NCERT Discrete 10.5.2 -15

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Question: For what value of *n*, are the *nth* terms of two A.Ps: 63, 65, 67,... and 3, 10, 17,... equal?
Solution:

A sequence is said to be in Arithmetic Progression when it is in the form of

$$a, a + d, a + 2d, a + 3d, ...$$

where a is first term and d is common difference. When there are n terms, the sequence becomes

$$a, a + d, a + 2d, a + 3d, ..., a + (n - 1)d.$$
$$T_n = a + (n - 1)d.$$

which is nth term. In the given question, there are two sequences.

$$63, 65, 67.... \tag{1}$$
$$3, 10, 17.... \tag{2}$$

for the sequence (1), let x(n) be *nth* term,

$$a = 63$$
$$a + d = 65$$
$$d = 2$$
$$x(n) = 63 + (n - 1) \times 2$$
$$x(n) = 61 + 2n \tag{3}$$

for sequence (2) , let y(n) be *nth* term,

$$a = 3$$
$$a + d = 10$$
$$d = 7$$
$$y(n) = 3 + (n - 1) \times 7$$
$$y(n) = 7n - 4 \tag{4}$$

given, x(n) = y(n)

$$\therefore 61 + 2n = 7n - 4 \tag{5}$$
$$5n = 65$$
$$n = 13 \tag{6}$$
$$So, x(n) = 61 + 2 \times 13 = 87 \text{ and}$$
$$y(n) = 7 \times 13 - 4 = 87$$

$\therefore$ 13th terms of given two APs are equal.

To find X(z) and Y(z)(i.e. the 'z' transforms):

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \times z^{-n} \tag{7}$$

$$\text{and } Y(z) = \sum_{n=-\infty}^{\infty} y(n) \times z^{-n} \tag{8}$$

$$\text{i.e. } X(z) = \sum_{n=1}^{\infty} (61 + 2n) \times z^{-n}$$

when $|z| > 1$ this can be resolved to:

$$X(z) = 61(z - 1)^{-1} + 2(2z - 1)(z - 1)^{-2} \tag{9}$$

$$\text{and } Y(z) = \sum_{n=1}^{\infty} (-4 + 7n) \times z^{-n}$$

when $|z| > 1$ this can be resolved to:

$$Y(z) = -4(z - 1)^{-1} + 7(2z - 1)(z - 1)^{-2} \tag{10}$$