NCERT Discrete 10.5.2 -15

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Question: For what value of n, are the nth terms of two A.Ps: 63, 65, 67,... and 3, 10, 17,... equal?

Solution:

variable	description	value
$x_1(0)$	1^{st} term of sequence 63,65,67 ...	63
$x_2(0)$	1^{st} term of sequence 3,10,17 ...	3
$x_1(n)$	n^{th} term of sequence 63,65,67 ...	$63\mathrm{u}(n) + 2n\mathrm{u}(n)$
$x_2(n)$	n^{th} term of sequence 3,10,17 ...	$3\mathrm{u}(n) + 7n\mathrm{u}(n)$
$U(z)$	z-transform of $u(n)$	$z \cdot (z-1)^{-1}\ \forall \lvert z\rvert > 1$
$U_n(z)$	z-transform of $nu(n)$	$z \cdot (z-1)^{-2}\ \forall \lvert z\rvert > 1$
$X_1(z)$	z-transform of sequence 63,65,67 ...	$63z(z-1)^{-1} + 2z(z-1)^{-2}\ \forall \lvert z\rvert > 1$
$X_2(z)$	z-transform of sequence 3,10,17 ...	$3z(z-1)^{-1} + 7z(z-1)^{-2}\ \forall \lvert z\rvert > 1$

TABLE I
PARAMETERS USED

A sequence is said to be in Arithmetic Progression when it is in the form of

$$a, a+d, a+2d, a+3d, \ldots \tag{1}$$

where a is first term and d is common difference. When there are n terms, the sequence becomes

$$a, a+d, a+2d, a+3d, \ldots, a+(n)d. \tag{2}$$
$$t_n = a+(n)d. \tag{3}$$

which is nth term. In the given question, there are two sequences.

$$63, 65, 67 \ldots \tag{4}$$
$$3, 10, 17 \ldots \tag{5}$$

let u(n) be unit step function.

$$u(n) = \begin{cases} 1, & if n \geq 0, \\ 0, & if n < 0. \end{cases} \tag{6}$$

For finding U(z):

$$U(z) = \sum_{n=-\infty}^{\infty} u(n) \tag{7}$$

$$\implies U(z) = \sum_{n=0}^{\infty} z^{-n} \tag{8}$$

this is infinite G.P. sequence with common ratio z^{-1}.
So, for this to converge $|z| > 1$. This is called region of convergence.

$$\implies U(z) = \sum_{n=0}^{\infty} z^{-n} \tag{9}$$

$$\therefore U(z) = z \cdot (z-1)^{-1}\ \forall |z| > 1 \tag{10}$$

similarly,

$$nu(n) = \begin{cases} n, & if n \geq 0, \\ 0, & if n < 0. \end{cases} \tag{11}$$

For finding $U_n(z)$:

$$U_n(z) = \sum_{n=-\infty}^{\infty} nu(n) \tag{12}$$

$$\implies U_n(z) = \sum_{n=1}^{\infty} nz^{-n} \tag{13}$$

this is infinite A.G.P. sequence with common ratio z^{-1} and common difference 1.
So, for this to converge $|z| > 1$. This is called region of convergence.

$$\implies U_n(z) = \sum_{n=1}^{\infty} z^{-n} \tag{14}$$

$$\therefore U_n(z) = z \cdot (z-1)^{-2}\ \forall |z| > 1 \tag{15}$$

for an A.P. with n^{th} term x(n) given by:

$$x(n) = x(0) + nd \tag{16}$$

$$so, X(z) = \sum_{n=-\infty}^{\infty} x(n) \cdot z^{-n} \tag{17}$$

$$\implies X(z) = \sum_{n=-\infty}^{\infty} \left(x(0)u(n) + dn \cdot u(n)z^{-n}\right) \tag{18}$$

$$from (10) and (15):$$

$$\boxed{X(z) = x(0)z(z-1)^{-1} + dz(z-1)^{-2}}\ \forall |z| > 1 \tag{19}$$

1) for the sequence (4), $let x_1(n)$ be nth term,
   a) Finding $x_1(n)$ for sequence (4)

$$x_1(0) = 63 \tag{20}$$
$$x_1(0) + d = 65 \tag{21}$$
$$\implies x_1(n) = 63 + 2n \tag{22}$$
$$\therefore x_1(n) = 63u(n) + 2n \cdot u(n) \tag{23}$$

   b) To find $X_1(z)$:

$$x_1(0) = 63 \tag{24}$$
$$d = 2 \tag{25}$$
$$from(19)X_1(z) = 63z(z-1)^{-1} + 2z(z-1)^{-2}\,\forall|z| > 1 \tag{26}$$

2) for sequence (5) , let $x_2(n)$ be nth term
   a) Finding $x_2(n)$ for (5)

$$x_2(0) = 3 \tag{27}$$
$$x_2(0) + d = 10 \tag{28}$$
$$\implies x_2(n) = 7n + 3 \tag{29}$$
$$\therefore x_2(n) = 3u(n) + 7n \cdot u(n) \tag{30}$$

   b) To find $X_2(z)$:

$$x_2(0) = 3 \tag{31}$$
$$d = 7 \tag{32}$$
$$from(19)X_2(z) = 3z(z-1)^{-1} + 7z(z-1)^{-2}\,\forall|z| > 1 \tag{33}$$

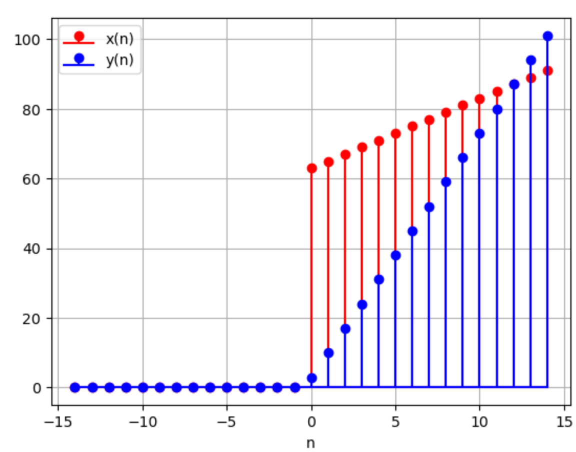

Fig. 1. Graphs of x(n)and y(n)

given, $x_1(n) = x_2(n)$

$$\therefore 63 + 2n = 7n + 3 \tag{34}$$
$$5n = 60 \tag{35}$$
$$\implies n = 12 \tag{36}$$
$$\tag{37}$$

$\therefore$ 13th terms of given two APs are equal.