CS215 ASSIGNMENT 2

Utkarsh Varun

Question 6

Contents

1 Introduction

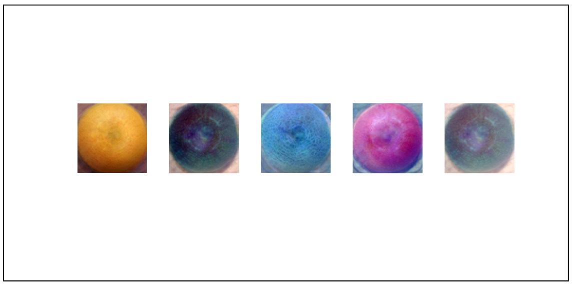
The Code for this question is in Q6.m file which is in code folder

We have also included data_fruit folder in code folder so that everytime when Q6.m executes it can take the input from the data_fruit folder

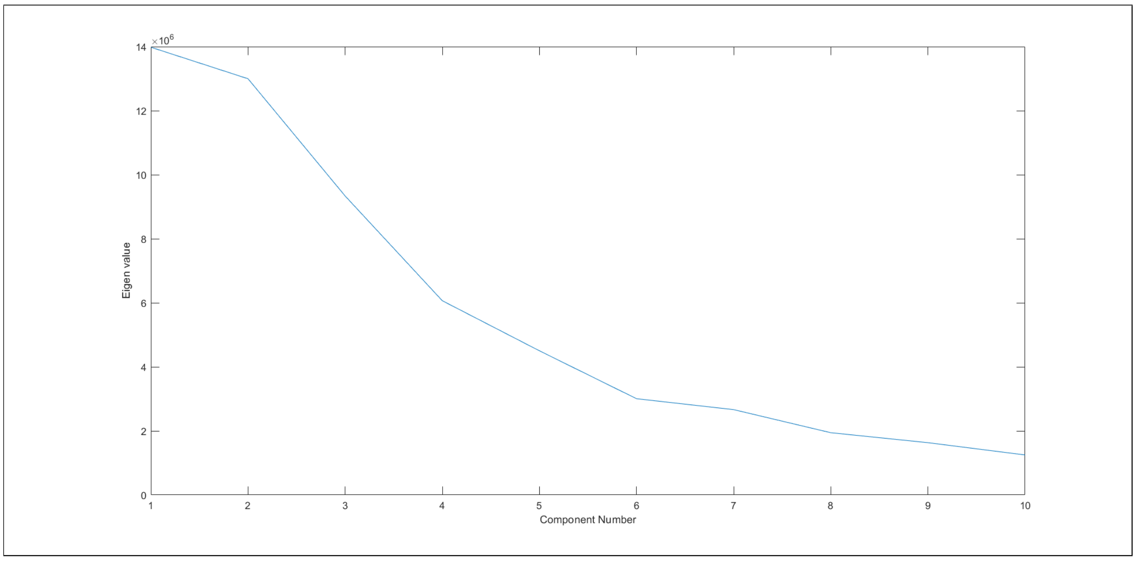
In results folder we are submitting 21 images 2 of which are for part a , 16 of which are for part b and remaining 3 are for part c which generates when Q6.m is executed

2 Part a

In this part we calculated the mean and top 4 principle eigen vectors and showed them as images in the same plot below



In the above plot the leftmost image is of the mean and the next four images are of the images of top 4 principle eigen vectors



For the above plot we have found the top 10 eigen values and sorted them in descending order and plotted them

3 Part b

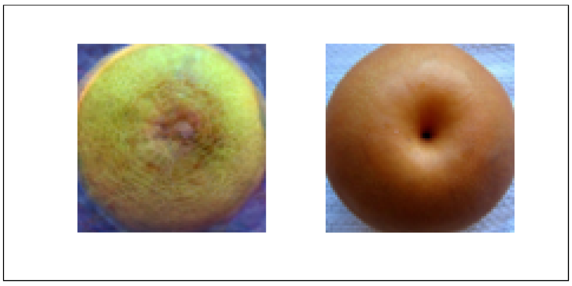
In this part we have found the closest representation of each image in the data_fruit folder and showed both of them side by side

To find the closest representation we have used the concept that eigen vectors are orthogonal to each other and we defined the newly formed as mean + linear combination of the four eigen vectors. For finding the coefficients of eigen vectors we have multiplied the transposed matrix of the original image's resized vector with the eigen vector to whom we are finding the coefficient.

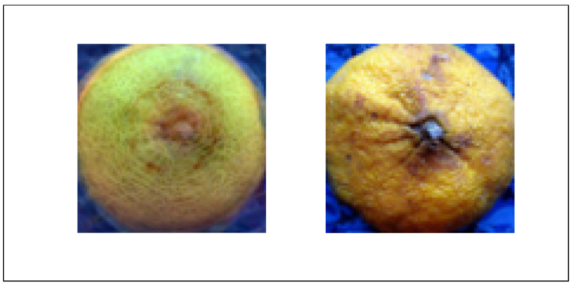
In all the pictures below the left side image is the one which we generated through the above described algorithm and the right side is the given image and as we can see that that the generated fruits are not that close to the original fruit because we are just using the top 4 eigen vectors out of 19200 available for generating the image

Also some handwritten proof for algorithm is at the end of report.

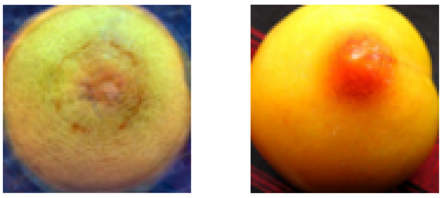
3.1 Fruit 1



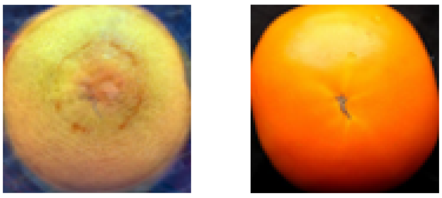
3.2 Fruit 2



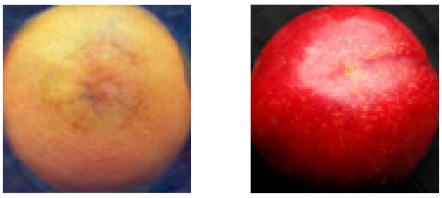
3.3 Fruit 3



3.4 Fruit 4



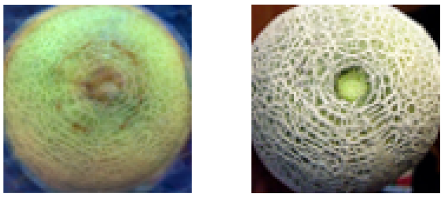
3.5 Fruit 5



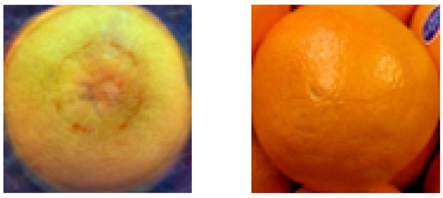
3.6 Fruit 6

3.7 Fruit 7

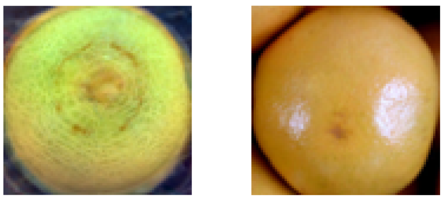
3.8 Fruit 8



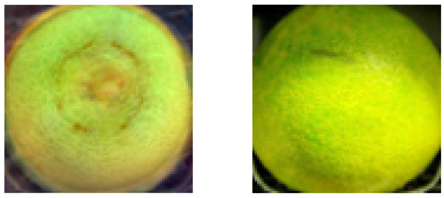
3.9 Fruit 9



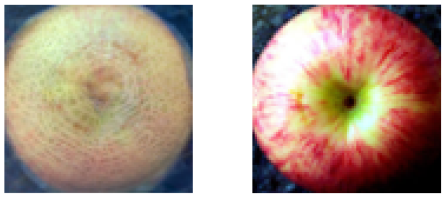
3.10 Fruit 10



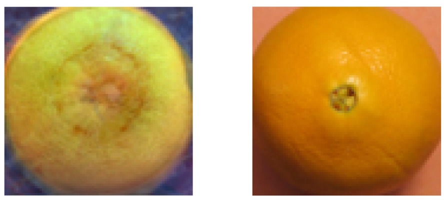
3.11 Fruit 11



3.12 Fruit 12



3.13 Fruit 13



3.14 Fruit 14

3.15 Fruit 15

3.16 Fruit 16

4 Part c

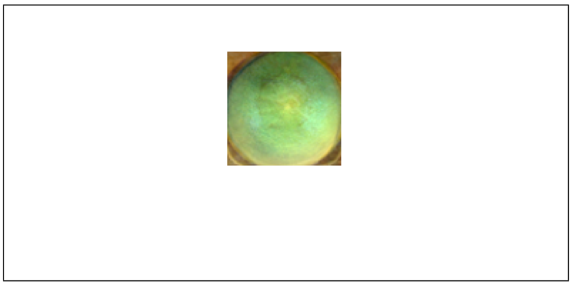
In this part we generated 3 new images or fruits by using mean and top 4 eigen vectors

For this we have defined a function named new_fruit in our code and in that function we defined a new vector as linear combination of mean and top 4 eigen vectors and we have kept the coefficient of mean to be very very small and fixed because if it's not small the contribution of it would dominate all eigen vectors and all the newly generated fruits would look same so we have done like that. For coefficients of eigen vectors we defined another function named const which generates random number between negative of the standard deviation to standard deviation of all the elements present in the eigen vector to which we are calculating the coefficient.

Also some handwritten proof for algorithm is at the end of report.

4.1 New Fruit 1

4.2 New Fruit 2



4.3 New Fruit 3

b) let A be the original vector and A_1 be the newly generated vector

$$A_1 = \mu + c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4$$

mean is added to the linear combination of eigen vectors.

c_1, c_2, c_3, c_4 are constants
v_1, v_2, v_3, v_4 are eigen vectors
μ is mean

To use Forbenius norm as measure of closeness is nothing but $\|A - A_1\|_2$ to be minimum.

As eigen vectors v_1, v_2, v_3, v_4 are orthogonal to each other

$$\| A - (\mu + c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4) \|^2$$

$$= tr\left(\langle (A - \mu) - (c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4), \right.$$
$$\left. (A - \mu) - (c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4) \rangle \right)$$

[as eigen vectors are orthogonal
$\langle v_i \; v_j \rangle = v_i^T v_j = 0$]

$$= tr\left(\langle A - \mu \;\; A - \mu \rangle + \sum_{i=1}^{4} \left[c_i^2 (\langle v_i \; v_i \rangle) - c_i \langle v_i \; A - \mu \rangle - c_i \langle A - \mu \; v_i \rangle \right] \right)$$

$$= tr\left(\langle A - \mu \;\; A - \mu \rangle + \sum_{i=1}^{4} c_i^2 - 2 c_i \langle A - \mu \; v_i \rangle \right)$$

to be minimum

$$\frac{d}{dc} \left(\sum_{i=1}^{4} c_i^2 - 2 c_i \langle A - \mu \;\; v_i \rangle \right) = 0$$

$$\Rightarrow c_i = \langle A - \mu \;\; v_i \rangle$$

$$\Rightarrow c_i = (A - \mu)^T v_i$$

$\therefore \quad C = (A-\mu)^T V$

V = eigen vector matrix

C = coefficient matrix

c) In third Part the algorithm is simple we took a new vector and call its as A_2.

let $A_2 = a \times \mu + c_1 v_1 + c_2 v_2 + c_3 v_3 + c_4 v_4$

a, c_1, c_2, c_3, c_4 are constants

$\mu \Rightarrow$ mean

v_1, v_2, v_3, v_4 are eigen vectors.

As μ dominates A_2 because of it's values in it are higher we are keeping a tobe very very small and c_1, c_2, c_3, c_4 are taken in the range of $-\sqrt{\lambda_i}$ to $\sqrt{\lambda_i}$ where λ_i is the variance of values in eigen vector V_i.

These c_1, c_2, c_3, c_4 are choosen randomly to generate new fruit.