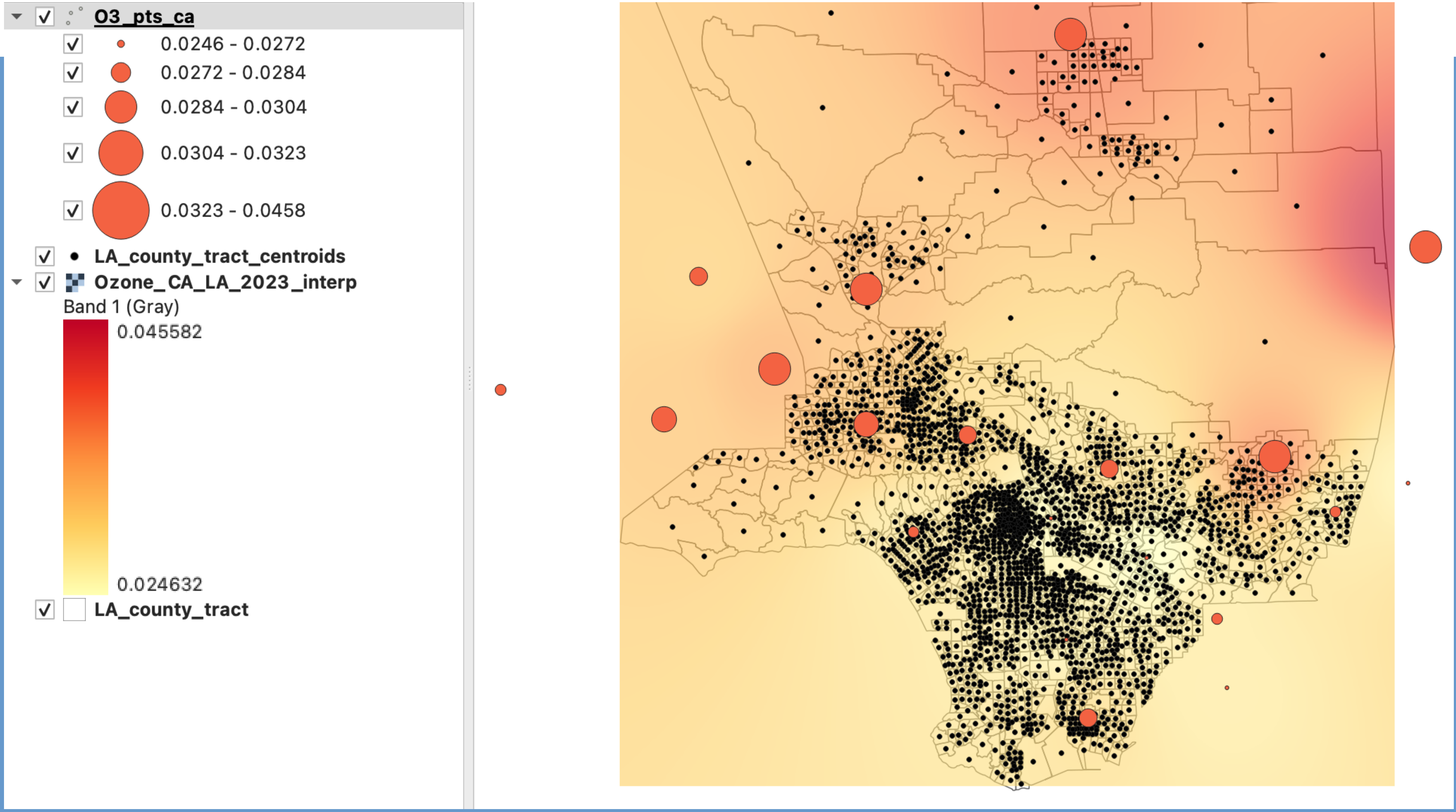
Spatial Interpolation

Lecture #20 | GEOG 510
GIS & Spatial Analysis in Public Health

Varun Goel

Outline

- Interpolation
  - Point-based methods
    - Nearest neighbor
    - Trend surface
    - IDW
    - Kriging (classical, EB)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0.0272 - 0.0284
0.0284 - 0.0304
0.0304 - 0.0323
0.0323 - 0.0458
LA_county_tract_centroids
Ozone_CA_LA_2023_interp
Band 1 (Gray)
0.045582
0.024632
LA_county_tract

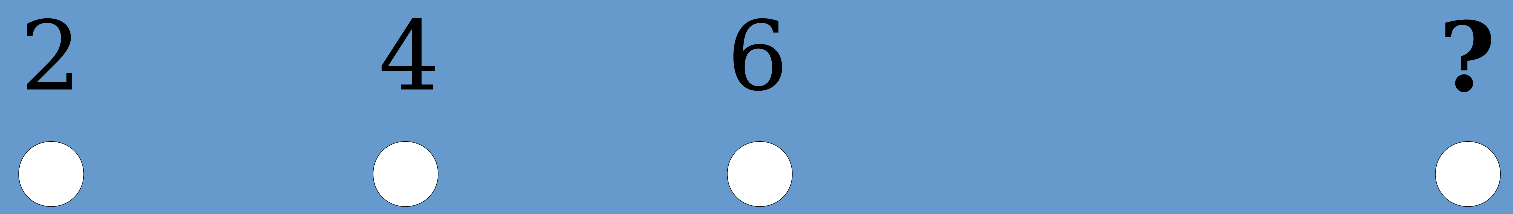
What is the missing value?

10 20 ? 40 50

What is the missing value?

10 20 **30** 40 50

What is the missing value?



What is the missing value?

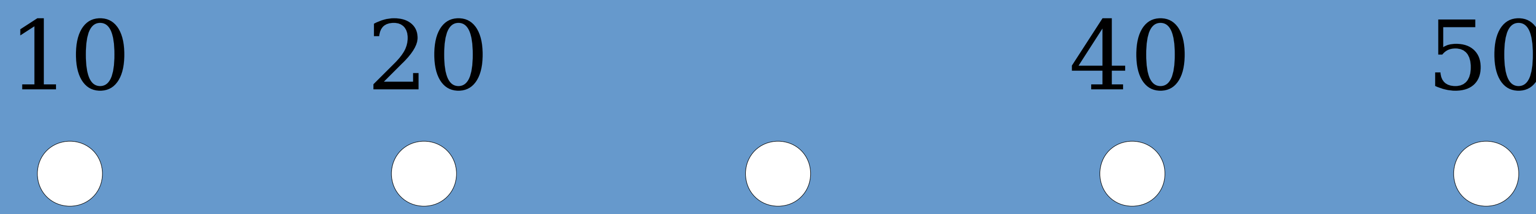
2 4 6 **10**

What is the missing value?

2 4 6 4 ?

What is the missing value?

2 4 6 4 **2**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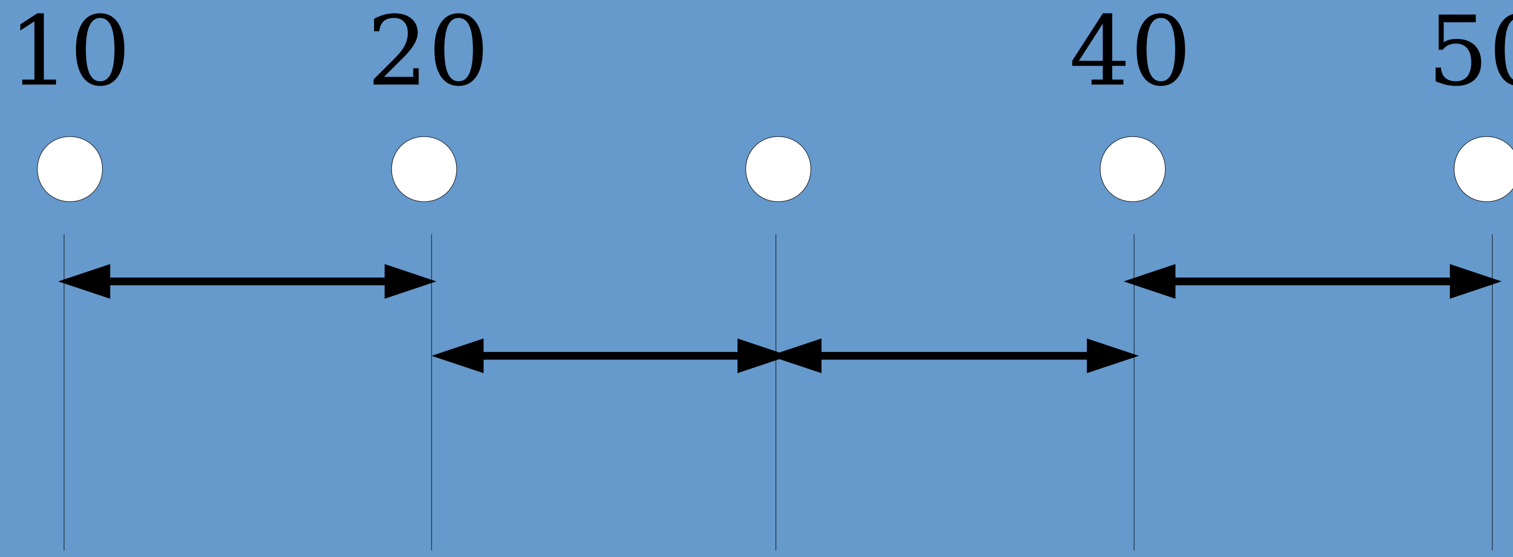
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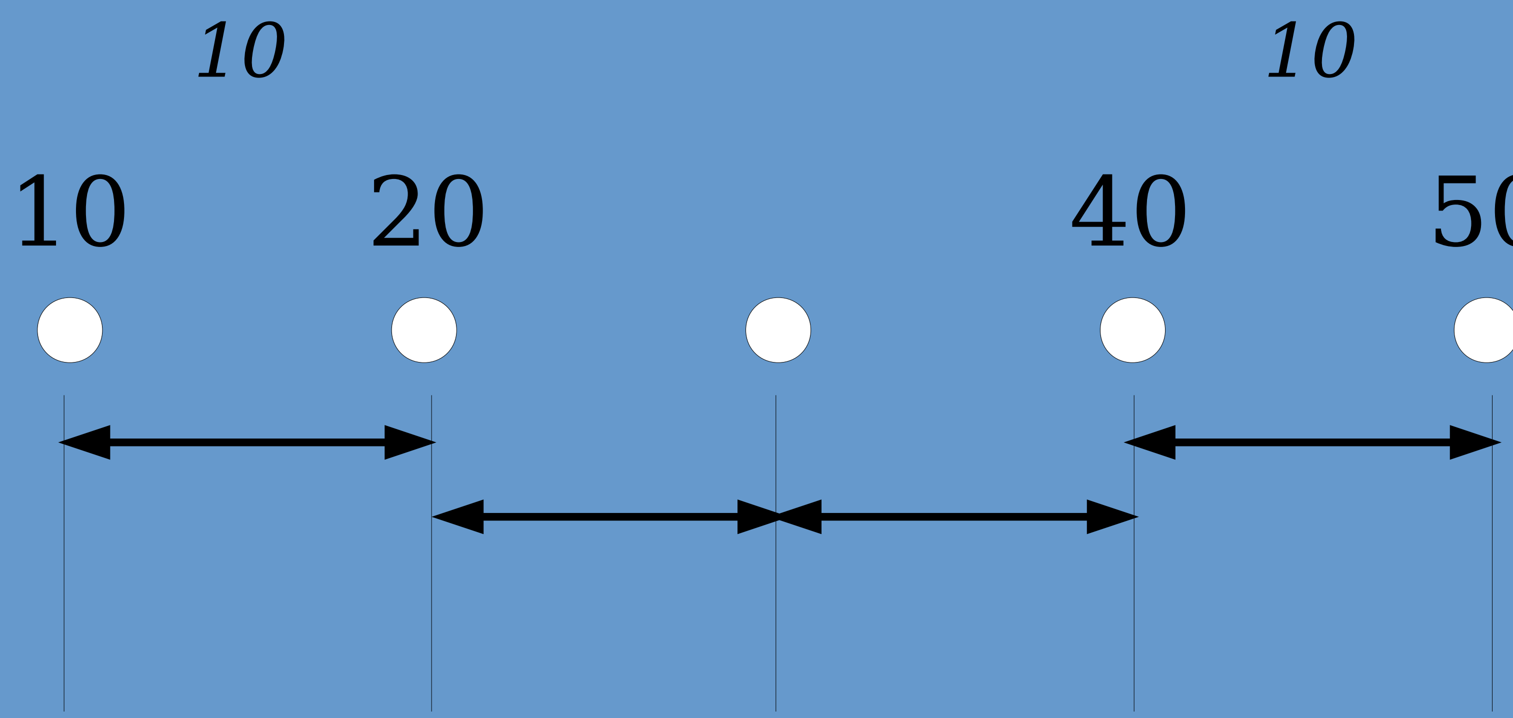

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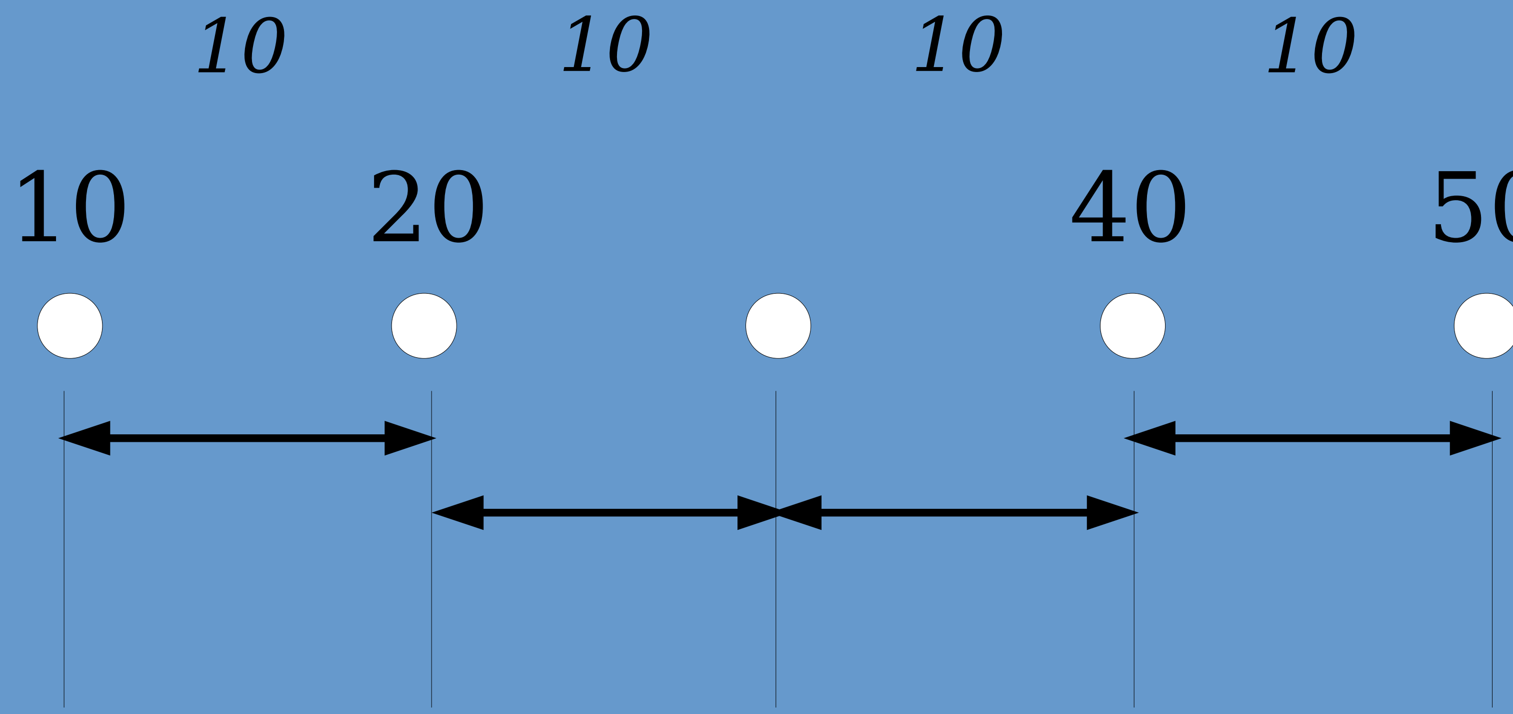
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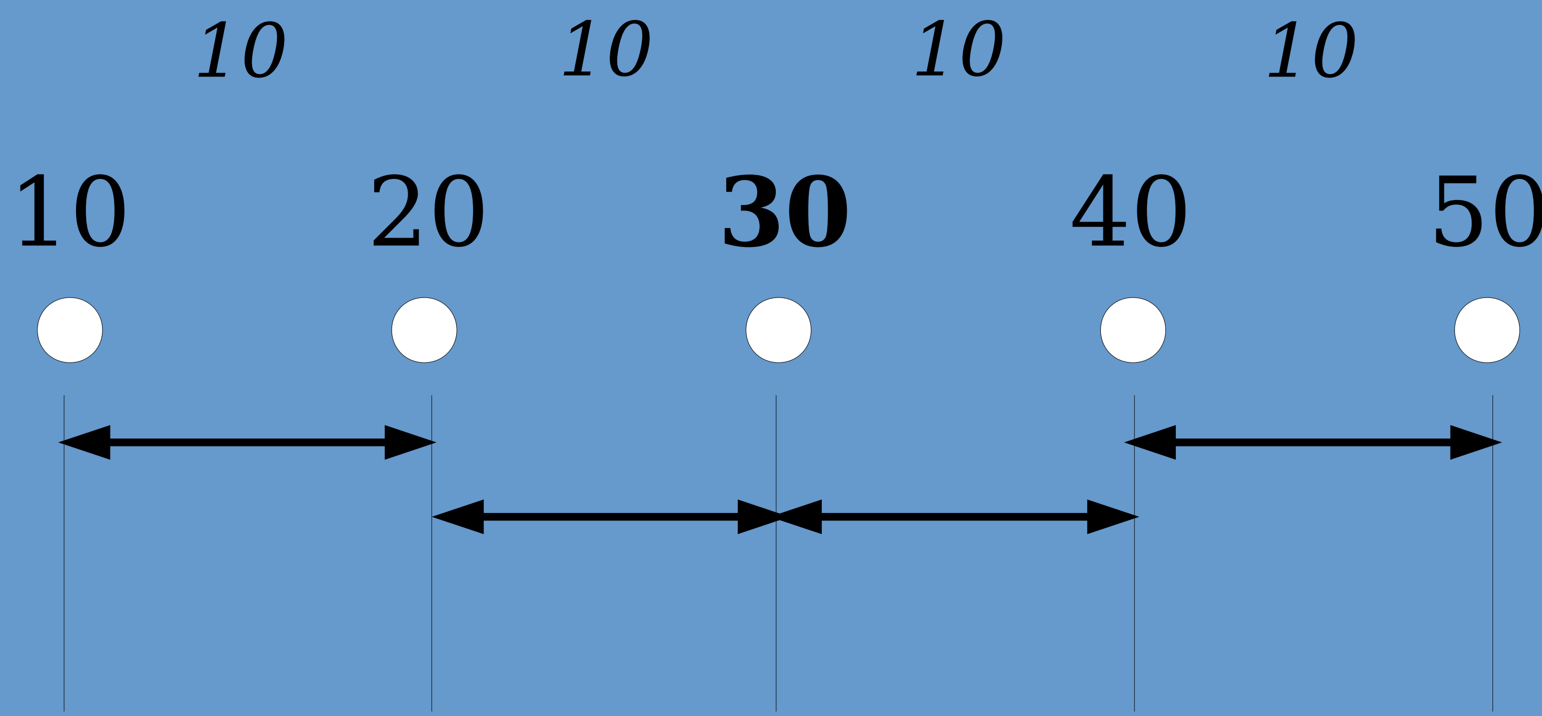
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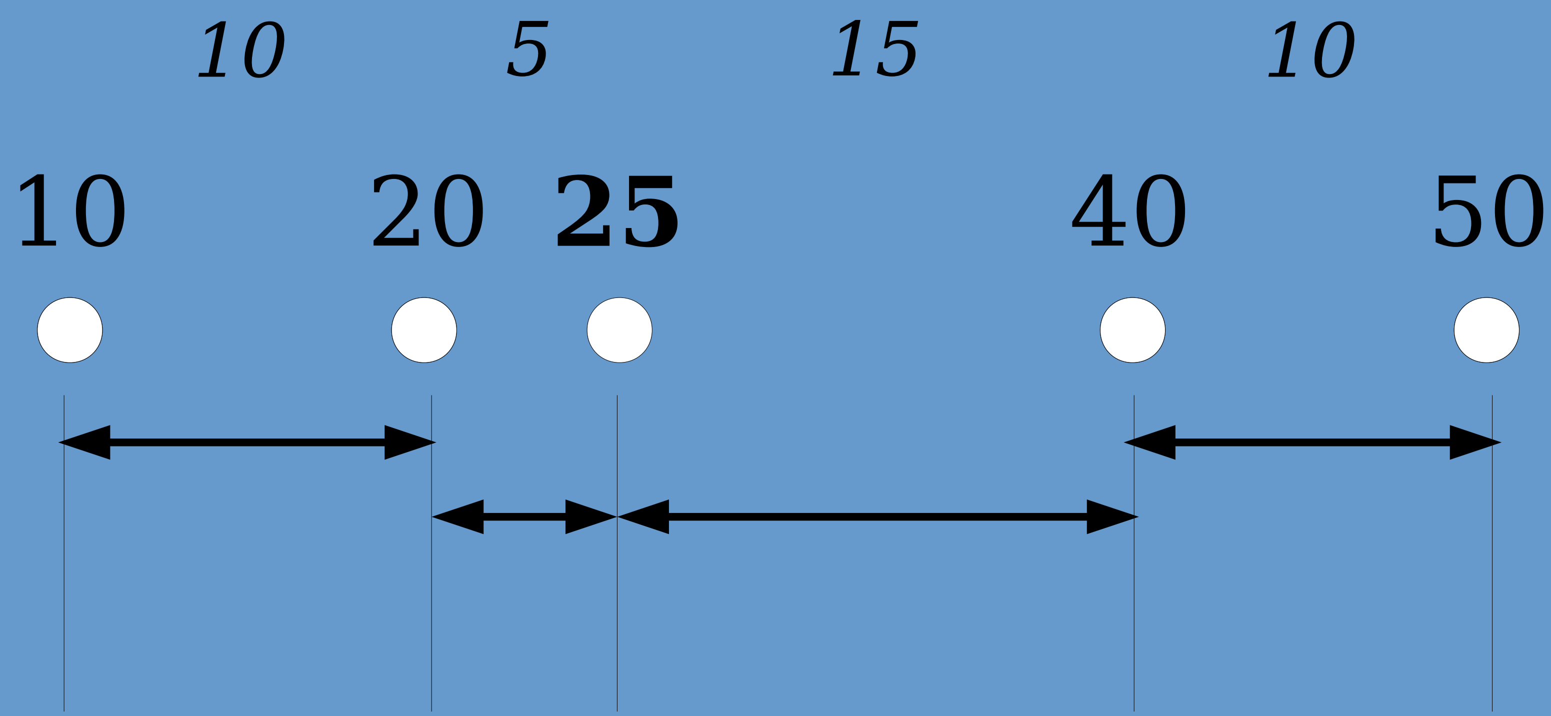
What is the missing value?


10
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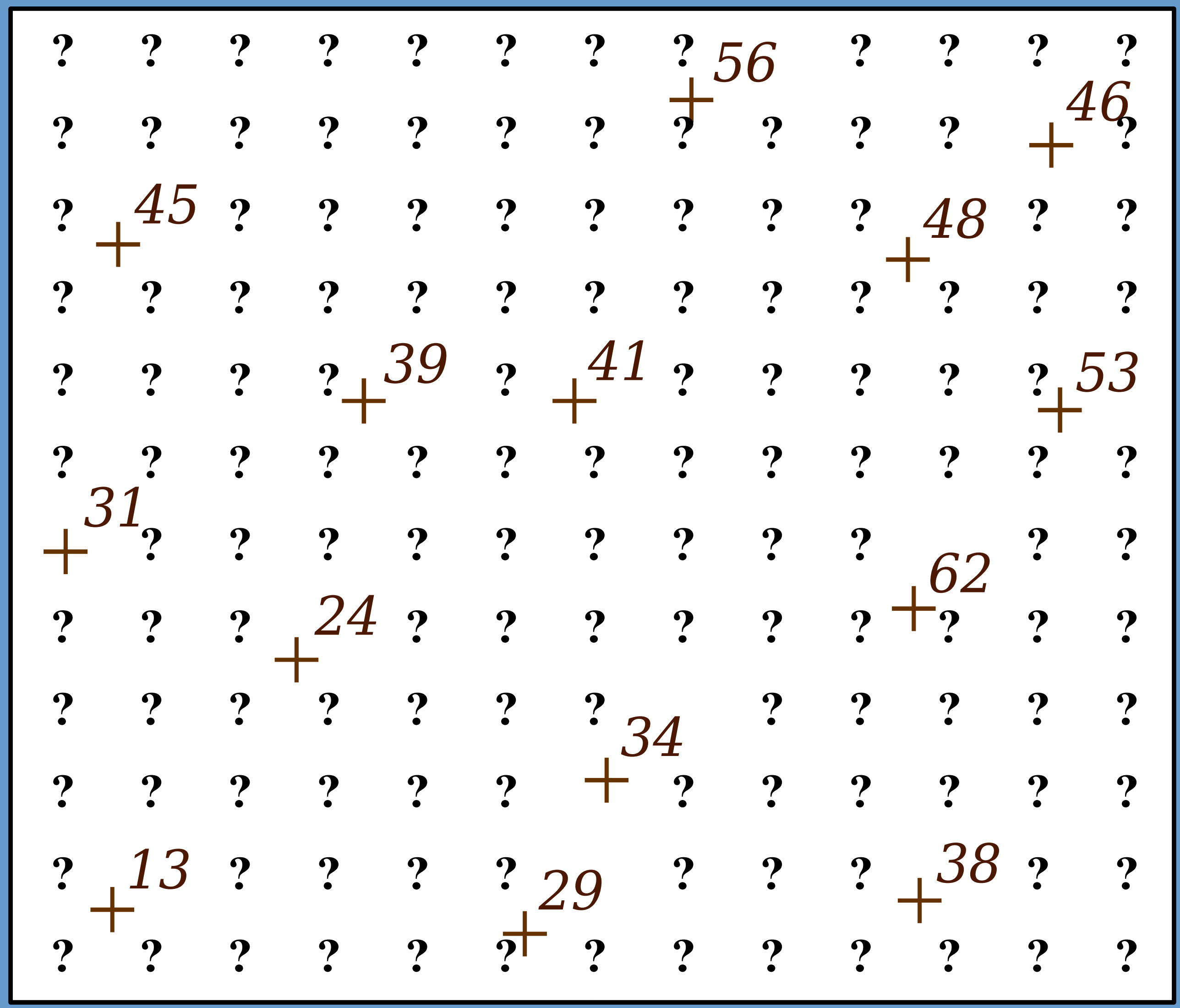
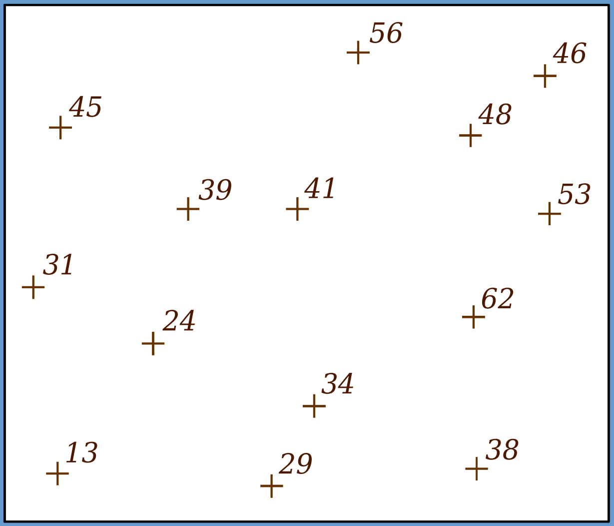
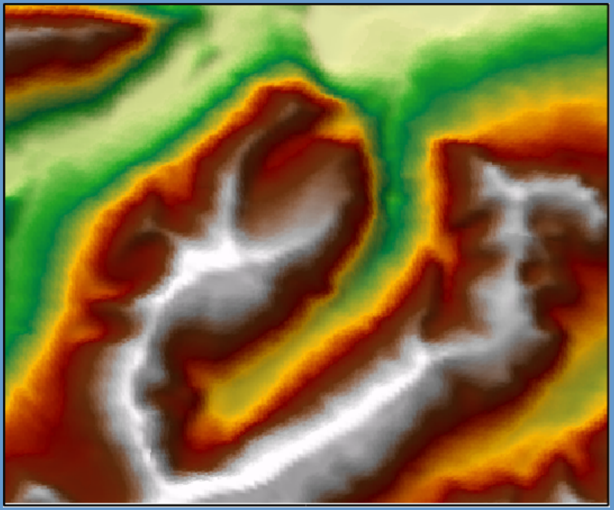
What are the missing values?



Interpolation

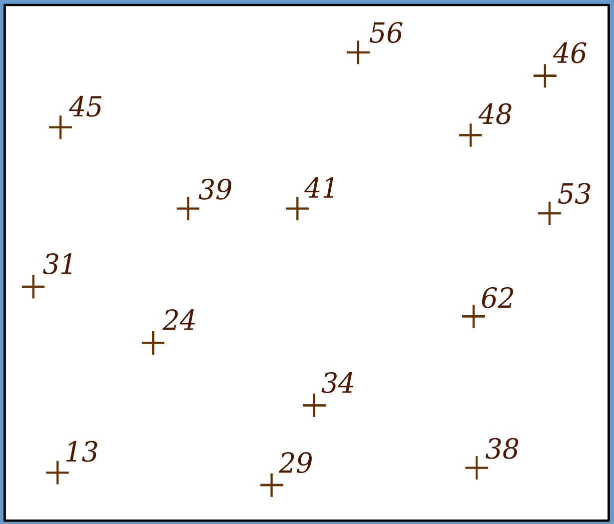
- *Interpolation* is the process of estimating attribute values at unmeasured locations
  - Especially useful when we only have *samples* distributed throughout space
  - Works for a single variable
    - Known values at specific locations
    - Interpolation "fills in the blanks"

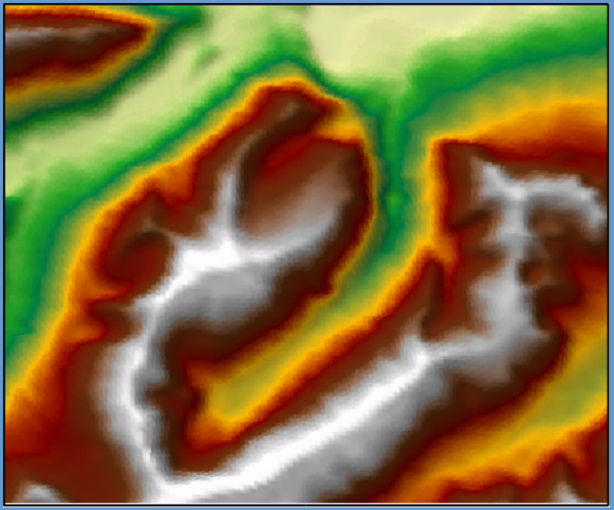




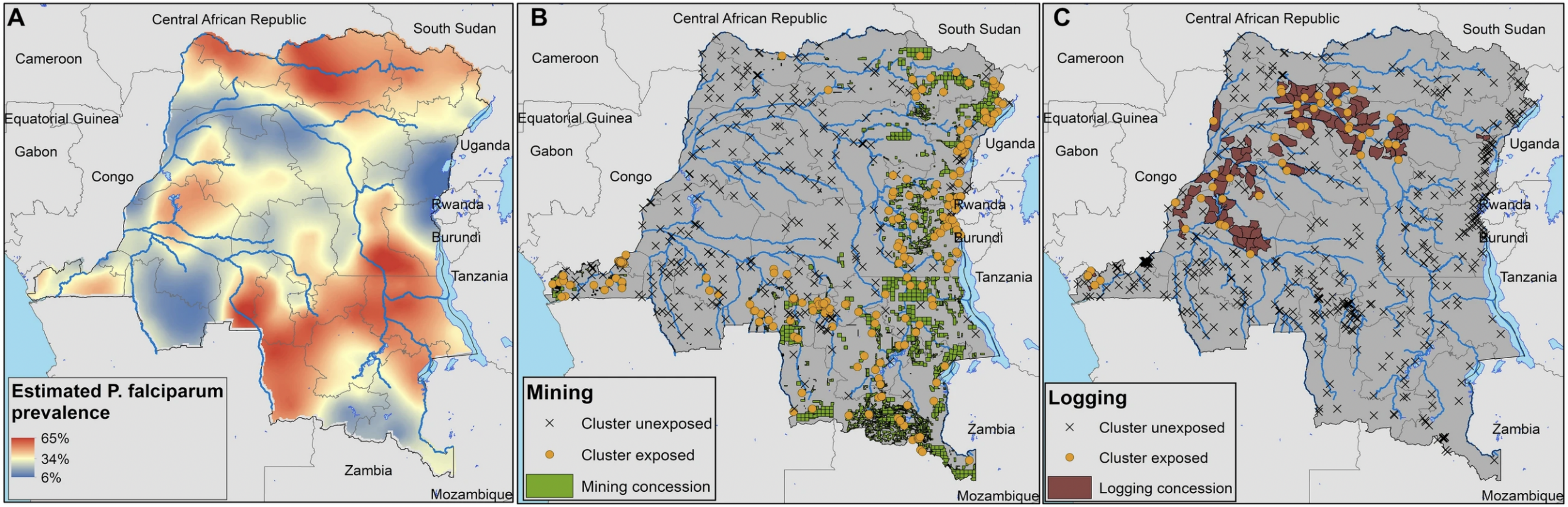
Interpolation

- *Interpolation* is the process of estimating attribute values at unmeasured locations
  - Takes advantage of our understanding of how values near to each other tend to be similar
    - Spatial autocorrelation





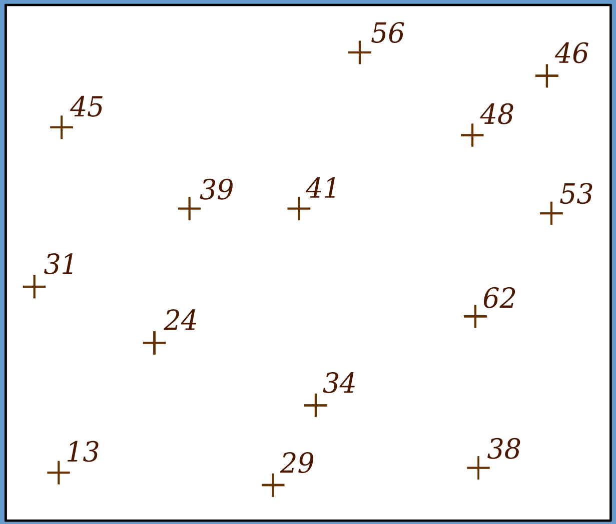
From: Impact of extractive industries on malaria prevalence in the Democratic Republic of the Congo: a population-based cross-sectional study

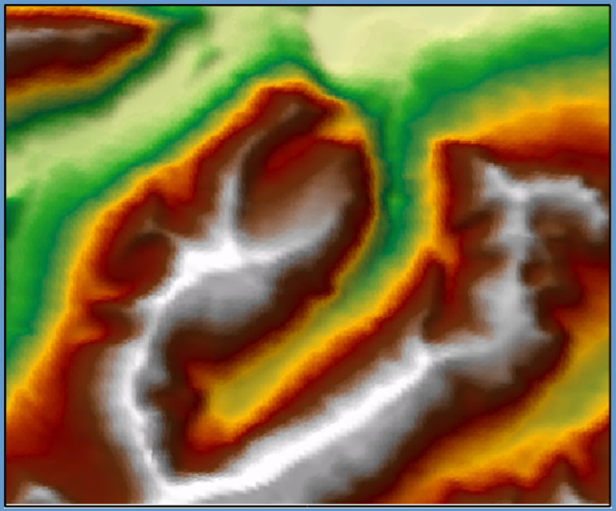


Mitchell et al., 2022

Interpolation

- *Interpolation* is the process of estimating attribute values at unmeasured locations
  - Assumes the underlying values are measured on a continuous scale
  - Assumes the underlying distribution is continuous across space





Cow Test!

Thanks to Dr. Tim Leslie

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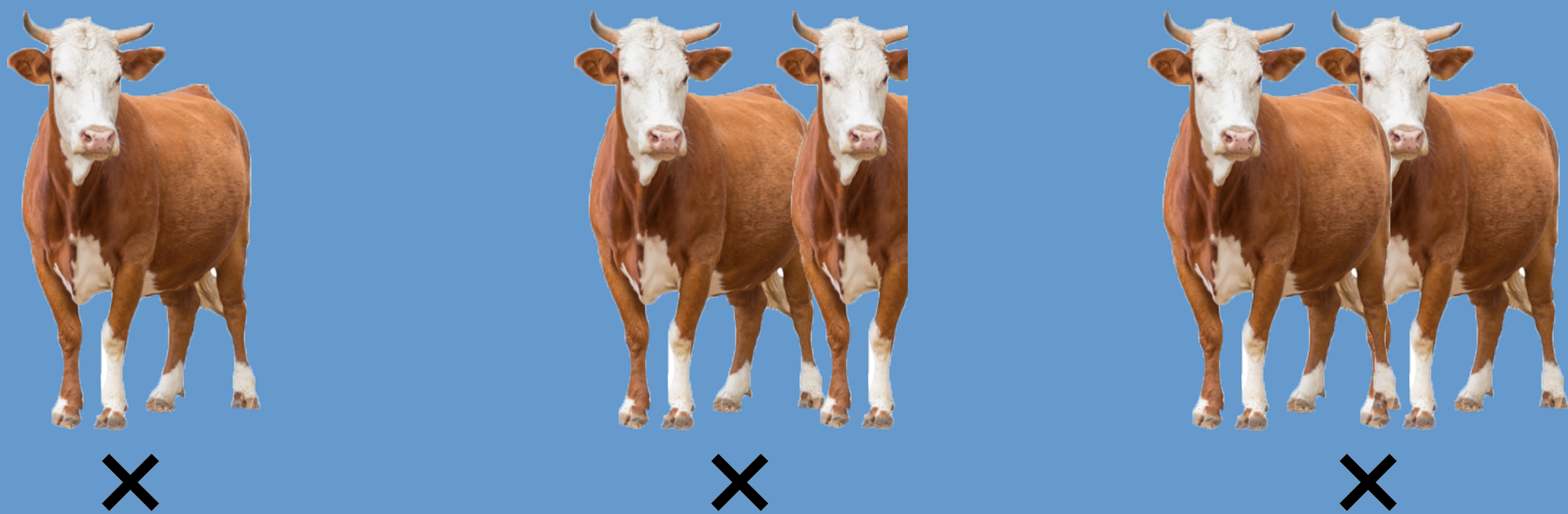
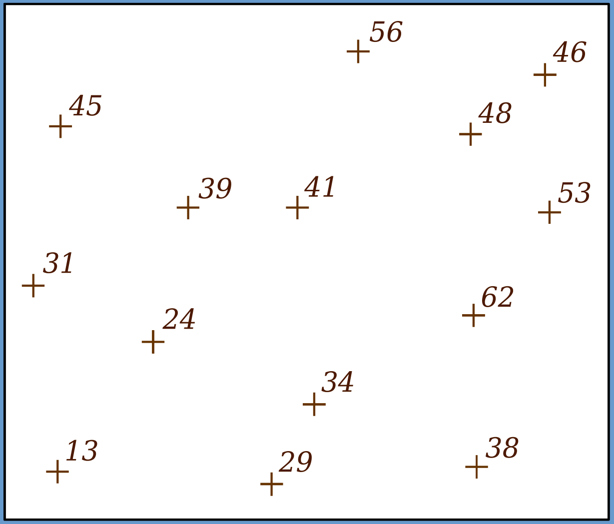
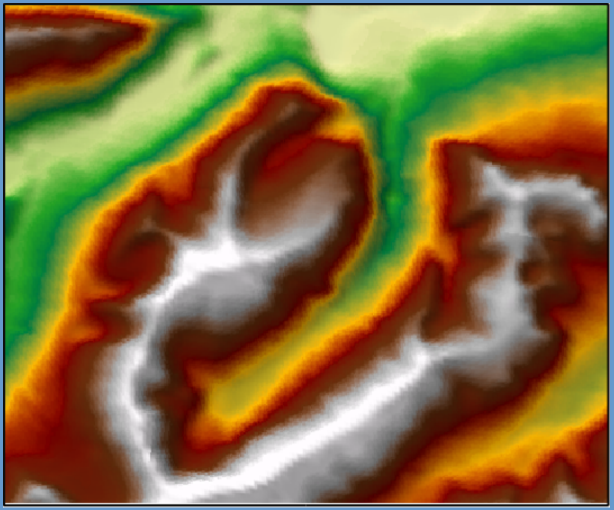


Does 1.5 cows make sense?

Interpolation

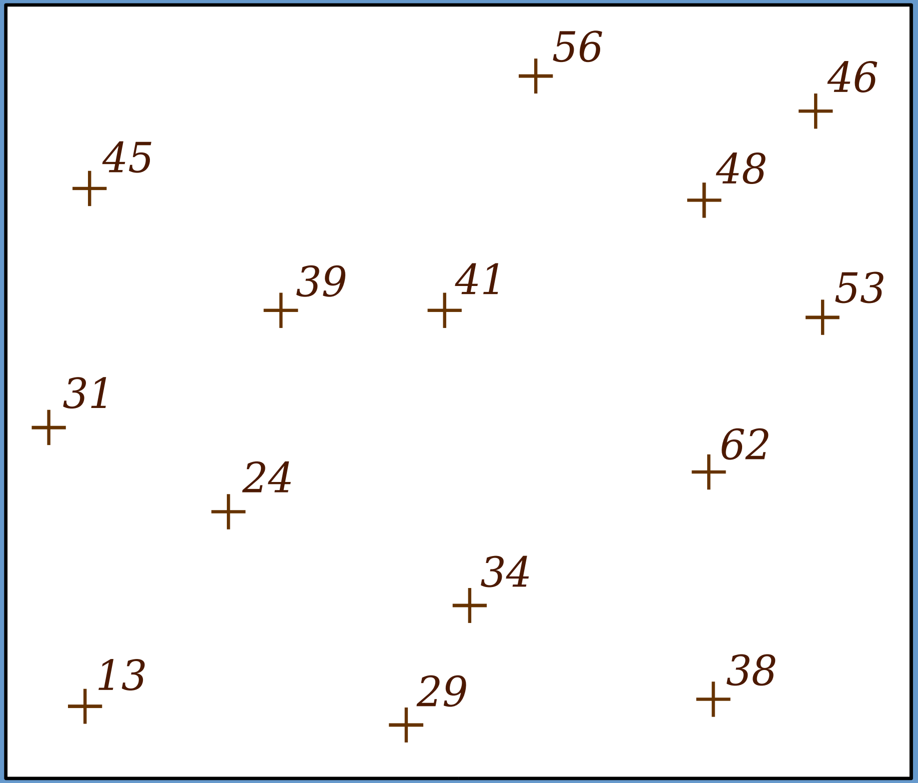
- *Interpolation* is the process of estimating attribute values at unmeasured locations
  - Useful for estimating the value of hazards at unknown locations
    - e.g., air pollution
  - Collection is only at discrete locations (points) but we want to estimate at non-sampled locations





Interpolation

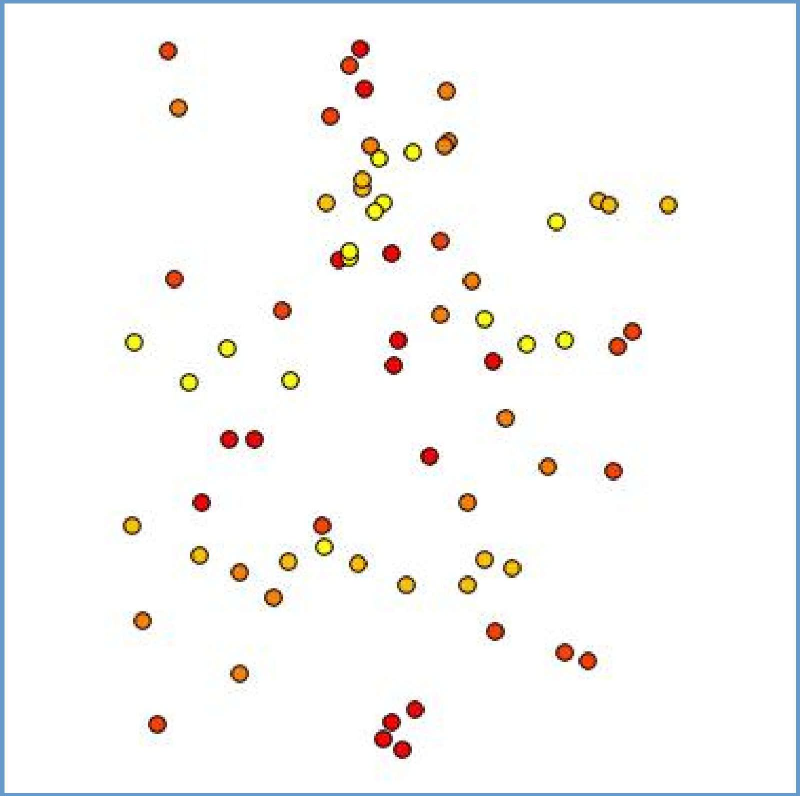
- Point-based approaches
  - Nearest neighbor
  - Trend surface
  - Inverse distance weighted (IDW)
  - Kriging



Input: set of point observations with measured attribute values
Output: continuous distribution of attribute values throughout a region

Nearest Neighbor

- Assigns each location to be interpolated the same value as the nearest sample point
  - Gives a stair-step shape to the surface (discontinuities)
- *Thiessen Polygons* are created when each location on the map is assigned to the nearest sample point



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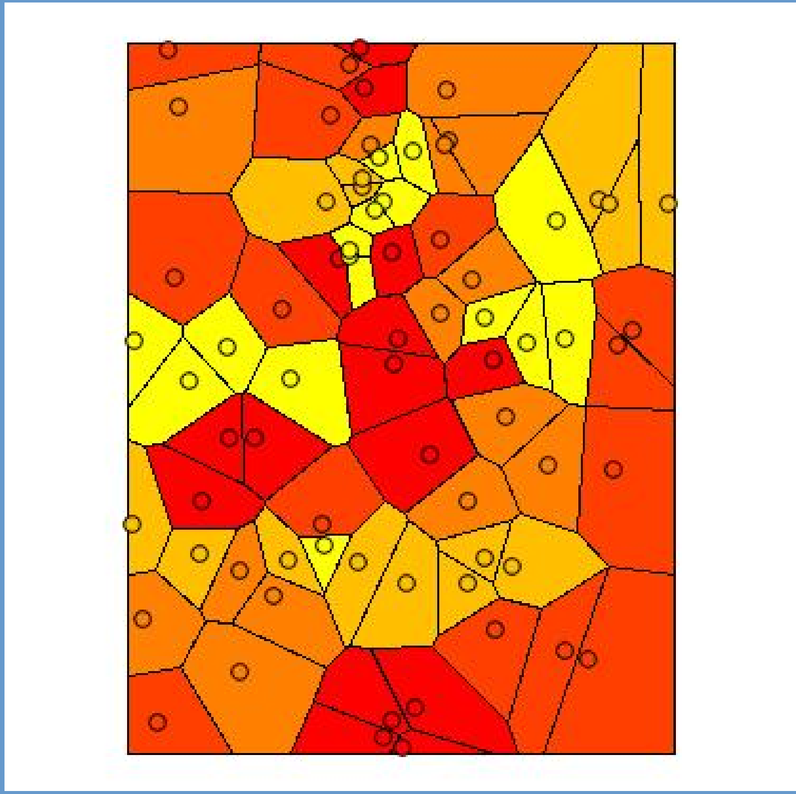
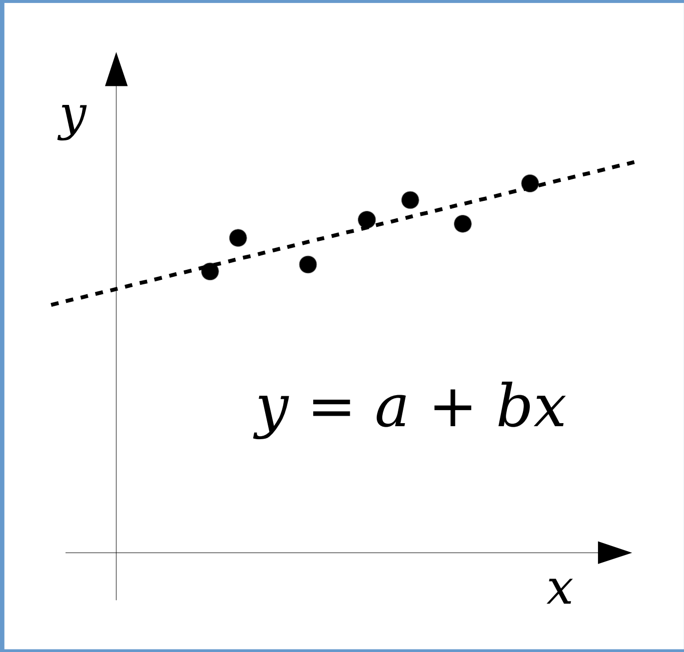


Nearest Neighbor

- Output features: polygons
- Estimated values are constrained to the input values
- At sample locations, estimated value will always equal sample value
- Results in a “shelved” surface

Trend Surface Analysis

- Fit a plane or other surface through the observed points
  - Similar to fitting a regression line to a set of observations



Trend Surface Analysis

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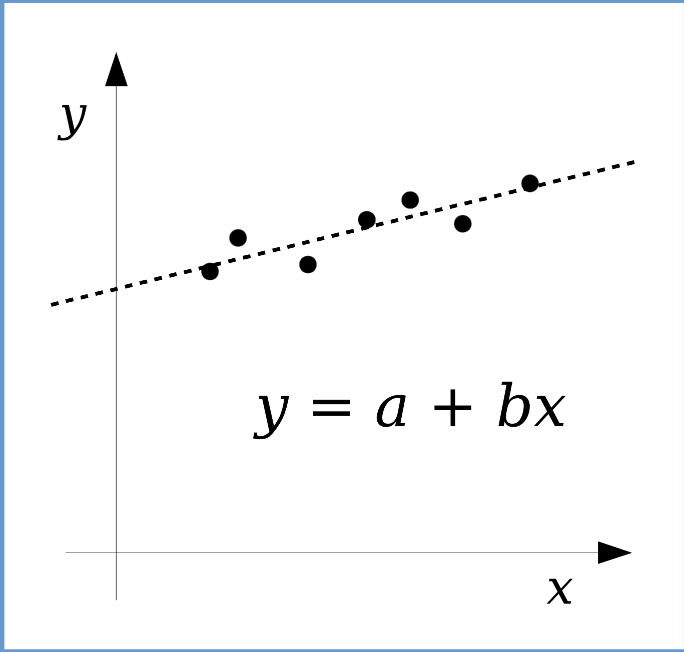

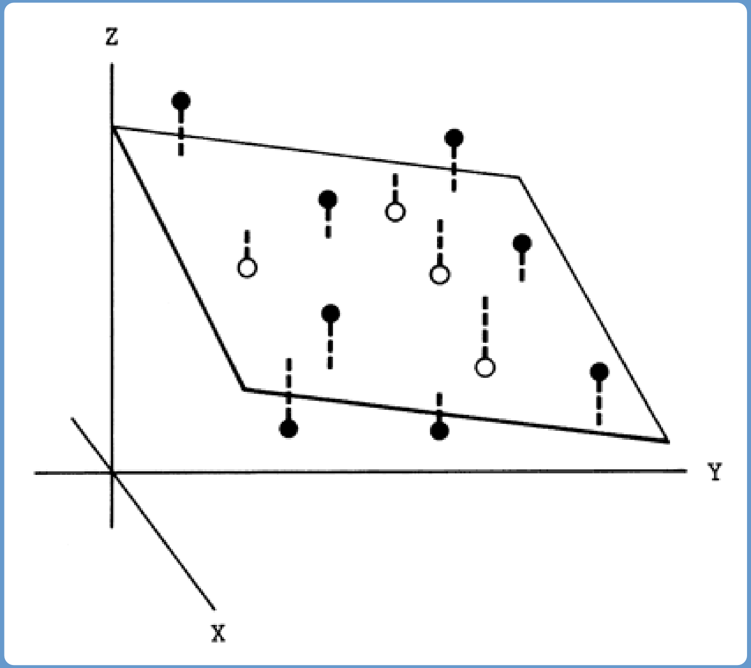

- First order polynomial equation

$$z(x,y) = a + bx + cy$$

Trend Surface Analysis

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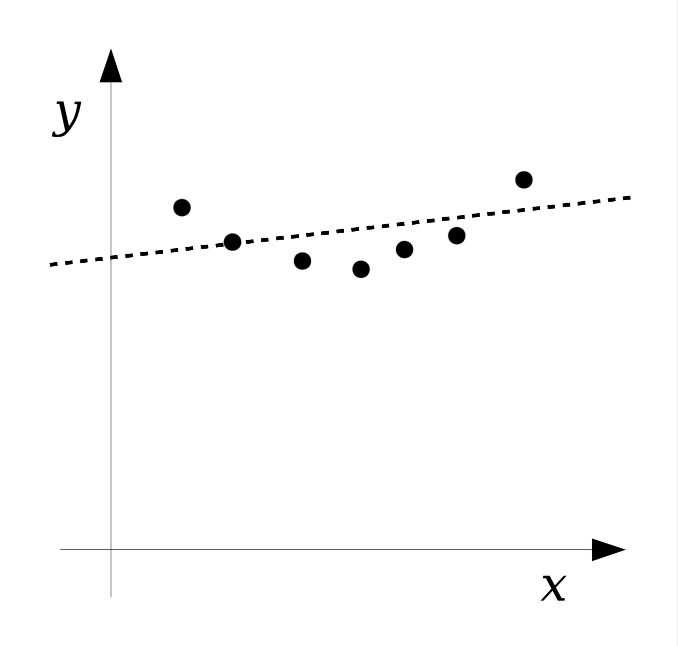
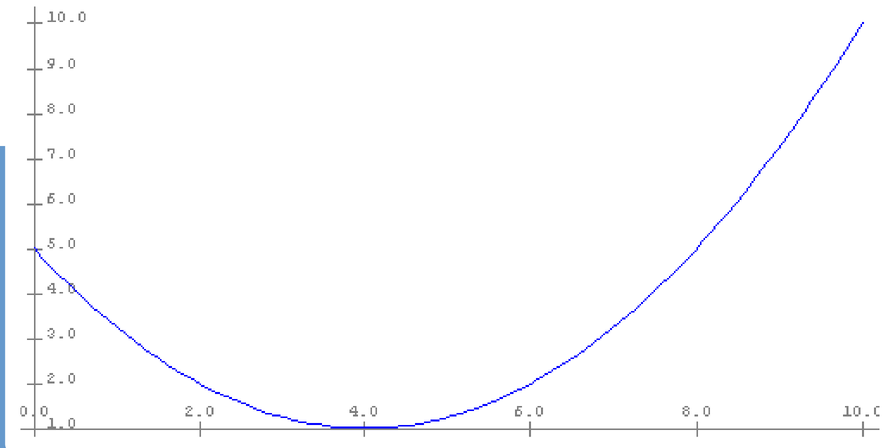
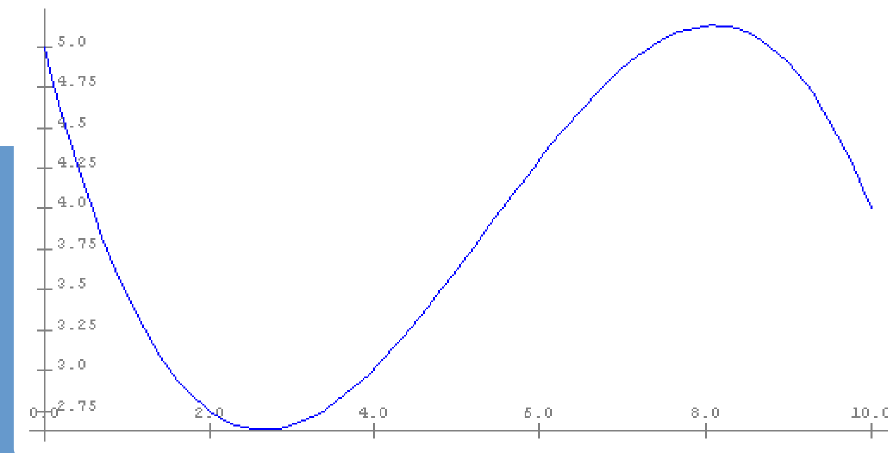
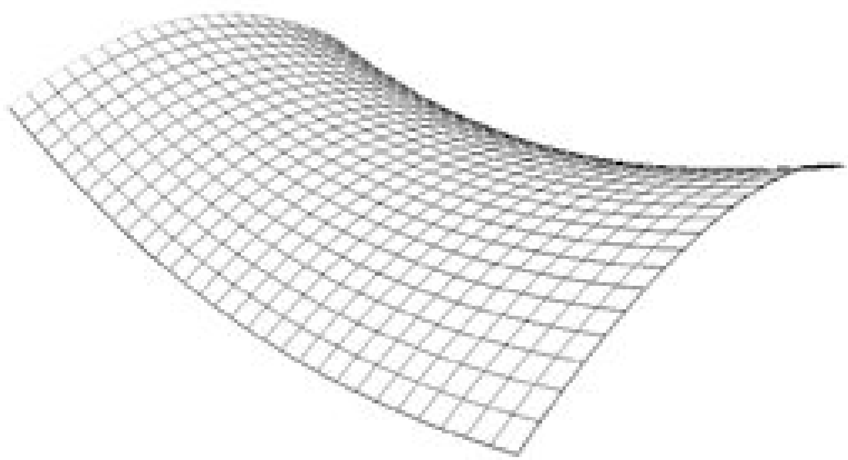
  - Similar to fitting a regression line to a set of observations

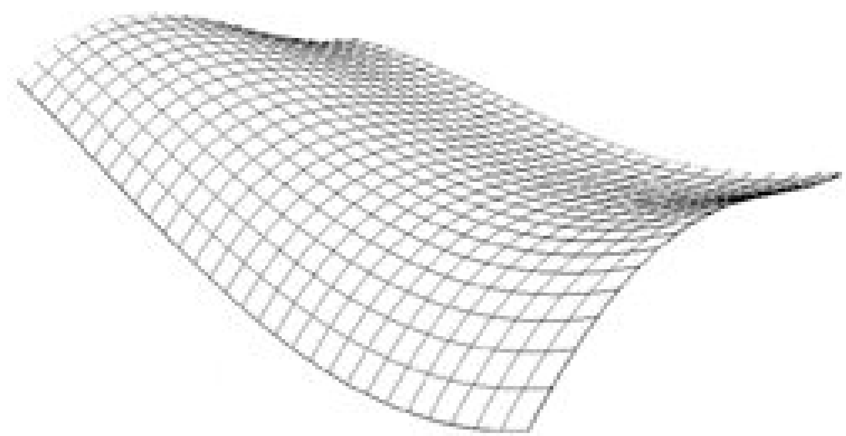








d) 2nd order trend surface



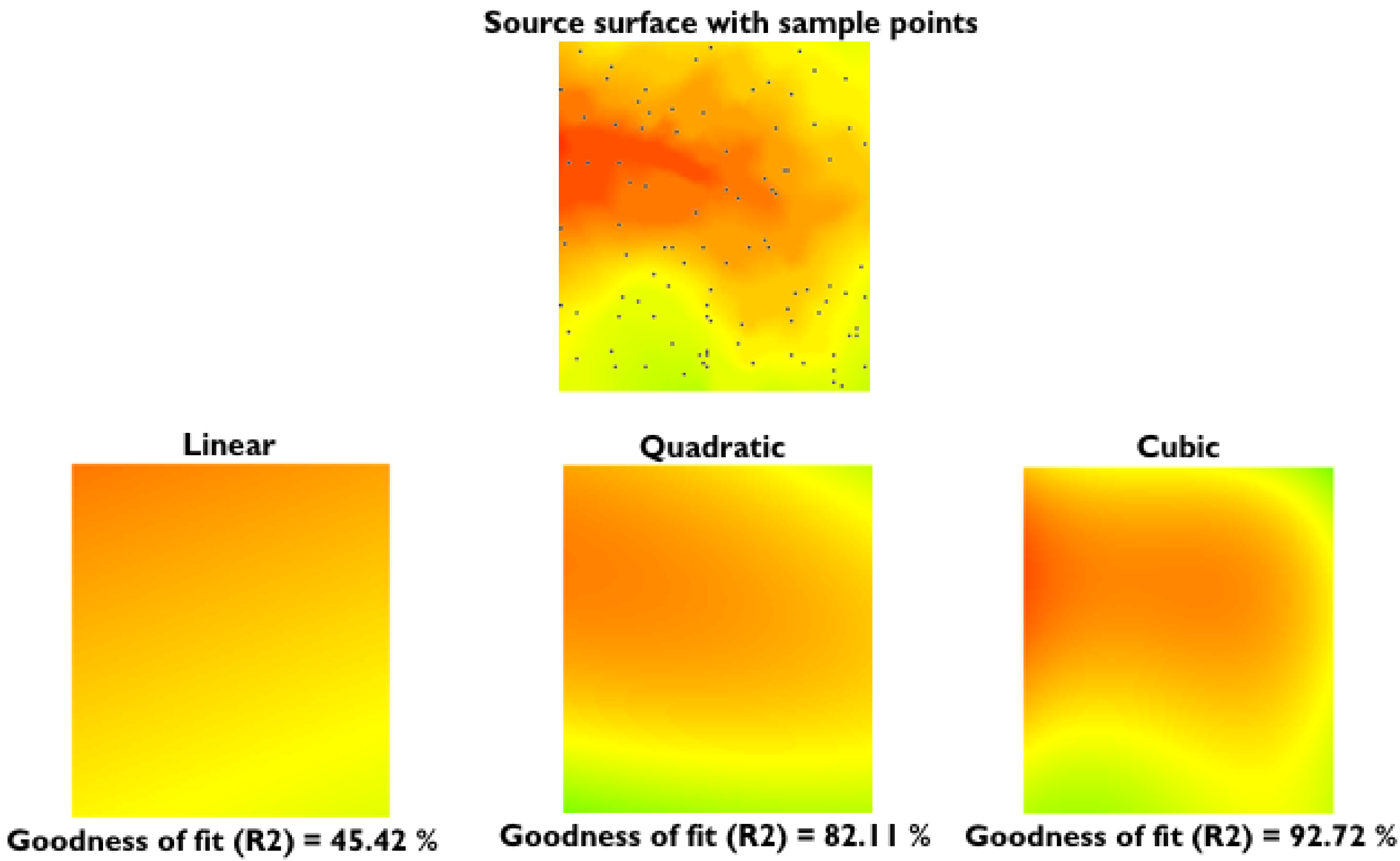
e) 3rd order trend surface

- Second order polynomial equation

$$z(x,y) = a + bx + cy + dxy + ex^2 + fy^2$$

Trend Surface Analysis

- Fitting a trend surface to observations
- Order of the equation and sample points
  - Limit on the number of coefficients: you cannot have more than the number of sample points
    - Just because you can doesn't mean you should!
  - Surface fit and model residuals
    - Regression diagnostics
      - R^2, *f*, residual plot



www.geog.leeds.ac.uk/courses/level2/geog2750/geog2750_14.ppt

Trend Surface Analysis

- Output features: raster (in reality, a function)
- Estimated values are not constrained to the input values
- At sample locations, estimated value does not always equal sample value
- Results in a very smooth surface

IDW

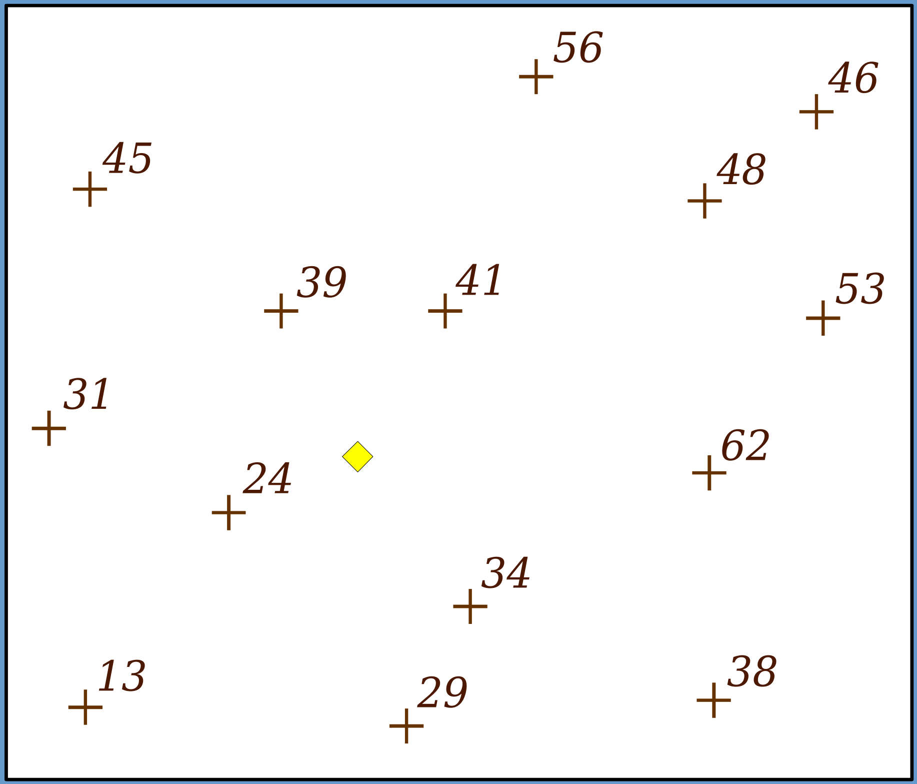
- Inverse Distance Weighted interpolation
  - Value for non-sampled locations is calculated as the weighted average of nearby values
  - Weighted average often takes the form:

$$z_j = \frac{\sum_{i=1}^{n} \frac{z_i}{d_i^2}}{\sum_{i=1}^{n} \frac{1}{d_i^2}}$$

  - *Requires*: neighborhood to be defined (n)
    - Specific number of points or specified *d* (radius)
  - *Key assumption*: the value at a point is assumed to be related to the value at other points such that the similarity declines at a rate of $1/d^x$
    - $1/d^x$ is only a guess at how nearby values are related ($1/d^2$ is commonly used)

IDW Calculation

- Calculate value at the yellow diamond
  - $n = 6$



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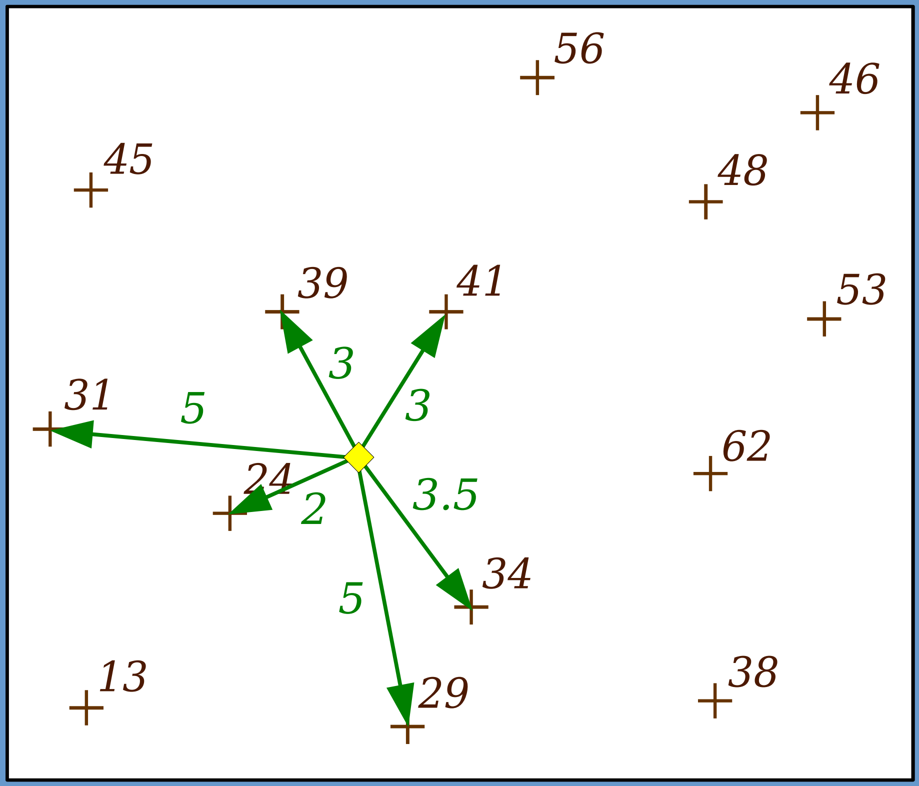

$$z_j = \frac{\frac{24}{2^2} + \frac{39}{3^2} + \frac{41}{3^2} + \frac{34}{3.5^2} + \frac{31}{5^2} + \frac{29}{5^2}}{\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{3^2} + \frac{1}{3.5^2} + \frac{1}{5^2} + \frac{1}{5^2}}$$

$$z_j = \frac{20.0644}{0.6339}$$

$$z_j = 31.6546$$

IDW Calculation

- Calculate value at the yellow diamond
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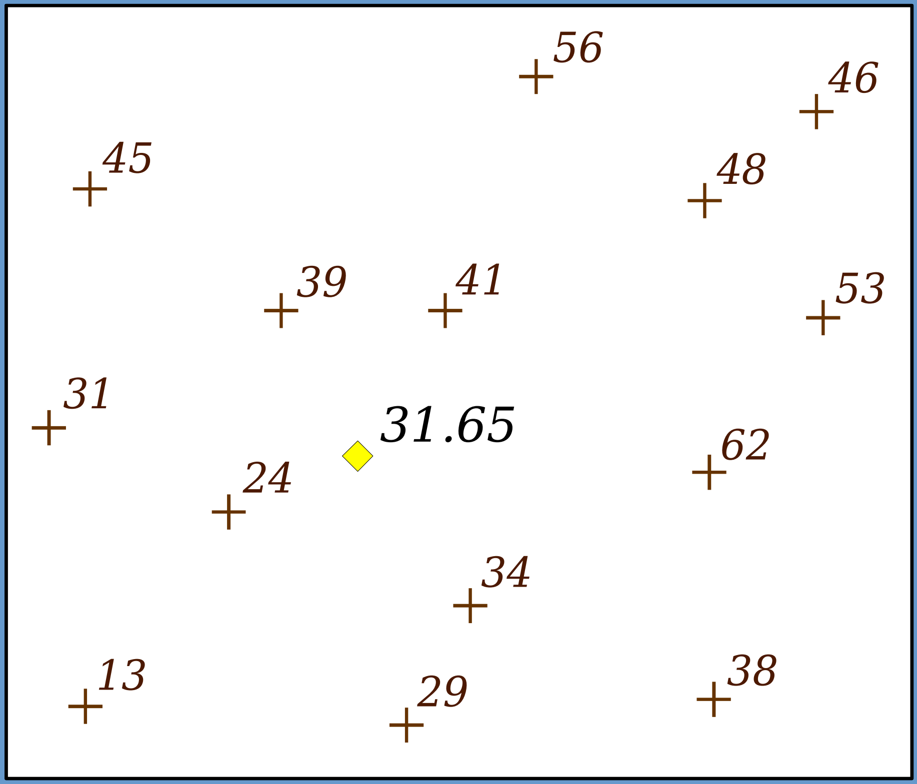

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IDW Calculation

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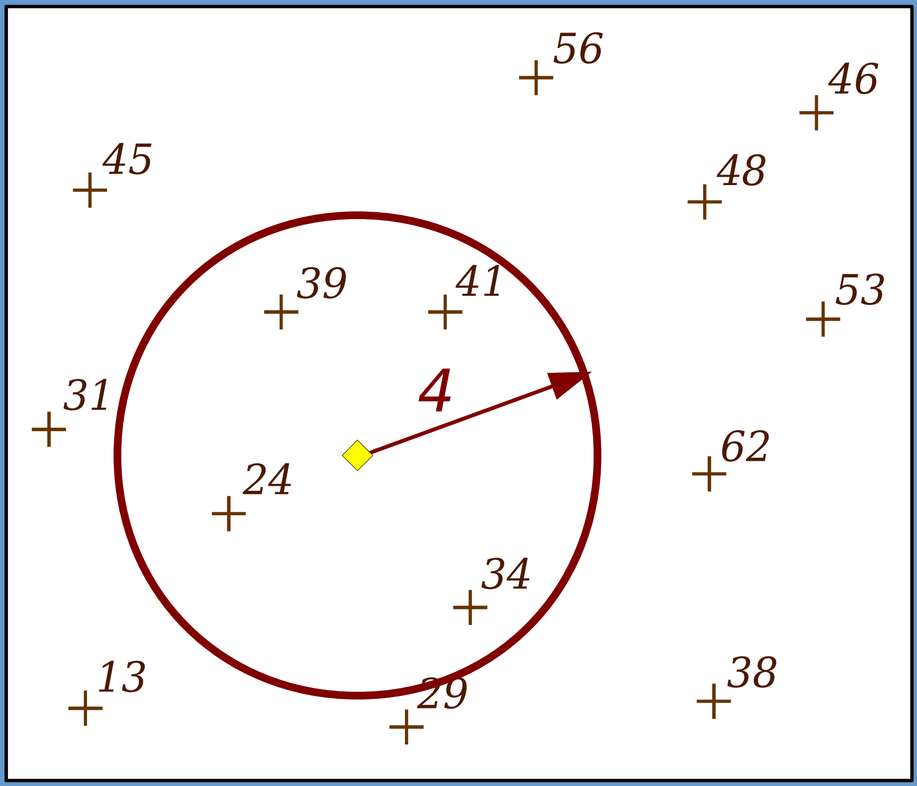
  - $d = 4$



IDW Calculation

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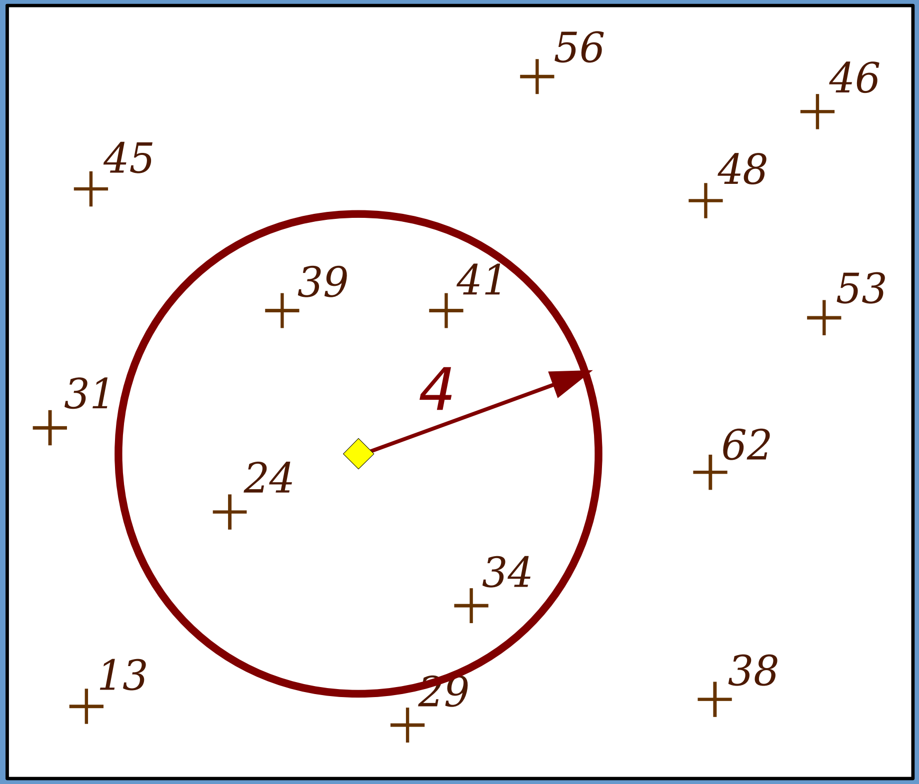

$$z_j = \frac{\frac{24}{2^2} + \frac{39}{3^2} + \frac{41}{3^2} + \frac{34}{3.5^2}}{\frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{3^2} + \frac{1}{3.5^2}}$$

$$z_j = \frac{17.6644}{0.5539}$$

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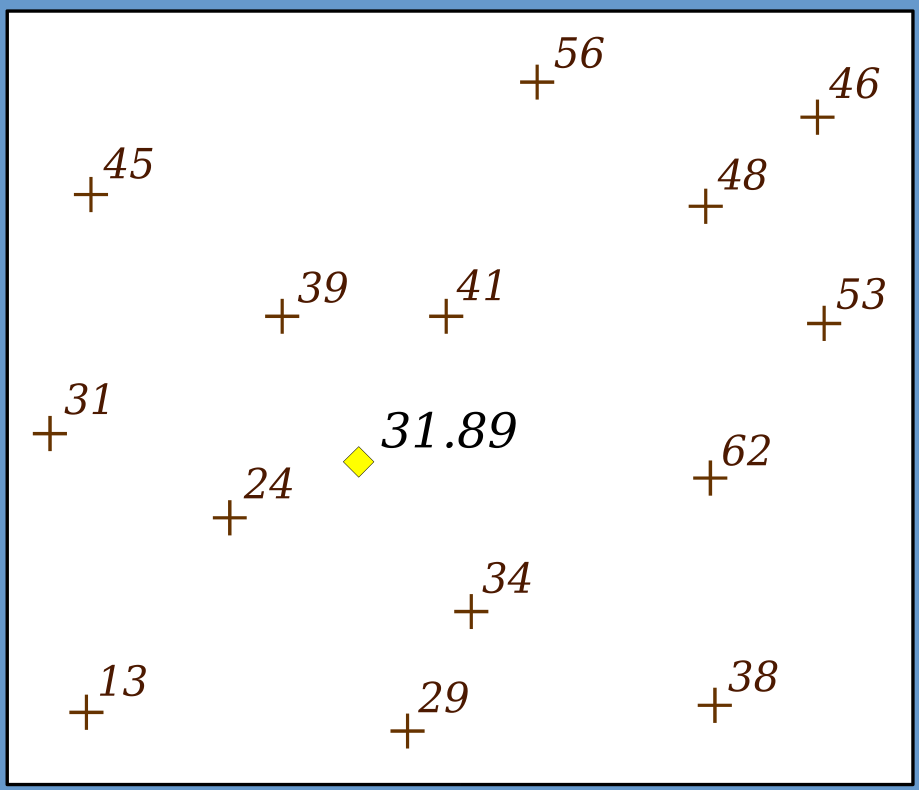

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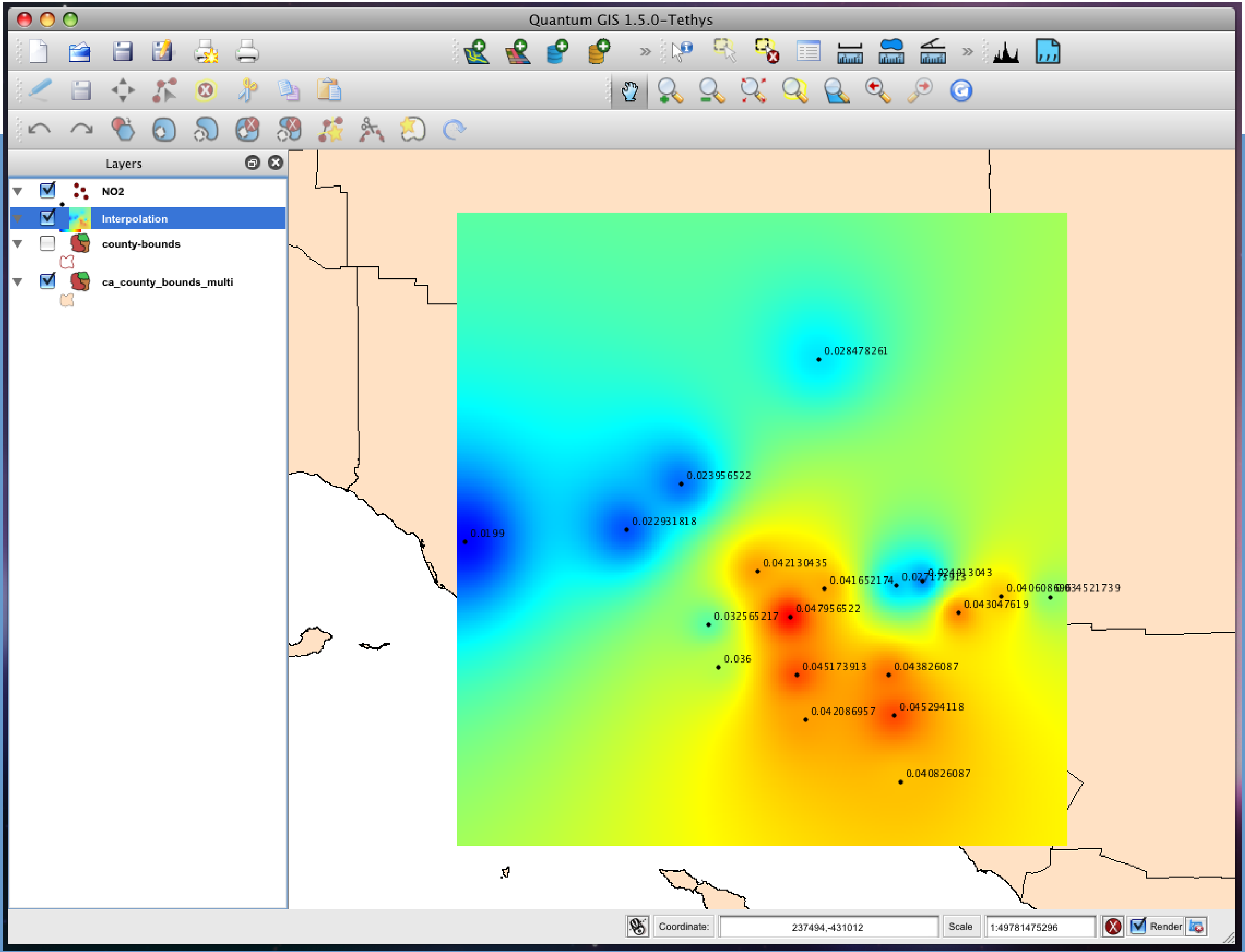
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0.043826087
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Scale
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Render

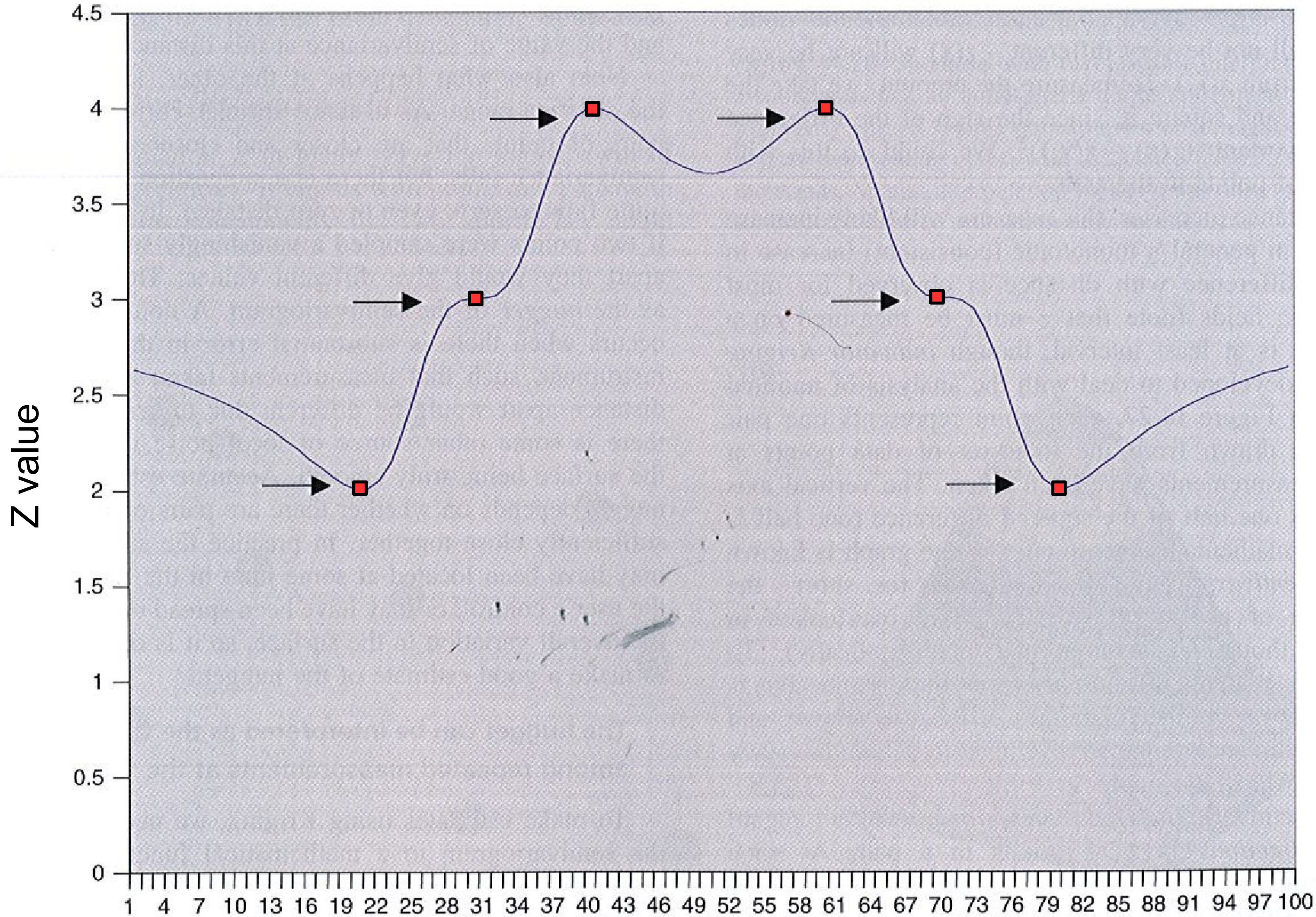
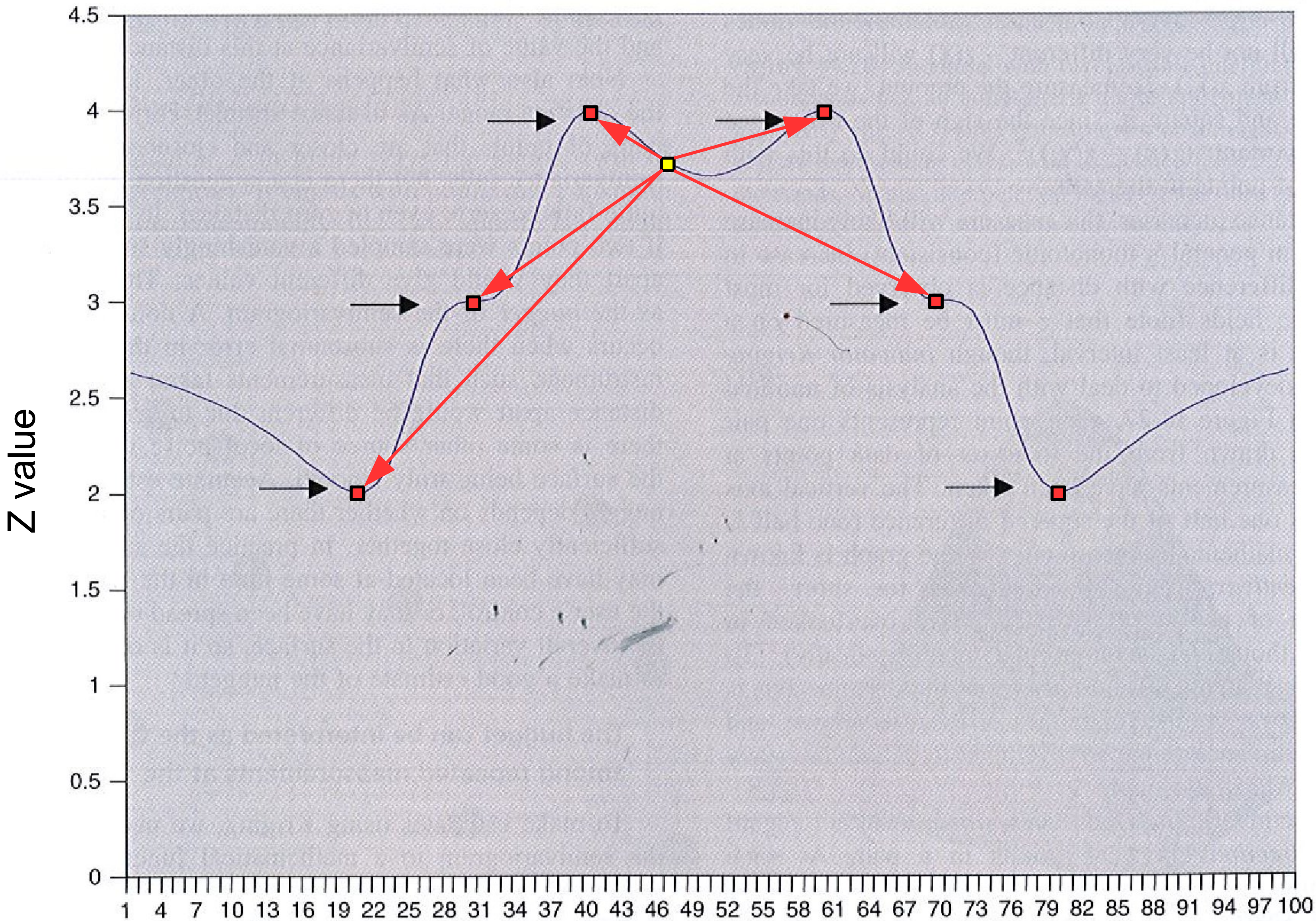


Figure 14.26 Potentially undesirable characteristics of IDW interpolation. Data points located at 20, 30, 40, 60, 70, and 80 have measured values of 2, 3, 4, 4, 3, and 2 respectively. The interpolated profile shows a pit between the two highest values, and regression to the overall mean value of 3 outside the area covered by the data



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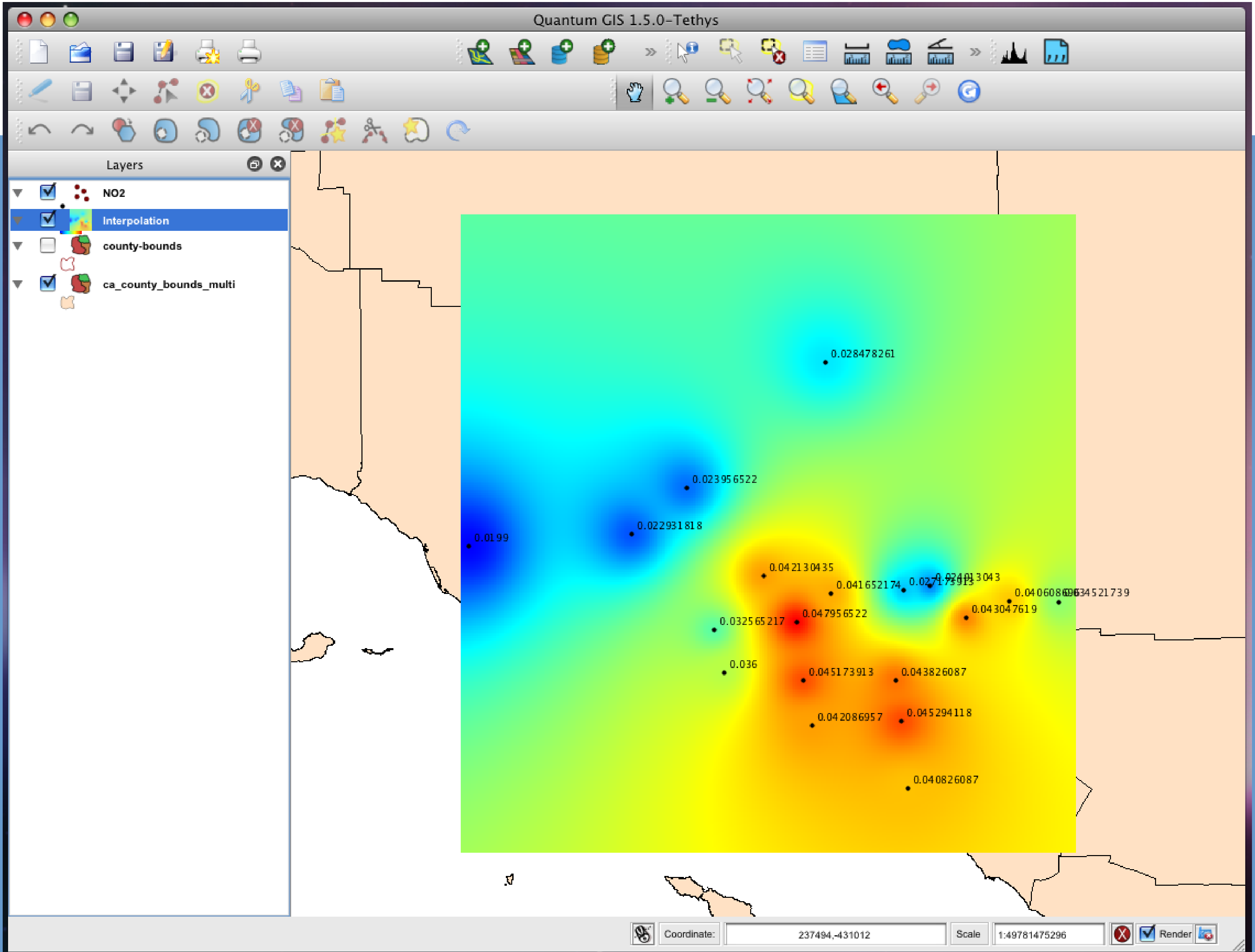

Quantum GIS 1.5.0-Tethys
Layers
NO2
Interpolation
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ca_county_bounds_multi
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0.023956522
0.022931818
0.0199
0.042130435
0.041652174
0.032565217
0.047956522
0.043047619
0.036
0.045173913
0.043826087
0.042086957
0.045294118
0.040826087
Coordinate:
237494,-431012
Scale
1:49781475296
Render

IDW

- Output features: raster
- Estimated values are constrained to the range of the input values
- At sample locations, estimated value always equals sample value
- Results in a pitted surface (local highs and lows)

Kriging

- Uses a *distance weighted average* method to calculate the value at unsampled locations
  - Weights are estimated from the data using the *semivariogram*
    - *Rather than guessing at the relationship* between similarity of values and distance (like we do when we guess at the exponent in IDW)

Kriging

- Called a *geostatistical* method
  - Parameters based on input data
    - Observed similarity as a function of distance
  - Statistical, not mathematical
    - Provide a measure of uncertainty

Semivariance

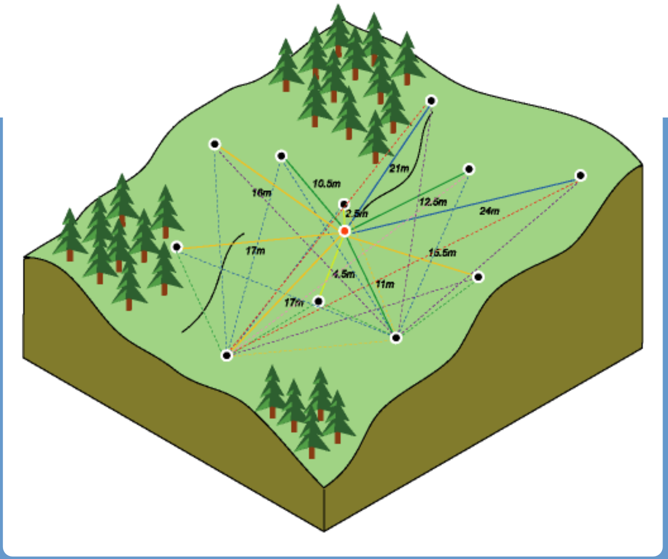
- *Semivariance*: the observed average variation between values measured at some separation distance (h)

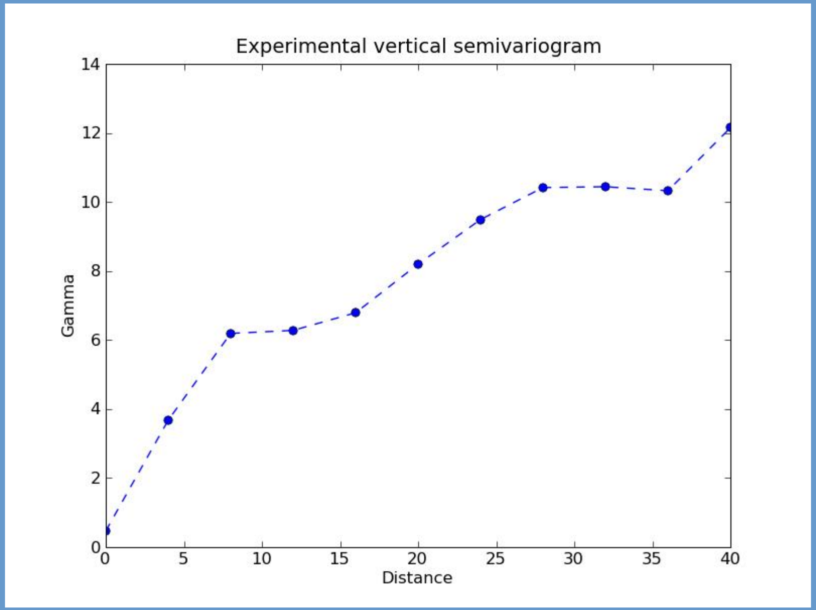
$$\gamma(h) = \frac{1}{2n} \sum_{i=1}^{n} \left[z(x_i) - z(x_i + h) \right]^2$$

- *Semivariogram*: the observed relationship between semivariance and distance (h)
  - Provides a description of spatial structure
  - Called "omnidirectional" when calculated regardless of any directional effects

Semivariogram

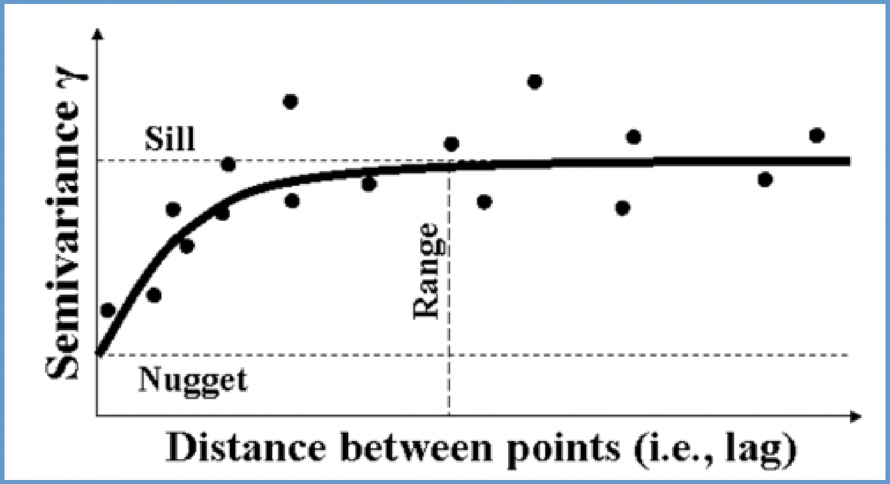
- In order to know how similarity changes with distance we calculate the empirical semivariogram from the sample data
  - Its shape tells us something about the general form of the surface (or continuous field)
  - Typically shows that difference between values increases with distance

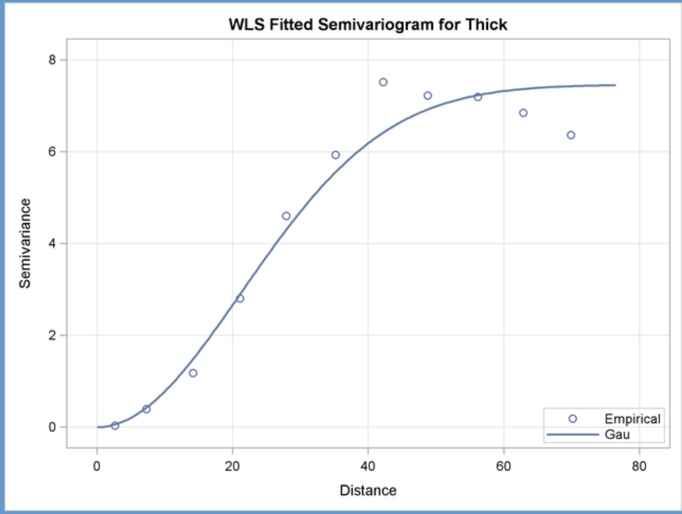




Semivariogram

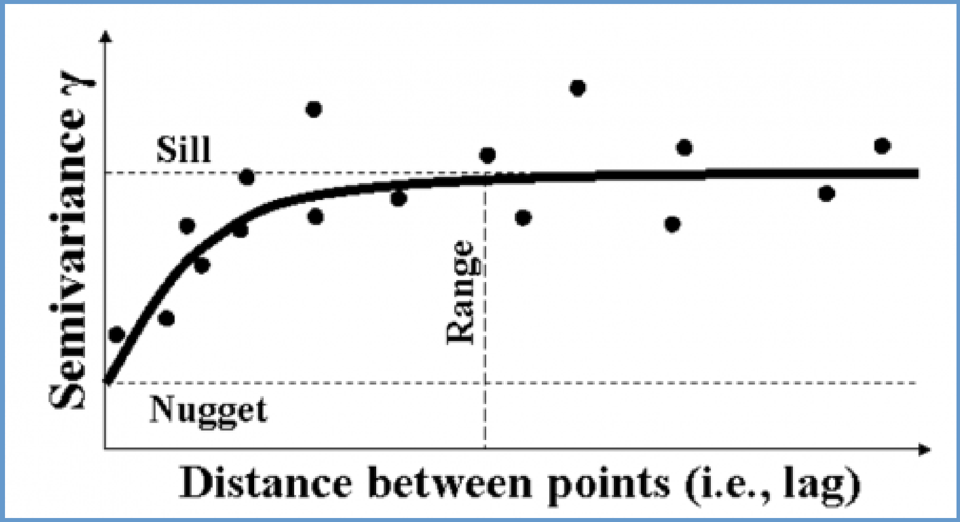
- Modeling the semivariogram
  - Describe the relationship between difference of values (variation) and distance using a mathematical function
  - Several different types of functions can be used, each with a different form to the distance function





Modeling the Semivariogram

- Described quantitatively by sill, range, and nugget and using different function forms

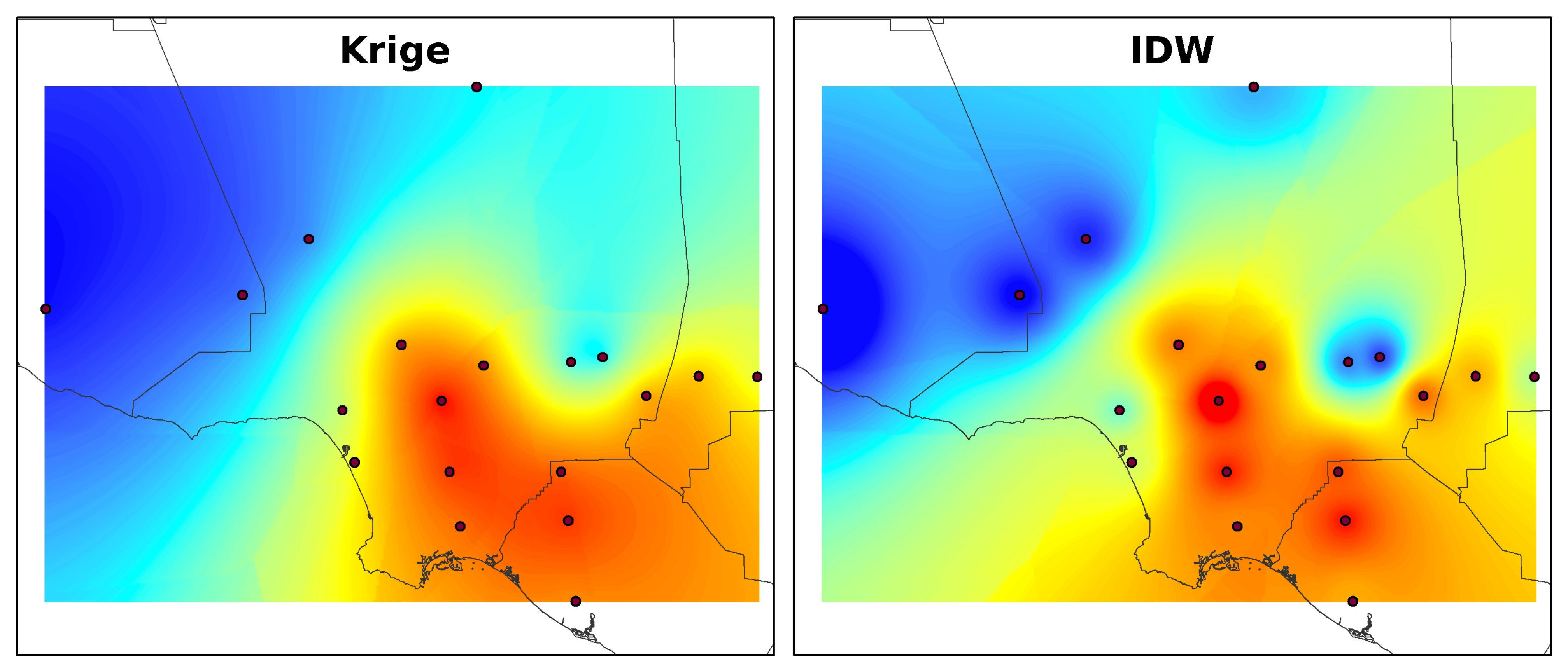


- *Range*: distance beyond which values are not spatially autocorrelated
- *Sill*: the true variance of the variable, observed between variables separated by a distance > range
- *Nugget*: the level of variance observed when the semivariogram curve is projected back to zero distance

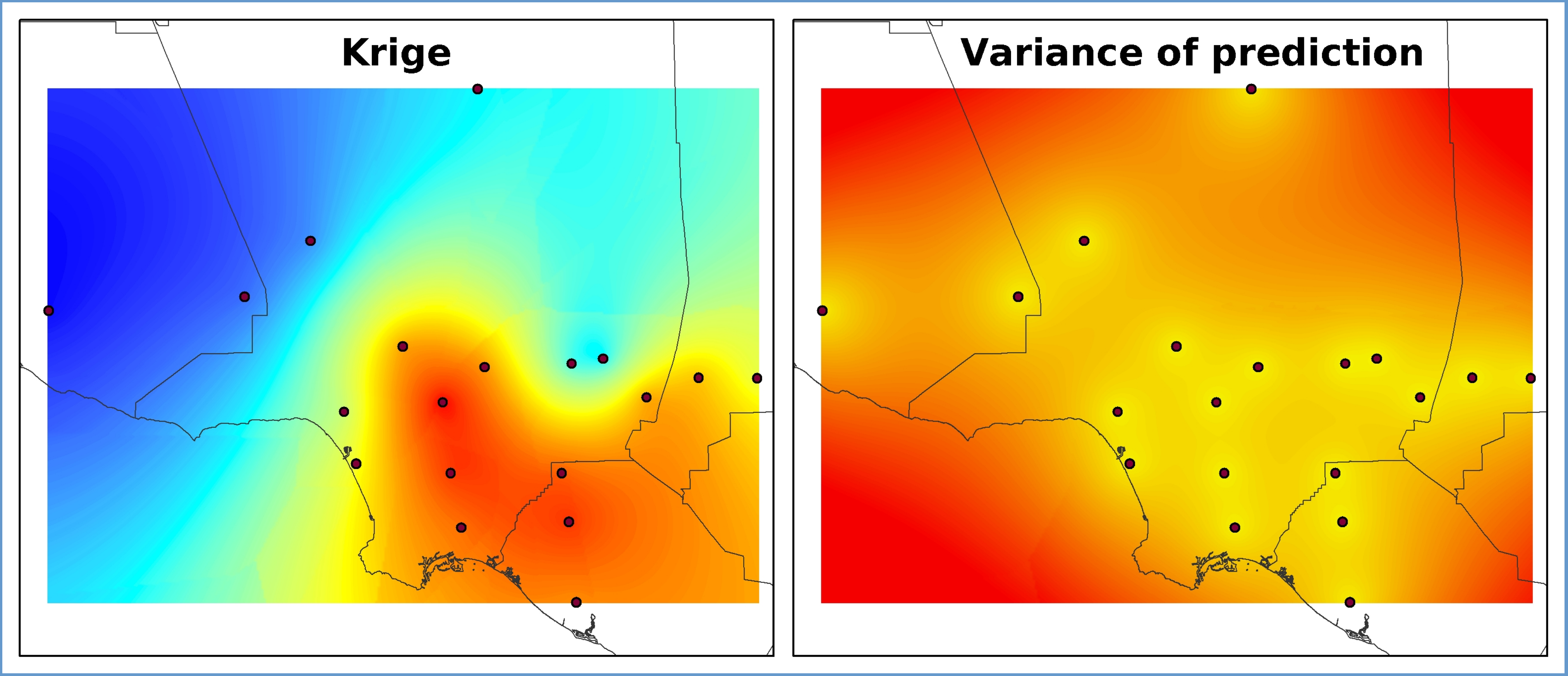
Kriging

- Estimates the weights based on distance
  - Using mathematical function derived from the semivariogram
- Calculates the weighted average of neighboring values for each location
  - Neighbors values as modified by distance and weight (associated with distance)
  - Kriging can be very computationally intensive
- *Provides interpolated surface plus an surface of uncertainty of prediction*

Kriging



Kriging



Kriging

- Output features: raster (estimated value and uncertainty)
- Estimated values are not constrained to the range of the input values
- At sample locations, estimated value not always equal to sample value
- Results in a smoother surface (than IDW)

Non-stationarity

- Thus, far the methods assume that there is a global spatial autocorrelation structure in the data
  - e.g., 1 / *d* applied to all places in IDW
  - e.g., global semivariogram function in Kriging
- This may not be appropriate for variables/phenomena that have a highly variable spatial structure
  - e.g., Think about how temperature varies in plains vs mountains (orographic effects)

Empirical Bayesian Kriging

- Does not assume a stationary process
- Accounts for error introduced by estimating a global semivariogram model
  - Estimates numerous local semivariogram models
    - Captures variation in the spatial dependence from place to place in the study region

Empirical Bayesian Kriging

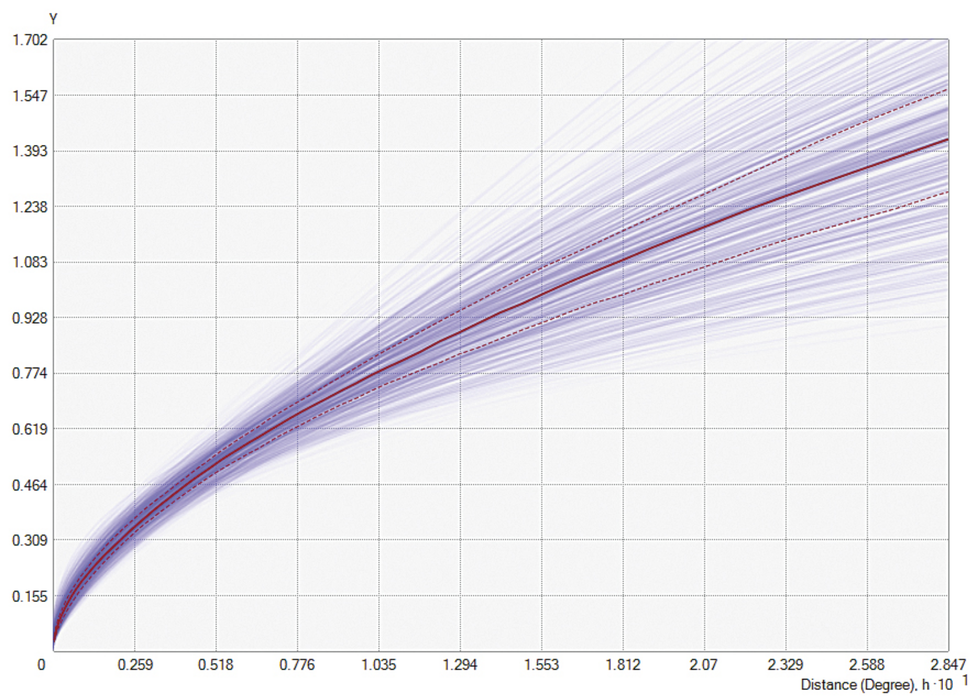
- Subset observed data to local region
  - Estimate initial semivariogram model from observed data values
  - Use initial semivariogram model to simulate a value at each original data point
  - Estimate new semivariogram model from simulated data values
    - Calculate the likelihood that the original data values were generated from the new semivariogram and assign weight (Bayes' rule)

Repeated Simulations

Empirical Bayesian Kriging

- Creates a large number of semivariograms for each subset (local region)
  - When plotted together, the result is a distribution of semivariograms

Example of distribution of Semivariograms: The darker the blue color, the more semivariogram models are present



Empirical Bayesian Kriging

- Prediction: for each unmeasured location, calculate a new semivariogram distribution using likelihood-based sampling of individual semivariograms
  - Using weight values
  - Can be drawn from multiple spatial subsets
    - For example, if a prediction location has neighbors with observed values in two different subsets, the prediction will be calculated using simulated semivariograms from both of the subsets
    - Semivariograms are chosen based on their likelihood values

EB Kriging

- Output features: raster (estimated value and uncertainty)
- Estimated values are not constrained to the range of the input values
- At sample locations, estimated value not always equal to sample value
- Results in a different surface (than classical kriging), depending on whether non-stationary

Keywords

- Interpolation
- Samples
- Unmeasured locations
- Cow test
- Point-baed
- Thiessen polygons
- Trend surface
- IDW
- Kriging
- Semivariogram
- Geostatistical
- Uncertainty