Assignment_2

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EE2703: Applied Programming
Author: Varun Sundar, EE16B068

1 Introduction

The assignment discusses various built in methods for integration and compares it to a common numerical technique, the trapezoidal rule. For the sake of this assignment, we consider the derivative arctan x function, $1/(1+t^2)$.

Conventions

1. We are using Python 3, GCC for C
2. Underscore naming vs Camel Case
3. PEP 25 convention style.
4. Chosen to stick with importing *matplotlib* and *numpy* seperately, rather than use *scipy's pylab*.

2 Question 1

We define a vectorised version of our function under consideration.

```
In [4]: import numpy as np

In [5]: def f(t):
            # t could be a vector
            return 1/(1+np.square(t))
```

3 Question 2

Now, we assign a variable to a given evenly spaced array of values. Since *numpy's* present implementation of *linspace* contains no spacing argument, we choose to play nice with it. Rather calculate the requisite spacing as $\frac{Range}{Spacing}+1$.

```
In [6]: # Create a spacing of 0.1
        x=np.linspace(0.0, 5.0, num=51)
```

4 Question 3

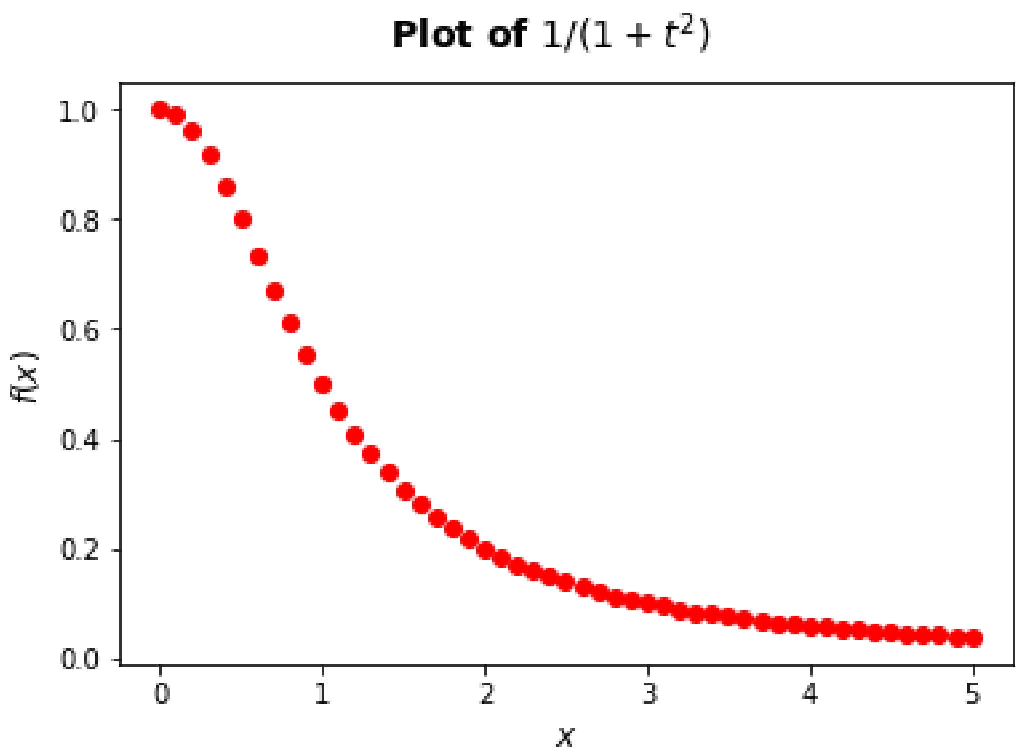
We now plot our integrand. This has been done for a range of 0.0 to 5.0, uniformly sampled at 0.1.

```
In [7]: import matplotlib.pyplot as plt
        %matplotlib inline

        plt.suptitle(r'Plot of $1/(1+t^{2})$', fontsize=14,fontweight="bold")

        plt.xlabel('$x$',fontsize=12)
        plt.ylabel('$f(x)$',fontsize=12)

        plt.plot(x,f(x),'ro')
        plt.show()
```



5 Question 4

We now utilise *scipy's* integration module for a quadratic approximated integration. Proceeding which we compare it to *numpy's* arctan function.

A semilogy plot (log versus linear scale) has also been plotted depicting the error.

```
In [8]: from scipy.integrate import quad
        quad?
```

Let's print out a few values of both our integrated output and the ground reality.

```
In [102]: from pprint import pprint
          integrated_arctan=np.array([quad(f,0,a)[0] for a in x])
          # A Few values
          pprint(integrated_arctan[:8])
```

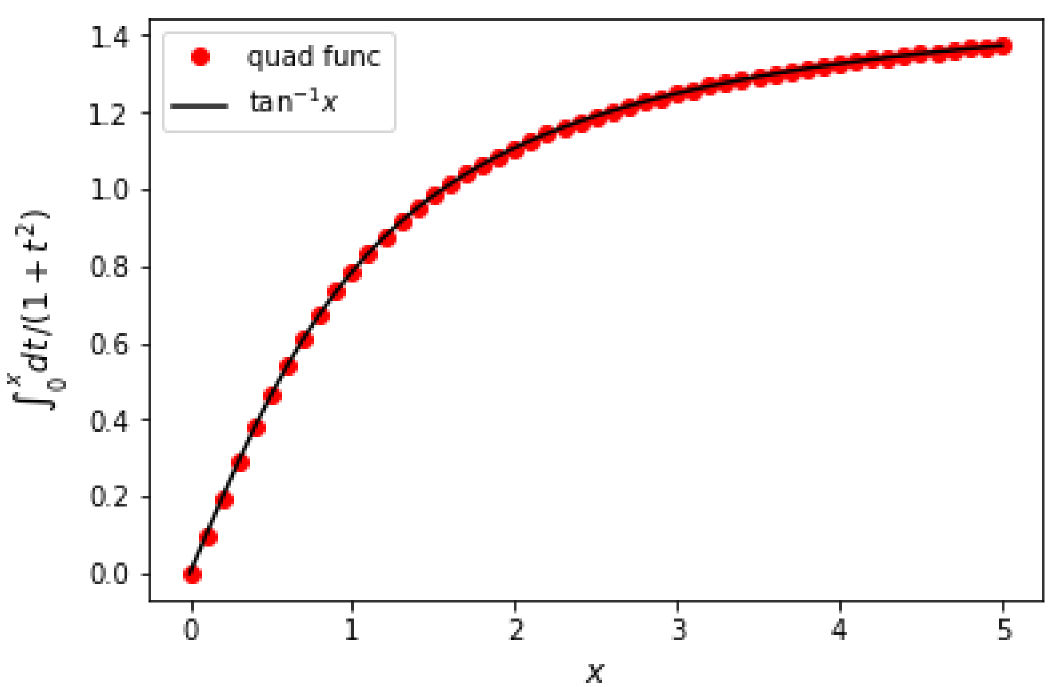
```
array([ 0.        ,  0.09966865,  0.19739556,  0.29145679,  0.38050638,
        0.46364761,  0.5404195 ,  0.61072596])
```

```
In [103]: np_arctan=np.arctan(x)
          # A Few Values
          pprint(np_arctan[:10])
```

```
array([ 0.        ,  0.09966865,  0.19739556,  0.29145679,  0.38050638,
        0.46364761,  0.5404195 ,  0.61072596,  0.67474094,  0.7328151 ])
```

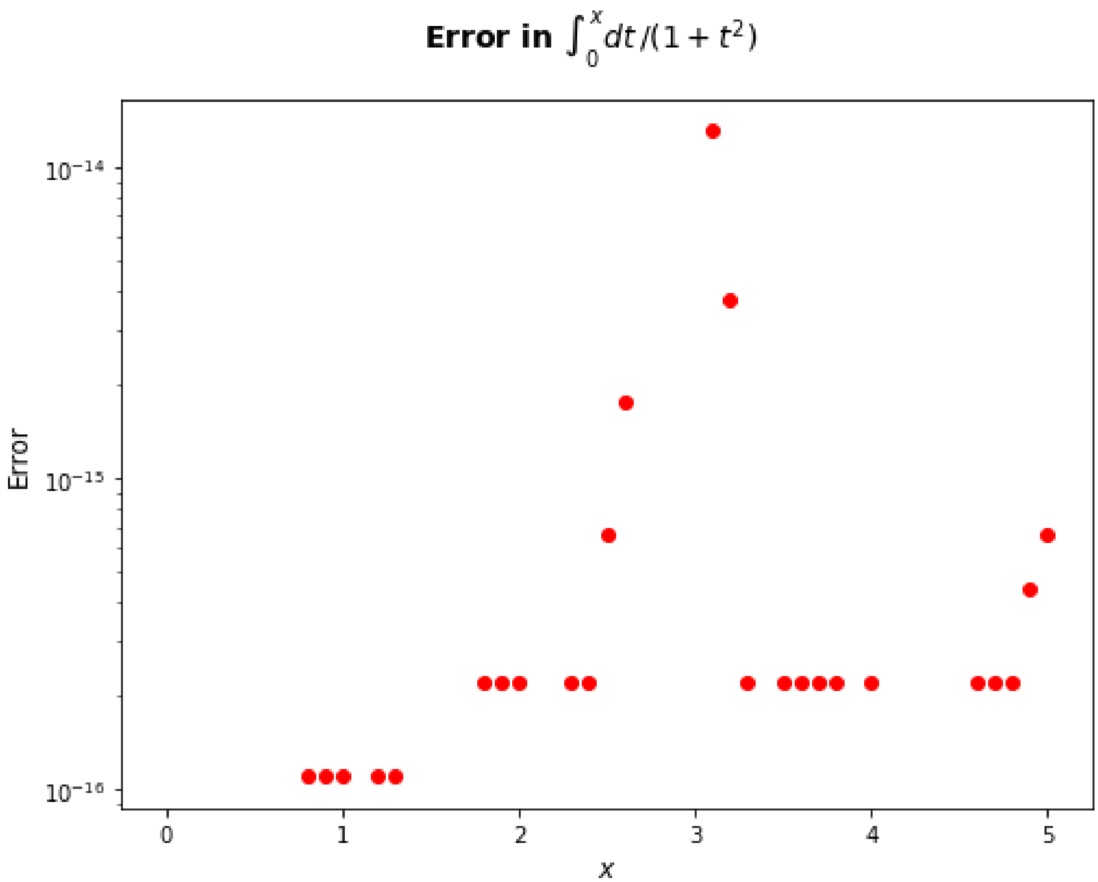
Using, $\tan^{-1} x$ instead of $\arctan x$

```
In [11]: plt.plot(x,np_arctan, 'ro',x, integrated_arctan,'k-')
         plt.legend(("quad func",r"$\tan^{-1}{x}$"))
         plt.xlabel('$x$',fontsize=12)
         plt.ylabel('$\int_{0}^{x} dt\,/(1+t^{2})$',fontsize=12)
         plt.show()
```



In [12]:
```
# Set plot sizes
import matplotlib
matplotlib.rcParams['figure.figsize']=[8,6]
```

In [13]:
```
plt.semilogy(x,np.abs(np_arctan - integrated_arctan), 'ro')
plt.xlabel('$x$',fontsize=12)
plt.ylabel("Error",fontsize=12)
plt.suptitle(r'Error in $\int_{0}^{x}dt\,/(1+t^{2})$', fontsize=14,fontweight="bold")
plt.show()
```



6 Question 5

We now perform a numerical method for integration; quadratic approximation via trapezoidal rule. It would be useful to compare this to the error obtained under *scipy's* implementation and interpret the corresponding results.

Trapezoidal Rule:
$I_i = h * (\sum_{j=1}^{i-1} f(x_j) - 2(f(x_1) + f(x_i))$
for the i'th integral.

In [28]:
```python
# Define a single integral
def integral(f,x,i):
    # f is the function
    # x is your array of values
    # i is the desired index (from 0 to n-2)
    # h is the trapezoidal step
    h=(np.amax(x)-np.amin(x))/len(x)
    j=i+1
    return h*(np.sum(f(x[:j]))-1/2*(f(x[0])+f(x[j])))

integral(f,x,1)
```

Out[28]:
```
0.098953899914878352
```

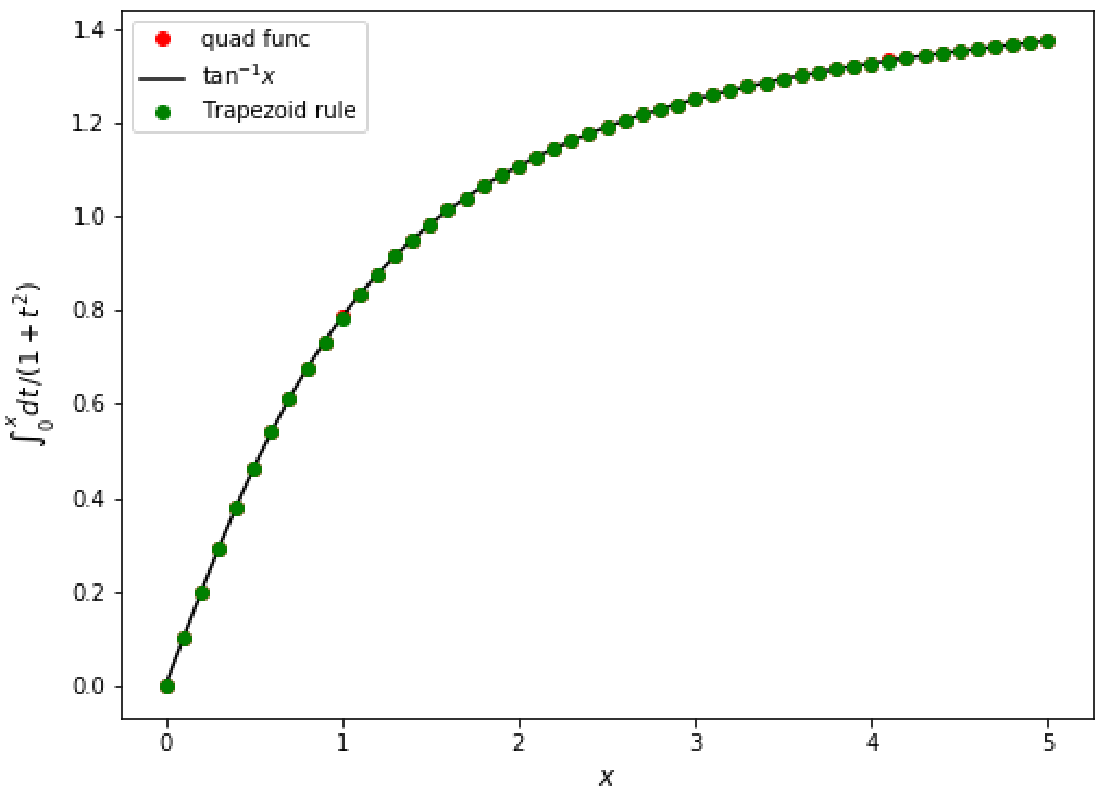
A vectorised implementation ...

In [48]:
```python
def integral_vectorised(f,x):
    # f is the function
    # x is your array of values
    # i is the desired index (from 0 to n-2)
    # h is the trapezoidal step
    h=(np.amax(x)-np.amin(x))/(len(x)-1)
    n=len(x)
    return np.array([h*(np.cumsum(f(x))[j]-1/2*(f(x[0])+f(x[j]))) for j in range(n)])

integral_vectorised(f,x)
```

Out[48]:
```
array([ 0.        ,  0.09950495,  0.19708682,  0.29103531,  0.38001031,
        0.46311376,  0.53987847,  0.61020022,  0.67424507,  0.73235719,
        0.7849815 ,  0.83260593,  0.87572217,  0.91480133,  0.95028059,
        0.98255709,  1.01198665,  1.03888507,  1.06353099,  1.08616943,
        1.10701542,  1.12625756,  1.14406135,  1.16057212,  1.17591769,
        1.19021069,  1.20355055,  1.21602521,  1.22771268,  1.23868228,
        1.24899578,  1.25870832,  1.26786925,  1.27652286,  1.28470897,
        1.29246345,  1.29981869,  1.30680403,  1.31344605,  1.31976891,
        1.3257946 ,  1.33154319,  1.337033  ,  1.34228082,  1.34730204,
        1.35211077,  1.35672003,  1.36114179,  1.3653871 ,  1.36946616,
        1.37338844])
```

In [49]:
```python
plt.plot(x,np_arctan, 'ro',x, integrated_arctan,'k-',x,integral_vectorised(f,x),'go')
plt.legend(("quad func",r"$\tan^{-1}{x}$","Trapezoid rule"))
plt.xlabel('$x$',fontsize=12)
plt.ylabel('$\int_{0}^{x} dt\,/(1+t^{2})$',fontsize=12)
plt.show()
```



Let's define a handy function for computing a cross error.

```
In [50]: def error(u,v):
             # Return maximum error between two numpy arrays
             return np.amax(np.abs(u-v))
```

And now use this to find out absolute errors.

```
In [75]: h_array=[10**(-j) for j in range(1,5)]
         x_array=[np.linspace(0.0, 5.0, num=5/(h)+1) for h in h_array]
         [error(np.arctan(x),integral_vectorised(f,x)) for x in x_array ]

/Users/Ankivarun/anaconda3/envs/tf_python3/lib/python3.6/site-packages/ipykernel_launcher.py:2:

Out[75]: [0.00054103142650707703,
          5.4126137517540585e-06,
          5.4126577109236962e-08,
          5.4126414461563854e-10]
```

We note that $h < 10^{-4}$ is gives us error tolerences $\delta < 10^{-8}$. So we ad-hoc start from $h = 10^{-2}$ and descend to $h = 10^{-4}$; and note down requisite error.

In [104]:
```
# A bottom threshold on the value of h
h_sweet=5*10**(-4)
```

In [83]:
```
i=4*10**(-2)
h_array=[]
for j in range(8):
    h_array.append(i/2)
    i=i/2
h_array
```

Out[83]:
```
[0.02, 0.01, 0.005, 0.0025, 0.00125, 0.000625, 0.0003125, 0.00015625]
```

In [84]:
```
x_array=[np.linspace(0.0, 5.0, num=5/(h)+1) for h in h_array]
error_exact_array=[error(np.arctan(x),integral_vectorised(f,x)) for x in x_array ]
error_exact_array
```

```
/Users/Ankivarun/anaconda3/envs/tf_python3/lib/python3.6/site-packages/ipykernel_launcher.py:1:
  """Entry point for launching an IPython kernel.
```

Out[84]:
```
[2.1650937672923476e-05,
 5.4126137517540585e-06,
 1.3531503878505546e-06,
 3.3829131440565874e-07,
 8.4572799208260108e-08,
 2.1143197970197036e-08,
 5.2857990207044736e-09,
 1.3214487282198206e-09]
```

We define our *estimated error* as follows:
For every integral with the trapezoidal width h, consider one with $\frac{h}{2}$ to be the truth for the test.
To facilitate this, we define a function called *cross-error* which computes the corrresponding error. Obviously, we compared only sampled values which are same.

In [82]:
```
def cross_error(h1,h2):
    # Return between h1, h2
    # Assume h2 is h1/2
    h_array=[h1,h2]
    x1,x2=[np.linspace(0.0, 5.0, num=5/(h)+1) for h in h_array]
    i1=integral_vectorised(f,x1)
    i2=integral_vectorised(f,x2)
    return error(i1,i2[::2])

cross_error(0.02,0.01)
```

```
/Users/Ankivarun/anaconda3/envs/tf_python3/lib/python3.6/site-packages/ipykernel_launcher.py:5:
  """
```

Out[82]:
```
1.6238323921169417e-05
```

```
In [88]: cross_tuples=[(h_array[i],h_array[i+1]) for i in range(len(h_array)-1)]
         estimated_errors=[cross_error(h_tuple[0],h_tuple[1]) for h_tuple in cross_tuples]
         pass
```

```
/Users/Ankivarun/anaconda3/envs/tf_python3/lib/python3.6/site-packages/ipykernel_launcher.py:5:
  """
```

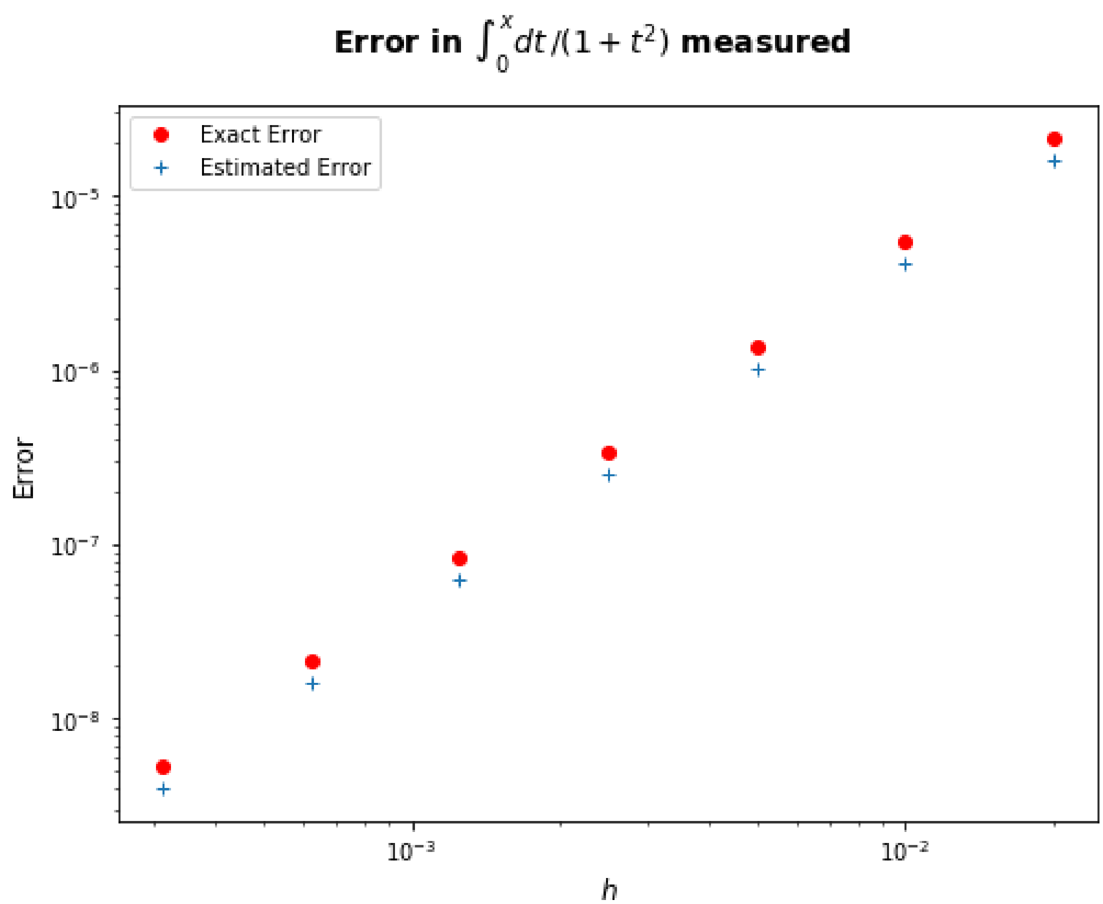
```
In [72]: estimated_errors
```

```
Out[72]: [1.6238323921169417e-05,
          4.0594678555327945e-06,
          1.0148632708650851e-06,
          2.5371851519739863e-07,
          6.3429601238063071e-08,
          1.5857398949492563e-08,
          3.9643504035069554e-09]
```

```
In [74]: error_exact_array
```

```
Out[74]: [2.1650937672923476e-05,
          5.4126137517540585e-06,
          1.3531503878505546e-06,
          3.3829131440565874e-07,
          8.4572799208260108e-08,
          2.1143197970197036e-08,
          5.2857990207044736e-09,
          1.3214487282198206e-09]
```

```
In [93]: # Now to plot errors
         plt.loglog(h_array[:7],error_exact_array[:7],'ro',h_array[:7],estimated_errors[:7],'+')
         plt.xlabel('$h$',fontsize=12)
         plt.ylabel("Error",fontsize=12)
         plt.legend(("Exact Error",r"Estimated Error"))
         plt.suptitle(r'Error in $\int_{0}^{x}dt\,/(1+t^{2})$ measured' , fontsize=14,fontweight
         plt.show()
```



7 Results and Discussion

We have implemented a numerical method for integration : a quadratic approximation via the trapezoidal method. We also have tried out different spacing values to attain a given tolerance in error.

We note, under our given error tolerances, that the numerical approach converges to the ground truth for the function, $\arctan x$.