NCERT Discrete 10.5.2 -15

EE23BTECH11057 - Shakunaveti Sai Sri Ram Varun

Question: For what value of n, are the *nth* terms of two A.Ps: 63, 65, 67,... and 3, 10, 17,... equal?

Solution: A sequence is said to be in Arithmetic Progression when it is in the form of

$$a, a+d, a+2d, a+3d, \ldots \tag{1}$$

where a is first term and d is common difference. When there are n terms, the sequence becomes

$$a, a+d, a+2d, a+3d, \ldots, a+(n)\,d. \tag{2}$$

$$T_n = a+(n)\,d. \tag{3}$$

which is nth term. In the given question, there are two sequences.

$$63, 65, 67 \ldots \tag{4}$$

$$3, 10, 17 \ldots \tag{5}$$

let u(n) be unit step function.

$$u(n) = \begin{cases} 1, & if n \geq 0, \\ 0, & if n < 0. \end{cases} \tag{6}$$

1) for the sequence (4), let x(n) be *nth* term,

a) Finding x(n) for sequence (4)

$$a = 63 \tag{7}$$

$$a+d = 65 \tag{8}$$

$$d = 2 \tag{9}$$

$$x(n) = 63+(n)\cdot 2 \tag{10}$$

$$x(n) = 63+2n \tag{11}$$

$$\boxed{x(n) = 63u(n) + 2n \cdot u(n)}$$

b) To find X(z):

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \times z^{-n} \tag{12}$$

$$X(z) = \sum_{n=-\infty}^{\infty} (63u(n)) + 2n \cdot u(n)\, z^{-n} \tag{13}$$

$$X(z) = \sum_{n=0}^{\infty} (63+2n)\, z^{-n} \tag{14}$$

For the above sum to be convergent: by using ratio test:

$$\frac{(n+1)\, z^{-n-1}}{(n)\, z^{-n}} < 1 \tag{15}$$

$$\frac{n+1}{n \cdot z} < 1 \tag{16}$$

$$\therefore |z| > 1 \tag{17}$$

This is called region of convergence.

$$X(z) = \sum_{n=0}^{\infty} (63)\, z^{-n} + \sum_{n=1}^{\infty} (2n)\, z^{-n} \tag{18}$$

$$X(z) = 63z\,(z-1)^{-1} + 2\,(2z-1)\,(z-1)^{-2} \tag{19}$$

$$\boxed{X(z) = 63z\,(z-1)^{-1} + 2\,(2z-1)\,(z-1)^{-2}} \forall |z| > 1 \tag{20}$$

2) for sequence (5) , let y(n) be *nth* term

a) Finding y(n) for (5)

$$a = 3 \tag{21}$$

$$a+d = 10 \tag{22}$$

$$d = 7 \tag{23}$$

$$y(n) = 3+(n)\cdot 7 \tag{24}$$

$$y(n) = 7n+3 \tag{25}$$

$$y(n) = 3u(n) + 7n \cdot u(n) \tag{26}$$

b) To find Y(z) :

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)\, z^{-n} \tag{27}$$

$$Y(z) = \sum_{n=-\infty}^{\infty} (3u(n) + 7nu(n))\, z^{-n} \tag{28}$$

$$Y(z) = \sum_{n=0}^{\infty} (3+7n)\, z^{-n} + 0 \tag{29}$$

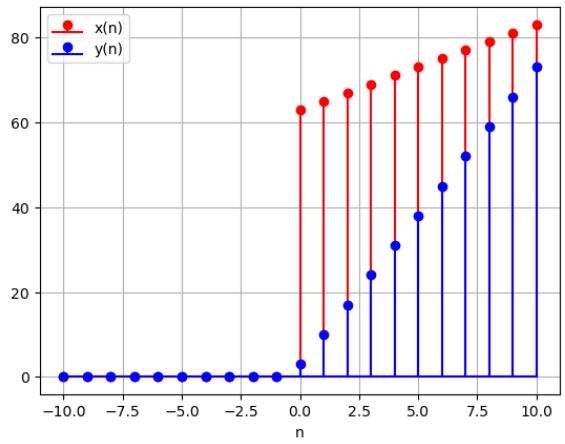

Fig. 2. Graphs of x(n)and y(n)

even for y(n), R.O.C is $|z| > 1$

$$Y(z) = \sum_{n=0}^{\infty} (3)\, z^{-n} + \sum_{n=1}^{\infty} (7n)\, z^{-n} \tag{30}$$

$$Y(z) = 3z\,(z-1)^{-1} + 7\,(2z-1)\,(z-1)^{-2} \tag{31}$$

$$\boxed{Y(z) = 3z\,(z-1)^{-1} + 7\,(2z-1)\,(z-1)^{-2}} \forall |z| > 1 \tag{32}$$

given, x(n) = y(n)

$$\therefore 63 + 2n = 7n + 3 \tag{33}$$

$$5n = 60 \tag{34}$$

$$n = 12 \tag{35}$$

$$So, x(n) = 63 + 2 \cdot 12 = 87 and \tag{36}$$

$$y(n) = 7 \cdot 12 + 3 = 87 \tag{37}$$

$\therefore$ 13th terms of given two APs are equal.

variable	description	value
a	first term of an AP	none
x(n)	n^{th} term of sequence (4)	equation (11)
y(n)	n^{th} term of sequence (5)	equation (26)
X(z)	z-transform of sequence (4)	equation (20)
Y(z)	z-transform of sequence (5)	equation (32)

TABLE 2
PARAMETERS USED