GATE 2022 -AE 63

EE23BTECH11057 - Shakunaveti Sai Sri Ram Varun

Question: A uniform rigid prismatic bar of total mass m is suspended from a ceiling by two identical springs as shown in figure. Let ω_1 and ω_2 be the natural frequencies of mode I and mode II respectively ($\omega_1 < \omega_2$). The value of $\frac{\omega_2}{\omega_1}$ is ____ (rounded off to one decimal place). (GATE AE 2022 QUESTION 63)

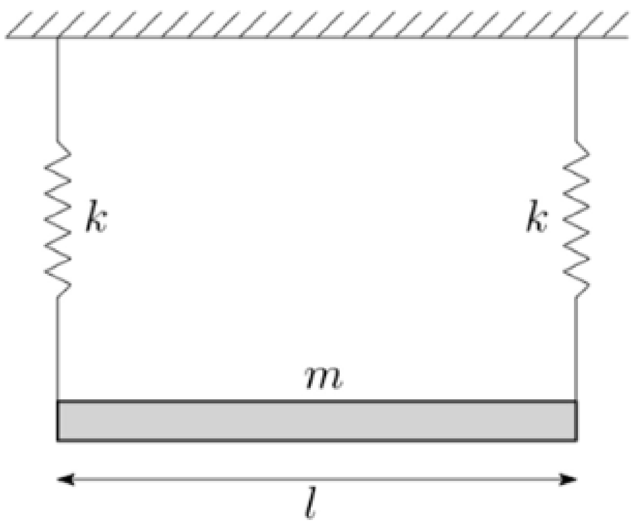

Fig. 1. Figure given in question

Solution:

Parameter	Description	Value
$X(s)$	position in laplace domain	$X(s)$
$\Theta(s)$	angle rotated in laplace domain	$\Theta(s)$
$x(t)$	position of mass w.r.t time	$x(t)$
$\theta(t)$	angle rotated by mass w.r.t time	188.5 $\theta(t)$
$\alpha(t)$	angular acceleration of mass w.r.t time	$\alpha(t)$
k	spring constant	k
m	mass of buoy	m
ω_o	initial angular velocity of mass	ω_o
$v(0)$	initial velocity of mass	$v(0)$
I	moment of inertia of rod	$\frac{mL^2}{12}$
L	length of rod	L

TABLE I
INPUT VALUES

i: For vertical oscillations: from Fig. 2,

$$m\frac{d^2x(t)}{dt^2} + 2kx(t) = 0 \tag{1}$$

Assuming the bar is at mean position and has non-zero intitial velocity, we can write it's laplace transform as:

$$s^2mX(s) - v(0) + 2kX(s) = 0 \tag{2}$$

$$\implies X(s) = \frac{\frac{v(0)}{m}}{s^2 + \frac{2k}{m}} \tag{3}$$

On taking inverse laplace transform we get,

$$x(t) = v(0)\sqrt{\frac{1}{2mk}}\sin\sqrt{\frac{2k}{m}}t \tag{4}$$

$$\therefore \omega_1 = \sqrt{\frac{2k}{m}} \tag{5}$$

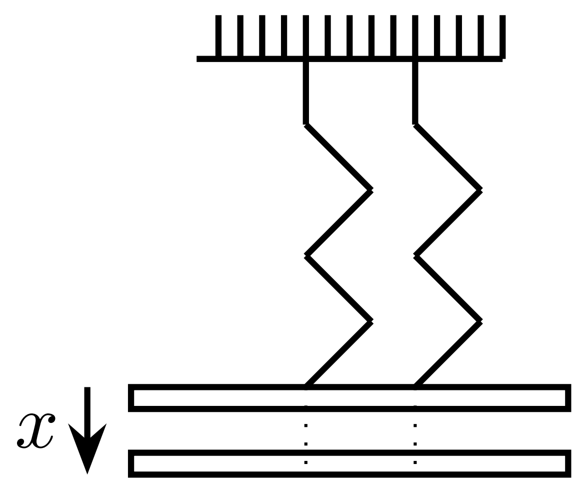

Fig. 2. Figure for Vertical strain

ii: For torsional strain from Fig. 3,:

$$I\alpha(t)(t) = k\left(x - \frac{L\theta(t)}{2}\right)\frac{L}{2} - k\left(x + \frac{L\theta(t)}{2}\right)\frac{L}{2} \tag{6}$$

$$I\alpha(t) = -\frac{kL^2\theta(t)}{2} \tag{7}$$

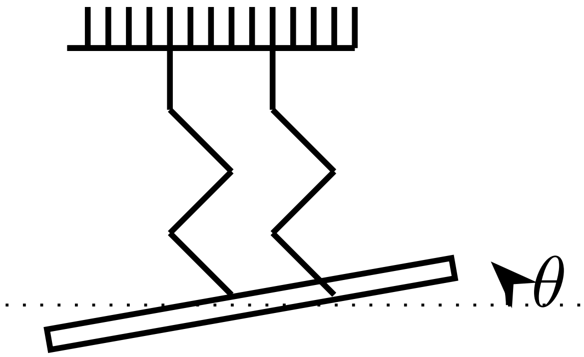

Fig. 3. Figure for Torsional strain

Assuming it is at mean position and having no zero angular velocity we can write it's laplace transform as:

$$s^2 I\Theta(s) - \omega_o + \frac{kL^2\theta(t)(s)}{2} = 0 \tag{8}$$

substituting values from Table I:

$$\Theta(s) = \frac{\frac{12\omega_o}{m}}{s^2 + \frac{6k}{m}} \tag{9}$$

On taking inverse laplace transform we get,

$$\theta(t) = \omega_o \sqrt{\frac{24}{mk}} \cos \sqrt{\frac{6k}{m}} t \tag{10}$$

$$\therefore \omega_2 = \sqrt{\frac{6k}{m}} \tag{11}$$

From (5) and (11) we see that

$$\frac{\omega_2}{\omega_1} = \sqrt{3} \tag{12}$$