GATE -BM 16

EE23BTECH11057 - Shakunaveti Sai Sri Ram Varun

Question: For the circuit given below, choose the angular frequency ω_0 at which voltage across capacitor has maximum amplitude?

Solution:

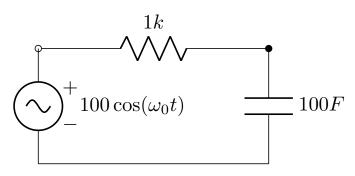


Fig. 1. circuit

Writing differential equation for circuit,

$ \qquad \qquad 1k $	•
$ _{-}^{+} 100 \frac{s}{s^2 + \omega_0^2} $	$\frac{10^4}{s} F$

Fig. 2. circuit in s-domain

on applying inverse Laplace transform,

$$V_c(t) = \frac{10^4 cos(\omega_o t)}{10^2 + \omega_o^2} + \frac{10^3 \omega_o sin(\omega_o t)}{10^2 + \omega_o^2} - \frac{10^4 e^{-10t}}{10^2 + \omega_o^2}$$

Parameter	Description	Value
R	Resistance in circuit	$1k\Omega$
C	Capacitace in circuit	100μ <i>F</i>
$v_i(t)$	Input voltage in circuit	$100\cos(\omega_o t)$
$v_c(t)$	Potential difference across Capacitor	$V_{c}\left(t\right)$
$V_{c}\left(s\right)$	Potential difference across Capacitor in s-domain	$V_{c}\left(s\right)$
$V_i(s)$	Input voltage in s-domain	$\frac{100s}{s^2+\omega_o^2}$
ω_o	angular frequency of input voltage	ω_o
	TABLE I	

INPUT VALUES

$$v_i(t) = 10^{-1} \frac{dv_c(t)}{dt} + v_c(t)$$
 (1)

Writing in s-domain (Laplace transform)

$$\mathcal{L}[v_i(t)] = \mathcal{L}[10^{-1} \frac{dv_c(t)}{dt}] + \mathcal{L}[v_c(t)] \quad (2)$$

$$\implies V_i(s) = s10^{-1}V_c(s) + V_c(s)$$
 (3)

$$\therefore V_c(s) = \frac{1000s}{(s^2 + \omega_o^2)} \frac{1}{(s+10)}$$
 (4)

splitting $V_c(s)$ into partial fractions,

$$V_c(s) = \frac{10^4 s + 10^3 \omega_o^2}{\left(10^2 + \omega_o^2\right) \left(s^2 + \omega_o^2\right)} - \frac{10^4}{\left(s + 10\right) \left(10^2 + \omega^2\right)}$$
(5)

The last term is natural response, we can ignore it.

Now, the amplitude can be computed by,

$$|v_c(t)| = \frac{10^3}{\sqrt{10^2 + \omega_o^2}} \tag{7}$$

we can see the highest amplitude is obtained when $\omega_o = 0$.