NCERT Discrete 10.5.2 -15

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Question: For what value of n, are the nth terms of two A.Ps: 63, 65, 67,... and 3, 10, 17,... equal?

Solution:

variable	description	value
$x_1(0)$	1^{st} term of 1^{st} A.P.	63
$x_2(0)$	1^{st} term of 2^{nd} A.P.	3
$x_1(n)$	n^{th} term of 1^{st} A.P.	63u(n) + 2nu(n)
$x_2(n)$	n^{th} term of 2^{nd} A.P.	3u(n) + 7nu(n)

TABLE I
PARAMETERS USED

$$\boxed{X(z) = x(0)\,z(z-1)^{-1} + dz(z-1)^{-2}\,\forall|z| > 1} \quad (1)$$

1) For the $1st$ A.P. $let x_1(n)$ be nth term:

a) Finding $x_1(n)$ for the $1st$ A.P:

$$x_1(0) = 63 \quad (2)$$
$$x_1(0) + d = 65 \quad (3)$$
$$\implies x_1(n) = 63 + 2n \quad (4)$$
$$\therefore x_1(n) = 63u(n) + 2n \cdot u(n) \quad (5)$$

b) To find $X_1(z)$:

$$x_1(0) = 63 \quad (6)$$
$$d = 2 \quad (7)$$
$$\therefore X_1(z) = 63z(z-1)^{-1} + 2z(z-1)^{-2}\,\forall|z| > 1 \quad (8)$$

2) For the $2nd$ A.P. $let x_2(n)$ be nth term:

a) Finding $x_2(n)$ for the for the $1st$ A.P:

$$x_2(0) = 3 \quad (9)$$
$$x_2(0) + d = 10 \quad (10)$$
$$\implies x_2(n) = 7n + 3 \quad (11)$$
$$\therefore x_2(n) = 3u(n) + 7n \cdot u(n) \quad (12)$$

b) To find $X_2(z)$:

$$x_2(0) = 3 \quad (13)$$
$$d = 7 \quad (14)$$
$$\therefore X_2(z) = 3z(z-1)^{-1} + 7z(z-1)^{-2}\,\forall|z| > 1 \quad (15)$$

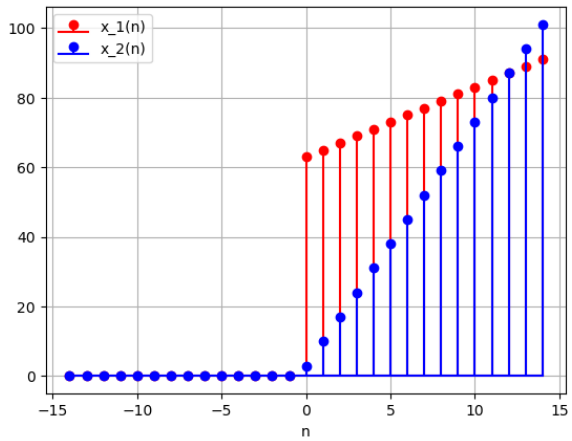

Fig. 1. Graphs of $x_1(n)$ and $x_2(n)$ and both are equal at $n = 12$

given, $x_1(n) = x_2(n)$

$$\therefore 63 + 2n = 7n + 3 \quad (16)$$
$$5n = 60 \quad (17)$$
$$\implies n = 12 \quad (18)$$
$$(19)$$

$\therefore$ 13th terms of given two APs are equal.