

# Lab 6

CS 418: Interactive Computer Graphics

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# Fog

Fog is an example of participating media  
these are typically some sort of particulate  
scatterers and otherwise changes light

there are a number of sophisticated ways to render such phenomena  
we'll use a simple one

for something complex, check out

<http://www.gdcvault.com/play/1023519/Fast-Flexible-Physically-Based-Volumetric>

# Distance Fog

Just mix a fragment color with a fog color

The farther the fragment from the viewer, more fog color

...and less original color



# Fog in WebGL and GLSL

- We implement fog in the fragment shader
- Compute the distance from fragment to camera

```
float fogCoord = (gl_FragCoord.z/gl_FragCoord.w);
```

- And define a fog color
  - White is a good choice...especially with a white background

```
vec4 fogColor = vec4(1.0, 1.0, 1.0, 1.0);
```

# Mixing in Fog

Use linear interpolation to mix fog with the fragment color

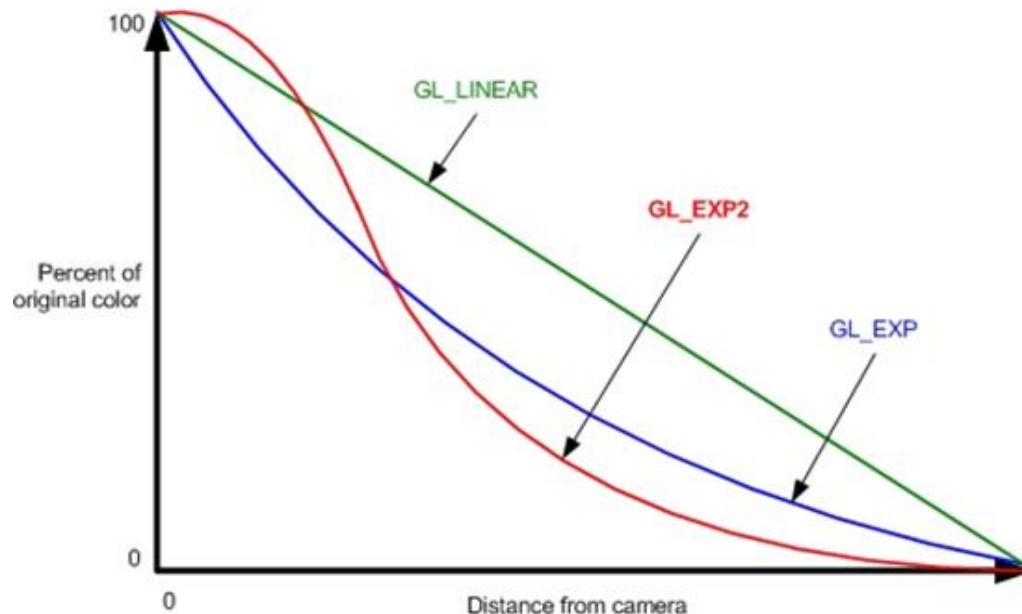
```
gl_FragColor = mix(fogColor, fragColor, fogFactor );
```

- `fragColor` is the color computed by your shading equation
  - Probably Blinn-Phong
- The `mix` function is built-in GLSL function that performs lerp
- But what is `fogFactor`?

# Fog Factor

```
const float LOG2 = 1.442695;  
float fogDensity = 0.0005  
float fogFactor = exp2( -fogDensity * fogDensity * fogCoord * fogCoord * LOG2 );  
fogFactor = clamp(fogFactor, 0.0, 1.0);
```

- Fog factor determines how much of the color is fog
  - A value of 1 means all fog in this case...a value of 0 means no fog
- You can see the curve below...we're computing GL\_EXP2

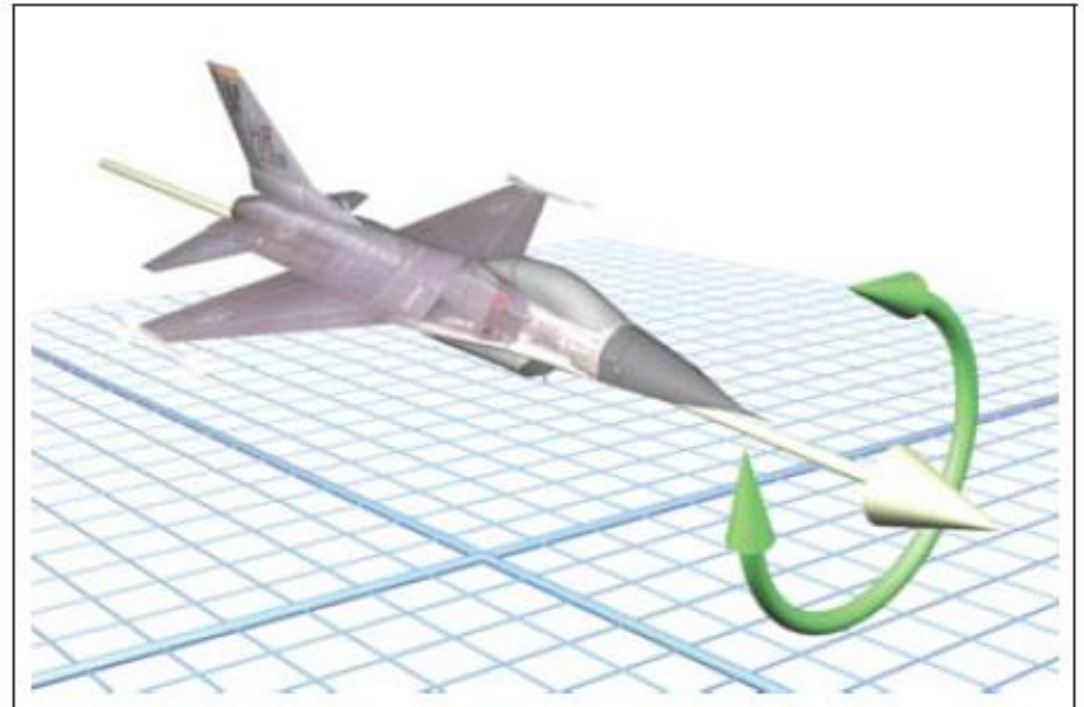
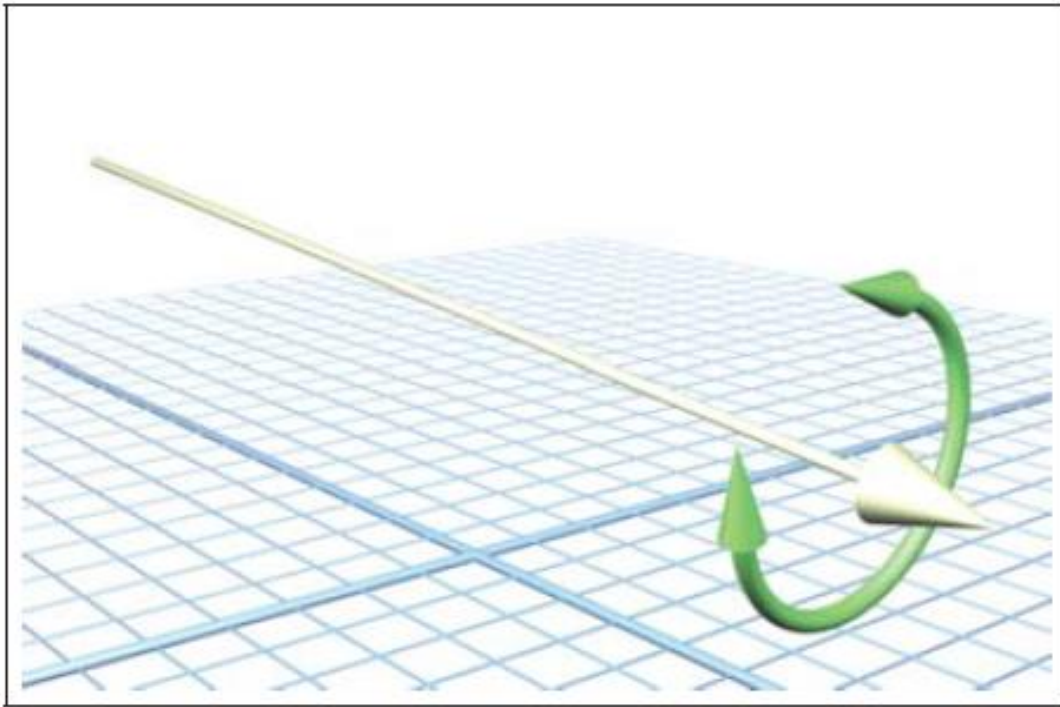


# Fog Factor Debugging

- In case you have trouble, try just implementing the linear curve for the fogFactor...see if that works.
- Can also render your fogCoord values
  - Compute  $z = \text{fogCoord} / \text{farClipDistance}$ ;
  - Set `gl_FragColor` to `(z,z,z, 1.0)`
  - See if the image makes sense



# Representing Orientations



- Rotating a vector around itself does not change it
- Rotating an object around its principal direction changes its orientation
- Orientations require more information to represent than a vector



# Representing Orientations

No simple means of representing a 3D orientation

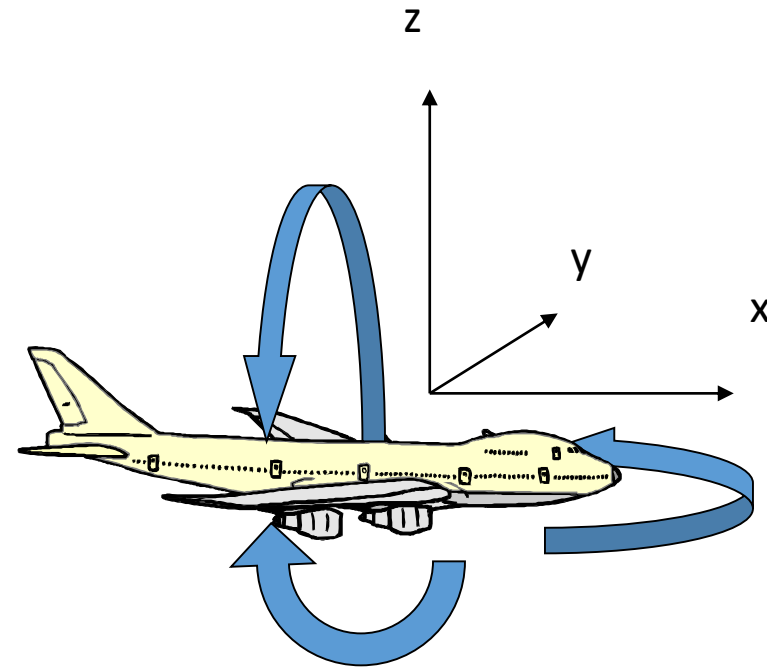
- Unlike position and Cartesian coordinates

There are several popular options:

- Euler angles
- Rotation vectors (axis/angle)
- 3x3 matrices
- Quaternions

# Euler Angles

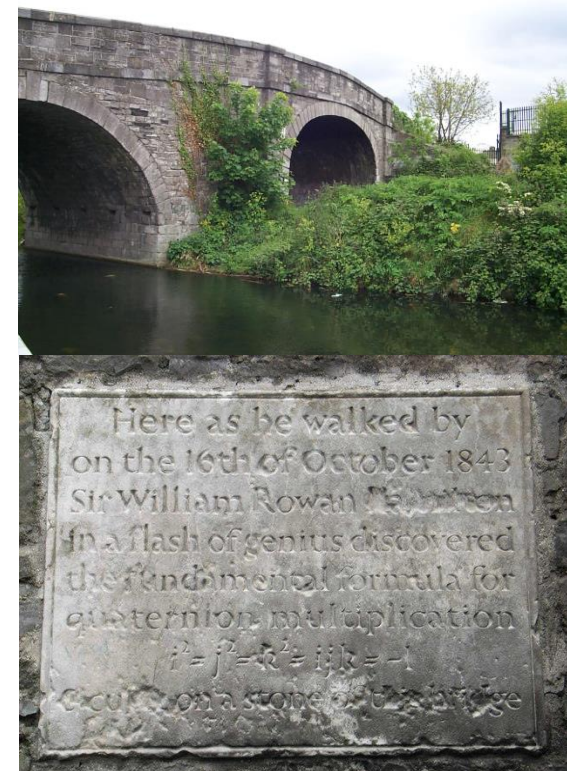
- Airplane orientation
  - Roll
    - rotation about x
    - Turn wheel
  - Pitch
    - rotation about y
    - Push/pull wheel
  - Yaw
    - rotation about z
    - Rudder (foot pedals)
- Airplane orientation
  - $R_x(\text{roll})$   $R_y(\text{pitch})$   $R_z(\text{yaw})$



# Quaternions

- Developed by Sir William Rowan Hamilton [1843]
- Quaternions are 4-D numbers
- With one real axis
- And three imaginary axes: **i,j,k**
- Imaginary-style multiplication rules

$$\mathbf{q} = q_0 + iq_1 + jq_2 + kq_3$$



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# Quaternions

- Introduced to Computer Graphics by Shoemake [1985]
- Given an angle and axis, easy to convert to and from quaternion
  - Euler angle conversion to and from arbitrary axis and angle difficult
- Quaternions allow stable and constant interpolation of orientations
  - Cannot be done easily with Euler angles

# Unit Quaternions

- For convenience, we will use only unit length quaternions

$$|\mathbf{q}| = \sqrt{q_0^2 + q_1^2 + q_2^2 + q_3^2} = 1$$

- These correspond to the set of 4D vectors
- They form the ‘surface’ of a 4D hypersphere of radius 1

# Quaternions as Rotations

- A quaternion can represent a rotation by angle  $\theta$  around a unit vector **a**:

$$\mathbf{q} = \begin{bmatrix} \cos \frac{\theta}{2} & a_x \sin \frac{\theta}{2} & a_y \sin \frac{\theta}{2} & a_z \sin \frac{\theta}{2} \end{bmatrix}$$

*or*

$$\mathbf{q} = \left\langle \cos \frac{\theta}{2}, \mathbf{a} \sin \frac{\theta}{2} \right\rangle$$

- If **a** is unit length, then **q** will be also



# Rotation using Quaternions

- Let  $q = \cos(\theta/2) + \sin(\theta/2) \mathbf{u}$  be a unit quaternion:  $|q| = |\mathbf{u}| = 1$
- Let point  $\mathbf{p} = (x,y,z) = x \mathbf{i} + y \mathbf{j} + z \mathbf{k}$
- Then the product  $q \mathbf{p} q^{-1}$  rotates the point  $\mathbf{p}$  about axis  $\mathbf{u}$  by angle  $\theta$
- Inverse of a unit quaternion is its **conjugate**

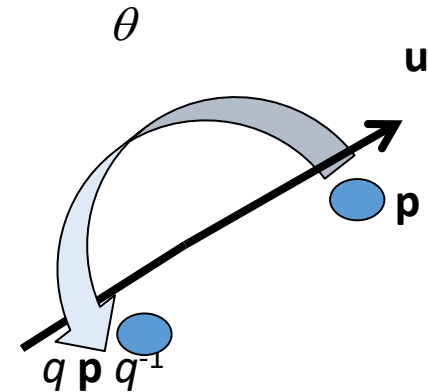
(negate the imaginary part)

$$\begin{aligned} q^{-1} &= (\cos(\theta/2) + \sin(\theta/2) \mathbf{u})^{-1} \\ &= \cos(-\theta/2) + \sin(-\theta/2) \mathbf{u} \\ &= \cos(\theta/2) - \sin(\theta/2) \mathbf{u} \end{aligned}$$

- Composition of rotations  $q_{12} = q_1 q_2 \neq q_2 q_1$

We haven't talked about how to multiply quaternions yet, but don't worry about that for now...

$$q = \cos \frac{\theta}{2} + \mathbf{u} \sin \frac{\theta}{2}$$



# Quaternion to Matrix

Again, why do we want to be able to do this?

- To convert a quaternion to a rotation matrix:

$$\begin{bmatrix} 1 - 2q_2^2 - 2q_3^2 & 2q_1q_2 + 2q_0q_3 & 2q_1q_3 - 2q_0q_2 \\ 2q_1q_2 - 2q_0q_3 & 1 - 2q_1^2 - 2q_3^2 & 2q_2q_3 + 2q_0q_1 \\ 2q_1q_3 + 2q_0q_2 & 2q_2q_3 - 2q_0q_1 & 1 - 2q_1^2 - 2q_2^2 \end{bmatrix}$$

# Quaternion Multiplication

- We can perform multiplication on quaternions
  - we expand them into their complex number form

$$\mathbf{q} = q_0 + iq_1 + jq_2 + kq_3$$

- If  $\mathbf{q}$  represents a rotation and  $\mathbf{q}'$  represents a rotation,  $\mathbf{q}\mathbf{q}'$  represents  $\mathbf{q}$  rotated by  $\mathbf{q}'$
- This follows very similar rules as matrix multiplication (i.e., non-commutative)

$$\begin{aligned} \mathbf{q}\mathbf{q}' &= (q_0 + iq_1 + jq_2 + kq_3)(q'_0 + iq'_1 + jq'_2 + kq'_3) \\ &= \langle ss' - \mathbf{v} \times \mathbf{v}', s\mathbf{v}' + s'\mathbf{v} + \mathbf{v} \wedge \mathbf{v}' \rangle \end{aligned}$$

# Quaternion Multiplication

- Two unit quaternions multiplied together results in another unit quaternion
- This corresponds to the same property of complex numbers
- Remember(?) multiplication by complex numbers is like a rotation in the complex plane
- Quaternions extend the planar rotations of complex numbers to 3D rotations in space

# Flight: Orientation

- Lots of ways to implement flight...
- MP requires the use of quaternions
- One option to change orientation:
  - Set up an initial view using `mat4.lookat`
  - Keep a quaternion that records current orientation
  - Each frame:
    - Capture key presses as Euler angles
    - Construct a temporary quaternion based on the Euler angles.
      - **quat.fromEuler in glmatrix library...get current release!**
    - Update the orientation quaternion using the temp
      - How?
    - Update the view matrix using the orientation quaternion
      - How?

# Flight: Moving Forward

- Keep a user set speed factor
  - User can adjust
- Move in the direction of the current orientation
  - By  $\text{speedFactor} * \text{directionVector}$
  - You'll need to experiment to find appropriate speedFactor
- What's the current directionVector?
  - Think about how you could compute it....