C and DATA STRUCTURES

Buereddy Varuni

Presentation-1

GATE2021 QP CS-1/CS,Set-1/Q.41

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Question:- GATE2021 QP CS-1/CS,Set-1/Q.41

An articulation point in a connected graph is a vertex such that removing the vertex and its incident edges disconnects the graph into two or more connected components. Let T be a DFS tree obtained by doing DFS in a connected undirected graph G. Which of the following options is/are correct?

1. Root of T can never be an articulation point in G.
2. Root of T is an articulation point in G if and only if it has 2 or more children.
3. A leaf of T can be an articulation point in G
4. If u is an articulation point in G such that x is an ancestor of u in T and y is a descendent of u in T, then all paths from x to y in G must pass through u.

Solution

Answer : 2

Explanation

Option 2 - Root of T is an articulation point in G if and only if it has 2 or more children.

Case 1:- If root is articulation point then root will have 2 or more children

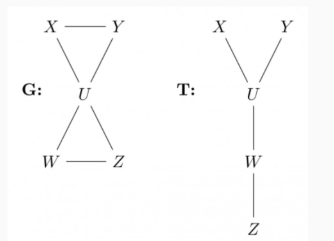
If root is an articulation point. Removal of root node will disconnect the graph. And there is no path between the disconnected components. Therefore, while constructing the DFS tree for graph G, Root vertex is first visited and will have two or more children.

Solution

Case 2:- If root vertex has 2 or more children then it is articulation point
Lets say in an undirected graph if root has 2 children then it is true that there is no path between the vertices in left sub-tree and right sub-tree of vertex V (w.r.t DFS traversal tree).Therefore, root of T is the articulation point because removal of T disconnects the graph into 2 or more parts.
Option 2 is correct implies Option 1 is wrong
Option 3 - A leaf of T can be an articulation point in G. This is FALSE because if some vertex is leaf of tree T then all the vertices to which it connects are already been visited which indicates that even without using this leaf vertex there exists path between all of its neighbors and hence it can not be an articulation point.

Solution

Option 4 - If u is an articulation point in G such that x is an ancestor of u in T and y is a descendent of u in T, then all paths from x to y in G must pass through u.
There is a counter example in the figure, U is the articulation point in G. And also X is the ancestor of U and Y is the descendent of U. But all the paths from X and Y in G does not pass through U.



Answer

Option 2 is correct.

Solving circuits using graphs

Kirchoff's voltage law

For any lumped electrical network, at any time the net sum (taking into account the orientations) of the voltages around a loop (i.e. circuit) is zero. In terms of the corresponding digraph , for the r^{th} circuit we must have

$$[B][V] = 0 \tag{1}$$

$$\sum_{k=1}^{e} b_{rk} v_k(t) = 0 \tag{2}$$

Where b_{rk} is the rk^{th} entry of the circuit matrix B of G and $v_k(t)$ is the amount of voltage across the k^{th} edge of G.

Solving circuits using graphs

Kirchoff's current law

For any lumped electrical network, at any time the net sum (taking into account the orientations) of all the currents leaving any node or vertex is zero. That is at r^{th} vertex of the corresponding digraph,we must have

$$[A][I] = 0 \tag{3}$$

$$\sum_{k=1}^{e} a_{rk} i_k(t) = 0 \tag{4}$$

Where a_{rk} is the rk^{th} entry of the incidence matrix A of G and $i_k(t)$ is the amount of current flowing through the k^{th} edge of G.

Application of graph theory in network equilibrium equations

Steps to be followed:-

1. Find the equivalent graph of the circuit.
2. Construct the tie-set matrix(B).
3. Apply equivalent KVL/KCL for the graph.
4. Solve the matrix equations

Let a voltage V_{sk} be the source voltage in branch k having impedance z_k and carrying current i_k, we can write $v_k = z_k i_k + V_{sk}$. In matrix form this can be written as $[V_b] = [Z_b][I_b]+[V_s]$ where $[Z_b]$ is the branch impedance matrix,$[I_b]$ is the column vector of branch currents and $[V_s]$ is the column vector of source voltage.

Application of graph theory in network equilibrium equations

Now Kirchhoff's Voltage law in matrix form is given by

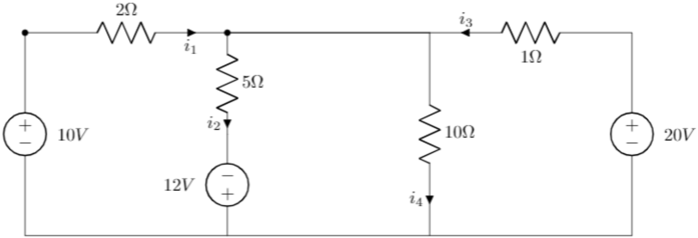
$$[B][V_b] = 0 \tag{5}$$

$$[B]\{[Z_b][I_b] + [V_s]\} = 0 \tag{6}$$

$$[B][Z_b][I_b] = -[B][V_s] \tag{7}$$

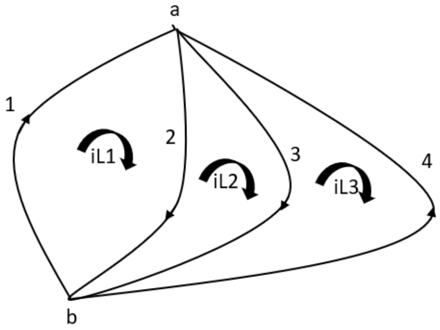
Example

Consider the example circuit as shown in the figure.



Example

The obtained equivalent graph is shown in the figure.,



Example

There are two vertices and four branches in the graph. We need to first find the B matrix(tie-set matrix). The size of matrix B is number of loops x number of branches(3x4).B_{ij}=1 if j^{th} branch current is in the direction of i^{th} loop current.

$$\text{tie-set matrix} = B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & -1 \end{bmatrix}$$

$$B^T = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix}$$

Example

Now find the circuit impedance matrix [zb]. The size of [zb] is number of branches x number of branches(4x4),

$$zb = \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 \\ 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Vs is the external source voltage, $Vs = \begin{bmatrix} -10 \\ -12 \\ 0 \\ 20 \end{bmatrix}$

The loop currents I_L has dimension (number of loops x 1) (3x1).

$$I_L = \begin{bmatrix} i_{L1} \\ i_{L2} \\ i_{L3} \end{bmatrix}$$

Example

From the equation.7,

$[\mathrm{B}][Z_b][B^T][I_L]$ = -$[\mathrm{B}][V_s]$

$$\begin{bmatrix} 2 & 5 & 0 & 0 \\ 0 & -5 & 10 & 0 \\ 0 & 0 & -10 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ i_{L3} \end{bmatrix} = \begin{bmatrix} 22 \\ -12 \\ -20 \end{bmatrix}$$

$$\begin{bmatrix} 7 & -5 & 0 \\ -5 & 15 & -10 \\ 0 & -10 & 11 \end{bmatrix} \begin{bmatrix} i_{L1} \\ i_{L2} \\ i_{L3} \end{bmatrix} = \begin{bmatrix} 22 \\ -12 \\ -20 \end{bmatrix}$$

Solving the above equation, we get

$$i_{L1} = -1.2777A \tag{8}$$

$$i_{L2} = -6.188A \tag{9}$$

$$i_{L3} = -7.444A \tag{10}$$

The End