NCERT Discrete 10.5.2 -15

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Question: For what value of n, are the *nth* terms of two A.Ps: 63, 65, 67,... and 3, 10, 17,... equal?

Solution:

variable	description	value
x(n)	n^{th} term of sequence 63,65,67 ...	63u(n) + 2nu(n)
y(n)	n^{th} term of sequence 3,10,17 ...	3u(n) + 7nu(n)
X(z)	z-transform of sequence 63,65,67 ...	$63z(z-1)^{-1}+2(2z-1)(z-1)^{-2}$
Y(z)	z-transform of sequence 3,10,17 ...	$3z(z-1)^{-1}+7(2z-1)(z-1)^{-2}$

TABLE I
PARAMETERS USED

A sequence is said to be in Arithmetic Progression when it is in the form of

$$a, a+d, a+2d, a+3d, \dots \tag{1}$$

where a is first term and d is common difference. When there are n terms, the sequence becomes

$$a, a+d, a+2d, a+3d, \dots, a+(n)d. \tag{2}$$
$$T_n = a+(n)d. \tag{3}$$

which is nth term. In the given question, there are two sequences.

$$63, 65, 67 \dots \tag{4}$$
$$3, 10, 17 \dots \tag{5}$$

let u(n) be unit step function.

$$u(n) = \begin{cases} 1, & if n \geq 0, \\ 0, & if n < 0. \end{cases} \tag{6}$$

1) for the sequence (4), let x(n) be *nth* term,
   a) Finding x(n) for sequence (4)

$$x(0) = 63 \tag{7}$$
$$x(0)+d = 65 \tag{8}$$
$$\implies x(n) = 63+2n \tag{9}$$
$$\therefore x(n) = 63u(n)+2n\cdot u(n) \tag{10}$$

   b) To find X(z):

$$X(z) = \sum_{n=-\infty}^{\infty} x(n)\times z^{-n} \tag{11}$$

$$\implies X(z) = \sum_{n=-\infty}^{\infty} (63u(n)) + 2n\cdot u(n)\, z^{-n} \tag{12}$$

$$\implies X(z) = \sum_{n=0}^{\infty} (63+2n)\, z^{-n} \tag{13}$$

For the above sum to be convergent:
by using ratio test:

$$\frac{(n+1)\, z^{-n-1}}{(n)\, z^{-n}} < 1 \tag{14}$$
$$\implies \frac{n+1}{n\cdot z} < 1 \tag{15}$$
$$\therefore |z| > 1 \tag{16}$$

This is called region of convergence.

$$X(z) = \sum_{n=0}^{\infty} (63)\, z^{-n} + \sum_{n=1}^{\infty} (2n)\, z^{-n} \tag{17}$$

$$\implies X(z) = 63z(z-1)^{-1} + 2(2z-1)(z-1)^{-2} \tag{18}$$

$$\boxed{X(z) = 63z(z-1)^{-1} + 2(2z-1)(z-1)^{-2}}\,\forall |z| > 1 \tag{19}$$

2) for sequence (5) , let y(n) be *nth* term

   a) Finding y(n) for (5)

$$y(0) = 3 \tag{20}$$
$$y(0)+d = 10 \tag{21}$$
$$\implies y(n) = 7n+3 \tag{22}$$
$$\therefore y(n) = 3u(n)+7n\cdot u(n) \tag{23}$$

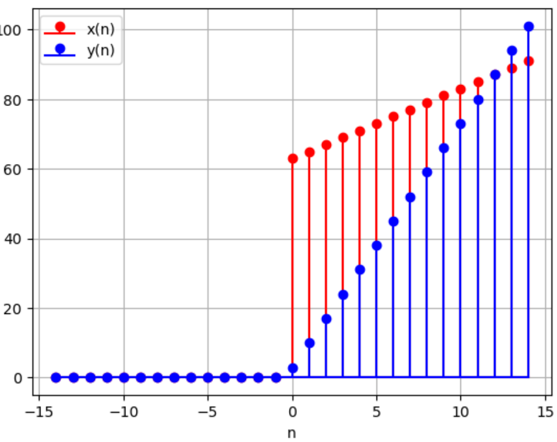

Fig. 1. Graphs of x(n)and y(n)

b) To find Y(z) :

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n} \tag{24}$$

$$\implies Y(z) = \sum_{n=-\infty}^{\infty} (3u(n) + 7nu(n)) z^{-n} \tag{25}$$

$$\implies Y(z) = \sum_{n=0}^{\infty} (3 + 7n) z^{-n} + 0 \tag{26}$$

even for y(n), R.O.C is $|z| > 1$

$$Y(z) = \sum_{n=0}^{\infty} (3) z^{-n} + \sum_{n=1}^{\infty} (7n) z^{-n} \tag{27}$$

$$\implies Y(z) = 3z(z-1)^{-1} + 7(2z-1)(z-1)^{-2} \tag{28}$$

$$\boxed{Y(z) = 3z(z-1)^{-1} + 7(2z-1)(z-1)^{-2}} \forall |z| > 1 \tag{29}$$

given, x(n) = y(n)

$$\therefore 63 + 2n = 7n + 3 \tag{30}$$

$$5n = 60 \tag{31}$$

$$\implies n = 12 \tag{32}$$

$$So, x(n) = 63 + 2 \cdot 12 = 87 and \tag{33}$$

$$y(n) = 7 \cdot 12 + 3 = 87 \tag{34}$$

$\therefore$ 13th terms of given two APs are equal.