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1) Solve the following recurrence relation.

a) $x(n) = x(n-1)+5$ for $n>1$, with $x(1)=0$

Sol) write down the first two terms to identify the pattern

$x(1) = 0$

$x(2) = x(1)+5 = 5$

$x(3) = x(2)+5 = 10$

$x(4) = x(3)+5 = 15$

2) Identify the pattern (or) the general term

-> the first term $x(1)=0$

the common difference $d=5$

the general formula for the nth term of an Ap is

$x(n) = x(1) + x(n-1)d$

Substituting the given value

$x(n) = 0+(n+1)5 = 5(n-1)$

The solution is $x(n) = 5(n-1)$

b) $x(n) = 3x(n-1)$ for $n>1$ with $x(1)=4$

① write down the first two terms to identify the pattern

$x(1) = 4$

$x(2) = 3x(1) = 3\cdot 4 = 12$

$x(3) = 3x(2) = 3\cdot 6$

$x(4) = 3(x)(5) = 10.8$

2) Identify the general term $x(1)=4$

the common ratio $r=3$

The general formula of the nth term of a gp is

$x(n) = x(1)\,r^{n-1}$

substituting the given value

$$x(n) = 4 \cdot 3^{n-1}$$

the solution is $x(n) = 4 \cdot 3^{n-1}$

5) $x(n) = x(n/2) + n$ for $n > 1$ with $x(1) = 1$ (solve for $n = 2^n$)

for $n = 2^k$, we can write recurrence in term of k

① Substitute $n = 2k$ in the recurrence

$$x(2k) = x(2^{k-1}) + 2k$$

② write down the first few terms to identify the pattern

$$x(1) = 1$$
$$x(2) = x(2^1) = x(1) + 2 = 1 + 2 = 3$$
$$x(4) = x(2^2) = x(2) + 4 = 3 + 4 = 7$$
$$x(8) = x(23) = x(3) + 8 = 7 + 8 = 15$$

3) Identify the general term by finding the pattern we observe that: $x(2^k) = x(2^{k-1}) + 2^k$

we sum the series:

$$x(2^k) = 2^k + 2^{k-1} + 2^{k-2} \dots$$

since

$$x(2k) = 2^k + 2^{k-1} + 2^{k-2} \dots$$

The execept for the additional +1 term

the sum of geometric series is with ratio $r = 2$ is given by $S = \frac{ar^n - 1}{r - 1}$

$$S = 2 \cdot \frac{2^k - 1}{2 - 1} = 2(2^k - 1) = 2^{k+1}$$

adding the +1 term

$$x(2^k) = 2^{k+1} - 2 + 1 = 2^{k+1} - 1$$

solution is

$$x(2^k) = 2^{k+1} - 1$$

(4) $x(n) = x(n/3)+1$ for with $(x(1)) = 1$ (Solve for $n=34$)

for $n=3^k$ we can write the recurrence in term of k

(1) Substitute $n=3^k$ in the recurrence

(2) write down the first few terms to identify the pattern

$x(1) = 1$

$x(3) = x(3^1) = x(1)+1 = 1+1 = 2$

$x(9) = x(3^2) = x(3)+1 = 2+1 = 3$

$x(27) = x(3^3) = (x(9)+1 = 3+1 = 4$

(3) identify the general term!

we observe that.

$x(3^k) = x(3^{k-1})+1$

Summing up the series

$x(3^k) = 1+1+1\cdots1+..$

$x(3^k) = k+1$

The solution is $x(3^k) = k+1$

2 Evaluate the following recurrence complexity.

i, $T(n) = T(n/2)+1$ where $n=2k$ for all $k \geq 0$

The recurrence relation can be solved using interaction method.

1, substitute $n=2^k$ in the recursnce.

2, iterate the reccurence

for $k=0$: $T(2^0) = T(1) = T(1)$

$k=1$: $T(2^1) = T(1)+1$

$k=2$: $T(2^2) = T(4) = T(n+1) = T(n)+2)+1 = T(1)+2$

$k=3$: $T(2^3) = T(8) = T(n)+1 = T(1)+2)+1 = T(1)+3$

3, generalize the pattern

$T(2^k) = T(1)+k$

Since $n=2^k$ $k = \log_2 n$

$T(n) = T(2^k) = T(1)+\log_2 n$

(ii) Assume $T(1)$ is a constant C

$$T(n) = C + \log_2 n$$

the solution is $T(n) = O(\log n)$

(ii) $T(n) = T(n/3) + T(2n/3) + n$ (where c is constant and 'n' is input) size

The recurrence can be solved using the master's teorem for divide and conquer recurrence of the form

$$T(n) = a\,T(n/b) + f(n)$$

where $a = 2$, $b = 3$ and $f(n) = cn$

let's determine the value of $\log_b^a$

$$\log_b^a = \log_3^2$$

using the properties of logarithms

$$\log_3^2 = \frac{\log 2}{\log 3}$$

Now we compare $f(n) = cn$ with $n \log_3^2$

$$f(n) = O(n)$$

$$n = n$$

since $\log_3^2$ we are in the third case of master's therem $f(n) = O(n^c)$ with $c > \log_b a$

the solution is

$$T(n) = O(f(n)) = O(cn) = O(n)$$

3) Consider the following recurrence algorithm

```
min (A[0 ... n-2])
    if n=1 return A[0]
  else
      term = min1 [A[0. ..n-2]]
          if temp <= A[n-1] return temp
     Return A[n-1]
```

(a) what is this algorithm compute.

5) The given algorithm $min(A(0...n-1))$ computes the min value in the array 'A' from index 0 to n-1 it does this by recursively finding the min value in the sub array $A(0...$ $...n-2]$ and two comparing it with the last element $A(n-1)$ to determin the overall min value.

b) Setup a recurrence relation for the algorithm basic operation count and solve it.

The solution is $T(n) = n$

this means the algorithm perform n basic operations for an input array of size 'n'

4) Analyze the order of growth

(i) $f(n) = 2^{n+5}$ and $g(n) = 7n$ use the $\Omega(g(n))$ notation.

To analyze the order the growth and use the Ω notation, we need to compare the given function $f(n)$ and $g(n)$

given function $f(n) = 2n^2+5$

$g(n) = 7n$

Order of growth using $\Omega(g(n))$ notation.

The notation $\Omega\, g(n))$ describes a lower bound on the growth rate that for sufficiency large n, $f(n)$ grows at least as for as $g(n)$

$f(n) \geq C \cdot g(n)$

$f(n) = 2n^2+5$ with respected to $g(n) = 7n$.

2) establish the inequality.

we want to find constants c and no such that:

$2n^2+5 \geq C \cdot 7n$ for all $n \geq n$

3, Simplify the inequality.

ignore the lower order term 5 for larger

$2n^2 \geq 7cn$

Divide both side by n

$2n \geq 7c$

Solve for n:

$n \geq 7c/2$

4, Choose constant

let $c = 1$

$n \geq \frac{7 \cdot 1}{2} = 3.5$

∴ for $n \geq n$ the inequality holds:

$2n^2 + 5 \geq 7n$ for all $n \geq n$

we have shown that there exist constant $c=1$ and $n_0 = n$

Such that for all $n \geq n_0$:-

$2n^2 + 5 \geq 7n$

thus we can conclude that:

$f(n) = 2n^2 + 5 = \Omega(7n)$

in Ω notation the dominat term $2n^2$ in

f(n) clearly grows fast than f(n). Hence

$f(n) = \Omega(n^2)$

However, for the specific comparision asked $f(n) = \Omega(7n)$

is also correct

Showing that (f(n)) grows at least as fast as $7n$.