GATE -BM 16

EE23BTECH11057 - Shakunaveti Sai Sri Ram Varun

Question: A buoy of virtual mass 30 kg oscillates in a fluid medium as a single degree of freedom system. If the total damping in the system is set as 188.5 N-s/m, such that the oscillation just ceases to occur, then the natural period of the system is ____ s (round off to one decimal place) (GATE MN 2023 question 63)

Solution:

Parameter	Description	Value
$X(s)$	position in laplace domain	$X(s)$
$x(t)$	position of buoy w.r.t time	$x(t)$
m	mass of buoy	30kg
λ	damping coeffecient of the system i.e $\left(\frac{188.5}{30}\right)$	$6.283 \approx 2\pi$
ω_o	natural angular frequency of the system	?
ω_d	damping frequency of the system	0 rad s^{-1}

TABLE I
INPUT VALUES

The differential equation of the system is:

$$\frac{d^2x(t)}{dt^2} + \lambda\frac{dx(t)}{dt} + \omega_0^2 x(t) = 0 \tag{1}$$

Taking laplace transform:

$$s^2X(s) + \lambda sX(s) + \omega_o^2X(s) = 0 \tag{2}$$
$$\implies s^2 + s\lambda + \omega_o^2 = 0 \tag{3}$$
$$\therefore s = \frac{-\lambda \pm \sqrt{\lambda^2 - 4\omega_o^2}}{2} \tag{4}$$

where $\sqrt{\lambda^2 - 4\omega_o^2}$ is ω_d.
From Table I,

$$\omega_d = 0 \tag{5}$$
$$\implies \lambda = 2\omega_o \tag{6}$$
$$\implies \omega_o \approx \pi \tag{7}$$
$$T_i = \frac{2\pi}{\omega_o} \tag{8}$$
$$\therefore t_i = 2 \text{ seconds} \tag{9}$$

To find x(t), we assume the initial amplitude of oscillations to be 1 meter and it is situated at extreme position.

$$s^2X(s) + \lambda sX(s) + \omega_o^2X(s) = sx(0) + x(0) \tag{10}$$
$$\implies X(s) = \frac{1+s}{s^2 + s\lambda + \omega_o^2} \tag{11}$$

substituting values from Table I and ω_o,

$$X(s) = \frac{1+s}{(s+\pi)^2} \tag{12}$$

Taking inverse laplace transform by method of partial fractions,

$$X(s) = \frac{1}{s+\pi} + \frac{1-\pi}{(s+\pi)^2} \tag{13}$$
$$\therefore x(t) = (1 + (1-\pi)t)e^{-\pi} \tag{14}$$