calculus practical - I

(1) $\lim_{x \to a} \left[\frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \right]$

$\lim_{x \to a} \left\{ \frac{\sqrt{a+2x} - \sqrt{3x}}{\sqrt{3a+x} - 2\sqrt{x}} \times \frac{\sqrt{a+2x} + \sqrt{3x}}{\sqrt{a+2x} + \sqrt{3x}} \times \frac{\sqrt{3a+x} + 2\sqrt{x}}{\sqrt{3a+x} + 2\sqrt{x}} \right.$

$\lim_{x \to a} \frac{(a+2x-3x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a+x-4x)(\sqrt{a+2x} + \sqrt{3x})}$

$\lim_{x \to a} \frac{(a-x)(\sqrt{3a+x} + 2\sqrt{x})}{(3a-3x)(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \lim_{x \to a} \frac{\cancel{(a-x)}(\sqrt{3a+x} + 2\sqrt{x})}{\cancel{(a-x)}(\sqrt{a+2x} + \sqrt{3x})}$

$\frac{1}{3} \frac{\sqrt{3a+a} + 2\sqrt{a}}{\sqrt{a+2a} + \sqrt{3a}}$

$\sqrt{4a} + 2\sqrt{a}$

$$\lim_{y\to 0}\left\{\frac{\sqrt{a+y}-\sqrt{a}}{y\sqrt{a+y}}\times\frac{\sqrt{a+y}+\sqrt{a}}{\sqrt{a+y}+\sqrt{a}}\right)$$

$$\lim_{y\to 0}\frac{\cancel{a}+y-\cancel{a}}{y\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$$

$$\lim_{y\to 0}\frac{\cancel{y}}{\cancel{y}\sqrt{a+y}\,(\sqrt{a+y}+\sqrt{a})}$$

$$\frac{1}{\sqrt{a+0}\,(\sqrt{a+0}+\sqrt{a})}$$

$$\frac{1}{\sqrt{a+0}\,(\sqrt{a}+\sqrt{a})}$$

$$\frac{1}{\sqrt{a}\,(2\sqrt{a})}=\frac{1}{2a}$$

$$\lim_{x\to\pi/6}\frac{\cos x-\sqrt{3}\sin x}{\pi-6x}$$

substituting $x-\pi/6=h$

$x=h+\pi/6$

$$\lim_{h\to 0}\frac{\cos(h+\pi/6)-\sqrt{3}\sin(h+\pi/6)}{\pi-6(h+\pi/6)}$$

$$\lim_{h\to 0}\frac{\cos h\cdot\cos\pi/6-\sin h(\sin\pi/6)-\sqrt{3}(\sin h\cos\pi/6+\cos h}{\pi-6\left(\frac{6h+\pi}{6}\right)}$$

$$\lim_{h\to 0}\frac{\cos h\cdot\frac{\sqrt{3}}{2}-\sin h\left(\frac{1}{2}\right)-\sqrt{3}(\sin h\sqrt{}}{\cancel{\pi}-6h\cancel{-\pi}}$$

$$\lim_{h\to 0}\frac{\cos h\,\frac{\sqrt{3}}{2}-\sin\frac{h}{2}-\sin\frac{3}{}}{-6h}$$

$$\lim_{h\to 0}\frac{\cancel{-}\sin\frac{4h}{2}}{\cancel{-}6h}=\sin$$

$$\lim_{h\to 0}\frac{\sin 4h}{3/2h}$$

$$\frac{1}{3}\lim_{h\to 0}\frac{\sin h}{h}=\frac{1}{3}$$

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-for both

$$\lim_{x\to\infty} \left\{ \frac{\sqrt{x^2+5}-\sqrt{x^2-3}}{\sqrt{x^2+3}-\sqrt{x^2+1}} \times \frac{\sqrt{x^2+5}+\sqrt{x^2-3}}{\sqrt{x^2+5}+\sqrt{x^2-3}} \times \sqrt{\ } \right.$$

$$\lim_{x\to\infty} \left[\frac{(x^2+5-x^2+3)\,(\sqrt{x^2+3}+\sqrt{x^2+1})}{(x^2+3-x^2-1)\,(\sqrt{x^2+5}+\sqrt{x^2-3})} \right]$$

$$\lim_{x\to\infty} \frac{4\cancel{8}\,(\sqrt{x^2+3}+\sqrt{x^2+1})}{1\cancel{2}\,(\sqrt{x^2+5}+\sqrt{x^2-3})}$$

$$4\lim_{x\to\infty} \frac{\sqrt{x^2\left(1+\frac{3}{x^2}\right)}+\sqrt{x^2\left(1+\frac{1}{x^2}\right)}}{\sqrt{x^2\left(1+\frac{5}{x^2}\right)}+\sqrt{x^2\left(1-\frac{3}{x^2}\right)}}$$

After Applying limit
we get,
$= \cancel{4}$

5) $f(x) = \dfrac{\sin 2x}{\sqrt{1-\cos 2x}}$, for $0 < \pi \leq \pi/2$
for
$= \dfrac{\cos x}{\pi - 2x}$, for $\pi/2 < x < \pi$ } at x:

$f(\pi/2) = \dfrac{\sin 2(\pi/2)}{\ (\pi/2)}$ $\therefore f(\pi/2) = 0$

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$\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} + \dfrac{\cos x}{\pi - 2x}$

By substituting method
$x - \pi/2 = h$
where $x = h + \pi/2$
$h \to 0$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{\pi - 2\left(h+\frac{\pi}{2}\right)}$$

$$\lim_{h\to 0} \frac{\cos(h+\pi/2)}{\pi - 2\left(\frac{2h+\pi}{2}\right)}$$

$$\lim_{h\to 0} \frac{\cos\left(h+\frac{\pi}{2}\right)}{-2h}$$ using $\cos(A+$ $\cos$

$$\lim_{h\to 0} \frac{\cos h\cdot\cos\frac{\pi}{2} - \sin h\cdot\sin\frac{\pi}{2}}{-2h}$$

$$\lim_{h\to 0} \frac{\cos h\cdot 0 - \sin h}{-2h}$$

$$\lim_{h\to 0} \frac{+\sin h}{+2h}$$

$$\frac{1}{2}\lim_{h\to 0} \frac{\sin h}{h}$$

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b $\lim_{x\to\pi/2} f(x) = \lim_{x\to\pi/2} \cdot \frac{\sin 2x}{\sqrt{1-\cos 2x}}$

$\lim_{x\to\pi/2} \frac{2\sin x \cdot \cos x}{\sqrt{2\sin^2 x}}$

$\lim_{x\to\pi/2} \frac{2}{\sqrt{2}} \frac{\sin x \cdot \cos x}{\sin x}$

$\lim_{x\to\pi/2} \frac{2}{\sqrt{2}} \cos x$

$\cos x$

$\frac{2}{\sqrt{2}} \lim_{x\to\pi/2}$

$\therefore$ L.H.L $\neq$ R.H.L

f is not continuous at $x = \pi/2$

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11. $f(x) = \frac{x^2-9}{x-3}$ $0 < x < 3$

$= x+3$ $3 \le x \le 6$

$= \frac{x^2-9}{\cancel{x+3}}$ $6 \le x < 9$

at $x=3$ and $x=6$

at $x=3$

$f(3) = \frac{x^2-9}{x-3} = 6$

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11. $\lim_{x\to3} f(x) = \lim_{x\to3^-} x+3$

$f(3) = x+3 = 3+3 = 6$

f is define at $x=3$

$\lim_{x\to3^+} f(x) = \lim_{x\to3^+} (x+3) = 6$

$\lim_{x\to3^-} f(x) = \lim_{x\to3^-} \frac{x^2-9}{x-3} = \frac{(x-3)(x+3)}{(x-3)}$

$\therefore$ L.HL = R.H.L

f is continuous at $x=3$

for $x=6$

$f(6) = \frac{x^2-9}{x+3} = \frac{36-9}{6+3} = \frac{2+3}{a} = 3$

2. $\lim_{x\to6^+} \frac{x^2-9}{x+3}$

$\lim_{x\to6^+} \frac{(x-3)(x+3)}{(x+3)}$

$\lim_{x\to6^+} (x-3) = 6-3 = 3$

~~$x \to$~~

$\lim_{x\to6^-} x+3 = 3+6 = 9$

$\therefore$ L.H.L $\neq$ R.H.L

$\therefore$ function is not continuous.

) $f(x) = \frac{1-\cos 4x}{x^2}$ $x<0$, $x=0$ at x

sol^n.

f is continuous at $x=0$.

$\lim_{x\to 0} f(x) = f(0)$

$\lim_{x\to 0} \frac{1-\cos 4x}{x^2} = K$

$\lim_{x\to 0} \frac{2\sin^2 2x}{x^2} = K$

$2\lim_{x\to 0} \frac{\sin^2 2x}{x^2} = K$

$2\lim_{x\to 0} \left(\frac{\sin 2x}{x^2}\right)^2 = K$

$2(2)^2 = K$

$K = 8$

11. $f(x) = (\sec^2 x)^{\cot^2 x}$, $x \neq 0$
$= K$, $x = 0$ $\}$ at $x = 0$

Sol^n $f(x) = (\sec^2 x)^{\cot^2 x}$

using

$\sec^2 - \tan^2 x - \sec^2 x = 1$

$\sec^2 x = 1 + \tan^2 x$

and

$\cot^2 x = \frac{1}{\tan^2 x}$

$\lim_{x\to 0} (\sec^2 x)^{\cot^2 x}$

$\lim_{x\to 0} (1+\tan^2 x)^{\frac{1}{\tan^2 x}}$

we know that

$\lim_{x\to 0} (1+px)^{\frac{1}{px}} = e$

$= e$

$K = e$

iii. $f(x) = \frac{\sqrt{3} - \tan x}{\pi - 3x}$, $x \neq \pi/3$
$= K$, $x = \pi/3$ $\}$ at $x = \frac{\pi}{3}$

$x = \frac{\pi}{3} - h$

$x = h + \frac{\pi}{3}$

where $h \to 0$

$f(\pi/3 + h) = \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\lim_{h\to 0} \frac{\sqrt{3} - \tan(\pi/3 + h)}{\pi - 3(\pi/3 + h)}$

$\lim_{h\to 0} \frac{\sqrt{3} - \frac{\tan\frac{\pi}{3} + \tanh}{1 - \tan\frac{\pi}{3}\cdot\tanh}}{-3h}$

$\lim_{h\to 0} \frac{\sqrt{3}\left(1 - \tan\frac{\pi}{3}\cdot\tanh\right) - \ldots}{1 - \tan\frac{\pi}{3}\cdot\tanh}$

$\lim_{h\to 0} \left(\sqrt{3} - \frac{\sqrt{3}\times\sqrt{3}\ \tan h}{1-\tan \pi/3 \cdot \tan h}\right) - \frac{(\sqrt{3}+\tanh)}{-3h} \lim_{h\to 0}$

$\lim_{h\to 0} \left(\sqrt{3} - \frac{3\tan h}{1-\sqrt{3}\cdot \tan h}\right) - (\sqrt{3} + \tanh)$
$-3h$

$\lim_{h\to 0} \left(\sqrt{3} - \frac{3\tan h - \sqrt{3} \tan h}{1-\sqrt{3}}\right) - \tanh)$
$-3h$

$\lim_{h\to 0} \left(\sqrt{3} - \frac{3\tan h}{1-\sqrt{3}} + \tan h\right)$
$-3h$

$\lim_{h\to 0} \frac{-4\tan h}{-3h\,(1-\sqrt{3}\tanh)}$

$\lim_{h\to 0} \frac{4\tan h}{3h\,(1-\sqrt{3}\tanh)}$

$\frac{4}{h} \lim_{h\to 0} \frac{\tanh}{h} \lim_{h\to 0} \frac{1}{(1-\sqrt{3}\tanh)}$,

$= \frac{4}{3} \frac{1}{(1-\sqrt{3}(0)}$

$= \frac{4}{3} \left(\frac{1}{1}\right) = \frac{4}{3}$

7. $f(x) = \frac{1 - \cos 3x}{x \tan x}$

$f(x) = \frac{1 - \cos x}{x \tan x}$

$\lim_{x\to 0} = \frac{2\sin^2 3/2x}{x\tan x}$

$= \frac{2\sin^2 \frac{3x}{2}}{x^2} \times x^2$

$\frac{}{x \cdot \frac{\tan x}{x^2}} \times x^2 = 2\times \frac{9}{4} = \frac{9}{2}$

$= 2 \lim_{x\to 0} \frac{\left(\frac{3}{2}\right)^2}{1}$

$\lim_{x\to 0} f(x) = \frac{9}{2} \quad 9 = f(0)$

f is not continuous at $x = 0$

Redefine function

$f(x) = \left\{ \frac{1-\cos 3x}{x \tan x} \quad \begin{matrix} x \neq 0 \\ x = 0 \end{matrix} \right\}$

Now $\lim_{x\to 0} f(x) = f(0)$

f has removable discontinuity at $x = 0$

$f(x) = \left\{ \frac{(e^{3x}-1)\sin x^{\circ}}{x^2} \quad \begin{matrix} x \neq 0 \\ x = 0 \end{matrix} \right\}$ at x

$= \pi/6$

$\lim_{x\to 0} \frac{(e^{3x}-1)\sin\left(\frac{\pi x}{180}\right)}{x^2}$

$\lim_{x\to 0} \frac{(e^{3x}-1)}{x} \sin x$ ($\lim_{x\to 0} \sin$

$\lim_{x\to 0} 3 \cdot \frac{e^{3x}-1}{3x} \lim_{x\to 0} \sin \left(\frac{\pi x}{18}\right.$

$3 \lim_{x\to 0} \frac{e^{3x}-1}{3x} \lim_{x\to 0} \sin\left(\frac{\pi}{}\right.$

$\log e \ \pi/180 = \pi/60 = f(0)$

f is continuous at $x =$

is contin...

given, f is continuous at $x = 0$

$\lim_{x \to 0} f(x) = f(0)$

$= \lim_{x \to 0} \frac{e^{x^2} - \cos x}{x^2} = f(0)$

$= \lim_{x \to 0} \frac{e^{x^2} - \cos x - 1 + 1}{x^2}$

$= \lim_{x \to 0} \frac{e^{x^2} - 1}{x^2} + \lim_{x \to 0} \frac{1 - \cos x}{x^2}$

$\log e + \lim_{x \to 0} \frac{2\sin^2 x/2}{x^2}$

$\log e + 2 \lim_{x \to 0} \left(\frac{\sin x/2}{x}\right)^2$

multiply with 2 in num and

$= 1 + 2 \times \frac{1}{4} = 8/2 = f(0)$

9. $f(x) = \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x}$ $x \neq \pi/2$

$f(0)$ is continuous at $x = \pi/2$

$\lim_{x \to \pi/2} \frac{\sqrt{2} - \sqrt{1+\sin x}}{\cos^2 x} \times \frac{\sqrt{2} + \sqrt{1+\sin x}}{\sqrt{2} + \sqrt{1+\sin x}}$

$\lim_{x \to \pi/2} \frac{2 - 1 + \sin x}{\cos^2 x (\sqrt{2} + \sqrt{1+\sin x})}$

$= \lim_{x \to \pi/2} \frac{1 + \sin x}{1 - \sin^2 x (\sqrt{2} + \sqrt{1+\sin x})}$

$\lim_{x \to \pi/2} \frac{1+\sin x}{(1-\sin x)(1+\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$\lim_{x \to \pi/2} \frac{1}{(1-\sin x)(\sqrt{2} + \sqrt{1+\sin x})}$

$= \frac{1}{2(\sqrt{2}+\sqrt{2})}$

$= \frac{1}{2(2\sqrt{2})} = \frac{1}{4\sqrt{2}}$

$= f(\pi/2) = \frac{1}{4\sqrt{2}}$ //

Practical - 02

Derivative

1) Show that the following function from $\mathbb{R}$ to $\mathbb{R}$ are differentiable

1) $\cot x$

$f(x) = \cot x$

$Df(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\cot x - \cot a}{x - a}$

$= \lim_{x \to a} \frac{\frac{1}{\tan x} - \frac{1}{\tan a}}{x - a}$

$= \lim_{x \to 0} \frac{\tan a - \tan x}{(x-a)\tan x \tan a}$

put $x - a = h$

$x = a + h$

Now $x \to a$ $h \to 0$

put $Df(h) = \lim_{h \to 0} \frac{\tan a - \tan(a+h)}{h \times \tan(a+h) \tan a}$

formula $\tan(a-a) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$

$= \lim_{h \to 0} \frac{(a-a-h) - (1+\tan a \tan a)}{h \times \tan(a+h) \tan a}$

$\tan h \times 1 + \tan a \tan$

$-1 \times 1 + \frac{\tan^2 a}{\tan^2 a}$

$= -\frac{\sec^2 a}{\tan^2 a}$

$= \frac{-\frac{1}{\cos^2 a}}{} \times \frac{\cos^2 a}{\sin^2 a}$

$= -\operatorname{cosec}^2 a$

$Df(a) = -\operatorname{cosec}^2 a$

f is differentiable $\forall a \in \mathbb{R}$.

1) $\operatorname{cosec} x$

$f(x) = \operatorname{cosec} x$

$Df(x) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$= \lim_{x \to a} \frac{\operatorname{cosec} x - \operatorname{cosec} a}{x - a}$

$= \lim_{x \to a} \frac{\frac{1}{\sin x} - \frac{1}{\sin a}}{x - a}$

$= \lim_{x \to a} \frac{\sin a - \sin x}{(x-a) \sin a \sin x}$

put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$Df(h) = \lim_{h \to 0} \frac{\sin a - \sin(a+h)}{\cancel{a}\,(a+h-a) \sin a \sin(a+h)}$

formula

$\sin C - \sin D = 2\cos\left(\frac{C+D}{2}\right) \sin($

$= \lim_{h \to 0} \frac{2\cos\left(\frac{a+a+h}{2}\right) \sin(}{h \times \sin a \sin(a+h)}$

$h/2$

$$= -\frac{1}{2} \times \frac{2\cos(2a+0)}{\sin(a+0)}$$

$$= \frac{-\cos a}{\sin^2 a} = -\cot a \operatorname{cosec} a$$

iii) $\sec x$

$f(x) = \sec x$

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x-a}$$

$$= \lim_{x \to a} \frac{\sec x - \sec a}{x-a}$$

$$= \lim_{x \to a} \frac{\cos a - \cos x}{(x-a)\cos a \cos x}$$

Put $x - a = h$

$x = a + h$

as $x \to a$, $h \to 0$

$$f'(h) = \lim_{h \to 0} \frac{\cos a - \cos(a+h)}{h \times \cos a \cos(a+h)}$$

Formula: $-2\sin\left(\frac{C+D}{2}\right)\sin\left(\frac{C-D}{2}\right)$

$$= \lim_{h \to 0} \frac{-2\sin\left(\frac{a+a+h}{2}\right)\sin\left(\frac{a-a-h}{2}\right)}{h \times \cos a \cos(a+h)}$$

$$\lim_{h \to 0} \frac{-2\sin\left(\frac{2a+h}{2}\right)\sin\left(-\frac{h}{2}\right)}{\cos a \cos(a+0) \times -\frac{h}{2}} \times -\frac{1}{2}$$

$$= -\frac{1}{2} \times \frac{-2\sin\left(\frac{2a+0}{2}\right)}{\cos a \cos(a+0)}$$

$$= -\frac{1}{2} \times \frac{-2\sin a}{\cos a \times \cos a}$$

$$= \tan a \sec a$$

2) If $f(x) = 4x+1$, $x \le 2$

$= x^2 + 5$, $x > 0$, at $x = 2$, then find function is differentiable or

solution

L.H.

$$Df(2^-) = \lim_{x \to 2^-} \frac{f(x) - f(2)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-(4 \times 2+1)}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x+1-9}{x-2}$$

$$= \lim_{x \to 2^-} \frac{4x-8}{x+2}$$

$$= \lim_{x \to 2^-} \frac{4(x-2)}{(x-2)}$$

$Df(2^-) = 4$

$$Df(2^+) = \lim_{x \to 2^+} x(2-$$

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$= \lim_{x \to 2^+} \frac{x^2-4}{x-4}$

$= \lim_{x \to 2^+} \frac{(x+2)(x-2)}{(x-2)}$

$= 2+2 = 4$

$Df(2^+) = 4$

RHD = LHD

f is differentiable at $x=2$

Q3) If $f(x) = 4x+7$, $x<3$
$= x^2+3x+1$, $x \geq 3$ at $x=3$
then find f is differentiable at

$\Rightarrow$ RHD $Df(3^+) = \lim_{x \to 3^+} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x \to 3^+} \frac{x^2+3x+1-(x^2+3x\ldots}{x-3}$

$\lim_{x \to 3^+} \frac{x^2+3x+1-19}{x-3}$

$\lim_{x \to 3^+} \frac{x^2+3x-18}{x-3}$

$\lim_{x \to 3^+} \frac{x^2+6x-3x-18}{x-3}$

$\lim_{x \to 3^+} \frac{x(x+6)-3(x+6)}{x-3}$

$\lim_{x \to 3^+} \frac{(x+6)(x-5)}{(x-3)} = 3+6$

$Df(3^+) = 9$

LHD $= Df(3^-)$

$= \lim_{x \to 3^-} \frac{f(x)-f(3)}{x-3}$

$= \lim_{x \to 3^-} \frac{4x+7-19}{x-3}$

$= \lim_{x \to 3^-} \frac{4(x-3)}{x-3}$

$Df(3^-) = 4$

RHD $\neq$ LHD

f is not differentiable at $x=3$

Q.4) If $f(x) = 8x-5$, $x \leq 2$
$= 3x^2-4x+7$, $x > 2$ at $x=2$ then
find f is differentiable or not

$\Rightarrow f(2) = 8 \times 2 - 5 = 16-5 = 11$

RHD $Df(2^+) = \lim_{x \to 2^+} \frac{f(x)-f(2)}{x-2}$

$\lim_{x \to 2^+} \frac{3x^2-4x+7-11}{x-2}$

$\lim_{x \to 2^+} \frac{3x^2-4x-4}{x-2}$

$\lim_{x \to 2^+} \frac{3x^2-6x+2x-4}{x-2}$

$\lim_{x \to 2^+} \frac{3x(x-2)+2(x-2)}{x-2}$

$\lim_{x \to 2^+} \frac{(3x+2)(x-2)}{(x-2)}$

No.	
(1)	[illegible]
(2)	[illegible]
(3)	
(4)	
(5)	
(6)	
(7)	
(8)	
(9)	
(10)	

and f is decreasing iff $f'(x) < 0$

$3x^2 - 5 < 0$

$3(x^2 - 5/3) < 0$

$3(x - \sqrt{5}/3)(x + \sqrt{5}/3) < 0$

2) $f(x) = x^2 - 4x$

$f'(x) = 2x - 4$

$f(x)$ is increasing iff $f'(x) > 0$

$2x - 4 > 0$

$2(x - 2) > 0$

$x - 2 > 0$

$x \in (2, \infty)$

and f is decreasing iff $f'(x) < 0$

$2x - 4 < 0$

$2(x - 2) < 0$

$x - 2 < 0$

$x \in (-\infty, 2)$

3) $f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

f is increasing iff $(f'(x)) > 0$

$6x^2 + 2x - 20 > 0$

$2(3x^2 + x - 10) > 0$

$3x^2 + x - 10 > 0$

$3x^2 + 6x - 5x - 10 > 0$

$3x(x+2) - 5(x+2) > 0$

$(x+2)(3x-5) > 0$

and f is decreasing iff $f'(x) < 0$

$6x^2 + 2x - 20 < 0$

$2(3x^2 + x - 10) < 0$

$3x^2 + x - 10 < 0$

$3x^2 + 6x - 5x - 10 < 0$

$3x(x+2) - 5(x+2) < 0$

$(x+2)(3x-5) < 0$

$x \in (-2, 5/3)$

-2 $\quad 5/3$

4) $f(x) = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

f is decreasing iff $f'(x) > 0$

$3(x^2 - 9) > 0$

$(x-3)(x+3) > 0$

$x \in (-\infty, -3) \cup (3, \infty)$

and f is decreasing iff $f'(x) < 0$

$3x^2 - 27 < 0$

$3(x^2 - 9) < 0$

$(x-3)(x+3) < 0$

$\therefore x \in (-3, 3)$

-3 $\quad 3$

5) $f(x) = 2x^3 - 9x^2 - 24x + 69$

$f'(x) = 6x^2 - 18x - 24$

f is decreasing iff $f'(x) > 0$

$6x^2 - 18x - 24 > 0$

$6(x^2 - 3x - 4) > 0$

$x^2 - 4x + x - 4 > 0$

$x(x-4) + 1(x-4) > 0$

$(x-4)(x+1) > 0$

$x \in (-\infty, -1) \cup (4, \infty)$

and f is ~~differe~~ decreasing iff f'(x) < 0

$6x^2 - 18x - 24 < 0$

$6(x^2 - 3x - 4) < 0$

$x^2 - 4x + x - 4 < 0$

$(x-4)(x+1) < 0$

$x \in (-1, 4)$

Q.2

1) $y = 3x^2 - 2x^3$

$f(x) = 3x^2 - 2x^3$

$f'(x) = 6x - 6x^2$

$f''(x) = 6 - 12x$

f is concave upward if f''(x) > 0

$(6 - 12x) > 0$

$12(6/12 - x) > 0$

$x - 1/2 > 0$

$x > 1/2$

$f''(x) > 0$

$x \in (1/2, \infty)$

$y = x^4 - 6x^3 + 12x^2 + 5x + 7$

$f'(x) = 4x^3 - 18x^2 + 24x + 5$

$12x^2 - 36x + 24$ f''(x) > 0

$12x^2 - 36x + 24 > 0$

$12(x^2 - 3x + 2) > 0$

$x^2 - 2x - x + 2 > 0$

$(x-2)(x-1) > 0$

3) $y = x^3 - 27x + 5$

$f'(x) = 3x^2 - 27$

$f''(x) = 6x$

f is concave upwards iff f''(x) > 0

$= 6x > 0$

$x > 0$

$x \in (0, \infty)$

4) $y = 69 - 24x - 9x^2 + 2x^3$

$f(x) = 2x^3 - 9x^2 - 24x + 64$

$f'(x) = 6x^2 - 18x - 24$

$f''(x) = 12x - 18$

f is concave upward iff

$12x - 18 > 0$

$12(x - 18/12) > 0$

$x - 3/2 > 0$ $x > 3/2$

5) $y = 2x^3 + x^2 - 20x + 4$

$f(x) = 2x^3 + x^2 - 20x + 4$

$f'(x) = 6x^2 + 2x - 20$

$f''(x) = 12x + 2$

f is concave upward

$f''(x) > 0$

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$f''(x) > 0$

$12x + 2 > 0$

~~$f'(x) = 6x^2 + 2x - 2 6$~~

~~$f''(x) = 12x + 2$~~

$f''(x) > 0$

$12x + 2 > 0$

$12(x + 2/12) > 0$

$x + 1/6 > 0$

$x < -1/6$

$f''(x) > 0$

There exist no interval.

20/01/2.20

Practical 4

048

Q.1)

1) $f(x) = x^2 + \frac{16}{x^2}$

$f'(x) = 2x - \frac{32}{x^3}$

now consider

$f'(x) = 0$

$2x - \frac{32}{x^3} = 0$

$2x = \frac{32}{x^3}$

$x^4 = \frac{32}{2}$

$x^4 = 16$

$x = \pm 2$

$f''(x) = 2 + \frac{96}{x^4}$

$f''(x) = 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

f has minimum value at $x = 2$

$\therefore f(2) = 2^2 + \frac{16}{2^2}$

$= 4 + \frac{16}{4}$

$= 4 + 4$

$= 8$

$f''(-2) = 2 + \frac{96}{-2^4}$

$= 2 + \frac{96}{16}$

$= 2 + 6$

$= 8 > 0$

$\therefore$ f has minimum value at $x = -2$

$\therefore$ Function reaches minimum value at $x = 2$, and $x = -2$

$f(x) = 3 - 5x^3 + 3x^5$

$f'(x) = -15x^2 + 15x^4$

consider,

$f'(x) = 0$

$-15x^2 + 15x^4 = 0$

$15x^4 = 15x^2$

$x^2 = 1$

$x = \pm 1$

$f''(x) = -30x + 60x^3$

$f''(1) = -30 + 60$

$= 30 > 0$

f has minimum value at $x = 1$

$\therefore f(1) = 3 - 5(1)^3 + 3(1)^5$

$= 6 - 5$

$= 1$

$f''(-1) = -30(-1) + 60(-1)^3$

$= 30 - 60$

$= -30 < 0$

$\therefore$ f has maximum value at $x = -1$

$\therefore f(-1) = 3 - 5(-1)^3 + 3(-1)^5$

$= 3 + 5 - 3$

$= 5$

$\therefore$ f has the maximum value 5 at $x = -1$ and has the minimum value 1 at $x = 1$

$f(x) = x^3 - 3x^2 + 1$

$f'(x) = 3x^2 - 6x$

consider,

$f'(x) = 0$

$3x^2 - 6x = 0$

$3x(x - 2) = 0$

$\therefore 3x = 0 \quad x - 2 = 0$

$x = 0$ or $x = 2$

$f''(x) = 6x - 6$

$f''(0) = 6(0) - 6$

$= -6 < 0$

f has maximum value at $x = 0$

$f(0) = (0)^3 - 3(0)^2 + 1$

$= 1$

$f''(2) = 6(2) - 6$

$= 12 - 6$

$= 6 > 0$

f has minimum value at $x = 2$

$f(2) = (2)^3 - 3(2)^2 + 1$

$= 8 - 3(4) + 1$

$= 9 - 12$

$= -3$

f has maximum value 1 at $x = 0$ and f has minimum value -3 at $x = 2$.

$f(x) = 2x^3 - 3x^2 - 12x + 1$

$f'(x) = 6x^2 - 6x - 12$

consider:

$f'(x) = 0$

$6x^2 - 6x - 12 = 0$

$6(x^2 - x - 2) = 0$

$x^2 - x - 2 = 0$

$x^2 + x - 2x - 2 = 0$

$x(x+1) - 2(x+1) = 0$

$(x-2)(x+1) = 0$

$x = 2$ or $x = -1$

$f''(x) = 12x - 6$

$f''(2) = 12(2) - 6$

$= 24 - 6$

$= 18 > 0$

f has minimum value of $x = 2$

$f(x) = 2(2)^3 - 5(2)^2 - 12(2) + 1$

$= 2(8) - 3(4) - 24 + 1$

$= 16 - 12 - 24 + 1$

$f''(-1) = 12(-1) - 6$

$= -12 - 6$

$= -18 < 0$

f has ~~minimum~~ maximum value at $x = -1$

$f(-1) = 2(-1)^3 - 3(-1)^2 - 12(-1) + 1$

$= -2 - 3 + 12 + 1$

$= 8$

f has maximum value 8 at ... and

f has minimum value -19 and ...

(2.2) $f(x) = x^3 - 3x^2 - 55x + 9.5$

$f'(x) = 3x^2 - 6x - 55$

By Newton's method,

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$x_1 = 0 + \frac{9.5}{55}$

$x_1 = 0.1727$

$f(x_1) = (0.1727)^3 - 3(0.1727)^2 - 55(0.1727) + 9.5$

$= 0.0051 - 0.0895 - 9.4985 + 9.5$

$= -0.0829$

$f'(x_1) = 3(0.1727)^2 - 6(0.1727) - 55$

$= 0.08925 - 1.0362 - 55$

$= -55.9467$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$= 0.1727 - \frac{0.0829}{55.9467}$

0.1712

$f(x) = (0.1712)^3 - 3(0.1712)^2$ ...

$= 0.0050 - 0.0879 -$ ...

$= 0.0011$

$f'(x) = 3(0.1712)^2 - 6(0.$ ...

$= 0.0879 - 1.0272$ ...

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 0.1712 + \frac{0.0011}{55.9393}$

$= 0.1712$

The root of the equation is 0.1712

ii) $f(x) = x^3 - 4x - 9$

$f'(x) = 3x^2 - 4$

$f(2) = 2^3 - 4(2) - 9$
$= 8 - 8 - 9$
$= -9$

$f(3) = 3^3 - 4(3) - 9$
$= 27 - 12 - 9$
$= 6.$

let $x_0 = 3$ be the intial approximation

$\therefore$ By newton method.

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$

$3 - \frac{6}{23}$

$= 2.9392$

$f(x_1) = (2.7392)^3 - 4(2.7392) - 9$
$= 20.5528 - 10.9568 - 9$
$= 0.596$

$f'(x_1) = 3(2.7392)^2 - 4$
$= 22.5096 - 4$
$= 18.5096$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$2.7392 - \frac{0.596}{18.5096}$

$= 2.7071$

$f(x_2) = (2.7071)^3 - 4(2.7071) - 9$
$19.8386 - 10.8284 - 9$
$= 0.0102$

$f'(x_2) = 3(2.7071)^2 - 4$
$= 21.9851 - 4$
$= 17.9851$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$

$= 2.7071 - \frac{0.0102}{17.9851}$

$= 2.7071 - 0.0056$

$= 2.7015.$

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No.
(1)
(2)
(3)
(4)
(5)

$f(x_3) = (2.7015)^3 - 10.8006 - 9 \cdot 4$
$= 19.788$
$= -0.0901$

$f'(x_3) = 3(2.7015)^2 - 4$
$= 21.8943 - 4$
$= 17.8943$

$x_4 = 2.7015 + \frac{0.099}{17.8943}$

$2.7015 + 0.0050$
2.7005

$f(x) = x^3 - 1.8x^2 - 10x + 17$
$f'(x) = 3x^2 - 3.6x - 10$
$f(1) = (1)^3 - 1.8(1)^2 - 10(1) + 17$
$= -1.8 - \cancel{1.0}\ 10 + 17$
$= 6.2$
$f(2) = (2)^3 - 1.8(2)^2 - 10(2) + 17$
$= 8 - 7.2 - 20 + 17$
$= -2.2$

let $x_0 = 2$ be initial approximation
By Newtons method
$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$x_0 - \frac{f(x_2)}{f'(x_2)}$

$2 - \frac{2.2}{5.5} = 2 - 0.4230$
$= 1.577$

$f(x_1) = (1.577)^3 - 1.8(1.577)^2 - 10(1.577)$
$= 3.9219 - 4.4764 - 15.77 + 17.052$
$= 0.6755$

$f'(x_1) = 3(1.577)^2 - 3.6(1.577) - 10$
$= 7.4608 - 5.6772 - 10$
$= -8.2164$

$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$

$1.577 + \frac{0.6755}{8.2164}$

$1.577 + 0.0822$
$= 1.6592$

$f(x_1) = (1.6592)^3 - 1.8(1.6692)^2 - 10(1.6592) + 1.7$
$= 4.5677 - 4.9553 - 16.592 + 17$
$= 0.0204$

$f'(x_2) = 3(1.6592)^2 - 3.6(1.6592) -$
$= 8.2588 - 5.97312 - 10$
$= -7.7143$

$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$
$= 1.6592 + \frac{0.204}{7.7143}$
$= 1.6592 + 0.0026$
$= 1.6592$

$f(x_3) = (1.6618)^3 - 1.8(1. + 17$
$= 4.5892 - 4.9708 -$
$= 0.0004$

$f'(x_3) = 3(1.6618)^2 - 3.6$

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$= \dots$

$x_4 = x_3 - \frac{f(x_3)}{f'(x_3)}$

$= 1.6618 + \frac{0.0004}{7.6477}$

$= 1.6618$

The root of the equation is

2/1/2020

Practical - 5 :

Q.1) solve the following integration.

(i) $\int \frac{dx}{\sqrt{x^2}}$

(ii) $\int (4e^{3x}+1)\,dx$

(iii) $\int (3x^2 - 3\sin x + 5\sqrt{x})\,dx$

(iv) $\int \frac{x^3+3x+4}{\sqrt{x}}$ (v) $\int t^7 \sin(2t+4)\,dt$

(vi) $\int \sqrt{x}\,(x^2-1)\,dx$ (vii) $\int \frac{1}{x^3}\sin\left(\frac{1}{x^2}\right)dx$

(viii) $\int \frac{\cos x}{\sqrt[3]{\sin^2 x}}\,dx$ (ix) $\int e^{\cos^2 x}\sin x\,dx$

(x) $\int \left(\frac{x^2-2x}{x^3-3x^2+1}\right)dx$.

solution :

(i) $\int \frac{1}{\sqrt{x^2+2x-3}}\,dx$

$\int \frac{1}{\sqrt{x^2+2x+1-4}}\,dx$

$\int \frac{1}{\sqrt{(x+1)^2-4}}\,dx$ [$\because a^2+2ab+$

Scanned with CamScanner

No. (1) (2) (3) (4) (5) 6) 7) 8)

put $x+1=t$

$dx = \frac{1}{t} \times dt$ where $t = x+1$

$= \int \frac{1}{\sqrt{t^2-4}}\, dt$

using formula

$\int \frac{1}{\sqrt{x^2-a^2}}\, dx = \ln(|x - \sqrt{x^2-a^2}|)$

$= \ln(|t + \sqrt{t^2-4}|)$

Return $t = x+1$

$= \ln(|x+1+\sqrt{(x+1)^2-4}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|)$

$= \ln(|x+1+\sqrt{x^2+2x-3}|) + C$

2) $\int 4e^{3x}+1\, dx$

$I = \int 4e^{3x}\, dx + \int 1\, dx$

$= 4\int e^{3x}\, dx + \int 1\, dx$

$= \frac{4e^{3x}}{3} + x + C$

(3) $\int (2x^2 - 3\sin x + 5\sqrt{x})\, dx$

$= \int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$= \int 2x^2 - 3\sin(x) + 5\sqrt{x}\, dx$

$= \int 2x^2\, dx - \int 3\sin(x)\, dx + \int 5x^{1/2}\, dx$

$= \frac{2x^3}{3} + 3\cos x + \frac{10x\sqrt{x}}{3} + C$

$= \frac{2x^3 + 10x\sqrt{x}}{3} + 3\cos x + C$

4) $\int \frac{x^3+3x+4}{\sqrt{x}}\, dx$

$= \int \frac{x^3+3x+4}{x^{1/2}}\, dx$

split the denominator

$= \int \frac{x^3}{x^{1/2}} + \frac{3x}{x^{1/2}} + 4/x^{1/2}\, dx$

$= \int x^{5/2} + 3x^{1/2} + 4/x^{1/2}\, dx$

$= \int x^{5/2}\, dx + \int 3x^{1/2}\, dx + \int 4/x^{1/2}\, dx$

$= \frac{x^{5/2+1}}{5/2+1}$

$= \frac{2x^3\sqrt{x}}{7} + 2x\sqrt{x} + 8\sqrt{x} + C$

5) $\int t^7 \times \sin(2t^4)\, dt$

put $u = 2t^4$

$du = 2 \times 4t^3$

$= \int t^7 \times \sin(2t^4) \times 1/2\times4t$

$= \int t^4 \times \sin(2t^4) \times 1/2\times$

$= \int t^4 \sin(2t^4) \times 1/8\, du$

substitute t^4 with $u/$

$= \int \frac{u}{2} \times \sin(x)\, du$

$= \int \frac{u \times \sin(u)}{2} / 8\, du$

$= \int \frac{u \times \sin(u)}{16}\, du$

$= 1/16 \int u \times \sin(u)\, du$

$\int u\,dv = uv - \int v\,du$

where $u = v$

$dv = \sin(u) \times du$ $v = -\cos(u)$

$du = 1du$

$= 1/16\ (u \times (-\cos(u)) - \int \cos(u)\,du$

$= 1/16 \times (u \times (-\cos u)) + \int \cos(u)\,du$

$= 1/16 \times (u \times$

$\int \cos x\,dx = \sin(x)$

$= 1/16 \times (u \times (-\cos(u)) + \sin(u))$

Return the substitute on $u = 2t^4$

$= 1/16 \times (2t^4 \times (-\cos(2t^4)) + \sin(2t^4)$

$= -\frac{t^4 \times \cos(2t^4)}{8} + \frac{\sin(2t^4)}{16} + C$

vii) $\int \sqrt{x}\,(x^2-1)\,dx$

$= \int \sqrt{x}\,x^2 - \sqrt{x}\,dx$

$= \int x^{1/2} \times x^2 - \sqrt{x}^{1/2}\,dx$

$= \int x^{5/2} - x^{1/2}\,dx$

$= \int x^{5/2}\,dx - \int x^{1/2}\,dx$

$= I_1\ \frac{5/2+1}{5/2+1} = \frac{x^{7/2}}{7/2} = \frac{2x^{7/2}}{7} = \frac{2\sqrt{x^7}}{7}$

$I_2 = \frac{x^{1/2}+1}{1/2+1} = \frac{x^{3/2}}{3/2} = \frac{2x^{3/2}}{3/2} = \frac{2\sqrt{x^3}}{3}$

$= \frac{2x^3\sqrt{x}}{7} + \frac{2\sqrt{x}}{3} + C$

$\int \frac{\cos x}{\sqrt[3]{\sin(x)^2}}\,dx$

$= \int \frac{\cos x}{\sin x^{2/3}}\,dx$

put $t = \sin(x)$

$t = \cos x$

$= \int \frac{\cos(x)}{\sin(x)^{1/2}} \times \frac{1}{\cos(x)}\,dt$

$= \frac{1}{\sin x^{3/2}}\,dt$

$= \frac{1}{t^{2/3}}\,dt$

$I = \int \frac{1}{t^{2/3}}\,dt = -\frac{1}{(2/3 - 1)\,t^{2/3}}$

$= \frac{-1}{-1/3\,t^{2/3-1}} = \frac{1}{1/3\,t^{-1/3}} = \frac{t^{1/3}}{1/3}$

Return substitution

$= 3\sqrt[3]{\sin(x)} + C$

(x) $\int \frac{x^2-2x}{x^3-3x^2+1}\,dx$

Put $x^3-3x^2+1 = dt$

$I = \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-3\times 2x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3x^2-6x}\,dt$

$= \int \frac{x^2-2x}{x^3-3x^2+1} \times \frac{1}{3(x^2-2x)}\,dt$

$= \int \frac{1}{x^3-3x^2+1} \times \frac{1}{3}\,dt$

$= \int \frac{1}{3(x^3-3x^2+1)}\,dt = \int \frac{1}{3t}\,dt$

$= \frac{1}{3} \int \frac{1}{t}\,dt \qquad \int \frac{1}{x}\,dx = \ln x$

$\neq$ $\cancel{\frac{1}{3}}$

$= \frac{1}{3} \times \ln(|x^3-3x^2+1|) + C$

AK
06/01/2020

Topic: Practical no 6
Application of integration and numerical integration

Find the length of the following curve

$x = t\sin t$ for t, $y = 1 - \cos t$ belong to $[0, 2\pi]$ $t \in [0, 2\pi]$.

Find the t

$$\int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$$

$x = t = \sin t$

$x \neq 1t$

$\frac{dx}{dt} = t - \cos t$

$y = 1 - \cos t$

$\frac{dy}{dt} = \sin t$

$$L = \int_0^{2\pi} \sqrt{(t - \cos t + \sin t)^2 + (\sin t)^2}\,dt$$

$$= \int_0^{2\pi} \sqrt{1 - \cancel{2}\times\cos t + 1}\,dt$$

$$= \int_0^{2\pi} \sqrt{2 - 2\cos t}\,dt$$

$$= \int_0^{2\pi} 2\left|\sin \frac{t}{2}\right|\,dt \qquad \sin^2 \frac{t}{2} \cdot \frac{1 - \cos t}{2}$$

$$= \int_0^{2\pi} 2\sin t/2\,dt$$

$= \left(-4\cos(t/2) \right)_0^{2\pi} = [-4(\cos\pi) - ($

$= 4+4$

$= 8$

$y = \sqrt{4-x^2} \quad x \in [-2,2]$

$2 = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dx}{dt}\right)^2}\, dt$

$= 2\int_0^2 \sqrt{\left(1 + \left(\frac{-x}{\sqrt{4-x^2}}\right)^2\right.}\, dx$

$= 2\int_0^2 \sqrt{1 + \frac{x^2}{4-x^2}}\, dx$

$= 4\int_0^2 \frac{1}{\sqrt{4-x^2}}\, dx$

$= 4\left(\sin^{-1}(x/2)\right)_0^2$

$= 2\pi$

$y = x^{3/2} \quad \text{in } [0,4]$

$f(x) = 3/2\, x^{1/2}$

$[f'(x)]^2 = 9/4\, x$

$L = \int_a^b \sqrt{1 + [f'(x)]^2}\, dx$

$= \int_0^x \sqrt{1 + \frac{9}{4}x}\, dx$

put $u = 1 + 9/4\, x$, $du = 9/4\, dx$

$L = \int_1^{1+9/4x} 4/9 \sqrt{u}\, du = \left[\frac{4}{9} \cdot 2/3\, (u^{3/2})\right]_1^{1+9/4x}$

$= 8/27 \left[(1 + 9x/4) - 1 \right]$

4) $x = 3\sin t$, $y = 3\cos t$

solution:

$dy/dx = 3\cos t$

$dy/dt = -3\sin t$

$L = \int_0^{2\pi} \sqrt{(dy/dt)^2 + (dx/dt)^2}\, dt$

$= \int_0^{2\pi} \sqrt{(3\cos t)^2 + (-3\sin t)^2}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} \sqrt{9\sin^2 t + 9\cos^2 t}\, dt$

$= \int_0^{2\pi} 3\sqrt{x}\, dt$

$= 3\int_0^{2\pi} x\, dt$

$= 3[x]_0^{2\pi}$

$= 3[x]_0^{2\pi}$

$= 3[2\pi - 0]$

$L = 6\pi$ units.

5) ...

solution:

$\frac{dx}{dy} = \frac{y^2}{2} - \frac{1}{2y^2}$

$\frac{dx}{dy} = \frac{y^4-1}{2y^2}$

$L = \int_1^2 \sqrt{1 + (\frac{dx}{dy})^2}\, dy$

$= \int_1^2 \sqrt{\frac{(y^4+1)}{(2y)^2}}\, dy$

$= \int_1^2 \frac{y^4+1}{2y^2}\, dy$

$= 1/2 \int_1^2 y^2\, dy + 1/2 \int_1^2 y^{-2}\, dy$

$= 1/2 [y^3/3 - y^{-2}/{-1}]_1^2$

$= 1/2 [8/3 - 1/2 - 1/3 + 1]$

$= 1/2 [7/3 - 1/2]$

$= 17/2$ units.

7) $\int_0^9 ex^2 dx$ with $n=4$

solution

$ex^2 dx = 16.04526$

each case the with of the sub interval the $\Delta x = \frac{2-0}{4} = 1/2$

and so the sub intervals will be $[1, 1.5]$ $[1.5, 2]$

By simpson rule

$\int^2 ex^2 dx = \frac{1/2}{3} (y_0 + 4y_1 + 2y_2 + 4y_3 + y_4)$

$\approx 1/2/3 [e^{0^2} + 4e^{(0.5)^2} + 2e^{(1)^2} + 4e^{(1.5)^2} + e^{(2)^2}]$

≈ 17.3586

2) $\int_1^4 x^0 dx\, n = 4$

$\Delta x = \frac{4-0}{4} = 1$

$\int_a^b f(x)\, dx = \frac{\Delta x}{3} [y_0 + 4y_1 + 2y_2 + 4y_3$

$= 1/3 [y(0) + 4(1)^2 + 2(2)^2 + 4(3)^2 + 1$

$= 1/3 [0 + 4(1)^2 + 2(2)^2 + 4(3$

$= 64/3$

3) $\int_0^{\pi/3} \sqrt{\sin x}\, dx$ $n = 6$

solution

$\Delta x = \frac{b-a}{n}$

$\Delta x = \frac{b-a}{n} = \frac{\pi/3 - 0}{6} = \pi/18$

x	0	$\pi/18$	$2\pi/18$	$3\pi/18$	$4\pi/18$
y	0	0.4167	0.584	0.707	0.801
	y_0	y_1	y_2	y_3	y_4

$\int_0^{\pi/3} \sqrt{\sin x}\, dx \approx \frac{\Delta x}{3} (y_0 + 4(y_1 + y_3 + \ldots) + 2(y_2 + y_4) \ldots$

$= \frac{\pi/18}{3} (0 + 4(0.4167 + 0.875) + 0.\ldots$

$\approx 2(0.584 + 0.801)) + 0.0\ldots$

≈ 0.681

Practical 7

(1) $x\frac{dy}{dx} + \frac{1}{x} y = \frac{e^x}{x}$

$P(x) = \frac{1}{x}$ $Q(x) = \frac{e^x}{x}$

$IF = e^{\int \frac{1}{x} dx}$

$= x$

$y(IF) = \int Q(x)(IF)\,dx + C$

$= \int \frac{e^x}{x} \cdot x \, dx + C$

$= \int e^x \, dx + C$

$xy = e^x + C$

(2) $e^x \frac{dy}{dx} + 2e^x y = 1$

solution : $dy/dx + \frac{2e^x}{e^x} = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = \frac{1}{e^x}$

$\frac{dy}{dx} + 2y = e^{-x}$

$P(x) = 2$ $Q(x) = e^{-x}$

$$IF = e^{\int P(x)\,dx} = e^{\int 2x}$$

$$= e^{2x}$$

$$y\,(IF) = \int Q(x)\,(IF)\,dx + c$$

$$= \int e^{x}\,dx \cdot e^{2x} + c$$

$$= e^{y}x + c$$

$$\frac{dy}{dx} = \cos \frac{\cot x}{x} - 2y$$

solution:-

$$x\frac{dy}{dx} = \frac{\cot x}{x} - 2y$$

$$\frac{dy}{dx} + \frac{2y}{x} = \cos \frac{\cot x}{x^2}$$

$$P(x) = 2(x) \quad Q(x) = \frac{\cot x}{x^2}$$

$$IF = e^{\int P(x)\,dx}$$

$$= e^{\int 2x\,dx}$$

$$y\,(IF) = \int Q(x)\,(IF)\,dx + c$$

$$= \int \frac{\cot x}{x^2} - x^2\,dx + c$$

$$= \int \cot x + c$$

$$x^2 y = \sin x + c.$$

solution

$$dy/dx + 3y/x = \frac{\sin x}{x^3}$$

$$P(x) = 3/x \quad Q(x) = \sin x / x^3$$

$$P(x) = \int 3/x\,dx$$

$$= x^3$$

$$IF = e^{\int P(x)\,dx}$$

$$= x^3$$

$$y\,(IF) = \int Q(x)\,(IF)\,dx + c$$

$$= \int \frac{\sin x}{x^3}\,x^3\,dx + c$$

$$= \int \sin x + c$$

$$x^3 y = -\cos x + c$$

5) $e^{2x}\,dy/dx + 2e^{2x}y = 2x$

solution -

$$dy/dx + 2y = \frac{2x}{e^{2x}}$$

$$P(x) = 2$$

$$Q(x) = \cancel{\frac{2x}{e}} = 2xe^{-2x}$$

$$(IF) = e^{\int P(x)\,dx}$$

$$= e^{\int 2\,dx}$$

$$= e^{2x}$$

$$y\,(IF) = \int Q(x)\,(IF)\,dx + c$$

$$= \int 2x\,e^{-2x}\,e^{2x} + c$$

$$y\,e^{2x} = \int 2x + c = x^2 + c$$

$\sec^2 x \tan y \, dx + \ldots \tan y$

(c) $\sec^2 x \tan y \, dx - \sec^2 y \tan x \, dy$

sol -

$\sec^2 x \tan y \, dx = -\frac{\sec^2 x}{\tan y} dy$

$\frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 x}{\tan y} dy$

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 x}{\tan y} dy$

$\int \frac{\sec^2 x}{\tan x} dx = -\int \frac{\sec^2 x}{\tan y} dy$

$\log|\tan x| = -\log|\tan y| + c$

$\log|\tan x + \tan y| = c$

$\tan x + \tan y = e^c$

$dy/dx = \sin^2(x - y + 1)$

Put $x - y + 1 = v$

$x - y + 1 = v$.

$1 - \frac{dv}{dx} = \frac{dy}{dx}$

$1 - \frac{dy}{dx} = \frac{dy}{dx}$

$1 - \frac{du}{dx} = \sin^2 u$

$\frac{du}{dx} = 1 - \sin^2 v$

$\frac{du}{dx} = \cos^2 x$

$\frac{du}{dx} = \cos^2 x$

$\frac{du}{\cos^2 x} = dx$

$\int \sec^2 x \, dv = \int dx$

$\tan v = x + c$ $\tan(x - y + 1) = \underline{x + c}$

(s) $\frac{dy}{dx} = \frac{2x + 3y - 1}{6x + 9y + 1}$

Put $2x + 3y = v$

$2 + 3\frac{dy}{dx} + \frac{dy}{dx}$

$\frac{dy}{dx} = \frac{1}{3}\left(\frac{du}{dx} - 2\right)$

$= \frac{1}{3}\left(\frac{du}{dx} - 2\right) = \frac{1}{3}\left(\frac{v-1}{v+}\right.$

$\frac{du}{dx} = \frac{v - 1 + 2v + 0}{v + 2}$

$= 3\frac{v + 3}{v + 2}$

$= \frac{3\cancel{v}+3}{v}$ $\quad \frac{3(v+1)}{v+2}$

$= \int \frac{v+2}{v+1} dv = 3dx$

$= \int \frac{v+1}{v} dx + \int \frac{1}{v+1}$ c

$v \log|v| = 3x + C$

$2x + 3y + \log|2x + 3y + 1| = 3x$

$3y = x - \log|2x + 3y + 1|$

Practical - 8

Topic: Euler's method

1) $\frac{dy}{dx} = y + e^{x} - 2$ $y(0) = 2$ $h = 0.5$ find $y(2) = ?$

solution-

$f(x) = y + e^{x} - 2$ $x_0 = 0$ $y(0) = 2$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	2	2.1487	2.5
1	0.5	2.5	4.2929	3.5743
2	1	3.5743	8.2021	5.7205
3	1.5	5.2705		9.8219
4	2	9.8219		

$y(2) = 9.8219$

Q.2) $dy/dx = 1 + y^2$, $y(0) = 1$, $h = 0.2$ find $y(1) = ?$

$y_0 = 0$, $y = 0$ & $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
				0.2
0	0	0.2	1	0.408
1	0.2	0.408	1.04	0.6412
2	0.4	0.6412	1.664	0.9234

Scanned with CamScanner

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Q(1)

$y_0 = 2 \quad x_0 = 1 \quad h = 0.25$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2		
1	1.25	3	5.6875	4.4218
2	1.5	4.4218	9	19.3360
3	1.75	19.3360	9.6599	
4	2	99.9966	1122.5625	299.9996

$y(2) = 299.9966$

Q.3) $dy/dx = \sqrt{xy}$

$x_0 = 1 \quad +2 \quad y(1) = 1 \quad h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	1		
1	1.2	3.6	3	

$y_0 = 1 \quad h = 0.2$

$y(1.2) = 3.6$

$y(1) = 1.2939$

Q3) $dy/dx = \sqrt{\frac{x}{y}}$ $y(0) = 1$ $h = 0.2$

find $y(1) = ?$

$x_0 = 0,\ y(0) = 1$ $h = 0.2$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	0	1	0	1
1	0.2	1	0.4472	1.089
2	0.4	1.0894	0.6054	1.2105
3	0.6	1.3513	0.7040	1.3513
4	0.8	1.5051	0.7097	1.5051
5	1			

$y(1) = 1.5051$

$dy/dx = 3x^2 + 1$ $y(1) = 2$ find $(y)(2)$ $h = 0.5$

$h = 0.25$

$y_0 = 2$ $x_0 = 1$ $h = 0.5$

n	x_n	y_n	$f(x_n, y_n)$	y_{n+1}
0	1	2	4	4
1	1.5	7.875	7.57	7.895
2				

Practical - 9

1) $\lim \dfrac{x^3 - 3y + y^2 - 1}{xy + 5}$

At $(-4, -1)$, Denominator $\neq 0$

$\therefore$ By applying limit

$= \dfrac{(-4)^3 - 3(-1) + 3(-1) + (-1)^2 - 1}{-4(-1) + 5}$

$= \dfrac{-64 + 3 + 1 - 1}{4 + 5}$

$= -\dfrac{61}{9}$

2) $\lim_{(x,y)\to(2,0)} \dfrac{(y+1)(x^2 + y^2 - 4x)}{x + 3y}$

At $(2, 0)$, Denominator $\neq 0$

$\therefore$ By applying limit

$= \dfrac{(0+1)(2^2 + 0^2 - 4(2))}{2 + 3(0)}$

$= \dfrac{1(4-8)}{2}$

$= -\dfrac{4}{2}$

$= -2$

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iii) $\lim \dfrac{x^2 - y^3}{x^2 - x^2 y^2}$

At $(1,1,1)$, Denominator 0

$\therefore \lim \dfrac{x^2 - y^2 z^2}{x^2 - x^2 y^2}$

$= \lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{(x-yz)}{x^2} \dfrac{(x+yz)}{(x-yz)}$

$= \lim\limits_{(x,y,z)\to(1,1,1)} \dfrac{x+yz}{x^2}$

On Applying limit

$= \dfrac{(1 + 1 \cdot (1))}{(1)^2}$

$= \dfrac{2}{1}$

$= 2$

Q.2) i) $f(x,y) = xye^{x^2} + y^2$

$f(x) = \dfrac{\partial}{\partial x} f((x,y))$

$= \dfrac{\partial}{\partial x} (xye^{x^2} + y^2)$

$\therefore f_x = 2xye^{x^2} + y^2$

$f_y = \dfrac{\partial}{\partial y} (f(x,y))$

$= \dfrac{\partial}{\partial y} (xy e^{x^2} + y^2)$

$= \dfrac{\partial}{\partial y}$ $(2y)$

ii) $f(x,y) = e^x \cos y$

$f(x) = \dfrac{\partial}{\partial x} (f(x,y))$

$= \dfrac{\partial}{\partial x} (e^x \cos y)$

$f_x = e^x \cos y$

$f_y = \dfrac{\partial}{\partial y} (f(x,y))$

$= \dfrac{\partial}{\partial y} (e^x \cos y)$

$f_y = -e^x \sin y$

iii) $f(x,y) = x^3y^2 - 3x^2y + y^3 + 1$

$f_x = \dfrac{\partial}{\partial x} (f(x,y))$

$= \dfrac{\partial}{\partial x} (x^3y^2 - 3x^2y + y^3 + 1)$

$f_x = 3x^2y^2 - 6xy$

$f_y = \dfrac{\partial}{\partial y} (f(x,y))$

$= \dfrac{\partial}{\partial y} (x^3y^2 - 3x^2y + y^3 + 1)$

$f_y = 2x^3y - 3x^2 + 3y^2$

Q.3) (i) $f(x,y) = \dfrac{2x}{1+y}$

$f_x = \dfrac{\partial}{\partial x} \left(\dfrac{2x}{1+y^2}\right)$

$= \dfrac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x}{(1+y^2)^2}$

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$$= \frac{2 + 2y^2 - 0 \quad (1+y^2)}{2 \quad (1+y^2)(1+y^2)}$$

$$= \frac{2}{1+y^2}$$

At $(0,0) = \frac{2}{1+0}$

$= 2$

$f_y = \frac{\partial}{\partial y}\left(\frac{2x}{1+y^2}\right)$

$$= \frac{1+y^2 \frac{\partial}{\partial x}(2x) - 2x \frac{\partial}{\partial x}(1+y^2)}{(1+y^2)^2}$$

$$= \frac{1+y^2(0) - 2x(2y)}{(1+y^2)^2}$$

At $(0,0)$

$$= \frac{-y(0)(0)}{(1+0)^2}$$

$= 0$

(1) (i) $f(x,y) = \frac{y^2 - xy}{x^2}$

$$f_x = \frac{x^2 \frac{\partial}{\partial x}(y^2 - xy) - (y^2 - xy)\frac{\partial}{\partial x}(x^2)}{(x^2)^2}$$

$$= \frac{x^2(-y) - (y^2 - xy)(2x)}{x^4}$$

$$= -x^2 y - \frac{2x}{x^4}(y^2 - xy)$$

$f_y = \frac{2y - x}{x^2}$

$f_{xx} = \frac{\partial}{\partial x}\left(-x^2 y - \frac{2x}{x^4}(y^2 - xy)\right)$

$$= \frac{x^4\left(\frac{\partial}{\partial x}(-x^2 y - 2xy^2 + 2x^2 \ldots\right.}{(x^4)^2}$$

$(-x^2y - 2xy + 2x^2y$

$$= x^4\left(\frac{-2xy - 2y^2 + 4xy}{x^6}\right) - 4x^3 (-$$

$f_{yy} = \frac{\partial}{\partial y}\left(\frac{2y - x}{x^2}\right)$

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$f_{xy} = \frac{\partial}{\partial y}\left(-\frac{x^2 y - 2xy^2 + 2x^2 y}{x^4} \right)$

$= -x^2 - \frac{4xy}{x^4} + 2x^2$

$f_{yx} = \frac{\partial}{\partial x}\left(\frac{2y - x}{x^2} \right)$

$= \frac{x^2 \frac{d}{dx}(2y - x) - (2y - x)\frac{d}{dx}(}{(x^2)^2}$

$= -x^2 - \frac{4xy}{x^4} + 2x^2$

from ③ and ④

$f_{xy} = f_{yx}$

(ii) $f(x, y)\ x^3 + 3x^2y^2 - \log(x^2+1)$

$f_x = \frac{\partial}{\partial x}(x^3 + 3x^2y^2 - \log(x^2+1))$

$= 3x^2 + 6xy^2 - \frac{2x}{x^2+1}$ $\quad f_y = \frac{\partial}{\partial y}(x^3 + 3x^2 y^2 - \log(x^2+1))$

$= 0 + 6x^2 \cdot 2y - 0$

$= 6x^2 y$

$f_{xx} = 6x + 6y^2 - \left(\frac{(x^2+1)\frac{d}{dx}(2x) - 2x\frac{d}{dx}(x^2+1)}{(x^2+1)^2} \right)$

$x \cos(xy) + \cos(xy) + e^{x+y}$

$f_{yx} = \frac{\partial}{\partial x}$ $(x \cos(xy) + \cos(xy) + e^{xy})$

$= -x^2 \sin x \,(xy)$

from (i) and (ii)

$f_{xy} \neq f_{yx}$

(ii) $f(x,y) = \sqrt{x^2+y^2}$ at $(1,1)$

$f(1,1) = \sqrt{(1)^2+(1)^2} = \sqrt{2}$

$f_x = \dfrac{1}{2\sqrt{x^2+y^2}}(2x)$ $f_y = \dfrac{1}{2\sqrt{x^2+y^2}}(2y)$

$= \dfrac{y}{\sqrt{x^2+y^2}}$

f_x at $(1,1) = \frac{1}{\sqrt{2}}$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1) + \frac{1}{\sqrt{2}}(y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x-1+y-1)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}(x+y-2)$

$= \sqrt{2} + \frac{1}{\sqrt{2}}x + \frac{1}{\sqrt{2}}y - \frac{2}{\sqrt{2}}$

$= \dfrac{x+y}{\sqrt{2}}$

(ii) $f(x,y) = 1-x+y\sin x$ at $(\pi/2, 0)$

$f(\pi/2, 0) = 1-\pi/2 - 0 = 1-\pi/2$

$f_x = -1 + y\cos x$

f_x at $(\pi/2, 0) = -1+0 = -1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 1-\pi/2 - x + \pi/2 + y$

$= 1-x+y$

(iv) $f(x,y) = \log x + \log y$ at $(1,1)$

$f(1,1) = \log(1) + \log(1) = 0$

f_x at $(1,1) = 1$

$L(x,y) = f(a,b) + f_x(a,b)(x-a) + f_y(a,b)(y-b)$

$= 0 + 1(x-1) + 1(y-1)$

$= x-1+y-1$

$= x+y-2$

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Practical - 10

Find the directional derivative of the following function at given points and in the direction of given vector

$f(x,y) = x + 2y - 3$, $a = (1,-1)$, $u = 3i - j$

$f(x,y) = y^2 - 4x + 1$, $a = (3,4)$, $u = i + 5j$

$f(x,y) = 2x + 3y$, $a = (1,2)$, $u = 3i + 4j$

Find gradient vector for the following function at point

$f(x,y) = x^y + y^x$, $a = (1,1)$

$f(x,y) = (\tan^{-1} x) \cdot y^2$, $a = (1,-1)$

$f(x,y) = xy^2 - e^{x+y+z}$, $a = (1,-1,0)$

3) Find the equation of tangent line normal to each of following

$x^2 \cos y + e^{xy} = 2$ at $(1,0)$

$x^2 + y^2 - 2x + 3y + 2 = 0$ at $(2,-2)$

4) Find the equation tangent and normal line

$x^2 - 2y^2 + 3y + xz = 7$ at $(2,1,0)$

$3xy^2 - x - y + z = -4$ at $(1,-1,2)$

5) Find the local maxima and minima following function

1) $f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$2x^4 + 3x^2y - y^2$

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1) $f(x,y) = x + 2y - 3$, $a = (1,-1)$, $u = 3i - j$

Here, $u = 3i - j$ is not a unit vector

$|u| = \sqrt{(3)^2 + (-1)^2} = \sqrt{9+1} = \sqrt{10}$

unit vector along u is $\dfrac{u}{|u|} = \dfrac{1}{\sqrt{10}}(3,-1) = \left(\dfrac{3}{\sqrt{10}}, \dfrac{-1}{\sqrt{10}}\right)$

$f(a+hu) = f(1,-1) + h\left(\dfrac{3}{\sqrt{10}}, \dfrac{-1}{\sqrt{10}}\right)$

$f(a+hu) = \left(1+\dfrac{3h}{\sqrt{10}}\right) + 2\left(-1-\dfrac{h}{\sqrt{10}}\right) - 3$

$= \left(1+\dfrac{3h}{\sqrt{10}}\right) - 2 - \dfrac{2h}{\sqrt{10}} - 3$

$= -4 + \dfrac{h}{\sqrt{10}}$

$D_u f(a) = \lim_{h \to 0} \dfrac{f(a+hu) - f(a)}{h}$

$= \lim_{h \to 0} \dfrac{-4 + h/\sqrt{10} + 4}{h}$

$= 1/\sqrt{10}$

ii) $f(x) = y^2 - 4x + 1$, $a = (3,4)$, $u = i + 5j$

Here, $u = i + 5j$ is not unit vector

$|\bar{u}| = \sqrt{(1)^2 + (5)^2} = \sqrt{26}$

unit vector along u is $\dfrac{u}{|u|} = \left(\dfrac{1}{\sqrt{26}}, \dfrac{5}{\sqrt{26}}\right)$

$f(a) = f(3,4) = (4)^2 - 4(3) + 1 \cdot 4$

$f(a+hu) = f(3,4) + h\left(\frac{1}{\sqrt{26}}, \frac{5}{\sqrt{26}}\right)$

$= \left(4 + \frac{5h}{\sqrt{26}}\right)^2 - 4\left(3 + \frac{h}{\sqrt{20}}\right) + 1$

$= 16 + \frac{25h^2}{26} + \frac{40h}{\sqrt{26}} - 12 + 1$

$= \frac{25h^2}{25} - \frac{4h}{\sqrt{26}} + \frac{40}{\sqrt{26}}h - \frac{4h}{\sqrt{26}} + 5$

$D_u f(a) = \lim_{h \to 0} \dfrac{\frac{25h^2}{26} + \frac{36h}{\sqrt{26}}}{h}$

$= h\left(\frac{25h}{26} + \frac{36}{\sqrt{26}}\right)$

$= \frac{25h}{26} + \frac{36}{\sqrt{26}}$

iii) $2x + 3y$ $\quad a = (1,2)$ $\quad u = 3i + 4j$

Here $u = 3i + 4j$ is not a unit vector

$|u| = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$

unit vector along $\frac{u}{|u|} = \frac{1}{5}(3, 4)$

$= \left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a) = f(1,2) = 2(1) + 3(2) = 8$

$f(a+hu) = f(1,2) + h\left(\frac{3}{5}, \frac{4}{5}\right)$

$f(a+hu) = 2 + \frac{6h}{5} + 6 + \frac{12h}{5}$

$= \frac{18h}{5}$

$D_u f(a) = \lim_{h \to 0} \dfrac{\frac{18h}{5} + 8 - 8}{h}$

$= \frac{18}{5}$

Q-2) a) $f(x,y) = x^4 + y^x$ $\quad a = (1,1)$

$f_x = y^x \log y + 4x^3$

$f_y = x y^{x-1}$

$Df(x,y) = (f_x, f_y)$

$= (y^x \log y + 4x^3, x y^{x-1})$

$f(1,1) = (1+0, 1)$

$f(x,y) = (\tan^{-1} x) \cdot y^2$ $\quad a$

$f_x = \frac{1}{1+x^2} \cdot y^2$

$f_y = 2y \tan^{-1} x$

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$+2(y) + d = 0$ at $(\ldots)$

$= -1(x) + 2y + d = 0$

$-x + 2y + (-2) + d = 0$

$-2 + 2 - 4 + d = 0$

$-6 + d = 0$

$d = 6$

Q.4]

(1) $x^2 - 2yz + 3y + x^2 = 7$

at $(2, 3, 0)$ $0 + 0 + 7$

$f_x = 2x$

$f_x = 2x + 2$ $= 2 + 3 + 0$

$f_y = 0 - 2 + 2 + 3$ $+ 0 + x$

$f_z = 0 - 2y$

$= -2y + x$

$(x_0, y_0, z_0) = (2, 1, 0)$ $x(0-2)$

$z_0 = 0$

$f_x (x_0, y_0, z_0) = 2(2) + 0 = 4$

$f_y (x_0, y_0, z_0) = 2(0 + 3) = 3$

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$= -2(1) + 2 = 0$

$f_z = (x_0, y_0, z_0) =$

Equation of tangent

$f_x (x - x_0) + f_y (y - y_0) + f_z (z - z_0) = 0$

$4(x - 2) + 3(y - 1) + 0 (z - 0) = 0$

$= 4x - 8 + 3y - 3$

$= 4x + 3y - 11 = 0$ → This is required tangent

at Equation of normal at $(4, 3, 1)$

$\frac{x - x_0}{f_x} = \frac{y - y_0}{f_y} = \frac{z - z_0}{f_z}$

$= \frac{x - 2}{4} = \frac{y - 1}{3} = \frac{z + 1}{0}$

(ii) $3xy^2 - x - y + z = -4$

$3xy^2 - x - y + z + 4 = 0$

$f_x = 3y^2 - 1 - 0 + 0 + 0$

$= 3y^2 - 1 - 0 - 0 + 0$

$f_y = 3x^2 \ldots$

$= 3x^2 - 1$

$f_z = 3xy - 0 - 0 + 1 + 0$

$= 3xy + 1$

$f_x (x_0, y_0, z_0) = 3(-1)(2) - 1 = -7$

$f_y (x_0, y_0, z_0) = 3(+1) \ldots (2) - 1$

$f_z (x_0, y_0, z_0) = 3(1)(-1) \ldots$

(iii) $f(x, y) = 2x^4 + 3x^2 y - y^2$

$f_x = 8x^3 + 6xy$

$f_y = 3x^2 - 2y$

$f_x = 0$

$8x^3 + 6xy = 0$

$2x(4x^2 + 3y) = 0$

$4x^2 + 3y = 0$

$f_y = 0$

$\therefore 3x^2 - 2y = 0$ with

multiply equation ①

$12x^2 + 9y = 0$

$12x^2 - 8y = 0$

$y = 0$

sub value of y in eq ①

$4x^2 + 3(0) = 0$

$4x^2 = 0$

$x = 0$

critical point $(0, 0)$

$r = f_{xx} = 24x^2 + 6x = 6(0) = 0$

$s = f_{xy} = 6x - 0 = 6x$

$t = f_{yy} = -2$

at $(0, 0)$

$24(0)^2 + 6(0)$

$= 0$

$r = 0$

$rt - s^2 = 0(-2) - 0^2$

$= 0 - 0$

$= 0$

$r = 0 \quad rt - s^2 = 0$

$f(x, y)$ at $(0, 0)$

$(0)^4 + 3(0)^2(0) - (0)^2$

$0 + 0 - 0$

$= 0$

(ii) $f(x, y) = x^2 - y^2 + 2x + 8y - 70$

$f_x = 2x + 2$

$f_y = -2y + 8$

$f_x = 0$

$f_y = 0$

critical point is $(-1, 4)$

$r = f_{xx} = 2$

$s = f_{xy} = 0$

$r > 0$

equation of tangent

$(x+1)+7 \quad (y+1)-2(\ldots)$

$= -7x + 5y \ldots + 16 = 0$

$= -7x$

This is required equation

Tangent and normal equation at $(5,-2)$

$(-7, \ldots)$

$x \frac{-6}{50} = y - \frac{y_0}{7y}$

$= \frac{x-1}{-9} = \frac{y+1}{5} = \frac{2-2}{-2}$

(Q.7) (1)

$f(x,y) = 3x^2 + y^2 - 3xy + 6x - 4y$

$f_x = 6x + 0 - 3y + 6 = 6x - 3y + 6$

$f_y = 0 + 2y - 3x + 0 = 2y - 3x - 4$

$f_x = 6$

$6x - 3y + 6 = 6$

$3(2x - y + 2) = 6$

$2x - y + 2 = 0$

$-y = -2$

$2y - 3x - 4 = 0$ ... (2)

multiply eq (1) by 2

$4x - 2y = -4$

$-3x + 2y = 4$

$x = 0$

subst. value of x in (1)

$2(0) - y = -2$

$y = 2$

critical point $(0,2)$

$r = f_{xx} = 6$

$t = f_{yy} = 2$

$s = f_{xy} = -3$

Here, $r > 0$

$rt - s^2$

$= 6(2) - (-3)^2$

$= 12 - 9$

$= 3 > 0$

is max at $(0,2)$

$f = 3x^2 + y^2 - 3xy + 6x - 4y$

at $(0,2)$

$3(0)^2 + (2)^2 - 3(0)(2)$

$xt - y^2 = 2(-2) - (0)^2$

$= -4 - 0$

$= -4 < 0$

$f(x,y)$ at $(-1,4)$

$= (-1)^2 - (4)^2 + 2(-1) + 8(4)$

$= 1 + 16 - 2 + 32 - 70$

$= 17 + 30 - 70$

$= 37 - 70$

$= -33$

AB

2 Marks