NCERT Discrete 10.5.2 -15

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Question: For what value of n, are the nth terms of two A.Ps: 63, 65, 67,... and 3, 10, 17,... equal?

Solution:

(i) A sequence is said to be in Arithmetic Progression when it is in the form of

$$a, a + d, a + 2d, a + 3d, ...$$

where a is first term and d is common difference.
When there are n terms, the sequence becomes

$$a, a + d, a + 2d, a + 3d, ..., a + (n - 1)d.$$
$$T_n = a + (n - 1)d.$$

which is nth term. In the given question, there are two sequences.

$$63, 65, 67.... \tag{1}$$
$$3, 10, 17.... \tag{2}$$

for the sequence (1), let x(n) be nth term,

$$a = 63$$
$$a + d = 65$$
$$d = 2$$
$$\text{x(n)} = 63 + (n - 1) \times 2$$
$$\text{x(n)} = 61 + 2n \tag{3}$$

for sequence (2) , let y(n) be nth term,

$$a = 3$$
$$a + d = 10$$
$$d = 7$$
$$\text{y(n)} = 3 + (n - 1) \times 7$$
$$\text{y(n)} = 7n - 4 \tag{4}$$

given, x(n) = y(n)

$$\therefore 61 + 2n = 7n - 4 \tag{5}$$
$$5n = 65$$
$$n = 13 \tag{6}$$
$$So, \text{x(n)} = 61 + 2 \times 13 = 87 \text{ and}$$
$$\text{y(n)} = 7 \times 13 - 4 = 87$$

$\therefore$ 13th terms of given two APs are equal.

(ii) To find X(z) and Y(z) (i.e. the 'z' transforms): let f(n) be unit step function.

$$\text{i.e. f(n)} = \begin{cases} 1, & \text{n} >= 0 \\ 0, & \text{n} < 0 \end{cases} \tag{7}$$

$$\text{x(n)} = 61 \times \text{f(n)} + 2n \times \text{f(n)} \tag{8}$$

$$\text{X(z)} = \sum_{n=-\infty}^{\infty} x(n) \times \text{z}^{-n} \tag{9}$$

$$\text{X(z)} = \sum_{n=-\infty}^{\infty} (61\text{f(n)} + 2\text{nf(n)})\text{z}^{-n}$$

$$\text{X(z)} = \sum_{n=1}^{\infty} (61 + 2n)\text{z}^{-n} + 0 \tag{10}$$

$$\text{X(z)} = \lim_{n\to\infty} [61(1 - z^{-n})(z - 1)^{-1} + 2(z - 1)^{-1}$$
$$+2(z^{n-1} - 1)(z^{1-n})(z - 1)^{-1} - 2[(n - 1)z + 1]z^{n-1}] \tag{11}$$

$$\boxed{\text{X(z)} = 61(\text{z} - 1)^{-1} + 2(2z - 1)(z - 1)^{-2} \forall |z| > 1} \tag{12}$$

$$\text{and Y(z)} = \sum_{n=-\infty}^{\infty} y(n)\text{z}^{-n} \tag{13}$$

$$\text{Y(z)} = \sum_{n=-\infty}^{\infty} (-4\text{f(n)} + 7n\text{f(n)})\text{z}^{-n}$$

$$\text{Y(z)} = \sum_{n=1}^{\infty} (-4 + 7n)\text{z}^{-n} + 0$$

$$\text{Y(z)} = \lim_{n\to\infty} [-4(1 - z^{-n})(z - 1)^{-1} + 7(z - 1)^{-1}$$
$$+7(z^{n-1} - 1)(z^{1-n})(z - 1)^{-1} - 7[(n - 1)z + 1]z^{n-1}] \tag{14}$$

$$\boxed{\text{Y(z)} = -4(z - 1)^{-1} + 7(2z - 1)(z - 1)^{-2} \forall |z| > 1} \tag{15}$$