GATE -BM 16

EE23BTECH11057 - Shakunaveti Sai Sri Ram Varun

Question: For the circuit given below, choose the angular frequency ω_0 at which voltage across capacitor has maximum amplitude?

Solution:

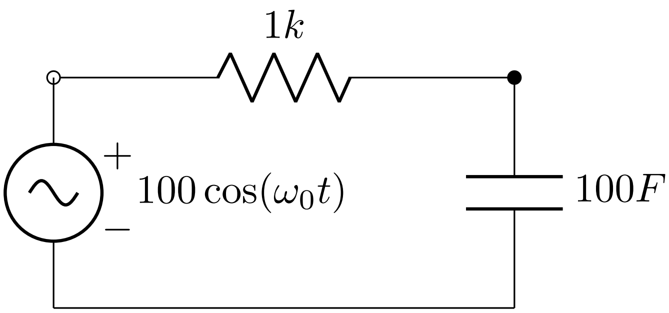

Fig. 1. circuit

Writing differential equation for circuit,

Parameter	Description	Value
R	Resistance in circuit	$1k\Omega$
C	Capacitace in circuit	$100\mu F$
$v_i(t)$	Input voltage in circuit	$100\cos(\omega_o t)$
$v_c(t)$	Potential difference across Capacitor	$V_c(t)$
$V_c(s)$	Potential difference across Capacitor in s-domain	$V_c(s)$
$V_i(s)$	Input voltage in s-domain	$\frac{100s}{s^2+\omega_o^2}$
ω_o	angular frequency of input voltage	ω_o

TABLE I
INPUT VALUES

$$v_i(t) = 10^{-1}\frac{dv_c(t)}{dt} + v_c(t) \tag{1}$$

Writing in s-domain (Laplace transform)

$$\mathcal{L}[v_i(t)] = \mathcal{L}[10^{-1}\frac{dv_c(t)}{dt}] + \mathcal{L}[v_c(t)] \tag{2}$$

$$\implies V_i(s) = s10^{-1}V_c(s) + V_c(s) \tag{3}$$

$$\therefore V_c(s) = \frac{1000s}{(s^2+\omega_o^2)}\frac{1}{(s+10)} \tag{4}$$

splitting $V_c(s)$ into partial fractions,

$$V_c(s) = \frac{10^4 s + 10^3\omega_o^2}{(10^2+\omega_o^2)(s^2+\omega_o^2)} - \frac{10^4}{(s+10)(10^2+\omega^2)} \tag{5}$$

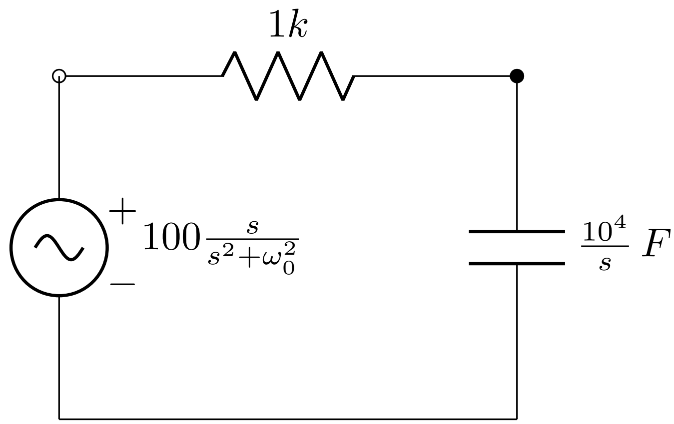

Fig. 2. circuit in s-domain

on applying inverse Laplace transform,

$$V_c(t) = \frac{10^4\cos(\omega_o t)}{10^2+\omega_o^2} + \frac{10^3\omega_o\sin(\omega_o t)}{10^2+\omega_o^2} - \frac{10^4 e^{-10t}}{10^2+\omega_o^2} \tag{6}$$

The last term is natural response, we can ignore it.
Now, the amplitude can be computed by,

$$|v_c(t)| = \frac{10^3}{\sqrt{10^2+\omega_o^2}} \tag{7}$$

we can see the highest amplitude is obtained when $\omega_o = 0$.