GATE -BM 16

EE23BTECH11057 - Shakunaveti Sai Sri Ram Varun

Question: For the circuit given below, choose the angular frequency ω_0 at which voltage across capacitor has maximum amplitude?

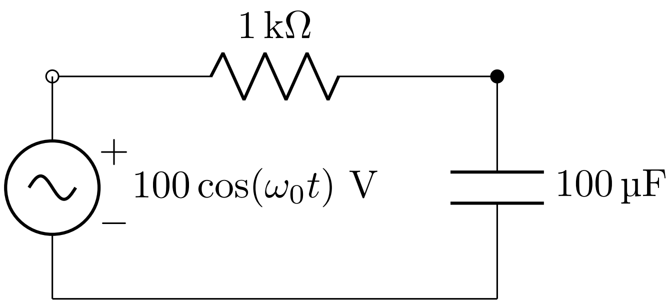

Fig. 1. circuit

(A) 1000

(B) 100

(C) 1

(D) 0

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Solution:
Writing in s-domain (Laplace transform)

Parameter	Description	Value
$v_i(t)$	Input voltage in circuit	$100\cos(\omega_o t)$ Volts
V_o	Amplitude of input voltage	100 Volts
R	Resistance in circuit	1 kΩ
C	Capacitace in circuit	100 µF
$V_c(s)$	Potential difference across Capacitor	$V_c(s)$
$V_i(s)$	Input voltage	$\frac{100s}{s^2+\omega_o^2}$
ω_o	angular frequency of input voltage	ω_o

TABLE I
INPUT VALUES

$$V_i(s) = sRCV_c(s) + V_c(s) \tag{1}$$

$$\implies V_c(s) = \frac{V_i(s)\frac{1}{RC}}{\frac{1}{RC}+s} \tag{2}$$

$$\therefore V_c(s) = \frac{V_o s\frac{1}{RC}}{(s^2+\omega_o^2)}\frac{1}{(s+\frac{1}{RC})} \tag{3}$$

Splitting $V_c(s)$ into partial fractions,

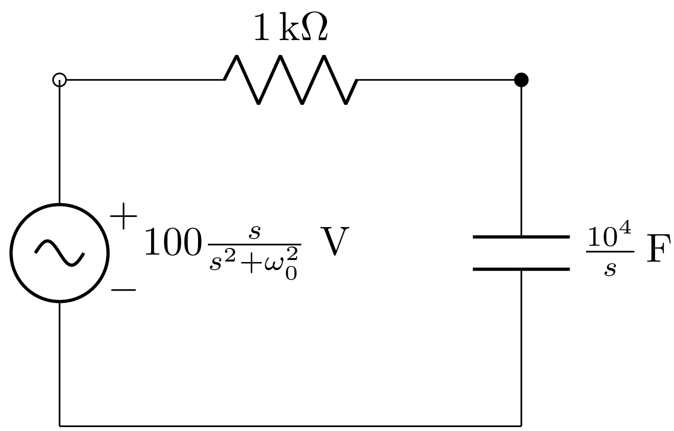

Fig. 2. circuit in s-domain

$$V_c(s) = \frac{V_o s + V_o RC\omega_o^2}{\left(1+(\omega_o RC)^2\right)(s^2+\omega_o^2)} - \frac{V_o}{\left(1+(\omega_o RC)^2\right)\left(s+\frac{1}{RC}\right)} \tag{4}$$

On applying inverse Laplace transform,

$$v_c(t) = \frac{V_o\cos(\omega_o t)}{1+(\omega_o RC)^2} + \frac{V_o RC\omega_o^2\sin(\omega_o t)}{1+(\omega_o RC)^2} + \frac{V_o e^{\frac{-t}{RC}}}{1+(\omega_o RC)^2} \tag{5}$$

The last term is natural response, we can ignore it.

Now, the amplitude can be computed by,

$$|v_c(t)| = \frac{V_o}{\sqrt{1+(\omega_o RC)^2}}\text{ V} \tag{6}$$

From values in Table I

$$|v_c(t)| = \frac{10^3}{\sqrt{10^2+\omega_o^2}}\text{ V} \tag{7}$$

We can see the highest amplitude is obtained when $\omega_o = 0$.