

NCERT Discrete 10.5.2 -15

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Question: For what value of n , are the n th terms of two A.Ps: 63, 65, 67,... and 3, 10, 17,... equal?

Solution:

A sequence is said to be in Arithmetic Progression when it is in the form of

$$a, a + d, a + 2d, a + 3d, \dots$$

where a is first term and d is common difference.

When there are n terms, the sequence becomes

$$a, a + d, a + 2d, a + 3d, \dots, a + (n - 1)d.$$

$$T_n = a + (n - 1)d.$$

which is n th term. In the given question, there are two sequences.

$$63, 65, 67, \dots \quad (1)$$

$$3, 10, 17, \dots \quad (2)$$

for the sequence (1), let $x(n)$ be n th term,

$$a = 63$$

$$a + d = 65$$

$$d = 2$$

$$x(n) = 63 + (n - 1) \times 2$$

$$x(n) = 61 + 2n \quad (3)$$

for sequence (2), let $y(n)$ be n th term,

$$a = 3$$

$$a + d = 10$$

$$d = 7$$

$$y(n) = 3 + (n - 1) \times 7$$

$$y(n) = 7n - 4 \quad (4)$$

given, $x(n) = y(n)$

$$\therefore 61 + 2n = 7n - 4 \quad (5)$$

$$5n = 65$$

$$n = 13 \quad (6)$$

$$\text{So, } x(n) = 61 + 2 \times 13 = 87 \text{ and}$$

$$y(n) = 7 \times 13 - 4 = 87$$

\therefore 13th terms of given two APs are equal.

To find $X(z)$ and $Y(z)$ (i.e. the 'z' transforms):

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) \times z^{-n} \quad (7)$$

$$\text{and } Y(z) = \sum_{n=-\infty}^{\infty} y(n) \times z^{-n} \quad (8)$$

$$\text{i.e. } X(z) = \sum_{n=1}^{\infty} (61 + 2n) \times z^{-n}$$

$$X(z) = 61(z - 1)^{-1} + 2(2z - 1)(z - 1)^{-2} \forall |z| > 1 \quad (9)$$

$$\text{and } Y(z) = \sum_{n=1}^{\infty} (-4 + 7n) \times z^{-n}$$

$$Y(z) = -4(z - 1)^{-1} + 7(2z - 1)(z - 1)^{-2} \forall |z| > 1 \quad (10)$$