

GATE -BM 16

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Question: For the circuit given below, choose the angular frequency ω_0 at which voltage across capacitor has maximum amplitude?

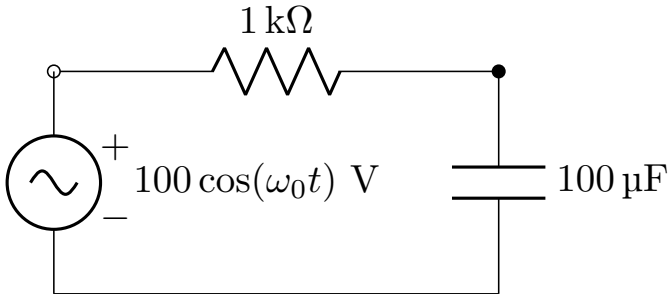


Fig. 1. circuit

- (A) 1000
- (B) 100
- (C) 1
- (D) 0

Solution:

Writing in s-domain (Laplace transform)

Parameter	Description	Value
$v_i(t)$	Input voltage in circuit	$100 \cos(\omega_o t)$ Volts
V_o	Amplitude of input voltage	100 Volts
R	Resistance in circuit	1 k Ω
C	Capacitance in circuit	100 μ F
$V_c(s)$	Potential difference across Capacitor	$V_c(s)$
$V_i(s)$	Input voltage	$\frac{100s}{s^2 + \omega_o^2}$
ω_o	angular frequency of input voltage	ω_o

TABLE I
INPUT VALUES

$$V_i(s) = sRCV_c(s) + V_c(s) \quad (1)$$

$$\Rightarrow V_c(s) = \frac{V_i(s) \frac{1}{RC}}{\frac{1}{RC} + s} \quad (2)$$

$$\therefore V_c(s) = \frac{V_o s \frac{1}{RC}}{(s^2 + \omega_o^2)(s + \frac{1}{RC})} \quad (3)$$

Splitting $V_c(s)$ into partial fractions,

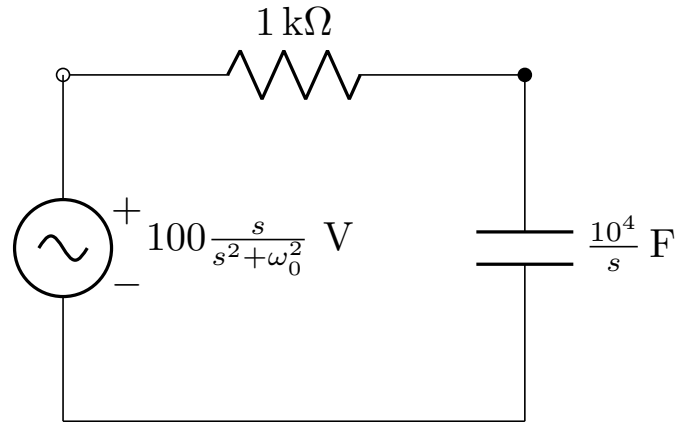


Fig. 2. circuit in s-domain

$$V_c(s) = \frac{V_o s + V_o RC \omega_o^2}{(1 + (\omega_o RC)^2)(s^2 + \omega_o^2)} - \frac{V_o}{(1 + (\omega_o RC)^2)(s + \frac{1}{RC})} \quad (4)$$

On applying inverse Laplace transform,

$$v_c(t) = \frac{V_o \cos(\omega_o t)}{1 + (\omega_o RC)^2} + \frac{V_o RC \omega_o^2 \sin(\omega_o t)}{1 + (\omega_o RC)^2} + \frac{V_o e^{-\frac{t}{RC}}}{1 + (\omega_o RC)^2} \quad (5)$$

The last term is natural response, we can ignore it.

Now, the amplitude can be computed by,

$$|v_c(t)| = \frac{V_o}{\sqrt{1 + (\omega_o RC)^2}} \quad (6)$$

From values in Table I

$$|v_c(t)| = \frac{10^3}{\sqrt{10^2 + \omega_o^2}} \quad (7)$$

We can see the highest amplitude is obtained when $\omega_o = 0$.