NCERT Discrete 10.5.2 -15

EE23BTECH11057 - Shakunaveti Sai Sri Ram Varun

Question: For what value of n, are the nth terms of two A.Ps: 63, 65, 67,... and 3, 10, 17,... equal?

Solution: A sequence is said to be in Arithmetic Progression when it is in the form of

$$a, a+d, a+2d, a+3d, ...$$

where a is first term and d is common difference. When there are n terms, the sequence becomes

$$a, a+d, a+2d, a+3d, ..., a+(n-1)d.$$
$$T_n = a+(n-1)d.$$

which is nth term. In the given question, there are two sequences.

$$63, 65, 67.... \quad (1)$$
$$3, 10, 17.... \quad (2)$$

let f(n) be unit step function.

$$\text{i.e. f(n)} = \begin{matrix} 1, & \text{n >= 0} \\ 0, & \text{n < 0} \end{matrix} \quad (3)$$

(i) for the sequence (1), let x(n) be nth term,

$$a = 63$$
$$a + d = 65$$
$$d = 2$$
$$\text{x(n)} = 63 + (n-1) \times 2$$
$$\text{x(n)} = 61 + 2n \quad (4)$$

$$\boxed{\text{x(n)} = 61\text{f(n-1)} + 2n\text{f(n-1)}}$$

To find X(z)(i.e. the 'z' transform) : (5)

$$\text{X(z)} = \sum_{n=-\infty}^{\infty} x(n) \times \text{z}^{-n} \quad (6)$$

$$\text{X(z)} = \sum_{n=-\infty}^{\infty} (61\text{f(n-1)} + 2n\text{f(n-1)})\text{z}^{-n}$$

$$\text{X(z)} = \sum_{n=1}^{\infty} (61 + 2n)\text{z}^{-n} + 0 \quad (7)$$

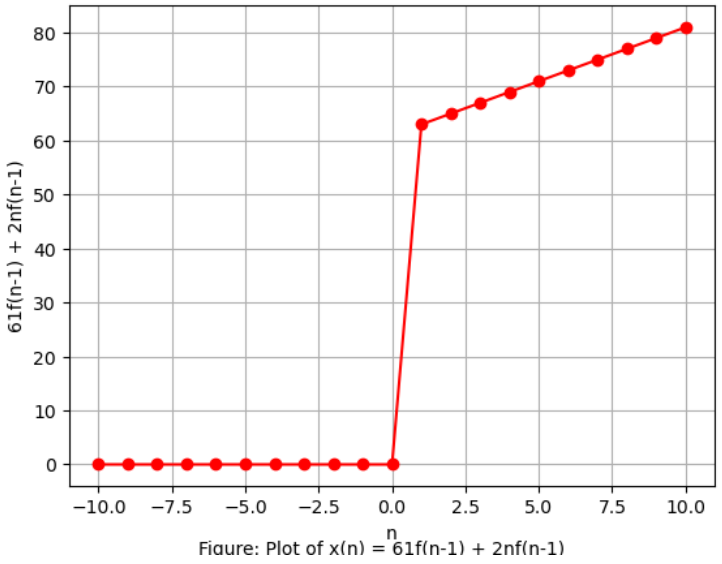

Figure: Plot of x(n) = 61f(n-1) + 2nf(n-1)

$$\text{X(z)} = \lim_{n\to\infty}[61(1-z^{-n})(z-1)^{-1} + 2(z-1)^{-1}$$
$$+2(z^{n-1}-1)(z^{1-n})(z-1)^{-1} - 2[(n-1)z+1]z^{n-1}] \quad (8)$$

$$\boxed{\text{X(z)} = 61(\text{z}-1)^{-1} + 2(2z-1)(z-1)^{-2} \forall |z| > 1} \quad (9)$$

(ii) for sequence (2) , let y(n) be nth term,

$$a = 3$$
$$a + d = 10$$
$$d = 7$$
$$\text{y(n)} = 3 + (n-1) \times 7$$
$$\text{y(n)} = 7n - 4 \quad (10)$$
$$\boxed{\text{y(n)} = -4\text{f(n-1)} + 7n\text{f(n-1)}} \quad (11)$$

To find Y(z) (the 'z' transform):

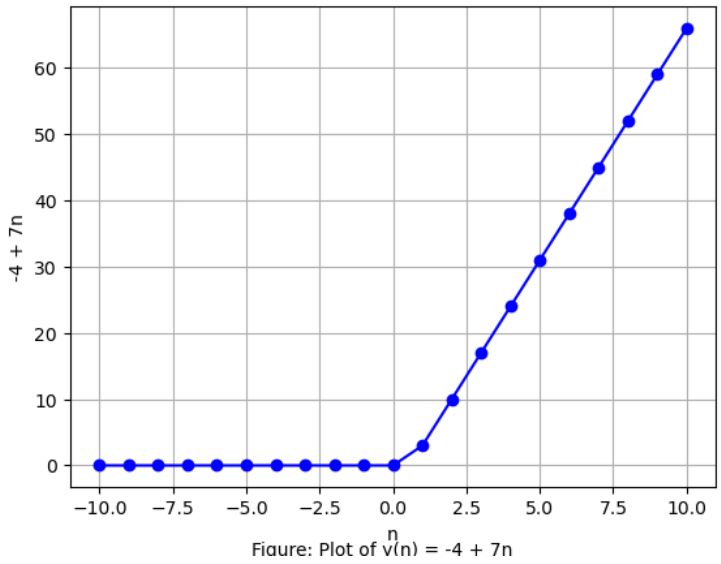

Figure: Plot of y(n) = -4 + 7n

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n)z^{-n} \tag{12}$$

$$Y(z) = \sum_{n=-\infty}^{\infty} (-4f(n-1) + 7nf(n-1))z^{-n}$$

$$Y(z) = \sum_{n=1}^{\infty} (-4 + 7n)z^{-n} + 0$$

$$Y(z) = \lim_{n\to\infty}[-4(1 - z^{-n})(z - 1)^{-1} + 7(z - 1)^{-1}$$
$$+7(z^{n-1} - 1)(z^{1-n})(z - 1)^{-1} - 7[(n - 1)z + 1]z^{n-1}] \tag{13}$$

$$\boxed{Y(z) = -4(z - 1)^{-1} + 7(2z - 1)(z - 1)^{-2} \forall |z| > 1} \tag{14}$$

given, x(n) = y(n)

$$\therefore 61 + 2n = 7n - 4 \tag{15}$$
$$5n = 65$$
$$n = 13 \tag{16}$$
$$So, x(n) = 61 + 2 \times 13 = 87 \text{ and}$$
$$y(n) = 7 \times 13 - 4 = 87$$

$\therefore$ 13th terms of given two APs are equal.