distinguish between the viewpoints of Einstein and Bohr. Who was correct?

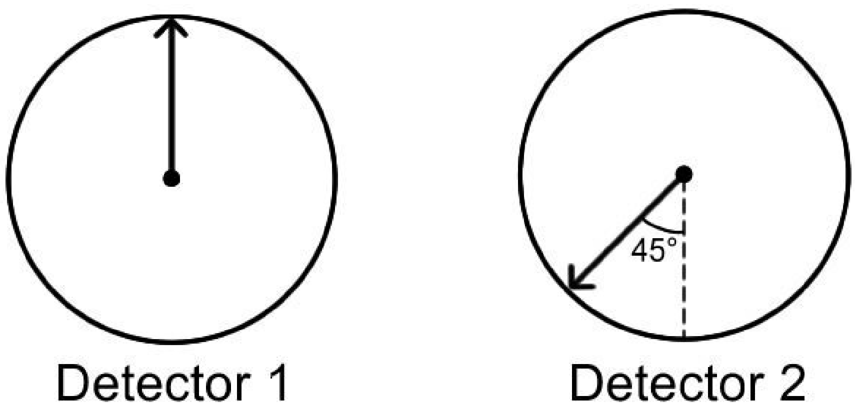
Bell's theorem

In the decades which followed, the theory of quantum mechanics brought one success after another, and it assumed its position as one of the bedrock theories in physics. Applications of quantum mechanics included semiconductors (leading to the computer age), and lasers (which introduced CDs and DVDs). For most physicists, the successes of quantum mechanics meant they were more than happy to use the basic equations to produce results, without spending too much time wondering about the philosophical interpretations. The discussions between Bohr and Einstein over EPR were largely forgotten.

However, in the 1960s, an Irish physicist by the name of John Bell became fascinated by the EPR paradox and decided to see if it was possible to design an experiment which could distinguish between the viewpoints of Einstein and Bohr. Instinctively, Bell felt that Einstein must be right — that there was an objective reality even in the absence of observation. Bell believed that the properties of particles had to have fixed values — even before those property values were measured. In an effort to prove Einstein correct, John Bell proposed an ingenious experiment.

As in the case of the EPR paradox, Bell's experiment required two experimenters to measure the spin value of two entangled particles. As with EPR, if one experimenter measures "spin up" then the other experimenter has to measure "spin down". According to the Copenhagen interpretation, after the spin of the first particle is measured, it is as if quantum entanglement works over a great distance to "pull" the spin of the second particle into the precisely opposite alignment. It does not matter how the spins of the particles were aligned before observation: after observation quantum mechanics "pulls" the spin of the second particle to point in precisely the opposite direction to the measured spin of the first particle.

Bell's brilliant innovation was to consider the situation when one of the detectors was not precisely aligned either up or down. What if the second detector is aligned at a 45° angle?



In that case, there would not be such a precise correlation between the results of the two experimenters. But the "pulling" effect of quantum mechanics (via quantum entanglement) ensures that there is still a surprisingly high level of correlation between the two results. In fact, according to quantum mechanics, the level of agreement between the two experimenters is predicted to be:

$$\frac{1}{2}(1+\cos\theta)$$

where θ is the angle by which the second detector is offset. So for an offset angle of 45°, this formula predicts a level of agreement between the two experimenters of approximately 85% (if you want to check the maths, you can enter the 45° value into the previous formula to check you get 85%).

But what happens if quantum mechanics is incorrect? What if Einstein was right, and the particle spins have fixed values before observation?

In that case, there is no quantum manipulation at a distance, and the level of correlation between the two experimenters simply falls off linearly as the offset angle increases. An offset angle of 45° represents a quarter of the distance from 0° (100% correlation) to 180° (0% correlation). So, as there is a linear relation, an offset angle of 45° represents 75% correlation.

So, in the case of fixed spin values (i.e., the situation preferred by Einstein), we find there should be only a 75% probability of agreement between the two experimenters.

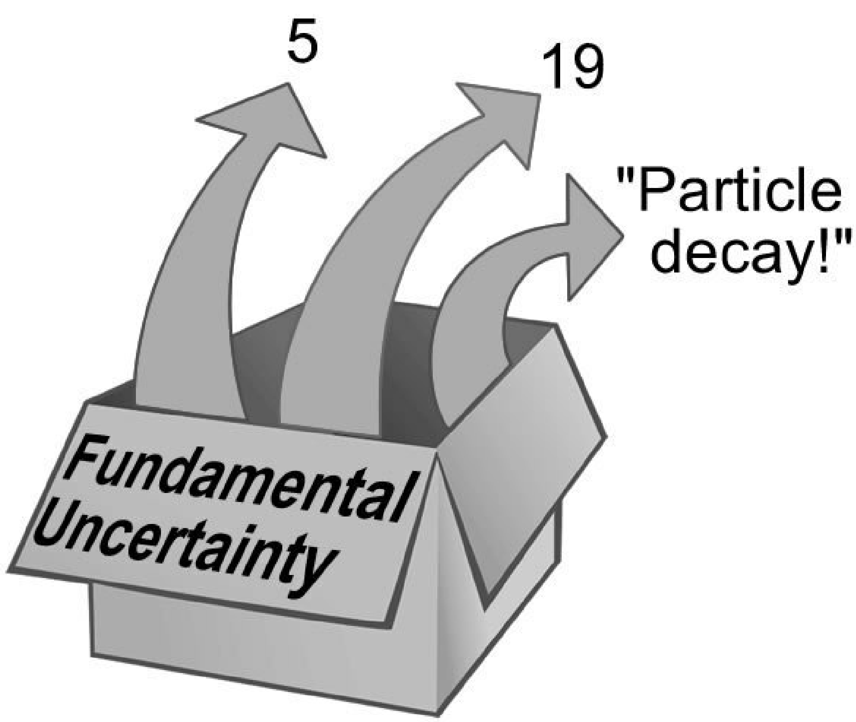
To sum up, if quantum mechanics is correct, we should expect an 85% level of correlation between the two experimenters. But if quantum mechanics is incorrect — and Einstein is right — we should expect to find a level of correlation no higher than 75%.

So it was in the early 1970s that several groups of experimenters started to test the level of correlation in entangled particles. When the results were analysed, the result was startling. It was shown that the level of correlation was, indeed, 85% — the value predicted by quantum mechanics. This was a higher value than the value which should have been possible (75%) if the particles had fixed property values. This meant that Bohr was right all along and Einstein was wrong: particles could not have fixed property values. There had to be some form of faster-than-light spooky signalling, the effect of which was to increase the correlation between the particles. This was an astonishing result, which *Physics World* magazine called "the most profound discovery of science."

Unfortunately, John Bell died suddenly of a stroke in 1990. In that same year he had been nominated for a Nobel Prize for his discovery. Bell would probably have won the prize, but it is never awarded posthumously (which hardly sounds fair). However, for his work in revealing the true extraordinary nature of quantum mechanics, Bell is recognised as one of the greatest physicists of the 20th century.

Fundamental uncertainty

We will end this chapter with a discussion of a very important concept, a concept which will be repeated throughout this book. It is the concept of *fundamental uncertainty*. We encountered fundamental uncertainty earlier in this chapter with our discussion of the quantum roulette wheel. It was explained how the process of making a quantum measurement is like playing a game of roulette. However, there is fundamentally no way of determining where the ball will stop, no way of calculating the value in advance. We might illustrate fundamental uncertainty using the roulette wheel analogy, or we might simply express it by using a box which produces random numbers. There is no way of determining which number will be produced by the box, no cause we can detect, no hidden mechanism we can investigate, and there is no pattern to the numbers.



Essentially, this is "God playing dice".

The diagram also shows a command being issued for a particle to decay. Again, there is fundamentally no way to determine when this command will be issued: the timing is completely random.

However, this type of behaviour — this fundamental uncertainty — seems very alien to us. There is nothing in our human-level macroscopic world which behaves in this way. We only ever encounter outcomes which are clearly defined, effects uniquely determined by their causes. In our macroscopic world we are always able to dig-deeper to find the cause of any observed behaviour. But at the lowest quantum level, this option of digging deeper is not available to us. It is called fundamental uncertainty because "fundamental" implies that there is no deeper layer.

And it is because this behaviour seems so alien to us that the possibility of this behaviour receives so much resistance, so much hostility. It is as if there is an inflexibility to accept that such behaviour could exist, as if people cannot "get their heads round it". As we have seen, even the great Albert Einstein seemed to have an inflexible resistance to the concept, a resistance which was to damage the remainder of his career.

We will see in Chapter Five how this aversion to fundamental uncertainty has resulted in a proposal which includes multiple parallel universes: the so-called Many Worlds interpretation of quantum mechanics. No matter how alien the concept of fundamental uncertainty might appear, it is surely more plausible than the suggestion of an infinity of parallel universes!

It is perhaps the hardest task in physics — and the most important skill — to have a flexible mind capable of accepting ideas which appear counter-intuitive or which do not agree with everyday experience, such as the concept of fundamental uncertainty. However, the skill is to only consider experimental data, the results of experiments, and **listen to Nature talking**.

So remember the name: fundamental uncertainty. It is a crucial concept which will crop up again in this book.

But why should Nature be behaving in such an unusual manner? Why do the laws of quantum mechanics work this way? It is interesting to present some speculation about why this might be the case.

In my first book, I asked the question as to why the laws of Nature take the form they do. It is a question which has been asked many times before, probably most famously when Einstein said: "What I am really interested in is whether God could have made the world in a different way; that is, whether the necessity of logical simplicity leaves any freedom at all." Do the laws of Nature arise from logical necessity? In other words, is it at all possible that the laws of Nature could have taken a different form? If that is not the case, if the laws could not have been any other way, then we would appear to have found a solution to our question of why the laws take the form they do.

Perhaps we can find an answer from the most remarkable moment in the history of the universe: the moment of the Big Bang, approximately 13.7 billion years ago. Our theories are not yet capable of predicting what precisely happened in the earliest moments of the existence of the universe, in the so-called *Planck epoch*, from 0 up to $10^{—27}$ seconds. However, if we wind the clock back, it is believed that the universe was compressed to a size smaller than an atomic nucleus, an incredibly hot and dense point. The condition of the universe at this moment is called the *boundary condition*. In order to be topical, I can quote Stephen Hawking's character at the end of the recently-released movie *The Theory of Everything*: "There ought to be something very special about the boundary conditions of the universe". Surely we can learn a lot from this condition.

If you read my first book, you will know that we should not treat the event of the Big Bang as the reason the universe exists (all times are equally real in a block universe: the last moments of the universe are just as real as the first moments). However, the very peculiar situation of the Planck epoch poses a tremendous challenge for classical deterministic physics. According to determinism, every effect has a clearly-defined unique cause. But if the universe did not exist before the Big Bang, then how can determinism link cause and effect over the period? Also, how can the classical law of conservation of energy possibly handle a situation in which no energy (before the Big Bang) is transformed into a great deal of energy (after the Big Bang)? Classical deterministic physics is inflexible in these matters. Determinism inevitably breaks down.

But if determinism breaks down, then indeterminism (in the form of quantum mechanics) is perfectly capable of stepping up to the mark. Quantum indeterminism breaks the strict laws of cause-and-effect. And, according to the uncertainty principle, it is even possible that a small amount of energy can spontaneously appear from empty space in a so-called quantum fluctuation, but only for a very short time. In fact, if you had to design a theory aimed at coping with the craziness of the earliest moments of the universe, you could not do much better than "anything goes" quantum mechanics.

So maybe at the lowest levels of reality there must be indeterminism and uncertainty, otherwise there could be no way that the universe could exist. Maybe every conceivable universe has to be fundamentally non-deterministic at its base level? Maybe this makes indeterminism and uncertainty a logical necessity in the laws of Nature.

We shall return to consider this point later in the book.

4

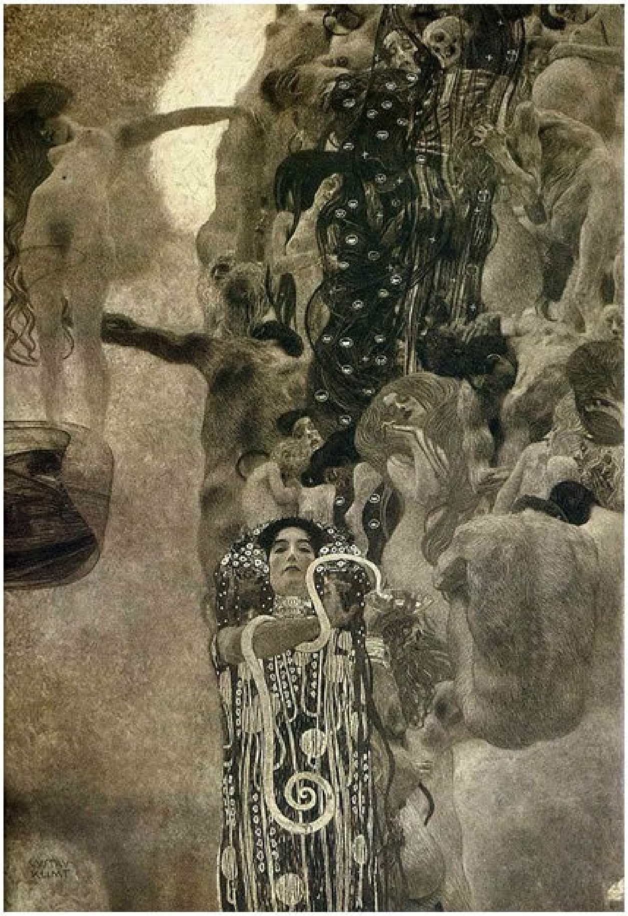
VIENNA

At the start of the 20th century, the city of Vienna was the capital of the Austro-Hungarian empire, a city of two million souls, and one of the most important centres of culture and philosophy in the world. The Austro-Hungarian empire consisted of 15 nations, and the citizens of those nations created a cultural melting-pot in Vienna. The city was a centre of the modernist movement, buzzing with the new ideas of psychoanalyst Sigmund Freud and philosopher Ludwig Wittgenstein who both chose to make the city their home. The centres of intellectual debate were the Viennese coffee houses, where arguments raged through the day. The clientele of the coffee houses included Adolf Hitler, Joseph Stalin, Leon Trotsky, and Sigmund Freud who were all living in the city within a few miles of each other. [6]

According to Professor Dave Cliff on his BBC TV programme *The Joy of Logic*: "Despite its grace and gentility, Vienna can lay justifiable claim — perhaps more than any other city — to be the birthplace of the modern. For it was here in art, design, philosophy, science, and psychology, that people most boldly challenged the tired conventions and assumptions of the 19th century."

I am the proud possessor of a postcard from Vienna dated 1904. It is a beautiful thing. The picture on the card gives a good impression of what street life must have been like in that era:

About this time, the University of Vienna commissioned a series of paintings for the ceiling of its great hall. The paintings were intended to celebrate the great recent technological and scientific advances, and the seemingly endless increase in knowledge. The artist they commissioned was Gustav Klimt. However, when Klimt presented his finished paintings to the professors of the university, they were shocked. Instead of a series of optimistic visions, Klimt presented a pessimistic, dark, hopeless view of naked men and women drifting aimlessly in voids.



This was certainly not what the professors wanted. The paintings seemed to challenge their view of a brave new world in which human knowledge would continue to boundlessly increase. Klimt's paintings were instantly rejected.

According to Professor Dave Cliff: "Klimt's paintings seemed to reject the fashionable notion that science and mathematics would provide us with complete knowledge founded on **absolute provable truth**." The emphasis on provable truth is important, because, as we shall see, it was the concept of provable mathematical truth which would later be severely challenged by one of the students of the University of Vienna.

The certainty of mathematics

What do we mean when we say a statement is "true" or "false"? Well, the statement might refer to some factual aspect of the material world, e.g., "The cat is on the mat". That statement could be evaluated as being either true or false by inspecting the mat and determining if the cat is sitting on it. In the field of logic, we would call a statement such as "the cat is on the mat" a *proposition*. A proposition is a statement which can be either true or false.

The rules of logic allows us to use reasoning to combine propositions. For example, we could combine the proposition "all men are mortal" with the proposition "Socrates is a man" to derive the proposition "Socrates is mortal". We can then say that we have used the rules of logic to **prove** that Socrates is mortal, and the steps we used to prove that proposition would be called a *proof*.

In mathematics, if a proposition is true and it is particularly useful or important, then it is called a *theorem*. Not only is a mathematical theorem true, but it will be true under all circumstances, in any conceivable world. This differs from a proposition about the material world. Propositions about the material world such as "the cat is on the mat" could conceivably be either true or false. We could certainly imagine an alternative world in which the other outcome is true (i.e., "the cat is **not** on the mat"). In mathematics, however, there is no uncertainty: a true proposition (theorem) would have to be true in any possible world.

So a mathematical proof is a valuable thing. It seems to reveal some insight into "the way things have to be". In his book *The Study of Mathematics*, Bertrand Russell said: "Mathematics takes us into the region of absolute necessity, to which not only the actual world, but every possible world, must conform."

Let us consider an example of a theorem from geometry (geometry being a branch of mathematics). The Pythagorean theorem ("On a right-angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides") has at least 370 known proofs. However, only one proof would be required to establish the truth of the theorem.

And because mathematical proof reveals an insight into "the way things have to be" we realise that the Pythagorean theorem would have to be true in any possible world we could imagine. For example, aliens on other planets would at some stage of their technological evolution undoubtedly independently discover their own version of the Pythagorean theorem. The great mathematician Carl Friedrich Gauss suggested that the diagram of the Pythagorean theorem should be drawn on the Siberian tundra so that Martian observers would recognise that there was intelligent life on Earth. The outlines of the shapes were to be ten-mile-wide strips of pine forest. The interiors of the shapes were to be giant fields of yellow wheat.

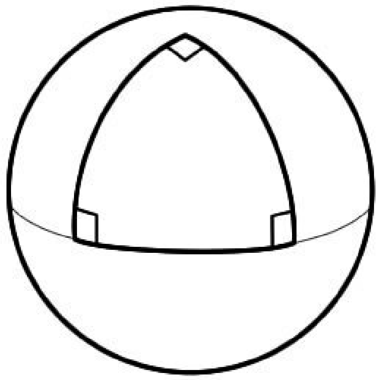
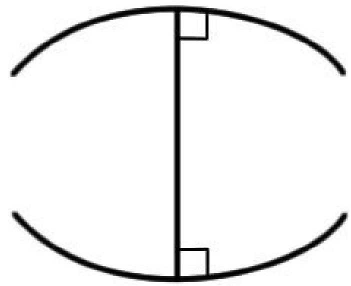
We can feel certain about the universal truth of mathematical theorems because they are derived — in a sequence of logical steps — from simple *axioms*. Axioms can be considered the foundation stones of mathematics. They are simple propositions which are considered as being "obviously true". For example, the axioms of classical geometry were defined by the Greek mathematician Euclid. These five axioms are:

1. It is possible to draw a straight line from any point to any other point.
2. It is possible to extend a straight line indefinitely.
3. It is possible to draw a circle with any centre and any radius.
4. All right angles are equal to one another.
5. Parallel lines never meet — no matter how long they are.

Though these axioms might appear trivially simple, all of classical geometry (including the Pythagorean theorem) can be derived by logical reasoning (building up) from these five axioms. In fact, it is the simplicity of the axioms which gives them their strength: the truth of these axioms appears intuitively obvious. And it is the obvious truth of its axioms which provides mathematics with its universal certainty.

However, it was known that the last of Euclid's five axioms — the *parallel postulate* — was not quite as intuitively obvious as the other four axioms. Attempts to prove the parallel postulate from the other four axioms failed. Did the parallel postulate necessarily have to be true?

Euclidean geometry is also called *plane geometry* because it assumes that the geometry is being constructed on a flat surface. However, in the 19th century it was realised that geometry could also be performed on curved surfaces, e.g., the surface of a sphere. In that case, the parallel postulate does not hold: parallel lines can eventually meet. Lines which are parallel on one part of the sphere might converge to a point. As an example, the north-south lines of longitude on the surface of the Earth are parallel at the Equator but converge to a point at the North and South Poles.

The resultant form of geometry — which can deal with curved surfaces — is called *non-Euclidean geometry*. In the early years of the 20^{th} century, the real-world application of non-Euclidean geometry was revealed when Einstein's theory of general relativity showed that space itself had curvature, so only a non-Euclidean geometry could correctly describe space.

So the introduction of non-Euclidean geometry showed that our certainty of mathematics is only as secure as our certainty of the axioms on which mathematics is based. Suddenly, mathematics seemed less than secure. As John Barrow says in his book *Pi in the Sky*: "The discovery that Euclidean geometry was not a unique, inevitable, and absolute truth about the world came therefore as a stunning blow. Its impact was far-reaching and irreversible. It undermined absolutist views about human knowledge across a vast spectrum of human thinking. Prior to the coming of non-Euclidean geometry, there was a unity, a confidence, and a certainty to our knowledge of the world. Afterwards, the one unassailable truth about the nature of the physical world had been eroded and so, along with it, had centuries of confidence in the existence and knowability of unassailable truths about the universe. How are the mighty fallen."

The Barber of Seville

In the early years of the 20^{th} century, there was a great deal of interest in a number of logical paradoxes which threatened to undermine the foundations of mathematics. As an example, Epimenides — a Greek philosopher of the sixth century — once said: "All Cretans are liars". This might appear rather a racist comment, however, it should be mentioned that Epimenides was from

Crete himself. So what are we to make of Epimenides' statement? If all Cretans really are liars, then Epimenides is a liar himself. In which case, we should not believe his statement. Therefore, Epimenides might be a truthful Cretan. In which case, all Cretans are liars, etc.

The liar's paradox also featured in a 1967 episode of *Star Trek* (http://tinyurl.com/liarsparadox). Captain Kirk and the crew manage to defeat an android simply by telling it "I am lying". The android finds this so illogical (if he is lying, then he must be telling the truth) that its circuits blow and smoke pours out of its head.

The liar's paradox is not written in the language of mathematics, it is written in English which is a language not notable for its logical consistency. Hence, this paradox is considered to be nothing more than a linguistic oddity, and not a threat to mathematics. However, other paradoxes were discovered which were phrased in the language of mathematics, and these paradoxes posed more serious problems. One of these paradoxes is known as the paradox of the Barber of Seville: "A man of Seville is shaved by the Barber of Seville if and only if the man does not shave himself. Does the Barber shave himself?"

This paradox is slightly more complex than the previous liar's paradox. It refers to the mathematical structure known as a *set*, which is a collection of objects. The paradox considers the set of all men who do not shave themselves (and are therefore shaved by the barber):

Should the barber be in the set? In other words, does the barber shave himself? If we assume the barber does **not** shave himself, then we should put him in the set. But if we put him in the set, then the barber **should** shave himself because the barber shaves all the men in the set.

So if the barber shaves himself, then he does not shave himself. But if the barber does **not** shave himself, then he **does** shave himself.

Hence, this is another paradox. But unlike the previous liar's paradox — which was expressed in the English language — sets are mathematical structures which can be expressed mathematically. So the tale of the Barber of Seville represents a true mathematical paradox.

This paradox was discovered by the English philosopher and logician Bertrand Russell in 1903. Russell was so disturbed by mathematical paradoxes that he embarked on a project aimed at restoring perfect certainty to mathematics.

Russell started writing a book. A very, very, long book.

Principia Mathematica

Although mathematicians believed they were completely rigorous in their approach, Russell realised that this was not the case. Often, mathematicians used terms which had not been rigorously defined, or used reasoning — perhaps unconsciously — which was logically inconsistent. Russell decided that what was needed was to derive all of mathematics using pure logic. Nothing would be left to chance. There would be no room for differing interpretations, or human emotion. Essentially, the whole process could be performed by a computer, relentlessly building-up endless theorems and printing out the results.

However, as there was a shortage of computers in the first decade of the 20^{th} century, the task fell solely on the shoulders of Russell and his occasional collaborator Alfred Whitehead. After 360 pages of writing, their first major result was definitively proved: 1+1=2

It might seem rather excessive to spend such a long time to prove such an apparently trivial result, but it revealed Russell's determination to build mathematics on a solid logical foundation and thereby eliminate all paradoxes and contradictions.

After almost ten years of work, the first volume of Russell's magnum opus *Principia Mathematica* was released in 1910. The book was so large that

Russell and Whitehead had to transport the book to their publisher's office in a wheelbarrow. Two further volumes followed in 1912 and 1913.

Unfortunately, the writing of the Principia required an almost obsessive approach, with no room for creativity or inspiration. Russell remarked that the mental effort had irretrievably drained his mental abilities. Talking of his experience, Russell said: "The effort was so severe that at the end we both turned aside from mathematical logic with a kind of nausea."

Russell's work influenced the German mathematician David Hilbert who was also working to eliminate uncertainty from mathematics. In 1920, Hilbert proposed a research program (which is now known as Hilbert's Program), the aim of which was to formalize mathematics by divorcing it from any meaning about the real world, and just considering the mathematical symbols and the strict rules which could be applied to those symbols. Such a system was called a *formal system*. The optimistic Hilbert believed that this rigorous approach would create a mathematics capable of solving any problem. But, more than that, Hilbert believed it should be possible to **prove** that the approach could solve any problem.

However, Hilbert's program to ensure the certainty and completeness of mathematics was about to be derailed in spectacular fashion.

Gödel's incompleteness theorem

Let us now return to Vienna. We have read how the paintings of Gustav Klimt "seemed to reject the fashionable notion that science and mathematics would provide us with complete knowledge founded on **absolute provable truth**." Indeed, one of the students in the University of Vienna at that time presented the greatest challenge to the notion of provable truth.

Kurt Gödel became interested in mathematical logic while studying as an undergraduate at the university. One year after completing his doctorate, in 1931 while he was still living in Vienna, he published the work for which he is most famous. Indeed, it is the work which sent shockwaves throughout mathematics and many other fields far removed from mathematical logic. Gödel showed that there are mathematical statements which are true, but which cannot be proved to be true. This meant that mathematics was not *complete*: there are some things you cannot prove using mathematics. This effectively proved the death knell of David Hilbert's program to eliminate uncertainty from mathematics.

This result is known as *Gödel's incompleteness theorem*.

So, how did Gödel arrive at his result? The method was actually very similar to the liar's paradox which we considered earlier. If you remember, the liar's paradox essentially consisted of the English language sentence "This sentence is false". If the sentence is false, then the sentence must be true. But if the sentence is true, then the sentence must be false. Hence the contradiction.

However, as was explained in the discussion about the liar's paradox, English is a language with poorly-defined rules, and it is not a mathematical language. As such, no sentence phrased in English can pose a threat to mathematics. I can basically say whatever I like in English. If I say "Mathematics is flawed" then that poses no actual threat to mathematics.

But Gödel's brilliant stroke of genius was to show how any statement could be converted into a mathematical statement. He achieved this translation by using a code. The code converted a statement into a single whole number (usually a very large whole number). The whole number is called the *Gödel number*. The code worked on the basis that any number can be calculated by multiplying prime numbers together in one — **and only one** — way. For example, 51 = 3 × 17. Because there is only one way of calculating a number in this manner, that meant that there is a one-to-one relationship between a string of prime numbers and a unique Gödel number. If each prime number is then taken as representing a symbol in a statement, then this allows a statement to be converted into a Gödel number.

This conversion meant that any statement **about** mathematics could then be imported **into** mathematics. Most importantly, this allowed a mathematical statement to refer to itself. This is how Gödel achieved that …

Gödel considered the liar's paradox, and wondered if it had a corresponding equivalent in mathematics. But instead of considering the statement "This sentence is false", Gödel considered the statement "This statement cannot be proved to be true". More precisely, Gödel considered the statement "The statement with Gödel number *x* cannot be proved to be true". What Gödel then did was to encode **that entire statement** into a Gödel number, and replace the letter *x* in the statement with that Gödel number. In that way, the statement forms a strange kind of loop, looping back to refer to itself. And the end result is that the final statement refers to itself and so does, indeed, end up representing the statement "This statement cannot be proved to be true".

Let us now consider that statement "This statement cannot be proved to be true". If that statement is false, then we **can** prove it to be true — a contradiction. But if the statement is true, then we have a true statement which we cannot prove!

So either way — if the statement is true or the statement is false — mathematics is in trouble. By converting the statement into a mathematical form using his coding approach, Gödel revealed that a statement could exist which was either a contradiction (a paradox), or else there existed a true statement which could never be proved to be true.

Essentially, Gödel's result showed that all the work done by Russell and Whitehead in their *Principia Mathematica* was doomed. It was simply never possible to prove all theorems by building up from axioms — the method proposed by Russell and Whitehead. *Principia Mathematica* aimed to create a mathematics which was perfectly consistent and complete. It now emerged that all that work was for nothing. Gödel had showed that there could never be **complete** certainty in mathematics, because mathematics itself was not complete.

In his book *Impossibility*, John Barrow presents a very nice analogy to incompleteness: "If all truths in a logical system can be deduced from its axioms, it is called **complete**. As an illustration, consider a board game like chess or Go. Incompleteness would mean that there were configurations of pieces on the board that could not have been reached from the starting layout by following the rules of the game."

Douglas Hofstadter wrote an extraordinary Pulitzer Prize-winning book about the incompleteness theorem entitled *Gödel, Escher, Bach*. In an extract from that book, Hofstadter describes the impact of the incompleteness theorem on the general public: "Modern readers may not be as nonplussed by this as readers of 1931 were, since in the interim our culture has absorbed Gödel's theorem, along with the conceptual revolutions of relativity and quantum mechanics, and their philosophically disorienting messages have reached the public, even if cushioned by several layers of translation (and usually obfuscation). There is a general mood of expectation, these days, of 'limitative' results — but back in 1931, this came as a bolt from the blue."

Towards the end of his life, Gödel unfortunately became quite paranoid. In 1947, Gödel applied for American citizenship to join his good friend Einstein in Princeton. Einstein was to appear at the hearing as a character witness for Gödel.

On the night before the hearing, Gödel became quite distressed when he believed he had found a logical loophole in the constitution which would enable a dictatorship to be created in America. Einstein only just managed to calm him down enough in order to appear at the hearing, and he told Gödel not to mention the loophole under any circumstance.

The next day, the hearing seemed to go smoothly. The judge was impressed by Einstein's testimony as character witness. In summing up, the judge turned to Gödel and noted that, up to this point, Gödel had held German citizenship. The judge noted that Germany had been under an evil dictatorship, something that was "simply not possible in America". On hearing the word "dictatorship", Gödel burst into life and started explaining to the judge why he has wrong and, what, what was more, "I can prove it!". It took the efforts of both Einstein and the judge to calm Gödel down, and he was eventually awarded American citizenship, joining Einstein at the Institute for Advanced Study.

Gödel and physics

It is often said that mathematics is the "language of Nature", with mathematics proving to be an uncanny match to the laws of physics. Indeed, discoveries in mathematics sometimes anticipate corresponding discoveries in physics (the most recent example of this being the discovery of the Higgs boson — after its existence was predicted by mathematical reasoning back in 1964). Bearing this in mind, we might reasonably ask the question as to whether Gödel's incompleteness theorem has some corresponding equivalent in the laws of physics? Does uncertainty in mathematics lead to a corresponding uncertainty in physics?

This is not an unreasonable question because it is already known that the incompleteness theorem has implications for computing. The British mathematician Alan Turing showed that there were certain mathematical problems which could not be solved by a computer. The principle behind these so called *uncomputable* problems was directly analogous to the incompleteness theorem.

So, if the universe can be considered as behaving like a computer, then the incompleteness theorem clearly has implications for its behaviour. In his book *Pi in the Sky*, John Barrow compares the operation of the universe to the processing of a computer, saying that there is: "The image of the universe as a great computer program, whose software consists of the laws of Nature which run on a hardware composed of the elementary particles of Nature."

If the universe really can be considered as being a form of computer, then the laws of mathematics should certainly apply to all its processes. However, in his book *Gödel, Escher, Bach*, Douglas Hofstadter compares the universe with a formal system of mathematics and realises the following: "Can all of reality be turned into a formal system? One could suggest, for instance, that reality is itself nothing but one very complicated formal system. Its symbols do not move around on paper, but rather in a three-dimensional vacuum (space); they are the elementary particles of which everything is composed. The 'typographical rules' are the laws of physics. So the theorems of this grand formal system are the possible configurations of particles at different times in the history of the universe. The sole axiom was the original configuration of all the particles at the beginning of time. However, quantum mechanics casts at least some doubts on even the theoretical worth of this idea. Basically, we are asking if the universe operates deterministically."

As we have discussed in the previous chapter, quantum mechanics reveals that — at the core of reality — there lies fundamental indeterminism and uncertainty. So, as Hofstadter suggests, it would appear that the functioning of the universe can never resemble the predictable processing of a digital computer.

However, in his 2002 lecture entitled *Gödel and the End of Physics*, Stephen Hawking presented the following insight: "In the standard approach to the philosophy of science, physical theories live rent free in a Platonic heaven of ideal mathematical models. That is, a model can be arbitrarily detailed and can contain an arbitrary amount of information without affecting the universes they describe. But we are not angels, who view the universe from outside. Instead, we and our models are both part of the universe we are describing. Thus a physical theory is self-referencing, like in Gödel's theorem. One might therefore expect it to be either inconsistent or incomplete. The theories we have so far are both inconsistent and incomplete."

Hawking's insight that we are "trapped" inside the universe and so are inevitably limited in what truths we can know about Nature. We will be returning to consider the remarkable implications of this idea in Chapter Six.

The uncertain century

In Chapter One, it was described how the end of the 19th century was a period of great achievement, and there was great optimism and certainty about continued progress in science. However, once the 20th century arrived, discoveries in science and mathematics (quantum mechanics and Gödel's theorem) seemed to undermine this certainty, and it is uncanny how this loss of confidence seemed to mirror a loss of direction in other areas of human endeavour.

We might regard David Hilbert's effort to formalise mathematics in the 1920s as a last-gasp attempt to ensure certainty and stability in a world on the brink of chaos. As David Leavitt says (in his biography of Alan Turing): "It is hard not to read into Hilbert's program an attempt, through mathematics, to ward off the coming nightmare, just as it is hard not to read into Kurt Gödel's subsequent derailing of that program both the death knell of prewar idealism and the advent of a bloody, off-kilter epoch in which the prevailing metaphors would be of chaos and night, not order and morning. Hilbert hoped to establish once and for all the security of the mathematical landscape (and by extension, the security of Europe)."

However, it was those other citizens of Vienna — Hitler and Stalin — who were to have the greatest influence on the 20th century. Once aimed solely at the good of mankind, science was turned to the creation of weapons. During the First World War, the Jewish director of Berlin's Institute for Chemistry, Fritz Haber, turned his attention to the creation of chemical weapons. He developed a mathematical formula — known as *Haber's law* — which gave the relationship between the concentration of a poisonous gas and how long the gas had to be breathed to achieve death.

Haber was awarded the Nobel Prize for Chemistry in 1918.

Progress in science and technology was being warped so that it was no longer the servant of humanity. Instead, science became the enemy of millions. Haber became the first director of the corporation which produced the poisonous gas Zyklon B, later responsible for the deaths of millions in the gas chambers during the Second World War.

Meanwhile, in America, the greatest physics project of all time — which at its peak was greater than the entire American automobile industry — was devoted not to the benefit of mankind but to the development of the ultimate weapon of mass destruction: the atomic bomb.

Finally, in Chapter One it was described how the iron steamship the Great Eastern was built in the 19th century and became the largest passenger ship ever built. The ship laid the first telegraph cable across the Atlantic, and became a symbol of the confidence and certainty of the era. However, the record of the largest passenger ship was beaten early in the 20^{th} century by a steamship whose selling-point was certainty: the perfect certainty of safety and security.

That ship was the Titanic.

Welcome to the uncertain century.

5

THE COASTLINE OF BRITAIN

In 1961, Edward Lorenz was a mathematician and meteorologist working at MIT. He was attempting to use a computer for the first time as a tool for long-range weather prediction. In theory, the computer seemed an ideal tool for weather forecasting, capable of performing endless repetitive calculations, applying Newton's deterministic laws of motion to millions of small pockets of air. In this way, it was supposed that the weather could be predicted in much the same way as the motion of the planets could be predicted with great accuracy.

In order to create his simulation of the movement of air in the atmosphere, Lorenz had constructed a simple model based on *convection*. Convection occurs when a volume of air is heated from below. As far as the weather is concerned, this is the situation when the Sun heats the ground and the air immediately above the ground becomes warm. Warm air is less dense than cold air, so this warm air rises to the higher levels of the atmosphere where it cools. This cooled air then descends again to replace the newly-warmed air at ground level. Hence, over time a circular convection current is formed.

Lorenz used three simple equations to model this convection current. While Lorenz's equations predicted that the system would always tend to form a simple circular current, they also revealed that this stability was deceptive. The equations also showed that sometimes — in a completely unpredictable way — the direction of circulation could slow down and then reverse.

This was a remarkable discovery. It had always been assumed that complex or unpredictable behaviour could only arise from a complicated system. You would certainly not expect random behaviour to be generated by a simple deterministic equation (an equation is deterministic as you can be completely certain of the output of the equation for a particular input). However, Lorenz's discovery showed that very simple equations can generate behaviour which appears random.

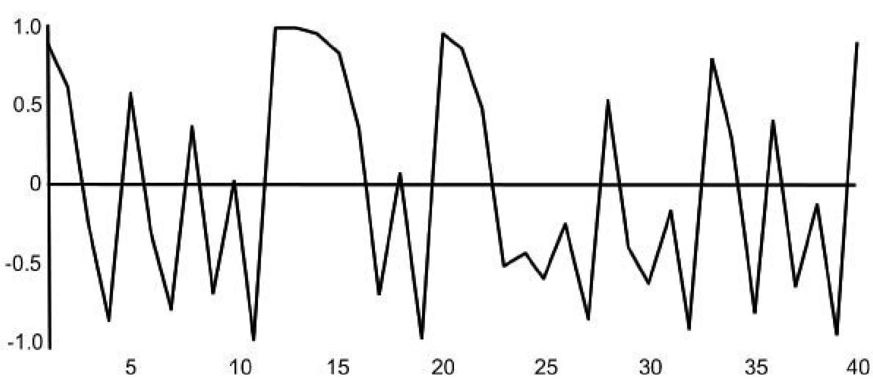
In order to illustrate this principle, Professor Ian Stewart in his book *Does*

God Play Dice? presents a very simple mathematical expression: $2x^2-1$. We will discover how a simple expression such as this can produce complex behaviour which appears random.

Pick a fractional value between 0 and 1 and substitute that value for x in the expression (use your calculator — there should be an x^2 button on your calculator which makes the calculation of $2x^2-1$ a simple process). As an example, we will start with the value 0.9. You will calculate the result of the expression as being 0.62. This then becomes your new value of x. So repeat the process, this time substituting your new value of 0.62 for x in the expression. You will find you get -0.2312. Repeat this process several times and you will get the following sequence:

0.9
0.62
-0.2312
-0.8930931
0.5952306
-0.2914009
-0.8301709
0.3783676
-0.7136758
0.0186665
-0.9993031
0.9972134

Here is a plot of the sequence extended to 40 numbers:



The values and the plot appears completely random! This is bizarre: a simple, deterministic equation has generated behaviour which appears completely random. Certainly, if you encountered this sort of behaviour in Nature then you would probably assume that there was some complex mechanism underlying it, such as the interactions of a large population of animals. You would not imagine that such randomness could be generated by something as simple as $2x^2-1$.

But still, at least this appears like a predictable process. We have a simple equation, and as long as we know the starting value we should be able to plot the rest of the sequence with perfect accuracy. If the weather really does behave like this, then it appears we have discovered the secret of accurate long-term weather forecasting!

However, one day when he was comparing two plots of output data which were supposedly generated by the same input data, Lorenz got a shock. The two plots were very similar at first, but diverged after a few values until the two graphs became completely different. Lorenz was baffled. These plots were not hand drawn, they were computer-generated so there should be no room for error. If you give a computer the same data, and perform the same calculation repeatedly you should always get the same result. Computers are entirely predictable. This result seemed to challenge everything we know about how computers function.

Because of the predictable behaviour of computers, Lorenz immediately suspected his input data had to be different in the two circumstances. When he examined the second sequence he saw he had set the value at the start of the sequence to be 0.506. He had copied this value from the first sequence. However, when he examined the first sequence he found the values there actually had six decimal places: 0.506127. In order to save time, Lorenz had not bothered to copy all the decimal places — he did not think they were important.

Lorenz was surely justified in thinking these extra numbers — just one part in a thousand — were not important. You would certainly not expect small features like a tiny gust of wind to be able to have a huge effect on the large-scale weather pattern. But this appeared to be the case. Lorenz looked at the two graphs and, almost immediately, knew his quest to predict the weather was doomed to failure.

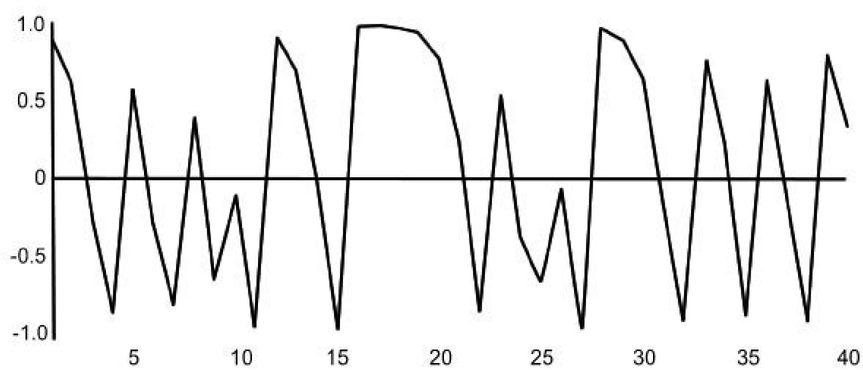
The butterfly effect

In order to duplicate Lorenz's discovery, let us return to our simple expression $2x^2-1$. Our first graph of values was generated from a starting value of 0.9. This time let us start with a very slightly different value of 0.9001. You might not imagine such a tiny difference — just one part in a thousand, as with Lorenz's experiment — would have much effect. In fact, you might expect this tiny variation to be barely visible on a graph.

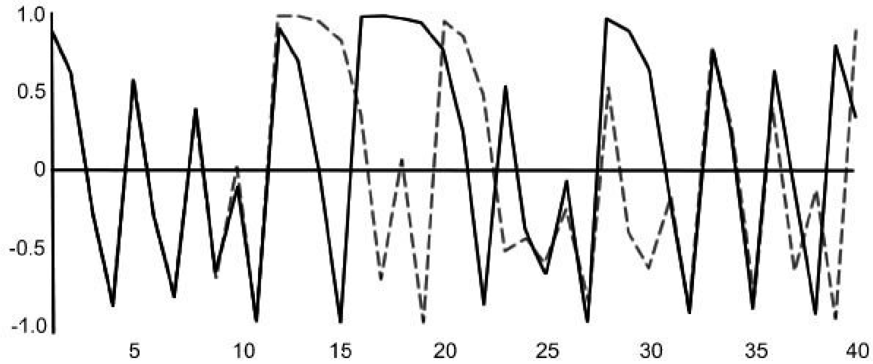
As before, you might like to calculate some of the values using your calculator, this time starting with 0.9001. I have calculated the first twelve numbers for you, but you might like to check:

0.9001
0.62036
-0.2303068
-0.8939174
0.5981768
-0.2843688
-0.8382687
0.405389
-0.6713194
-0.0986603
-0.9805322
0.9228871

And here is a plot of the sequence again extended to 40 numbers:



The graph appears very similar to the first graph (which was calculated starting with the value 0.9), but let us superimpose the first graph over the top of this second graph (the first graph is shown by the dotted line):



At the start, you will see that the two graphs are in very close agreement. However, after approximately ten points the two graphs start to diverge. After fifteen points the two graphs look completely different — despite starting with almost identical values.

This extreme sensitivity to initial conditions is a property of all deterministic systems which behave in this chaotic manner. This behaviour is called *chaos*.

Returning to Edward Lorenz's attempt to predict the weather, we find that if we knew the precise position and velocity of every tiny pocket of air at any one time, then we could produce long-term weather forecasts of perfect accuracy. However, for all practical purposes, it is simply impossible to obtain this perfect knowledge. We would have to know the value of all variables with perfect accuracy, to an effectively infinite number of decimal places. As we have seen in our experiment with $2x^2-1$, any slight error — no matter how small — would be magnified over time to produce huge later errors in our forecasts.

Even the tiniest fluttering of a butterfly's wings would affect the state of the current weather by a small amount, and that effect would become magnified over time. Lorenz called this principle the *butterfly effect*, the principle that a butterfly flapping its wings in China could potentially cause a hurricane next month in New York.

Lorenz was well aware of the implications of this discovery for long-range weather forecasting: "When our results concerning the instability of nonperiodic flow are applied to the atmosphere, which is ostensibly nonperiodic, they indicate that prediction of the sufficiently distant future is impossible by any method, unless the present conditions are known exactly. In view of the inevitable inaccuracy and incompleteness of weather observations, precise very-long-range forecasting would seem to be non-existent."

In Chapter Two we considered the uncertainty due to quantum mechanics and found that — at the very base of Nature — there was a fundamental uncertainty. Randomness seems built into the very fabric of Nature at a low level. However, the uncertainty we encounter due to chaos is a very different kind of uncertainty.

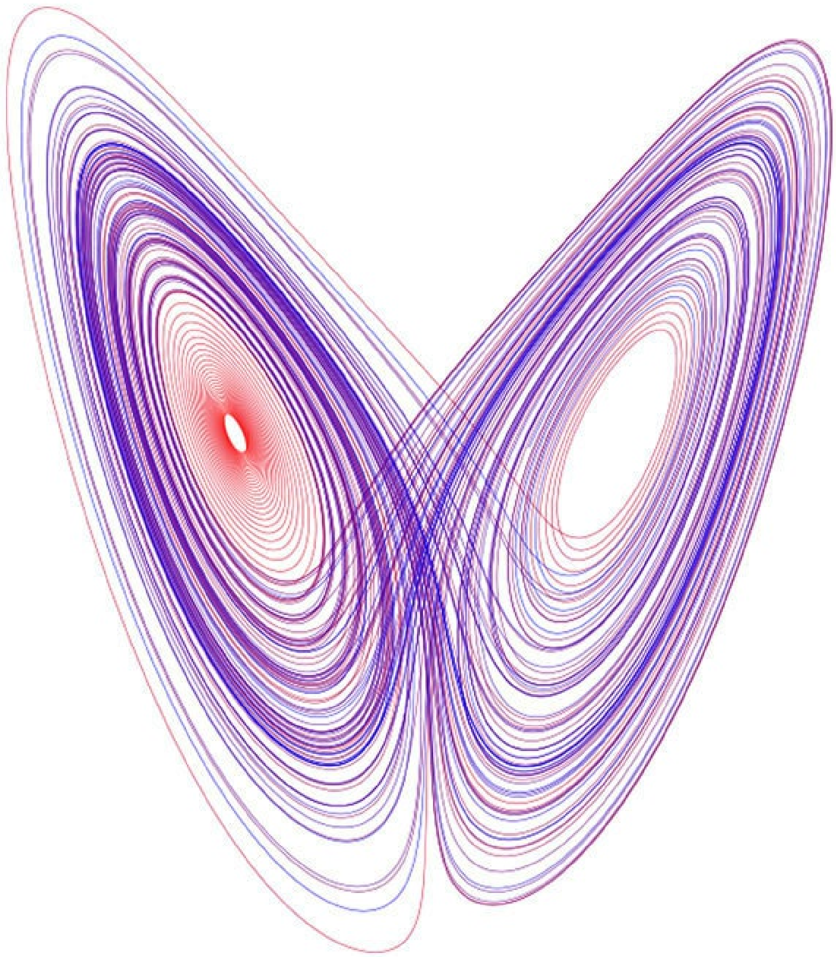
The processes which produce chaos are completely deterministic (unlike the case with quantum mechanics). Every pocket of air in the atmosphere moves in a perfectly deterministic manner according to Newtonian laws. **There is no randomness in this situation**, there is only the illusion of randomness. Chaos is produced by simple deterministic equations creating behaviour that appears random. From the point of view of this book, we have found another source of uncertainty, but this uncertainty is generated by a very unlikely source: determinism.

What is more, while the fundamental uncertainty of quantum mechanics only becomes significant at very small scales (for example, attempting to determine the velocity and momentum of a particle), we encounter the uncertainty of chaos at the far larger human-scale. Every time you feel turbulence in an airplane, or watch a flag flapping in the breeze, or simply look at the clouds drifting across the sky, you encounter the uncertainty of chaos. As James Gleick says in his book *Chaos*: "Twentieth-century science will be remembered for just three things: relativity, quantum mechanics, and chaos. Of the three, the revolution in chaos applies to the universe we see and touch, to objects at human scale."

Strange attractors

Lorenz's three simple equations had just three variables — x, y, and z — which denoted the current state of the system at any particular time, plus two fixed parameters: σ (which represented the viscosity of the fluid) and ρ (which represented the temperature difference between the top and bottom of the volume of air). These fixed parameters would be set to particular values before the simulation was run in order to obtain an interesting output.

In order to examine how the convection system developed over time, Lorenz took the x, y, and z values (which represented the state of the system at any particular time) and plotted a single point in a three-dimensional space, the coordinate of the point being the value (*x*, *y*, *z*). As the system developed over time, this single point moved in the three-dimensional space and plotted a curve. The resulting diagram is shown below (although it is a two-dimensional diagram, it is actually a rendering of a three-dimensional plot):



This method — by which the state of a system is represented by a single point moving in a multi-dimensional space — is a common approach in physics and engineering. The space is called a *phase space*.

This diagram is the most famous diagram in the history of chaos research, appearing in Lorenz's groundbreaking 1962 paper on chaos.[7] You can clearly see the diagram has two lobes. Depending on which lobe the point is currently orbiting, this determines whether the convection current is circulating clockwise or anticlockwise. If the point switches to orbiting the opposite lobe, then that represents a reversal in the direction of the convection current. These type of diagrams, which show the evolution of the state of a chaotic system over time, are called *strange attractors*. Strange attractors are objects which live in phase space. This particular diagram is called the *Lorenz attractor*.

These diagrams are called "attractors" because whatever random state a system might be in initially, the effect of chaos will always be to drag the state of that system towards the attractor. For example, in the case of a volume of air being heated from below, no matter what random configuration the air molecules are in initially, the effect of chaos is to create circular convection currents. Thus the state of the system is dragged towards the Lorenz attractor as if experiencing a "strange attraction". Hence, ironically, chaos operates to create structure, and that structure is the strange attractor.

The most important feature of the Lorenz attractor diagram is that the point never writes over the same point twice — the point continually writes a new curve through space, showing that the system is never in the same state twice. Hence, the system never repeats — it continually creates new convection patterns in a completely unpredictable way.

Strange attractors are therefore infinitely detailed: no matter how much you zoom in on them, you will always find more detail. This infinite detailing causes the extreme sensitivity to initial conditions in chaotic systems: even the slightest difference in position, the slightest movement from one pixel to the next, can have a huge effect on the later state of the system. Even the tiniest fluttering of a butterfly's wings can move the point on the attractor which represents the current weather — thus modifying the long-term behaviour of the weather.

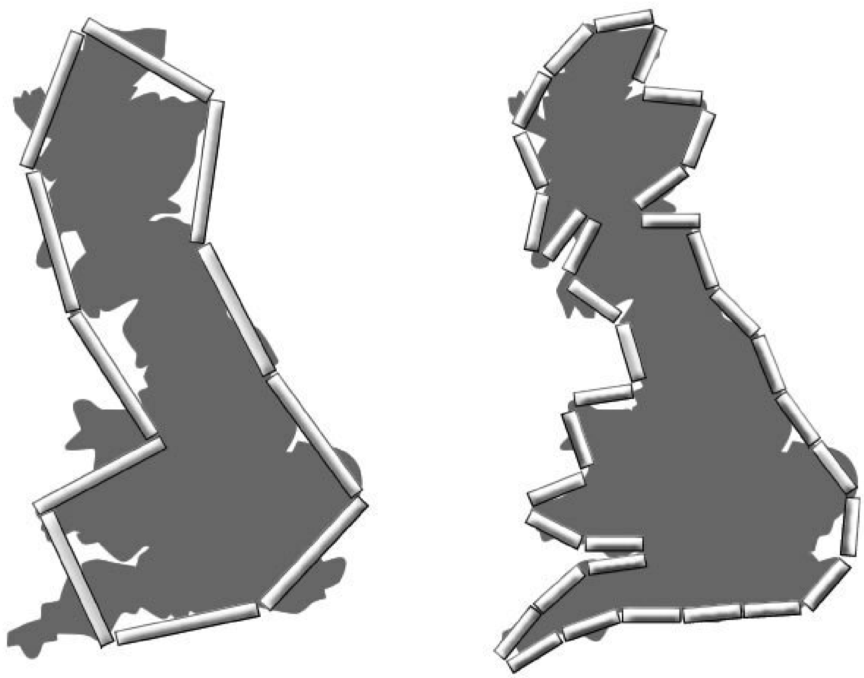
Structures which have this kind of infinite detail are called *fractals*. According to Ian Stewart: "It is now customary to define a strange attractor to be one that is fractal." Fractals are the geometry of chaos, and they are found throughout Nature.

Fractals

The French mathematician Benoit Mandelbrot was a scientific maverick who discovered the geometry of chaos, a geometry which we encounter throughout the natural world.

In 1960, Mandelbrot was working in the Thomas J. Watson Research Center, the pure research wing of IBM which was located 38 miles north of New York City. The scientists were effectively sheltered there, able to pursue whichever obscure research topic took their interest. IBM's philosophy was that if they got enough highly-intelligent people together in a single location then surely some great ideas would emerge which would benefit the company. Five Nobel Prize winners were to come out of the facility.

Legend has it that Mandelbrot became interested in unconventional forms of geometry when he wanted to obtain a value for the length of the coastline of Britain. On examining several encyclopedias, he found they all gave different values for the length. Mandelbrot realised that there was no single correct value for the length: it depended on how the length was measured. To be precise, it depended on the length of your measuring rod. If you imagine a giant measuring rod several miles in length, it would not be able to get into all the bays and inlets of the coast. Alternatively, a measuring rod only a few metres in length would be able to do a better job of measuring around all the bays. Hence, the shorter rod would measure a greater length for the coast of Britain than the longer rod.

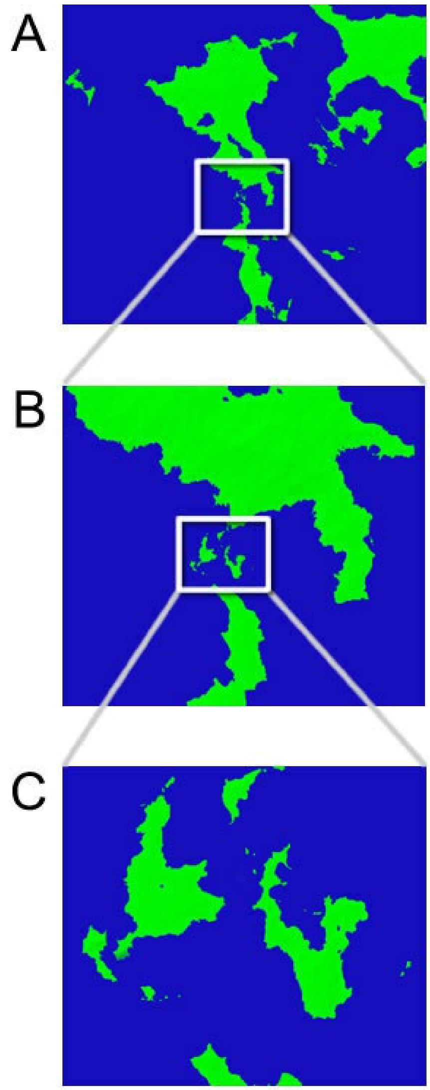


But which measurement should we accept as the accurate measurement of the length of the coast of Britain? You might intuitively accept the longer measurement — obtained with the shorter ruler — as the correct measurement. However, if we further reduce the length of our measuring rod so that it is only a few inches in length, we will find we get an even longer measurement (as we can now get our ruler into the smallest nooks and crannies of the coast). So it appears that the smaller our ruler, the longer our measurement of the length of the coast. So, remarkably, it emerges that there is no "correct" measurement of the length of the coast of Britain: the value is completely dependent on the length of the measuring ruler.

It would appear that — as we zoom into the coastline — we find an ever increasing range of detail. The further we zoom, the more detail we find. And it appears that we could keep zooming into the coastline without limit: our measurement for the length of the coast would just keep getting larger.

In order to see this peculiar property of the coastline, examine the next set of three images. The images show an area of coast at three increasing levels of zoom. A smaller rectangular area of image A is expanded to form the entirety of image B. Hence, image B is a zoomed version of image A. Similarly, a smaller rectangular area of image B is expanded to form the entirety of image C. Hence, image C is a zoomed version of image B.

If you examine image C you will see that the general characteristics of the coast — with its rocky bays and inlets — looks very much the same as the coast in image A. No matter how far you zoom into a coast, it looks very much the same. We could even imagine zooming down to the level of small rock pools — the basic shapes would still be similar. So the coastline looks similar at small and large scales.



As another similar example, consider a small rock — only a couple of inches high — which might be found on a mountain:

and compare that rock to the mountain on which the rock was found (in this case, Camelback Mountain near Phoenix, Arizona):

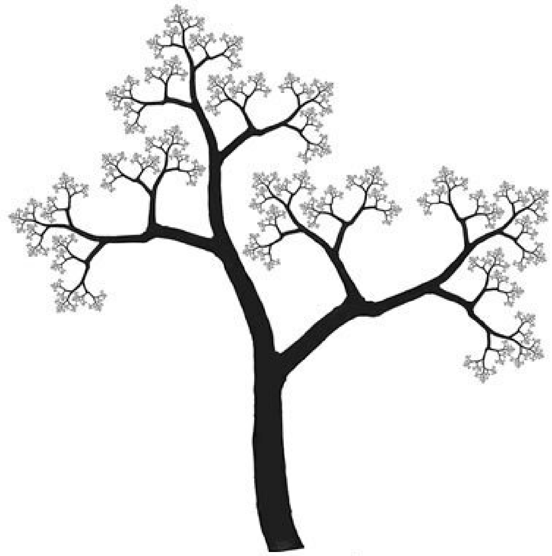
The similarity is clear. And you could even zoom further into the small rock and find a miniature rugged landscape which resembles a microscopic mountain range. So there is a similarity across scale between small rocks and large mountains just like there is a similarity of the coastline at small and large scales.

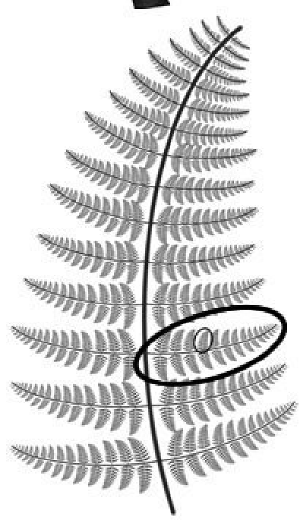
So there can be a similarity between a large object, and the smaller objects which comprise the large object. This is called *self-similarity*, and it is a common property of many natural objects.

Benoit Mandelbrot realised that these objects possessed a symmetry. But this was not the same kind of symmetry seen in normal geometric objects. Conventional symmetry meant an object looked the same when it was reflected in a mirror, or turned upside down. Mandelbrot realised that self-similarity resulted in a symmetry when an object was examined at different scales: if you zoomed into an object, it still looked the same.

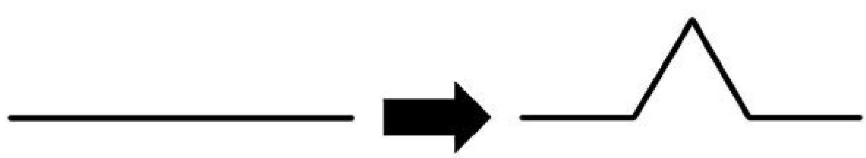
So we have seen that coastlines and rocks have self-similarity, but self-similarity can also be found in living objects. In the following diagram of a tree, you will see that each branch, and each twig, of the tree resembles the tree in its entirety. Hence, if you were to snap off a branch, it would resemble a small tree. And if you snapped a few twigs off that branch, it would again resemble a small tree.

So trees — and many other plants — possess self-similarity in abundance. In the following diagram, an example of a fern leaf is also shown, the leaf being composed of 30 segments. You will see that each of those 30 segments resembles the entire fern leaf. And each of those segments can be further subdivided into 30 sub-segments which each resemble the entire fern leaf:

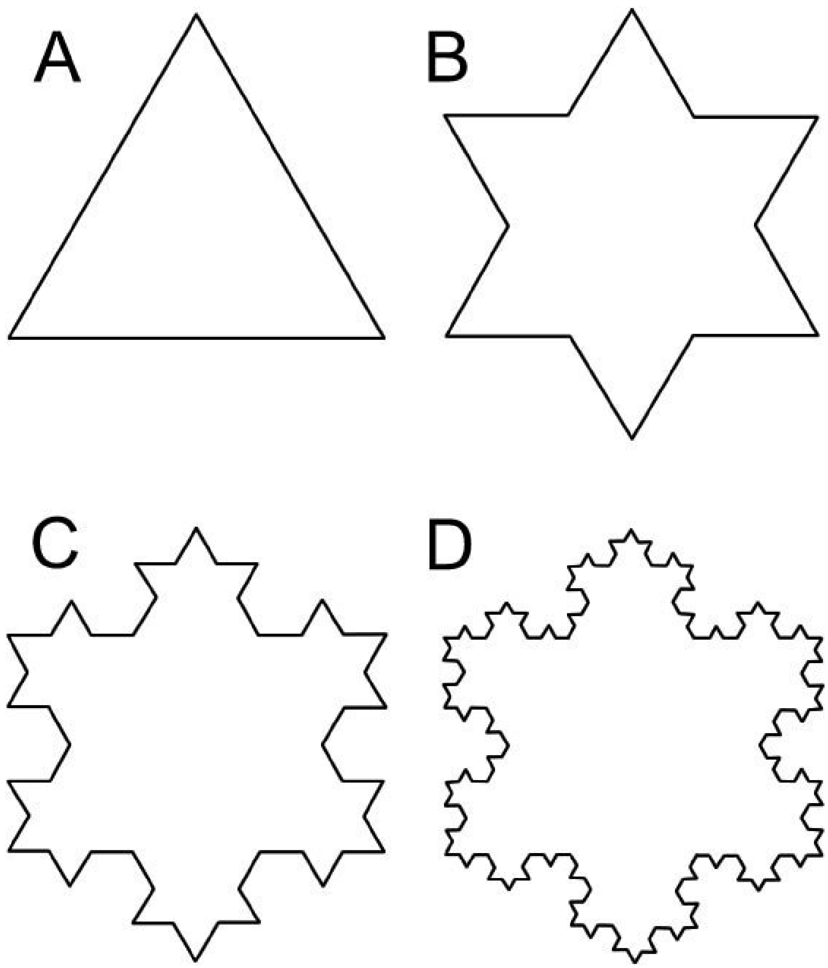




In order to see how Nature uses simple mechanisms to produce these strange zoomable structures, consider the following transformation:



What this simple transformation does is replace a straight line with a line with a triangular kink in the middle. In order to see how this transformation can be applied, consider the following four shapes:



Shape A shows a simple triangle. In shape B, we have used the previous transformation to replace each straight line of the triangle with a line with a triangular kink in it. Similarly, to generate shape C we have applied the transformation again to shape B, replacing each of its straight lines with a line with a triangular kink in it. Shape D is generated in a similar manner from shape C. This transformation could be applied repeatedly — continuing to infinity — producing an ever more elaborate final shape. This shape is called the *Koch snowflake*.

We could imagine the jagged edge of the Koch snowflake as representing a coastline: you can keep on zooming forever. So we find that by applying very simple construction rules repeatedly, Nature can produce structures of infinite complexity. Mandelbrot named these structures *fractals*.

As was considered in the previous chapter, classical geometry was defined two thousand years ago by the Greek mathematician Euclid. Euclidean geometry is based on straight lines, and regular geometric shapes such as circles, spheres, triangles, and cones. For most school pupils, this is the only type of geometry they will ever study. However, when we look around and consider the natural world, this is not the geometry we see. As Benoit Mandelbrot said in his book *The Fractal Geometry of Nature*: "Clouds are not spheres, mountains are not cones, coastlines are not circles, and bark is not smooth, nor does lightning travel in a straight line." The true geometry of Nature is fractal geometry, not Euclidean geometry.

Just as chaos results from simple rules which produce endlessly complex behaviour, so fractals result from simple rules which produce endlessly complex structures. Just as chaos is the true language for describing the behaviour of Nature, so fractals are the true language for describing the structure of Nature.

The Mandelbrot set

In discovering how Nature manages to produce these wonderfully intricate fractals, we have seen that simple processes are continuously applied in an iterative manner. In a similar example from mathematics, we might consider the *Mandelbrot set*, which is an incredibly beautiful, infinitely detailed fractal which is produced by a very simple formula.

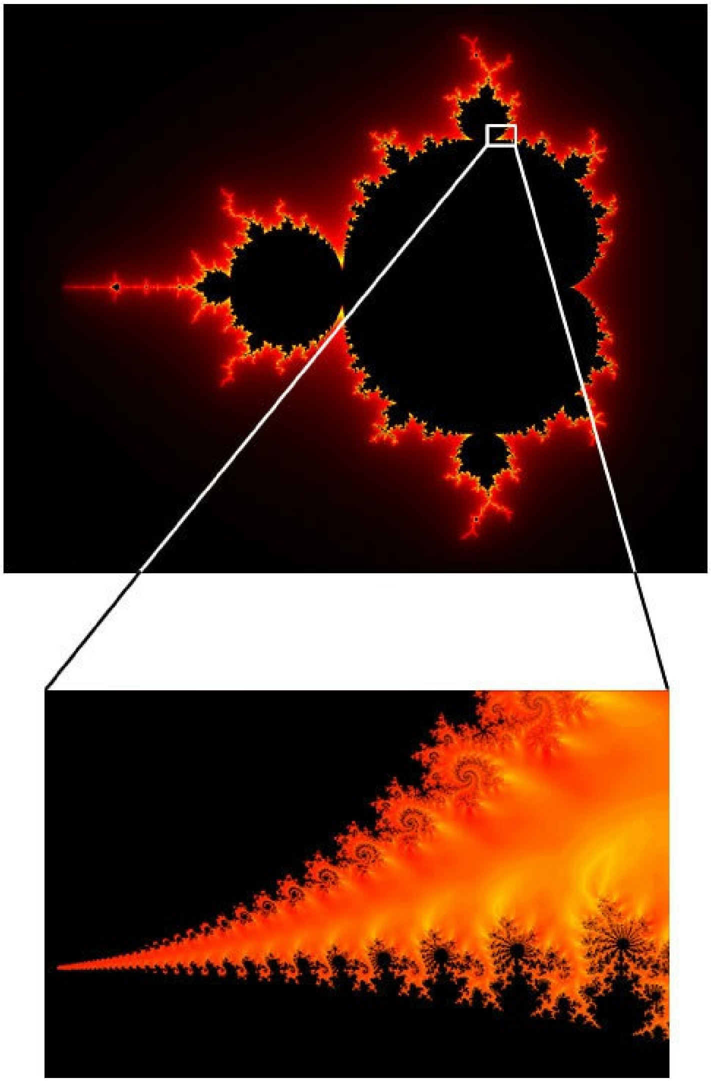
Benoit Mandelbrot discovered the Mandelbrot set by considering complex numbers, which we first encountered in Chapter Two in the discussion of Schrödinger's wavefunction. If you remember, a complex number has two parts: a *real* part (which is a conventional real number) and an *imaginary* part. The imaginary part is based on the square root of -1. This is rather strange because you might realise that the square of no conventional number can be a negative number. So how can -1 have a square root? Well, we don't worry about that — we just symbolise the square root of -1 by the letter i. The imaginary part of a complex number then consists of a number multiplying i. Hence, an example of a complex number might be: $6 + 3i$.

One of the most useful features of complex numbers is that they can be represented geometrically. It is possible to plot complex numbers on a two-dimensional graph with the real part being plotted along the horizontal x axis, and the imaginary part being plotted up the vertical y axis. This is called the *complex plane*. Hence, in the complex plane, the real axis is horizontal, and the imaginary axis is vertical.

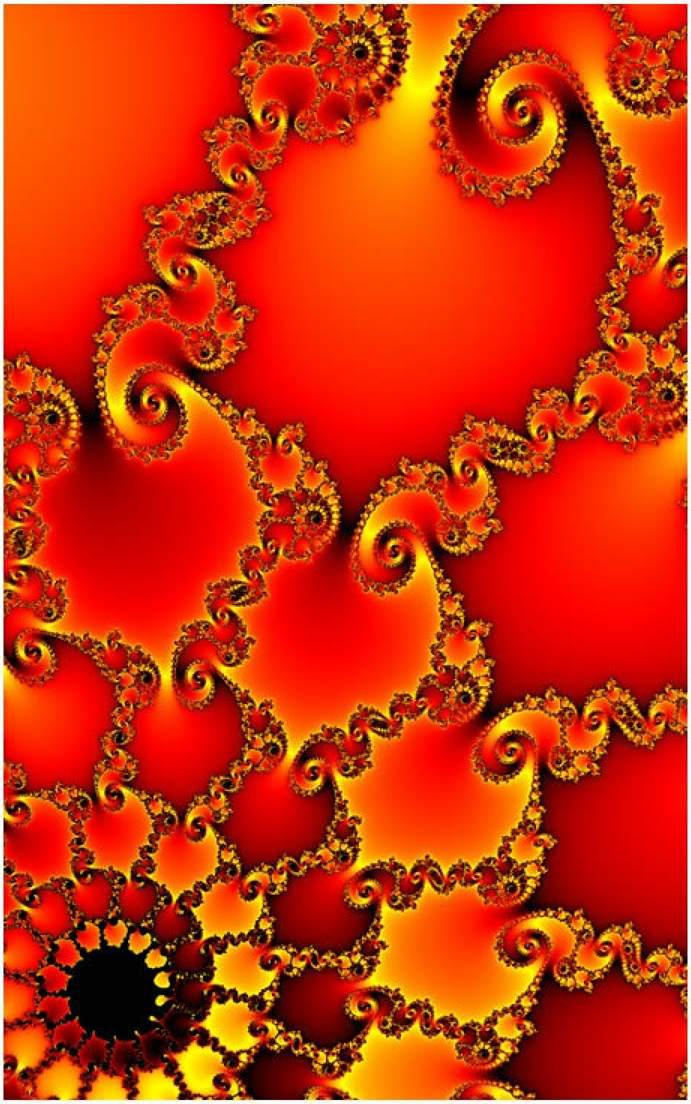
An image of the Mandelbrot set can be produced very simply. Consider a point in the complex plane, and the complex number represented by that point. You then apply a simple iterative process which involves taking the square of the number and adding the original number. You then square the result, and add the original number again. This process is performed several times. If the resultant number stays low (within a certain limit) then the point **is** in the Mandelbrot set, and the associated pixel is coloured black. However, if after a number of iterations the resultant number shoots off to a very large value, the point is **not** in the Mandelbrot set. In that case, instead of colouring the pixel black, the pixel is assigned a colour based on how many iterations were performed.

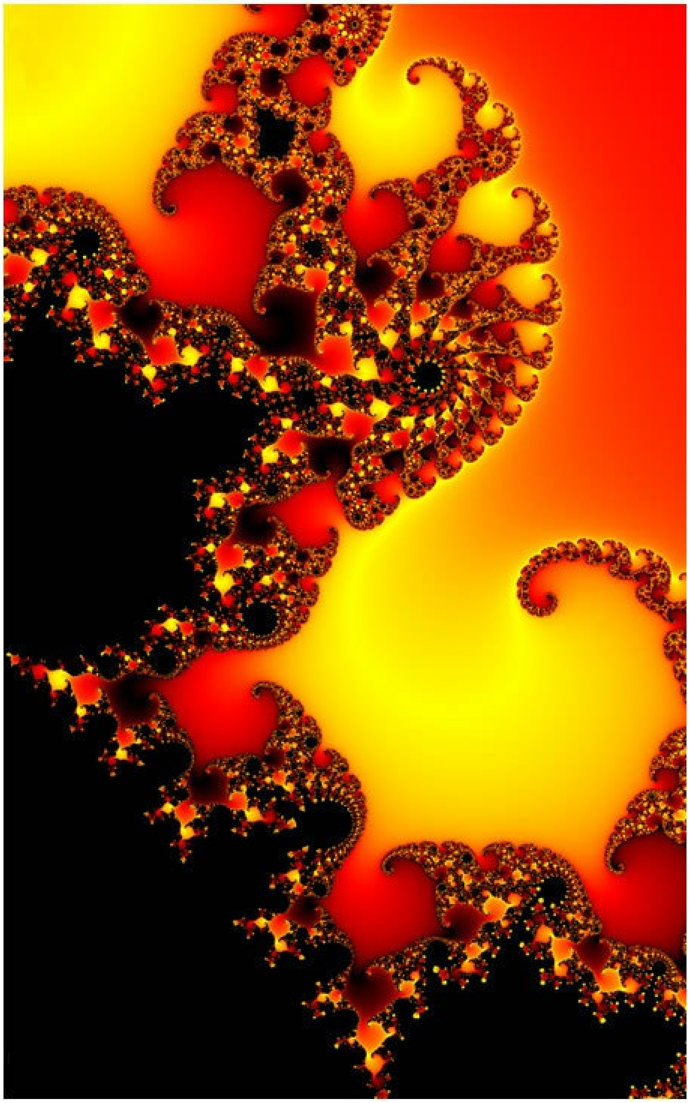
This can be coded as a very simple computer program (I have done it myself several times). You can also get various apps on your mobile phone and tablet to explore the Mandelbrot set. These allow you to zoom into the image very simply by using two fingers to pinch and zoom. As you zoom into the set, the full beauty of the Mandelbrot set is revealed. I believe the best app on an iOS device is called "Frax". On my Android tablet I used the free "Fractoid" app (very simple to use and well worth getting) to generate the following images of the Mandelbrot set.

The first image shows the full Mandelbrot set, featuring the characteristic large black blobs. You will notice a very small white rectangle has been drawn on the top border of the set. This is magnified in the lower image, so you can see how increasing levels of detail emerge as you zoom in. You will notice how this magnified image resembles the earlier image of a fractal fern, perhaps giving a clue as to how Nature produces fractal structures by repeatedly applying simple iterative processes.



The following two images were obtained using the Fractoid app, zooming in to various areas of the Mandelbrot set. I hope they give you an impression of the beauty of the Mandelbrot set. In the first image you might be able to identify areas which resemble turbulence in a fluid (such as smoke rising):





Fractal dimensions

When we consider a fractal border, such as the Koch snowflake, we find that as we zoom in to the fractal the border gets fractionally longer. This is a similar principle to the idea that if we measure the coastline of Britain with progressively shorter rulers we get an ever increasing value for the length of the coast. The conventional measurement of length clearly has no validity when measuring a fractal coastline, so Mandelbrot realised that another form of measurement was required: the *dimension*.

This is a surprise as we do not usually think of the number of dimensions of an object as a value of measurement. However, a square is a two-dimensional object, and a cube is a three-dimensional object, so the number of dimensions an object possesses is clearly a property of that object — just like its width and height.

Mandelbrot asked a simple question: what is the dimension of a ball of string? The answer is not as obvious as you might assume. From a great distance, the ball appears as a tiny point, and would therefore have zero dimensions. If we move closer to the ball of string, it appears as a sphere and therefore has three dimensions. However, if we actually consider the material of which the ball is made — the string — then we can consider it as only having a length: a one dimensional object. So the number of dimensions of the ball of string is dependent on your point of view, a number from zero to three.

So returning to the theme of this book, once again the old notion of certainty in mathematics has taken a blow. From considering objects as having a clear observer-independent number of dimensions, we now find that the number of dimensions is uncertain — dependent on the observer. Mandelbrot — a mathematician — was even so bold as to suggest a connection with developments about uncertainty in physics: "The notion that a numerical result should depend on the relation of object to observer is in the spirit of physics in this century and is even an exemplary illustration of it."

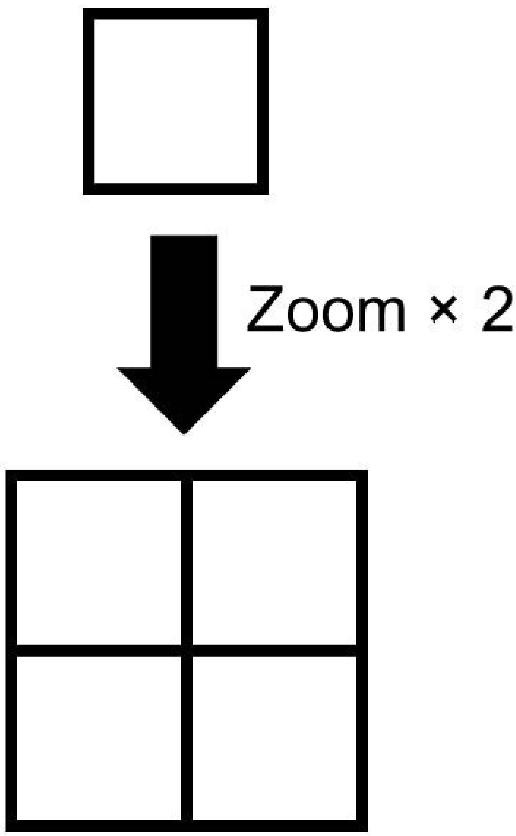
So a ball of string can be considered as having zero dimensions when viewed from far away, or three dimensions when viewed up close. But what about intermediate distances? What about values between zero and three dimensions? Mandelbrot came up with a remarkable new concept: fractional dimensions. He suggested we could consider an object as having 2.7 dimensions, for example, or 1.55 dimensions.

So how do we calculate how many dimensions a fractal has? Mandelbrot realised that the number of dimensions of a self-similar object (such as a fractal) could be calculated by a simple formula:

$$D = \frac{\log(N)}{\log(Z)}$$

where N is the total number of self-similar pieces in the fractal, and Z is the zoom factor. The formula basically reveals how many more self-similar pieces of the fractal do we see as we zoom out (as if we are zooming out of a coastline to reveal more detail).

In order to see how the formula works, let us consider one of the simplest self-similar objects: a square. Yes, surprisingly a square is a self-similar object as it can be divided into a number of smaller squares, each exactly the same shape (but not the same scale) as the larger square. Imagine we zoom out of a square by a factor of two, thus revealing more detail:

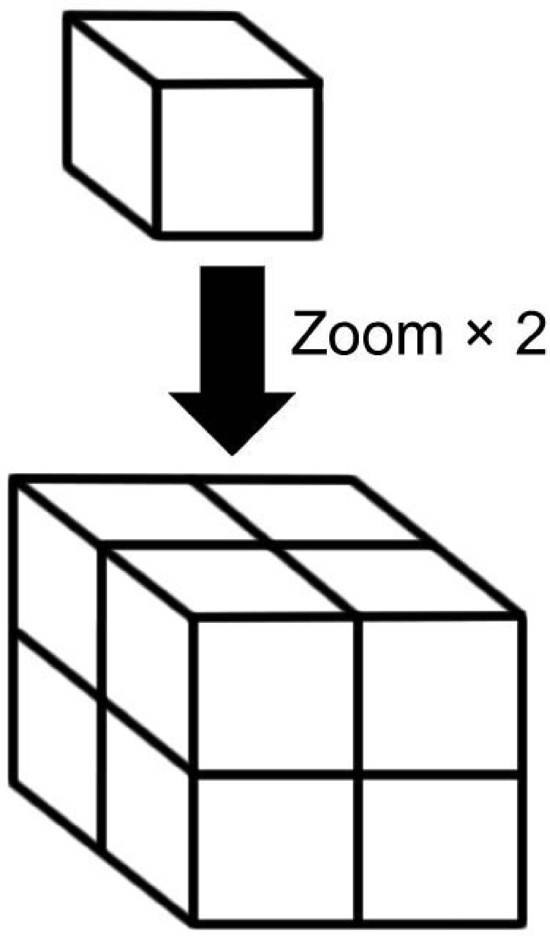


It can be seen that each side of the resultant larger square is twice as long (because of the zoom factor). However, it can also be seen that the larger square actually contains four self-similar smaller squares. So, from Mandelbrot's formula, we can calculate the dimension of the square as:

$$D = \frac{\log(4)}{\log(2)} = 2$$

So the formula states that a square has two dimensions: it is a two-dimensional object, which is the result we would expect.

Now let us use the formula to calculate the number of dimensions of a cube (yes, a cube is also a self-similar object as it is composed of a number of smaller cubes). As before, imagine we zoom out by a factor of two, revealing more detail:



As before, it can be seen that each side of the resultant larger cube is twice as long (because of the zoom factor). However, this time the resultant cube contains eight self-similar smaller cubes. So, according to Mandelbrot's formula, the number of dimensions of a cube is:

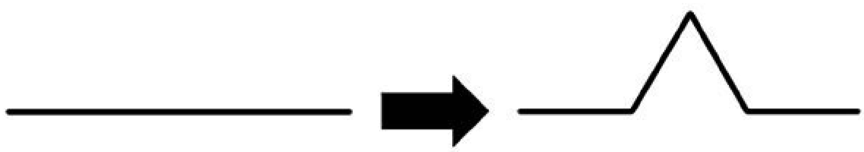
$$D = \frac{\log(8)}{\log(2)} = 3$$

So the formula states that a cube has three dimensions: it is a three-dimensional object, which is the result we would expect.

If we repeat this process for a straight line (yes, a line is also a self-similar object as it can be thought of as being composed of many smaller lines) then the formula tells us that a line has one dimension — which is again the value we would expect.

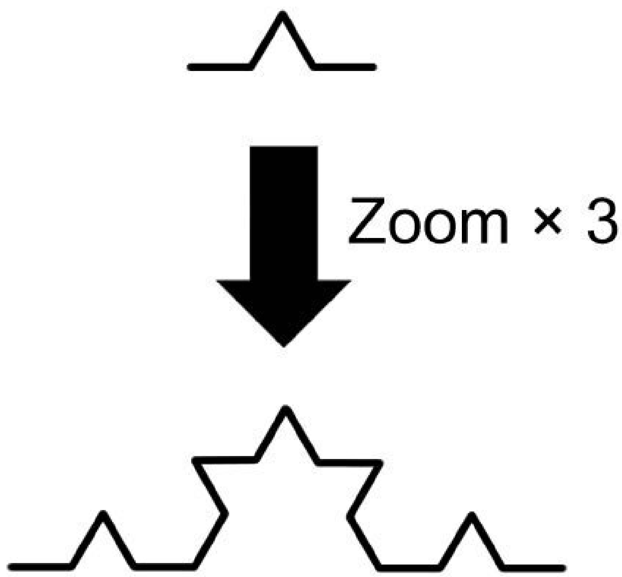
So when we consider the classical Euclidean shapes of the line, the square, and the cube, we find they all have an integer (whole number) dimensionality. But what do we find when we consider the dimensionality of a fractal? As an example, let us calculate the dimensionality of the Koch snowflake fractal which we considered earlier.

Remember how the Koch snowflake fractal was generated by a triangular line with a "kink" in the middle:



This type of kinky line which is used to form a fractal is called a *generator*.

Now let us zoom out of the Koch snowflake fractal in order to see more detail, but this time let us zoom out by a factor of three:



You will see that as we zoom out of the generator, more detail (the "coastline") starts to emerge. However, although we have zoomed-out by a factor of three, you will see that the second zoomed-out image is now composed of four of the initial generator shapes. So, from Mandelbrot's formula, we can calculate the dimension of the Koch snowflake as:

$$D = \frac{\log(4)}{\log(3)} = 1.26$$

So the formula is telling us that the Koch snowflake is an object with 1.26 dimensions!

It might seem bizarre to consider a structure as having 1.26 dimensions. Perhaps think of the Koch snowflake as being "rougher" than a line — which has a dimensionality of one — but not filling the entire two dimensional plane like a square would do (a square having two dimensions). So, on that basis, we might expect the dimensionality of a Koch snowflake to lie somewhere between 1 and 2.

When Mandelbrot considered the fractal coastline of Great Britain he found it had a dimensionality of approximately 1.24. In contrast, the coastline of South Africa — which is nearly circular — has a dimensionality of approximately one. It is clear that the rougher the fractal shape, the higher its dimensionality.

Mandelbrot was therefore finally able to answer his question "How long is the coastline of Britain?" with an answer presented in the only measure which made sense: the fractal dimension.

As we find fractal structures throughout the natural world here on Earth, we should perhaps not be so surprised to discover that the entire universe itself appears to have a self-similar fractal shape. Stars group together to form galaxies, galaxies group into clusters, and clusters group together into superclusters. There is some symmetry over scale here — just as in a fractal. In fact, the universe has been measured as being fractal on scales up to 350 million light years, with a fractal dimension of 1.2.

At the mercy of chaos

In this chapter we have seen how chaotic behaviour can emerge in simple mathematical systems, and also in natural systems such as the weather. The common theme in these systems is that a simple nonlinear process (as we have seen, the process could be as simple as $2x^2-1$) is iterated several times before the chaos emerges. If we now extend our study to consider human populations, with their complex behaviours and their millions of daily interactions, we should not be surprised to find chaotic behaviour emerging.

It was Benoit Mandelbrot (again) who first detected the signs of mathematical chaos in the behaviour of the stock market. When Mandelbrot was considering the behaviour of cotton prices over the previous century, he recognised a degree of self-similarity: the seemingly random jumps in price over a day resembled the shape of the jumps in price over a year. There was symmetry over scale. There was order within the disorder — the trademark of mathematical chaos.

However, this chaotic jumping of prices was contrary to conventional economics theory. According to orthodox economic theory (which I was taught as an undergraduate many moons ago — and I was dissatisfied with it then), prices should reach a steady equilibrium when supply of a product matches the demand for that product. But if equilibrium is the natural state of affairs, then how come markets are so turbulent?

As Mandelbrot says in his book *The (Mis)Behaviour of Markets*: "To me, all the power and wealth of the New York Stock Exchange or a London currency-dealing room are abstract; they are analogous to physical systems of turbulence in a sunspot or eddies in a river."

In his book, Mandelbrot firstly describes the conventional approach to predicting the behaviour of the financial markets: "There are many ways of handling risk. In the financial markets, the oldest is the simplest: 'fundamental' analysis. If a stock is rising, seek the cause in a study of the company behind it, or of the industry and economy around it. 'Because' is the key word here. The price of a stock, bond, derivative, or currency moves 'because' of some event or fact that more often than not comes from outside the market. World wheat prices rise because a heat wave desiccates Kansas or Ukraine. The dollar sinks because talk of war raises oil prices. Financial newspapers thrive on it; they sell news and rank the importance of all the 'becauses'. The implicit assumption in all this: if one knows the cause, one can forecast the event and manage the risk. Would it were so simple. In the real world, causes are usually obscure. The precise market mechanism that links news to price, cause to effect, is mysterious and seems inconsistent."

Mandelbrot then goes on to explain why it is simply not possible to find simple causes — the "becauses" — of complex market behaviour. He does this by explaining how a financial market is a nonlinear dynamic system which is effectively impossible to model and analyse accurately. Mandelbrot considered the interactions in a simulation of two groups of investors who behave differently: "In computer simulations by economists in Belgium, the two groups start interacting in unexpected ways, and price bubbles and crashes arise spontaneously. The market switches from a well-behaved linear system in which one factor adds predictably to the next, to a chaotic nonlinear system in which factors interact and yield the unexpected."

This switch from linearity to nonlinearity is crucial. A linear system can be broken into individual elements — "black boxes" — and the behaviour of the entire system can be analysed by considering the behaviour of each individual element. This is not possible with a nonlinear system which cannot be broken-down in the same way. Instead, it is only possible to analyse the system as a whole — a far more complex task.

The financial crisis of 2008 (which Mandelbrot considers in later versions of his book) is often blamed on the missellling of subprime mortgages in the United States. However, this implies that we can break down the market into individual units (one of those units being the selling of mortgages) and place the blame for the crisis on one particular unit — as if the selling of mortgages was the "cause" of the crash. But this is simply not the case in a nonlinear system. A nonlinear system cannot be subdivided. Its individual units cannot be considered in isolation. No single unit can be considered as being the cause of the overall behaviour.

A recent news story was that a 90-year-old famous man had "died from pneumonia". But, of course, he did not die of pneumonia: he died from being 90 years old. When you are 90 years old, even a common cold can kill you. But the true cause of death would not be the cold — it would be your age. To try to isolate one particular cause is to miss the bigger picture, it is to fail to see that the real problem lies with the entire nonlinear system (yes, a human body is another example of a nonlinear system). Once again, individual units cannot be considered in isolation.

As we saw with the weather, a butterfly flapping its wings in China could potentially cause a hurricane next month in New York. So it is in the financial markets: a trader waking up late one morning could cause a crash on Wall Street in three months time. To blame the selling of subprime mortgages is to miss the point: it can never be possible to identify a single cause and try to isolate that cause to eliminate the possibility of market chaos. The possibility of chaos is always latent in the system and can never be eliminated — it is built into the design of the system. It is an inherent property of nonlinear systems.

We have created a monster.

So just as we can never eliminate the possibility of storms from the weather, we can never eliminate the possibility of chaos in our financial markets. All we can do is try to insulate ourselves from the financial storms when they do eventually come. As Mandelbrot says: "For centuries, shipbuilders have put care into the design of their hulls and sails. They know that, in most cases, the sea is moderate. But they also know that typhoons arise and hurricanes happen. They design not just for the 95 percent of sailing days when the weather is clement, but also for the other 5 percent, when storms blow and their skill is tested. The financiers and investors of the world are, at the moment, like mariners who heed no weather warnings."

In this respect, humanity is at the mercy from the nonlinear systems it has created. But the system of financial markets is not the nonlinear system which poses the greatest threat for the stability of humanity. The complex military network around the world poses the greatest threat. When nations arm themselves, they might feel they are acting responsibly as they have created a strong defence against possible aggression. But all they are really doing is adding to the complexity of the existing worldwide military nonlinear system. They are simply making a bad situation worse. The die is cast — even before war breaks out. Mankind is no longer in control of the situation — there is only an illusion of control. The weather might appear calm for an extended period of time, but, from now on, mankind is at the mercy of chaos. And the butterfly effect tells us that there is always the possibility that a tiny disturbance will be responsible for a terrible storm. This principle was illustrated perfectly in Sarajevo in 1914 when a single assassin's bullet killed Archduke Franz Ferdinand of Austria and instigated the tragedy of the First World War.

According to the BBC TV programme about chaos entitled *High Anxieties* (available on YouTube): "When World War One broke out, the world was still determined to believe in a Newtonian world where even the complexities of human affairs **could** be predicted, and the war **would** be over by Christmas. They all wheeled out their artillery confident that reality could be made to follow their plans. After all, the same mathematics which predicted the orbits of the planets also governed the firing of artillery. The mathematics of ballistics told them exactly where the shells would land. The tragedy of the war was that no matter what carnage unfolded, they believed there was not only a science of ballistics, but a science of war: a way of predicting how many men per mile of front would win the objective. They thought if they understood the science of war they could control its outcome — they could simply overpower the chaos. In the end, they weren't fighting each other. Both sides were fighting the chaos of reality. In the end, the chaos defeated them all, and ten million men died."

6

HOLLYWOOD

Hollywood is the town where dreams become reality. We are told that every waitress wants to be an actress, every limo driver has written a screenplay. And dreams become reality in Hollywood movies as well. No one would pay to watch a purely realistic movie of dreary office jobs, or being stuck in interminable traffic jams. So movies portray escapism: an artificial reality, a synthetic universe of musicals, adventure, and imaginative science fiction. The advances in computer-generated imagery (CGI) have made it more cost effective to produce entire alien worlds in a computer rather than construct them in a studio. If you watch a Hollywood movie you know you will be transported into another, better — but completely artificial — universe.

The director James Cameron was well-aware of the importance of creating a convincing artificial world when he created the alien planet Pandora for his movie *Avatar*: "You've got to compete head-on with these other epic works of fantasy and fiction, the Tolkiens, and the Star Wars, and the Star Treks. People want a persistent alternate reality to invest themselves in and they want the detail that makes it rich and worth their time. They want to live somewhere else. Like Pandora."

However, we are not only transported to another world when we enter a movie theatre. Modern consumer society is based on creating artificial reality. Enter a modern shopping mall, which may have a dreary exterior, and you might find yourself entirely enclosed in an artificial retail environment which resembles a glamorous European catwalk. Go to a swimming pool and you might find yourself in a synthetic environment which resembles a tropical island. Go to Las Vegas and you might well find yourself in a simulation of anywhere on Earth. Play an immersive video game and you might well find yourself in a simulation of anywhere in the universe.

In 1981, the French philosopher Jean Baudrillard wrote a short book entitled *Simulacra and Simulation* which explored the idea of modern life being a simulation. Baudrillard picked the particular example of Disneyland as an extreme example of a completely immersive artificial reality. The zero-crime zero-litter policy of Disneyland is an attempt to generate an improved version of reality in much the same way that a Hollywood movie provides escapism. As Baudrillard says: "Disneyland is a perfect model of all the entangled orders of simulacra. It is first of all a play of illusions and phantasms: the Pirates, the Frontier, the Future World, etc. This imaginary world is supposed to ensure the success of the operation. But what attracts the crowds the most is without a doubt the social microcosm, the religious, miniaturized pleasure of real America, of its constraints and joys. One parks outside and stands in line inside, one is altogether abandoned at the exit."

Baudrillard is making the point that visitors to Disneyland mistakenly perceive Main Street in Disneyland as being an accurate — though improved — copy of a Main Street in a real American town outside the gates of Disneyland. Whereas, the truth is that old-fashioned perfect Main Street simply does not exist in modern America. So Disneyland is not a copy of anything: Disneyland is a completely original reality. It is not accurate to call Disneyland an "artificial reality" because that would infer that it was a copy of a reality that actually existed. No, in Disneyland (and in our modern consumer culture) **the simulation has become the reality**. It is literally the American dream. Baudrillard referred to this artificial reality, which has no basis or resemblance to actual reality, as *hyperreality*.

When we watch a Hollywood movie, we become entirely immersed in its alternate reality. We might become so emotionally involved in the movie that we even cry at a sad moment. However, we would rarely cry at the start of the movie — we would only cry near the end of the movie, because it takes a full two hours for us to become emotionally involved to the point at which we care about the characters on the screen. At that point, the artificial reality has become our genuine reality. We might even forget there is a real world outside the movie theatre. As with Disneyland, the simulation has become the reality.

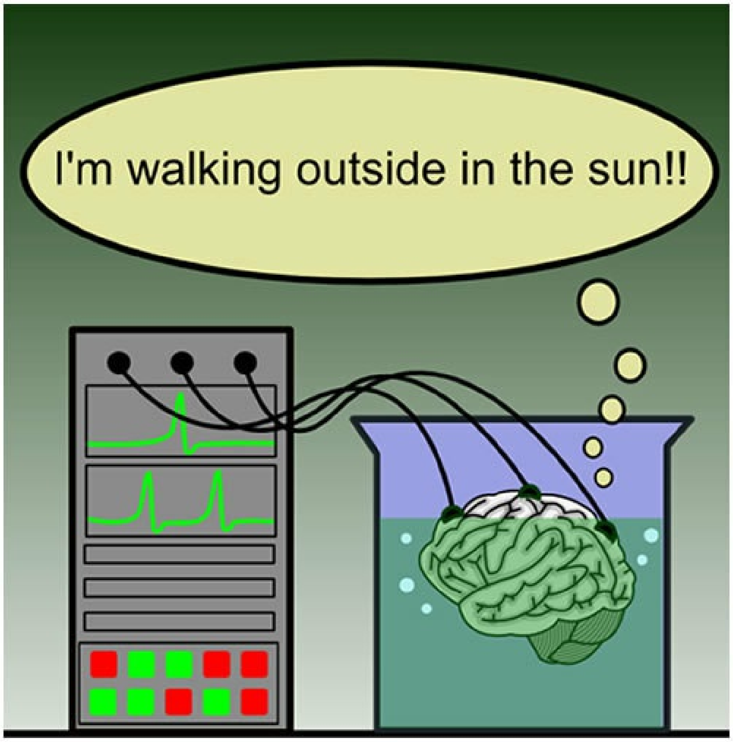
And that is when things become really interesting …

The Matrix

It is rather ironic that it was a Hollywood blockbuster movie which presented the work of Jean Baudrillard to the mass market. *The Matrix* was a science fiction movie released in 1999 which became something of a sensation, becoming the biggest-selling DVD of all time. About eight minutes into the movie, the character of Keanu Reeves takes a book down from a shelf. The book is hollow. It is a fake book, a simulation. In its hollow compartment it contains some computer disks. The book is none other than Baudrillard's *Simulacra and Simulation*. From this early statement of intent, it is clear this movie is going to be — in the words of the producers — an "intellectual action movie". [8]

In *The Matrix*, Keanu Reeves plays a computer hacker called Neo who is informed that the world in which he lives is unreal — a computer simulation. The illusion is being created by intelligent machines which now rule the Earth. Humans are being kept in a subdued state to provide heat and electrical activity as an energy source for the machines. The humans float in vats, while the illusory Matrix world is transmitted into their brains in order to subdue and pacify them. The simulation, of the world as it existed in 1999, is so accurate that it is indistinguishable from the real thing.

This idea that we might be living in a simulated reality caused quite a stir at the time, though it is far from an original idea. The idea dates back as far as 1641 when the French philosopher René Descartes suggested the possibility that the entire world was merely an illusion created by an evil demon. In that case, Descartes suggested that all sensations might be inserted directly into the mind, and even the human body might be an illusion. In the modern era, a computer-based version of the evil demon was provided by the American philosopher Hilary Putnam. Putnam proposed what is known as the brain-in-a-vat thought experiment. Putnam suggested that an evil mad scientist might have extracted your brain and was keeping it alive in a vat of liquid. All sensory neurons of your brain were connected to an external computer which was supplying an accurate simulation of an external world. The disembodied brain would have no idea of its predicament, and would probably be blissfully happy.



It would appear that there is no way of knowing whether we are brains-in-a-vat or if we are living in the Matrix, with our bodies suspended in fluid, our brains connected to a supercomputer. Might this impose fundamental limitations on our certainty?

However, there are weaknesses in these theories. In an article entitled *Why Make a Matrix? And Why You Might Be In One*, Oxford philosopher Nick Bostrom argues that there would be no obvious motive for an advanced civilisation to imprison our bodies in fluid. In the movie *The Matrix*, the human bodies are used to provide power, but as Bostrom points out: "Human brains may be many things, but efficient batteries they are not." Presumably, the human would have to fed as well, which would entail some form of energy input to the comatose human. The energy which could be generated by the entrapped humans could never be more than the energy supplied to the humans. The scenario might have made more sense if the humans were being used for food (after all, we feed cattle to supply food for ourselves, but we only get back a quarter of the energy present in the food which we feed to the cattle). But machine overlords would have no reason or desire to eat humans — I presume future computers would not have a digestive system!

And why should the advanced civilisation (or mad scientist) go to all the trouble of generating a simulated reality when the human brains could be kept subdued merely by the use of drugs? Basically, why bother creating the Matrix?

No, the whole brain-in-a-vat scenario in which we are suspended in a gloopy substance appears to make no sense. But there is another form of simulated reality which is more interesting, and which is being taken more seriously by the scientific community.

Simulacron-3

In 1964, the second book by science fiction author Daniel F. Galouye was published. The book was called *Simulacron-3*. It really was quite a remarkably foresighted work, and has been called a "virtual reality novel from a time before virtual reality". Not only was the novel ahead of its time in terms of technology, its style was also highly-influential, being called the first "cyberpunk" novel.

In *Simulacron-3* we meet our main protagonist Douglas Hall who is a computer developer specialising in the construction of simulated computer societies (Hall's field of expertise is described as "simulectronics"). Individual computer-generated "people" are programmed to model human behaviour. The purpose of these simulations was to discover the behaviour of communities in order to guide government policy, and also to provide feedback to marketers.

Hall describes the computer-generated "people" who populate the simulation: "We can electronically simulate a social environment. We can populate it with subjective analogs — reactional identity units. By manipulating the environment, by prodding the ID units, we can estimate behaviour in hypothetical situations."

Although these ID units behave like humans, we would imagine that they are merely unthinking automatons — just states in computer memory — and are certainly not conscious like you and me. Well, *Simulacron-3* takes this scenario a step further to imagine that these computer generated citizens are, indeed, conscious: "The reaction entities weren't merely ingenious circuits in a simulectronic complex, but instead were real, living, thinking personalities. They actually existed. In a solipsistic world, perhaps, but never suspecting that their past experiences were synthetic, that their universe wasn't a good, solid, firm, materialistic one."

You might find this a bold assertion — that mere states in a computer can become conscious — but we understand so little about the nature of consciousness that this is certainly not beyond the bounds of possibility. If something acts like a human, maybe it thinks like a human? There is a wonderful quote from one of the characters in *Simulacron-3*: "You can hardly stuff people into machines without starting to wonder about the basic nature of both machines and people."

So here we have an example of a simulated reality which exists merely as states in a computer. People are programmed into existence. Crucially, this is in contrast to the Matrix scenario, or the brain-in-a-vat scenario. In both of those scenarios an actual human brain in suspension is required.

This type of computer-based simulation is very similar to the type of artificial life simulations which are currently very popular such as *The Sims* which is the best-selling PC game of all time. In *The Sims*, virtual people go about their daily activities oblivious to the fact that they are being observed by the game player who retains complete control over the environment. The virtual people have individual personalities and experience desires and fears. There is no obvious goal to the lives of these "people" except to entertain the game player.

Again, as with the case of the brain-in-a-vat thought experiment, this seems to have implications for certainty and uncertainty. As Descartes realised back in the 17th century, if you cannot be certain as to whether or not you are in a simulation, then **you cannot be certain of anything**. This is because you are effectively isolated from reality. Only the simulation programmers would live in the real world: you would be merely living in an illusion of reality. It might appear to you that "the cat is on the mat", but in reality there is no cat and no mat: they would be states in a computer program. This type of general uncertainty — in which you can no longer be certain of anything — is called *Cartesian uncertainty* (called "Cartesian" after "Descartes"). [9]

The Thirteenth Floor

From this discussion, it would appear that the only people who would actually be living in reality — and could therefore say that "the cat is on the mat" with certainty — would be the simulation programmers. But is that really the case? In order to find out, let us now return to *Simulacron-3*, because the story is about to take a surprising twist.

We left Douglas Hall in charge of development of the total environment simulator called Simulacron-3. Expectations were high for the impending launch of this revolutionary product. However, a series of unusual and unnerving incidents makes Hall doubt the reality of his own world. Events reach a startling conclusion when Hall is driving in the desert one night and encounters a most unusual sight: "The road ended a hundred feet away. On each side of the strip, the very earth itself dropped off into an impenetrable barrier of stygian blackness. Out there were no stars, no moonlight — only the nothingness within nothingness that might be found beyond the darkest infinity."

Hall realises that his own universe is nothing more than a simulated reality generated by a computer at a higher level of reality to his own world. The road disappears because it has not yet been generated by the simulating computer (probably for reasons of computational economy). Hence, there are now three levels of reality: the simulated reality generated by Hall's computer, the reality in which Hall find himself, and the highest level of reality which is simulating Hall's world.

But the possible number of levels of reality is clearly not just limited to three levels. In 1999, the movie The *Thirteenth Floor* was released which was based on the novel *Simulacron-3*.[10] The plot of *The Thirteenth Floor* follows the plot of *Simulacron-3* fairly closely, but the number of possible levels of reality is increased from three to thirteen (hence the title). Of course, the number of possible levels is not limited to thirteen levels: we could consider an infinity of levels. The point is that — no matter which level you find yourself — you could never be certain that you were the highest level. Even at the level of the supposed simulation programmers there would always be uncertainty as to whether there was a level above. It would appear that uncertainty is a fundamental property of **all** universes.

The purpose of this discussion is most certainly not to convince you that we are living in a computer simulation. I am merely presenting this discussion as the closest modern-day technological equivalent of Descartes' "evil demon" argument. The demon has been replaced by a computer, so the technology has improved. However, I believe Descartes' argument — which was first presented in the 17th century — remains valid to this day. The argument reveals that there lies a fundamental, inescapable uncertainty at the heart of reality. If the idea of living in a computer simulation sounds too much like science fiction, then we can rephrase Descartes' argument along more scientific terms as "there will always be an uncertainty about any truth which lies outside our universe". I believe it is very hard to argue with this statement. As Stephen Hawking said in a quote presented earlier in this book: "We are not angels, who view the universe from outside". Uncertainty is inescapable, not just in our universe, but in any conceivable universe. In fact, I believe this could be considered a fundamental principle, to be added to the list of fundamental principles I have built in my previous books.

So when we discover some apparent fundamental uncertainty in Nature — as in quantum mechanics, for example — we should not be surprised, or throw up our arms in horror or denial. Because, way back in the 17th century, Descartes showed that uncertainty is fundamental and unavoidable in Nature.

Uncertainty is the only certainty.

The multiverse

Based on the discussion so far, there appears to be a fundamental uncertainty about any truth "outside the universe". Such questions would appear to be beyond scientific enquiry. We pursue physics as a means of understanding the universe with an assumption that there is "nothing outside the universe" otherwise our questions are meaningless and our pursuit is doomed. Are we living in a computer simulation? We assume that we are not.

However, there is a hypothesis — or family of hypotheses — which has gained in popularity over recent years which suggest that it is, indeed, possible to obtain information about universes which lie outside our own universe. In fact, these hypotheses suggest that there are a vast number of alternate universes. These parallel universes form what is known as the *multiverse*. The associated theories are called multiverse theories.

(If you read my first book, you will know I am not a fan of multiverse theories, whereas many physicists seem convinced by them. So bear that in mind in the discussion which follows. However, it is probably true that multiverse theories are nowhere near as popular with physicists as you might imagine by reading popular science books and magazines.)

A popular multiverse theory is the Many Worlds interpretation (MWI) of quantum mechanics. If you remember back to our discussion of quantum mechanics in Chapter Two of this book, it was explained how the act of taking a quantum mechanical measurement resembles playing a game of roulette. Before observation, the system behaves as though it is in a superposition of all possible states — as though a ball is rolling around a roulette wheel. But when the ball stops rolling, a single value is provided for the measurement. You cannot predict where the ball will stop, and you cannot predict what property value you will measure: that is where fundamental uncertainty enters into quantum mechanics.

The MWI denies this fundamental uncertainty of quantum mechanics. The MWI says there is no uncertainty — it knows precisely what is going to happen. The MWI says that all possible outcomes occur, but each of those outcomes only occurs in a different parallel universe. Multiple copies of the observer are generated and are placed in each of those parallel universes, so only one outcome is ever observed. Hence, the observed outcome appears random to each observer. According to the MWI, there is only an illusion of randomness, an illusion of uncertainty.

The MWI attempts to replace the fundamental uncertainty present in the Copenhagen interpretation of quantum mechanics. The MWI arises from a dislike of uncertainty and a desire for certainty. Instead of accepting that only one reality is chosen at random, the MWI states that all possible worlds exist. It is an attempt to do away with the random roulette wheel and to replace it with a cosy world where everybody wins. Unfortunately, life is not like that.

Mathematically, the MWI comes from an insistence that the evolution of a quantum system must always be "linear", an approach considered in my first book. Scientists prefer linearity over nonlinearity, as was made clear in the earlier chapter on chaos. Linearity is neater, more elegant, and — crucially — easier to analyse. But Nature tends to be determinedly nonlinear, as Benoit Mandelbrot made clear. And, as I said earlier in this book, the key to being a good physicist is to **listen to Nature talking**, and not to impose your own preconceptions on how Nature **should** behave.

In my view, what the MWI attempts to achieve is rather sneaky. In Chapter Two of this book, the fundamental, unavoidable uncertainty present in quantum mechanics was discussed. And so far in this chapter the fundamental, unavoidable uncertainty of Descartes' demon has been discussed (we cannot obtain knowledge outside our universe). The MWI very sneakily attempts to exchange the uncertainty of quantum mechanics for the uncertainty of Descartes' demon. This is a powerful approach because, whereas we can do nothing about the uncertainty present in quantum mechanics, we are free to imagine all sorts of crazy scenarios about

Descartes' demon. We can imagine we are living in a Matrix, or a computer simulation, or that there are multiple universes out there. No one can prove you wrong. However, this is not an approach that should be taken. Instead it should be accepted that the uncertainty of Descartes' demon is just as valid and unavoidable as the uncertainty of quantum mechanics. Both types of uncertainty are fundamental, and one should never be exchanged for the other.

On her excellent blog *Backreaction* (backreaction.blogspot.com), Sabine Hossenfelder features a quote from a famous (unnamed) physicist: "The multiverse, the simulation hypothesis, modal realism, or the Singularity — it's all the same nonsense, really."

The MWI emerged from the belief that there can be no such thing as fundamental uncertainty. If we have fundamental uncertainty we don't need many worlds — one universe is more than enough. In that case, the MWI dies overnight. We have seen throughout this book how many of the greatest minds in science and mathematics have been repelled by the notion of fundamental uncertainty. Einstein was so repelled by the idea that he constructed endless thought experiments to argue with Niels Bohr. Bertrand Russell tried to eliminate uncertainty from mathematics by writing three huge volumes of the *Principia Mathematica*. In the end, though, Bell's theorem proved that Einstein was wrong, and Kurt Gödel's incompleteness theorem proved that Russell was wasting his time. Some of the greatest minds have been made to look rather foolish in their attempts to eliminate fundamental uncertainty. The MWI is the latest in a long line of attempts to eliminate fundamental uncertainty. However, it has been shown time and time again that science only advances when fundamental uncertainty is firstly recognised and then accepted.

Anthropic reasoning

One of the most appealing features of multiverse theories is that they seem to explain the apparent fine-tuning of some of the fundamental constants of Nature. It appears that if some of these values were only slightly different then the emergence of life in the universe would have been impossible. As the great physicist John Wheeler said: "It is not only that man is adapted to the universe, the universe is adapted to man. Imagine a universe in which one or another of the fundamental dimensionless constants of physics is altered by a few percent one way or another. Man could never come into being in such a universe."

This seems to suggest that the values of the constants have been fine-tuned with the aim of producing an environment suitable for life — which represents something of a mystery.

One such apparently fine-tuned value is the proposed dark energy density of the universe, which essentially plays the role of Einstein's *cosmological constant* (considered in detail in my second book). It is believed that the dark energy density controls the rate at which the expansion of the universe is accelerating. The dark energy density appears to be set to the particular value which allows the universe to expand at just the correct rate to be amenable to life. If the universe expanded too fast, stars and galaxies would be unable to form. If the universe expanded too slowly, it would collapse back on itself before life could emerge. It appears that the required value for the dark energy density is extraordinarily small: in the order of a millionth of a billionth of a joule per cubic centimetre. This is equivalent to just six protons per cubic metre.

However, the predicted value of the quantum vacuum energy is calculated to be far larger (by a factor of 10^{120}) than this measured value. If the quantum vacuum energy really is the source of dark energy then there must be some mechanism which reduces its value, such as the energies of different types of symmetrical particles cancelling each other out. However, it seems bizarre that a cancelling mechanism should leave such a tiny value. If the value of dark energy was zero, not this strange tiny non-zero value, then it would be much easier to accept. It appears almost as if the value has been set specifically to allow the formation of life-supporting structures in the universe.

However, if we have a multiverse of different types of universe, with the fundamental constants set to different random values in each universe, then we just happen to find ourselves in a universe in which the values are amenable to life.

This type of reasoning is called *anthropic reasoning*, and is one of the most contentious aspects of multiverse theories. It is either one of the most attractive features, or one of the reviled features — depending on your point of view. On the plus side, it seems to solve apparently intractable problems regarding fine-tuning. On the negative side, it doesn't really solve anything at all: the values of the fundamental constants have not been determined uniquely by the theory. This multiverse-based reasoning is often criticised on the basis that a theory which predicts everything predicts nothing. If we just lazily accept multiverse-based solutions to these mysteries then it feels like "giving up". Paul Steinhardt reflects this opinion: "I think it is far too early to be so desperate. This is a dangerous idea that I am simply unwilling to contemplate."

If we resort to relying on anthropic reasoning to explain the value of the cosmological constant then that would be a good example of "giving up". However, if we don't give up and instead continue our hard work to better understand the laws of Nature then we might find alternative solutions which do not require fine-tuning, or maybe solutions which unambiguously determine the values of the physical constants.

Either way, let's not give up.

7

THE SAGA OF THE SOUTH POLE AND THE MULTIVERSE

We discussed the Many Worlds interpretation of quantum mechanics in the previous chapter, but in 2014 another multiverse-based theory hit the international news headlines. I would like to end this book with the saga of the South Pole and the Multiverse.

The Amundsen-Scott South Pole Station is a scientific research station which is the only continuously inhabited building at the South Pole. Getting to the station from the USA involves a 15-hour flight from California to New Zealand, then a 14-hour military flight to the McMurdo station in Antarctica, followed by another 3-hour flight to the South Pole — arriving on a plane which lands on skis. The South Pole Station rises above the flat polar plateau, a sleek modular metal building, raised on stilts to prevent the station from being buried under snow drifts.

The station is located on the high plateau of Antarctica, 9,300 feet above sea level. The clear skies, thin air (due to the altitude), and low water vapour makes this an ideal location for telescopes, particularly telescopes which make microwave observations.

BICEP2 (Background Imaging of Cosmic Extragalactic Polarization) is a refracting telescope located at the station. The telescope is designed to detect very weak microwave signals. The aperture of the telescope is only 12 inches wide, but it does not need to be large as it is trained on only one particular small patch of sky 24 hours a day.

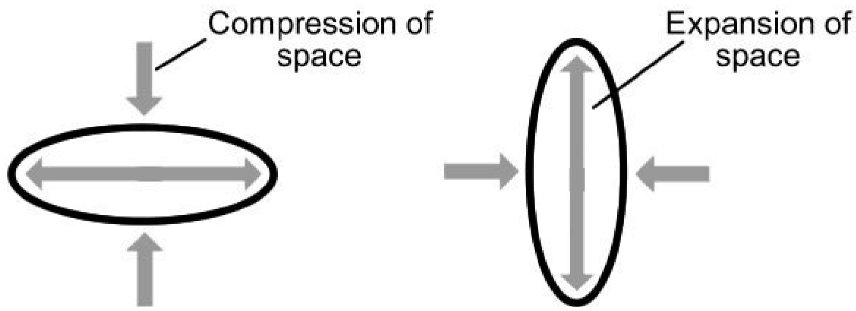
Each sensor inside the telescope is essentially an ultrasensitive thermometer. Each sensor is approximately one centimetre square, and is composed of a fine printed metal pattern on a circuit board — formed by the same photolithography technique used to manufacture integrated circuits. Essentially, it's a detector-on-a-chip. Any incident electromagnetic radiation which lands on the chip gets converted to heat, and this is detected as a change in the resistance of the metal element. The sensors are arranged in a square grid of 512 sensors.

If you are trying to detect a weak heat signal, you had better be sure that the signal is not completely drowned-out by the heat generated by the telescope. For this reason, the sensors are cooled to 0.25 kelvin: just a fraction above absolute zero. It is not easy to get something that cold, even at the South Pole. In fact, it took several days to cool the sensors. Liquid nitrogen was first used to cool the sensors to 77 kelvin. Then liquid helium was used to cool the sensors to 4 kelvin. Finally, a small amount of the rare isotope helium-3 — a byproduct of plutonium production — was used to cool the sensors to 0.25 kelvin.

The BICEP2 telescope is shown on the right of the following image:

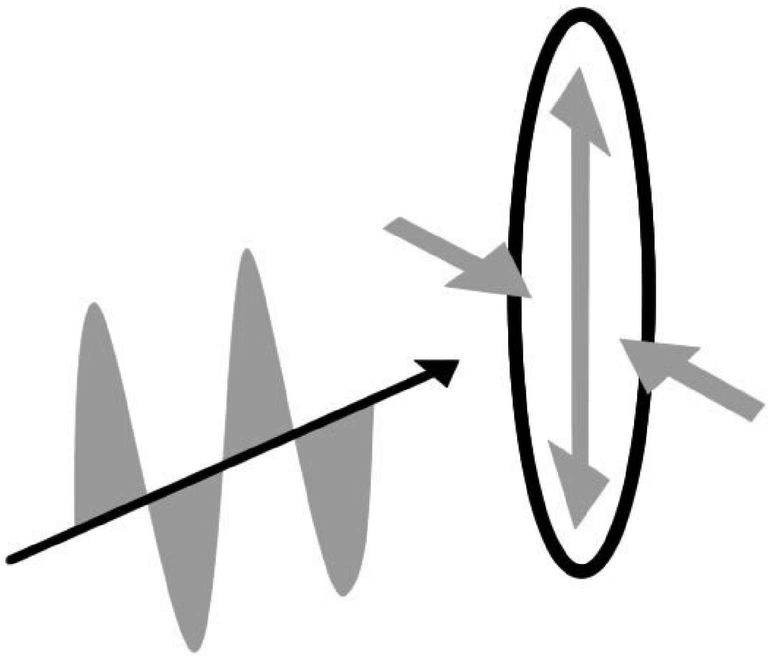
The purpose of the BICEP2 telescope is to detect microwave radiation from the cosmic microwave background (CMB) which is the most distant — and therefore the oldest — object it is possible to observe. This radiation has the potential to reveal the imprint of gravitational waves from the early universe. Virtually every physicist believes gravitational waves exist, but they have never been directly detected. If you have read my earlier books, you know that forces are transmitted through space in the form of fields. If an electric charge is moved, then that disturbance travels through the field: the effect is not felt instantaneously. It would seem logical to apply similar reasoning to the force of gravity. If the Sun is moved — or disappears — then the effect of the disturbance would not be felt instantaneously. Instead, it would take approximately seven minutes for the effect to be felt on Earth, the force reaching the Earth via gravitational waves.

It is predicted that gravitational waves will deform any space they pass through, squashing that space into an elliptical shape. Space would wobble like a jelly, first vertically and then horizontally:



Unfortunately, because gravity is such a weak force, these waves — and their deformation of space — would be very weak, and that makes them very difficult to detect.

Which brings us back to the BICEP2 telescope. It was predicted that gravitational waves from the enormous explosion of the Big Bang would have become frozen into the structure of the CMB. Due to the elliptical squashing of space, the radiation was expected to be polarised in the direction of the major axis of the ellipse. The following diagram shows a wave being polarised in the vertical direction by the compression of space (like a letter being posted through a letterbox):

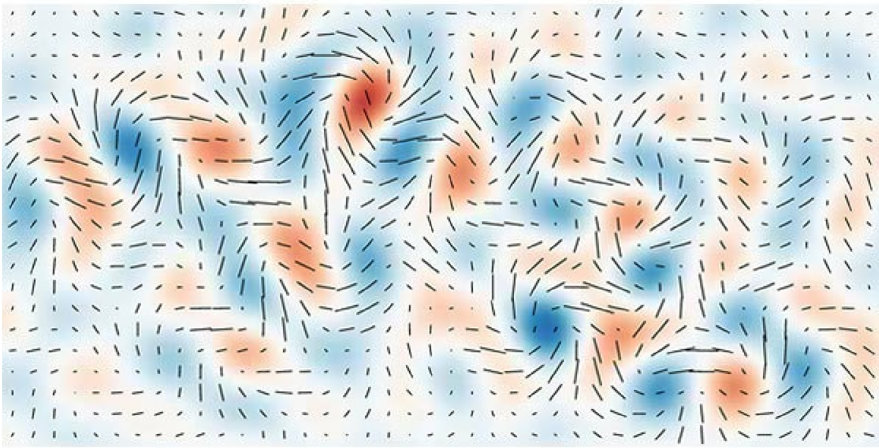


The particular type of polarised wave predicted to have been produced by the Big Bang was called *B-mode* polarisation. The possible detection of B-modes was considered to be so important because it would also represent a "smoking gun" for the inflation hypothesis. The term "smoking gun" was particularly apt because, according to inflation, there was an enormous explosion in the first moments in the existence of the universe. Inflation is supposed to have put the "bang" into the Big Bang. If BICEP could detect evidence of inflation, then it would be the most important physics discovery for decades.

And so the excitement began. In March 2014, a press release announced that the BICEP team were about to release their results at a media conference with the promise of "a major discovery". So on the morning of 17th March, I was one of many thousands of excited cosmologists and physicists eagerly sitting by their computer waiting to download the BICEP paper as soon as it was made available.

The paper certainly looked impressive.[11] It announced detection of B-modes with a 5 sigma confidence level. This meant that if the signal was due to chance, then the experiment would have to be repeated 3.5 million times before you would expect to find this result (basically, it was highly unlikely the result was due to chance).

The paper included a picture which showed the various twisty polarisation directions in the CMB, the "pinwheel" twists being characteristic of B-mode polarisation. The picture went around the world and was reprinted in many major newspapers. This looked extremely convincing:



But there was also something rather unnerving about the BICEP paper, and I was not the only one to feel that way. Scientists are notorious for downplaying their results, for being humble, and especially for being cautious. This paper was not like that. There was a certain triumphalism about the paper, a sense of hubris. The last sentence captured the tone: "The long search for tensor B-modes is apparently over, and a new era of B-mode cosmology has begun". Caution had been cast to the wind. It was the kind of bold triumphalist statement you can perhaps issue when your results have been independently confirmed, but it was also the kind of statement which — if you have screwed-up in any way — can leave you with an awful lot of egg on your face.

The impression was only reinforced when it emerged that the BICEP team had sent a video crew round to the house of Andrei Linde — the architect of eternal inflation, which we shall consider shortly — to capture his reaction when he heard the good news (http://tinyurl.com/inflationresult). This was science crossed with reality TV. It seemed aimed more at generating media buzz and column inches than providing a sober and cautious assessment of the result. This was not how science was supposed to be done.

On the internet, however, many bloggers such as Matt Strassler and Sean Carroll were stressing the need for caution. As Matt Strassler said: "As with any claim of a big discovery, you should view the BICEP2 result as provisional, until checked thoroughly by outside experts, and until confirmed by other experiments."[12] Later in his posting, he went further: "In physics, the assumption by the experts is that every claim of a scientific result, especially a major discovery, is wrong until proven right." Neil Turok, the director of the Perimeter Institute in Toronto, quoted Carl Sagan: "Extraordinary claims require extraordinary evidence." The general feeling was that we would have to wait for independent confirmation of the results from the European Space Agency's Planck satellite observatory team who were due to report their findings in October.

However, the BICEP news was generally received with tremendous excitement. It was recognised as a result of potentially monumental importance, certainly with the potential to be the most important scientific discovery of the 21st century. It appeared to be the strongest possible evidence in favour of inflation.

The explosion due to inflation was more accurately an expansion of space. It was proposed that the entire universe expanded virtually instantaneously — faster than the speed of light — from a size smaller than an atom to about the size of a grapefruit. Once inflation stopped, the universe continued to expand because of the "kick" it got from the explosion.

I covered inflation in detail in my second book, but it can be explained briefly how the inflation hypothesis solved two outstanding problems in cosmology:

1. **The horizon problem**. It was a mystery how the temperature of the CMB was so evenly distributed as distant regions were not causally connected (i.e., light had not had enough time to travel between the regions, equalising the temperature).

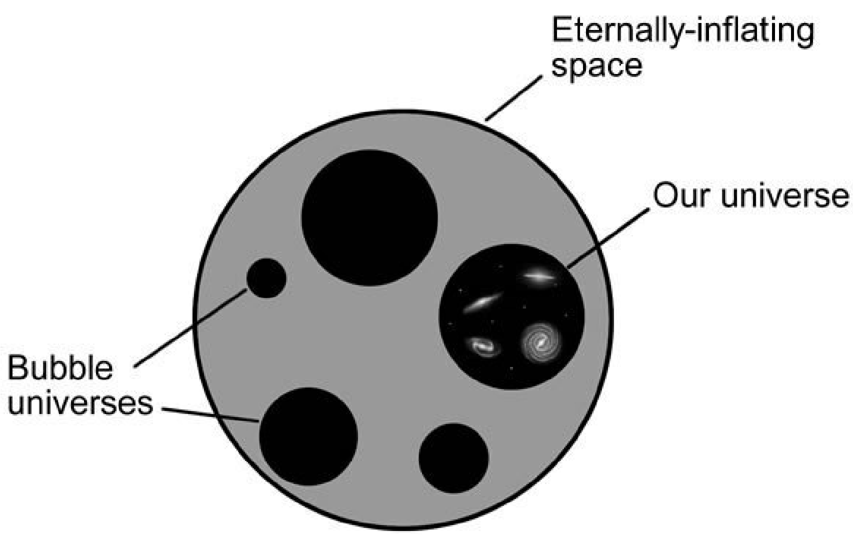
2. **The flatness problem**. The universe appears to be spatially flat. This is a mystery as, according to general relativity, the slightest initial variations from flatness would have been amplified over time.

Inflation appears to solve both these problems. The horizon problem is solved because temperature equalisation could have occurred before inflation, when the universe was so small. While the flatness problem is solved because inflation smooths and flattens the universe — like a wrinkly rubber sheet being stretched.

No one is sure why inflation started and, perhaps more importantly, no one knows why it ended. But the end condition is quite vital, because if inflation had not rapidly come to an end then the process of astonishingly rapid expansion would have continued indefinitely and our universe as we know it would not have had a chance to come into existence. Fortunately, inflation did stop. And at that point it had already provided the Big Bang explosion after which the universe could evolve at a more leisurely pace under the influence of gravity.

But if our universe was initially a tiny region of space, maybe that suggests that it was merely part of a much larger region of space. And maybe that region was undergoing unrestrained continuous inflation. Our universe would then just be a region in which the inflation process decayed and stopped. As Max Tegmark says in his book *Our Mathematical Universe*: "In other words, what we've called our Big Bang wasn't the ultimate **beginning**, but rather the **end** — of inflation in our part of space."

So maybe our universe has a much vaster region of continuously-inflating space outside it. In which case, maybe other universes are popping into existence continuously inside that larger region of eternally-inflating space:



Maybe you can see where this is going. Yes, it is another multiverse theory, this theory being called *eternal inflation*. This is a particularly important multiverse theory because of its close connection with the inflation hypothesis. It has been said that if inflation is true then it is very likely that eternal inflation is true. I think it is fair to say that the eternal inflation hypothesis is probably the most widely-accepted multiverse theory.

However, there are plenty of critics of eternal inflation. One of the most notable is Paul Steinhardt, a professor at Princeton University. Steinhardt is notable in this respect as, surprisingly, he is one of the originators of the inflation hypothesis. In an interview for *Scientific American* in 2014, Steinhardt produced a remarkable criticism of inflation and the multiverse, suggesting that inflation creates as many problems as it solves: "The whole point of inflation was to get rid of fine-tuning — to explain features of the original big bang model that must be fine-tuned to match observations. The fact that we had to introduce one fine-tuning to remove another was worrisome. This problem has never been resolved. But my concerns really grew when I discovered that, due to quantum fluctuation effects, inflation produces a multitude of patches (universes) that span every physically conceivable outcome (flat and curved, smooth and not smooth, isotropic and not isotropic, etc.). So we have not explained any feature of the universe by introducing inflation after all. To me, the accidental universe idea is scientifically meaningless because it explains nothing and predicts nothing. Also, it misses the most salient fact we have learned about large-scale structure of the universe: its extraordinary simplicity when averaged over large scales. Scientific ideas should be simple, explanatory, predictive. The inflationary multiverse as currently understood appears to have none of those properties." [13]

So a lot was riding on the BICEP result. And I had quite a considerable personal interest in the result as well. That was because my second book, imaginatively titled *Hidden In Plain Sight 2*, proposed an alternative hypothesis to the inflation hypothesis. For details, I would suggest you take a look at my second book, but I can briefly explain the hypothesis here.

If we consider that the universe must have zero total energy, then that suggested a quite ingenious modification to the theory of general relativity (gravity). Instead of attracting objects to an infinitely small distance (as is predicted by general relativity), the modified gravity hypothesis predicted that objects would be attracted until they were a certain distance apart: an equilibrium distance which reflected the zero energy condition. The value of this equilibrium distance is well known as the *Schwarzschild radius* (a distance equal to the event horizon of a black hole). Fortunately, for almost every object this equilibrium distance — its Schwarzschild radius — is an incredibly small distance. For example, the Schwarzschild radius of a human being is about the size of an atom. Crucially, this meant the hypothesis agreed with all existing measurements of general relativity, and explained why we only ever see gravity as an attractive force.

However, this subtle modification made quite extraordinarily accurate additional predictions. For the universe as a whole, the Schwarzschild radius is very close to the radius of the observable universe, so if the universe is expanding to its Schwarzschild radius then this would explain the observed radius of the universe, and the observed accelerated expansion of the universe (generally attributed to dark energy). It also agreed with the two main predictions of the inflation hypothesis. It solved the horizon problem, and predicted a flat universe (as spatial flatness arises naturally in a universe at its Schwarzschild radius). For the details, see my second book.

So, crucially, the modified gravity hypothesis predicted a flat universe without the need for the mysterious fine-tuned dark energy (considered in the last chapter), and so removed the need for a multiverse.

And in many ways, the predictions of the hypothesis went beyond the explanatory power of inflation. The hypothesis removed the singularities at the heart of black holes, presented a simple solution to the mystifying black hole information loss paradox, and it also predicted the correct value for black hole entropy.

However, what the hypothesis did not predict was an initial explosive expansion generating primordial gravity waves in the CMB. So the BICEP result appeared in conflict with the hypothesis. If I am honest, I was fairly disappointed when I heard the BICEP news — it looked like I had screwed-up big time. But I had such confidence in the modified gravity hypothesis that I was also quite shocked by the news. The hypothesis seemed simple and logical, with great explanatory power. It made inflation look positively messy in comparison!

Never mind, hypotheses live and die by experimental results. So, after being moderately depressed for a couple of days, the BICEP news meant that I had work to do. Though I was genuinely pleased that it appeared that physics had taken a great leap forward, I was less than thrilled at the thought of having to rewrite large chunks of my second book. I also felt I had let down my loyal readers who had invested time and money in my book.

However, very soon after the BICEP result was released, dissenting voices appeared. These voices were led by David Spergel, an astrophysicist from Princeton and a world expert in this field. Spergel had been a team member on the earlier successful WMAP mission (Wilkinson Microwave Anisotropy Probe), a space telescope launched in 2001 which provided accurate measurements of the CMB. On the same day in March that the BICEP result was released, Spergel posted on his Twitter account: "Reading the potentially very exciting BICEP paper, but am worried by many anomalies in the data". In later tweets on that same day, Spergel listed several suspicious anomalies in the data which his expert eye had identified. Spergel was concerned about "leakage" in the polarisation data. Matt Strassler joined in on Spergel's Twitter feed asking for clarification about the meaning of "leakage". Spergel replied that leakage occurred when temperature data mixed into the polarisation data. This revealed an early concern that the polarisation data had become contaminated by an external source in some way. Lisa Randall also posted on Spergel's Twitter feed that same day expressing her concerns. Perhaps you can get a feel of how fevered the day was, and the important role of social media in discussing the results.

The internet rumour mill continued. A particle physicist based in Paris called Adam Falkowski was posting under the pseudonym of Jester on his blog *Résonaances*. On the 12th May, Jester published the following post: "The rumours that have been arriving from the Planck camp are not encouraging, as they were not able to confirm the primordial B-mode signal. It seems that experts have now put a finger on what exactly went wrong in BICEP."[14] According to Jester, the Planck team believed the polarisation signal detected by BICEP came from very small grains of dust distributed throughout the galaxy. It is known that dust can radiate polarised microwaves. It was therefore vital to be certain that the effect of dust was too small to affect the signal.

It is possible to determine if a polarisation signal is coming from dust or gravitational waves by considering the signal at different frequencies. The amount of energy from dust increases at higher frequencies, whereas the amount of energy from gravitational waves decreases at higher frequencies. Unfortunately, BICEP was only equipped to measure at one frequency: 150 GHz. That was the ideal frequency for studying the CMB, but it was the wrong frequency for studying dust.

So the way the BICEP team estimated the amount of dust was by reinterpreting some earlier data from the Planck team (the Planck satellite could measure at 353 GHz where dust had its greatest effect). But the Planck team had not published all their data. Some of the Planck data had only been presented to the public as slides in conference talks. And it was here that Jester's rumour really had an impact. Jester suggested that the BICEP team had used a "screen scrape", taking one of the Planck PowerPoint slides and reading the data from it in order to obtain an estimate for the amount of dust.

Not only that, but the BICEP team had misinterpreted the slide. The slide showed polarisation for all foreground emissions — not just dust. Once you have corrected for that, and rescaled the results, the rumour was that dust could account for all of the BICEP signal.

Just a few days after Jester's posting, a talk was being held at Princeton University entitled *Searching for Simplicity*. The talk was organised by Neil Turok and Paul Steinhardt. One of the speakers at the talk, Raphael Flauger, had also scraped data from the same Planck slide used by the BICEP team. However, Flauger had come up with a considerably different conclusion. He believed that the significance of the result was much lower than stated in the BICEP paper. At the end of the talk, Professor Lyman Page was quite ruthless in his criticism: "This is a really peculiar situation. In that the best evidence for this not being a foreground, and the best evidence for foregrounds being a possible contaminant, both come from digitizing maps from PowerPoint presentations that were not intended to be used this way by teams just sharing the data. So this is not — we all know — this is not sound methodology. You can't bank on this, you shouldn't. And I may be whining, but if I were an editor I wouldn't allow anything based on this in a journal. Just this particular thing, you know. You just can't, you can't do science by digitizing other people's images."

Flauger went on to co-author a paper with David Spergel which suggested that the BICEP signal could be completely due to polarised dust. The story emerged that David Spergel had first realised the possibility of dust contamination back in March while travelling on a train to New York to give a lecture.

In June, the BICEP paper was finally published with some of its claims watered-down. The infamous last sentence had been replaced. In September, a paper published by the Planck team announced that the amount of dust in BICEP's "Southern Hole" was considerably greater than had been assumed. There were no clear regions in the sky where BICEP could have attained a signal clear of dust, but they were unfortunate to have chosen a particularly dusty window. Planck showed that the expected signal from dust was a good match for the BICEP signal.

Over on his *Résonaances* blog, Jester performed a final appraisal of the approach of the BICEP team, and was fairly scathing in not-very-scientific language: "There's no question that BICEP screwed-up big time. BICEP goofed it up and deserves ridicule, in the same way a person slipping on a banana skin does." [15]

In January 2015, the joint analysis by the Planck and BICEP teams made an official announcement that BICEP had just made an extremely precise measurement of dust.

For his role in identifying the problems with the BICEP analysis, *Nature* magazine made David Spergel one of its ten People of the Year for 2014.

It appears that the saga of the South Pole and the Multiverse has now come to a conclusion.

I may be wrong. We might be surrounded by an infinity of multiverses in which infinite copies of myself are writing slightly different versions of this book. Maybe I am wrong (and Descartes is wrong) in saying that uncertainty is fundamental. Maybe one day we will be able to detect parallel universes outside our own universe. Maybe Niels Bohr was wrong. Maybe one day we will uncover some deterministic mechanism in quantum mechanics. Yes, maybe the multiverse theorists will one day be proved right in saying that we can eliminate fundamental uncertainty (even though — as we have seen — the likes of Bertrand Russell, David Hilbert, and even Albert Einstein have failed in that particular quest).

However, in the meantime, I would politely encourage the proponents of multiverse theories to restrain from publishing sensationalist pop science articles in newspapers and magazines, to restrain from making doorstep videos of multiverse theorists, and to restrain from chasing publicity and column inches in which these highly-speculative theories are sometimes presented as established fact. Some of us have not yet completely given up on trying to explain the universe. It would be a shame if the most common response from the general public on hearing the latest physics breakthrough was to become "don't believe the hype". As David Spergel has said: "The negative side is, as one of my right-wing Facebook friends remarked: If the scientists get gravity waves wrong, why should we believe them on global warming?" [16]

In this book, a history of uncertainty in the 20th century has been presented. If the 20th century has taught us anything, it is that we should beware of those who speak loudest of absolutes, with absolute certainty. Instead, we should accept that Nature is based on relatives and uncertainty (relativity and quantum mechanics).

However, I am optimistic for the future. I believe good science will win through, and, more importantly, the truth will win through.

Because it always does.

FURTHER READING

A great selection this time, including some of the best popular science books ever written:

Gödel, Escher, Bach by Douglas R. Hofstadter
One of the finest non-fiction books ever written. A story of "strange loops" — including Gödel's incompleteness theorem. A book in which the form is as important as the content.

Chaos by James Gleick
The book which introduced chaos theory to the general public.

Does God Play Dice? by Ian Stewart
Another great book on chaos. More technical and less-wordy than Gleick's book.

Hunting the Hidden Dimension
An excellent NOVA documentary on fractals, available on YouTube.

Quantum by Manjit Kumar
A very detailed account of the development of quantum mechanics, and the arguments between the key figures.

The (Mis)Behaviour of Markets by Benoit Mandelbrot
If you have investments on the stock market, this is a book you cannot afford to ignore.

Pi in the Sky by John D. Barrow
An ambitious attempt to find the meaning of mathematics.

Impossibility by John D. Barrow
Examining the limits of science, including the effect of Gödel's incompleteness theorem.

The Newtonian Casino (published as *The Eudaemonic Pie* in the USA) by Thomas Bass
The account of how Norman Packard and Doyne Farmer defeated the casinos of Las Vegas by using their knowledge of Newtonian mechanics.

The Simulation Argument by Nick Bostrom
http://www.simulation-argument.com/
Plenty of articles on the theme that we might be living in the Matrix.

PICTURE CREDITS

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Photograph of Camelback Mountain is by Dru Bloomfield, published on Wikimedia Commons.

Photograph of BICEP2 telescope by Amble, published on Wikimedia Commons.

NOTES

[1] Interestingly, the development of the first mechanical computer, the laws of probability, and the invention of the game of roulette are all associated with the French mathematician Blaise Pascal.

[2] Another visionary quote from Lord Kelvin in 1895 was that "Heavier than air flying machines are impossible."

[3] The type of matrix which can be used for these quantum operations is called an *Hermitian* matrix.

[4] The great British mathematical physicist, Paul Dirac, later showed that Heisenberg's matrix mechanics and Schrödinger's wave mechanics were mathematically equivalent.

[5] The physical process behind the so-called "collapse of the wavefunction" is often presented as a great mystery, as something almost mystical. However, if you have read my first book you will know that great strides have been taken in understanding the process as a form of *environmental decoherence*.

[6] *1913: When Hitler, Trotsky, Tito, Freud and Stalin all lived in the same place*, BBC News, http://tinyurl.com/viennapeople

[7] Edward Lorenz, *Deterministic Nonperiodic Flow*, Journal of the Atmospheric Sciences, 1962.

[8] In fact, the producers of the movie, the Wachowskis, asked Keanu Reeves to read *Simulacra and Simulation* as preparation for his role. The book is completely impenetrable — I doubt Reeves thanked the Wachowskis for their decision. Baudrillard has since said that *The Matrix* misunderstands and distorts his work.

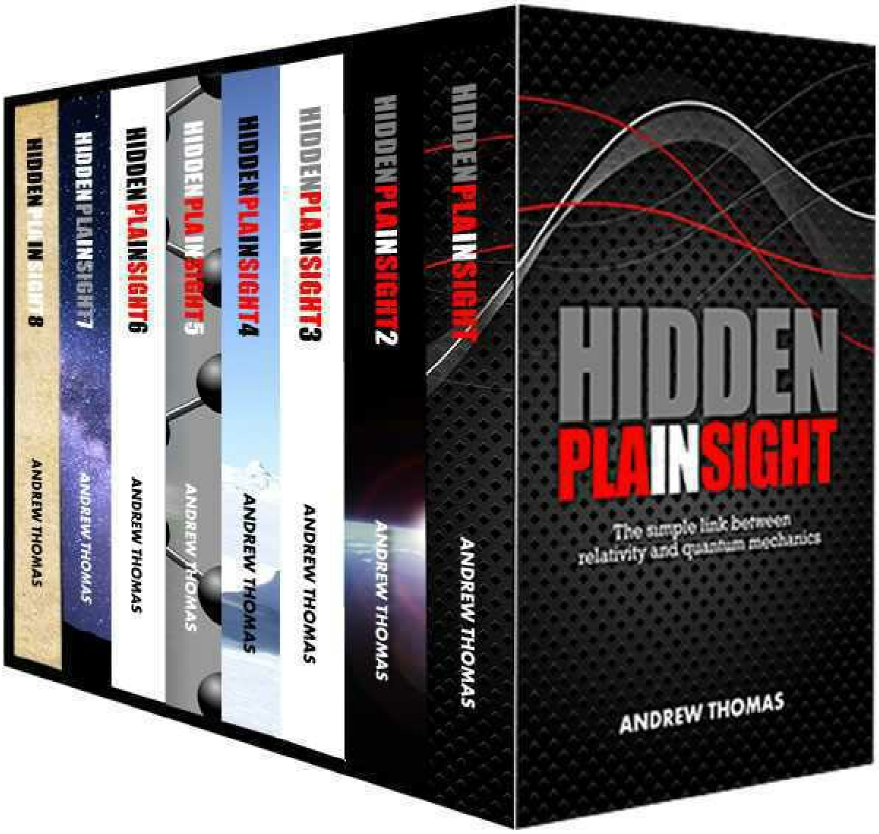
[9] If we cannot trust our senses, then Descartes reasoned that we could only trust the contents of our own minds. Hence his famous dictum *cogito, ergo sum* ("I think, therefore I am").

[10] *The Thirteenth Floor* was released just a few months after *The Matrix* was released, and so made nothing like the same impact. *The Thirteenth Floor* features an astonishingly convincing simulated reality of Los Angeles in the 1930s. You would swear it was real …

[11] *Detection of B-mode Polarisation at Degree Angular Scales*, http://arxiv.org/abs/1403.3985

[12] http://profmattstrassler.com/2014/05/19/will-bicep2-lose-some-of-its-

muscle
[13] http://tinyurl.com/inflationcriticism
[14] http://resonaances.blogspot.co.uk/2014/05/is-bicep-wrong.html
[15] http://resonaances.blogspot.co.uk/2014/09/bicep-what-was-wrong-and-what-was-right.html
[16] http://tinyurl.com/spergel-interview



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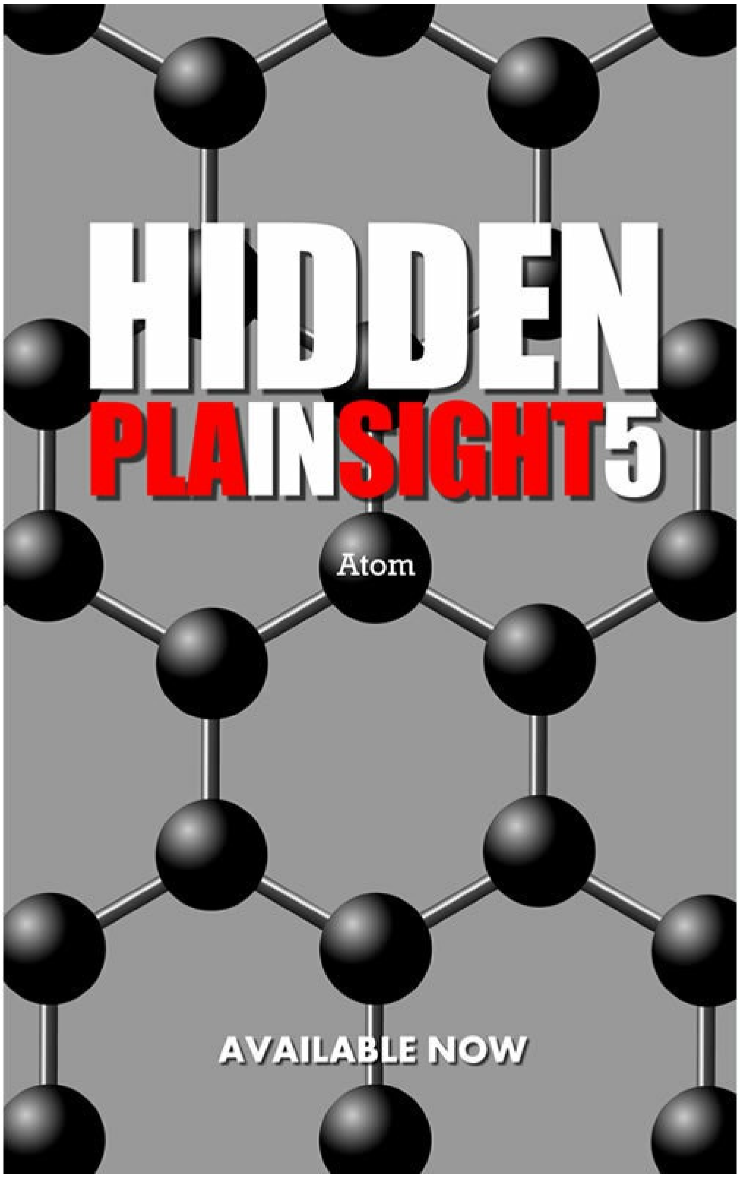
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