`Filter_Clusters` runtime

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Note that:

- $\Lambda(T)$ = leaf set of T
- $|\Lambda(A)| = |\Lambda(B)| = n$
- $\sigma(\pi)$ = set of side trees of A (after centroid decomposition)
- $T|C$ is the tree constructed by restricting T to the leaf set C
- $T||C$ is the tree constructed from $T|C$ by adding "special nodes"

From the FDCT paper, runtime of `Filter_Clusters`(A, B) can be written as:

$$T(\texttt{Filter\_Clusters}(A, B)) = O(n\ logn)\ +\sum_{\tau\in\sigma(\pi)} T(\texttt{Filter\_Clusters}(\tau, B||\Lambda(\tau))) \tag{1}$$

Original analysis:

Each τ is such that $|\Lambda(\tau)| \leq n/2$, thus there are $log(n)$ recursion levels, and so total runtime $= O(n\ log^2 n)$

Altered analysis:

$$\sum_{\tau\in\sigma(\pi)} |\Lambda(\tau)| = n - 1$$

Also, $|B||\Lambda(\tau)| = O(|\Lambda(\tau)|)$ [since $B||\Lambda(\tau)$ is constructed by adding at most one special node per node in $B|\Lambda(\tau)$]

Then, size of subproblem $\texttt{Filter\_Clusters}(\tau, B||\Lambda(\tau)) = O(\Lambda(\tau))$

Thus

$$T(\texttt{Filter\_Clusters}(A, B)) = T(n) = O(n\ logn)\ +\sum_{\tau\in\sigma(\pi)} T(|\Lambda(\tau)|) \tag{2}$$

Given the result that if $\sum m = n$ then $\sum m\ logm = O(n\ logn)$ and inductively assuming that $T(i) = O(i\ logi)$

$$T(n) = O(n\ logn) \tag{3}$$