NCERT Discrete 10.5.2 -15

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Question: For what value of n, are the *nth* terms of two A.Ps: 63, 65, 67,... and 3, 10, 17,... equal?

Solution: A sequence is said to be in Arithmetic Progression when it is in the form of

$$a, a + d, a + 2d, a + 3d, \ldots \qquad (1)$$

where a is first term and d is common difference. When there are n terms, the sequence becomes

$$a, a + d, a + 2d, a + 3d, \ldots, a + (n) d. \qquad (2)$$

$$T_n = a + (n) d. \qquad (3)$$

which is nth term. In the given question, there are two sequences.

$$63, 65, 67 \ldots \qquad (4)$$

$$3, 10, 17 \ldots \qquad (5)$$

let u(n) be unit step function.

$$u(n) = \begin{cases} 1, & if\, n \geq 0, \\ 0, & if\, n < 0. \end{cases} \qquad (6)$$

1) for the sequence (4), let x(n) be *nth* term,

a) Finding x(n) for sequence (4)

$$a = 63 \qquad (7)$$

$$a + d = 65 \qquad (8)$$

$$d = 2 \qquad (9)$$

$$x(n) = 63 + (n) \cdot 2 \qquad (10)$$

$$x(n) = 63 + 2n \qquad (11)$$

$$\boxed{x(n) = 63u(n) + 2n \cdot u(n)}$$

b) To find X(z):

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \times z^{-n} \qquad (12)$$

$$X(z) = \sum_{n=-\infty}^{\infty} (63u(n)) + 2n \cdot u(n) z^{-n} \qquad (13)$$

$$X(z) = \sum_{n=0}^{\infty} (63 + 2n) z^{-n} \qquad (14)$$

For the above sum to be convergent: by using ratio test:

$$\frac{(n + 1) z^{-n-1}}{(n) z^{-n}} < 1 \qquad (15)$$

$$\frac{n + 1}{n \cdot z} < 1 \qquad (16)$$

$$\therefore |z| > 1 \qquad (17)$$

This is called region of convergence.

$$X(z) = \sum_{n=0}^{\infty} (63) z^{-n} + \sum_{n=1}^{\infty} (2n) z^{-n} \qquad (18)$$

$$X(z) = 63z (z - 1)^{-1} + 2 (2z - 1) (z - 1)^{-2} \qquad (19)$$

$$\boxed{X(z) = 63z (z - 1)^{-1} + 2 (2z - 1) (z - 1)^{-2}} \, \forall |z| > 1 \qquad (20)$$

2) for sequence (5) , let y(n) be *nth* term

a) Finding y(n) for (5)

$$a = 3 \qquad (21)$$

$$a + d = 10 \qquad (22)$$

$$d = 7 \qquad (23)$$

$$y(n) = 3 + (n) \cdot 7 \qquad (24)$$

$$y(n) = 7n + 3 \qquad (25)$$

$$y(n) = 3u(n) + 7n \cdot u(n) \qquad (26)$$

b) To find Y(z) :

$$Y(z) = \sum_{n=-\infty}^{\infty} y(n) z^{-n} \qquad (27)$$

$$Y(z) = \sum_{n=-\infty}^{\infty} (3u(n) + 7nu(n)) z^{-n} \qquad (28)$$

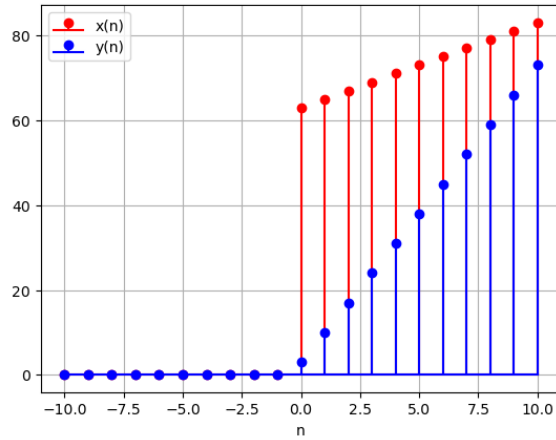$$Y(z) = \sum_{n=0}^{\infty} (3 + 7n) z^{-n} + 0 \qquad (29)$$

Fig. 1. Graphs of x(n)and y(n)

even for y(n), R.O.C is $|z| > 1$

$$Y(z) = \sum_{n=0}^{\infty} (3) z^{-n} + \sum_{n=1}^{\infty} (7n) z^{-n} \tag{30}$$

$$Y(z) = 3z(z-1)^{-1} + 7(2z-1)(z-1)^{-2} \tag{31}$$

$$\boxed{Y(z) = 3z(z-1)^{-1} + 7(2z-1)(z-1)^{-2}} \forall |z| > 1 \tag{32}$$

given, x(n) = y(n)

$$\therefore 63 + 2n = 7n + 3 \tag{33}$$
$$5n = 60 \tag{34}$$
$$n = 12 \tag{35}$$
$$So, x(n) = 63 + 2 \cdot 12 = 87 and \tag{36}$$
$$y(n) = 7 \cdot 12 + 3 = 87 \tag{37}$$

$\therefore$ 13th terms of given two APs are equal.

variable	description	value
a	first term of an AP	none
x(n)	n^{th} term of sequence (4)	equation (11)
y(n)	n^{th} term of sequence (5)	equation (26)
X(z)	z-transform of sequence (4)	equation (20)
Y(z)	z-transform of sequence (5)	equation (32)

TABLE I
PARAMETERS USED