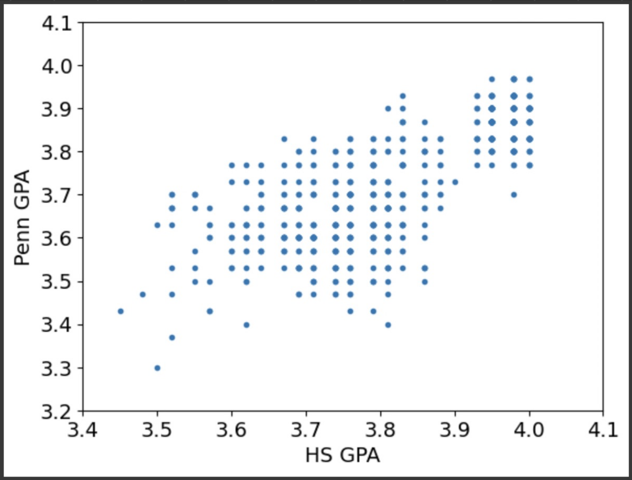
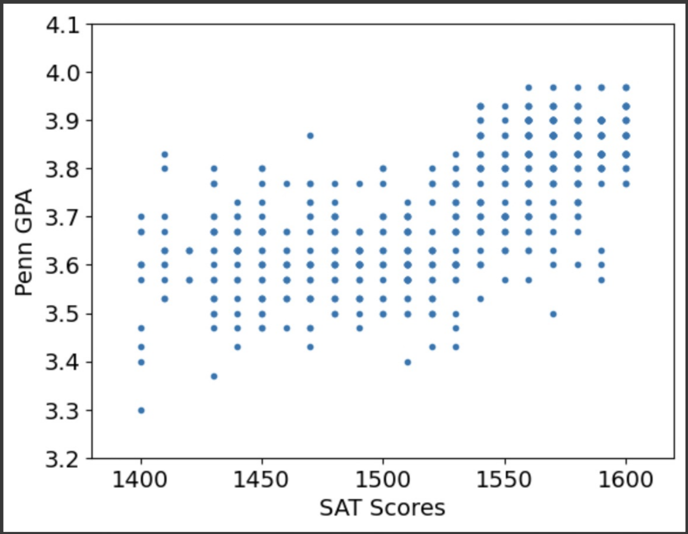
Lab 1 A

Task 1:

①



②



Task 2:

(1)

$$\alpha^* = \underset{\alpha}{\operatorname{argmin}} \frac{1}{2} MSE(\alpha)$$

$$\alpha^* = \underset{\alpha}{\operatorname{argmin}} \frac{1}{2N} \sum_{i=1}^{N} (y_i - \alpha x_i)^2$$

$$= \underset{\alpha}{\operatorname{argmin}} \frac{1}{2N} \sum_{i=1}^{N} y_i^2 - 2y\alpha x_i + \alpha x_i^2$$

$$= \frac{d}{d\alpha}\left(\frac{1}{2N} \sum_{i=1}^{N} y_i^2 - 2y\alpha x_i + \alpha^2 x_i^2 \right)$$

$$= \frac{1}{2N} \sum_{i=1}^{N} 2x_i^2\alpha - 2yx_i$$

$$= \frac{1}{2N}\left(\sum_{i=1}^{N} 2x_i^2\alpha - \sum_{i=1}^{N} 2yx_i \right) = 0$$

$$= \frac{1}{\cancel{2N}} \left(\cancel{2}\alpha\cancel{N} \sum_{i=1}^{N} x_i^2 - \cancel{2N} \sum_{i=1}^{N} yx_i \right) = 0$$

$$= \alpha \sum_{i=1}^{N} x_i^2 - \sum_{i=1}^{N} yx_i = 0$$

$$\boxed{\alpha^* = \frac{\sum_{i=1}^{N} yx_i}{\sum_{i=1}^{N} x_i^2}} \checkmark$$

(2)

```
numerator = torch.dot(hs_gpa, penn_gpa)
denominator = torch.dot(hs_gpa, hs_gpa)
alpha = numerator/denominator
alpha
```

```
tensor(0.9753)
```

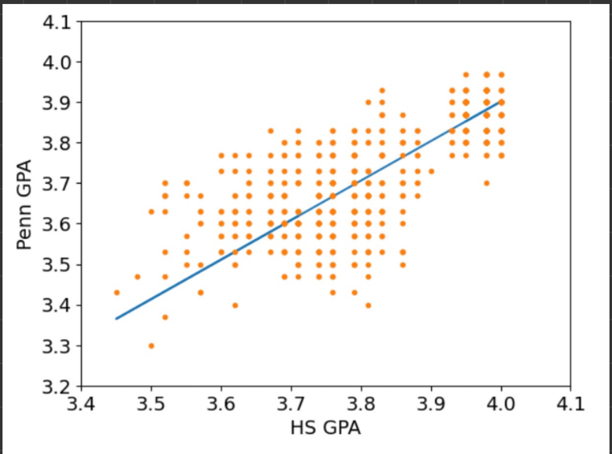
Task 3:

①

```
diff = penn_gpa - alpha * hs_gpa
rmse = math.sqrt(torch.dot(diff, diff).item()/penn_gpa.size(dim=0))
rmse
```

0.09352524560916761

②




```
# The value of RMSE is 0.09352524560916761, which seems to be relatively accurate, thus
# from what we can observe from the graph, the range of the GPA values is large (3.3 to 4) which implies that the value of RMSE which is close to 0.1,
# is not very accurate
```

Task 4:

① $$\alpha^* = \underset{\alpha}{\operatorname{argmin}} \frac{1}{2} MSE(\alpha)$$

$$\alpha^* = \underset{\alpha}{\operatorname{argmin}} \frac{1}{2N} \sum_{i=1}^{N} (y_i - \hat{y})^2$$

$$\hat{y} = w_1 x_1 + w_2 x_2 = w^T x$$

$$w^* = \underset{w}{\operatorname{argmin}} \frac{1}{2N} \sum_{i=1}^{N} (y_i - w^T x_i)^2$$

derive wrt to w & set = 0

$$= \frac{d}{dw}\left(\frac{1}{2N} \sum_{i=1}^{N} (y_i^2 - 2 y_i w^T x_i + w^{T^2} x_i^2) \right) = 0$$

$$= \frac{1}{2N} \sum_{i=1}^{N} -2 y_i x_i + 2 w^T x_i^2 = 0$$

$$= \frac{1}{\cancel{2N}} \left(\cancel{-2N} \sum_{i=1}^{N} y_i x_i + \cancel{2N} w^T \sum_{i=1}^{N} x_i^2 \right) = 0$$

$$= -\sum_{i=1}^{N} y_i x_i + w^T \sum_{i=1}^{N} x_i^2 = 0$$

$$= w^T \sum_{i=1}^{N} x_i^2 = \sum_{i=1}^{N} y_i x_i$$

$$\boxed{w^* = \frac{\sum_{i=1}^{N} x_i y_i}{\sum_{i=1}^{N} x_i x^T}}$$

②

Tensor with shape torch.Size([2, 1])

```
w = torch.mm(torch.linalg.inv(torch.mm(hs_sat_input.T, hs_sat_input)), torch.mm(hs_sat_input.T,penn_gpa.reshape(600,1)))
w
```

```
tensor([[0.6332],
        [0.0009]])
```

③

```python
diff = (penn_gpa - torch.mm(w.T, hs_sat_input.T)).resize(600)
rmse = math.sqrt(torch.dot(diff, diff).item()/penn_gpa.size(dim=0))
rmse
```

```
0.08760880007917757
```

④

```python
# The value of RMS, which is 0.08760880007917757, is still not very accurate, as it is high
# but it is relatively better compared to the last value of RMSE that we found
```