The Vector problems

Coding the Matrix, 2015

For auto-graded problems, edit the file The_Vector_problems.py to include your solution.

Vector Addition Practice

Problem 1: For vectors $\boldsymbol{v} = [-1, 3]$ and $\boldsymbol{u} = [0, 4]$, find the vectors $\boldsymbol{v} + \boldsymbol{u}$, $\boldsymbol{v} - \boldsymbol{u}$, and $3\boldsymbol{v} - 2\boldsymbol{u}$.

Problem 2: Given the vectors $\boldsymbol{v} = [2, -1, 5]$ and $\boldsymbol{u} = [-1, 1, 1]$, find the vectors $\boldsymbol{v} + \boldsymbol{u}$, $\boldsymbol{v} - \boldsymbol{u}$, $2\boldsymbol{v} - \boldsymbol{u}$, and $\boldsymbol{v} + 2\boldsymbol{u}$.

Problem 3: For the vectors $\boldsymbol{v} = [0, one, one]$ and $\boldsymbol{u} = [one, one, one]$ over $GF(2)$, find $\boldsymbol{v}+\boldsymbol{u}$ and $\boldsymbol{v}+\boldsymbol{u}+\boldsymbol{u}$.

Expressing one $GF(2)$ vector as a sum of others

Problem 4: Here are six 7-vectors over $GF(2)$:

$\mathbf{a} =$	1100000	$\mathbf{d} =$	0001100
$\mathbf{b} =$	0110000	$\mathbf{e} =$	0000110
$\mathbf{c} =$	0011000	$\mathbf{f} =$	0000011

For each of the following vectors $\boldsymbol{u}$, find a subset of the above vectors whose sum is $\boldsymbol{u}$, or report that no such subset exists. You should be able to do this without the help of a computer.

1. $\boldsymbol{u} = 0010010$
2. $\boldsymbol{u} = 0100010$

Problem 5: Here are six 7-vectors over $GF(2)$:

$\mathbf{a} =$	1110000	$\mathbf{d} =$	0001110
$\mathbf{b} =$	0111000	$\mathbf{e} =$	0000111
$\mathbf{c} =$	0011100	$\mathbf{f} =$	0000011

For each of the following vectors $\boldsymbol{u}$, find a subset of the above vectors whose sum is $\boldsymbol{u}$, or report that no such subset exists.

1. $\boldsymbol{u} = 0010010$
2. $\boldsymbol{u} = 0100010$

Problem 6: (You should be able to solve this problem without using a computer.) Find a vector $\boldsymbol{x} = [x_1, x_2, x_3, x_4]$ over $GF(2)$ satisfying the following linear equations:

$$\begin{aligned} 1100 \cdot \boldsymbol{x} &= 1 \\ 1010 \cdot \boldsymbol{x} &= 1 \\ 1111 \cdot \boldsymbol{x} &= 1 \end{aligned}$$

Verify for yourself that $\boldsymbol{x} + 1111$ also satisfies the equations.

Problem 7: Consider the equations

$$\begin{array}{ccccccccc} 2x_0 & + & 3x_1 & - & 4x_2 & + & x_3 & = & 10 \\ x_0 & - & 5x_1 & + & 2x_2 & + & 0x_3 & = & 35 \\ 4x_0 & + & x_1 & - & x_2 & - & x_3 & = & 8 \end{array}$$

Your job is not to solve these equations but to formulate them using dot-product. In particular, come up with three vectors `v1`, `v2`, and `v3` represented as lists so that the above equations are equivalent to

$$\begin{aligned} \texttt{v1} \cdot \boldsymbol{x} &= 10 \\ \texttt{v2} \cdot \boldsymbol{x} &= 35 \\ \texttt{v3} \cdot \boldsymbol{x} &= 8 \end{aligned}$$

where x is a 4-vector over $\mathbb{R}$.

Practice with Dot-Product

Problem 8: For each of the following pairs of vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ over $\mathbb{R}$, evaluate the expression $\boldsymbol{u} \cdot \boldsymbol{v}$:

(a) $\boldsymbol{u} = [1, 0], \boldsymbol{v} = [5, 4321]$

(b) $\boldsymbol{u} = [0, 1], \boldsymbol{v} = [12345, 6]$

(c) $\boldsymbol{u} = [-1, 3], \boldsymbol{v} = [5, 7]$

(d) $\boldsymbol{u} = [-\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}], \boldsymbol{v} = [\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}]$