CONTROL SYSTEMS - EE2227

GATE - 2019 problem

EE18BTECH11005

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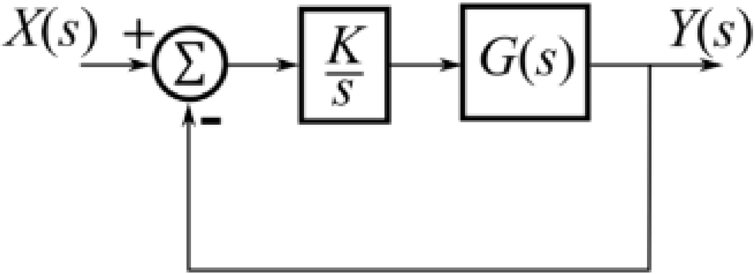
QUESTION - 42, EC:

Consider a unity feedback system as shown in the figure,shown with an integral compensator k/s and open-loop transfer function

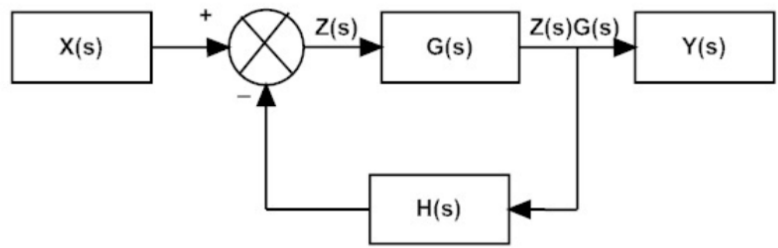
G(s) $= \frac{1}{s^2+3s+2}$

where k$>$0. The positive value of k for which there are two poles of unity feedback system on jω axis is equal to——(rounded off to two decimal places)


X(s)
+
Σ
-
K/s
G(s)
Y(s)

Transfer function of Negative feedback

A transfer function is the relative function between input and output.
In a negative feedback system an intermediate signal is defined as Z.



Y(s) $=$ Z(s).G(s)
Z(s) $=$ X(s) - Y(s).H(s) $=>$ X(s) $=$ Z(s)$+$Y(s).H(s)
X(s) $=$ Z(s)$+$Z(s).G(s).H(s)
$\frac{Y(s)}{X(s)} = \frac{Z(s).G(s)}{Z(s)+Z(s).G(s).H(s)}$
So,the transfer function of negative feedback is $\frac{G(s)}{1+G(s).H(s)}$
Since unit feedback H(s) $=$ 1
Now the transfer function of unity negative feedback is $\frac{G(s)}{1+G(s)}$

Net transfer function

The net transfer function in the given question is.....

$$\frac{Y(s)}{X(s)} = \frac{G(s)*k/s}{1+G(s)*k/s}$$

The characteristic equation is $1 + (\mathrm{G(s)x}\frac{k}{s}) = 0$
that is..,

$$\mathrm{C.E} = 1 + \frac{k}{s(s^2+3s+2)} = 0 => \mathrm{s}(\mathrm{s}^2 + 3s + 2) + k = 0$$

$$=> \mathrm{s}^3 + 3s^2 + 2s + k = 0$$

Routh-Hurwitz Criterion

This criterion is based on arranging the coefficients of characteristic equation intoan array called Routh array.

$q(s) = a_0 s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots.. + a_{n-1} s + a_n = 0$

$$\left|\begin{matrix} s^n \\ s^{n-1} \\ s^{n-2} \\ \vdots \end{matrix}\right| \left|\begin{matrix} a_0 & a_2 & a_4 & \cdots & \\ a_1 & a_3 & a_5 & \cdots & \\ b_1 & b_2 & b_3 & \cdots & \\ \vdots & \vdots & \vdots & \ddots & \vdots \ldots \end{matrix}\right| \text{where}$$

$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$ $b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$ $c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$ $c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$

For poles to lie on imaginary axis any one entire row of hurwitz matrix should be zero.

For the given characteristic equation $= s^3 + 3s^2 + 2s + k = 0$

$$\begin{vmatrix} s^3 \\ s^2 \\ s^1 \\ s^0 \end{vmatrix} \begin{vmatrix} 1 & 2 \\ 3 & k \\ \frac{6-k}{3} & 0 \\ k & 0 \end{vmatrix}$$

For poles on $j\omega$ axis any one of the row should be zero

$=> \frac{6-k}{3} = 0$ or $k = 0$

But given $k>0$ ...

therefore, $6-k=0 => k = 6$

To find the location of poles on jω axis

Auxillary equation of the given CE is $3s^2 + k = 0$
$=> 3s^2 + 6 = 0$
$=> s = \pm j2$