NCERT Discrete 10.5.2 -15

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Question: For what value of n, are the *nth* terms of two A.Ps: 63, 65, 67,... and 3, 10, 17,... equal?

Solution:

part-(i) A sequence is said to be in Arithmetic Progression when it is in the form of

$$a, a+d, a+2d, a+3d, ...$$

where a is first term and d is common difference. When there are n terms, the sequence becomes

$$a, a+d, a+2d, a+3d, ..., a+(n-1)d.$$
$$T_n = a+(n-1)d.$$

which is nth term. In the given question, there are two sequences.

$$63, 65, 67.... \tag{1}$$
$$3, 10, 17.... \tag{2}$$

for the sequence (1), let x(n) be *nth* term,

$$a = 63$$
$$a+d = 65$$
$$d = 2$$
$$\text{x(n)} = 63 + (n-1)\times 2$$
$$\text{x(n)} = 61 + 2n \tag{3}$$

for sequence (2) , let y(n) be *nth* term,

$$a = 3$$
$$a+d = 10$$
$$d = 7$$
$$\text{y(n)} = 3 + (n-1)\times 7$$
$$\text{y(n)} = 7n - 4 \tag{4}$$

given, x(n) = y(n)

$$\therefore 61 + 2n = 7n - 4 \tag{5}$$
$$5n = 65$$
$$n = 13 \tag{6}$$
$$So, \text{x(n)} = 61 + 2\times 13 = 87 \text{ and}$$
$$\text{y(n)} = 7\times 13 - 4 = 87$$

$\therefore$ 13th terms of given two APs are equal.

part-(ii) To find X(z) and Y(z) (i.e. the 'z' transforms):

$$\text{X(z)} = \sum_{n=-\infty}^{\infty} x(n)\times \text{z}^{-n} \tag{7}$$

$$\text{and Y(z)} = \sum_{n=-\infty}^{\infty} y(n)\times \text{z}^{-n} \tag{8}$$

$$\text{i.e. X(z)} = \sum_{n=1}^{\infty}(61+2n)\times \text{z}^{-n}$$

$$\text{X(z)} = \sum_{n=1}^{\infty}(61\times \text{z}^{-n}) + \sum_{n=1}^{\infty}(2n\times \text{z}^{-n})$$

$$\text{X(z)} = \sum_{n=1}^{\infty}(61\times \text{z}^{-n}) + \sum_{n=1}^{\infty}(2n\times \text{z}^{-n})$$

$$\text{X(z)} = \lim_{n\to\infty}[61\times(1-z^{-n})(z-1)^{-1} + 2(z-1)^{-1}$$
$$+2(z^{n-1}-1)(z^{1-n})(z-1)^{-1} - 2[(n-1)z+1]z^{n-1}] \tag{9}$$

$$\boxed{\text{X(z)} = 61(\text{z}-1)^{-1} + 2(2\text{z}-1)(\text{z}-1)^{-2} \forall |\text{z}|>1} \tag{10}$$

$$\text{and Y(z)} = \sum_{n=1}^{\infty}(-4+7n)\times \text{z}^{-n}$$

$$\text{Y(z)} = \sum_{n=1}^{\infty}(-4\times \text{z}^{-n}) + \sum_{n=1}^{\infty}(7n\times \text{z}^{-n})$$

$$\text{Y(z)} = \lim_{n\to\infty}[-4\times(1-z^{-n})(z-1)^{-1} + 7(z-1)^{-1}$$
$$+7(z^{n-1}-1)(z^{1-n})(z-1)^{-1} - 7[(n-1)z+1]z^{n-1}] \tag{11}$$

$$\boxed{\text{Y(z)} = -4(\text{z}-1)^{-1} + 7(2\text{z}-1)(\text{z}-1)^{-2} \forall |\text{z}|>1} \tag{12}$$