

NCERT Discrete 10.5.2 -15

EE23BTECH11057 - Shakunaveti Sai Sri Ram Varun

Question: For what value of n , are the n th terms of two A.Ps: 63, 65, 67, ... and 3, 10, 17, ... equal?

Solution: A sequence is said to be in Arithmetic

Progression when it is in the form of

$$a, a + d, a + 2d, a + 3d, \dots \quad (1)$$

where a is first term and d is common difference.

When there are n terms, the sequence becomes

$$a, a + d, a + 2d, a + 3d, \dots, a + (n) d. \quad (2)$$

$$T_n = a + (n) d. \quad (3)$$

which is n th term. In the given question, there are two sequences.

$$63, 65, 67 \dots \quad (4)$$

$$3, 10, 17 \dots \quad (5)$$

let $u(n)$ be unit step function.

$$u(n) = \begin{cases} 1, & \text{if } n \geq 0, \\ 0, & \text{if } n < 0. \end{cases} \quad (6)$$

1) for the sequence (4), let $x(n)$ be n th term,

a) Finding $x(n)$ for sequence (4)

$$a = 63 \quad (7)$$

$$a + d = 65 \quad (8)$$

$$\Rightarrow x(n) = 63 + 2n \quad (9)$$

$$\therefore x(n) = 63u(n) + 2n \cdot u(n) \quad (10)$$

b) To find $X(z)$:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \times z^{-n} \quad (11)$$

$$\Rightarrow X(z) = \sum_{n=-\infty}^{\infty} (63u(n)) + 2n \cdot u(n) z^{-n} \quad (12)$$

$$\Rightarrow X(z) = \sum_{n=0}^{\infty} (63 + 2n) z^{-n} \quad (13)$$

For the above sum to be convergent:
by using ratio test:

$$\frac{(n+1)z^{-n-1}}{(n)z^{-n}} < 1 \quad (14)$$

$$\Rightarrow \frac{n+1}{n \cdot z} < 1 \quad (15)$$

$$\therefore |z| > 1 \quad (16)$$

This is called region of convergence.

$$X(z) = \sum_{n=0}^{\infty} (63) z^{-n} + \sum_{n=1}^{\infty} (2n) z^{-n} \quad (17)$$

$$\Rightarrow X(z) = 63z(z-1)^{-1} + 2(2z-1)(z-1)^{-2} \quad (18)$$

$$\boxed{X(z) = 63z(z-1)^{-1} + 2(2z-1)(z-1)^{-2}} \quad \forall |z| > 1 \quad (19)$$

2) for sequence (5), let $y(n)$ be n th term

a) Finding $y(n)$ for (5)

$$a = 3 \quad (20)$$

$$a + d = 10 \quad (21)$$

$$\Rightarrow y(n) = 7n + 3 \quad (22)$$

$$\therefore y(n) = 3u(n) + 7n \cdot u(n) \quad (23)$$

b) To find $Y(z)$:

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n} \quad (24)$$

$$\Rightarrow Y(z) = \sum_{n=-\infty}^{\infty} (3u(n) + 7nu(n)) z^{-n} \quad (25)$$

$$\Rightarrow Y(z) = \sum_{n=0}^{\infty} (3 + 7n) z^{-n} + 0 \quad (26)$$

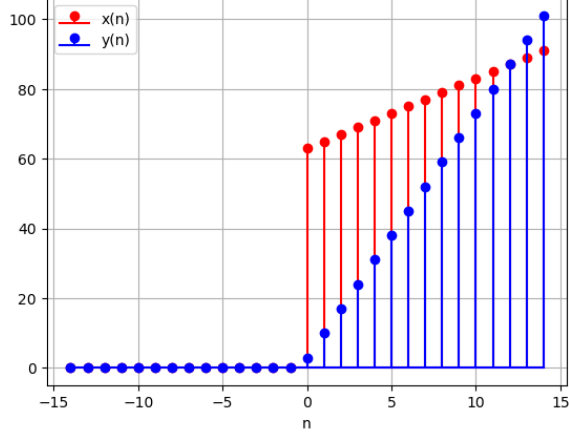


Fig. 1. Graphs of $x(n)$ and $y(n)$

even for $y(n)$, R.O.C is $|z| > 1$

$$Y(z) = \sum_{n=0}^{\infty} (3) z^{-n} + \sum_{n=1}^{\infty} (7n) z^{-n} \quad (27)$$

$$\Rightarrow Y(z) = 3z(z-1)^{-1} + 7(2z-1)(z-1)^{-2} \quad (28)$$

$$Y(z) = 3z(z-1)^{-1} + 7(2z-1)(z-1)^{-2} \quad \forall |z| > 1 \quad (29)$$

given, $x(n) = y(n)$

$$\therefore 63 + 2n = 7n + 3 \quad (30)$$

$$5n = 60 \quad (31)$$

$$\Rightarrow n = 12 \quad (32)$$

\therefore 13th terms of given two APs are equal.

variable	description	value
a	first term of an AP	none
$x(n)$	n^{th} term of sequence 63,65,67 ...	$63u(n) + 2nu(n)$
$y(n)$	n^{th} term of sequence 3,10,17 ...	$3u(n) + 7nu(n)$
$X(z)$	z-transform of sequence 63,65,67 ...	$63z(z-1)^{-1} + 2(2z-1)(z-1)^{-2}$
$Y(z)$	z-transform of sequence 3,10,17 ...	$3z(z-1)^{-1} + 7(2z-1)(z-1)^{-2}$

TABLE I

PARAMETERS USED